Classifying two-dimensional superfluids: why there is more to cuprate superconductivity than the condensation of charge -2*e* Cooper pairs

cond-mat/0408329, cond-mat/0409470, and to appear

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December 13, 2004

Experiments on the cuprate superconductors show:

• Tendency to produce "density" wave order near wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

• Proximity to a Mott insulator at hole density $\delta=1/8$ with long-range "density" wave order at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

• Vortex/anti-vortex fluctuations for a wide temperature range in the normal state

• STM studies of $Ca_{2-x}Na_xCuO_2Cl_2$ at low T, T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

• Measurements of the Nernst effect, Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

• STM studies of $Bi_2Sr_2CaCu_2O_{8+\delta}$ above T_c, M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

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- Vortex/anti-vortex fluctuations for a wide temperature range in the normal state

Needed: A quantum theory of transitions between superfluid/supersolid/insulating phases at fractional filling, and a deeper understanding of the role of vortices

Outline

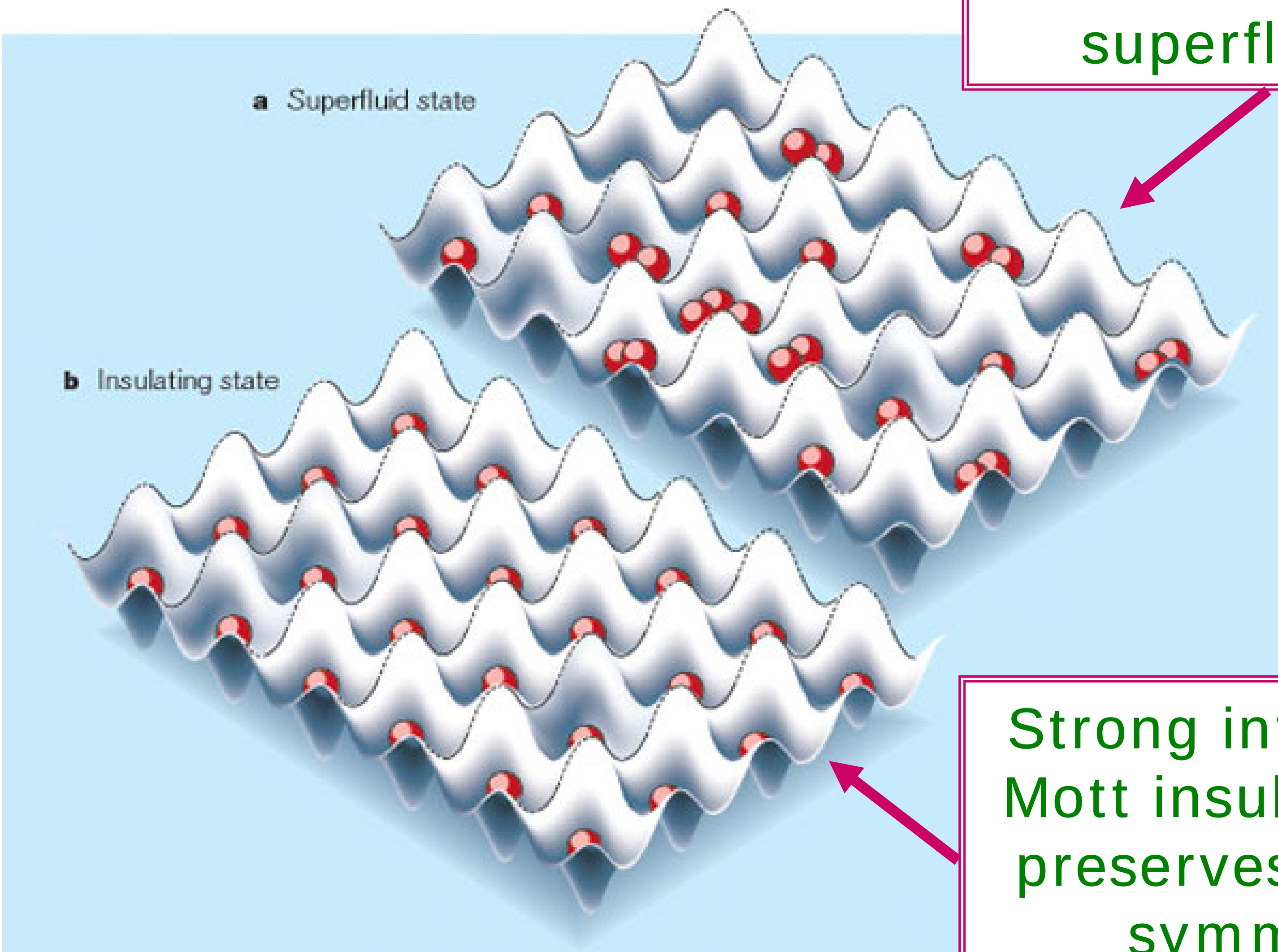
A. Superfluid-insulator transitions of bosons
on the square lattice at fractional filling
Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator at filling f

B. Extension to electronic models for the cuprate superconductors
Dual vortex theories of the doped
(1) Quantum dimer model
(2)"Staggered flux" spin liquid

A. Superfluid-insulator transitions of bosons on the square lattice at fractional filling

Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator at filling f

Bosons at filling fraction f = 1

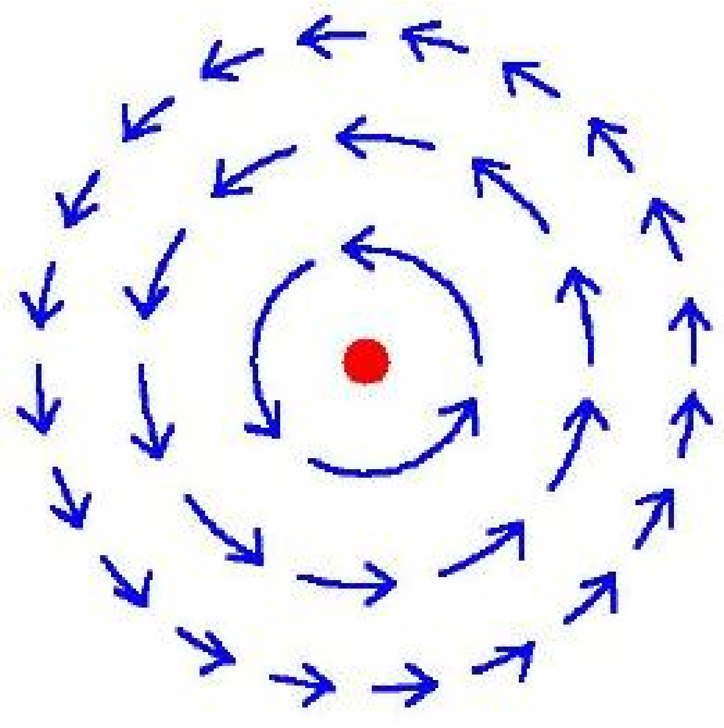


Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

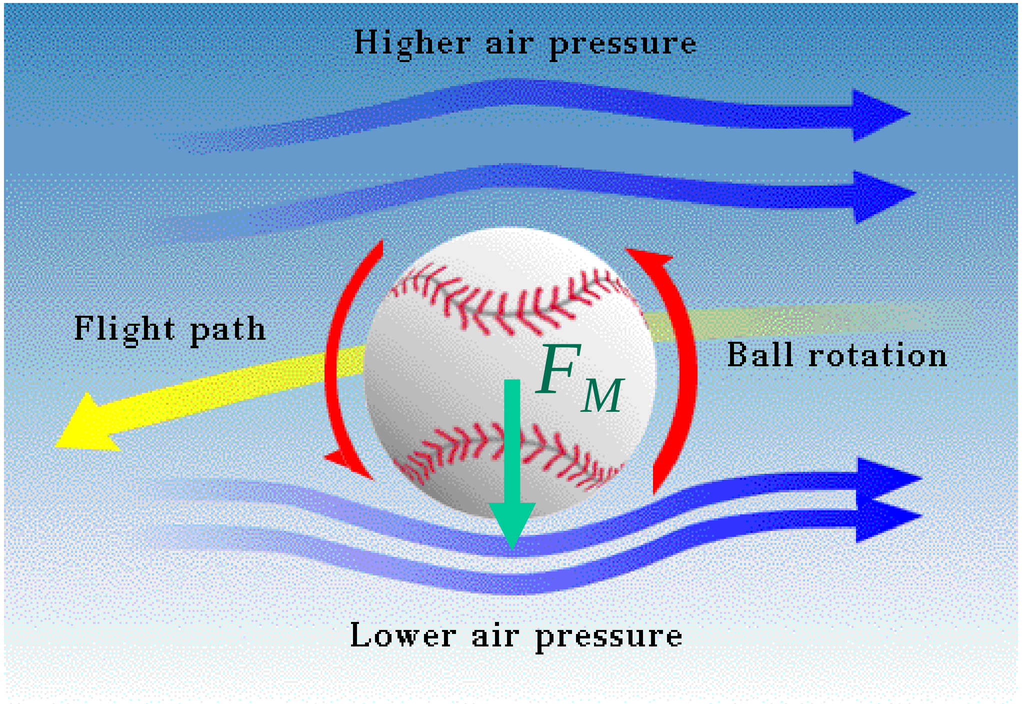
Excitations of the superfluid: **Vortices**



Vortices proliferate as the superfluid approaches the insulator.

In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these “particles” ?

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of fluid}) \bullet (\text{velocity}) \bullet (\text{circulation})$$

In a Galilean-invariant superfluid at $T = 0$, the Magnus force on a quantized vortex with vorticity m (m integer) is

$$F_M = mh\rho\left(\boldsymbol{v} - \frac{d\boldsymbol{r}}{dt}\right) \times \hat{z}$$

where ρ is the number density of bosons,

$\boldsymbol{v}$ is local superfluid velocity, and

$\boldsymbol{r}$ is the position of the vortex.

In the presence of a lattice, we must distinguish two physically distinct situations, and write

$$F_M = F_M^{(E)} + F_M^{(B)}$$

with

(1) A stationary vortex in a moving superfluid

$$F_M^{(E)} = m\boldsymbol{E} \text{ where } \boldsymbol{E} = h\rho\boldsymbol{v}\times\hat{z}$$

(2) A moving vortex in a stationary superfluid

$$F_M^{(B)} = m\frac{d\boldsymbol{r}}{dt}\times\boldsymbol{B} \text{ where } \boldsymbol{B} = -h\rho\,\hat{z}.$$

The expression for $F_M^{(E)}$ is basically correct (with $\rho \rightarrow \rho_s$), while that for $F_M^{(B)}$ is *not* correct. The latter is modified by the periodic potential of the lattice close to a Mott insulator................

$F_M^{(B)}$ can be re-interpreted as a Lorentz force on a vortex "particle" due to a "magnetic" field $B=h\rho$

So we need to consider the quantum mechanics of a particle moving in a "magnetic" field B and a periodic lattice potential --- the Hofstadter problem.

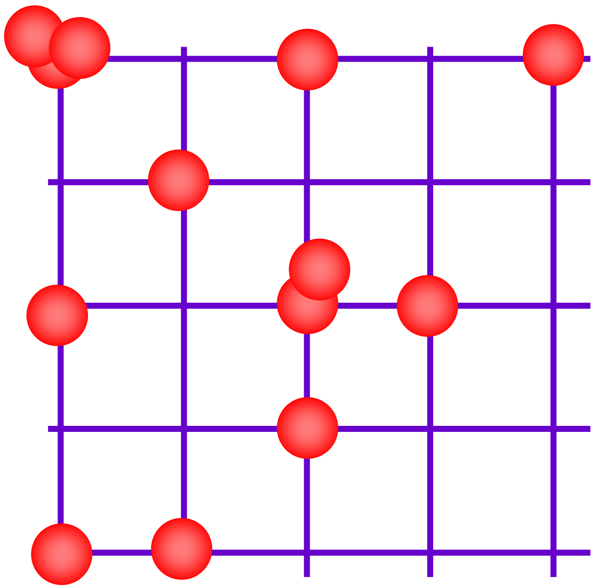
At filling fraction $f=1$, the B field is such that there is exactly one flux quantum per unit cell. Such a B field is "invisible", and the vortex "particle" moves in conventional Bloch waves $\Rightarrow \quad F_M^{(B)} = 0.$

At densities ρ close to the Mott insulator density ρ_{MI} the effective B field is

$$B = -h\left(\rho - \rho_{MI}\right)\hat{z}$$

where $\rho_{MI} = f/a_0^2 = 1/a_0^2$, and a_0 is the lattice spacing.

Bosons at filling fraction f = 1/2 (equivalent to *S*=1/2 AFMs)



$$\langle \psi \rangle \neq 0$$

Weak interactions: superfluidity

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev*. B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

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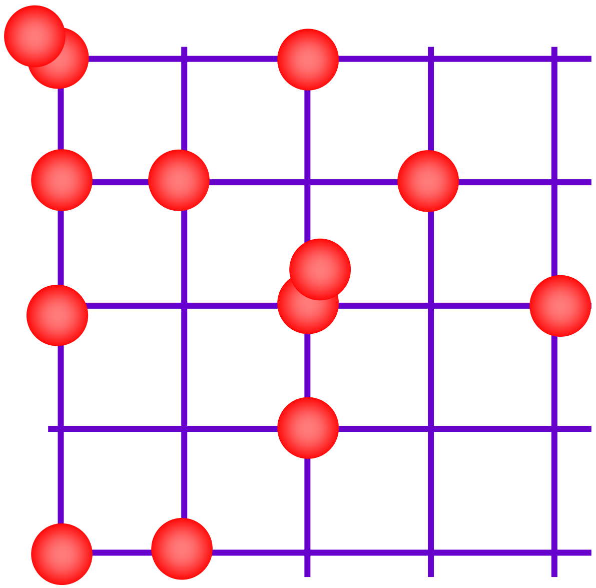

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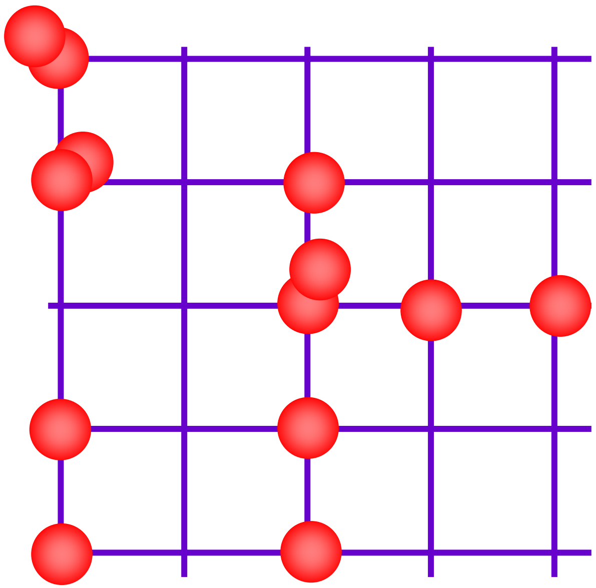

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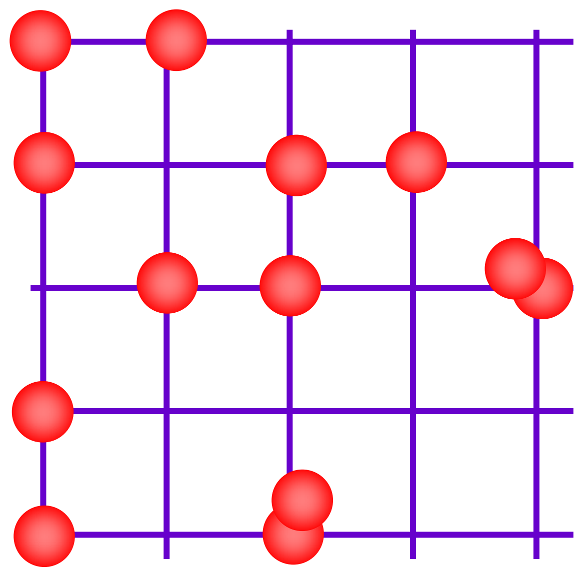

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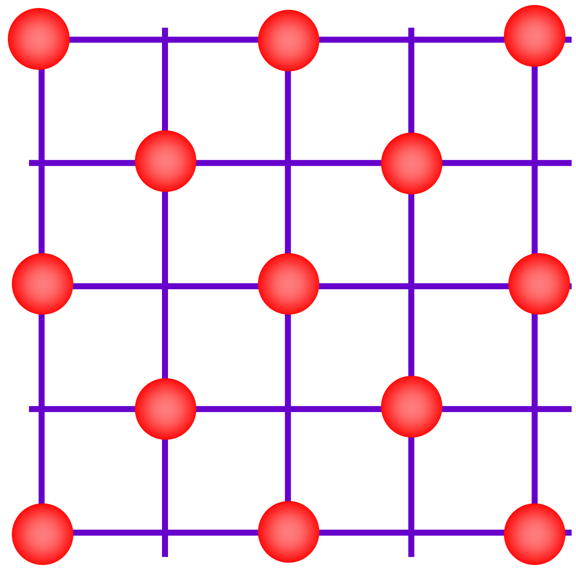

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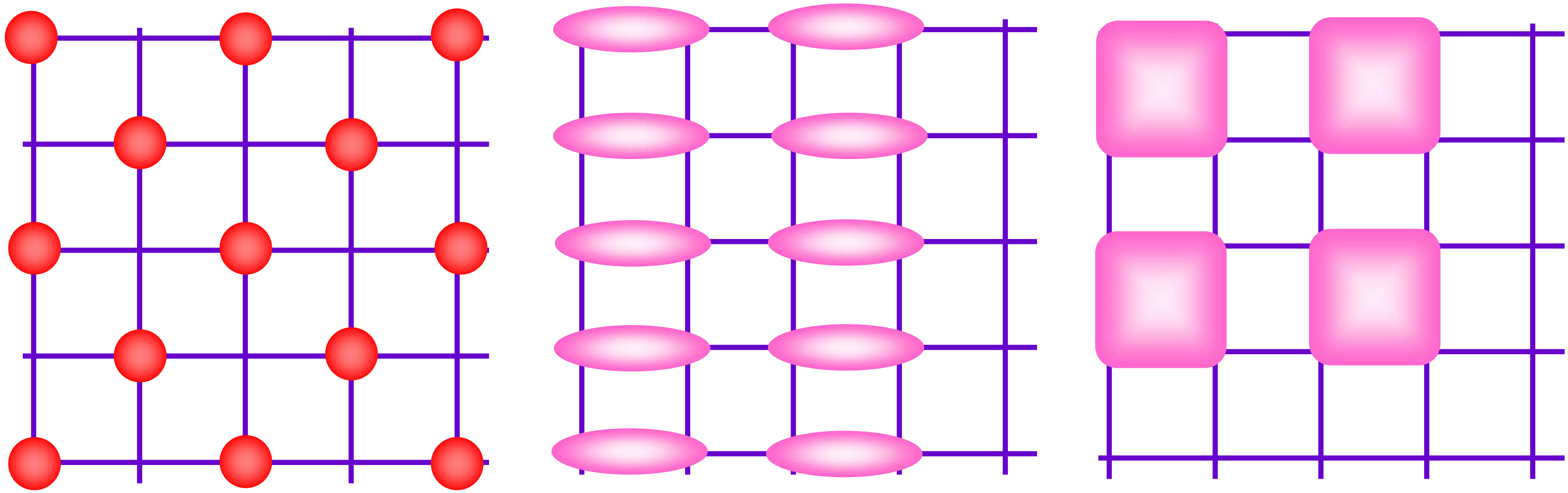

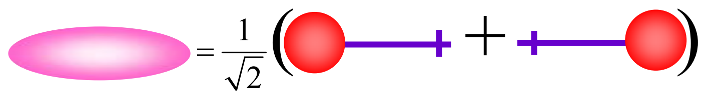

All insulating phases have "density" wave order $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$ with $\langle \rho_{\mathbf{Q}} \rangle \neq 0$

Strong interactions: insulator

$\langle \psi \rangle = 0$

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Vortices in a superfluid near a Mott insulator at filling f

Quantum mechanics of the vortex "particle" in a periodic potential with f flux quanta per unit cell

Space group symmetries of Hamiltonian:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

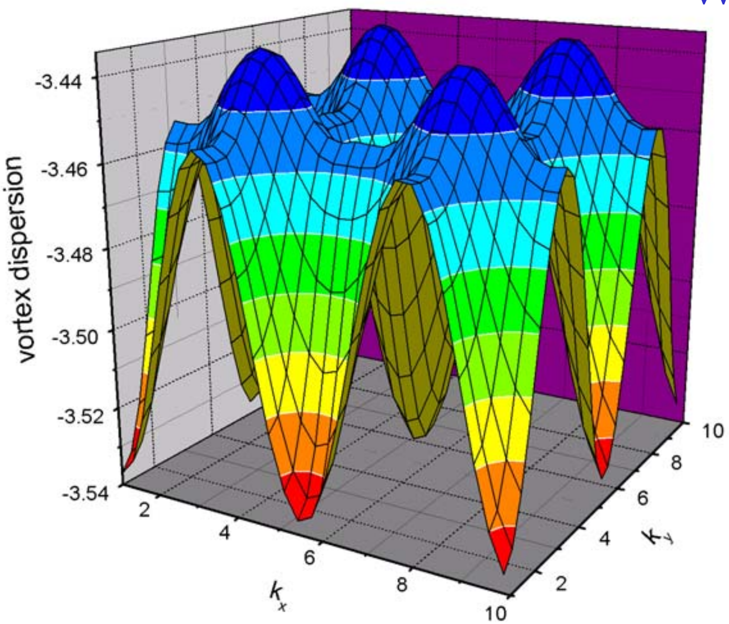
Magnetic space group:

$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

Hofstadter spectrum of the quantum vortex "particle" with field operator φ



At filling $f=p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell=1\ldots q$), associated with q gauge-equivalent regions of the Brillouin zone

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Vortices in a superfluid near a Mott insulator at filling *f=p/q*

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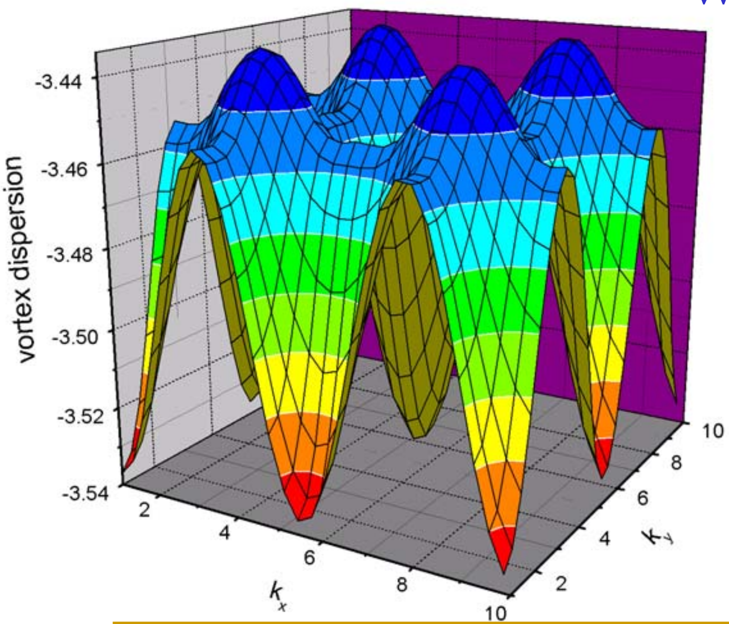

At filling $f = p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \ldots q$), associated with q gauge-equivalent regions of the Brillouin zone

The q vortices form a *projective* representation of the space group

$$T_x : \varphi_\ell \to \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \to e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \to \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

See also X.-G. Wen, *Phys. Rev.* B **65**, 165113 (2002)

Vortices in a superfluid near a Mott insulator at filling *f*=*p*/*q*

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

$$\underline{\text{Superconductor/insulator}} : \langle \varphi_\ell \rangle = 0 / \langle \varphi_\ell \rangle \neq 0$$

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

Density wave order:

Status of space group symmetry determined by density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$$

$$T_x : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{y}}$$

$$R : \rho(\mathbf{Q}) \to \rho(R\mathbf{Q})$$

Vortices in a superfluid near a Mott insulator at filling *f=p/q*

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

Vorticity modulations:

In the presence of an applied magnetic field, there are also modulations in the vorticity at the same wavevectors $\boldsymbol{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$V_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \left(\varphi_\ell^* \frac{\partial \varphi_{\ell+n}}{\partial \tau} - \frac{\partial \varphi_\ell^*}{\partial \tau} \varphi_{\ell+n} \right) e^{2\pi i \ell m f}$$

Field theory with projective symmetry

Degrees of freedom:

q complex φ_ℓ vortex fields

1 non-compact U(1) gauge field A_μ which mediates $F_M^{(E)}$ and $F_M^{(B)}$

$$\mathcal{S} = \int d^2x d\tau \Bigg[\sum_\ell \left\{ |(\partial_\mu - iA_\mu)\varphi_\ell|^2 + s|\varphi_\ell|^2 \right\} + \frac{K_\mu}{2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda - B\delta_{\mu\tau}\right)^2 + \sum_{\ell mn} \gamma_{mn}\varphi_\ell^*\varphi_{\ell+m}^*\varphi_{\ell+n}\varphi_{\ell+m-n} \Bigg]$$

where $K_{x,y} = 1/(4\pi^2\rho_s)$ and $K_\tau^{-1} = dn/d\mu$, and

$$B = -h\left(\rho - \rho_{MI}\right) \quad \text{with} \quad \rho_{MI} = \frac{f}{a_0^2} = \frac{p}{qa_0^2}.$$

Field theory with projective symmetry

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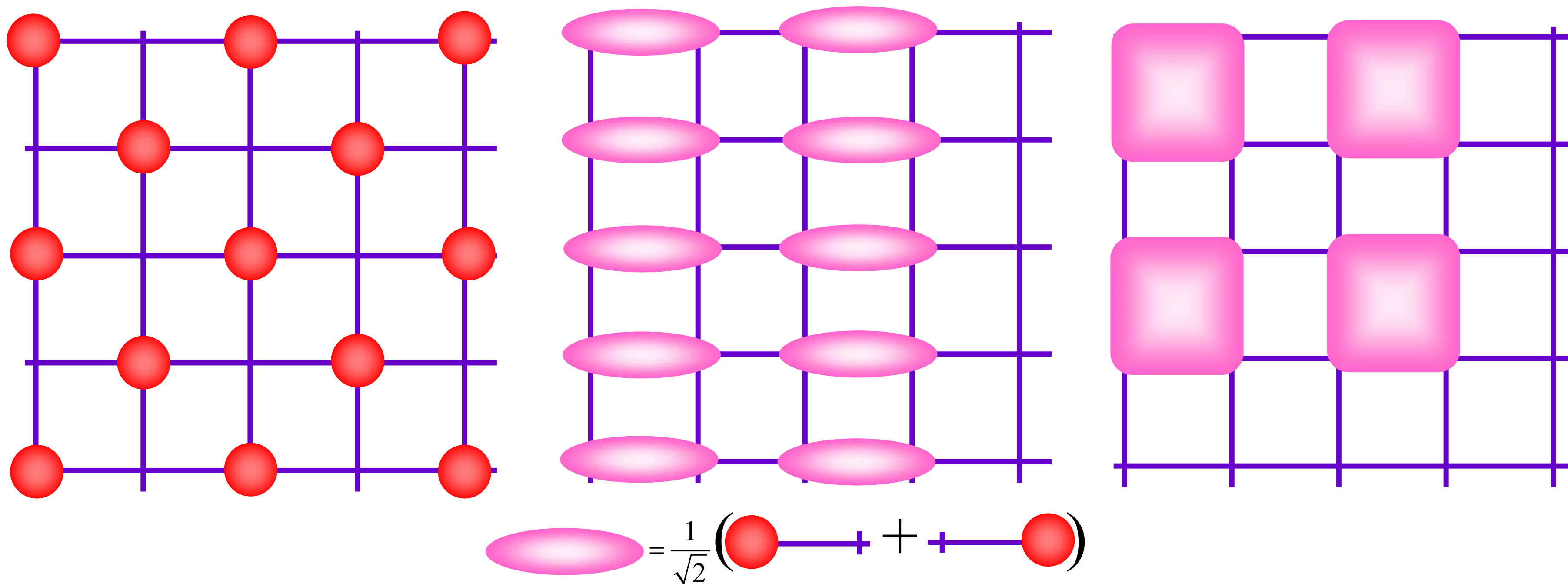
$$+ \frac{K_\mu}{2} (\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda - B\delta_{\mu\tau})^2 + \sum_{\ell mn} \gamma_{mn} \varphi^*_\ell \varphi^*_{\ell+m} \varphi_{\ell+n} \varphi_{\ell+m-n} \Bigg]$$

The projective symmetries constrain the couplings γ_{mn} to obey

$$\begin{aligned} \gamma_{mn} &= \gamma_{-m,-n} \;\; ; \;\; \gamma_{mn} = \gamma_{m,m-n} \;\; ; \;\; \gamma_{mn} = \gamma_{m-2n,-n} \\ \gamma_{\bar{m}\bar{n}} &= \frac{1}{q} \sum_{mn} \gamma_{mn} e^{-2\pi i f [n(\bar{m}-\bar{n}) + \bar{n}(m-n)]} \end{aligned}$$

Field theory with projective symmetry

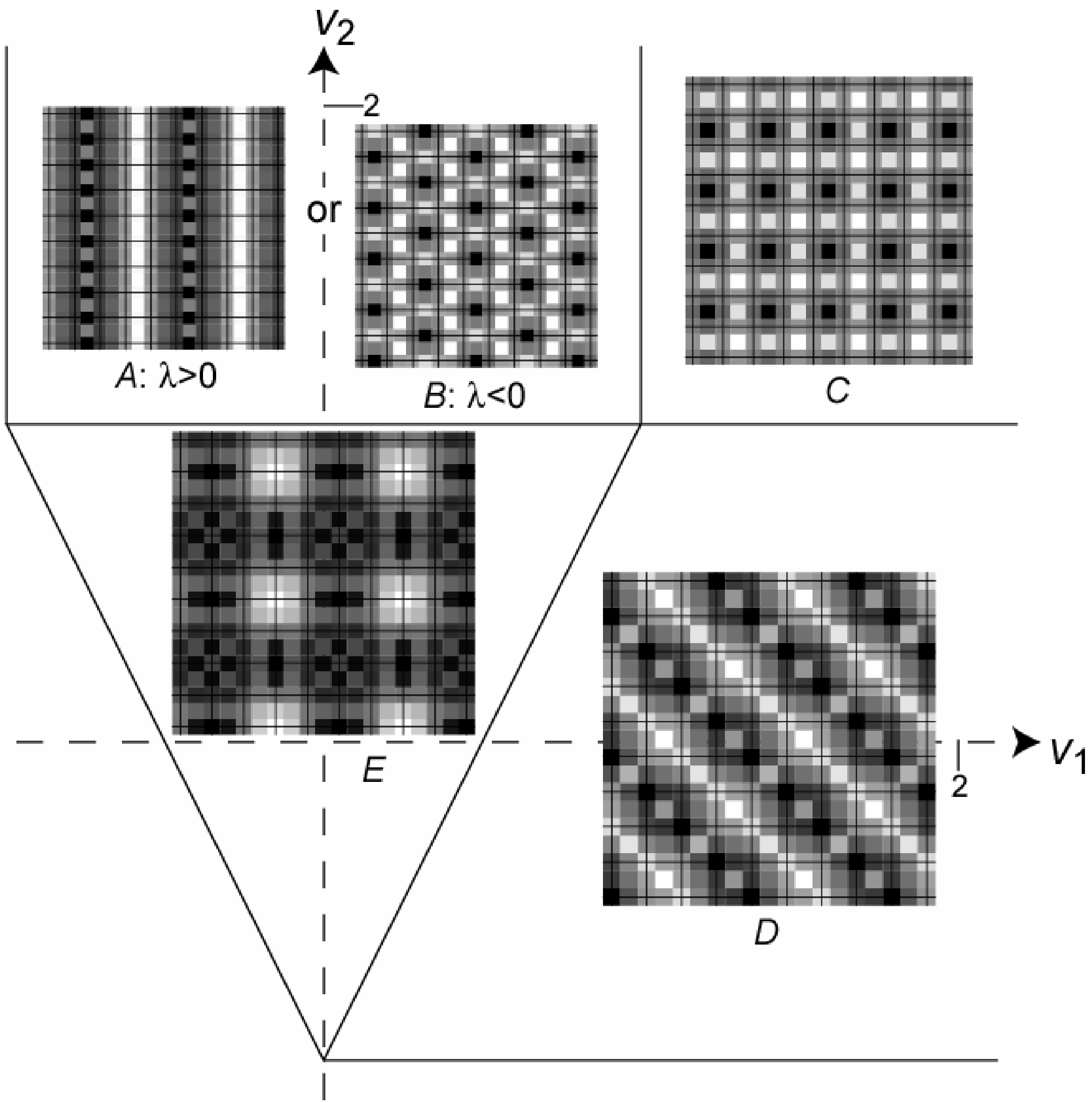
Spatial structure of insulators for $q=2$ ($f=1/2$)



All insulating phases have density-wave order $\rho(\boldsymbol{r}) = \sum_{\boldsymbol{Q}} \rho_{\boldsymbol{Q}} e^{i\boldsymbol{Q}.\boldsymbol{r}}$ with $\langle \rho_{\boldsymbol{Q}} \rangle \neq 0$

Field theory with projective symmetry

Spatial structure of insulators for q=4 (f=1/4 or 3/4)



$a \times b$ unit cells;

q/a, q/b, ab/q

all integers

Field theory with projective symmetry

Pinned vortices in the superfluid

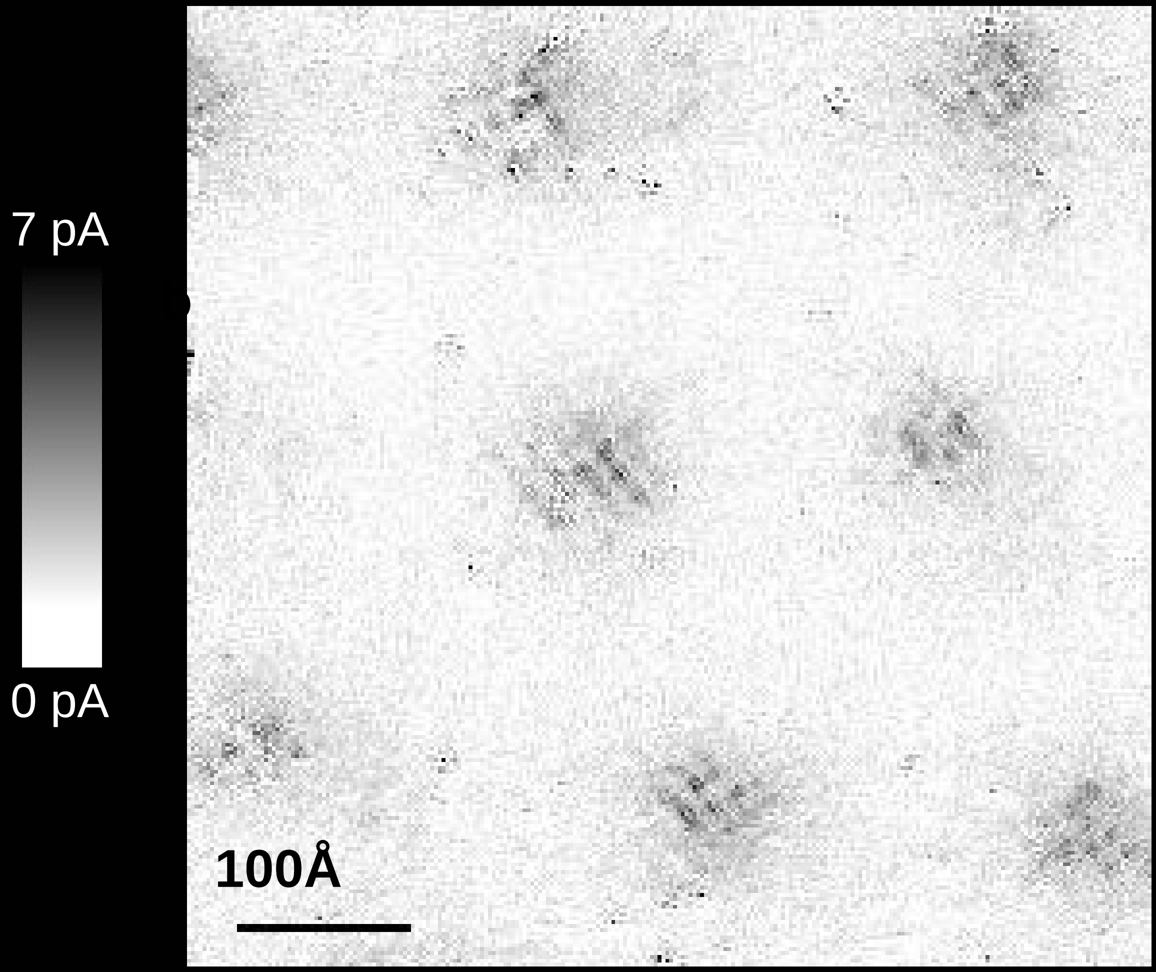
Any pinned vortex must chose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.

Density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_{\ell}^{*} \varphi_{\ell+n} e^{2\pi i \ell mf}$$

In the cuprates, assuming boson density=density of Cooper pairs we have $\rho_{\mathrm{MI}} = 7/16,$ and $q = 16$ (both models in part B yield this value of q). So modulation must have period $a \times b$ with $16/a,\ 16/b,$ and $ab/16$ all integers.

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

J. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).

Measuring the inertial mass of a vortex

Solve the equations of motion

$$m_v \frac{d^2\mathbf{r}}{dt^2} = F_M^{(E)} + F_M^{(B)}$$

for a triangular lattice of vortices in the harmonic approximation. We estimate that retardation effects can be neglected (the 'electric' interactions are instantaneous and obey the 'Coulomb' law). We define

$$u_{\rm rms} = \text{rms displacement of vortex from its equilibrium position,}$$

which can be determined from the LDOS modulations in the STM measurement. Then we find from the vortex 'magnetophonon' spectrum

$$\begin{aligned} m_v &= 0.0419 \frac{\hbar^2 A_0}{\rho_s u_{\rm rms}^4} F\left(\frac{u_{\rm rms}^2 B}{\hbar}\right) \\ F(x) &\approx 0.5039 + \sqrt{0.2461 + 0.4147 x^2} \end{aligned}$$

where A_0 is the area of a vortex lattice unit cell, and $B = -h(\rho - \rho_{MI})$.

Measuring the inertial mass of a vortex

Preliminary estimates for the BSCCO experiment:

Inertial vortex mass $m_v \approx 10 m_e$

Vortex magnetoplasmon frequency $\nu_\mathrm{p} \approx 1\ \mathrm{THz} = 4\ \mathrm{meV}$

Large uncertainty due to uncertainty in value of u_{rms}

Note: With nodal fermionic quasiparticles, m_v is expected to be dependent on the magnetic field *i.e.* vortex density

G. E. Volovik, JETP Lett. **65,** 217 (1997);
N. B. Kopnin, Phys. Rev. B **57**, 11775 (1998).

B. Extension to electronic models for the cuprate superconductors

Dual vortex theories of the doped

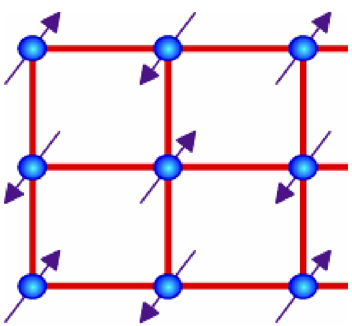
(1) Quantum dimer model

(2)"Staggered flux" spin liquid

(B.1) Phase diagram of doped antiferromagnets

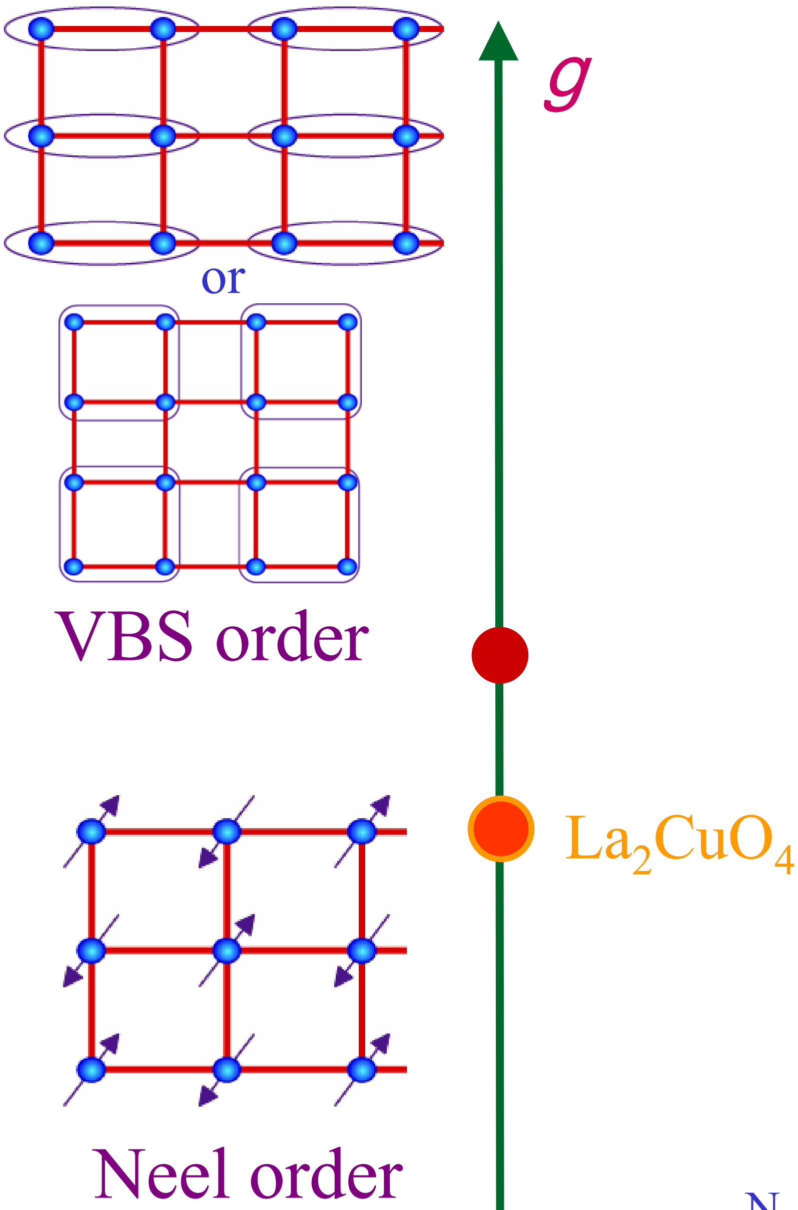
g = parameter controlling strength of quantum fluctuations in a semiclassical theory of the destruction of Neel order

La_2CuO_4



Neel order

(B.1) Phase diagram of doped antiferromagnets



N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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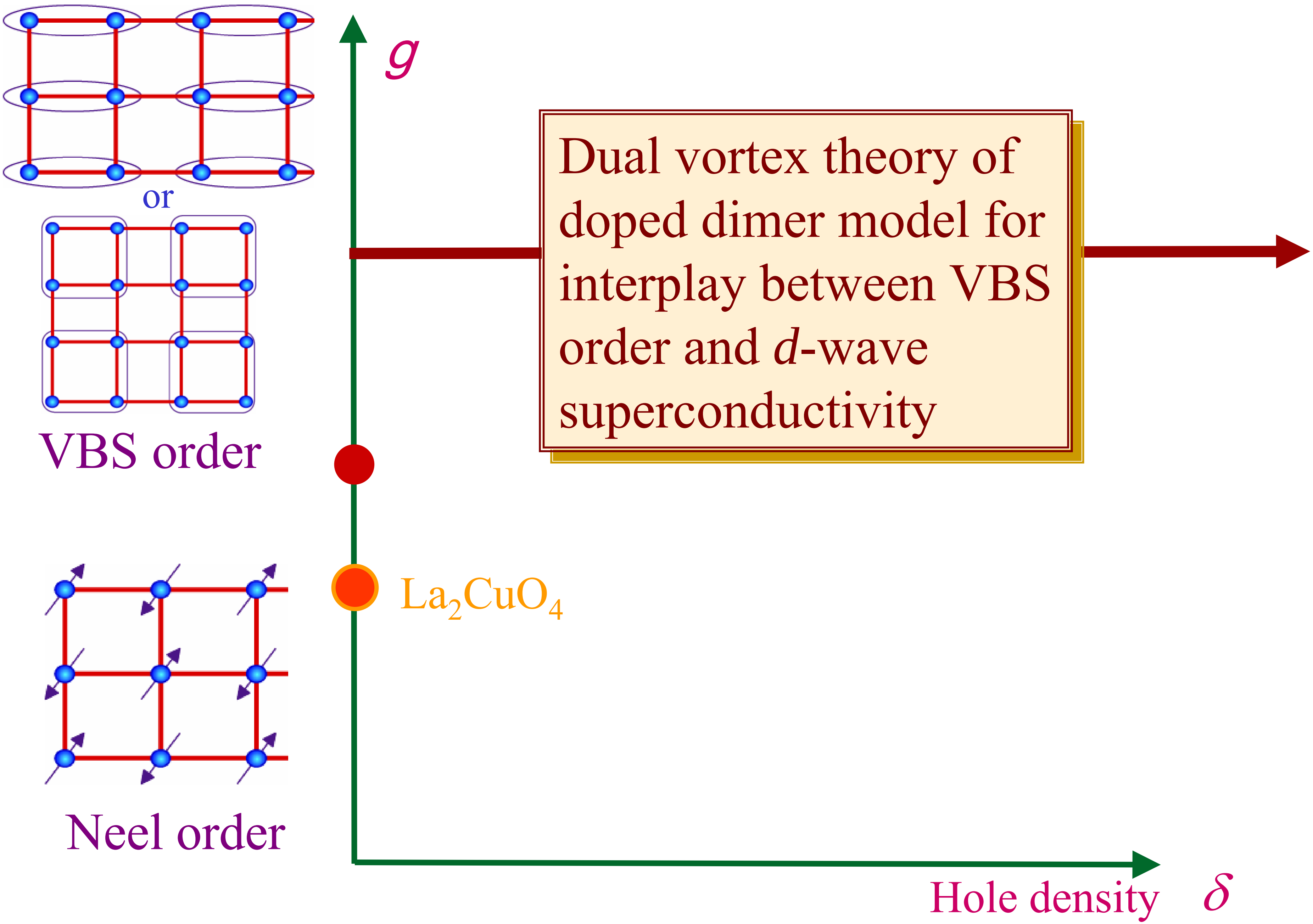
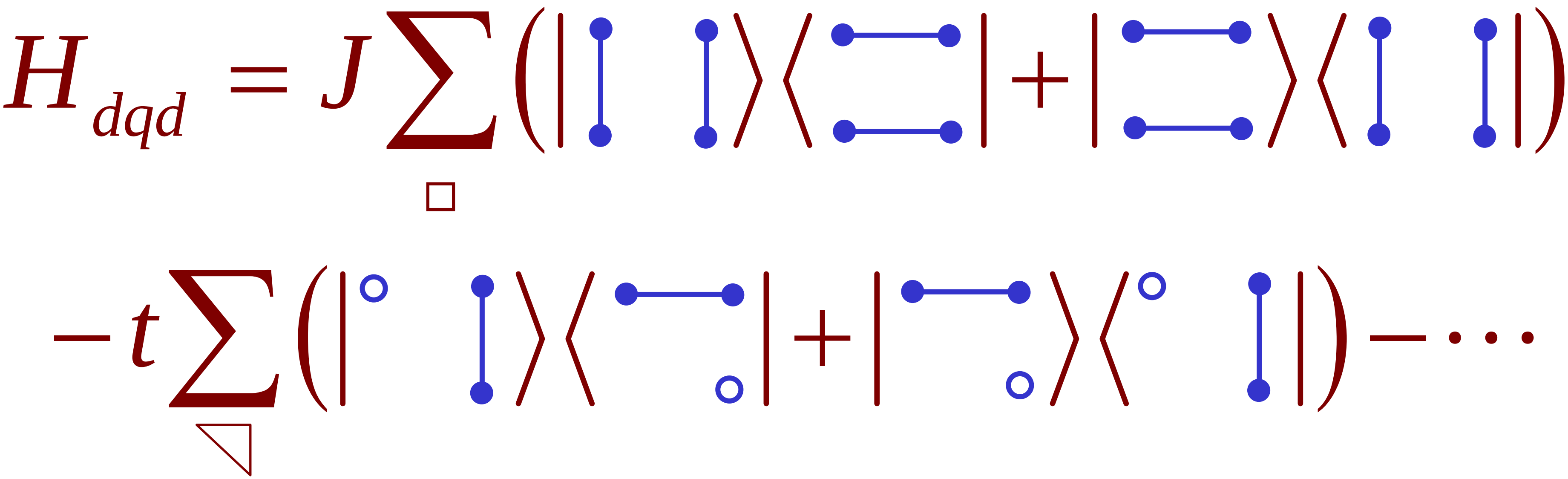


(**B.1**) Doped quantum dimer model



Density of holes = δ

E. Fradkin and S. A. Kivelson, *Mod. Phys. Lett.* B **4**, 225 (1990).

(B.1) Duality mapping of doped quantum dimer model shows:

Vortices in the superconducting state obey the magnetic translation algebra

$$T_x T_y = e^{2\pi i f} T_y T_x$$

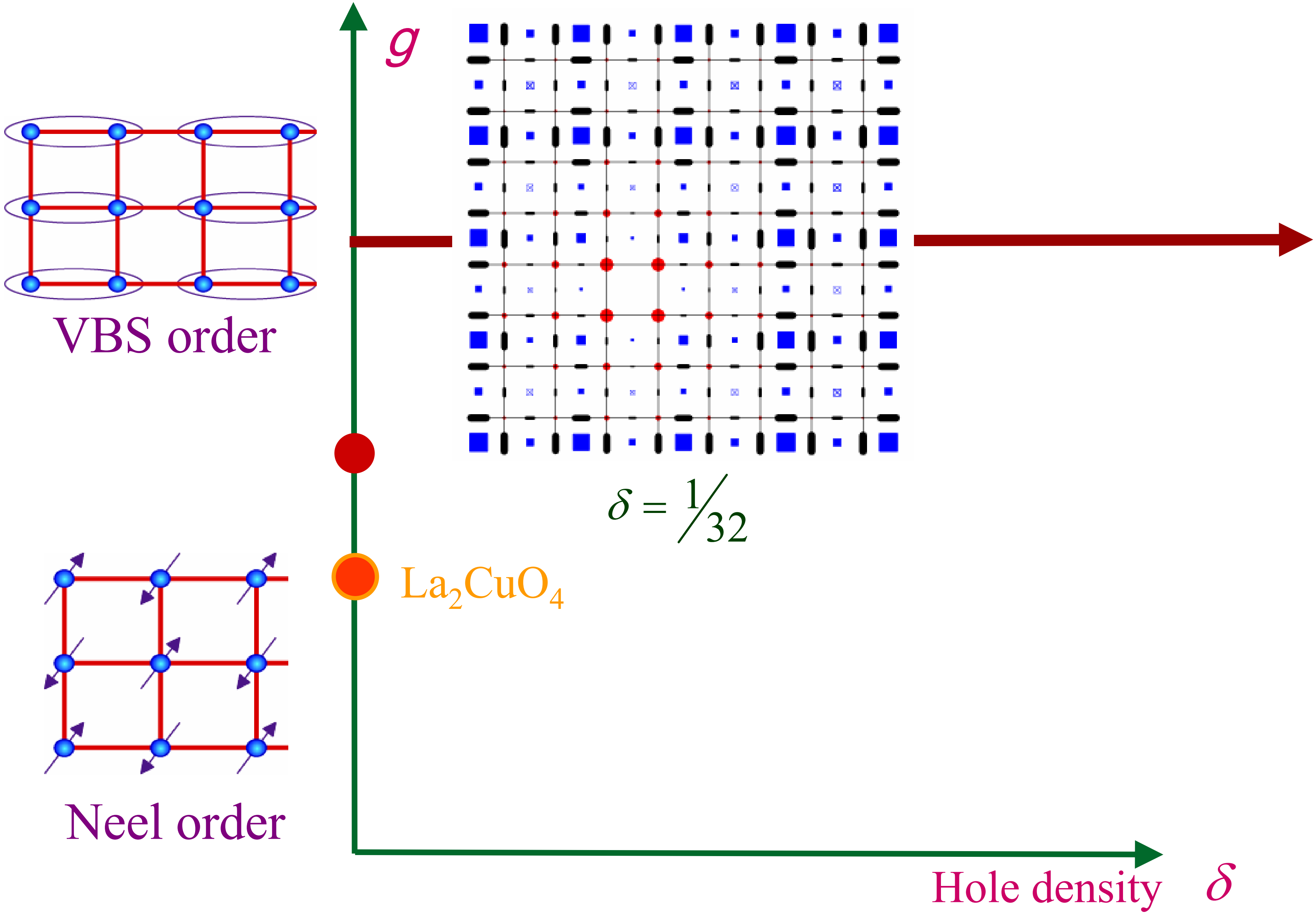
$$\text{with} \quad f = \frac{p}{q} = \frac{1-\delta_{MI}}{2}$$

where δ_{MI} is the density of holes in the proximate Mott insulator (for $\delta_{MI} = 1/8$, $f = 7/16 \Rightarrow q = 16$)

Note: f = density of Cooper pairs

Most results of Part A on bosons can be applied unchanged with q as determined above

(B.1) Phase diagram of doped antiferromagnets



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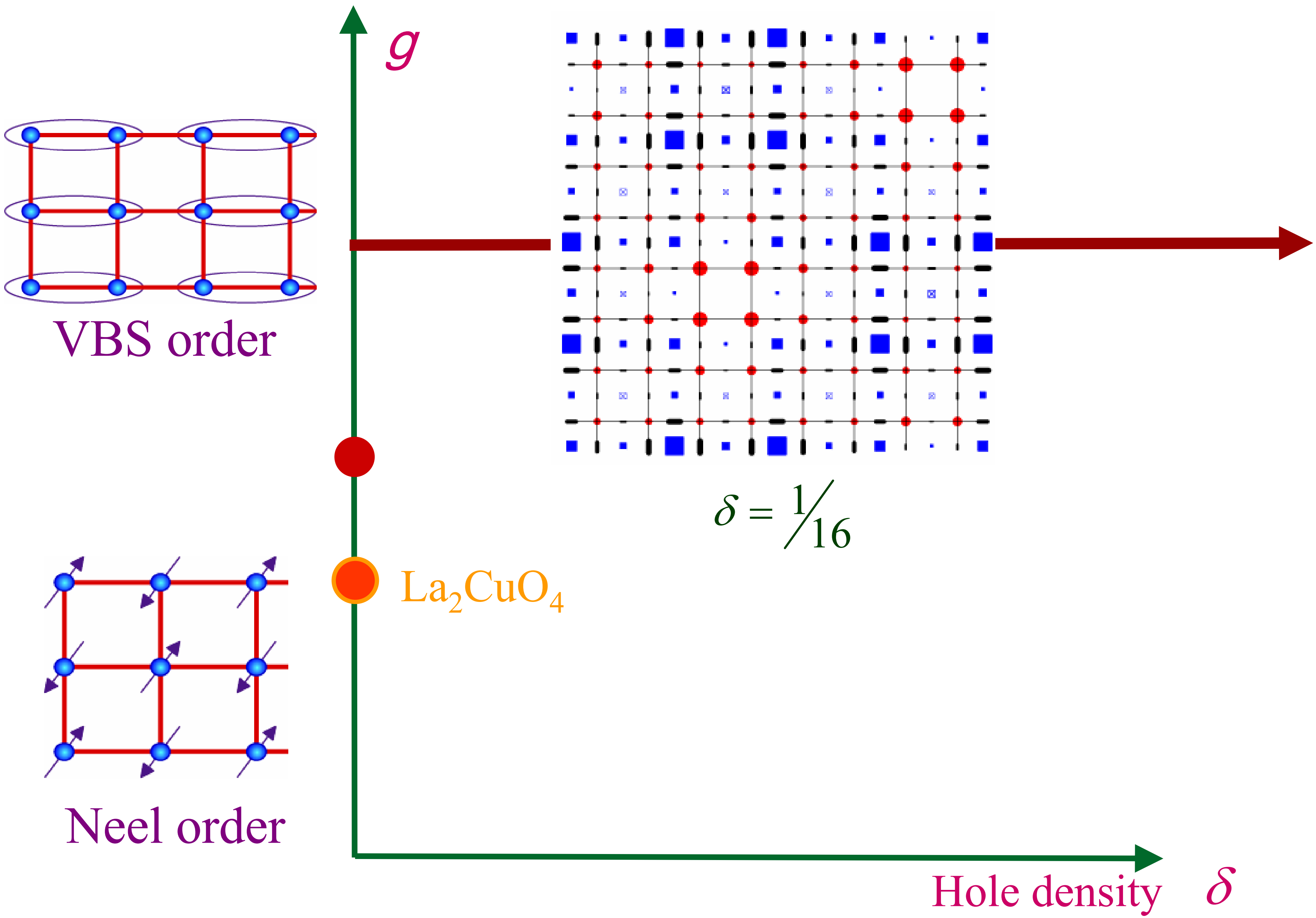

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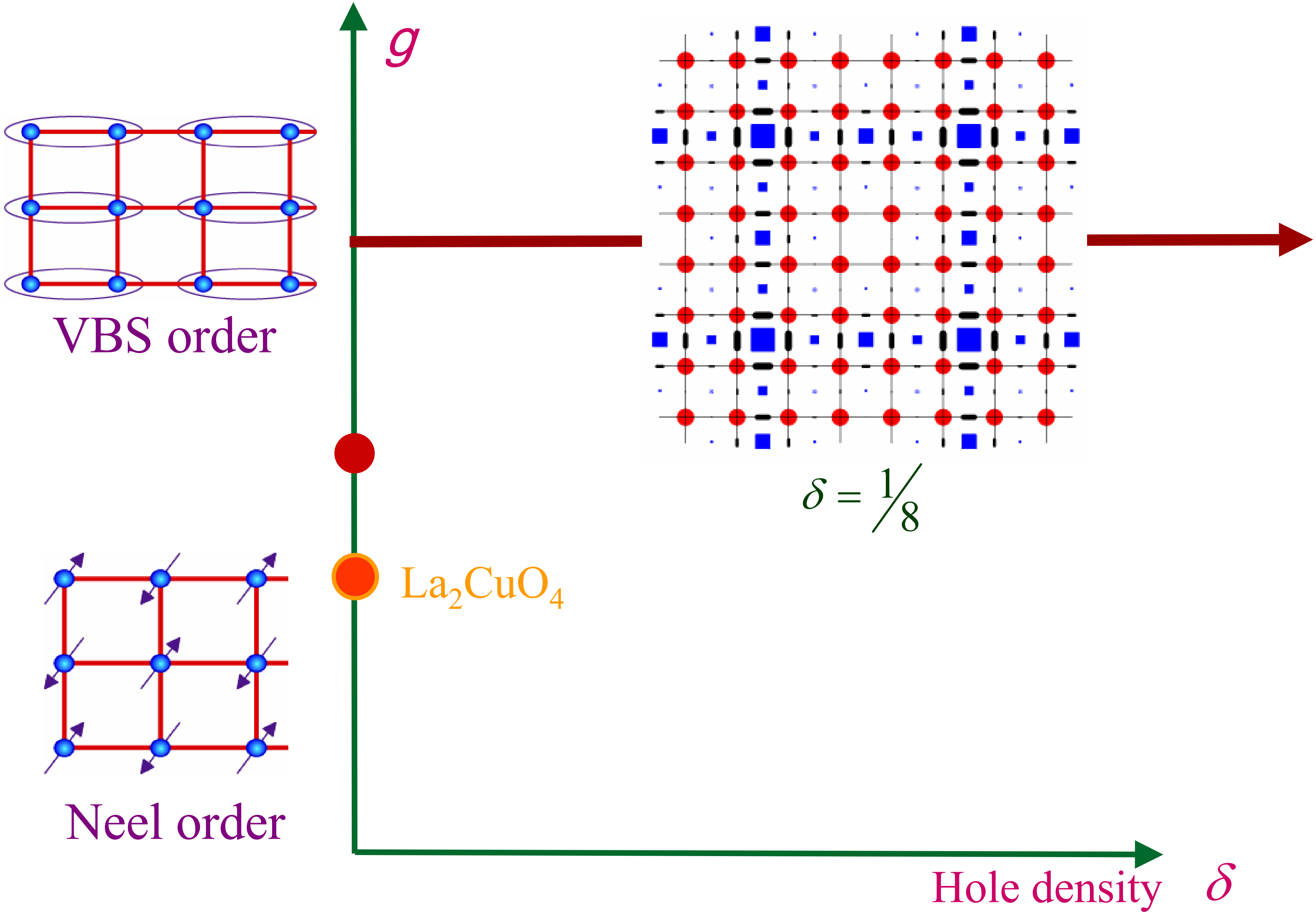

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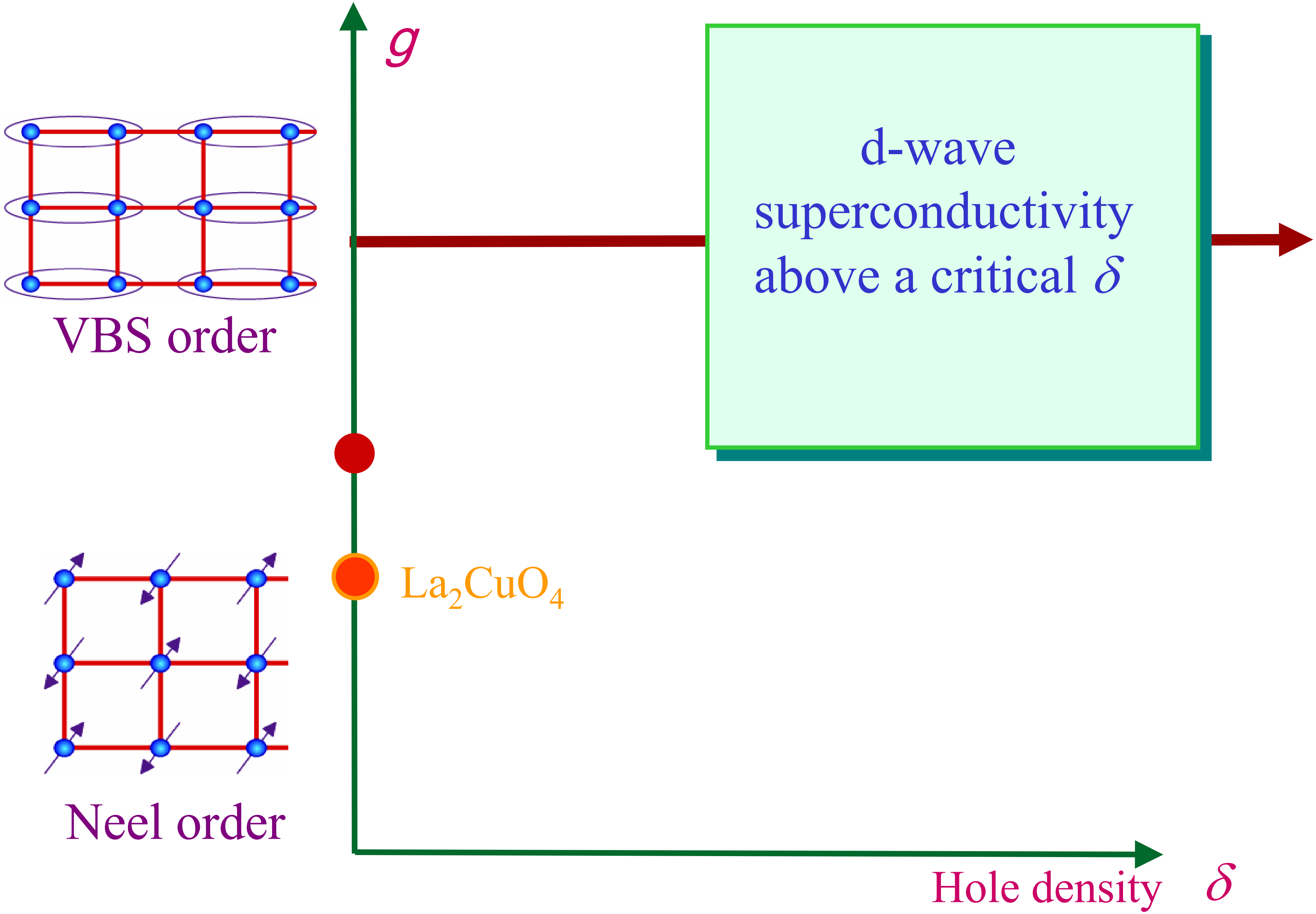

(B.2) Dual vortex theory of doped "staggered flux" spin liquid

Describe a d-wave superconductor as a doped "staggered flux" spin liquid in the SU(2) gauge theory formulation.
X.-G. Wen and P. A. Lee *Phys. Rev. Lett.* **76**, 503 (1996).

The theory is expressed in terms of neutral fermionic spinons ψ which are charged under a U(1) gauge field C_μ. The electrically charged carriers are represented by 2 boson species, b_1 and b_2, which carry opposite charges under C_μ.

We wish to describe quantum fluctuations in such a superconductor near a transition to a Mott insulator. The Mott insulator has hole density δ_{MI}, with

$$\frac{\delta_{MI}}{2} = \frac{p}{q},$$

with p, q relatively prime integers.

(B.2) Dual vortex theory of doped "staggered flux" spin liquid

It is essential to account for the quantum dynamics of the bosons b_1 and b_2, each near density p/q. We do this by applying the methods developed earlier for boson systems separately to both b_1 and b_2.

This yields a theory with a pair of q complex vortex fields $\varphi_{1\ell}$ and $\varphi_{2\ell}$, which are dual to the two species of bosons, b_1, b_2 of the SU(2) gauge theory. Each vortex carries physical magnetic flux $h/(2e)$. These vortices are coupled to 2 non-compact U(1) gauge fields: A_μ (responsible for the Magnus forces), and B_μ (whose Chern-Simons dual is coupled to the nodal fermions).

(B.2) Dual vortex theory of doped "staggered flux" spin liquid

The effective action for the theory is:

$$
\begin{aligned}
\mathcal{S}_{sf} &= \mathcal{S}_v + \mathcal{S}_A \\
\mathcal{S}_v &= \int d^2r d\tau \sum_{\ell=0}^{q-1} \Bigg[h_s (-1)^\ell \Bigg\{ \varphi^*_{1,\ell+q/2} \left(\frac{\partial}{\partial \tau} - iA_\tau - iB_\tau \right) \varphi_{1\ell} \\
&\qquad\qquad - \varphi^*_{2,\ell+q/2} \left(\frac{\partial}{\partial \tau} - iA_\tau + iB_\tau \right) \varphi_{2\ell} \Bigg\} \\
&\qquad + \left| (\partial_i - iA_i - iB_i) \varphi_{1\ell} \right|^2 + s \left| \varphi_{1\ell} \right|^2 \\
&\qquad + \left| (\partial_i - iA_i + iB_i) \varphi_{2\ell} \right|^2 + s \left| \varphi_{2\ell} \right|^2 \Bigg] \\
\mathcal{S}_A &= \int d^2r d\tau \left[\frac{K_\mu}{2} \left(\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda \right)^2 + \frac{i}{\pi} \epsilon_{\mu\nu\lambda} B_\mu \partial_\nu C_\lambda + \overline{\psi} \gamma_\mu (\partial_\mu - iC_\mu) \psi \right]
\end{aligned}
$$

There are also additional "monopole" terms which are not shown.

(B.2) Dual vortex theory of doped "staggered flux" spin liquid

Main results:

- Presence of the staggered flux makes the vortices "non-relativistic" and allows a theory of a dilute gas of vortices and anti-vortices.
- As the superfluid approaches the Mott insulator, the vortices and anti-vortices form "excitonic" bound states which condense first.
- This implies that a *supersolid* intervenes between the superfluid and the insulator.

Superfluids near Mott insulators

The Mott insulator has average Cooper pair density, f = p/q per site, while the density of the superfluid is close (but need not be identical) to this value

- Vortices with flux *h/(2e)* come in multiple (usually *q*) "flavors"
- The lattice space group acts in a projective representation on the vortex flavor space.
- These flavor quantum numbers provide a distinction between superfluids: they constitute a "quantum order"
- Any pinned vortex must chose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.