The metallic states of the cuprates: Fermi liquids, fractionalized Fermi liquids, and non-Fermi liquids

School on Strongly Coupled Field Theories
for Condensed Matter and Quantum Information Theory
International Institute of Physics
Natal, Brazil
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Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS
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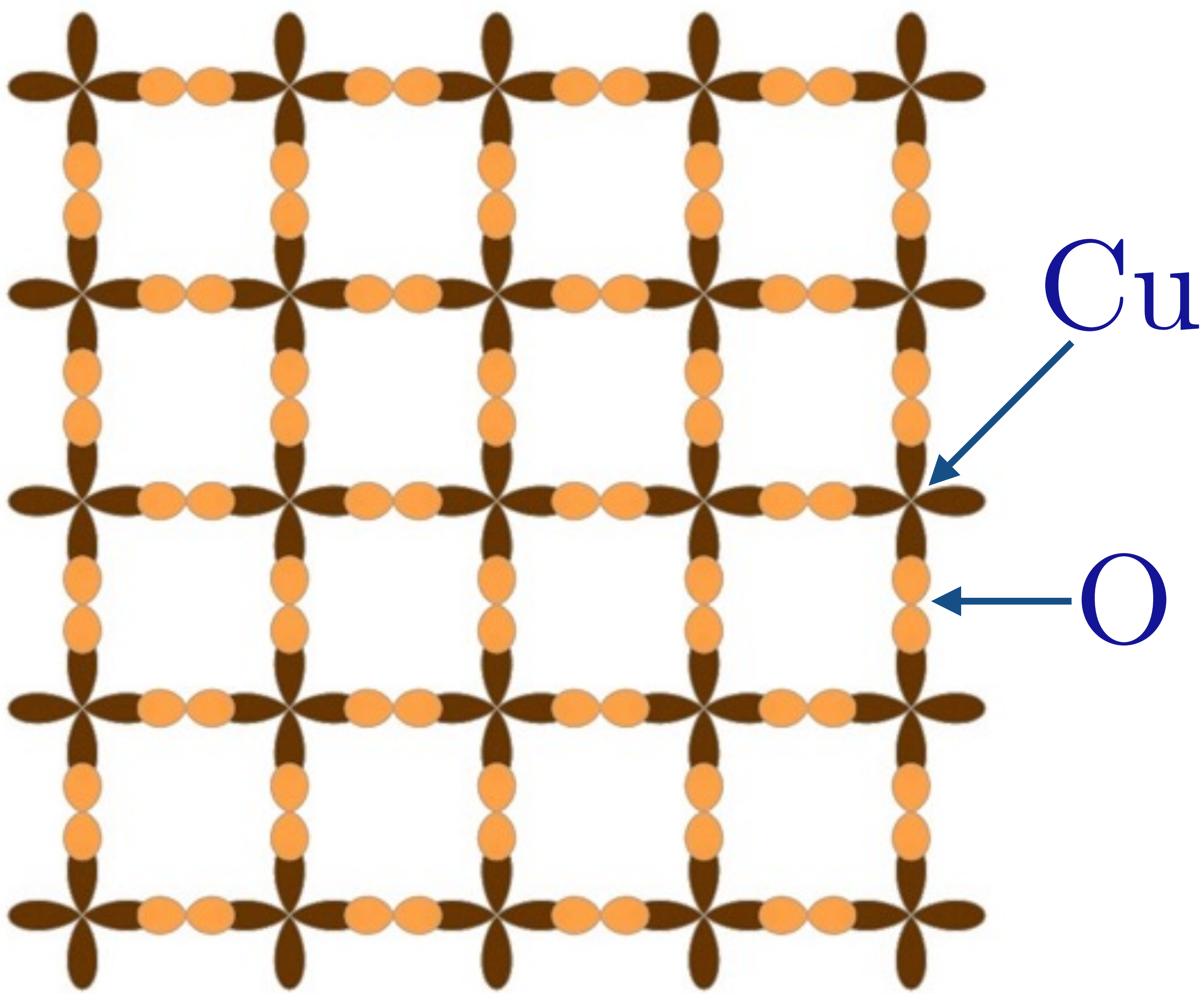
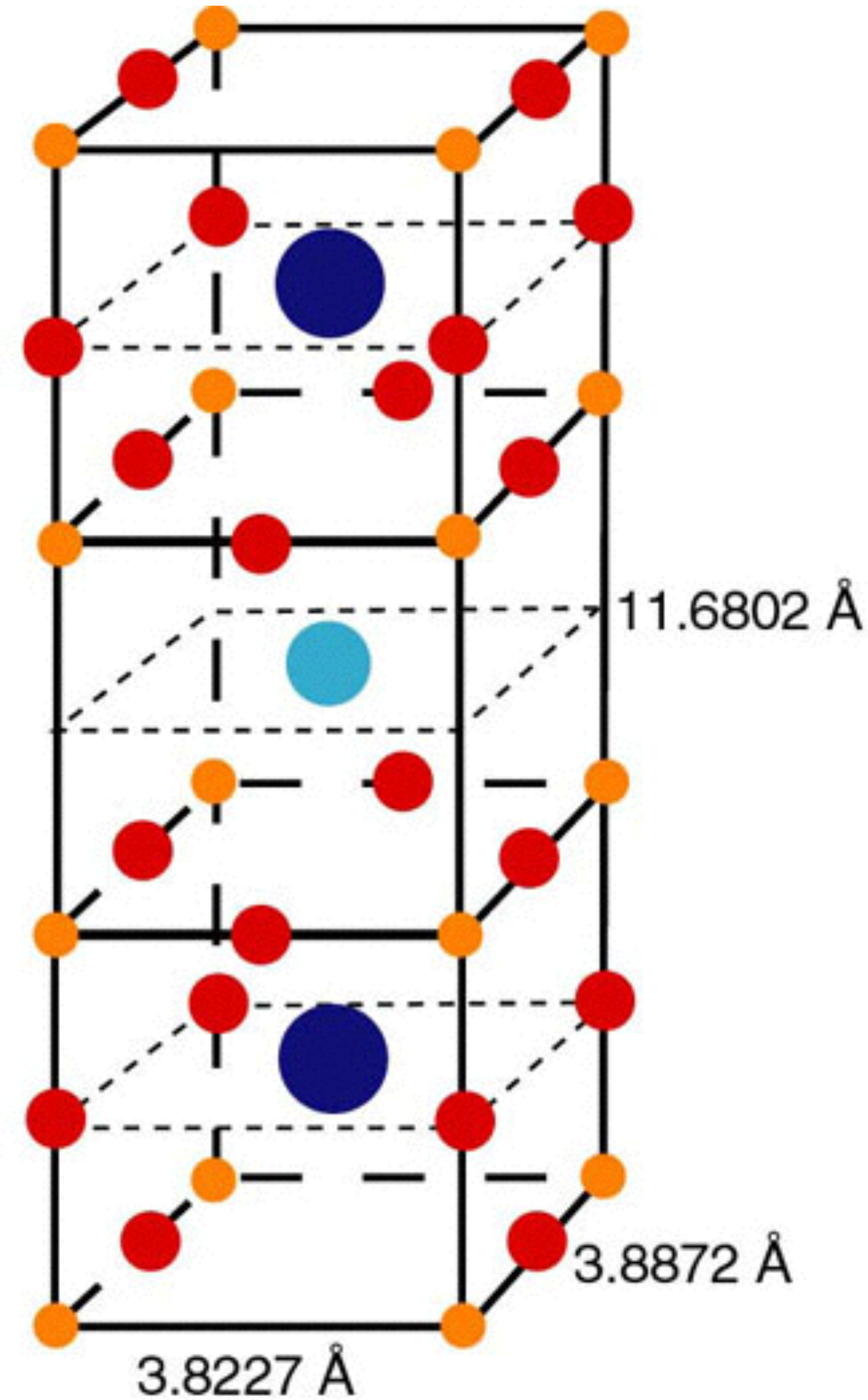
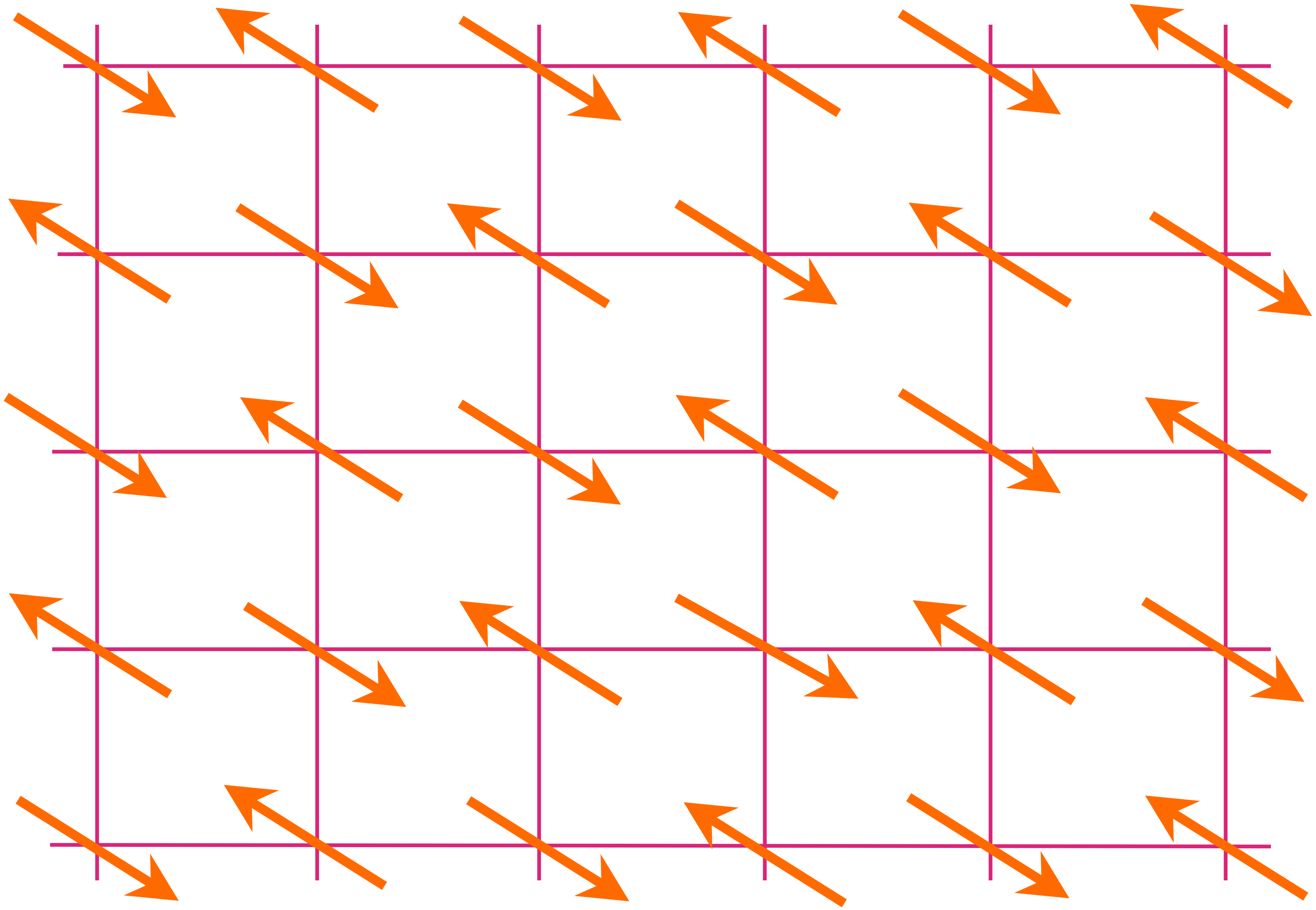
High temperature superconductors



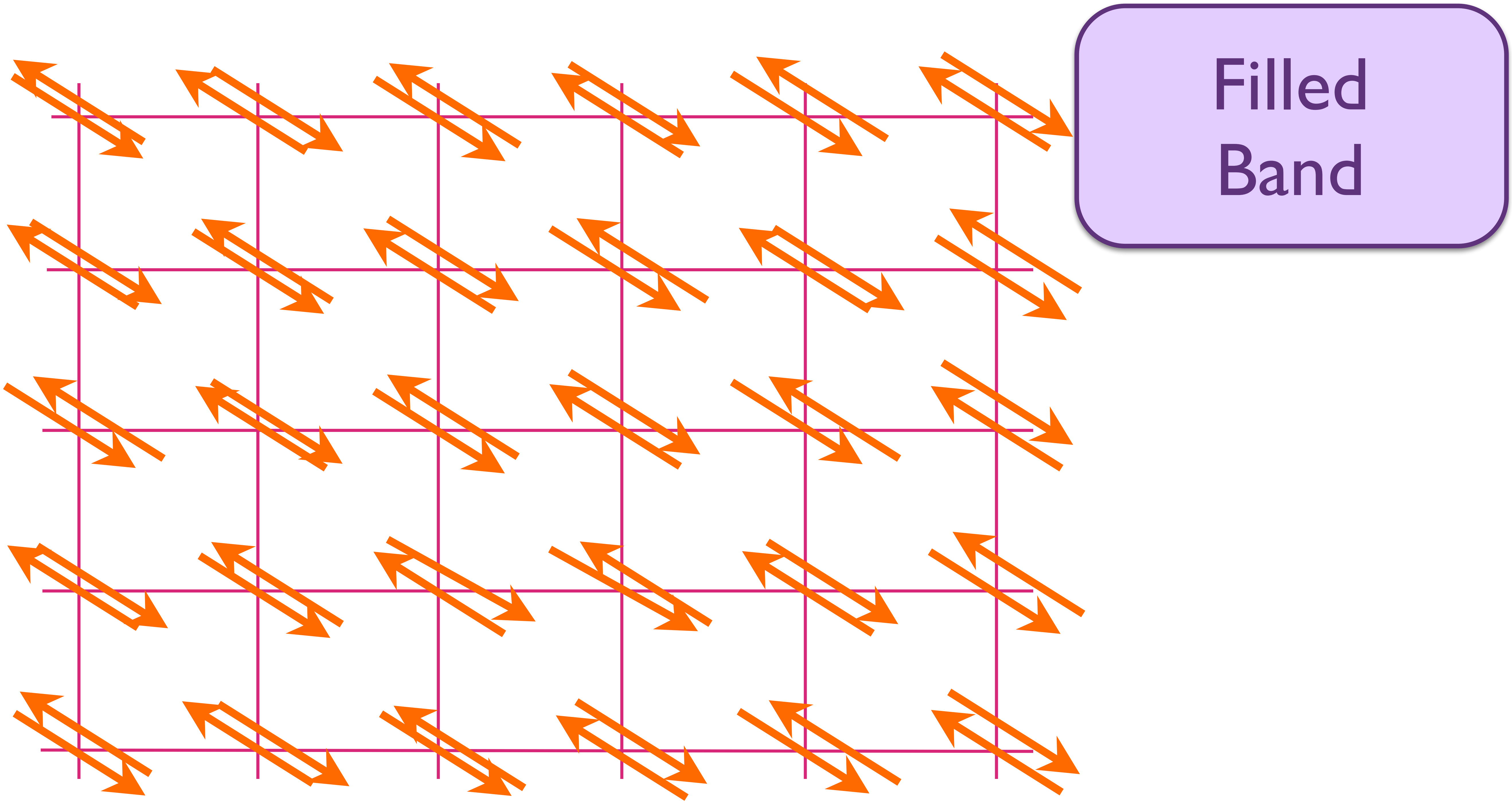
CuO_2 plane

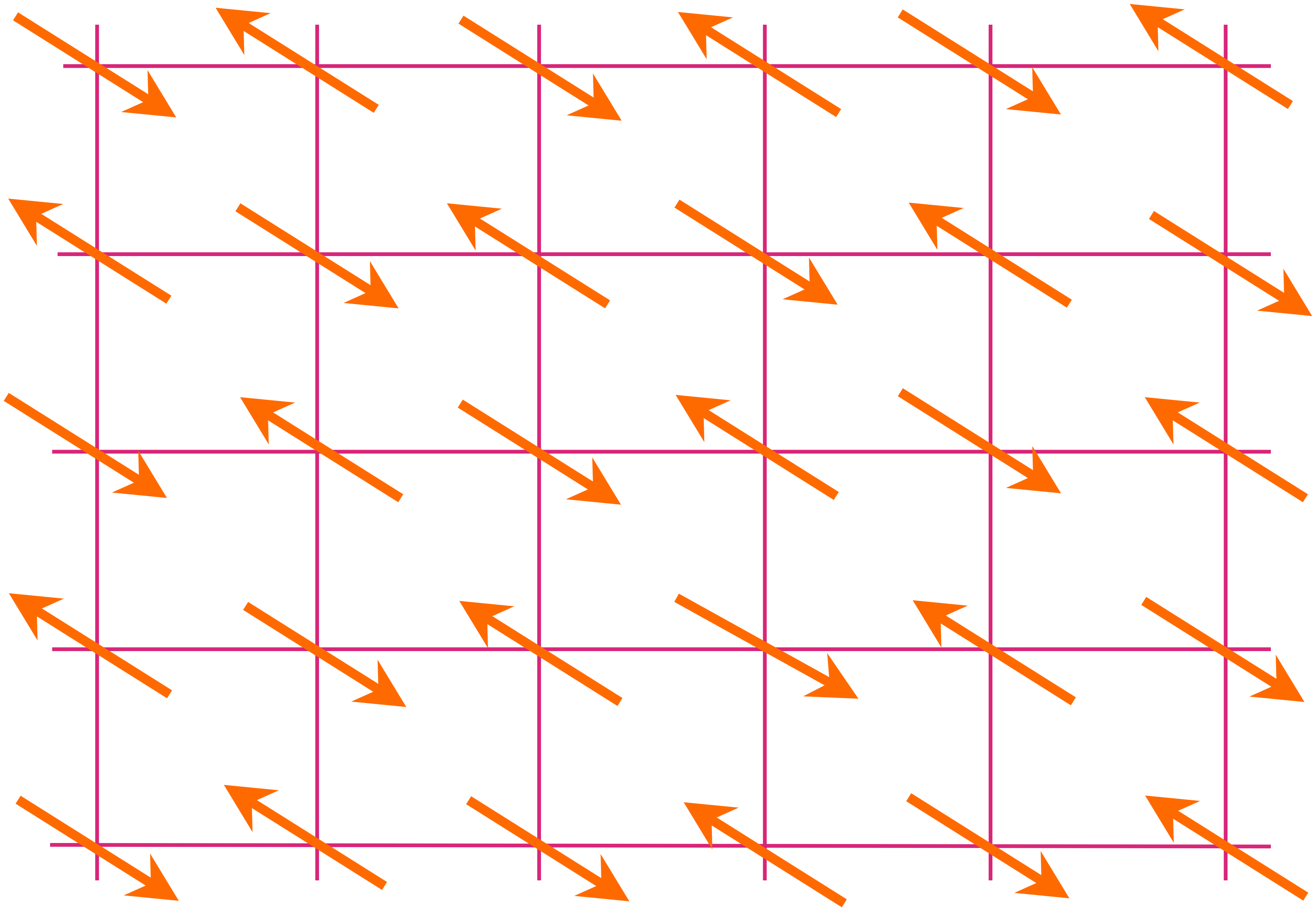


$YBa_2Cu_3O_{6+x}$



“Undoped” Anti-ferromagnet


Filled
Band



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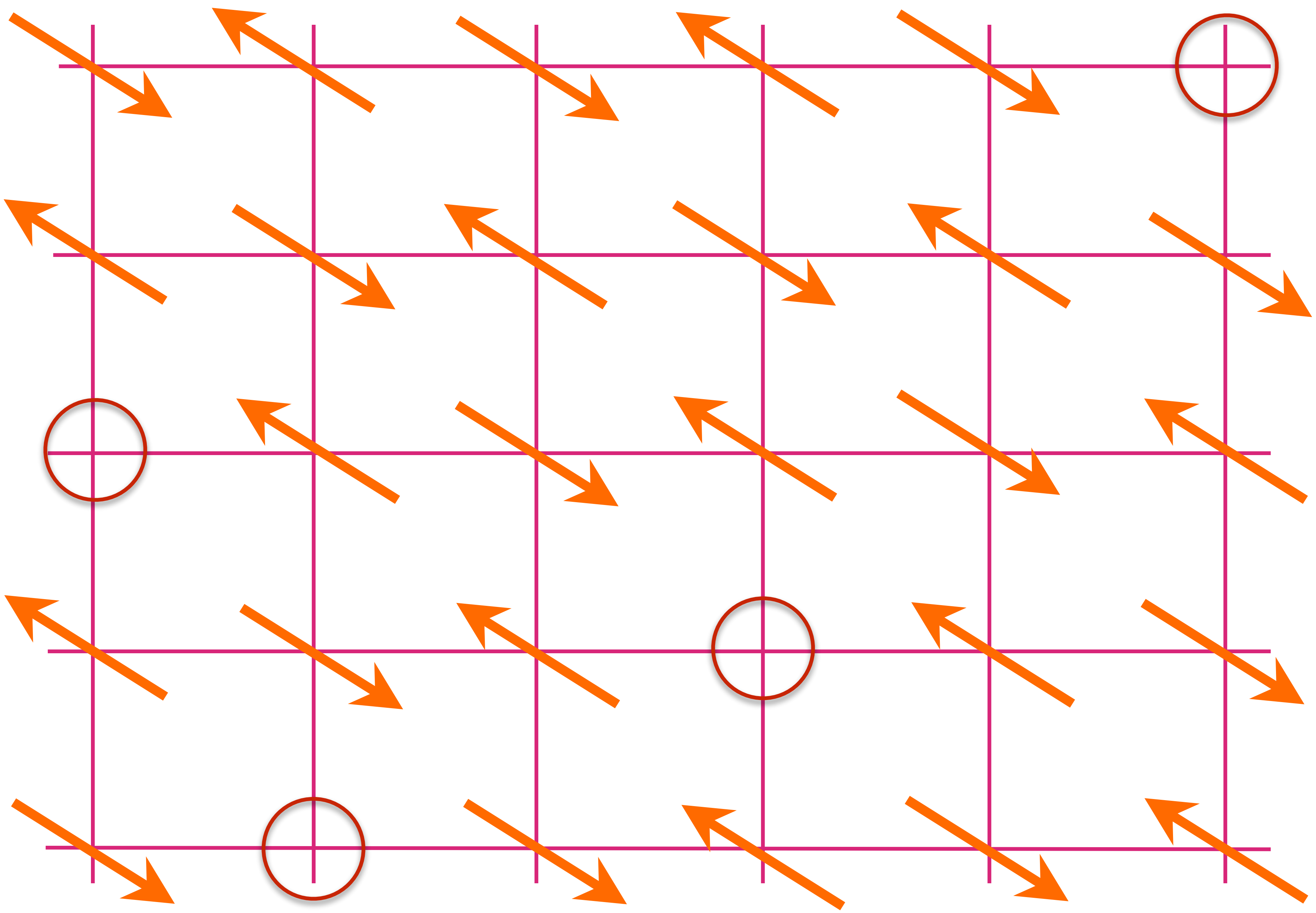
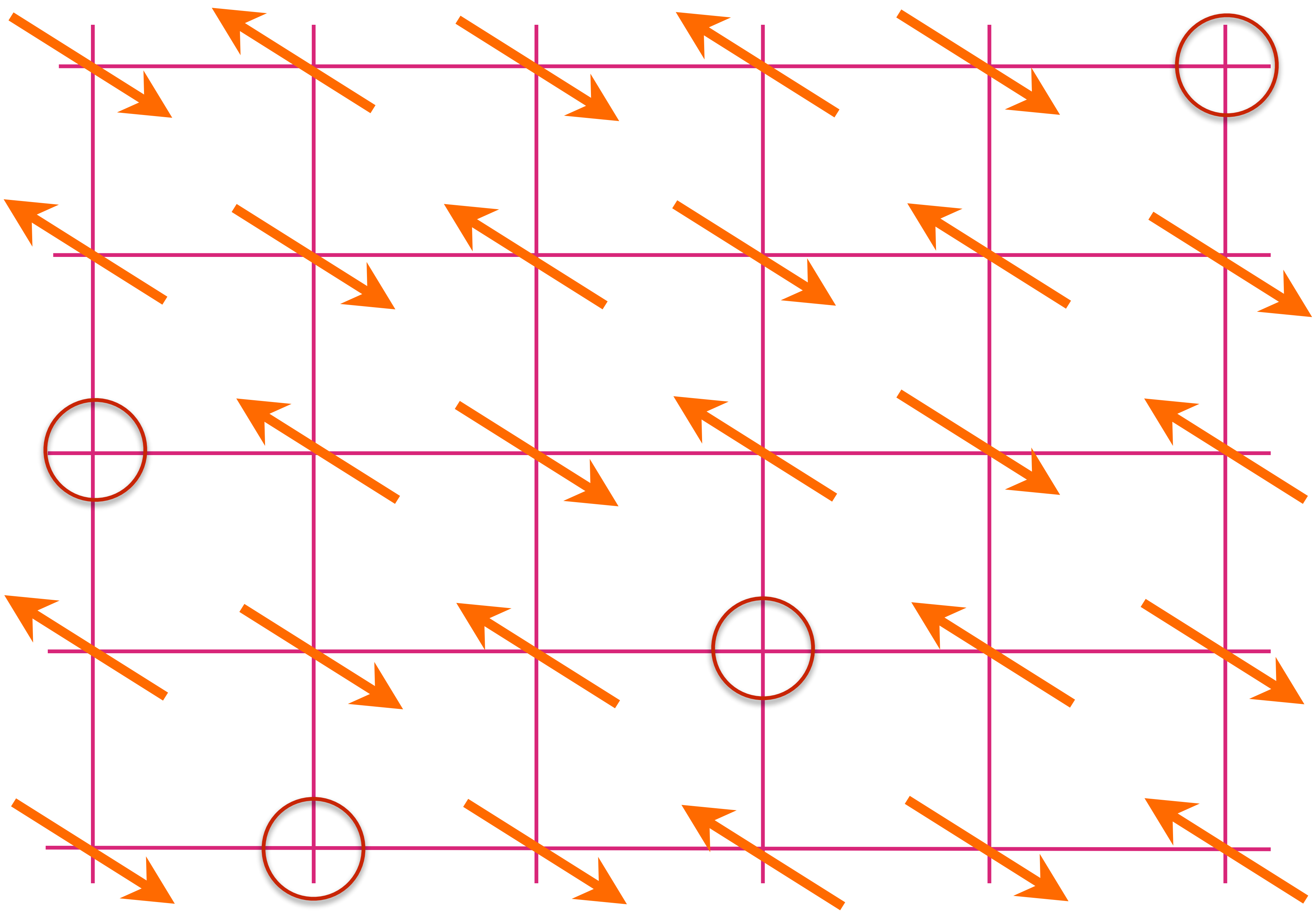
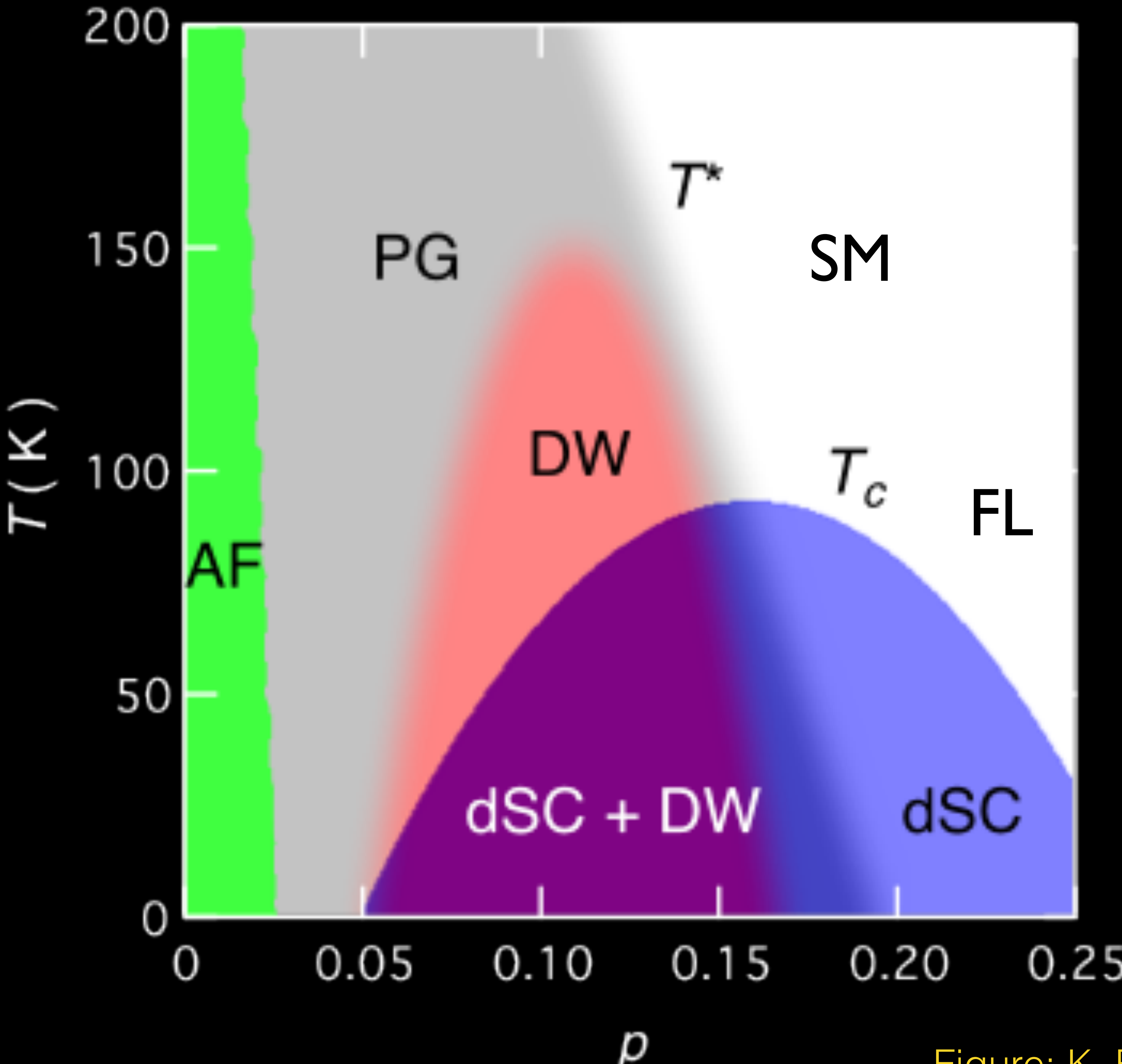


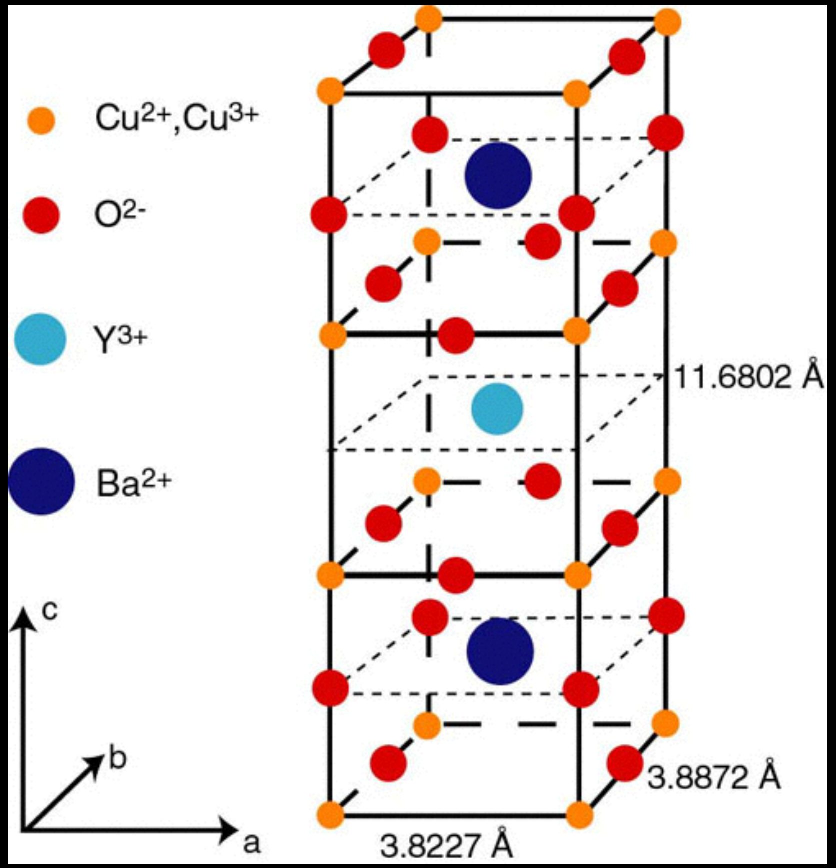
Anti-ferromagnet with p holes per square



Anti-ferromagnet with p holes per square

But relative to the band insulator, there are $1+p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1+p$

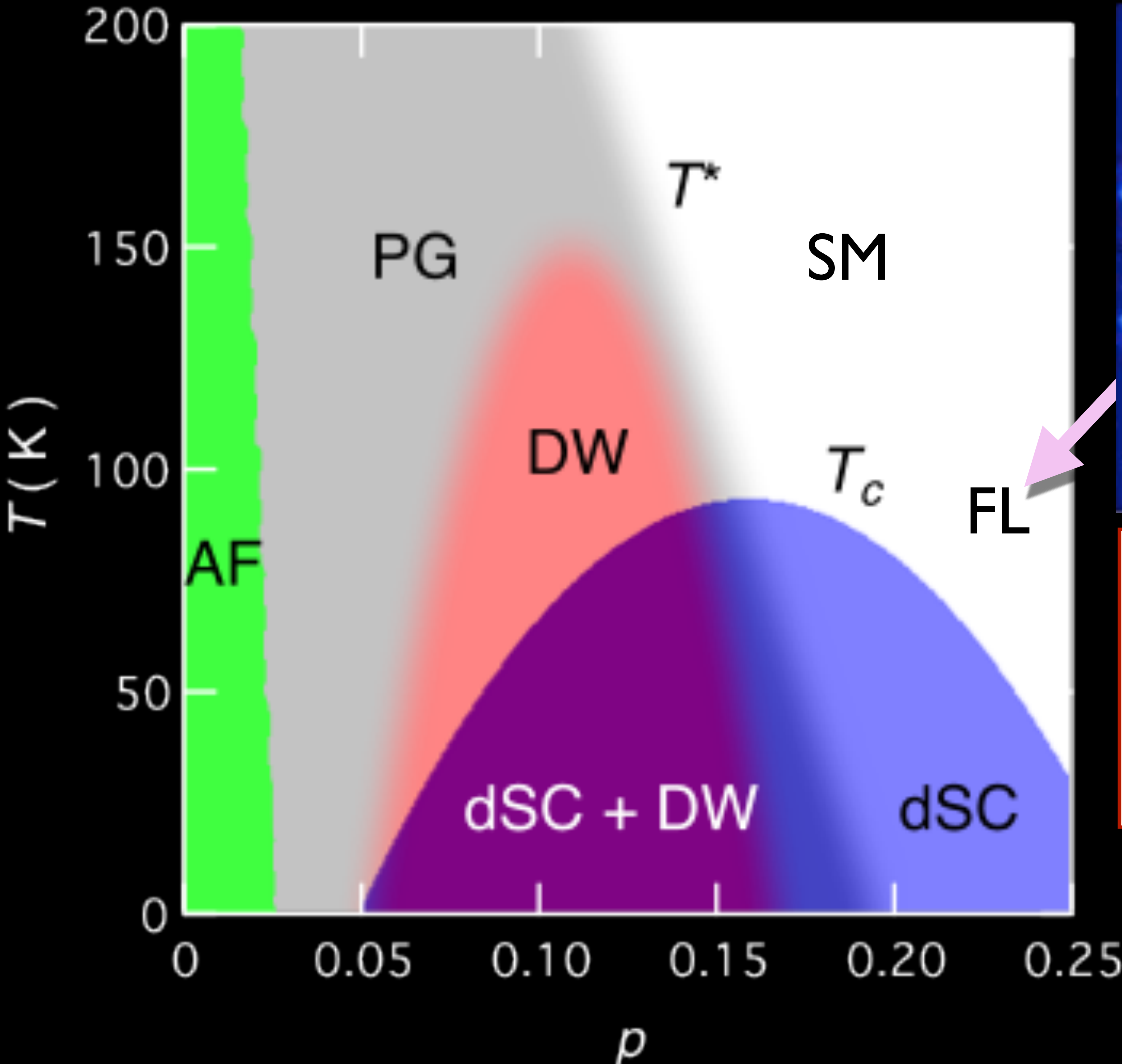


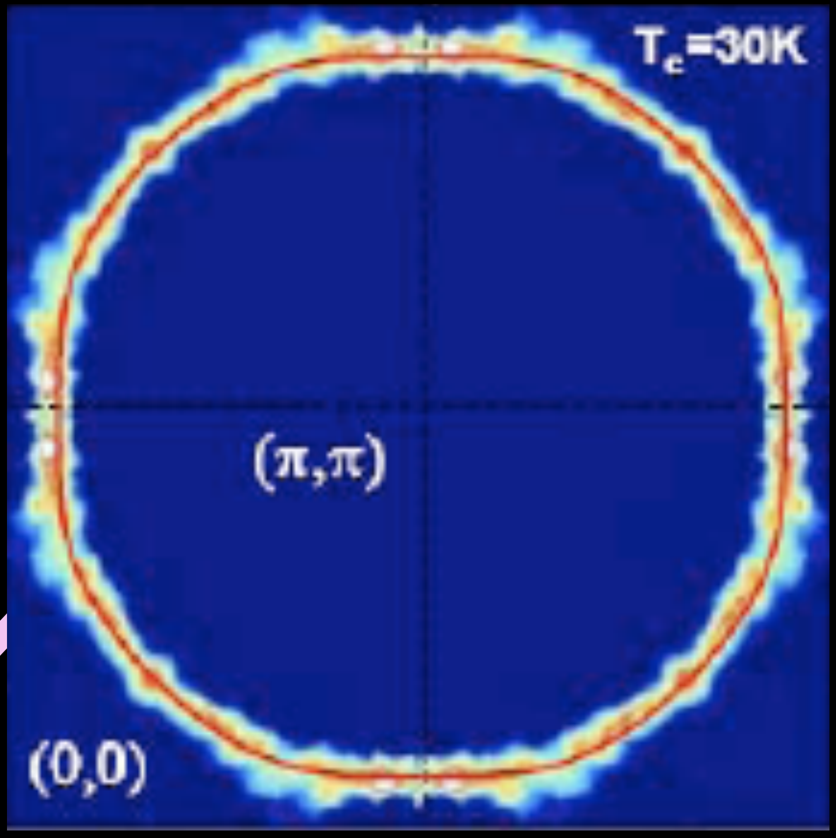


$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

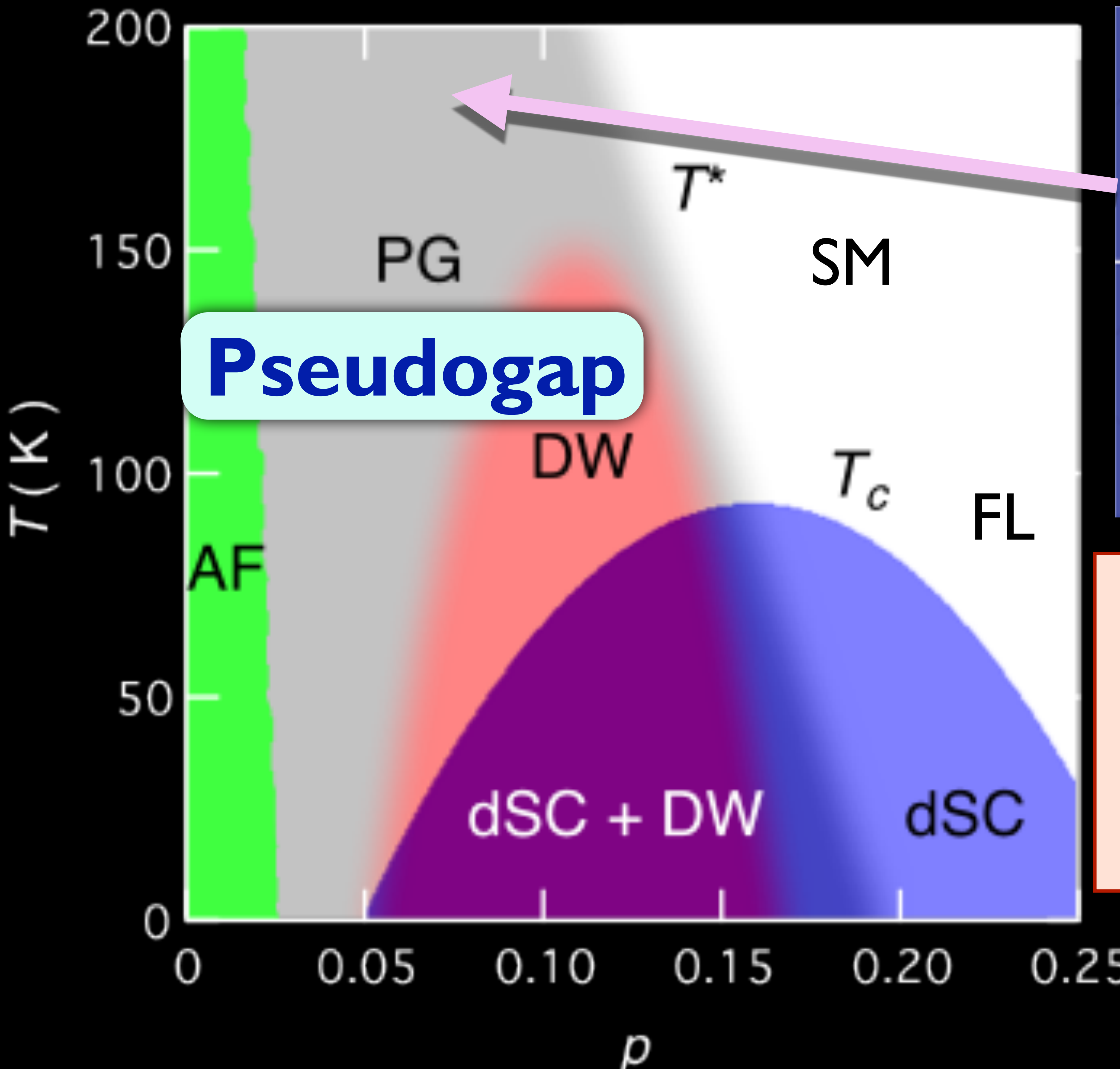


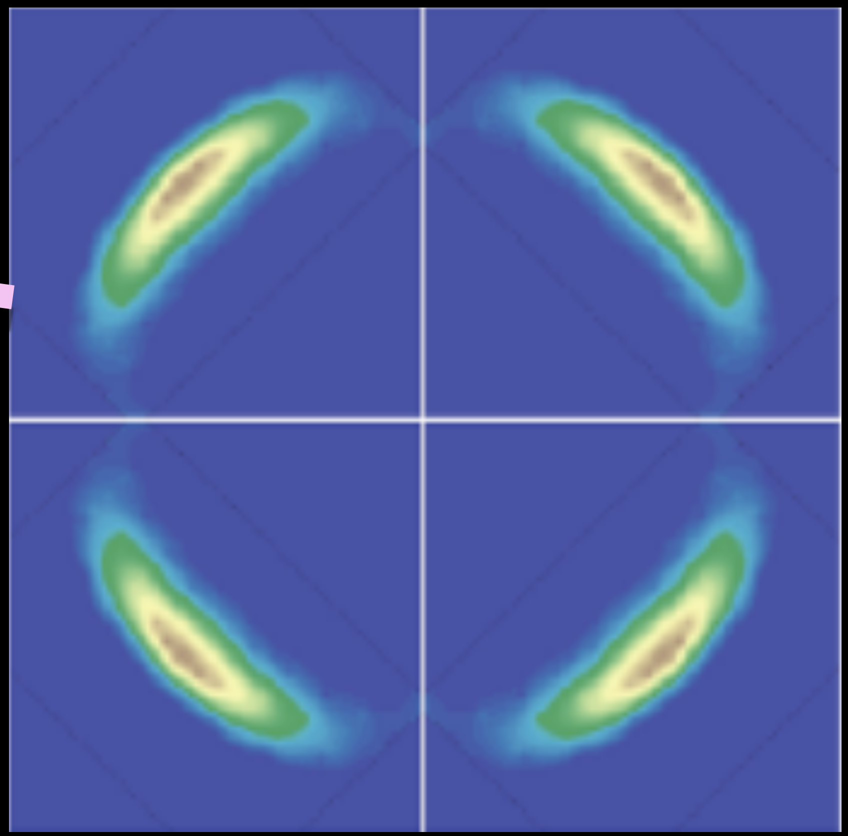


1. Conventional metal

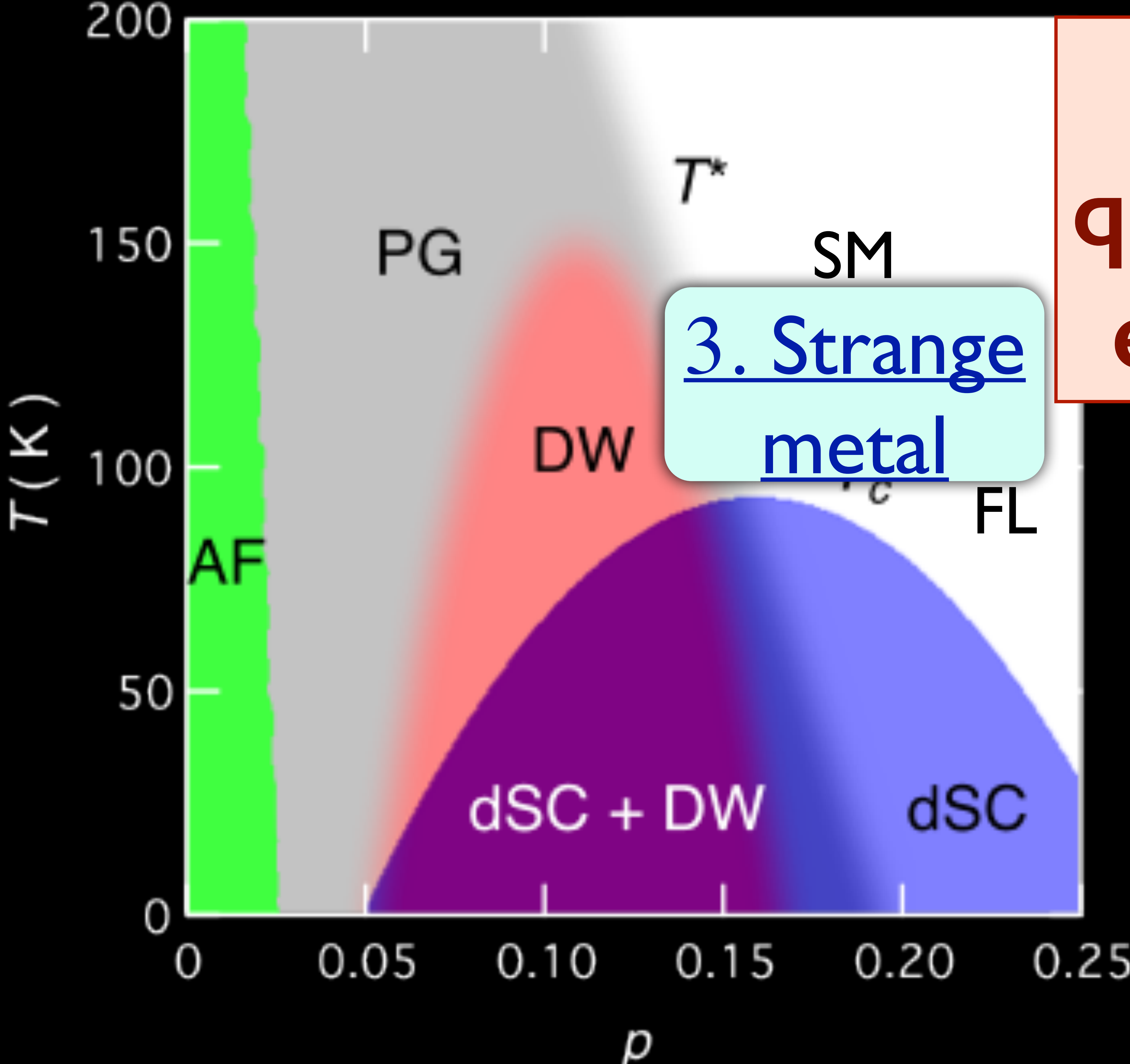
Area enclosed by Fermi surface =1+p

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)





2. Pseudogap metal at low p



3. Strange metal

No quasiparticle excitations

1. Review of Fermi liquid theory

Topological argument for the Luttinger theorem

2. Fractionalized Fermi liquid

A Fermi liquid co-existing with topological order for the pseudogap metal

3. Quantum matter without quasiparticles

(A) A mean-field model of a non-Fermi liquid, and charged black holes
(B) Field theory of a non-Fermi liquid (Ising-nematic quantum critical point)
(C) Theory of transport in strange metals: application to the (less) strange metal in graphene

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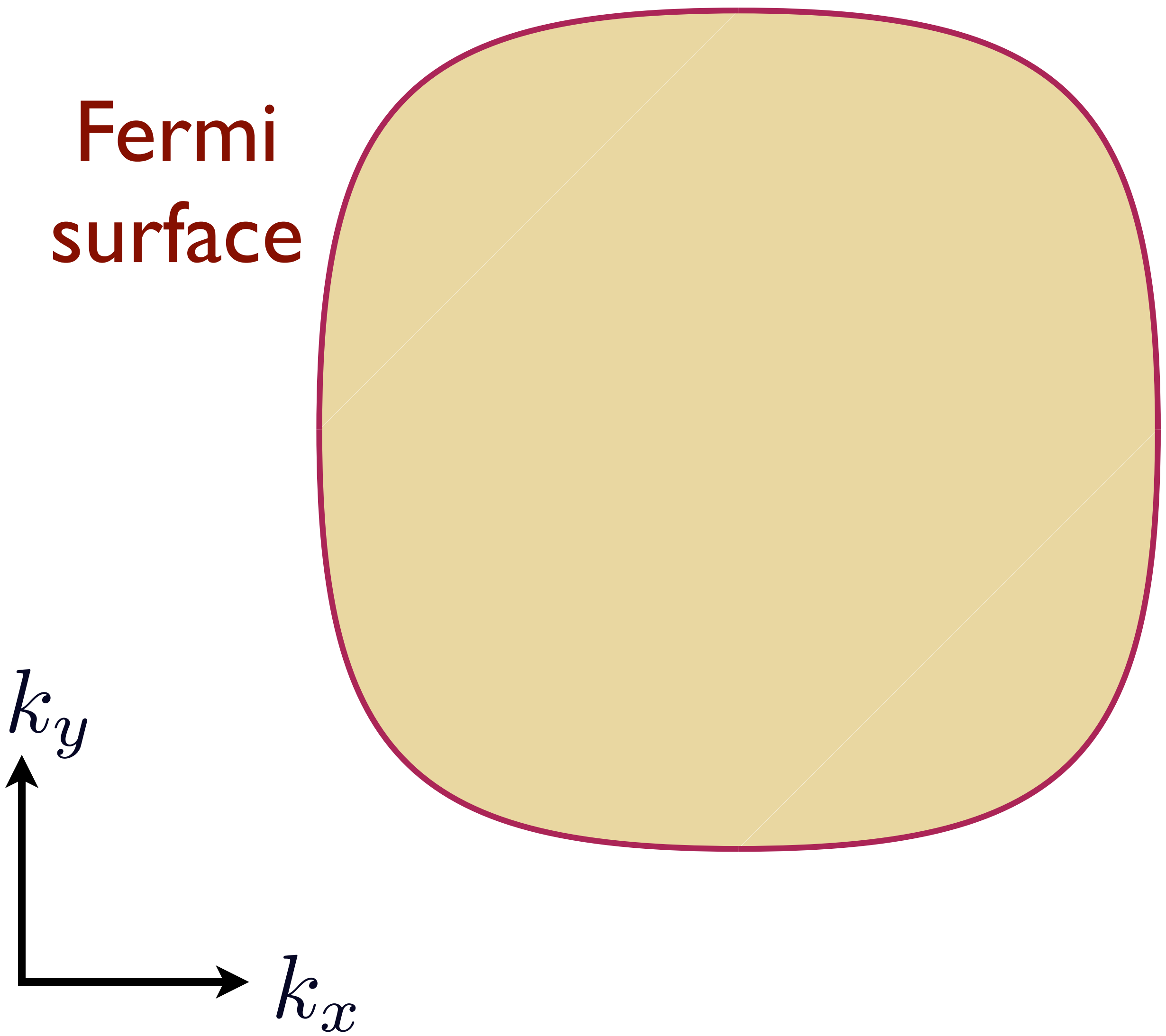
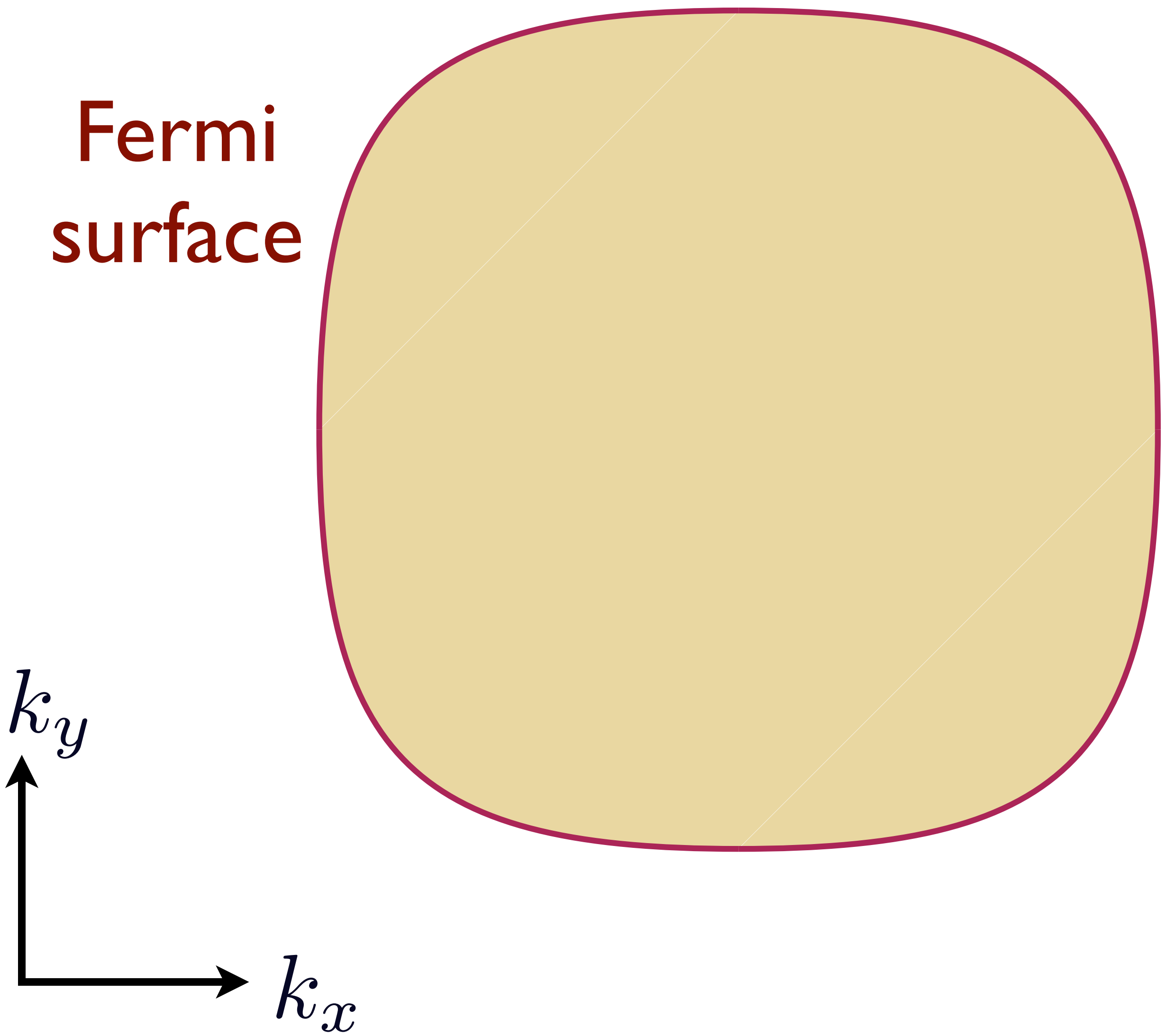
Ordinary quantum matter: the Fermi liquid (FL)



- Fermi surface separates empty and occupied states in momentum space.

Ordinary quantum matter: the Fermi liquid (FL)



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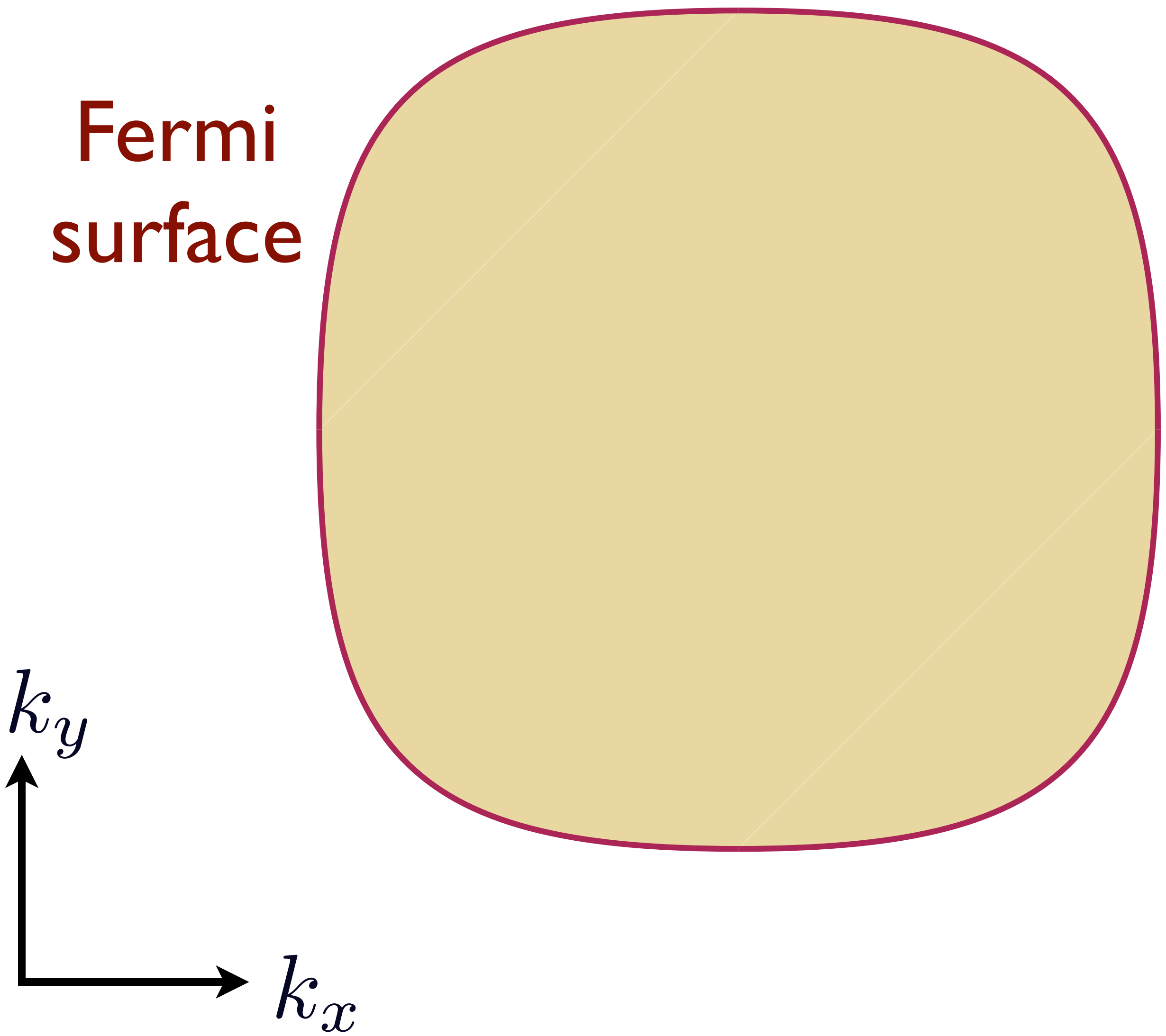

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- Density of electrons can be continuously varied at zero temperature.

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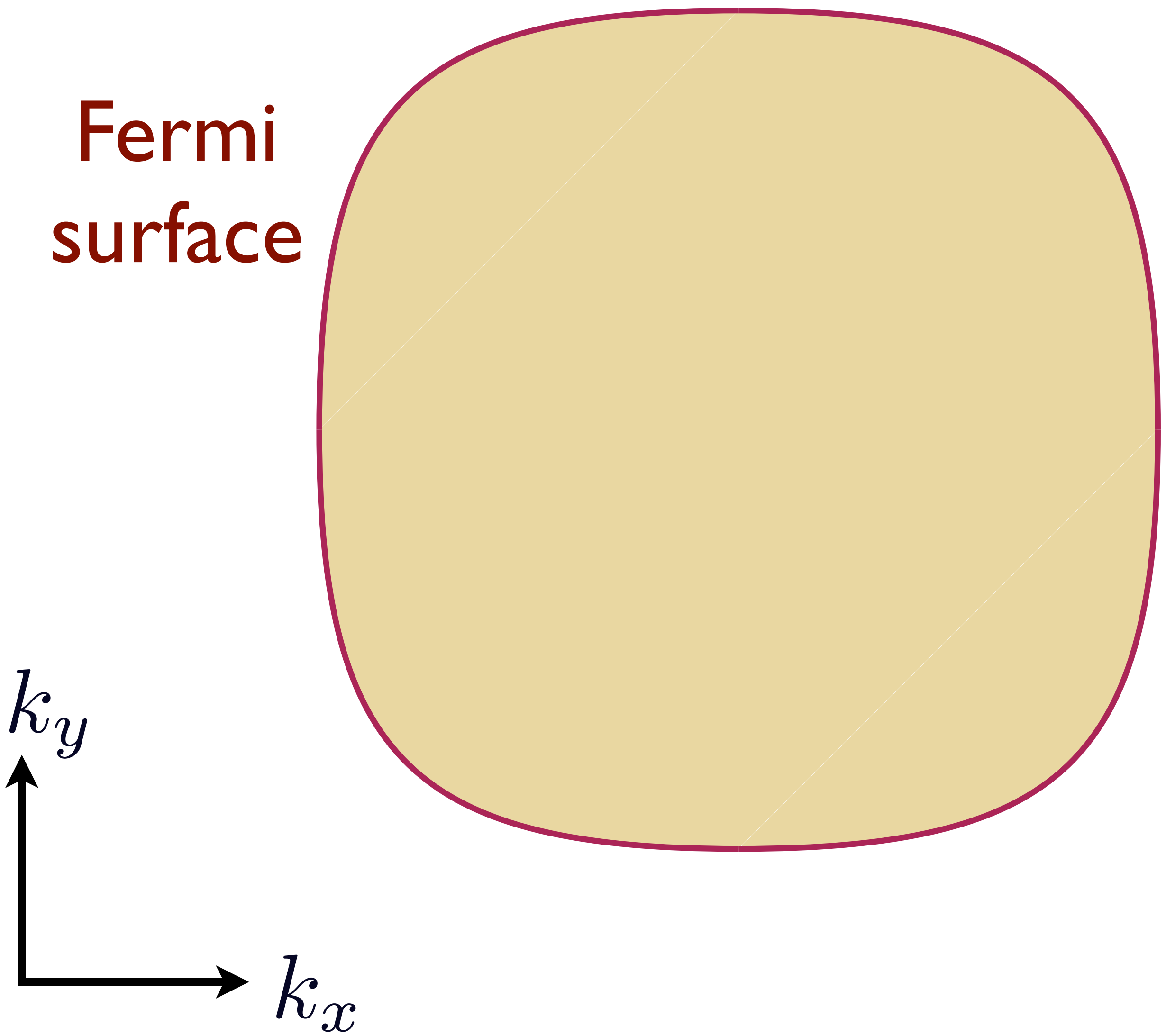


- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2)
- Density of electrons can be continuously varied at zero temperature.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

Fermi Liquid Theory

(Chapter 15, Bruus + Flensberg).

In Hartree-Fock theory

$$G(k, i\omega_n) = \frac{Z}{i\omega_n - v_F(k - k_F)}$$

where $v_F = \frac{k_F}{m^*}$

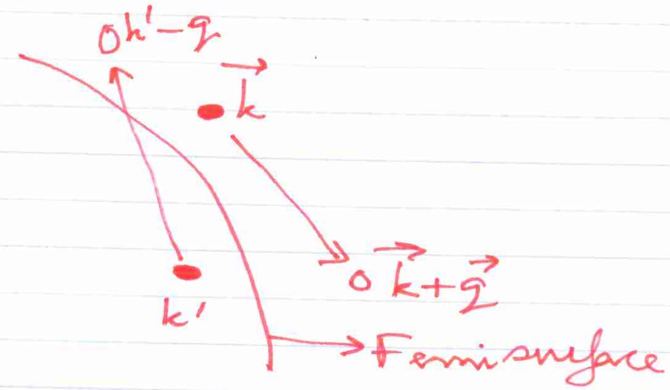
and $2\int^{k_F} \frac{d^3k}{8\pi^3} = N$ (Luttinger relation).

Beyond Hartree-Fock

Z and m^* will have corrections at order U^2.

However more significant is the imaginary of the self energy, representing decay of particle excitations.

By Fermi's Golden rule decay rate for a particle with momentum $\vec{k}$.



$$\frac{1}{\tau_k} = |U|^2 \int_{\vec{k}',\vec{q}} \left(1 - f(\varepsilon_{k+q})\right) f(\varepsilon_{k'}) \left(1 - f(\varepsilon_{k'-q})\right) \delta\left(\varepsilon_k + \varepsilon_{k'} - \varepsilon_{k+q} - \varepsilon_{k'-q}\right)$$

$$\sim |U|^2 [g(E_F)]^3 \int_{-\infty}^{0} d\varepsilon' \int_{0}^{\infty} d\varepsilon'' \, \Theta(\varepsilon_k + \varepsilon' - \varepsilon'').$$

$$\sim |U|^2 [g(E_F)]^3 \, \varepsilon_k^2$$

So lifetime $\to \infty$ as $1/\varepsilon^2$ as $\varepsilon \to 0$.

Particle-decay effects can be neglected near the Fermi surface!

Scattering rate of fermions vanishes on the Fermi surface.

$$\Rightarrow \text{Im}\,\Sigma(k_F, \omega \to 0) = 0.$$

$$\Rightarrow G^{-1}(k = k_F, \omega = 0) = 0$$

defines the Fermi surface for an interacting system.

In a Fermi liquid

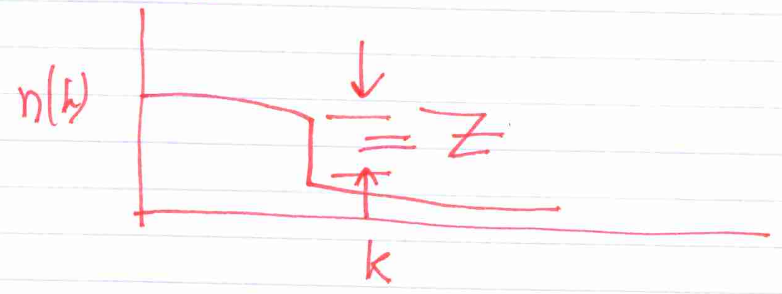
$$G(k, i\omega) = \frac{Z}{i\omega - v_F(k - k_F) + i\omega^2 \,\text{sgn}(\omega)}$$

As $k \to k_F$ and $\omega \to 0$.

Use this to compute

$$n(k) = \int \frac{d\omega}{2\pi} G(k, i\omega)\, e^{i\omega 0^+}$$

Then



Luttinger Relation.

The relationship

$$N = \oint 2\int^{k_F} \frac{d^3k}{8\pi^3} \quad \text{is exact!}$$

Write

$$N = +2\int \frac{d^3k}{8\pi^3}\frac{d\omega}{2\pi} G(k, i\omega)\, e^{i\omega 0^+}$$

$$= +2i\int \frac{d^3k}{8\pi^3}\frac{d\omega}{2\pi}\left[-i G_k(i\omega)\frac{\partial}{\partial\omega}\Sigma_k(i\omega) - \frac{\partial}{\partial\omega}\ln G(i\omega)\right]$$

where $G(i\omega) = \dfrac{1}{i\omega - \epsilon_k - \Sigma_k(i\omega)}$

We will show below that

$$\int \frac{d^3k}{8\pi^3}\frac{d\omega}{2\pi} G_k(i\omega)\frac{\partial}{\partial\omega}\Sigma_k(i\omega) = 0 \quad (*).$$

So

$$N = -2i \int \frac{d^3k\,d\omega}{(2\pi)^4} e^{i\omega 0^+} \frac{\partial}{\partial \omega} \ln G_k(i\omega)$$

$$= -2i \int \frac{d^3k}{8\pi^3} \int_{-\infty}^{0} \frac{dz}{2\pi} \frac{\partial}{\partial z} \ln \frac{G_k(z+i0^+)}{G_k(z+i0^-)}.$$

$$= -2i \int \frac{d^3k}{(2\pi)^4} \ln \frac{G_k(i0^+)}{G_k(i0^-)}.$$

$$= 2 \int \frac{d^3k}{(2\pi)^3} \theta\left(-\varepsilon_k - \Sigma_k(i0^+)\right)$$

$$= 2 \int^{k_F} \frac{d^3k}{8\pi^3}$$

where k_F is defined by

$$G^{-1}(k = k_F, \omega = 0) = 0.$$

↑ location of Fermi surface.

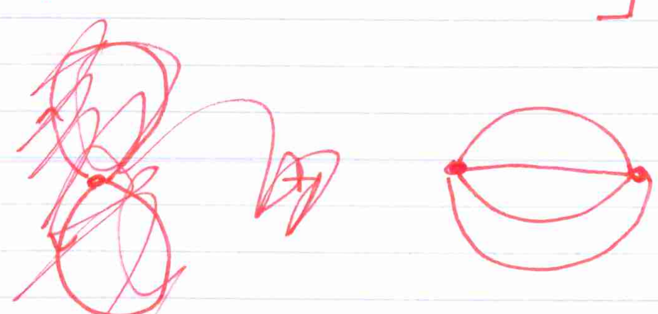
Finally we need to establish (☆).

(☆) can be shown if there exists a functional $\Phi[G_k(i\omega)]$ (Luttinger-Ward functional)

with 2 properties

(i) $\Sigma_k(i\omega) = \dfrac{\delta \Phi}{G_k(i\omega)}$

(ii) $\Phi[G_k(i\omega + i\varepsilon)] = \Phi[G_k(i\omega)]$

$\Phi =$  $+ \dots$

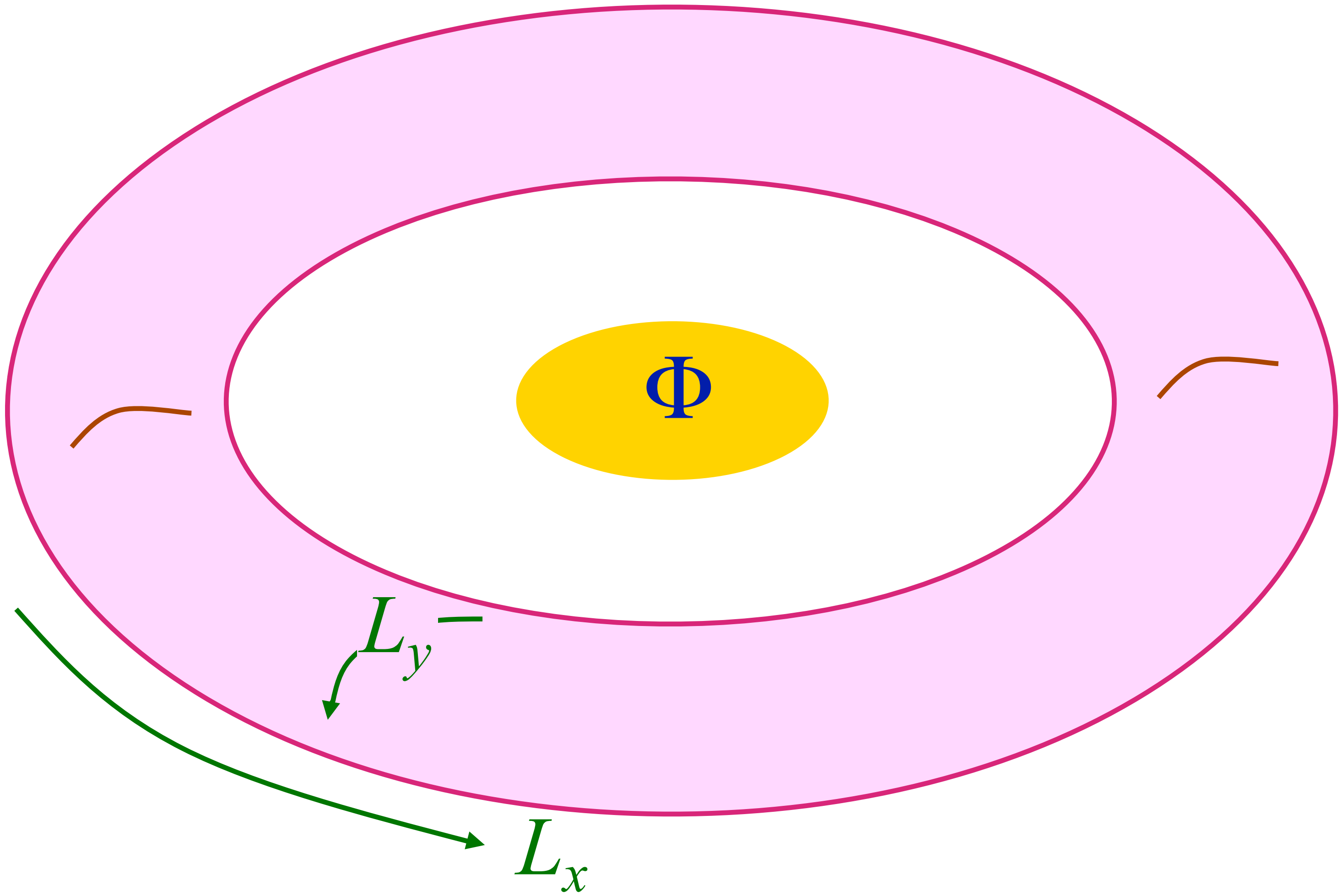
$= U\int GG + U^2 \int GGGG + \dots$

Sum of 2-particle irreducible graphs.

For more details, see cond-mat/0406671.

Oshikawa's non-perturbative proof of the Luttinger theorem

M. Oshikawa, PRL **84**, 3370 (2000)
A. Paramekanti and A. Vishwanath,
PRB **70**, 245118 (2004)



We take N particles, each with charge Q, on a $L_x \times L_y$ lattice on a torus.
We pierce flux $\Phi = hc/Q$ through a hole of the torus.
An exact computation shows that the change in crystal momentum of the many-body state due to flux piercing is

$$P_{xf} - P_{xi} = \frac{2\pi N}{L_x} (\mathrm{mod}\ 2\pi) = 2\pi\nu L_y (\mathrm{mod}\ 2\pi)$$

where $\nu = N/(L_x L_y)$ is the density.

Oshikawa's non-perturbative proof of the Luttinger theorem

Proof of
$$P_{xf} - P_{xi} = \frac{2\pi N}{L_x} (\text{mod } 2\pi) = 2\pi \nu L_y (\text{mod } 2\pi).$$

The initial and final Hamiltonians are related by a gauge transformation

$$\mathcal{U}_G H_f \mathcal{U}_G^{-1} = H_i \quad , \quad \mathcal{U}_G = \exp\left(i \frac{2\pi}{L_x} \sum_i x_i \hat{n}_i \right).$$

while the wavefunction evolves from $|\Psi_i\rangle$ to $\mathcal{U}_T |\Psi_i\rangle$, where $\mathcal{U}_T$ is the time evolution operator. We want to work in a fixed gauge in which the initial and final Hamiltonians are the same: in this gauge, the final state is $|\Psi_f\rangle = \mathcal{U}_G \mathcal{U}_T |\Psi_i\rangle$. Let $\hat{T}_x$ be the lattice translation operator. Then we can show that

$$\hat{T}_x |\Psi_i\rangle = e^{-iP_{xi}} |\Psi_i\rangle \quad , \quad \hat{T}_x |\Psi_f\rangle = e^{-iP_{xf}} |\Psi_f\rangle ,$$

using the easily established properties

$$\hat{T}_x \mathcal{U}_T = \mathcal{U}_T \hat{T}_x \quad , \quad \hat{T}_x \mathcal{U}_G = \exp\left(-i2\pi \frac{N}{L_x} \right) \mathcal{U}_G \hat{T}_x$$

Oshikawa's non-perturbative proof of the Luttinger theorem

$$\Delta P_x = 2\pi\nu L_y (\text{mod } 2\pi) \quad , \quad \Delta P_y = 2\pi\nu L_x (\text{mod } 2\pi)$$

Now we compute the momentum balance *assuming* that the only low energy excitations are quasiparticles near the Fermi surface, and these react like free particles to a sufficiently slow flux insertion. Then we can write

$$\Delta P_x = \left(\frac{2\pi}{L_x}\right)\frac{L_x L_y}{4\pi^2} V_{\rm FS} \quad , \quad \Delta P_y = \left(\frac{2\pi}{L_y}\right)\frac{L_x L_y}{4\pi^2} V_{\rm FS}$$

where $V_{\rm FS}$ is the volume of the Fermi surface. Actually, the quasiparticles are only defined near the Fermi surface, but by using Gauss's Law on the momentum acquired by quasiparticles near the Fermi surface, we can convert the answer to an integral over the volume enclosed by the Fermi surface, as shown above.

Now we equate these values to those obtained above, and obtain

$$N - L_x L_y \frac{V_{\rm FS}}{4\pi^2} = L_x m_x \quad , \quad N - L_x L_y \frac{V_{\rm FS}}{4\pi^2} = L_y m_y$$

for some integers m_x, m_y. By choosing L_x, L_y mutually prime integers we can now show

$$\nu = \frac{N}{L_x L_y} = \frac{V_{\rm FS}}{4\pi^2} + m$$

for some integer m: this is Luttinger's theorem.

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Topological argument for the Luttinger theorem

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A Fermi liquid co-existing with topological order for the pseudogap metal

3. Quantum matter without quasiparticles

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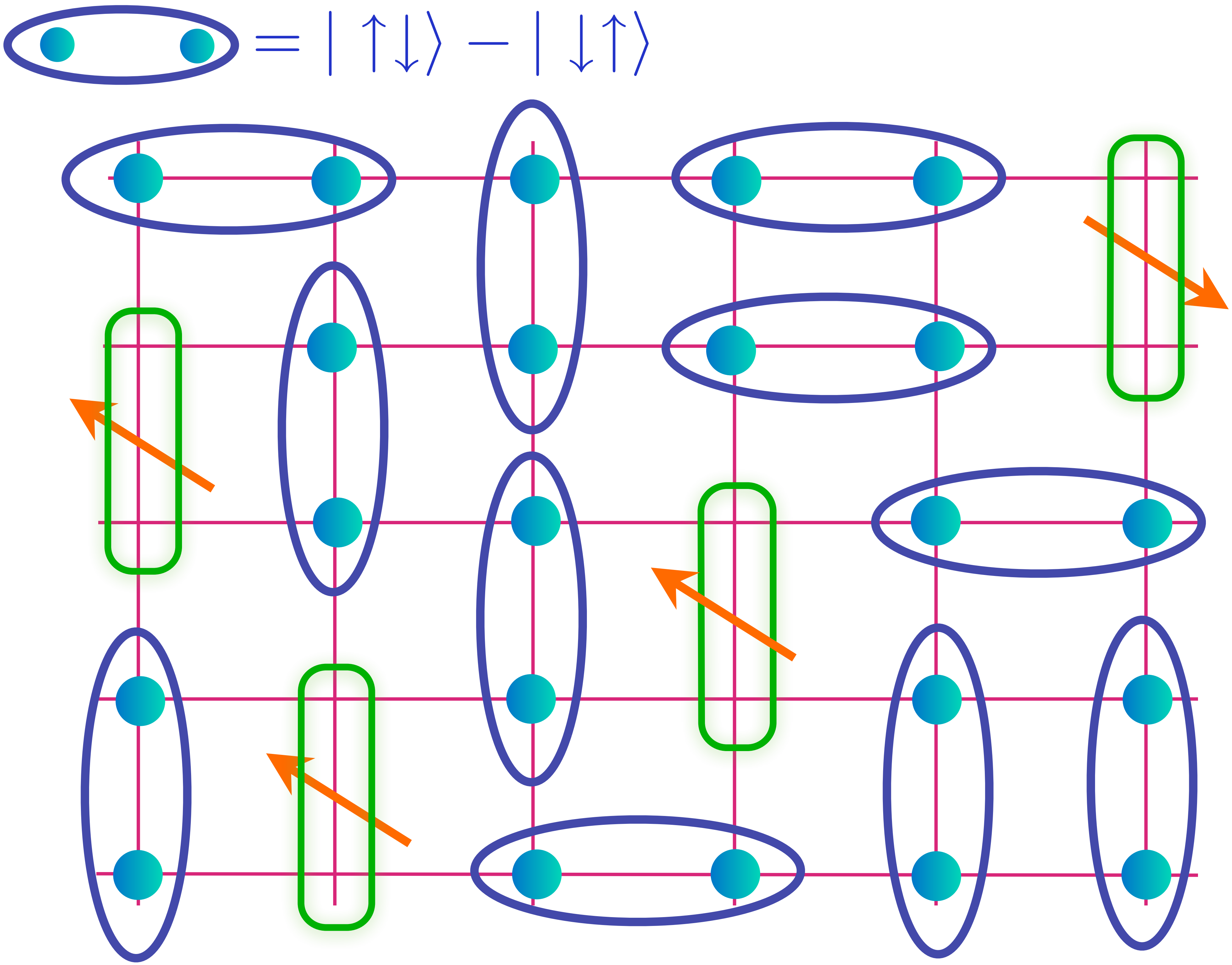
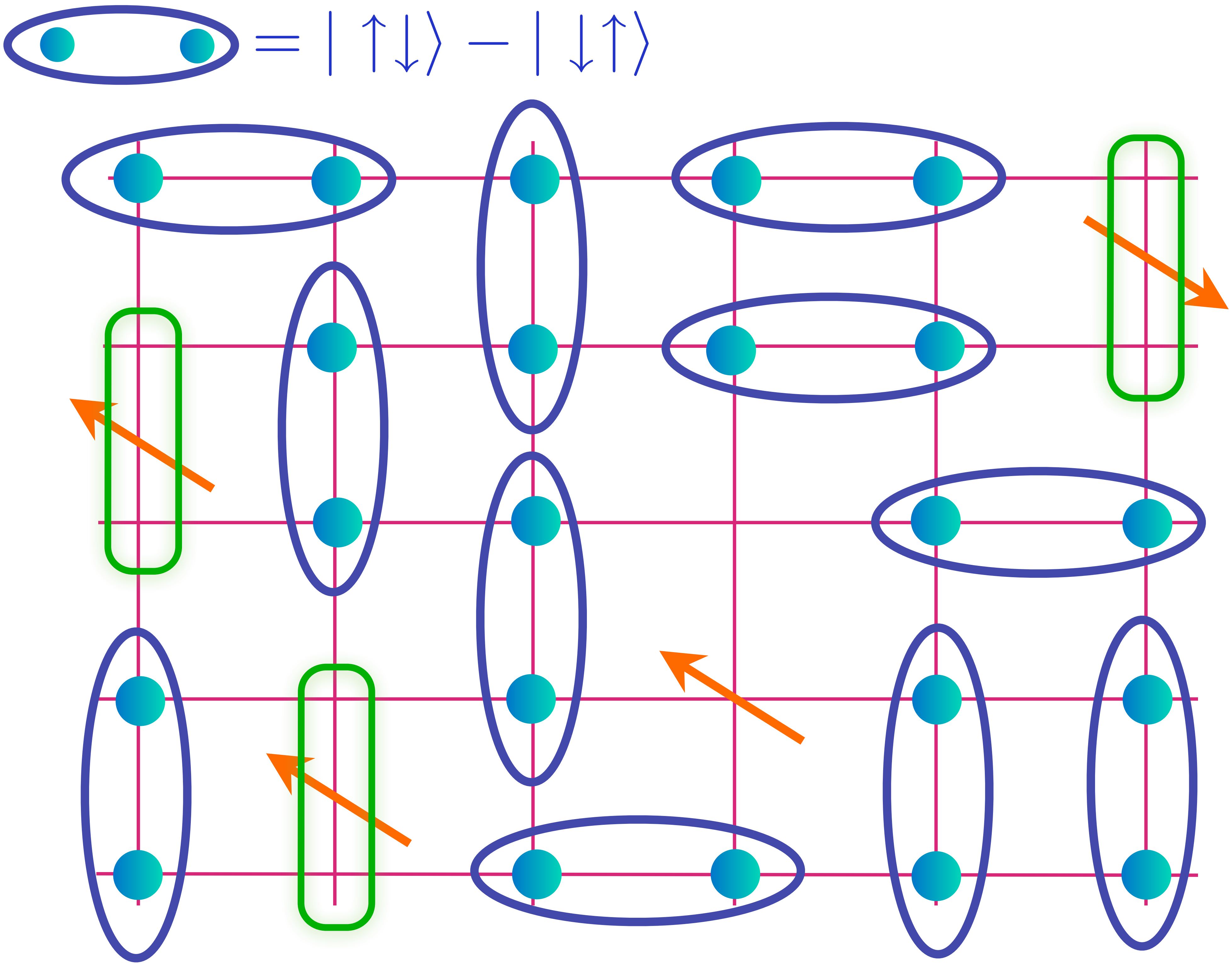
Fractionalized Fermi liquid (FL*)



Realizes a metal with a Fermi surface of area p co-existing with "topological order"

M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

Fractionalized Fermi liquid (FL*)



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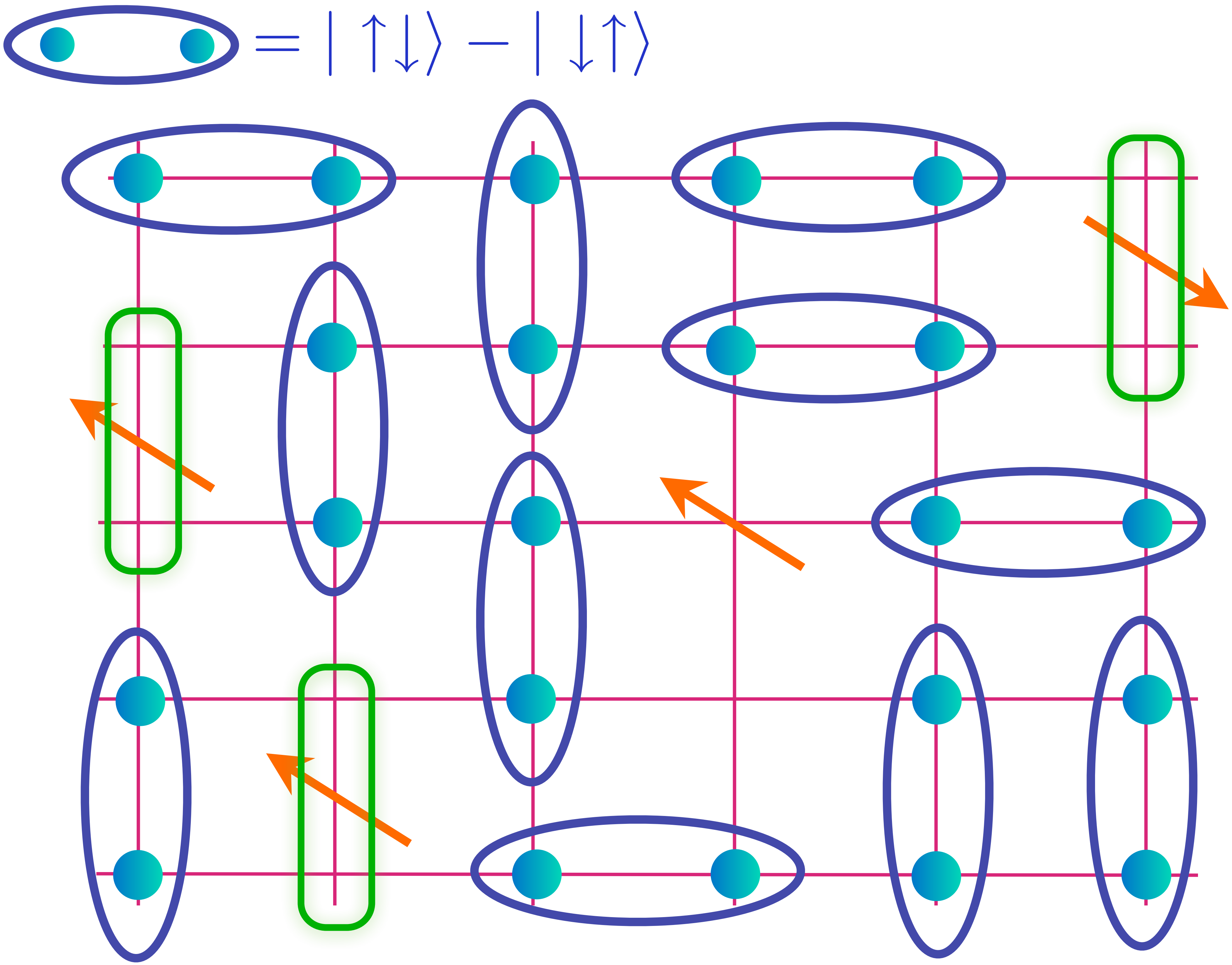

$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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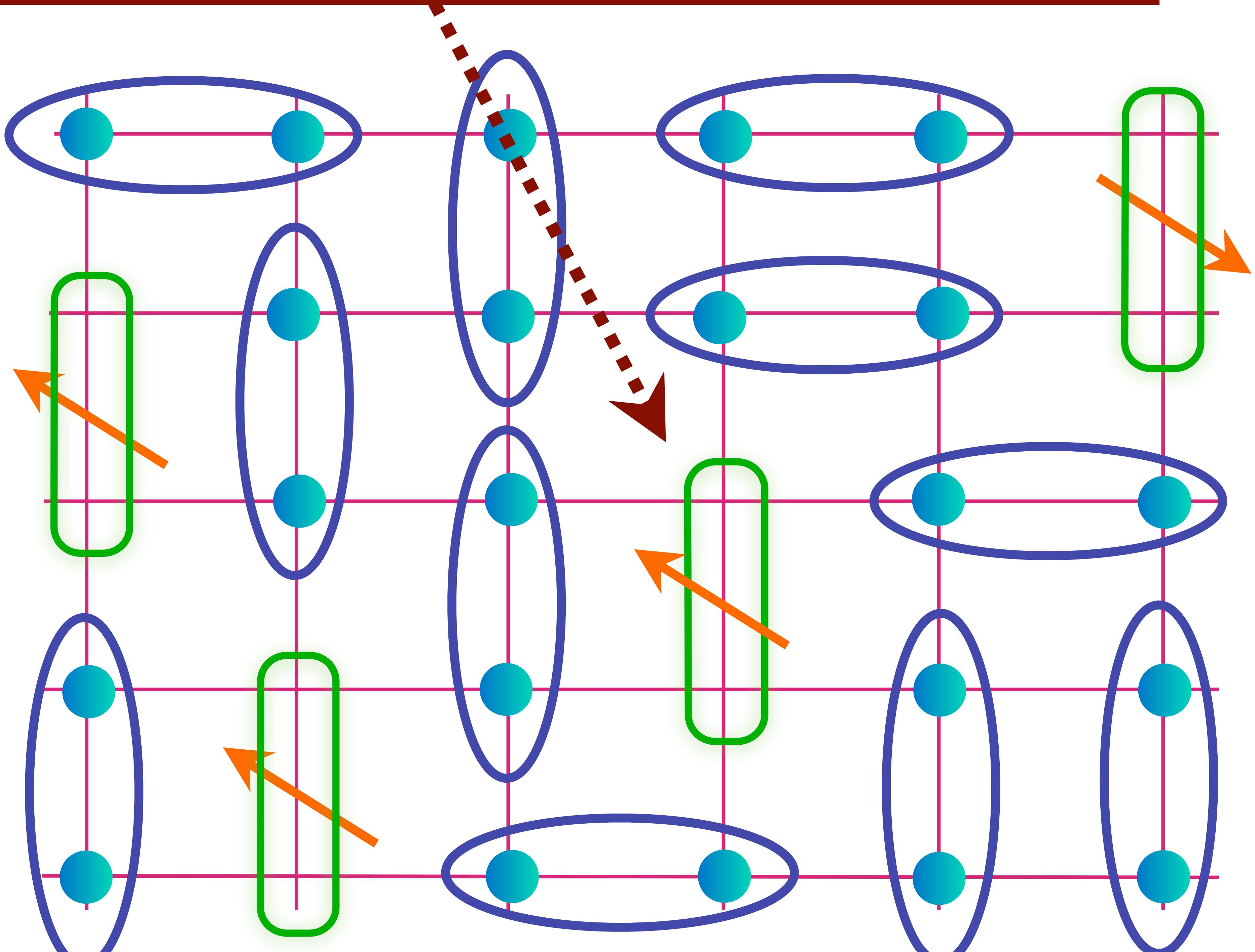
A fermionic "dimer" describing a "bonding" orbital between two sites



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Density of fermionic dimers = p;
density of holes relative to filled band = $1+p$

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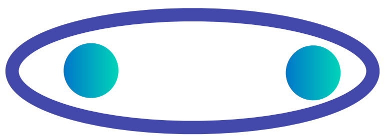
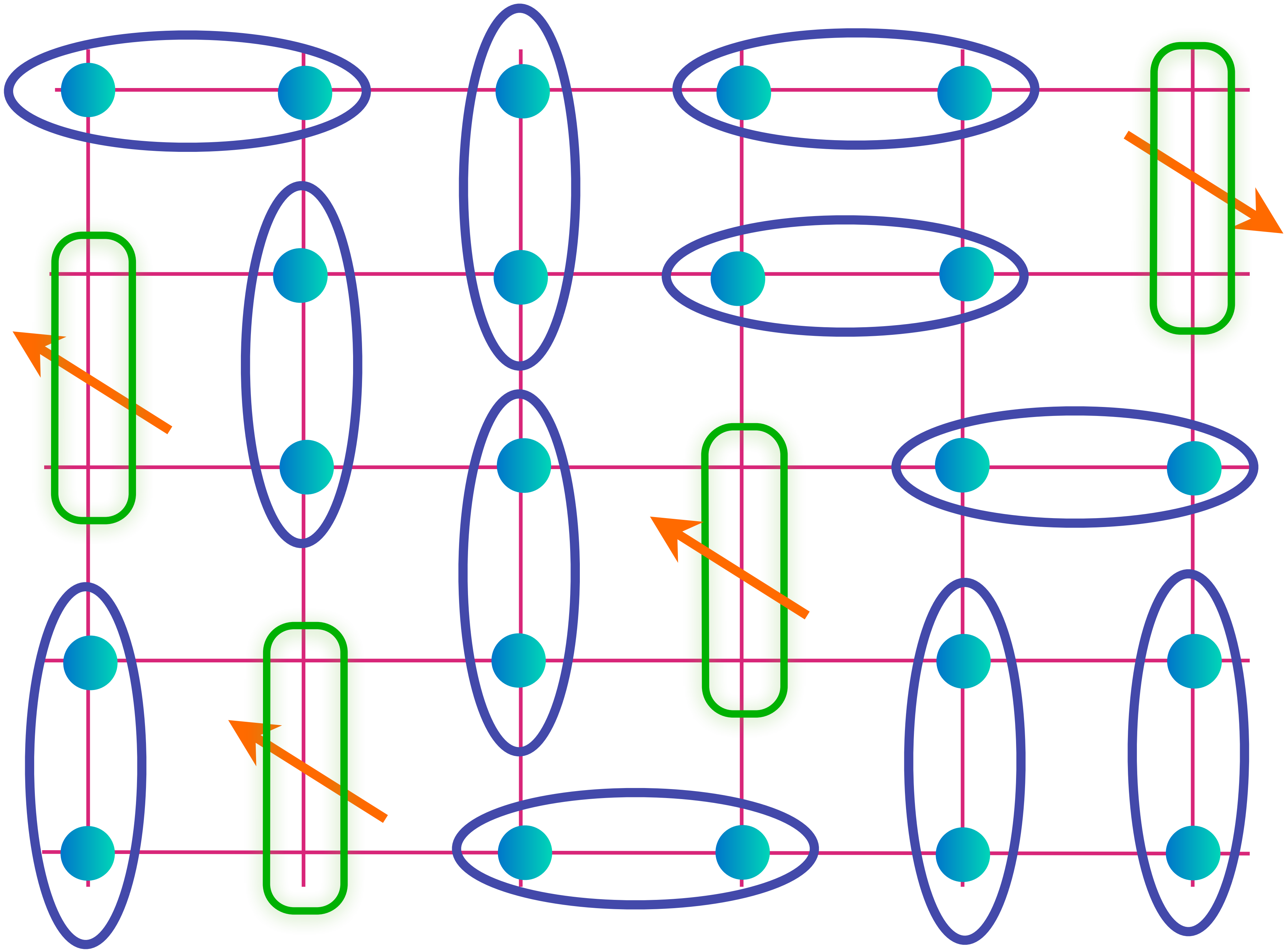
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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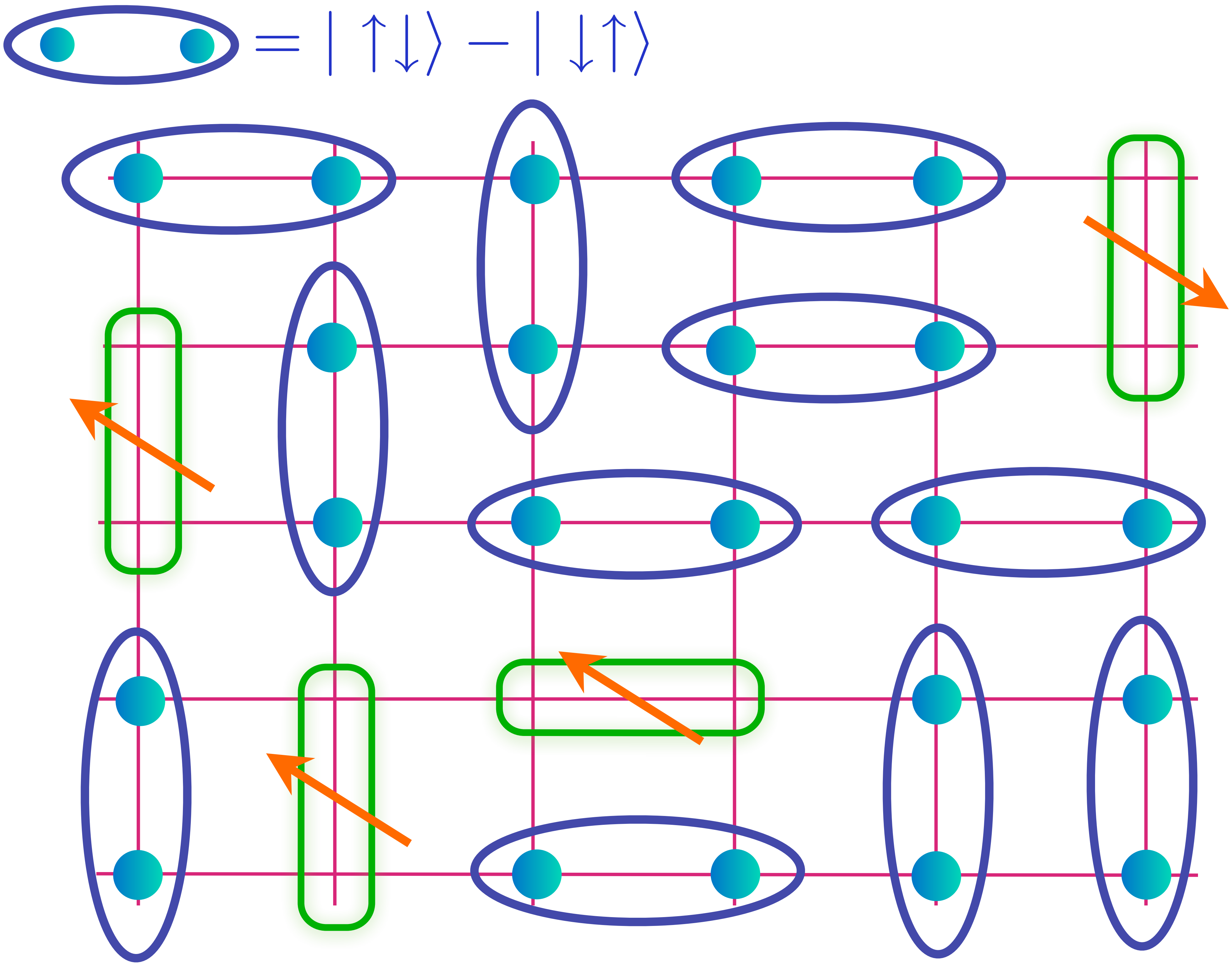

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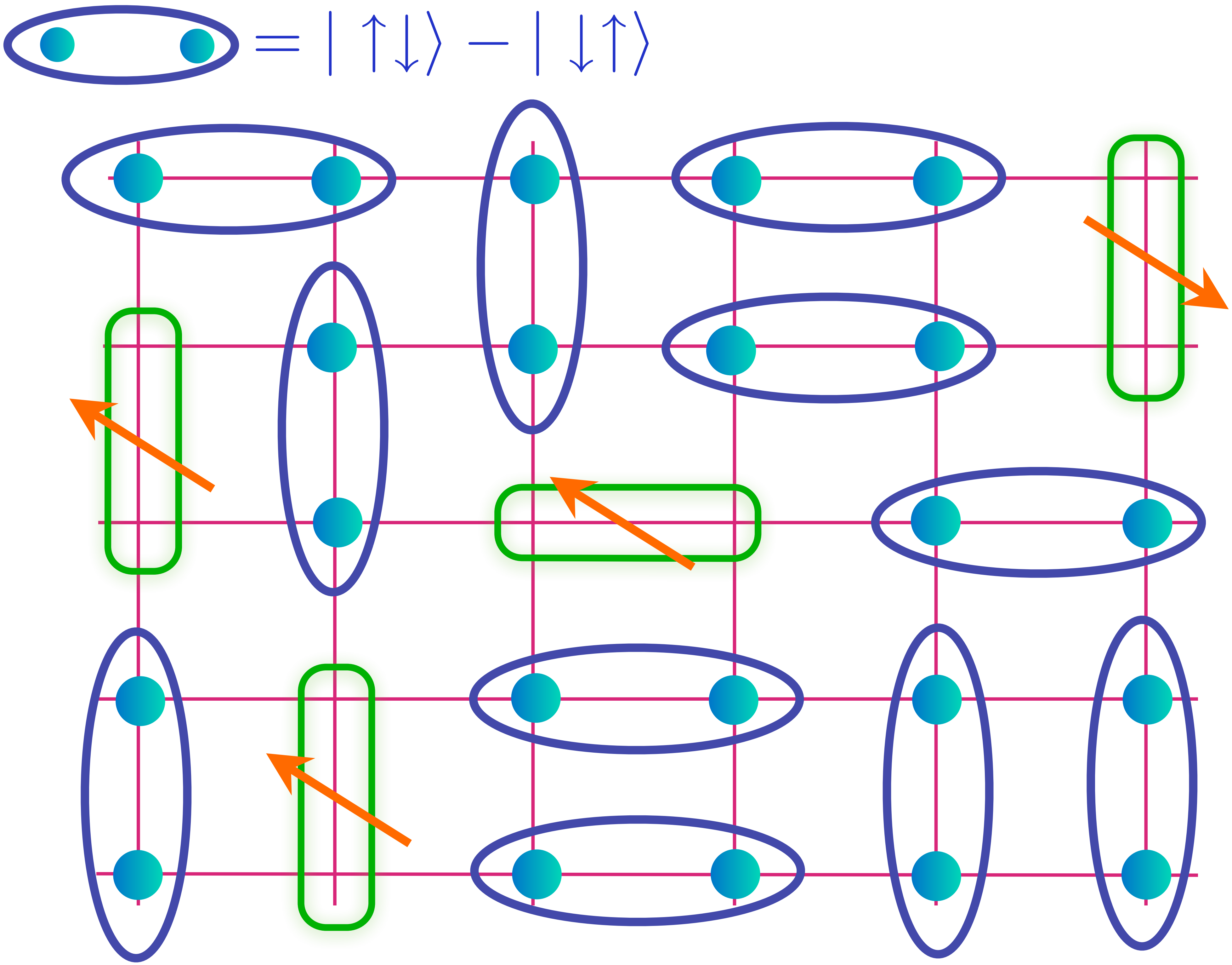

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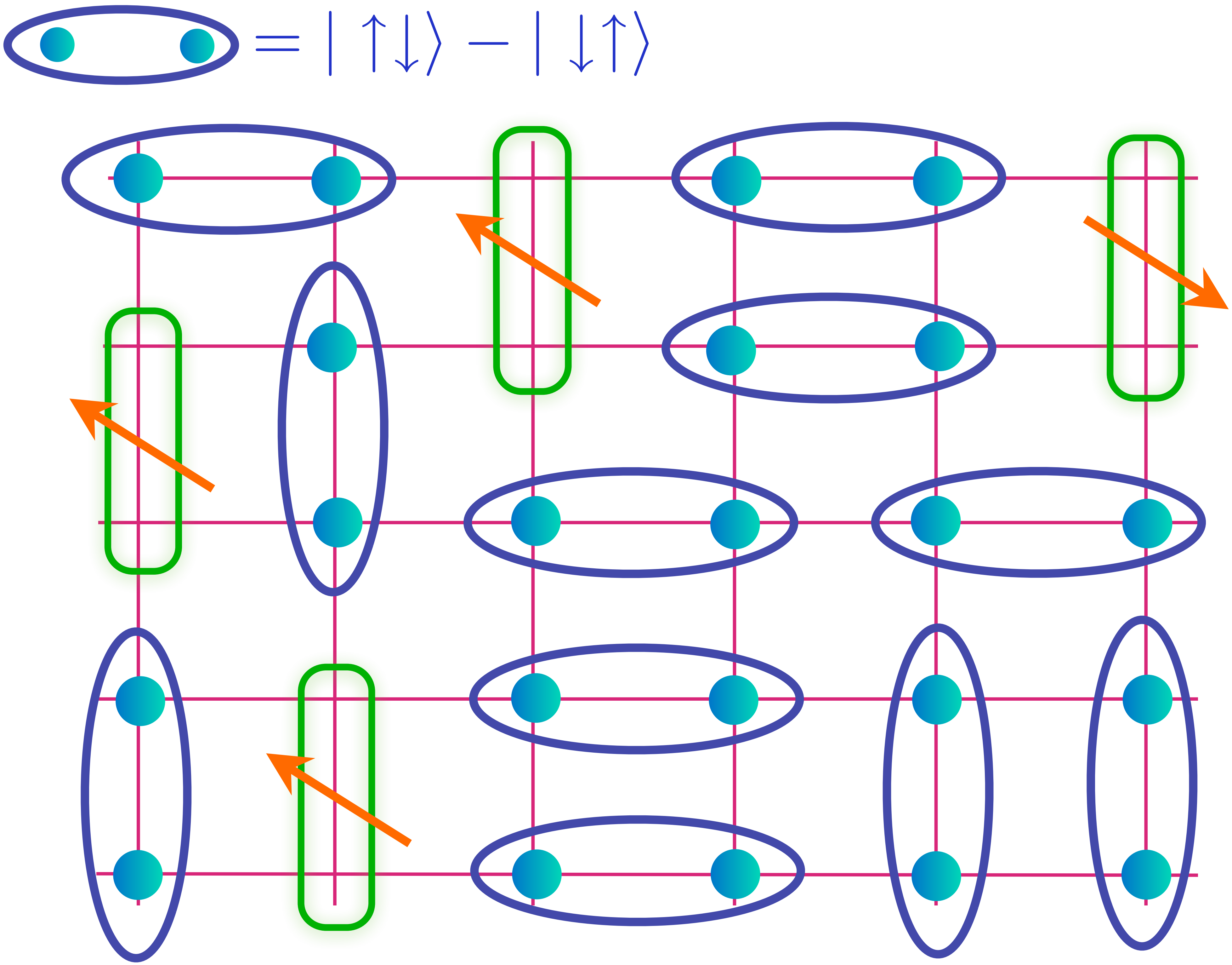

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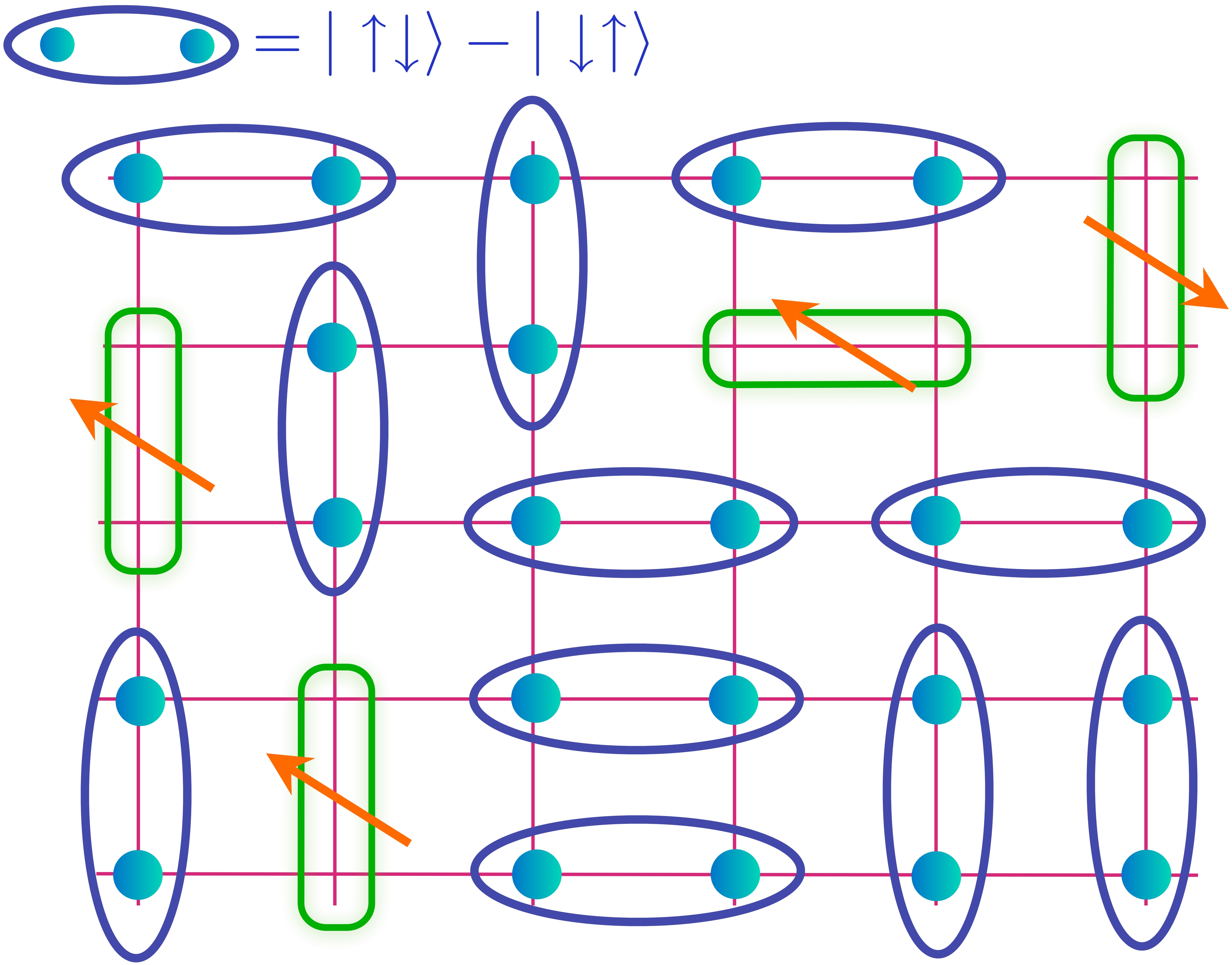

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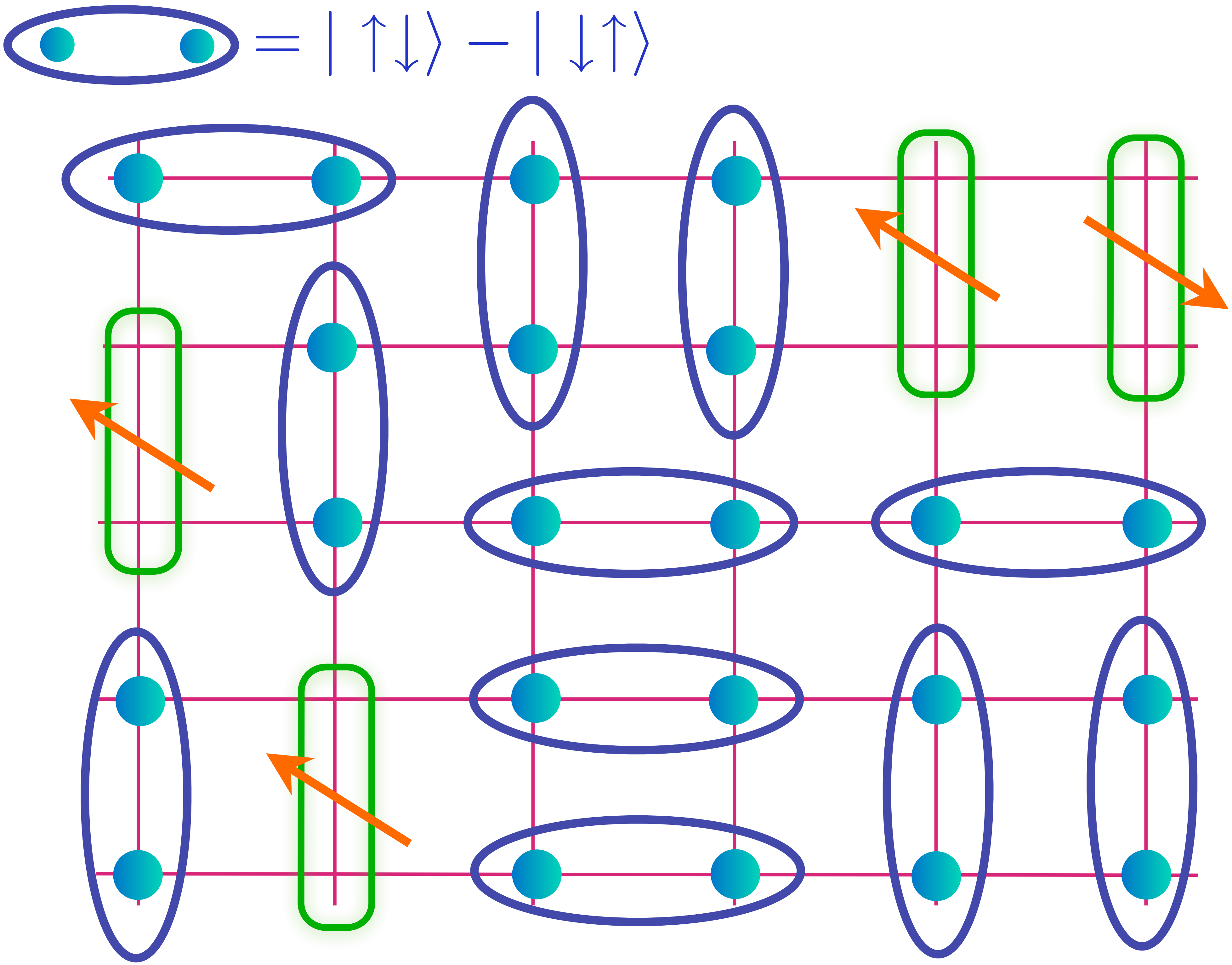

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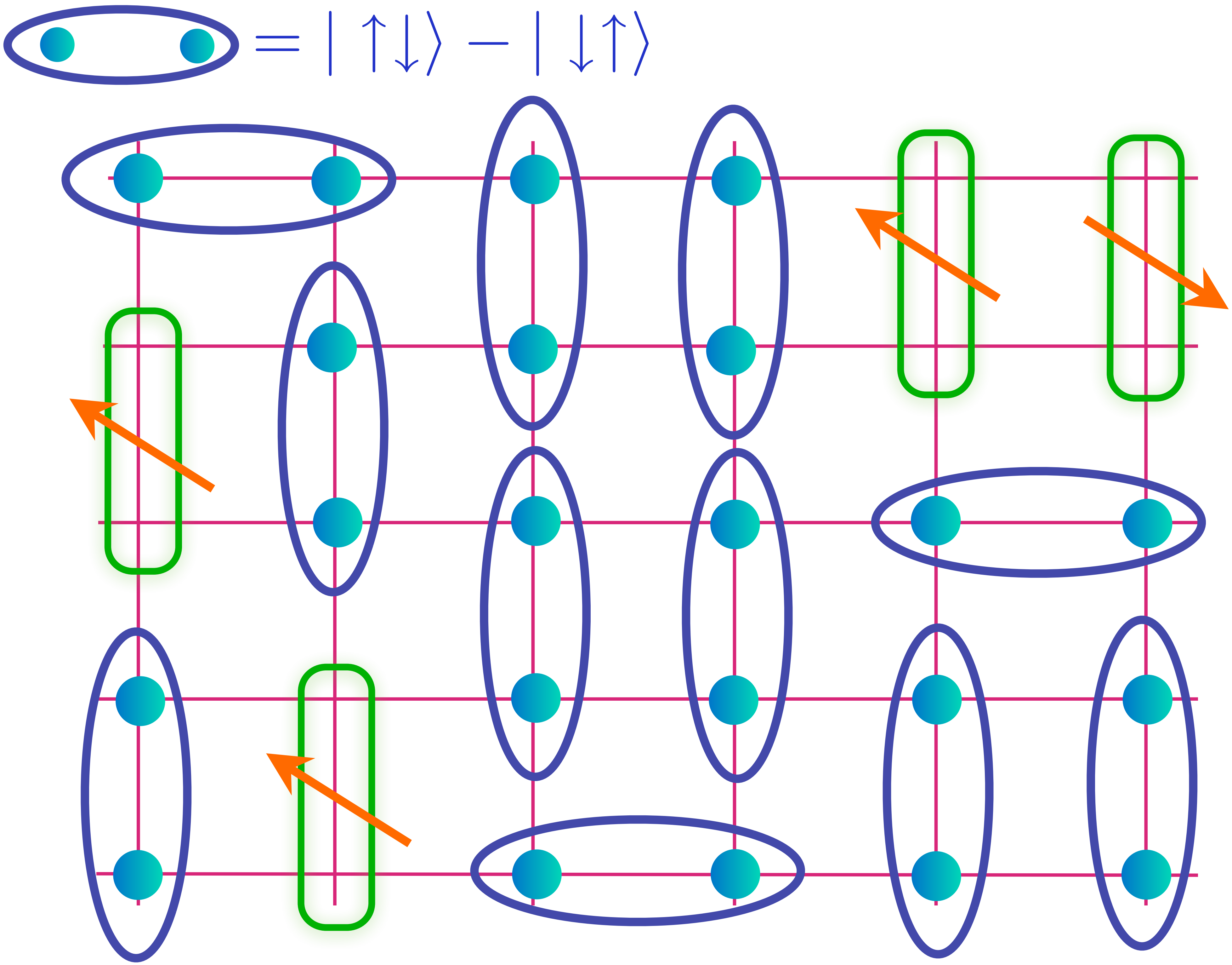

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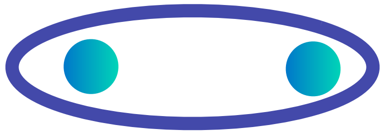
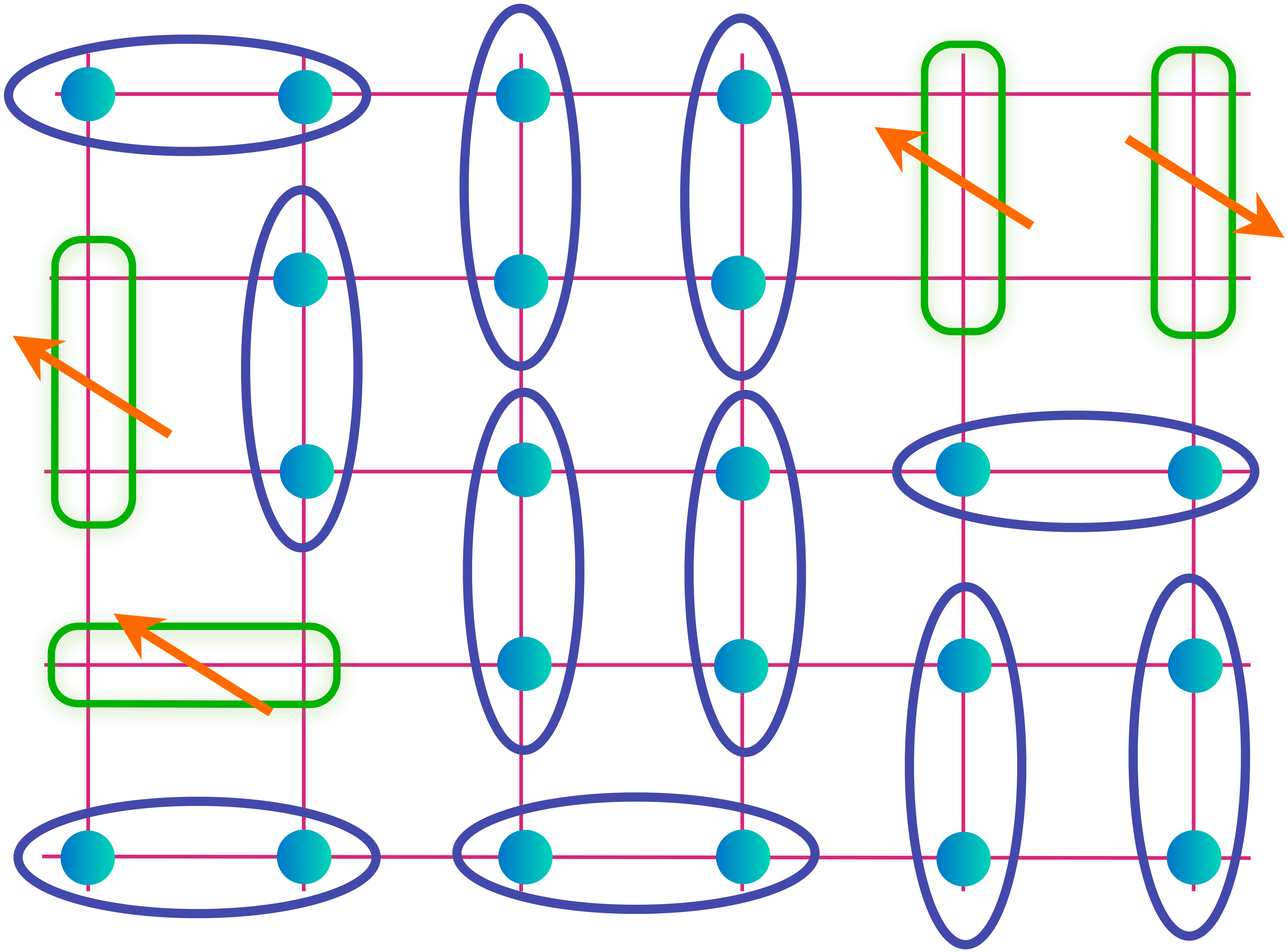
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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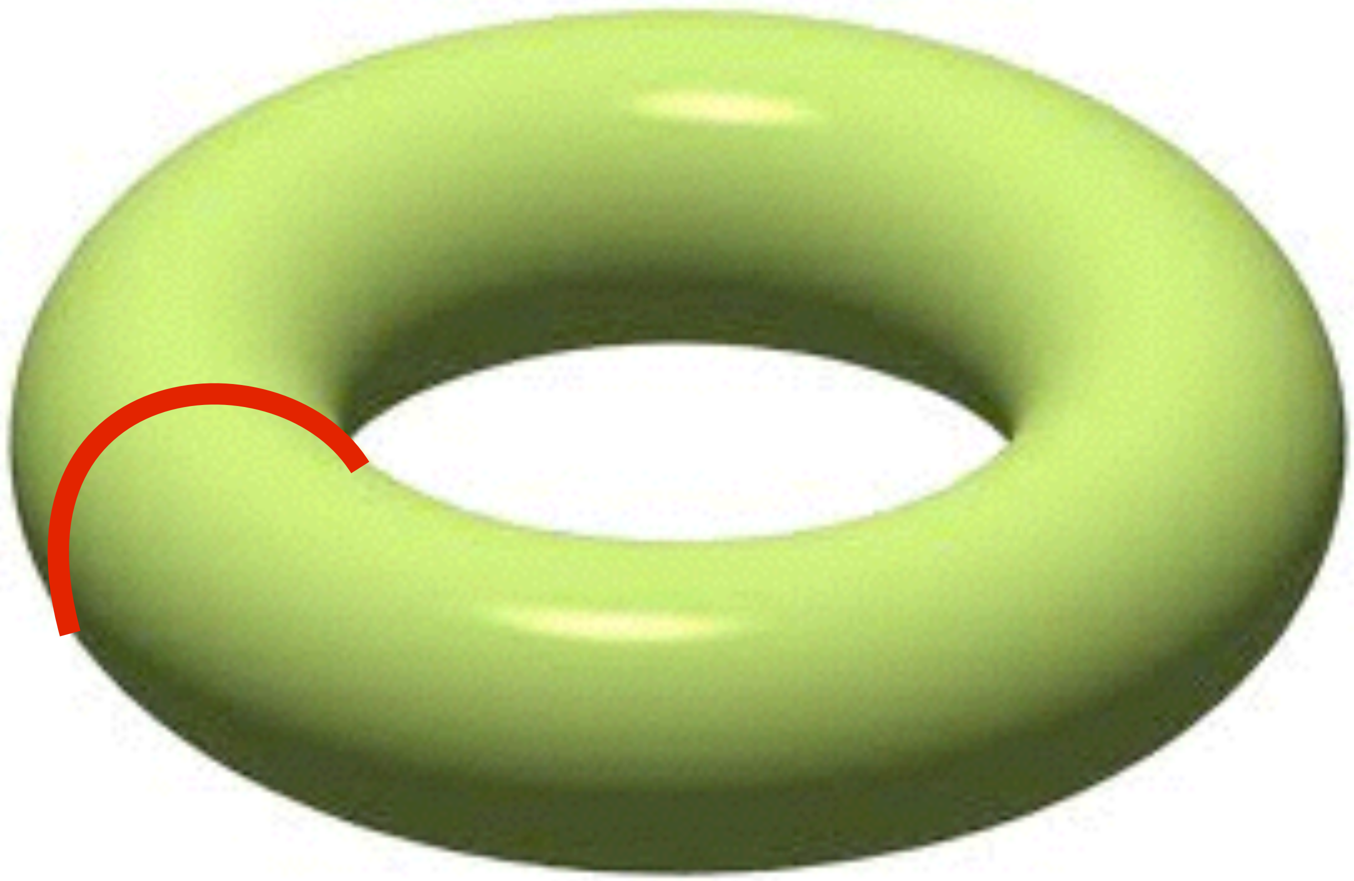
Topological order

Place pseudogap metal on a torus;

Topological order



Place pseudogap metal on a torus;

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is (almost) conserved modulo 2

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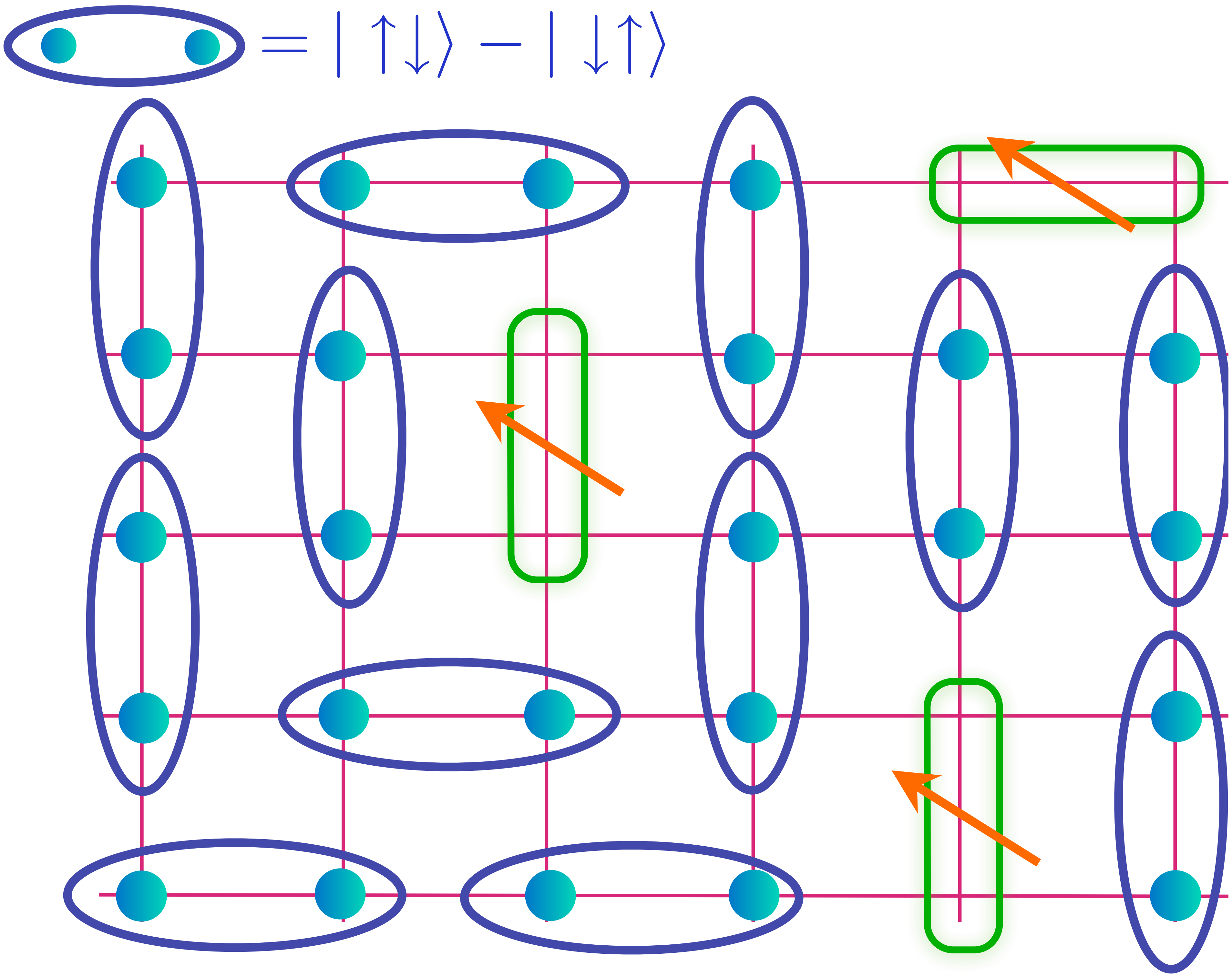

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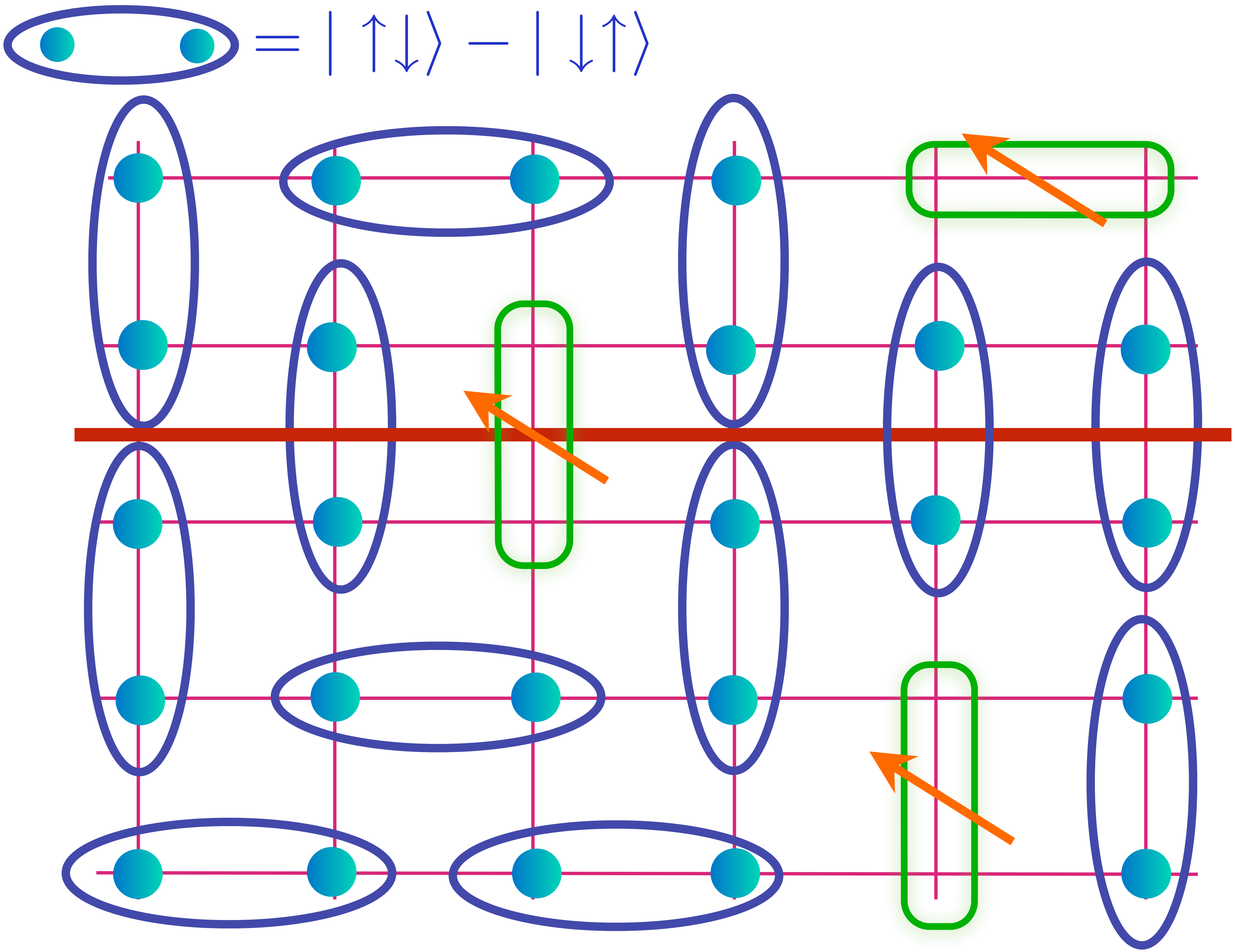

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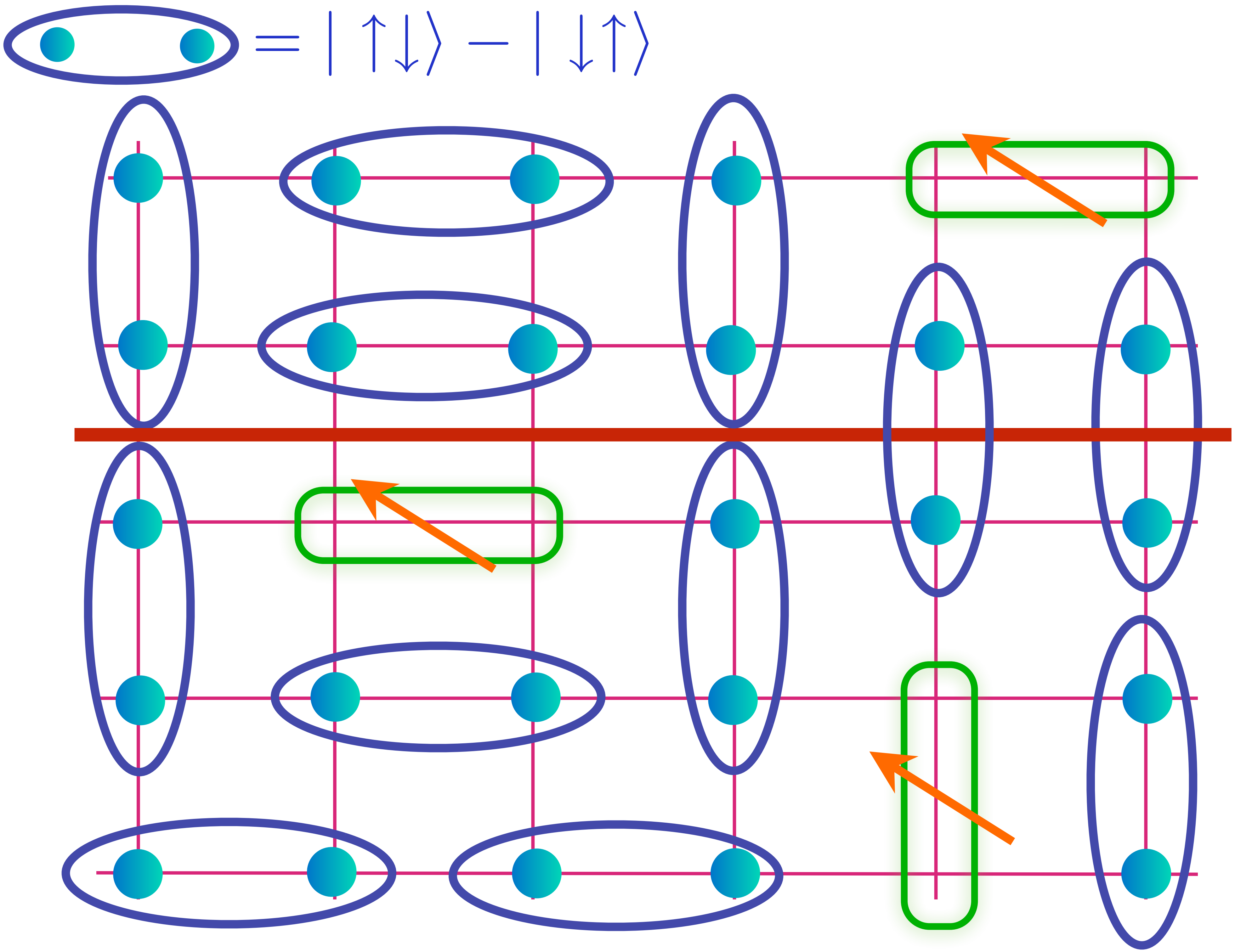

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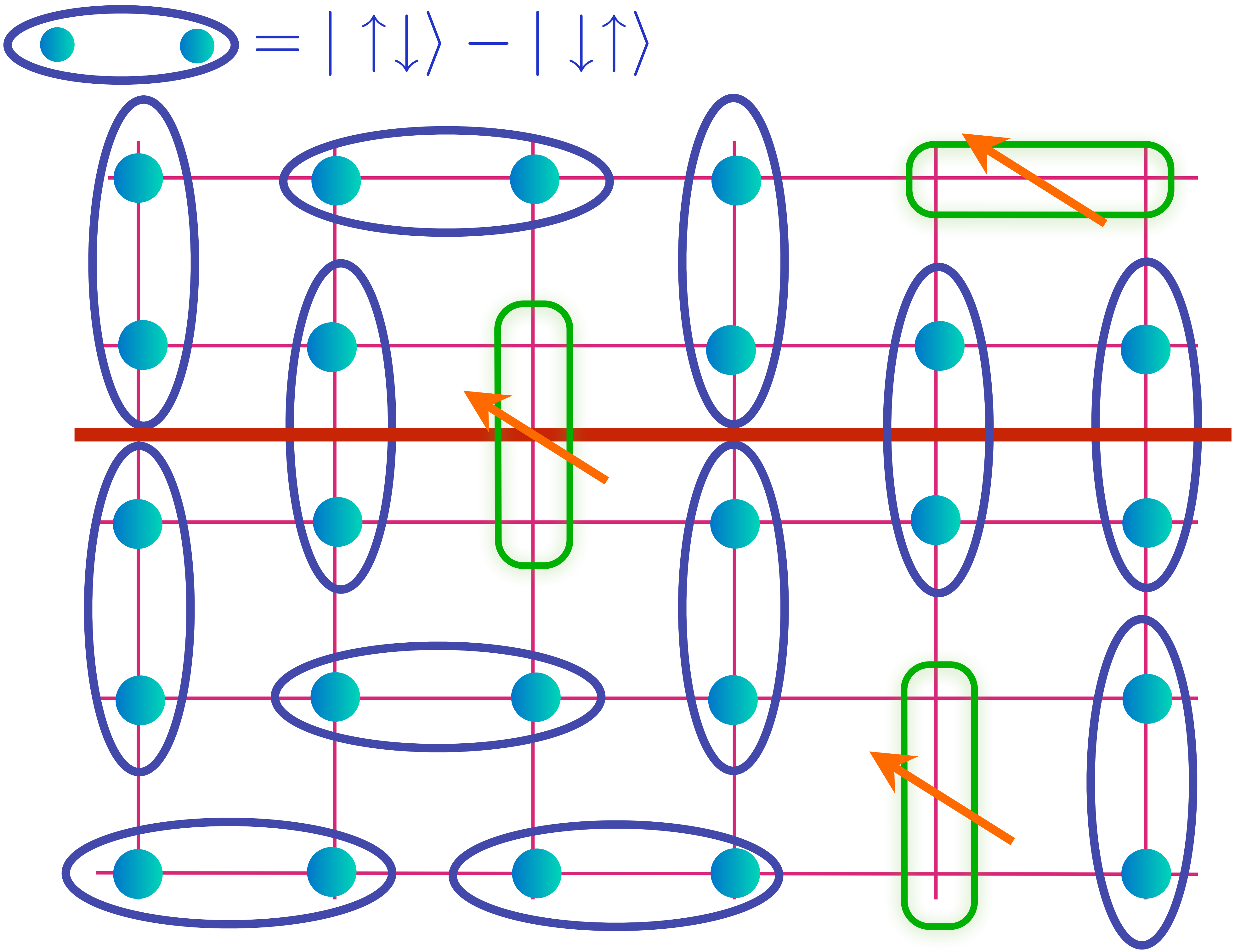

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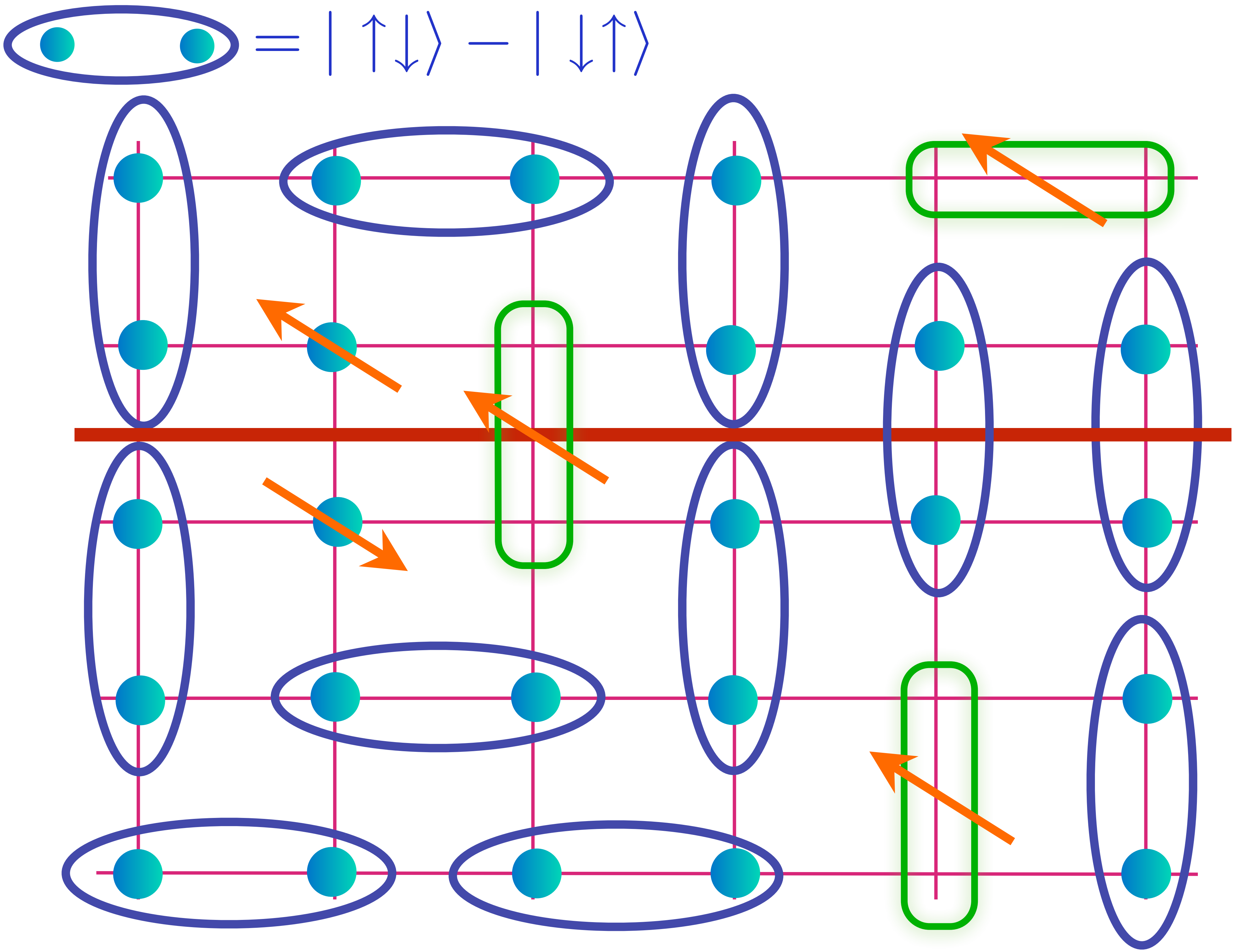
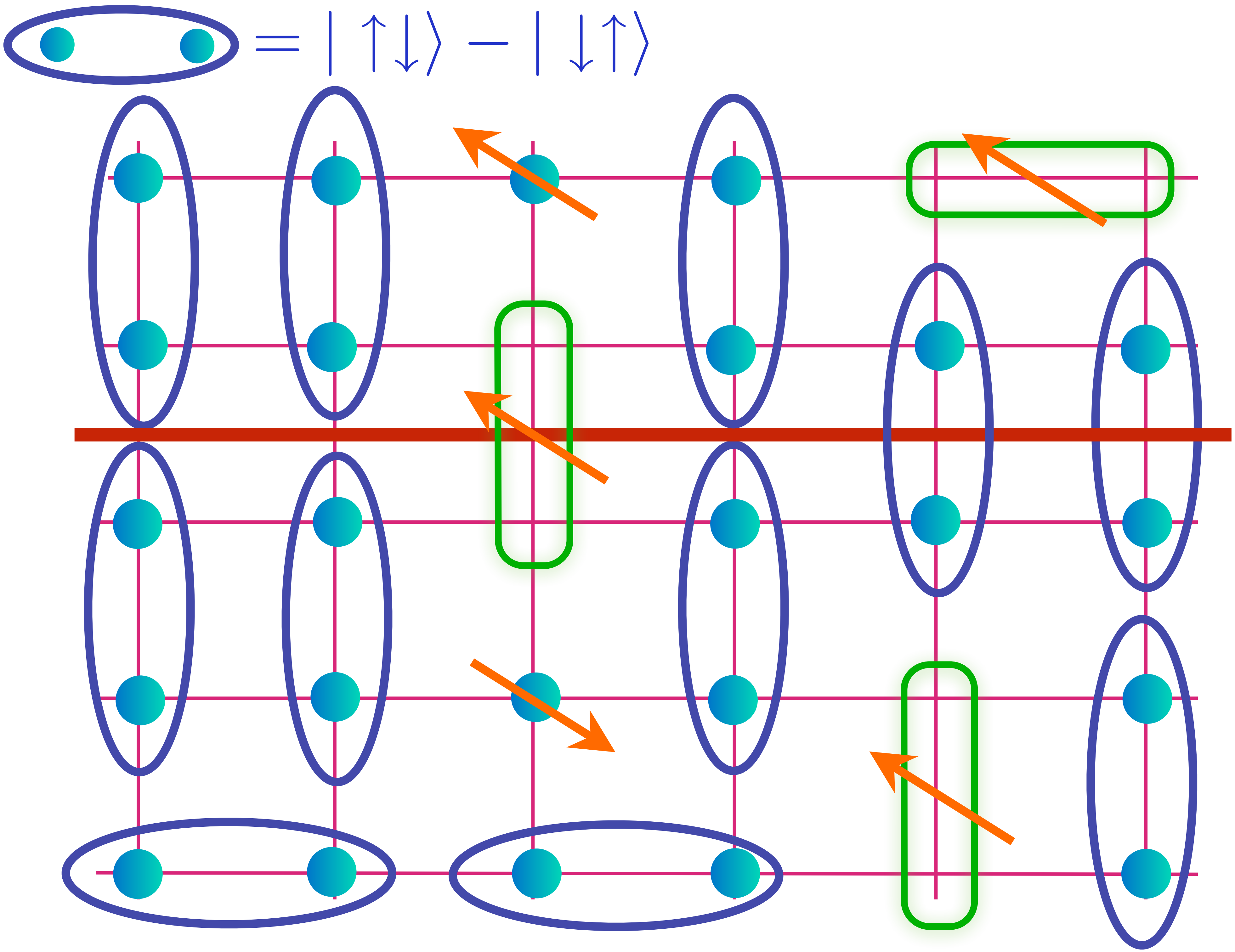


Place pseudogap metal on a torus;

to change dimer number parity across red line, it is necessary to create a pair of unpaired spins and move them around the sample.

Topological order



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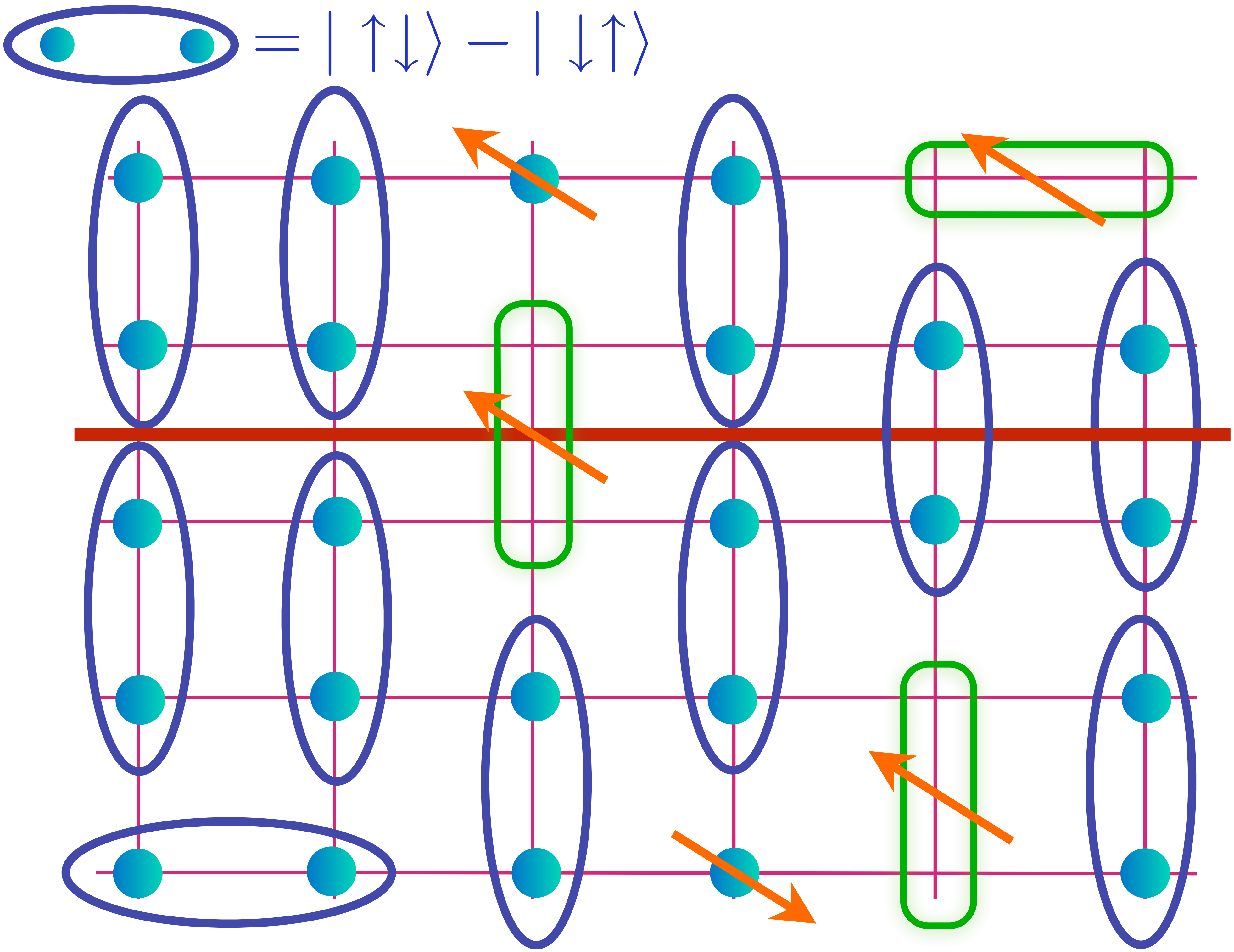

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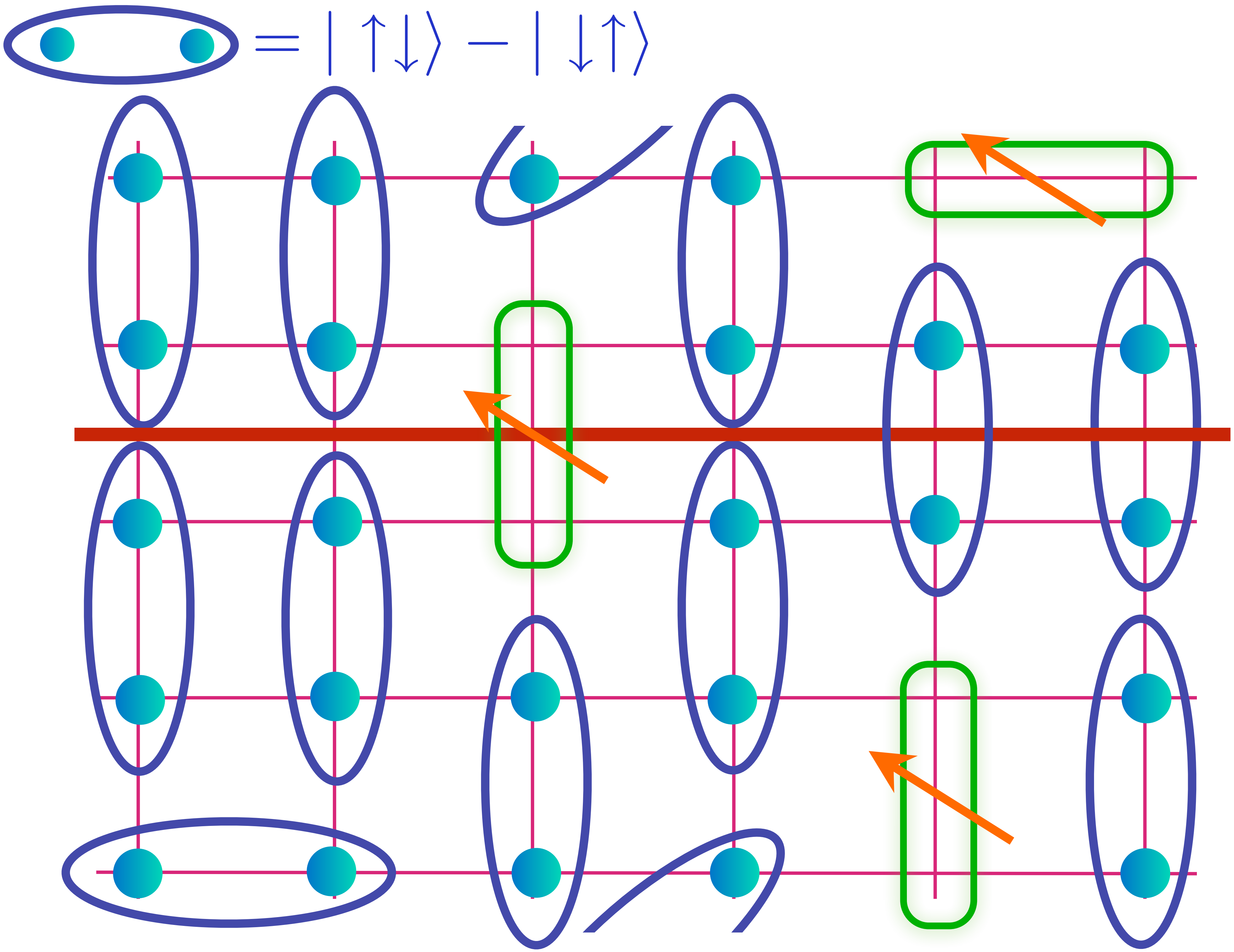
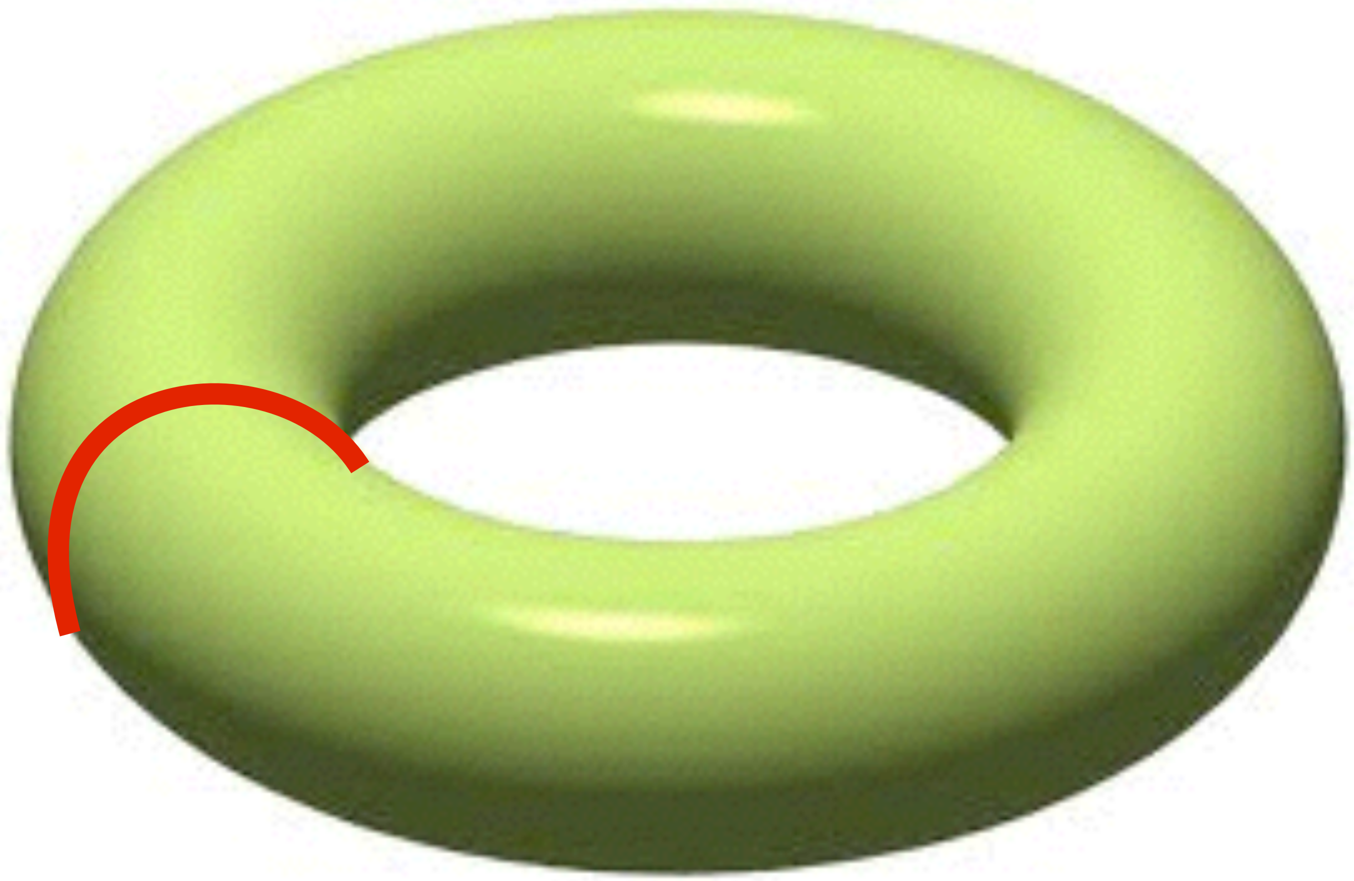


Place pseudogap metal on a torus;

to change dimer number parity across red line, it is necessary to create a pair of unpaired spins and move them around the sample.

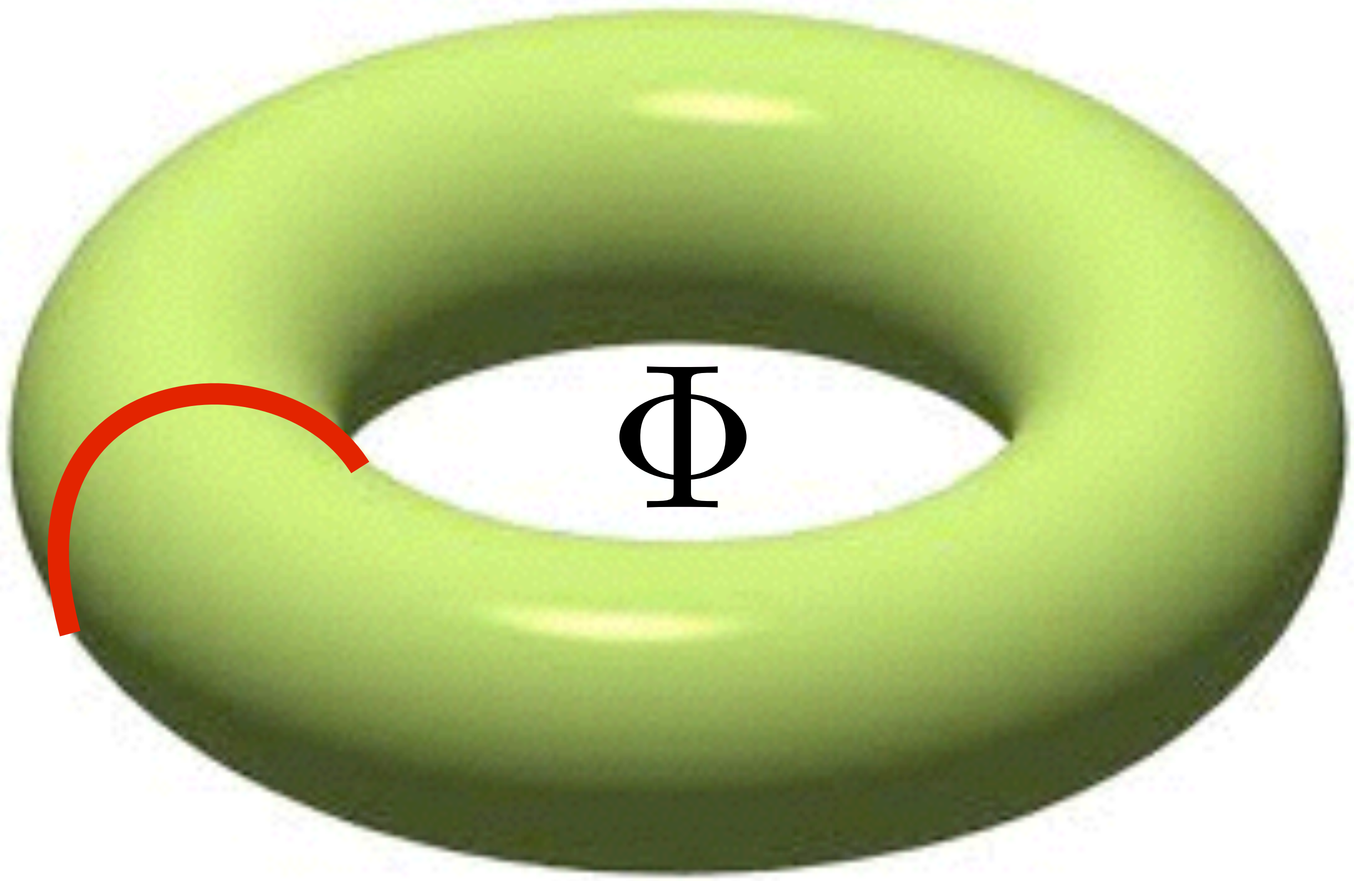
Topological order



Place pseudogap metal on a torus;

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is (almost) conserved modulo 2

Topological order: flux insertion



Upon inserting a flux quantum $\Phi = h/e$ coupling only to $\uparrow$ electrons (say), the states with an even (odd) number of dimers crossing the cut pick up a factor of $+1$ (-1). The hole of the torus has a "*vison*" after the flux insertion.

N. E. Bonesteel, PRB **40**, 8954 (1989).

Fractionalized Fermi liquid (FL*)

$$\Delta P_x = 2\pi\nu L_y (\text{mod } 2\pi) \quad , \quad \Delta P_y = 2\pi\nu L_x (\text{mod } 2\pi)$$

Momentum balance for a Z_2 fractionalized Fermi liquid:

Momentum acquired by quasiparticles near the Fermi surface:

$$(\Delta P_x)_{\text{quasiparticles}} = \left(\frac{2\pi}{L_x}\right) \frac{L_x L_y}{4\pi^2} V_{\text{FS}}$$

Momentum acquired by vison:

$$(\Delta P_x)_{\text{vison}} = \pi L_y$$

Using $\Delta P_x = (\Delta P_x)_{\text{quasiparticles}} + (\Delta P_x)_{\text{vison}}$ we obtain the Fermi surface volume:

$$\nu = \frac{N}{L_x L_y} = 2\frac{V_{\text{FS}}}{4\pi^2} + 2m + 1$$ for some integer m. For the cuprates, this implies a Fermi volume of density p and not $1 + p$.

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

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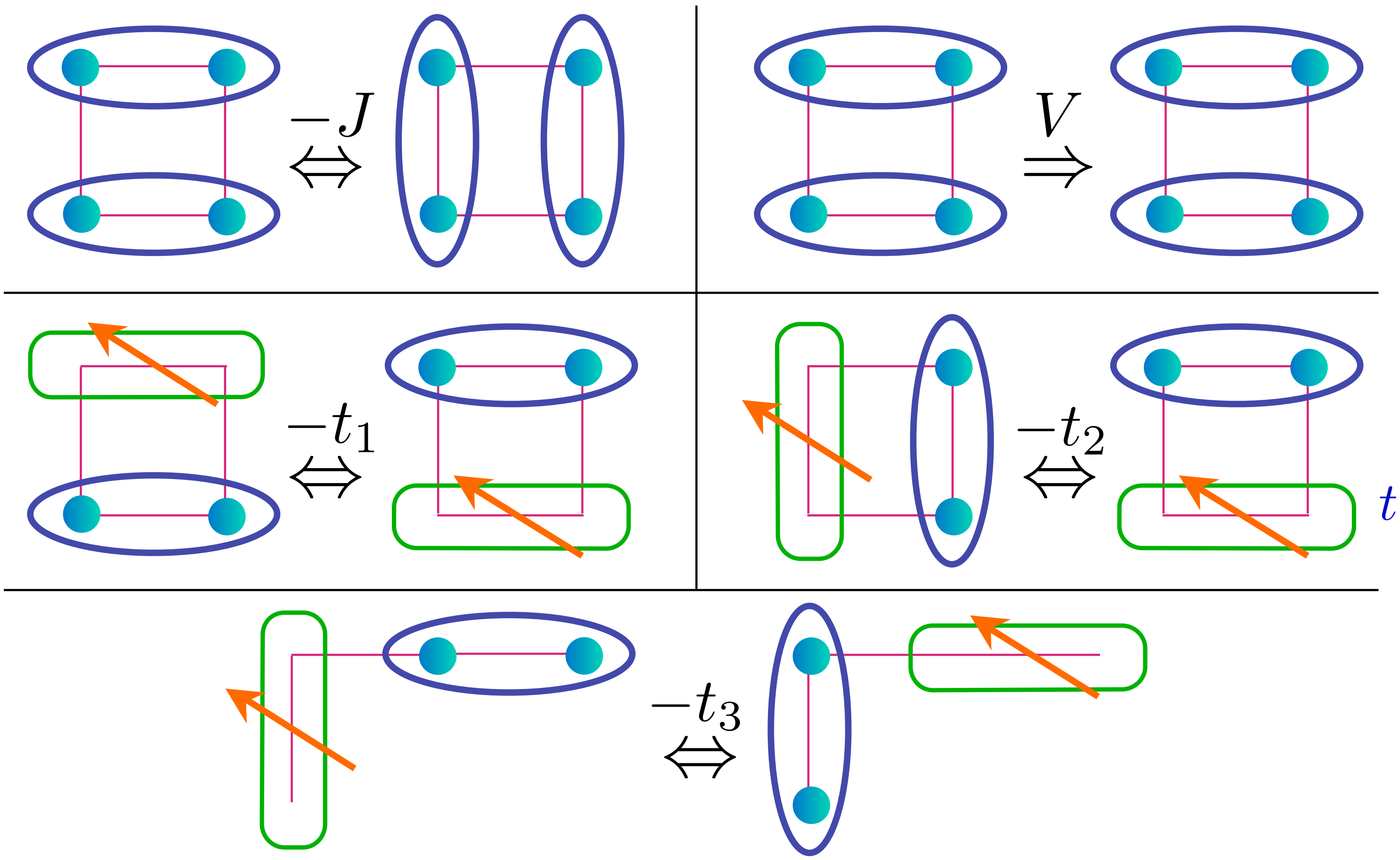
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

Quantum dimer model with bosonic and fermionic dimers

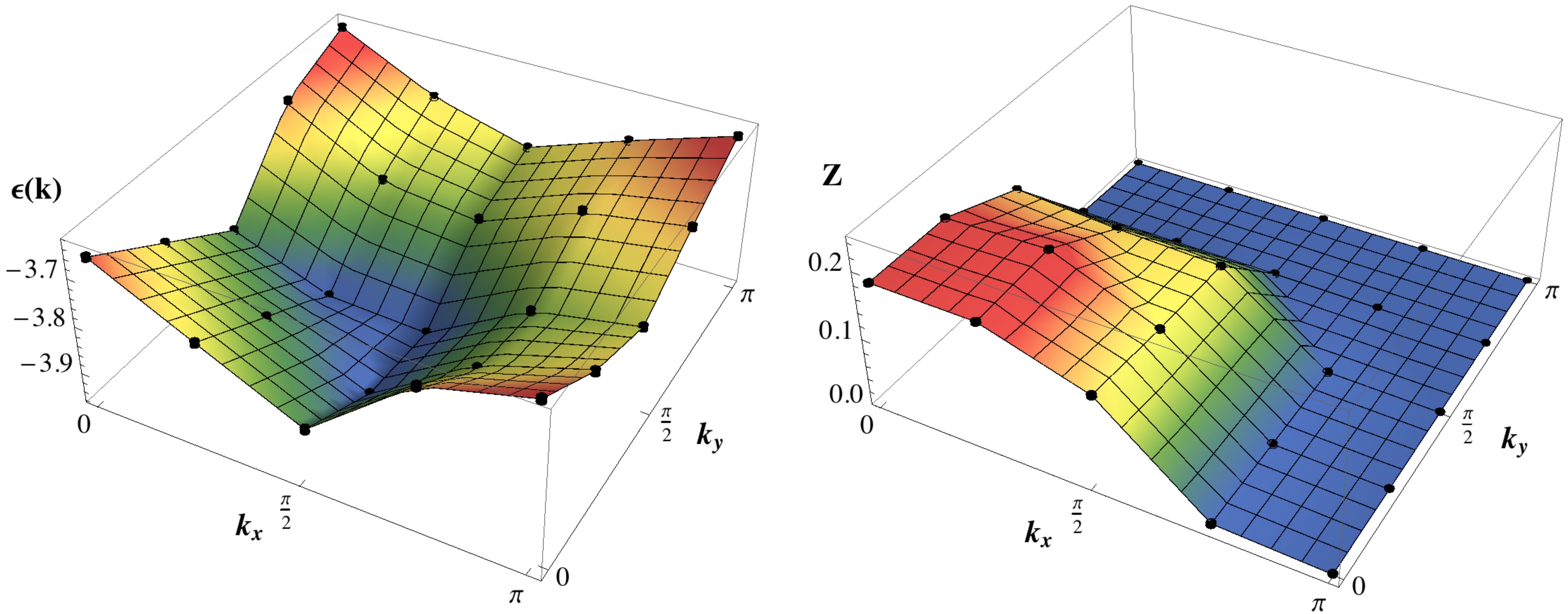


Connection to the t-t'-t''-J model:

$$t_1 = -(t+t')/2$$
$$t_2 = (t-t')/2$$
$$t_3 = -(t+t'+t'')/4$$

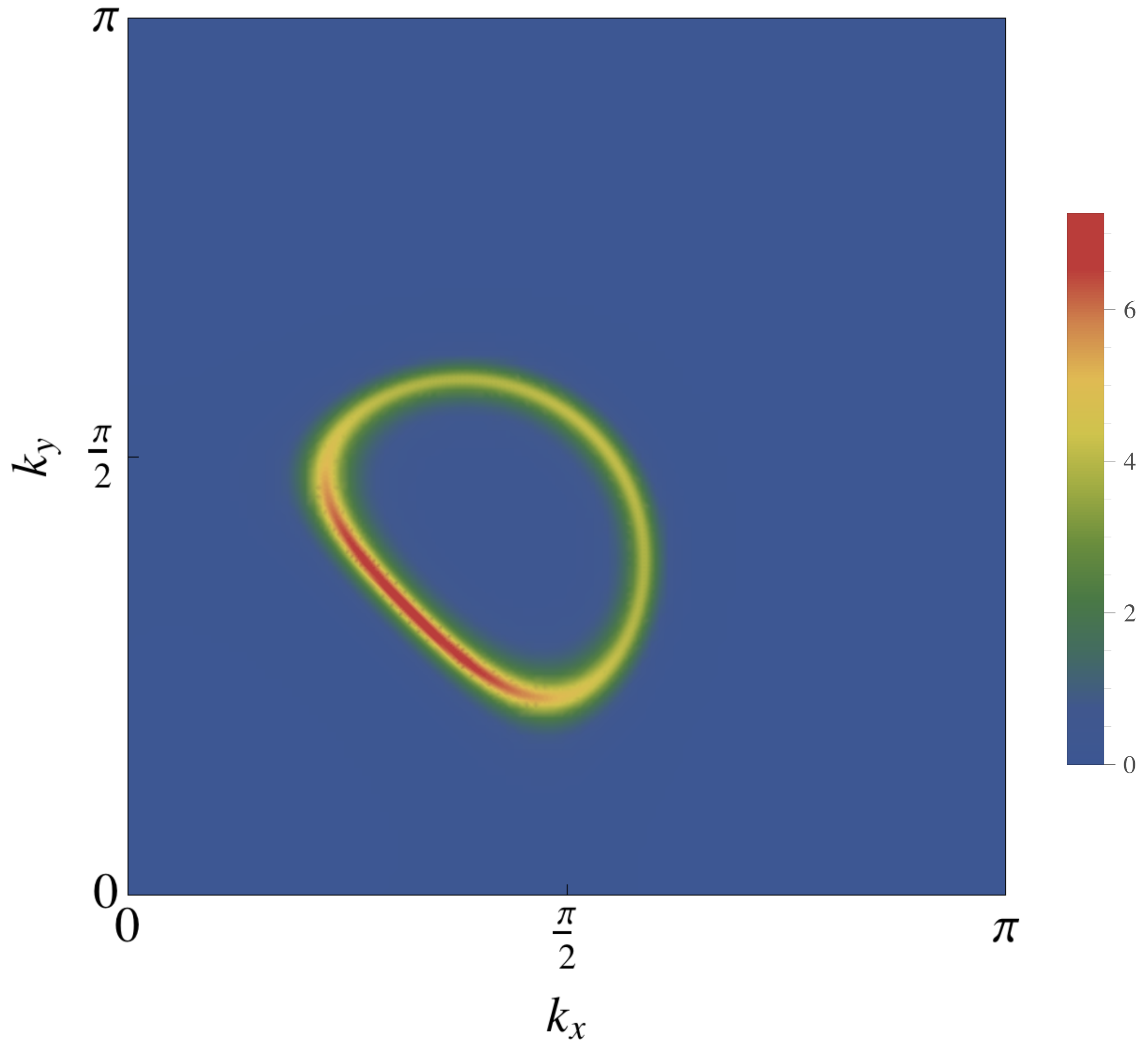
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

Quantum dimer model with bosonic and fermionic dimers

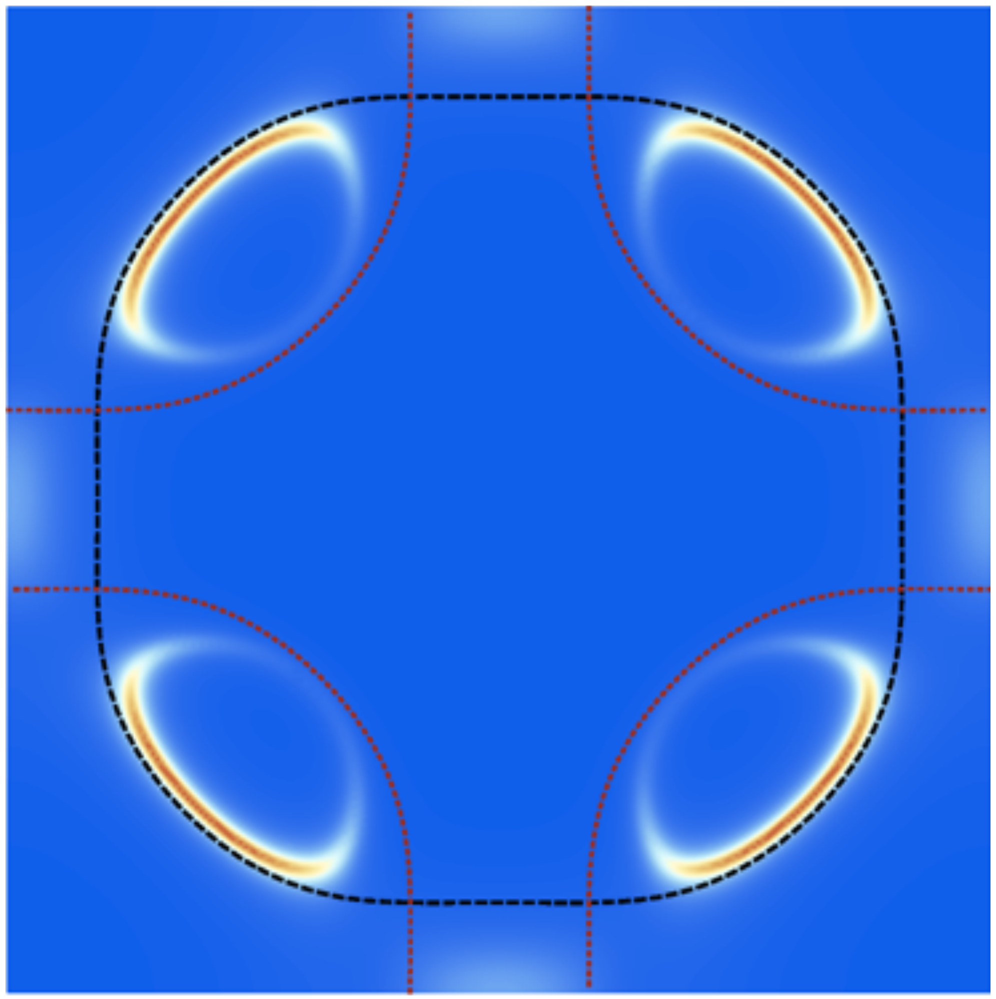


Dispersion and quasiparticle residue of a single fermionic dimer for $J = V = 1$, and hopping parameters obtained from the t-J model for the cuprates, $t_1 = -1.05$, $t_2 = 1.95$ and $t_3 = -0.6$, on a 8×8 lattice.

M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)



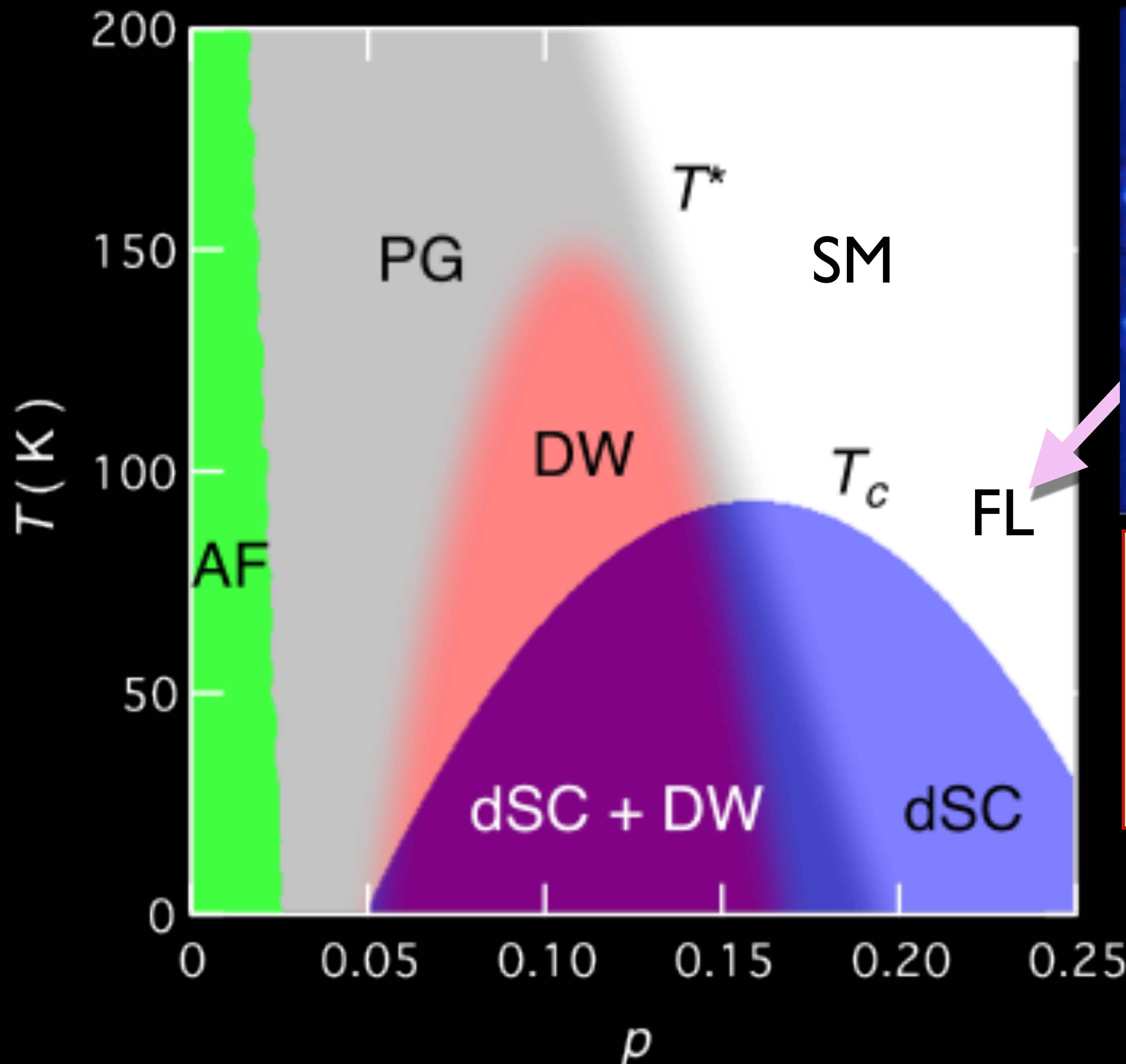
M. Punk, A. Allais, and S. Sachdev,
PNAS **112**, 9552 (2015)

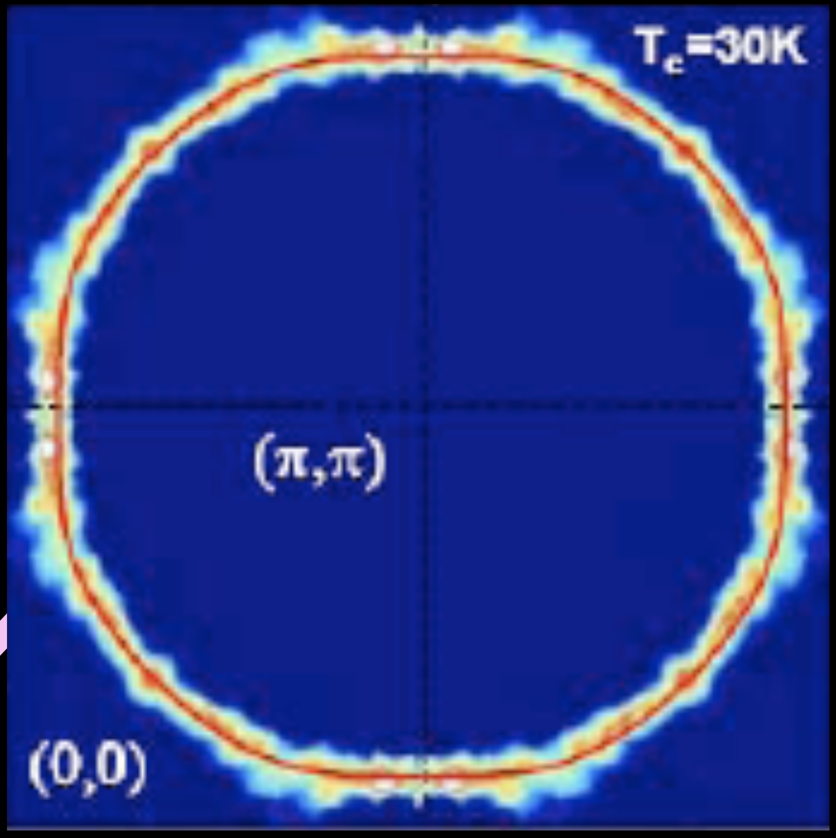


Y. Qi and S. Sachdev,
Phys. Rev. B **81**, 115129 (2010)

"Back side" of Fermi surface is suppressed for observables which change electron number in the square lattice

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

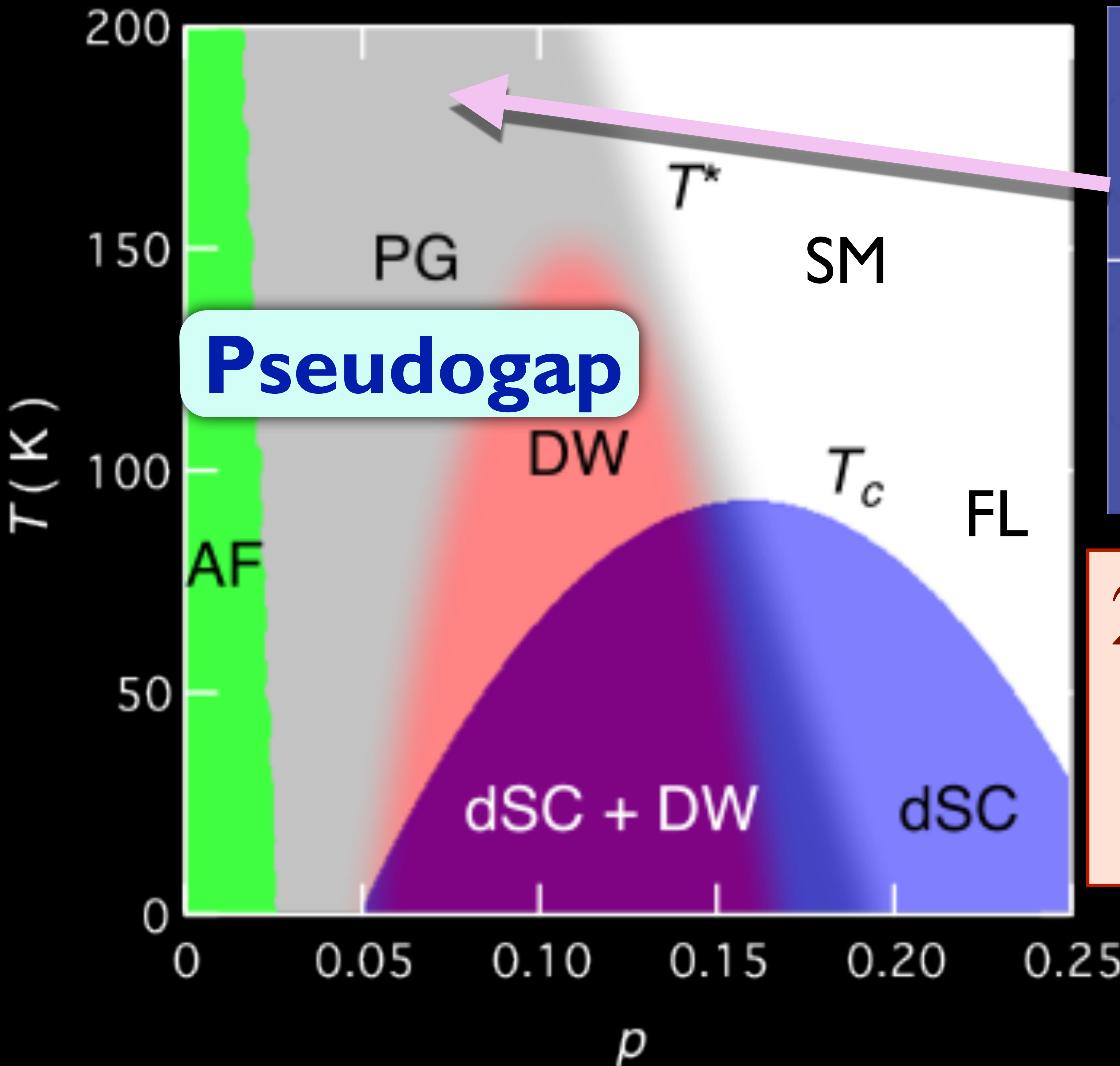


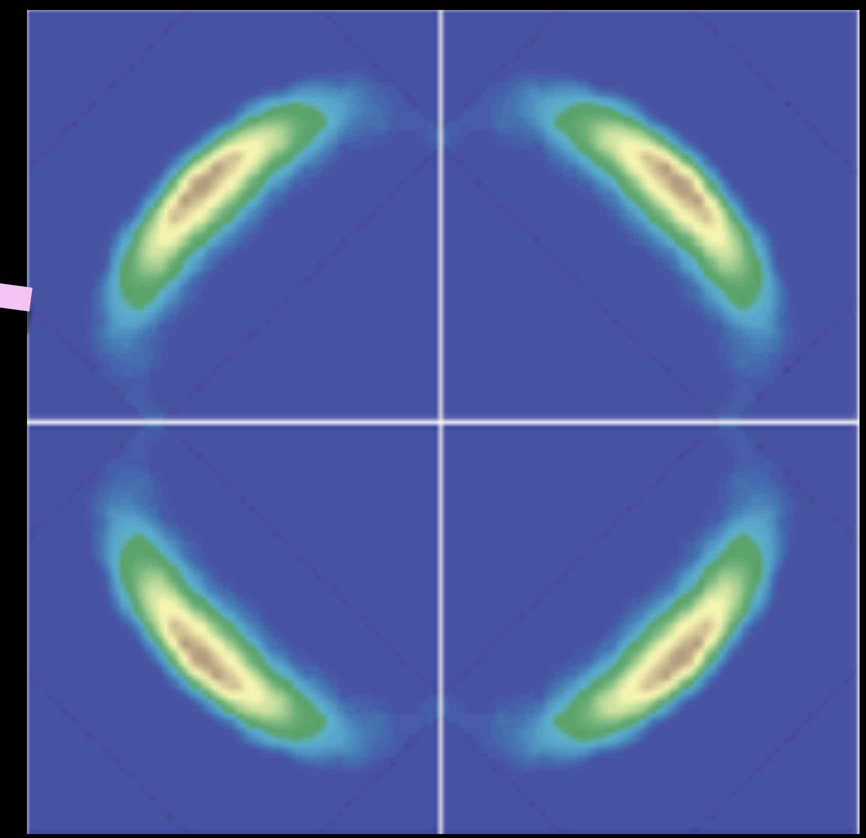


1. Conventional metal

Area enclosed by Fermi surface $=1+p$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)





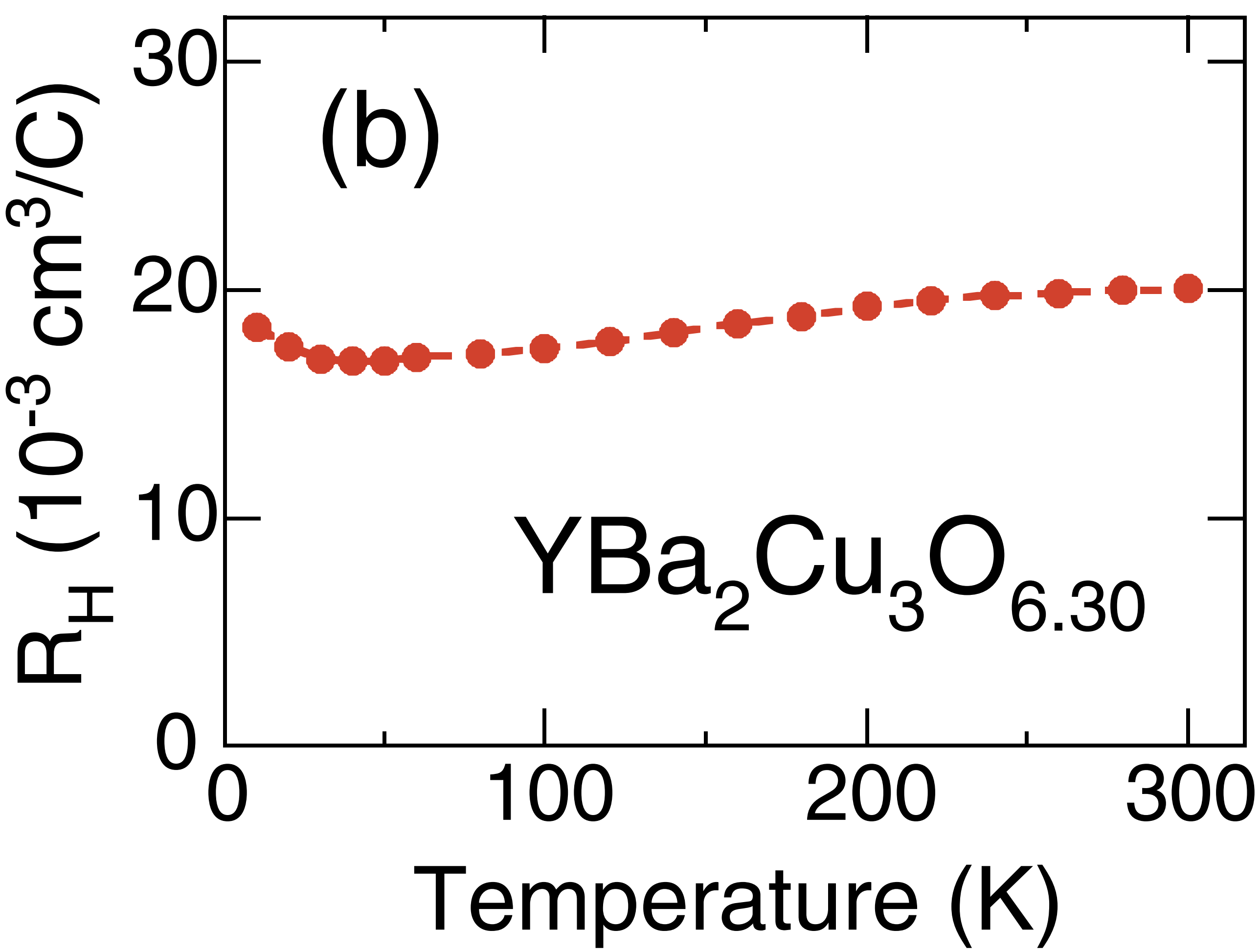
2. <u>Pseudogap metal</u> at low p

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density *p*

Evolution of the Hall Coefficient and the Peculiar Electronic Structure of the Cuprate Superconductors

Yoichi Ando,* Y. Kurita,† Seiki Komiya, S. Ono, and Kouji Segawa

PRL **92**, 197001 (2004)



T-independent Hall effect in a magnetic field of fermions of charge +e and density *p*

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density p

Spectroscopic evidence for Fermi liquid-like energy and temperature dependence of the relaxation rate in the pseudogap phase of the cuprates

Seyed Iman Mirzaei[a], Damien Stricker[a], Jason N. Hancock[a,b], Christophe Berthod[a], Antoine Georges[a,c,d], Erik van Heumen[a,e], Mun K. Chan[f], Xudong Zhao[f,g], Yuan Li[h], Martin Greven[f], Neven Barišić[f,i,j], and Dirk van der Marel[a,1]

PNAS 110, 5774 (2013)

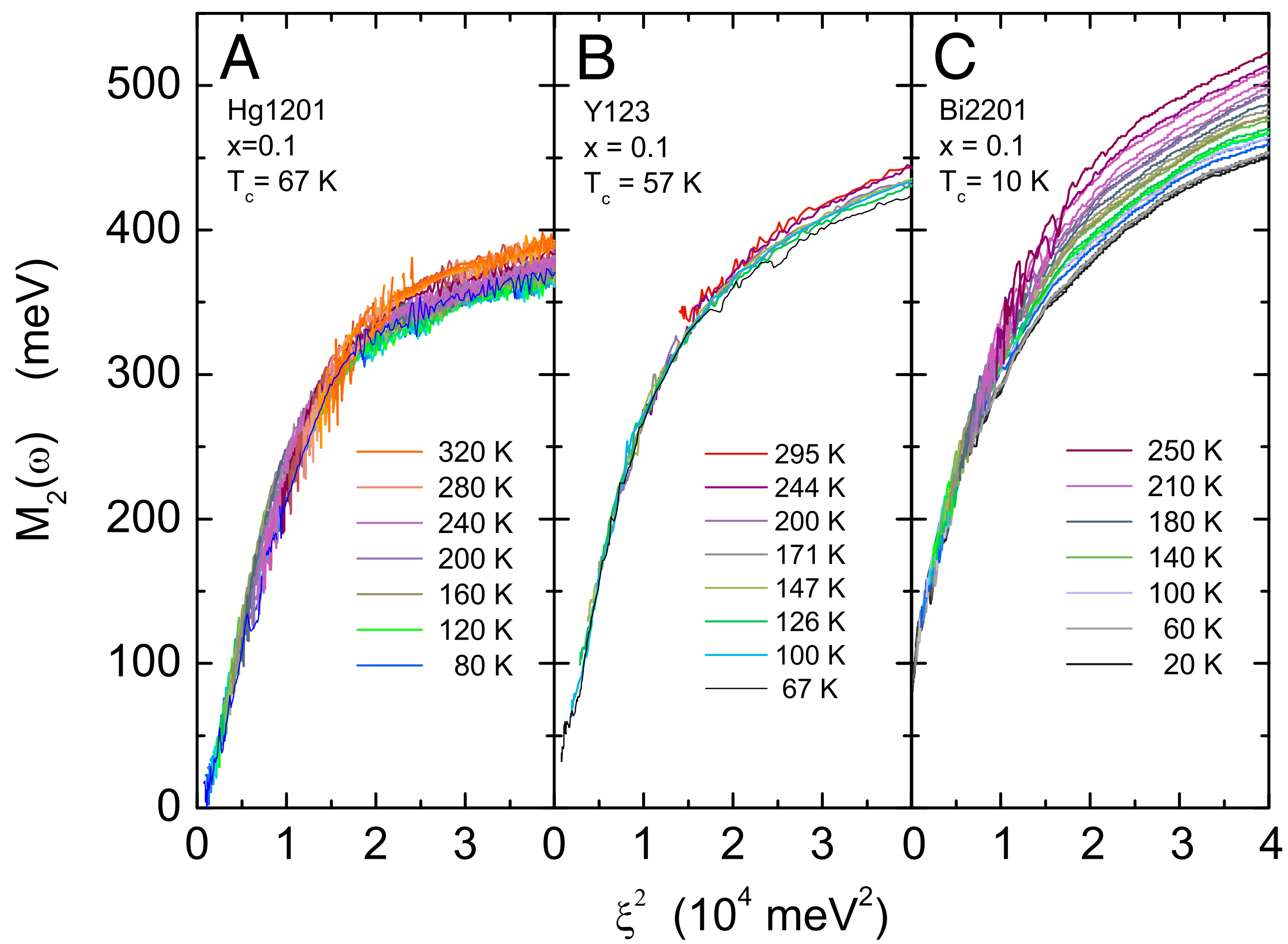


Fig. 6. Collapse of the frequency and temperature dependence of the relaxation rate of underdoped cuprate materials. Normal state $M_2(\omega, T)$ as a function of $\xi^2 \equiv (\hbar\omega)^2 + (p\pi k_B T)^2$

$$\sigma_{xx} \sim \frac{1}{(-i\omega + 1/\tau)}$$

$$\text{with } \frac{1}{\tau} \sim \omega^2 + T^2$$

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density p

In-Plane Magnetoresistance Obeys Kohler's Rule in the Pseudogap Phase of Cuprate Superconductors

M. K. Chan,[1,*] M. J. Veit,[1] C. J. Dorow,[1,†] Y. Ge,[1] Y. Li,[1] W. Tabis,[1,2] Y. Tang,[1] X. Zhao,[1,3]
N. Barišić,[1,4,5,‡] and M. Greven[1,§]
PRL **113**, 177005 (2014)

We report in-plane resistivity (ρ) and transverse magnetoresistance (MR) measurements for underdoped $HgBa_2CuO_{4+\delta}$ (Hg1201). Contrary to the long-standing view that Kohler's rule is strongly violated in underdoped cuprates, we find that it is in fact satisfied in the pseudogap phase of Hg1201. The transverse MR shows a quadratic field dependence, $\delta\rho/\rho_0 = aH^2$, with $a(T) \propto T^{-4}$. In combination with the observed $\rho \propto T^2$ dependence, this is consistent with a single Fermi-liquid quasiparticle scattering rate. We show that this behavior is typically masked in cuprates with lower structural symmetry or strong disorder effects.

$$\rho_{xx} \sim \frac{1}{\tau}(1 + aH^2\tau^2 + \ldots)$$

$$\text{with } \frac{1}{\tau} \sim T^2$$

Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size p ?

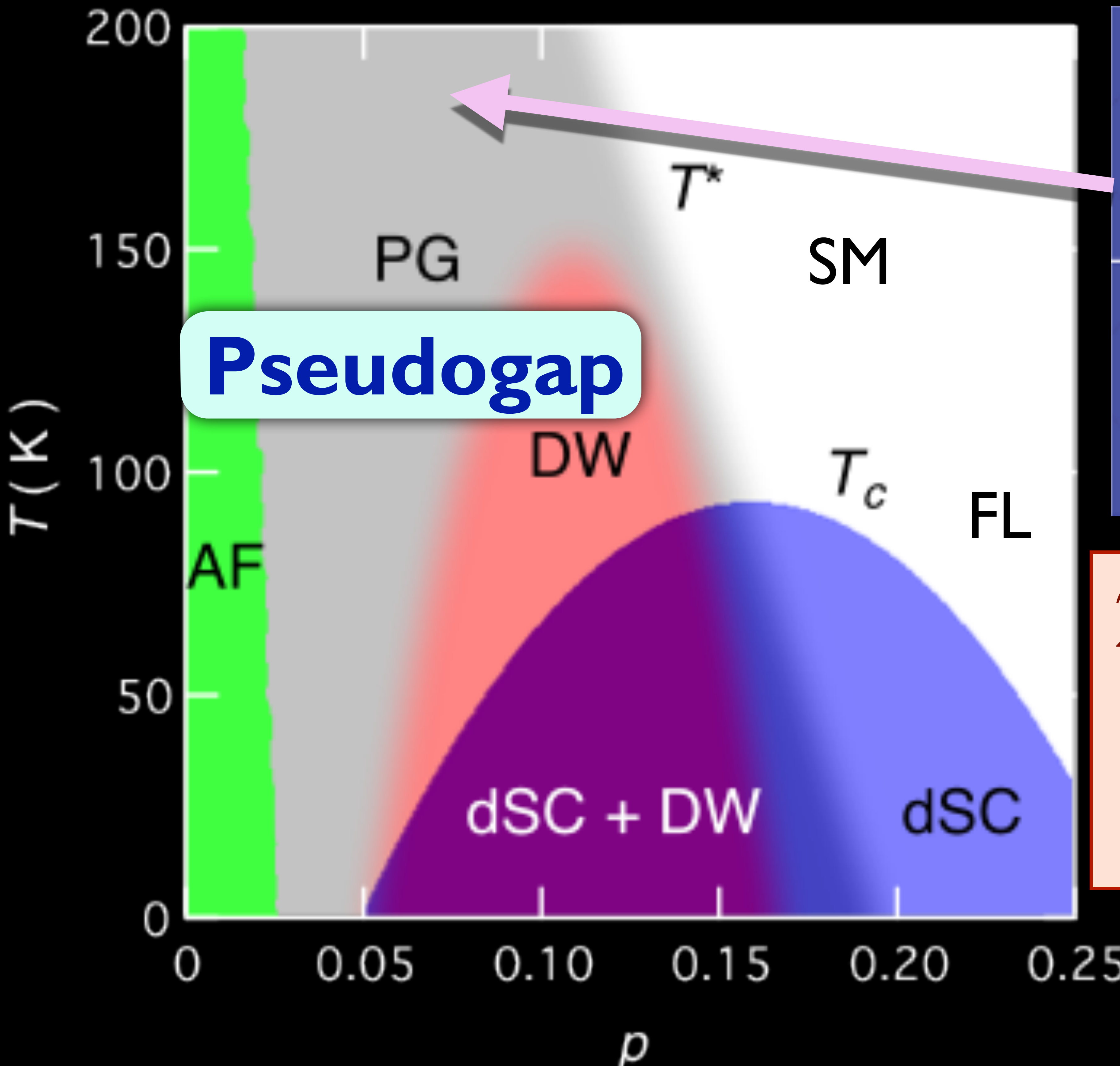
Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size p ?

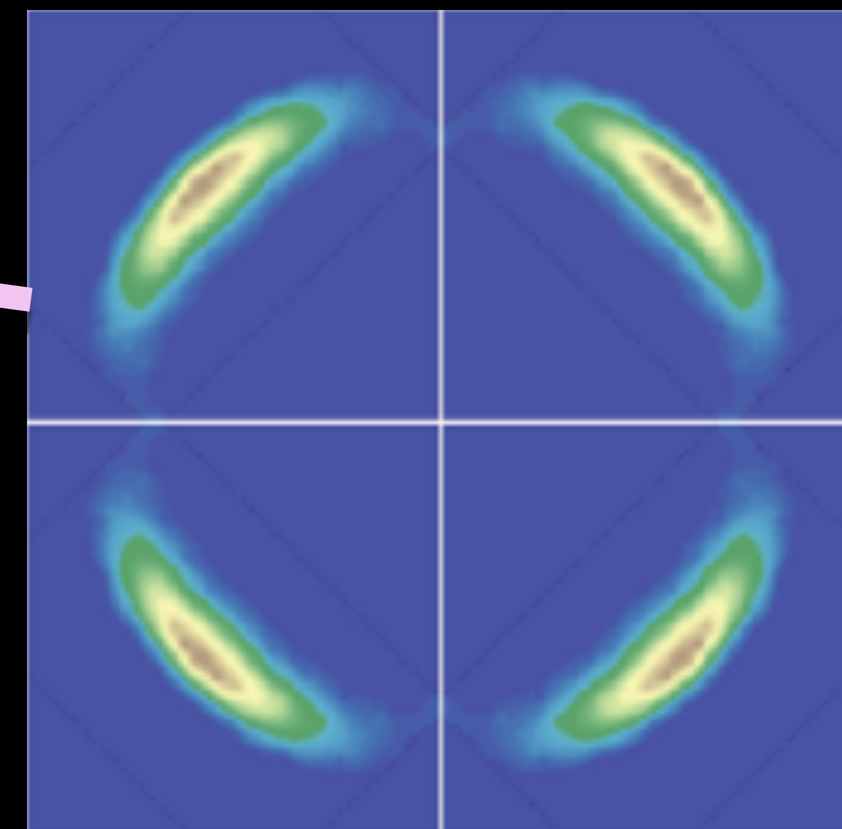
Answer: **Yes.**

There can be a Fermi surface of size p, but it must be accompanied by topological order, in a "fractionalized Fermi liquid".

At $T=0$, such a metal must be separated from a Fermi liquid (with a Fermi surface of size $1+p$) by a quantum phase transition

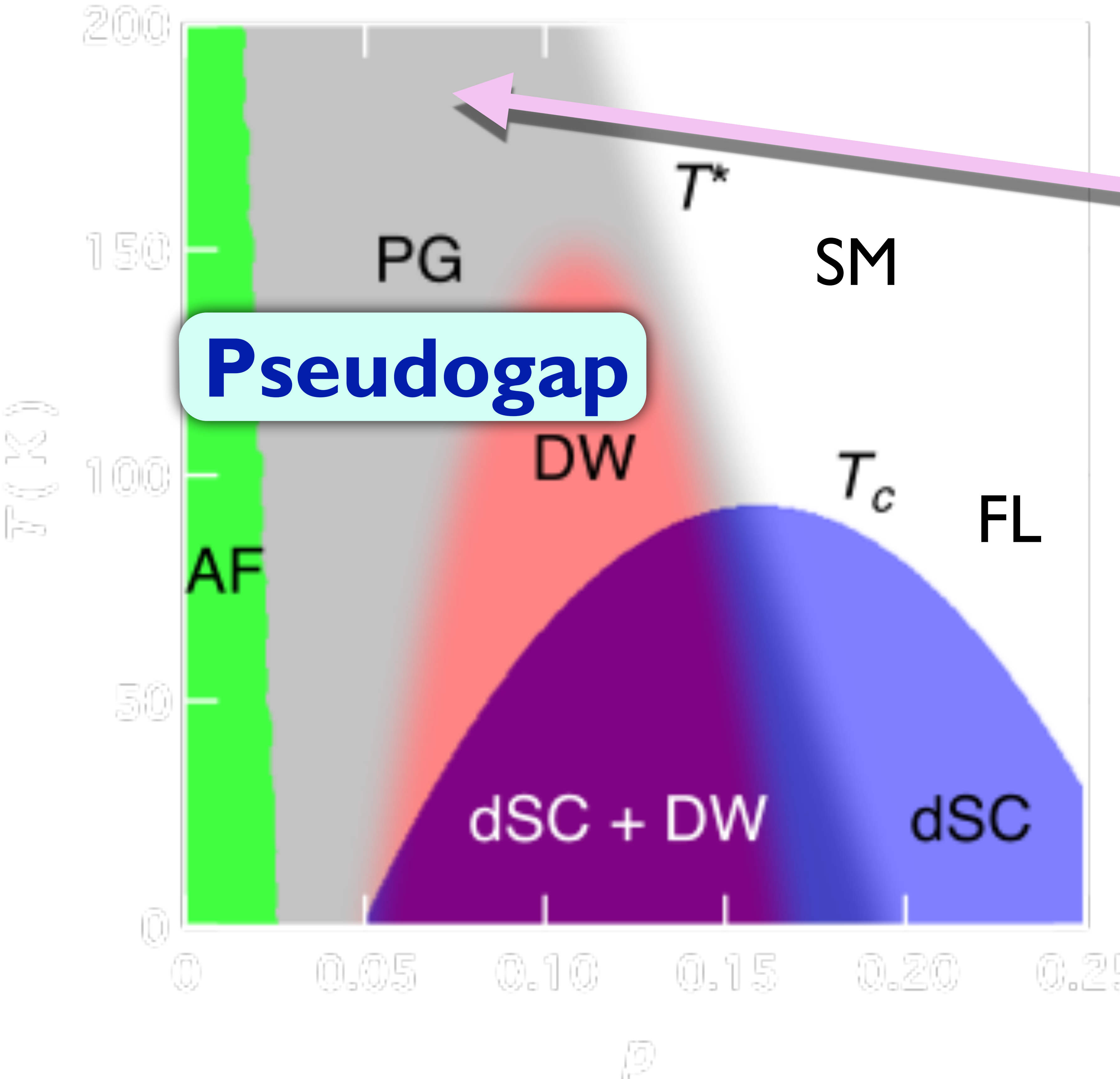
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

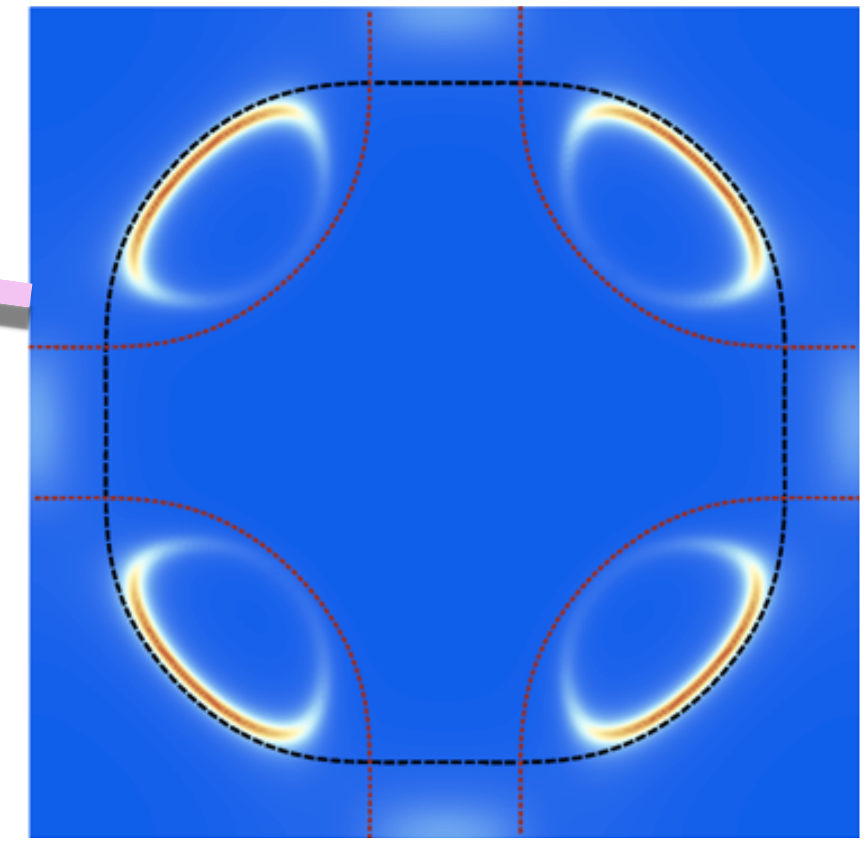




2. Pseudogap metal at low p

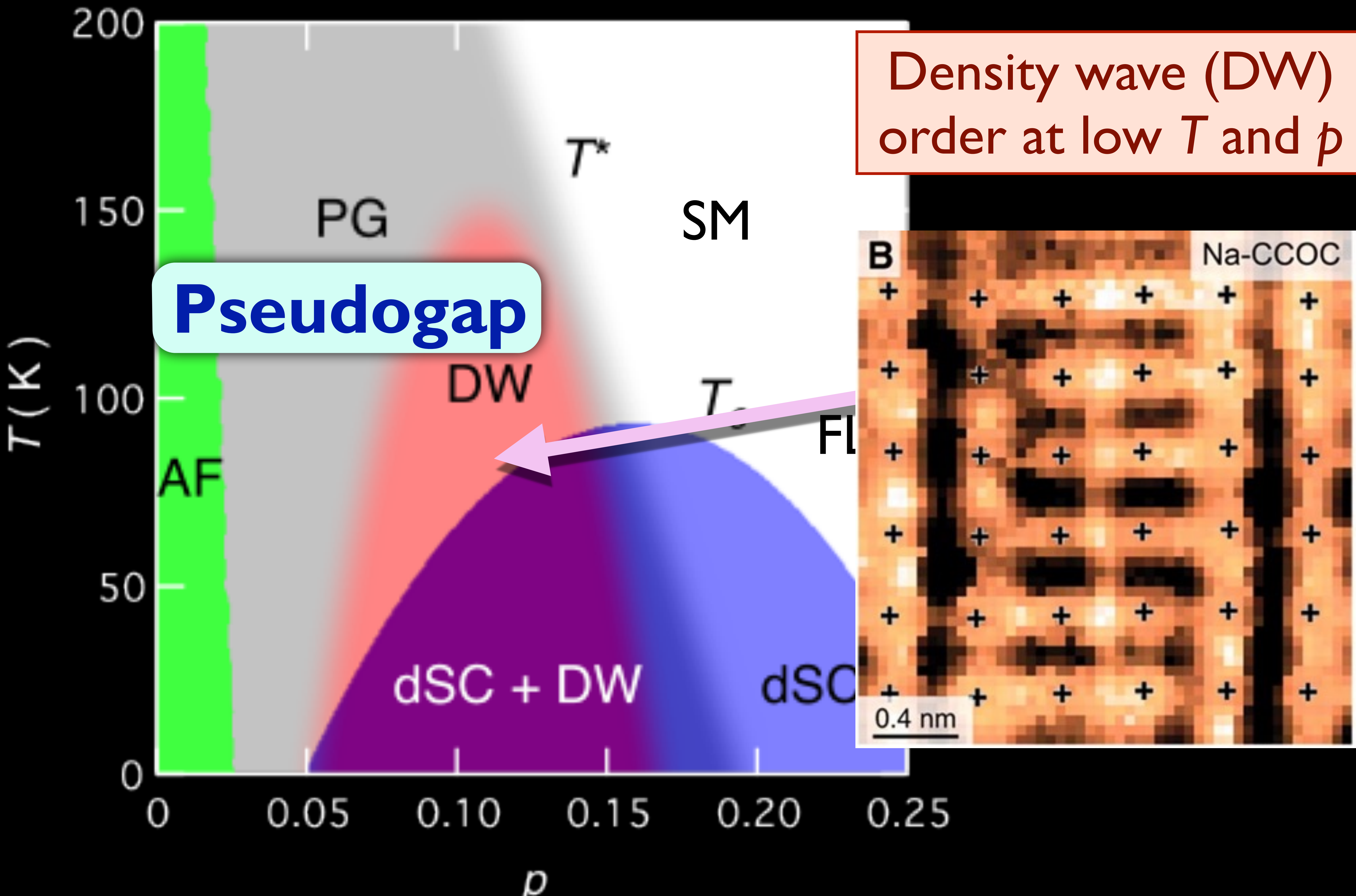
Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

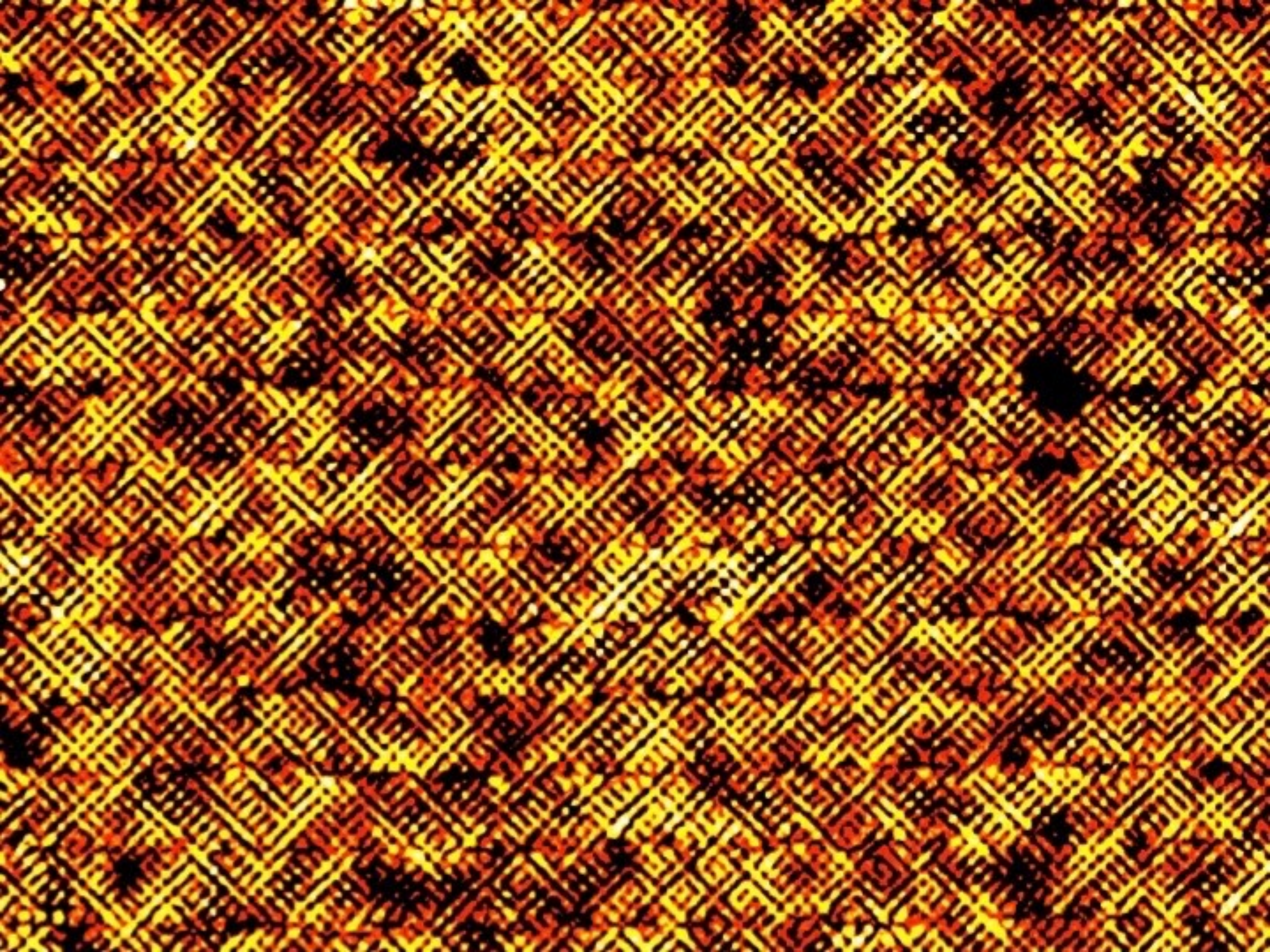




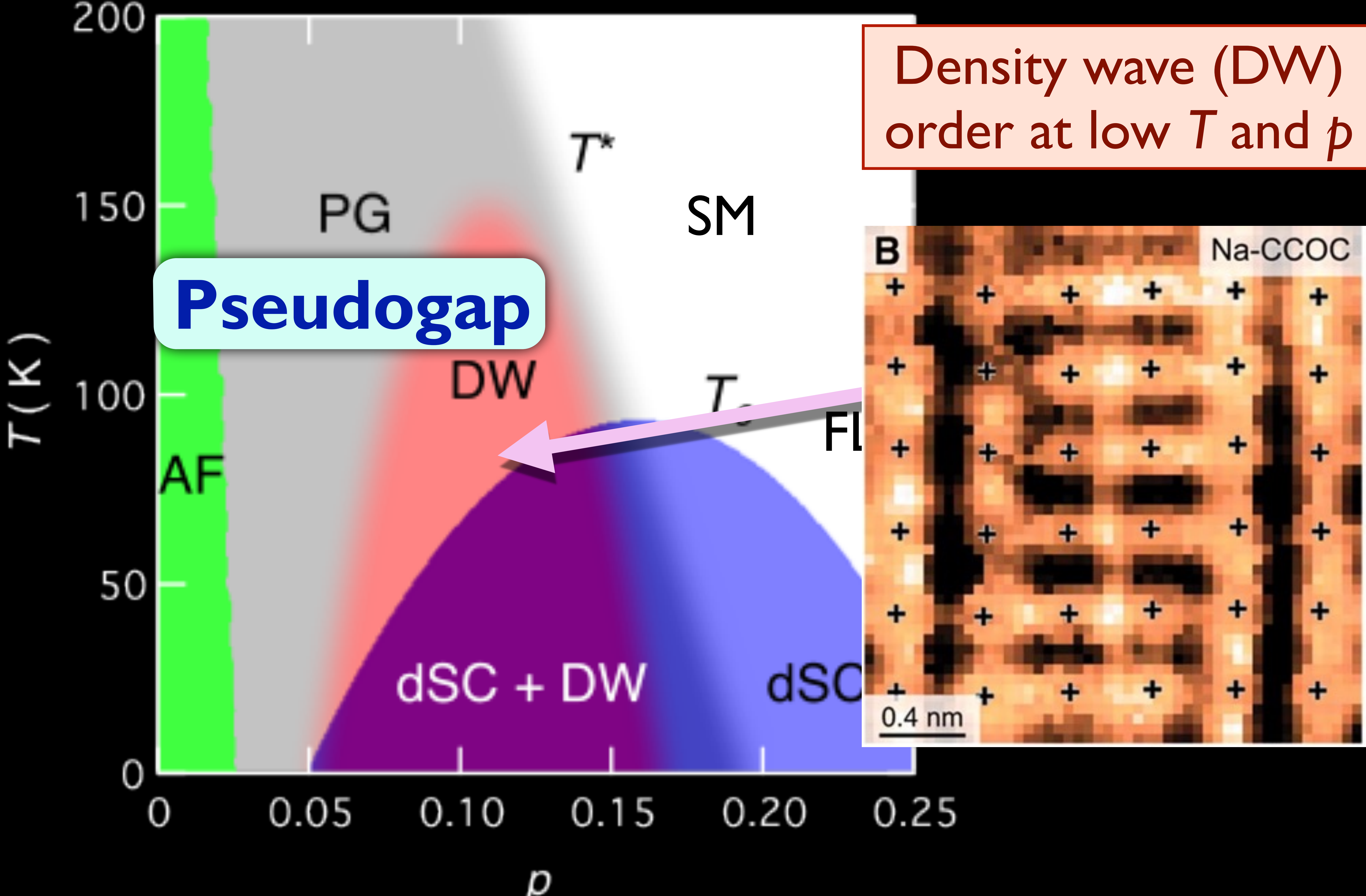
A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size p

Y. Kohsaka *et al.*, Science **315**, 1380 (2007)



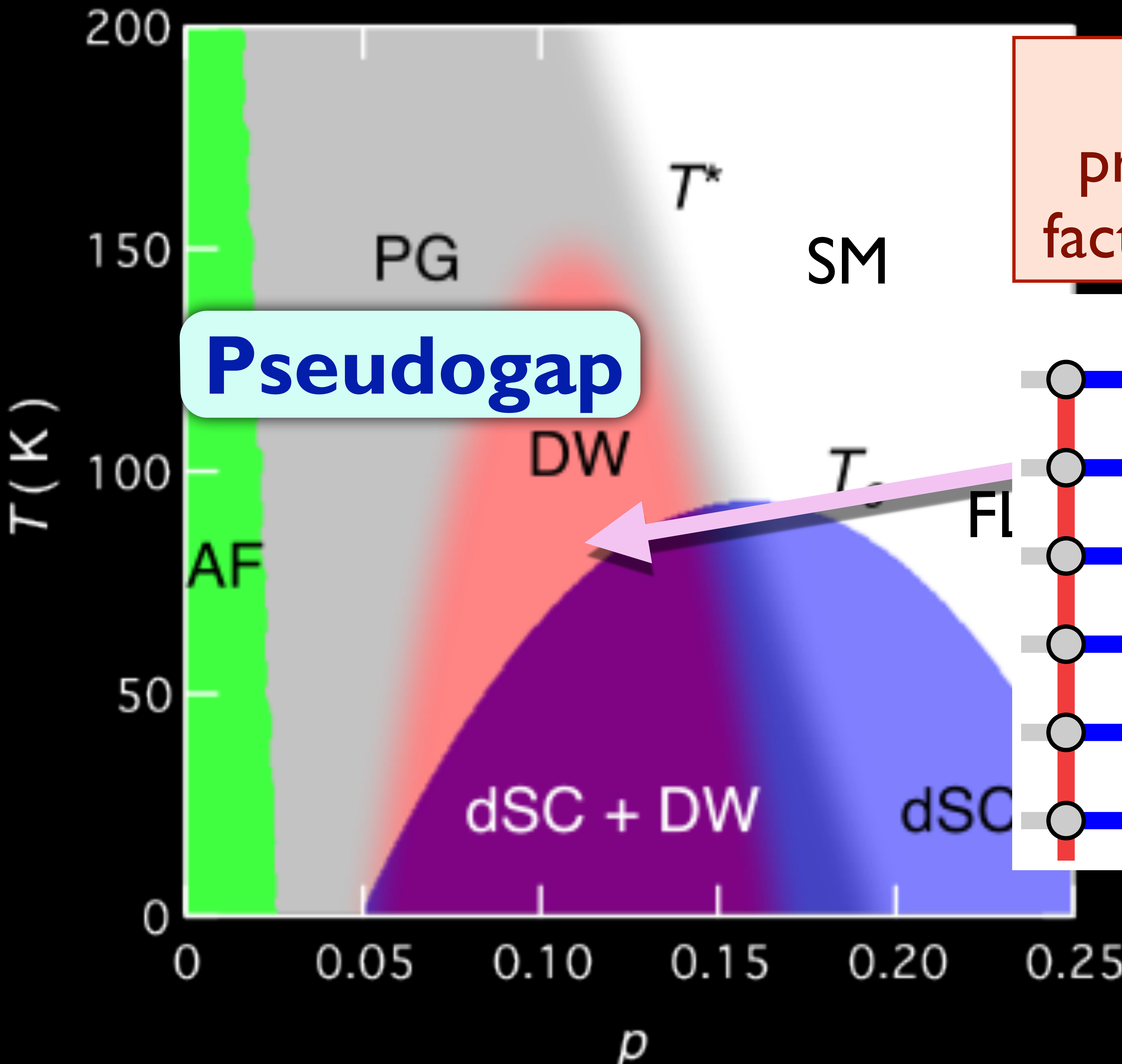


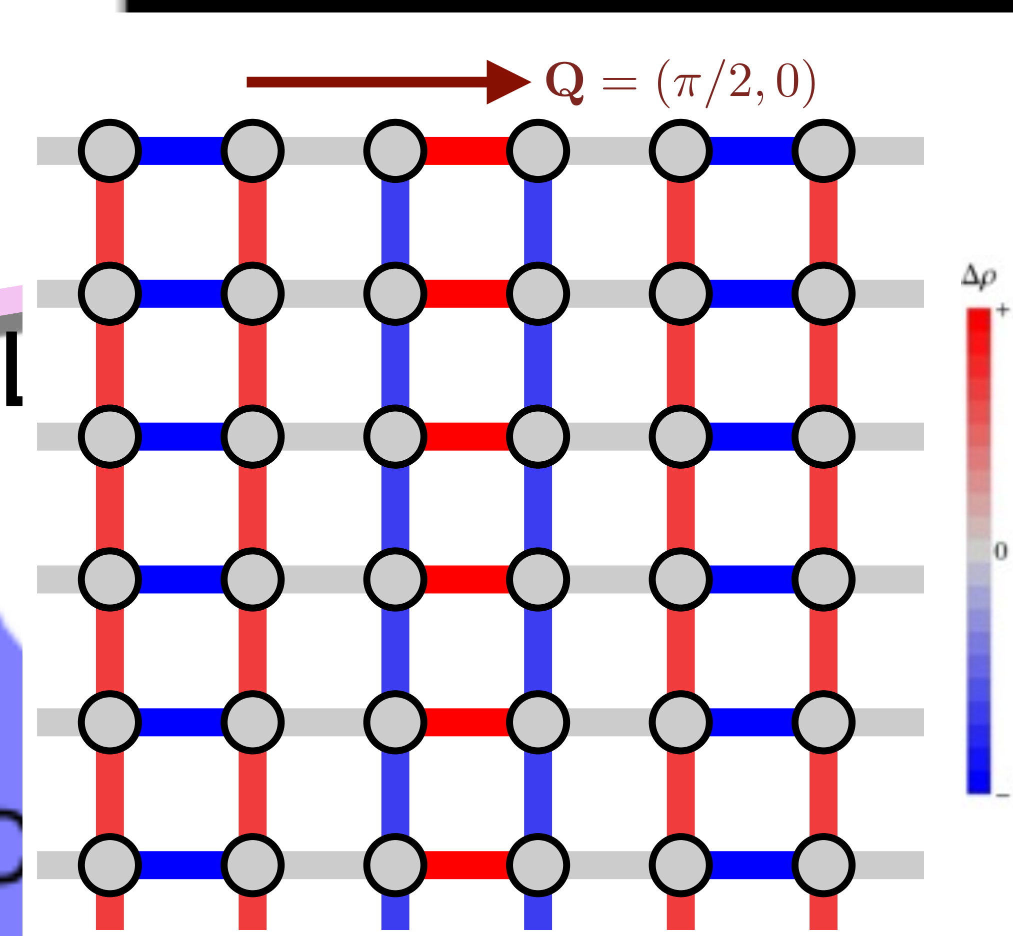
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

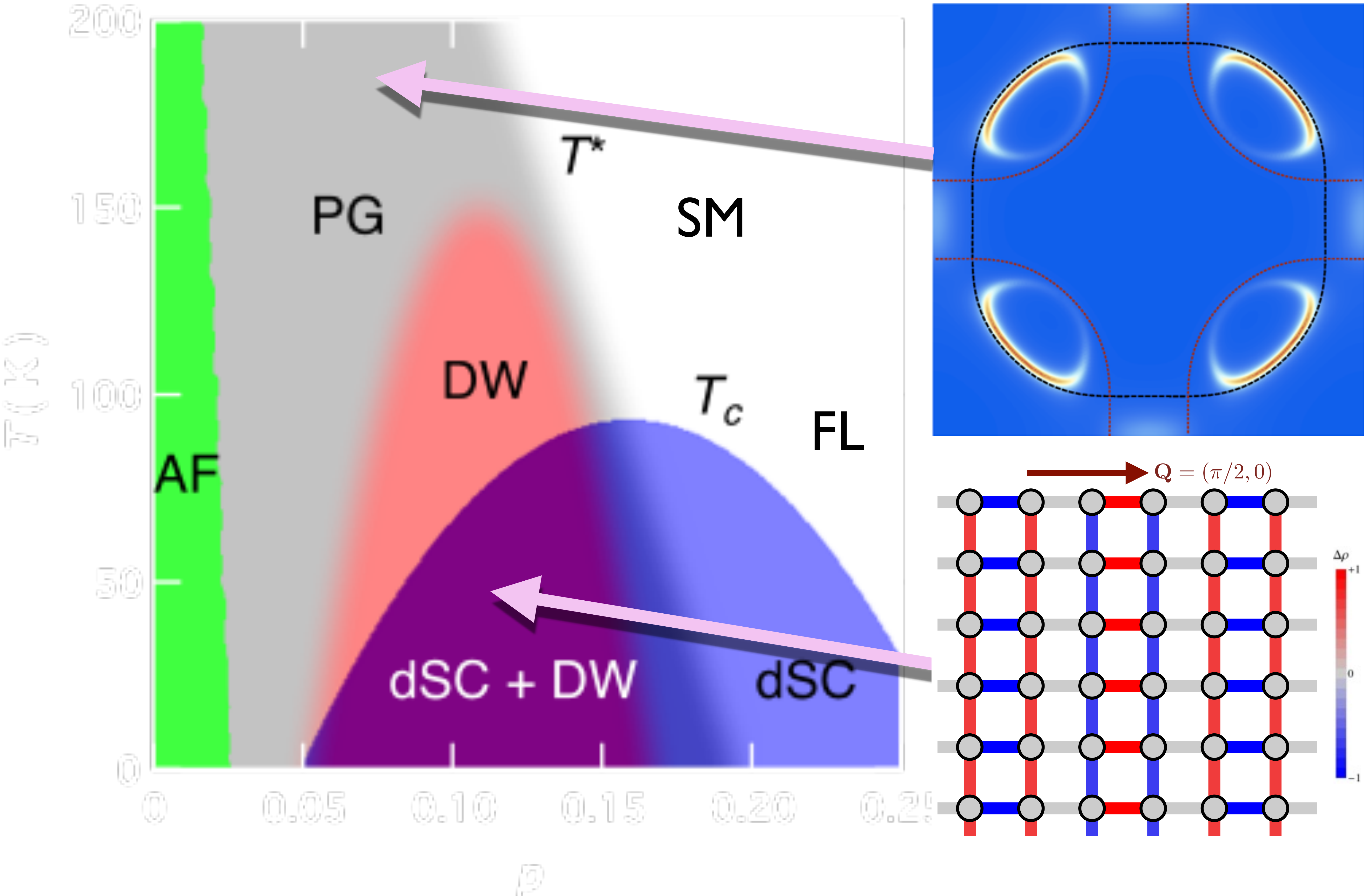


M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi,
H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)

Identified as a predicted "*d*-form factor density wave"



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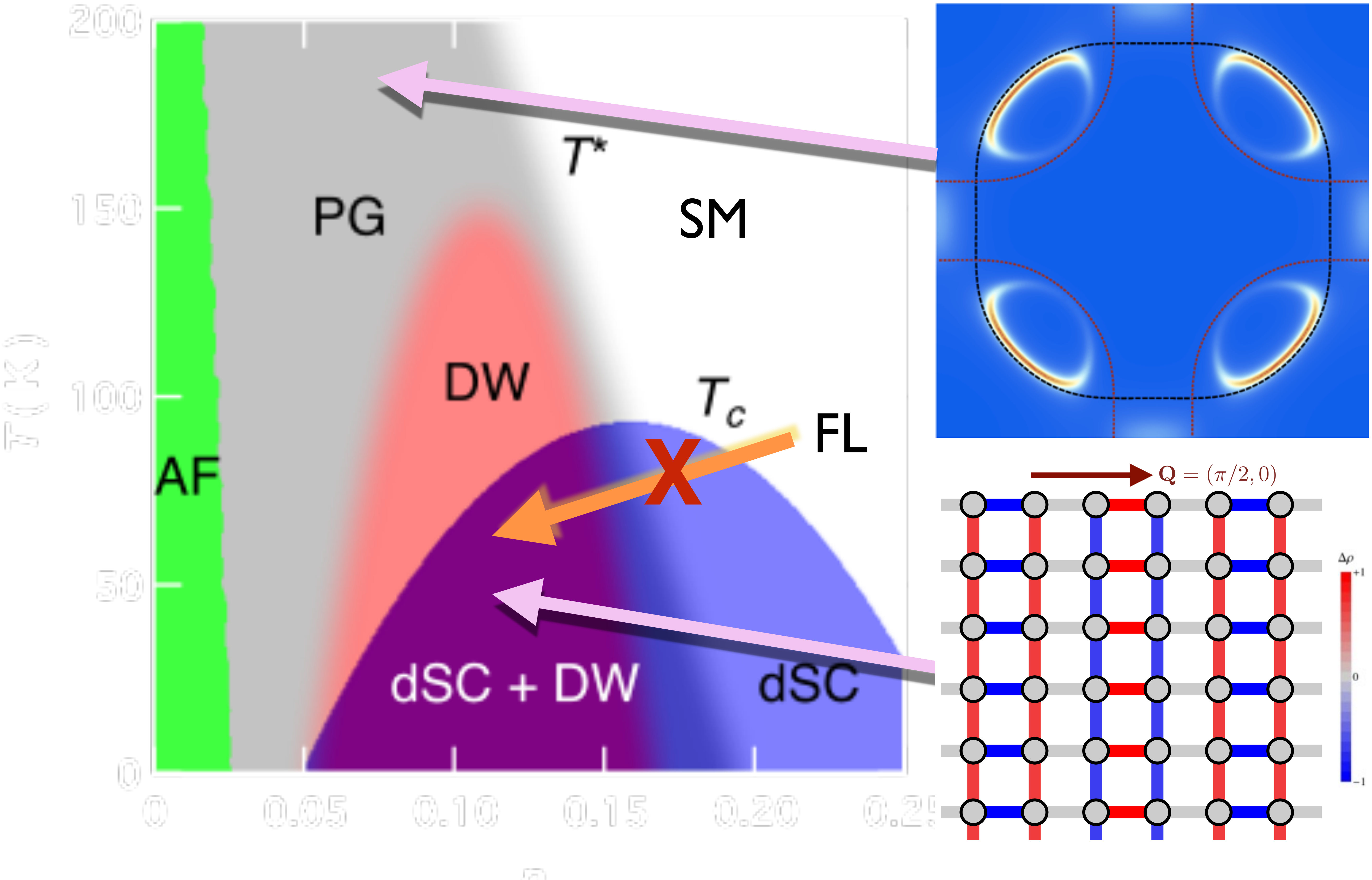

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dSC
Q = (π/2, 0)
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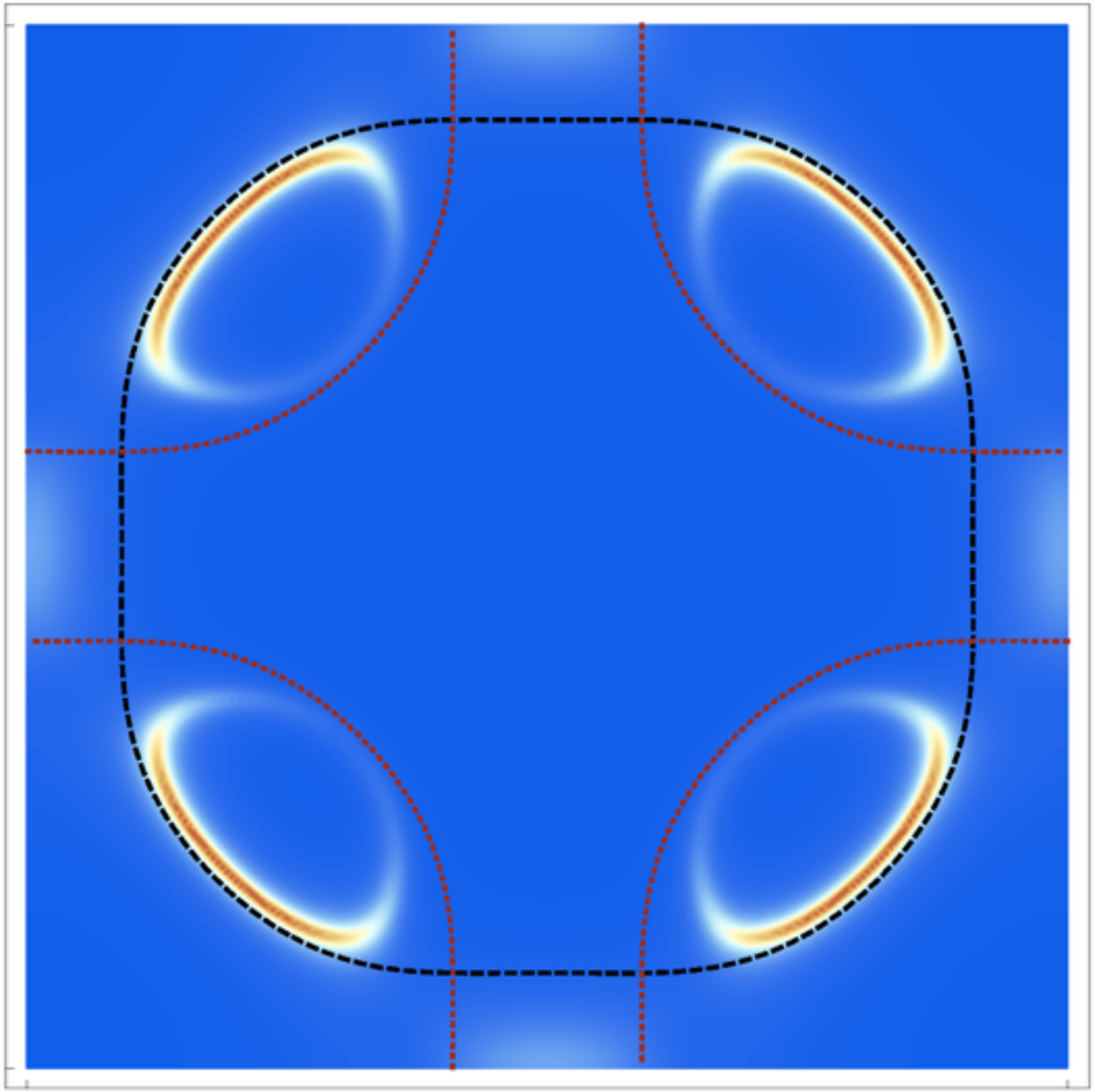
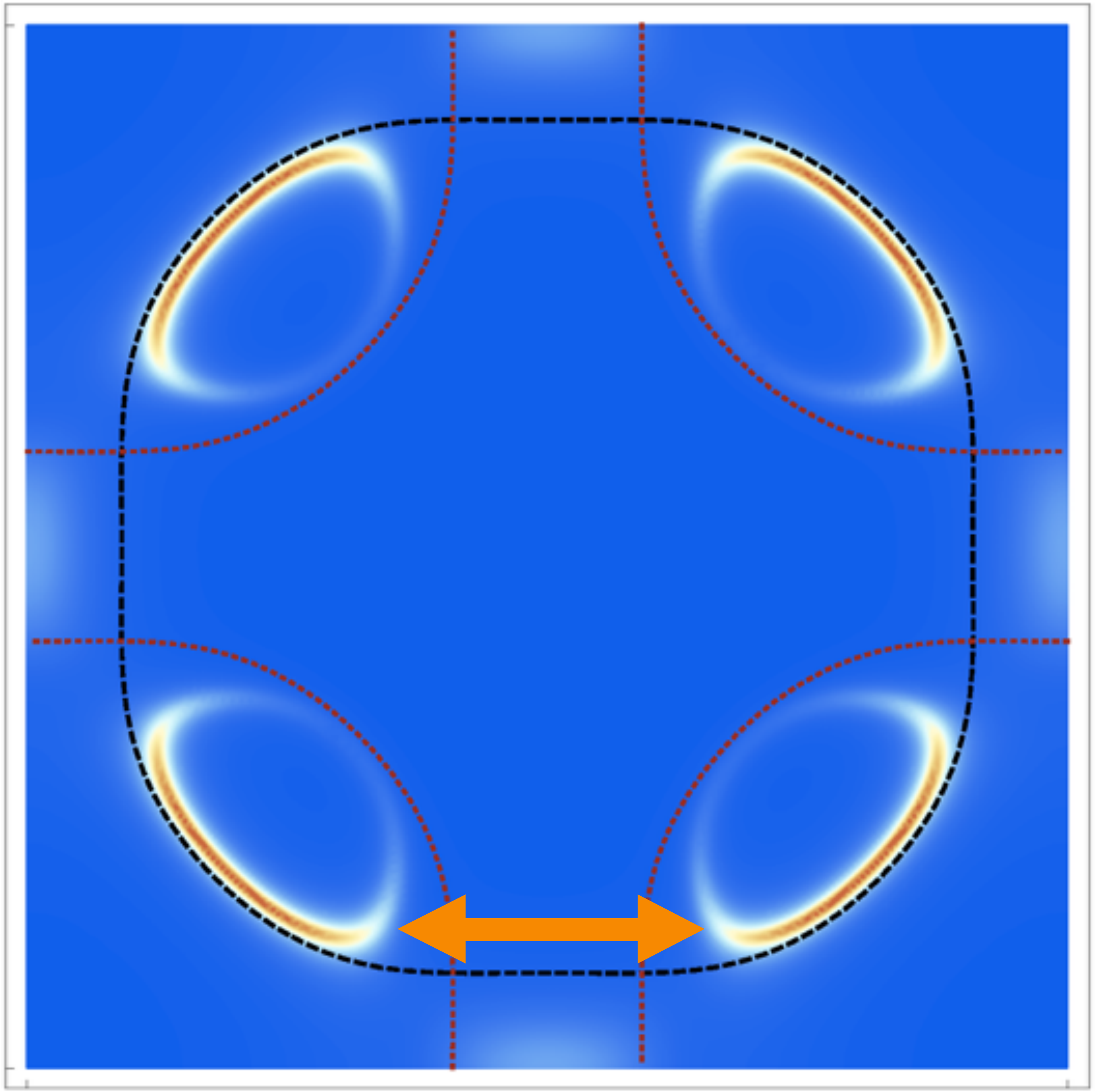


Fermi surface of a fractionalized Fermi liquid (FL*)

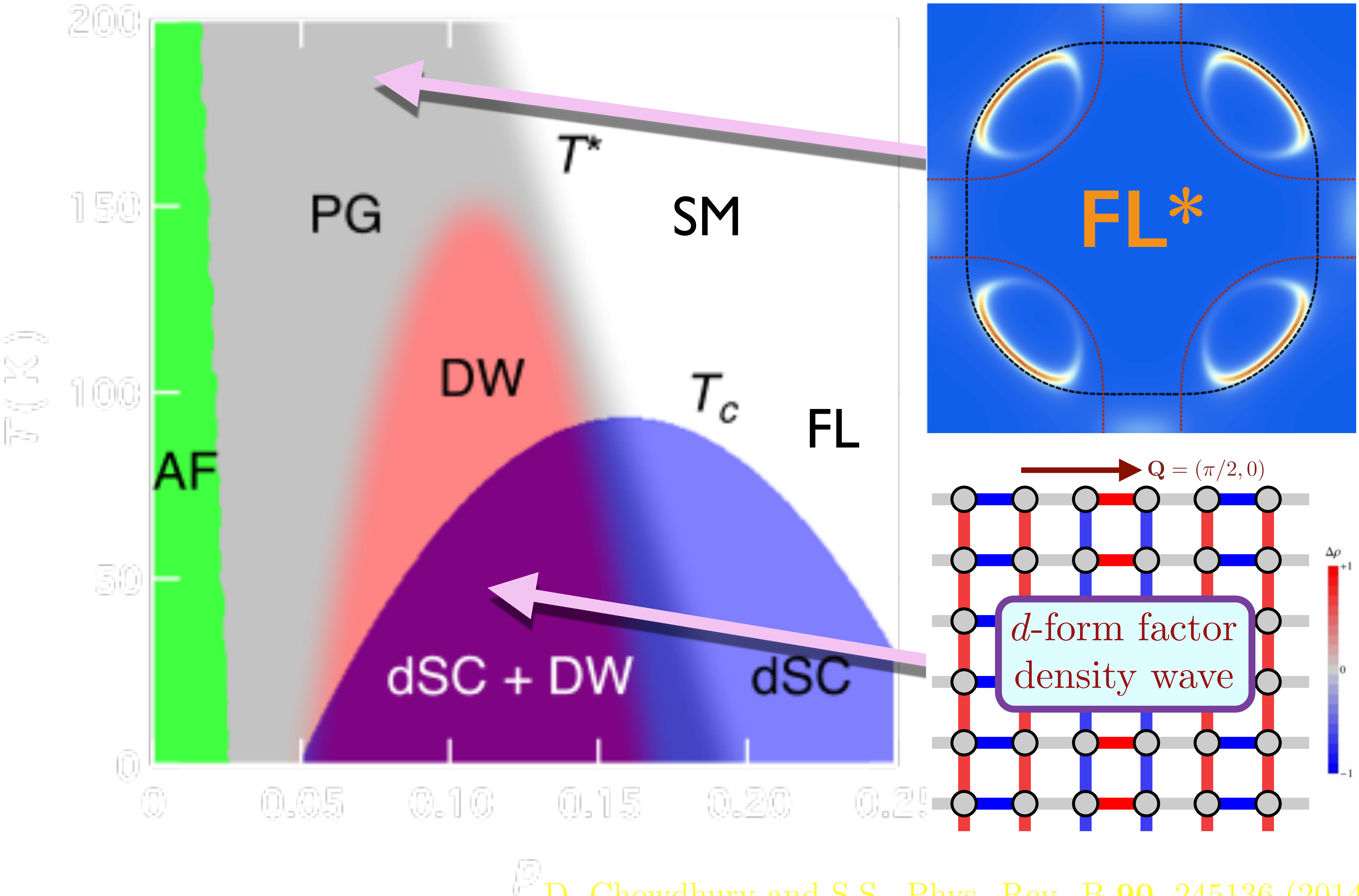
Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)



Density wave instability of FL* leads to the observed wavevector and form-factor

D. Chowdhury and S. Sachdev, Physical Review B **90**, 245136 (2014).

The high T FL* can help explain the "d-form factor density wave" observed at low T



D. Chowdhury and S.S., Phys. Rev. B **90**, 245136 (2014)

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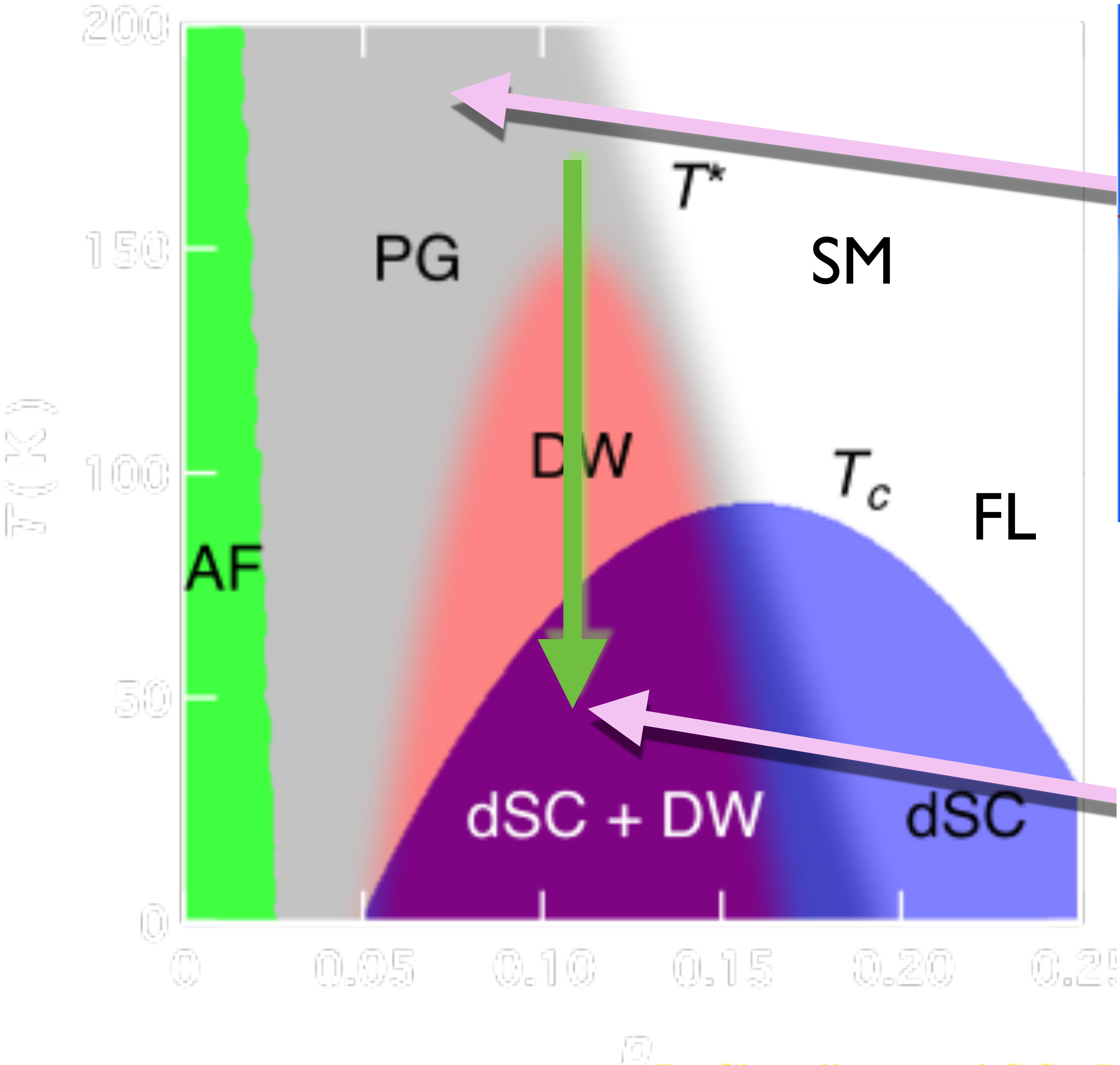

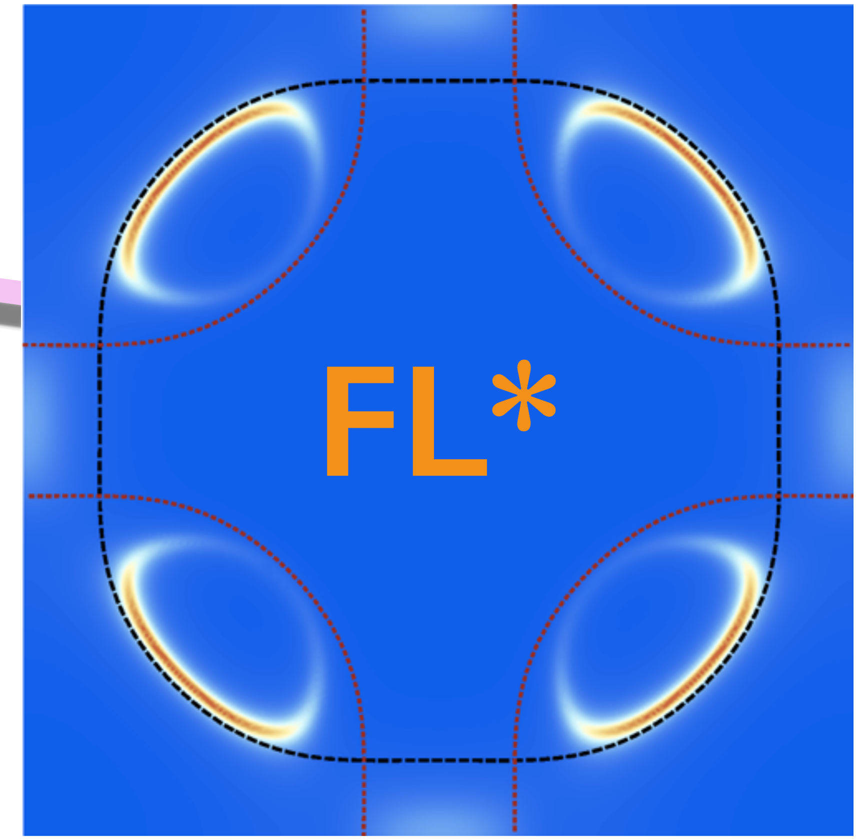

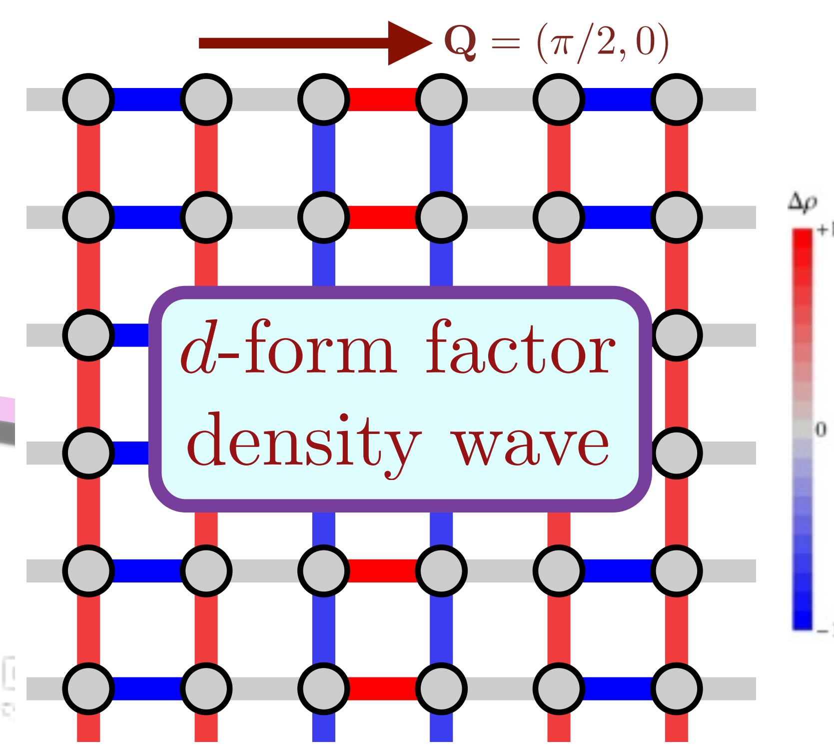

D. Chowdhury and S.S., Phys. Rev. B **90**, 245136 (2014)