$$H = -\sum_{i<j} w_{ij}\left(c_{i\alpha}^{\dagger}c_{j\alpha} + c_{j\alpha}^{\dagger}c_{i\alpha}\right) + \frac{U}{2}\sum_{i} n_i\left(n_i - 1\right) \quad ; \quad n_i = c_{i\alpha}^{\dagger}c_{i\alpha} \quad ; \quad \text{Vary } U/\overline{w}$$

A. *S=1/2* fermions (Si:P , 2DEG)

Metal-insulator transition

Evolution of magnetism across transition.

Phil. Trans. Roy. Soc. A **356**, 173 (1998) (cond-mat/9705074)

Pramana **58**, 285 (2002) (cond-mat/0109309)

Reznikov
Sarachik

B. *S=0* bosons (ultracold atoms in an optical lattice)

Superfluid-insulator transition

Mott insulator in a strong electric field - $Ea \approx U$

S. Sachdev, K. Sengupta, S.M. Girvin, cond-mat/0205169

Transport in coupled quantum dots
150000 dots

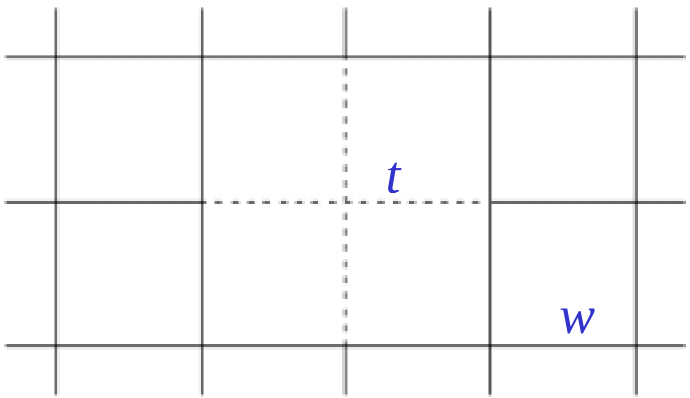
Talk online at
http://pantheon.yale.edu/~subir

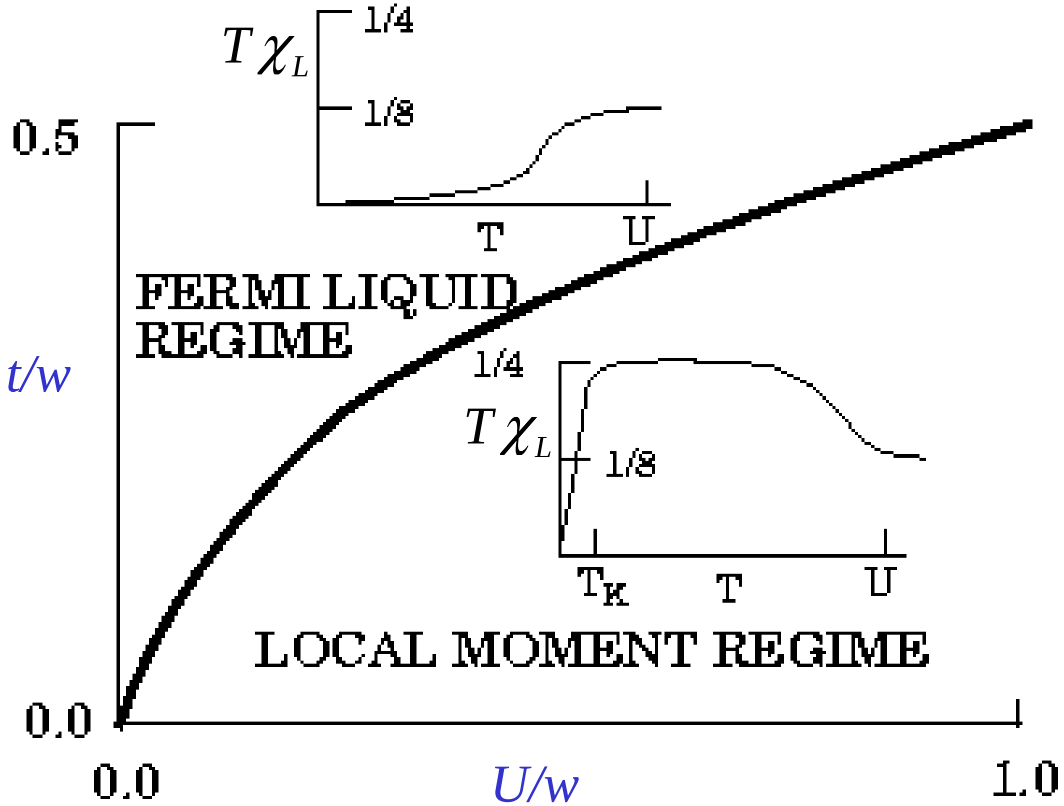
1701 YALE UNIVERSITY TERCENTENNIAL 2001

A. *S=1/2* fermions

$U/\bar{w}$ small ; charge transport "metallic"

Magnetic properties of a single impurity





Low temperature magnetism dominated by such impurities

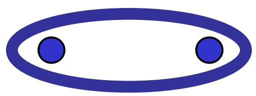
M. Milovanovic, S. Sachdev, R.N. Bhatt, *Phys. Rev. Lett*. **63**, 82 (1989).
R.N. Bhatt and D.S. Fisher, *Phys. Rev. Lett*. **68**, 3072 (1992).

A. $S=1/2$ fermions

$U/\bar{w}$ large ; charge transport "insulating"

$$H = \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad ; \quad J_{ij} \sim \exp\left(-\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|/a\right)$$

Spins pair up in singlets



$$= \tfrac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Strong disorder

Random singlet phase

$$P(J) \sim J^{-\alpha} \Rightarrow \chi \sim T^{-\alpha} \quad ; \ \alpha \approx 0.8$$

Weak disorder

Spin gap

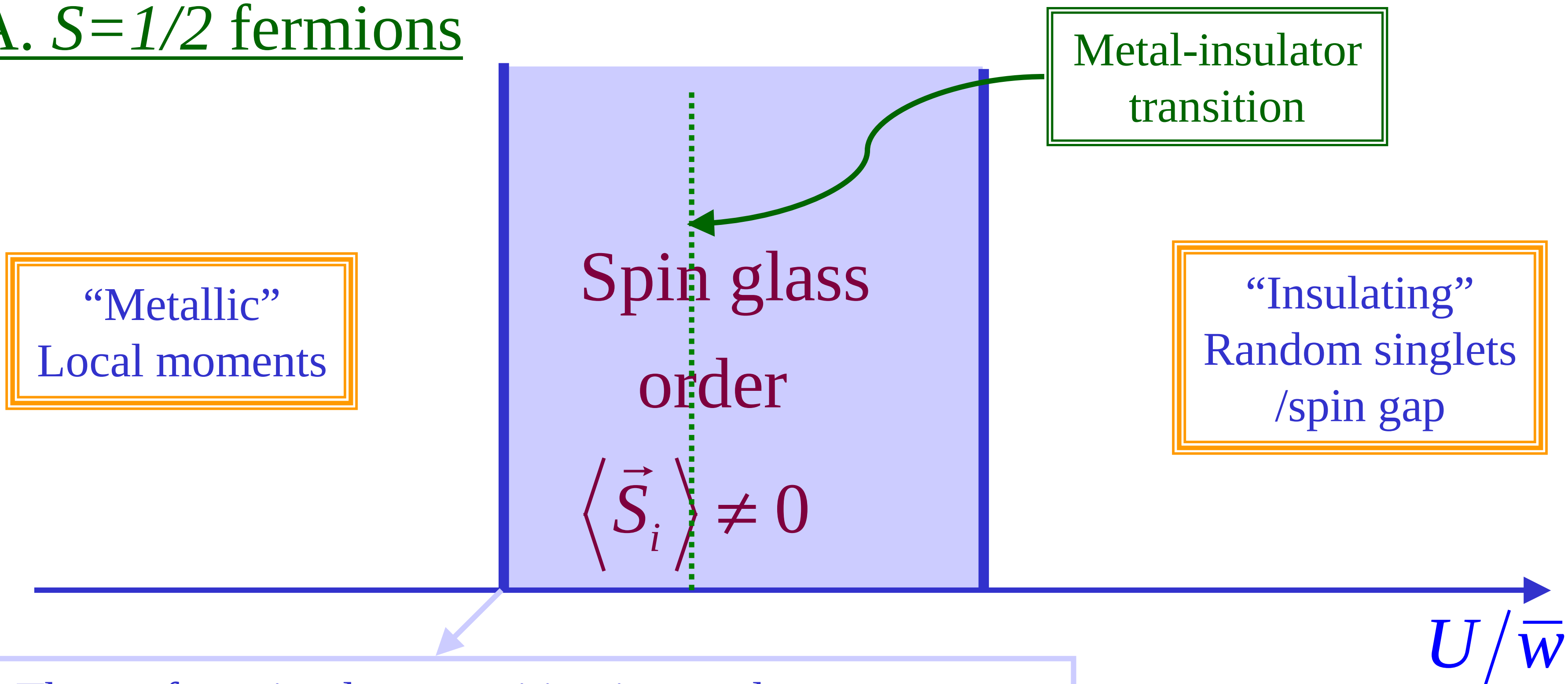
$$\chi \sim \exp\left(-\Delta/T\right)$$

R.N. Bhatt and P.A. Lee, *Phys. Rev. Lett.* **48**, 344 (1982).

B. Bernu, L. Candido, and D.M. Ceperley, *Phys. Rev. Lett.* **86**, 870 (2002).

G. Misguich, B. Bernu, C. Lhuillier, and C. Waldtmann, *Phys. Rev. Lett.* **81**, 1098 (1998).

A. *S=1/2* fermions



Theory for spin glass transition in metal

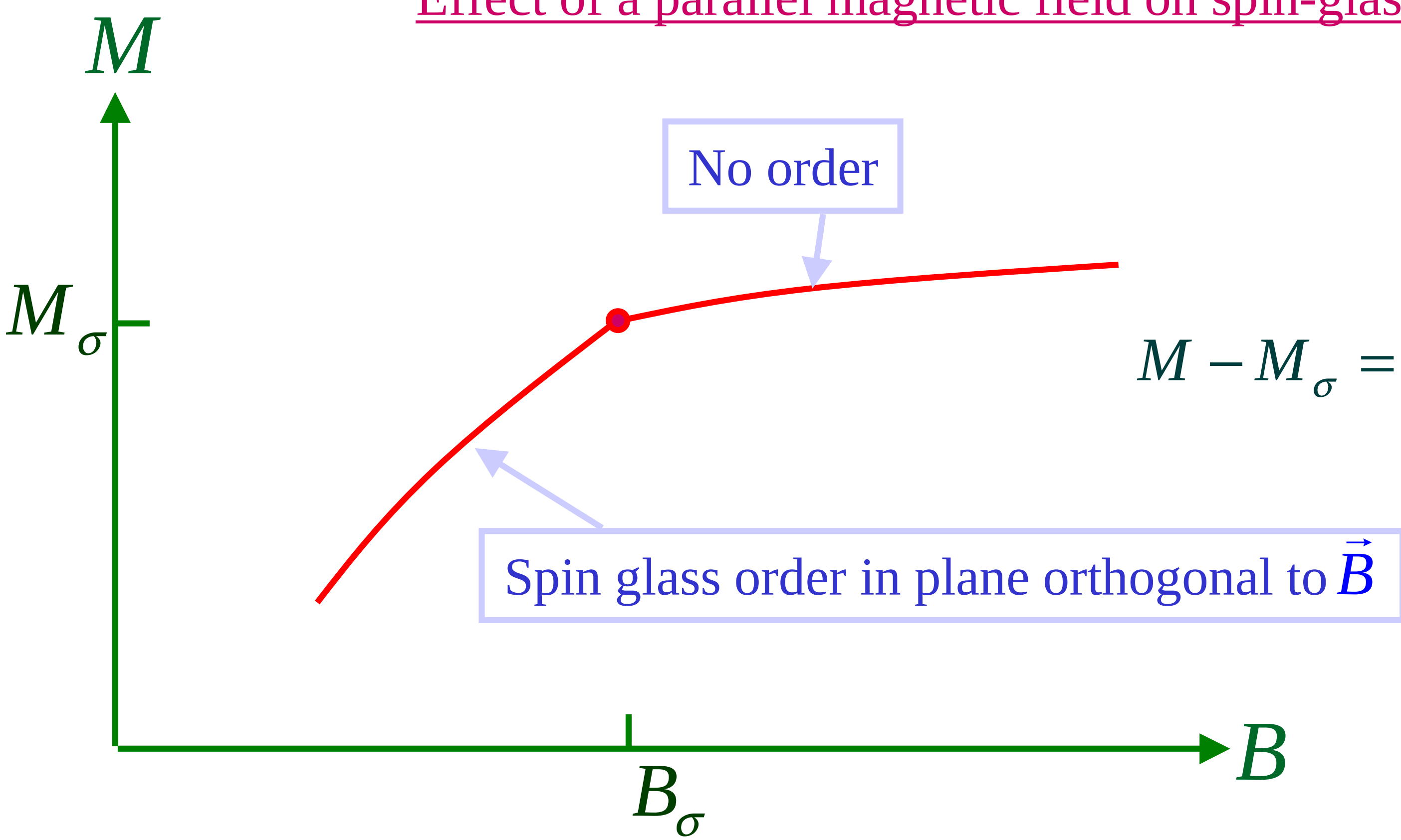
S. Sachdev, N. Read, R. Oppermann, *Phys. Rev* B **52**, 10286 (1995).
A.M. Sengupta and A. Georges, *Phys. Rev* B **52**, 10295 (1995).

Higher density of moments + longer range of exchange interaction induces spin-glass order. Also suggested by strong-coupling flow of triplet interaction amplitude in Finkelstein's (*Z. Phys. B* **56**, 189 (1984)) renormalized weak-disorder expansion.

Glassy behavior observed by S. Bogdanovich and D. Popovic, cond-mat/0106545.

S. Sachdev, *Phil. Trans. Roy. Soc.* **356A**, 173 (1998) (cond-mat/9705074);
S. Sachdev, *Pramana*. **58**, 285 (2002) (cond-mat/0109309).

Effect of a parallel magnetic field on spin-glass state.



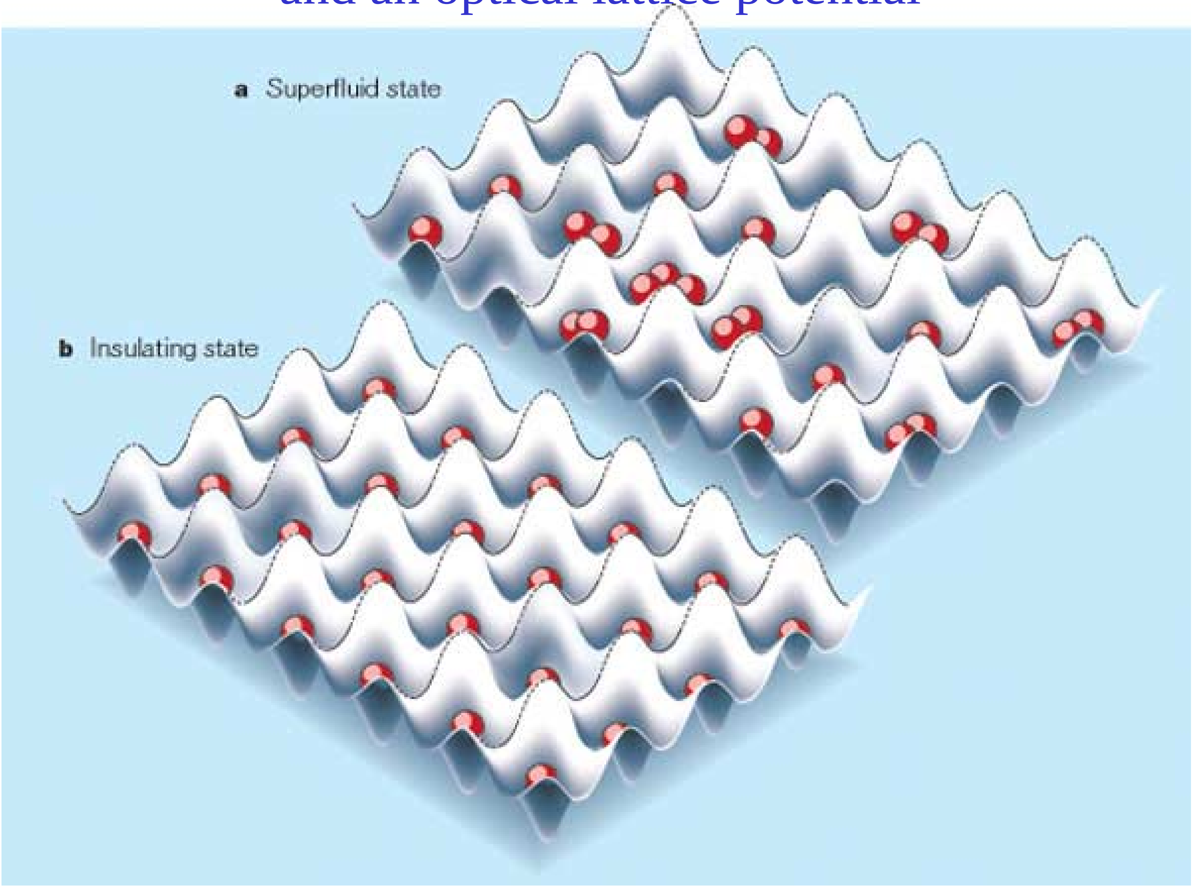
$$M - M_\sigma = T^{d/z} \varphi\left(\frac{B - B_\sigma}{T}\right)$$

Singular behavior at a critical field at *T=0*
Possibly related to observations of
S. A. Vitkalov, H. Zheng, K. M. Mertes, M. P. Sarachik, and T. M. Klapwijk, *Phys. Rev. Lett.* **8**7, 086401 (2001).

S. Sachdev, *Pramana*. **58**, 285 (2002) (cond-mat/0109309).
N. Read, S. Sachdev, and J. Ye, *Phys. Rev.* B **52**, 384 (1995);

B. *S=0* bosons

Superfluid-insulator transition of ^{87}Rb atoms in a magnetic trap and an optical lattice potential



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Related earlier work by C. Orzel, A.K. Tuchman, M. L. Fenselau, M. Yasuda, and M. A. Kasevich, *Science* **291**, 2386 (2001).

Detection method

Trap is released and atoms expand to a distance far larger than original trap dimension

$$\psi(\boldsymbol{R},T) = \exp\left(i\frac{m\boldsymbol{R}^2}{2\hbar T}\right)\psi(\boldsymbol{0},0) \approx \exp\left(i\frac{m\boldsymbol{R}_0^2}{2\hbar T} + i\frac{m\boldsymbol{R}_0\cdot\boldsymbol{r}}{\hbar T}\right)\psi(\boldsymbol{0},0)$$

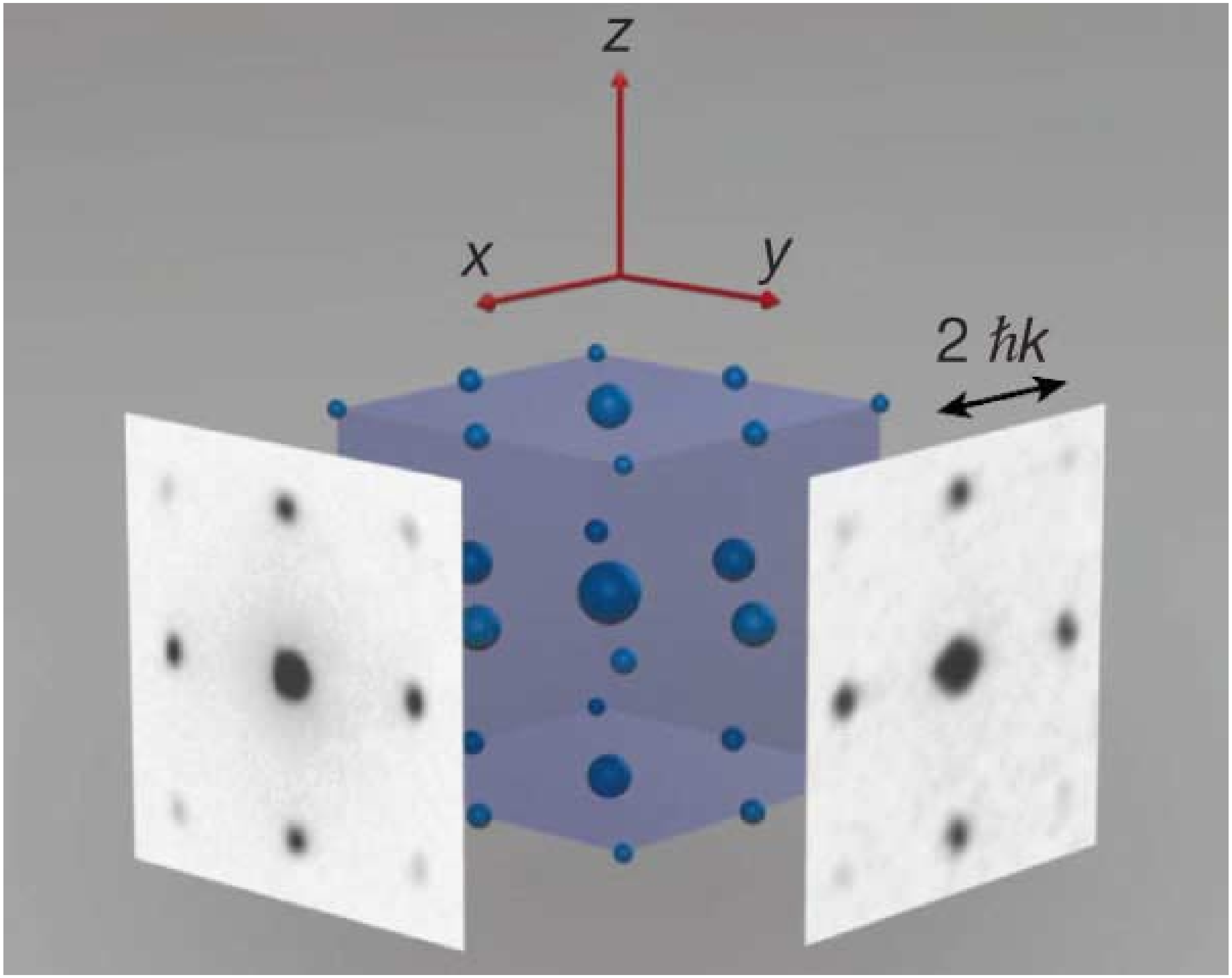
where $\boldsymbol{R} = \boldsymbol{R}_0 + \boldsymbol{r}$, with $\boldsymbol{R}_0$ = the expansion distance, and $\boldsymbol{r}$ = position within trap

In tight-binding model of lattice bosons b_i ,

detection probability $\propto \sum_{i,j}\left\langle b_i^\dagger b_j\right\rangle \exp\left(i\boldsymbol{q}\cdot\left(\boldsymbol{r}_i - \boldsymbol{r}_j\right)\right)$ with $\boldsymbol{q} = \dfrac{m\boldsymbol{R}_0}{\hbar T}$

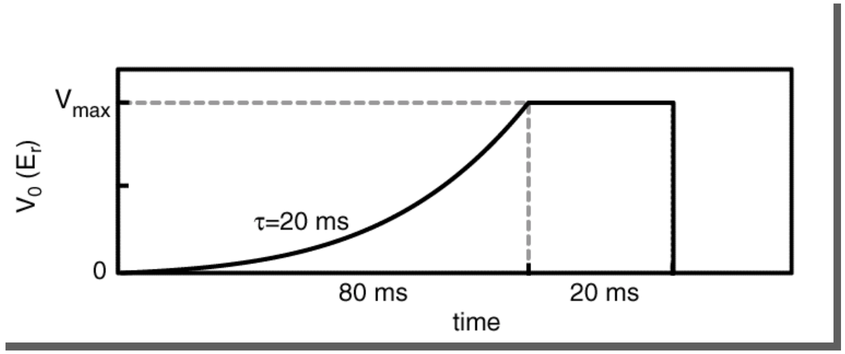
Measurement of momentum distribution function

Superfluid state

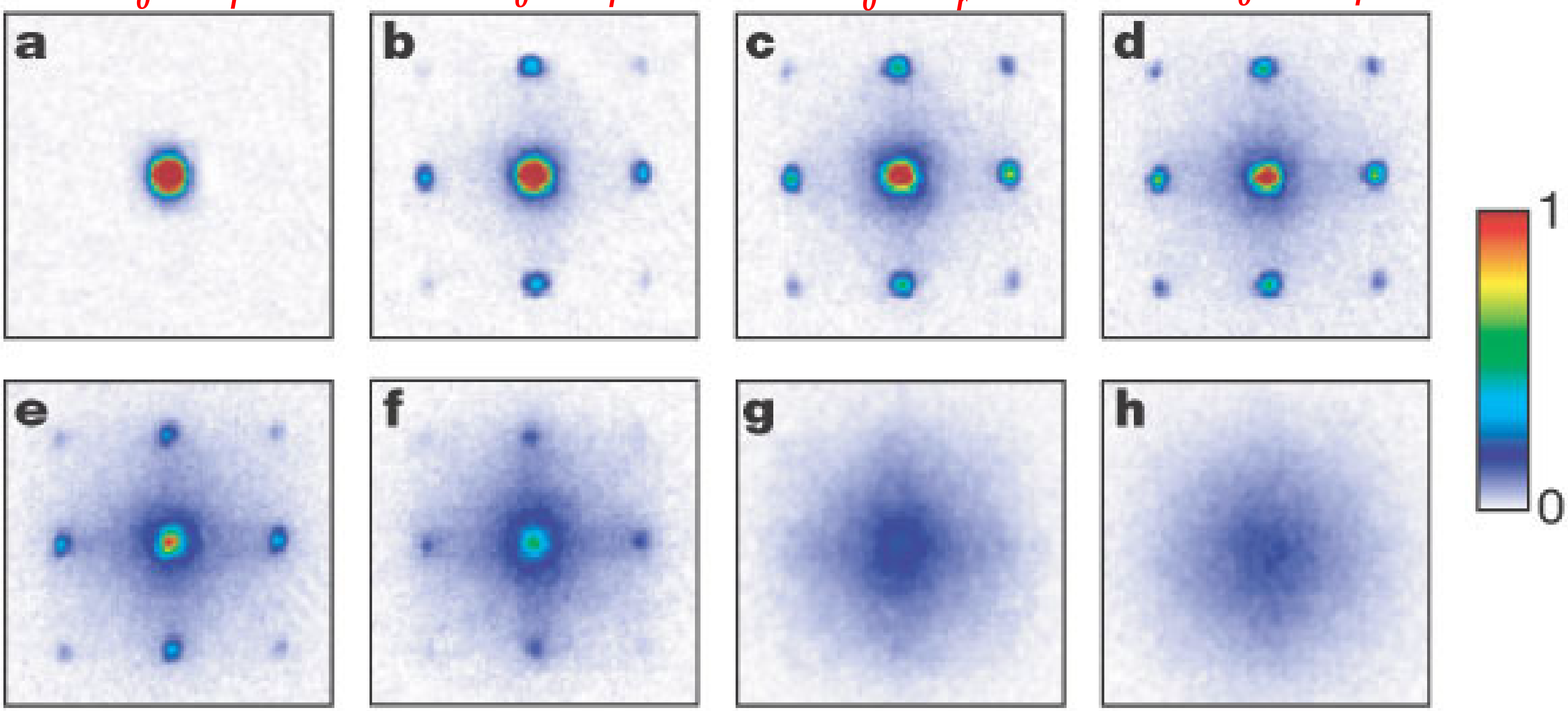


Schematic three-dimensional interference pattern with measured absorption images taken along two orthogonal directions. The absorption images were obtained after ballistic expansion from a lattice with a potential depth of $V_0 = 10\ E_r$ and a time of flight of 15 ms.

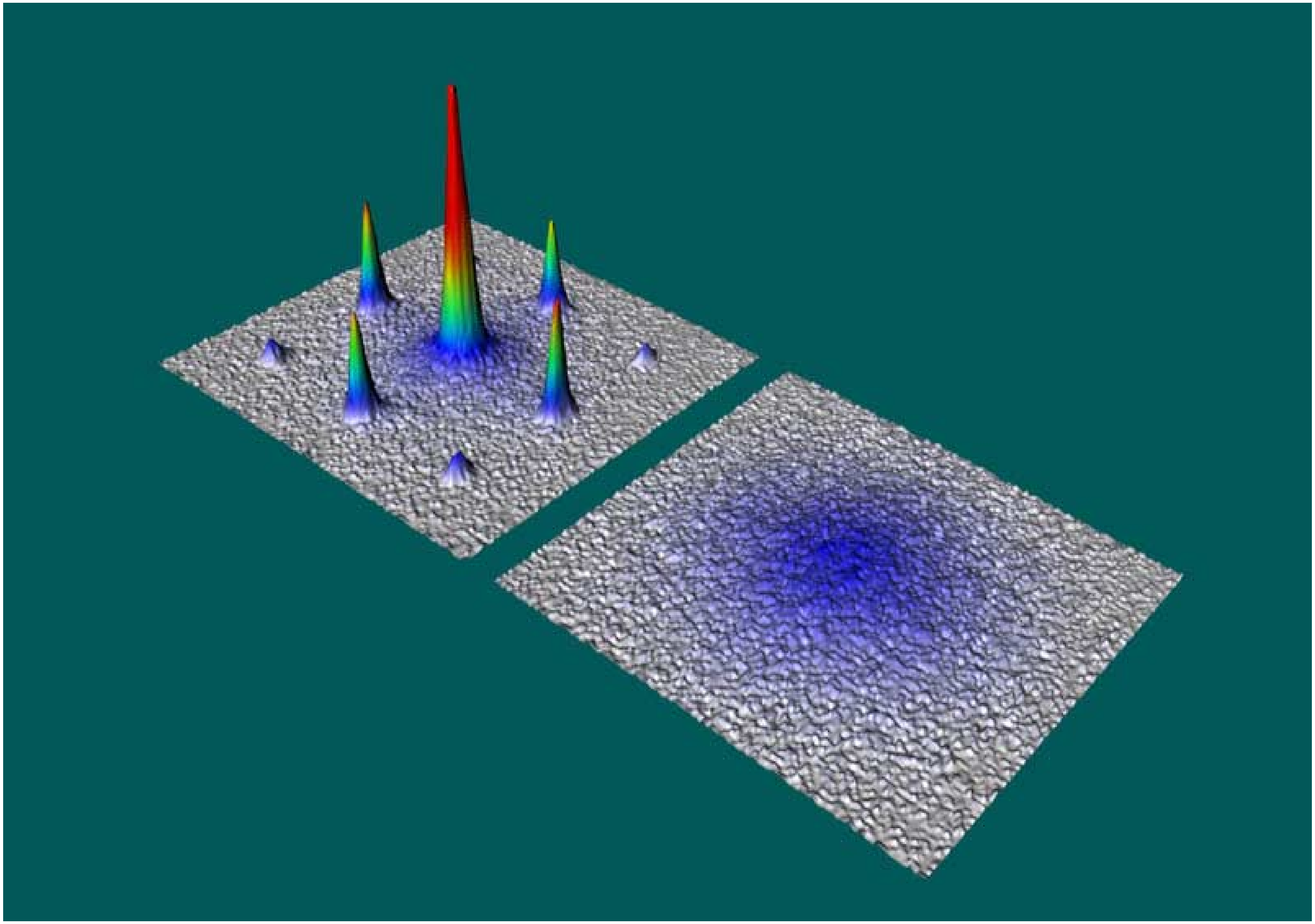
Superfluid-insulator transition



$V_0=0E_r$ $V_0=3E_r$ $V_0=7E_r$ $V_0=10E_r$

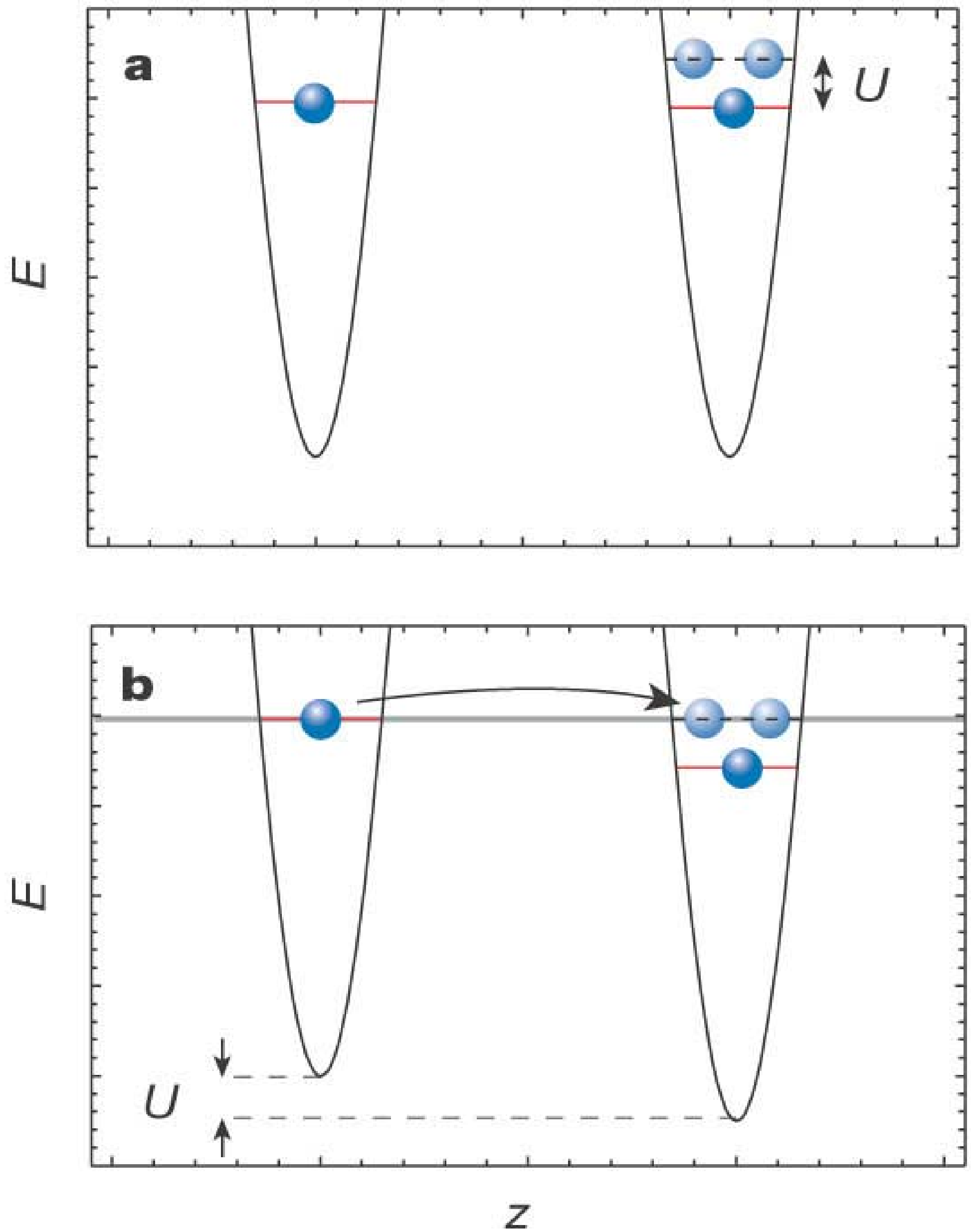


$V_0=13E_r$ $V_0=14E_r$ $V_0=16E_r$ $V_0=20E_r$

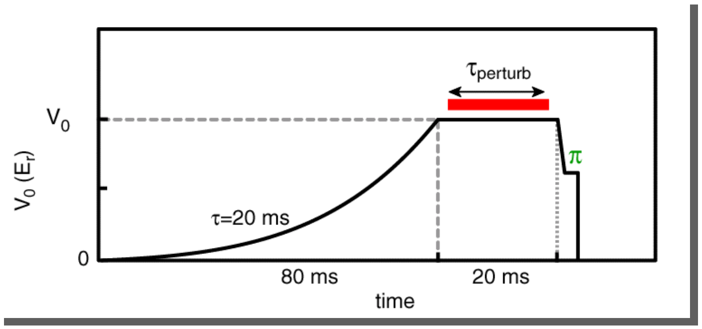


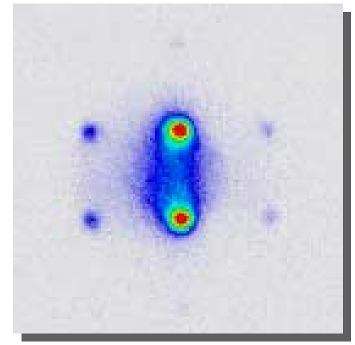
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

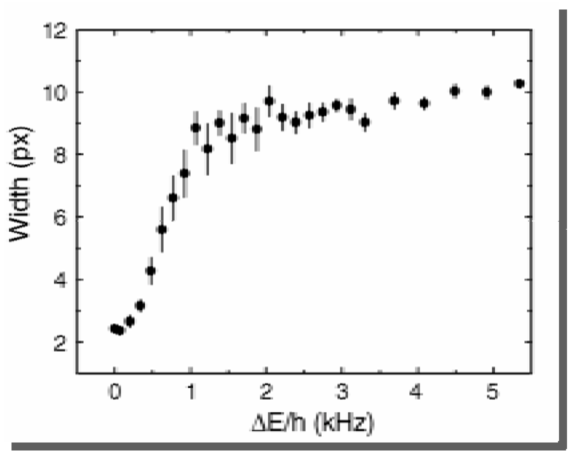
Applying an "electric" field to the Mott insulator



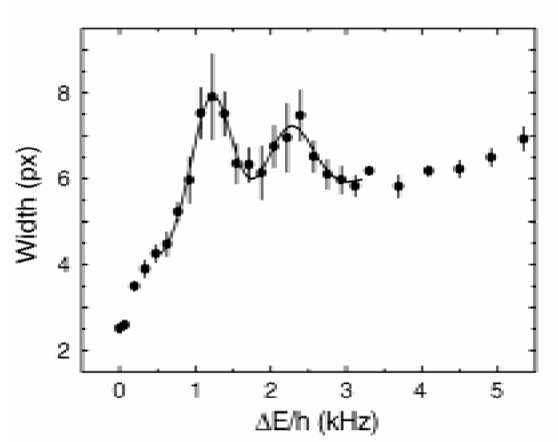
Coupled quantum dots !





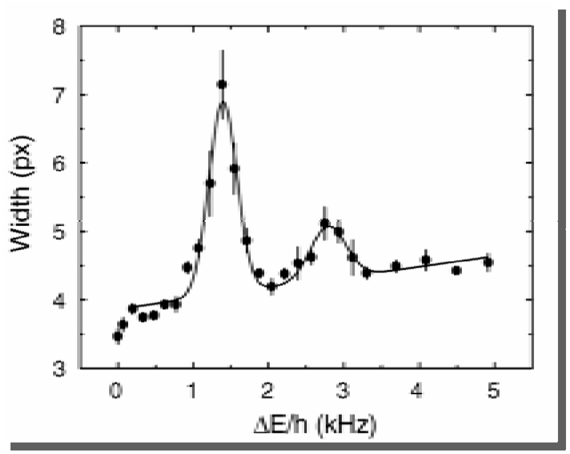


V_0=10 E_{recoil} $\tau_{perturb}$ = 2 ms

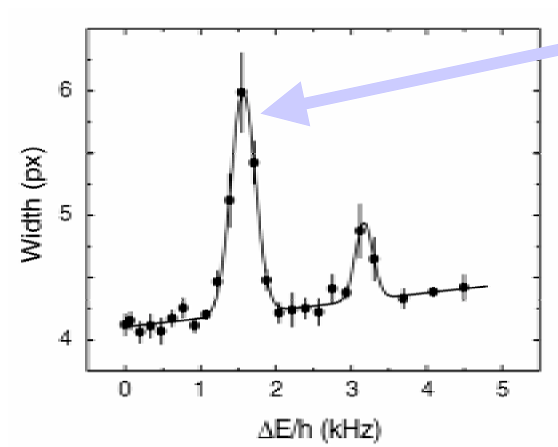


V_0= 13 E_{recoil} $\tau_{perturb}$ = 4 ms

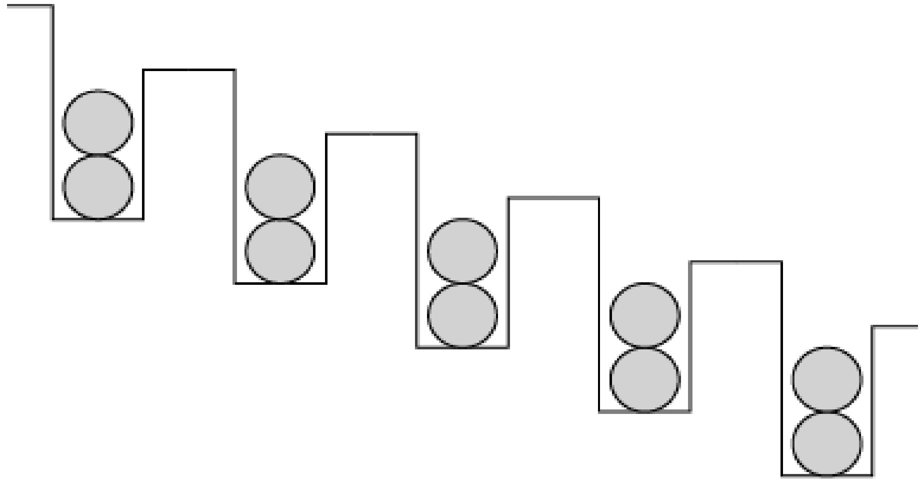
What is the quantum state here ?



V_0= 16 E_{recoil} $\tau_{perturb}$ = 9 ms



V_0= 20 E_{recoil} $\tau_{perturb}$ = 20 ms

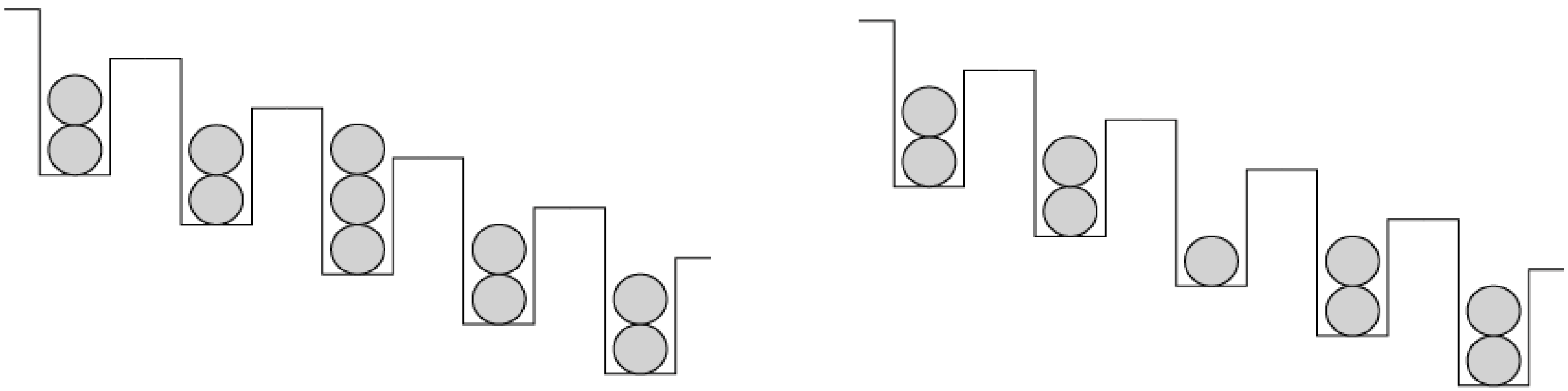


$$H = -w\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \sum_i \boldsymbol{E}\cdot\boldsymbol{r}_i n_i$$

$$n_i = b_i^\dagger b_i$$

$$\left|U - E\right|, w \ll E, U$$

Describe spectrum in subspace of states resonantly coupled to the Mott insulator



Effective Hamiltonian for a quasiparticle in one dimension (similar for a quasihole):

$$H_{\text{eff}} = -\sum_j \left[3w\left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) + Ejb_j^\dagger b_j \right]$$

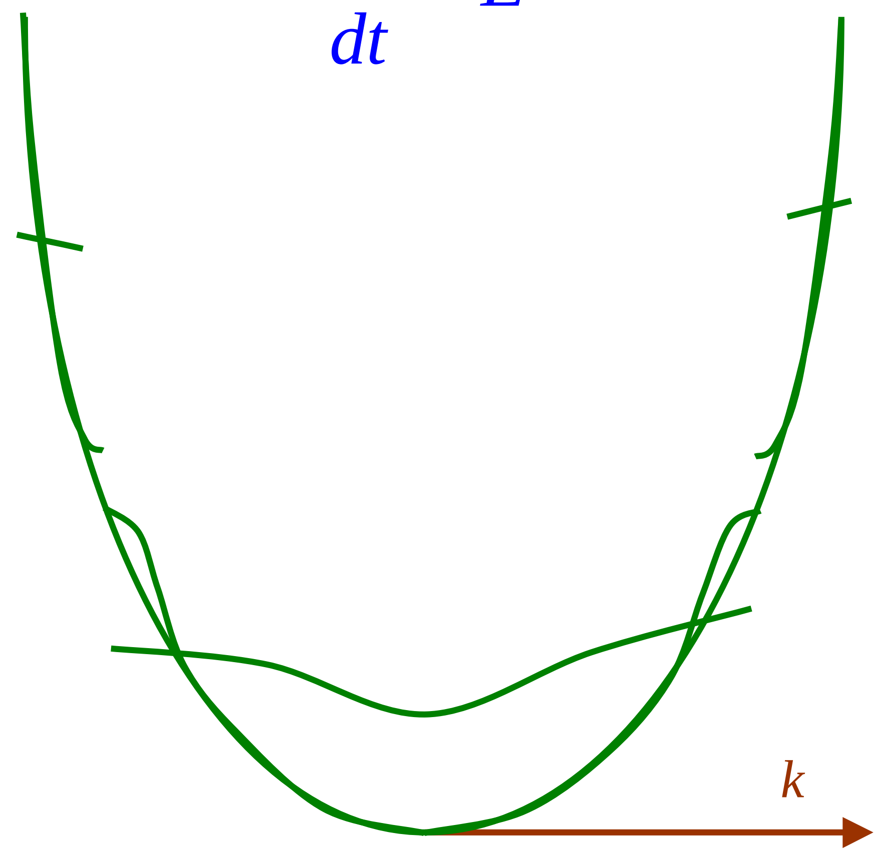
Exact eigenvalues $\varepsilon_m = Em \quad ; \quad m = -\infty \cdots \infty$

Exact eigenvectors $\psi_m(j) = J_{j-m}(6w/E)$

All charged excitations are strongly localized in the plane perpendicular electric field.
Wavefunction is periodic in time, with period *h/E* (Bloch oscillations)
Quasiparticles and quasiholes are not accelerated out to infinity

Semiclassical picture

$$\frac{dk}{dt} = E$$

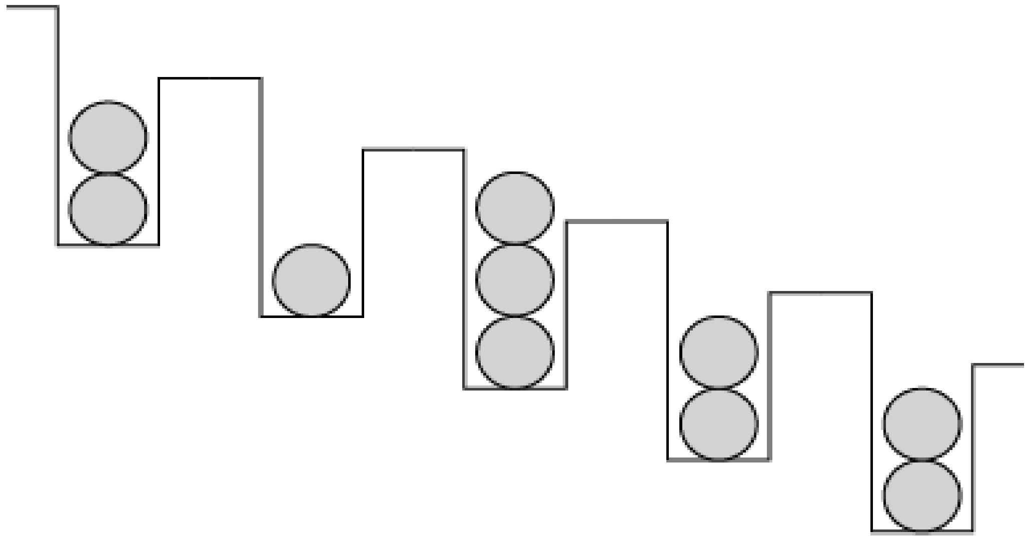


Free particle is accelerated to infinity

In a weak periodic potential, escape to infinity occurs via Zener tunneling across band gaps

Experimentalists' dream: Strong potential in which there is negligible Zener tunneling, and the particle undergoes Bloch oscillations

Important neutral excitations (in one dimension)

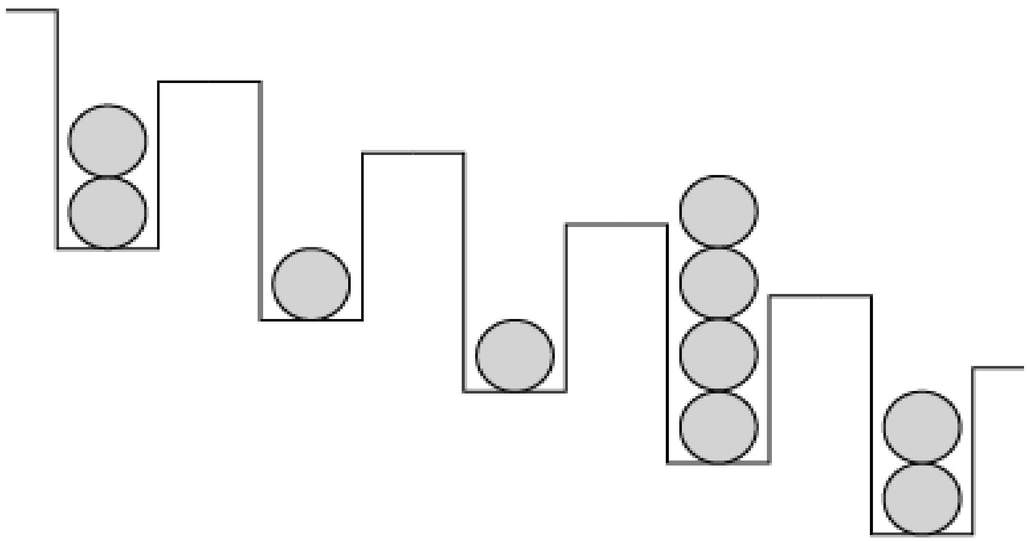


Nearest-neighbor dipole

Creating dipoles on nearest-neighbor links creates a state with relative energy *U-2E ;* such states are *not* part of the resonant manifold

Dipoles can appear resonantly on non-nearest-neighbor links. Within resonant manifold, dipoles have infinite on-link and nearest-link repulsion

A non-dipole state



State has energy *3(U-E)* but is connected to resonant state by a matrix element smaller than w^2/U

State is not part of resonant manifold

Hamiltonian for resonant dipole states (in one dimension)

$d_\ell^\dagger \Rightarrow$ Creates dipole on link ℓ

$$H_d = -\sqrt{6}w\sum_\ell \left(d_\ell^\dagger + d_\ell\right) + (U-E)\sum_\ell d_\ell^\dagger d_\ell$$

Constraints: $d_\ell^\dagger d_\ell \leq 1$; $d_{\ell+1}^\dagger d_{\ell+1} d_\ell^\dagger d_\ell = 0$

Determine phase diagram of H_d as a function of $(U\text{-}E)/w$

Note: there is no explicit dipole hopping term.

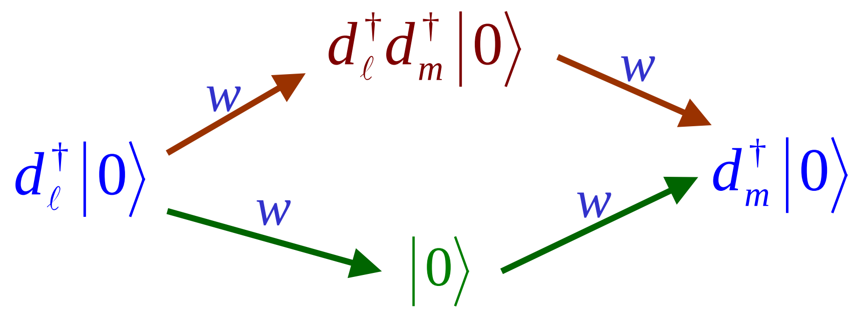
However, dipole hopping is generated by the interplay of terms in H_d and the constraints.

Weak electric fields: $(U\text{-}E) \gg w$

Ground state is dipole vacuum (Mott insulator) $|0\rangle$

First excited levels: single dipole states $d_\ell^\dagger |0\rangle$

Effective hopping between dipole states



If both processes are permitted, they exactly cancel each other.
The top processes is blocked when ℓ, m are nearest neighbors

$\Rightarrow$ A nearest-neighbor dipole hopping term $\sim \dfrac{w^2}{U-E}$ is generated

Strong electric fields: $(E-U) \gg w$

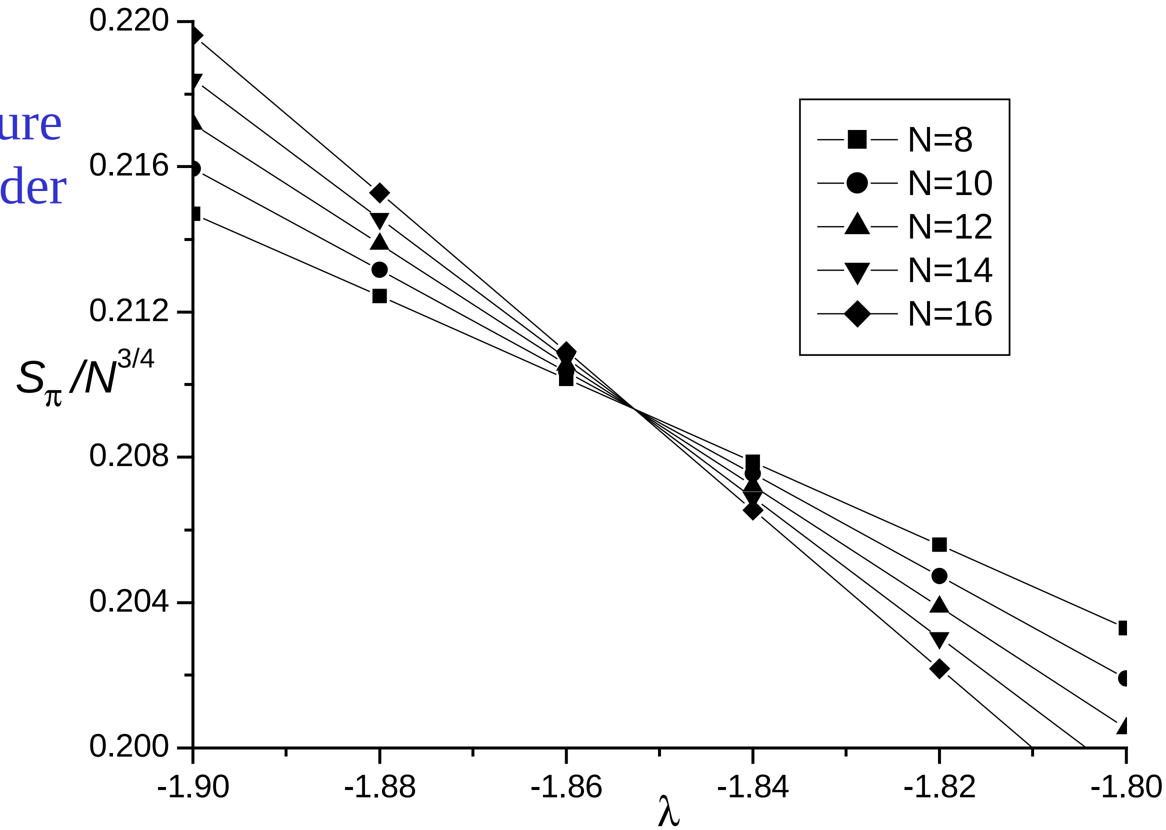
Ground state has maximal dipole number.

Two-fold degeneracy associated with Ising density wave order:

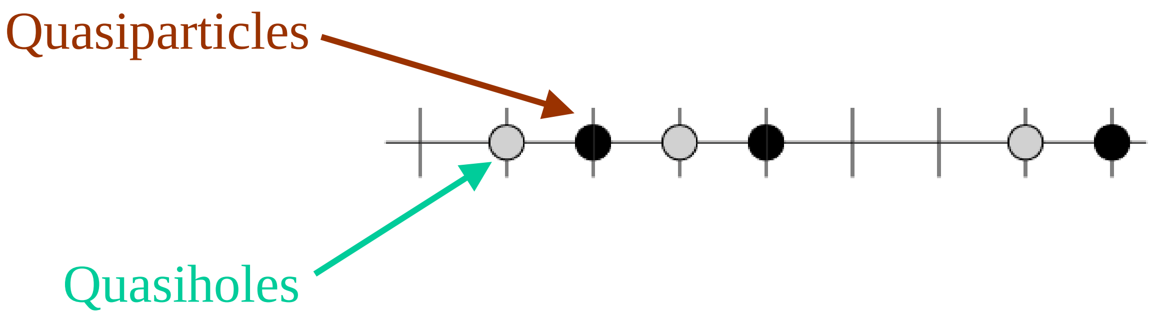
$$\cdots d_1^\dagger d_3^\dagger d_5^\dagger d_7^\dagger d_9^\dagger d_{11}^\dagger \cdots |0\rangle \quad or \quad \cdots d_2^\dagger d_4^\dagger d_6^\dagger d_8^\dagger d_{10}^\dagger d_{12}^\dagger \cdots |0\rangle$$

Ising quantum critical point at $E-U=1.08\ w$

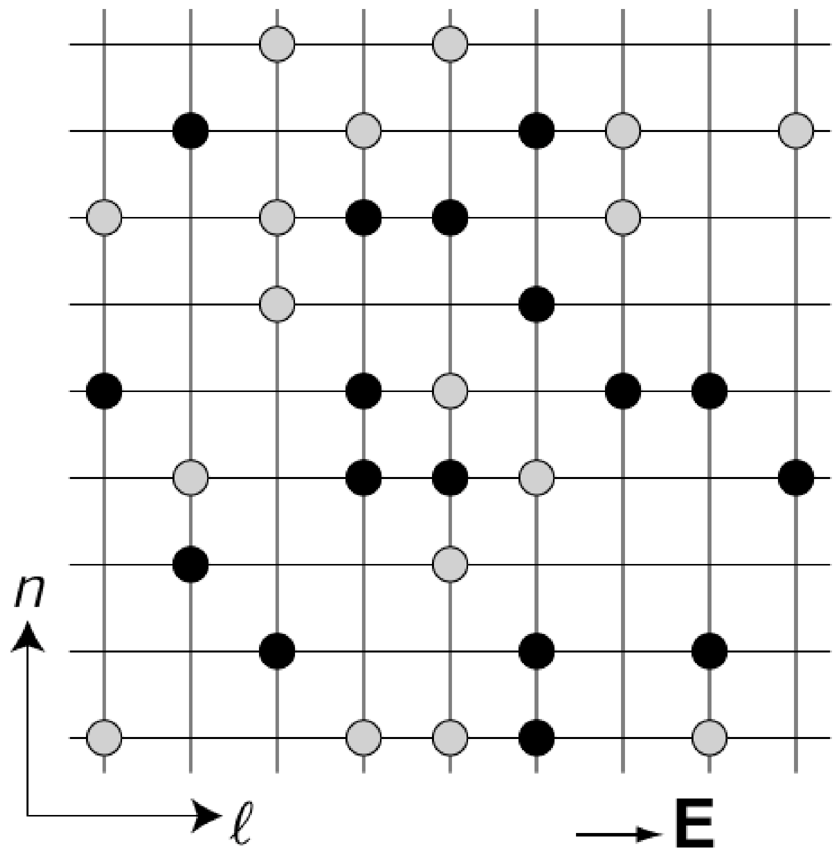
Equal-time structure factor for Ising order parameter



Resonant states in higher dimensions



Dipole states in one dimension



Quasiparticles and quasiholes can move resonantly in the transverse directions in higher dimensions.

Constraint: number of quasiparticles in any column = number of quasiholes in column to its left.

Hamiltonian for resonant states in higher dimensions

$p^{\dagger}_{\ell,n} \Rightarrow$ Creates quasiparticle in column ℓ and transverse position n

$h^{\dagger}_{\ell,n} \Rightarrow$ Creates quasihole in column ℓ and transverse position n

$$H_{ph} = -\sqrt{6}w\sum_{\ell,n}\left(p_{\ell+1,n}h_{\ell,n} + p^{\dagger}_{\ell+1,n}h^{\dagger}_{\ell,n} \right)$$

$$+\frac{(U-E)}{2}\sum_{\ell,n}\left(p^{\dagger}_{\ell,n}p_{\ell,n} + h^{\dagger}_{\ell,n}h_{\ell,n} \right)$$

Terms as in one dimension

$$-w\sum_{\ell,\langle nm\rangle}\left(2h^{\dagger}_{\ell,n}h_{\ell,m} + 3p^{\dagger}_{\ell,n}p_{\ell,m} + \text{H.c.}\right)$$

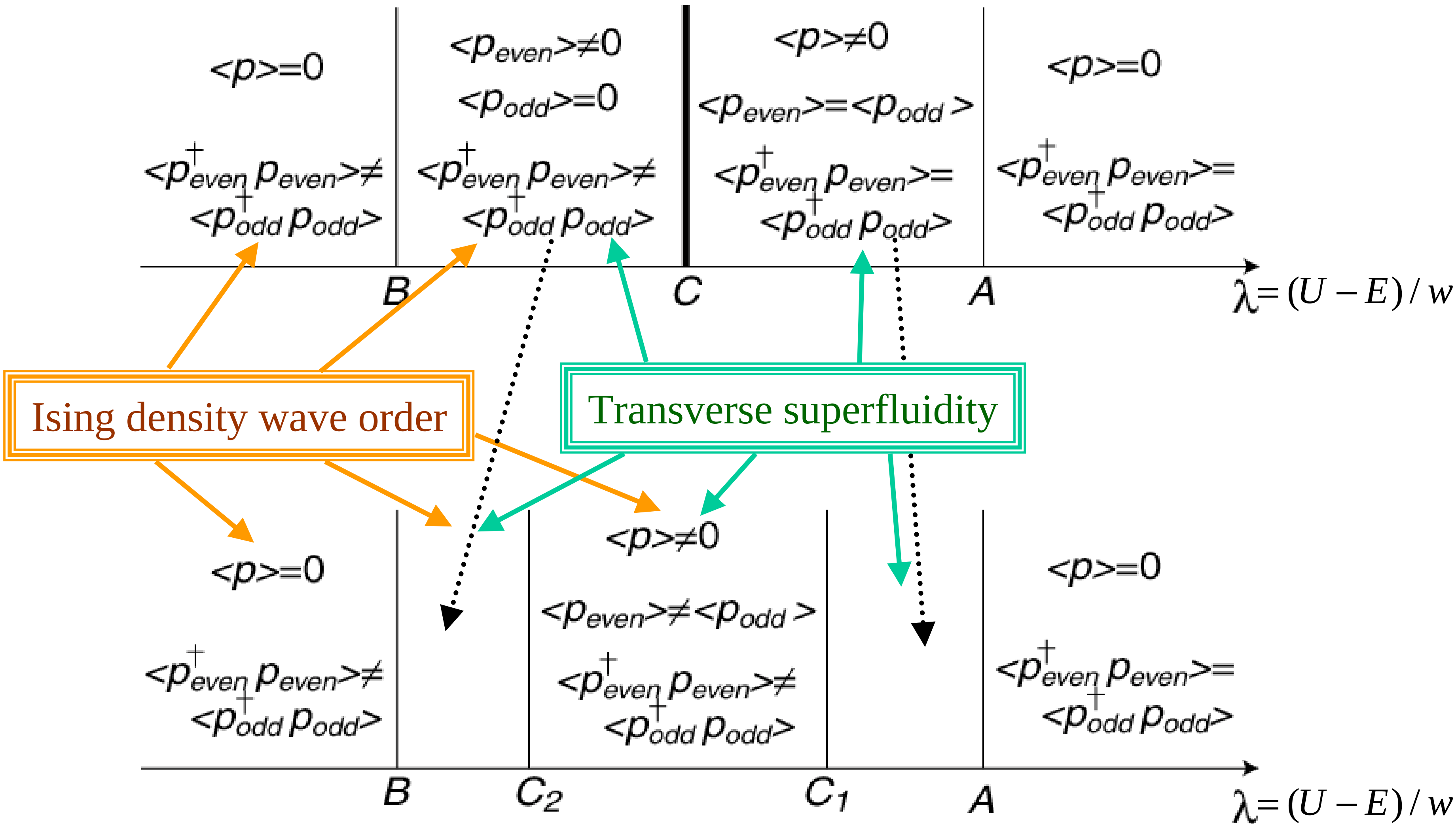
Transverse hopping

$$p^{\dagger}_{\ell,n}p_{\ell,n} \leq 1 \quad ; \quad h^{\dagger}_{\ell,n}h_{\ell,n} \leq 1 \quad ; \quad p^{\dagger}_{\ell,n}p_{\ell,n}h^{\dagger}_{\ell,n}h_{\ell,n} = 0$$

Constraints

New possibility: superfluidity in transverse direction (a smectic)

Possible phase diagrams in higher dimensions



Implications for experiments

•Observed resonant response is due to gapless spectrum near quantum critical point(s).

•Transverse superfluidity (smectic order) can be detected by looking for "Bragg lines" in momentum distribution function---bosons are phase coherent in the transverse direction.

•Present experiments are insensitive to Ising density wave order. Future experiments could introduce a phase-locked subharmonic standing wave at half the wave vector of the optical lattice---this would couple linearly to the Ising order parameter. The AC stark shift of the atomic hyperfine levels would differ between adjacent sites. The relative strengths of the split hyperfine absorption lines would then be a measure of the Ising order parameter.