The spin density wave quantum phase transition in two-dimensional metals

Unconventional Superconductivity workshop,
University of Minnesota,
April 24, 2011

Talk online: sachdev.physics.harvard.edu

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Max Metlitski

Sean Hartnoll

Diego Hofman

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Outline

1. Low energy theory of spin density wave quantum critical point

2. Instabilities near the quantum critical point:
   A. Unconventional superconductivity
   B. $2k_F$ bond-nematic ordering

3. Electron spectral function and optical conductivity at quantum critical point
   Scattering off composite operators

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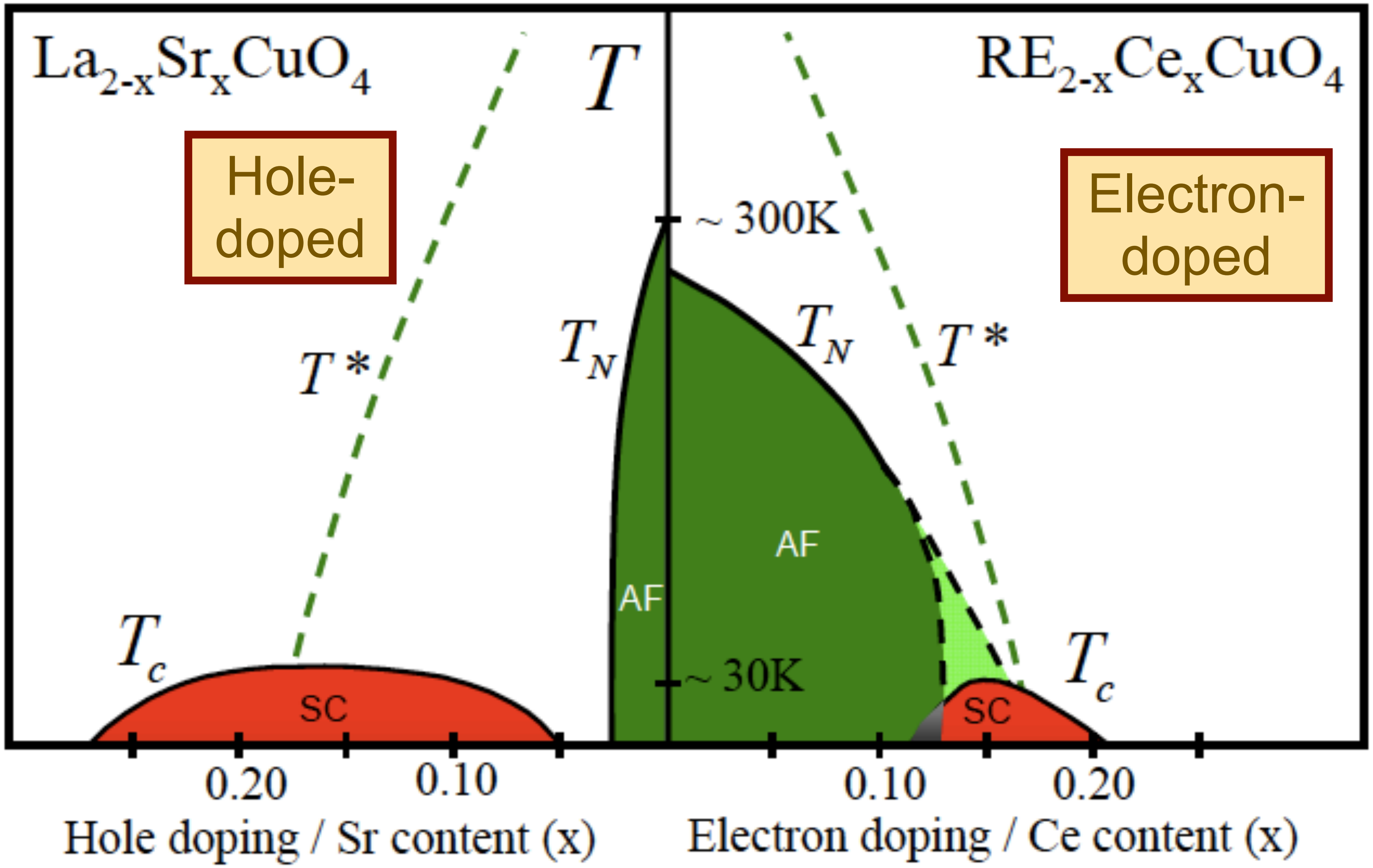
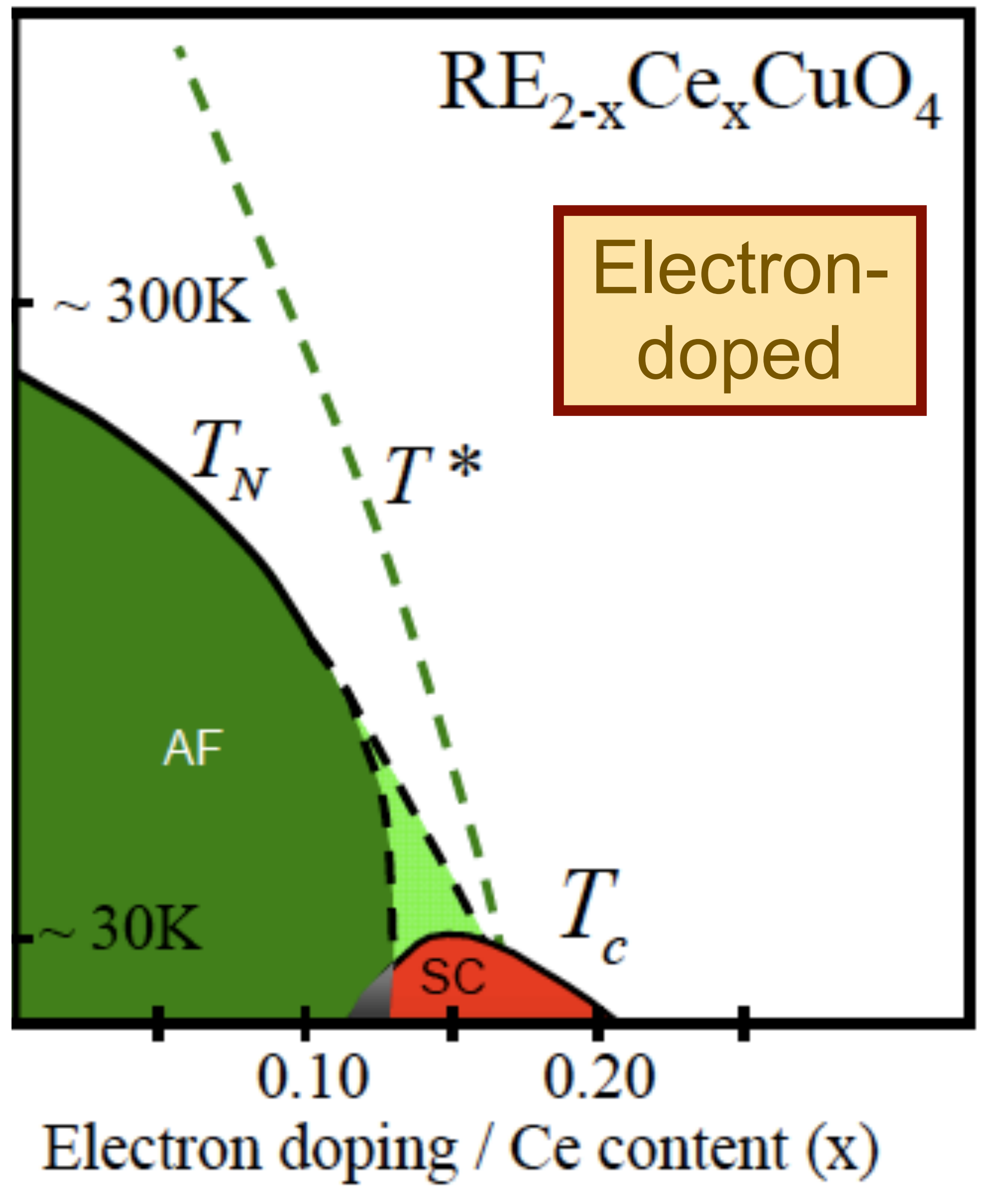


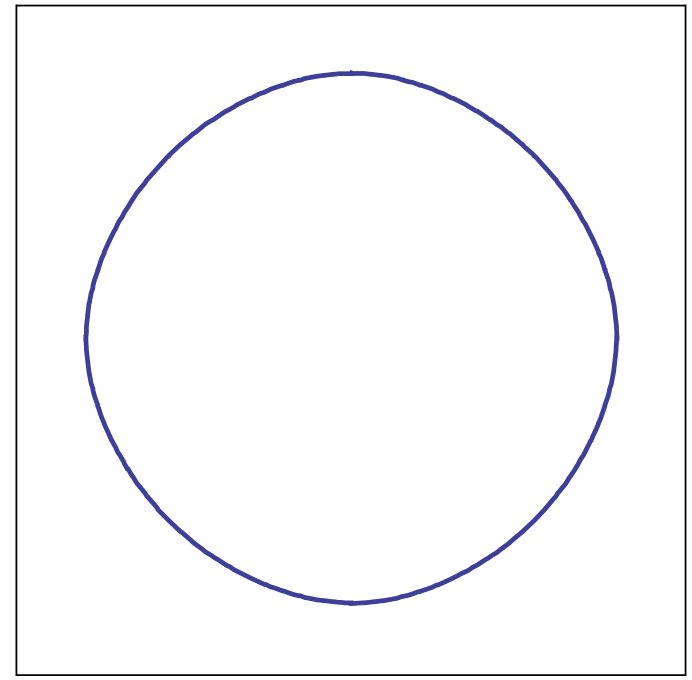
$La_{2-x}Sr_xCuO_4$
$RE_{2-x}Ce_xCuO_4$
Hole-doped
Electron-doped
T
~ 300K
~ 30K
T^*
T_N
T_c
AF
SC
0.20
0.10
Hole doping / Sr content (x)
Electron doping / Ce content (x)

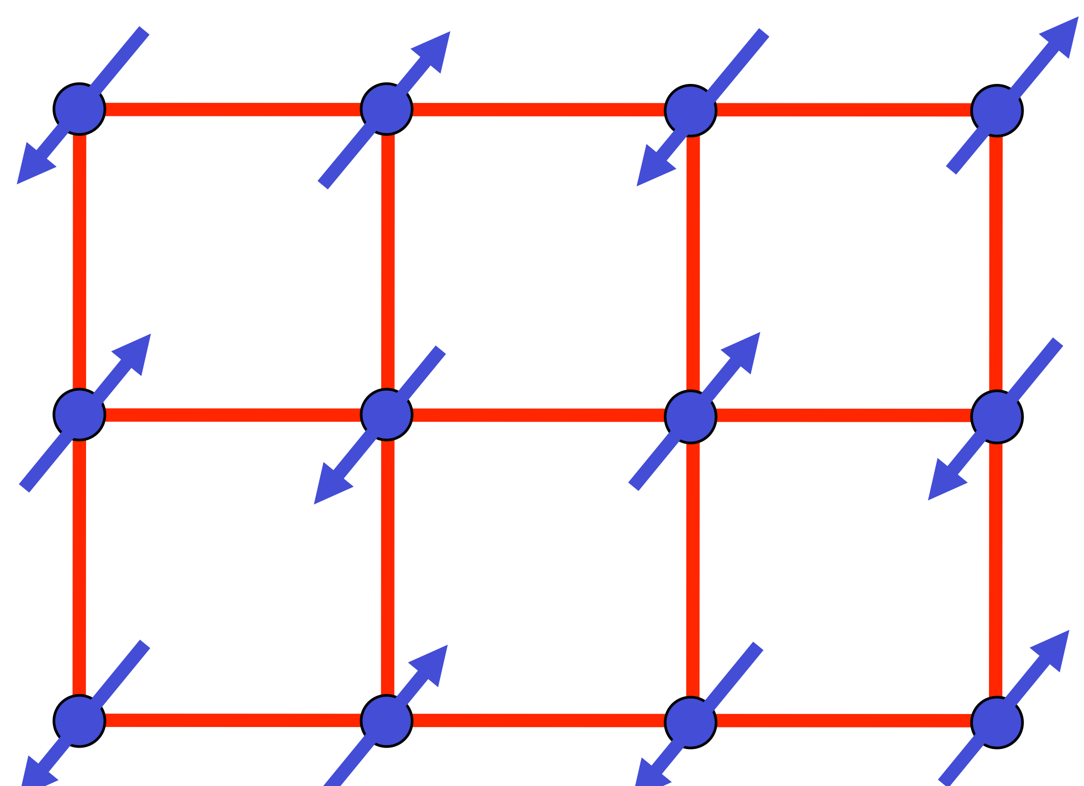
Electron-doped cuprate superconductors



Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



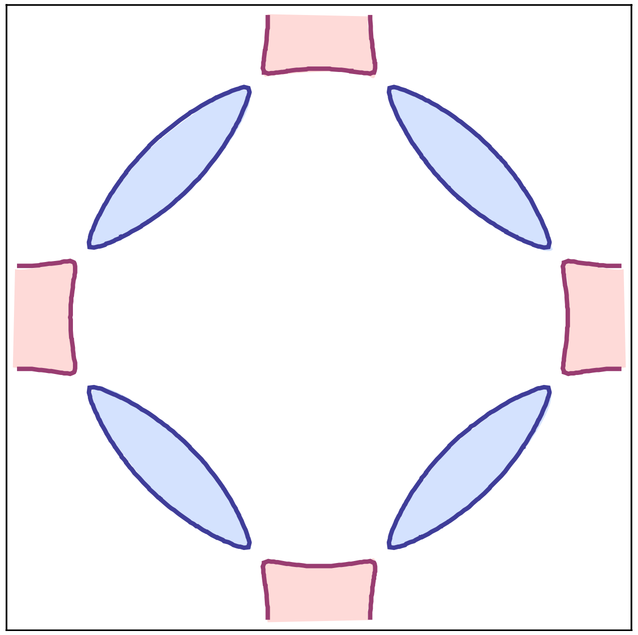


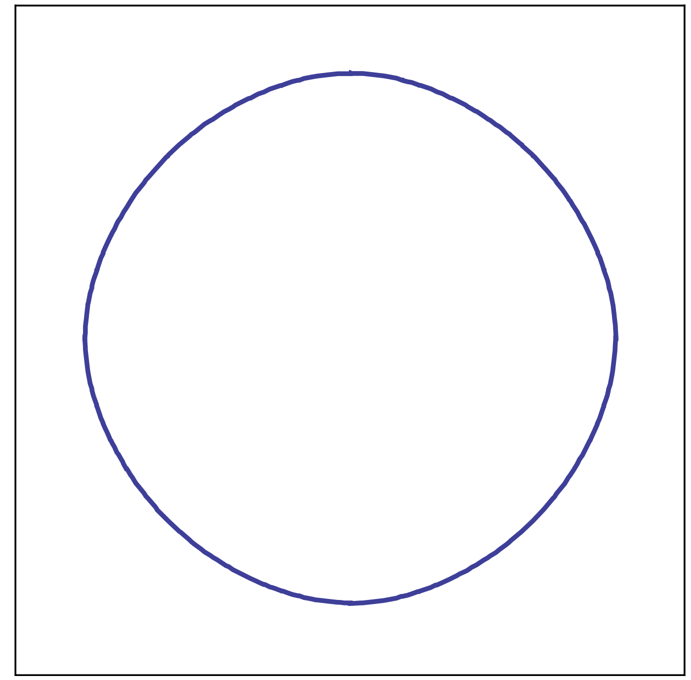
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism





$\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

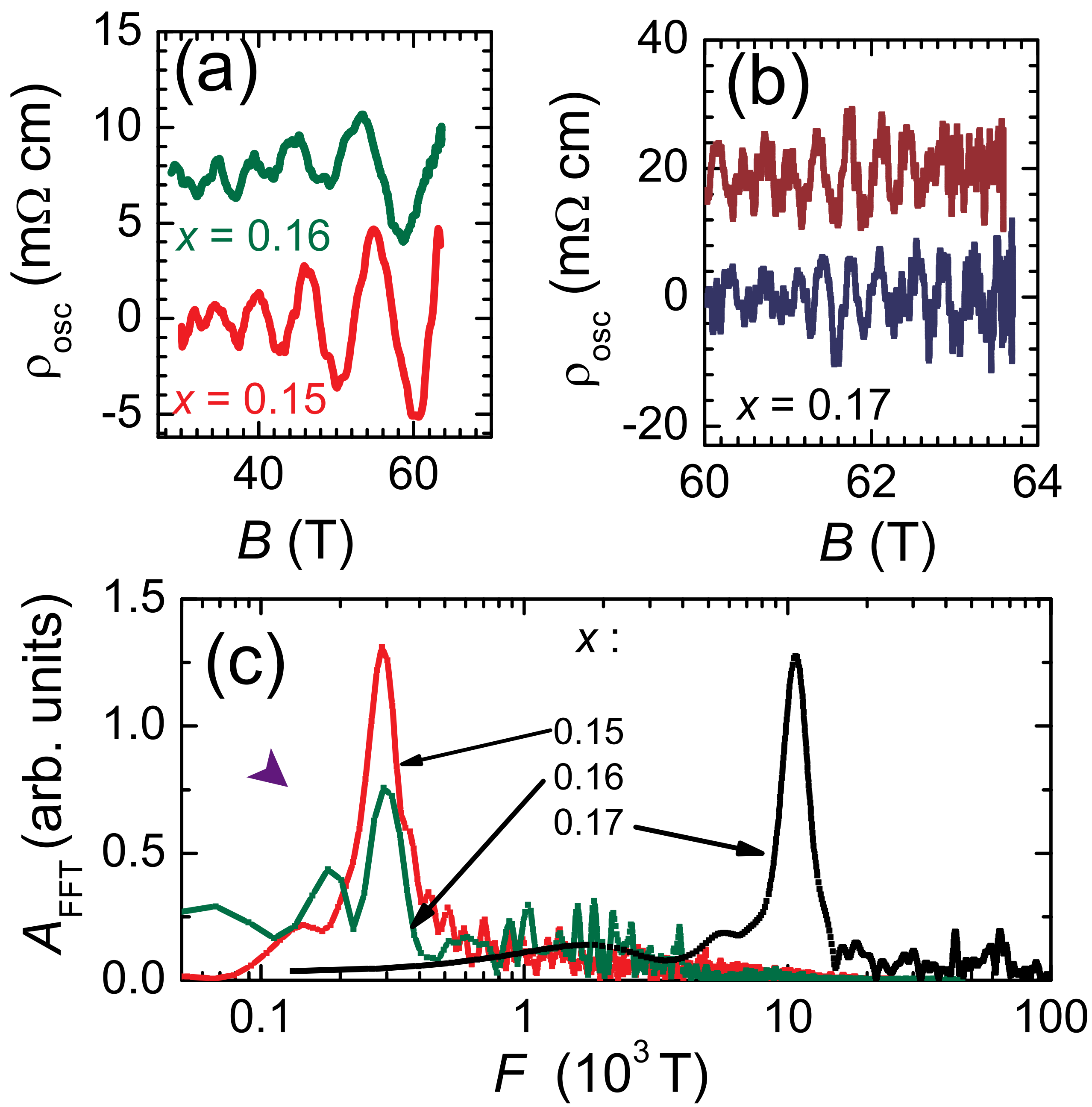
Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).



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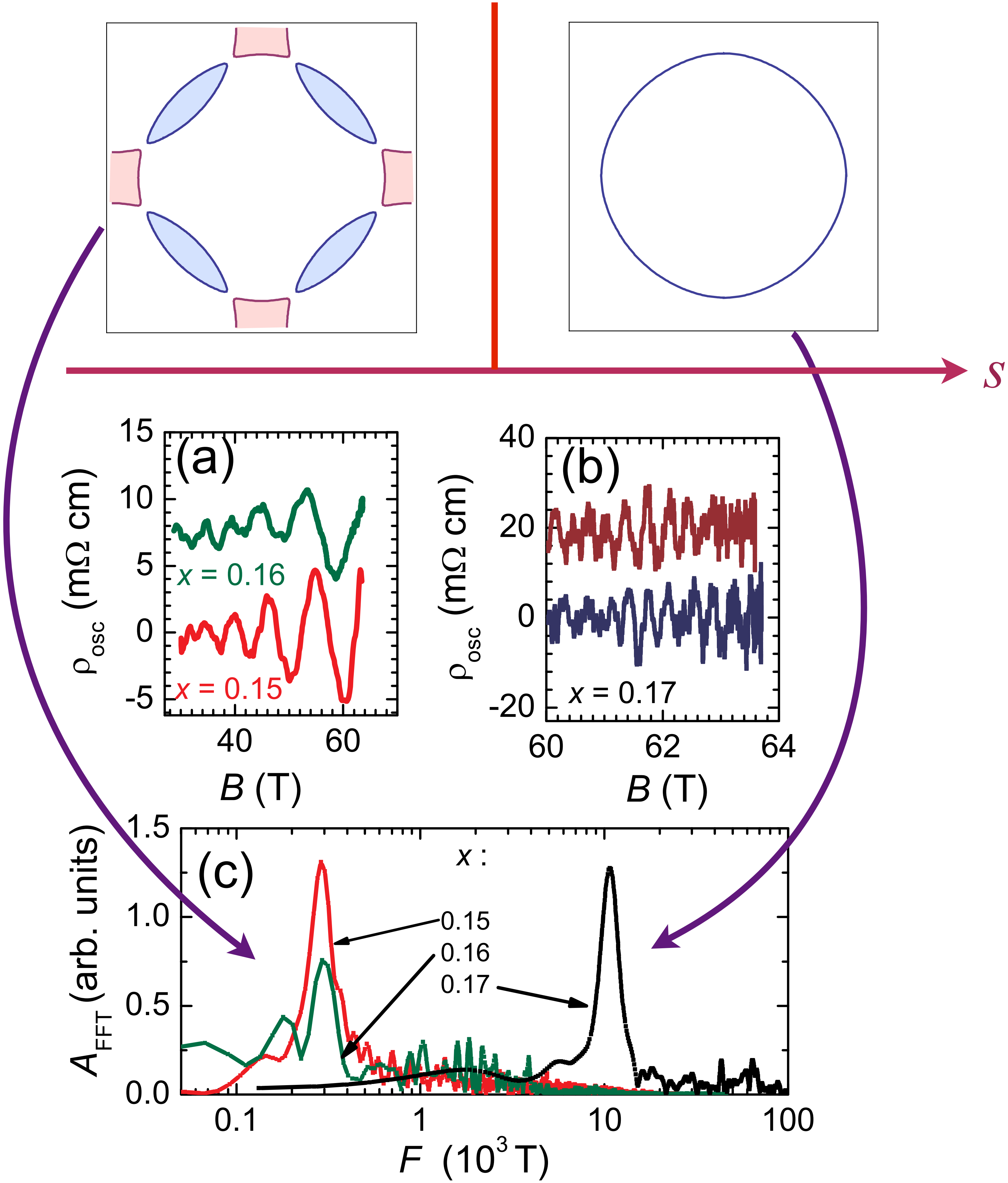

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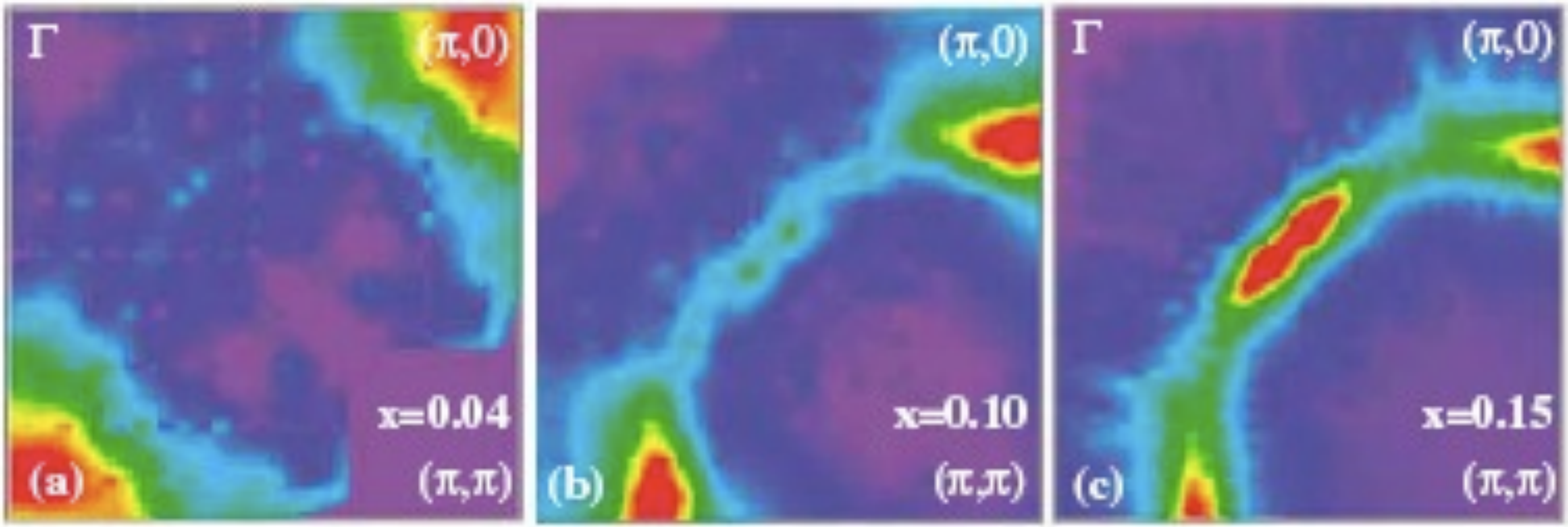
Photoemission in $Nd_{2-x}Ce_xCuO_4$



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Spin-fermion model: Elections with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \dots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{r}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

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Coupling between fermions and antiferromagnetic order: $\lambda^2 \sim U$, the Hubbard repulsion

A technical aside......

Hertz-Moriya-Millis theory

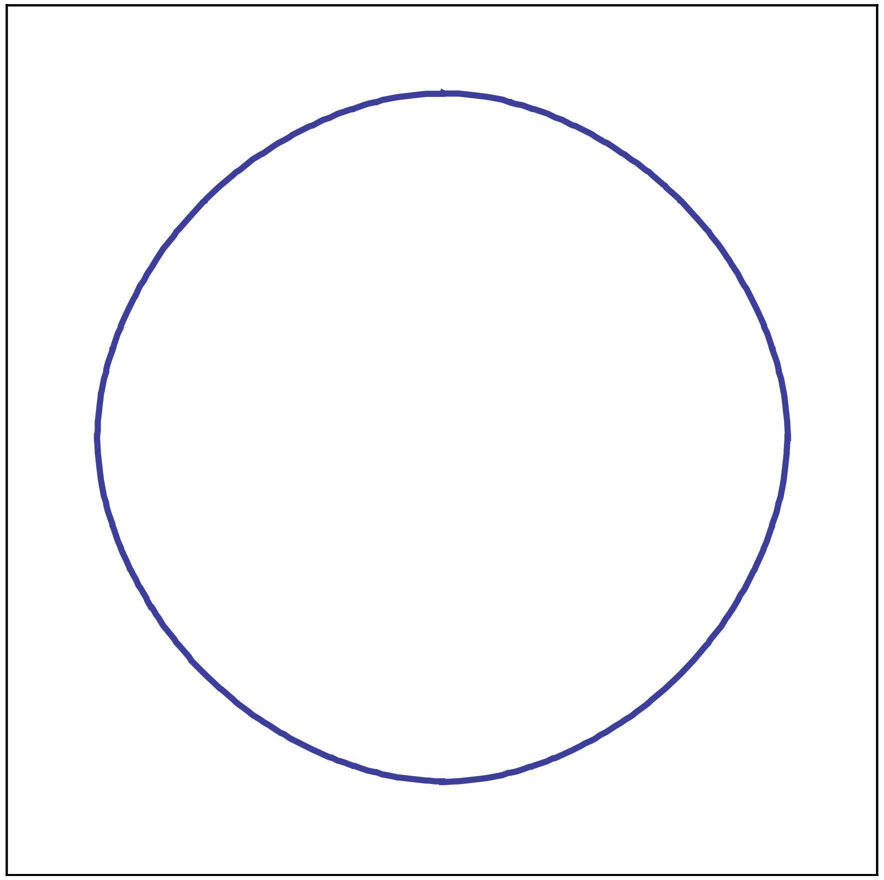
Integrate out fermions and obtain an effective action for the boson field $\vec{\varphi}$ alone. Because the fermions are gapless, this is potentially dangerous, and will lead to non-local terms in the $\vec{\varphi}$ effective action. Hertz focused on only the simplest such non-local term. However, there are an infinite number of non-local terms at higher order, and these lead to a breakdown of the Hertz theory in $d = 2$.

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

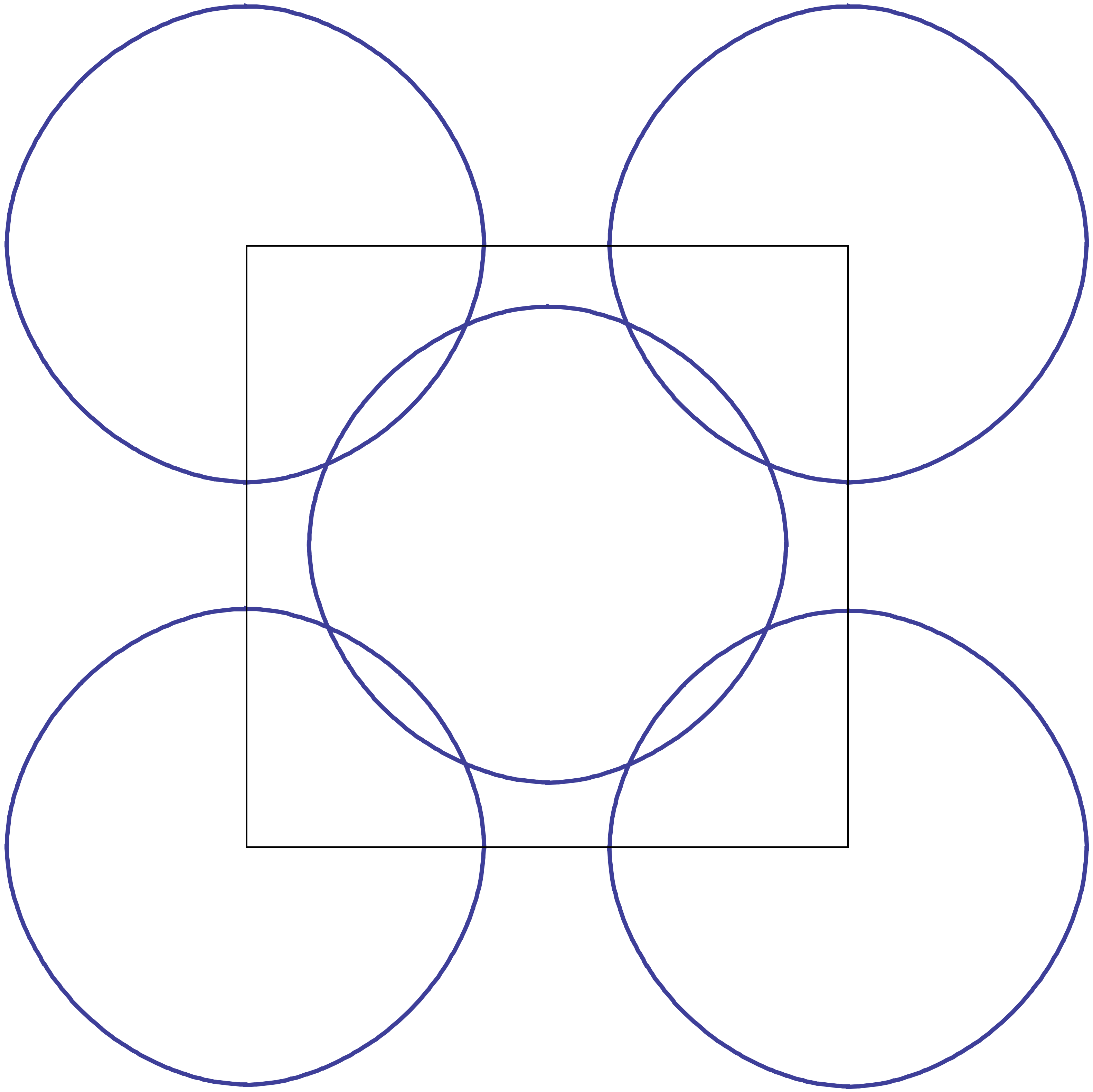
A technical aside......

We need to perform an RG analysis on a local theory of both the fermions and the $\vec{\varphi}$. It appears that such a theory can be analyzed using a $1/N$ expansion, where N is the number of fermion flavors. At two-loop order, the $1/N$ expansion is well-behaved, and we can determine consistent RG flow equations. However, at higher loops we find corrections to the renormalizations which require summation of all planar graphs even at the leading order in $1/N$, and the $1/N$ expansion appears to be organized as a genus expansion of random surfaces. But even this genus expansion breaks down in the renormalization of a quartic coupling of $\vec{\varphi}$. In the following, I will describe some of the two loop results.

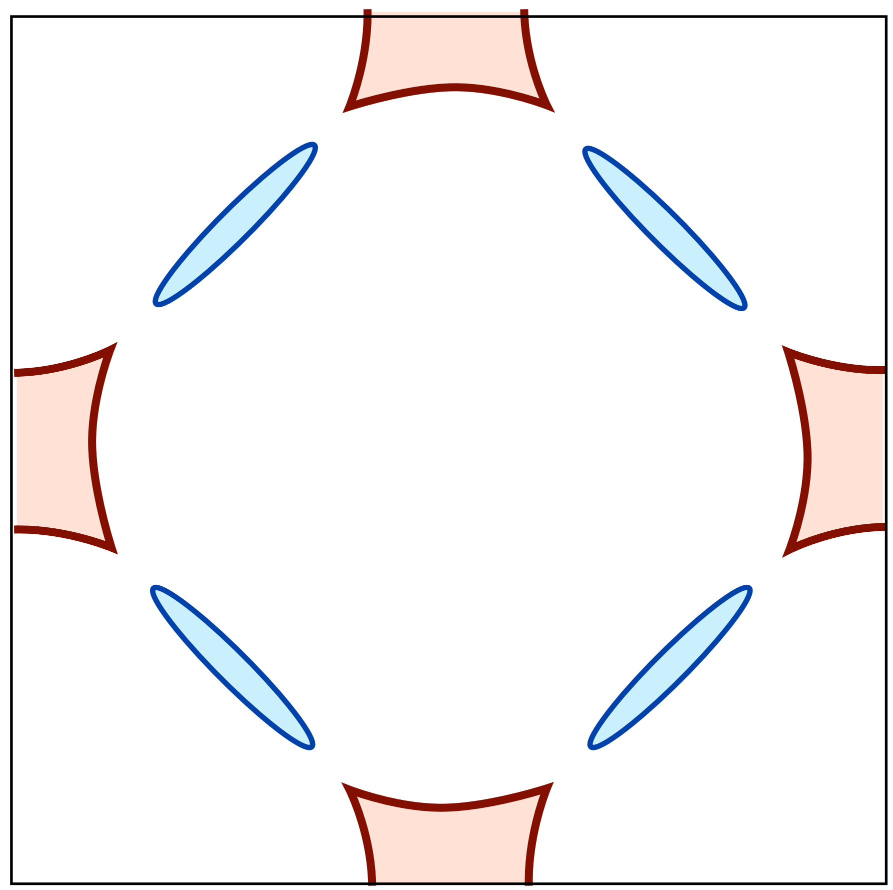
M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



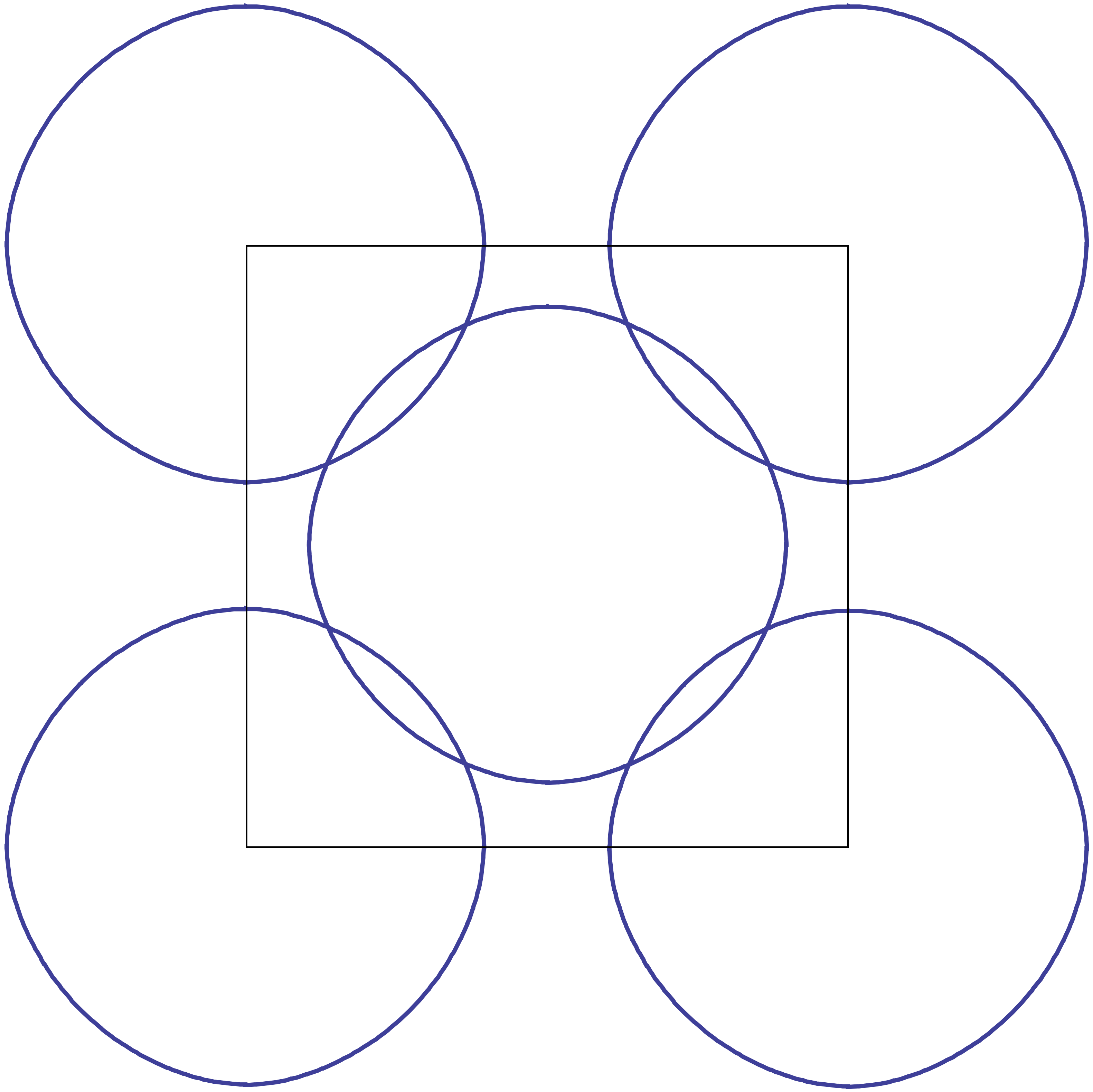
Metal with “large” Fermi surface



Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$



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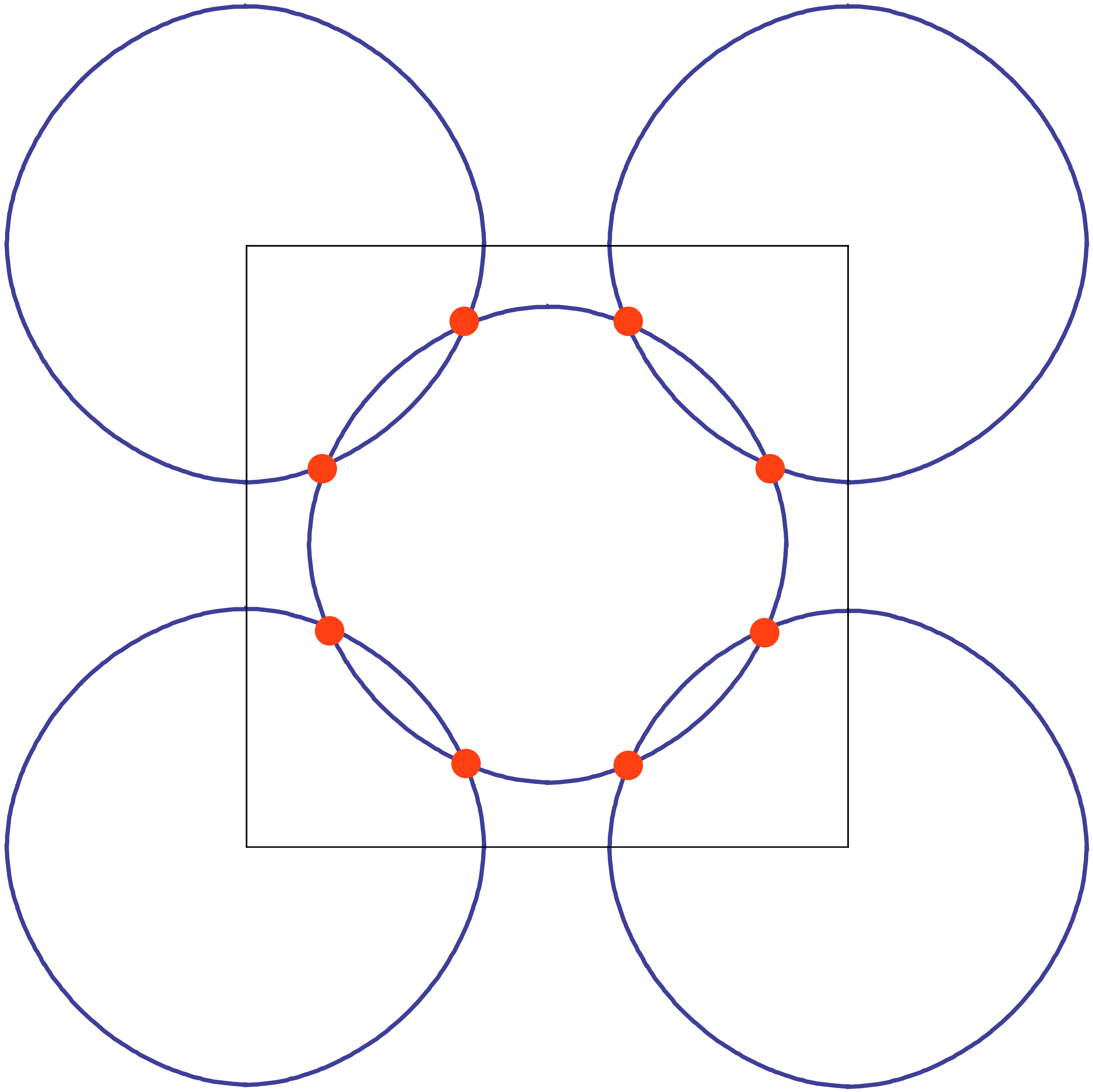
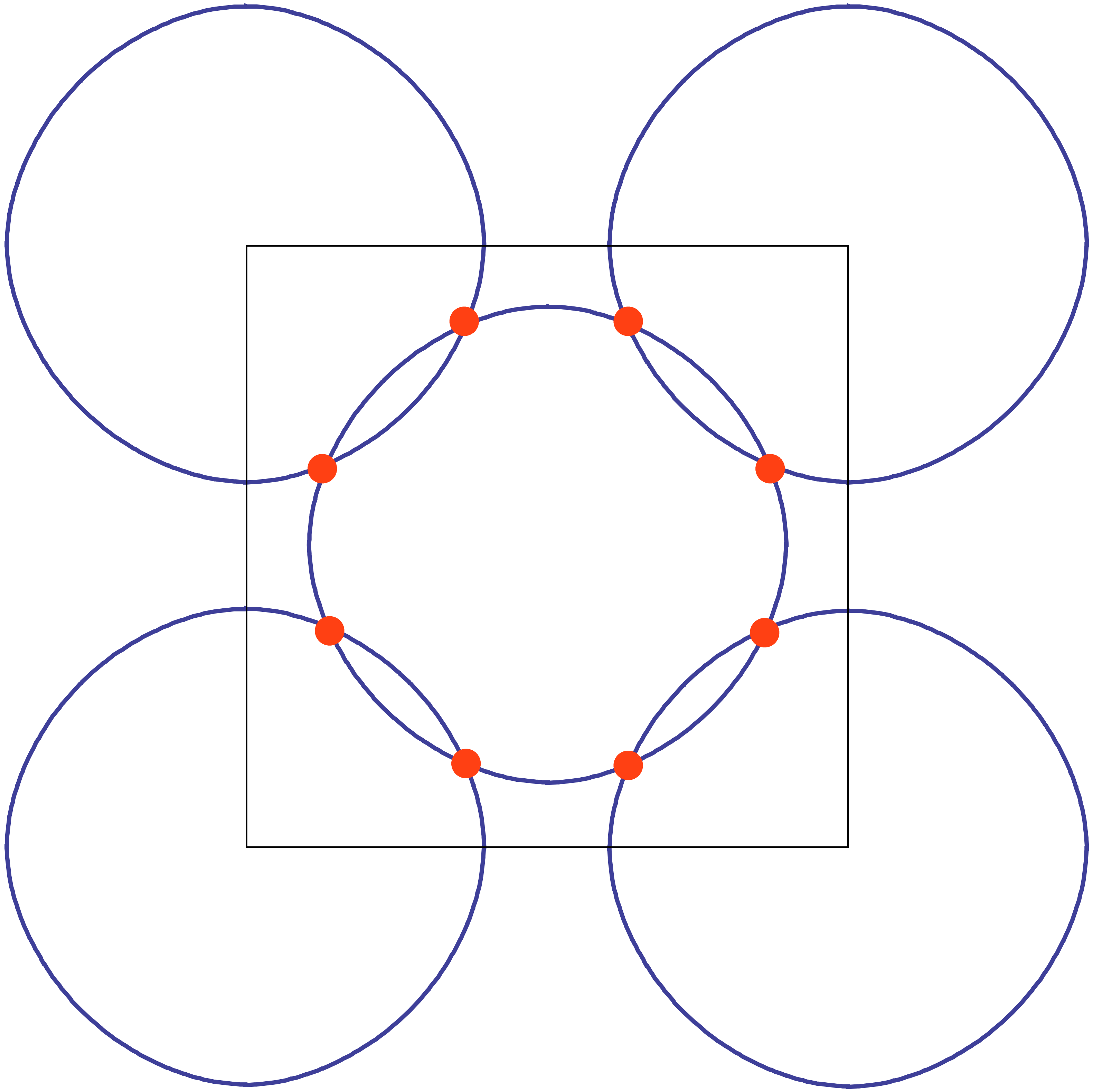
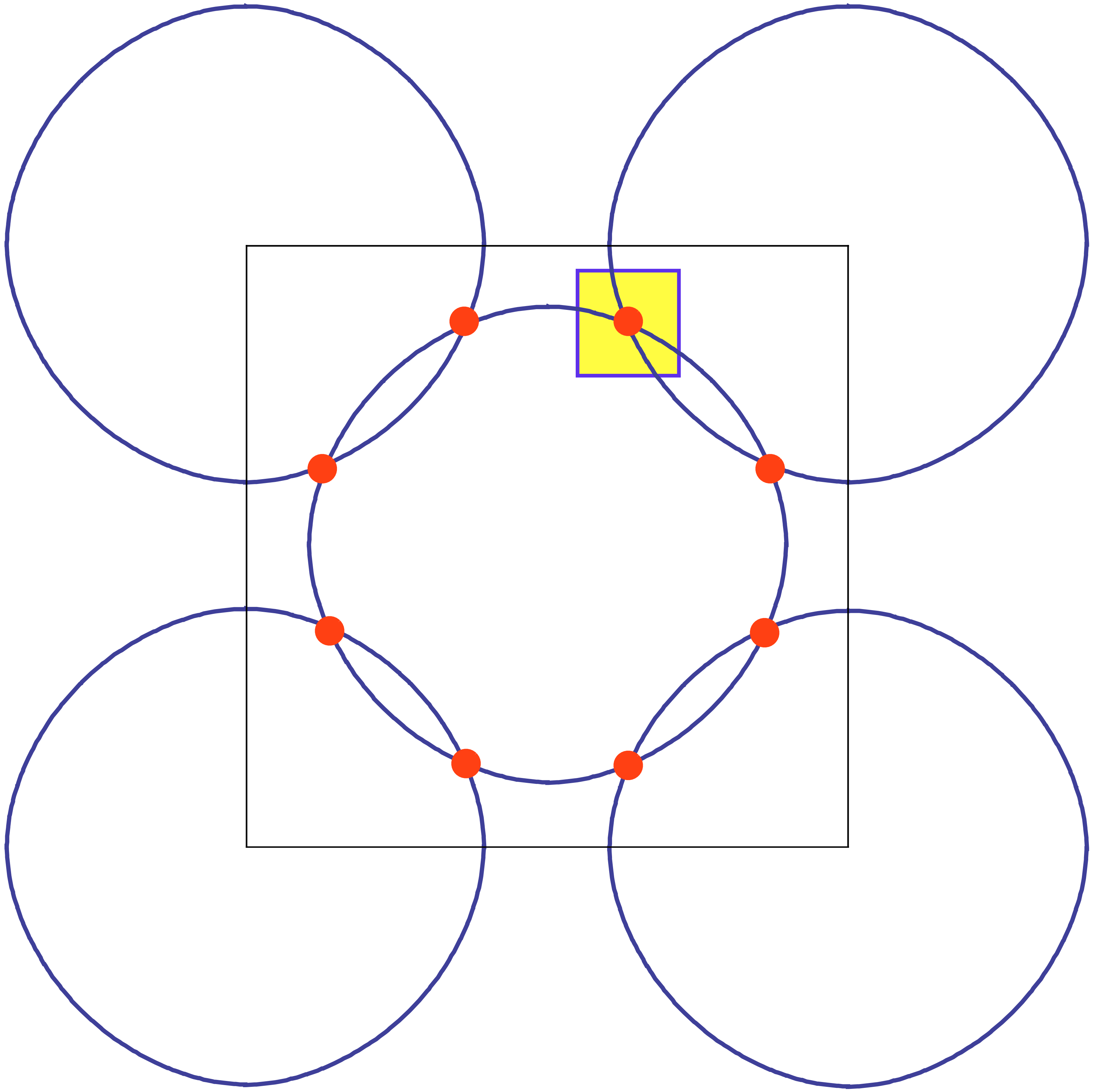


"Hot" spots

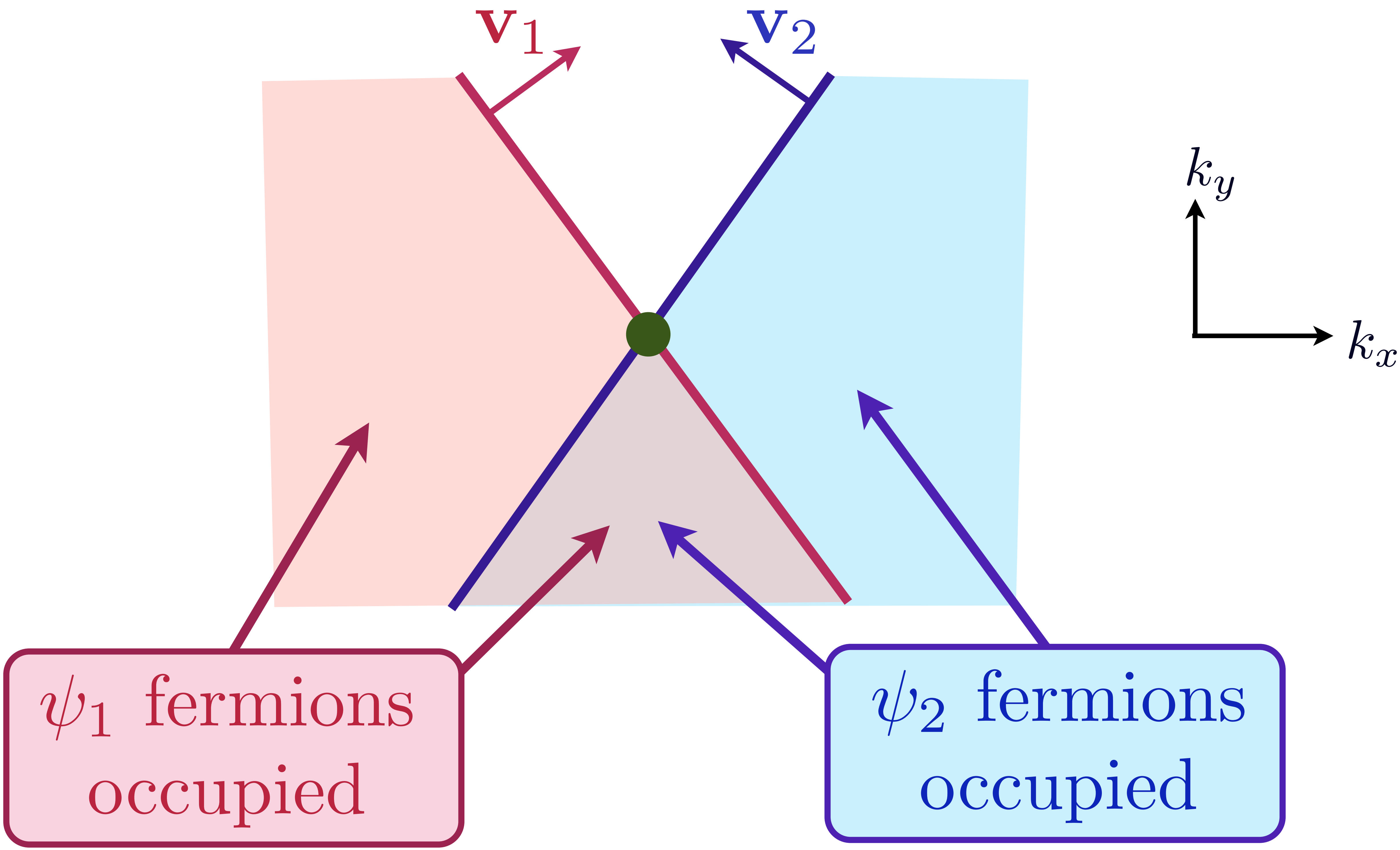


Low energy theory for critical point near hot spots

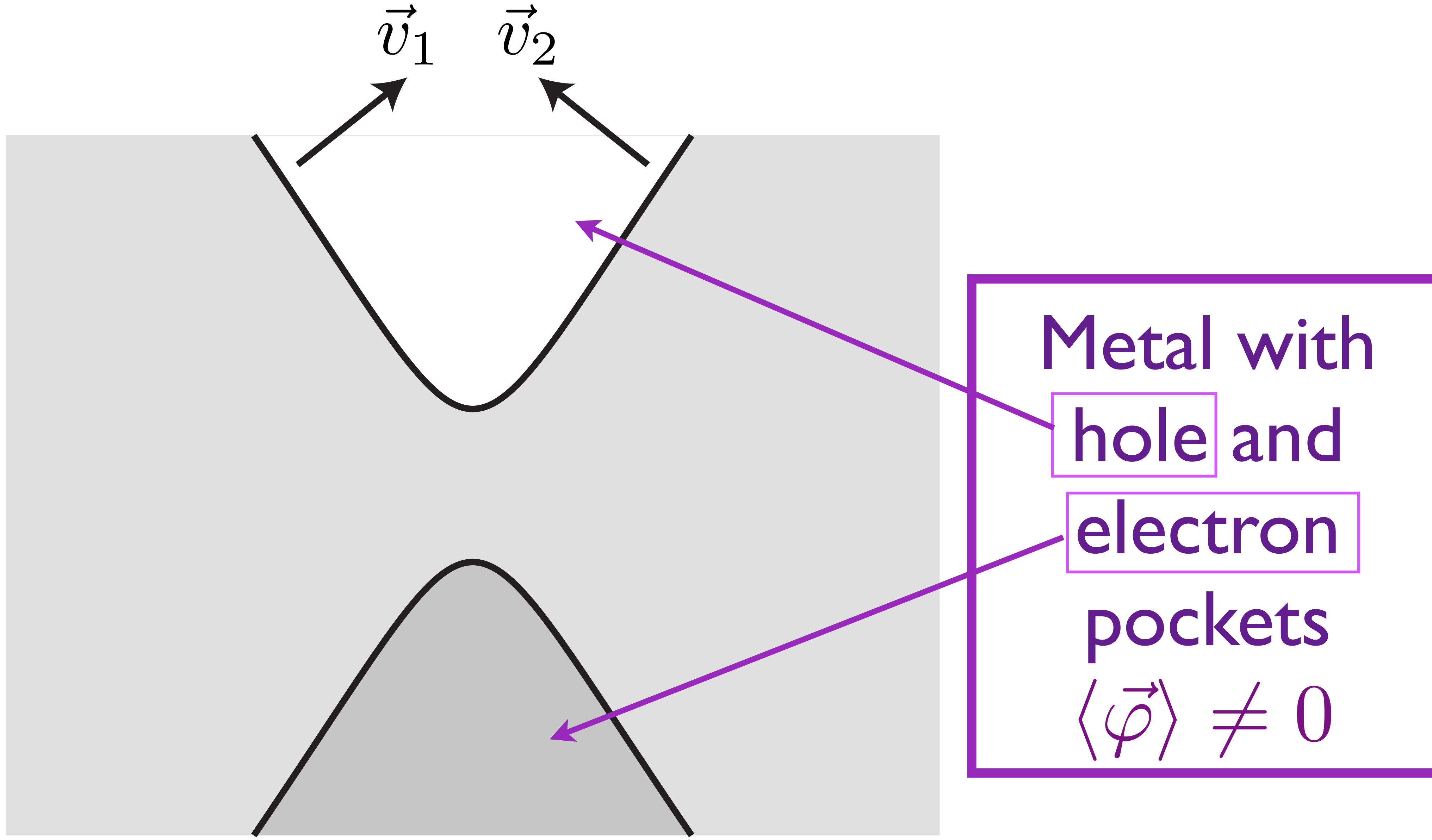


Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



Fermi lines reconnect in antiferromagnetic phase



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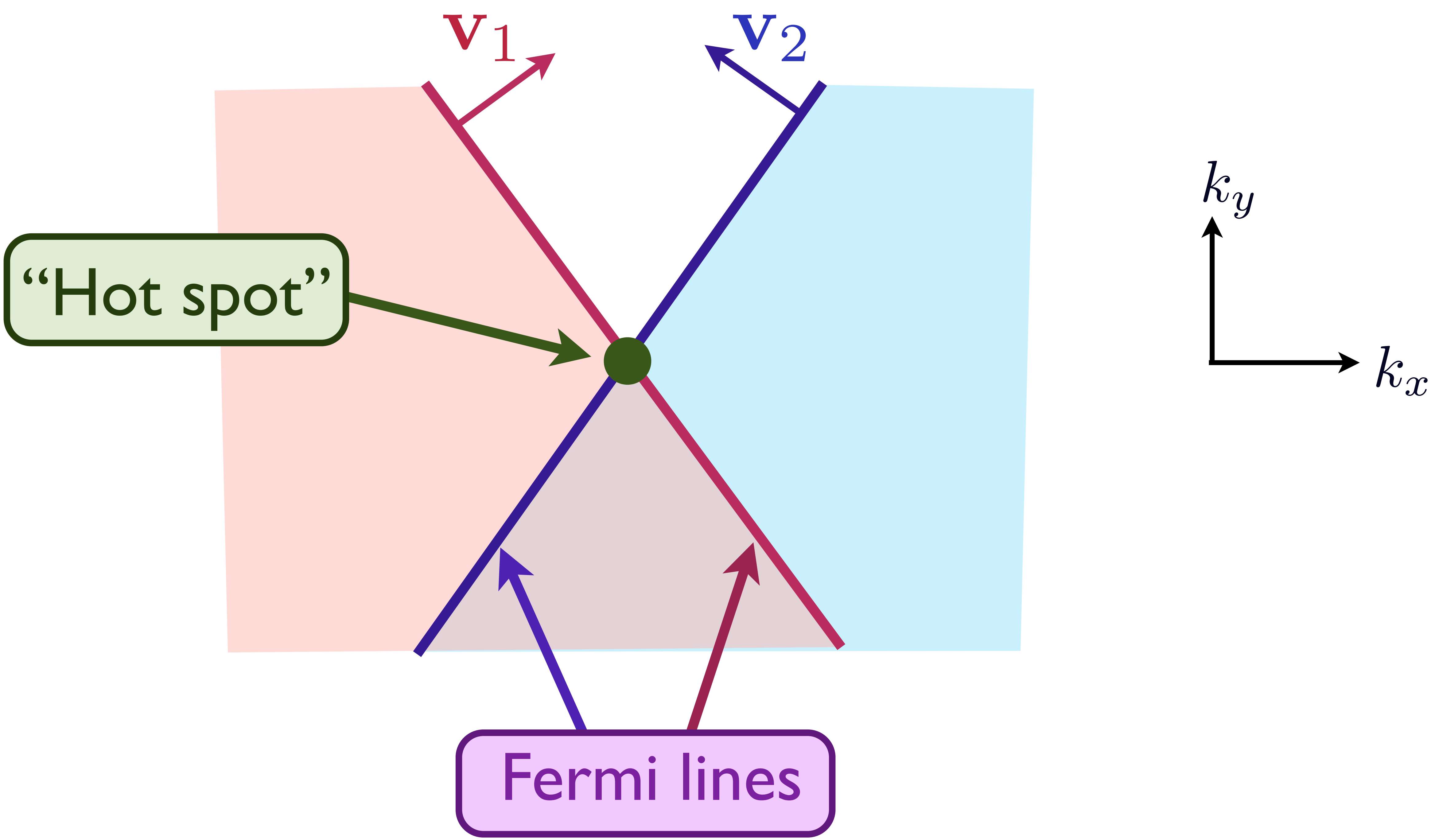
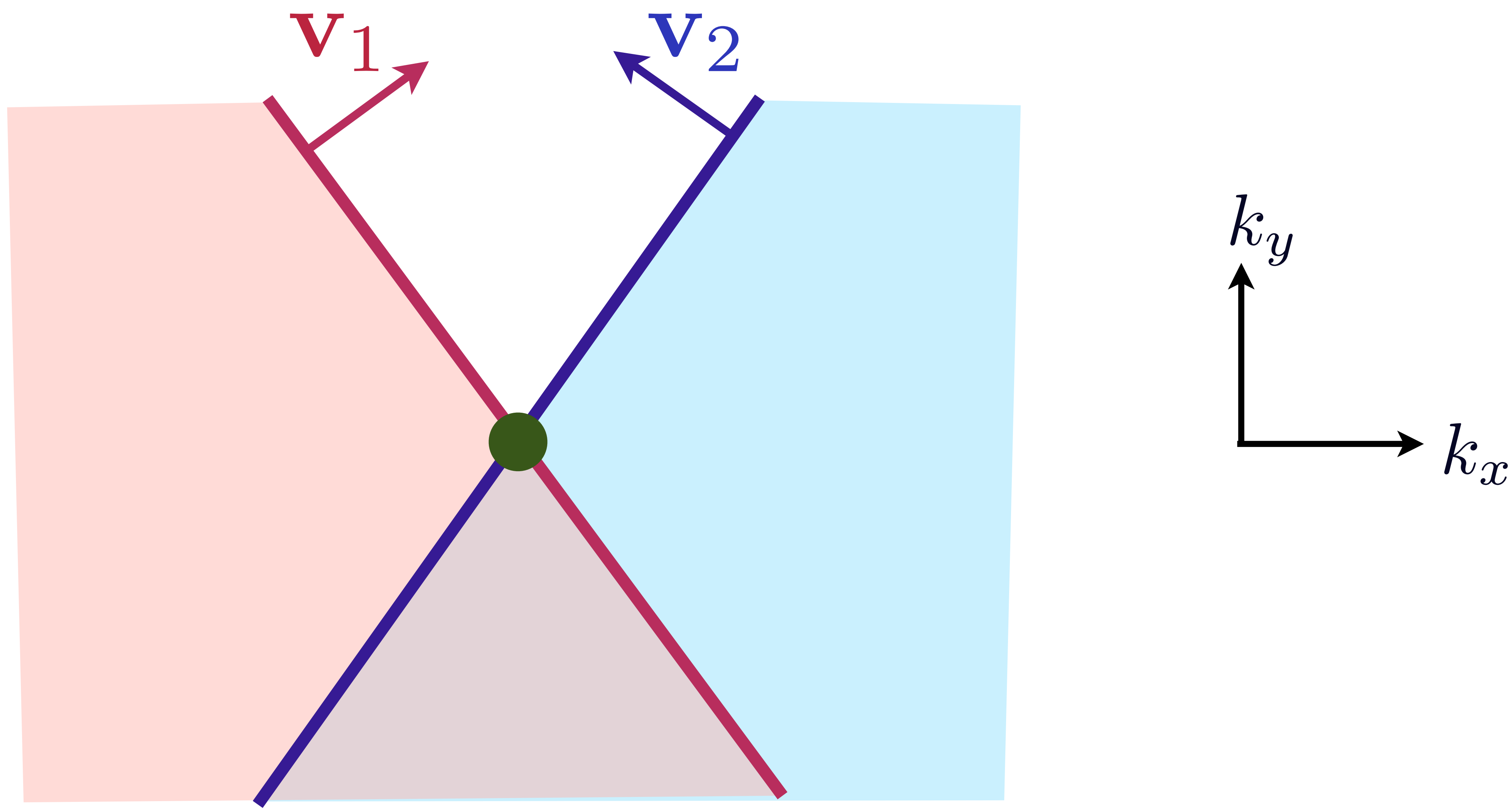


Critical point theory is strongly coupled in $d = 2$
Results are *independent* of coupling λ



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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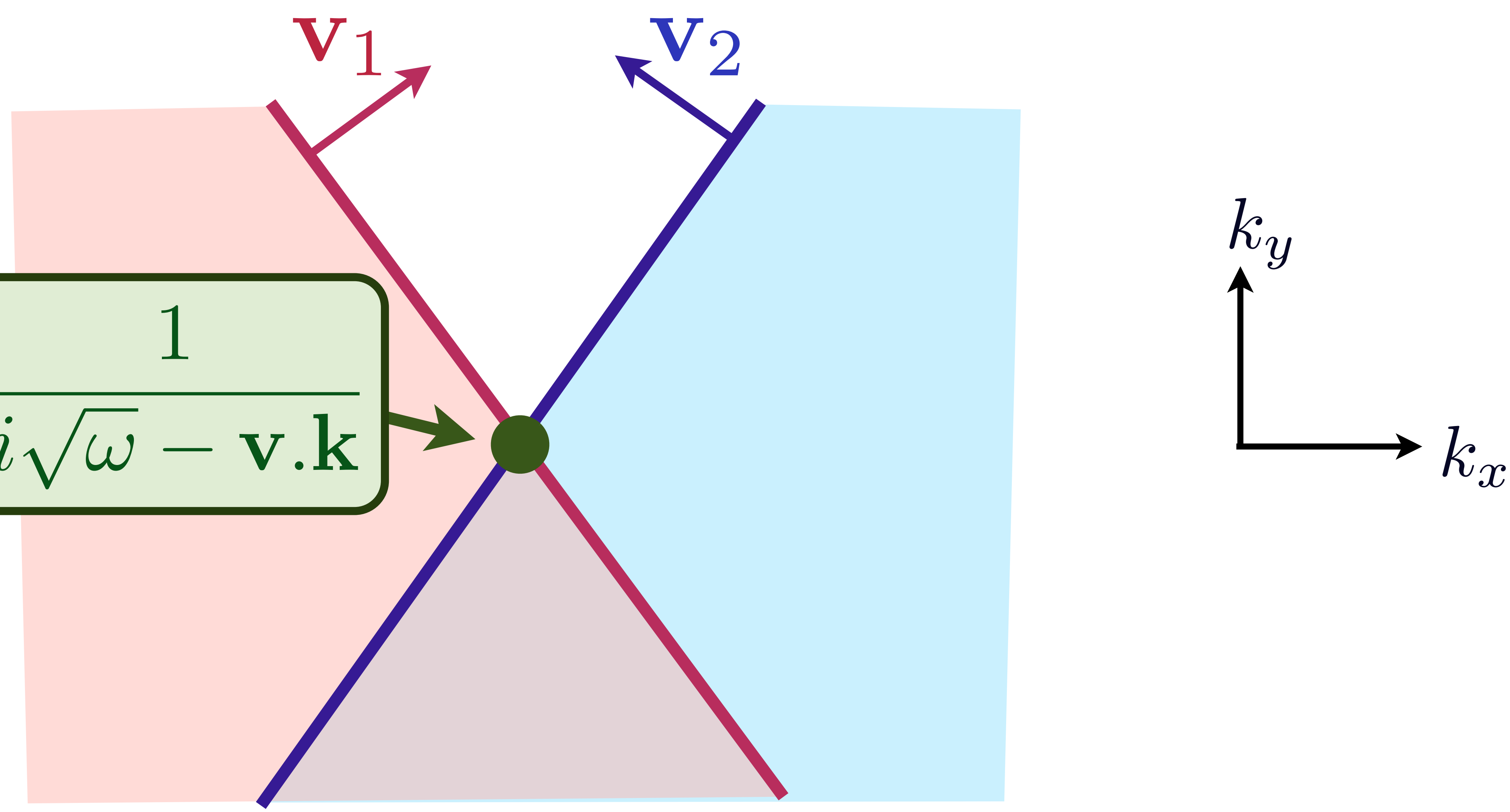

$$G_{\text{fermion}} \sim \frac{1}{i\sqrt{\omega} - \mathbf{v}.\mathbf{k}}$$

A. J. Millis, *Phys. Rev. B* **45**, 13047 (1992)
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004)

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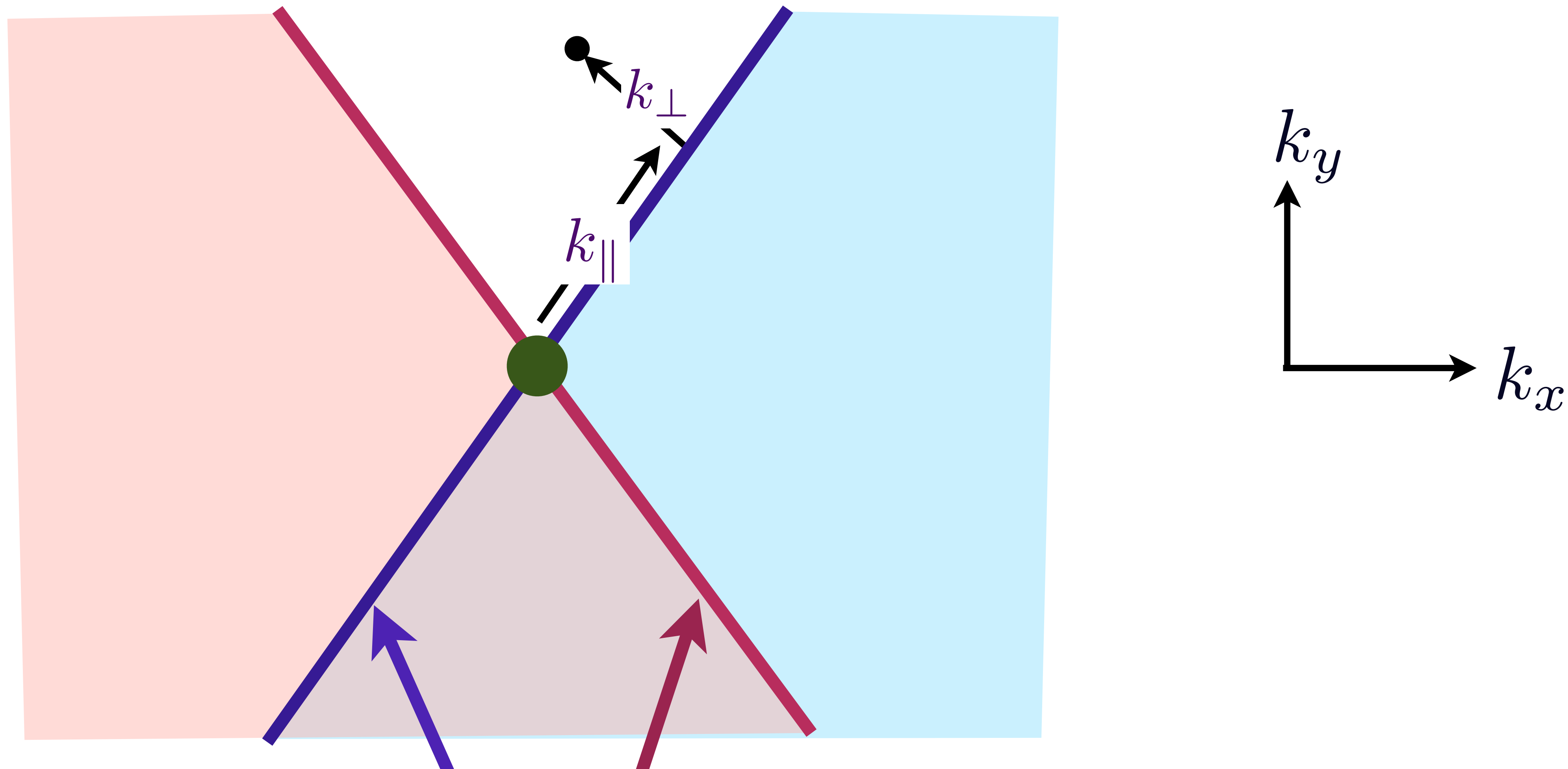

$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel)k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$

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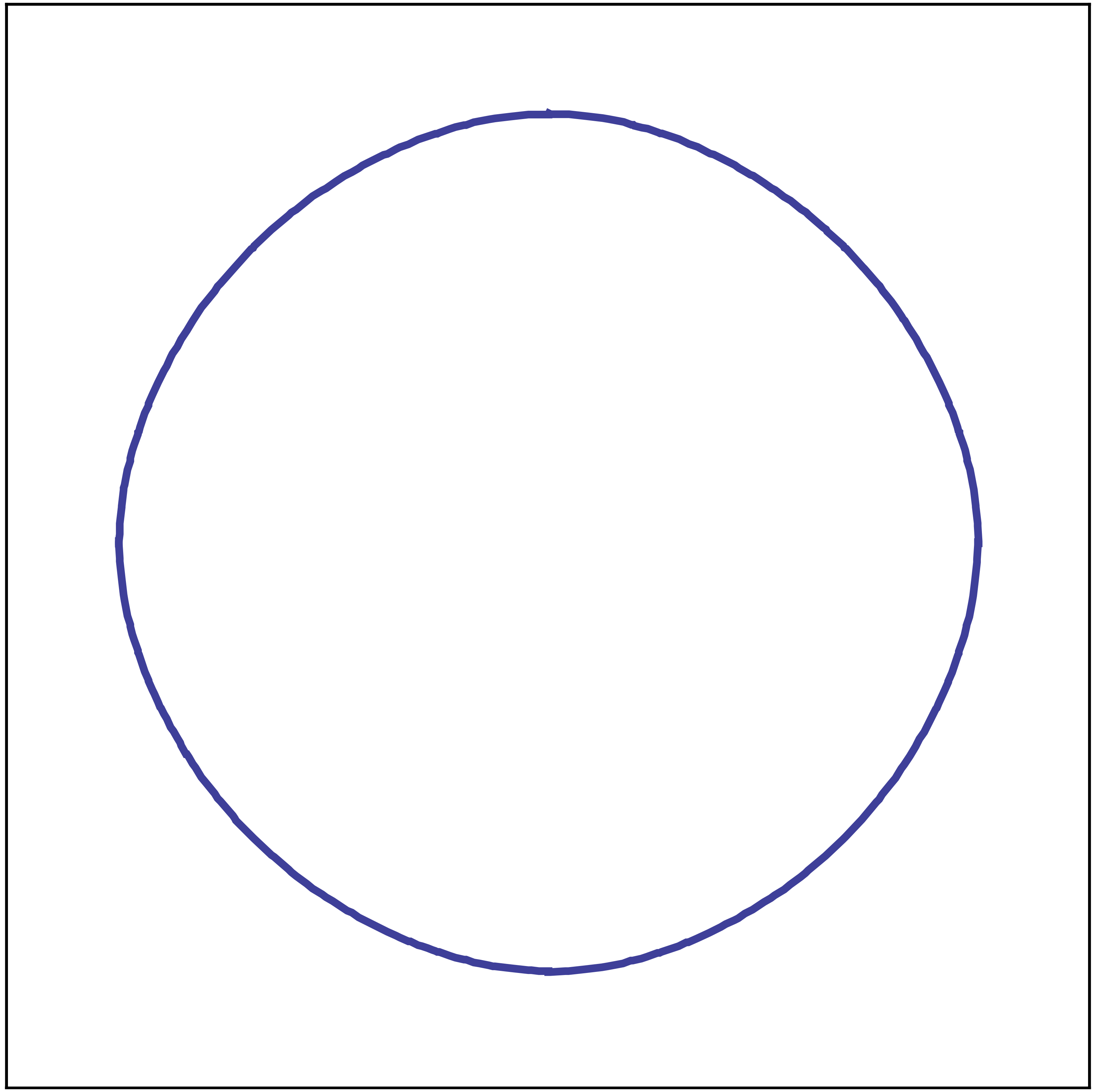

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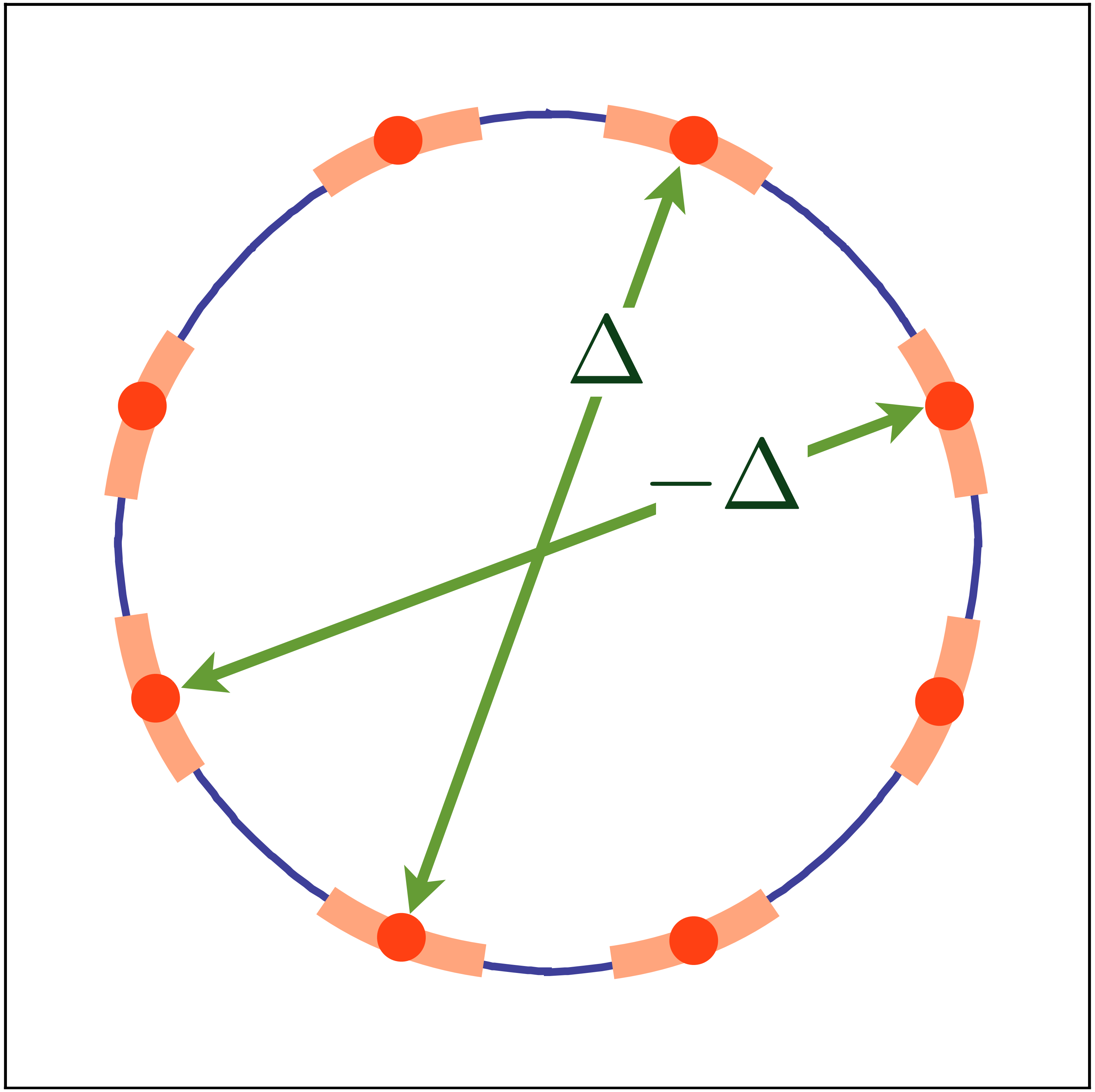
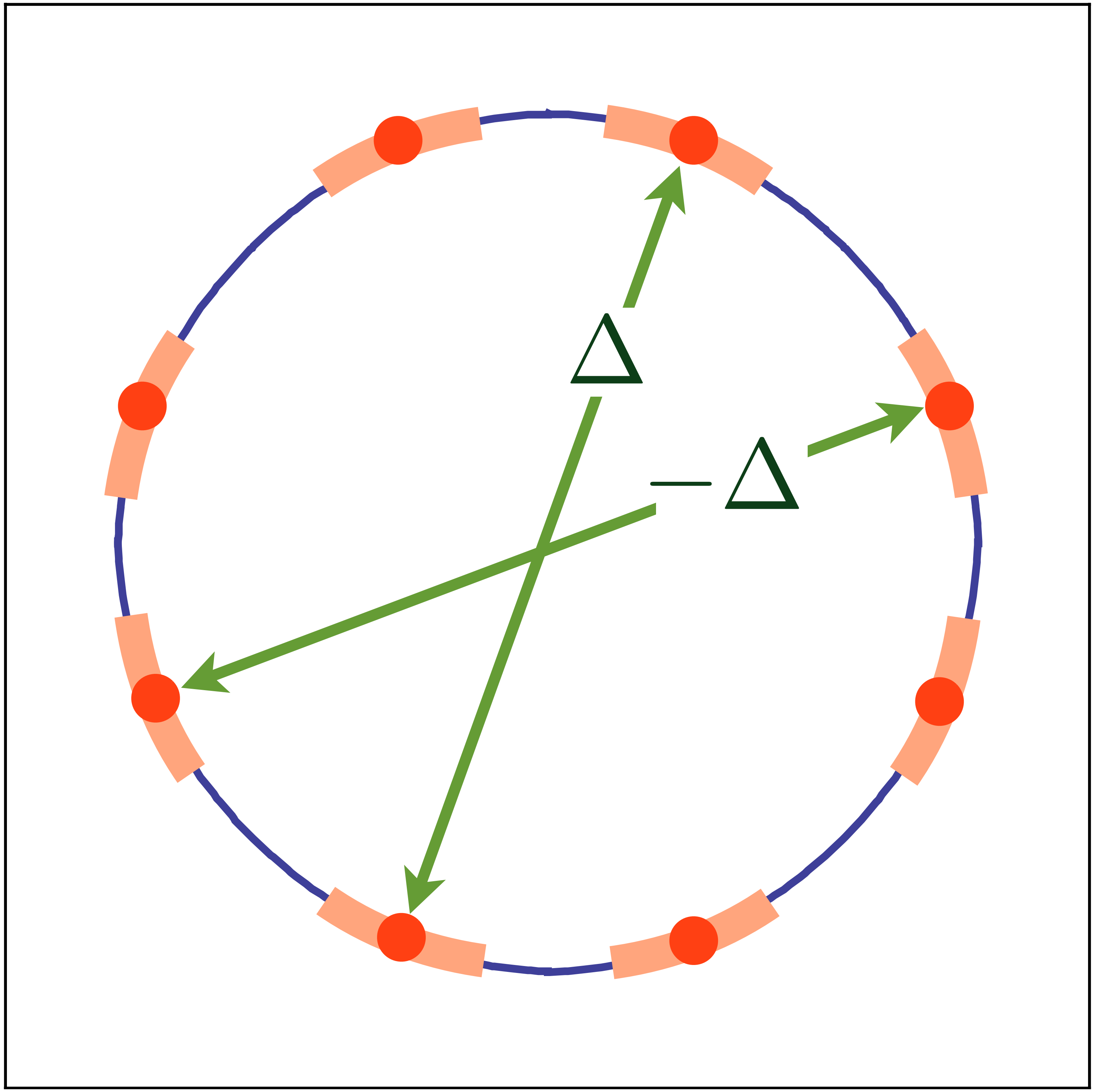


Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$

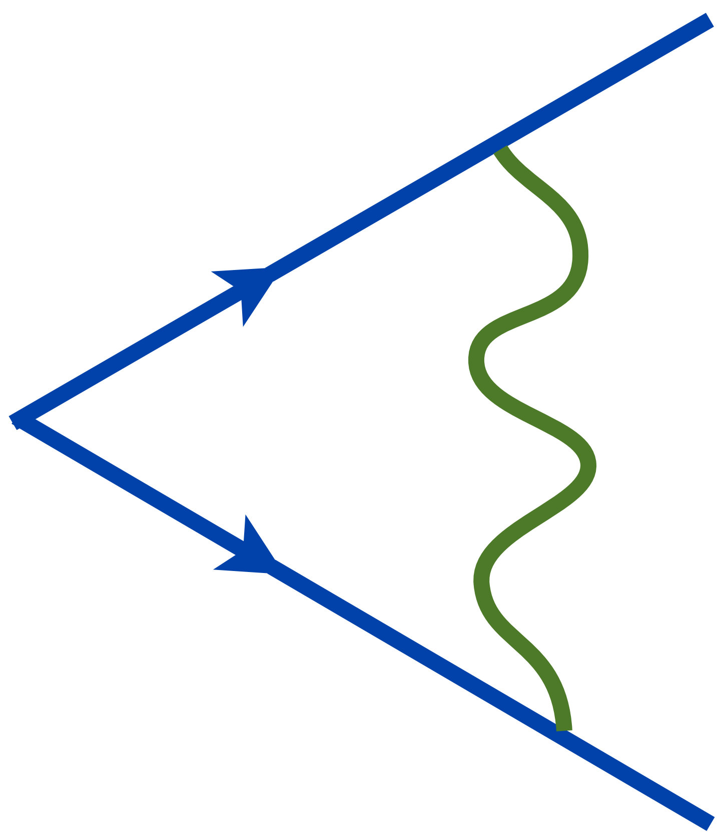


Unconventional pairing at *and near* hot spots

BCS theory

$$1 + \lambda_{\text{e-ph}} \log \left(\frac{\omega_D}{\omega} \right)$$

Cooper logarithm



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Electron-phonon coupling

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Electron-phonon coupling

Debye frequency

BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Electron-phonon coupling

Debye frequency

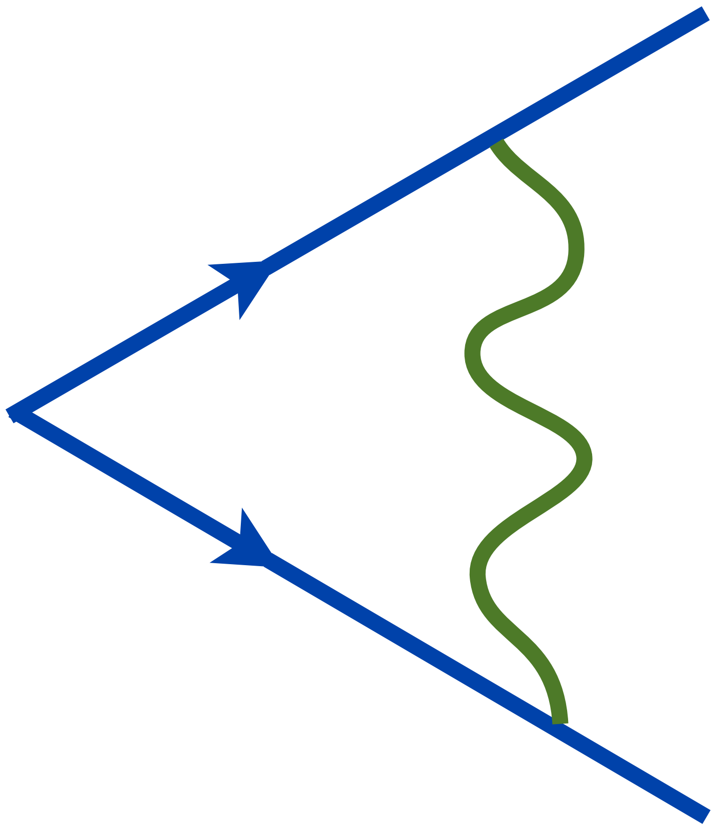
Implies

$$T_c \sim \omega_D \exp\left(-1/\lambda\right)$$

Enhancement of pairing susceptibility by interactions

Antiferromagnetic fluctuations: weak-coupling

$$1 + \left(\frac{U}{t}\right)^2 \underbrace{\log\left(\frac{E_F}{\omega}\right)}_{\text{Cooper logarithm}}$$



V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
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K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

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Fermi energy

Implies $T_c \sim E_F \exp\left(-\left(t/U\right)^2\right)$

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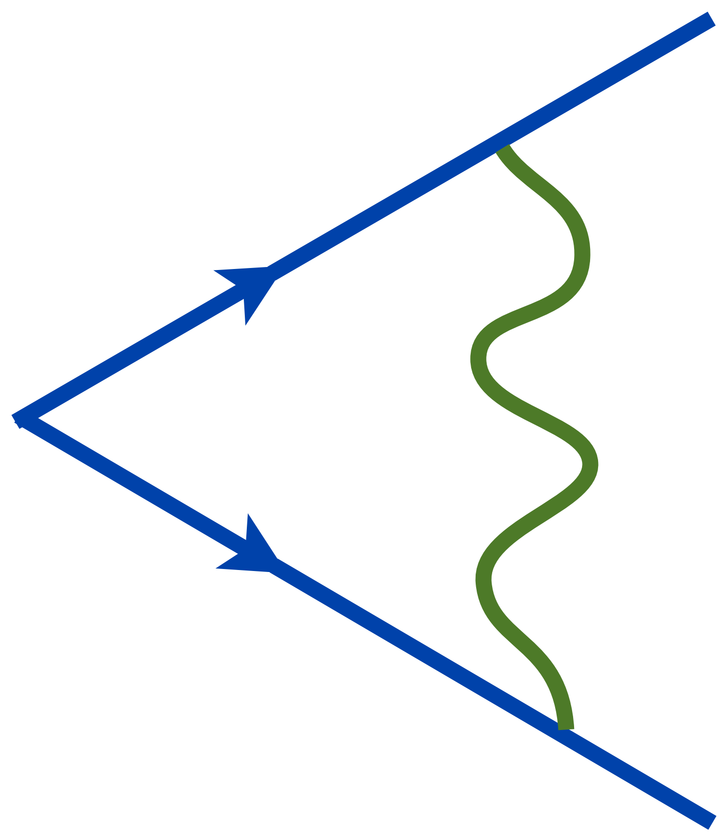
Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$



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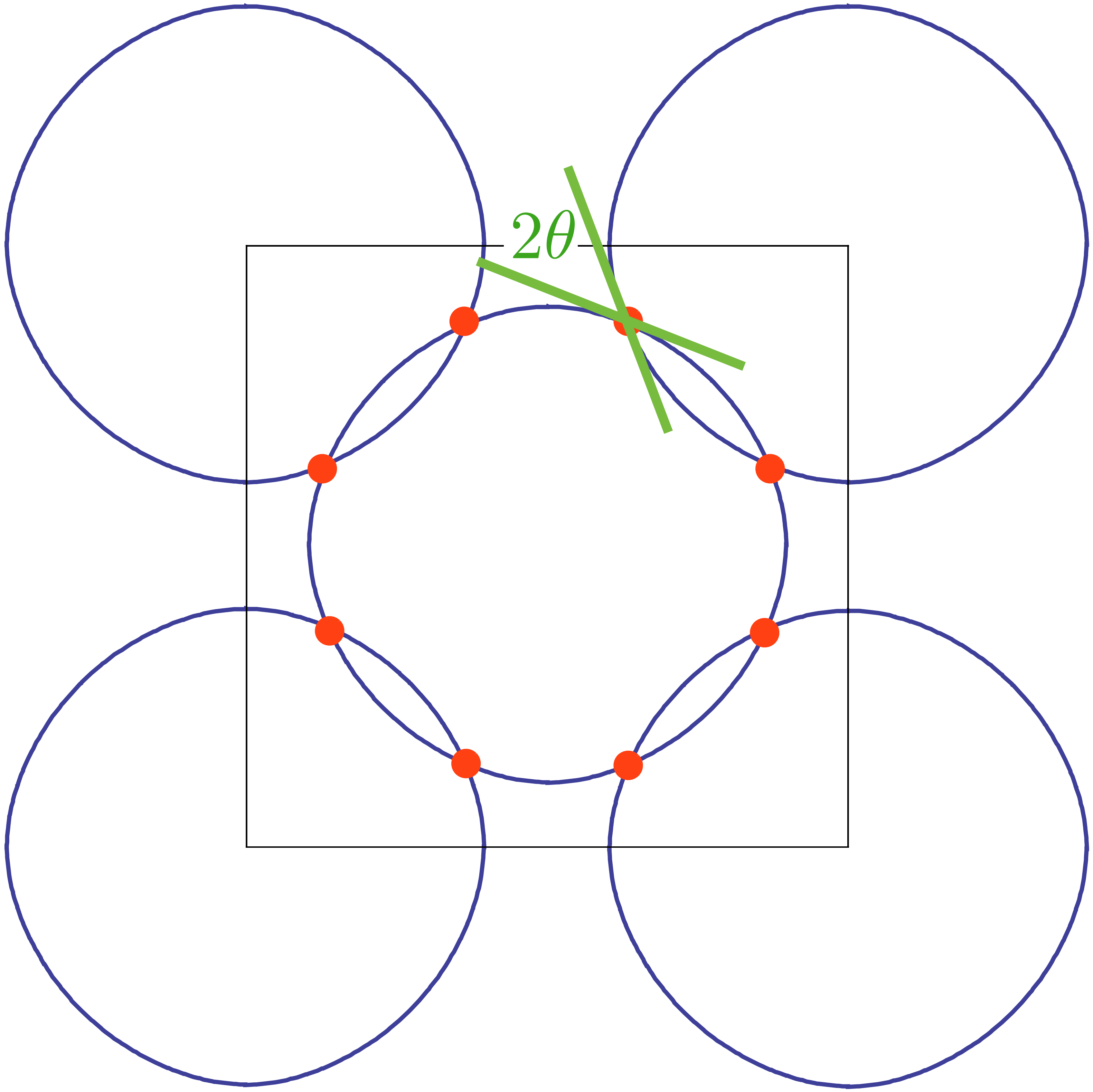
Fermi energy

$\alpha = \tan\theta$, where 2θ is the angle between Fermi lines. *Independent* of interaction strength U in 2 dimensions.

(see also Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)


2θ

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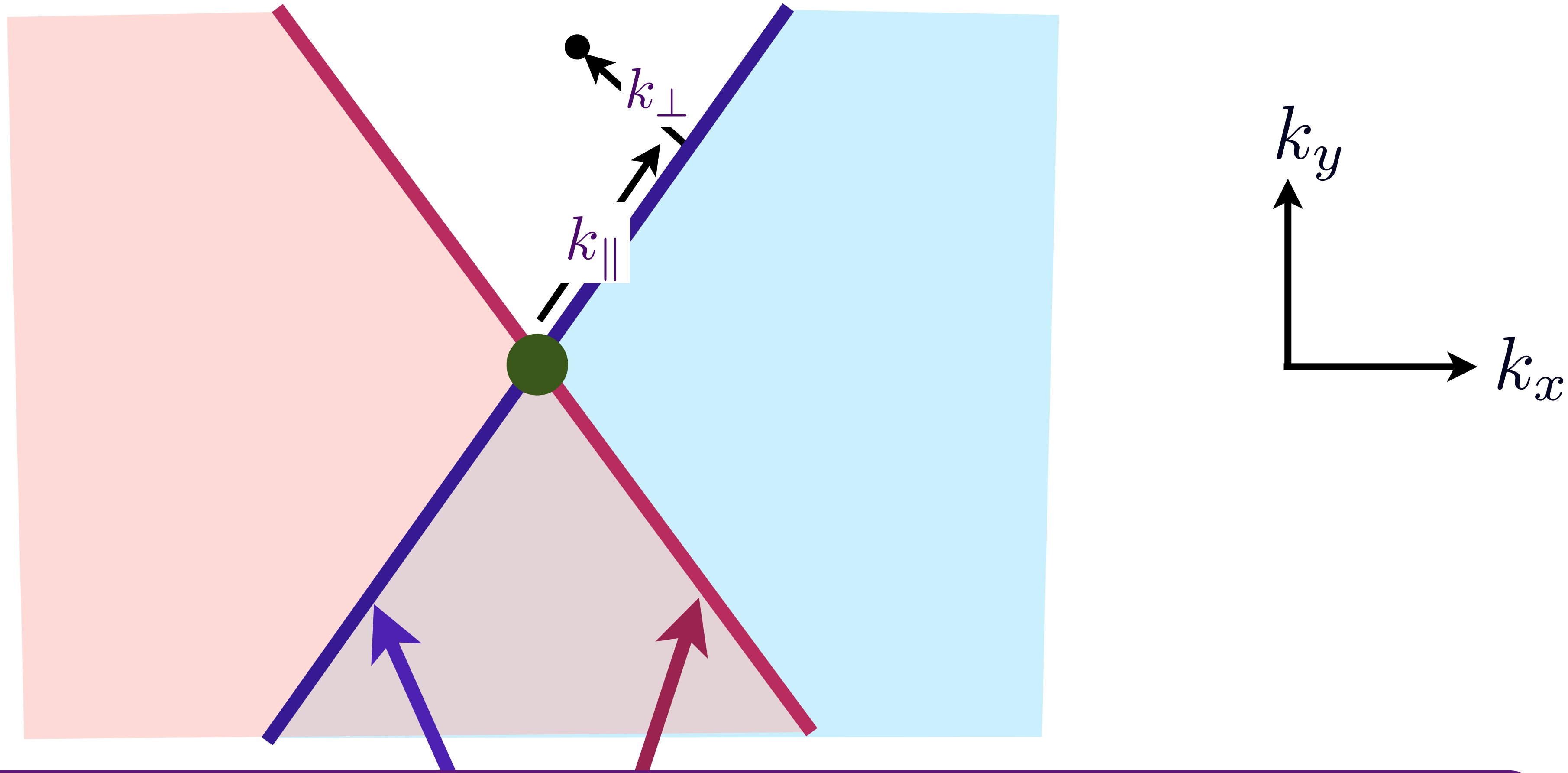

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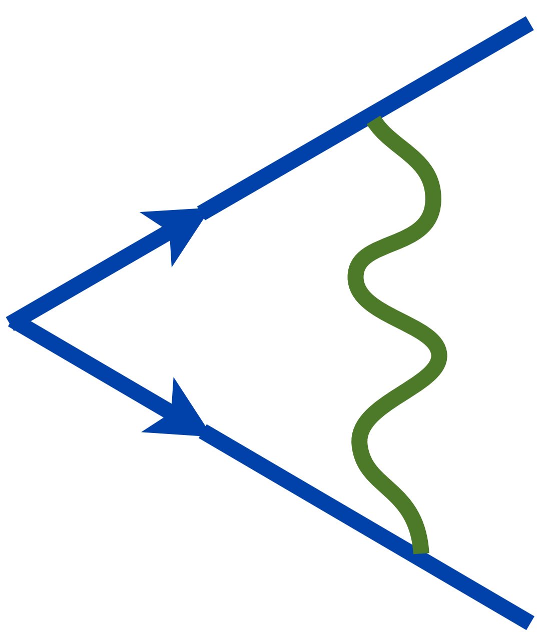

$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

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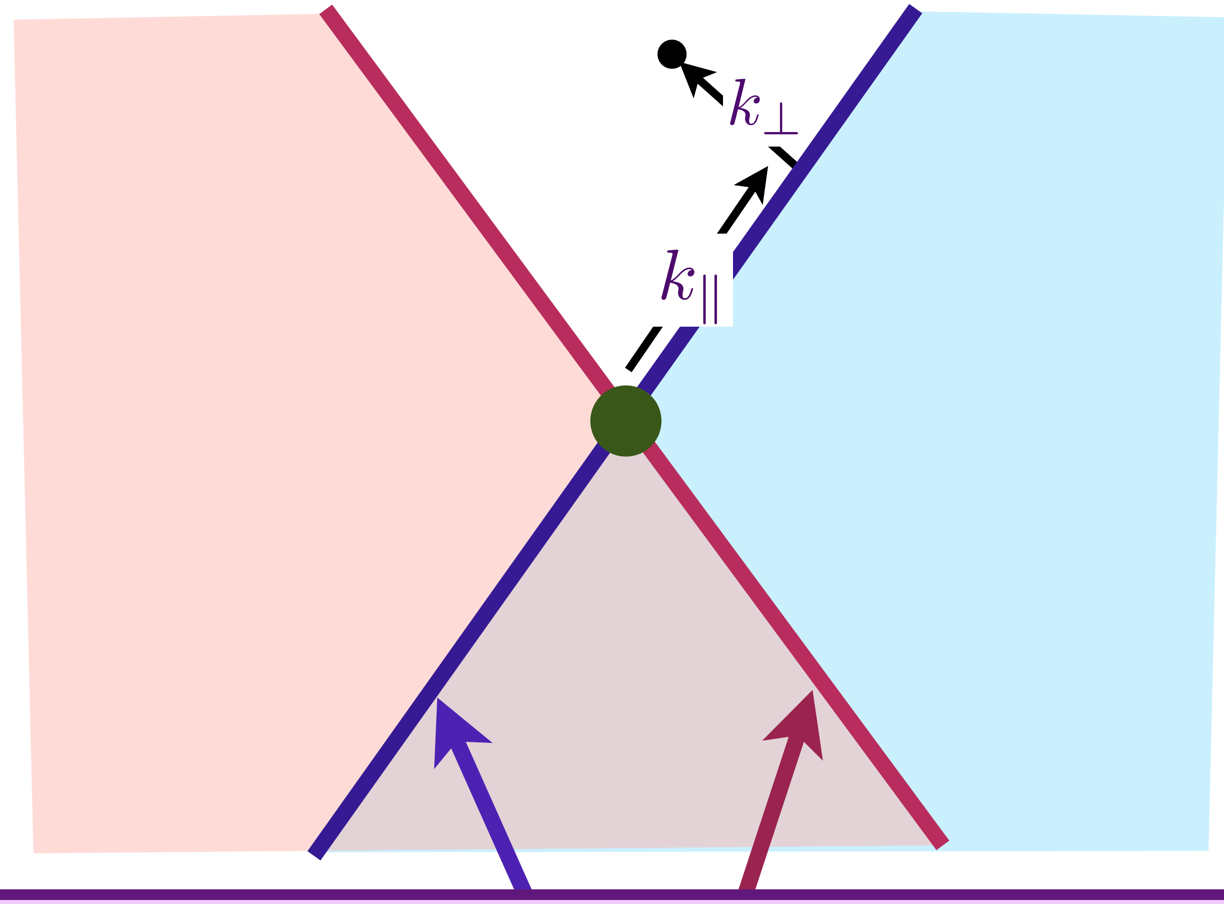

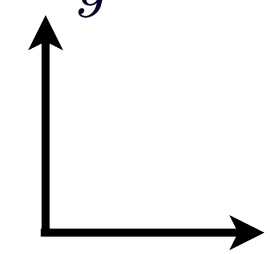

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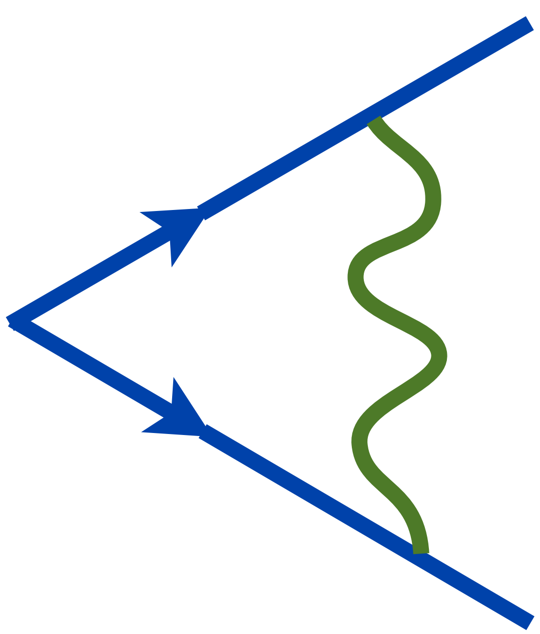

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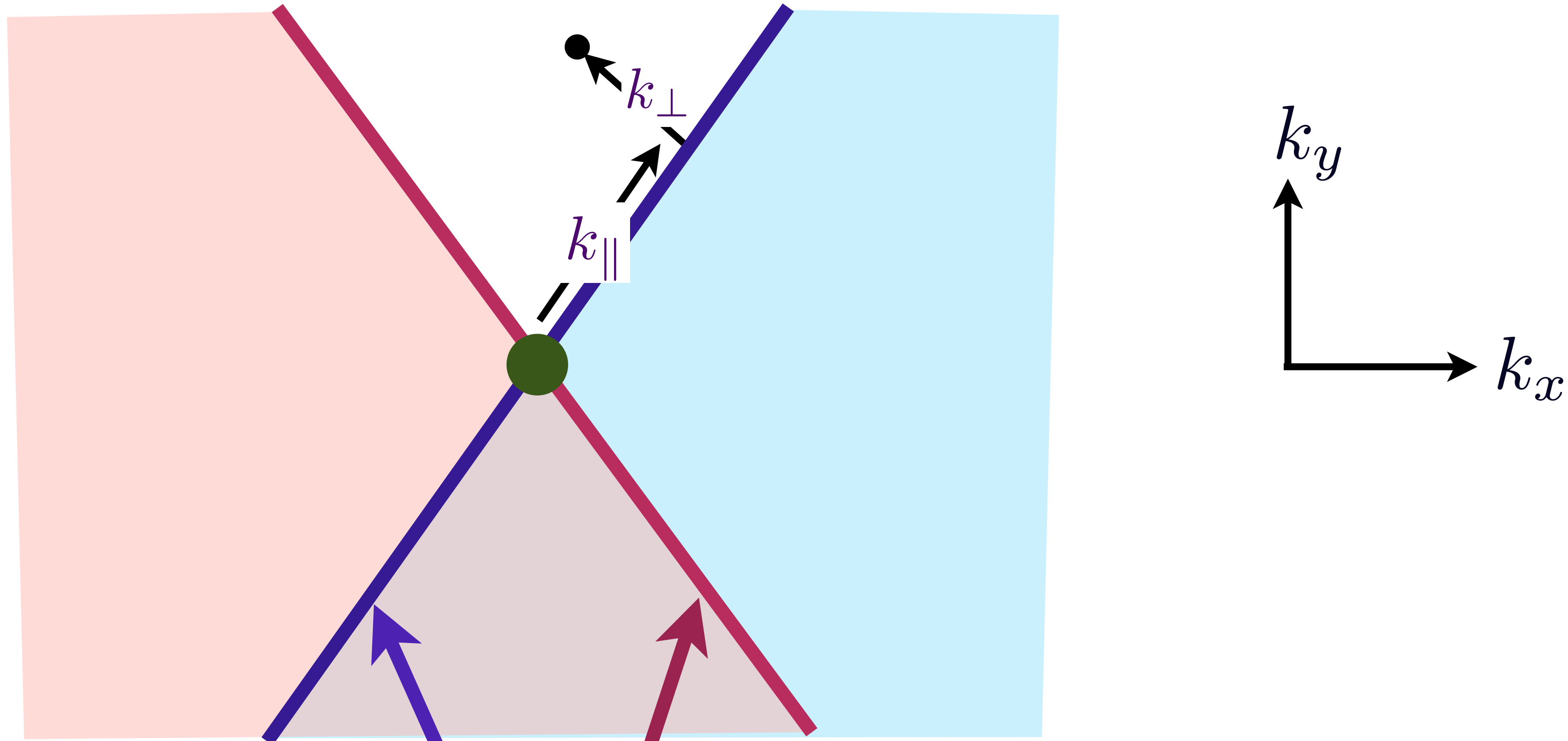
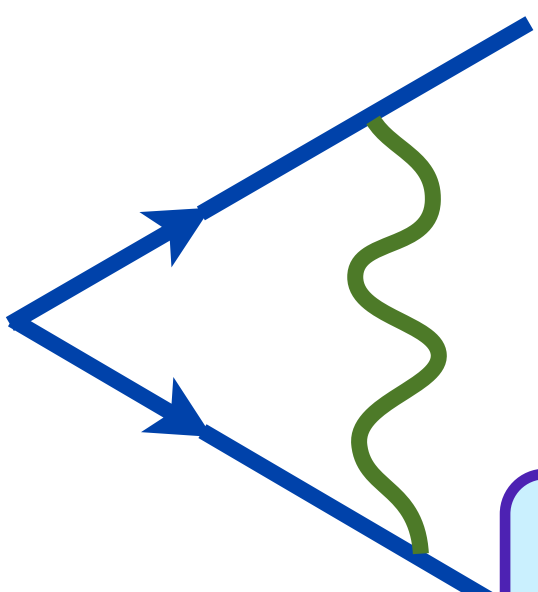
Cooper logarithm

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel)k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$



$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

Spin fluctuation propagator

Cooper logarithm

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

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M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.
- Not suppressed by $1/N$ factor in $1/N$ expansion.

Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Is there a log^2 for any other susceptibility ?

Is there a *log*^{2} for any other susceptibility ?

Only one other

Outline

1. Low energy theory of spin density wave quantum critical point

2. Instabilities near the quantum critical point:
   A. Unconventional superconductivity
   B. $2k_F$ bond-nematic ordering

3. Electron spectral function and optical conductivity at quantum critical point
   Scattering off composite operators

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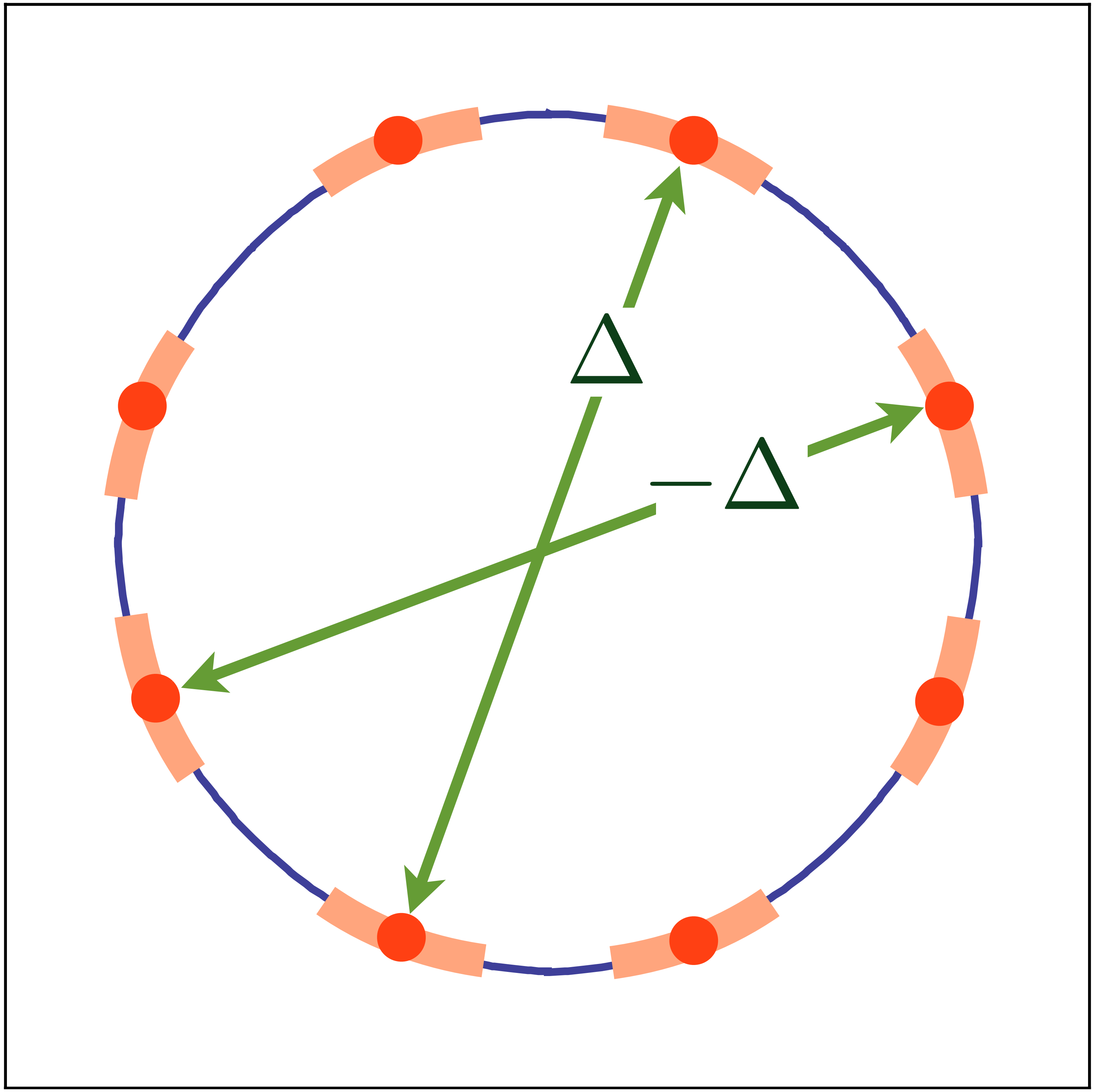
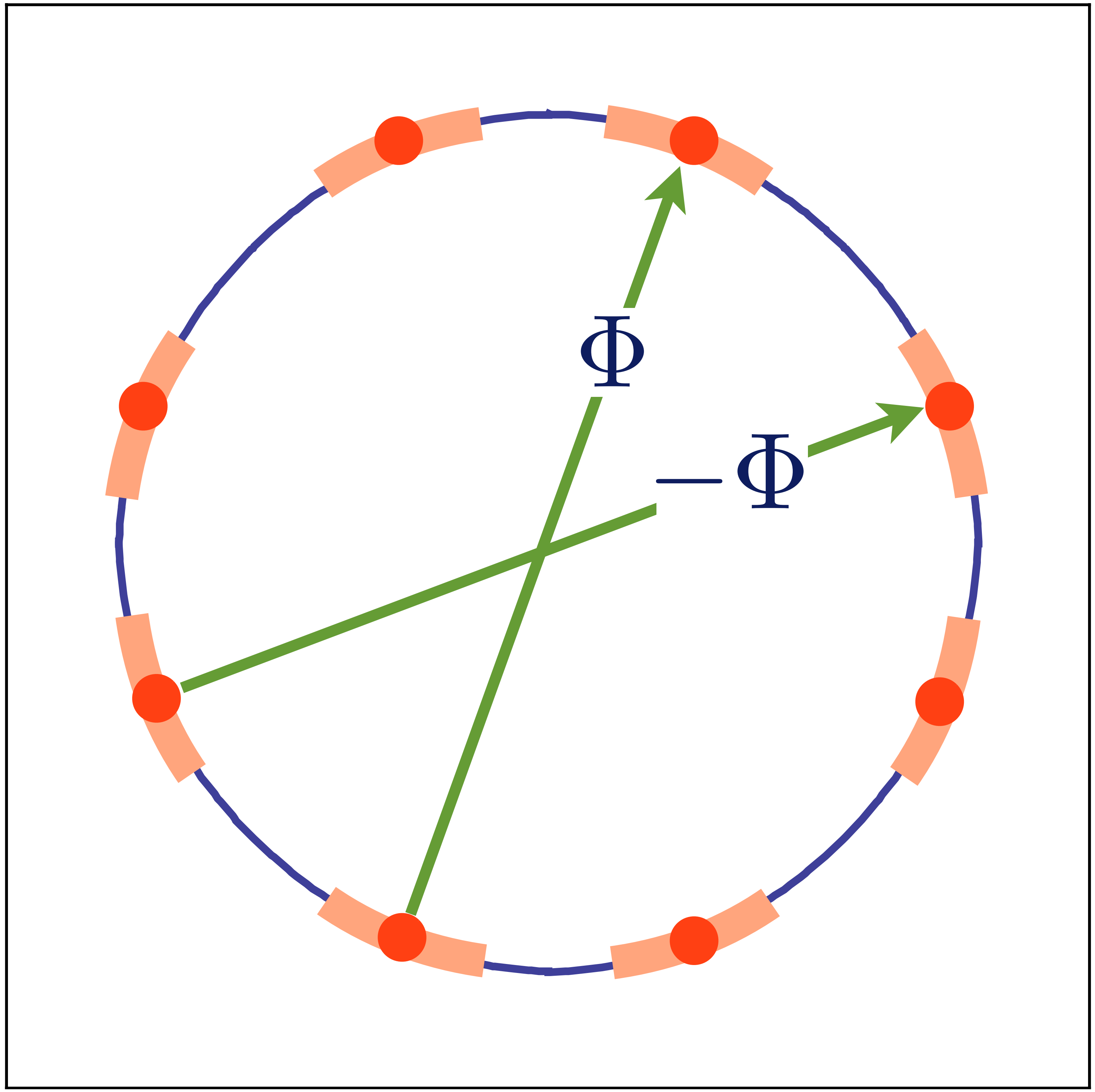
$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$



$\mathbf{Q}$ is '$2k_F$' wavevector

Unconventional particle-hole pairing at *and near* hot spots

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of Φ susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{3\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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$$1 + \frac{\alpha}{3\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

- Emergent pseudospin symmetry of low energy theory also induces $\log^2$ in a single "d-wave" particle-hole channel. Fermi-surface curvature reduces prefactor by 1/3.

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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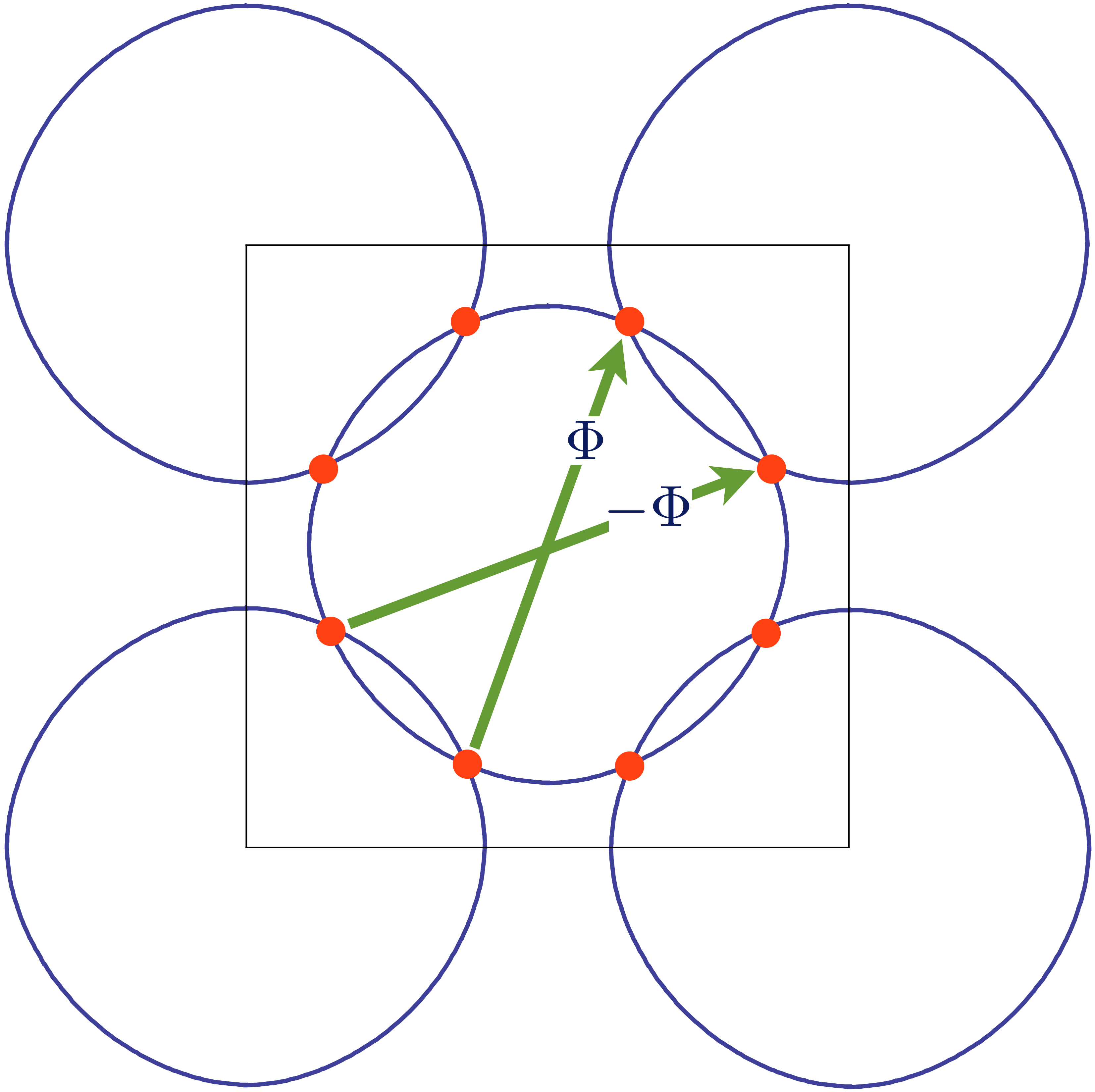
- Φ corresponds to a <u>$2k_F$ *bond-nematic order*</u>

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$2k_F$ bond-nematic order



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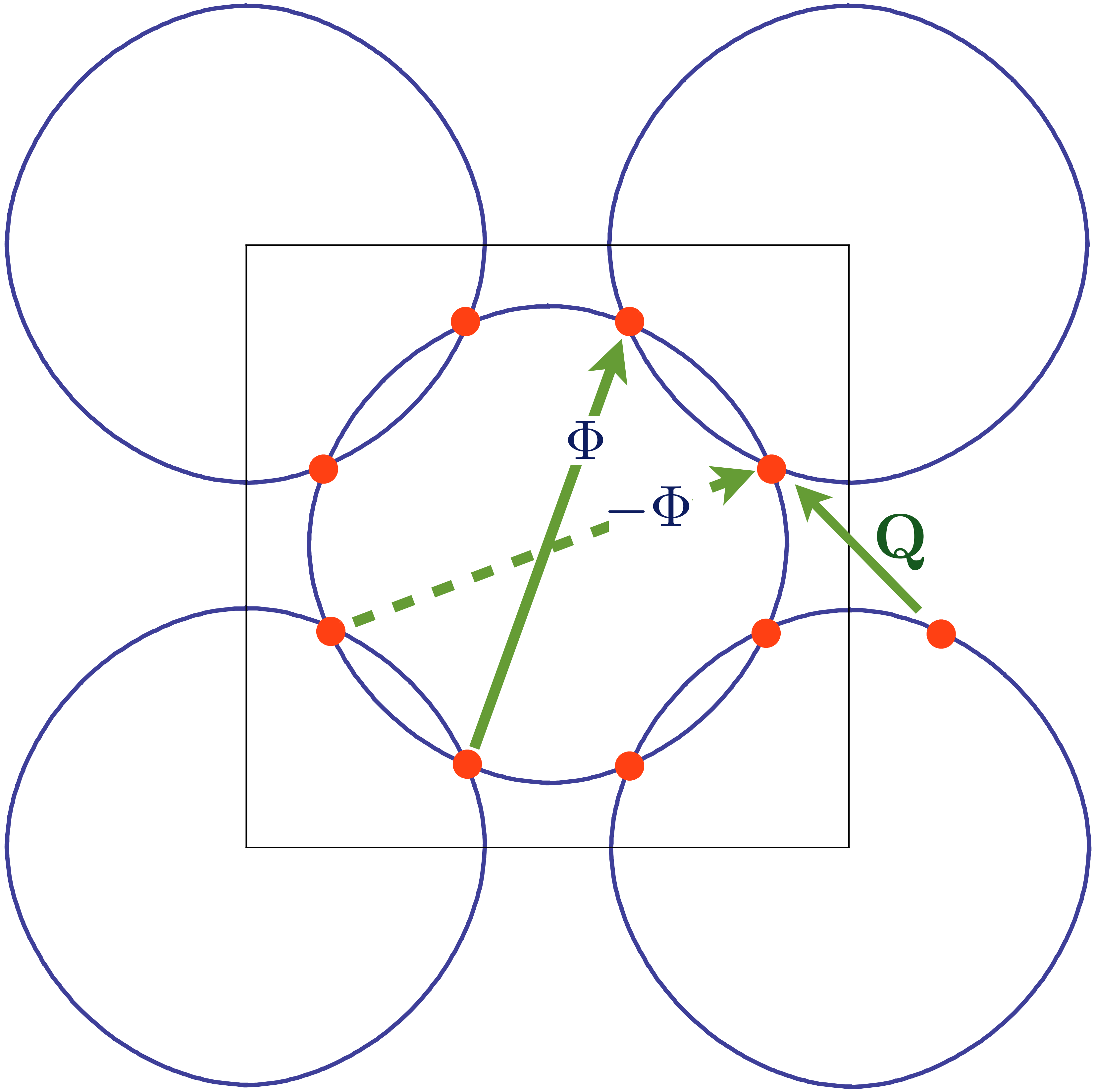
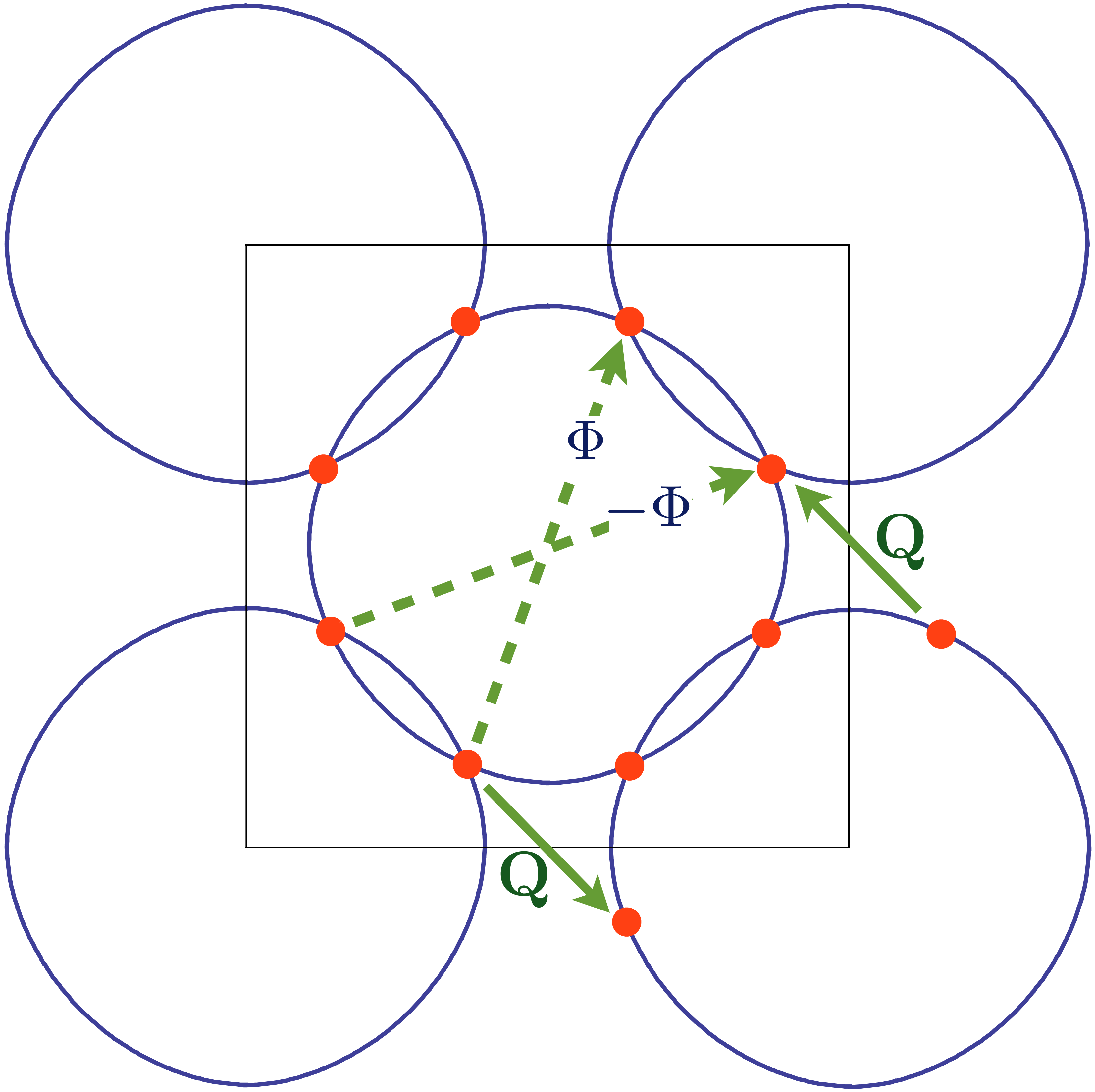


$2k_F$ bond-nematic order



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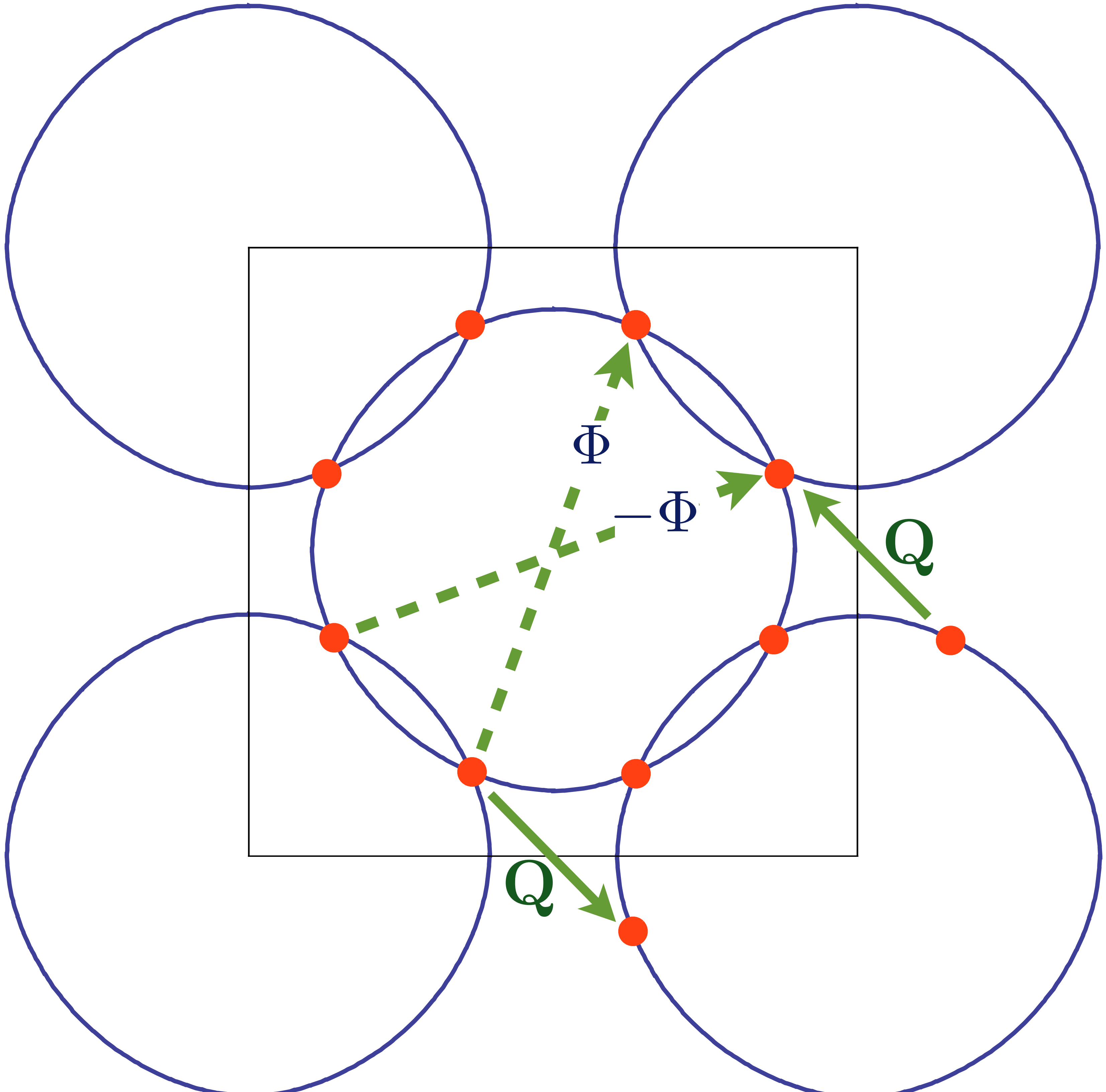
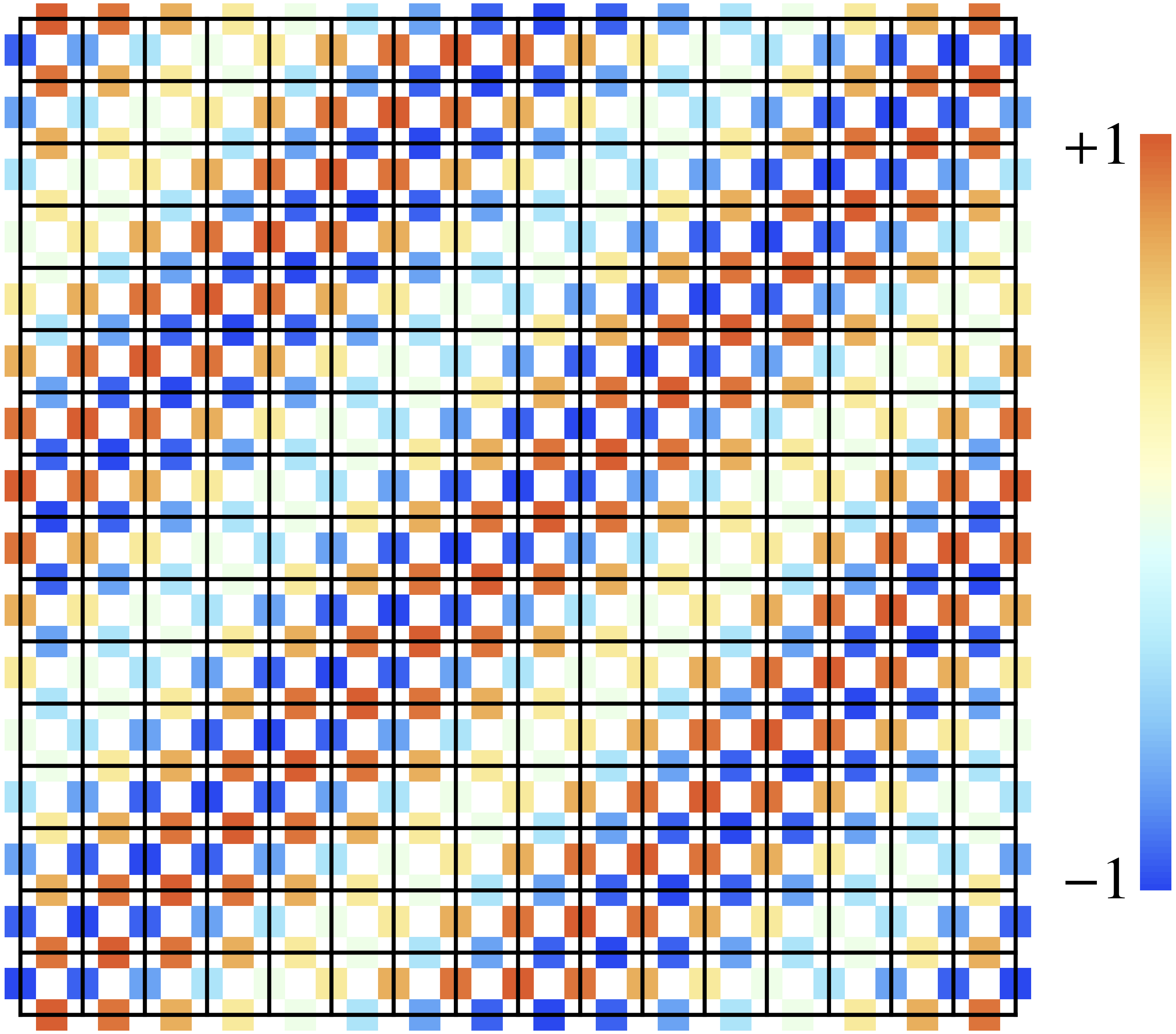


$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

$2k_F$ bond-nematic order



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

No modulations on sites, $\langle c^\dagger_{\mathbf{r}\alpha} c_{\mathbf{s}\alpha} \rangle$ is modulated only for $\mathbf{r} \neq \mathbf{s}$.

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

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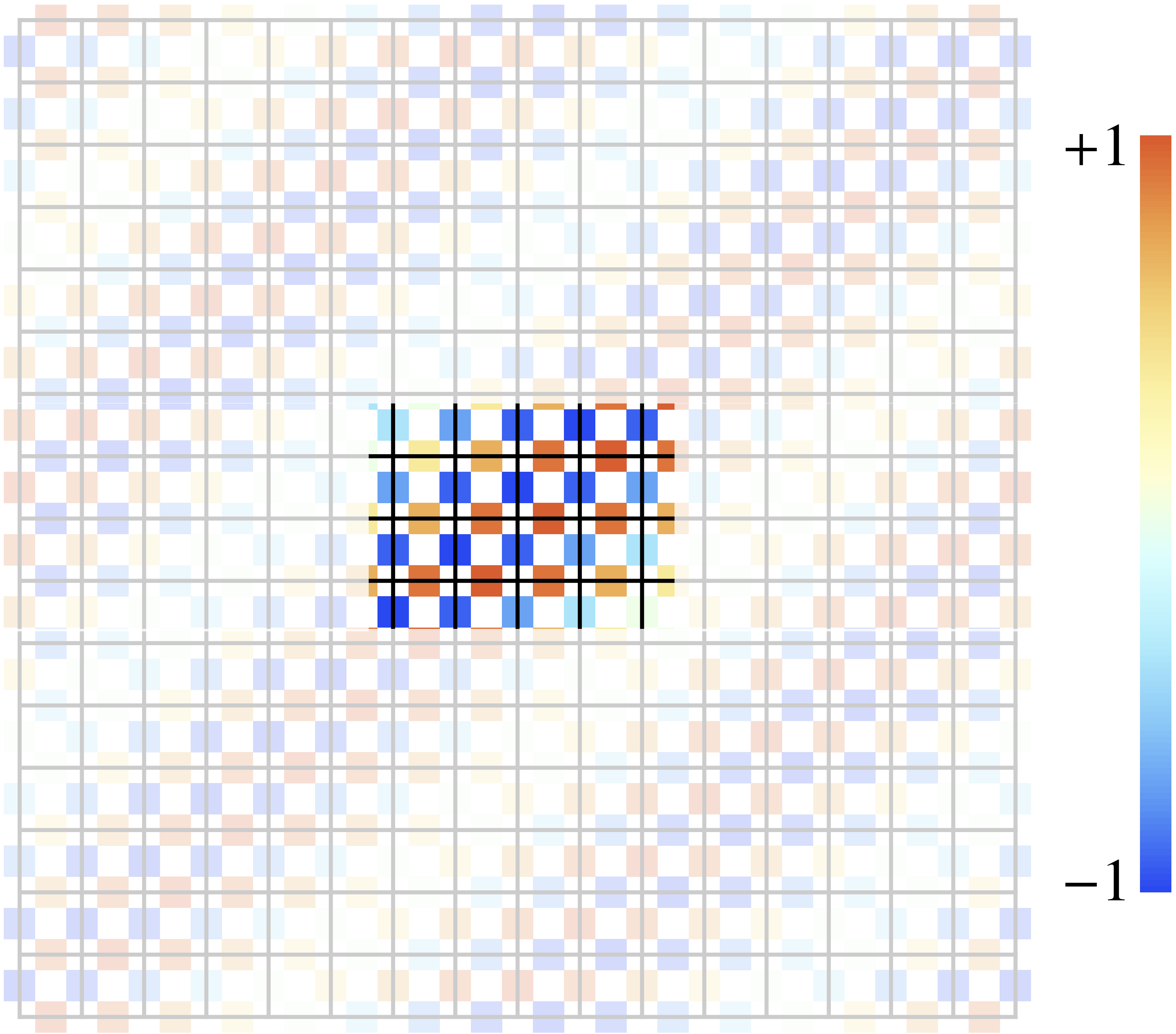

Local Ising nematic order with an envelope which oscillates

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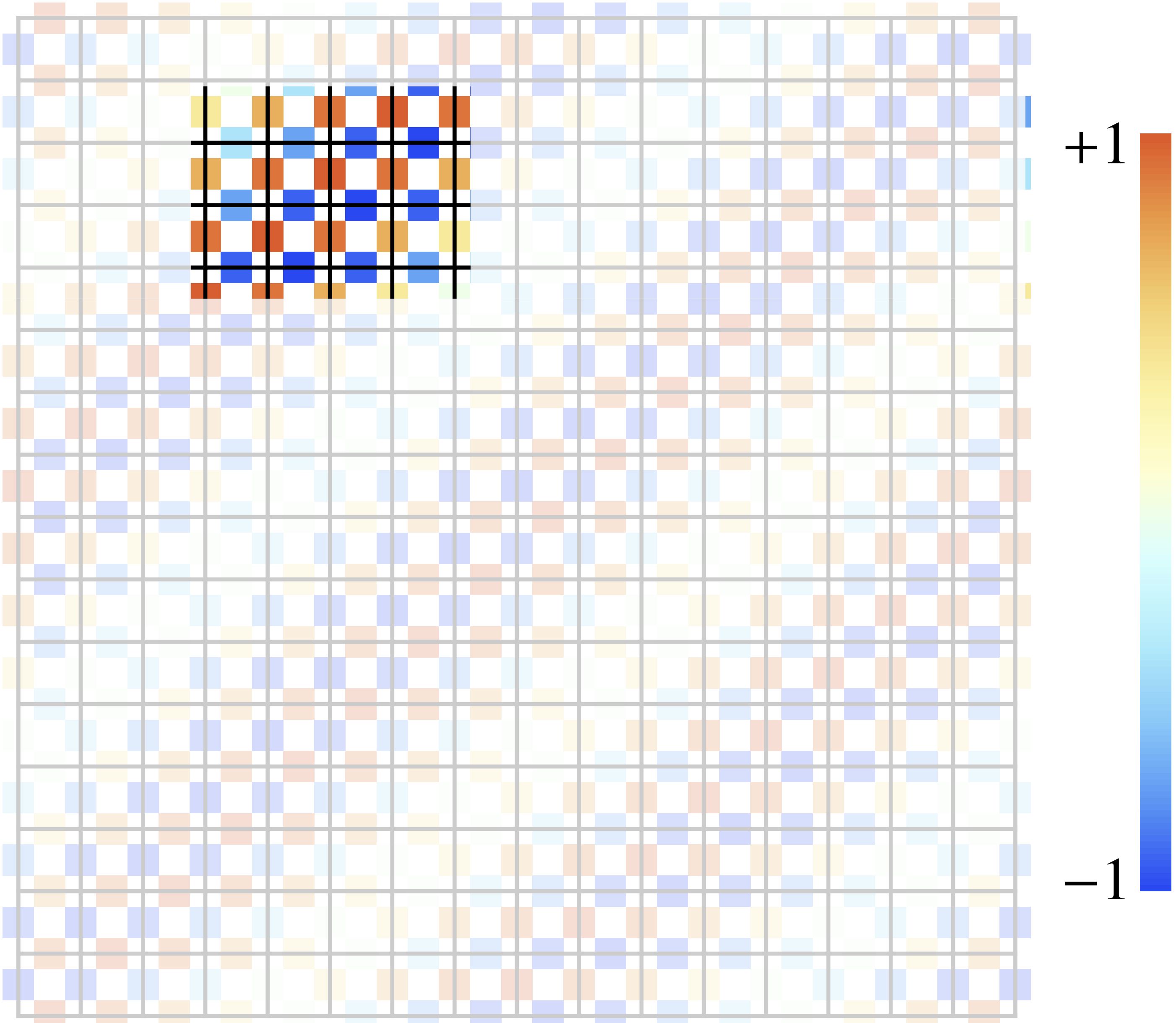

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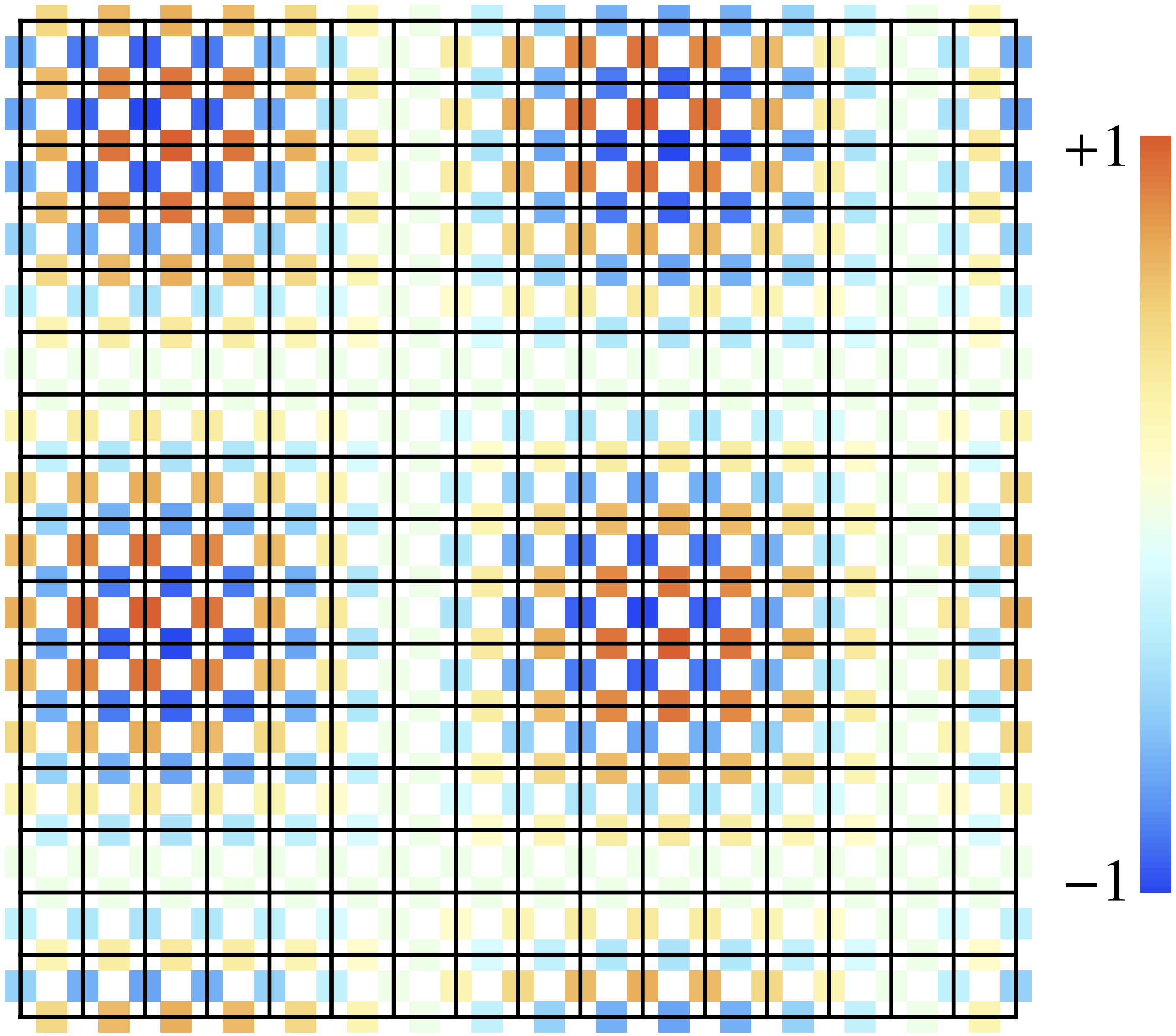

"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

No modulations on sites, $\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \rangle$ is modulated only for $\mathbf{r} \neq \mathbf{s}$.

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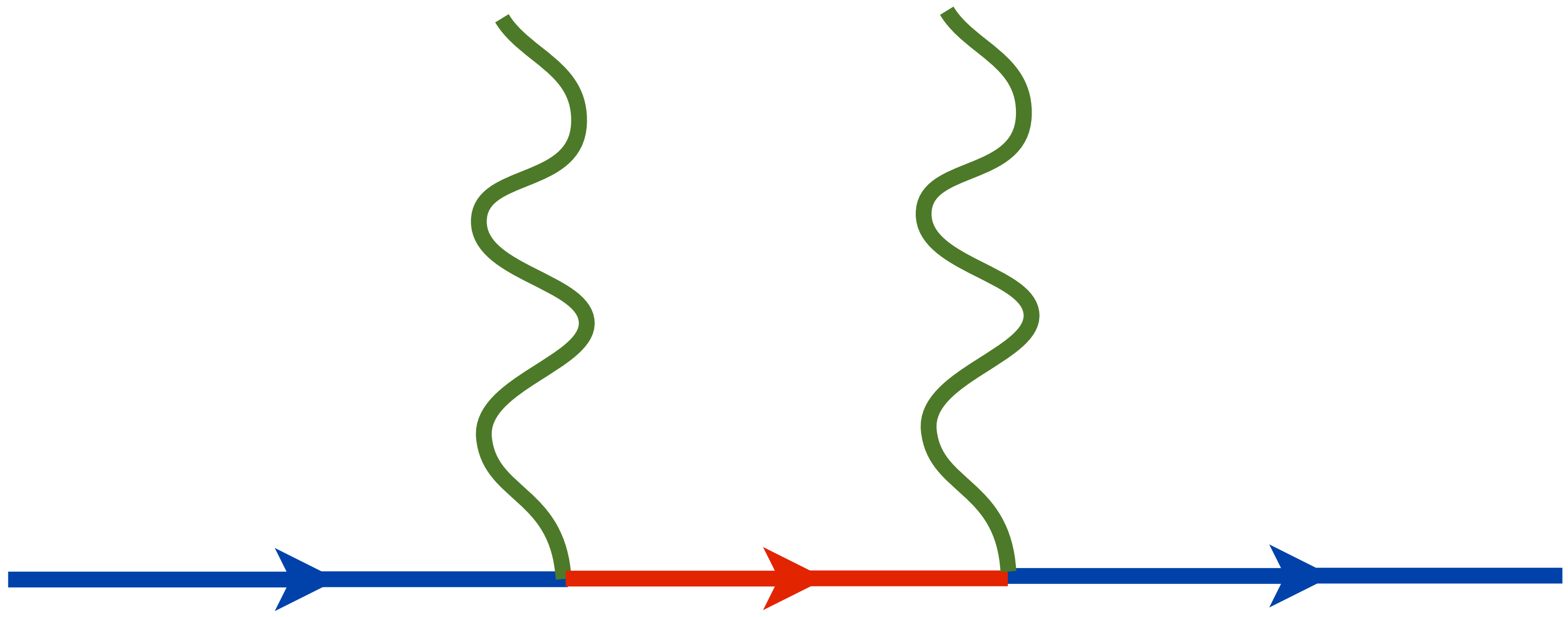
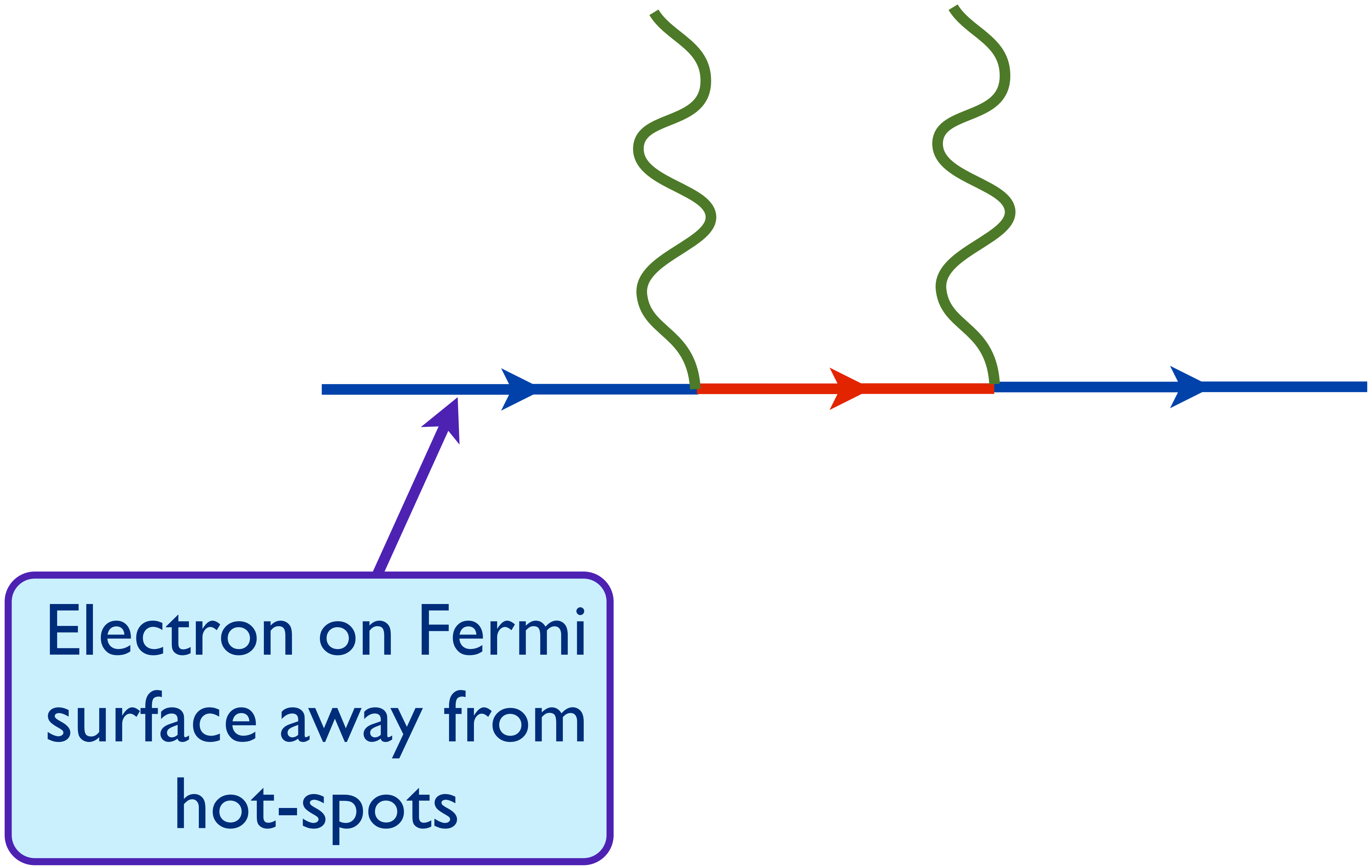
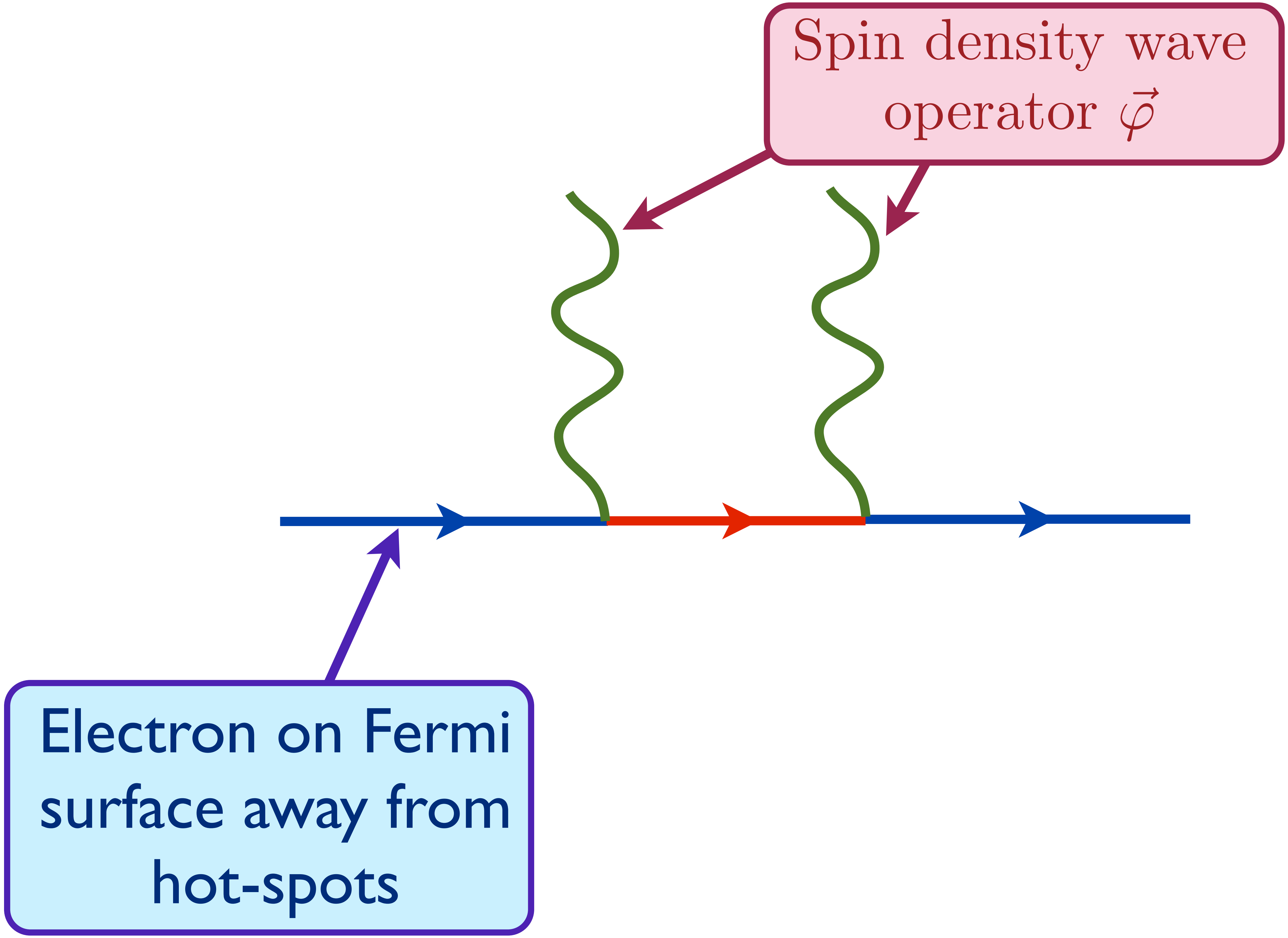
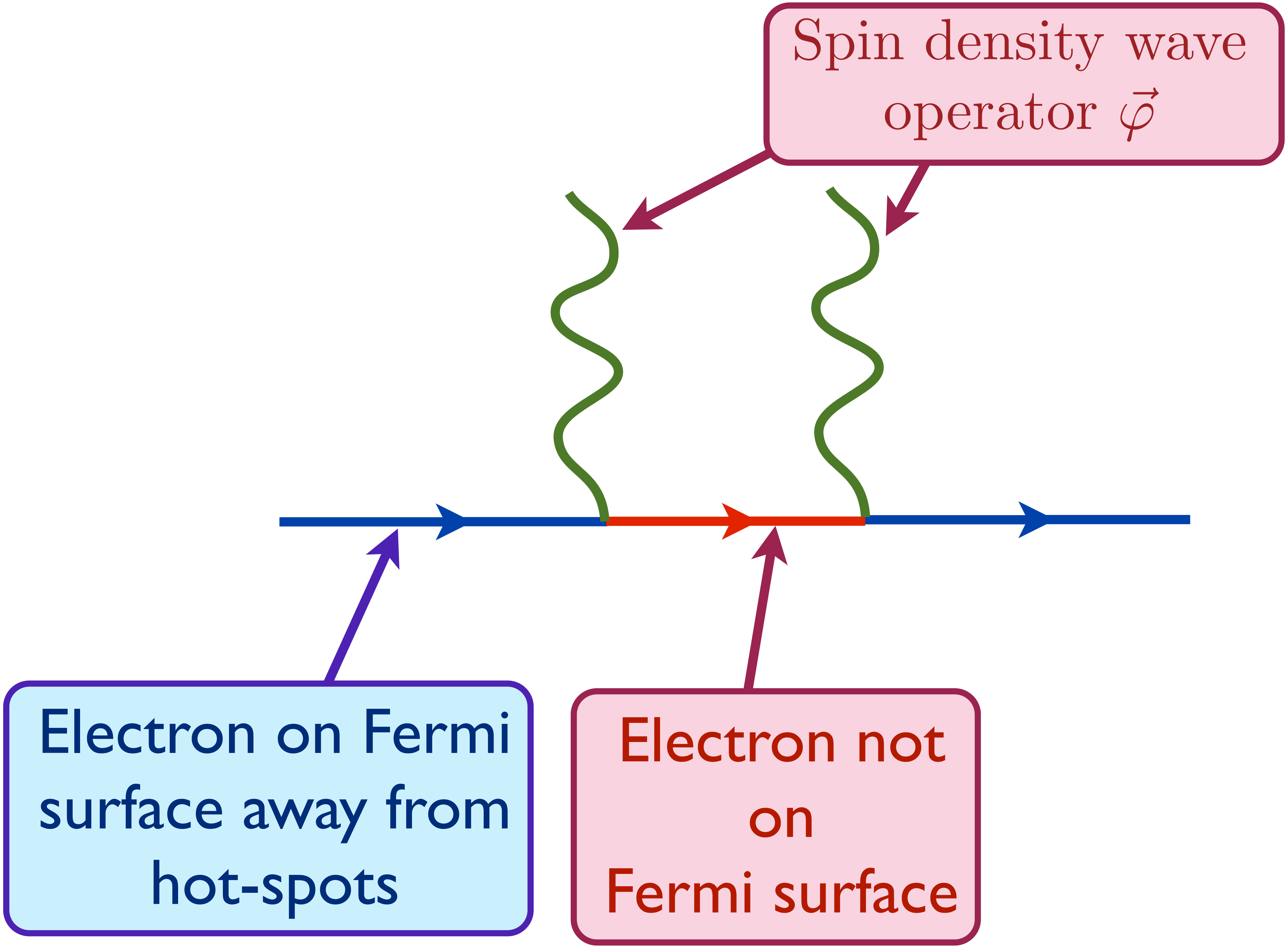
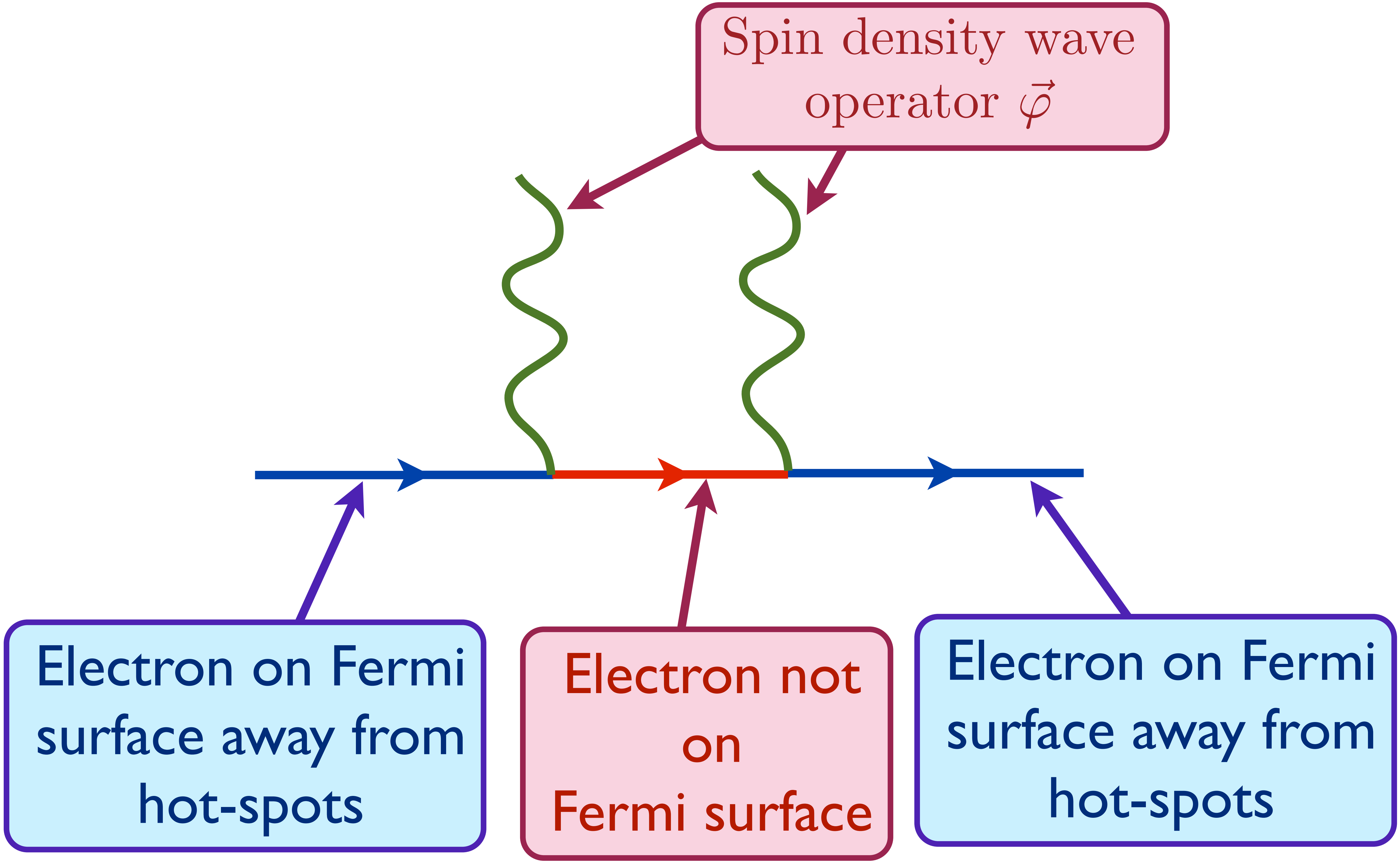
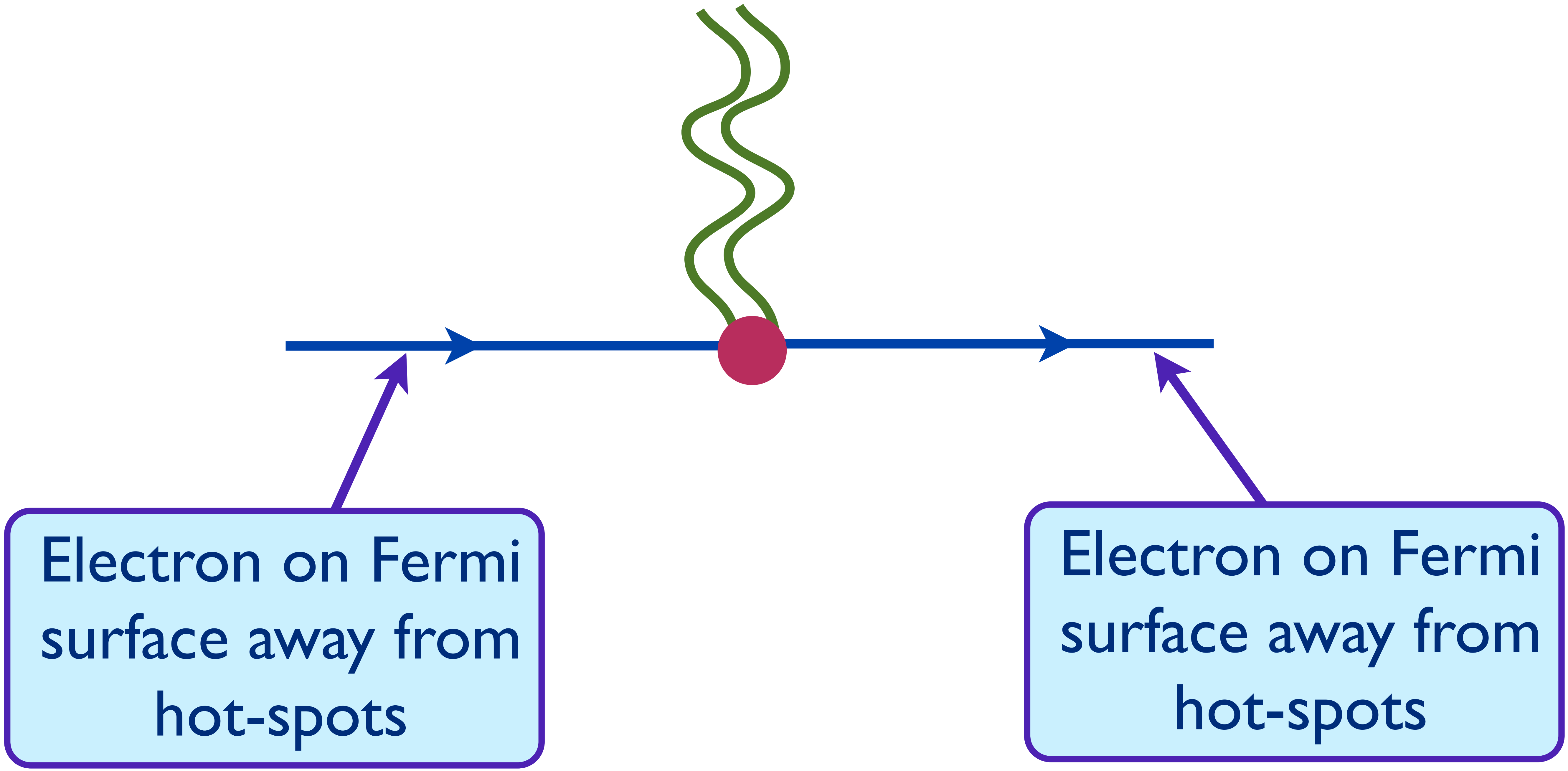


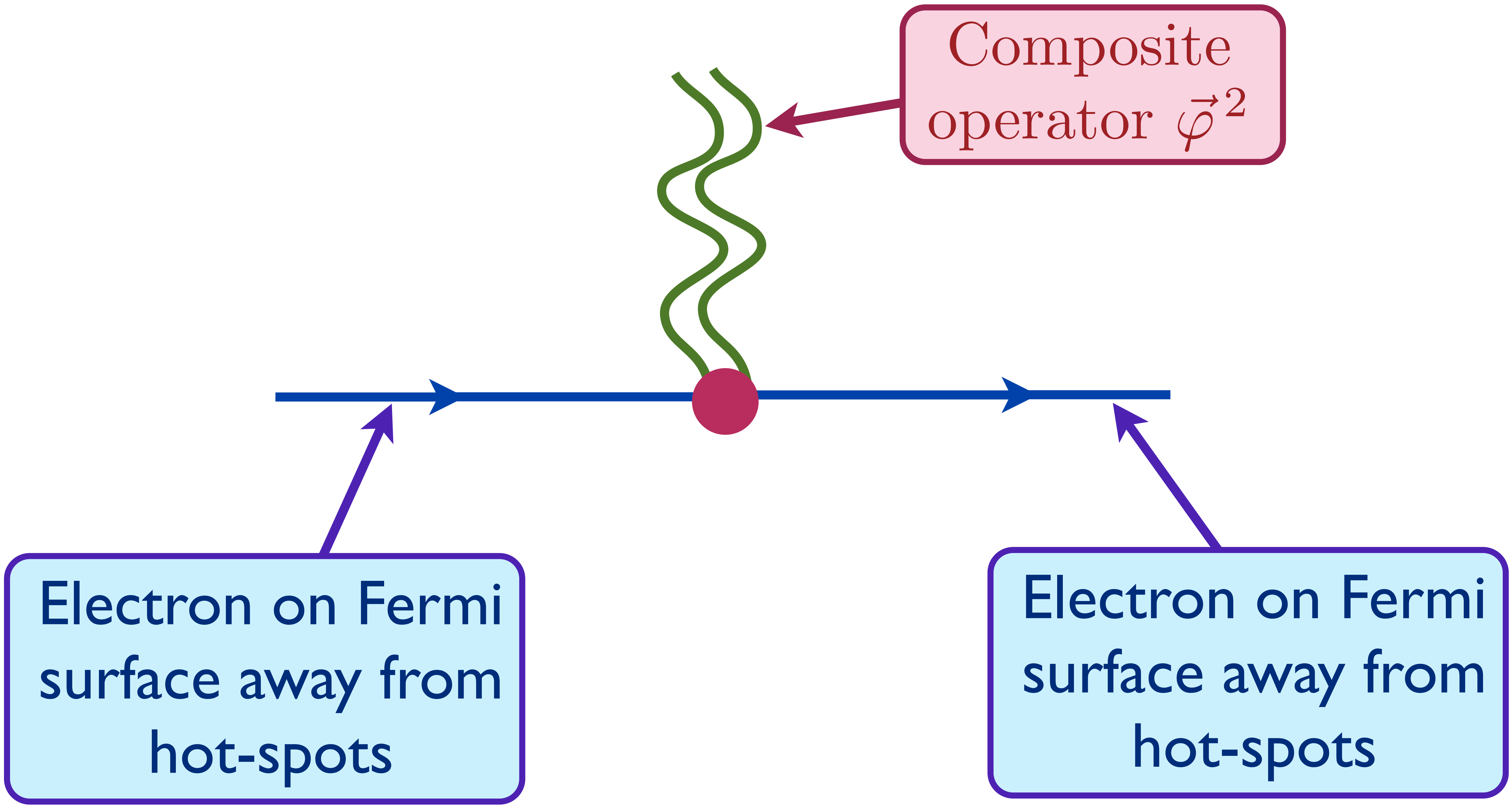

Electron on Fermi
surface away from
hot-spots

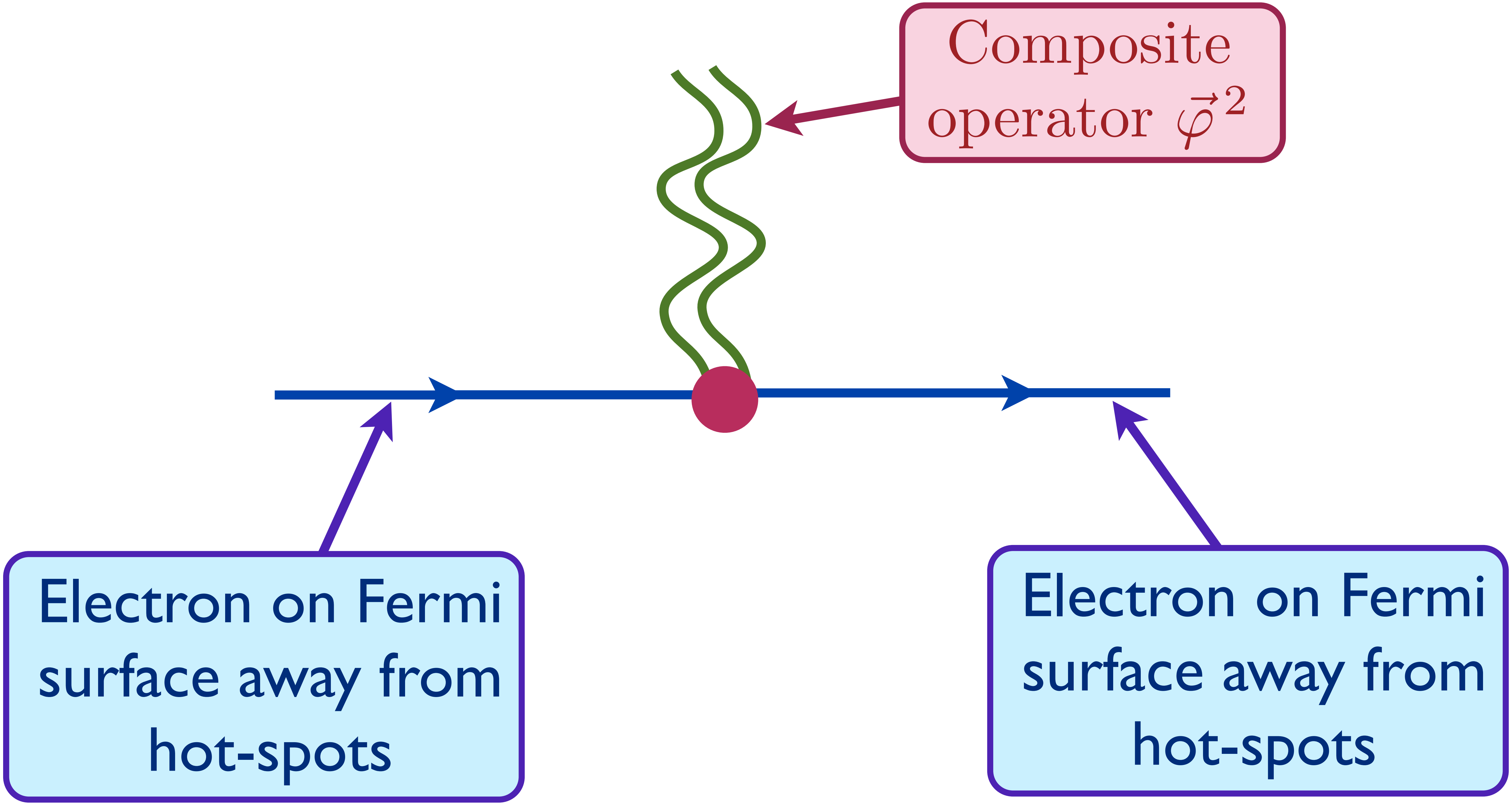

Spin density wave operator $\vec{\varphi}$
Electron on Fermi surface away from hot-spots

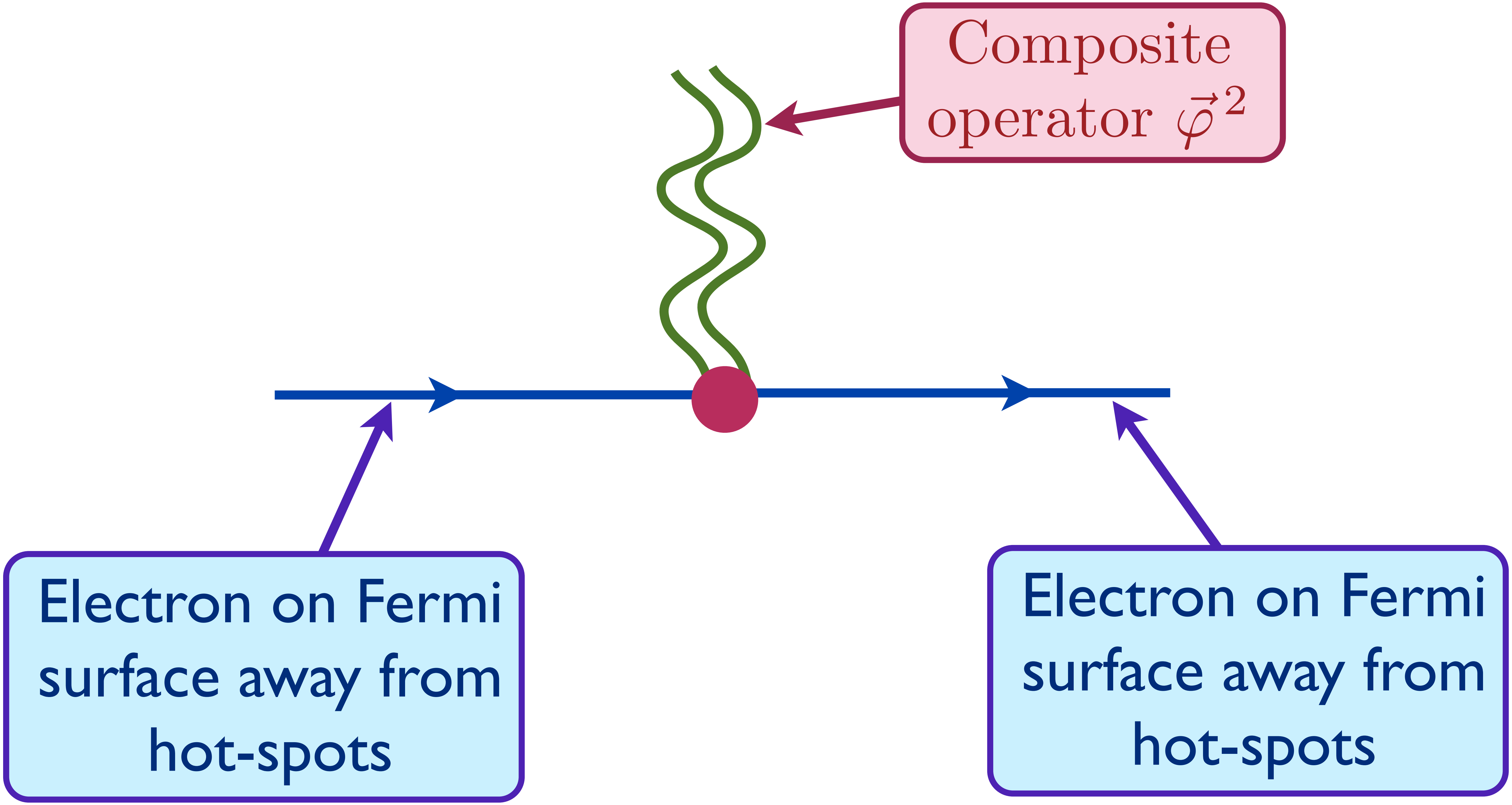

Spin density wave operator $\vec{\varphi}$
Electron on Fermi surface away from hot-spots
Electron not on Fermi surface


Spin density wave operator $\vec{\varphi}$
Electron on Fermi surface away from hot-spots
Electron not on Fermi surface
Electron on Fermi surface away from hot-spots

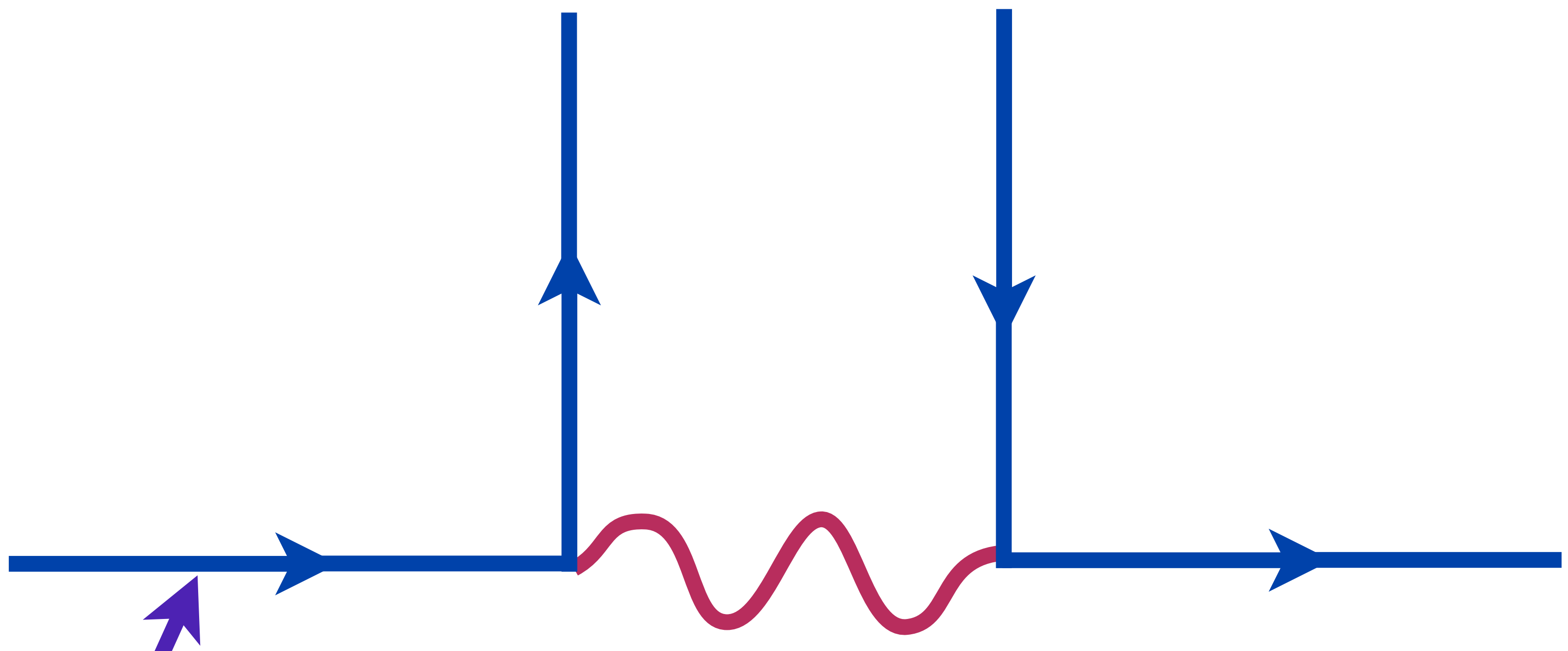

Electron on Fermi surface away from hot-spots
Electron on Fermi surface away from hot-spots


Composite operator $\vec{\varphi}^2$
Electron on Fermi surface away from hot-spots
Electron on Fermi surface away from hot-spots

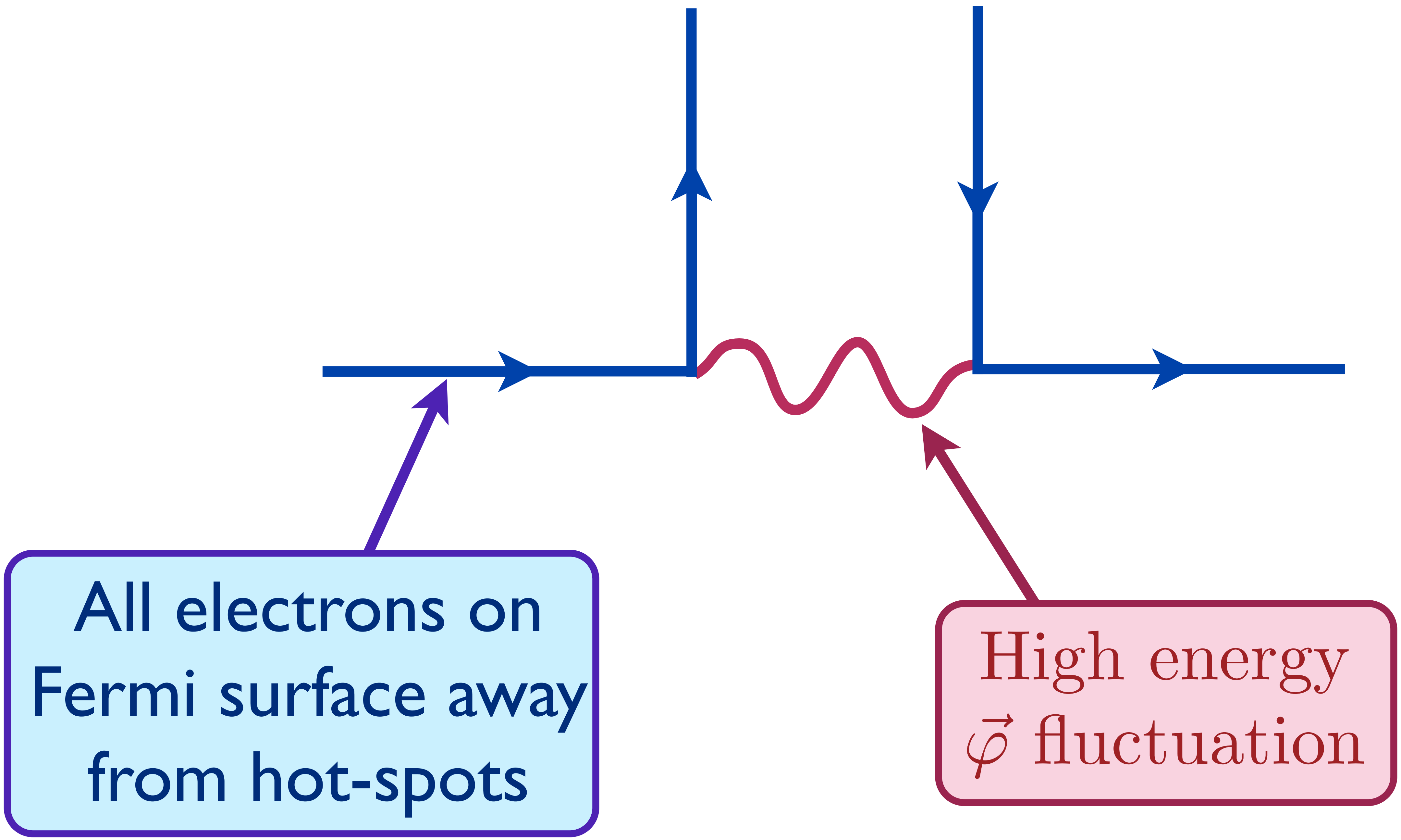

Composite operator $\vec{\varphi}^2$
Electron on Fermi surface away from hot-spots
Electron on Fermi surface away from hot-spots

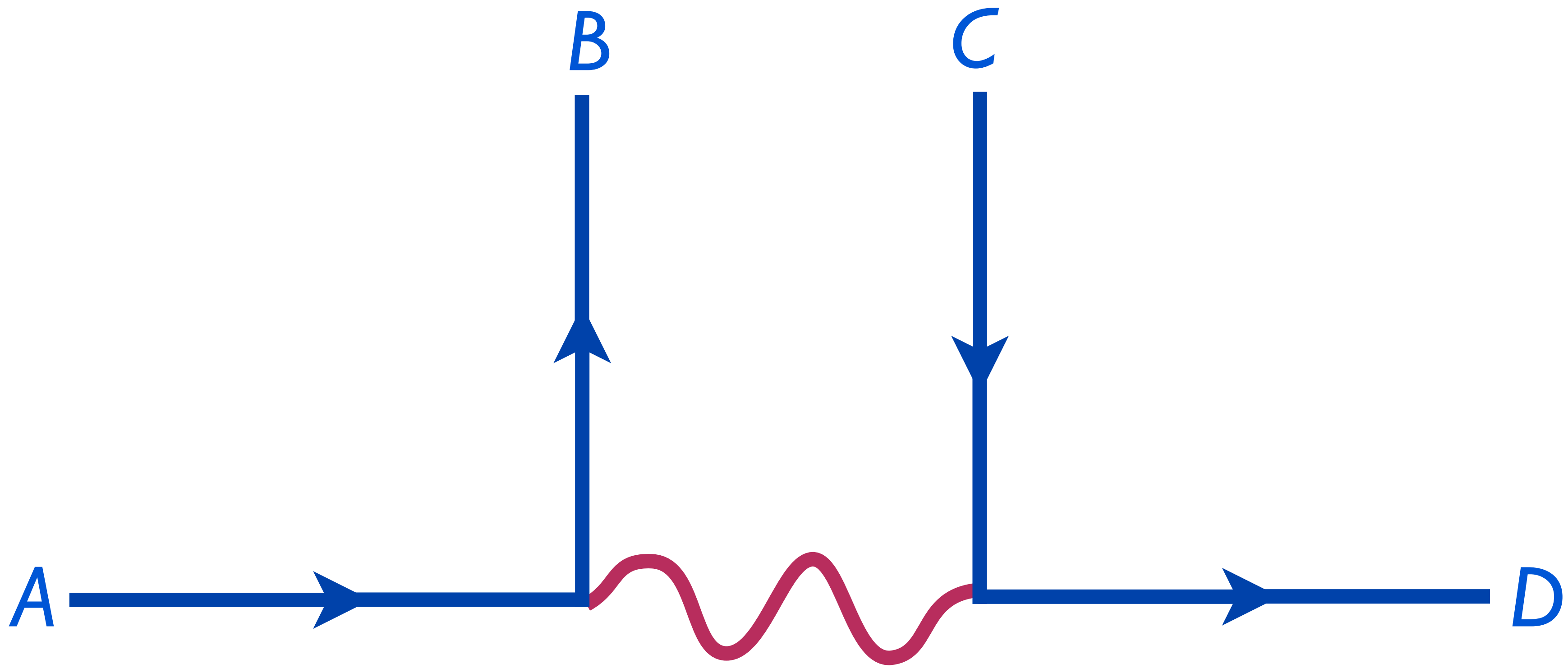


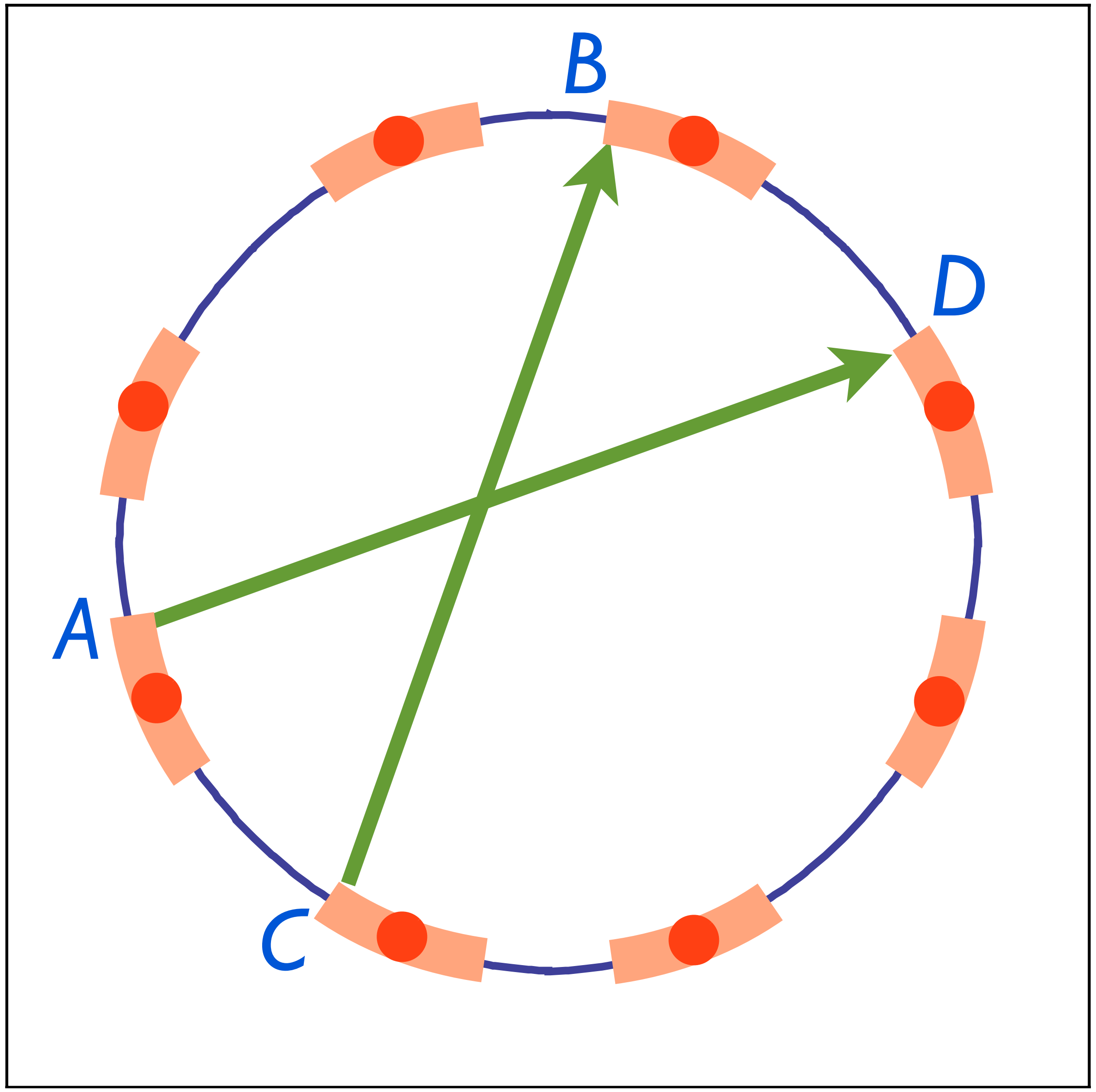
All excitations are low energy

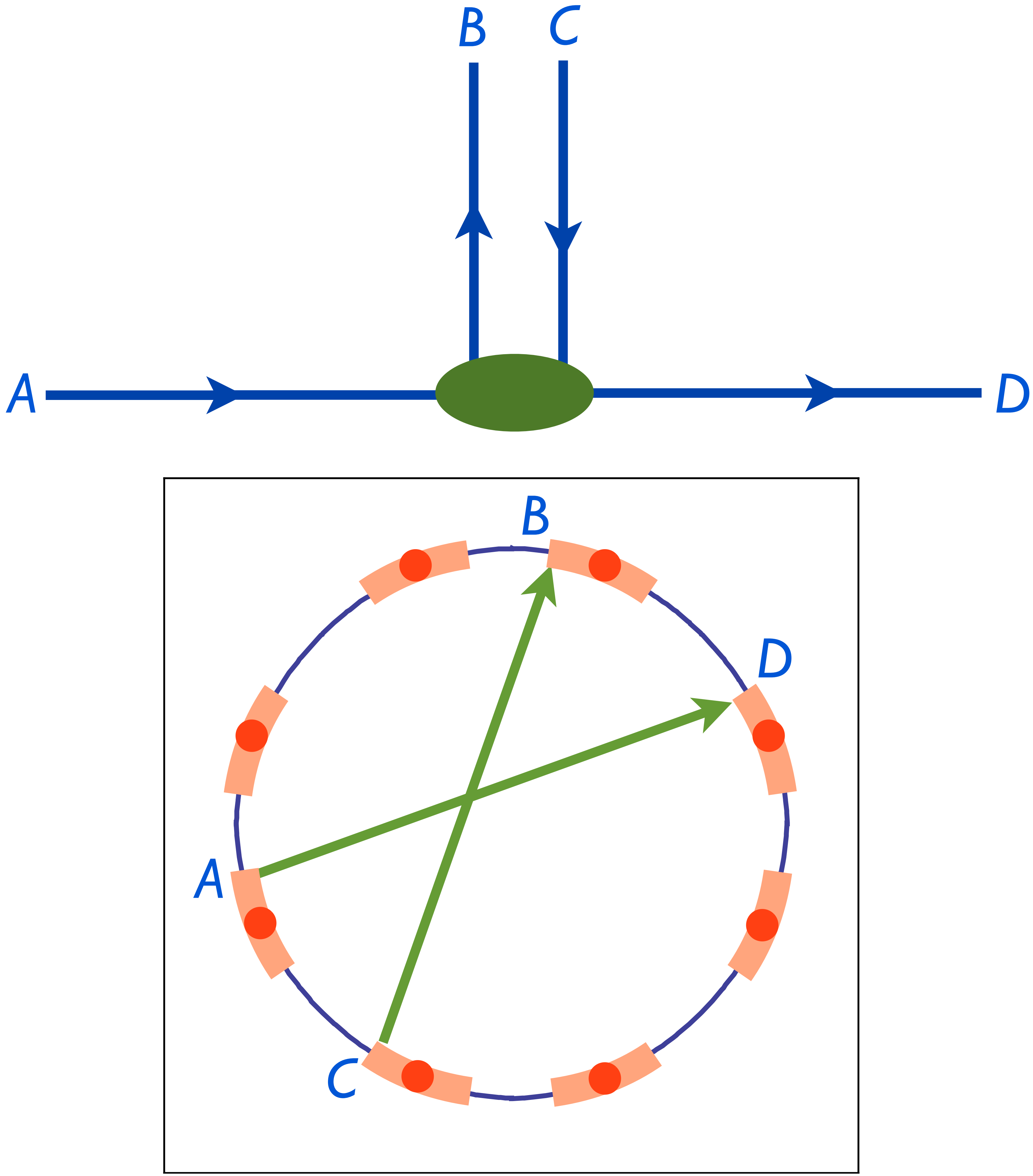


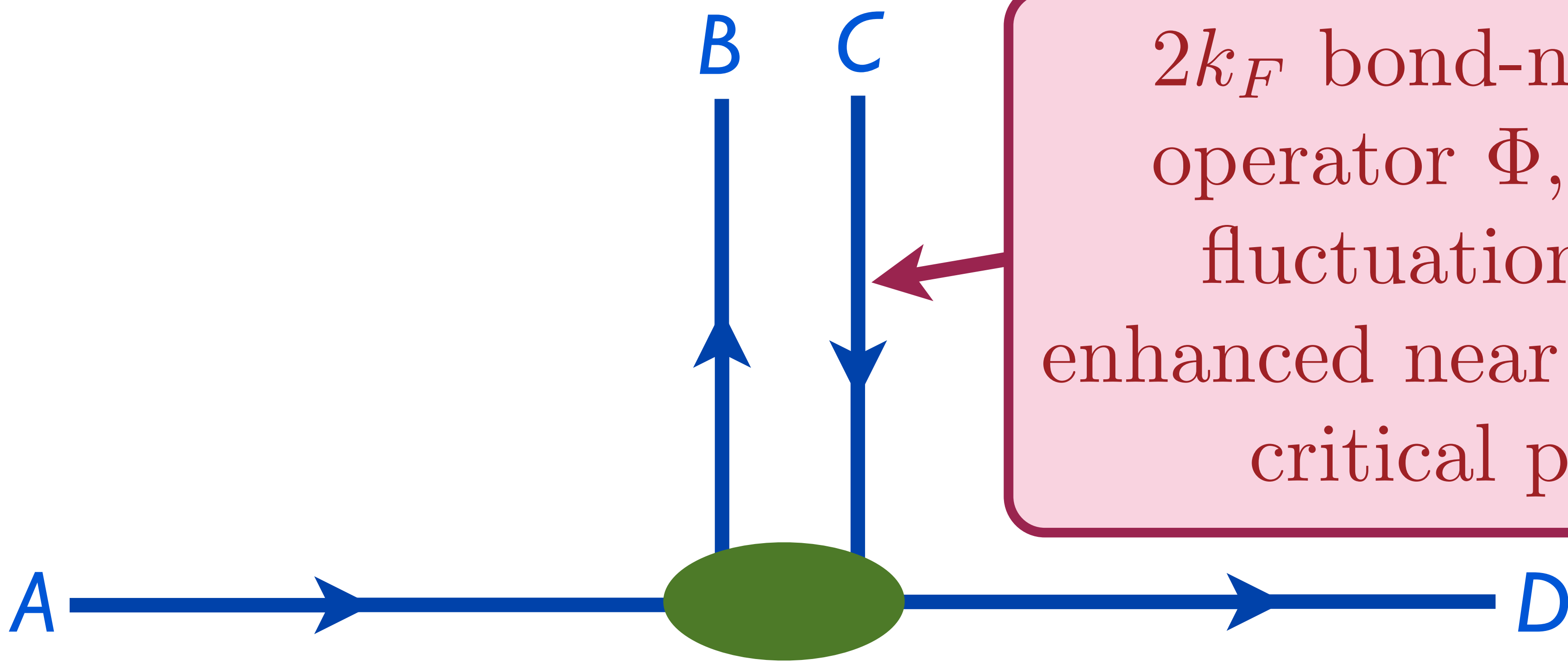
All electrons on Fermi surface away from hot-spots


All electrons on Fermi surface away from hot-spots
High energy $\vec{\varphi}$ fluctuation

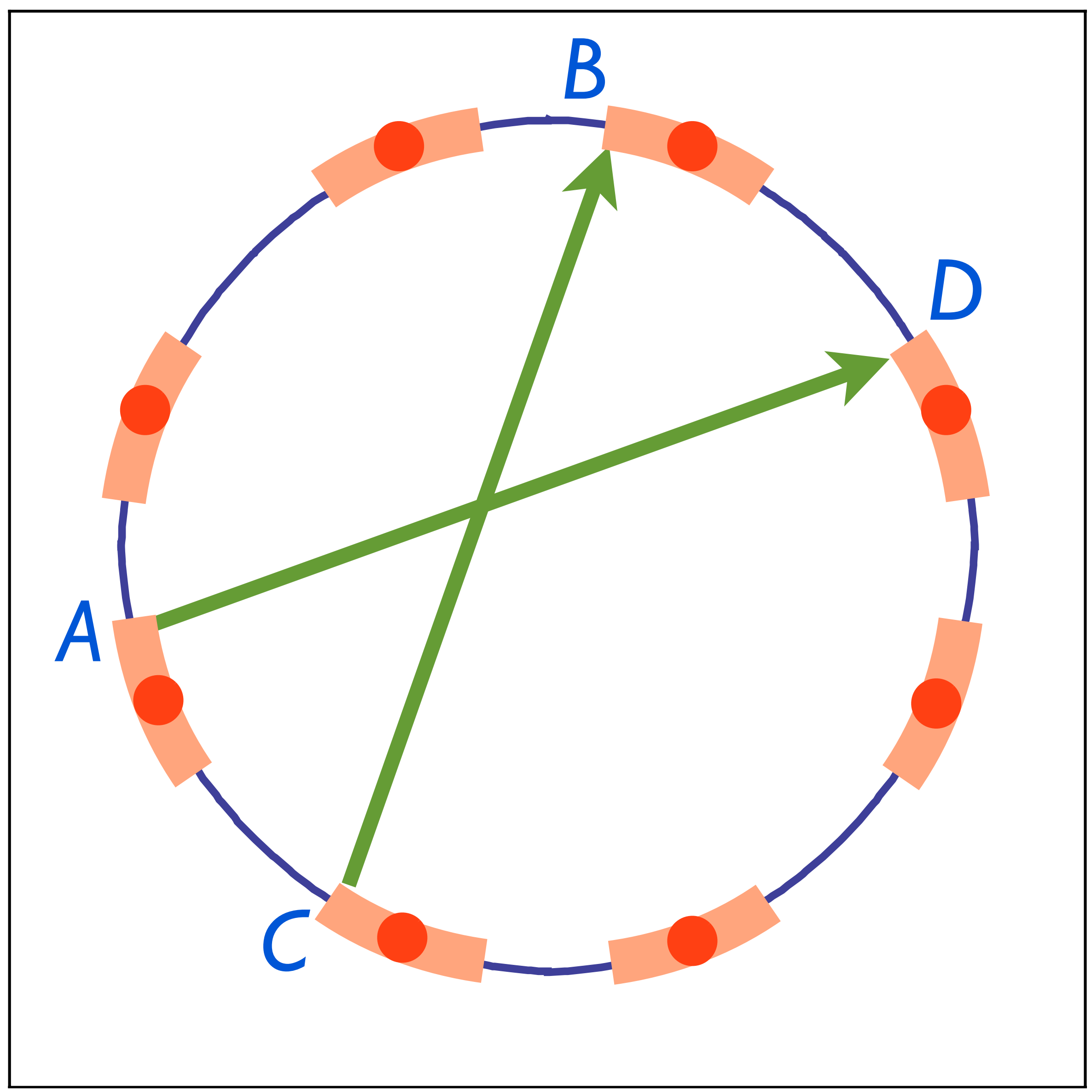

B
C
A
D

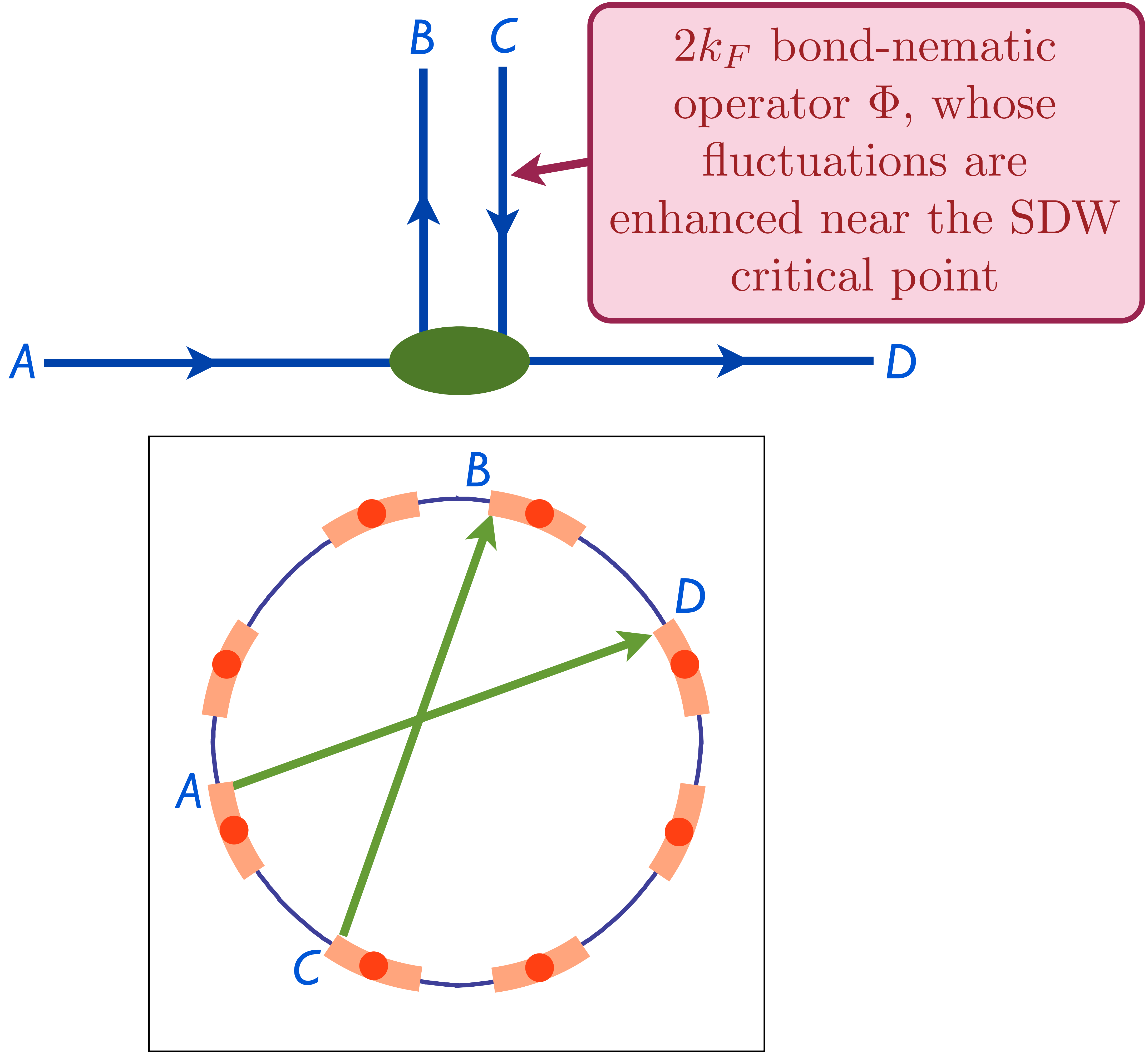

B
D
A
C

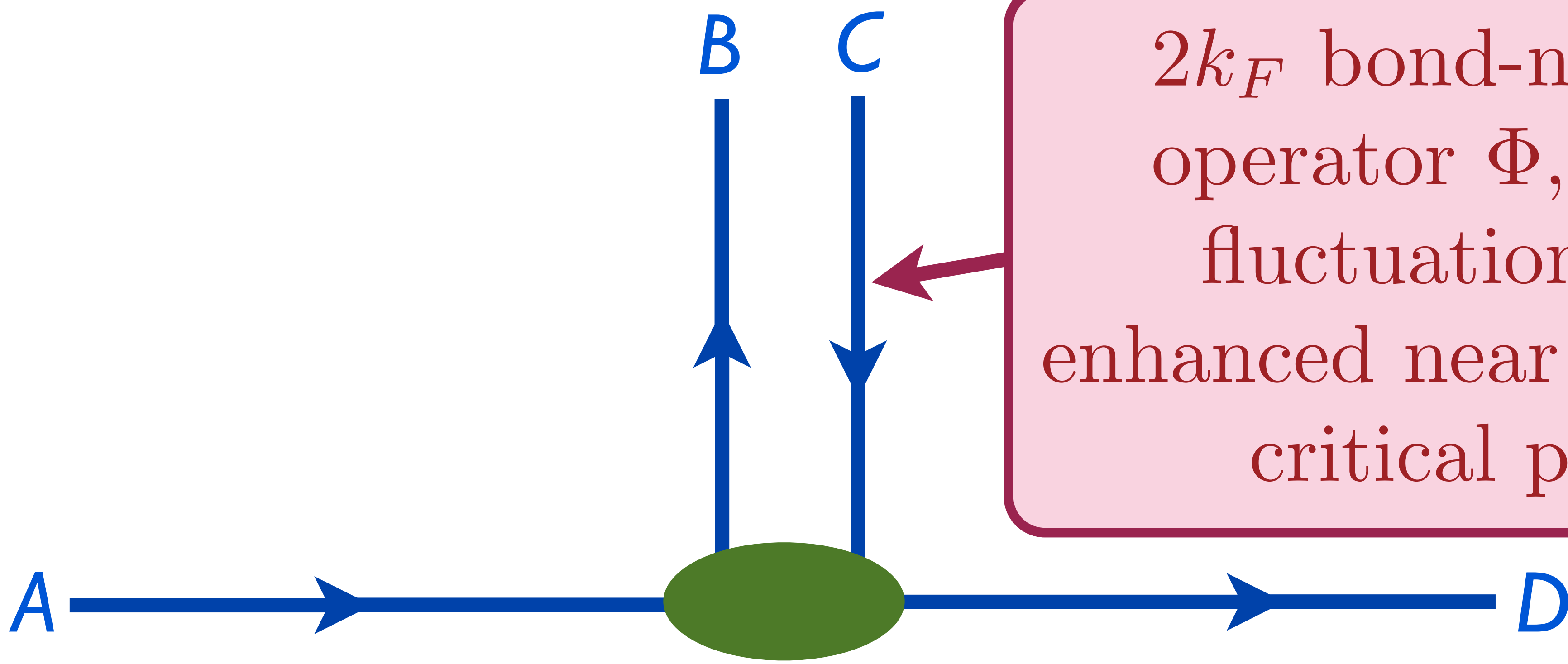

B
C
A
D
B
D
A
C



$2k_F$ bond-nematic operator Φ, whose fluctuations are enhanced near the SDW critical point




B
C
$2k_F$ bond-nematic operator Φ, whose fluctuations are enhanced near the SDW critical point
A
D
B
D
A
C



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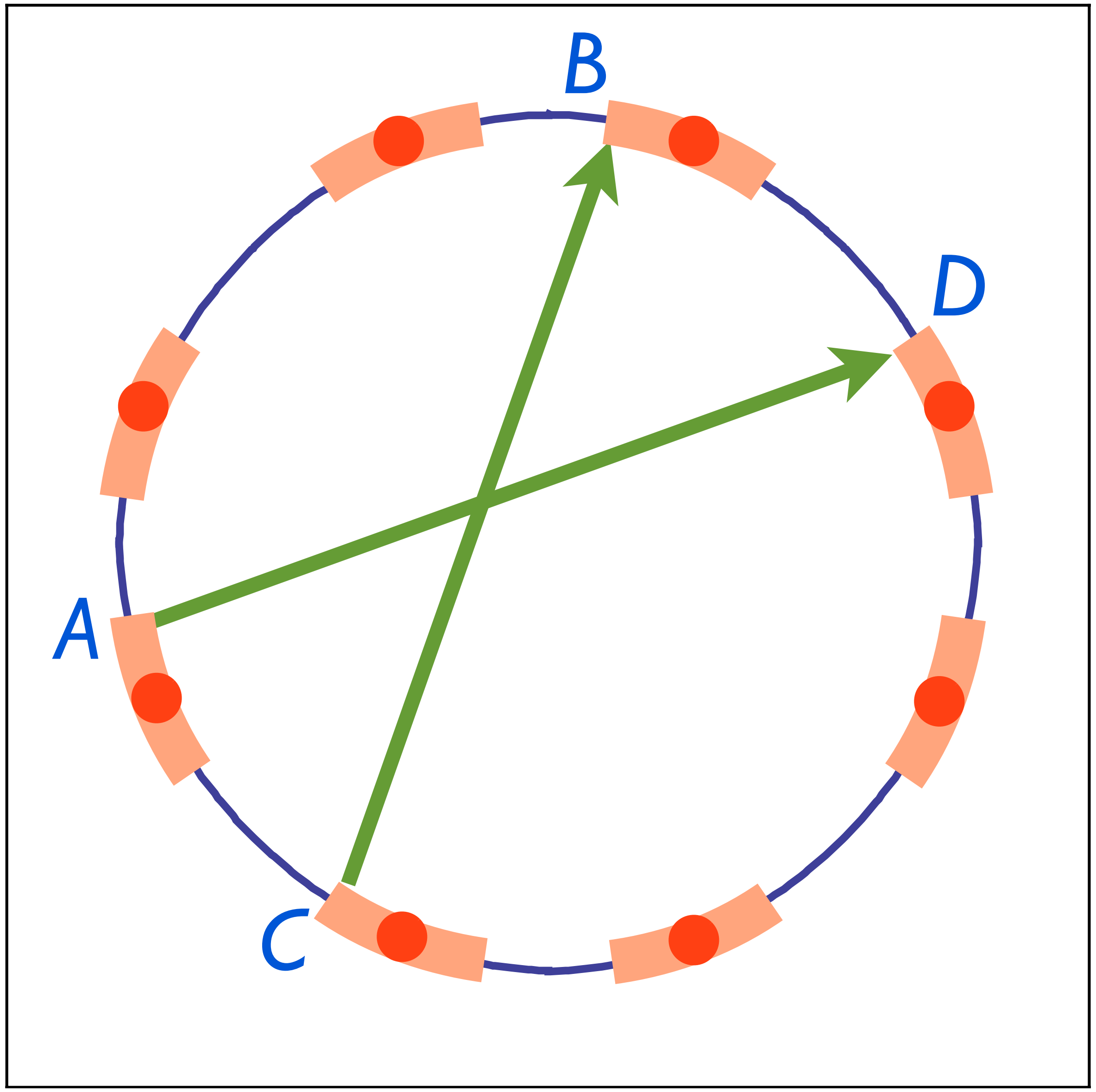

All low energy excitations in an umklapp process: this is important for transport properties

Consequences of composite operators

- Non-Fermi liquid spectral functions around *entire* Fermi surface.

- Scattering off $\vec{\varphi}$ and $\vec{\varphi}^2$ fluctuations leads to strong scattering of electronic excitations, but contribution to optical conductivity is suppressed by vertex corrections. Quasiparticles break down at the hot spots, but survive elsewhere (at leading order).

- Dominant contribution to optical conductivity, $\sigma(\omega) \sim \omega^{\mu}$, arises from $2k_F$ umklapp scattering.

Conclusions

The quantum critical point describing the onset of spin-density-wave order in metals is strongly coupled in two spatial dimensions, and displays universal non-Fermi liquid physics which is independent of electron interaction strength.

Conclusions

The quantum critical point has
an instability to
unconventional "*d*-wave" pairing, with a
universal *log-squared* enhancement
of the pairing susceptibility,
which is independent of
electron interaction strength.

Conclusions

The leading subdominant instability is to
a $2k_F$ bond-nematic ordering.
Its susceptibility also has a
universal *log-squared* enhancement ,
which is independent of
electron interaction strength.

Conclusions

Composite operators lead to non-Fermi liquid behavior around entire Fermi surface

Conclusions

Composite umklapp scattering operators dominate the optical conductivity