SYK gauge theories

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May 16, 2018
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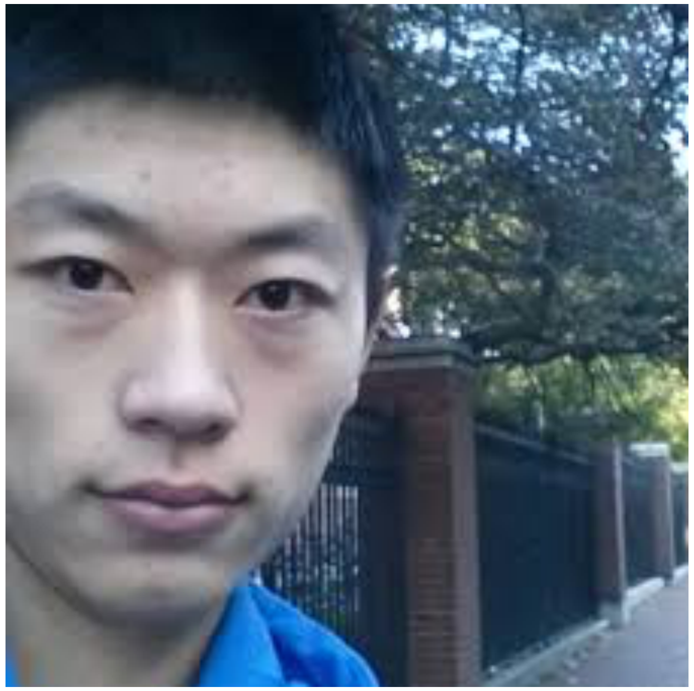
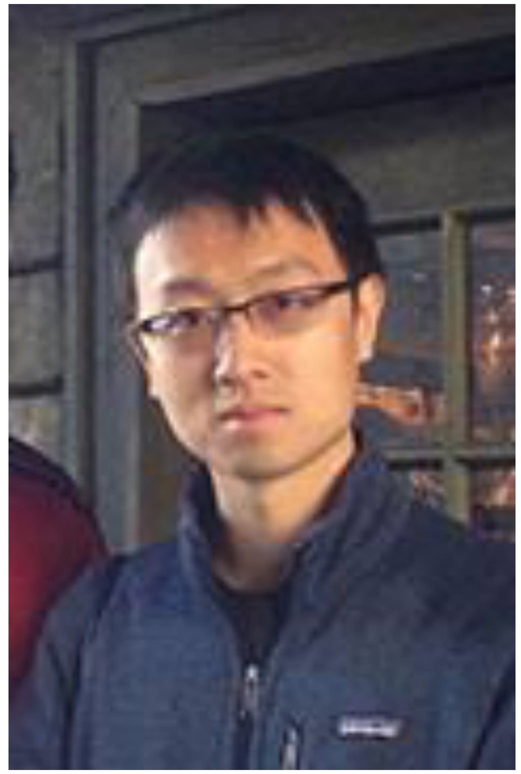
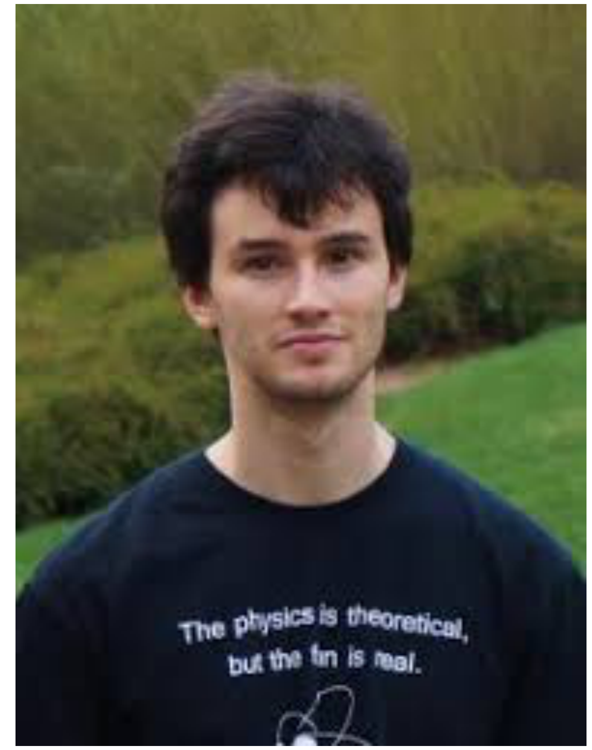


Wenbo Fu
Harvard



Yingfei Gu
Harvard



Grisha Tarnopolsky
Harvard

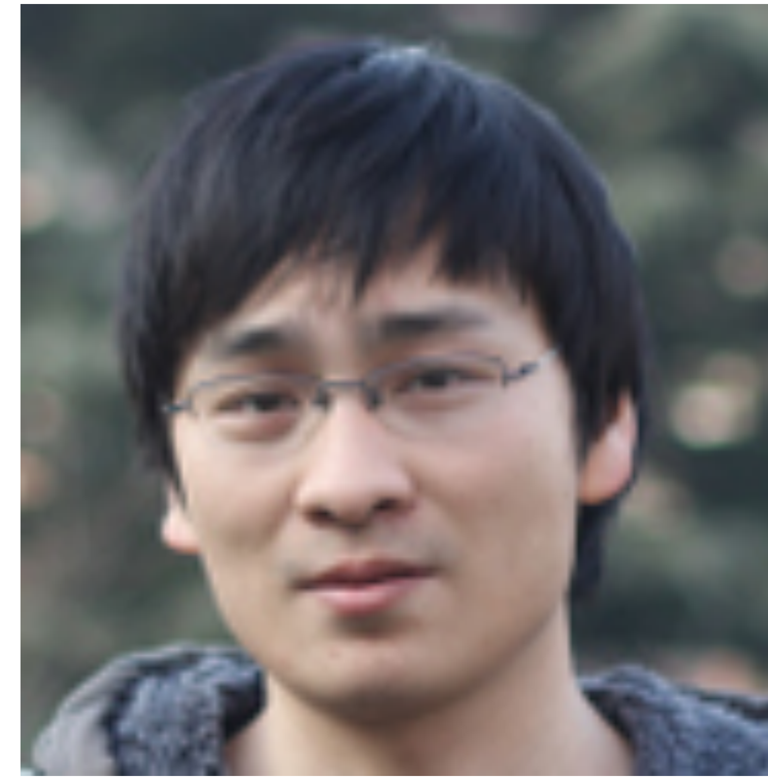
arXiv:1804.04130

Aavishkar Patel

To appear

Mathias Scheurer

Shubhayu Chatterjee



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arXiv:1711.09925

arXiv:1707.06602

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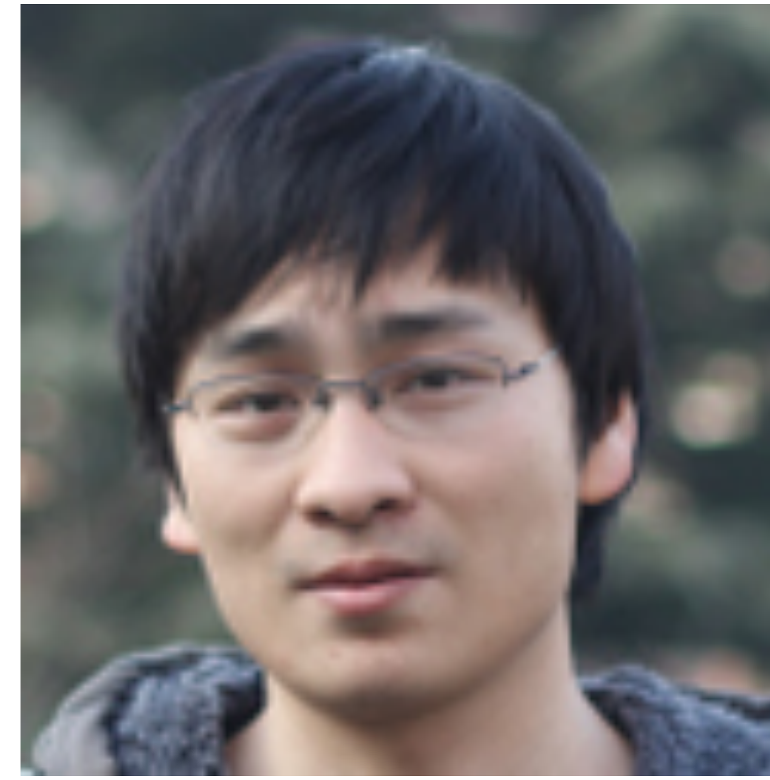

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Antoine Georges

1. The SYK Model

2. Gauge theories for the pseudogap and optimum-doping criticality in the cuprates

3. Z_2 Fractionalization in a SYK *t-J* model

4. SYK U(1) gauge theory

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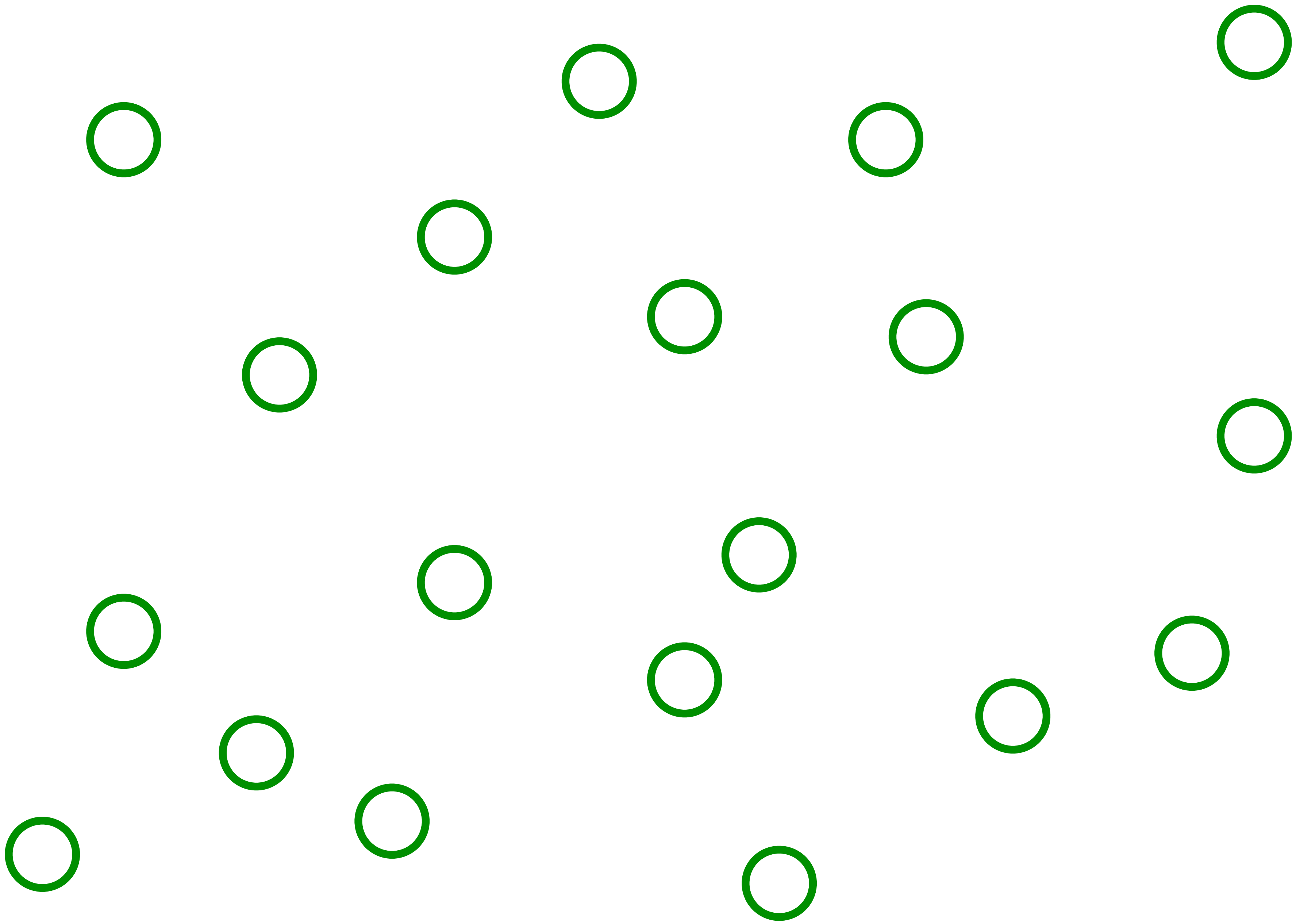
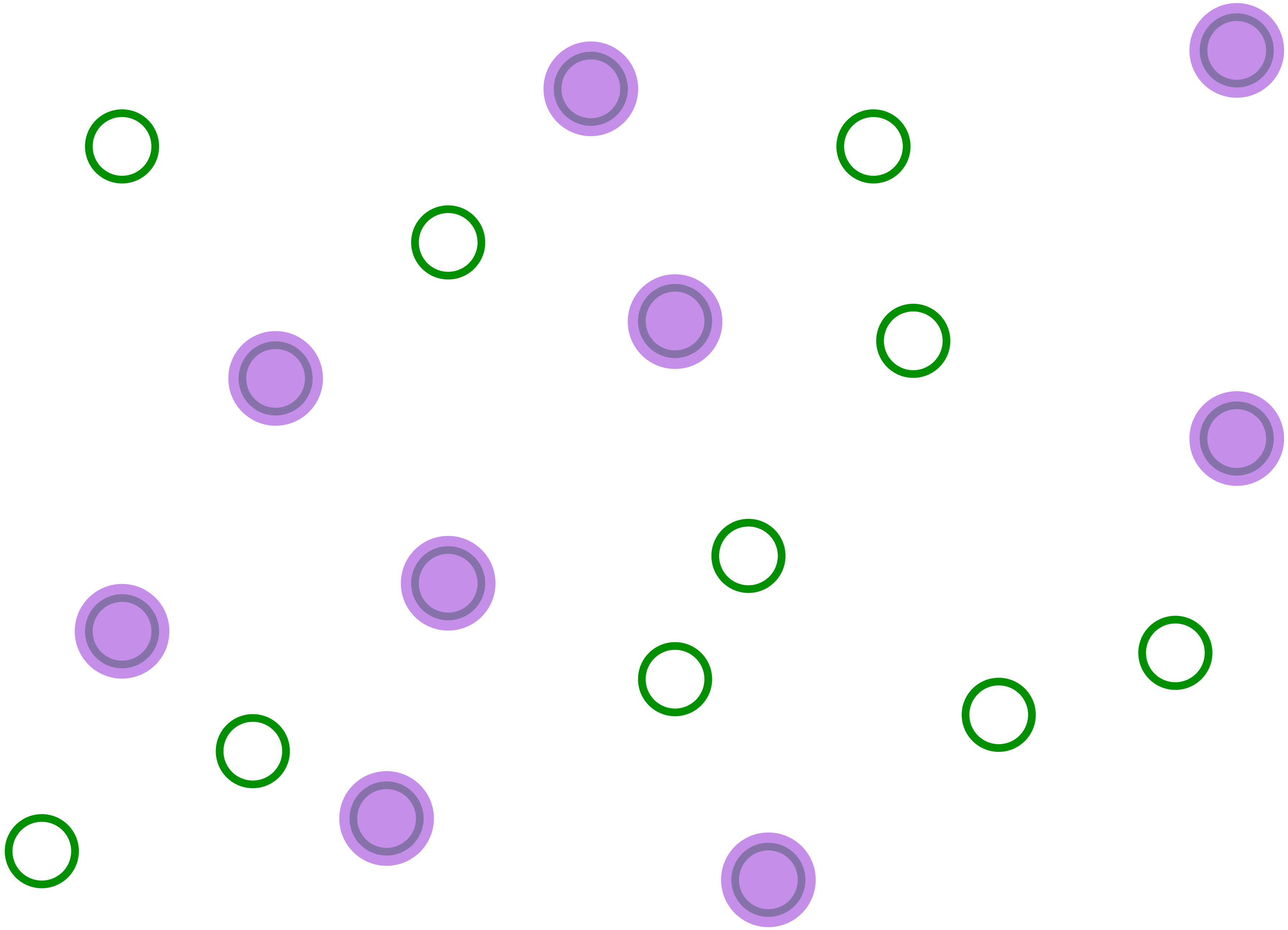
The Sachdev-Ye-Kitaev (SYK) model



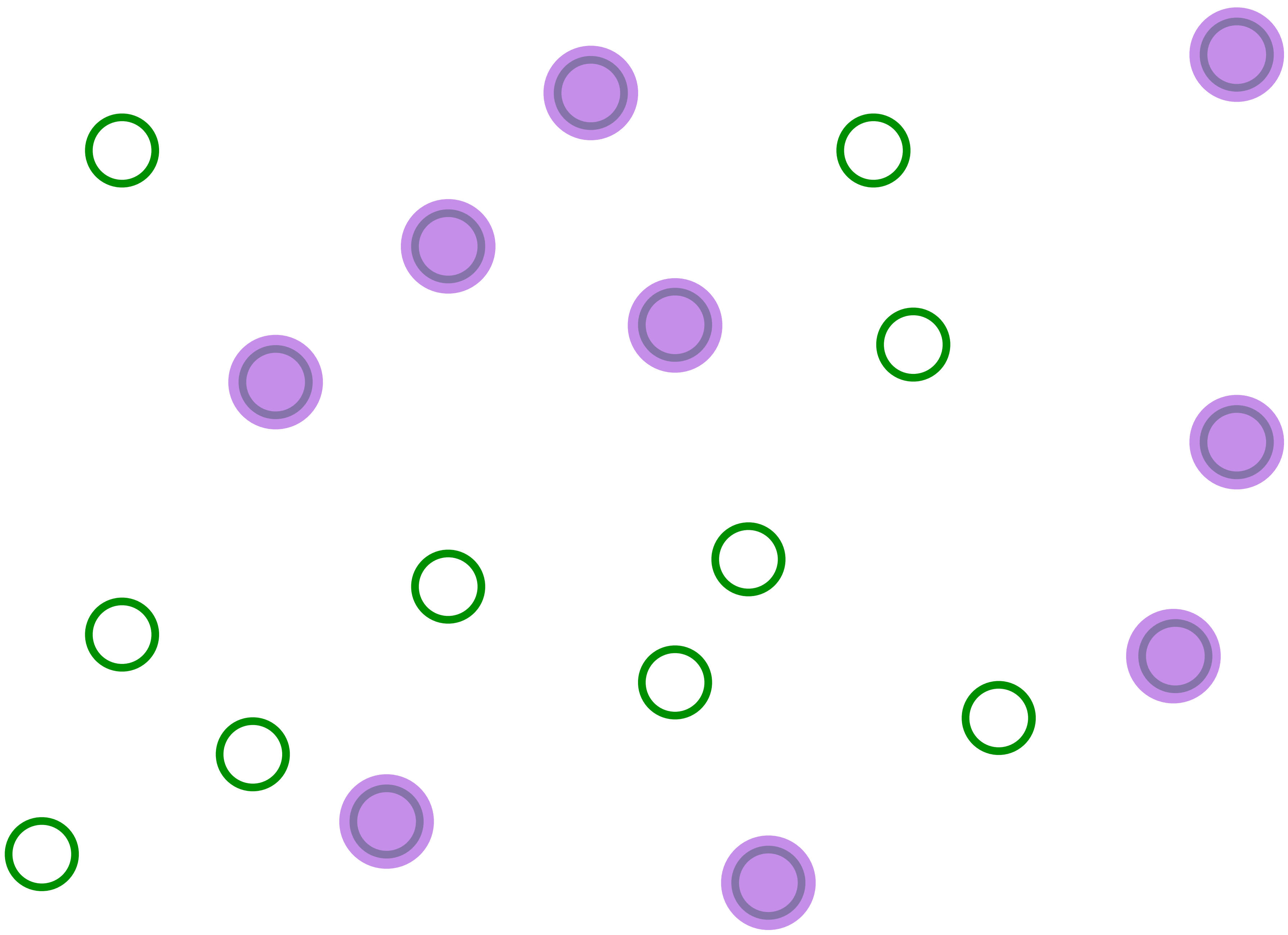
Pick a set of random positions

The SYK model



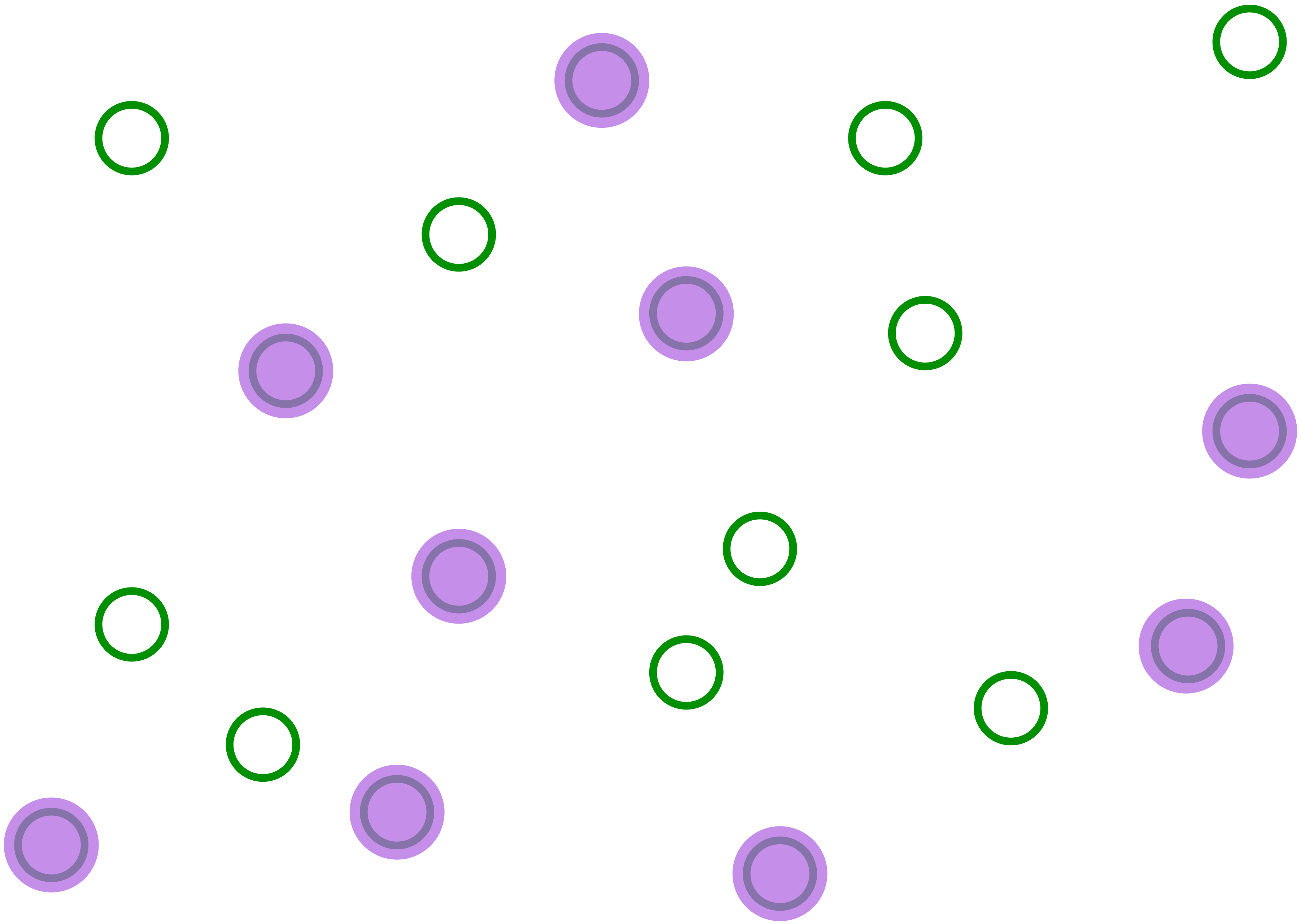
Place electrons randomly on some sites

The SYK model



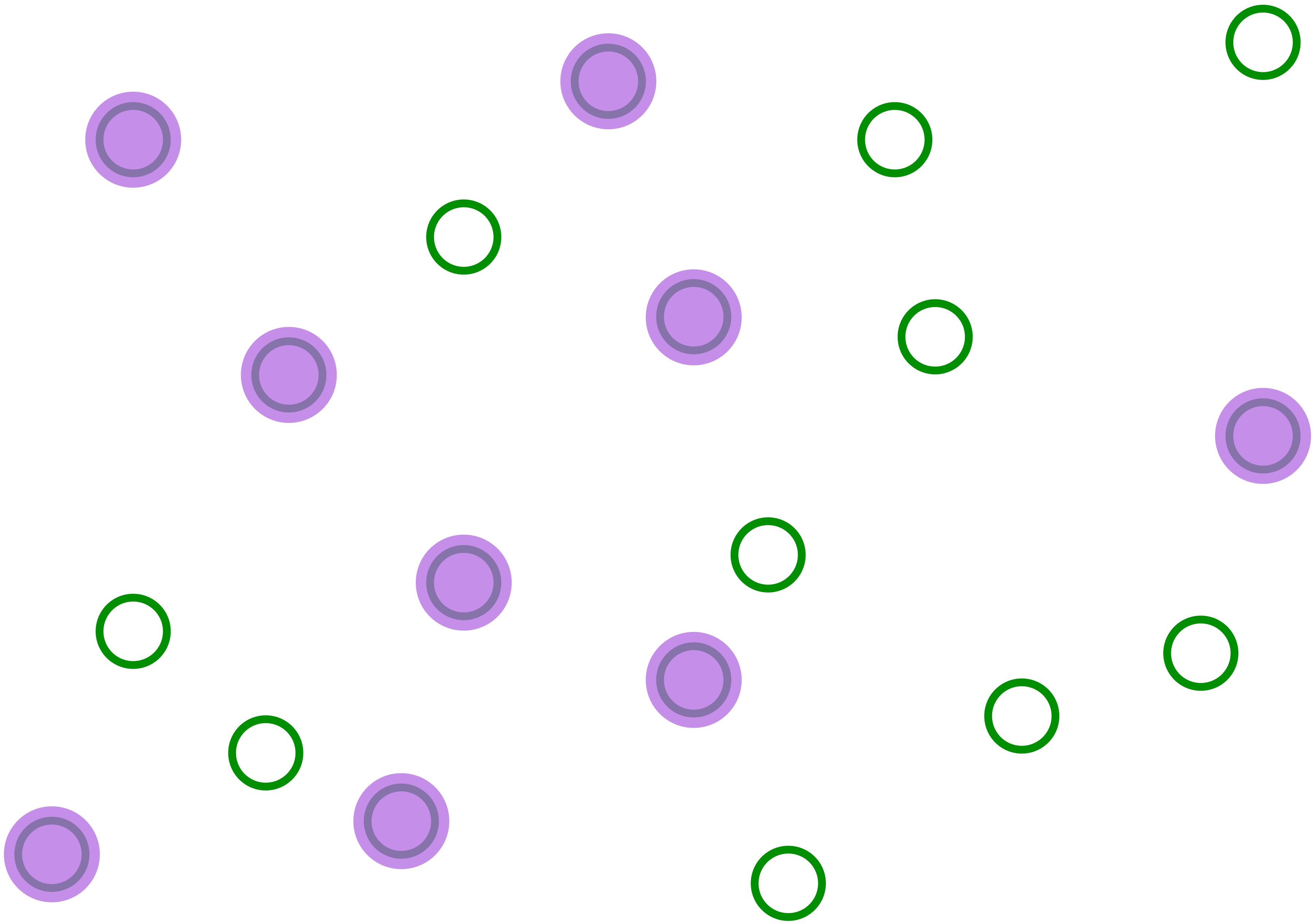
Entangle electrons pairwise randomly

The SYK model



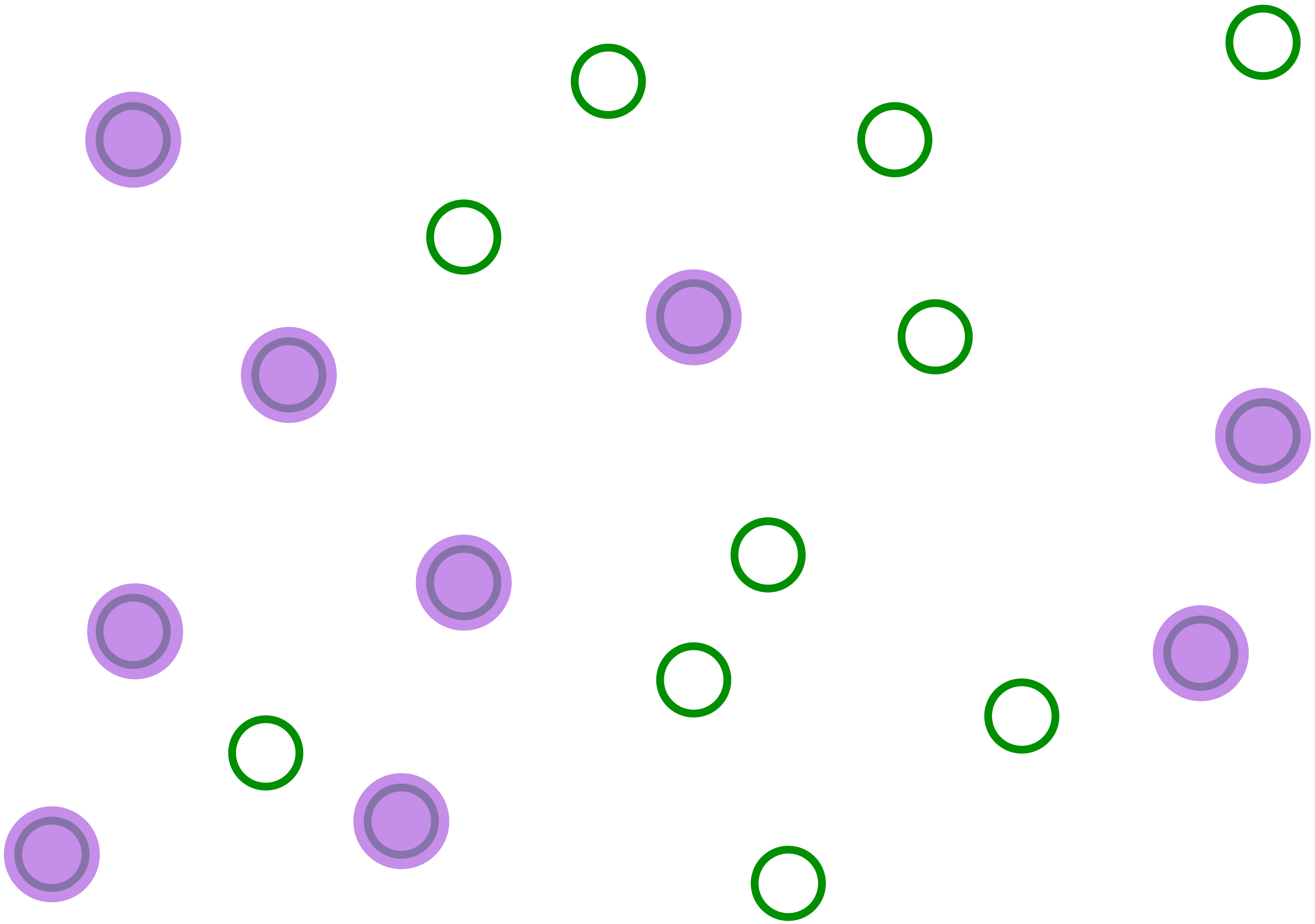
Entangle electrons pairwise randomly

The SYK model



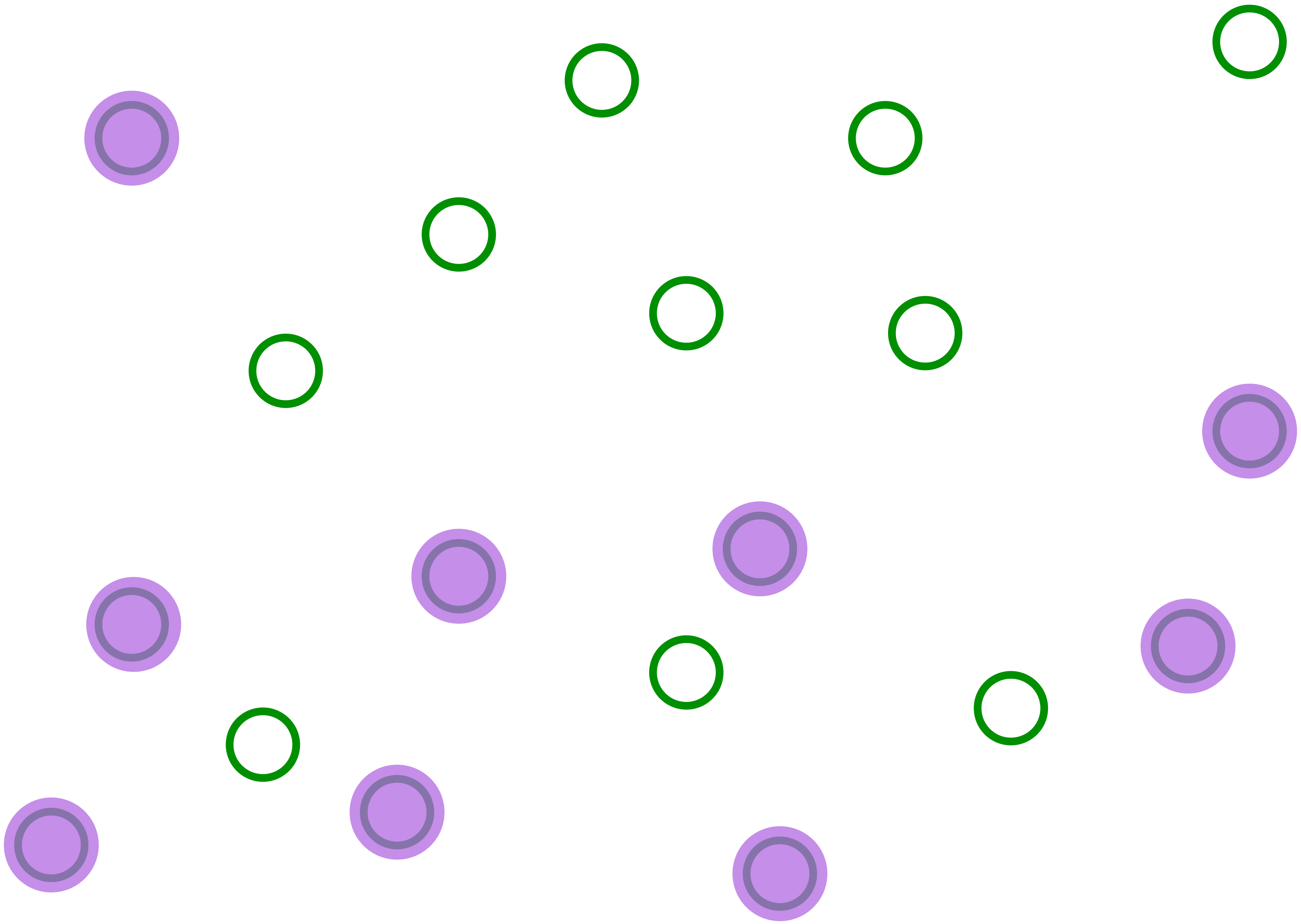
Entangle electrons pairwise randomly

The SYK model



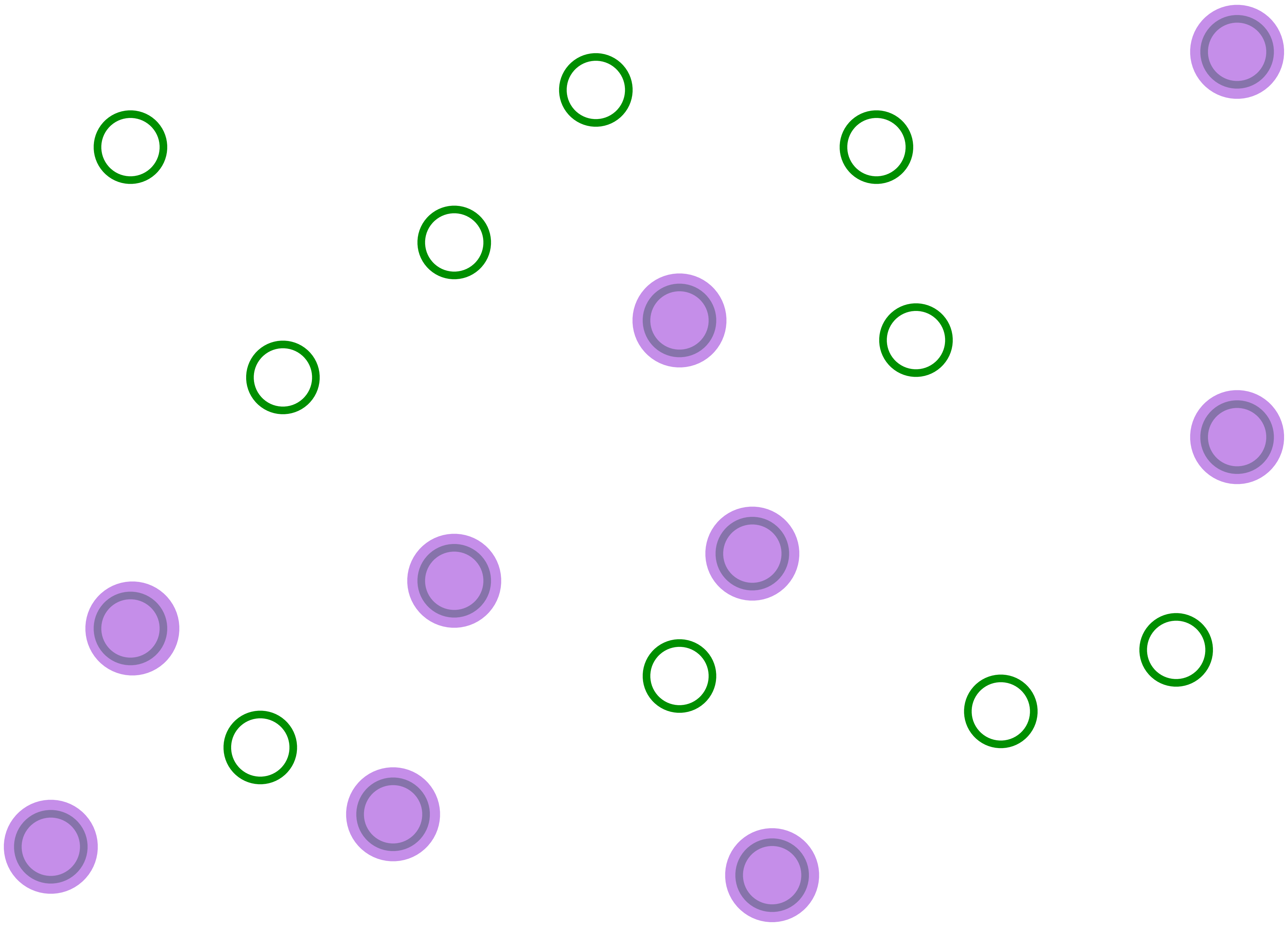
Entangle electrons pairwise randomly

The SYK model



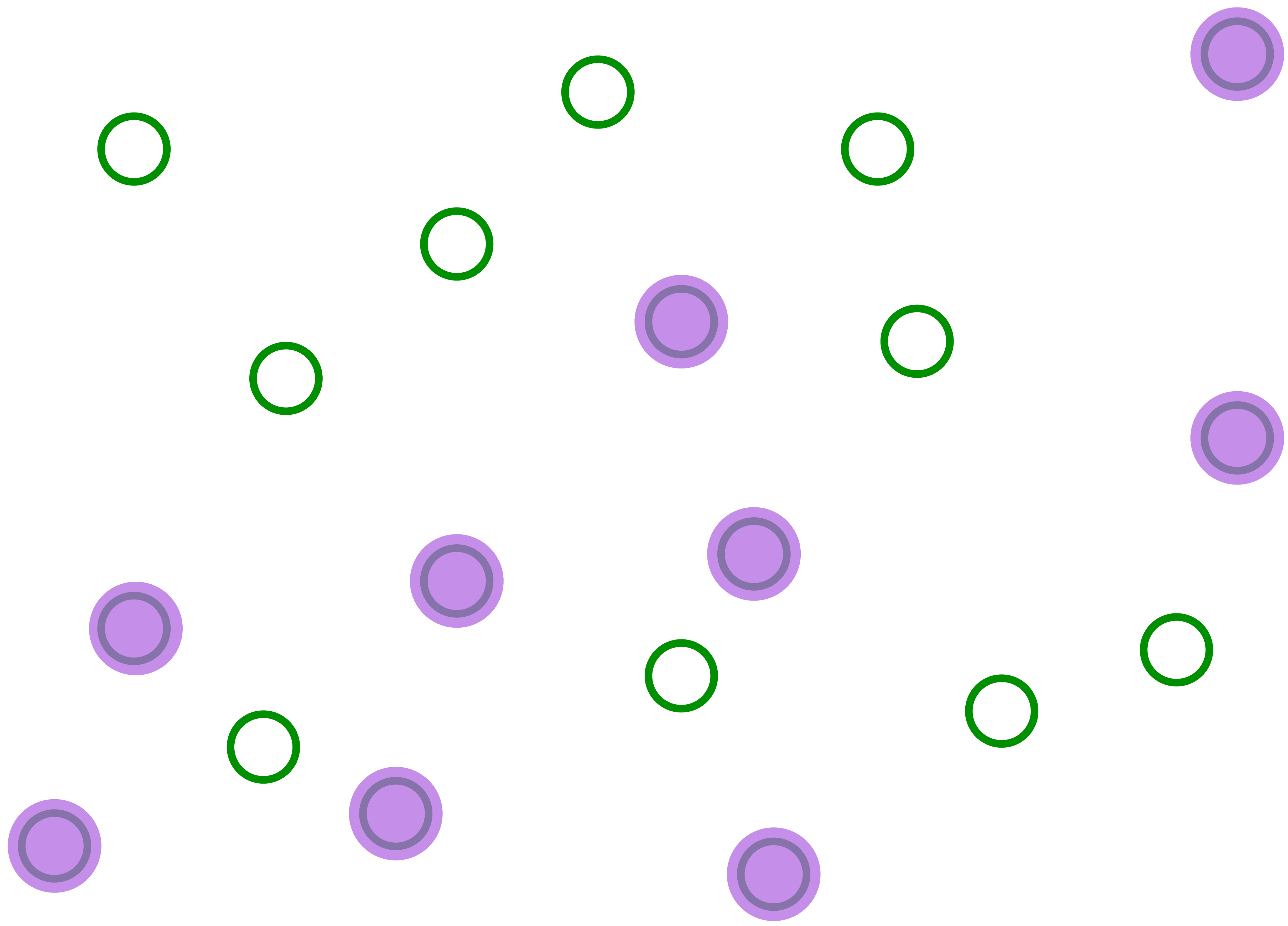
Entangle electrons pairwise randomly

The SYK model



Entangle electrons pairwise randomly

The SYK model



This describes both a strange metal and a black hole!

The SYK model

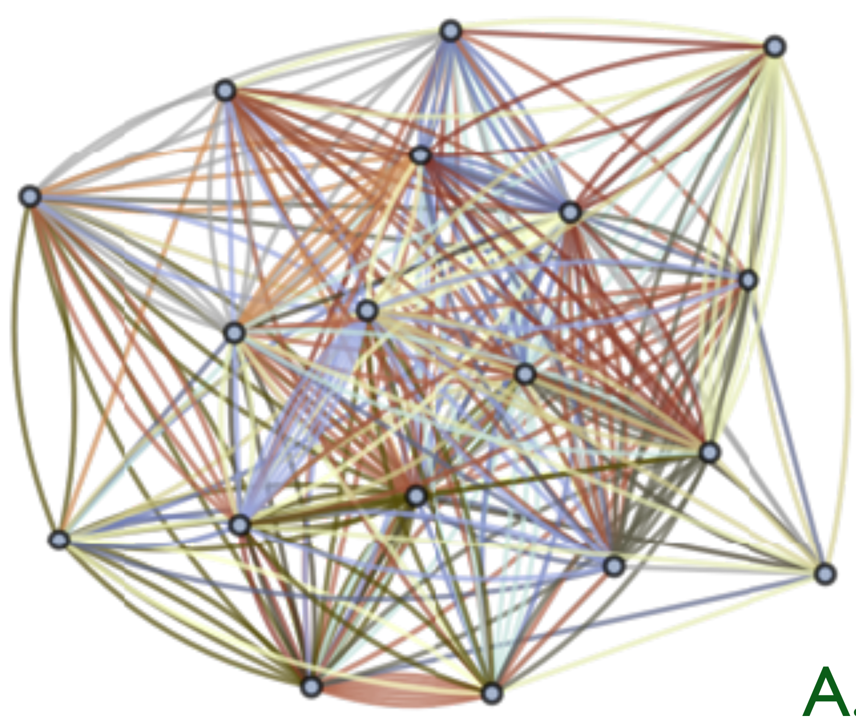
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

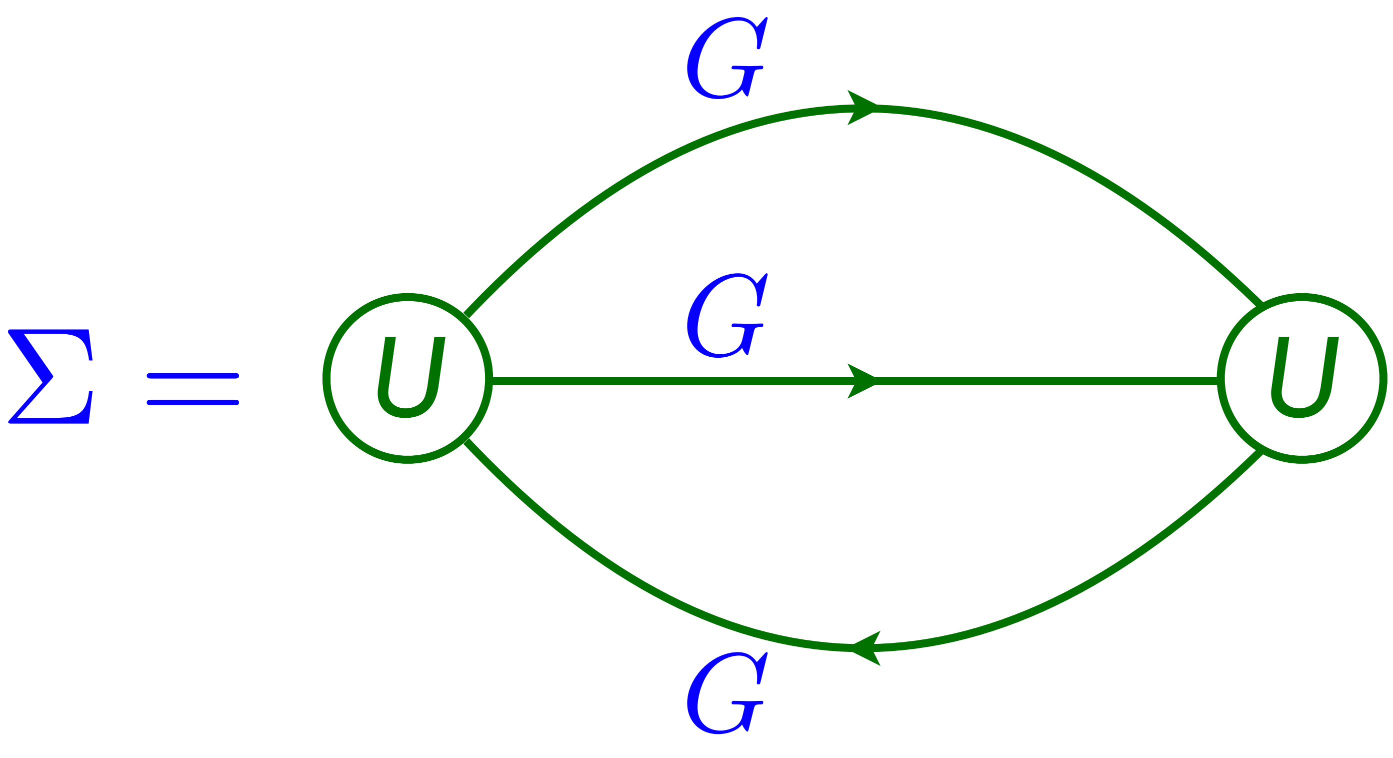
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$



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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

where $A = e^{-i\pi/4}(\pi/U^2)^{1/4}$ at half-filling. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK model: A solvable realization of quantum matter <u>without</u> quasiparticles

- Thermalization and many-body chaos in the <u>shortest possible</u> *'Planckian'* time of order $\hbar/(k_B T)$. Note that essentially all other solvable models (*e.g.* integrable lattice models in one dimension) do not exhibit many-body chaos.

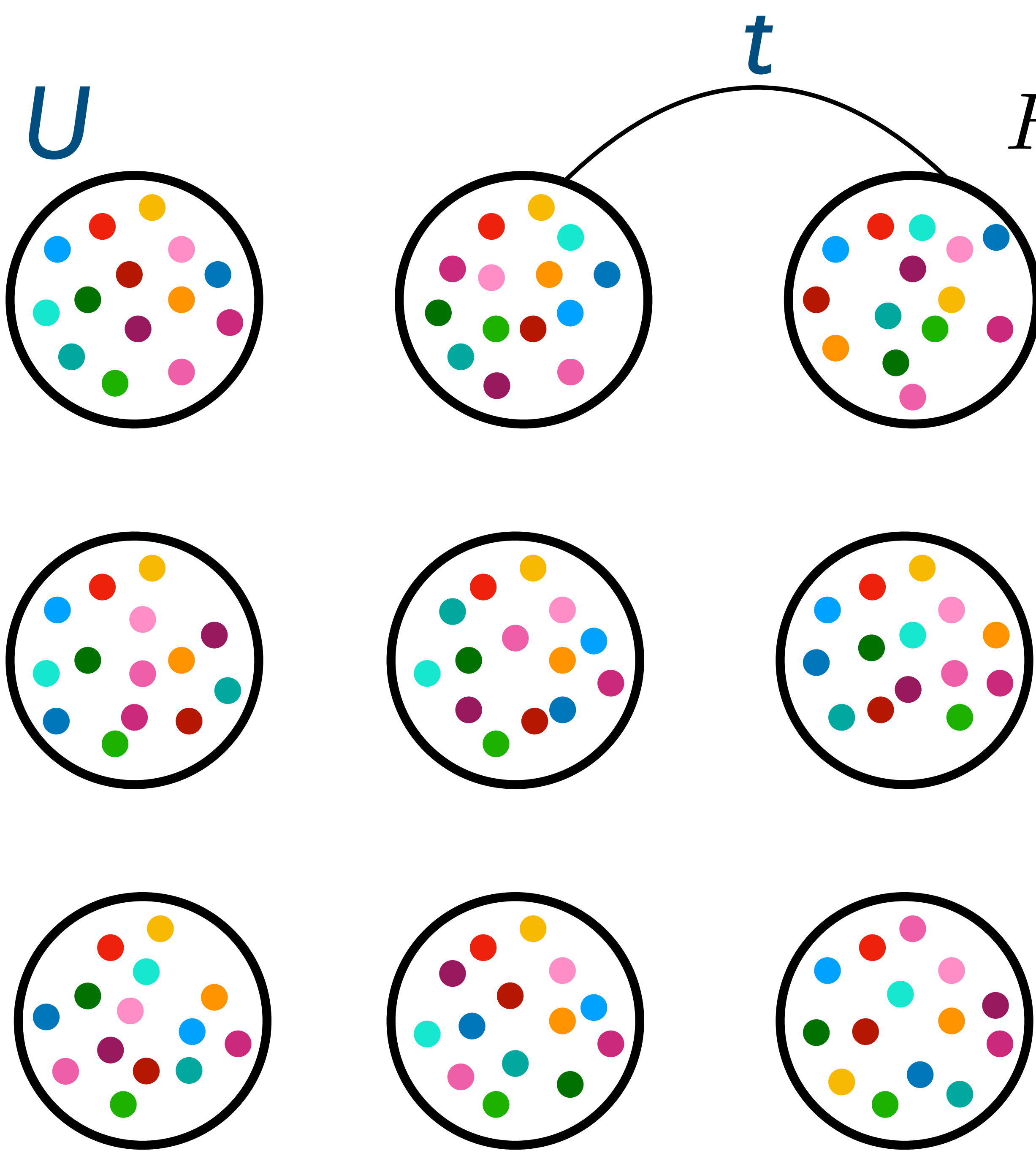
- The SYK models exhibit eigenstate thermalization (ETH), and yet many aspects are exactly solvable.

SYK model: A solvable realization of quantum matter without quasiparticles

- The SYK models are dual to gravitational theories in $1+1$ dimensions which have a black hole horizon. The connection between the SYK models and black holes with a near-horizon AdS_2 geometry was proposed by SS (2010), and made sharper by Kitaev/Maldacena/Stanford (2016). This connection has been used to examine aspects of the black hole information problem.

SYK building blocks for a strange metal

SYK quantum islands of electrons with random hopping between them.



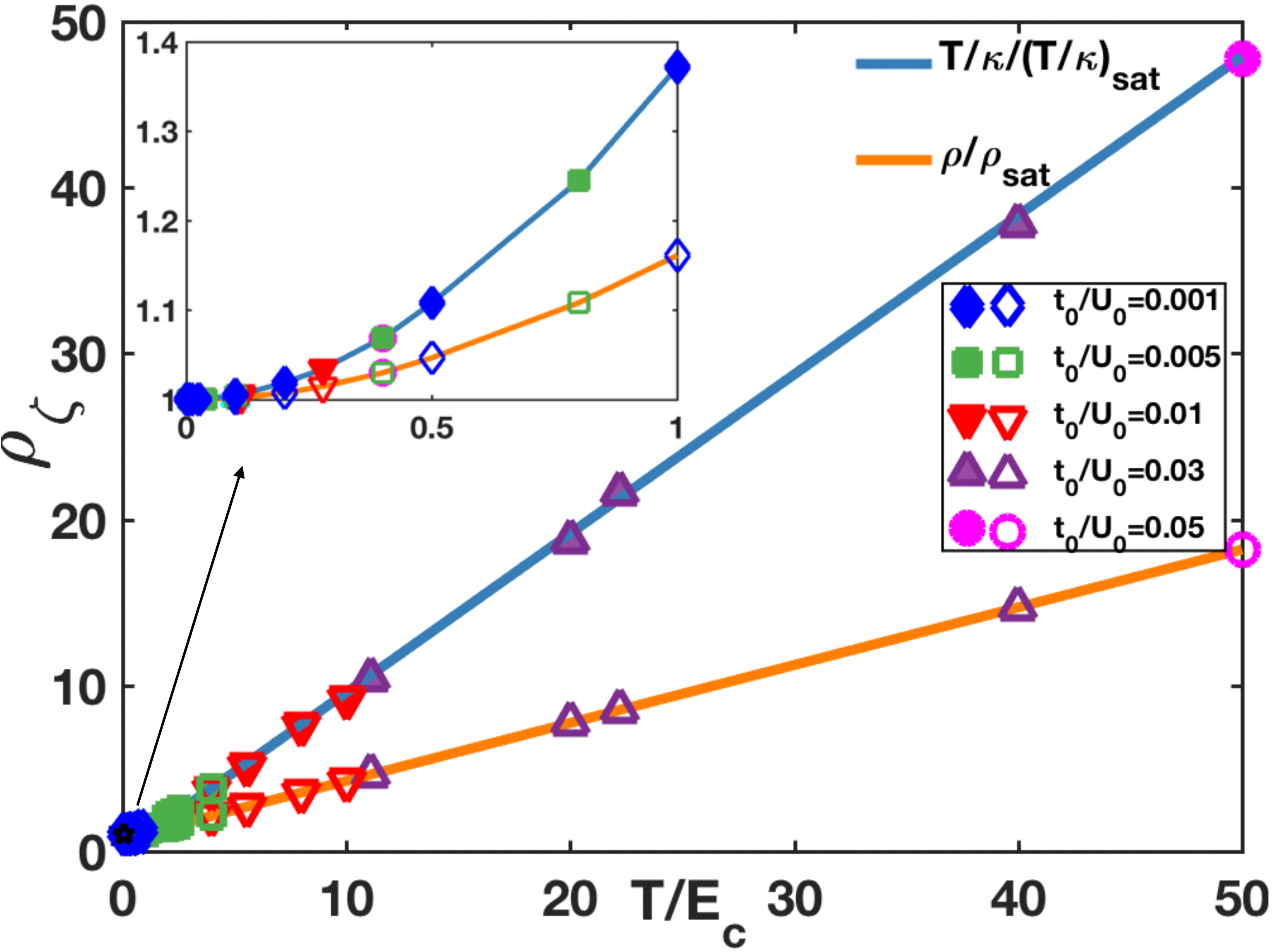
$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx}$$

$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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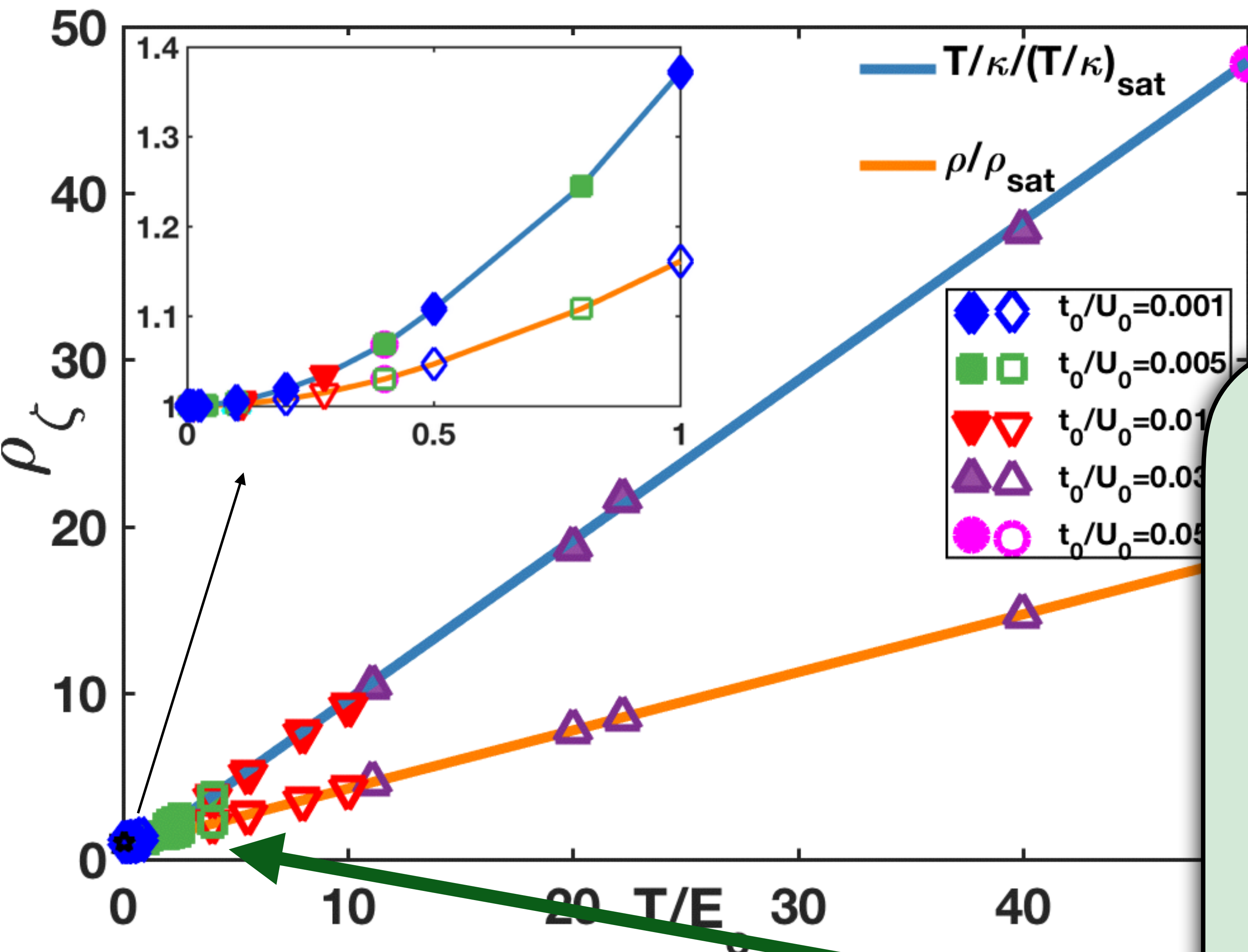


For $T < E_c$, the resistivity, ρ, and entropy density, s, are

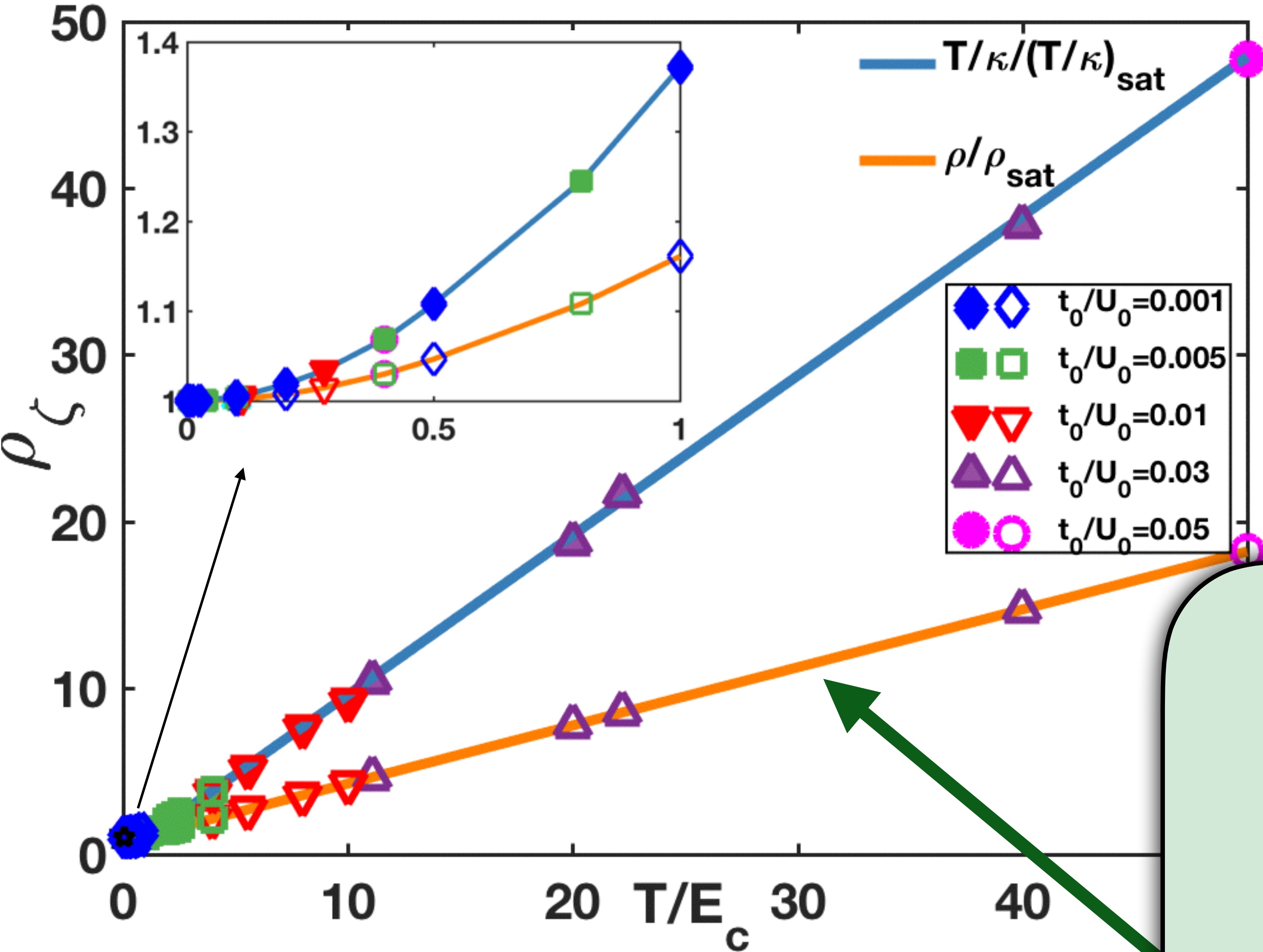
$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low 'coherence' scale

$$E_c \sim \frac{t_0^2}{U}$$



For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right), \quad s = s_0$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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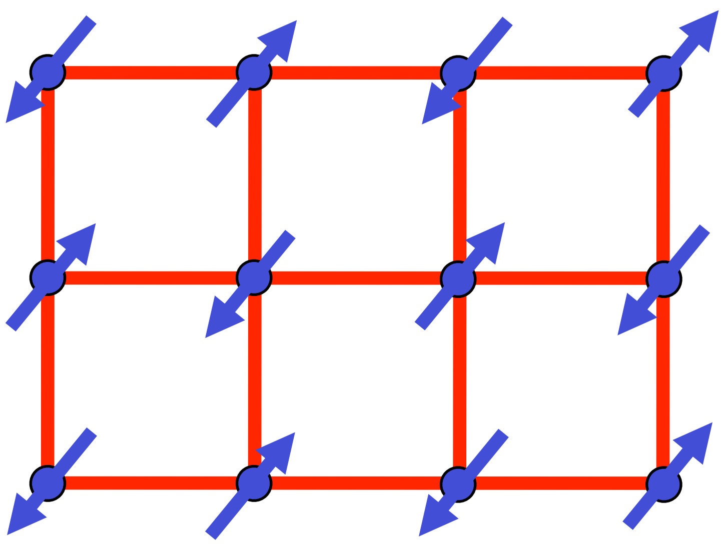
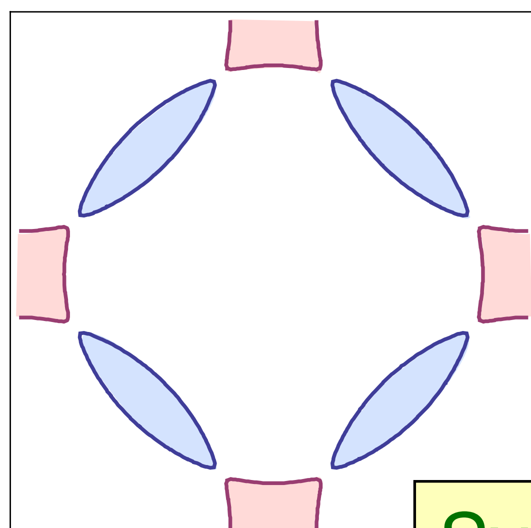
Antiferromagnetism in the Hubbard Model

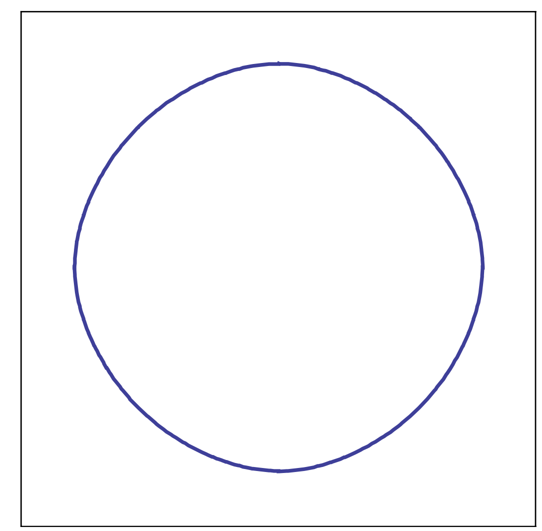
$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Mean-field theory with a spin density wave (SDW) order parameter $\vec{\Phi}_i = (-1)^{i_x+i_y} \left\langle c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta} \right\rangle /2$







Symmetry breaking phase transition

SDW LRO
Reconstructed Fermi surface
$\langle \vec{\Phi} \rangle \neq 0$

SDW SRO
Large Fermi surface.
$\langle \vec{\Phi} \rangle = 0$

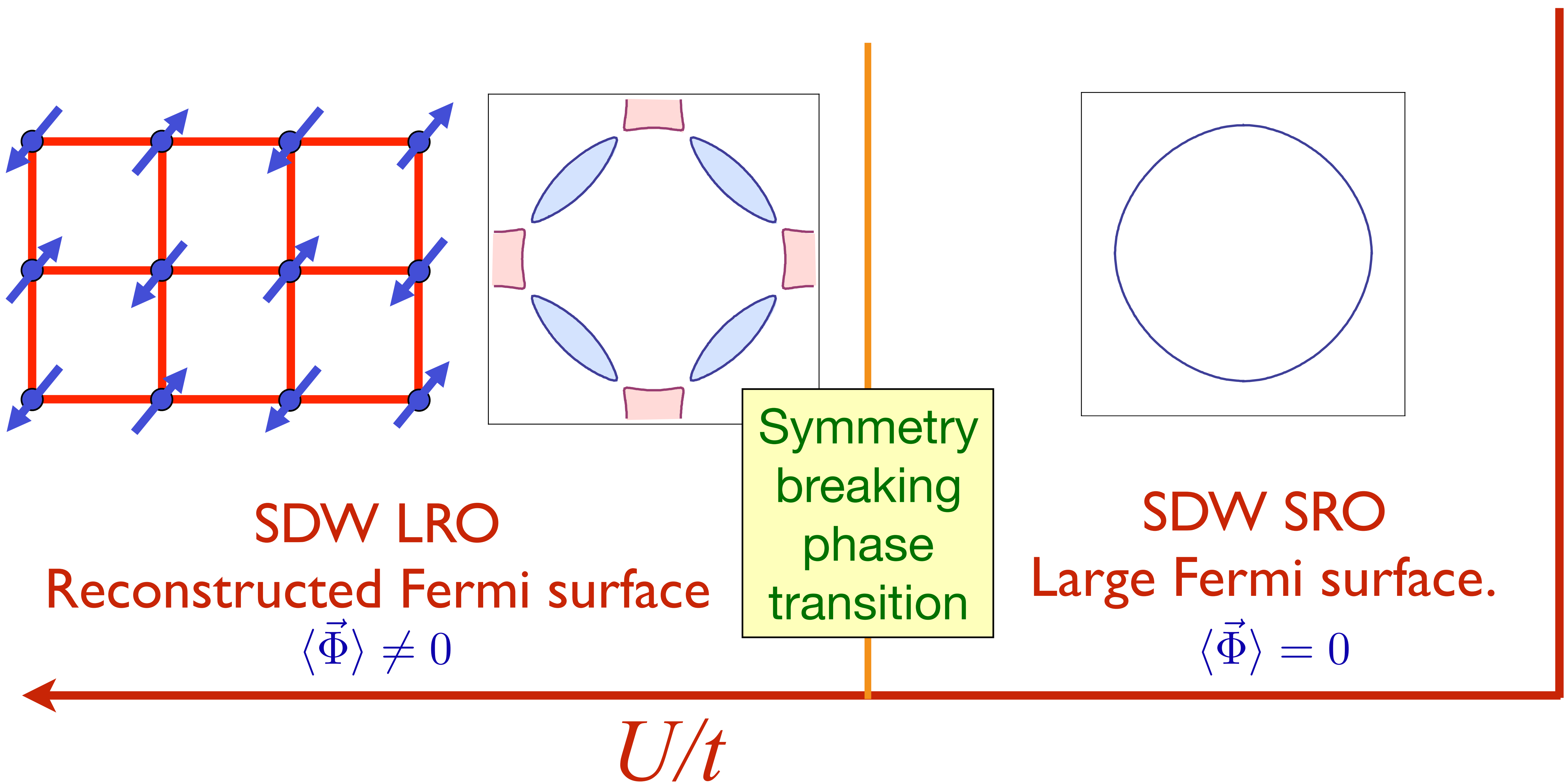
U/t

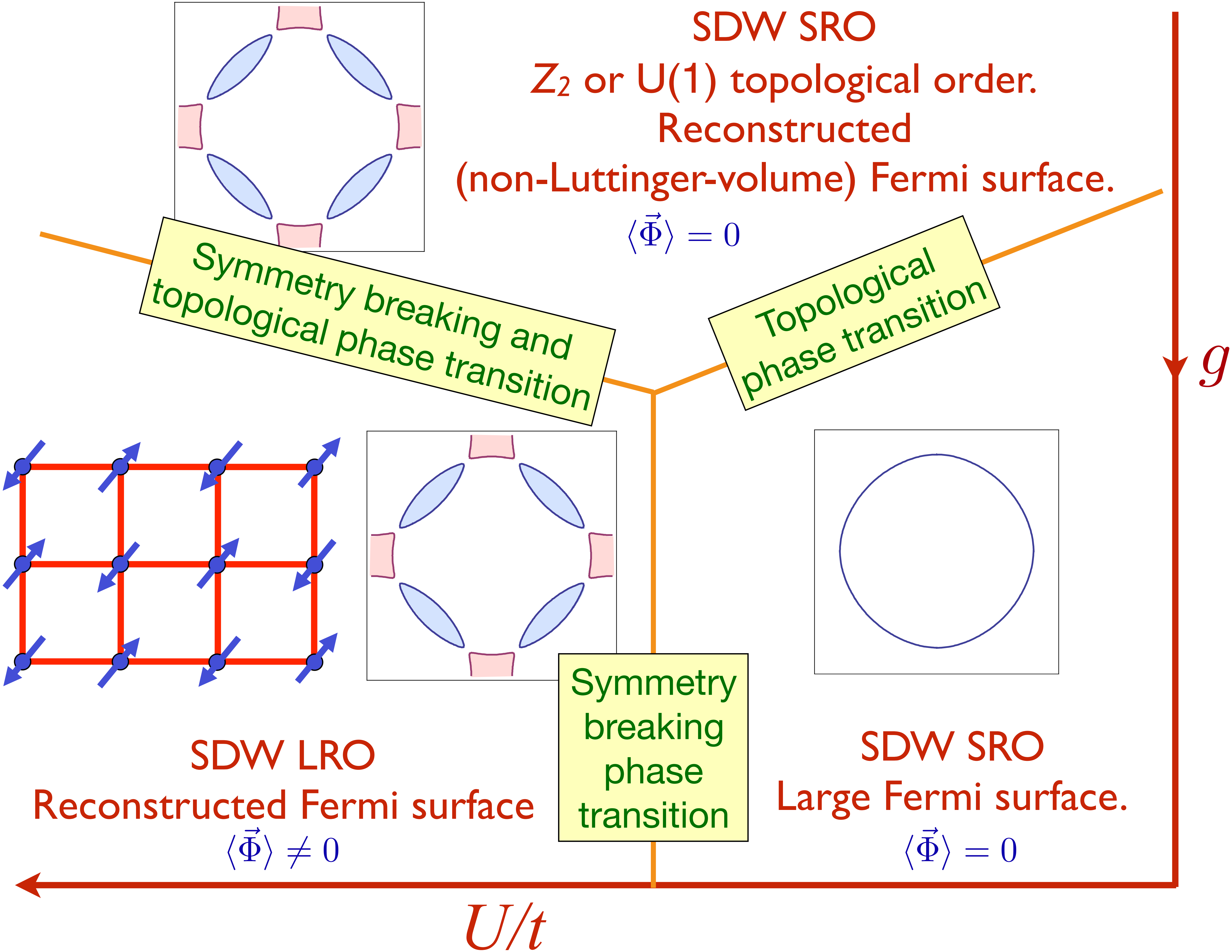
Antiferromagnetism in the Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_{i} c_{i\alpha}^{\dagger} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Both states have Luttinger volume Fermi surfaces




SDW SRO
Z_2 or U(1) topological order.
Reconstructed
(non-Luttinger-volume) Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
Symmetry breaking and topological phase transition
Topological phase transition
g
Symmetry breaking phase transition
SDW LRO
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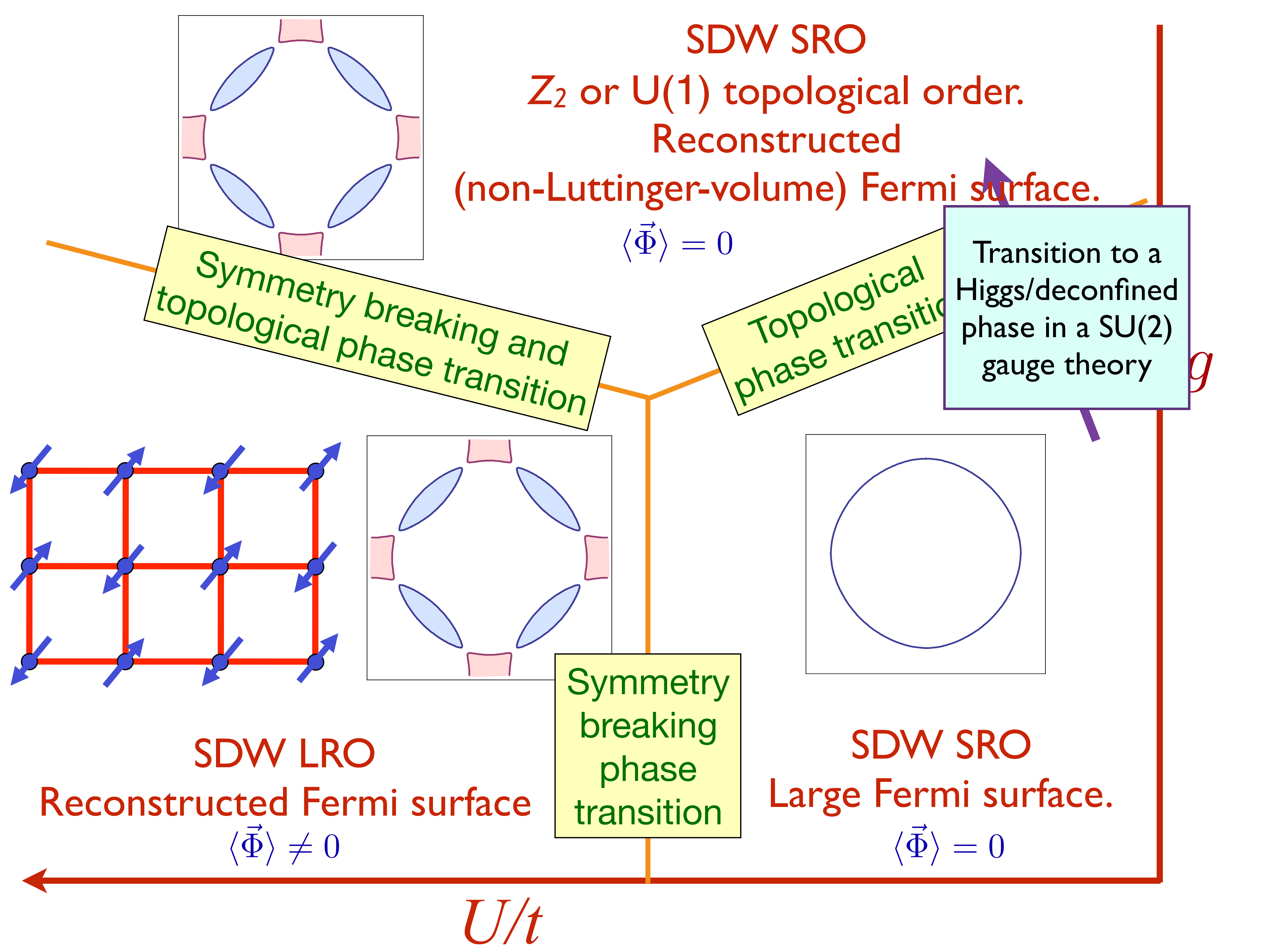
U/t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Transition to a Higgs/deconfined phase in a SU(2) gauge theory
g
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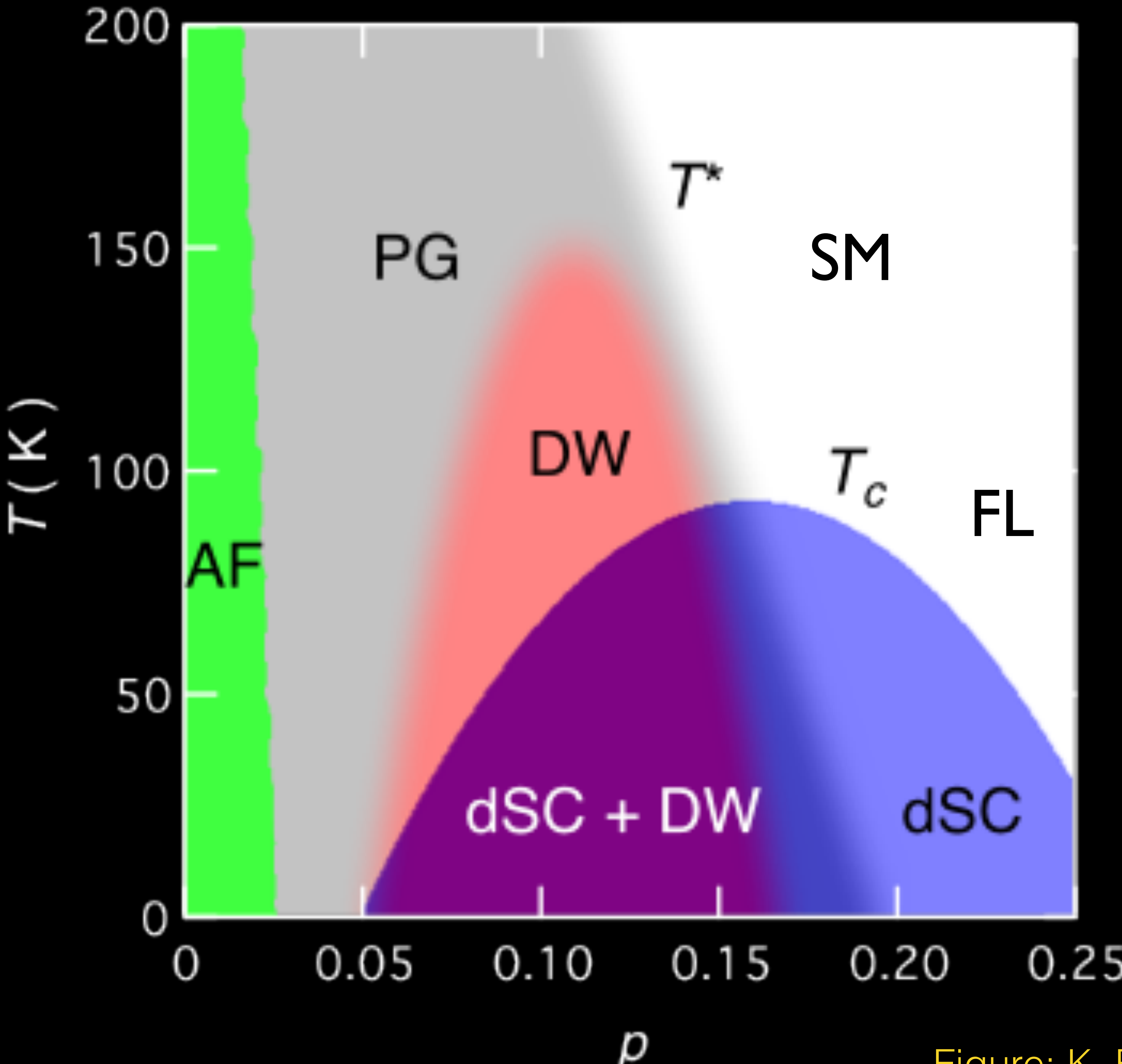
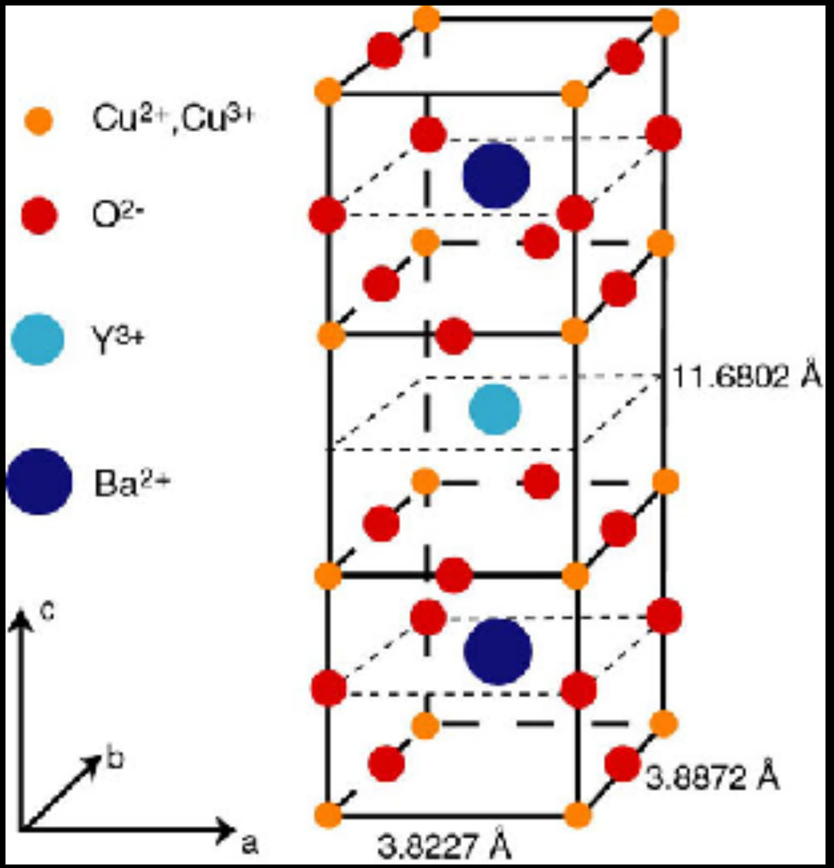
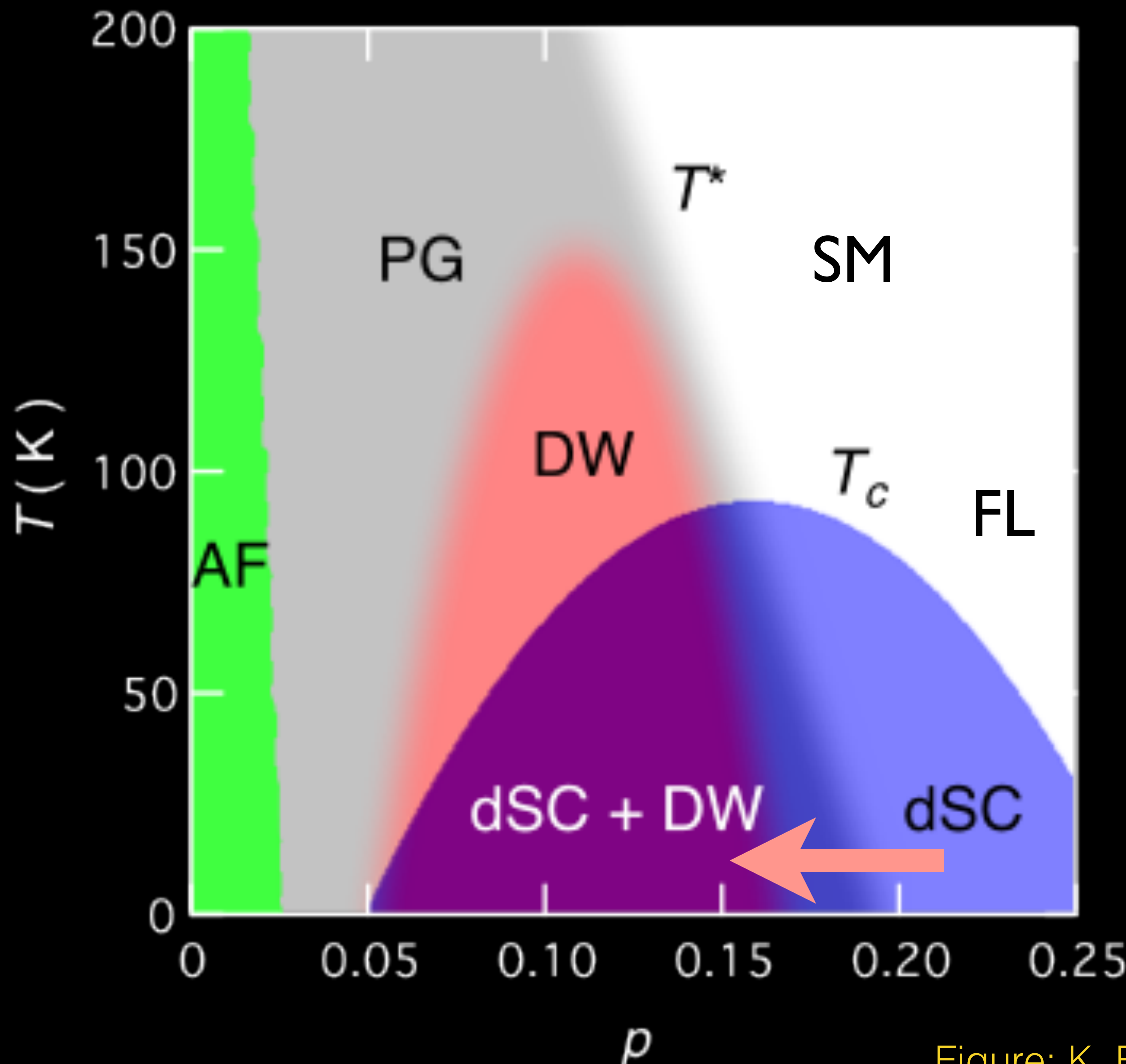
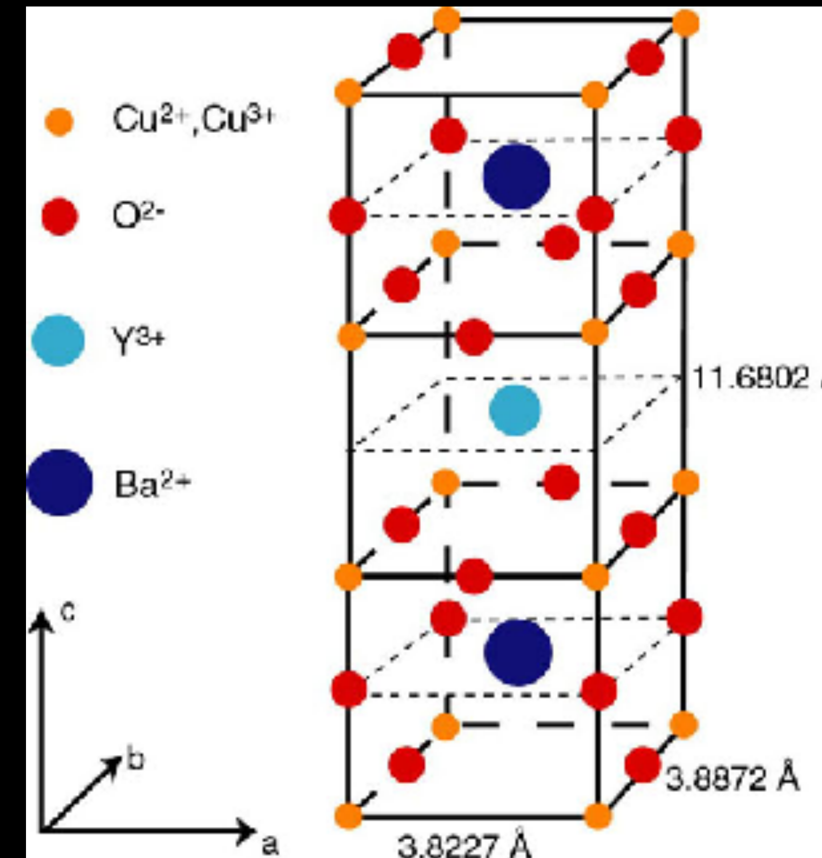




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis



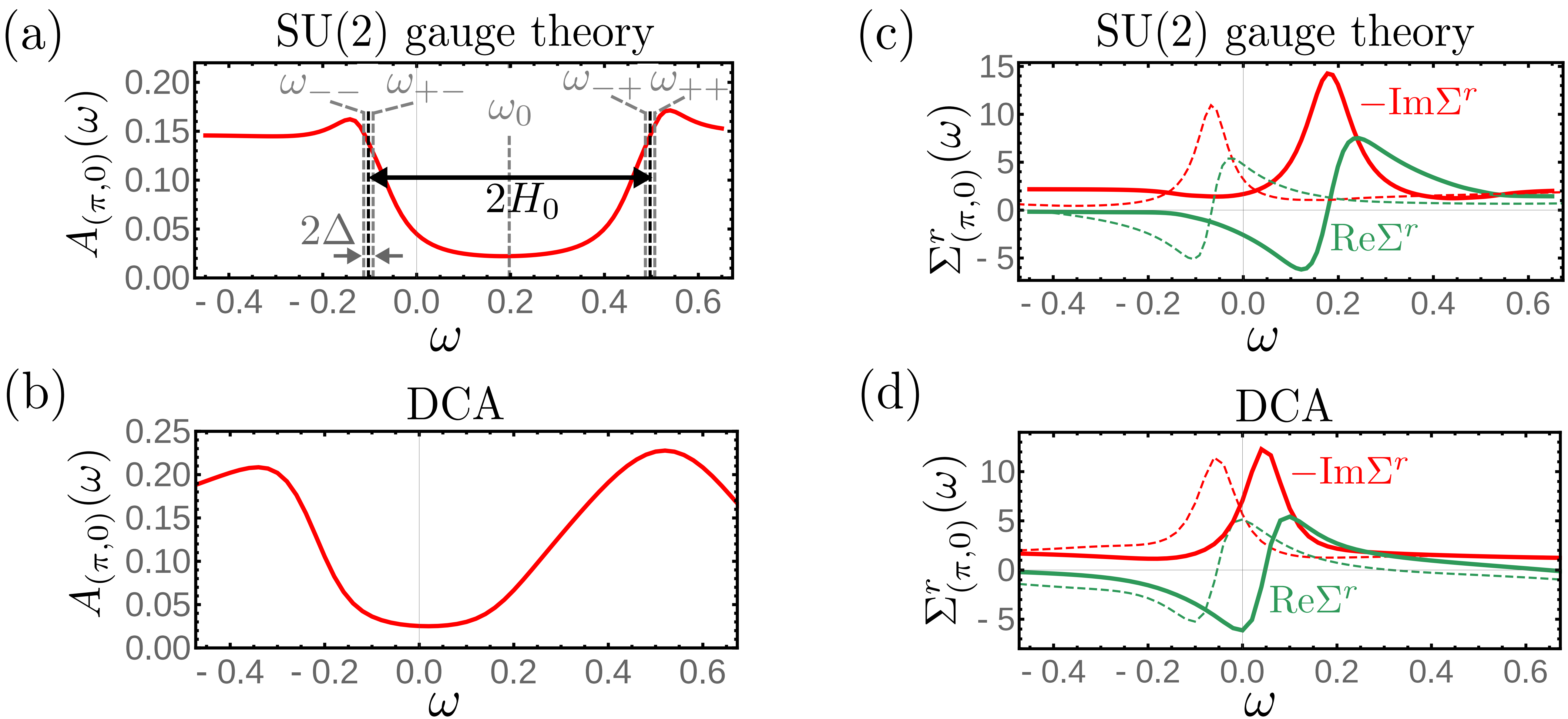


$YBa_2Cu_3O_{6+x}$

Transition to a Higgs/deconfined phase in a SU(2) gauge theory

Figure: K. Fujita and J. C. Seamus Davis

Electron Green's function in Higgs phase of SU(2) gauge theory



$$T = t/30 \quad , \quad U = 7t \quad , \quad p = 0.05$$

t' takes different negative values

Anti-nodal spectra compared to cluster DMFT

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev,
PNAS **115**, E3665 (2018) and PRX to appear

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Orthogonal metals

Fractionalize the electron $c_{i\alpha}$, $\alpha =\uparrow, \downarrow$ into an "orthogonal fermion" $f_{i\alpha}$ and an Ising spin $\sigma_i^z = \pm 1$:

$$c_{i\alpha} = \sigma_i^z \, f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\sigma_i^z \to \eta_i \sigma_i^z \quad , \quad f_{i\alpha} \to \eta_i f_{i\alpha}$$

The orthogonal fermion, f_α, carries both the spin and charge of the electron.
The Ising matter field, σ^z, is 'dark matter' carrying only energy, and a $\mathbb{Z}_2$ gauge charge.

R. Nandkishore, M. Metlitski, and T. Senthil, Phys. Rev. B 86, 045128 (2012)

Orthogonal metals

Fractionalize the electron $c_{ip\alpha}$, on sites $i = 1 \ldots N$, with spin $\alpha = 1 \ldots M$ and orbital index $p = 1 \ldots M'$ into an "orthogonal fermion" $f_{i\alpha}$ and a real scalar ϕ_{ip}:

$$c_{ip\alpha} = \phi_{ip} f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\phi_{ip} \to \eta_i \phi_{ip} \quad , \quad f_{i\alpha} \to \eta_i f_{i\alpha}$$

The orthogonal fermion $f_{i\alpha}$ carries both the spin and charge of the electron.
The scalar field, ϕ_{ip}, is 'dark matter' carrying only energy, and a $\mathbb{Z}_2$ gauge charge.

A solvable model

We examine the t-J model:

$$\mathcal{L} = \frac{1}{2g}\sum_{i,p}\left(\partial_\tau \phi_{ip}\right)^2 + \sum_{i,\alpha} f_{i\alpha}^\dagger \left(\frac{\partial}{\partial\tau} - \mu\right) f_{i\alpha} \\ + \frac{1}{\sqrt{NM}}\sum_{i,j,p,\alpha} t_{ij}\phi_{ip}\phi_{jp} f_{i\alpha}^\dagger f_{j\alpha} + \frac{1}{\sqrt{NM}}\sum_{i>j,\alpha\beta} J_{ij} f_{i\alpha}^\dagger f_{i\beta} f_{j\beta}^\dagger f_{j\alpha}\,,$$

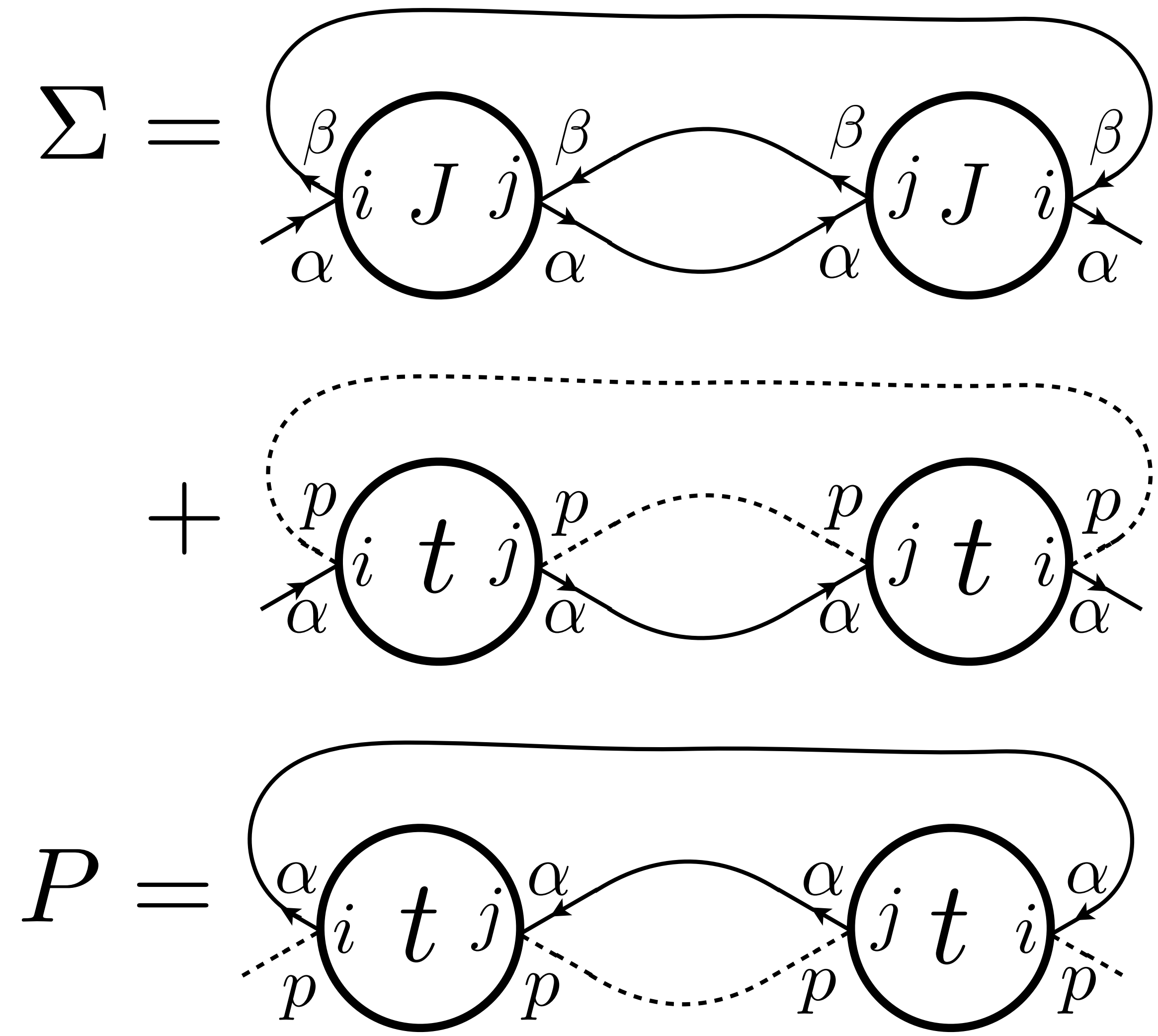
with the scalar field obeying the fixed length constraint

$$\sum_{p=1}^{M'} \phi_{ip}^2 = M'\,.$$

With t_{ij} and J_{ij} independent random numbers with zero mean, $\mathcal{L}$ is solvable in the limit of large number of sites, N, followed by the limit of large M and M' at fixed

$$k \equiv \frac{M'}{M}\,.$$

A solvable model



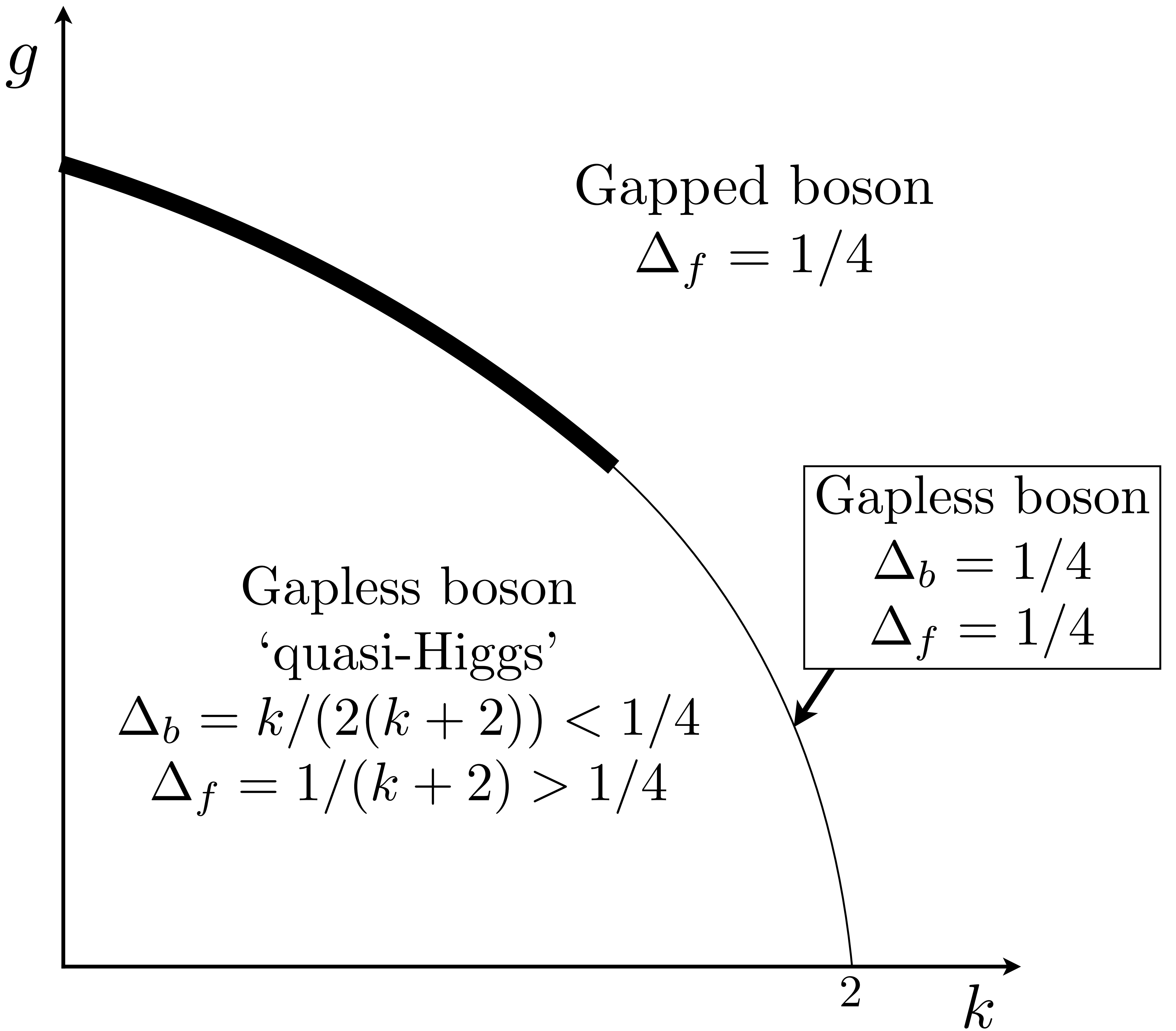
Equations for the fermion Green's function G and the boson Green's function χ:

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau) + k\, \tilde{t}^{\,2} G(\tau)\chi^2(\tau)$$

$$\chi(i\omega_n) = \frac{1}{\omega_n^2 + \chi_0^{-1} - P(i\omega_n) + P(i\omega_n = 0)} \quad , \quad P(\tau) = -2\,\tilde{t}^{\,2} G(\tau) G(-\tau)\chi(\tau)$$

where χ_0^{-1} is determined by solving $\chi(\tau = 0) = 1/g$, and $\tilde{t} = tJ$.

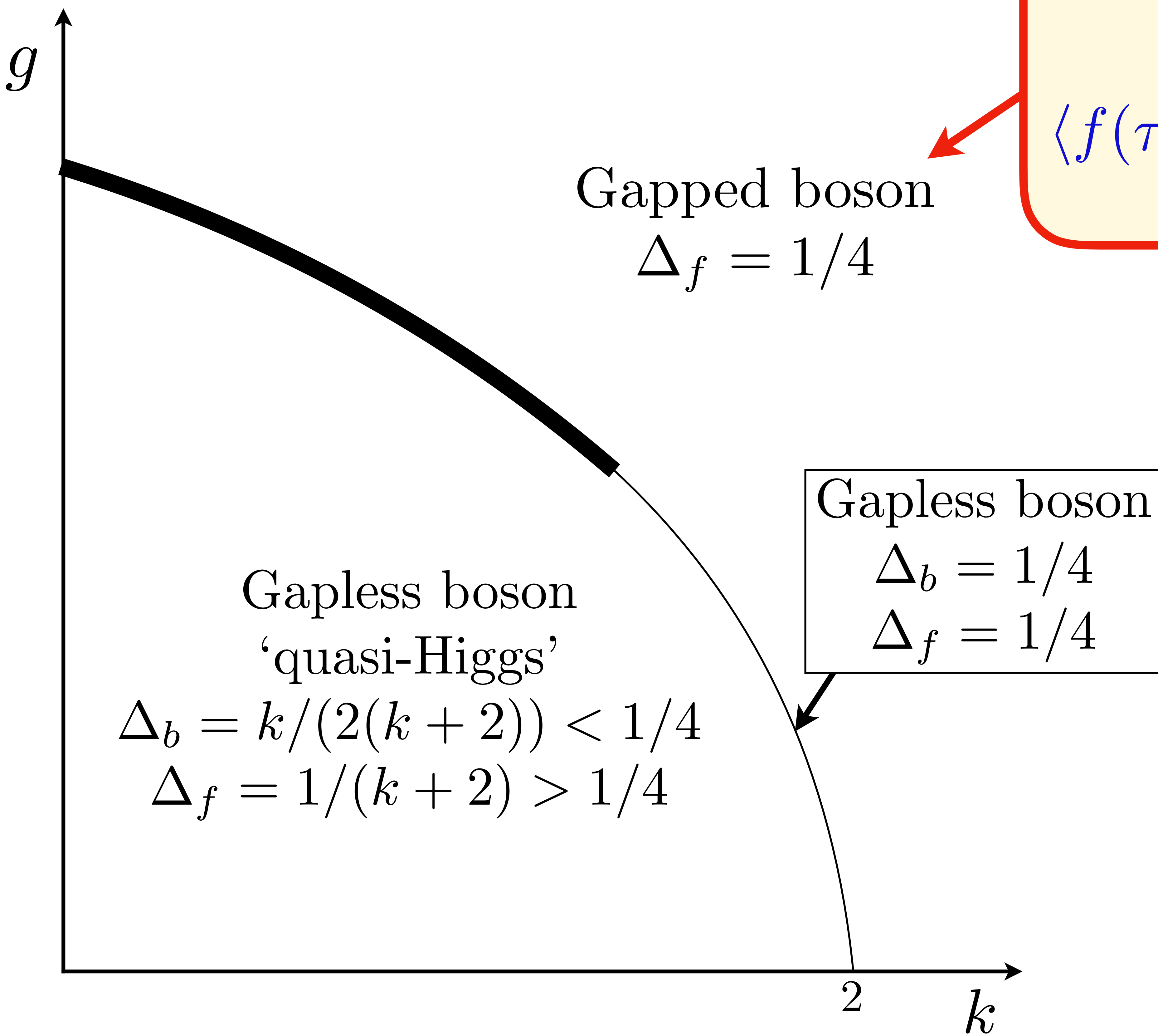
A solvable model



A solvable model

$$\langle \phi(\tau)\phi(0)\rangle \sim \frac{e^{-m|\tau|}}{\sqrt{\tau}}$$

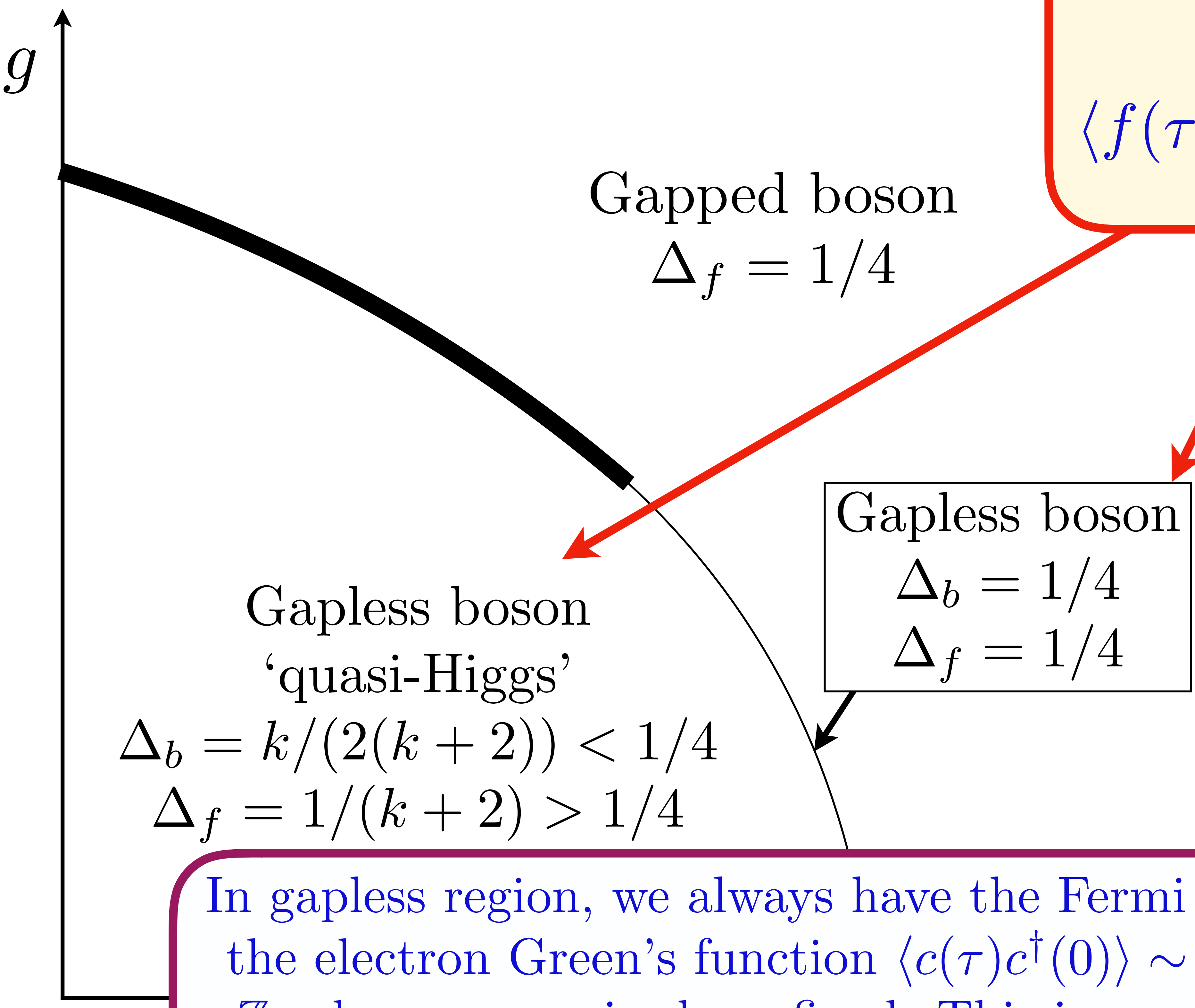
$$\langle f(\tau) f^\dagger(0)\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{2\Delta_f}}$$



A solvable model

$$\langle \phi(\tau)\phi(0)\rangle \sim \frac{1}{|\tau|^{2\Delta_b}}$$

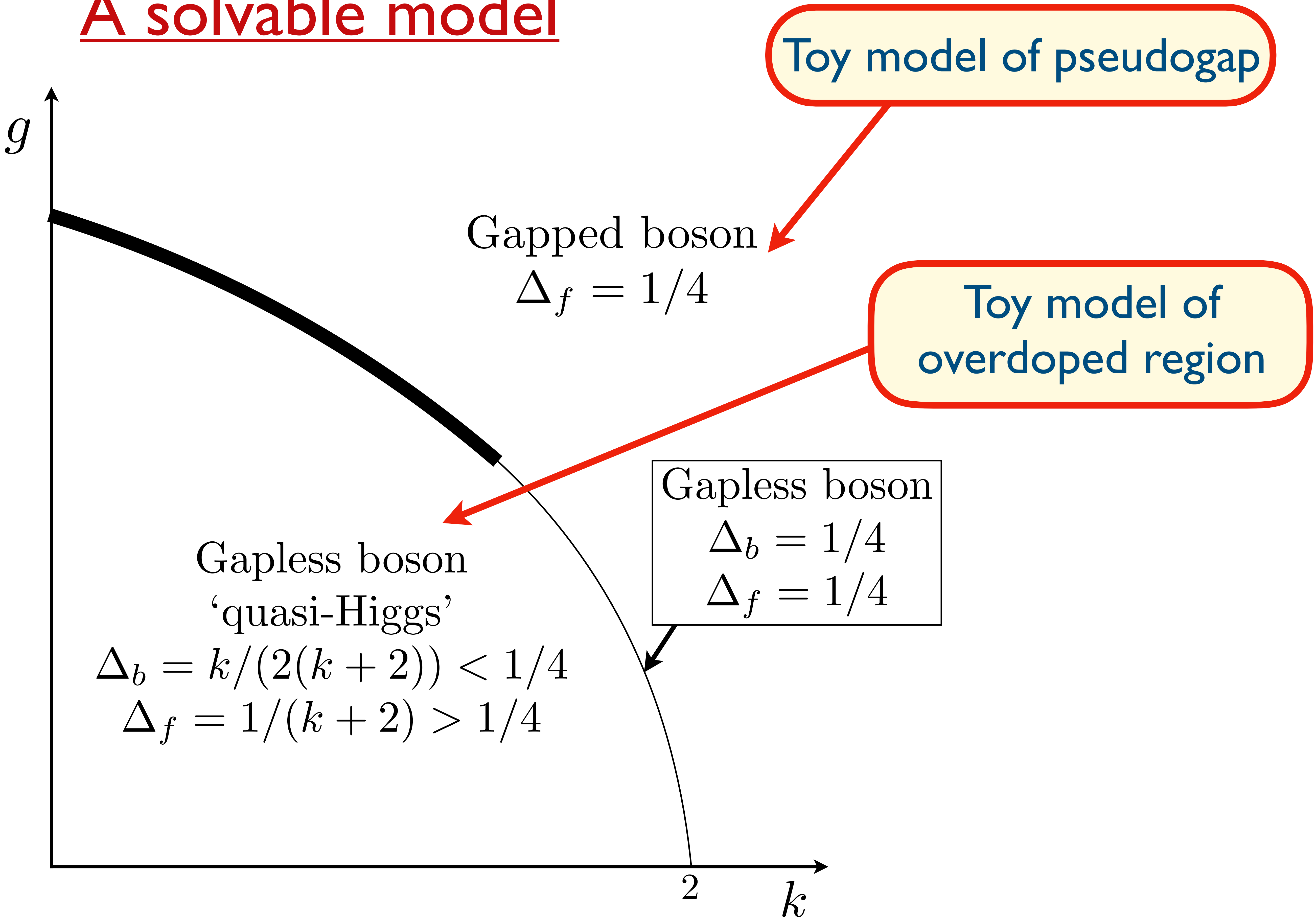
$$\langle f(\tau)f^\dagger(0)\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{2\Delta_f}}$$



$$\Delta_b + \Delta_f = 1/2$$

In gapless region, we always have the Fermi liquid form for the electron Green's function $\langle c(\tau)c^\dagger(0)\rangle \sim 1/\tau$, although $\mathbb{Z}_2$ charges remain deconfined. This is a consequences of the fixed point with a non-zero t term in the t-J model.

A solvable model



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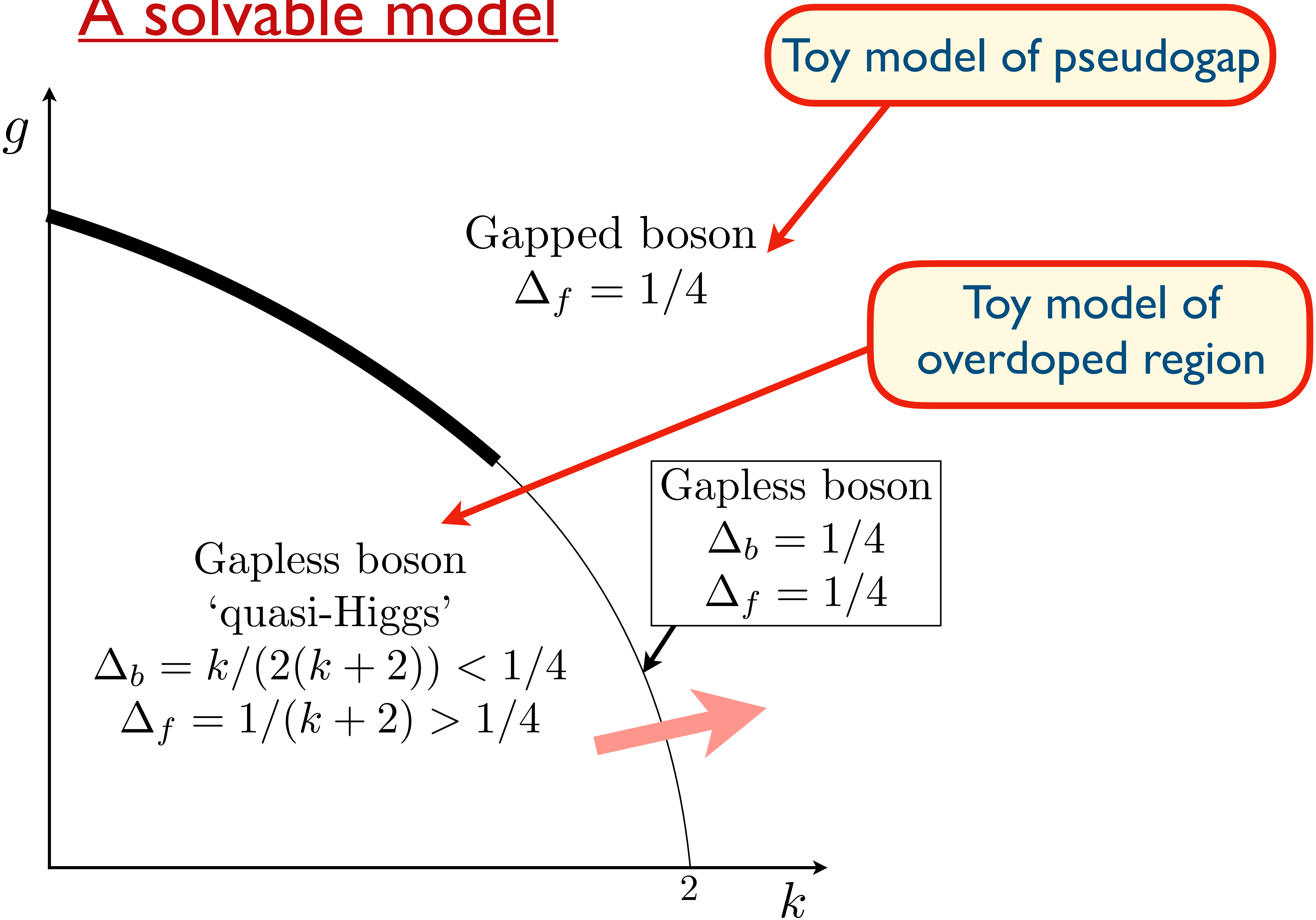

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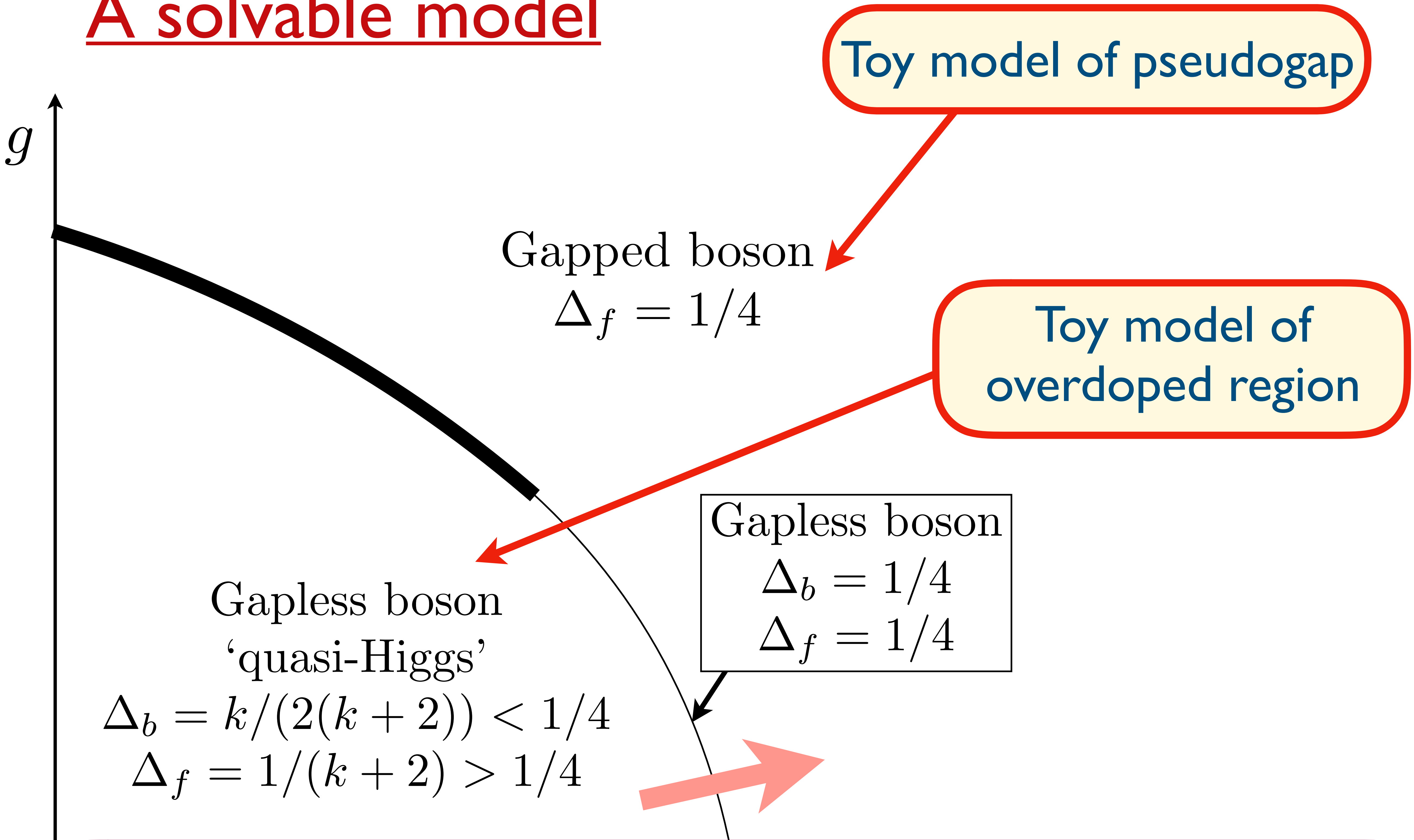


In the overdoped region, Fermi liquid electron spectral function, with anomalies in other properties, match recent observations in cuprates (Hussey, Bozovic, Armitage, Taillefer...)

1. The SYK Model

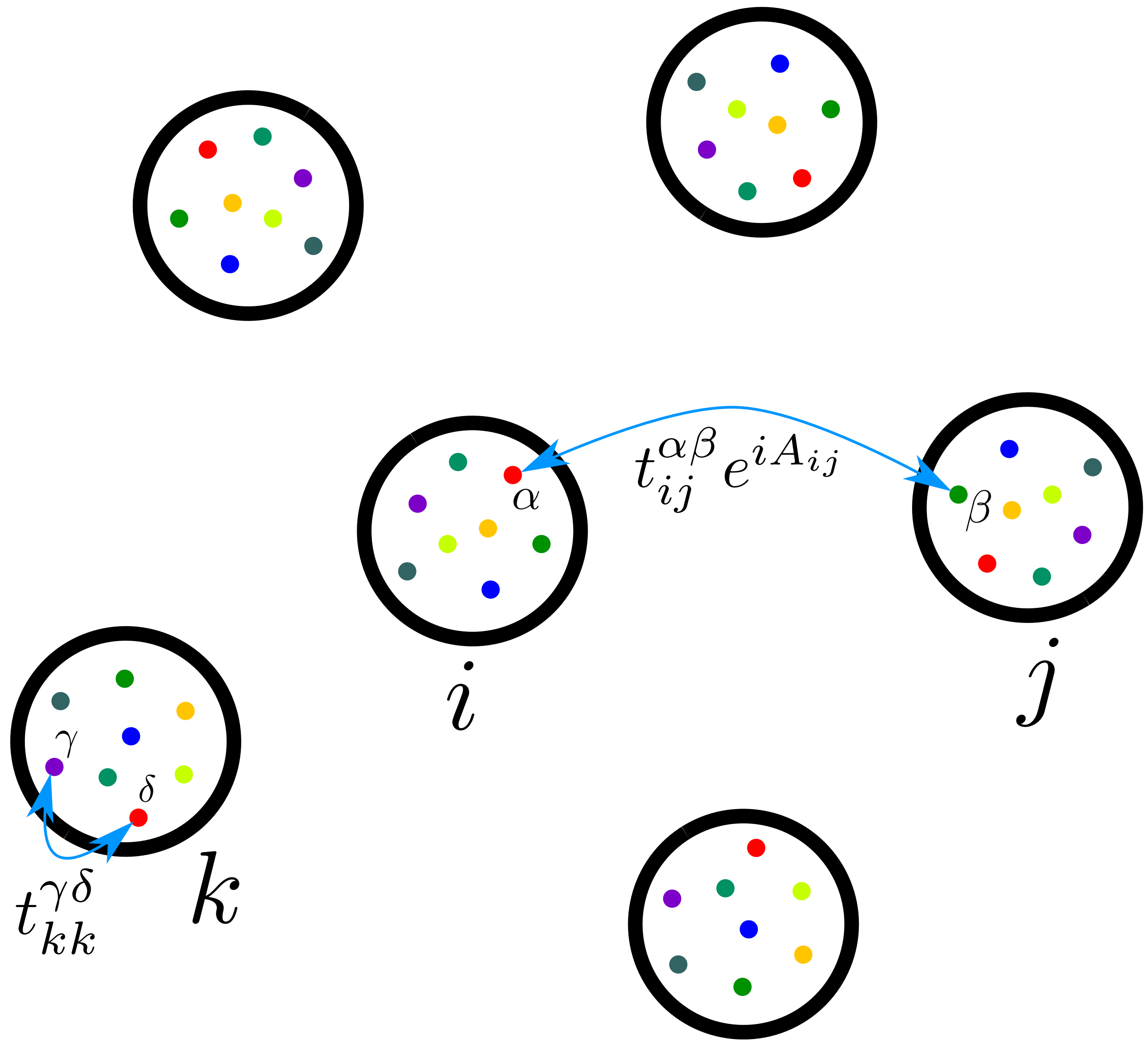
2. Gauge theories for the pseudogap and optimum-doping criticality in the cuprates

3. Z_2 Fractionalization in a SYK *t-J* model

4. SYK U(1) gauge theory

Aavishkar Patel

Fermions with random hopping coupled to a fluctuating U(1) gauge field



$$H = -\frac{1}{(MN)^{1/2}} \sum_{ij=1}^{N} \sum_{\alpha\beta=1}^{M} \left[t_{ij}^{\alpha\beta} e^{iA_{ij}} f_{i\alpha}^{\dagger} f_{j\beta} + (MN)^{1/2} \mu \delta_{ij}^{\alpha\beta} f_{i\alpha}^{\dagger} f_{i\alpha} \right]$$

$$\ll t_{ij}^{\alpha\beta} t_{ji}^{\beta\alpha} \gg = \ll |t_{ij}^{\alpha\beta}|^2 \gg = t^2, \quad A_{ji} = -A_{ij}.$$

Fermions with random hopping coupled to a fluctuating U(1) gauge field

$$\Sigma(i\omega_n) = t^2 G(i\omega_n) + t^2 T \sum_{\Omega_m \neq 0} \frac{G(i\omega_n + i\Omega_m) - G(i\omega_n)}{\Pi(i\Omega_m) - \Pi(i\Omega_m = 0)},$$

$$\Pi(i\Omega_m) = t^2 T \frac{M}{N} \sum_{\omega_n} G(i\omega_n) G(i\omega_n + i\Omega_m), \quad G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)}.$$

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General low energy solution

$$G(\tau > 0) = -\frac{C(\mathcal{E})}{t^{1-x}\tau^{1-x}}, \quad G(\tau < 0) = \frac{C(\mathcal{E}) e^{-2\pi\mathcal{E}}}{t^{1-x}|\tau|^{1-x}}.$$

where $\mathcal{E}$ is a parameter universally related to the filling fraction ($\mathcal{E} = 0$ at half-filling). The exponent x is the solution to

$$\frac{(1/x - 2)(\cosh(2\pi\mathcal{E}) - \cos(\pi x))}{\tan(\pi x)\sin(\pi x)} = \frac{M}{N}.$$

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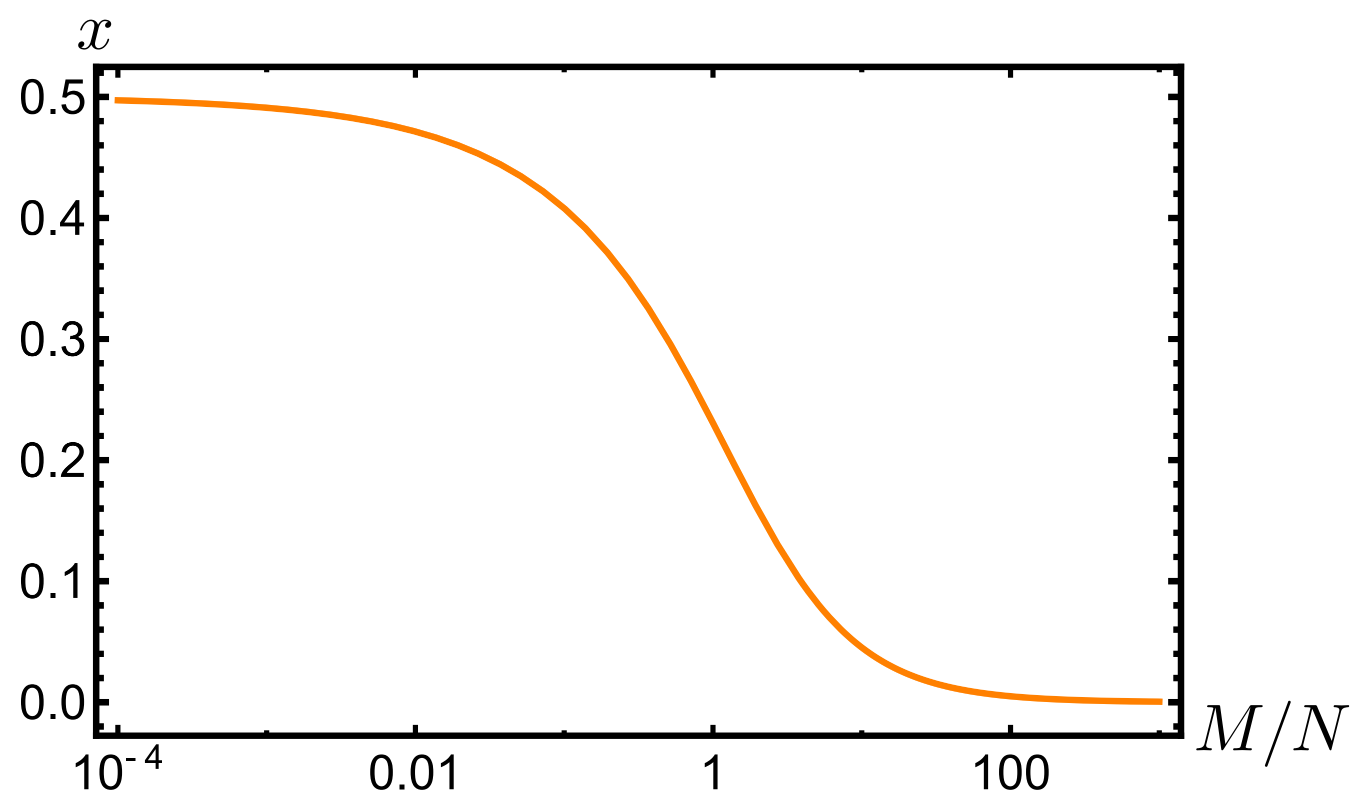

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Solvable models with fractionalization, variable fermion density, and disorder

1. Non-Fermi liquid regime of SYK models with linear-in-T resistivity

2. Z_2 Fractionalization in a SYK t-J model

Overdoped state described by state with fractionalization but with a Fermi liquid electron spectral function

3. SYK U(1) gauge theory