Quantum critical transport, duality, and M-theory

hep-th/0701036

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Talks online at http://sachdev.physics.harvard.edu

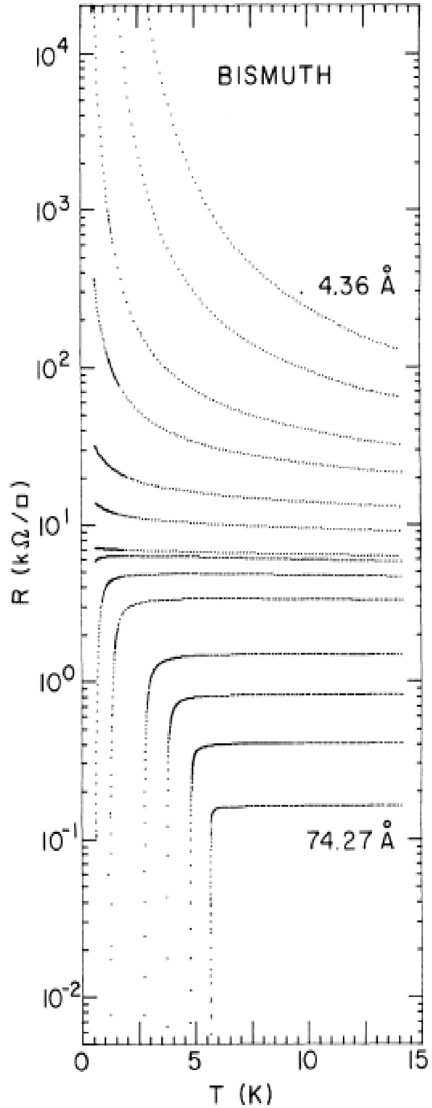


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

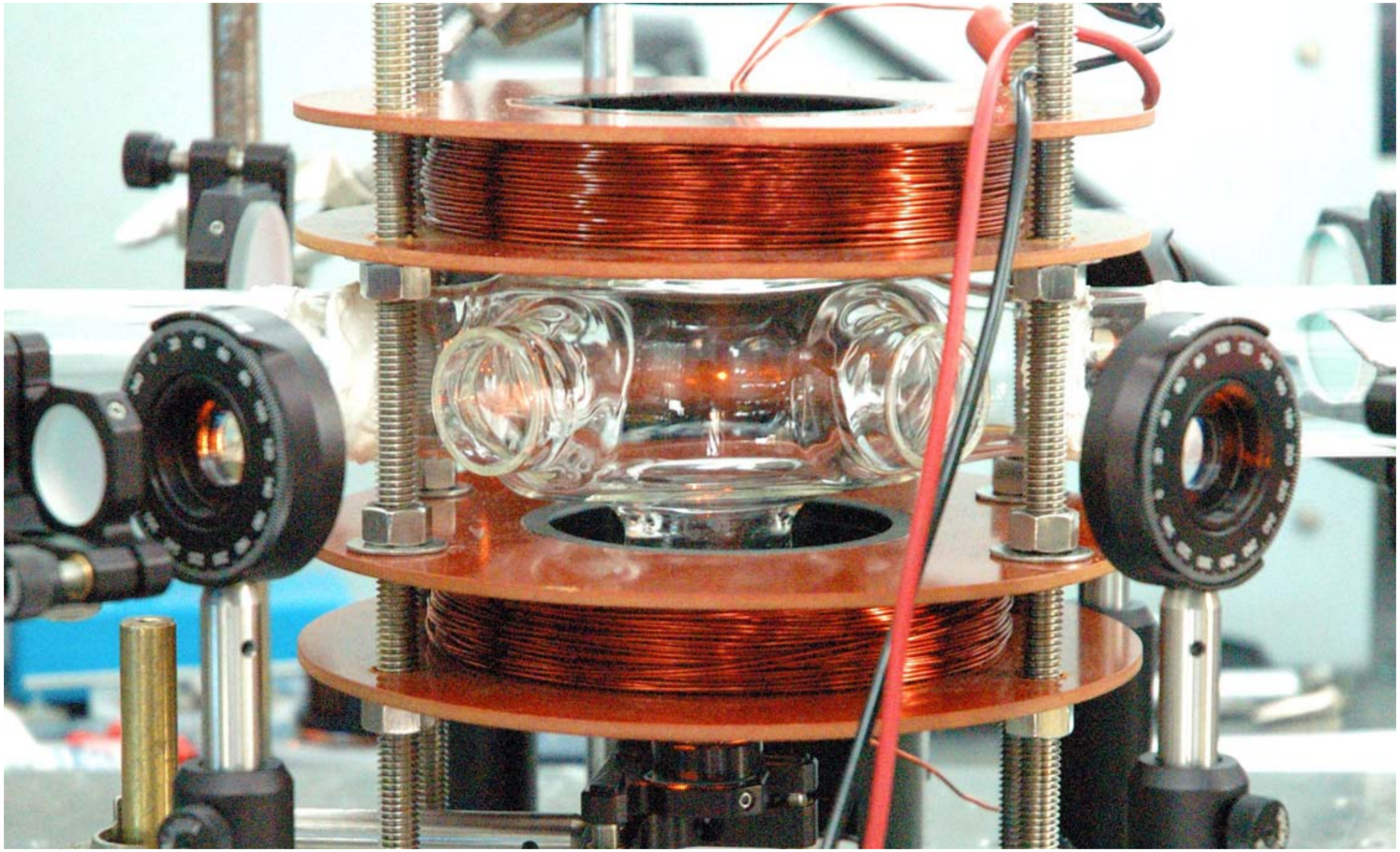
D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

Conductivity σ

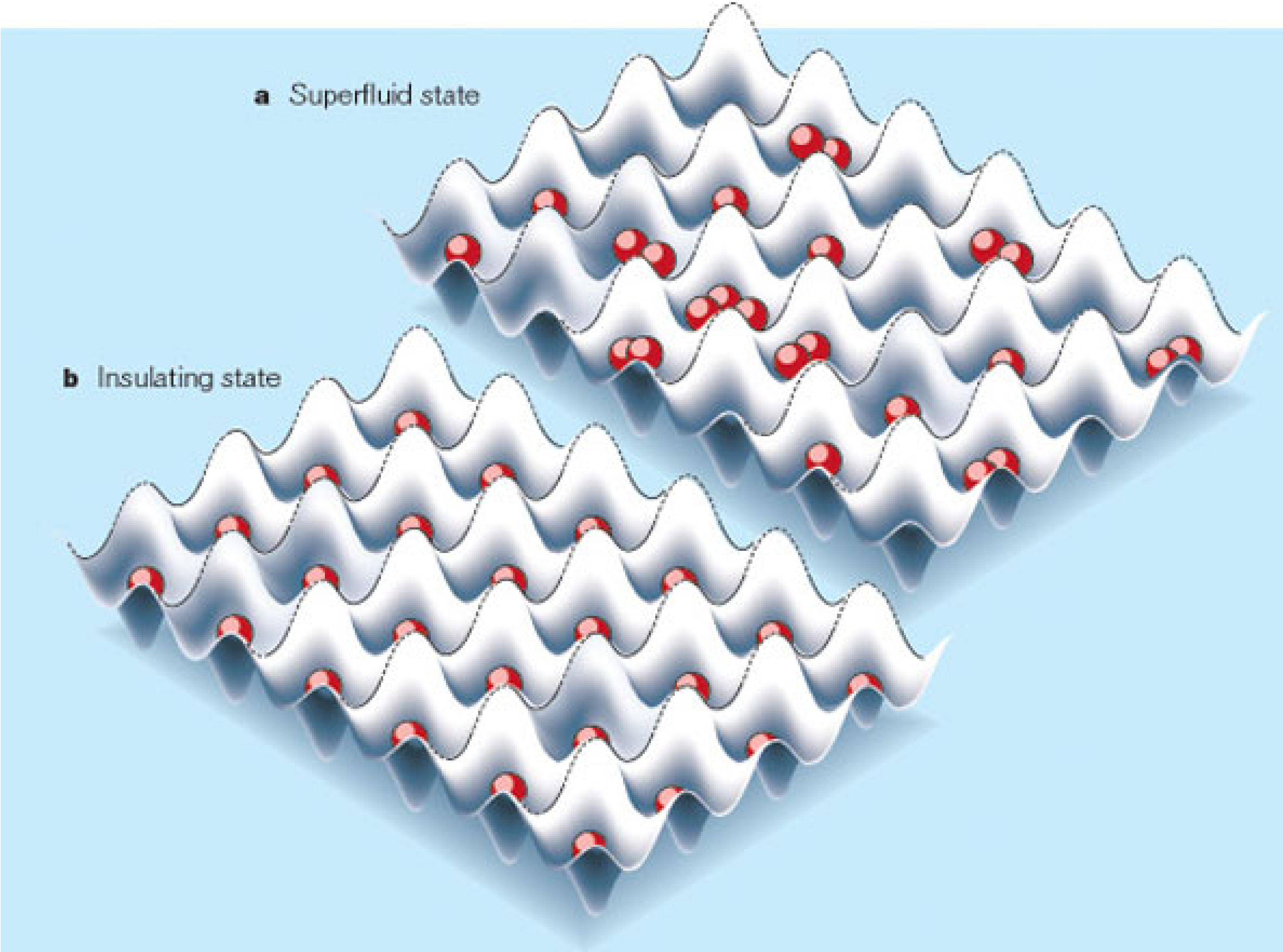
$$\sigma_{\text{Superconductor}}\left(T \to 0\right) = \infty$$

$$\sigma_{\text{Insulator}}\left(T \to 0\right) = 0$$

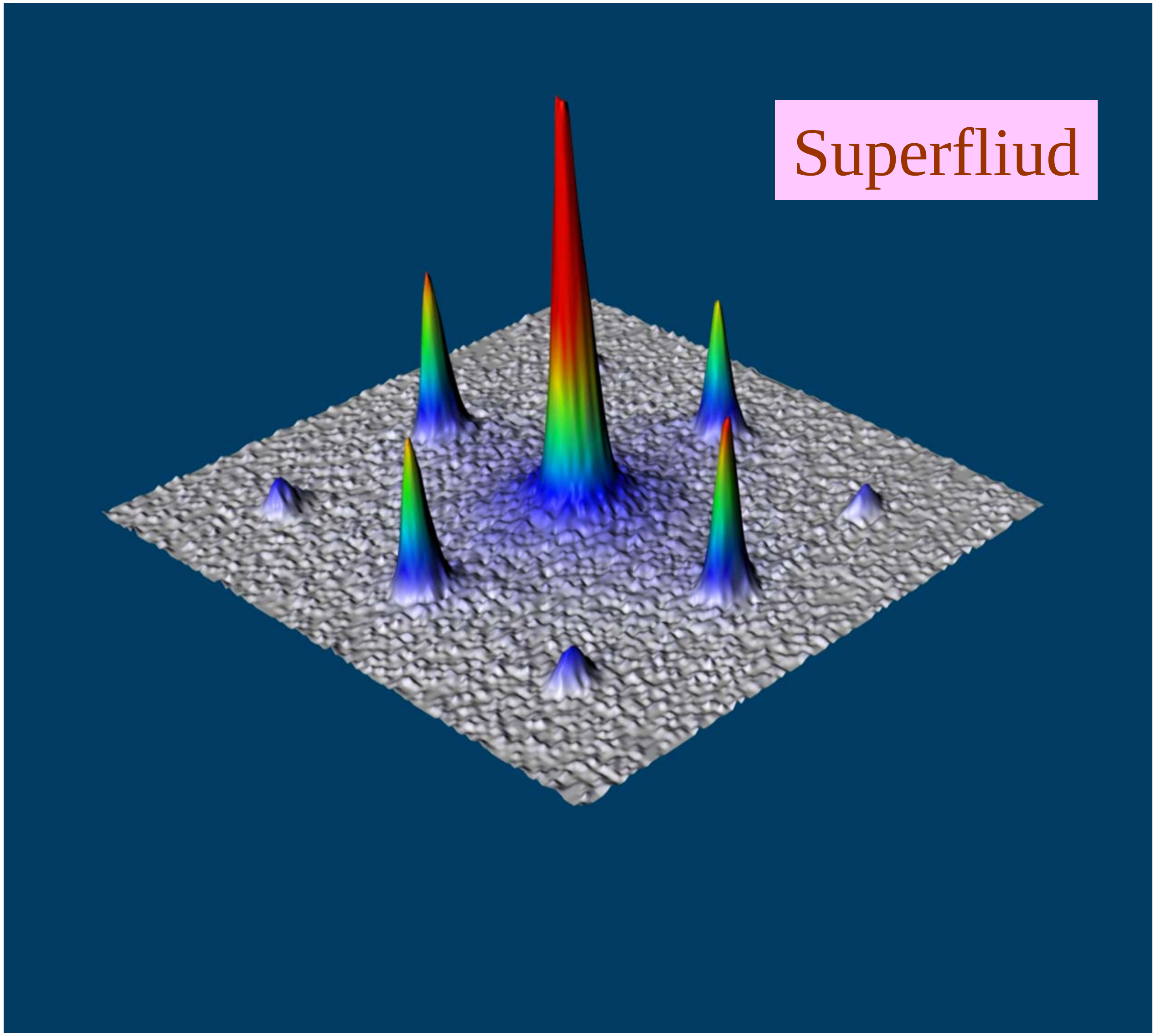
$$\sigma_{\text{Quantum critical point}}\left(T \to 0\right) \approx \frac{4e^2}{h}$$



Trap for ultracold ^{87}Rb atoms

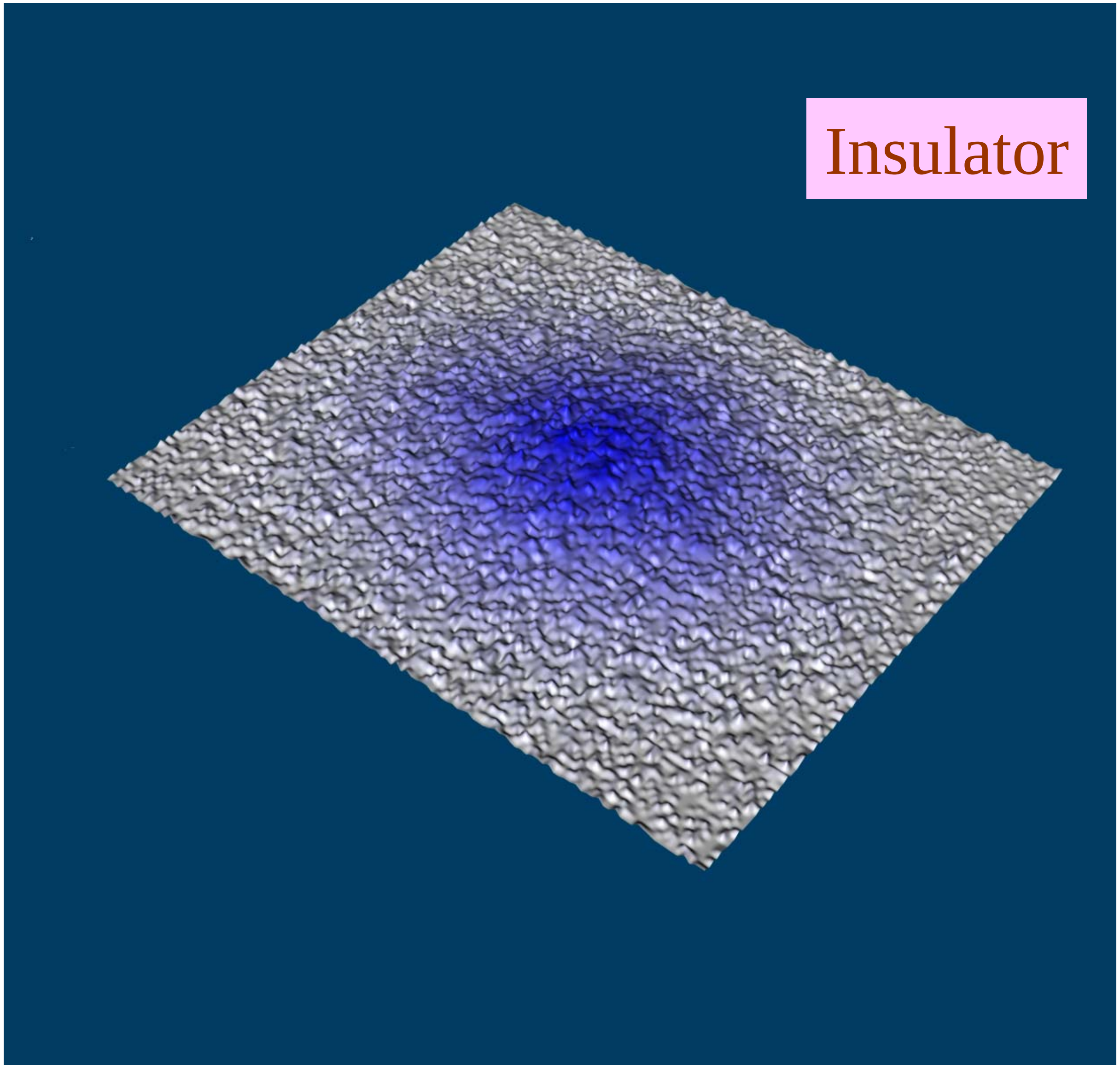

a Superfluid state
b Insulating state

Velocity distribution of ^{87}Rb atoms



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Velocity distribution of ^{87}Rb atoms



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Outline

1. Boson Hubbard model – conformal field theory (CFT)
2. Hydrodynamics of CFTs
3. Duality
4. SYM_3 with N=8 supersymmetry

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Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

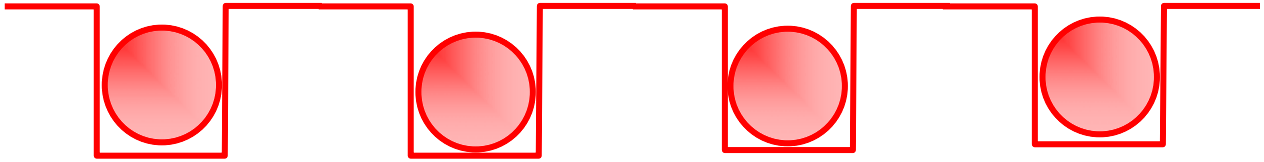
$$n_j \equiv b_j^\dagger b_j$$

M.PA. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher *Phys. Rev. B* **40**, 546 (1989).

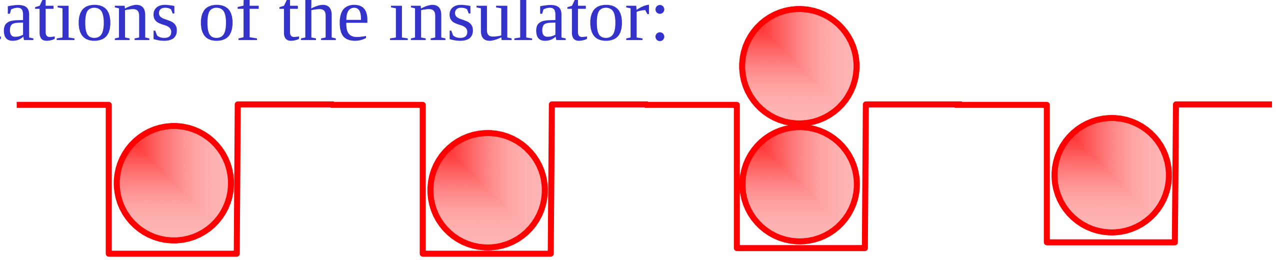
For small U/t, superfluid

For large U/t, insulator

The insulator:



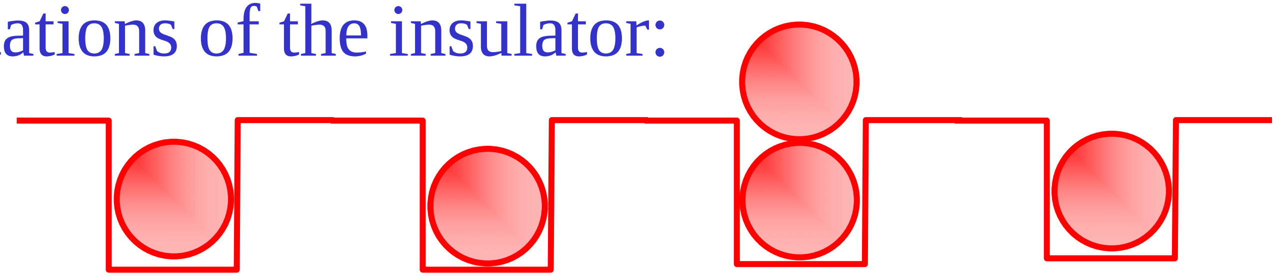
Excitations of the insulator:



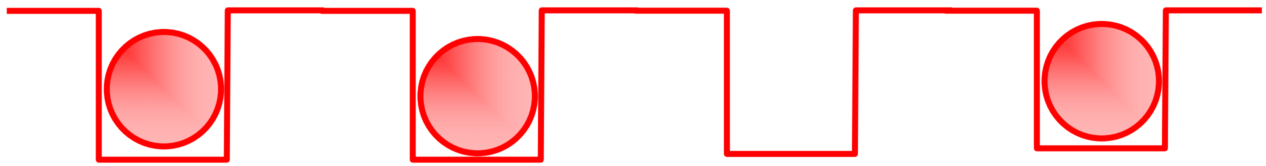
Particles ~ $\psi^\dagger$

Holes ~ ψ

Excitations of the insulator:



Particles ~ $\psi^\dagger$



Holes ~ ψ

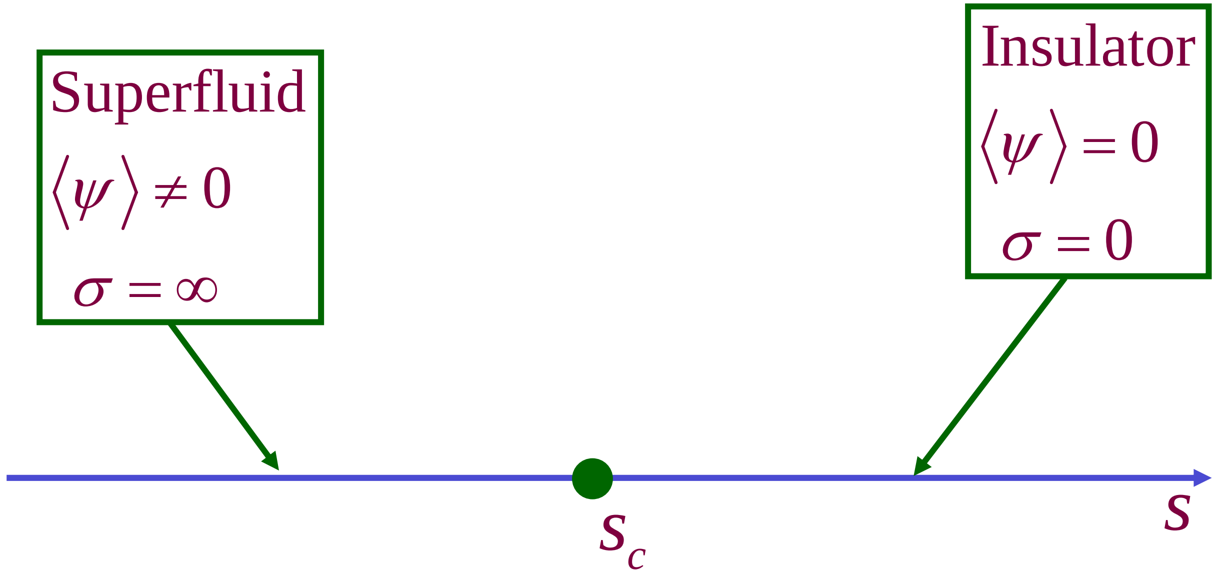
Density of particles = density of holes $\Rightarrow$

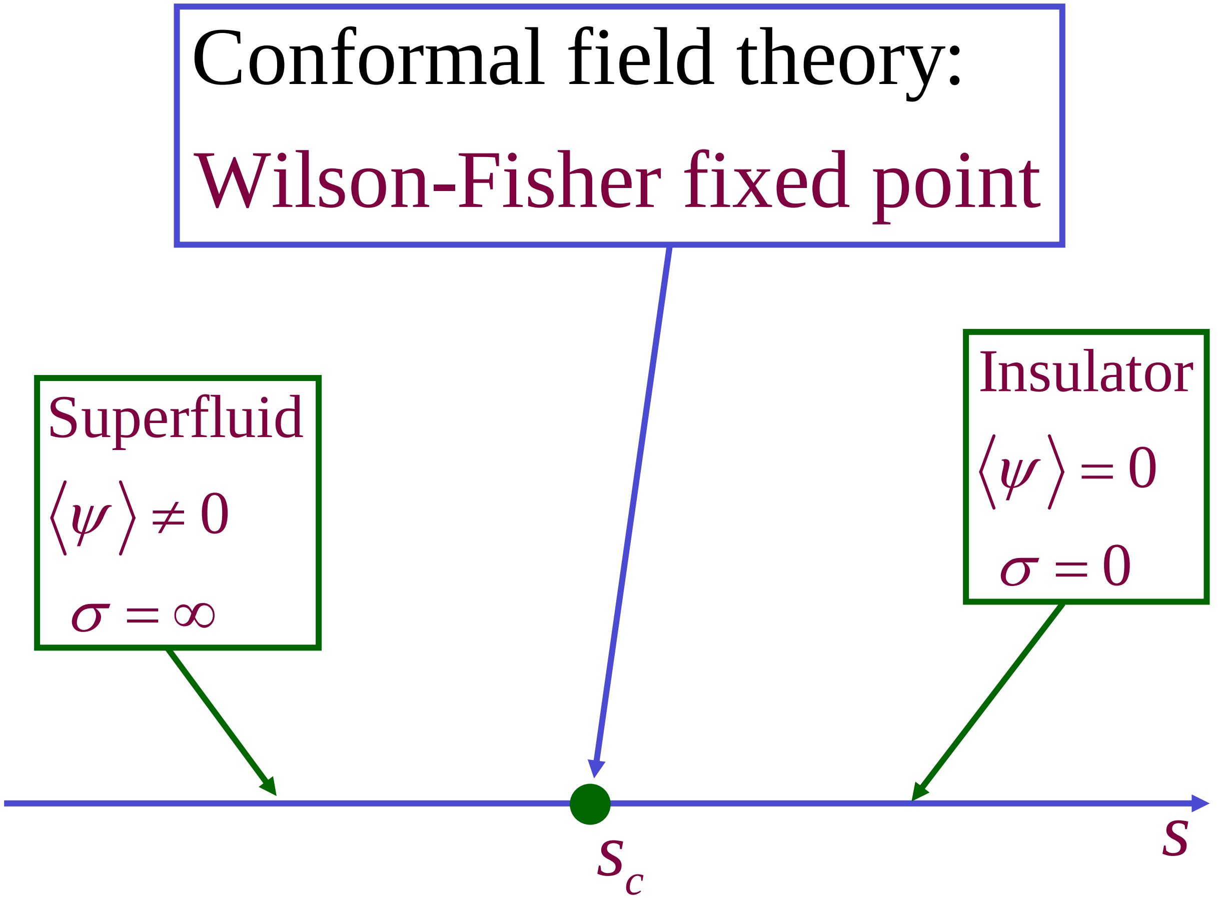
. "relativistic" field theory for ψ:

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$


Superfluid
$\langle \psi \rangle \neq 0$
$\sigma = \infty$
Insulator
$\langle \psi \rangle = 0$
$\sigma = 0$
s_c
s



Conformal field theory:
Wilson-Fisher fixed point
Superfluid
$\langle\psi\rangle \neq 0$
$\sigma = \infty$
Insulator
$\langle\psi\rangle = 0$
$\sigma = 0$
s_c
s

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1. Boson Hubbard model – conformal field theory (CFT)
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CFT correlator of $\mathrm{U}(1)$ current J_μ in 2+1 dimensions

$$\left\langle J_\mu(p) J_\nu(-p) \right\rangle = K\sqrt{p^2}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

K: a universal number analogous to the level number of the Kac-Moody algebra in 1+1 dimensions

Application of Kubo formula shows that

$$\sigma = \frac{4e^2}{h} 2\pi K$$

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

However: computation is at $T = 0$, with $\omega \to 0$,

while experimental measurements are for

$$\hbar\omega \ll k_B T$$

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while experimental measurements are for

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Does this matter ?

CFT correlator of U(1) current J_μ in 1+1 dimensions

Charge density correlation at $T = 0$:

$$\left\langle J_R(x,\tau) J_R(0) \right\rangle \sim \frac{1}{(\tau + ix)^2}$$

$$\left\langle J_t(k,\omega) J_t(-k,-\omega) \right\rangle \sim \frac{k^2}{k^2 - \omega^2}$$

CFT correlator of U(1) current J_μ in 1+1 dimensions

Charge density correlation at $T \geq 0$:

$$\left\langle J_R(x,\tau) J_R(0) \right\rangle \sim \frac{\pi^2 T^2}{\sin^2\left(\pi T\left(\tau + ix\right)\right)}$$

$$\left\langle J_t(k, i\omega_n) J_t(-k, -i\omega_n) \right\rangle \sim \frac{k^2}{k^2 + \omega_n^2}$$

Conformal mapping of plane to cylinder with circumference *1/T*

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Does this matter ?

For CFTs in 1+1 dimensions: NO

Correlators of conserved charges are independent of the ratio $\hbar\omega / k_B T$

No diffusion of charge, and no hydrodynamics

However: computation is at $T = 0$, with $\omega \to 0$,

while experimental measurements are for

$$\hbar\omega \ll k_B T$$

Does this matter ?

For CFTs in 2+1 dimensions: YES

However: computation is at $T = 0$, with $\omega \to 0$, while experimental measurements are for

$$\hbar\omega \ll k_B T$$

Does this matter ?

For CFTs in 2+1 dimensions: YES

$\hbar / k_B T$ is a characteristic *decoherence* or *collision* time

$\hbar\omega \gg k_B T$: Collisionless physics

$\hbar\omega \ll k_B T$: Hydrodynamic, collision-dominated transport

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

For $|\omega - k| \gg k_B T/\hbar$ we should have collisionless conformal behavior

$$\langle J_t(k,\omega) J_t(-k,-\omega)\rangle_{\rm ret} = K \frac{k^2}{\sqrt{k^2-\omega^2}}$$

while for $\omega, k \ll k_B T/\hbar$ we should have hydrodynamics:

$$\langle J_t(k,\omega) J_t(-k,-\omega)\rangle_{\rm ret} = \chi \frac{Dk^2}{-i\omega + Dk^2}$$

Conductivity obeys Einstein relation $(\hbar/4e^2)\sigma = D\chi \neq K$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

CFT correlator of $\mathrm{U}(1)$ current J_μ in 2+1 dimensions

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Application of Kubo formula shows that

$$\sigma = \frac{4e^2}{h} 2\pi K$$

CFT correlator of $\mathrm{U}(1)$ current J_μ at $T = 0$

$$\left\langle J_\mu(p) J_\nu(-p) \right\rangle = K \sqrt{p^2} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

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Application of Kubo formula shows that

$$\sigma\left(\omega/T = \infty \right) = \frac{4e^2}{h} 2\pi K$$

CFT correlator of $\mathrm{U}(1)$ current J_μ at $T>0$

$$\left\langle J_\mu(k,\omega) J_\nu(-k,-\omega)\right\rangle = \sqrt{k^2-\omega^2}\left(P_{\mu\nu}^T K^T(k,\omega) + P_{\mu\nu}^L K^L(k,\omega)\right)$$

The projectors are defined by

$$P_{ij}^T = \delta_{ij} - \frac{k_i k_j}{k^2} \text{ and } P_{\mu\nu}^L = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} - P_{\mu\nu}^T \quad ; \quad p=(k,\omega)$$

while $K^{L,T}(k,\omega)$ are universal functions of ω/T and k/T

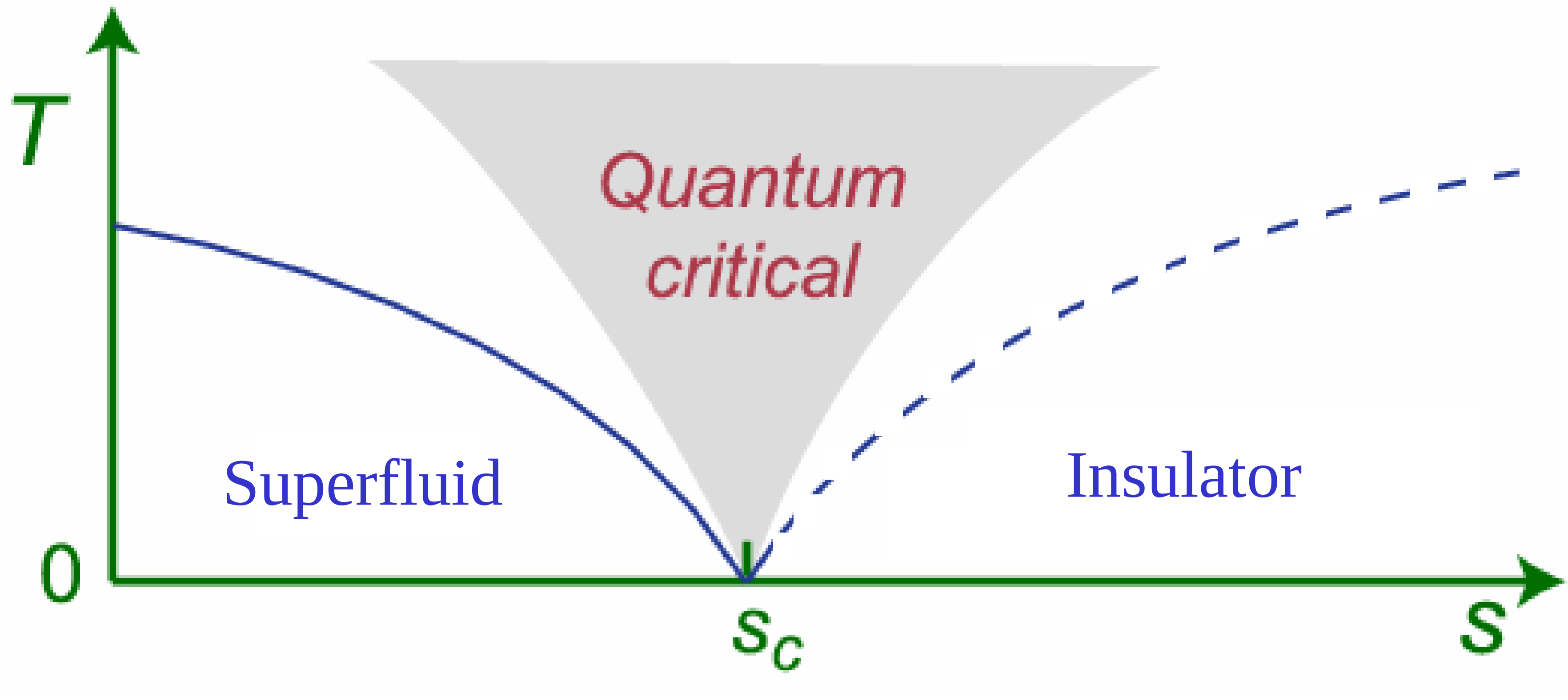
Application of Kubo formula shows that

$$\sigma\left(\omega/T\right) = \frac{4e^2}{h} 2\pi K^T(0,\omega) = \frac{4e^2}{h} 2\pi K^L(0,\omega)$$

$$\mathcal{S} = \int d^2x \int_0^{1/T} d\tau \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$



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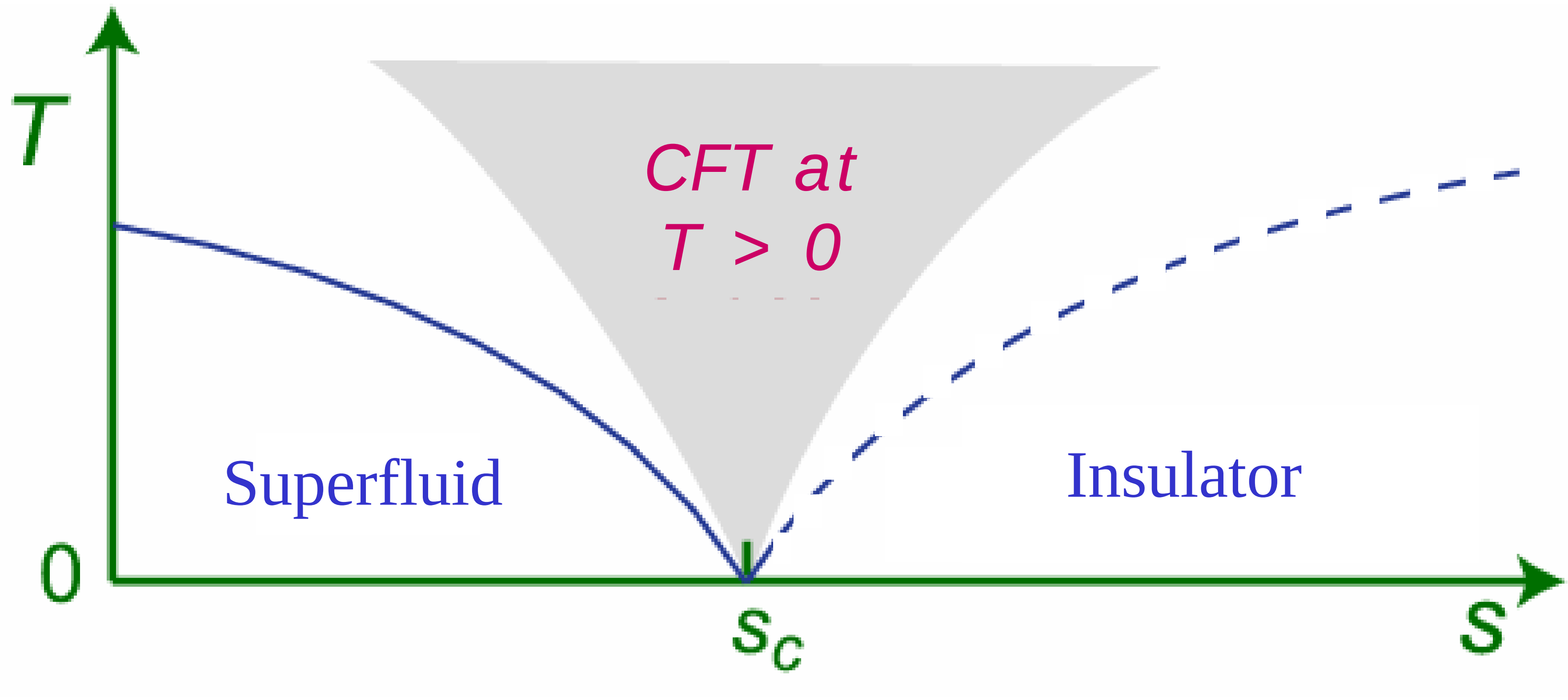

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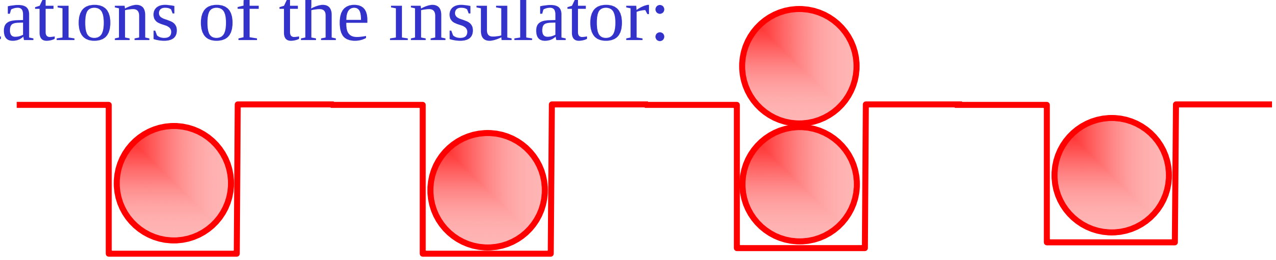
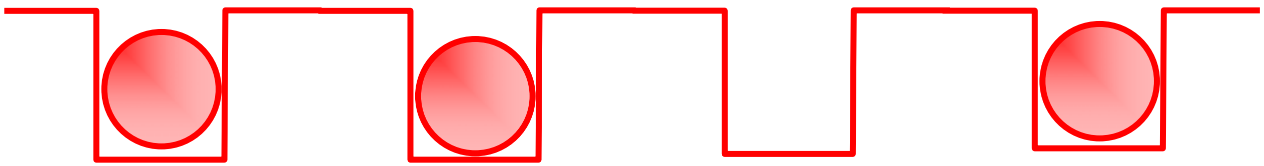
Excitations of the insulator:



Particles ~ $\psi^\dagger$



Holes ~ ψ

Density of particles = density of holes $\Rightarrow$

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Approaching the transition from the superfluid

Excitations of the superfluid: (A) **Spin waves**

With $\psi \sim e^{i\theta}$, action for spin waves is

$$\mathcal{S}_{sw} = \frac{\rho_s}{2} \int d^3x (\partial_\mu \theta)^2$$

Dual form: After a Hubbard-Stratonovich transformation, write

$$\mathcal{S}_{sw} = \int d^3x \left[\frac{1}{2\rho_s} J_\mu^2 + i J_\mu \partial_\mu \theta \right]$$

Integrating over θ yields $\partial_\mu J_\mu = 0$. Solve, by writing

$$J_\mu = \epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda$$

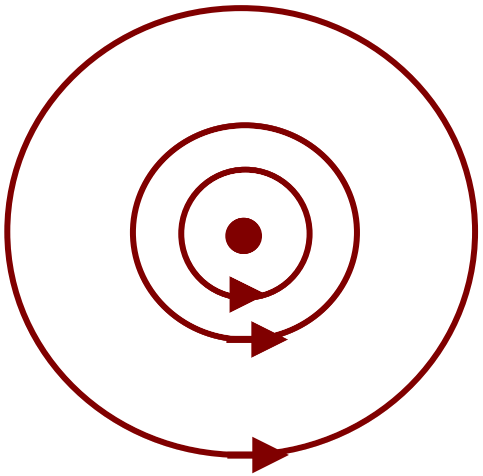
leading to

$$\mathcal{S}_{sw} = \int d^3x \left[\frac{1}{2\rho_s} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

Spin waves are dual to a U(1) gauge theory in 2+1 dimensions

Approaching the transition from the superfluid

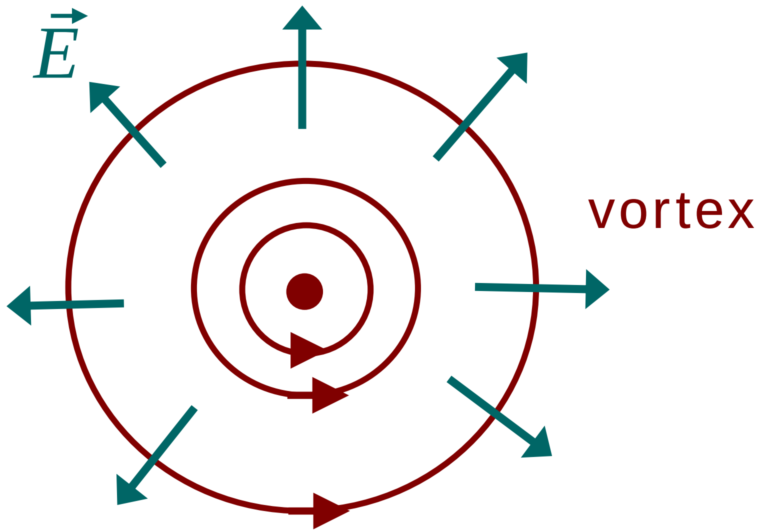
Excitations of the superfluid: (B) **Vortices**



A vortex is a point-like object. We can therefore define a local field operator, φ, which annihilates a vortex.

Approaching the transition from the superfluid

Excitations of the superfluid: (B) **Vortices**



A vortex is a point-like object. We can therefore define a local field operator, φ, which annihilates a vortex.

Each vortex is the source of an 'electric field' $\vec{E}$ associated with the U(1) gauge field A_μ.

Consequently, φ carries +1 U(1) gauge charge.

Approaching the transition from the superfluid

Excitations of the superfluid: **Spin wave and vortices**

φ: vortex annihilation operator.

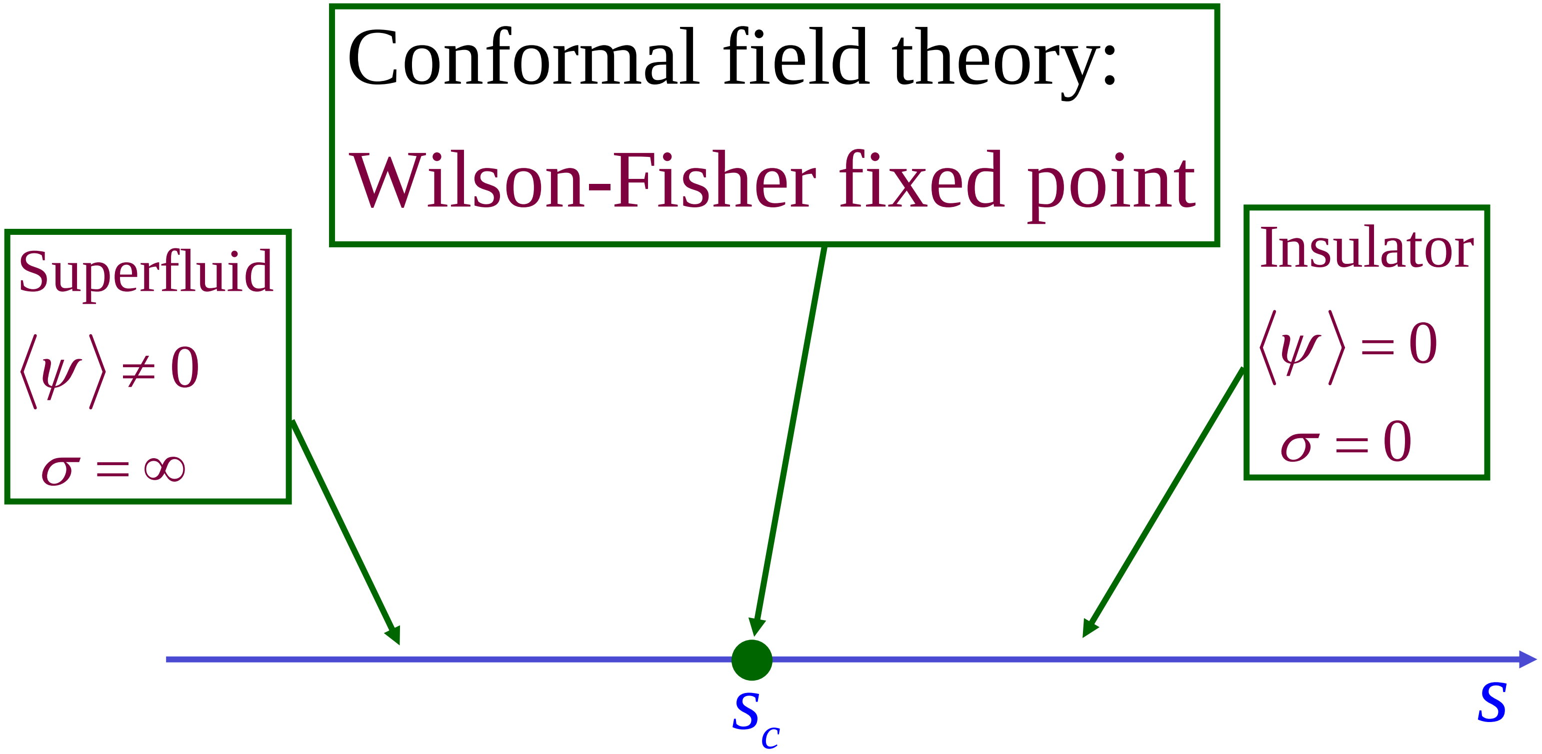
$\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$: boson current $\sim i\psi^*\partial_\mu\psi - i\partial_\mu\psi^*\psi$.

Density of vortices = density of antivortices $\Rightarrow$
"relativistic" field theory for φ:

$$\mathcal{S}_{\text{dual}} = \int d^3x \Big[|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2\rho_s}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

$$\text{Superfluid} \Leftrightarrow \langle\varphi\rangle = 0$$

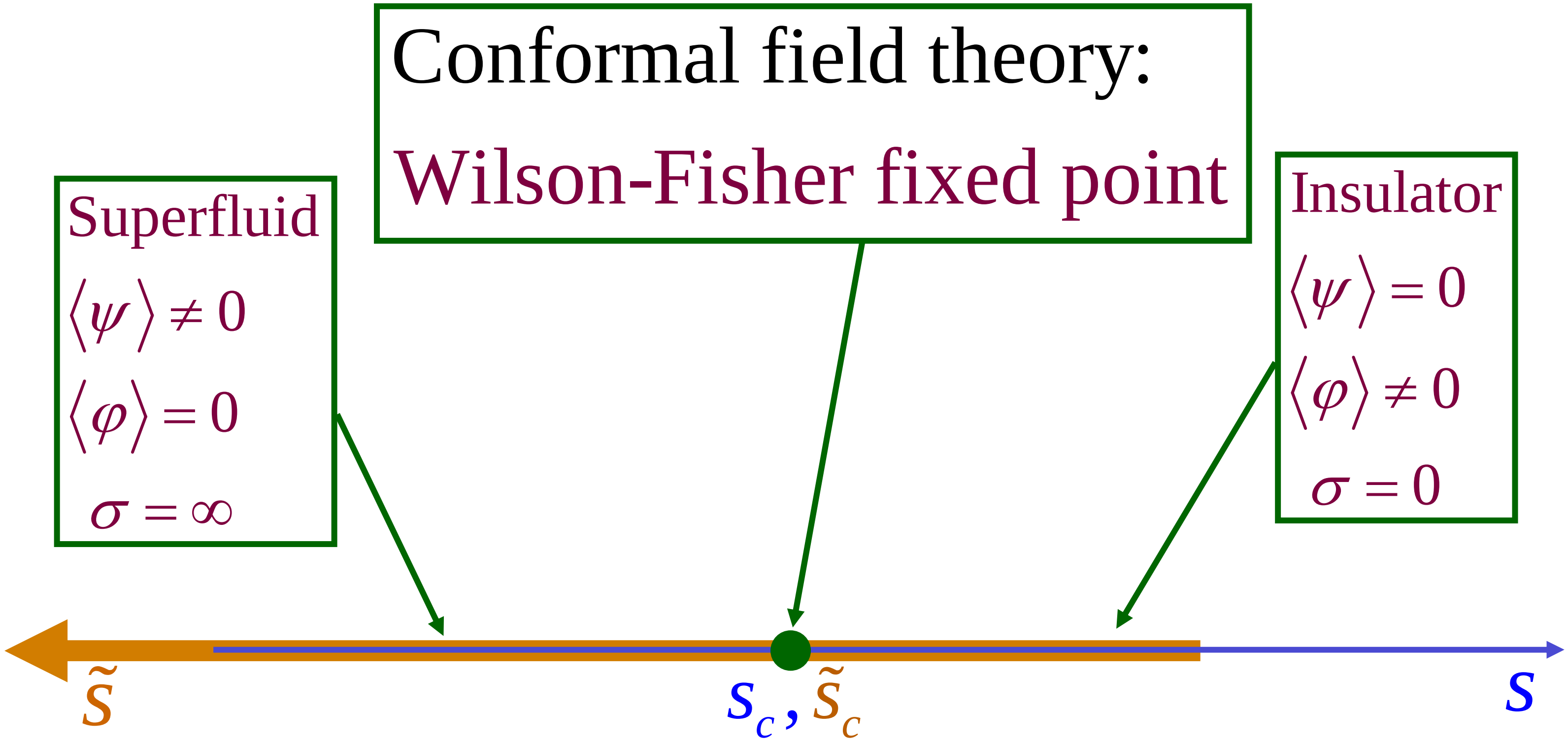
$$\text{Insulator} \Leftrightarrow \langle\varphi\rangle \neq 0$$



Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981)



Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

<u>is dual to</u>

Using the vortex quasiparticle excitations of the superfluid $\sim \varphi$

$$\mathcal{S}_{\text{dual}} = \int d^3x \left[|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981)

Consequences of duality on CFT correlators of $\mathrm{U}(1)$ currents

$$\langle J_\mu(k,\omega) J_\nu(k,\omega)\rangle_{\mathcal{S}} = \sqrt{k^2-\omega^2}\left(P^T_{\mu\nu}K^T(k,\omega) + P^L_{\mu\nu}K^L(k,\omega)\right)$$

$$\left\langle \widetilde{J}_\mu(k,\omega) \widetilde{J}_\nu(k,\omega)\right\rangle_{\mathcal{S}_{\mathrm{dual}}} = \sqrt{k^2-\omega^2}\left(P^T_{\mu\nu}\widetilde{K}^T(k,\omega) + P^L_{\mu\nu}\widetilde{K}^L(k,\omega)\right)$$

$$K^L(k,\omega)\widetilde{K}^T(k,\omega) = \frac{1}{4\pi^2}$$

$$K^T(k,\omega)\widetilde{K}^L(k,\omega) = \frac{1}{4\pi^2}$$

Application of Kubo formula shows that

$$\sigma\left(\omega/T\right) = \frac{4e^2}{h}2\pi K^T(0,\omega) = \frac{4e^2}{h}2\pi K^L(0,\omega)$$

C. Herzog, P. Kovtun, S. Sachdev, and D.T. Son, hep-th/0701036

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4. SYM_3 with N=8 supersymmetry

SYM$_3$ with $\mathcal{N} = 8$ supersymmetry and SU(N) gauge group

- Obtained by compactifying SYM$_{10}$ on $R^3 \times T^7$: has a global SO(7) R-charge symmetry
- Contains a single dimensionful gauge coupling constant, g_{3D}.
- It is believed that g_{3D} flows in the infrared to a fixed point which defines a SCFT. The fixed point has strong universal interactions (like the Wilson-Fisher fixed point) but no relevant perturbation which preserves all (super)symmetries (unlike the Wilson-Fisher fixed point).
- The fixed point has an emergent global SO(8) R-charge symmetry.

AdS/CFT correspondence

- SU(N) SYM_3 is the low energy theory of Type IIA strings on N co-incident D2 branes
- Flow to strong coupling is captured by the lift to M theory.
- The SU(N) SYM_3 SCFT is described by M theory on N M2 branes
- The large N limit of the SCFT is described by eleven dimensional supergravity on $\text{AdS}_4 \times S^7$ – the SO(8) R-charge symmetry is explicitly realized.
- The correspondence extends also to $T > 0$. The supergravity theory acquires a black hole, and Hawking temperature of black hole $\leftrightarrow$ temperature of SCFT

Correlations of SO(8) currents of the SYM$_3$ SCFT at $T > 0$

- Related to the dynamics of a 4-dimensional SO(8) gauge in the curved gravity background.
- SO(8) gauge field has a dimensionless coupling constant g_{4D} with the value

$$\frac{1}{g_{4D}^2} = \frac{\sqrt{2}}{6\pi} N^{3/2}$$

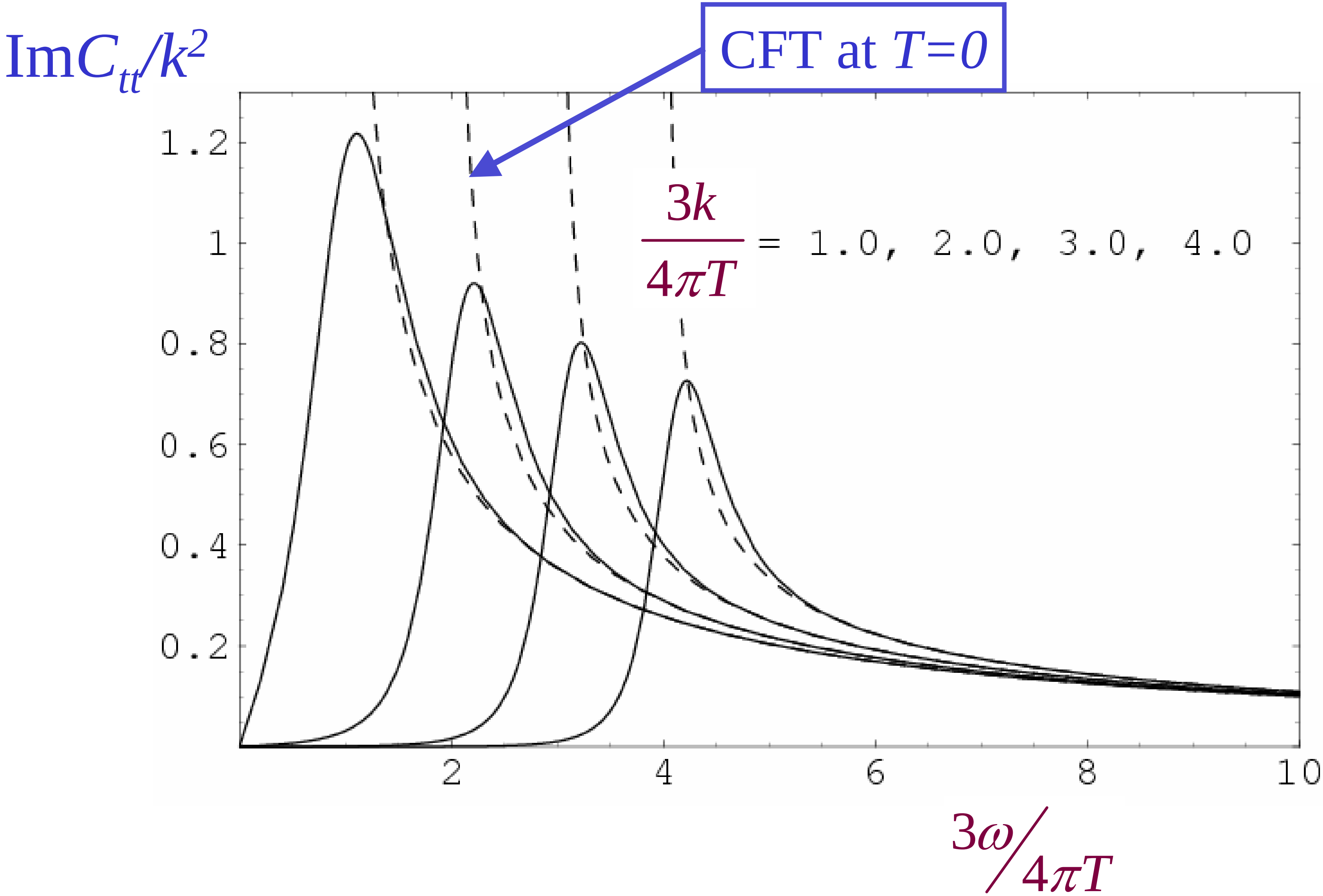
- SO(8) gauge field is weakly coupled in the large N limit and exact results can be obtained for all ω and k.

Correlations of SO(8) currents of the SYM$_3$ SCFT at $T > 0$

$$C_{tt}(k,\omega)\delta^{ab} = \left\langle J_t^a(k,\omega) J_t^b(-k,-\omega)\right\rangle_{\text{ret}}$$

For $|\omega - k| \gg T$ we have collisionless conformal behavior

$$C_{tt}(k,\omega) = \frac{1}{g_{4D}^2}\frac{k^2}{\sqrt{k^2-\omega^2}}$$



$\mathrm{Im}C_{tt}/k^2$
CFT at $T=0$
$\frac{3k}{4\pi T}$ = 1.0, 2.0, 3.0, 4.0
1.2
1
0.8
0.6
0.4
0.2
2
4
6
8
10
$3\omega/4\pi T$

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For $|\omega - k| \gg T$ we have collisionless conformal behavior

$$C_{tt}(k,\omega) = \frac{1}{g_{4D}^2}\frac{k^2}{\sqrt{k^2-\omega^2}}$$

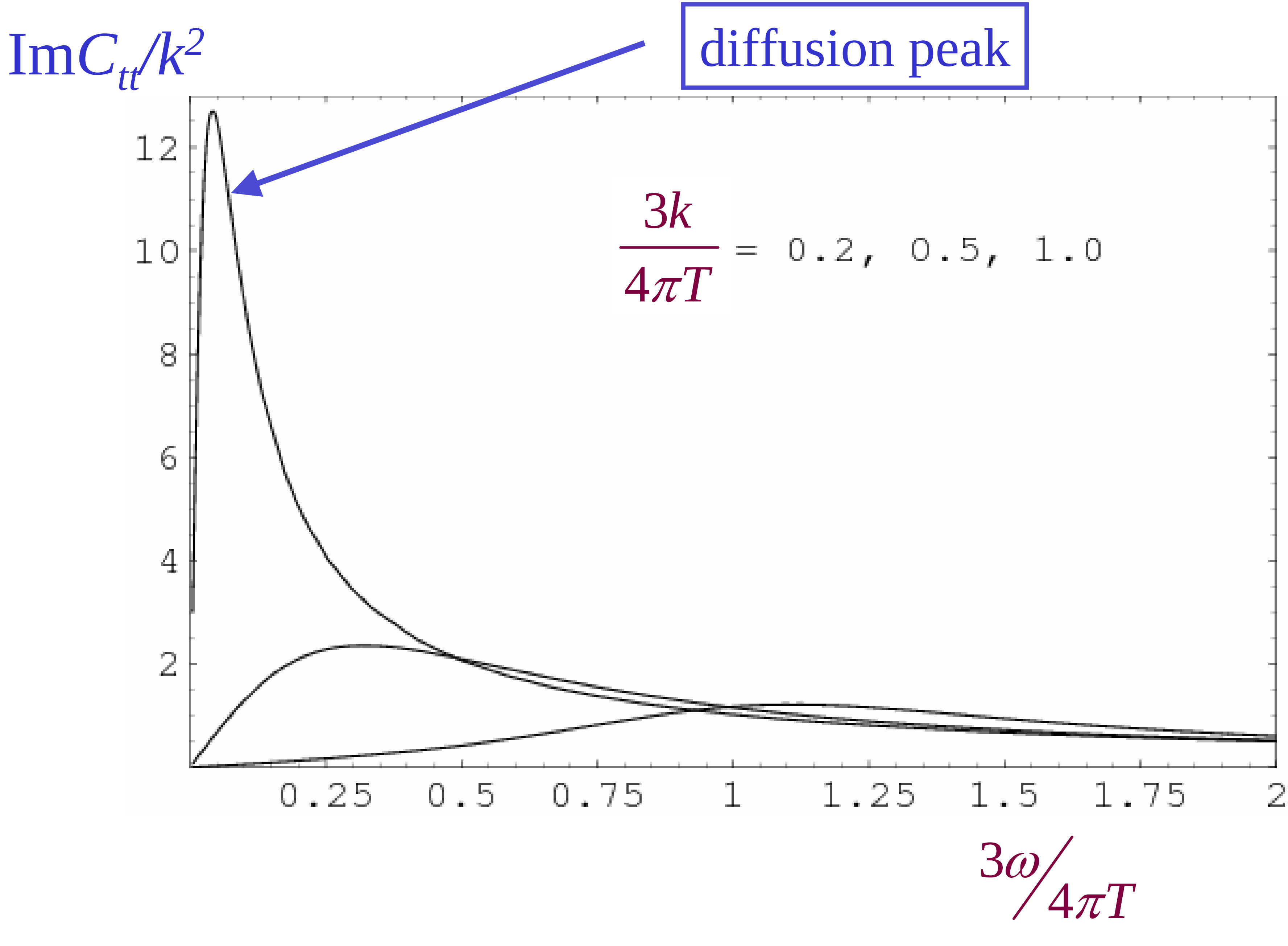
while for $\omega, k \ll T$ we have hydrodynamics:

$$C_{tt}(k,\omega) = \chi \frac{Dk^2}{-i\omega + Dk^2}$$

with $\chi = 4\pi T/(3g_{4D}^2)$ and

$$D = \frac{3}{4\pi T}$$

Note, *exact result for a diffusion constant.*



ImC_{tt}/k^2
diffusion peak
$\frac{3k}{4\pi T}$ = 0.2, 0.5, 1.0
12
10
8
6
4
2
0.25
0.5
0.75
1
1.25
1.5
1.75
2
$3\omega / 4\pi T$

Correlations of SO(8) currents of the SYM_3 SCFT at $T > 0$

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For $|\omega - k| \gg T$ we have collisionless conformal behavior

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$$C_{tt}(k,\omega) = \chi\frac{Dk^2}{-i\omega + Dk^2}$$

with $D = 3/(4\pi T)$ and $\chi = 4\pi T/(3g_{4D}^2)$.

Correlations of SO(8) currents of the SYM_3 SCFT at $T > 0$

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while for $\omega, k \ll T$ we have hydrodynamics:

$$C_{tt}(k,\omega) = \chi \frac{Dk^2}{-i\omega + Dk^2}$$

with $D = 3/(4\pi T)$ and $\chi = 4\pi T/(3g_{4D}^2)$.

Notice, however, that $D\chi = 1/g_{4D}^2$! This is *unexpected* and implies $\sigma(\omega/T \to 0) = \sigma(\omega/T \to \infty)$.

Correlations of SO(8) currents of the SYM_3 SCFT at $T > 0$

$$\left\langle J_\mu^a(k,\omega)\, J_\nu^b(-k,-\omega)\right\rangle = \delta^{ab}\sqrt{k^2-\omega^2}\left(P_{\mu\nu}^T K^T(k,\omega) + P_{\mu\nu}^L K^L(k,\omega)\right)$$

The self-duality of the 4D SO(8) gauge fields leads to

$$\boxed{K^L(k,\omega)K^T(k,\omega) = \frac{N^3}{18\pi^2}}$$

C. Herzog, P. Kovtun, S. Sachdev, and D.T. Son, hep-th/0701036

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Analyticity of correlations at $T > 0$ implies

$$K^T(0,\omega) = K^L(0,\omega),$$

and so the conductivity

$$\sigma(\omega/T) = K^T(0,\omega) = K^L(0,\omega) = \sqrt{\frac{N^3}{72\pi^2}}$$

is frequency independent.

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Holographic self-duality

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Open questions

1. Does $K^L K^T$ = constant (*i.e.* holographic self-duality) hold for SYM_3 SCFT at finite N ?

2. Is there any CFT_3 with an Abelian U(1) current whose conductivity can be determined by self-duality ? (unlikely, because global and topological U(1) currents are interchanged under duality).

3. Is there any CFT_3 solvable by AdS/CFT which is not (holographically) self-dual ?

4. Is there an AdS_4 description of the hydrodynamics of the O(N) Wilson-Fisher CFT_3 ? (can use $1/N$ expansion to control strongly-coupled gravity theory).