

# Quantum matter without quasiparticles

M.I.T.  
March 13, 2014

Subir Sachdev

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)

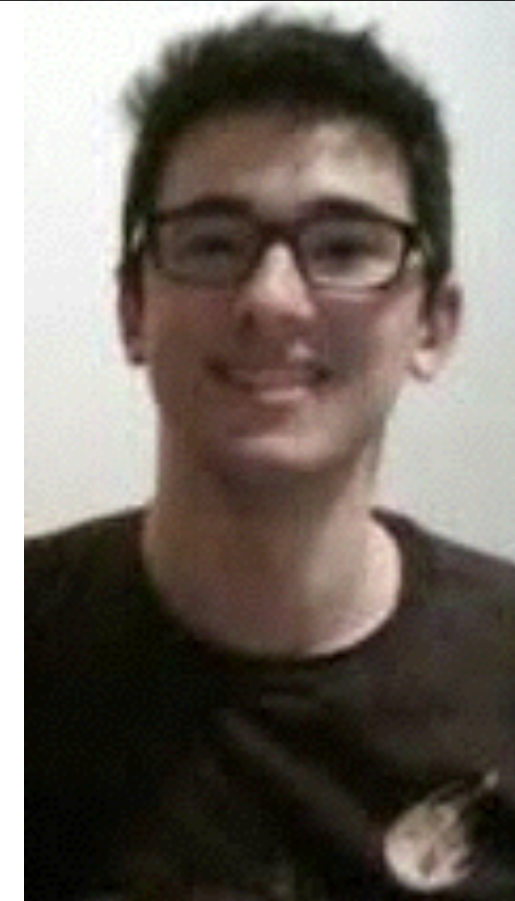




**William Witczak-Krempa**  
**Perimeter**



**Erik Sorensen**  
**McMaster**



**Andrew Lucas**  
**Harvard**



**Sean Hartnoll**  
**Stanford**



**Raghu Mahajan**  
**Stanford**



**Matthias Punk**  
**Innsbruck**

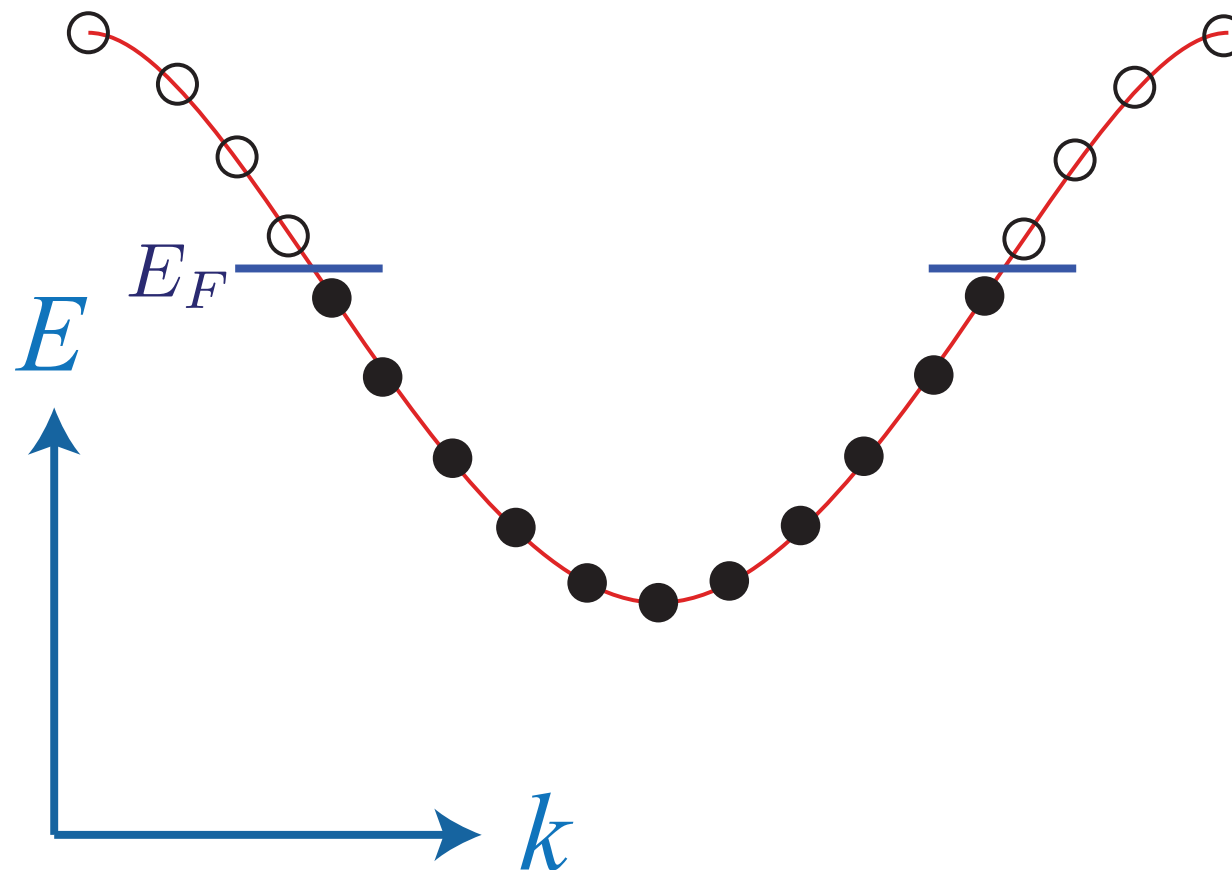


**Koenraad Schalm**  
**Leiden**

# *Foundations of quantum many body theory:*

## *I. Ground states connected adiabatically to independent electron states*

### Metals

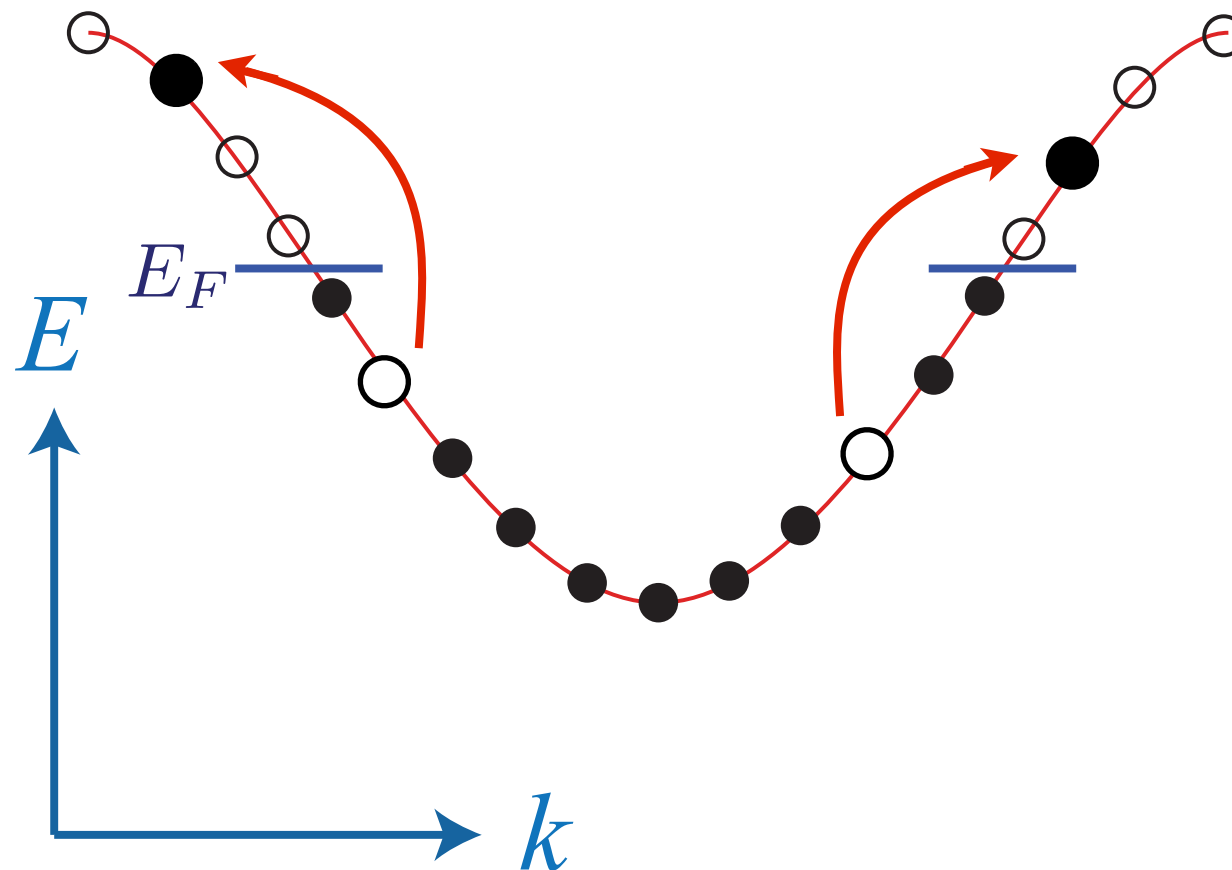


# *Foundations of quantum many body theory:*

*1. Ground states connected adiabatically to independent electron states*

*2. Boltzmann-Landau theory of quasiparticles*

## Metals





## Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. Boltzmann-Landau theory of quasiparticles*

## Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

## Modern phases of quantum matter:

1. *Ground states disconnected from independent electron states: many-particle entanglement*
2. *Boltzmann-Landau theory of quasiparticles*

## Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations (“phonons”) are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. No quasiparticles**

## Modern phases of quantum matter:

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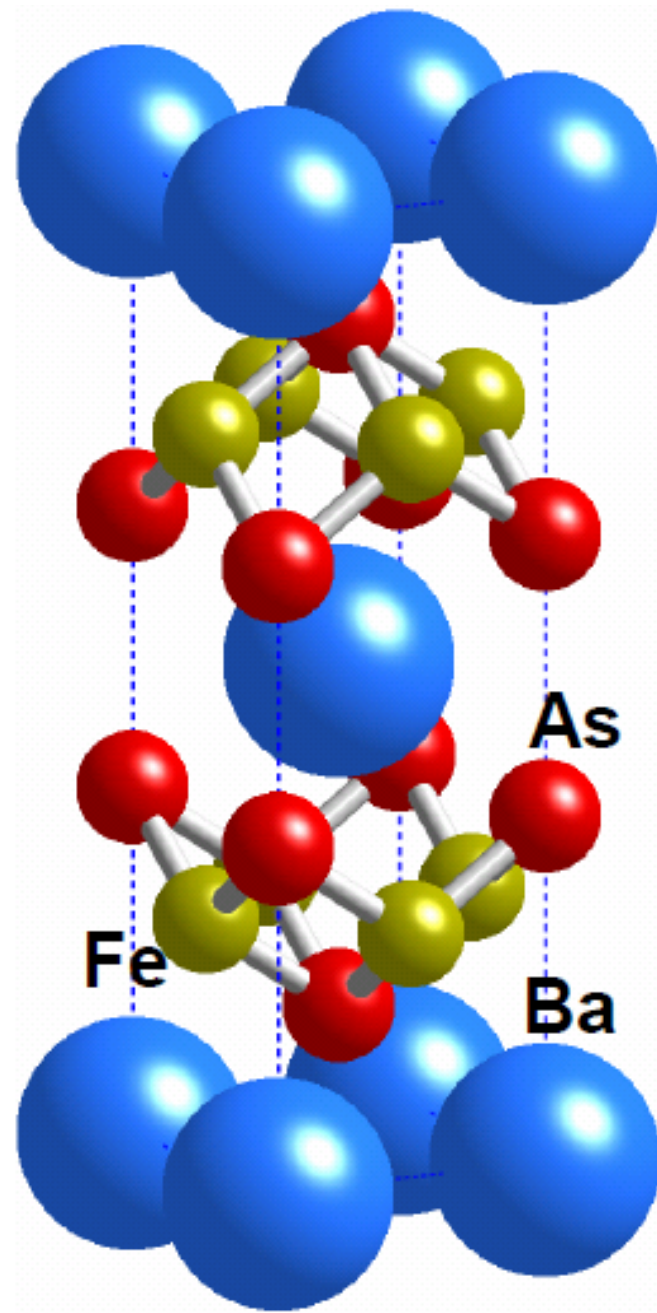
## Only 2 examples:

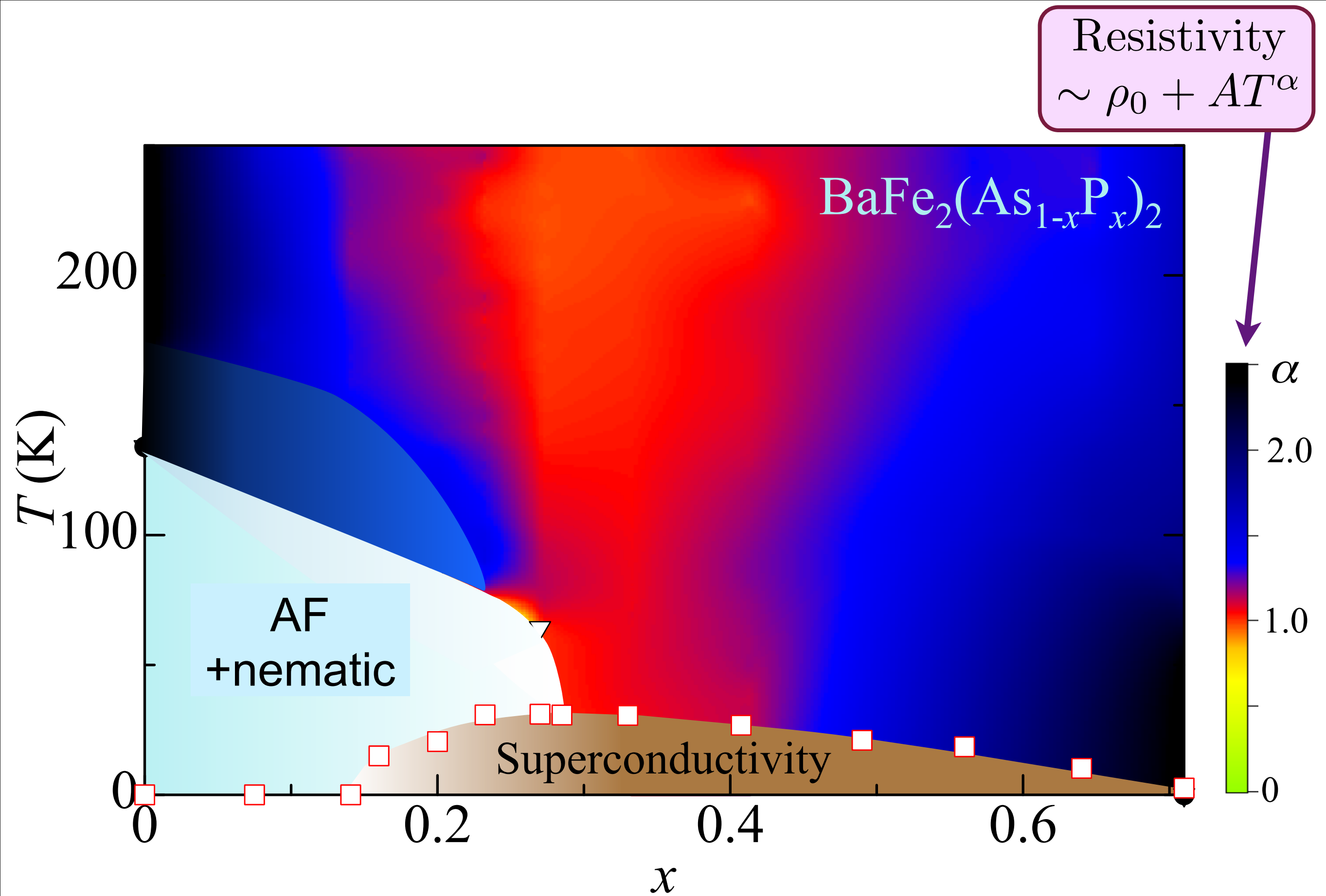
1. Conformal field theories in spatial dimension  $d > 1$
2. Quantum critical metals in dimension  $d=2$



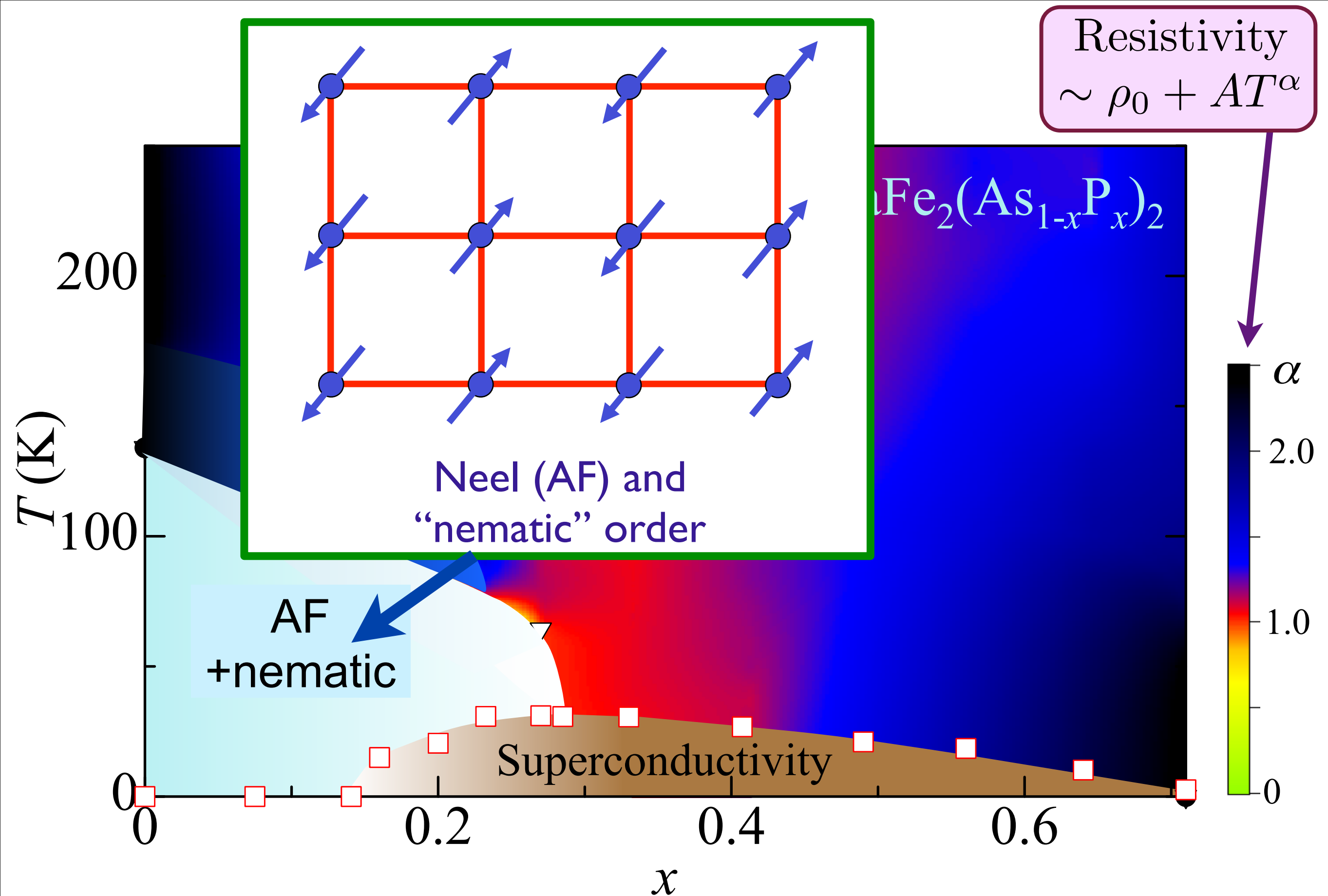
# Iron pnictides:

a new class of high temperature superconductors



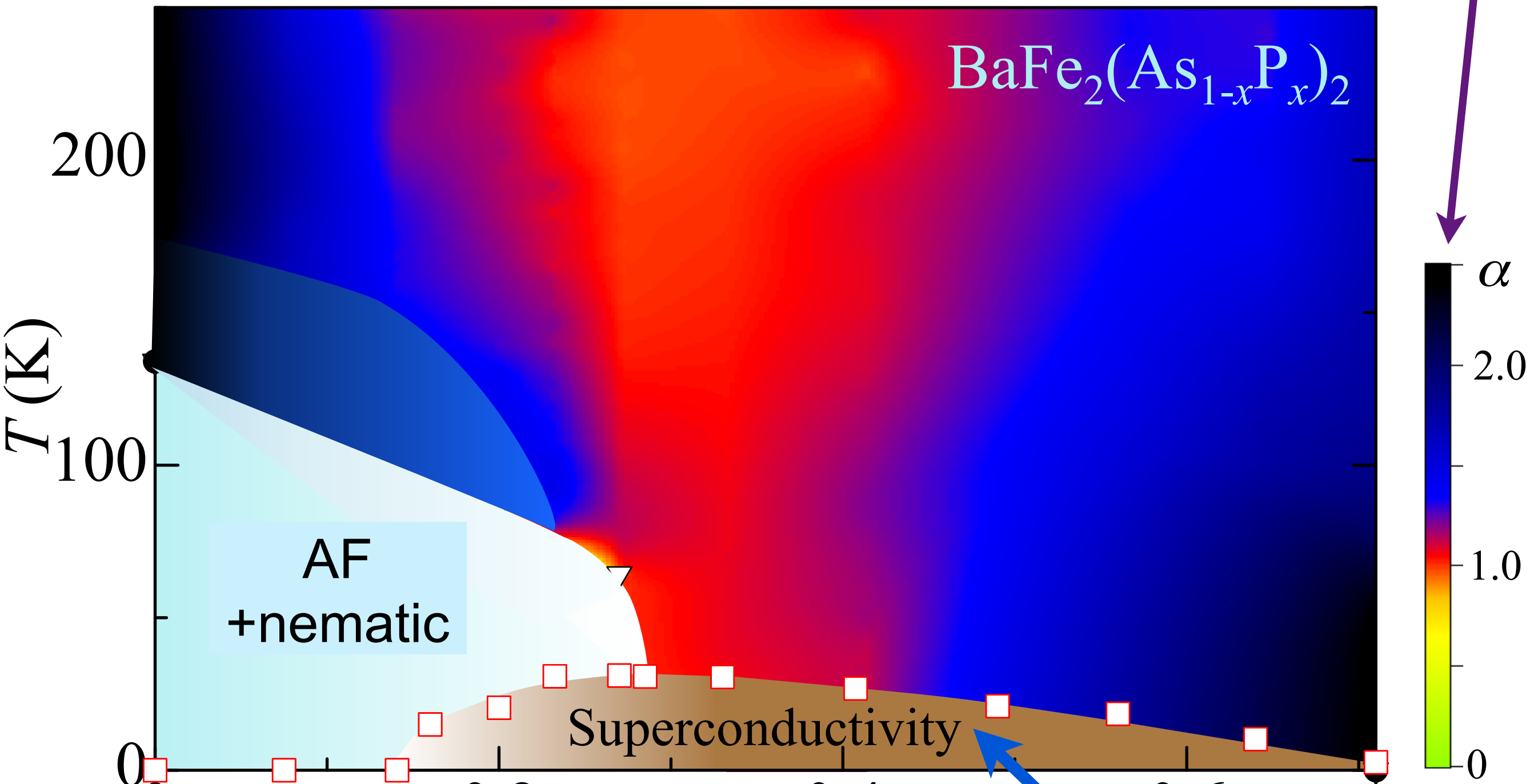


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,  
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,  
*Physical Review B* **81**, 184519 (2010)



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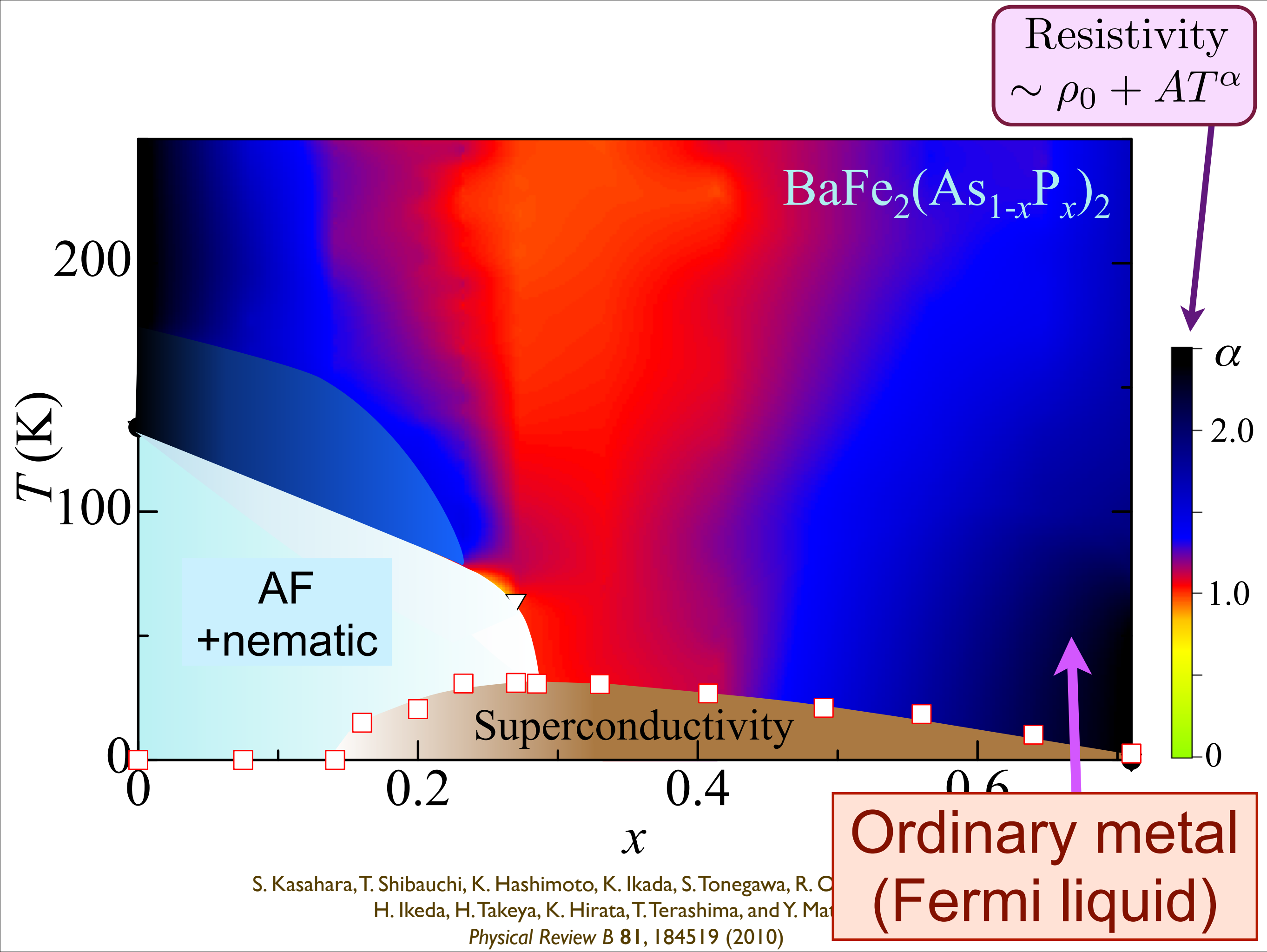
Resistivity  
 $\sim \rho_0 + AT^\alpha$



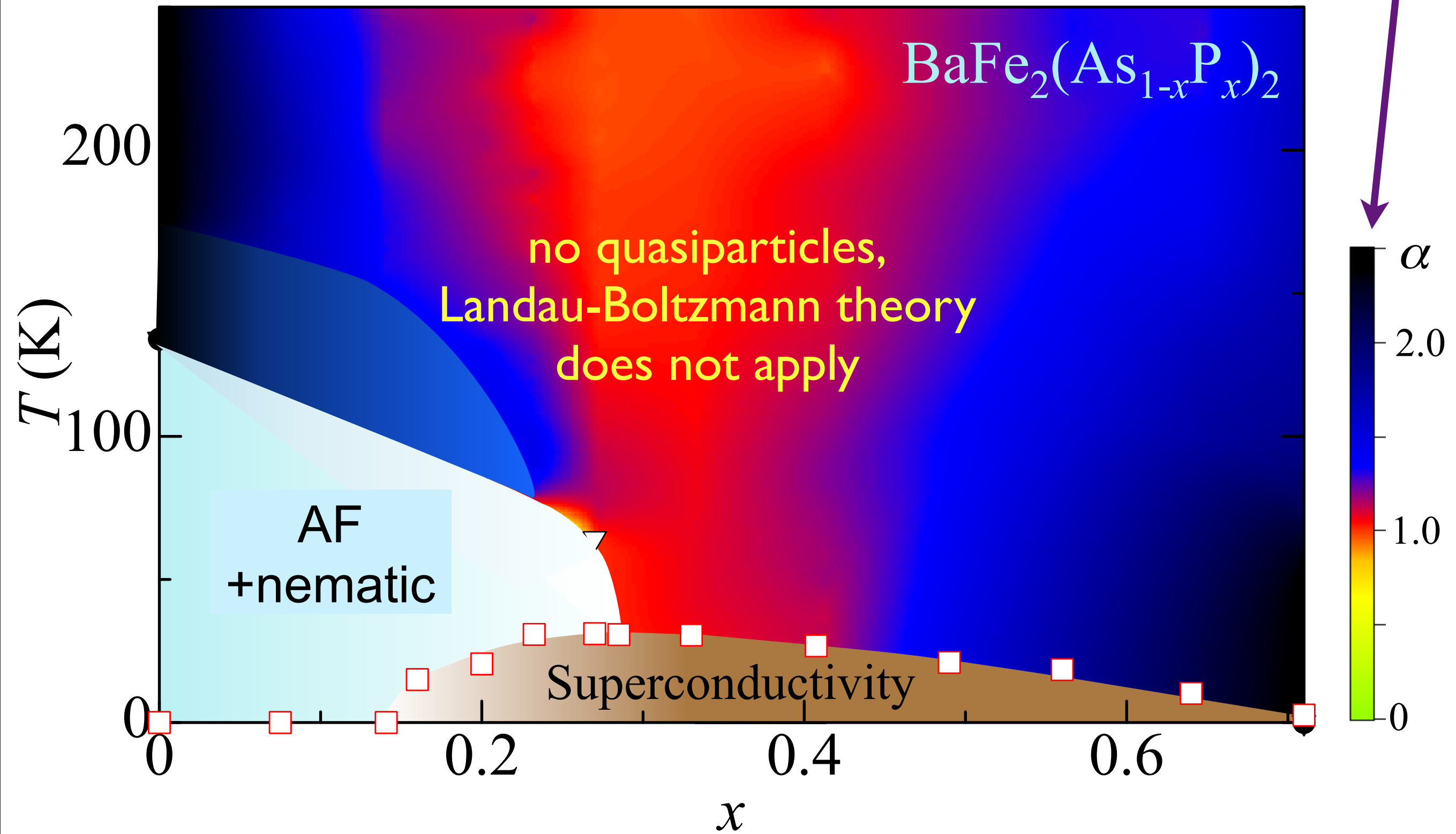
Superconductor  
Bose condensate of pairs of electrons

S. Kasahara, T. Shiba  
H. Ike



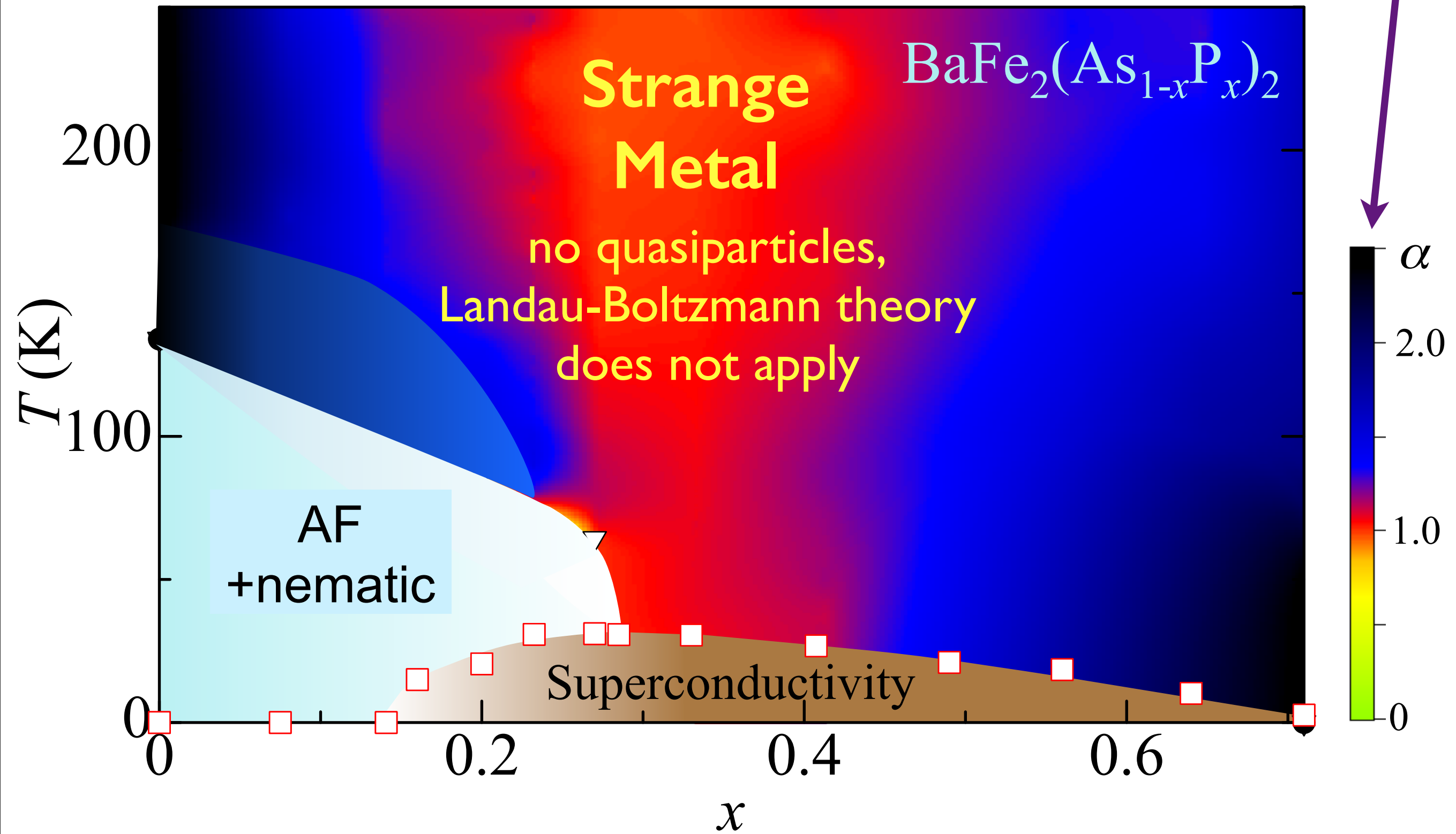


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# Outline

## 1. The simplest model without quasiparticles

*Superfluid-insulator transition*

*of ultracold bosonic atoms in an optical lattice*

*(Conformal field theories in  $2+1$  dimensions)*

## 2. Strange metals in the high $T_c$ superconductors

*Non-quasiparticle transport at the*

*Ising-nematic quantum critical point*



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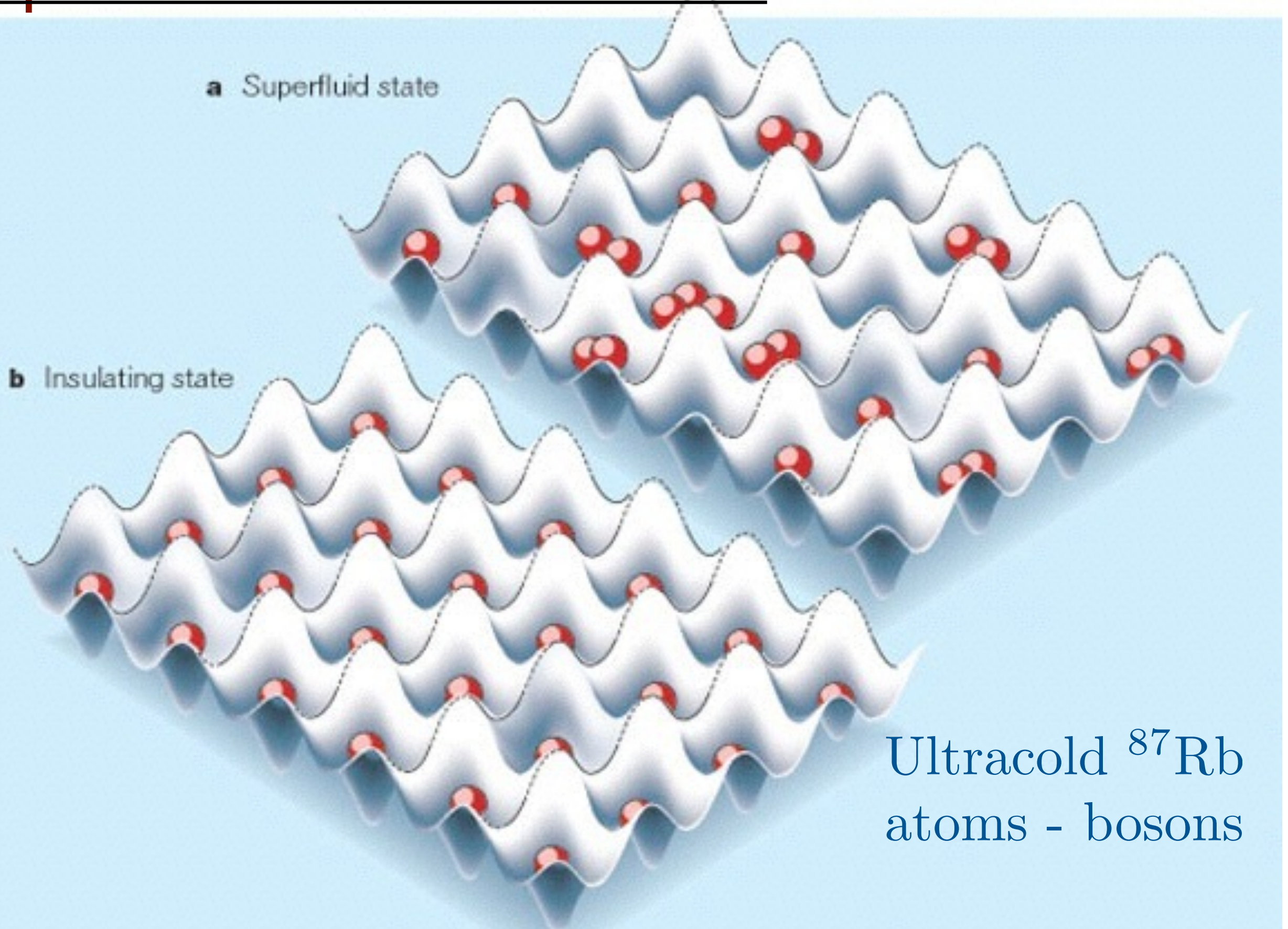
*(Conformal field theories in  $2+1$  dimensions)*

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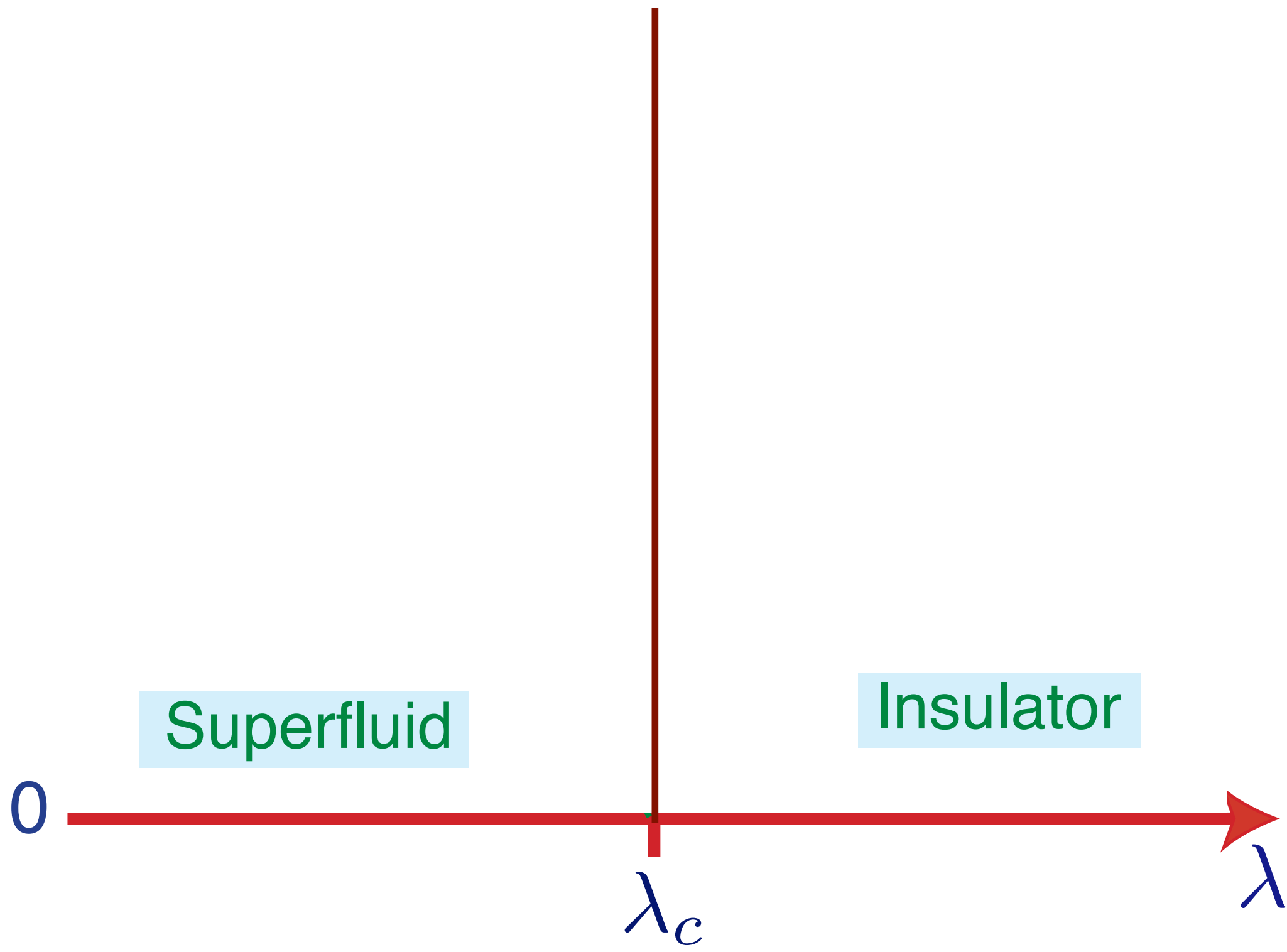
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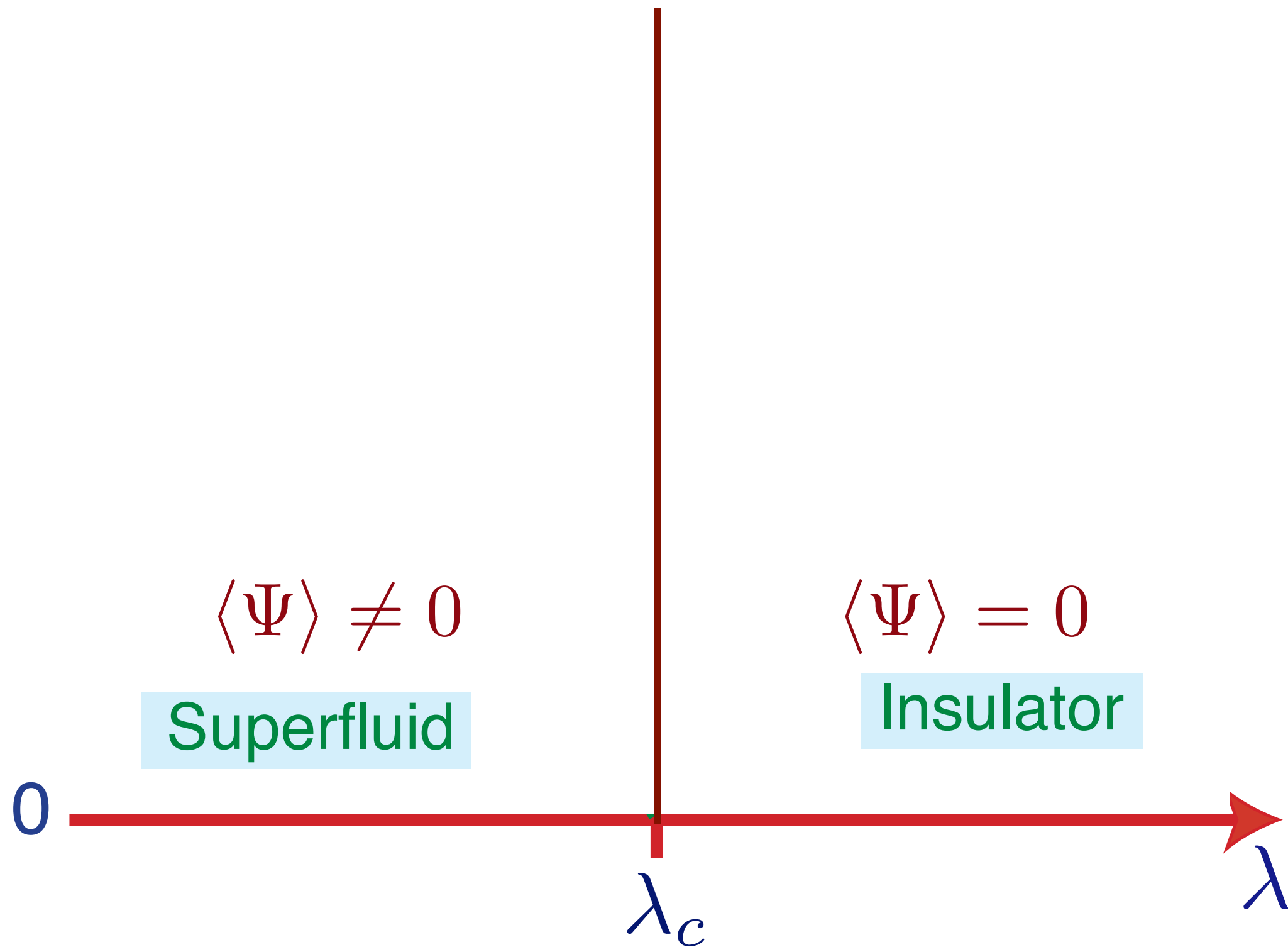
# Superfluid-insulator transition



Ultracold  $^{87}\text{Rb}$   
atoms - bosons



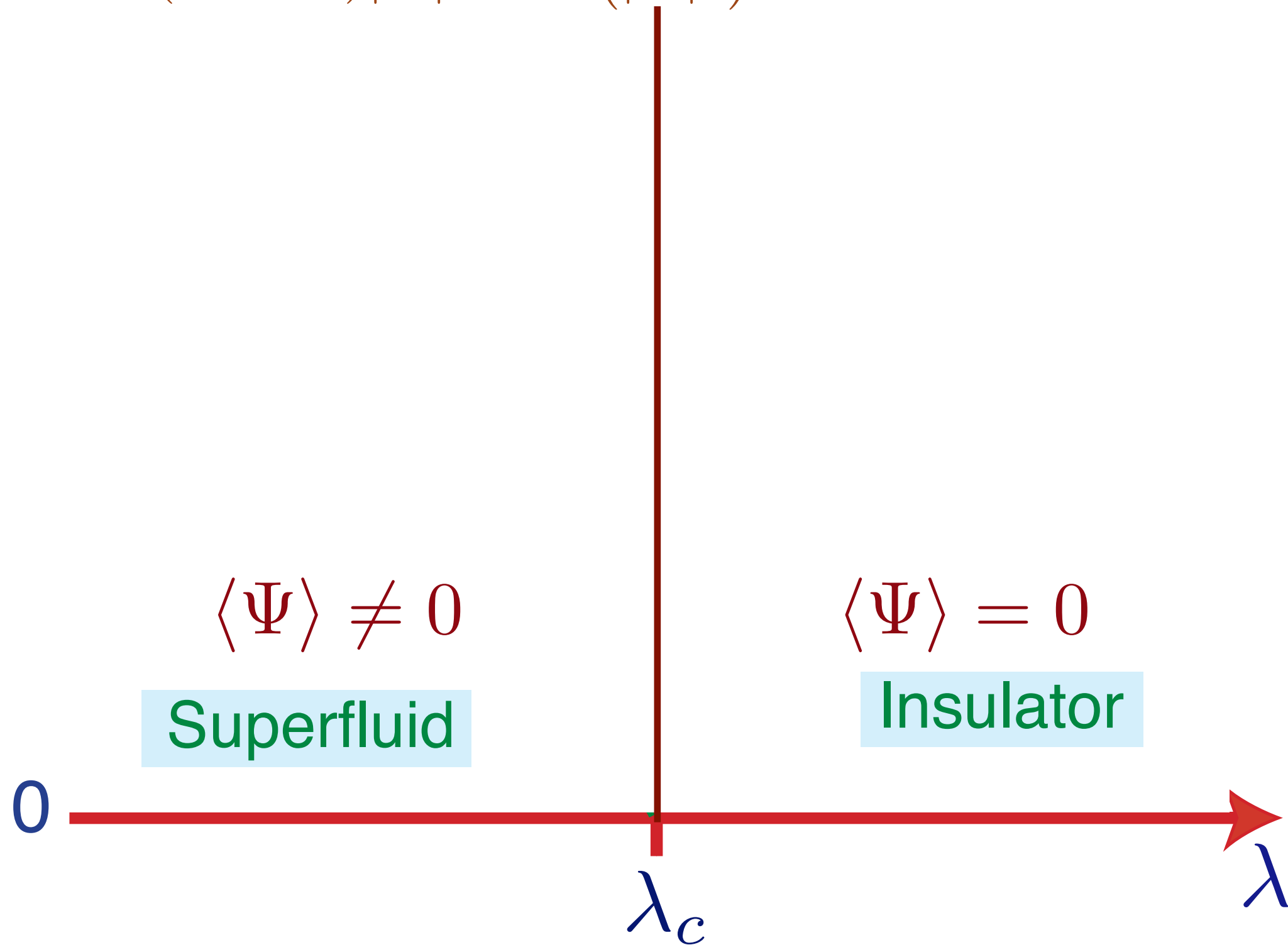
$\Psi \rightarrow$  a complex field representing the Bose-Einstein condensate of the superfluid





$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

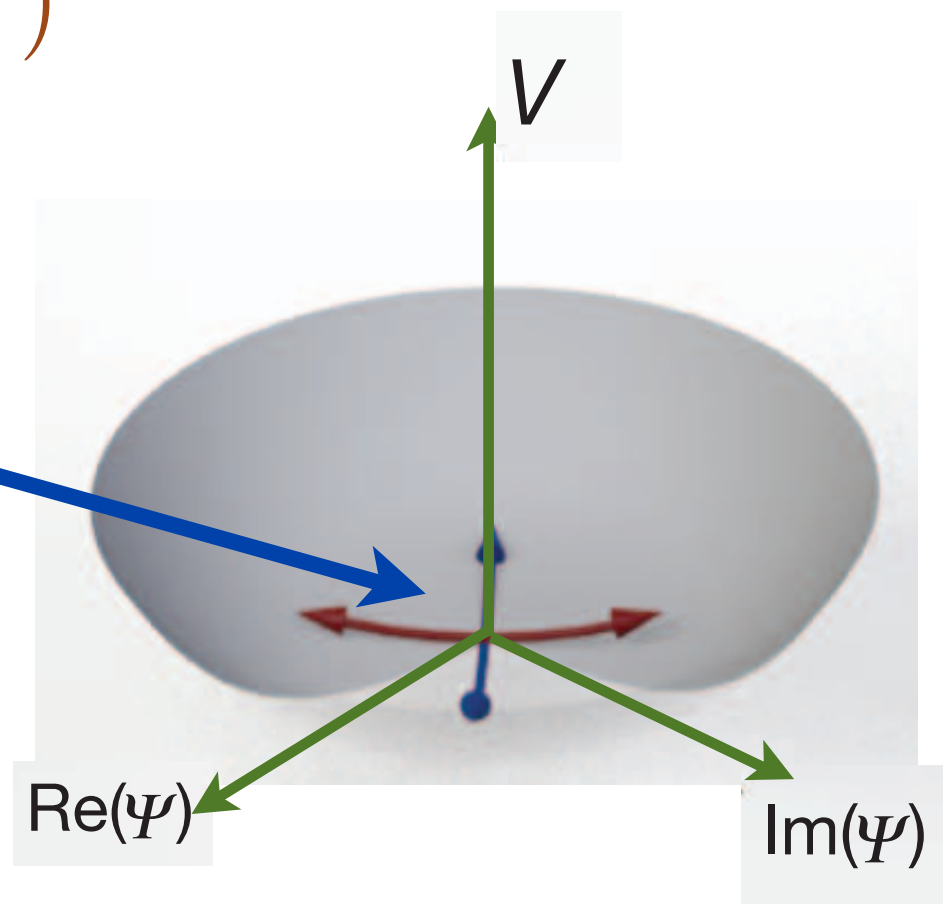
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



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Particles and holes correspond to the 2 normal modes in the oscillation of  $\Psi$  about  $\Psi = 0$ .



$$\langle \Psi \rangle \neq 0$$

Superfluid

$$\langle \Psi \rangle = 0$$

Insulator

0

$\lambda_c$

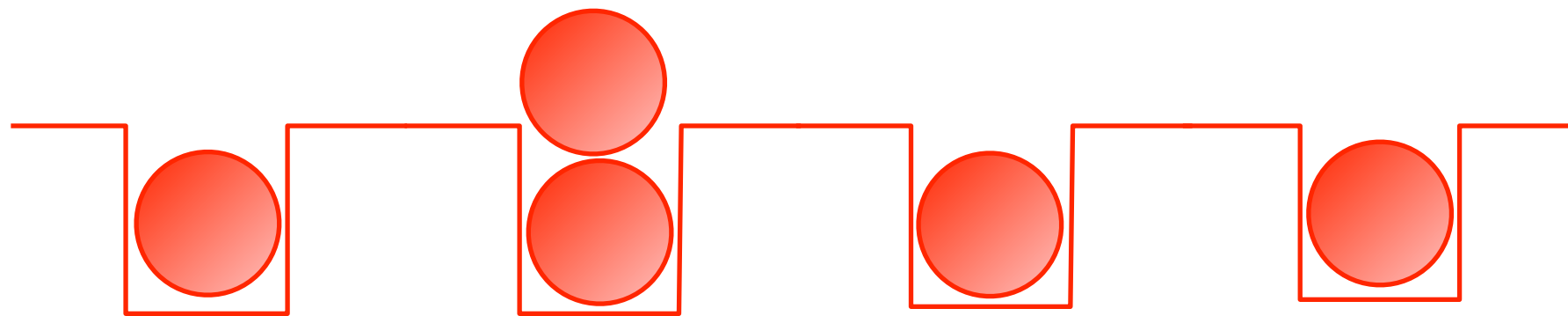
$\lambda$



Insulator (the vacuum)  
at large repulsion between bosons

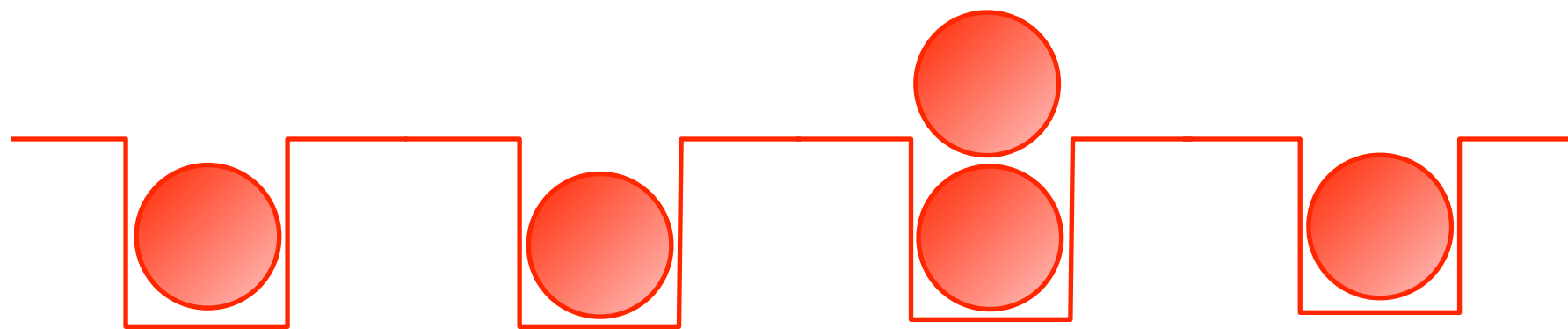
$$|\text{Ground state}\rangle = \prod_i b_i^\dagger |0\rangle$$

Excitations of the insulator:



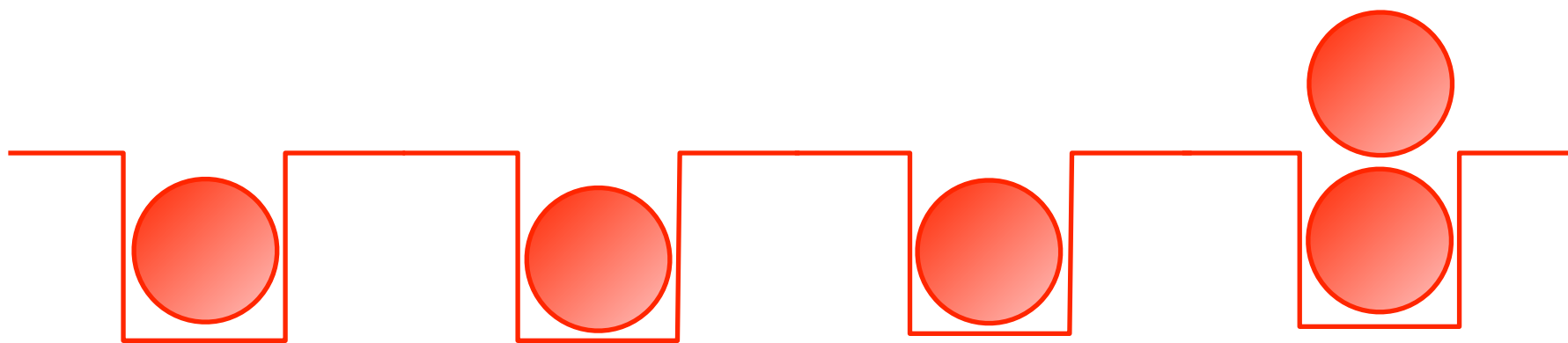
Particles  $\sim \psi^\dagger$

Excitations of the insulator:



Particles  $\sim \psi^\dagger$

Excitations of the insulator:



Particles  $\sim \psi^\dagger$



Excitations of the insulator:



Holes  $\sim \psi$

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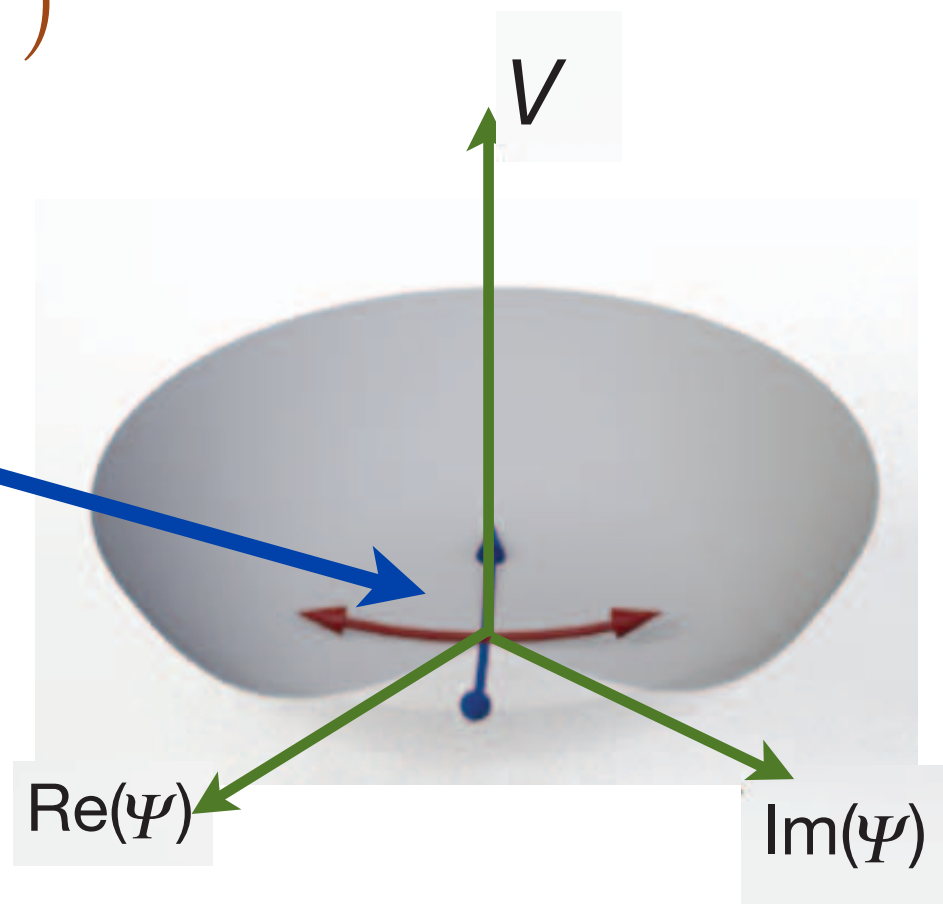


Holes  $\sim \psi$

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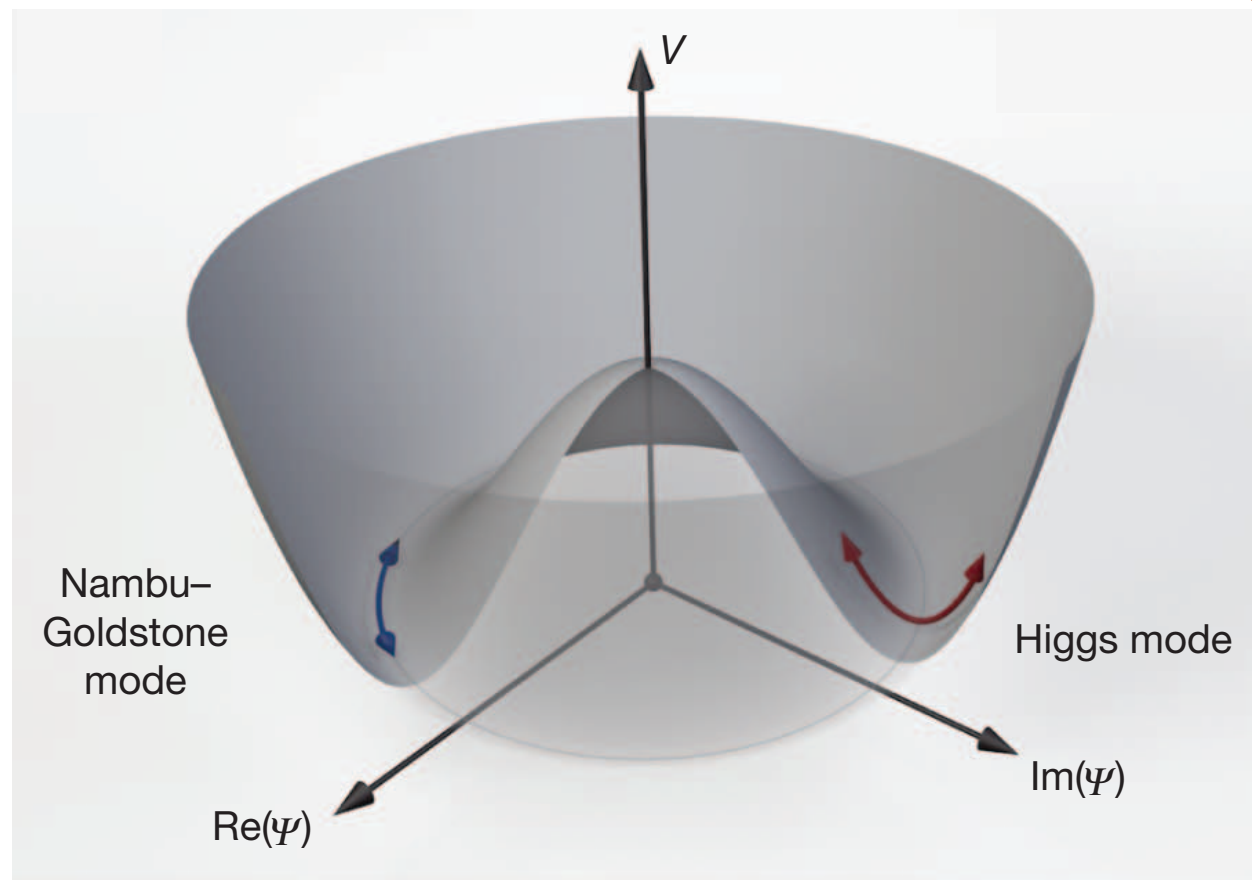
0

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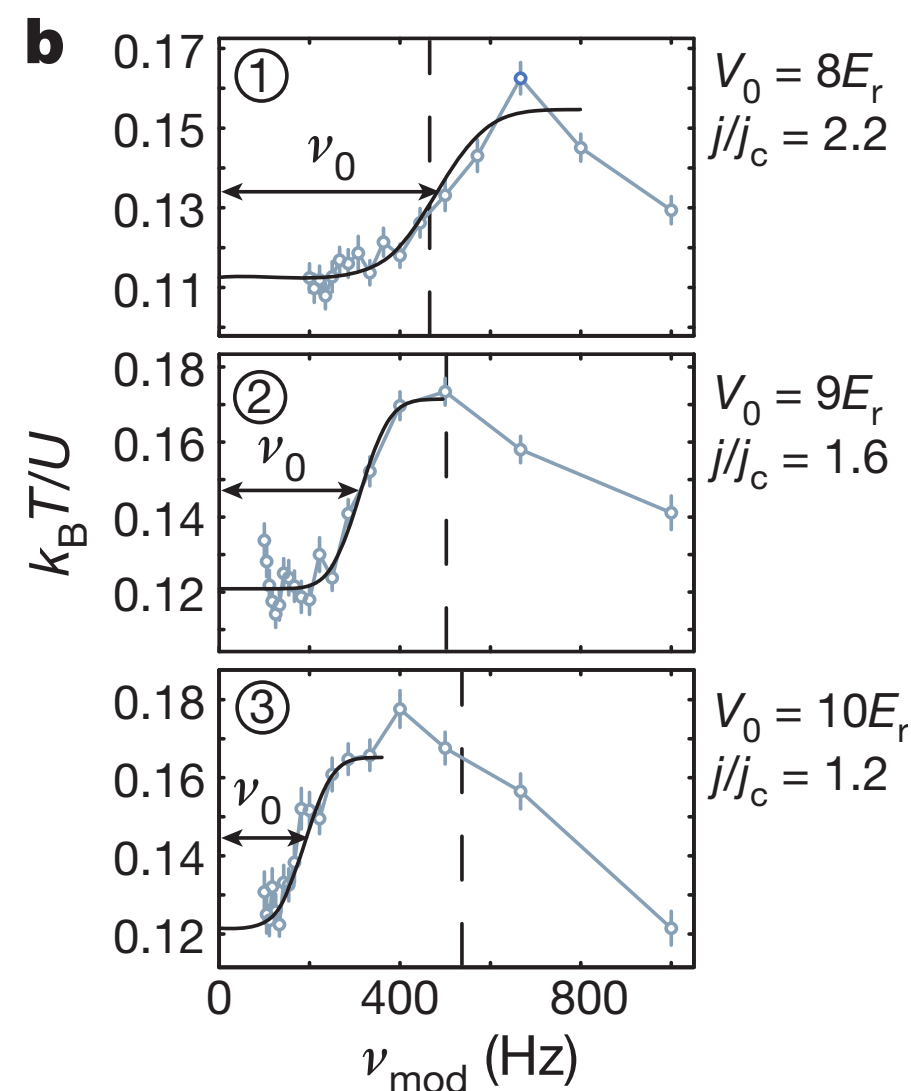
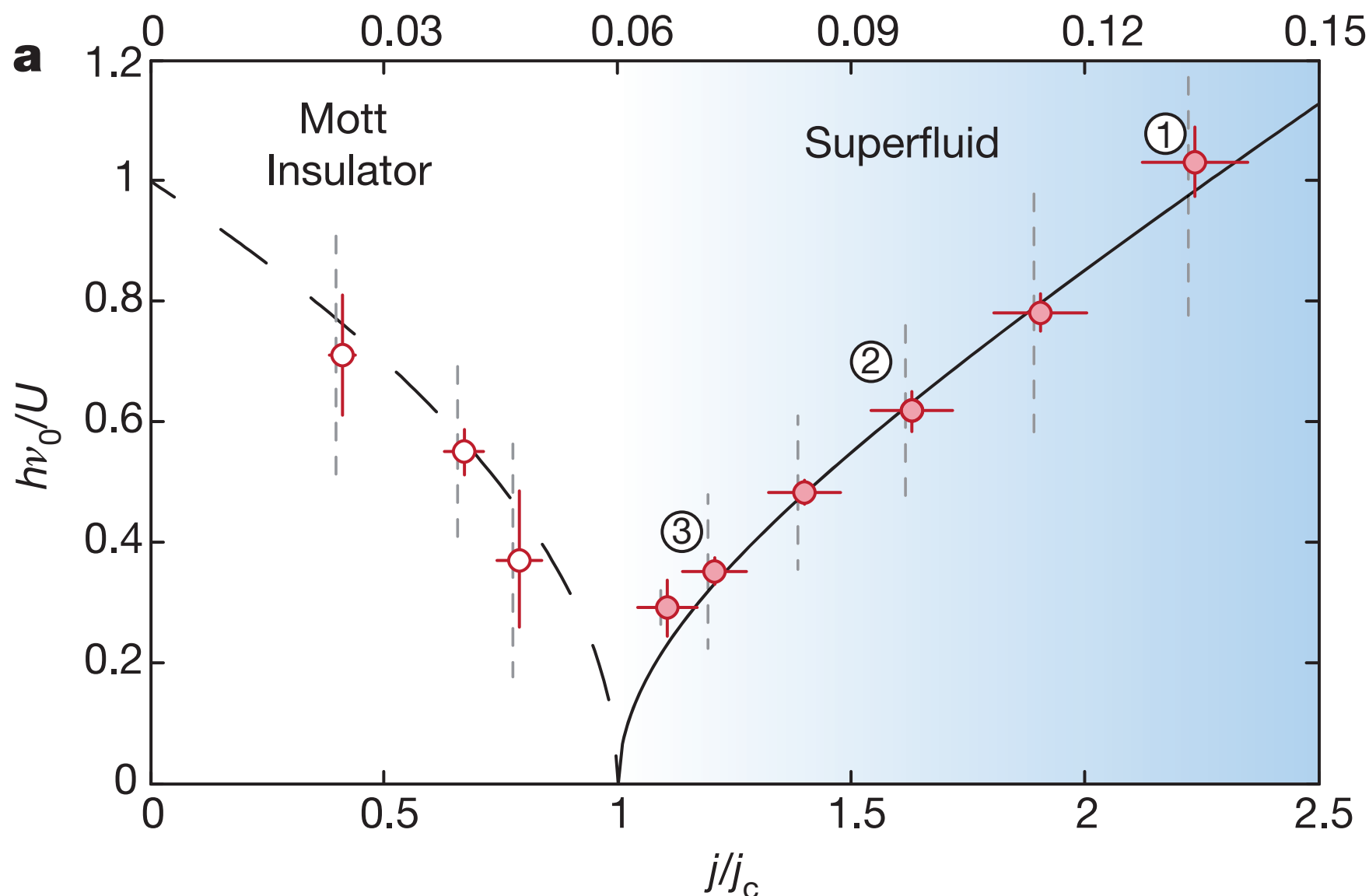
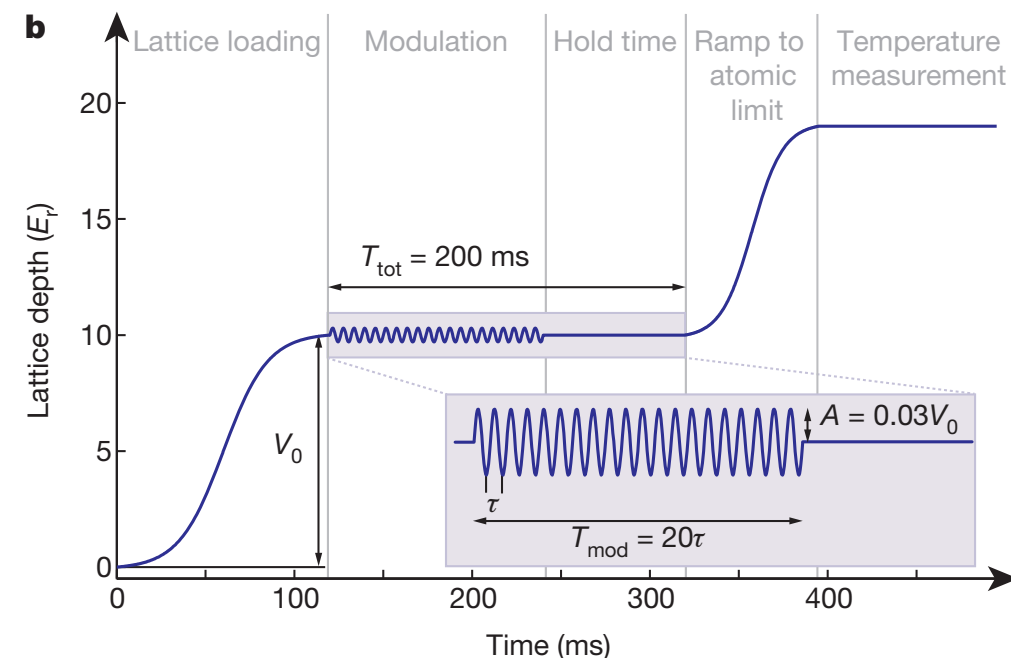
0

$\lambda_c$

$\lambda$

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole



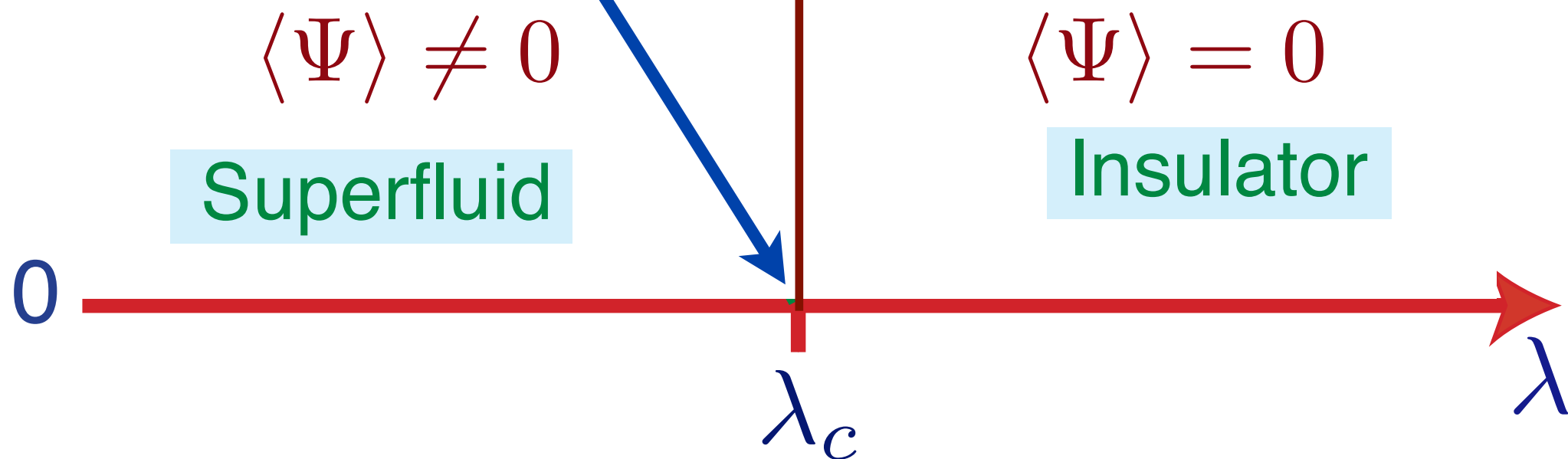
Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).



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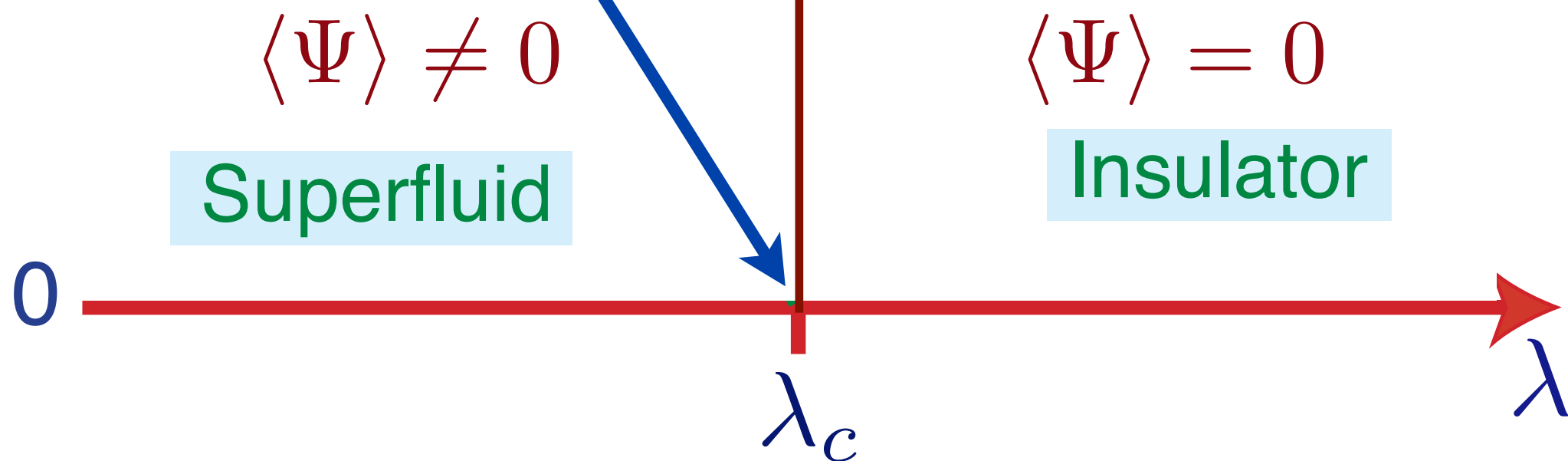
Quantum state with  
“long-range” quantum entanglement  
**and no quasiparticles.**

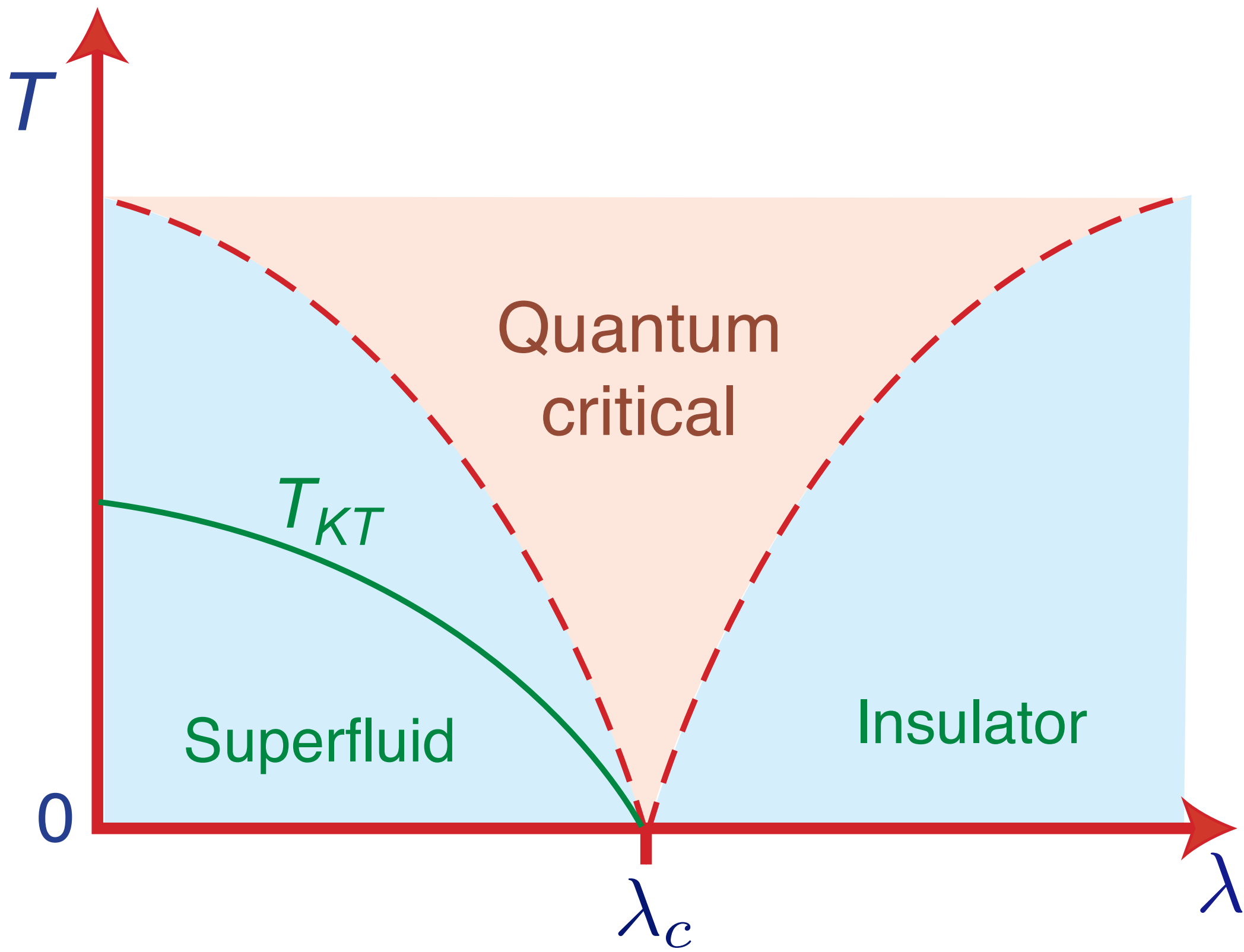


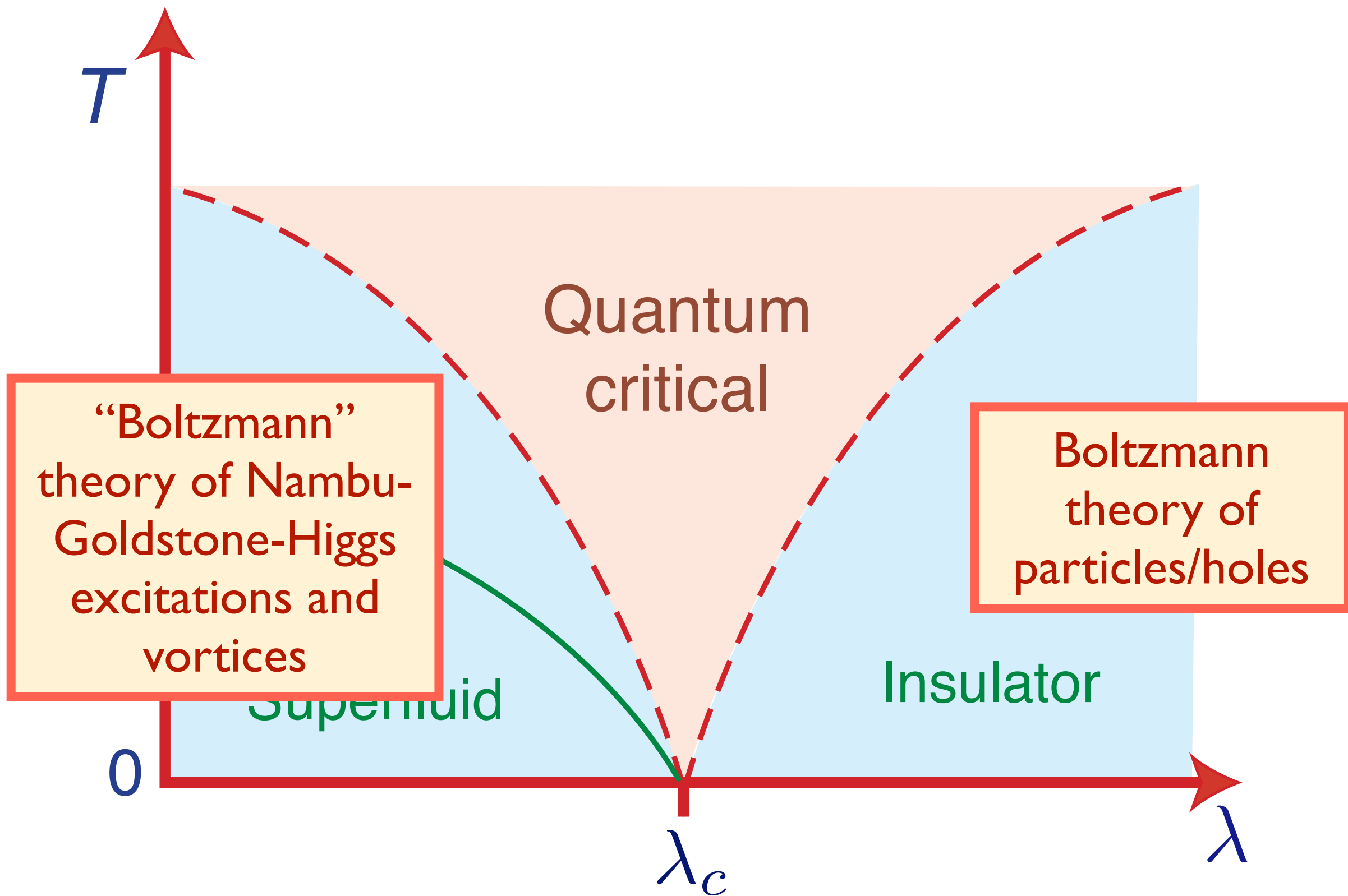
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A conformal field theory  
in 2+1 spacetime dimensions:  
a CFT3







$T$

0

$\lambda_c$

$\lambda$

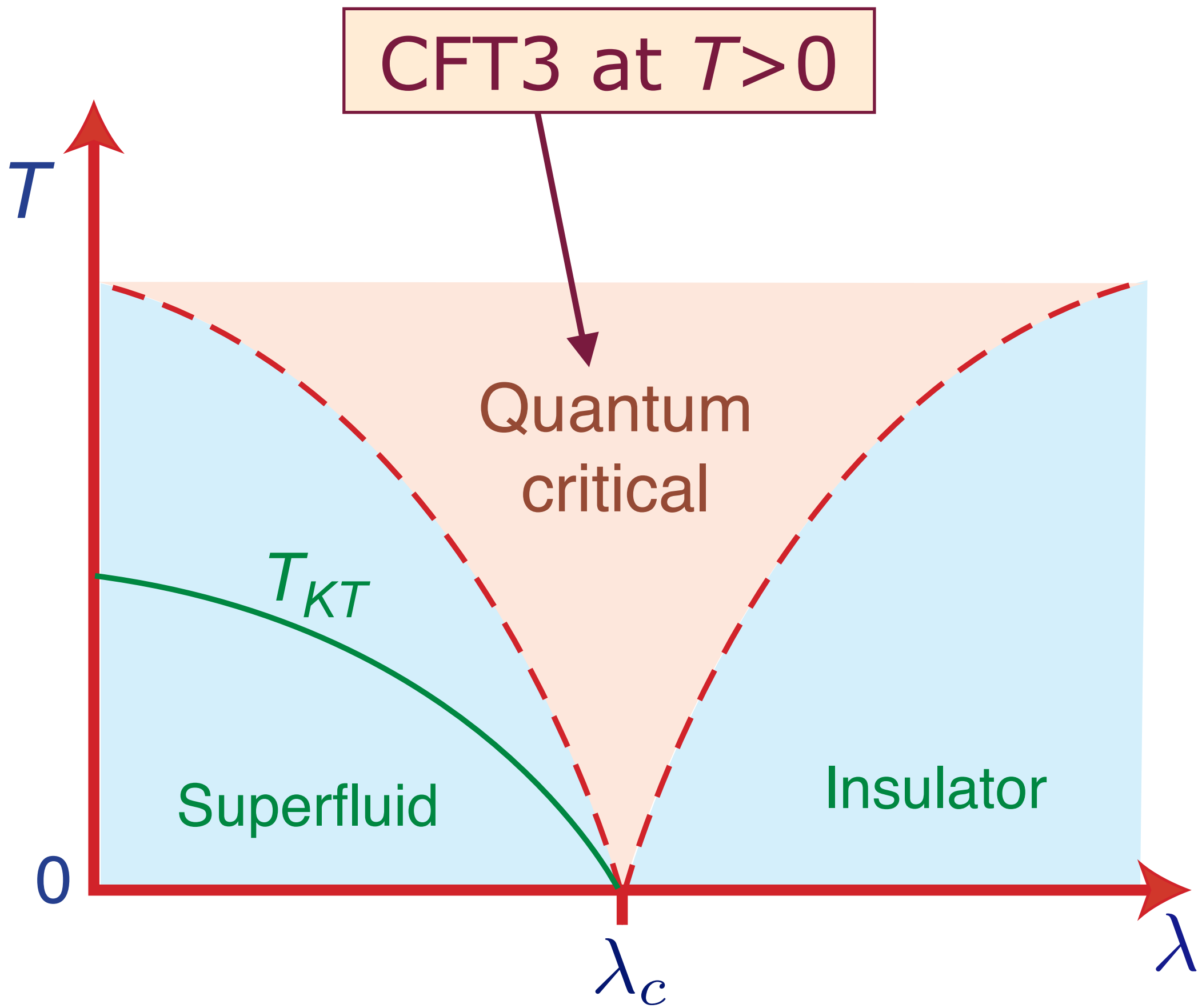
Quantum critical

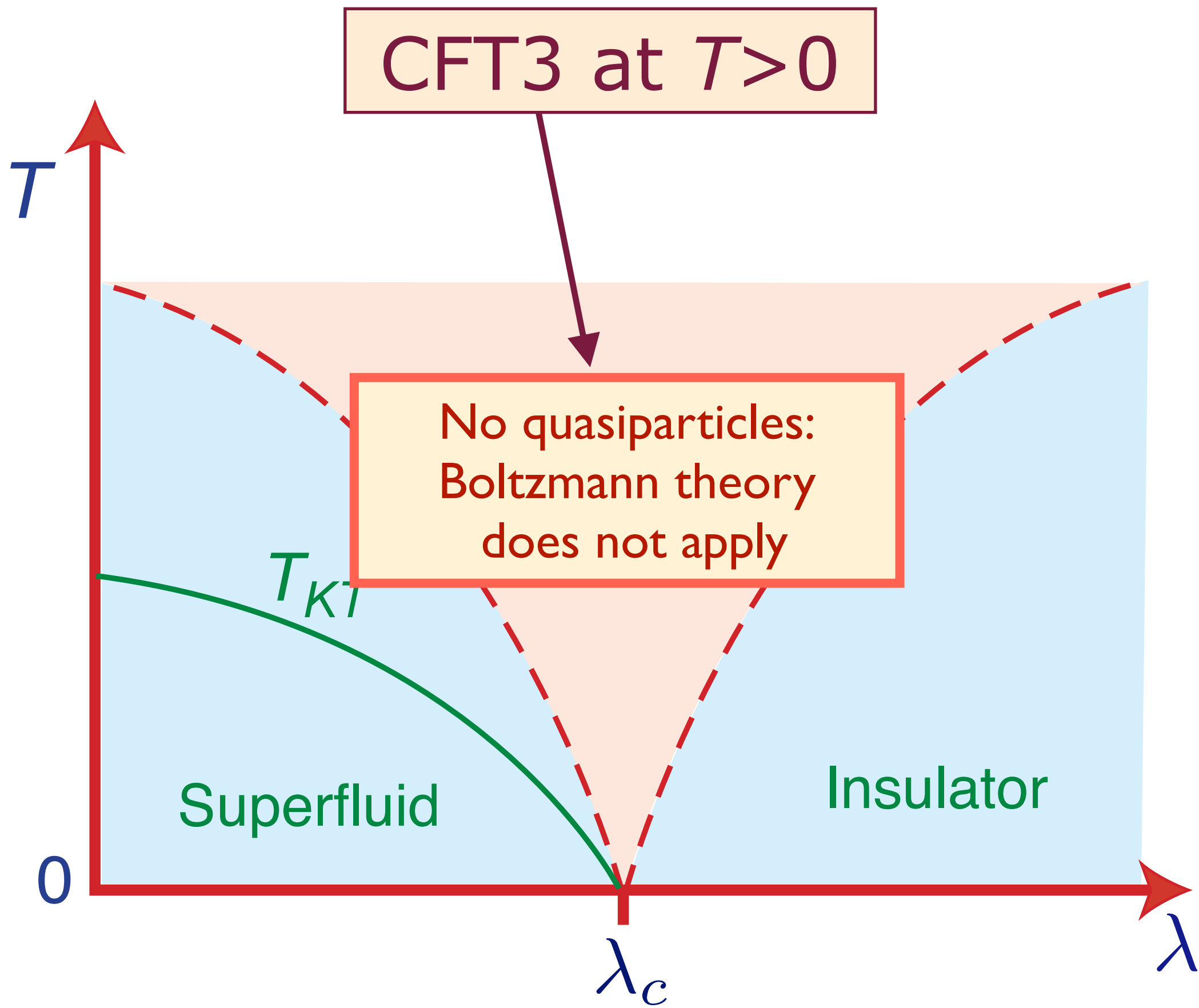
“Boltzmann” theory of Nambu-Goldstone-Higgs excitations and vortices

Boltzmann theory of particles/holes

Superfluid

Insulator





# Traditional CMT

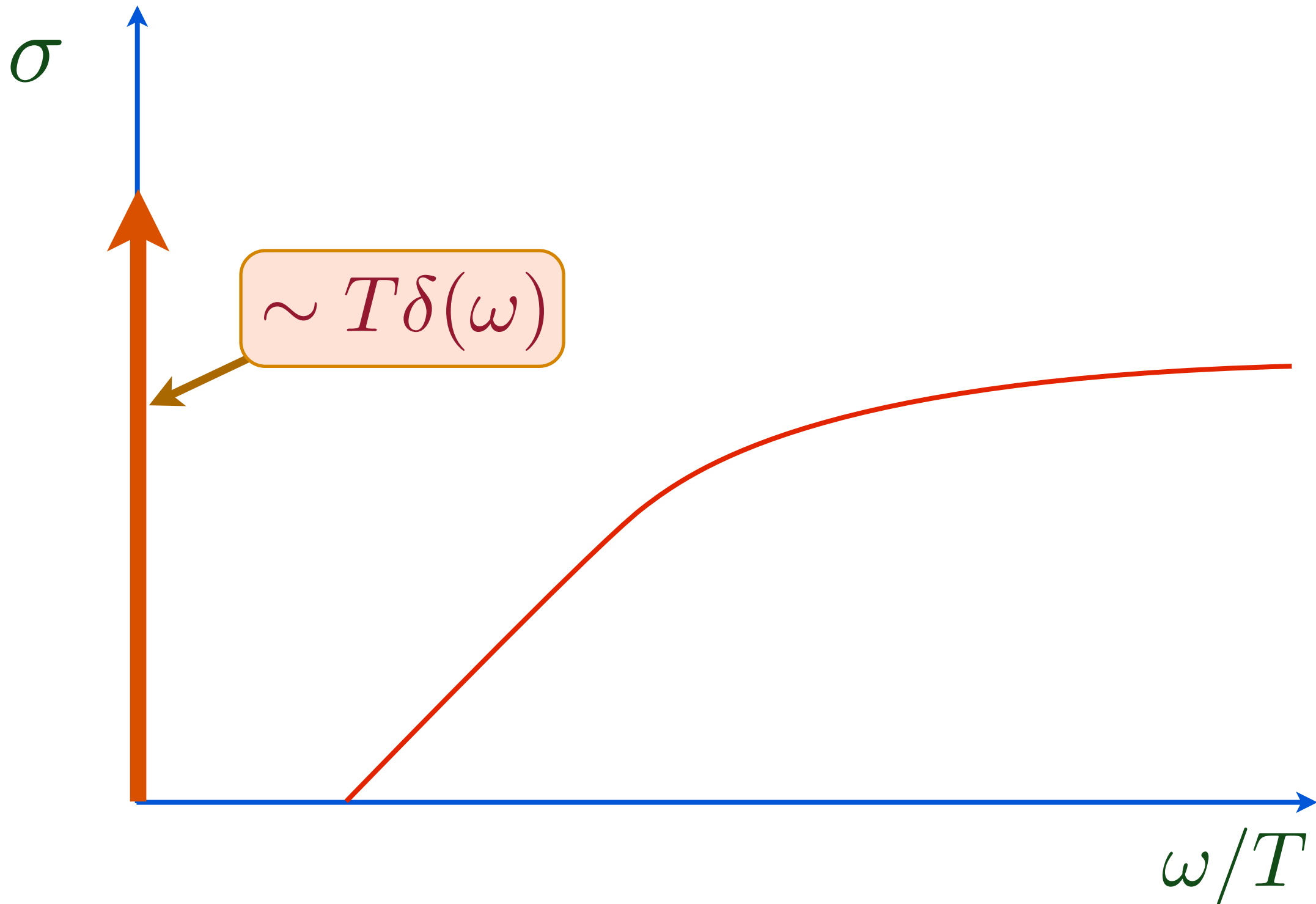
- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)



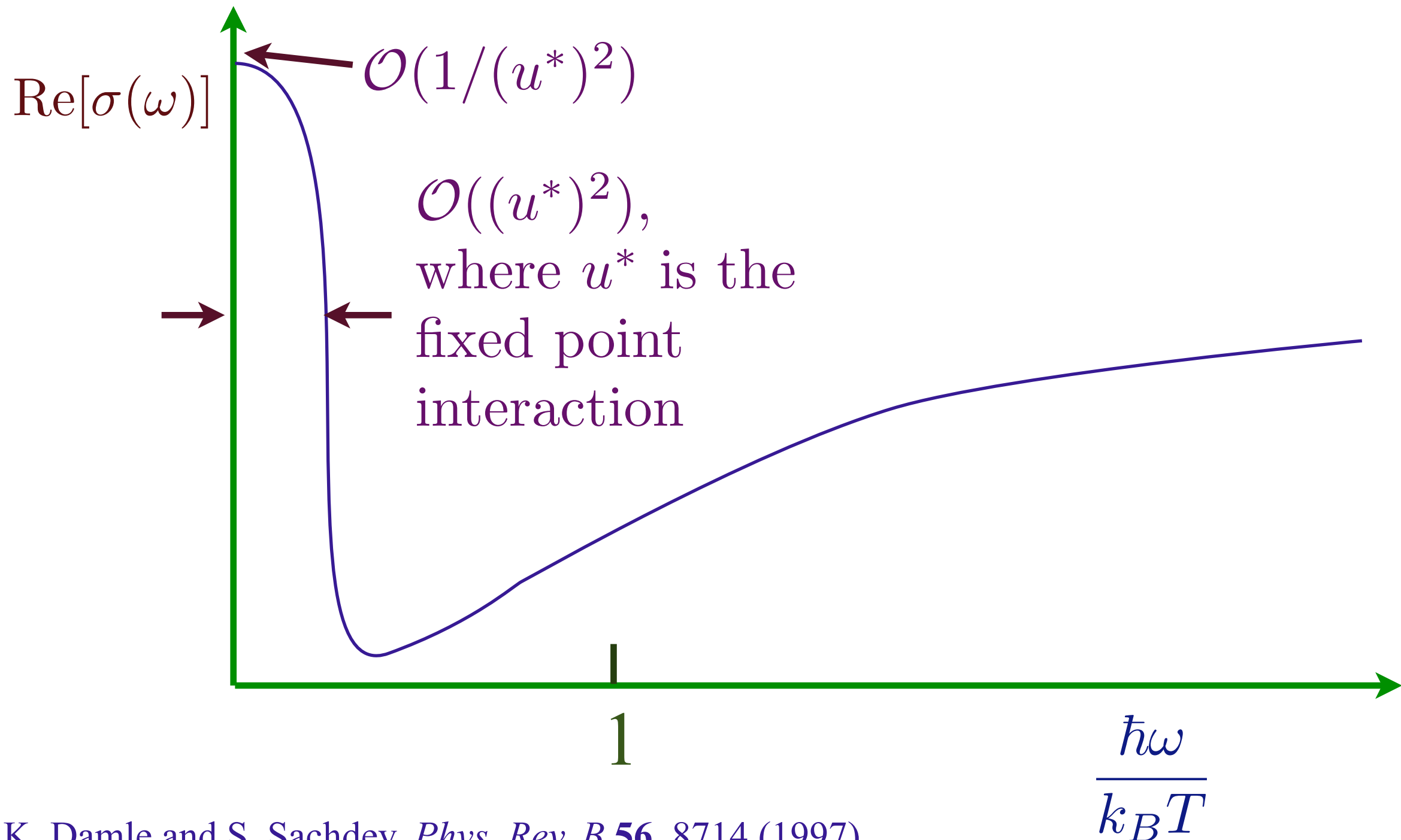
# Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

Quasiparticle view of quantum criticality (Boltzmann equation):  
Electrical transport for a free CFT3



# Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

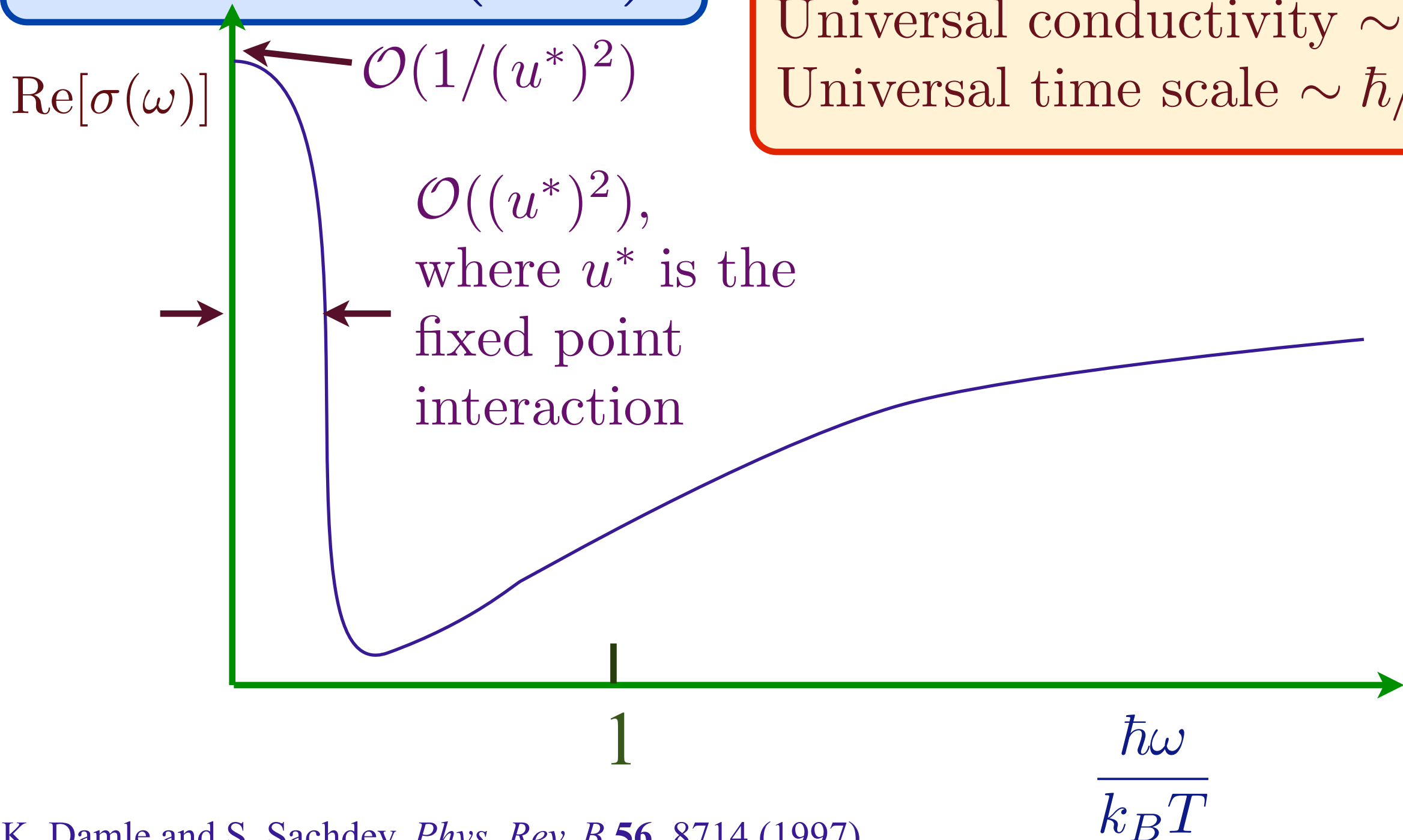


# Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left( \frac{\hbar\omega}{k_B T} \right)$$

$\Sigma \rightarrow$  a universal function

Universal conductivity  $\sim e^2/h$   
Universal time scale  $\sim \hbar/k_B T$



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## Dynamics without quasiparticles

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT

## Basic characteristics of CFTs

Ordinary quantum field theories are characterized by their particle spectrum, and the  $S$ -matrices describing interactions between the particles. The analog of these concepts for CFTs are the *primary operators*  $O_a(x)$  and their *operator product expansions* (OPEs). Each primary operator is associated with a scaling dimension  $\Delta_a$ , defined by the ( $T = 0$ ) expectation value (for the simplest case of scalar operators):

$$\langle O_a(x) O_b(0) \rangle = \frac{\delta_{ab}}{|x|^{2\Delta_a}}$$



## Basic characteristics of CFTs

The OPE describes what happens when two operators come together at a single spacetime point (considering scalar operators only)

$$\lim_{x' \rightarrow x} \langle O_a(x') O_b(x) O_c(0) \rangle = \frac{f_{abc}}{|x|^{\Delta_a + \Delta_b + \Delta_c}}$$

The values of  $\{\Delta_a, f_{abc}\}$  determine (in principle) all observable properties of the CFT, as constrained by a complex set of conformal Ward identities.

For the Wilson-Fisher CFT<sub>3</sub>, systematic methods exist to compute (in principle) all the  $\{\Delta_a, f_{abc}\}$ , and we will assume this data is *known*. This knowledge will be taken as an *input* to the computation of the finite  $T$  dynamics

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- Relate OPE co-efficients to couplings of an effective gravitational theory on AdS

# AdS/CFT correspondence at zero temperature

$\text{AdS}_4$

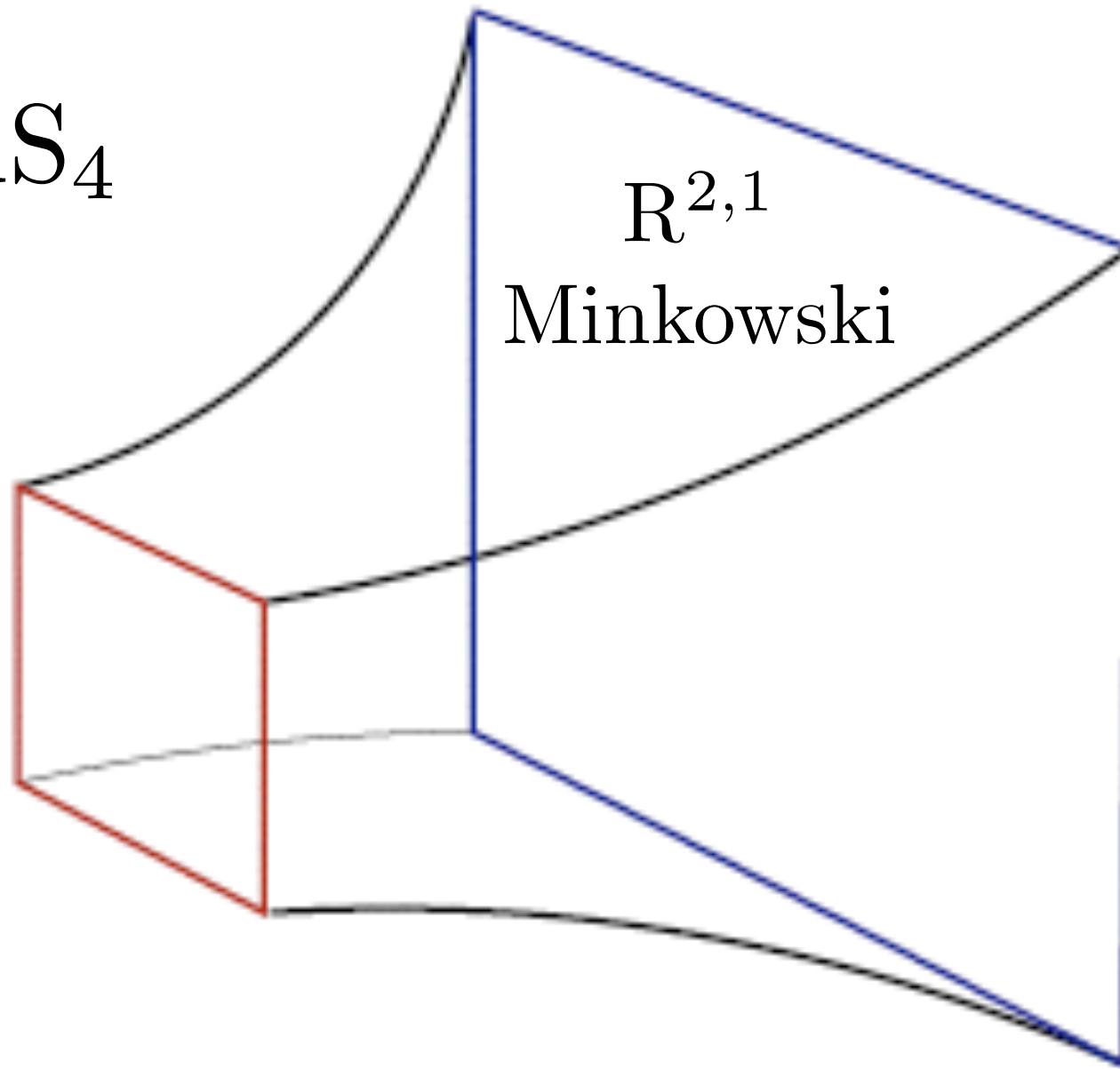
$\mathbb{R}^{2,1}$   
Minkowski

CFT3

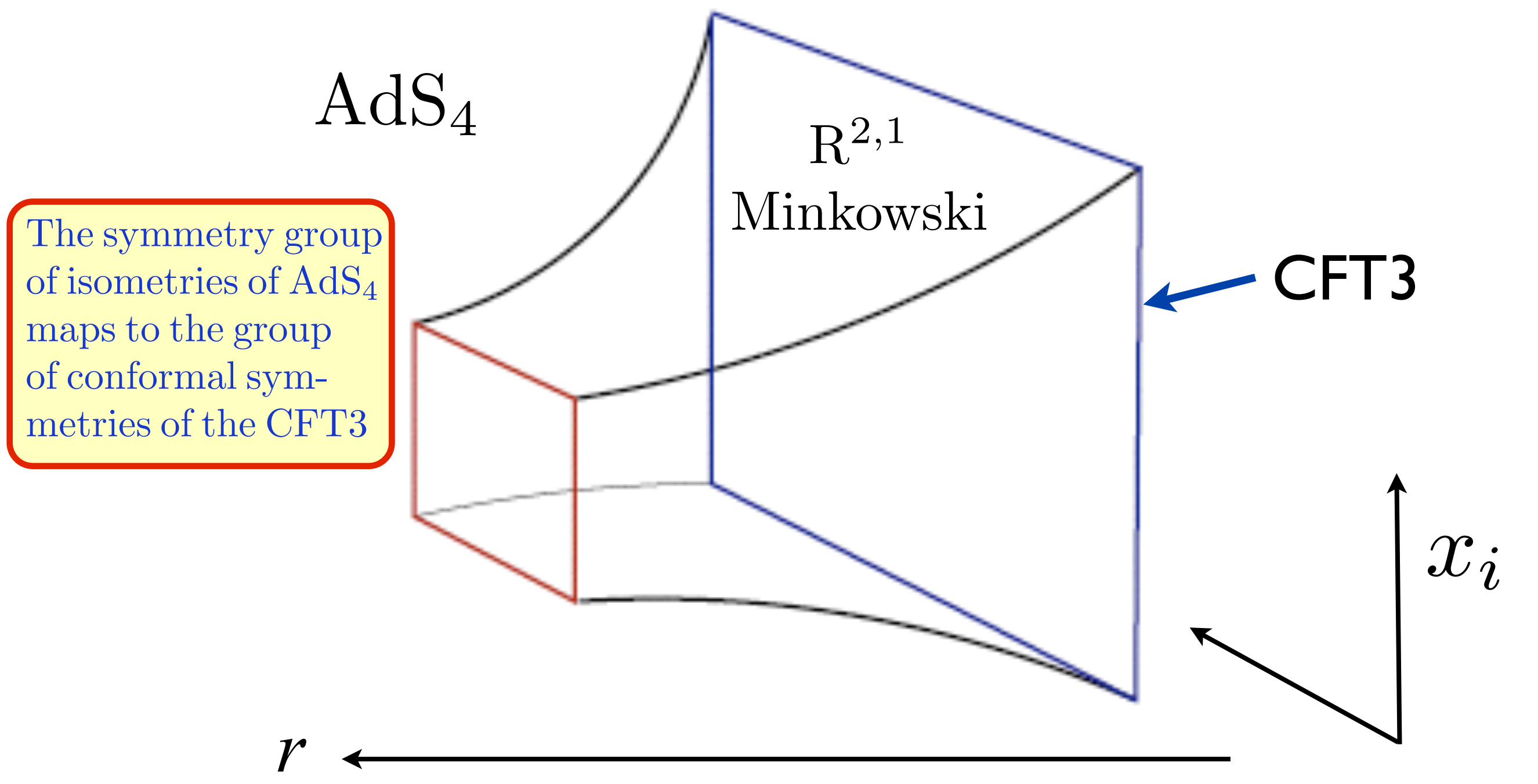
The symmetry group of isometries of  $\text{AdS}_4$  maps to the group of conformal symmetries of the CFT3

$r$

$x_i$



# AdS/CFT correspondence at zero temperature



The symmetry group of isometries of AdS<sub>4</sub> maps to the group of conformal symmetries of the CFT<sub>3</sub>

A classical gravitational theory on AdS<sub>4</sub> encodes the CFT<sub>3</sub> data of  $\{\Delta_a, f_{abc}\}$ , and allows computation of CFT<sub>3</sub> correlators consistent with all conformal Ward identities

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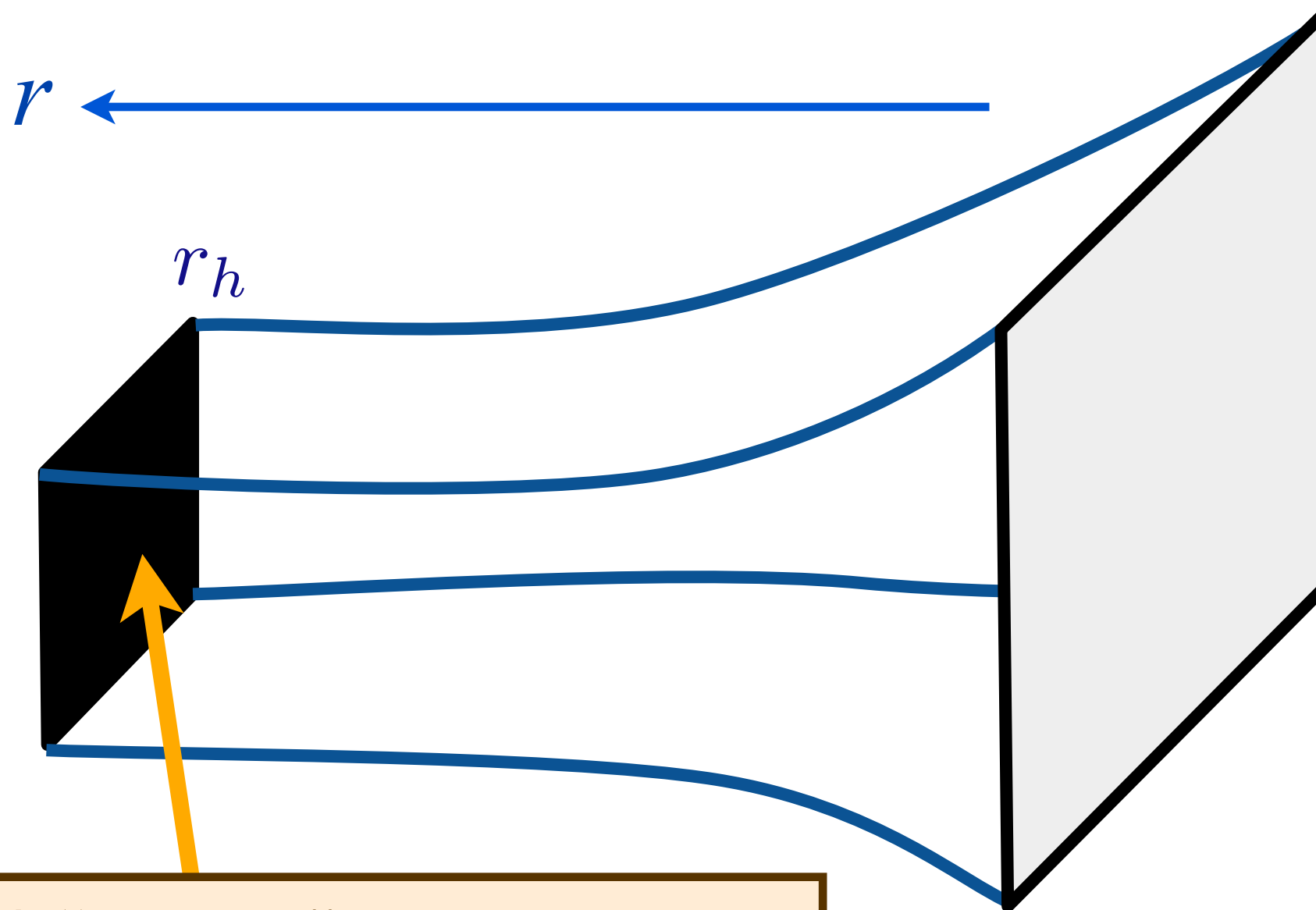
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- Non-zero  $T$  dynamics of CFT maps to dynamics of a “horizon” in (Einstein’s) gravitational theory

# Gauge-gravity duality at non-zero temperatures

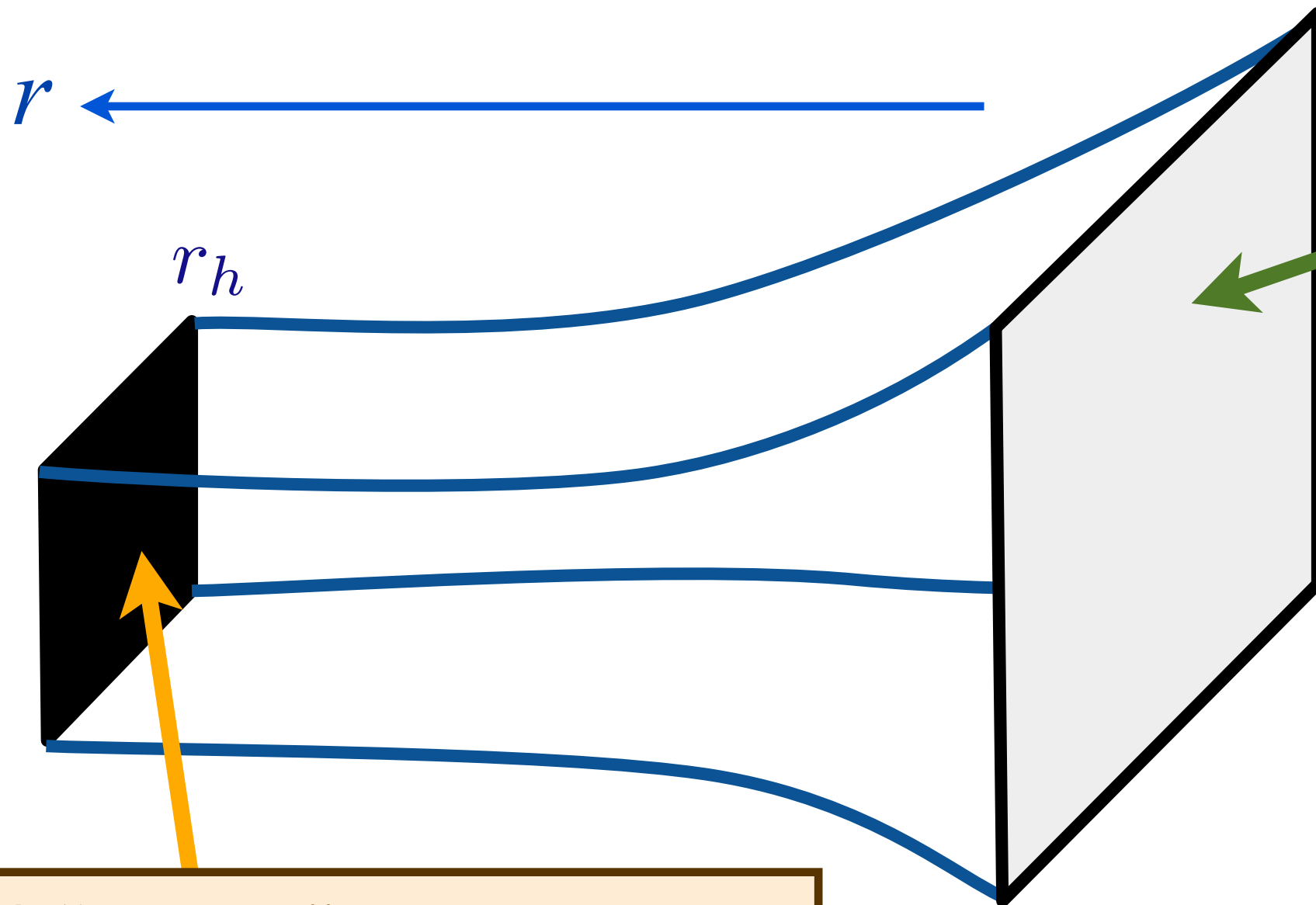


A “horizon”, similar to the surface of a black hole !

There is a family of solutions of Einstein's equations which are  $\text{AdS}_4$  as  $r \rightarrow 0$ , but which have horizons at  $r = r_h$ .



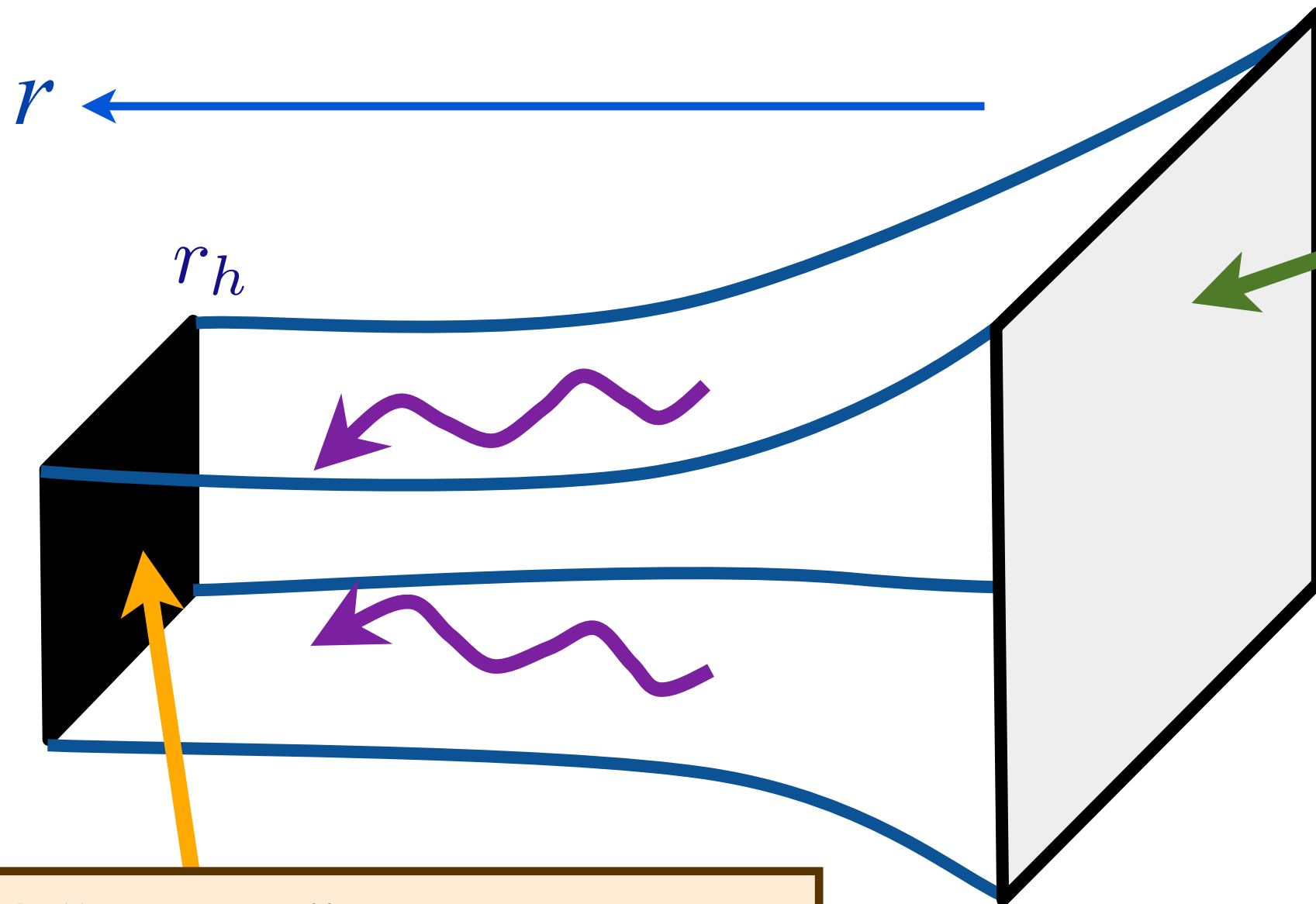
# Gauge-gravity duality at non-zero temperatures



A CFT3 at a temperature  $T \sim 1/r_h$  equal to the Hawking temperature of the horizon.

A “horizon”, similar to the surface of a black hole !

# Gauge-gravity duality at non-zero temperatures



A CFT3 at a temperature  $T \sim 1/r_h$  equal to the Hawking temperature of the horizon.

A “horizon”, similar to the surface of a black hole !

Dissipation and friction in the CFT3 = waves falling past the horizon

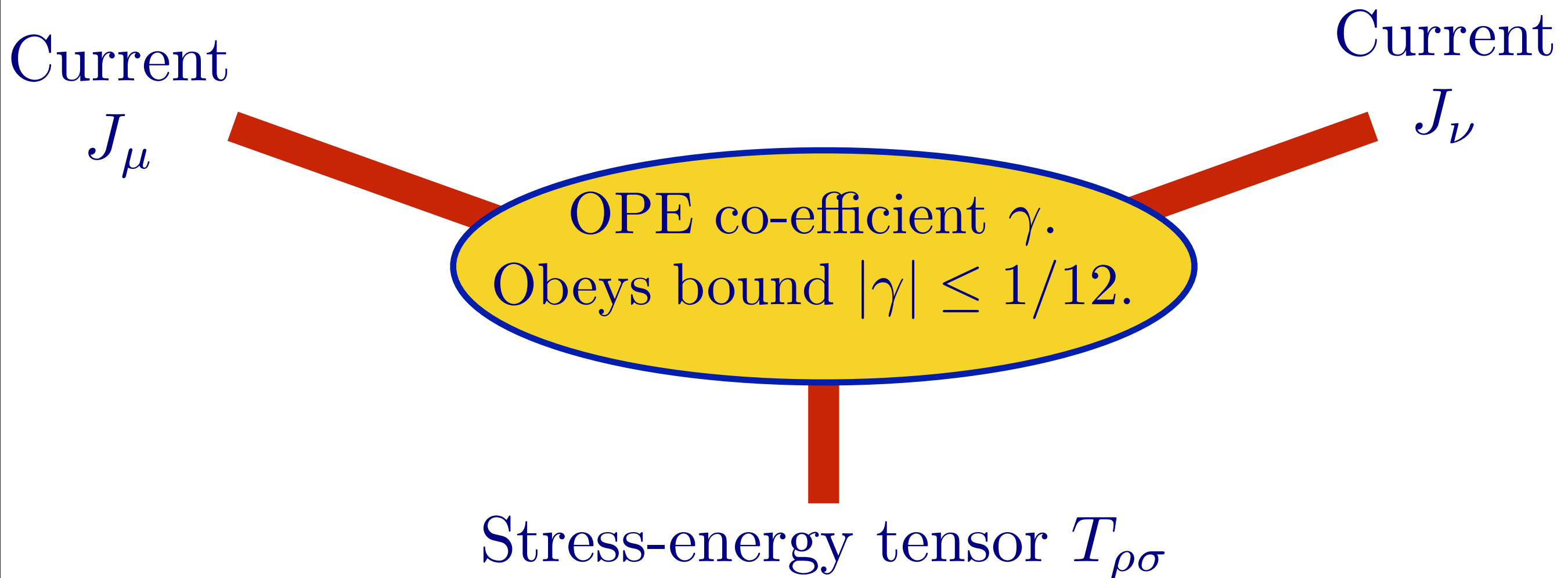
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# Physical picture of electrical transport in a CFT3



Conductivity at  $T > 0$  determined by  
“scattering” of current by  
thermal stress-energy tensor.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Physical Review B* **87**, 085138 (2013).

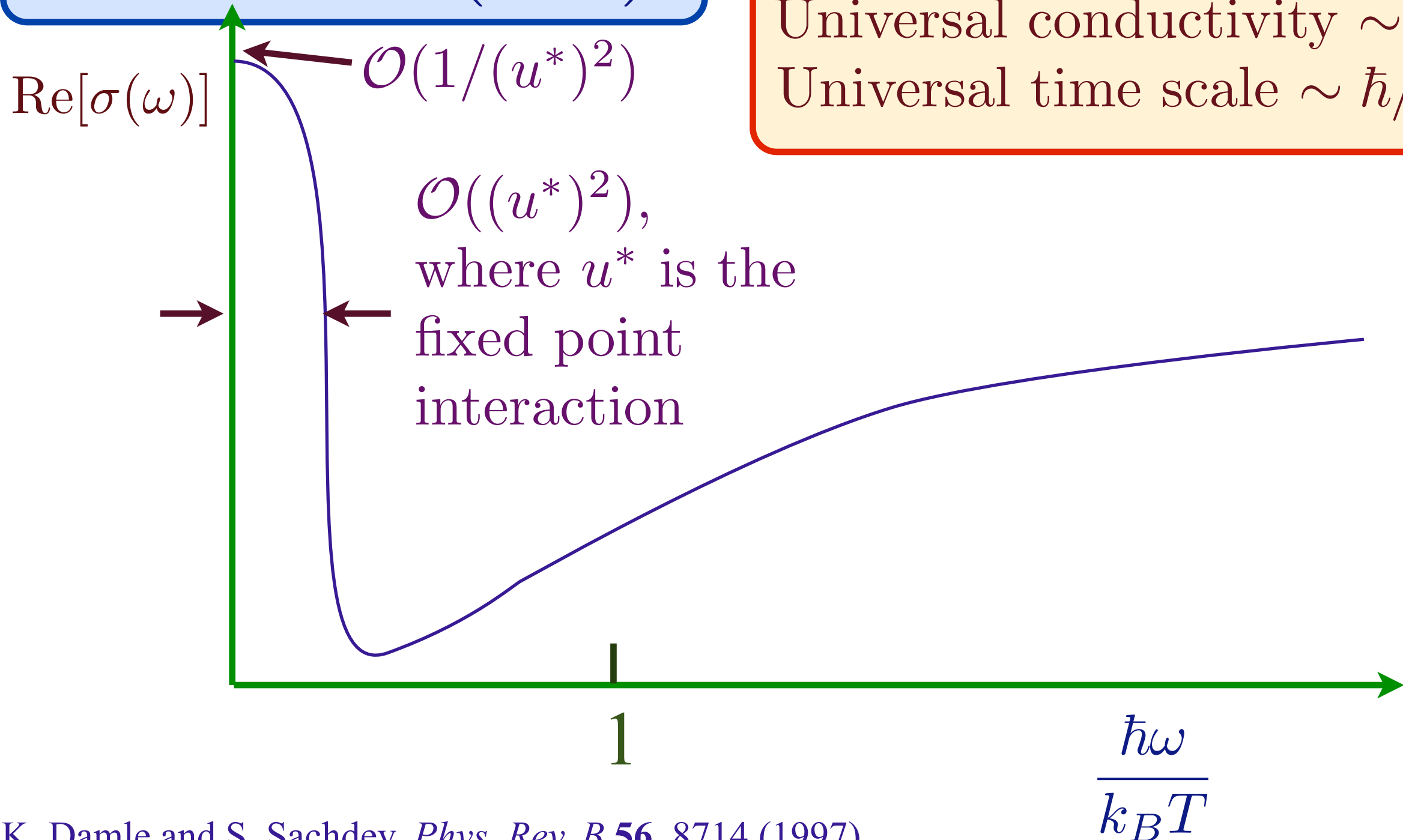
Quasiparticle view of quantum criticality (Boltzmann equation):  
Electrical transport for a (weakly) interacting CFT3

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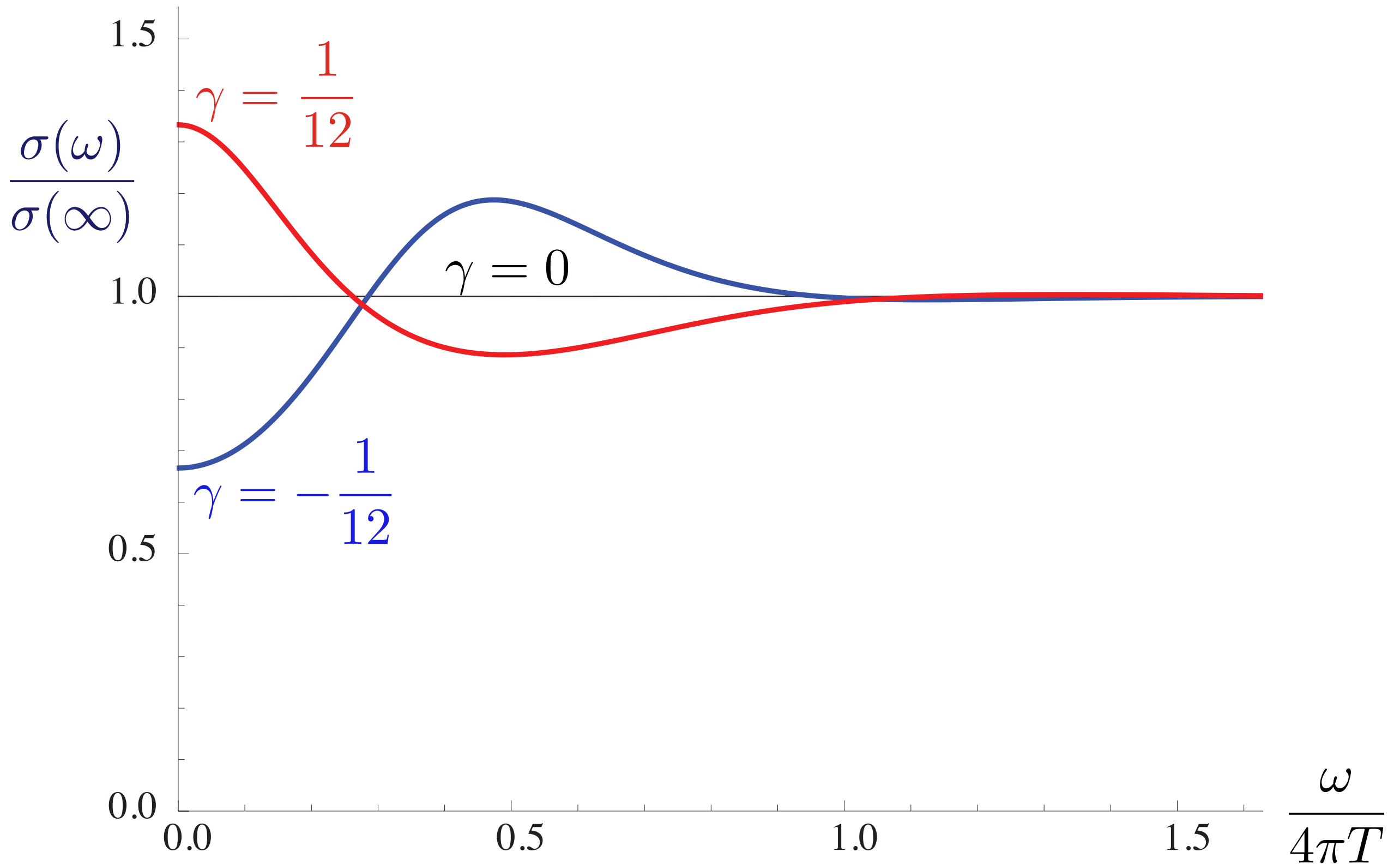
$\Sigma \rightarrow$  a universal function

Universal conductivity  $\sim e^2/h$

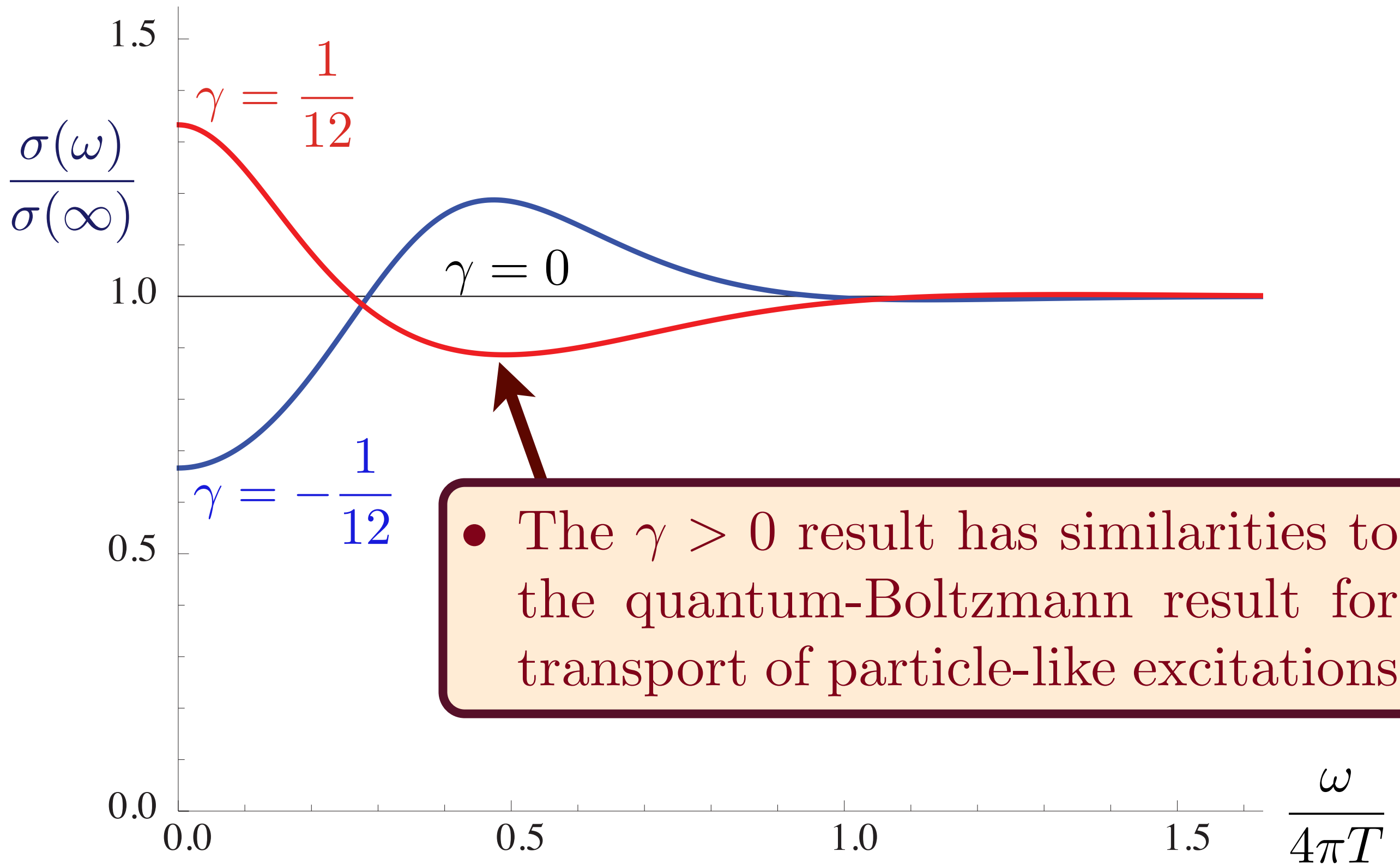
Universal time scale  $\sim \hbar/k_B T$



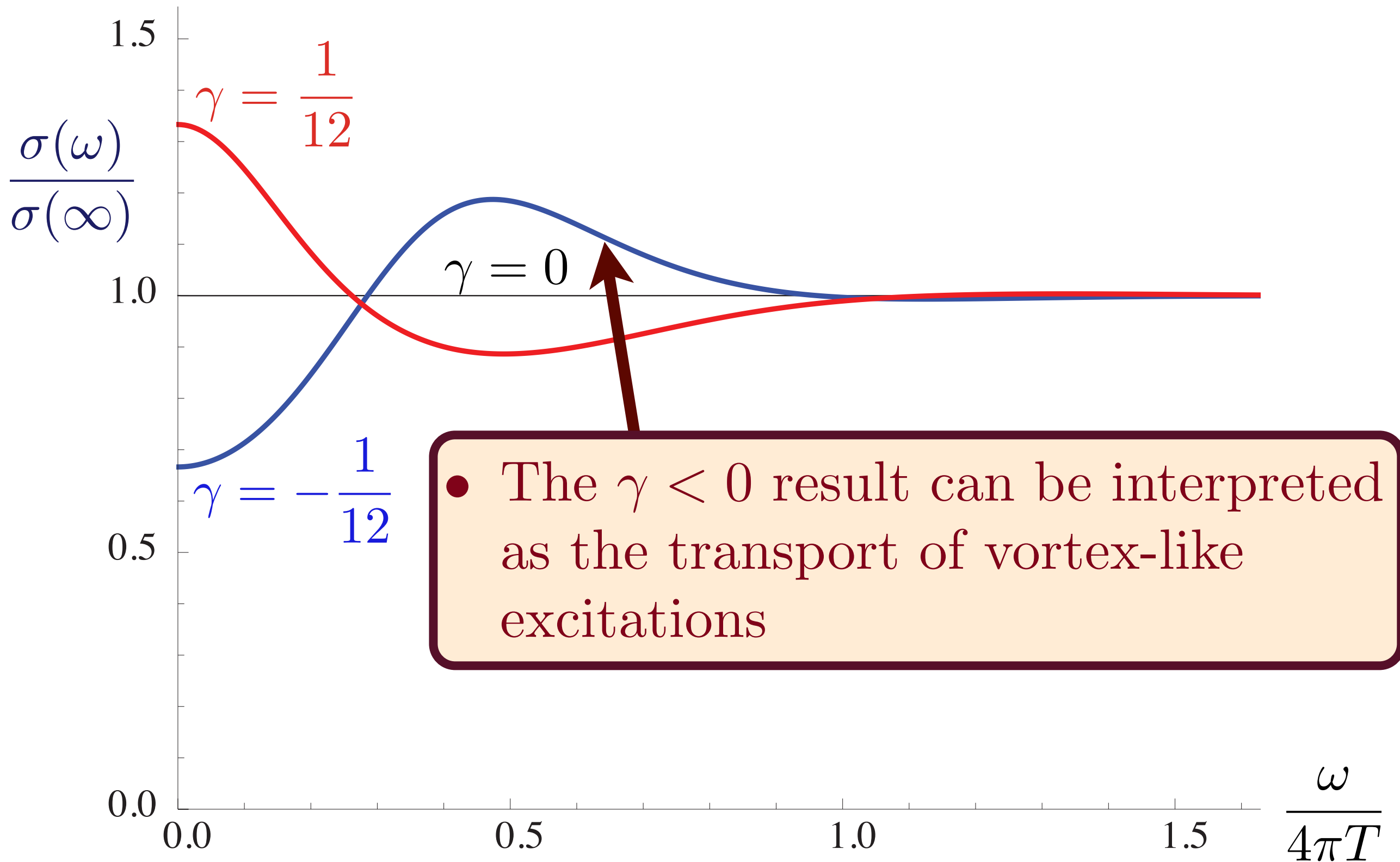
# AdS<sub>4</sub> theory of quantum criticality



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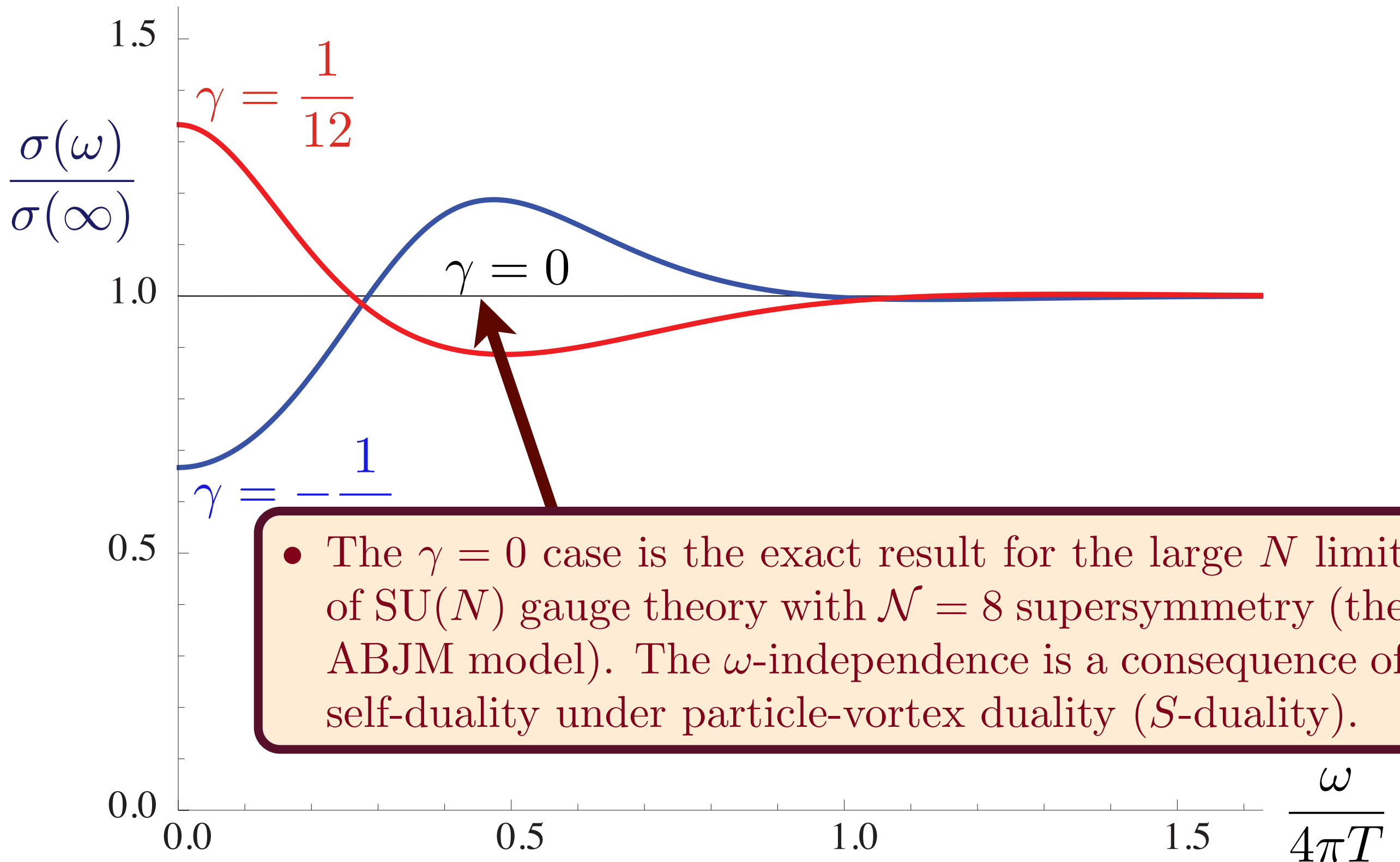


# AdS<sub>4</sub> theory of quantum criticality

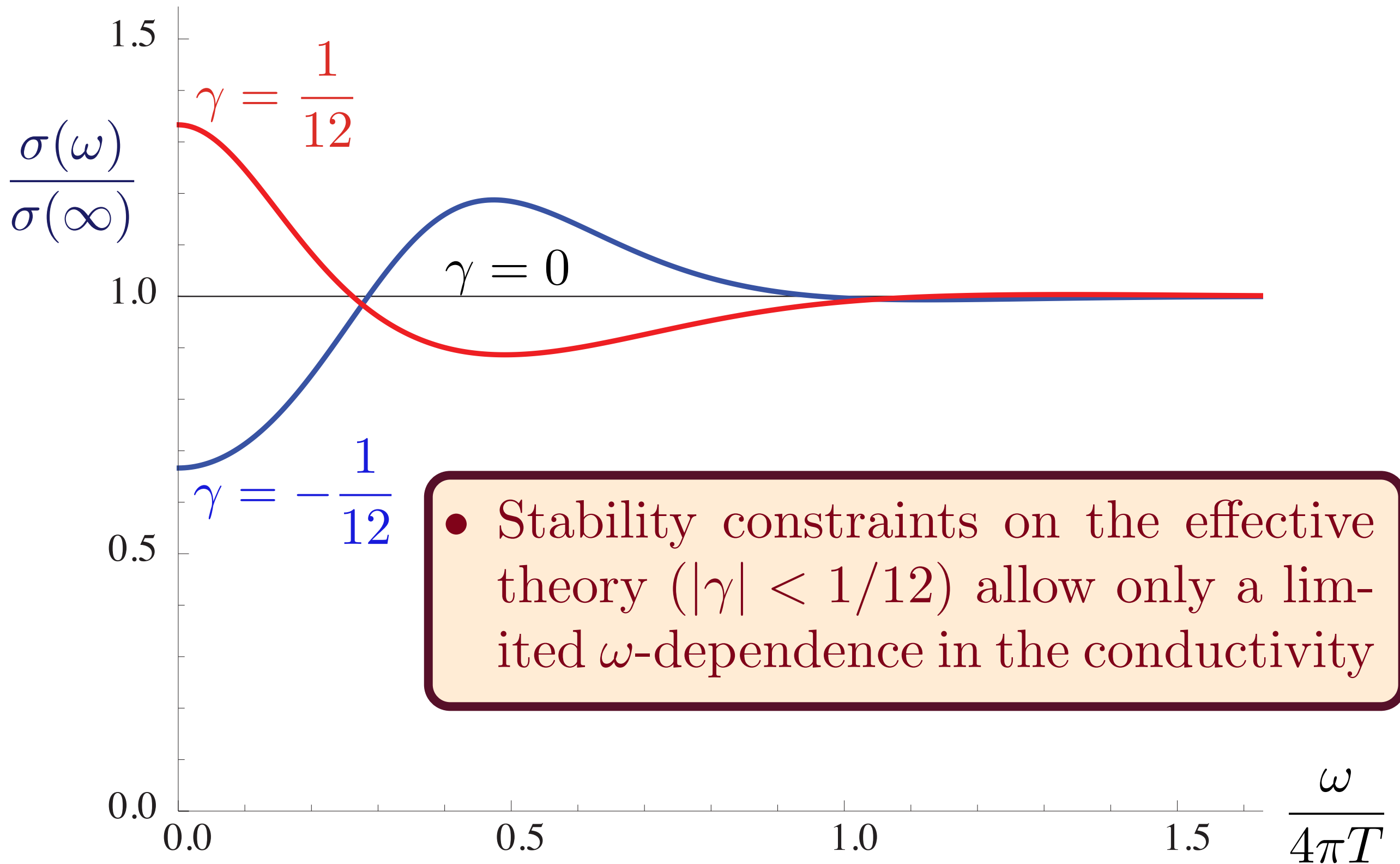




# AdS<sub>4</sub> theory of quantum criticality



# AdS<sub>4</sub> theory of quantum criticality



# The dynamics of quantum criticality via Quantum Monte Carlo and holography

William Witczak-Krempa, Erik Sorensen, Subir Sachdev

(Submitted on 11 Sep 2013 (v1), last revised 29 Nov 2013 (this version, v2))

Understanding the real time dynamics of quantum systems without quasiparticles constitutes an important yet challenging problem. We study the superfluid-insulator quantum-critical point of bosons on a two-dimensional lattice, a system whose excitations cannot be described in a quasiparticle basis. We present detailed quantum Monte Carlo results for two separate lattice realizations: their low-frequency conductivities are found to have the same universal dependence on imaginary frequency and temperature. We then use the structure of the real time dynamics of conformal field theories described by the holographic gauge/gravity duality to make progress on the difficult problem of analytically continuing the Monte Carlo data to real time. Our method yields quantitative and experimentally testable results on the frequency-dependent conductivity near the quantum critical point, and on the spectrum of quasinormal modes in the vicinity of the superfluid-insulator quantum phase transition. Extensions to other observables and universality classes are discussed.

# Universal Conductivity in a Two-dimensional Superfluid-to-Insulator Quantum Critical System

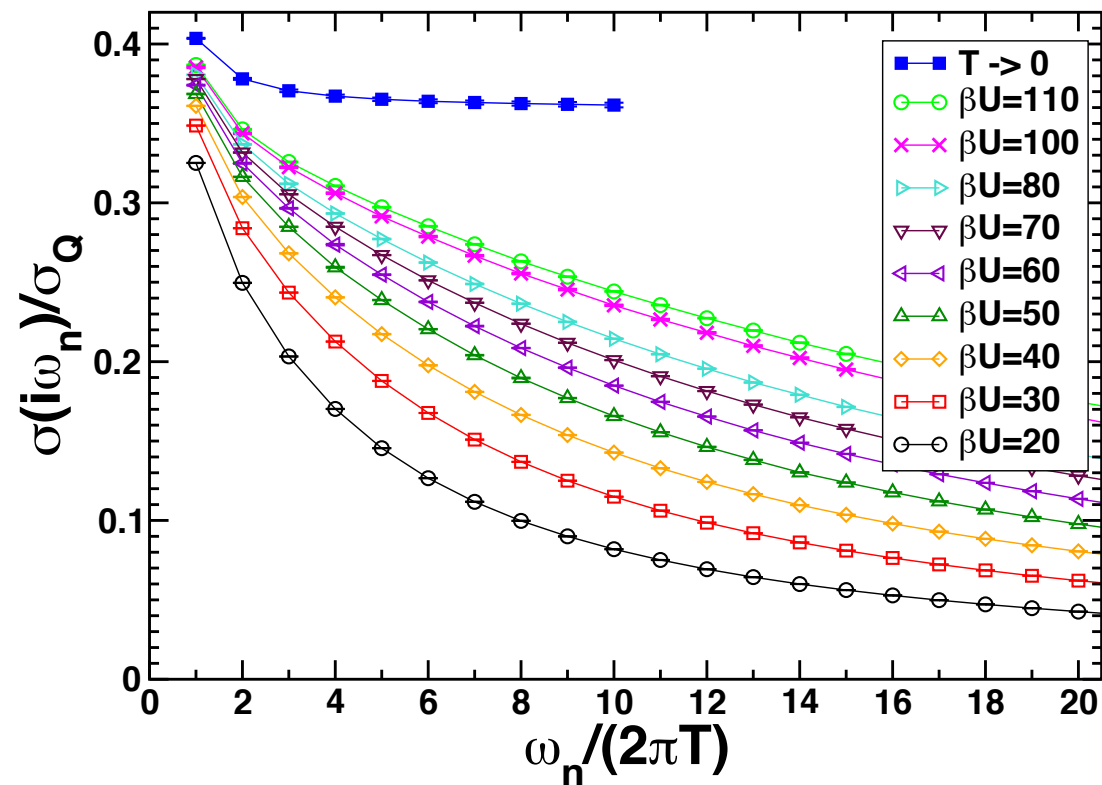
Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, Nikolay Prokof'ev

(Submitted on 22 Sep 2013)

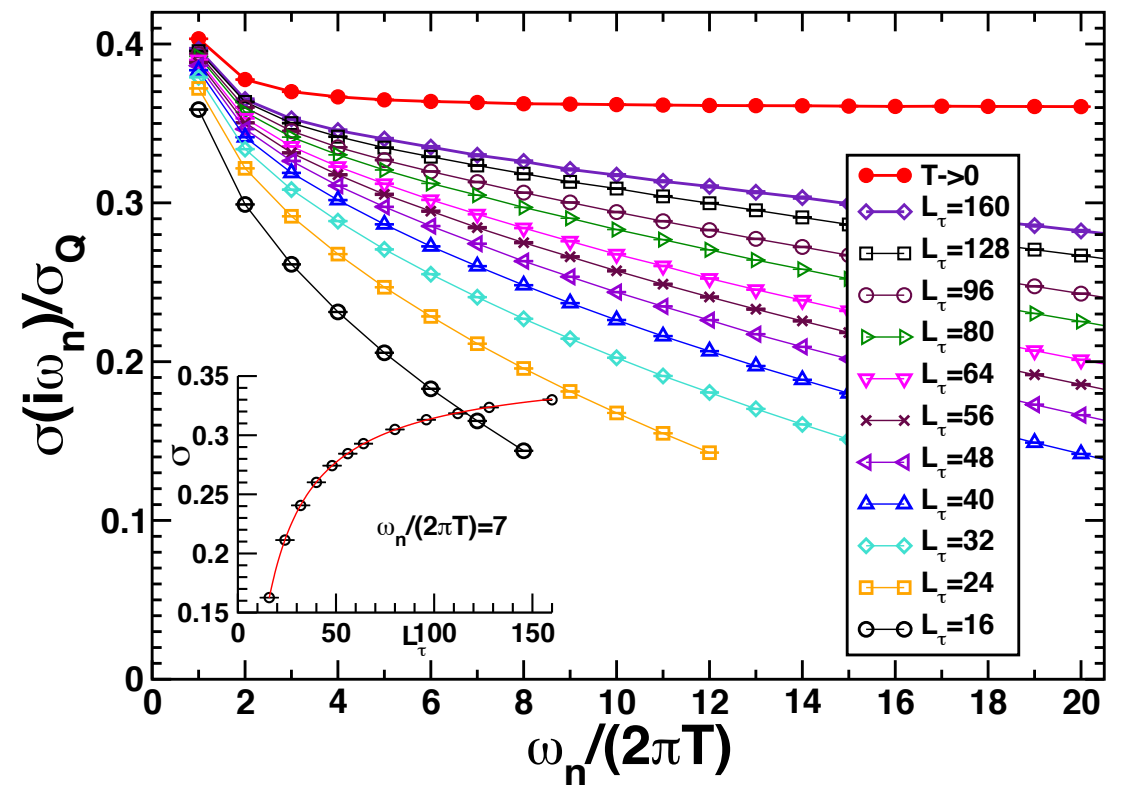
We compute the universal conductivity of the (2+1)-dimensional XY universality class, which is realized for a superfluid-to-Mott insulator quantum phase transition at constant density. Based on large-scale Monte Carlo simulations of the classical (2+1)-dimensional  $J$ -current model and the two-dimensional Bose-Hubbard model, we can precisely determine the conductivity on the quantum critical plateau,  $\sigma(\infty) = 0.359(4)\sigma_Q$  with  $\sigma_Q$  the conductivity quantum. The universal conductivity is the schoolbook example of where the AdS/CFT correspondence from string theory can be tested and made to use. The shape of our  $\sigma(i\omega_n) - \sigma(\infty)$  function in the Matsubara representation is accurate enough for a conclusive comparison and establishes the particle-like nature of charge transport. We find that the holographic gauge/gravity duality theory for transport properties can be made compatible with the data if temperature of the horizon of the black brane is different from the temperature of the conformal field theory. The requirements for measuring the universal conductivity in a cold gas experiment are also determined by our calculation.



# Quantum Monte Carlo for lattice bosons



(a)



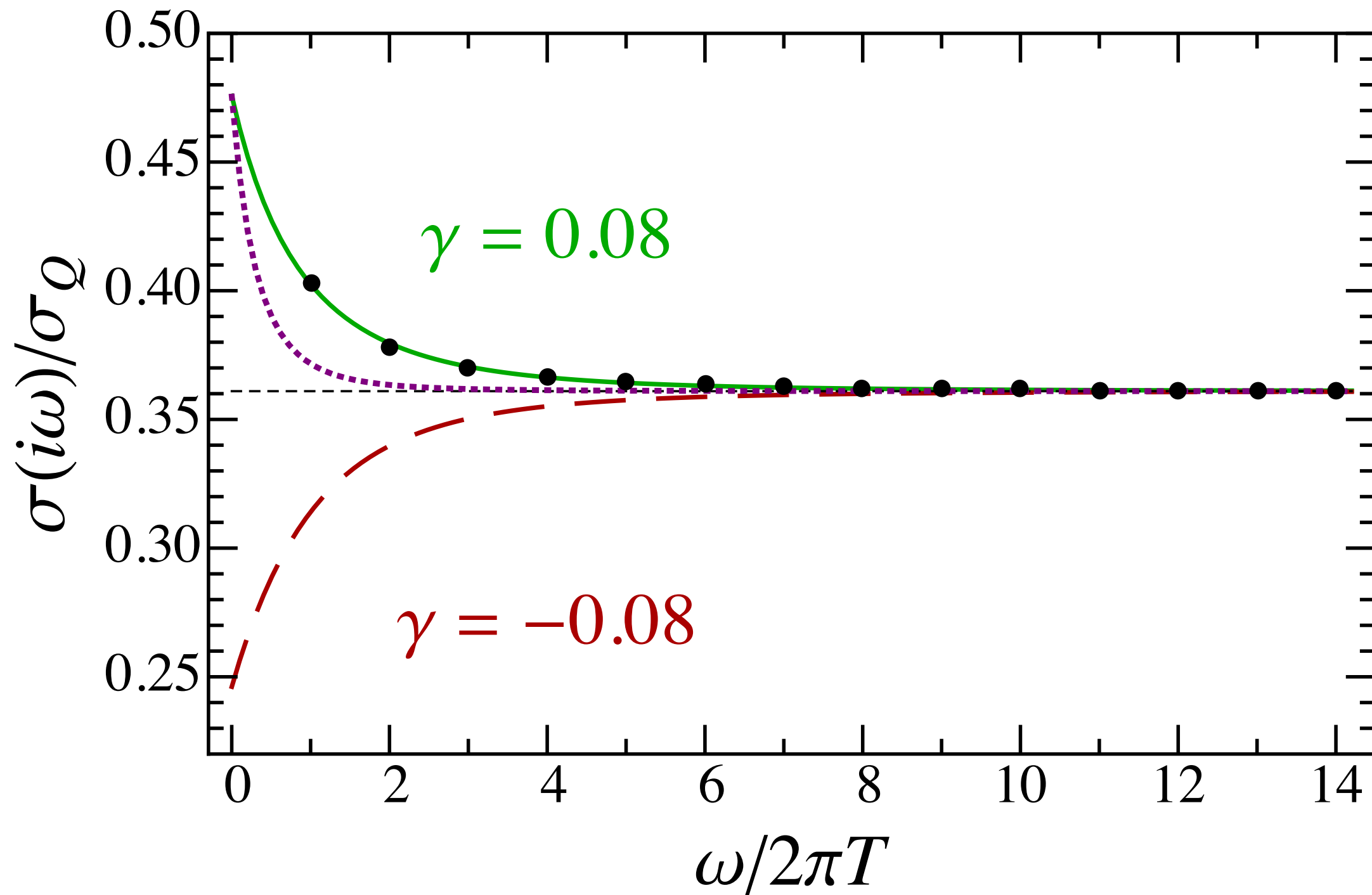
(b)

FIG. 2. **Quantum Monte Carlo data** (a) Finite-temperature conductivity for a range of  $\beta U$  in the  $L \rightarrow \infty$  limit for the quantum rotor model at  $(t/U)_c$ . The solid blue squares indicate the final  $T \rightarrow 0$  extrapolated data. (b) Finite-temperature conductivity in the  $L \rightarrow \infty$  limit for a range of  $L_\tau$  for the Villain model at the QCP. The solid red circles indicate the final  $T \rightarrow 0$  extrapolated data. The inset illustrates the extrapolation to  $T = 0$  for  $\omega_n/(2\pi T) = 7$ . The error bars are statistical for both a) and b).

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

# AdS<sub>4</sub> theory of quantum criticality

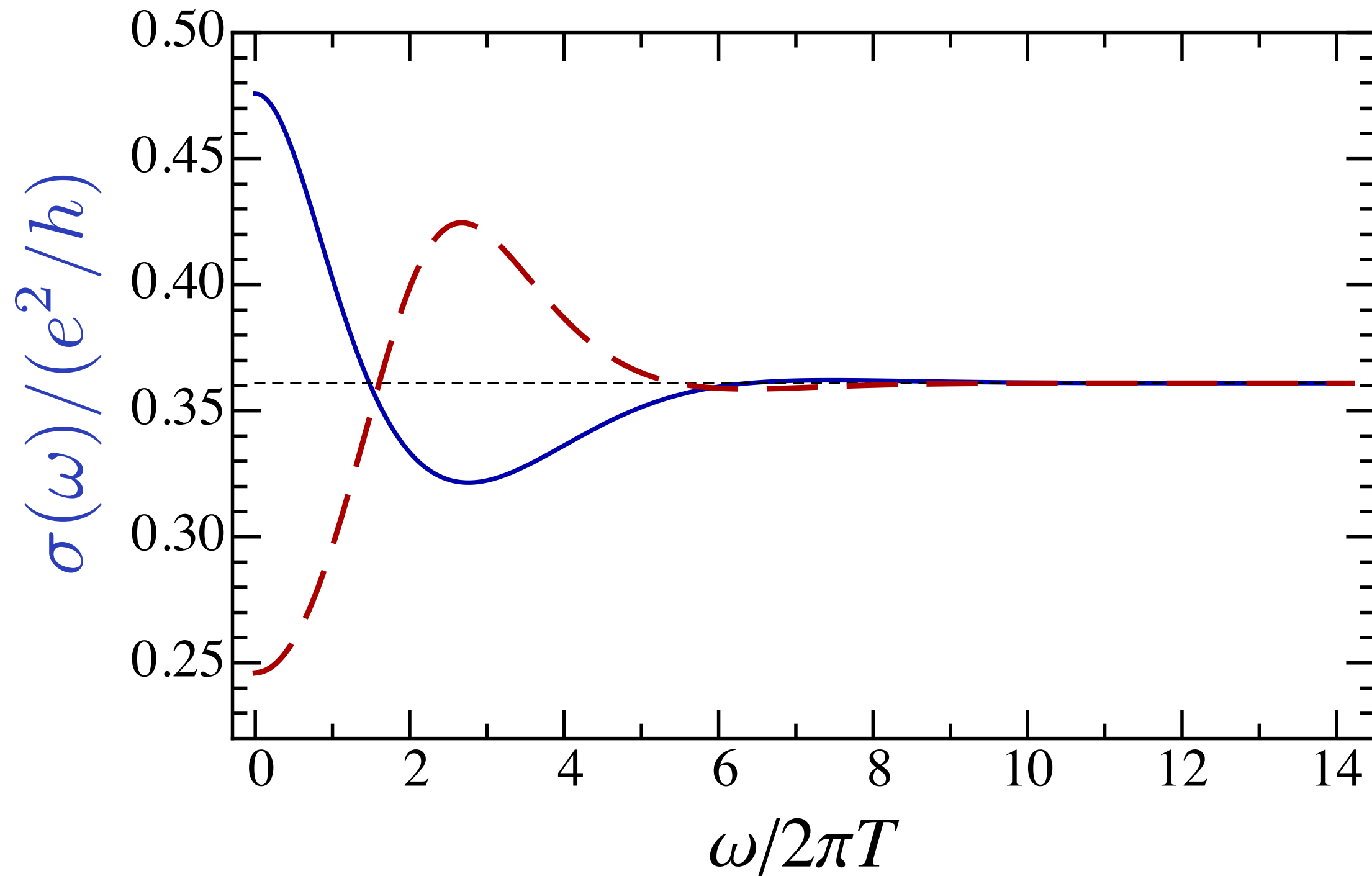


Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective  $T$  and taking  $\sigma_Q = 1/g_M^2$ .

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

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# AdS<sub>4</sub> theory of quantum criticality



Predictions of holographic theory,  
after analytic continuation to real frequencies

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

# Outline

## 1. The simplest model without quasiparticles

*Superfluid-insulator transition*

*of ultracold bosonic atoms in an optical lattice*

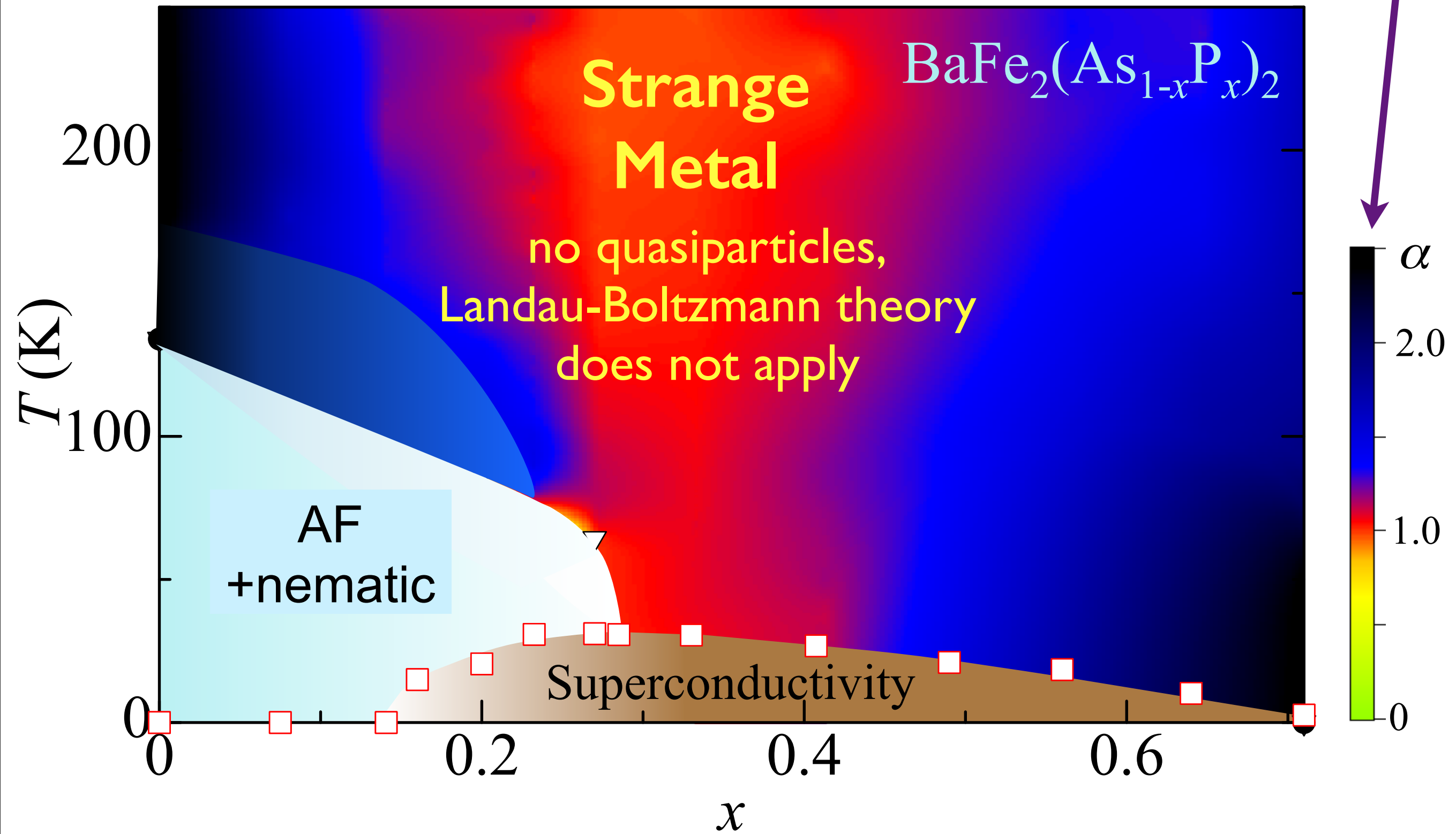
*(Conformal field theories in  $2+1$  dimensions)*

## 2. Strange metals in the high $T_c$ superconductors

*Non-quasiparticle transport at the*

*Ising-nematic quantum critical point*

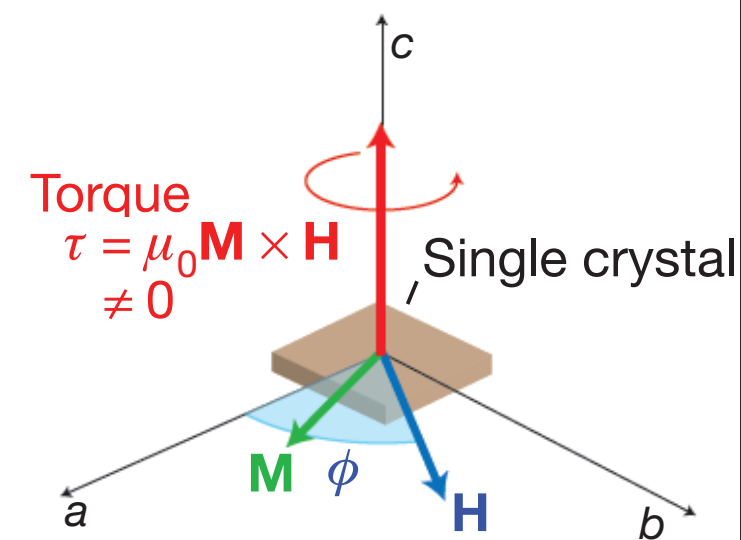
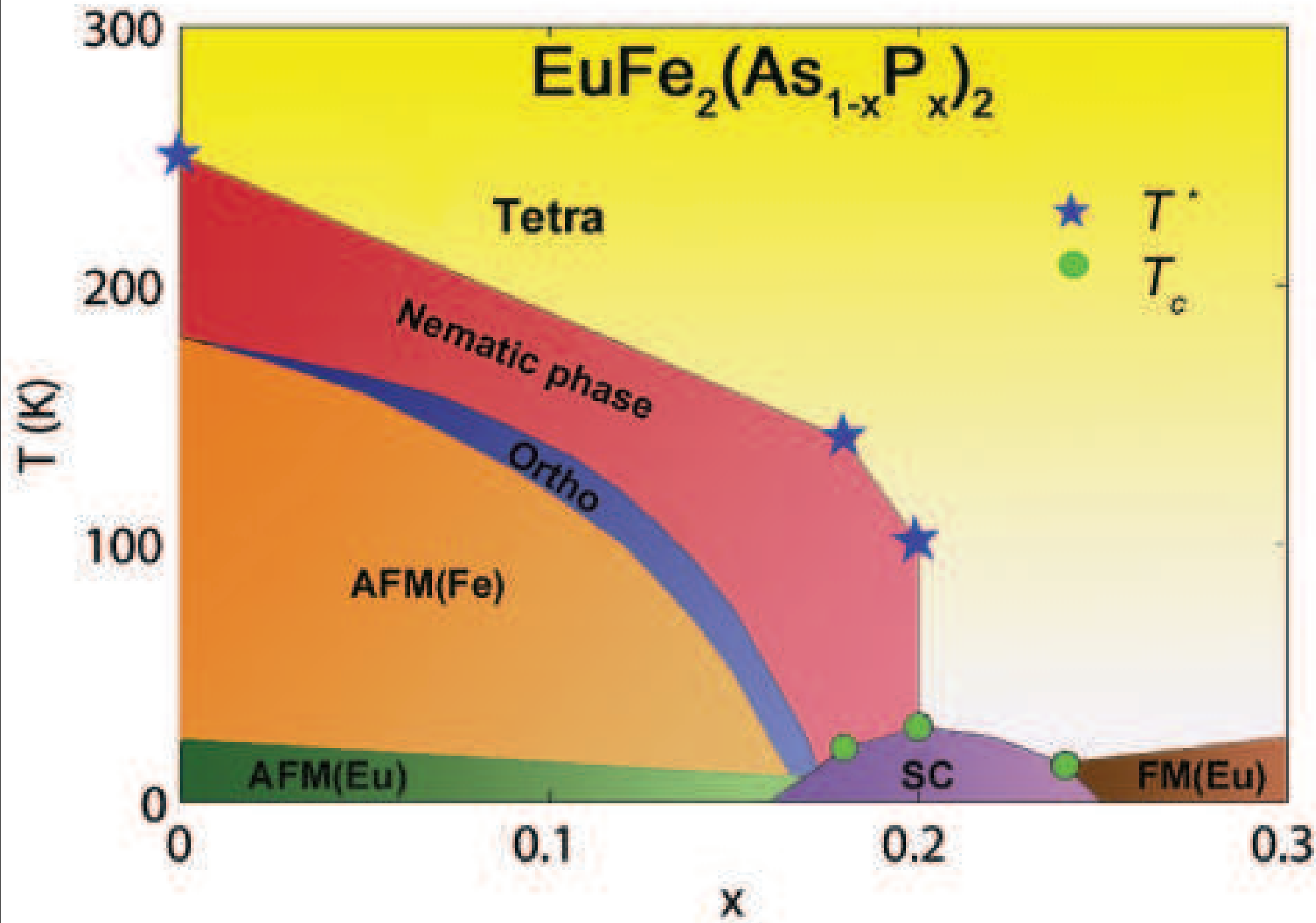
Resistivity  
 $\sim \rho_0 + AT^\alpha$



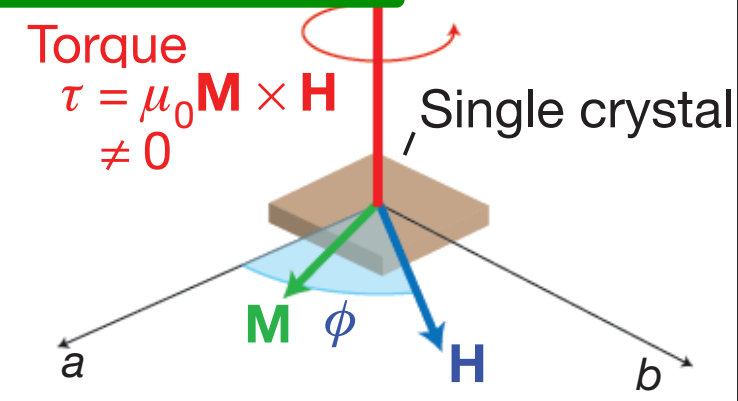
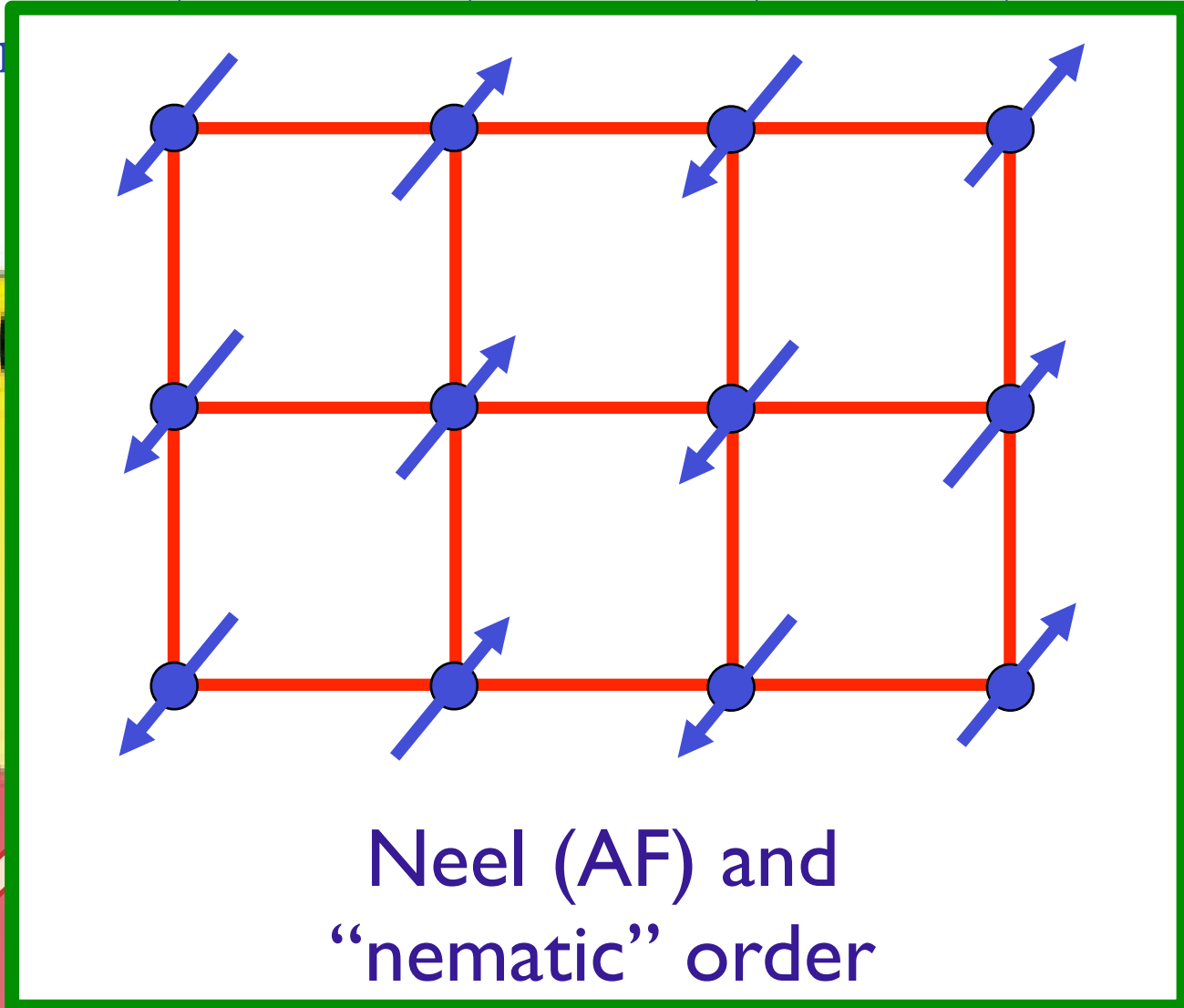
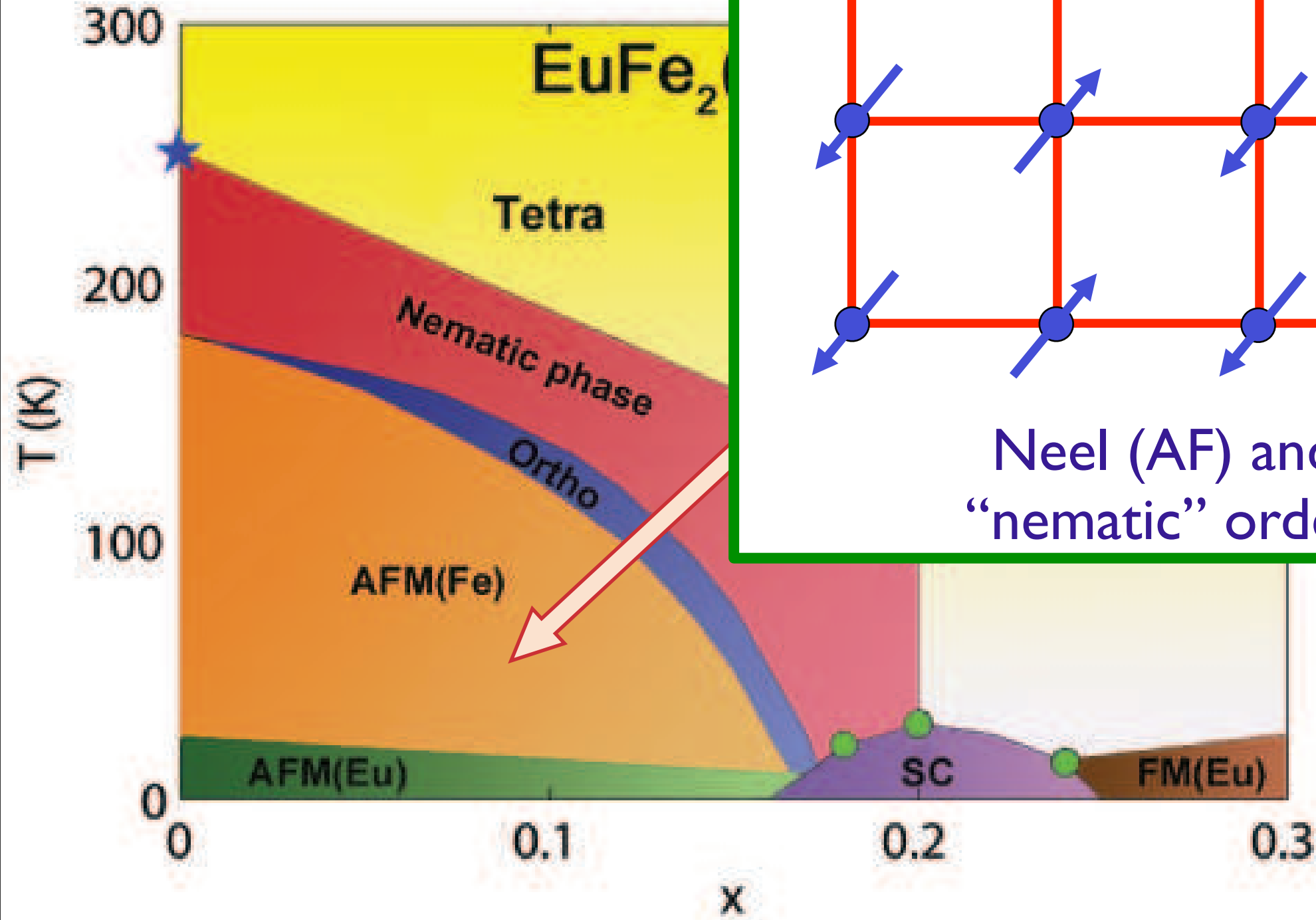
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



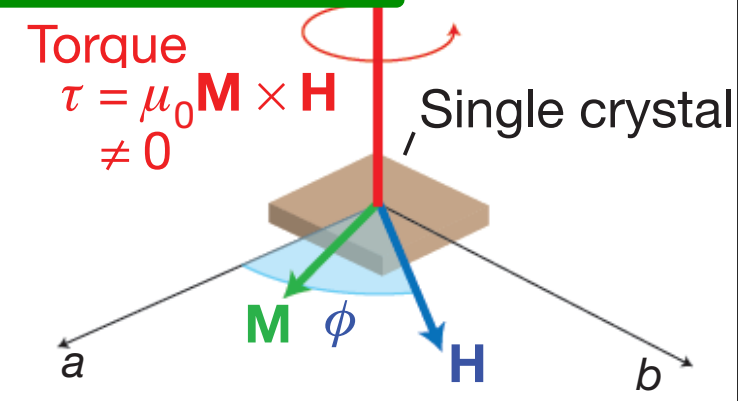
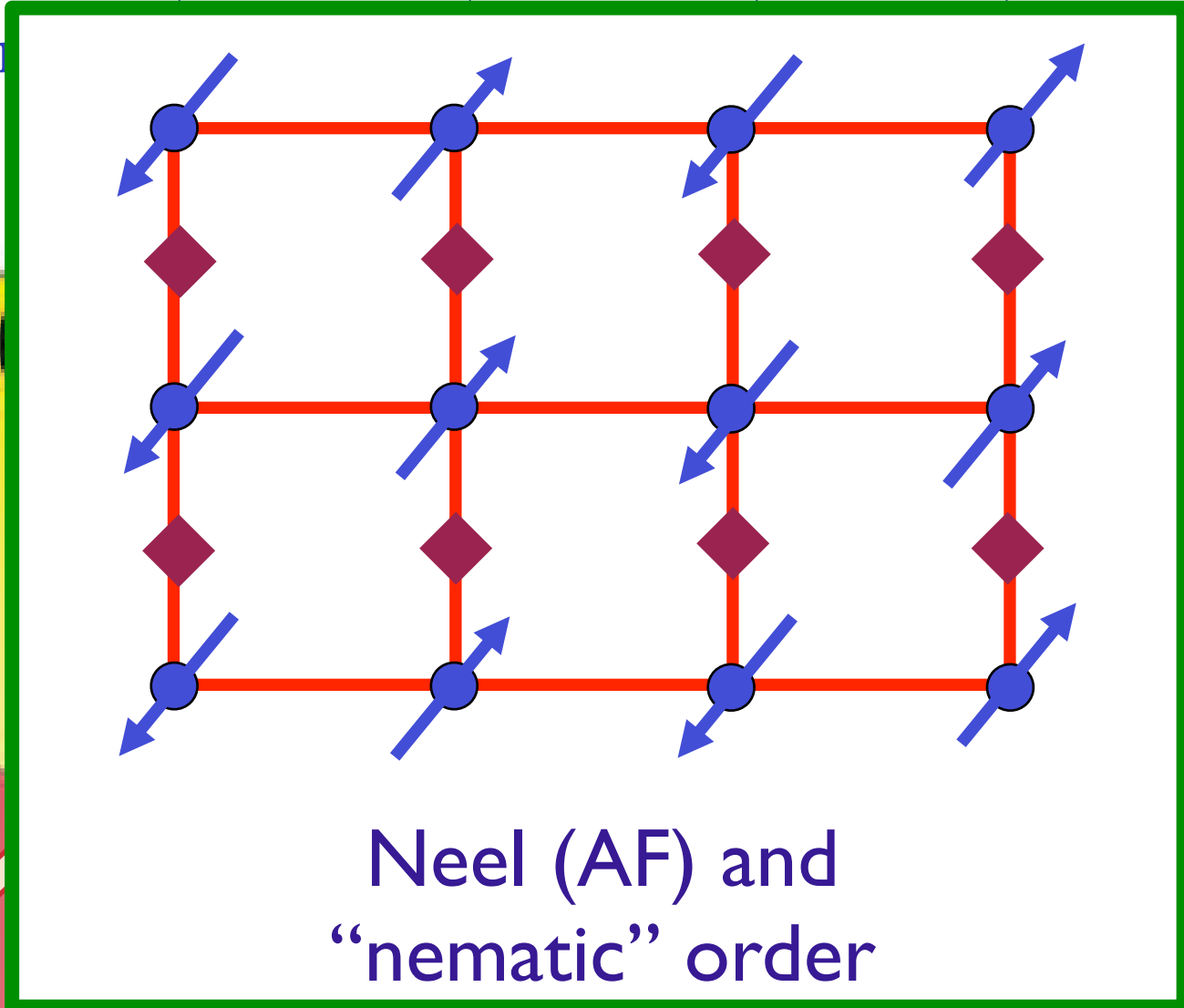
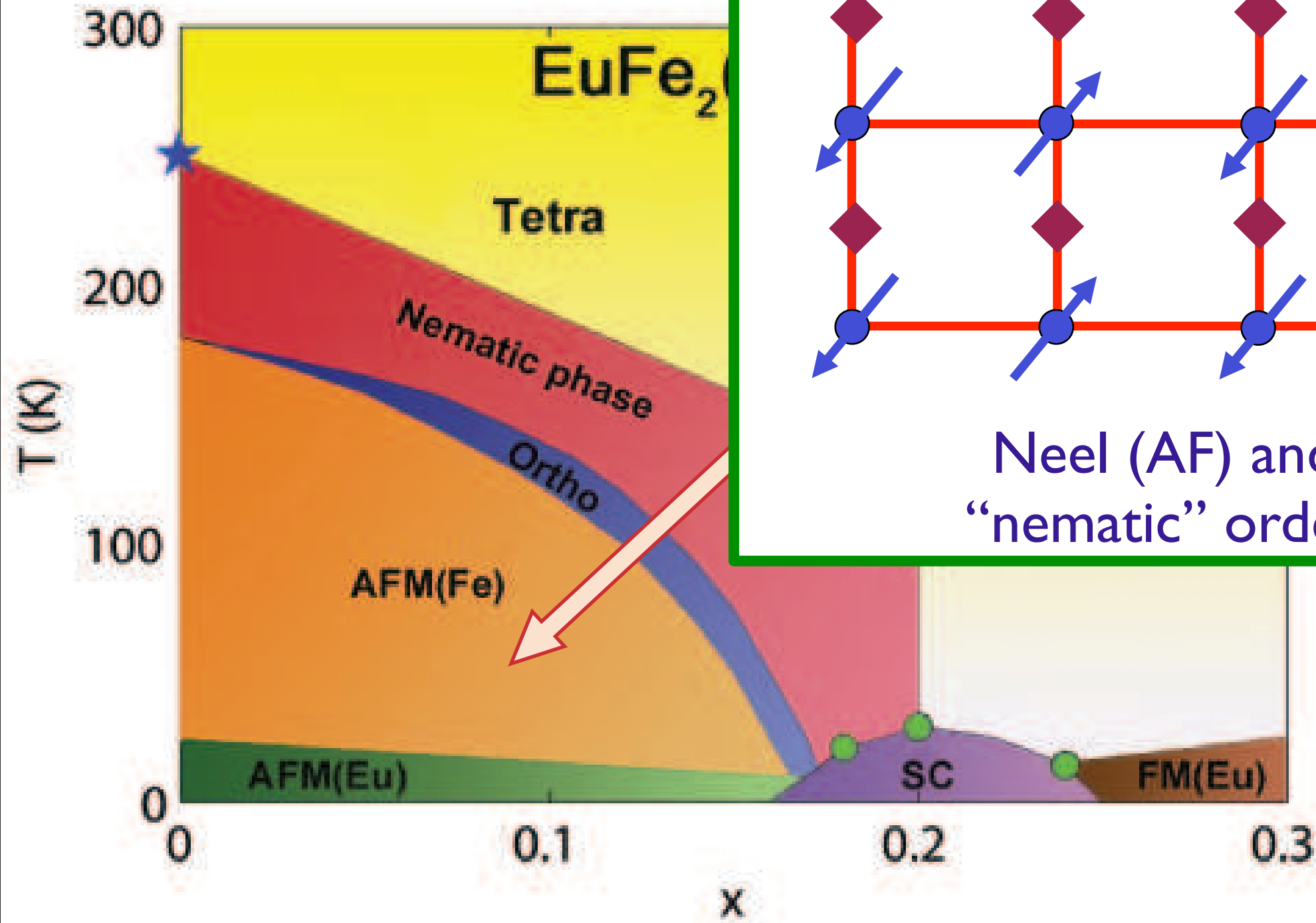
Xiaofeng Xu, W. H. Jiao, N. Zhou, Y. K. Li, B. Chen, C. Cao, Jianhui Dai,  
 A. F. Bangura, and Guanghan Cao, arXiv:1402.4124



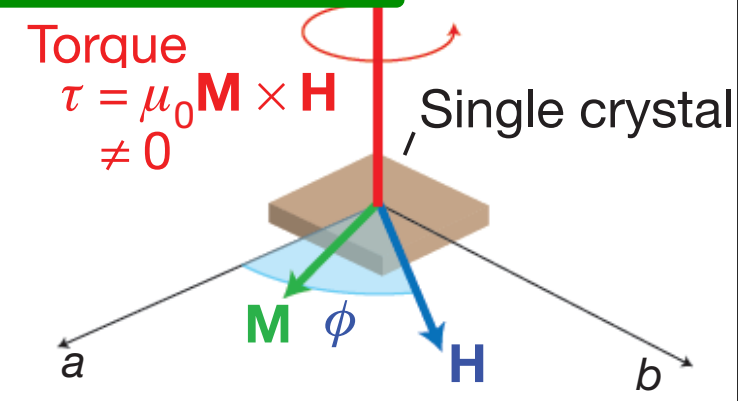
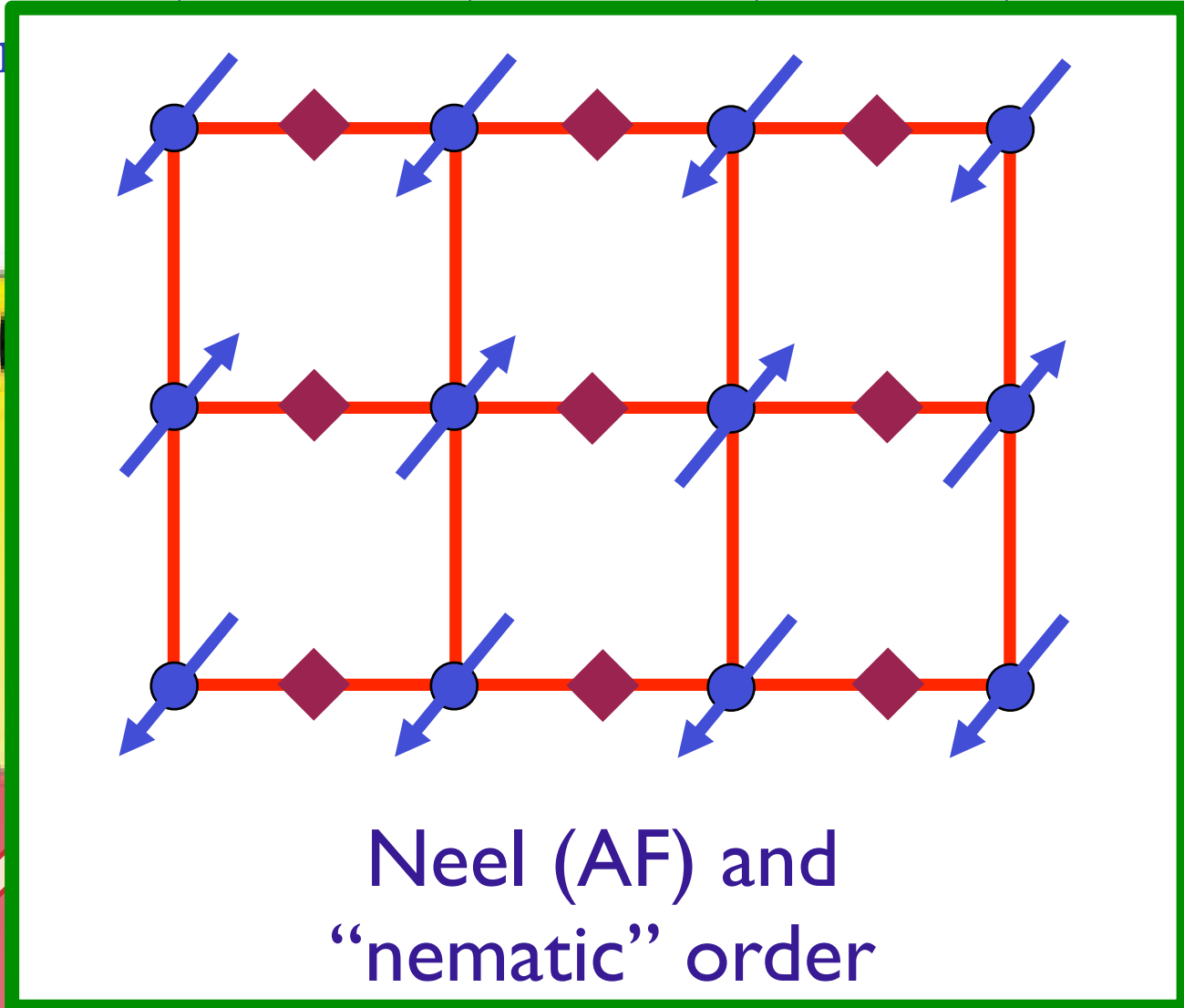
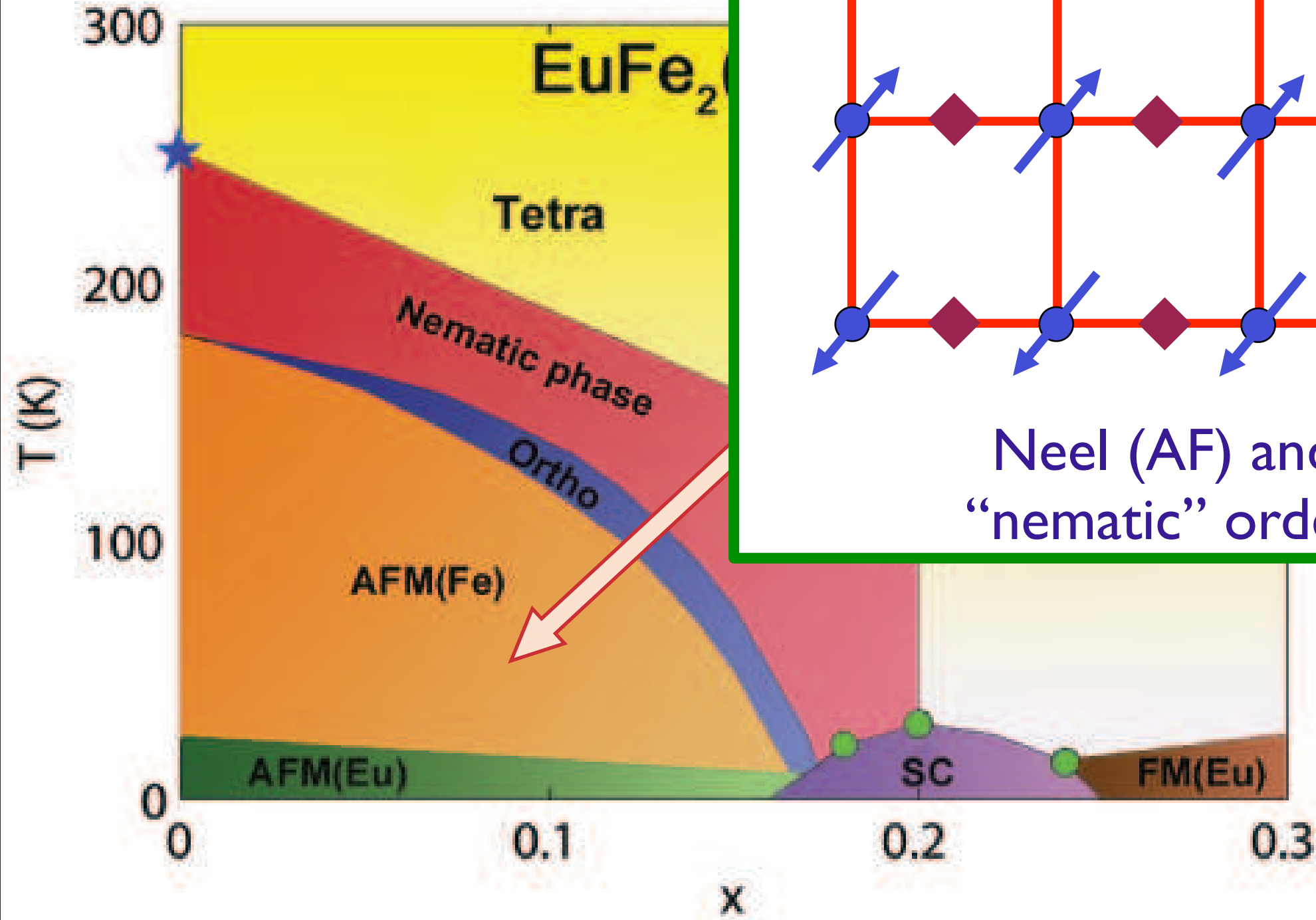
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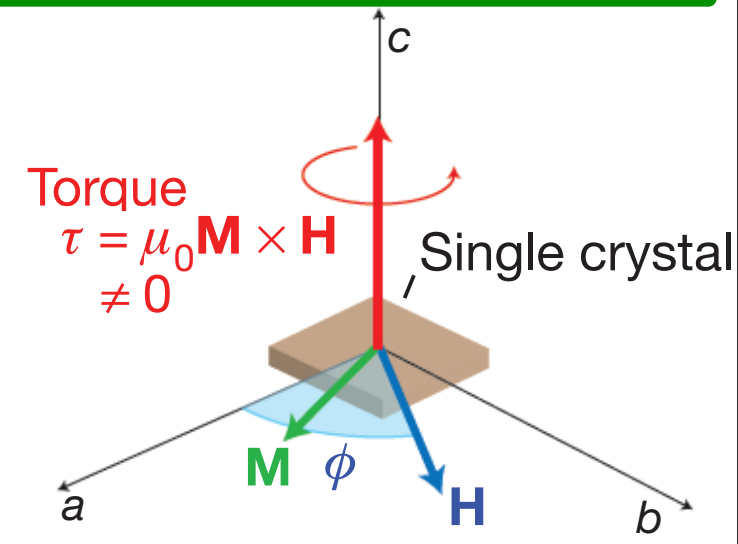
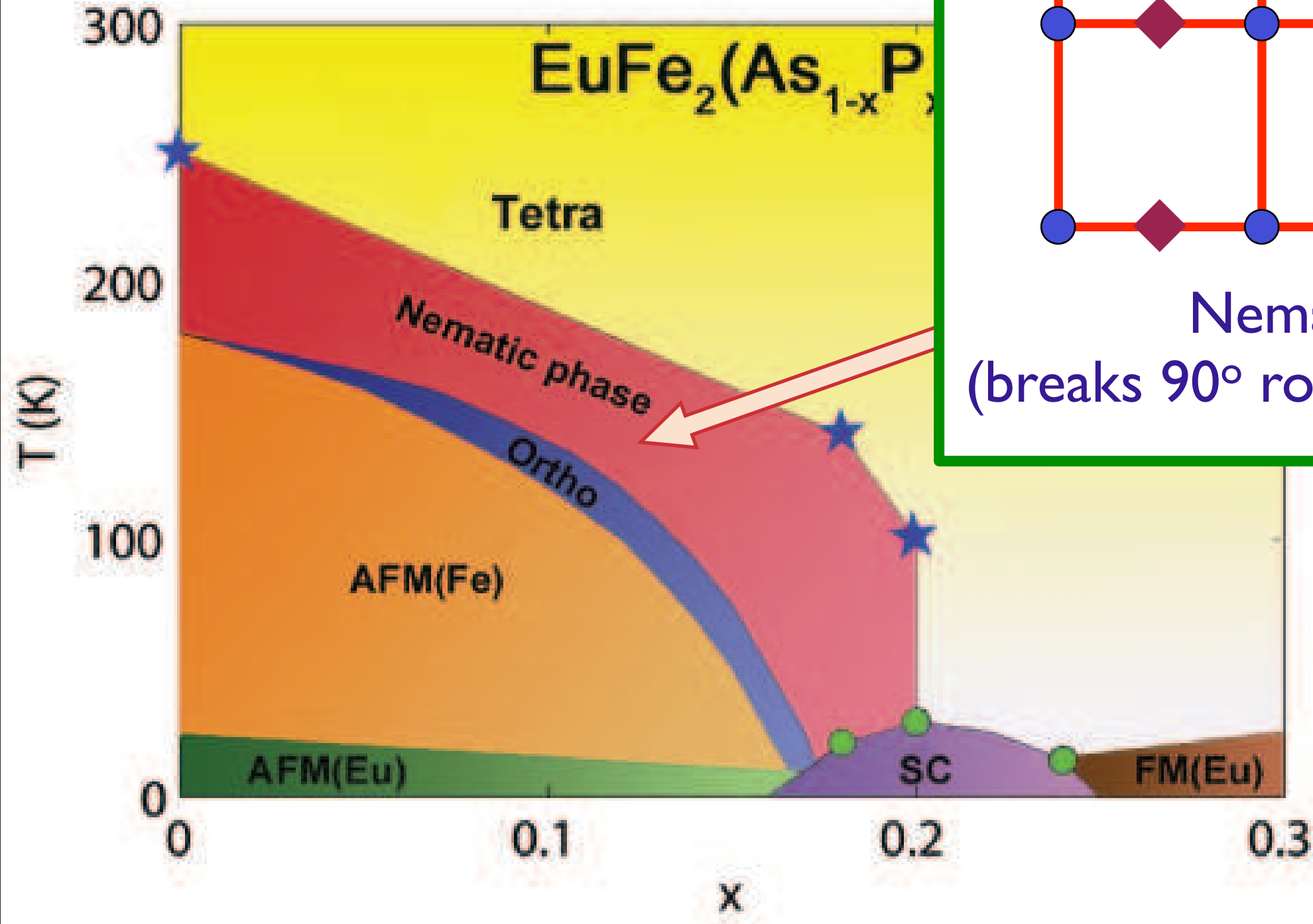
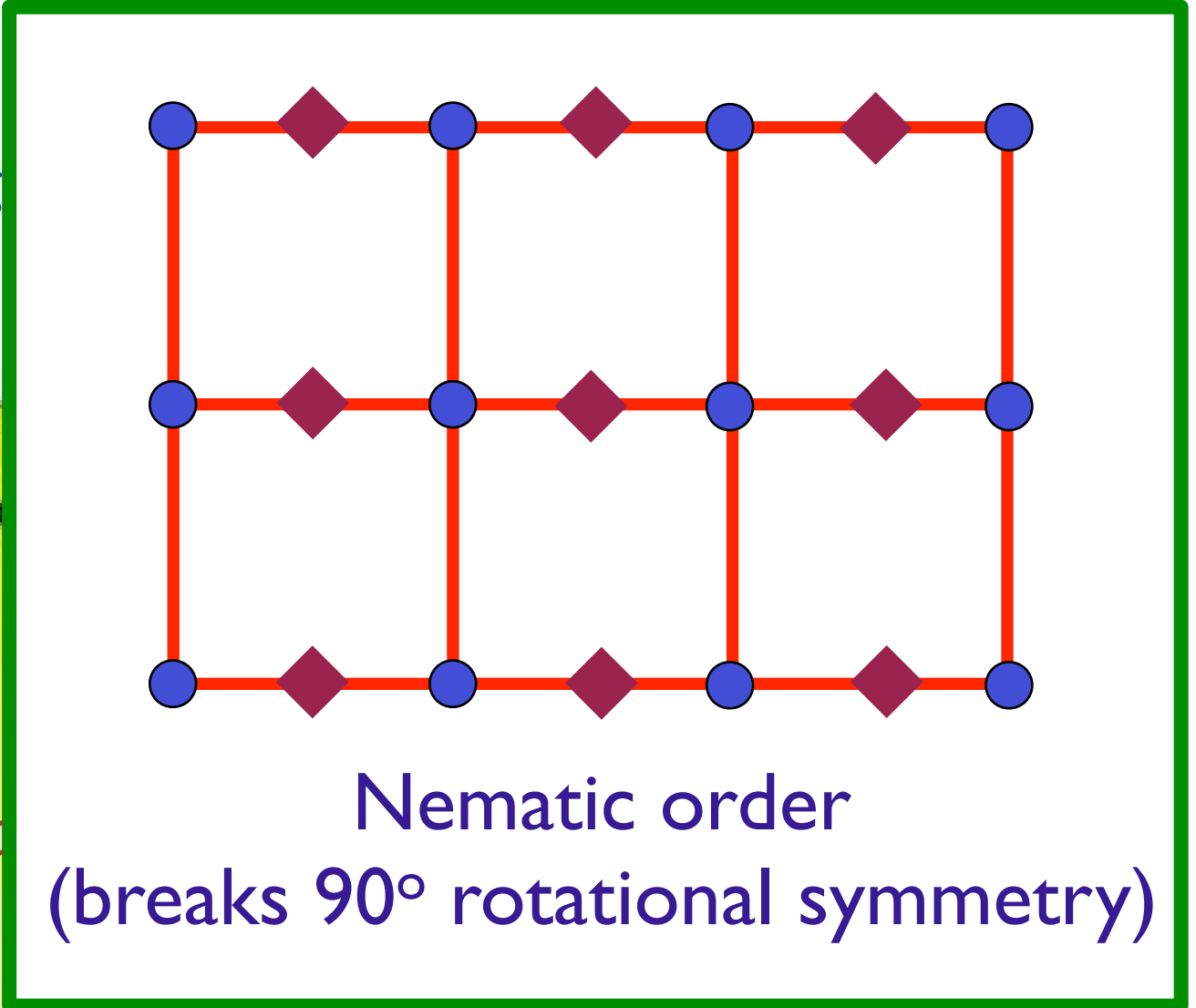
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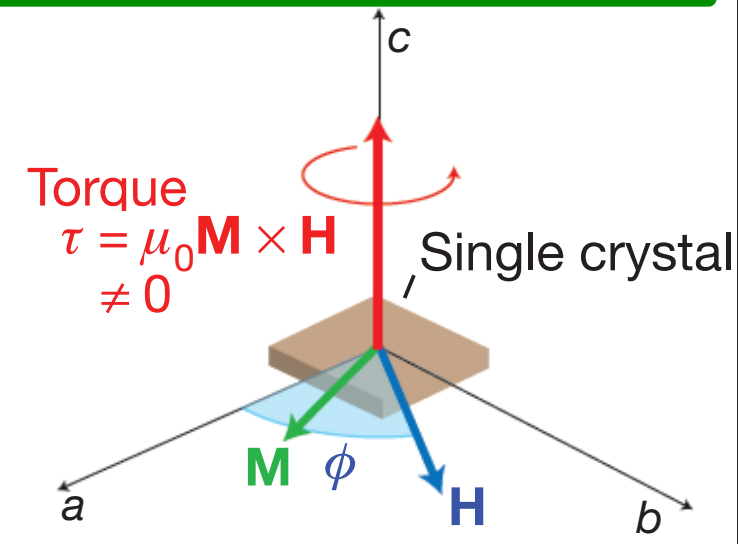
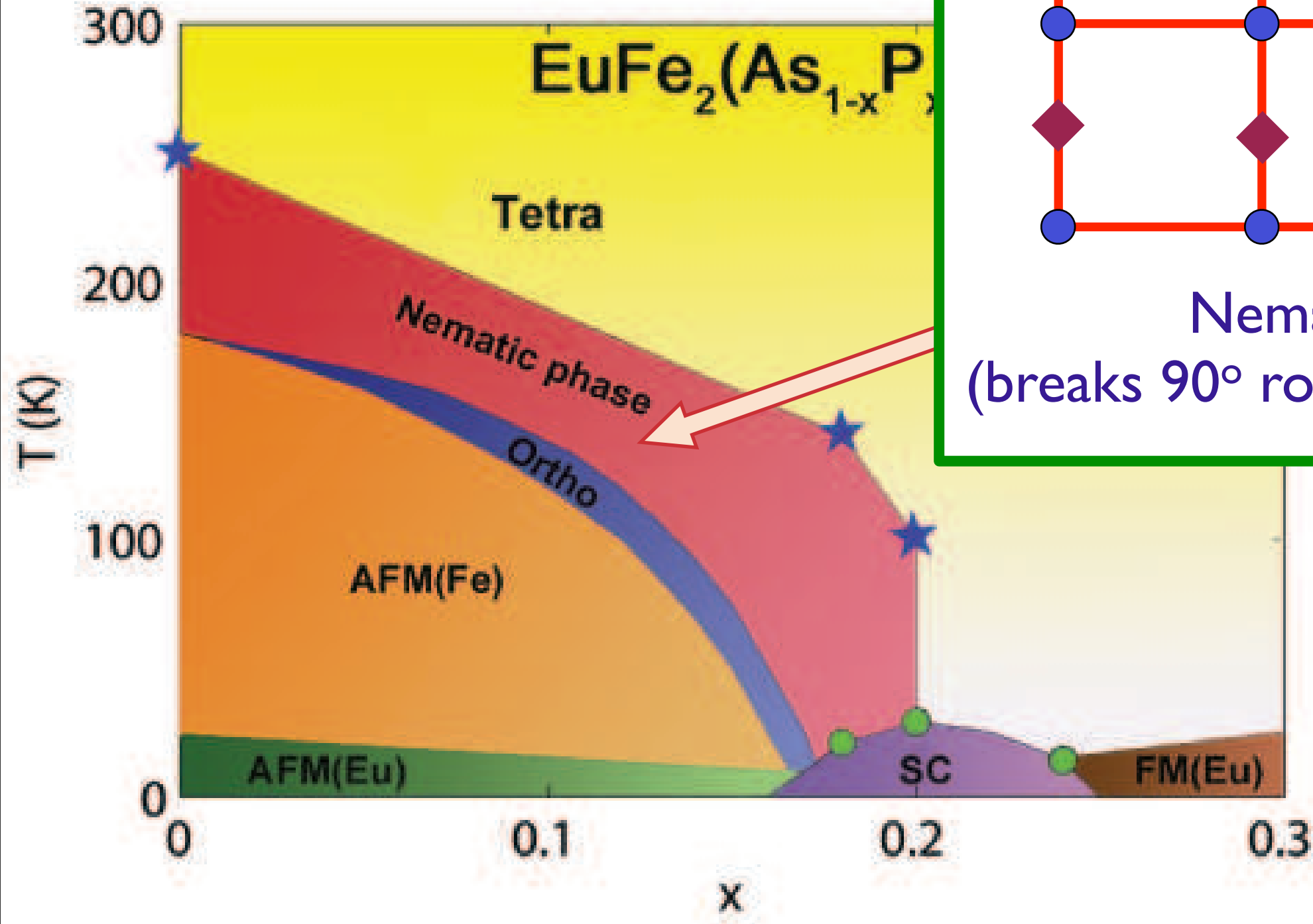
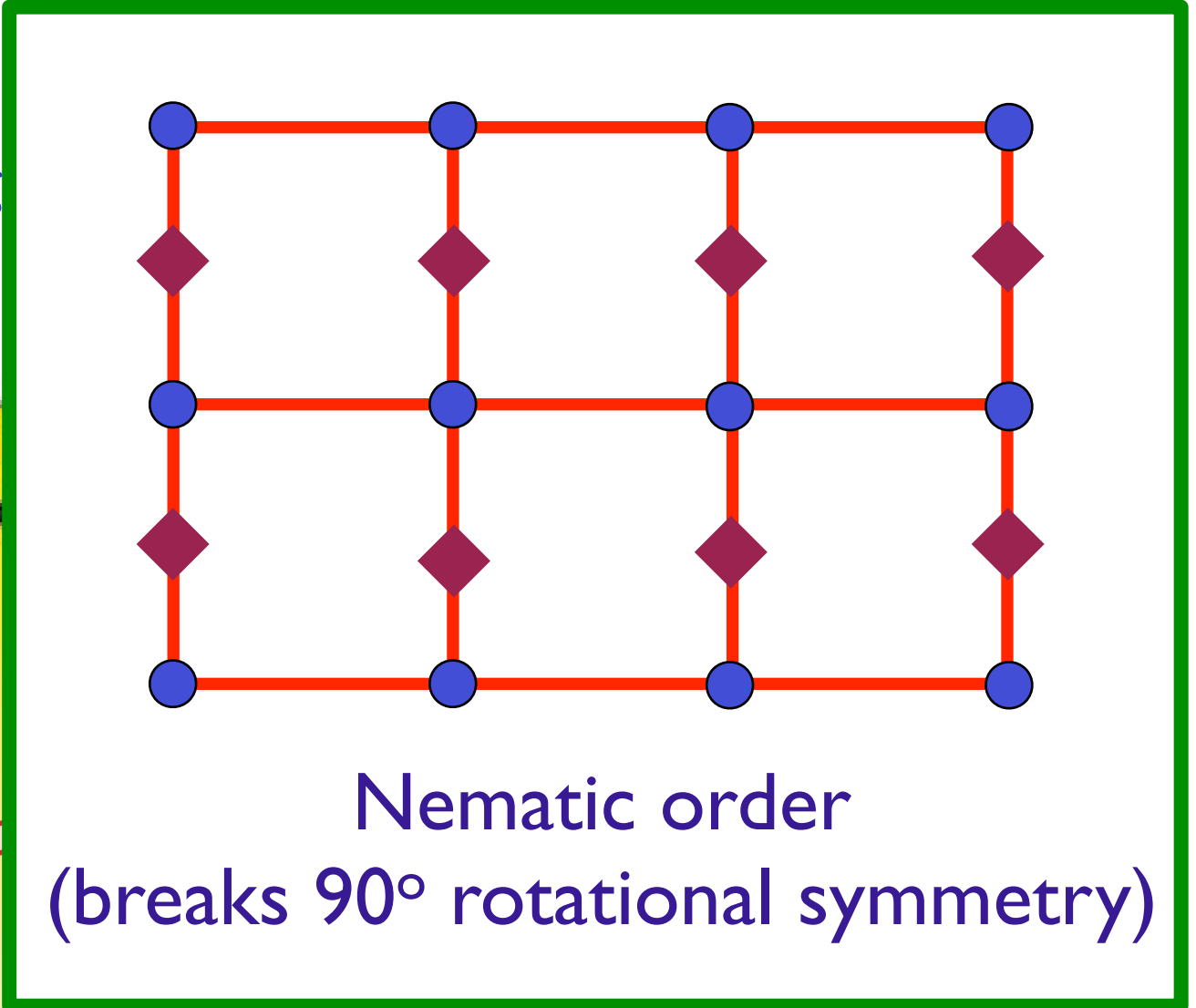
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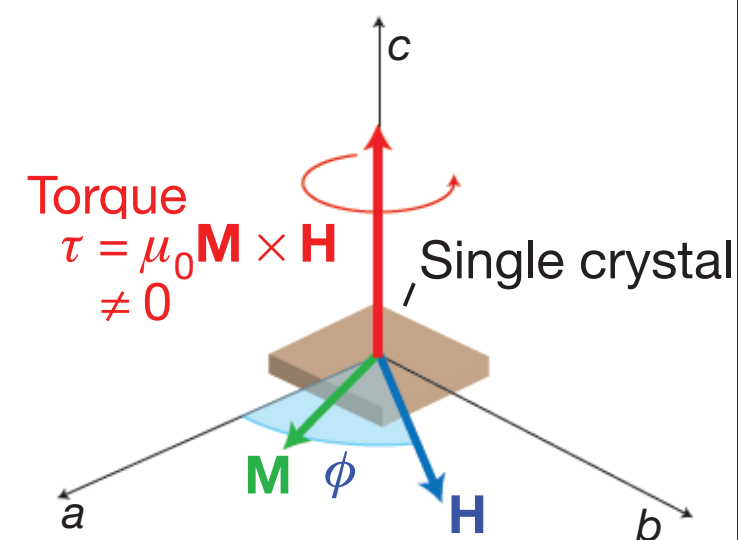
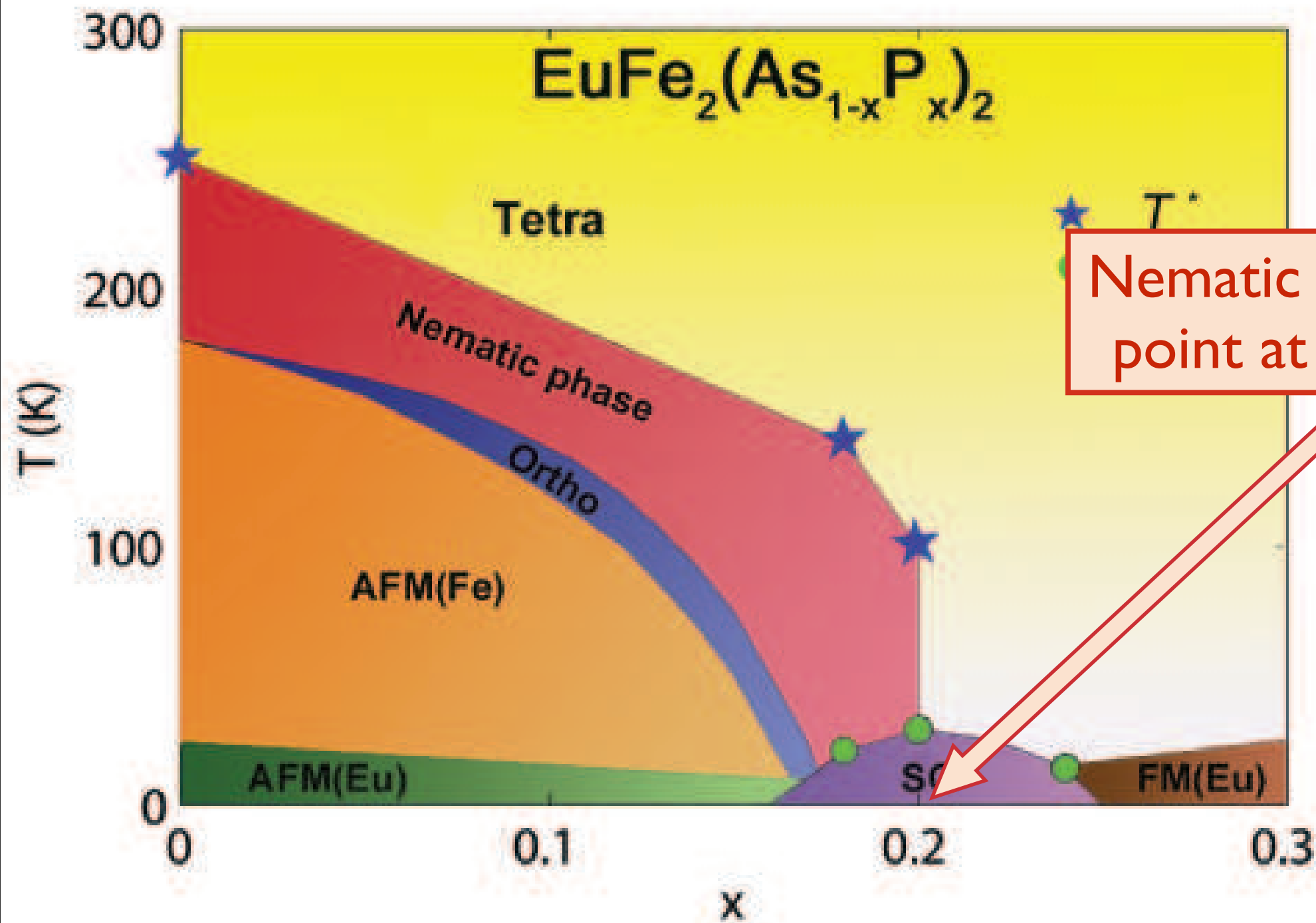
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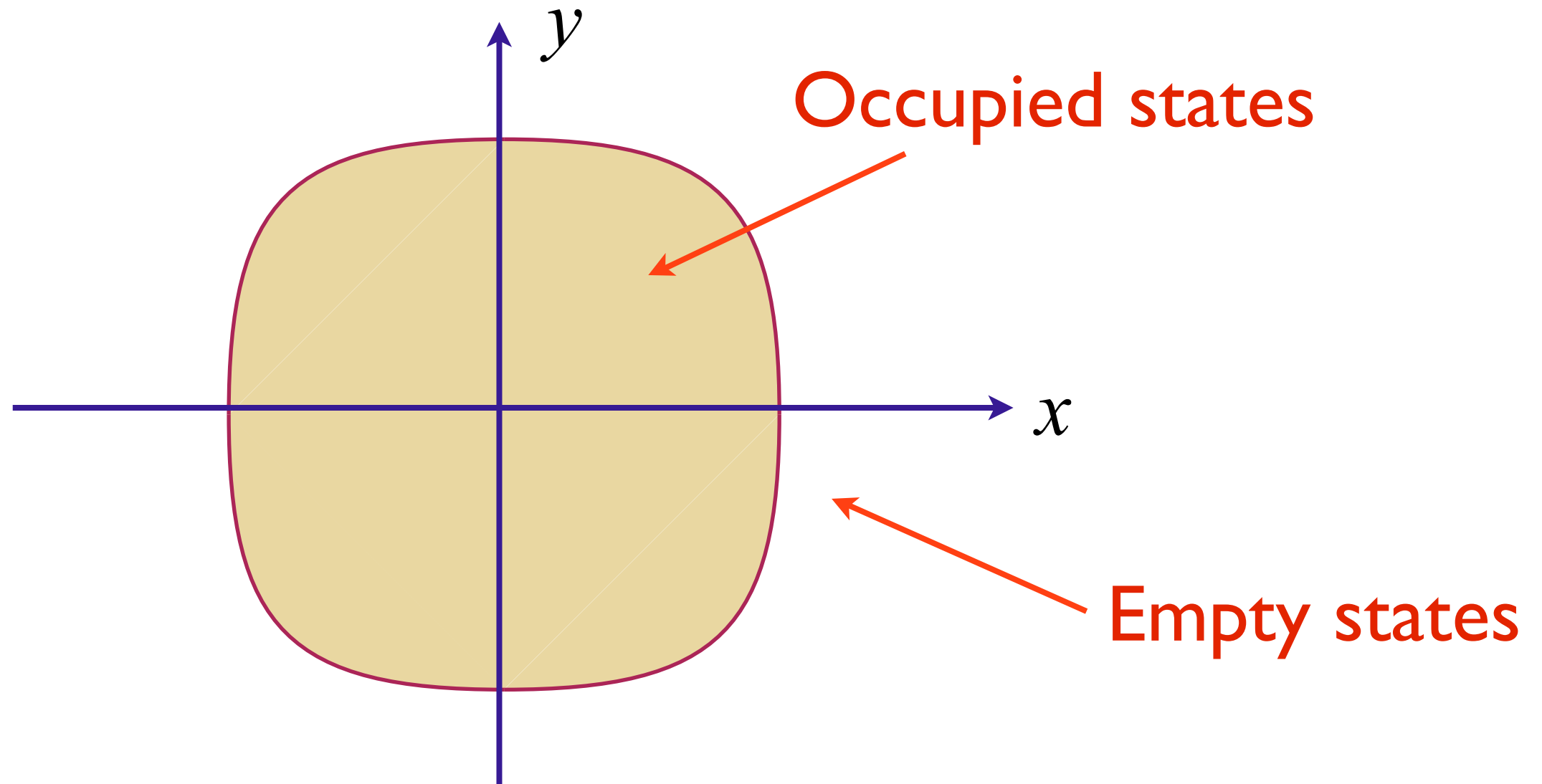
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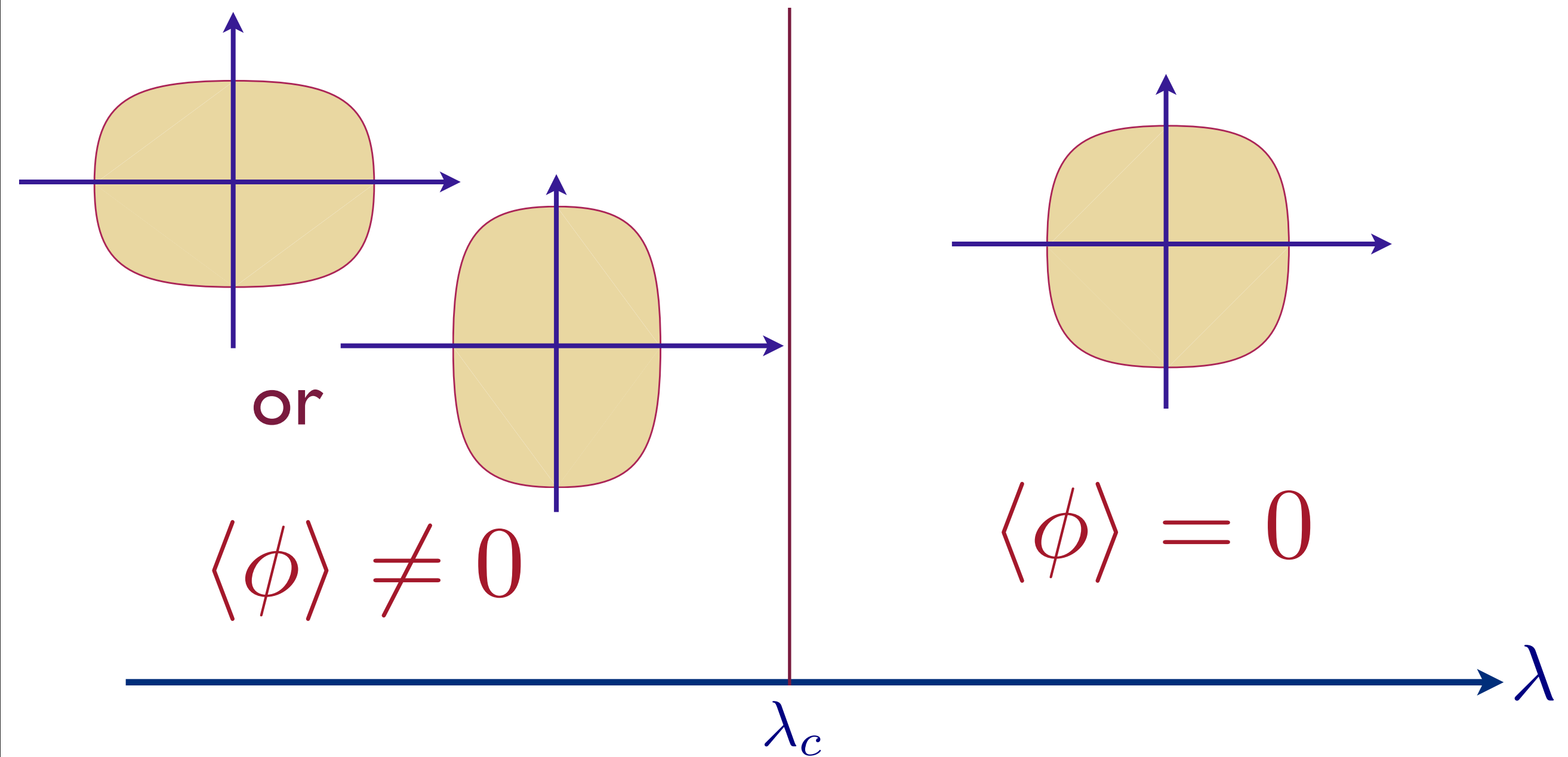
# Quantum criticality of Ising-nematic ordering in a metal



A metal with a Fermi surface  
with full square lattice symmetry

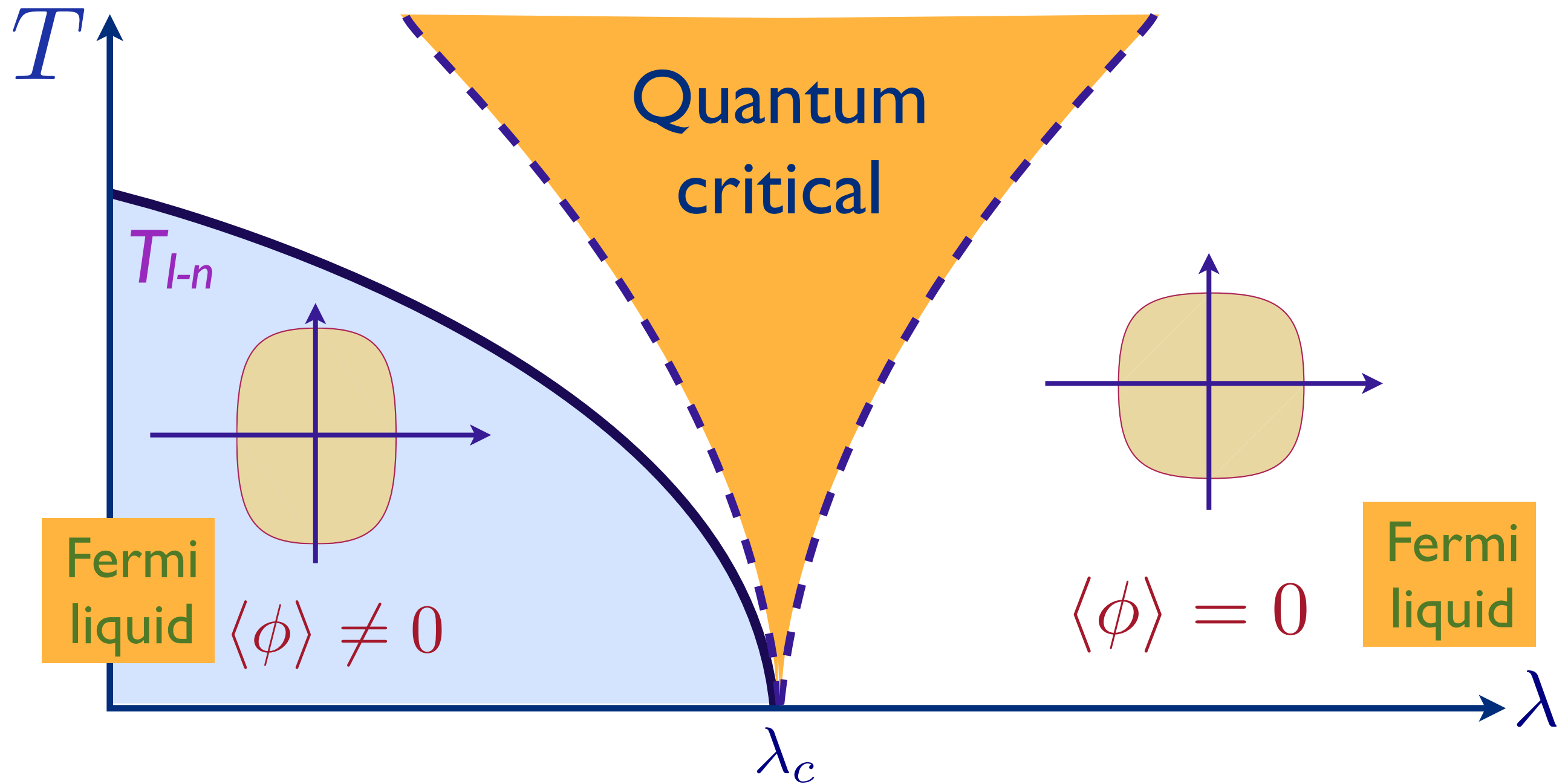


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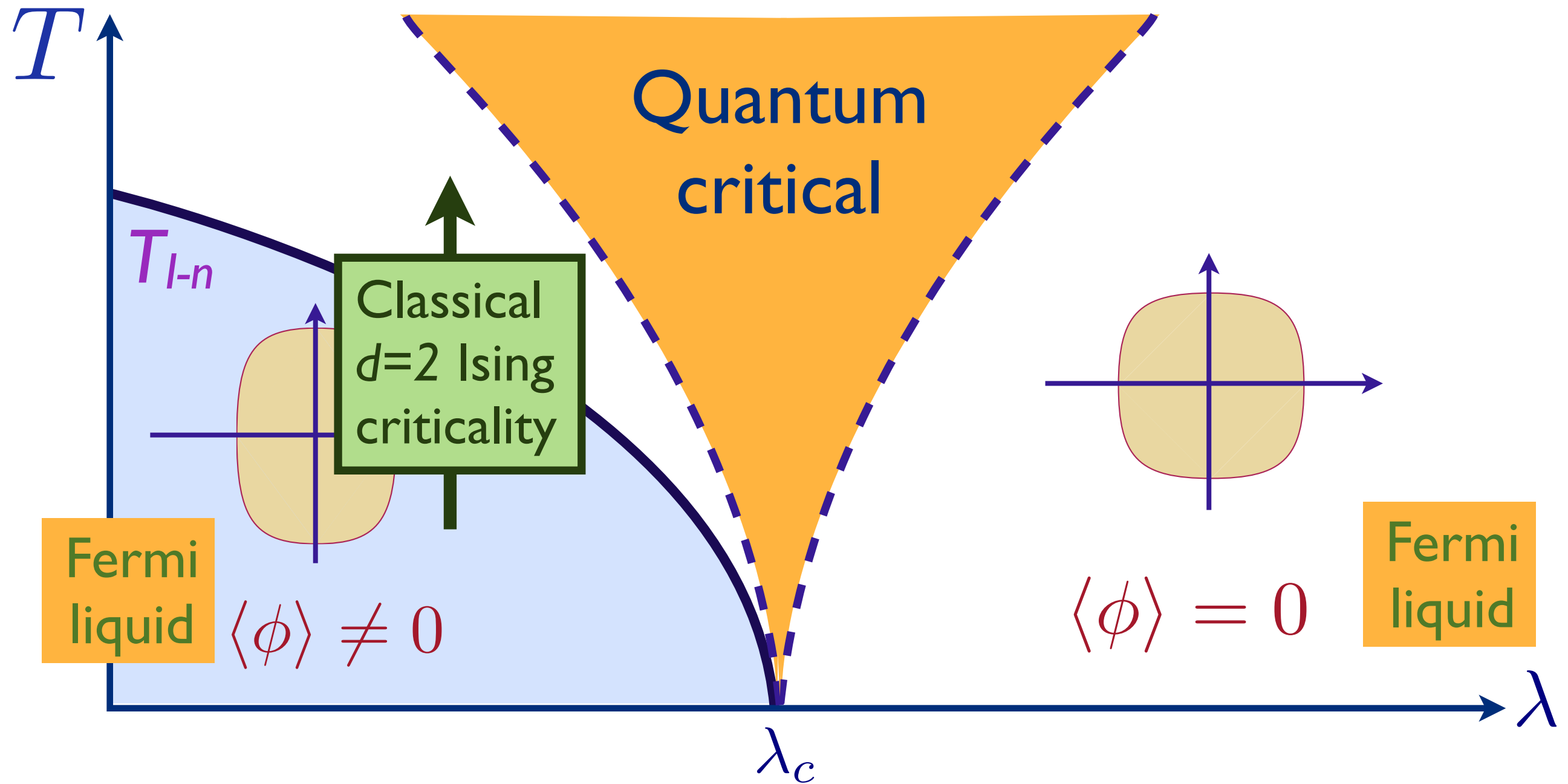
Pomeranchuk instability as a function of coupling  $\lambda$

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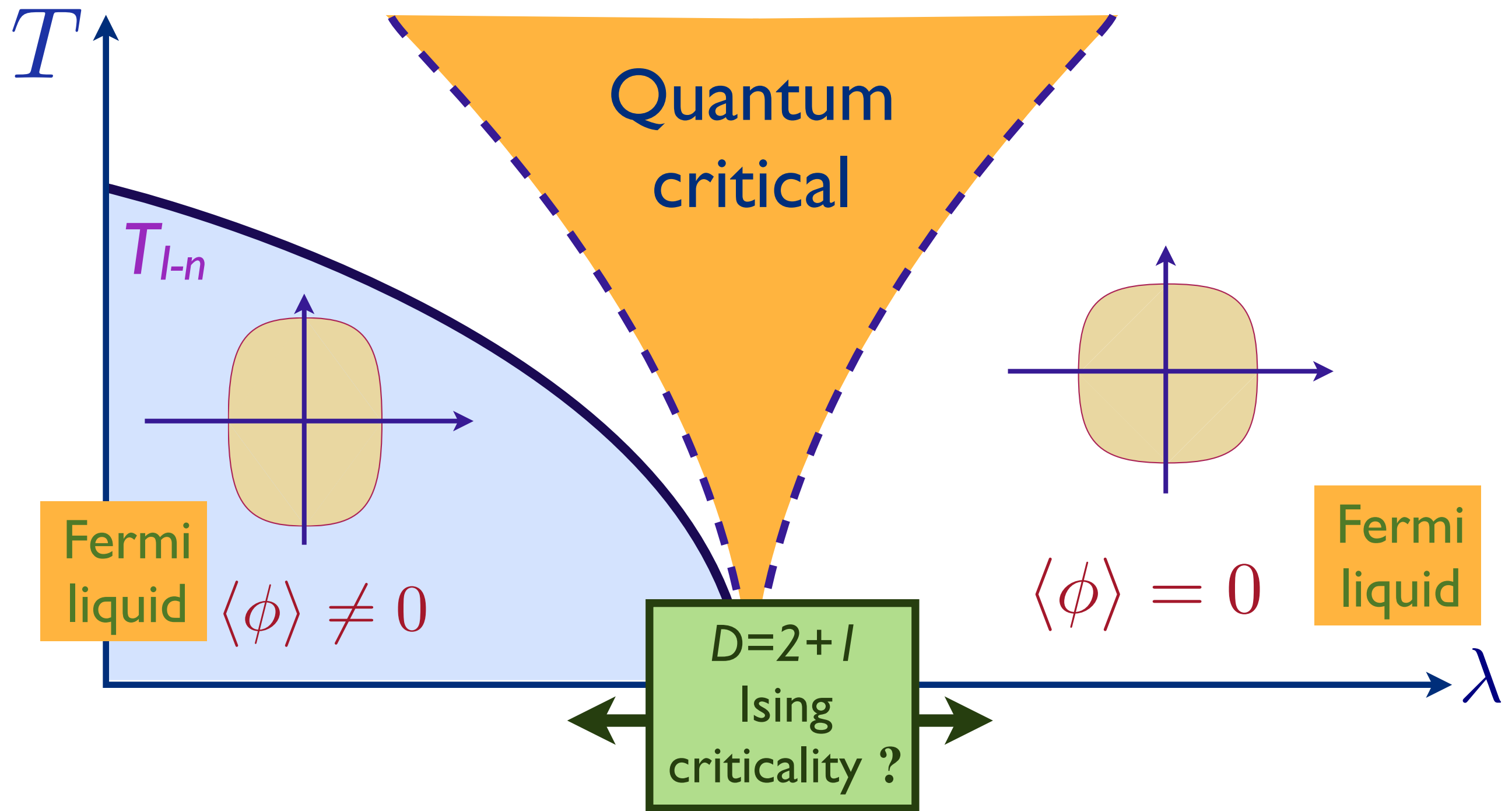
Phase diagram as a function of  $T$  and  $\lambda$

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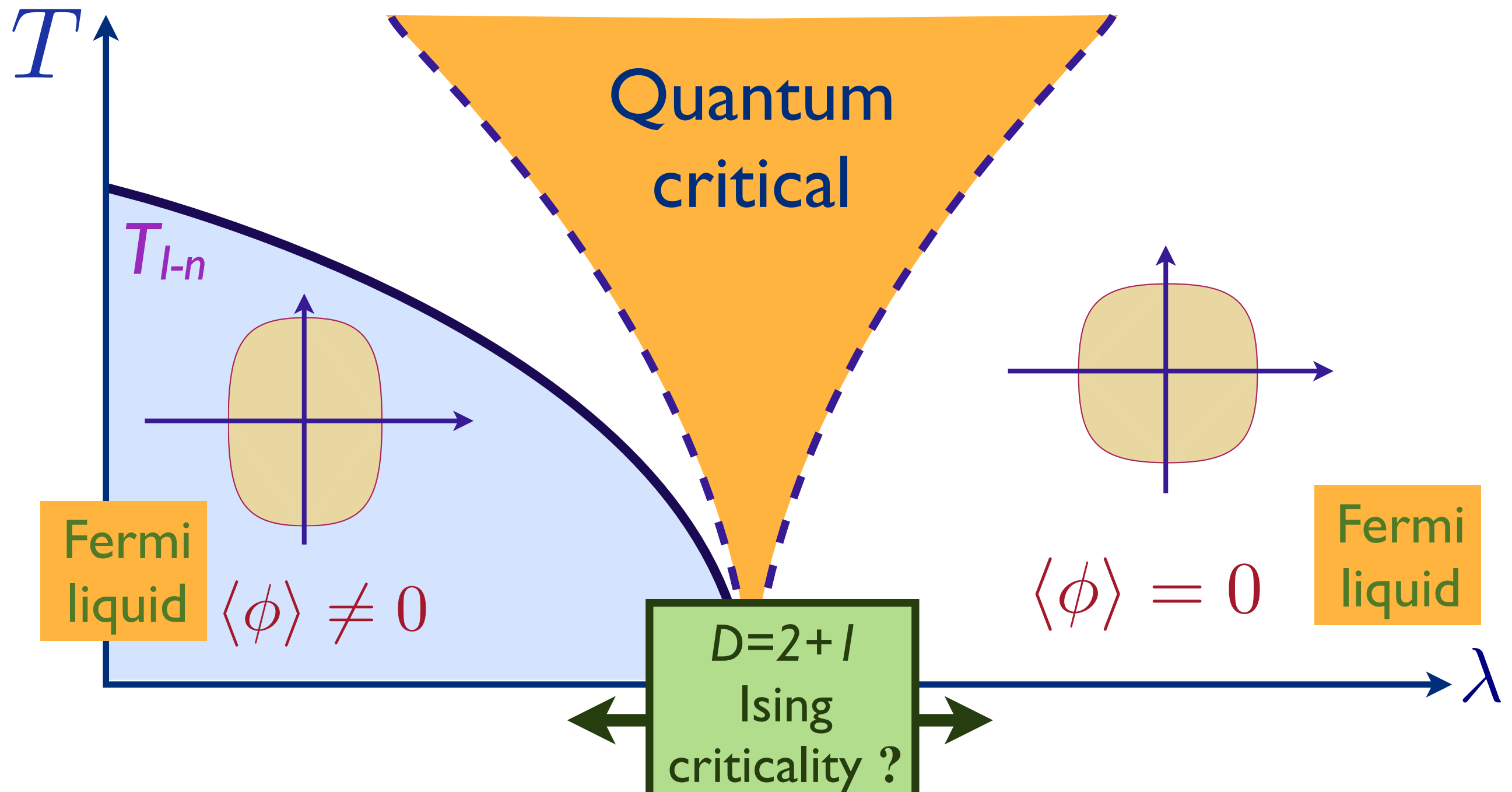
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Phase diagram as a function of  $T$  and  $\lambda$

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Only at higher energies; at the lowest energy bosonic  $\phi$  fluctuations are strongly coupled to fermionic excitations near Fermi surface.

Phase diagram as a function of  $T$  and  $\lambda$

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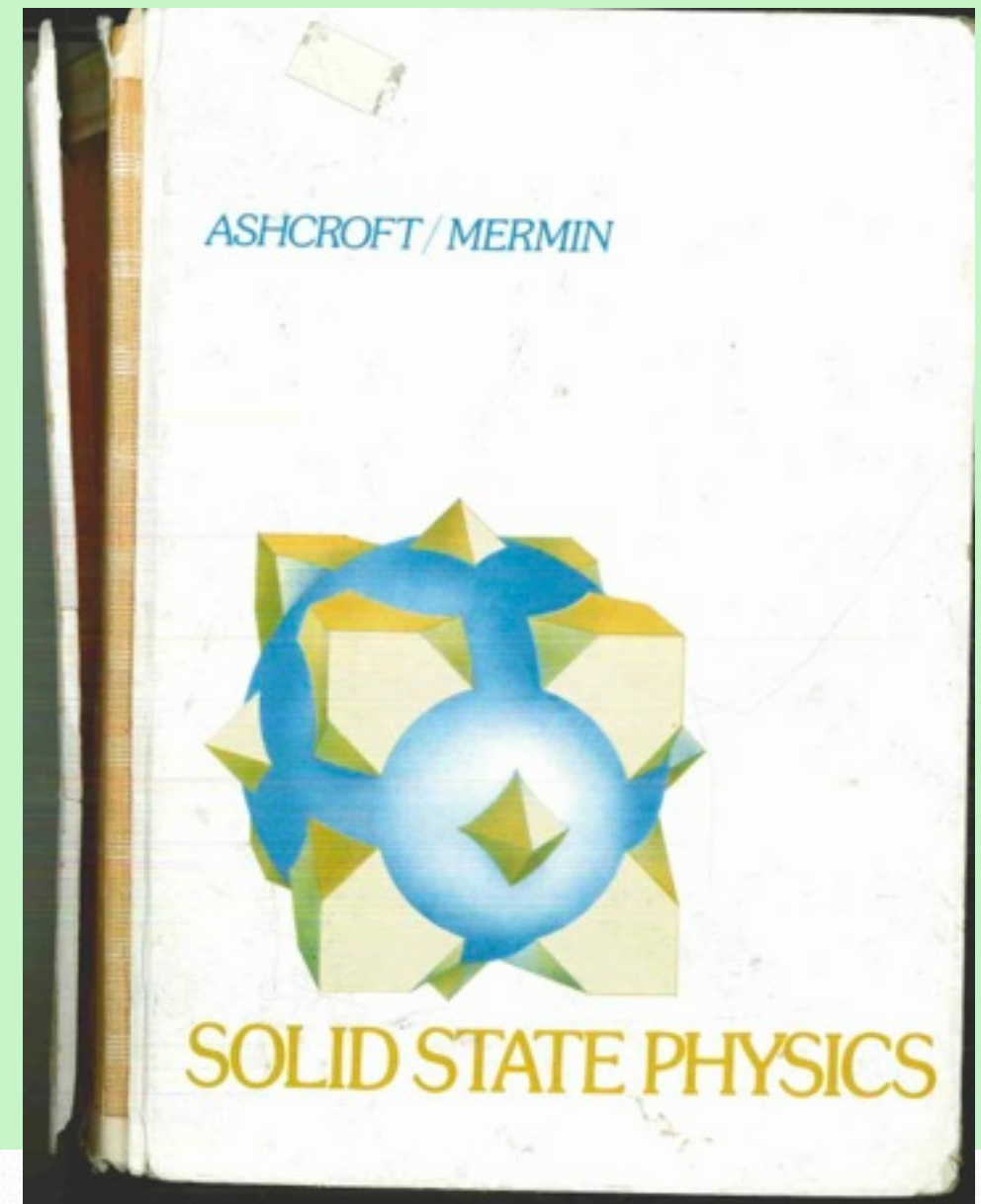
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## PHONON DRAG

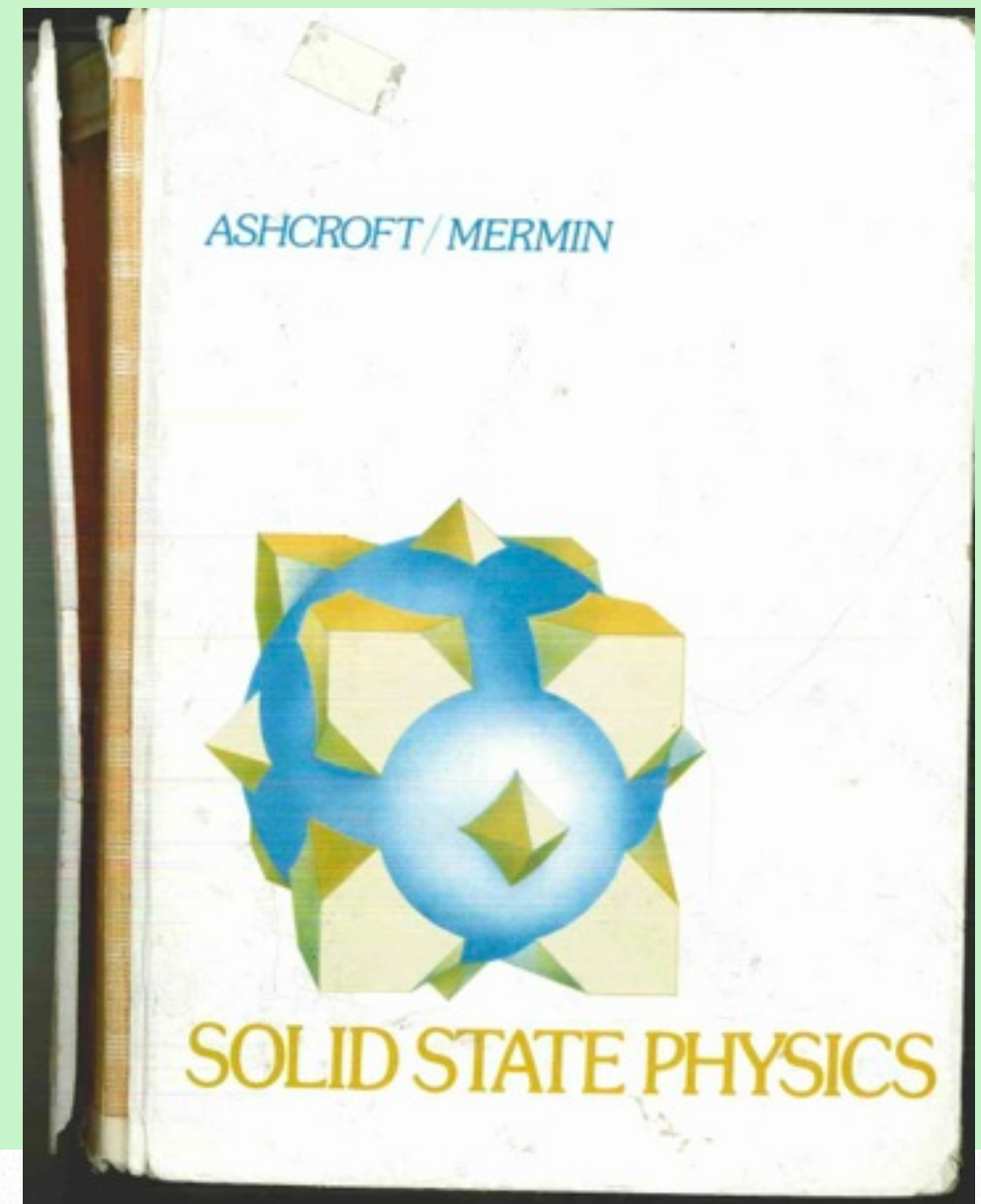
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## PHONON DRAG

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# Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

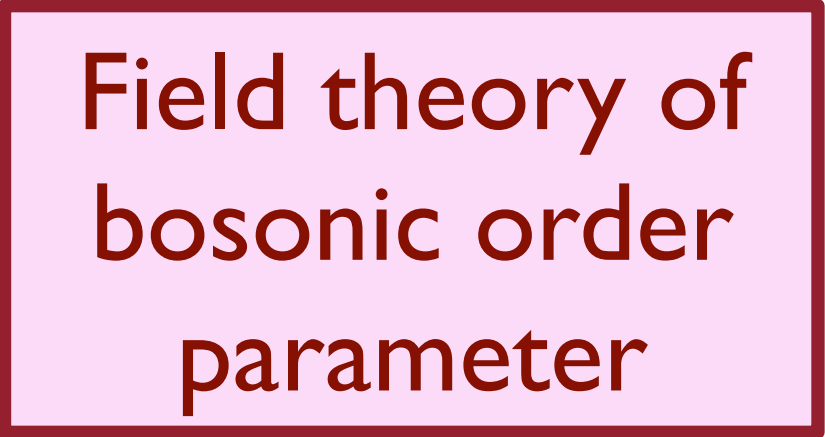
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Field theory of  
bosonic order  
parameter

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Electrons with a  
Fermi surface:  $\varepsilon_{\mathbf{k}} =$   
 $-2t(\cos k_x + \cos k_y) - \mu \dots$

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“Yukawa”  
coupling  
between bosons  
and fermions



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$$\mathcal{S}_{\phi c} = -g \int d^2r d\tau \sum_{\alpha=1}^{N_f} \phi \left[ c_\alpha^\dagger \{ (\partial_x^2 - \partial_y^2 + \dots) c_\alpha \} \right. \\ \left. + \{ (\partial_x^2 - \partial_y^2 + \dots) c_\alpha^\dagger \} c_\alpha \right]$$

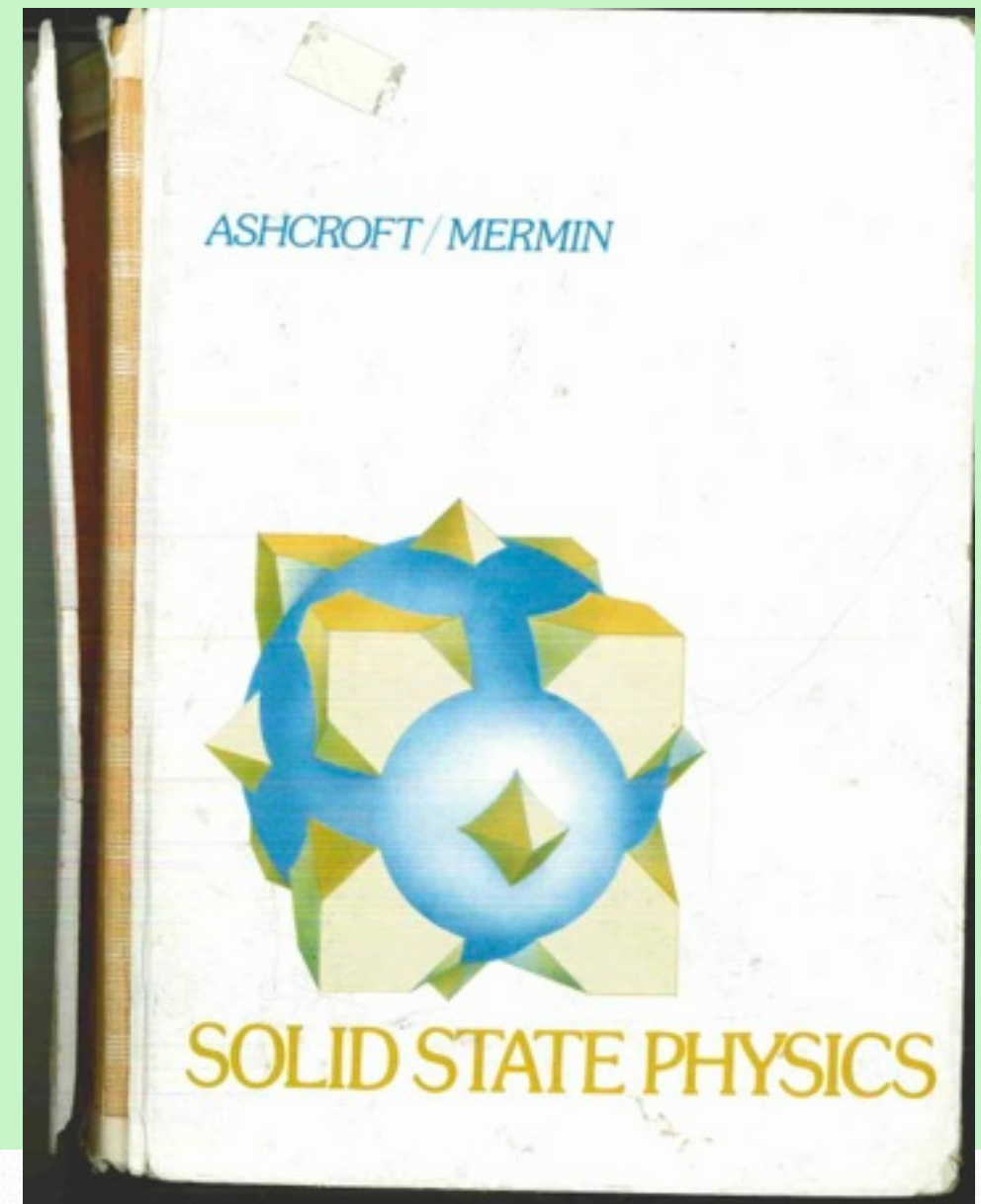
This continuum theory has a conserved momentum  $\mathbf{P}$ , and  $\chi_{\mathbf{J}, \mathbf{P}} \neq 0$ , and so the resistivity  $\rho(T) = 0$ .



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- However, Bloch’s law ignores conservation of total momentum, or **phonon drag**.
- The field theory for the Ising-nematic critical point has strong electron– $\phi$  scattering, and no quasi-particle excitations. Nevertheless, because of the central importance of the analog of phonon drag, it has  $\rho(T) = 0$ .

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- Focus on the interplay between  $J_\mu$  and  $T_{\mu\nu}$  !

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The most-probable state with a non-zero current **J** has a non-zero momentum **P** (and vice versa).

At non-zero density, **J** “drags” **P**.

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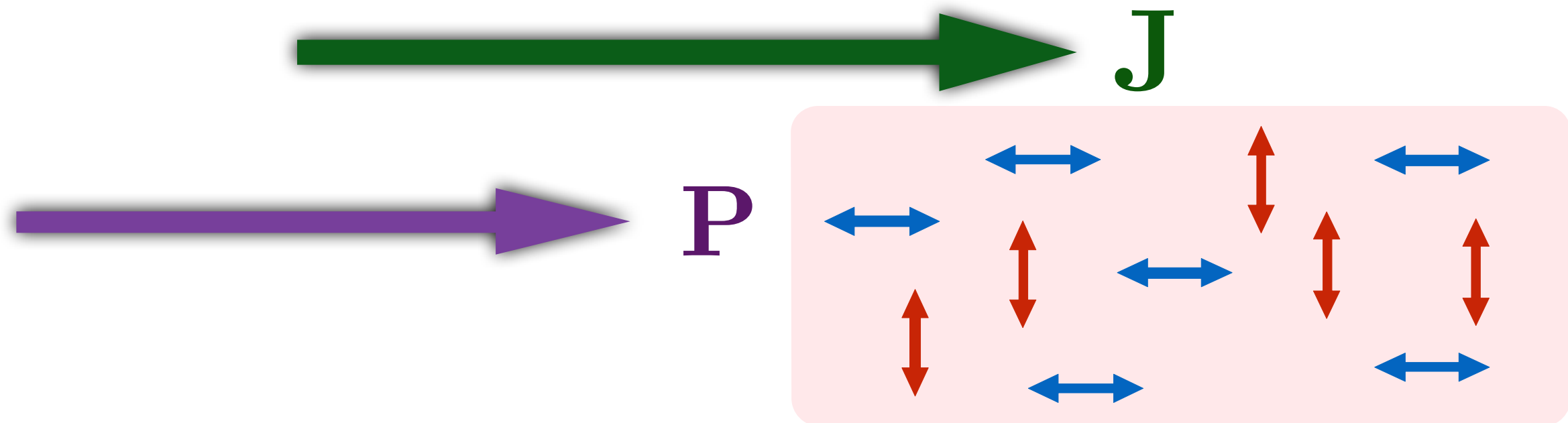
At non-zero density,  $\mathbf{J}$  “drags”  $\mathbf{P}$ .

The resistivity of this metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic

# Quantum criticality of Ising-nematic ordering in a metal

## Transport without quasiparticles:

- Focus on the interplay between  $J_\mu$  and  $T_{\mu\nu}$  !



The dominant momentum loss occurs via the scattering of the neutral bosonic  $\phi$  excitations off random fields

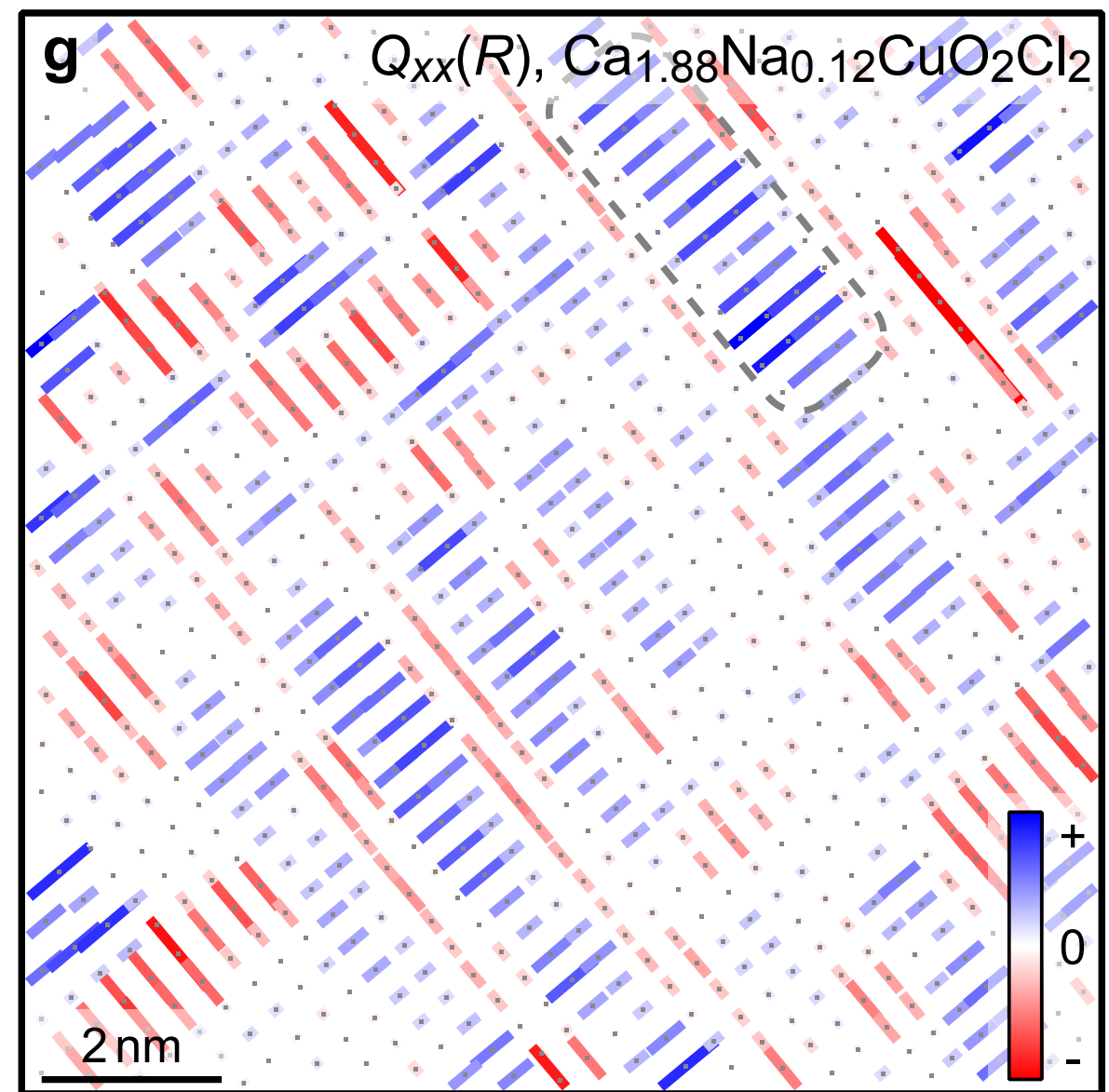
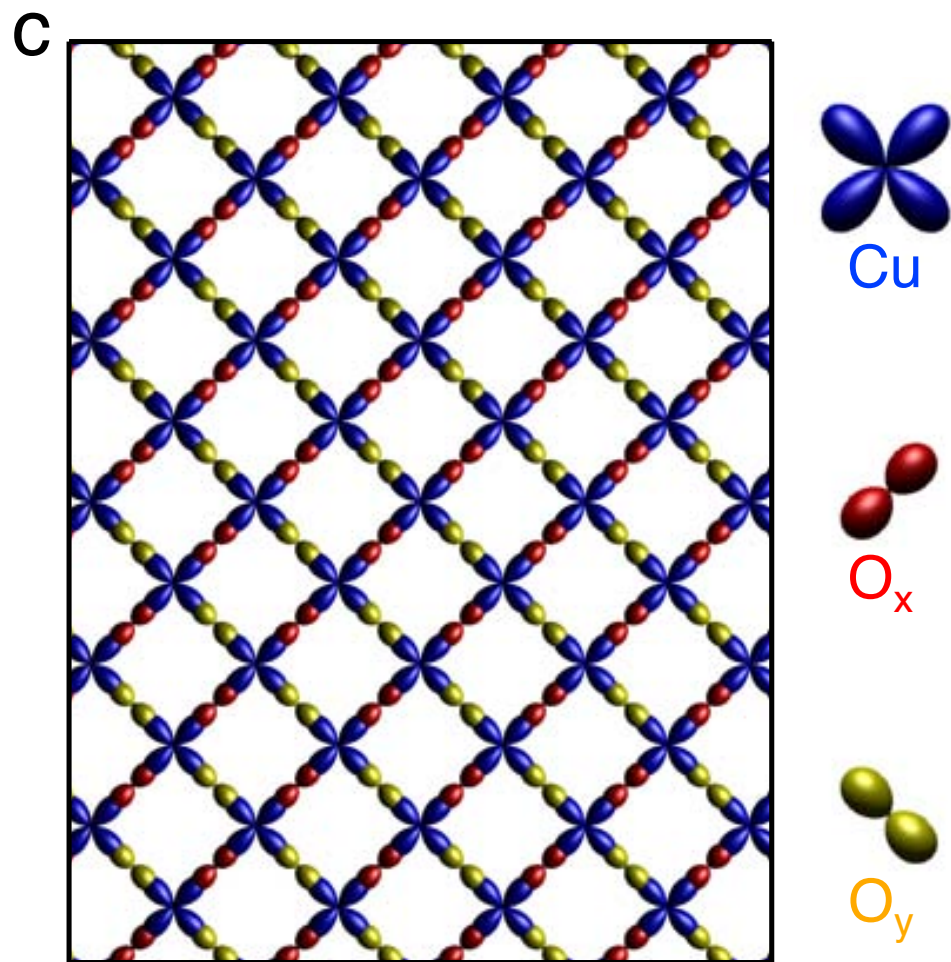
S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, arXiv:1401.7012.

A. Lucas, S. Sachdev, and K. Schalm, arXiv:1401.7933



# Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

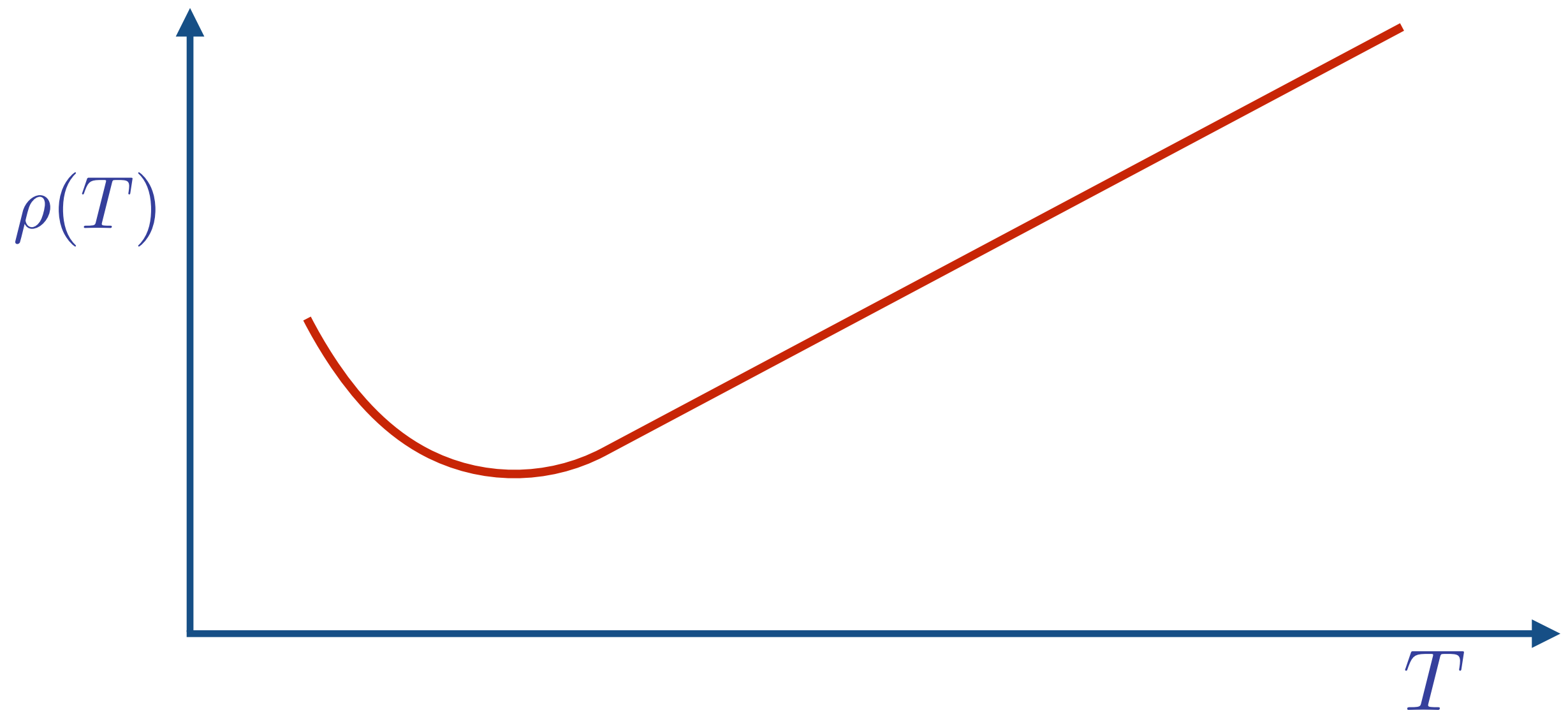
Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi  
*Nature Physics*, 8, 534 (2012).



Evidence for “nematic” order (*i.e.* breaking of  $90^\circ$  rotation symmetry) in  $\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$ .

# Quantum criticality of Ising-nematic ordering in a metal

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The dominant momentum loss occurs via the scattering of the neutral bosonic  $\phi$  excitations off random fields

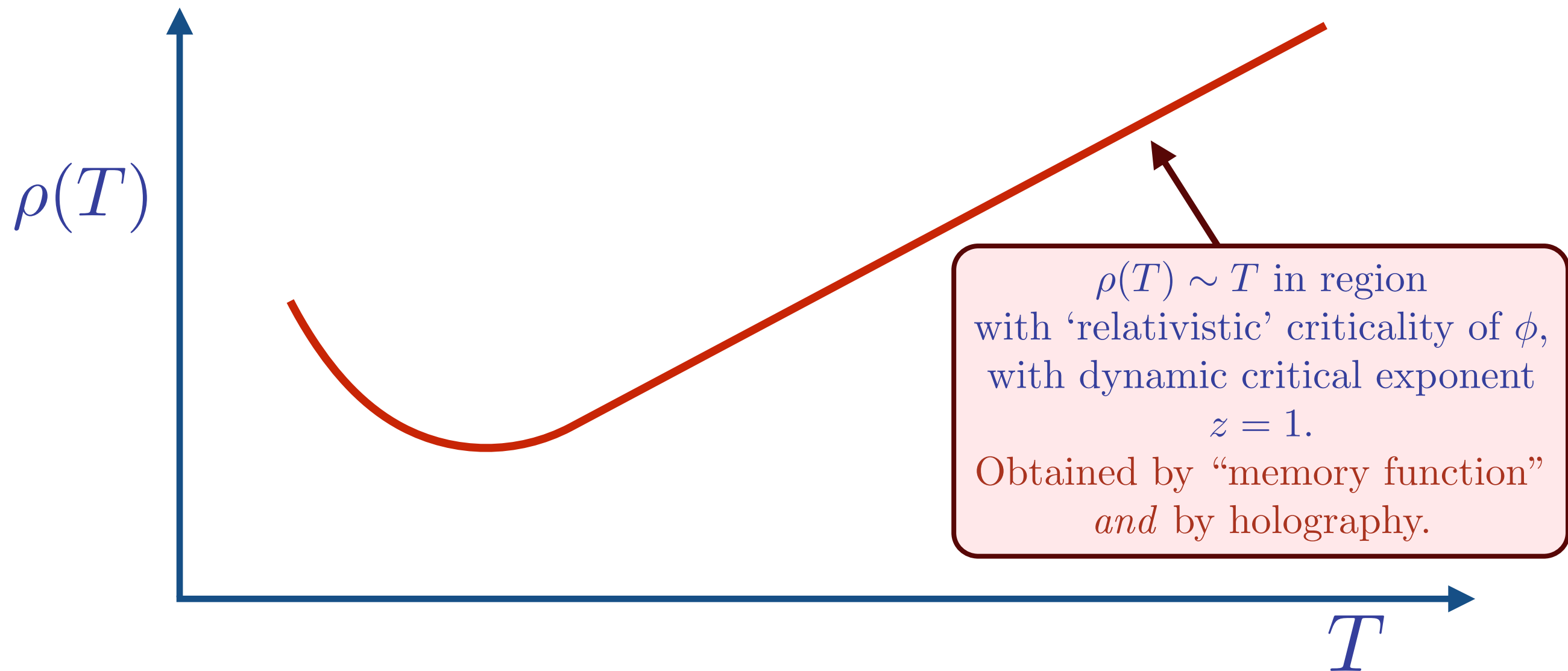
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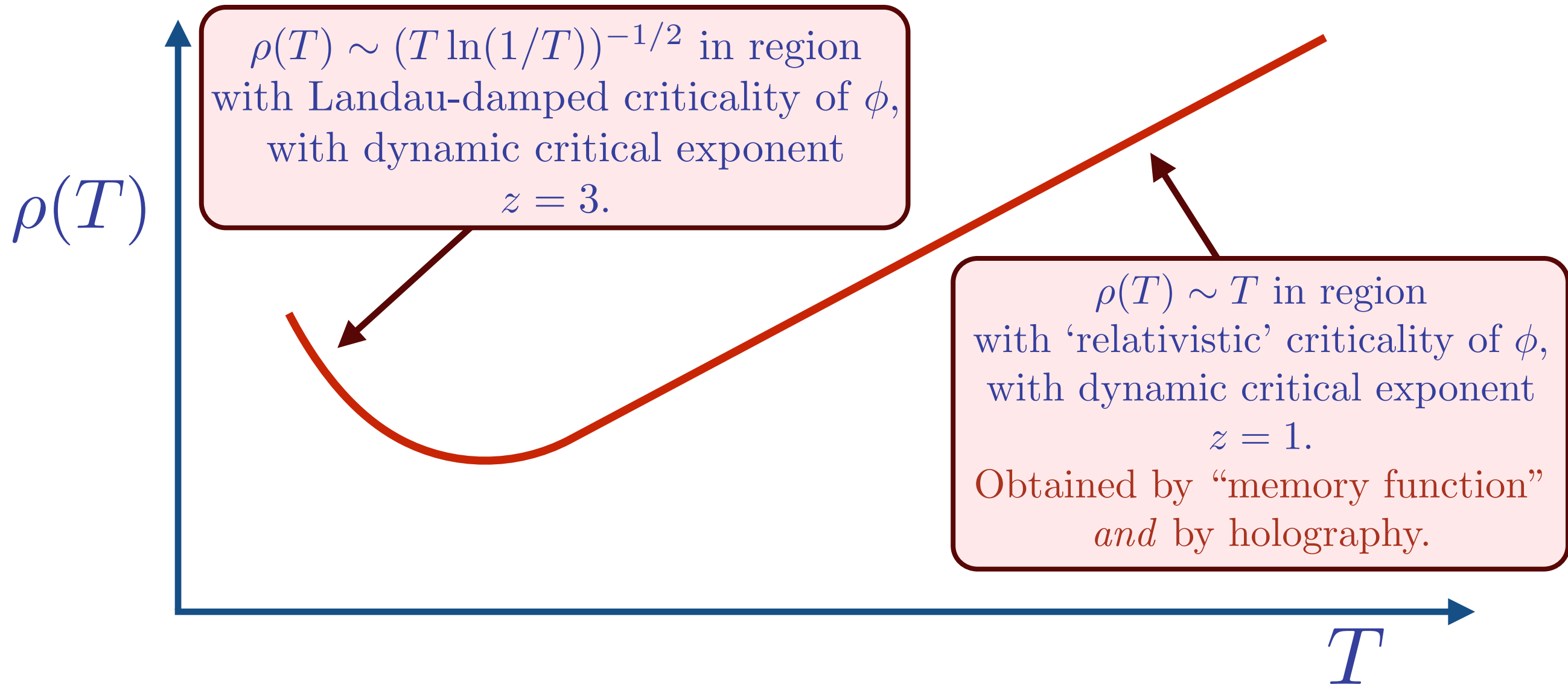
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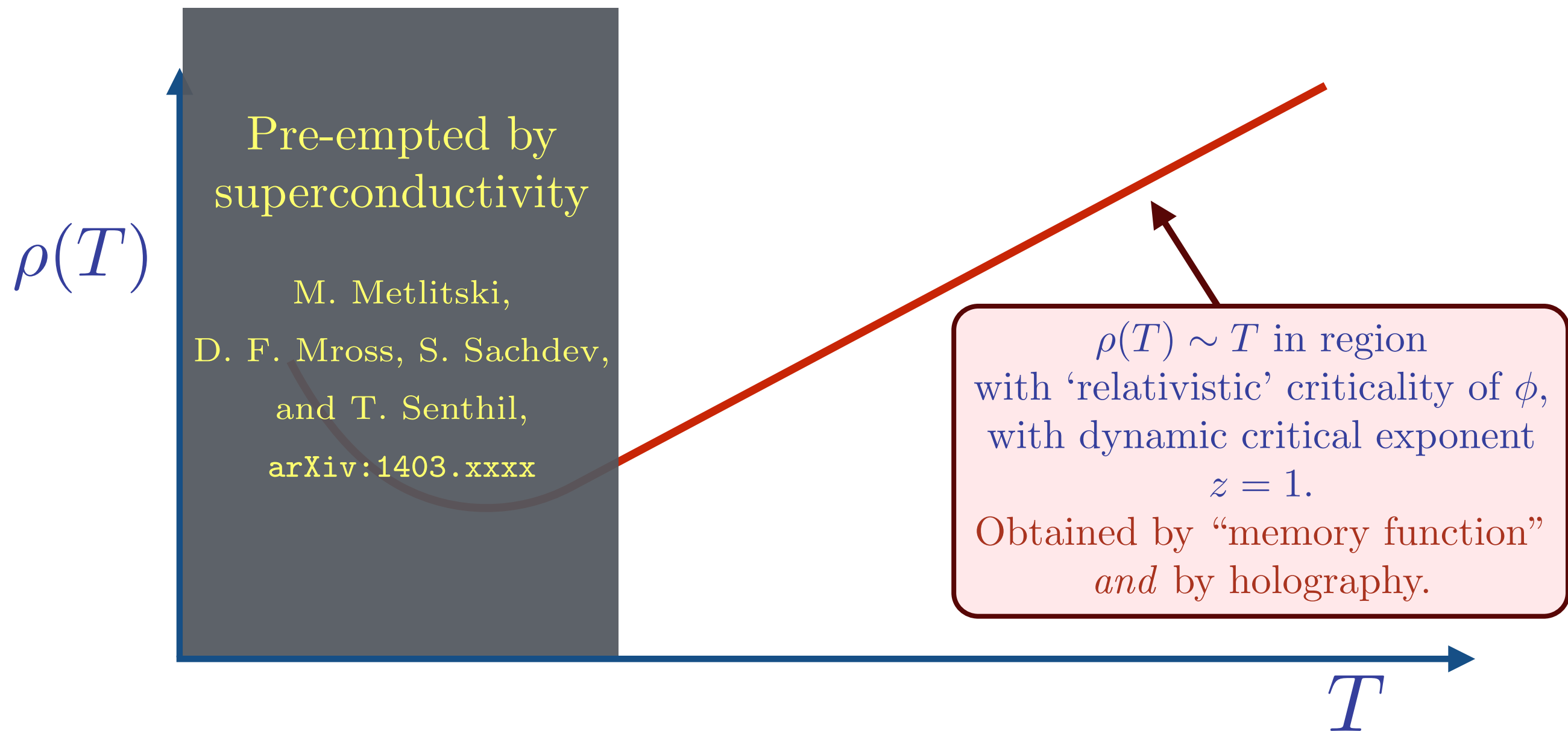
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- Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography.