

# Gauge theory of optimal doping criticality in the cuprates

EPIQS Investigator Symposium  
Monterey, CA  
October 13, 2018

Subir Sachdev and Mathias Scheurer



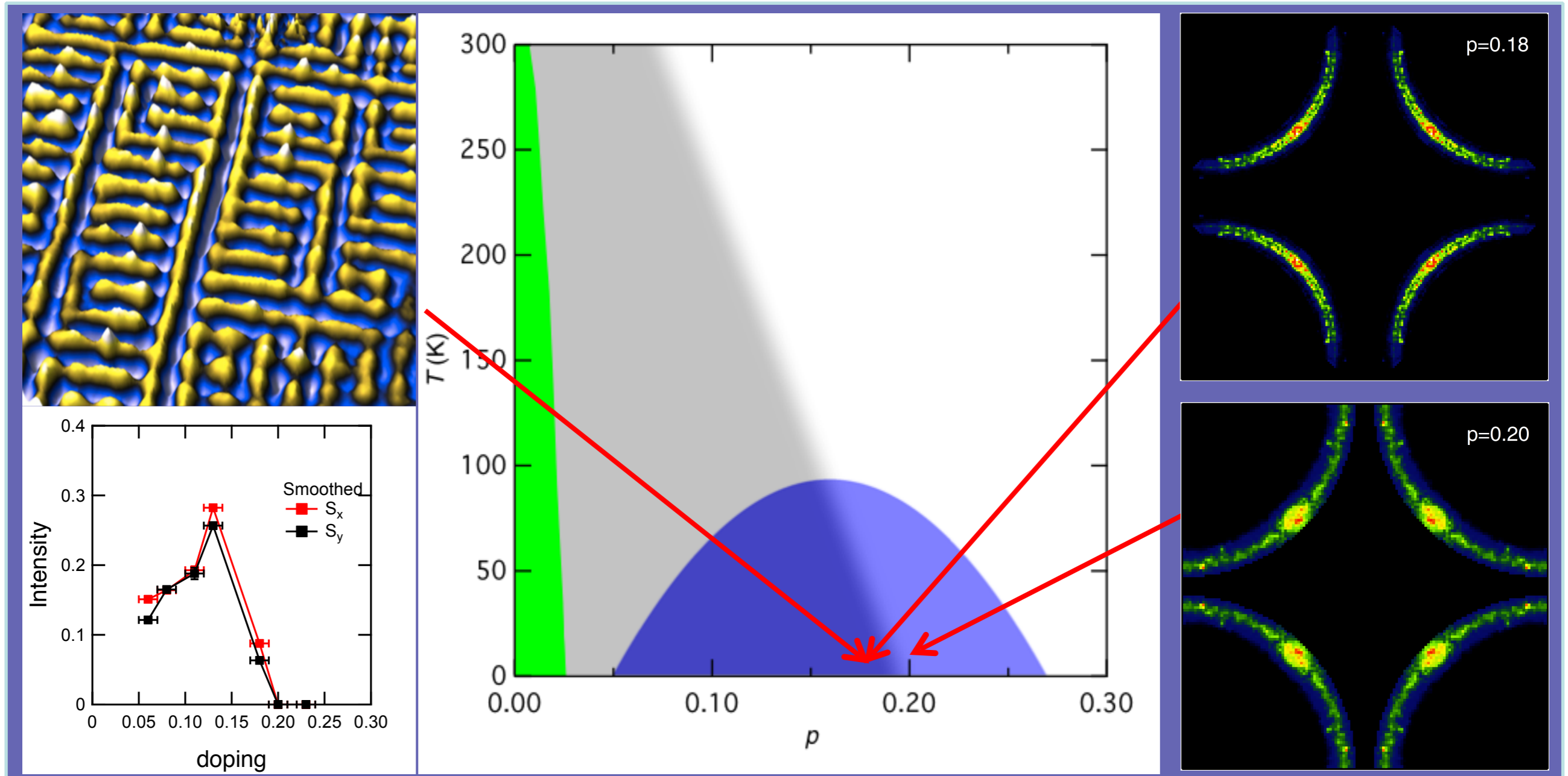
Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)

# Fermi Surface and Pseudogap Evolution in a Cuprate Superconductor

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, *Science* **344**, 608 (2014)

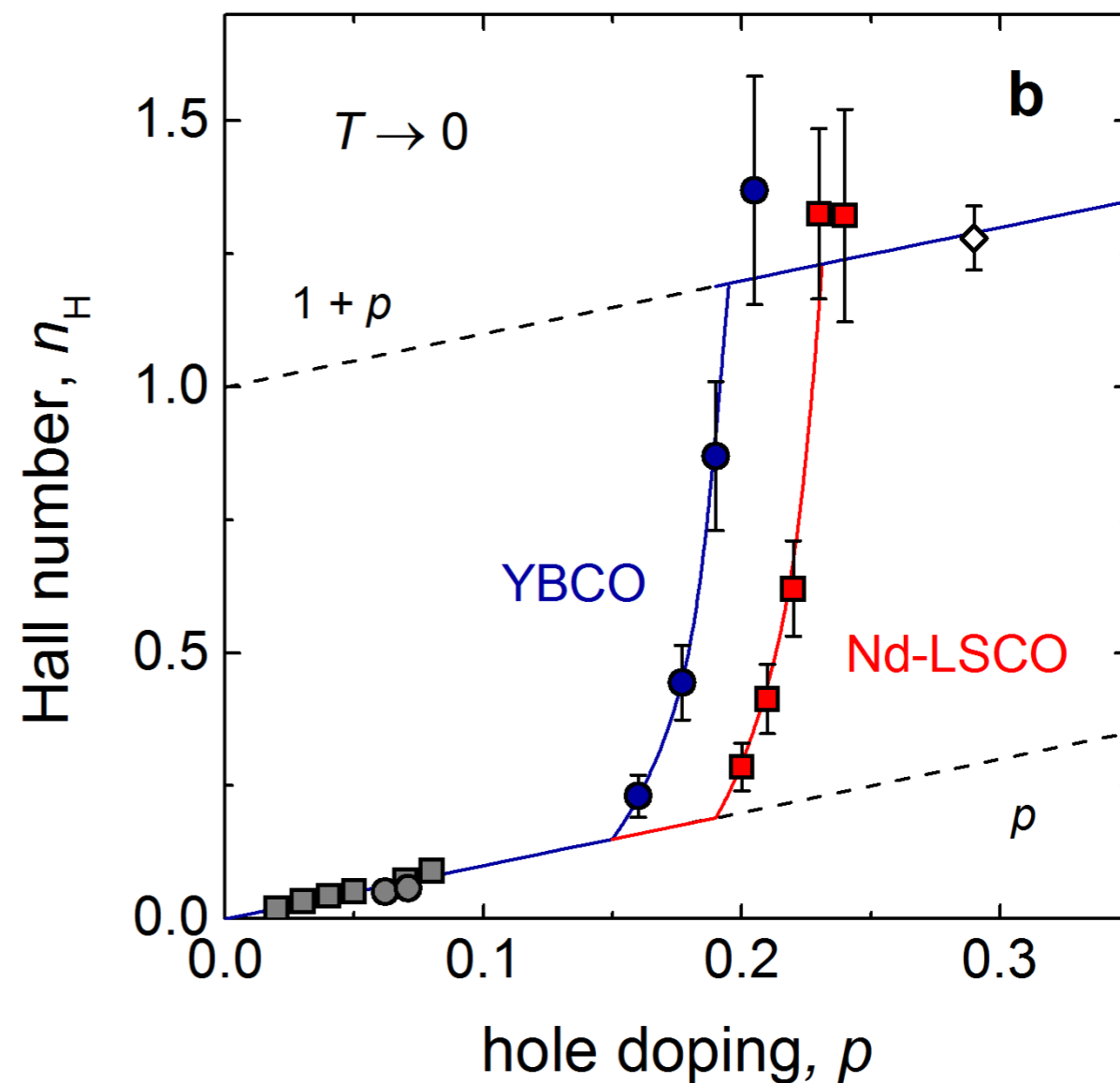
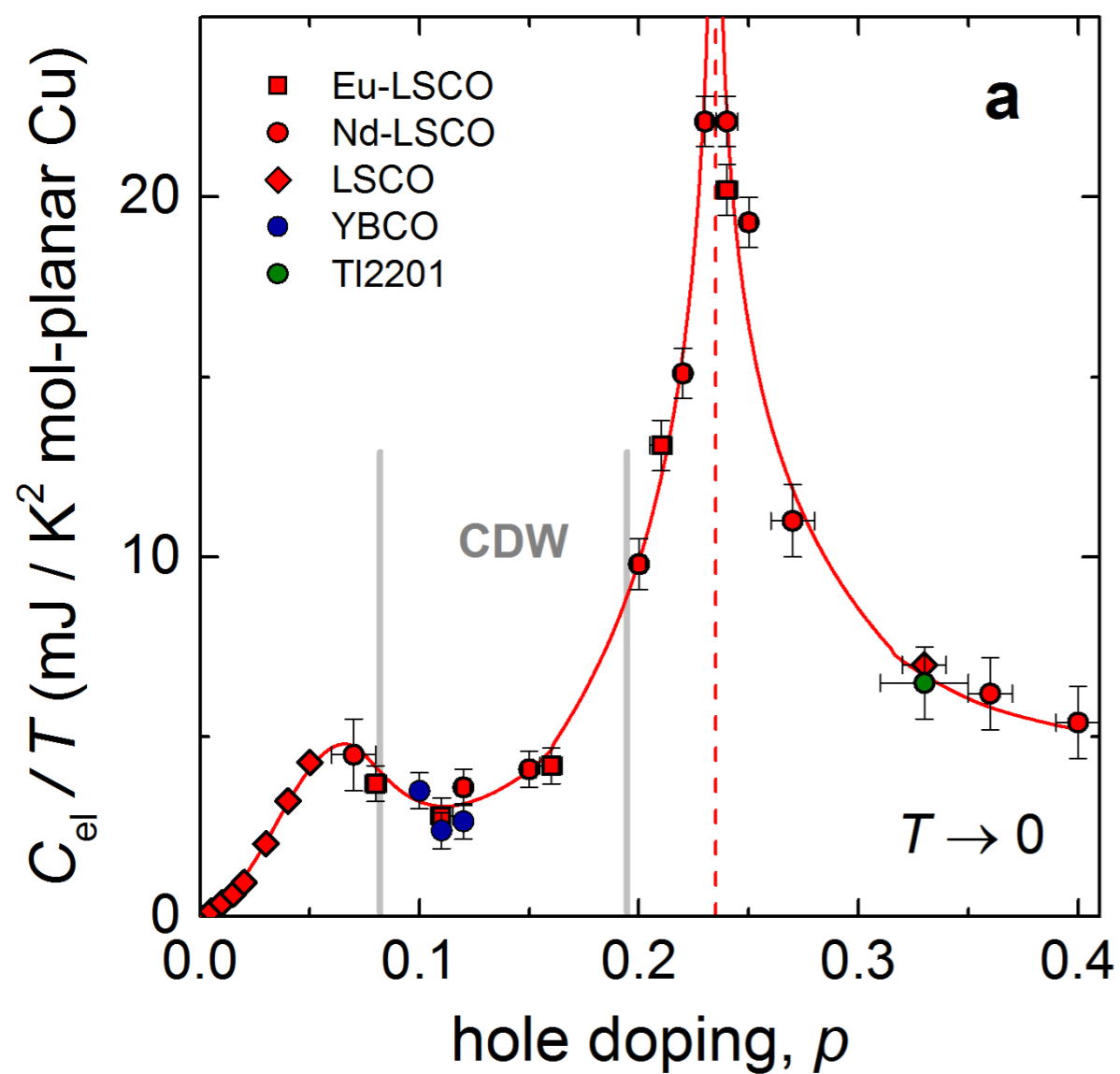
# Simultaneous Transitions in Cuprate Momentum-Space Topology and Electronic Symmetry Breaking

K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, *Science* **344**, 612 (2014)



# The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507



# Theoretical framework

We can (exactly) transform the Hubbard model to the “spin-fermion” model:

**electrons**  $c_{i\alpha}$  on the square lattice with dispersion

$$\mathcal{H}_c = - \sum_{i,\rho} t_\rho \left( c_{i,\alpha}^\dagger c_{i+\mathbf{v}_\rho,\alpha} + c_{i+\mathbf{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}}$$

are coupled to a magnetic moment order parameter  $\Phi^p(i)$ ,  $p = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \Phi^p(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^p c_{i,\beta} + V_\Phi$$

# Theoretical framework

For (fluctuating) SDW SRO, we transform to a **rotating reference frame** using the SU(2) rotation  $R_i$

$$\begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} = R_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix},$$

in terms of fermionic “chargons”  $\psi_s$  and a **Higgs field**  $H^a(i)$

$$\sigma^p \Phi^p(i) = R_i \sigma^a H^a(i) R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

# Theoretical framework

We obtain different numbers of adjoint Higgs scalars,  $N_h$ , depending upon the spatial dependence of the local spin correlations:

Neel correlations:  $N_h = 1$ ,

$$\mathbf{K} = (\pi, \pi),$$

$$H^a(i) = H_1^a(\mathbf{r}) e^{i\mathbf{K} \cdot \mathbf{r}_i}$$

Canted antiferromagnetic correlations:  $N_h = 2$ ,

$$\mathbf{K} = (\pi, \pi),$$

$$H^a(i) = H_1^a(\mathbf{r}) e^{i\mathbf{K} \cdot \mathbf{r}_i} + H_2^a(\mathbf{r})$$

Unidirectional incommensurate correlations:  $N_h = 2$ ,

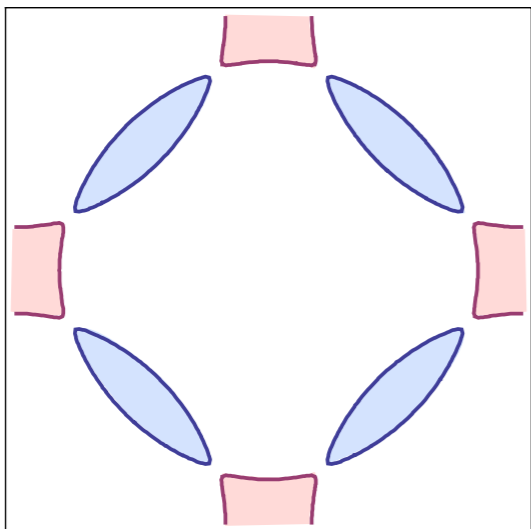
$$\mathbf{K} = (\pi, \pi - \delta),$$

$$H^a(i) = \text{Re} \left\{ [H_1^a(\mathbf{r}) + iH_2^a(\mathbf{r})] e^{i\mathbf{K} \cdot \mathbf{r}_i} \right\}$$

Bidirectional incommensurate correlations:  $N_h = 4$ ,

$$\mathbf{K}_y = (\pi, \pi - \delta), \quad \mathbf{K}_x = (\pi - \delta, \pi),$$

$$H^a(i) = \text{Re} \left\{ [H_1^a(\mathbf{r}) + iH_2^a(\mathbf{r})] e^{i\mathbf{K}_x \cdot \mathbf{r}_i} + [H_3^a(\mathbf{r}) + iH_4^a(\mathbf{r})] e^{i\mathbf{K}_y \cdot \mathbf{r}_i} \right\}$$



SDW SRO

# Higgs phase

Fractionalized Fermi Liquid (FL\*)

Emergent deconfined  $Z_2$  gauge field/  
CDW/Ising-nematic

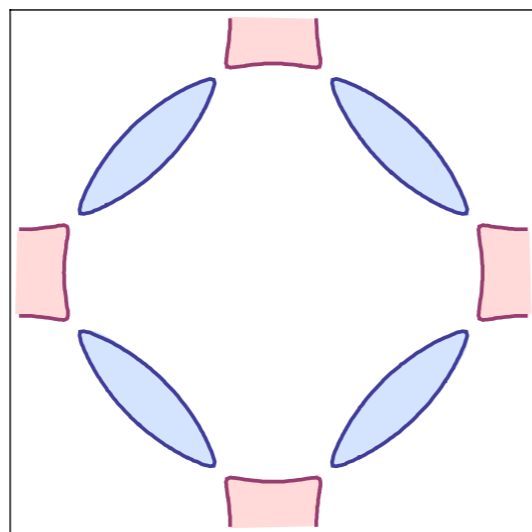
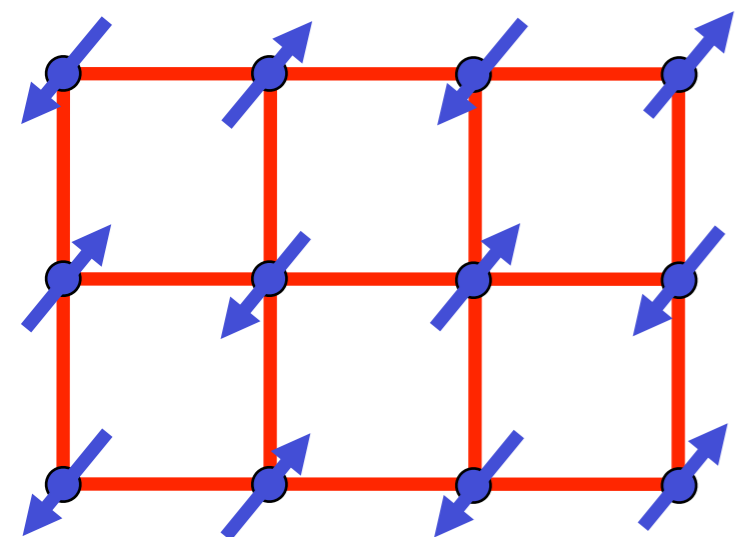
$$\langle H^a \rangle \neq 0 \quad \langle \Phi^p \rangle = 0$$

$$\langle R \rangle = 0$$

SU(2) gauge theory with  
 $N_f = 1, 2, 4$  adjoint Higgs fields

$$\langle \Phi^p \rangle \neq 0$$

$$\langle H^a \rangle \neq 0, \quad \langle R \rangle \neq 0$$



SDW LRO

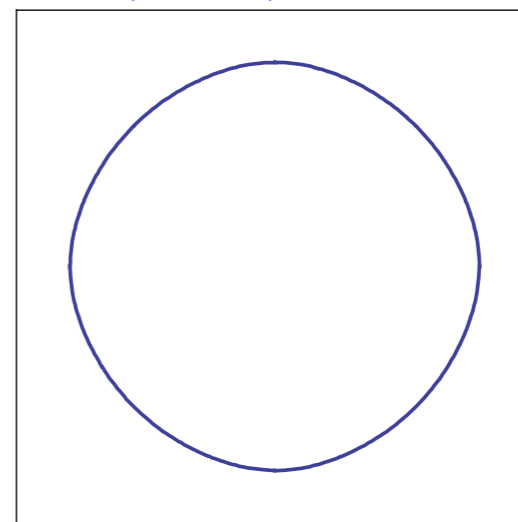
SDW SRO

# Confinement

No topological order.

$$\langle H^a \rangle = 0, \quad \langle R \rangle \neq 0$$

$$\langle \Phi^p \rangle = 0$$



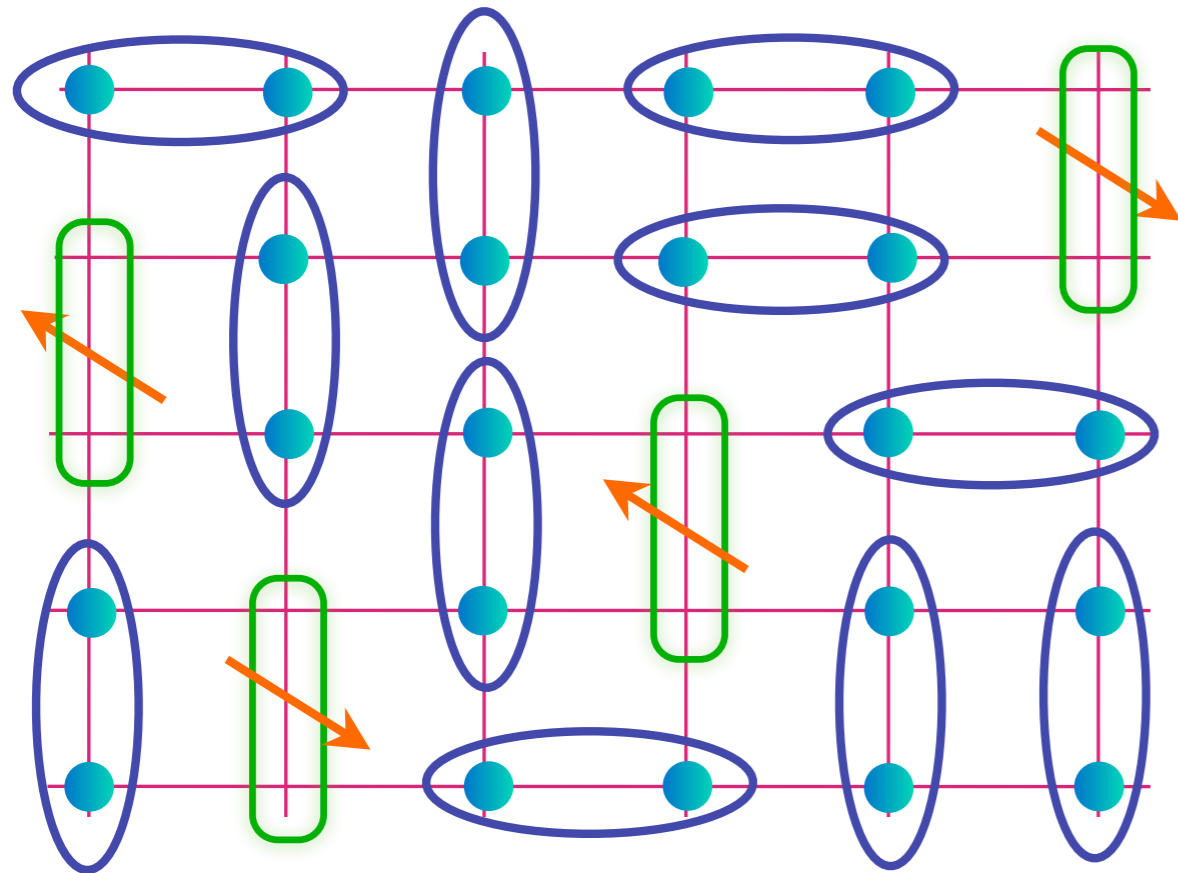
$g$

$U/t$

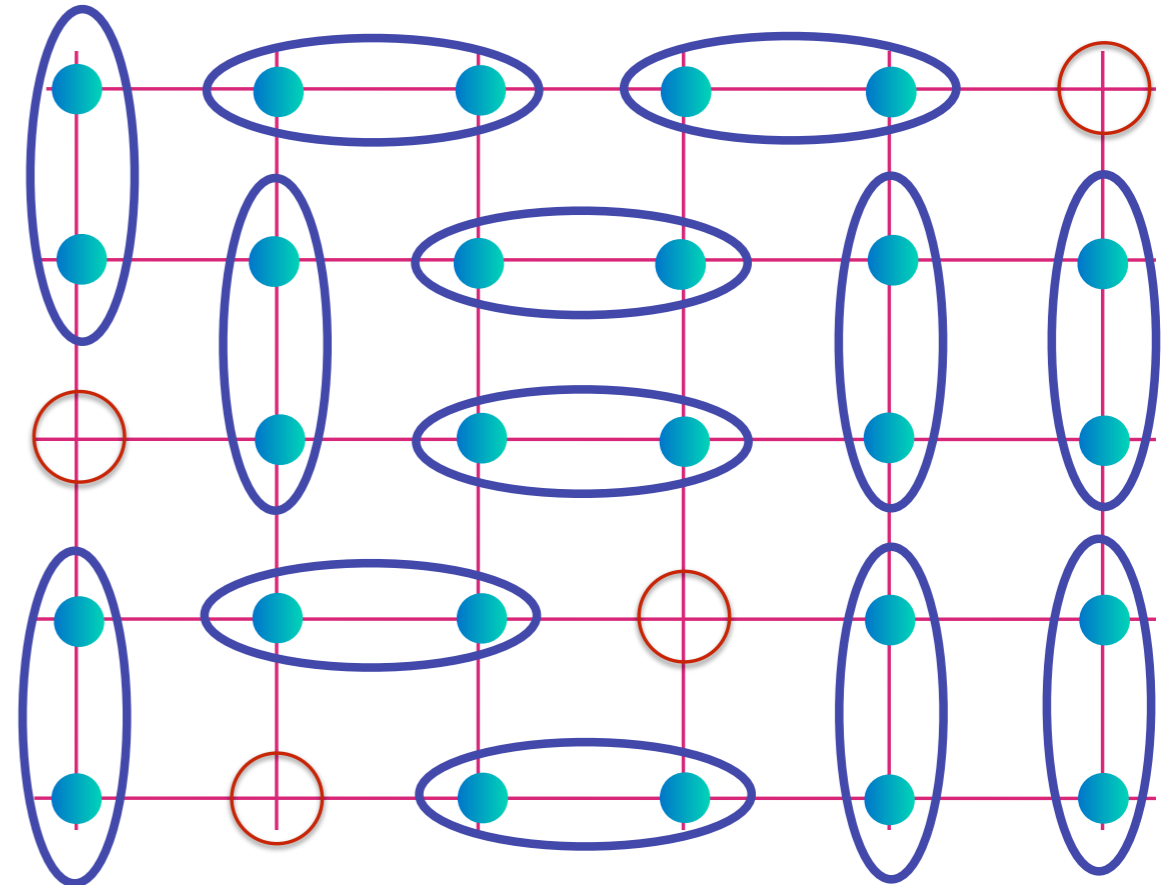
# Theory of the pseudogap phase

$$\text{Green oval with orange arrow} = (|\uparrow \circ\rangle + |\circ \uparrow\rangle) / \sqrt{2}$$

$$\text{Blue oval with two dots} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$



(a) FL\*



(b) ACL

Metals with emergent gauge fields (the blue dimers):

(a) FL\*: electron-like (b) ACL: spinless charginos  
with quasiparticles on a Fermi surface of size  $p$

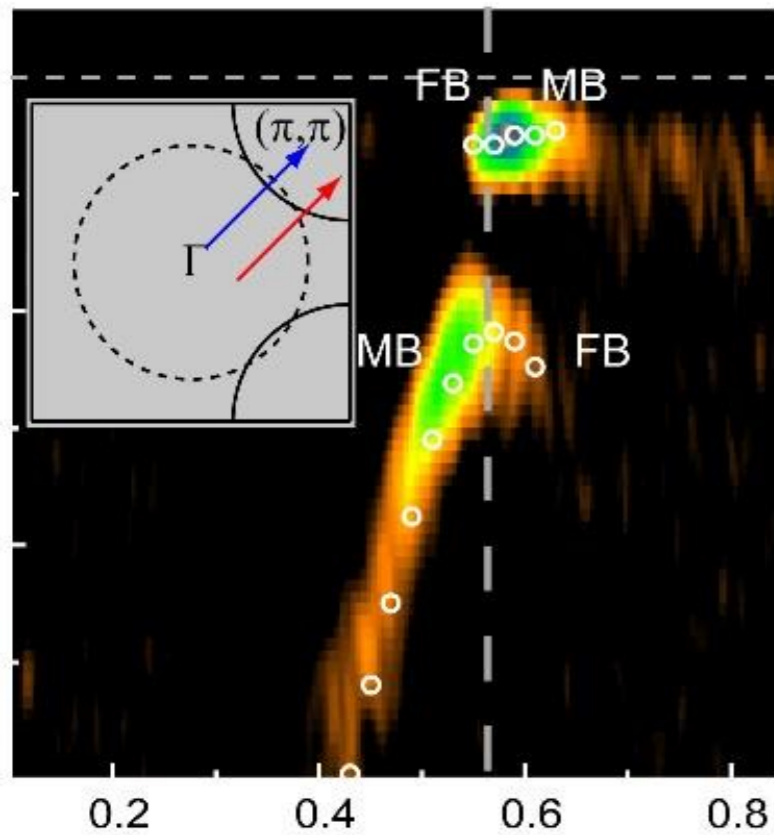


# Theory of the pseudogap phase

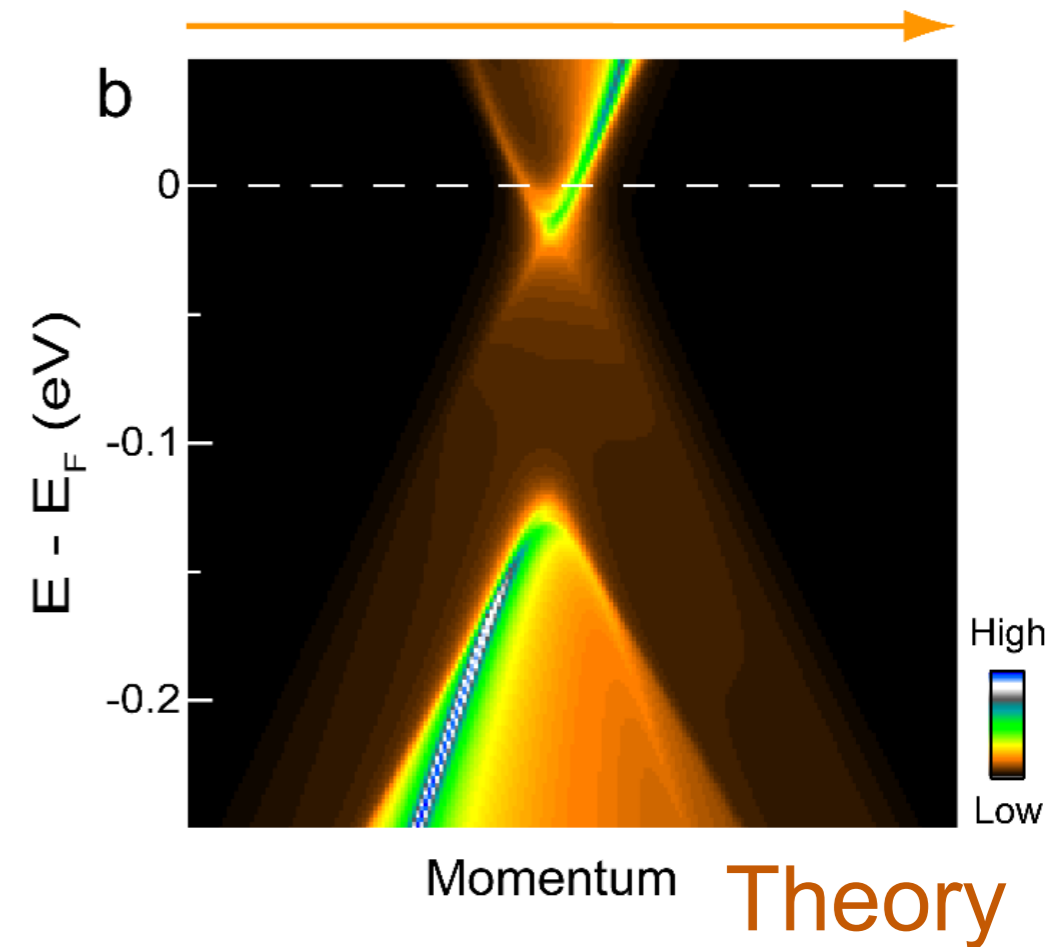
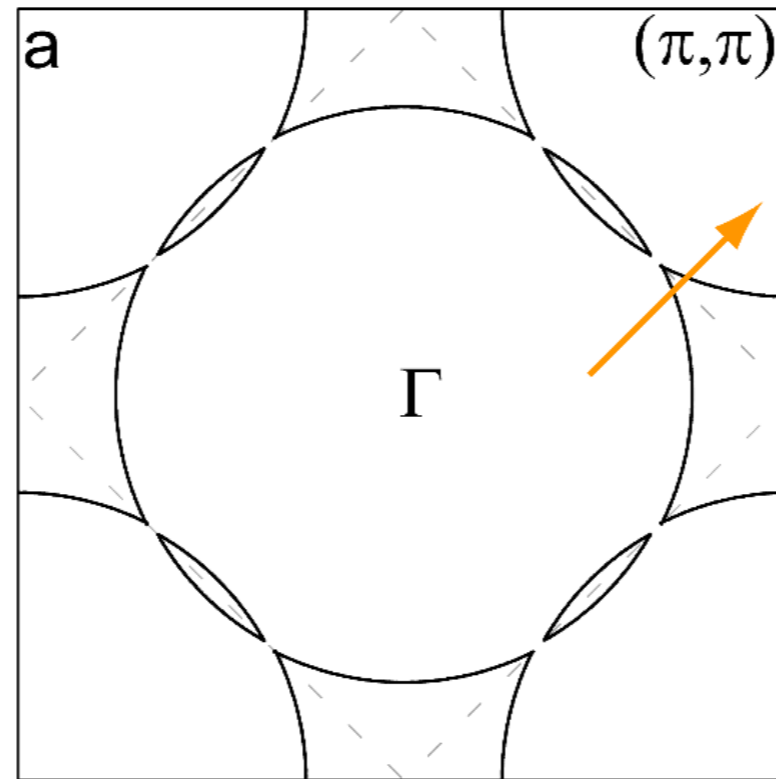
Field	Symbol	Statistics	$SU(2)_{\text{gauge}}$	$SU(2)_{\text{spin}}$	$U(1)_{\text{e.m.charge}}$
Electron	$c$	fermion	<b>1</b>	<b>2</b>	-1
AF order	$\Phi$	boson	<b>1</b>	<b>3</b>	0
Chargon	$\psi$	fermion	<b>2</b>	<b>1</b>	-1
Spinon	$R$ or $z$	boson	$\bar{\mathbf{2}}$	<b>2</b>	0
Higgs	$H$	boson	<b>3</b>	<b>1</b>	0

*SU(2) gauge theory:* fractionalize the SDW order parameter into the Higgs field ( $H$ ) and the spinons ( $R$ ); fractionalize the electron ( $c$ ) into chargons ( $\psi$ ) and spinons ( $R$ ). When the Higgs field is condensed, the  $\psi$  fermions and  $R$  bosons are deconfined particles in an algebraic charge liquid (ACL) state, but with a strong residual attractive interaction from the  $t_{ij}$ . These can bind to form Fermi surfaces of quasi-particles with the same quantum numbers as an electron in a fractionalized Fermi liquid (FL\*) state: the Fermi surfaces in the ACL/FL\* states are reconstructed by the Higgs condensate, in a manner similar to a state with long-range SDW order. In the present  $SU(2)$  gauge theory, we ignored the interactions between the  $\psi$  and  $R$  in an ACL state, and obtained results in good agreement at intermediate temperatures with cluster DMFT, and with photoemission observations on NCCO.

# Theory of the pseudogap phase



Experiment



Theory

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen,  
Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order,  
preprint

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev,  
Topological order in the pseudogap metal  
PNAS **115**, E3665 (2018)

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero,  
Pseudogap and Fermi surface topology in the two-dimensional Hubbard model  
Physical Review X **8**, 021048 (2018)

# Theory of optimal doping criticality

Field	Symbol	Statistics	$SU(2)_{\text{gauge}}$	$SU(2)_{\text{spin}}$	$U(1)_{\text{e.m.charge}}$
Electron	$c$	fermion	<b>1</b>	<b>2</b>	-1
AF order	$\Phi$	boson	<b>1</b>	<b>3</b>	0
Chargon	$\psi$	fermion	<b>2</b>	<b>1</b>	-1
Spinon	$R$ or $z$	boson	<b><math>\bar{2}</math></b>	<b>2</b>	0
Higgs	$H$	boson	<b>3</b>	<b>1</b>	0

$SU(2)$  gauge theory for quantum criticality: Assume the  $R$  spinons and  $\psi$  chargons are gapped, and all the low energy fermionic excitations are electron-like across the quantum critical point. Integrate out the spinons  $R$ , the chargons  $\psi$ , and the high energy  $c_\alpha$  to obtain effective theory for  $H_\ell^a$  and the low energy  $c_\alpha$  near the Fermi surface.

# Theory of optimal doping criticality

## SU(2) gauge theory

SU(2) gauge theory with  $N_h$  adjoint Higgs fields  $H_\ell^a$  ( $a = 1, 2, 3$ ,  $\ell = 1 \dots N_h$ )  
with potential  $V(H_\ell^a)$  and SU(2) gauge field  $A_\mu^a$ ,  
with a *quartic* coupling to gauge-invariant fermions  
with a large Fermi surface.

The quartic coupling is (assumed) innocuous, unlike a Yukawa coupling.

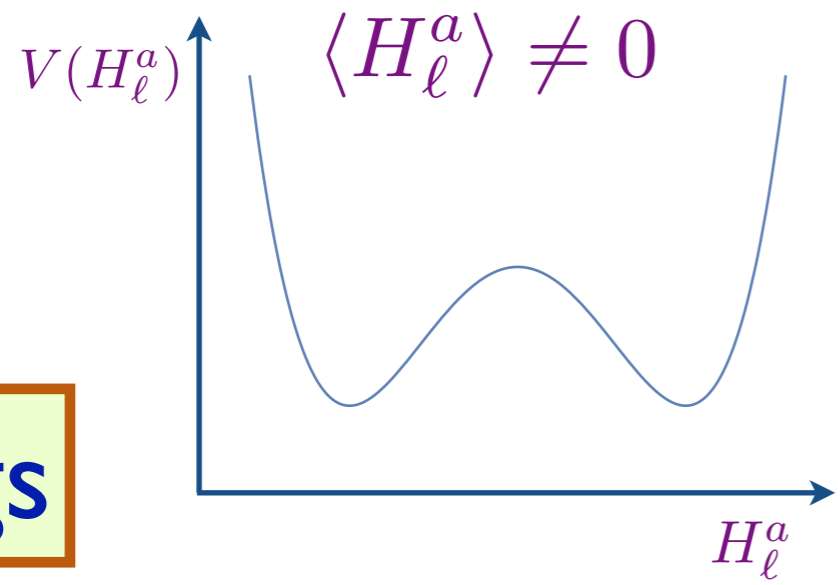
$$\begin{aligned} & (1/2) (\partial_\mu H_\ell^a - \epsilon_{abc} A_\mu^b H_\ell^c)^2 + V(H_\ell^a) \\ & - \sum_{i,\rho} t_\rho \left( c_{i,\alpha}^\dagger c_{i+\mathbf{v}_\rho,\alpha} + c_{i+\mathbf{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} \\ & + \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} H^a(i) H^a(i) \end{aligned}$$

$$V(H_\ell^a) = s H_\ell^a H_\ell^a + u_1 (H_\ell^a H_\ell^a)^2 + u_2 H_\ell^a H_m^a H_\ell^b H_m^b + \dots$$

$$N_f = 1$$

# Phase diagrams of SU(2) gauge theory

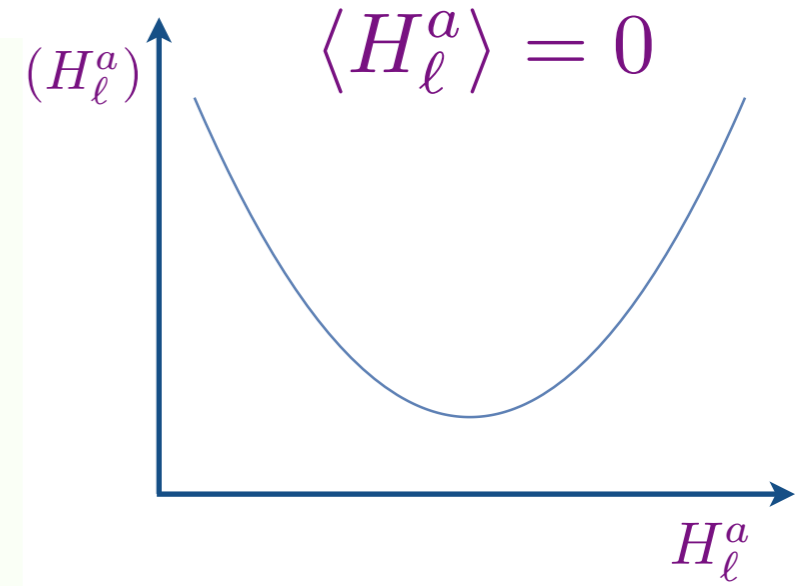
Higgs



Exponentially large  
confinement length

Reconstructed Fermi surfaces at  
distances smaller than the  
confinement length

Crossover



Confinement

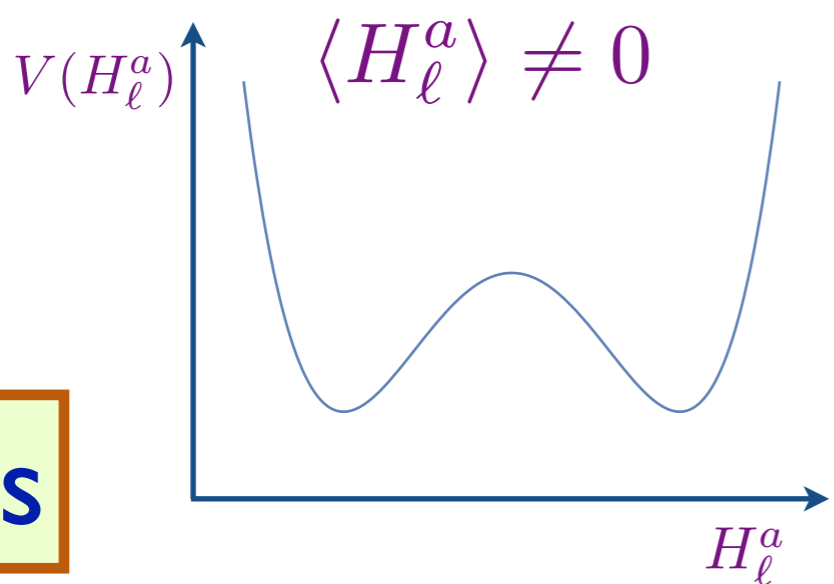
Fermi liquid with  
large Fermi surface



$$N_f = 4$$

# Phase diagrams of SU(2) gauge theory

**Higgs**



$$u_2 > 0:$$

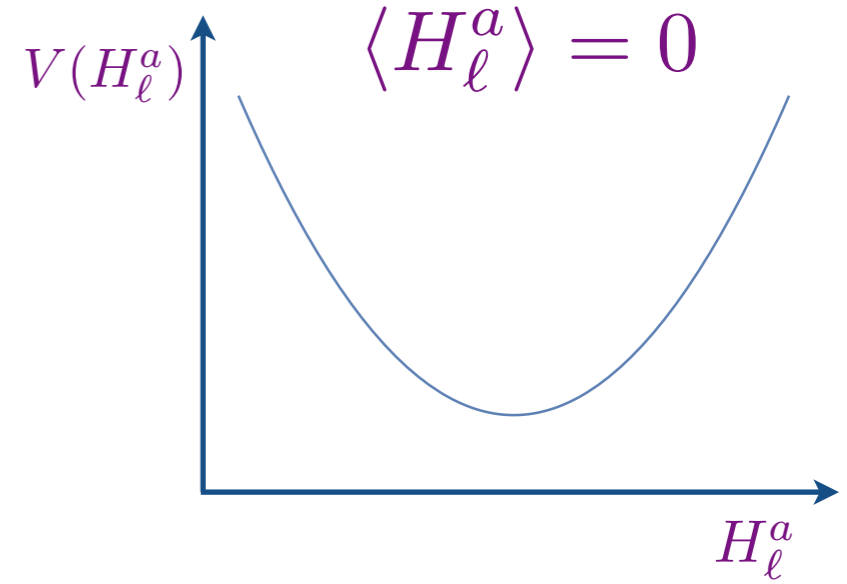
Emergent deconfined  $\mathbb{Z}_2$  gauge field with Ising-nematic order and large CDW susceptibility

or

$$u_2 < 0:$$

Confining U(1) gauge field with charge density wave order

Reconstructed (FL\*) Fermi surfaces, with large length scale confinement in the  $u_2 < 0$  case



**Confinement**

Fermi liquid with large Fermi surface

Large  $N$  expansion yields  $u_2 > 0$  near the quantum critical point. Influence of fermions can change sign of  $u_2$  in the Higgs phase.

Deconfined critical SU(2) gauge theory