

Planckian metals from spatially random interactions

Open Challenges in the Theory of Strongly
Correlated Electron Systems,
William I. Fine Theoretical Physics Institute,
University of Minnesota,
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Talk online: sachdev.physics.harvard.edu



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PHYSICS



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1. The SYK model

2. Fermi surface coupled to a critical boson
3. Adding spatially random interactions to the critical Fermi surface
4. Random t - J model

The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

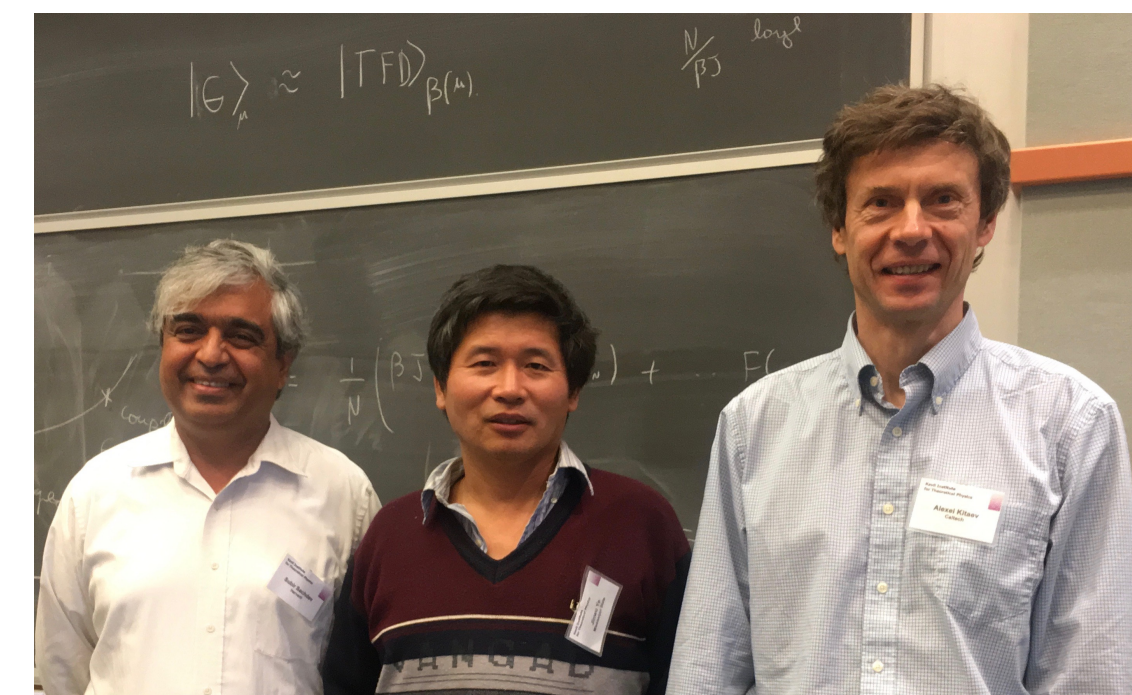
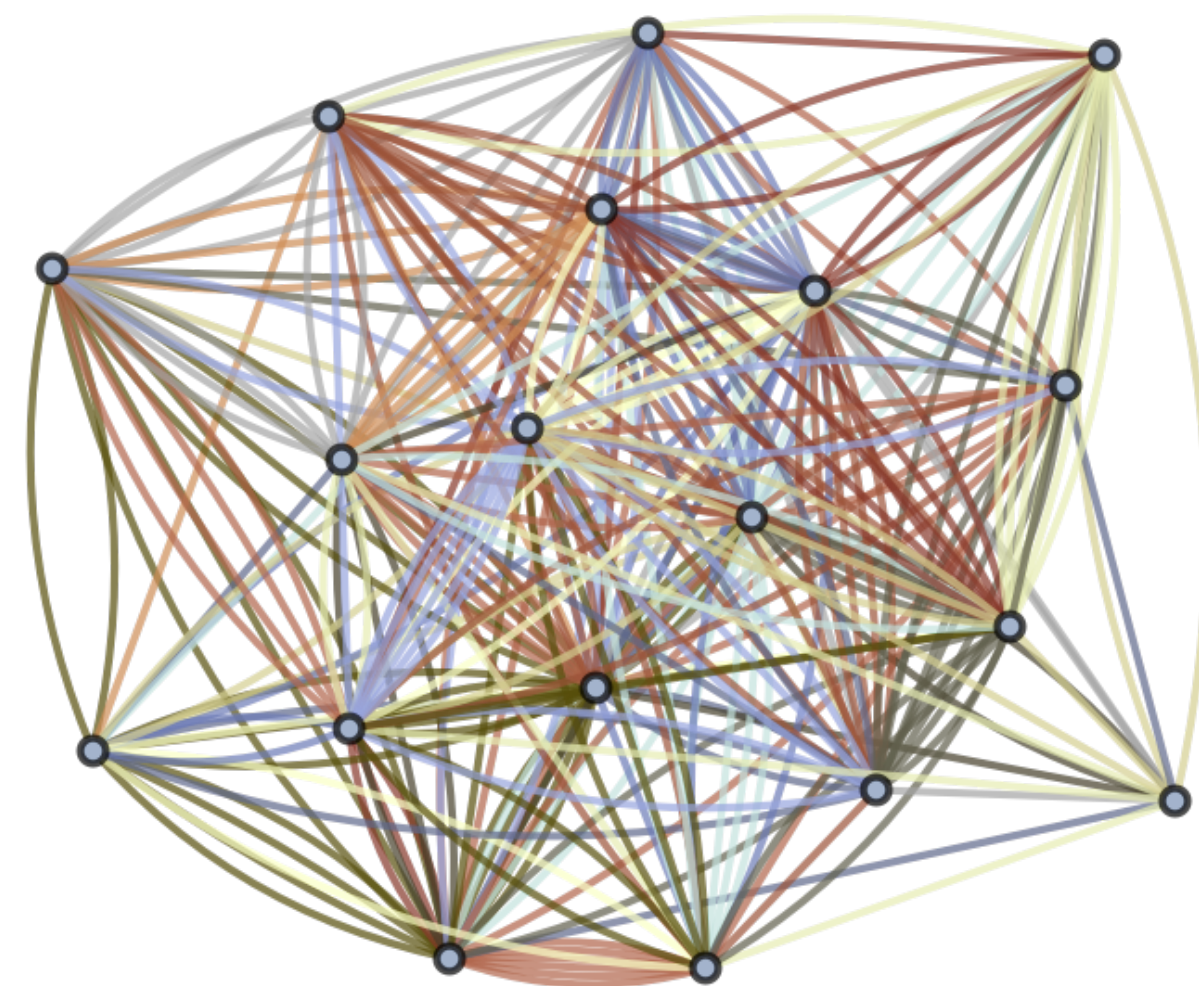
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$Q = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



G-Σ Theory

The theory self-averages, and the average partition function can be written exactly as a ‘G-Σ’ theory involving a path integral over bilocal in time Green’s function $G(\tau_1, \tau_2)$ and self energy $\Sigma(\tau_1, \tau_2)$

$$\bar{Z} = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln \det [\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)] \\ + \int d\tau_1 d\tau_2 [\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)]$$

The large N saddle point yields $G(\tau_1 - \tau_2)$ and $\Sigma(\tau_1 - \tau_2)$ which obey

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad ; \quad \Sigma(\tau) = -U^2 G^2(\tau)G(-\tau)$$

Exact Solution at small ω :

$$\Sigma(i\omega) \sim -i \operatorname{sgn}(\omega) \sqrt{|\omega|}, \quad G(i\omega) = \frac{-1}{\Sigma(i\omega)}$$

where the co-efficient is known exactly.

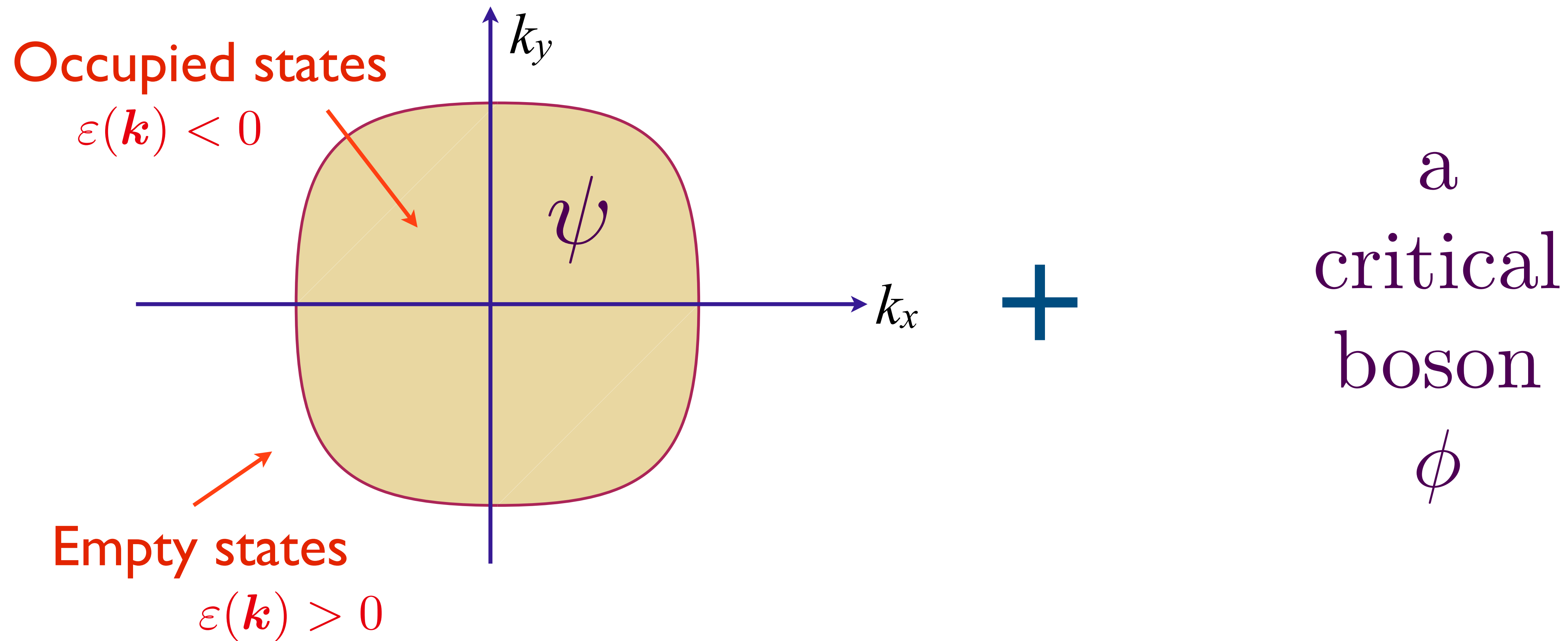
1. The SYK model

2. Fermi surface coupled to a critical boson

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4. Random t - J model

Fermi surface coupled to a critical boson



Fermi surface coupled to a critical boson

“Yukawa” coupling: $g \int d^2r d\tau \psi^\dagger(r, \tau) \psi(r, \tau) \phi(r, \tau)$

Yields a state without quasiparticle excitations, but the theory is not systematic at large N

Sung-Sik Lee (2009)

Fermi surface coupled to a critical boson

“Yukawa” coupling: $\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Main idea:

Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

Ilya Esterlis, J. Schmalian, PRB **100**, 115132 (2019)

Yuxuan Wang and A.V. Chubukov, PRR **2**, 033084 (2020)

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G - Σ - D - Π Theory

The theory self-averages, and the average partition function can be written exactly as a ‘ G - Σ ’ theory involving a path integral over *bilocal in spacetime*. We introduce the spacetime co-ordinate $X \equiv (\tau, x, y)$, and all Green’s functions and self energies in the path integral are functions of two spacetime co-ordinates X_1 and X_2 .

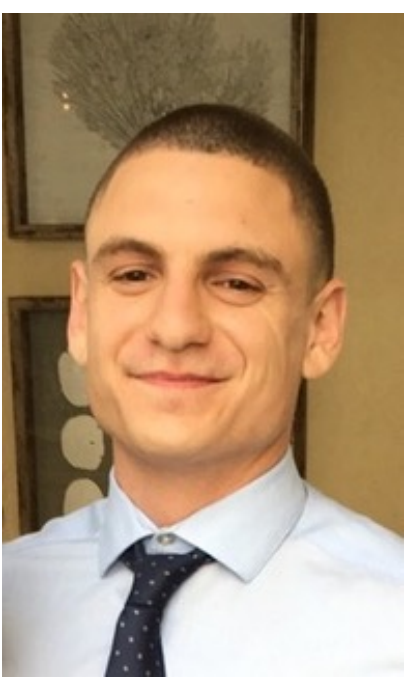
$$\bar{\mathcal{Z}} = \int \mathcal{D}G(X_1, X_2) \mathcal{D}\Sigma(X_1, X_2) \mathcal{D}D(X_1, X_2) \mathcal{D}\Pi(X_1, X_2) \exp [-NI(G, \Sigma, D, \Pi)] .$$

The G - Σ - D - Π action is now

$$\begin{aligned} I(G, \Sigma, D, \Pi) = & \frac{g^2}{2} \text{Tr} (G \cdot [GD]) - \text{Tr}(G \cdot \Sigma) + \frac{1}{2} \text{Tr}(D \cdot \Pi) \\ & - \ln \det [(\partial_{\tau_1} + \varepsilon(-i\nabla_1)) \delta(X_1 - X_2) + \Sigma(X_1, X_2)] \\ & + \frac{1}{2} \ln \det [(-\partial_{\tau_1}^2 - \nabla_1^2 + s) \delta(X_1 - X_2) - \Pi(X_1, X_2)] . \end{aligned}$$

where we have introduced notation

$$\text{Tr} (f \cdot g) \equiv \int dX_1 dX_2 f(X_2, X_1) g(X_1, X_2) .$$



G-Σ-D-Π Theory

The saddle point equations are

$$\Sigma(\mathbf{r}, \tau) = g^2 \lambda D(\mathbf{r}, \tau) G(\mathbf{r}, \tau),$$

$$\Pi(\mathbf{r}, \tau) = -g^2 G(-\mathbf{r}, -\tau) G(\mathbf{r}, \tau),$$

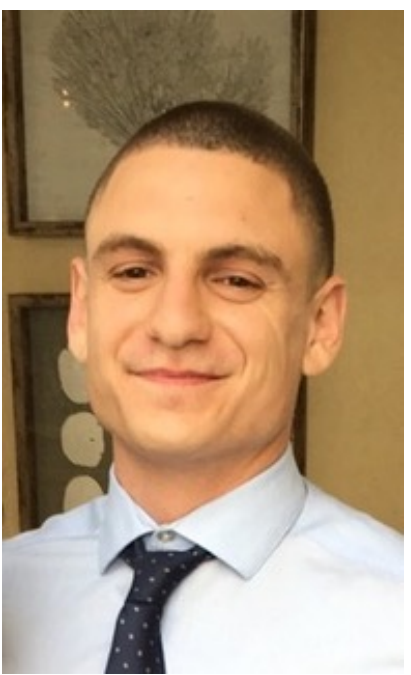
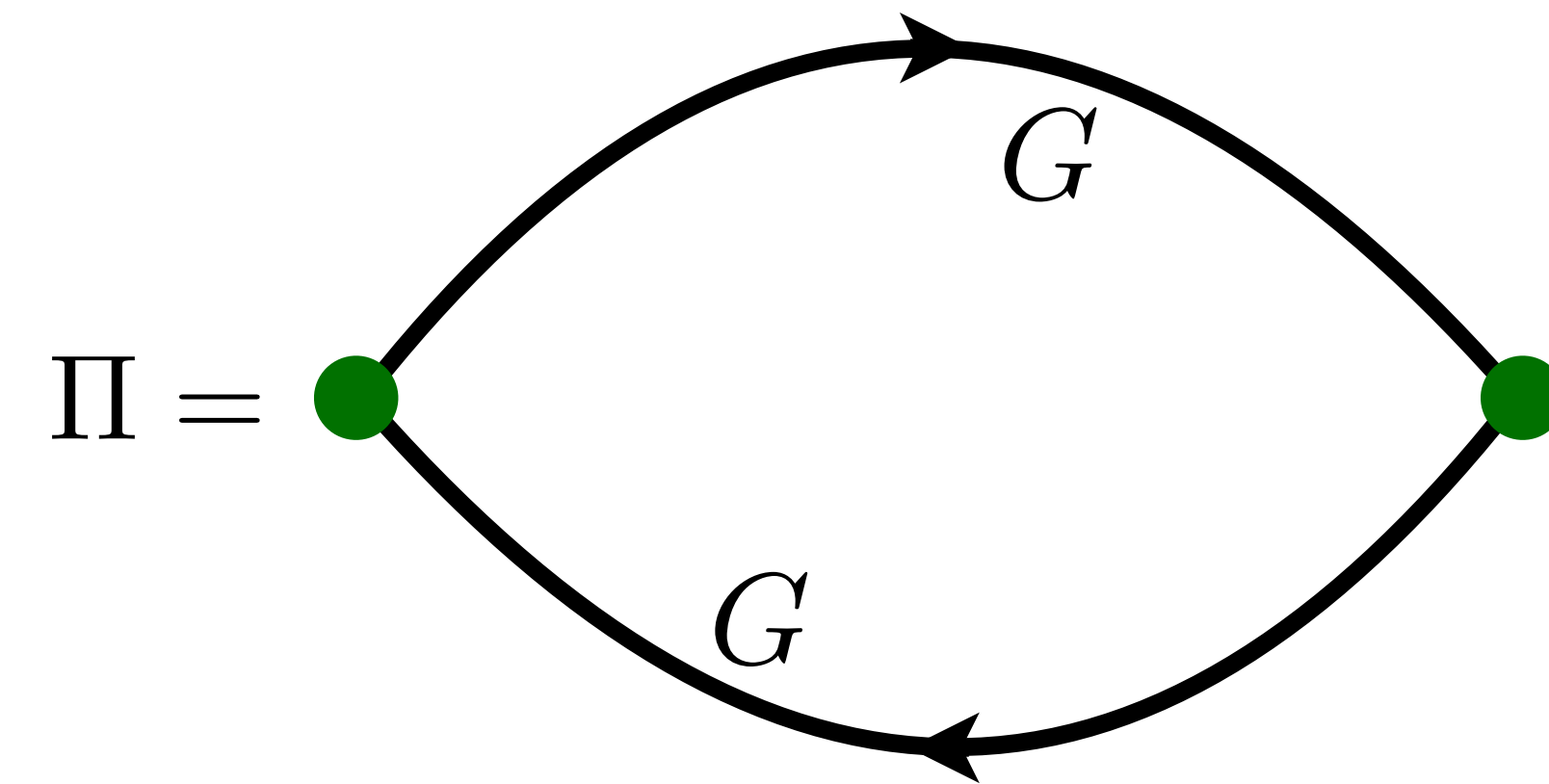
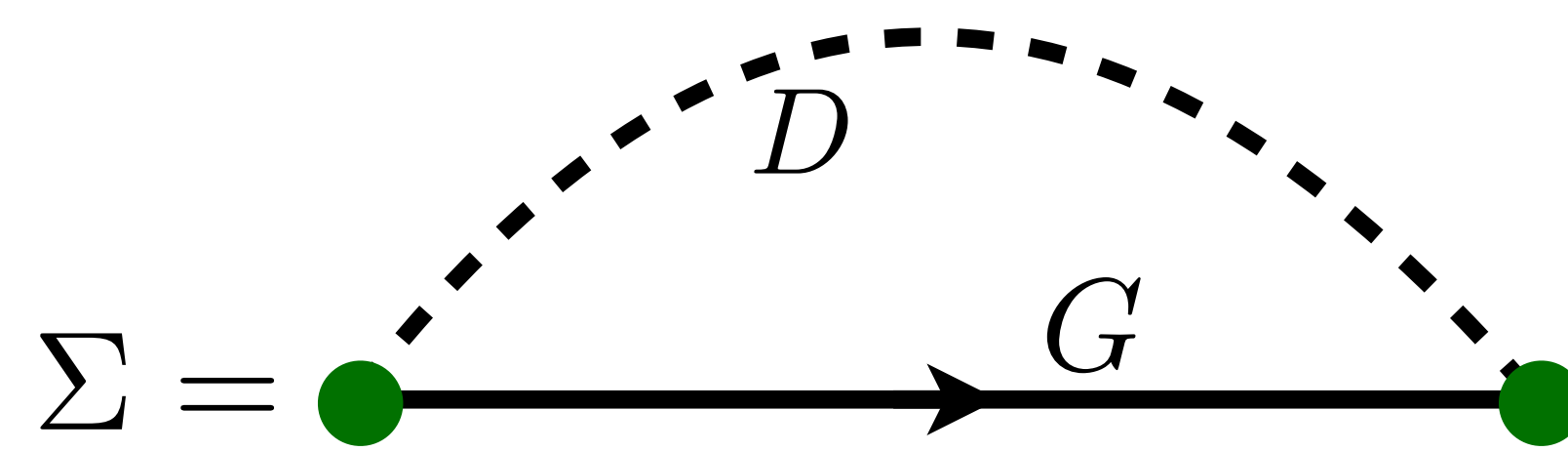
$$G(\mathbf{k}, i\omega_n) = \frac{1}{i\omega_n - \varepsilon(\mathbf{k}) - \Sigma(\mathbf{k}, i\omega_n)},$$

$$D(\mathbf{q}, i\Omega_m) = \frac{1}{\Omega_m^2 + q^2 + s - \Pi(\mathbf{q}, i\Omega_m)}.$$

Exact Solution at small ω :

$$\Sigma(\hat{\mathbf{k}}, i\omega) \sim -i \text{sgn}(\omega) |\omega|^{2/3}, \quad G(\mathbf{k}, i\omega) = \frac{-1}{\varepsilon(\mathbf{k}) + \Sigma(\hat{\mathbf{k}}, i\omega)}$$

where the co-efficient is known exactly in terms of the Fermi velocity and Fermi surface curvature at the Fermi surface point along the direction $\hat{\mathbf{k}}$.



G-Σ-D-Π Theory

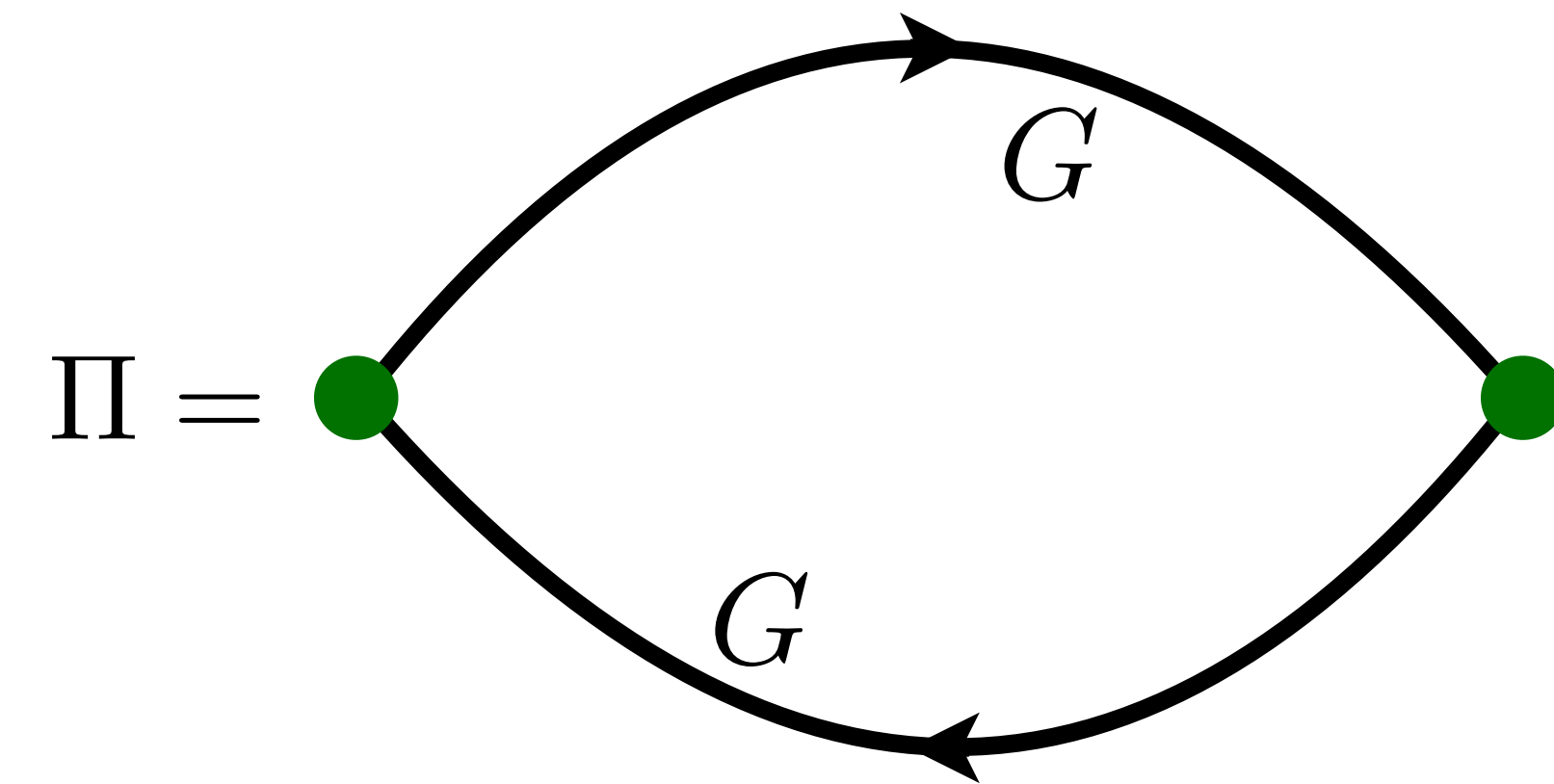
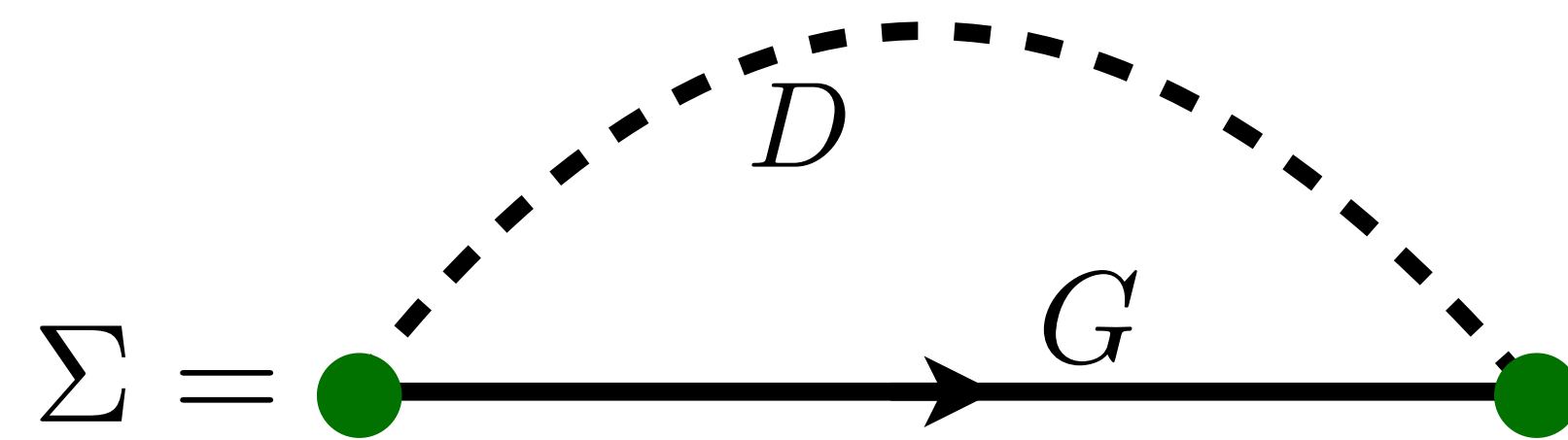
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$$D(\mathbf{q}, i\Omega_m) = \frac{1}{\Omega_m^2 + q^2 + s - \Pi(\mathbf{q}, i\Omega_m)}.$$



- There is many-body quantum chaos in the out-of-time-order correlator (OTOC) with maximal Lyapunov exponent $\lambda_L = 2\pi k_B T / \hbar$.

Fermi surface coupled to a critical boson

“Yukawa” coupling:
$$\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$$
$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Conservation of momentum implies the d.c. conductivity is infinite

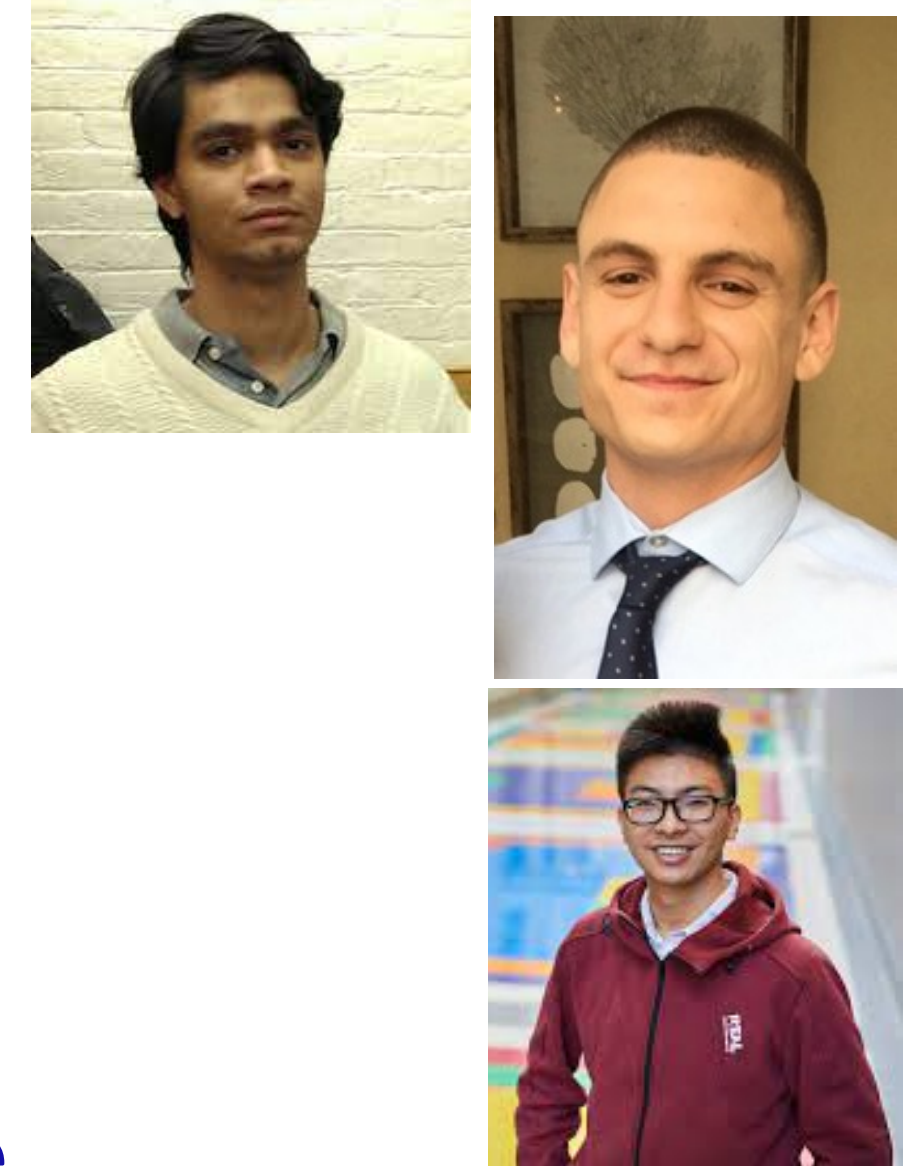
S. A. Hartnoll, R. Mahajan, M. Punk, and S. Sachdev, PRB **89**, 155130 (2014)

A. Eberlein, I. Mandal, and S. S., PRB **94**, 045133 (2016)

$$\text{Re } \sigma(\omega) = D\delta(\omega) + \text{Re } \sigma_{\text{reg}}(\omega)$$

$$\text{Re } \sigma_{\text{reg}}(\omega, T = 0) \sim \frac{1}{\omega^{2/3}}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994)



1. The SYK model

2. Fermi surface coupled to a critical boson

3. Adding spatially random interactions to the critical Fermi surface

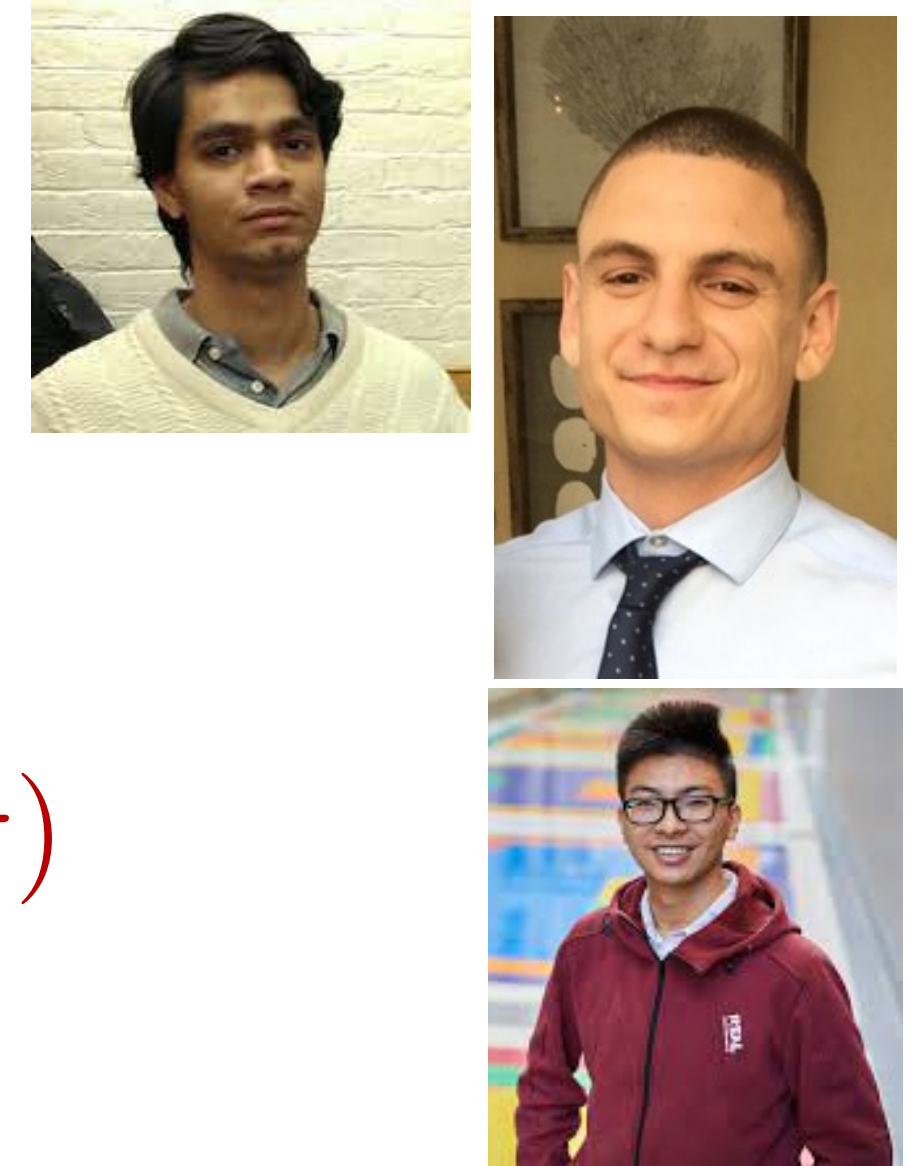
4. Random t - J model

Fermi surface coupled to a critical boson with spatial disorder

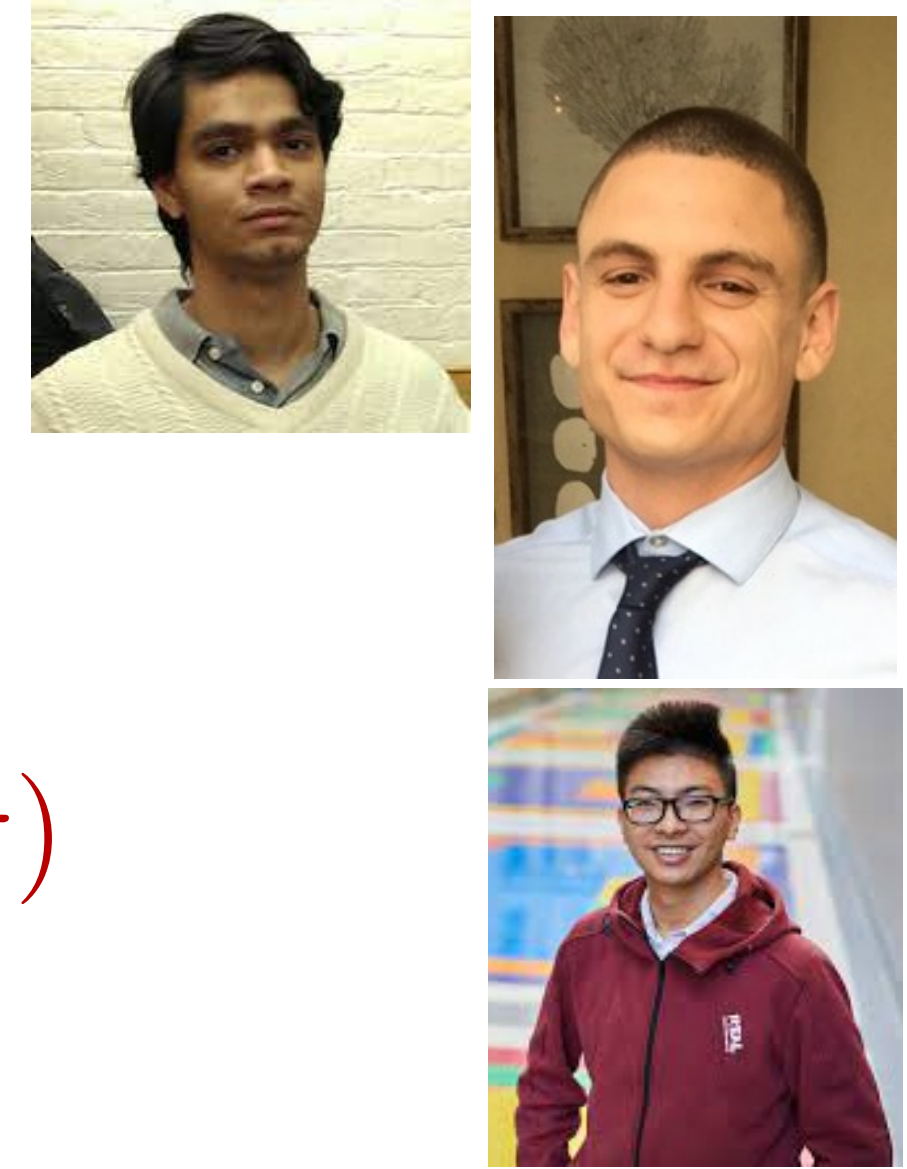
“Yukawa” coupling: $\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$

Random potential: $+\frac{1}{\sqrt{N}} \int d^2r d\tau v_{ij}(r) \psi_i^\dagger(r, \tau) \psi_j(r, \tau)$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g_{ijl}^* g_{abc}} = g^2 \delta_{ia} \delta_{jb} \delta_{lc} \quad , \quad \overline{v_{ij}(r)} = 0 \quad , \quad \overline{v_{ij}^*(r) v_{lm}(r')} = v^2 \delta(r - r') \delta_{il} \delta_{jm}$$



Fermi surface coupled to a critical boson with spatial disorder



“Yukawa” coupling: $\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$

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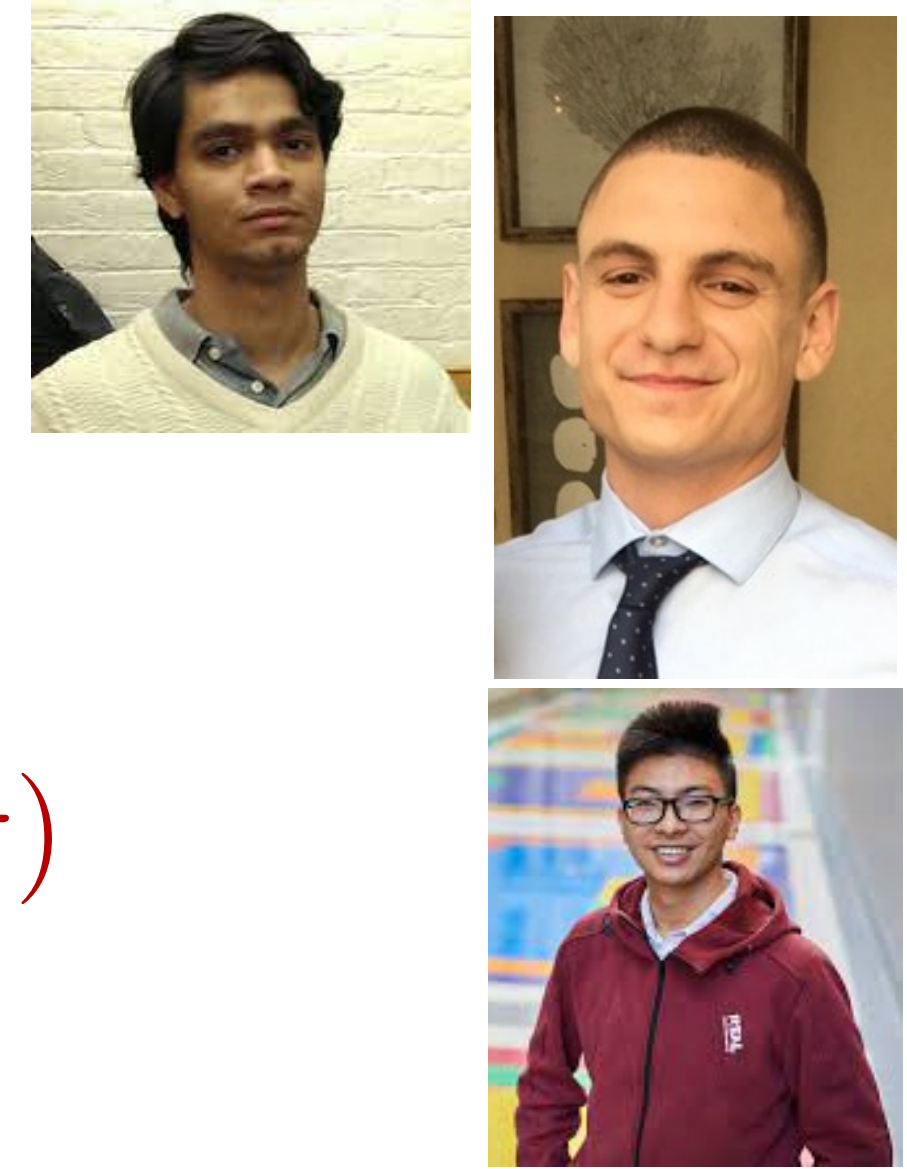
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$$\text{Boson self energy: } \Pi \sim -\frac{g^2}{v^2} |\Omega|, \quad D(q, i\Omega) = \frac{1}{q^2 + \gamma |\Omega|}$$

$$\text{Fermion self energy: } \Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\frac{g^2}{v^2} \omega \ln(1/|\omega|)$$

Marginal Fermi liquid self energy and $T \log T$ specific heat

Fermi surface coupled to a critical boson with spatial disorder



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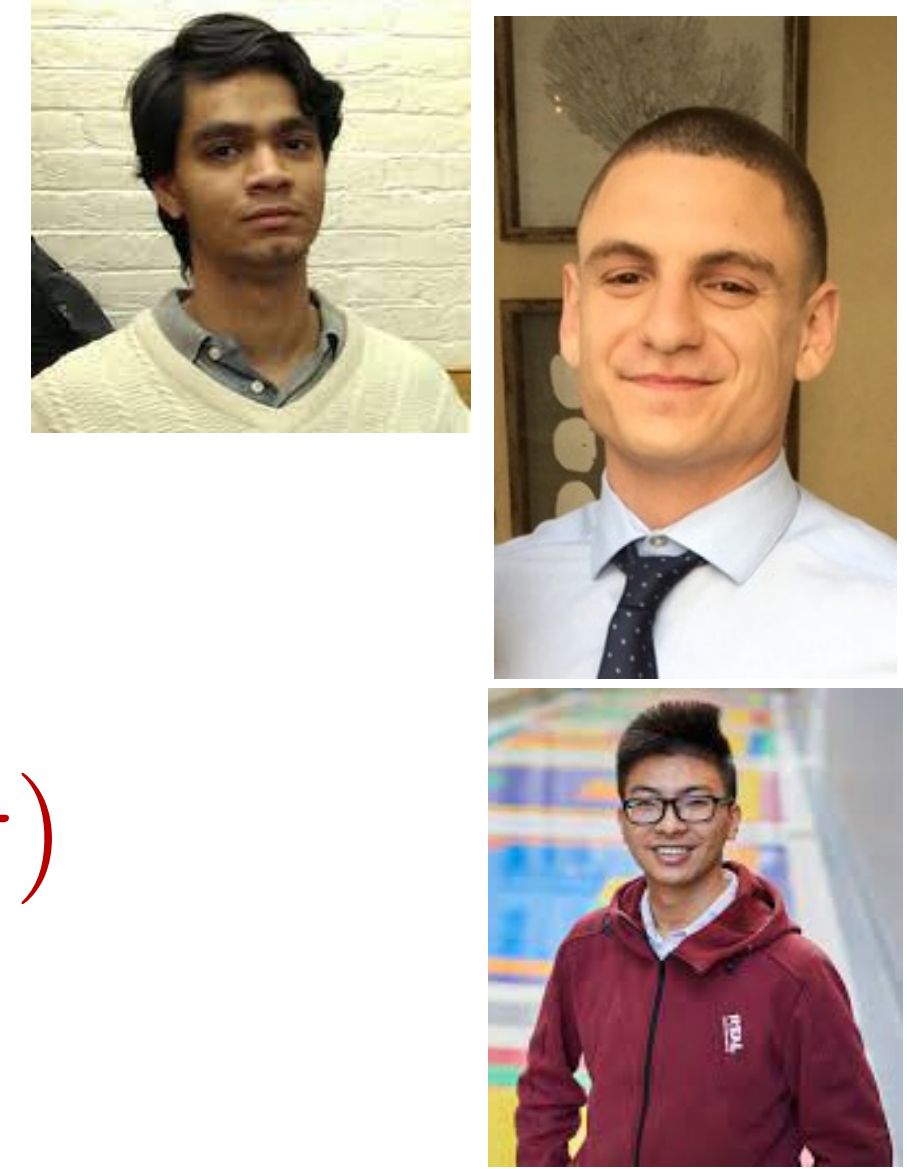
$$\text{Fermion self energy: } \Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\frac{g^2}{v^2} \omega \ln(1/|\omega|)$$

The g^2 log term does not contribute to transport

Fermi surface coupled to a critical boson with spatial disorder

“Yukawa” coupling: $\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$

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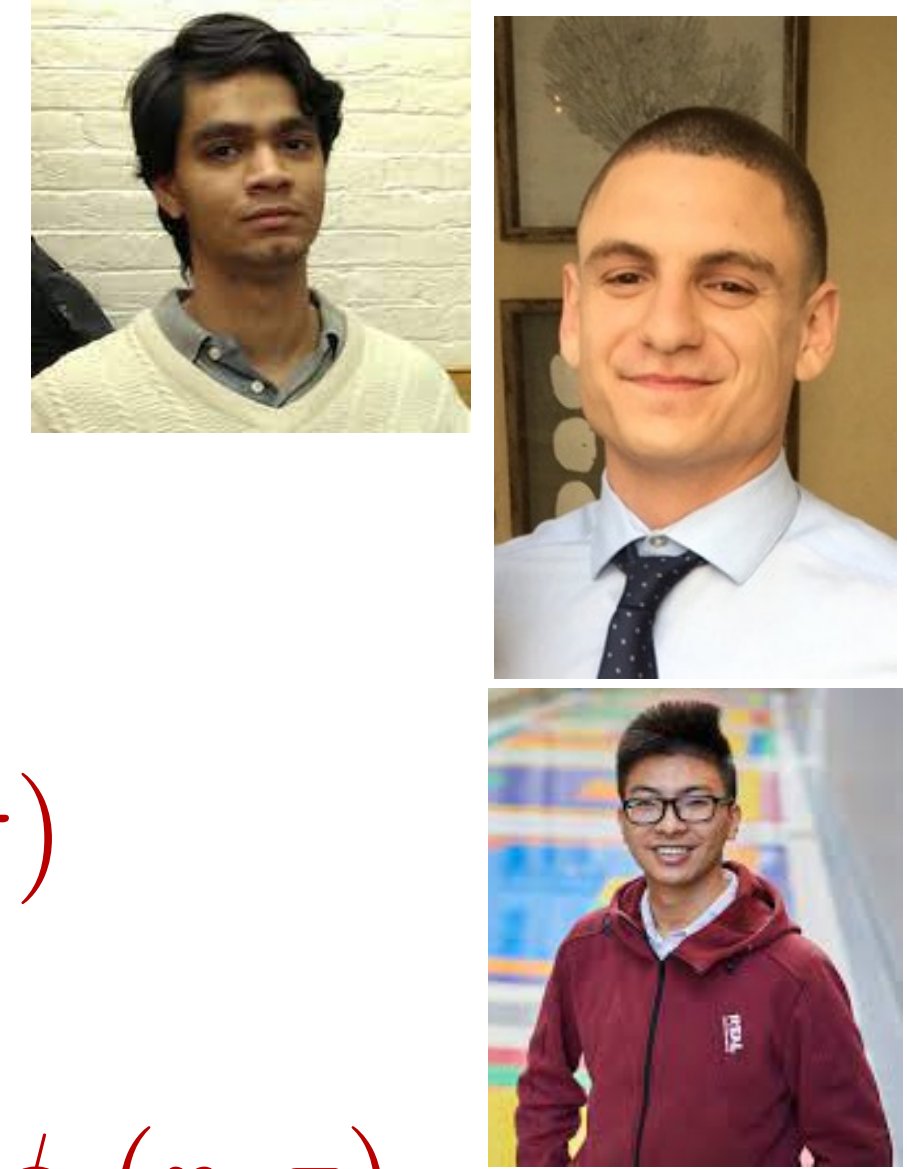
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With g and v non-zero, we obtain a non-zero residual resistivity and Fermi liquid like corrections

$$\rho(T) = \rho(0) + AT^2 + \dots$$

with $1/\rho(0) \sim 1/\tau_{\text{trans}} \sim v^2$.

Fermi surface coupled to a critical boson with spatial disorder



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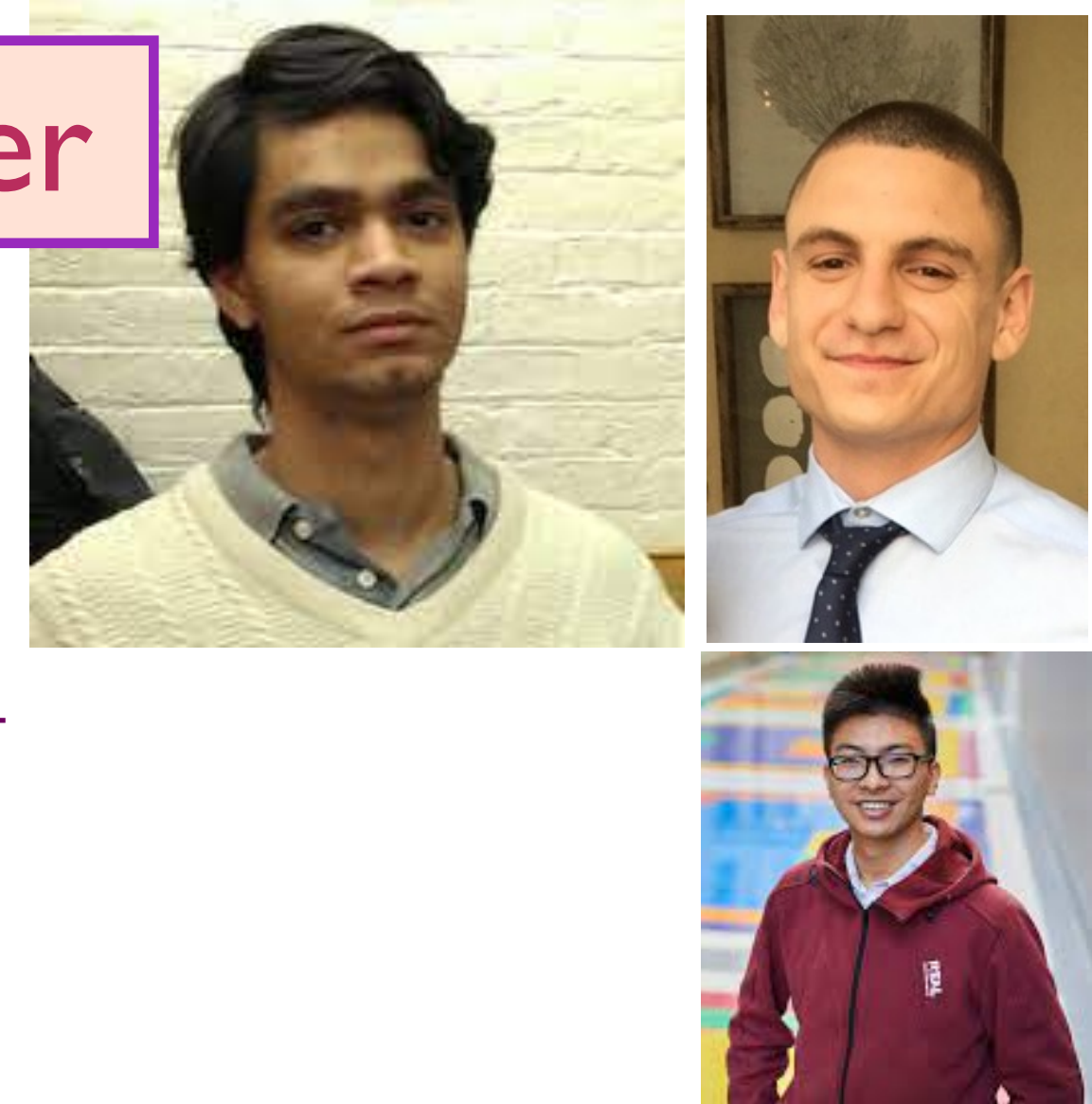
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Fermi surface coupled to a critical boson with spatial disorder

Boson self energy: $\Pi = \Pi_g + \Pi_{g'}$

$$\Pi_g(i\Omega) \sim -\frac{g^2}{v^2}|\Omega|, \quad \Pi_{g'}(i\Omega) \sim -g'^2|\Omega|, \quad D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|}$$



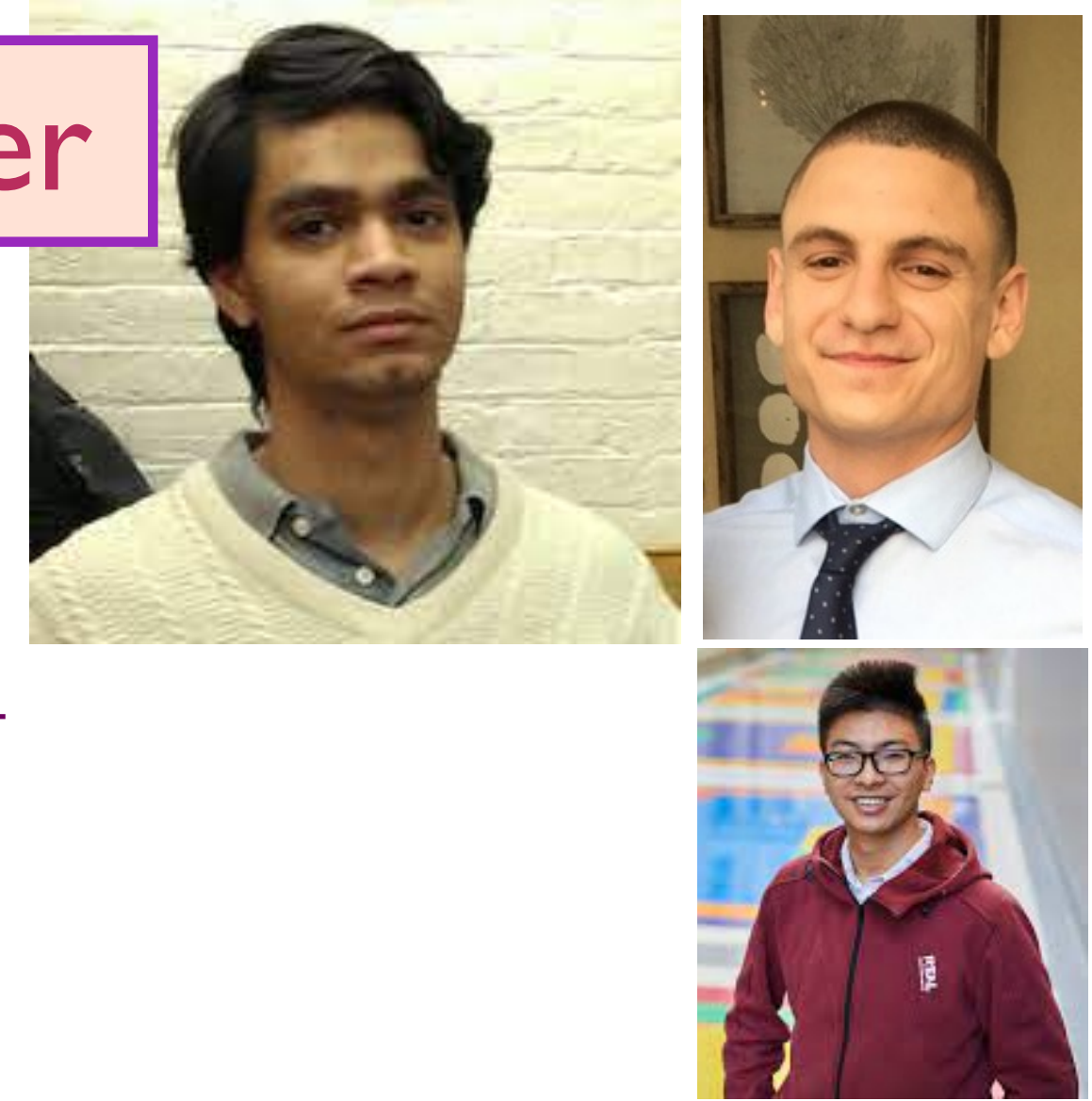
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Fermion self energy: $\Sigma = \Sigma_v + \Sigma_g + \Sigma_{g'}$

$$\Sigma_v(i\omega) \sim -iv^2\text{sgn}(\omega), \quad \Sigma_g(i\omega) \sim -i\frac{g^2}{v^2}\omega \ln(1/|\omega|), \quad \Sigma_{g'}(i\omega) \sim -ig'^2\omega \ln(1/|\omega|)$$



Fermi surface coupled to a critical boson with spatial disorder

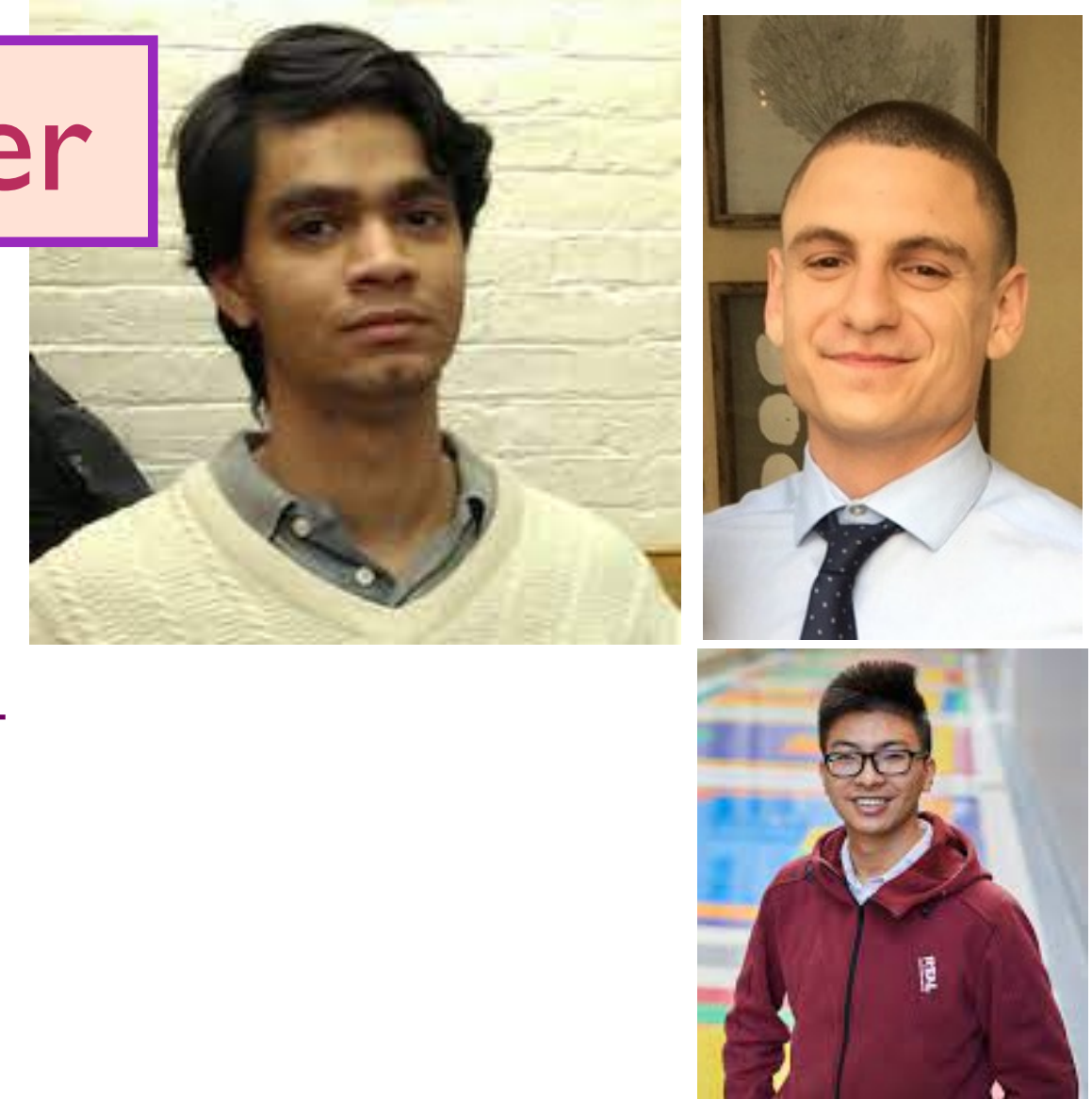
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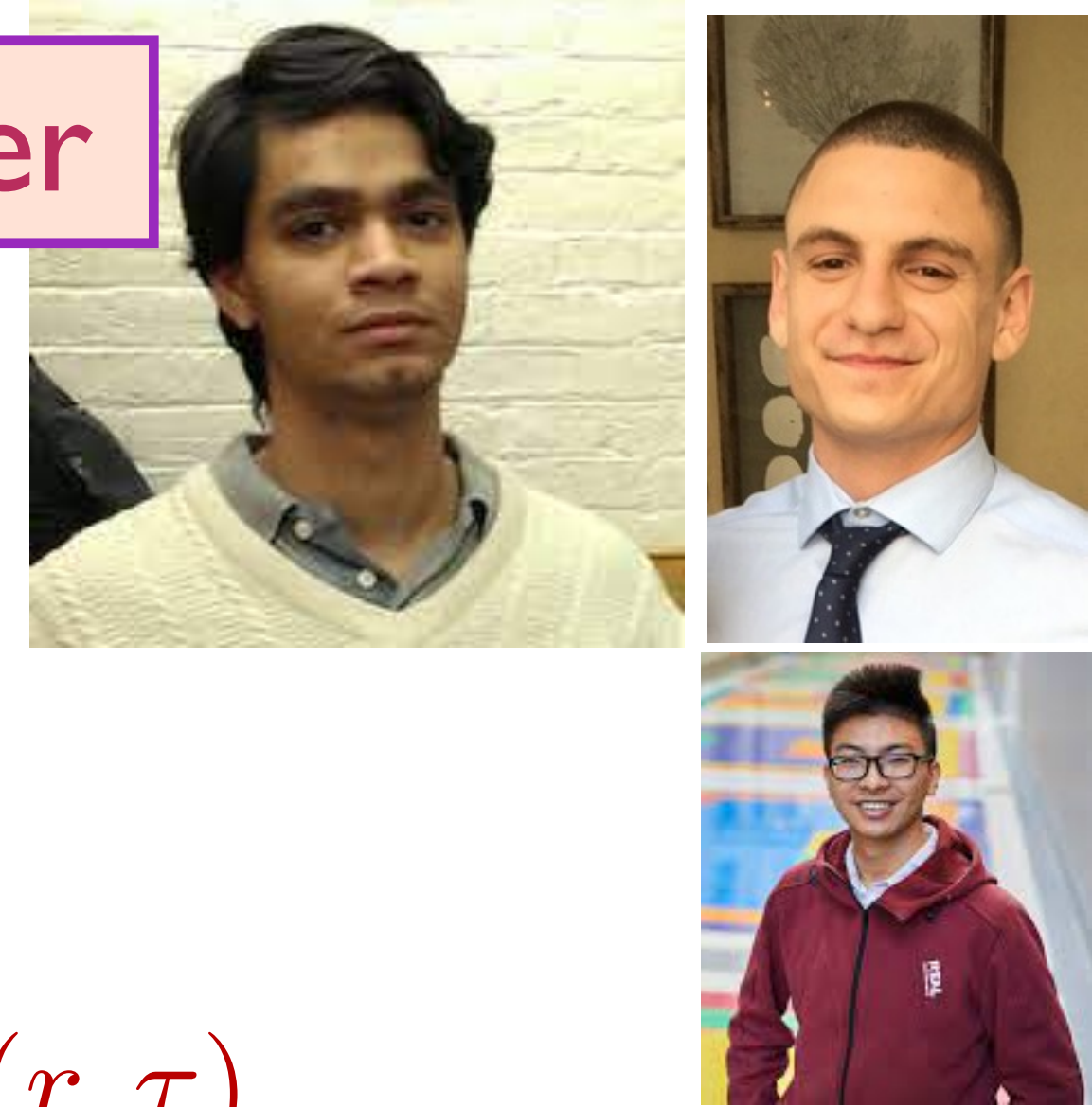
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The g^2 log term does not contribute to transport
but the g'^2 log term does!



Fermi surface coupled to a critical boson with spatial disorder



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$$\text{Conductivity: } \sigma(\omega) \sim \tau_{\text{trans}}(\omega)$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega|$$

Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 .

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4. Random t - j model

Random t - J model doped with hole density p

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

\mathcal{P}_d projects out doubly-occupied sites.

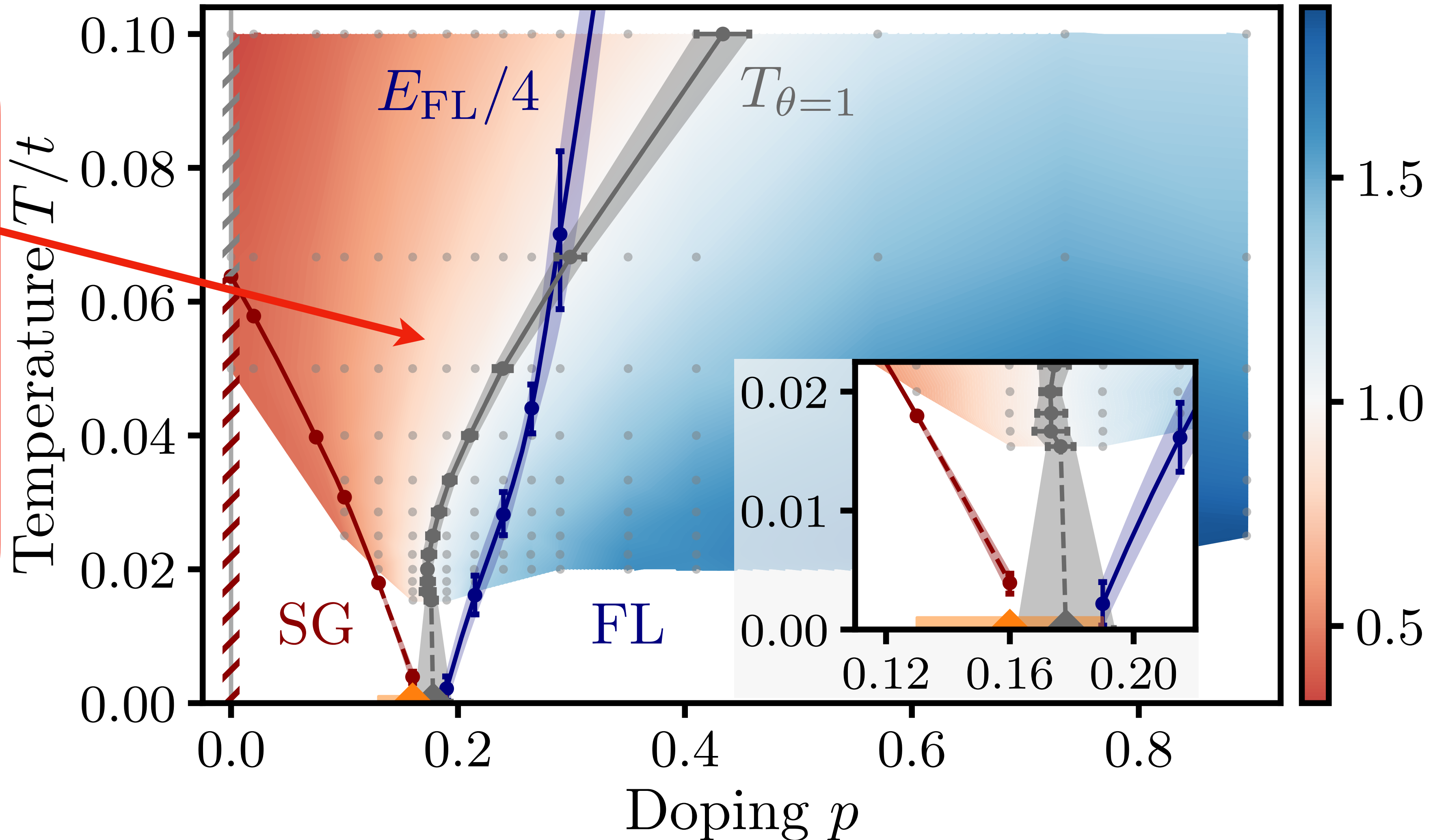
$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

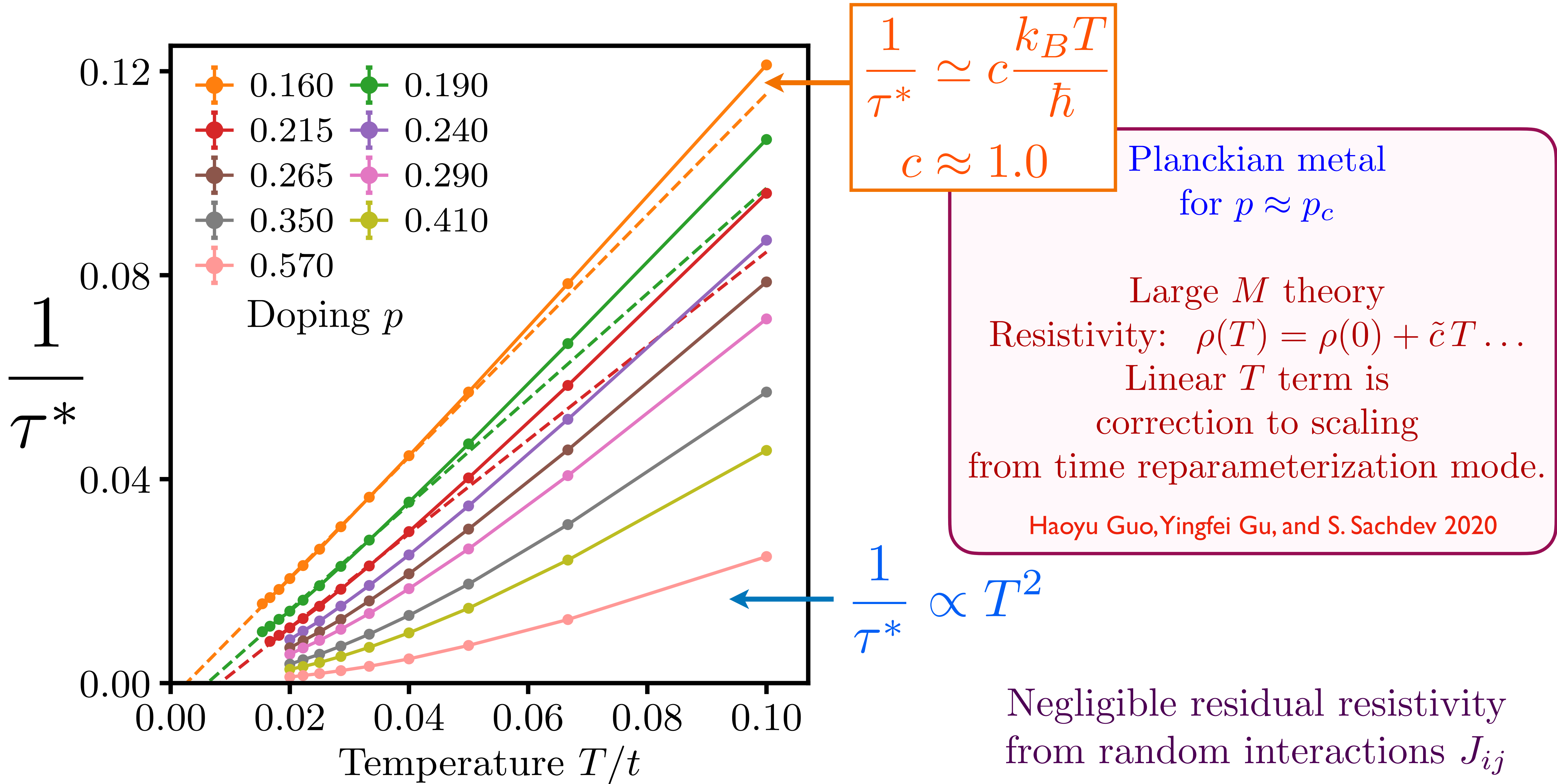
$J \Rightarrow$ two-particle interaction, similar to that in SYK

$t \Rightarrow$ one-particle hopping, can be regular or random

Numerical solution of t - J model on a fully-connected cluster
with all-to-all and random t_{ij} and J_{ij}

Planckian
metal with
SYK dynamics
of
fractionalized
spinon
excitations





Summary

- Two-dimensional Fermi surface coupled to a critical boson has no quasiparticle excitations, and exhibits Planckian time dynamics and maximal chaos with Lyapunov exponent $2\pi k_B T/\hbar$. In the presence of spatial disorder, linear- T resistivity arises from the first subleading operator of random interactions, while the leading operator of random potential leads to the residual resistivity. The spatial disorder theory also leads to a $T \ln T$ specific heat.

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- Random t - J model captures many aspects of the cuprates over a wide intermediate temperature range, including the Planckian metal behavior. The linear- T resistivity arises from the first subleading operator of random interactions.

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- Random t - J model captures many aspects of the cuprates over a wide intermediate temperature range, including the Planckian metal behavior. The linear- T resistivity arises from the first subleading operator of random interactions.