The quantum phases of matter and gauge-gravity duality

University of Michigan, Ann Arbor, March 13, 2013

Subir Sachdev

PHYSICS HARVARD

Sommerfeld-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

Modern phases of quantum matter

Not adiabatically connected

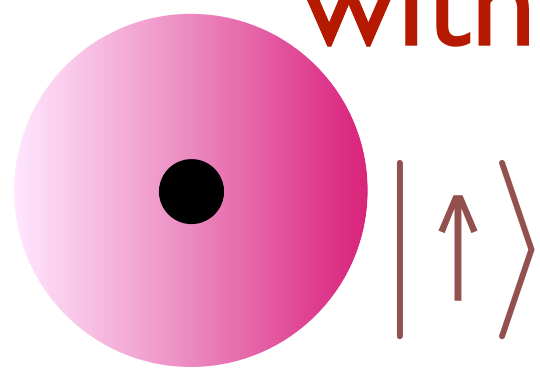
to independent electron states:

many-particle

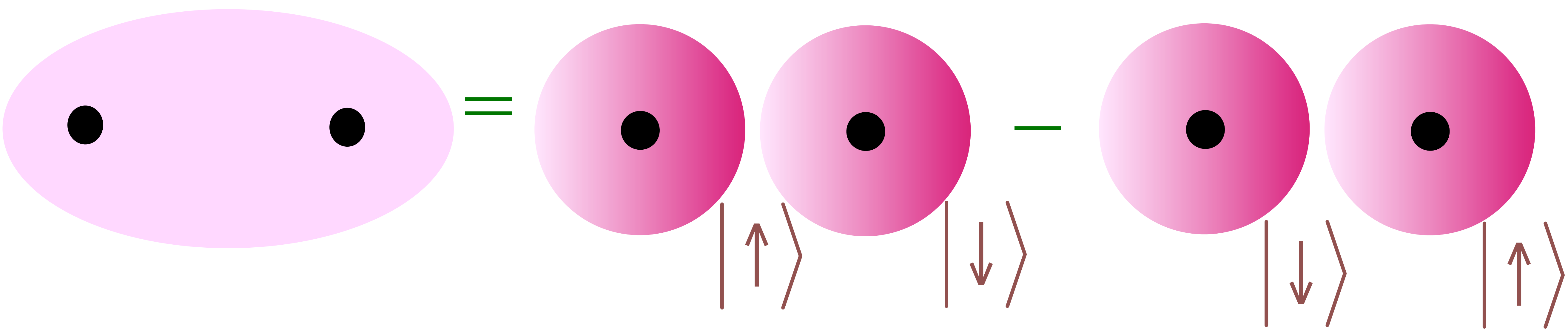
quantum entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:



Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Superposition of two electron states leads to non-local correlations between spins

Quantum Entanglement: quantum superposition with more than one particle

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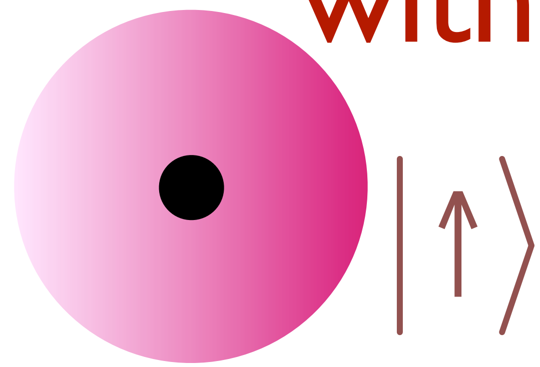

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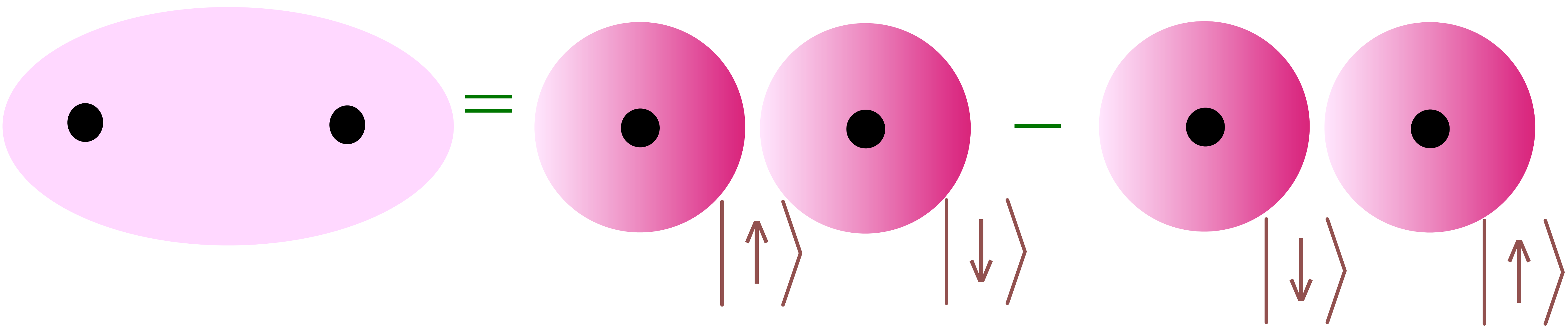

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Einstein-Podolsky-Rosen "paradox": Non-local correlations between observations arbitrarily far apart

Outline

1. Z_2 Spin liquid in the kagome antiferromagnet

2. Superfluid-insulator transition of ultracold atoms in optical lattices:
Quantum criticality and conformal field theories

3. Holography and the quasi-normal modes of black-hole horizons

4. Strange metals:
What lies beyond the horizon ?

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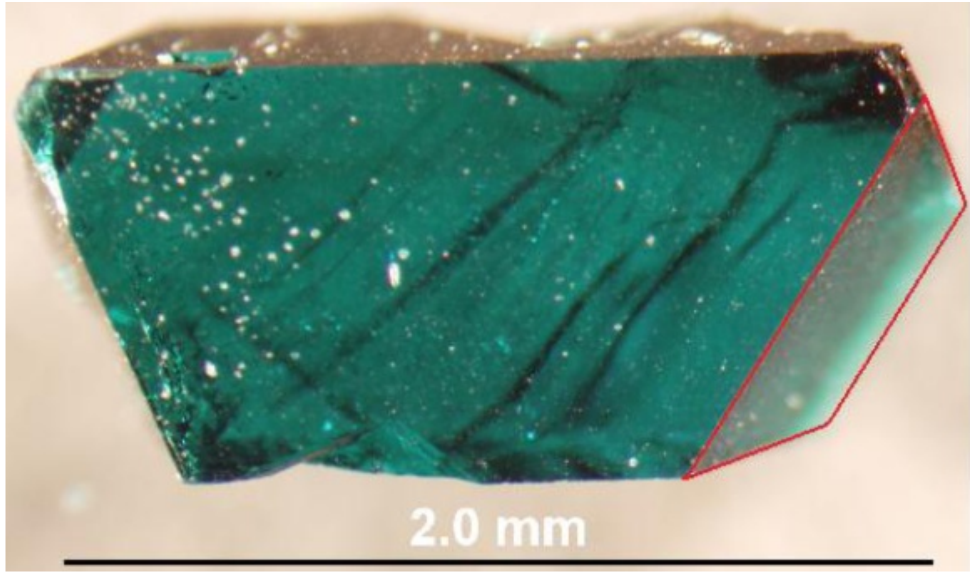
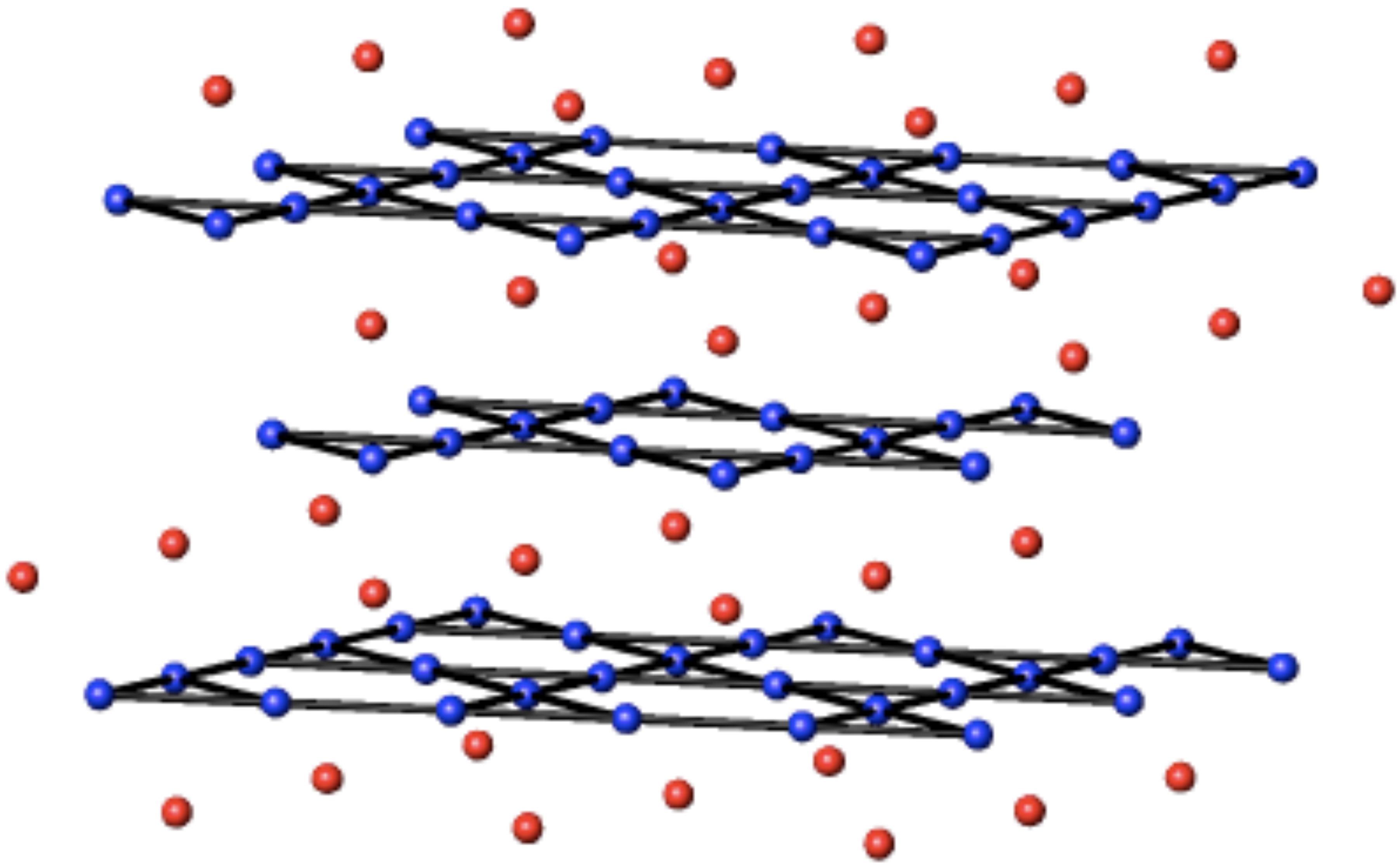


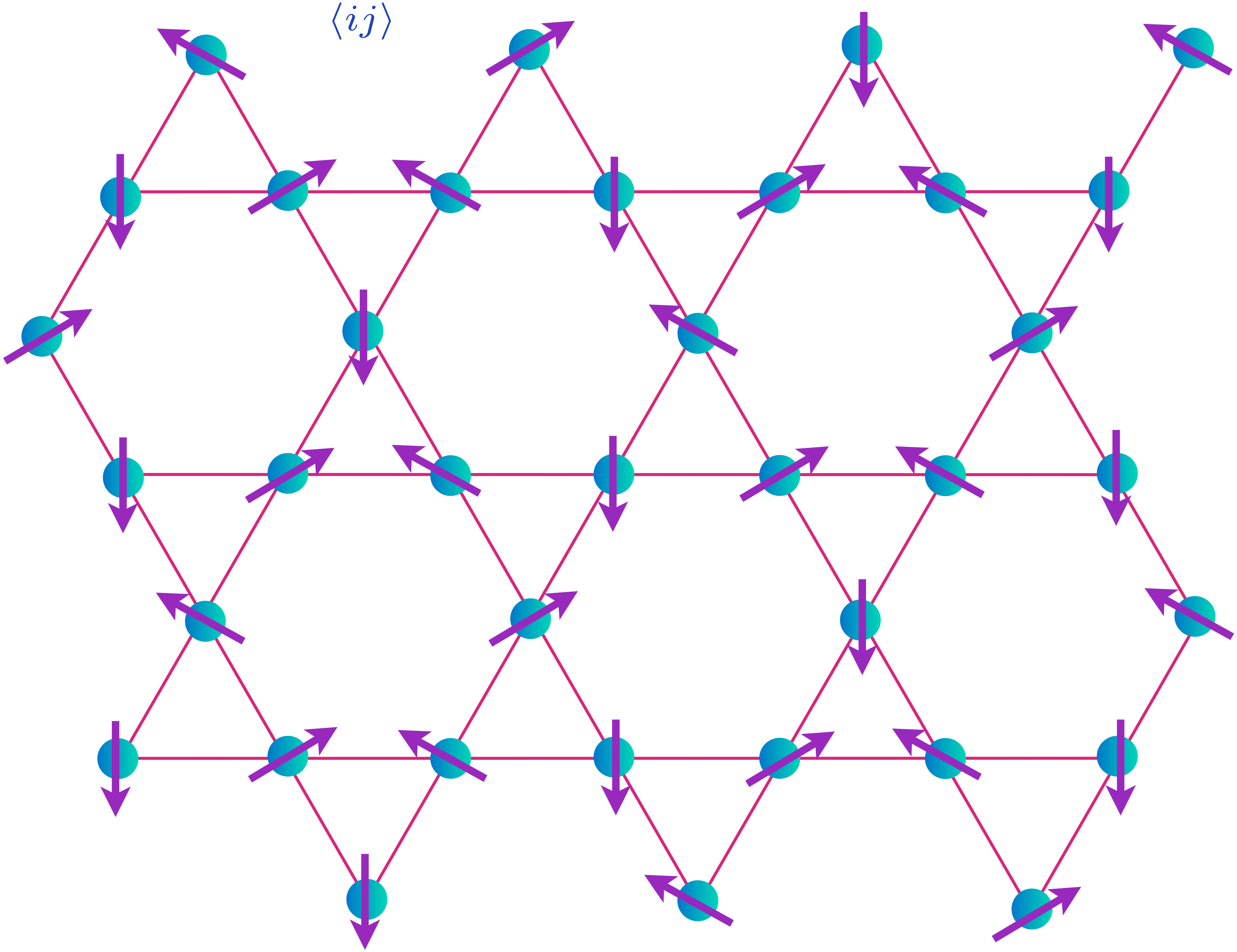
$ZnCu(OH)_6Cl_2$

herbertsmithite single crystals



Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



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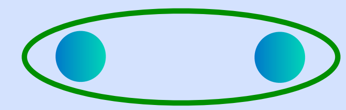
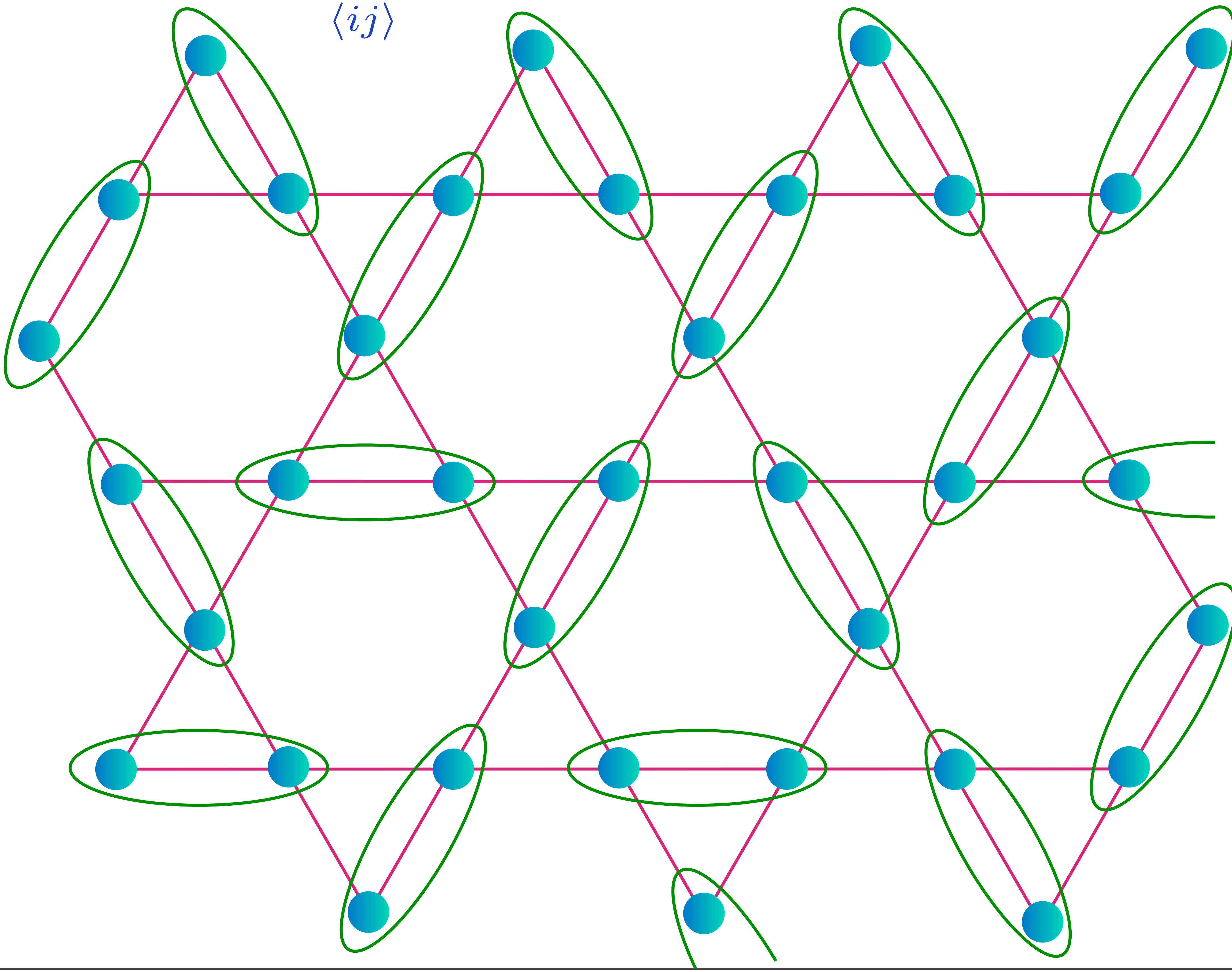
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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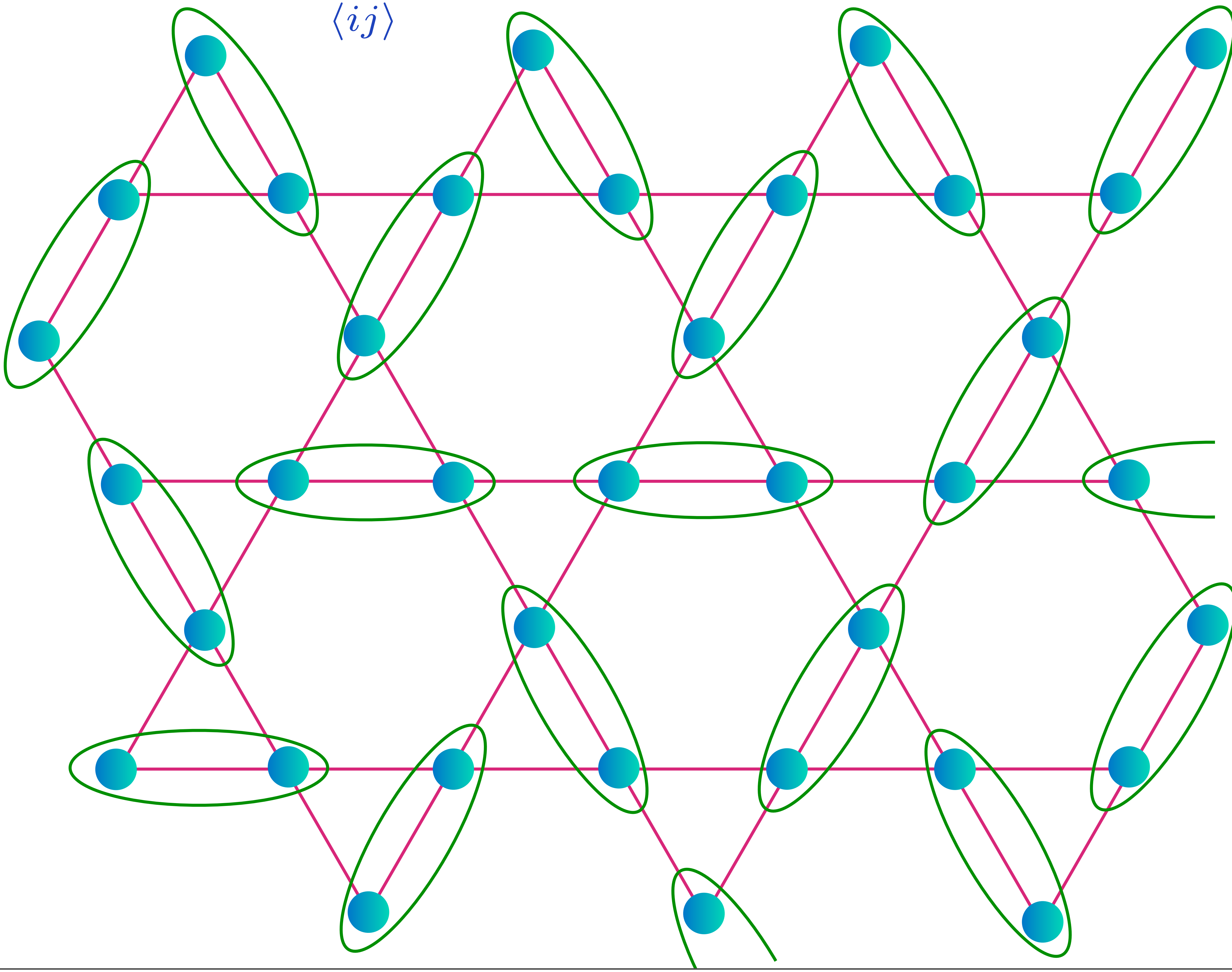

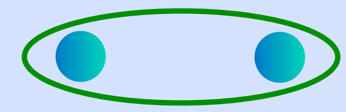
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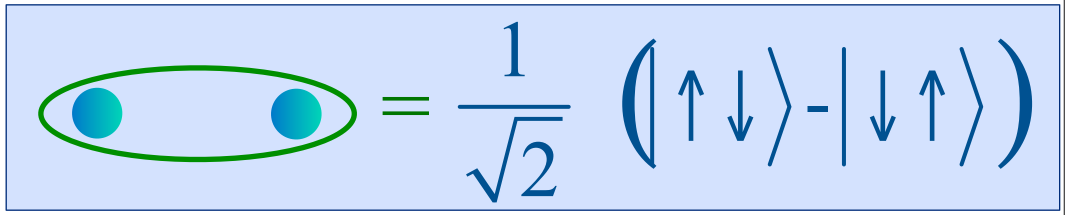

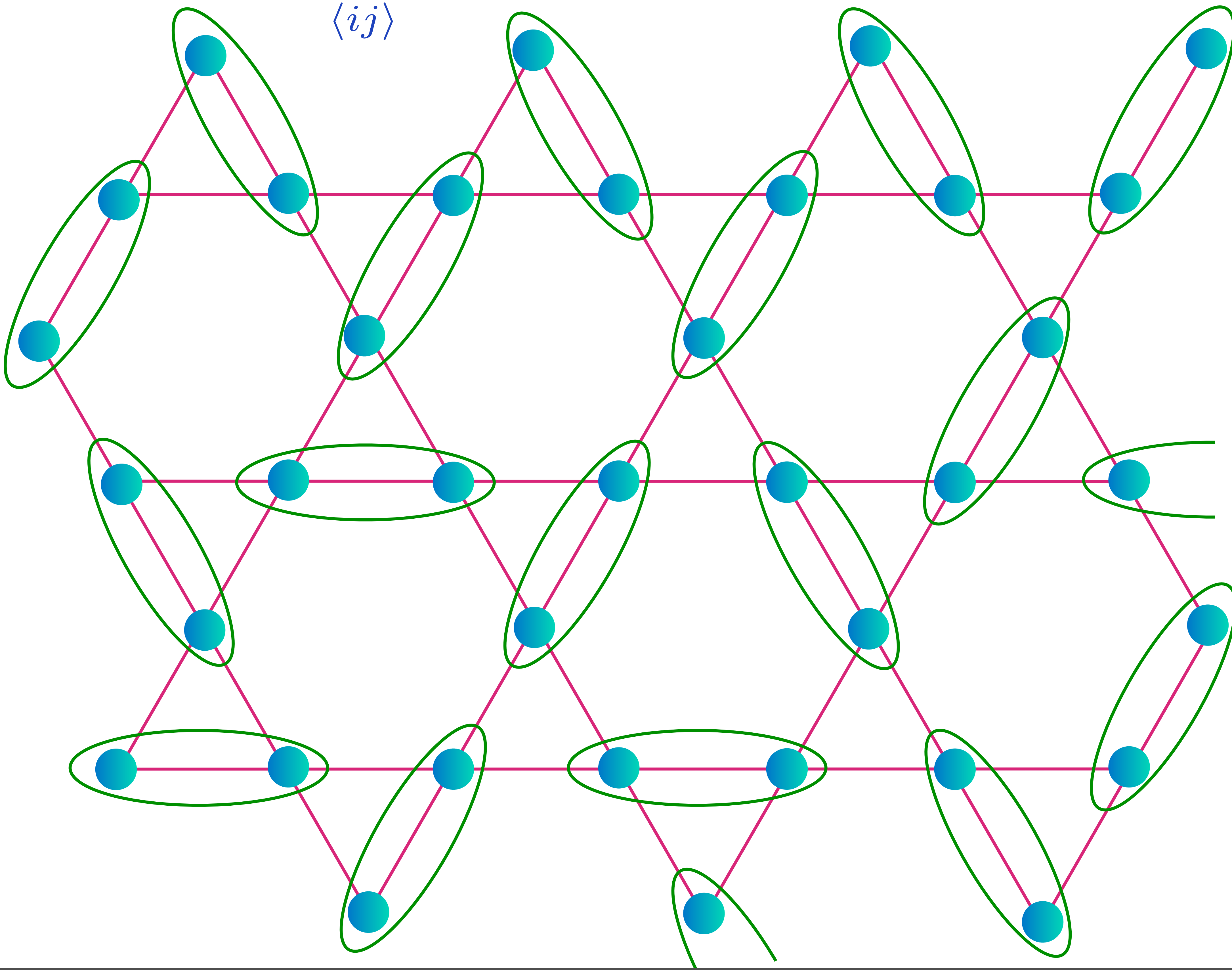

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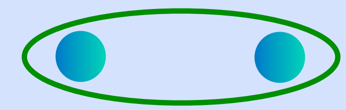

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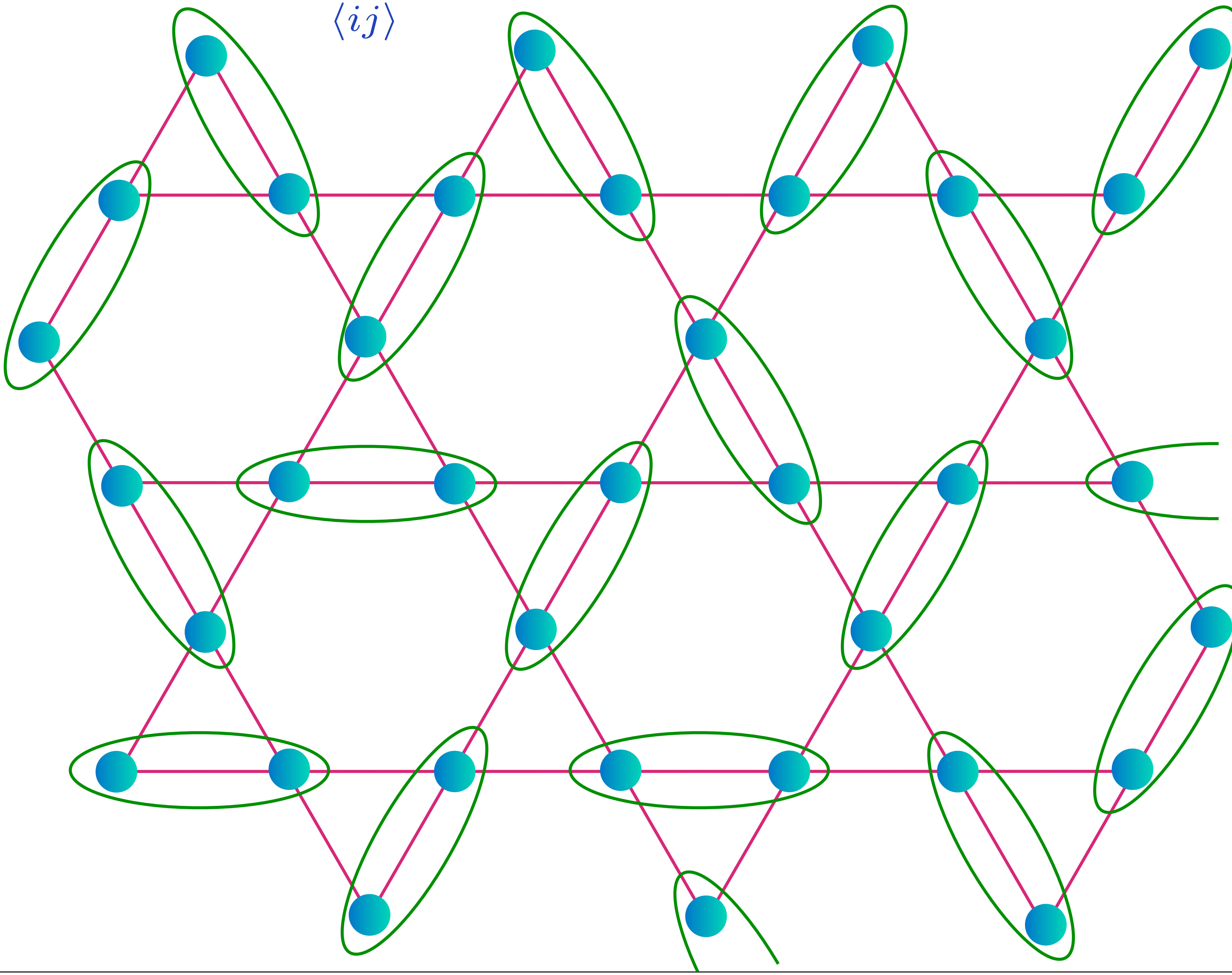

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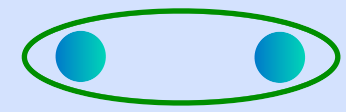

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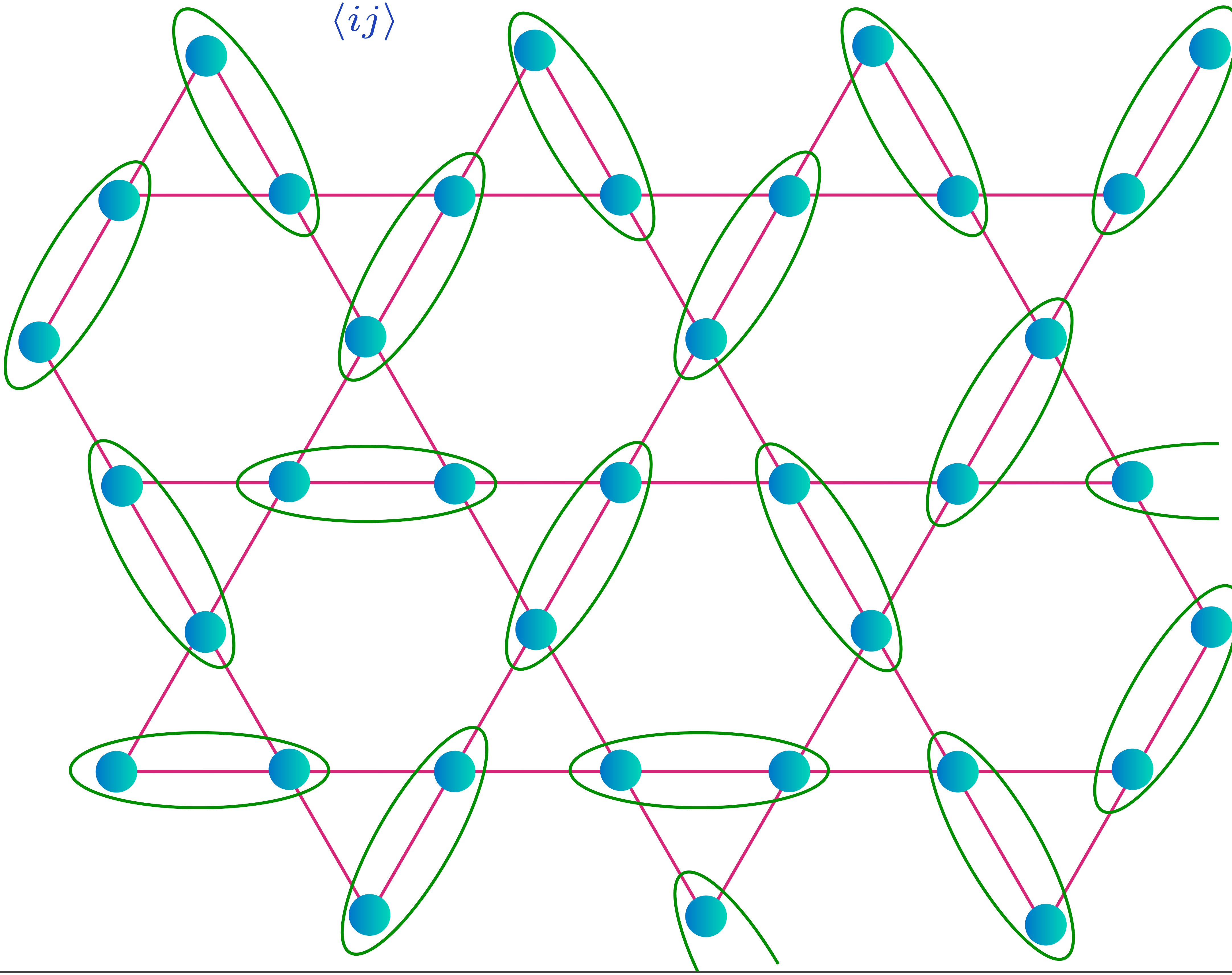

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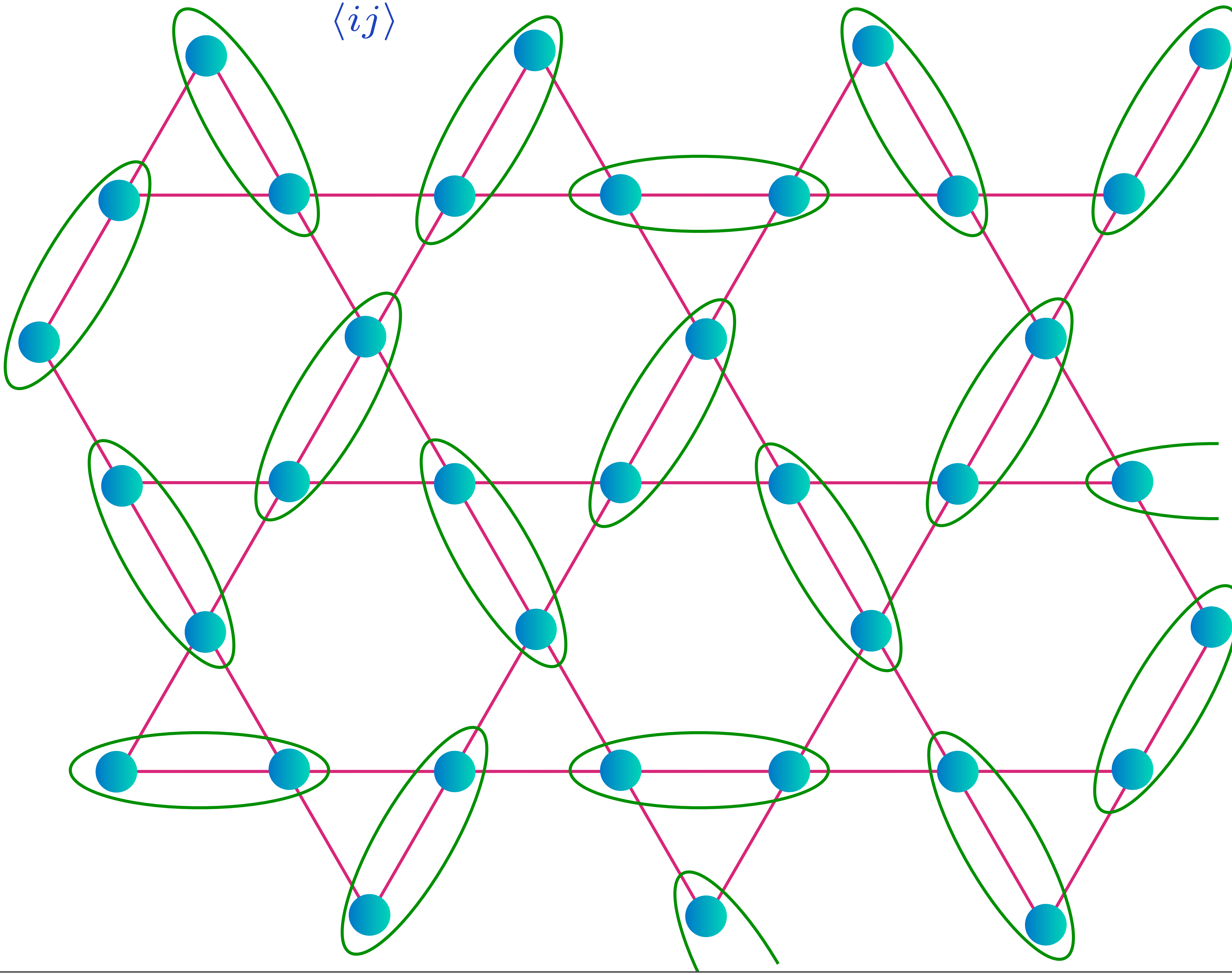

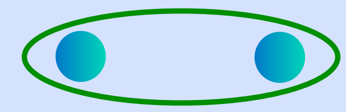

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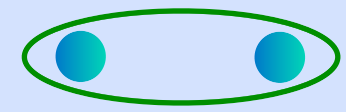

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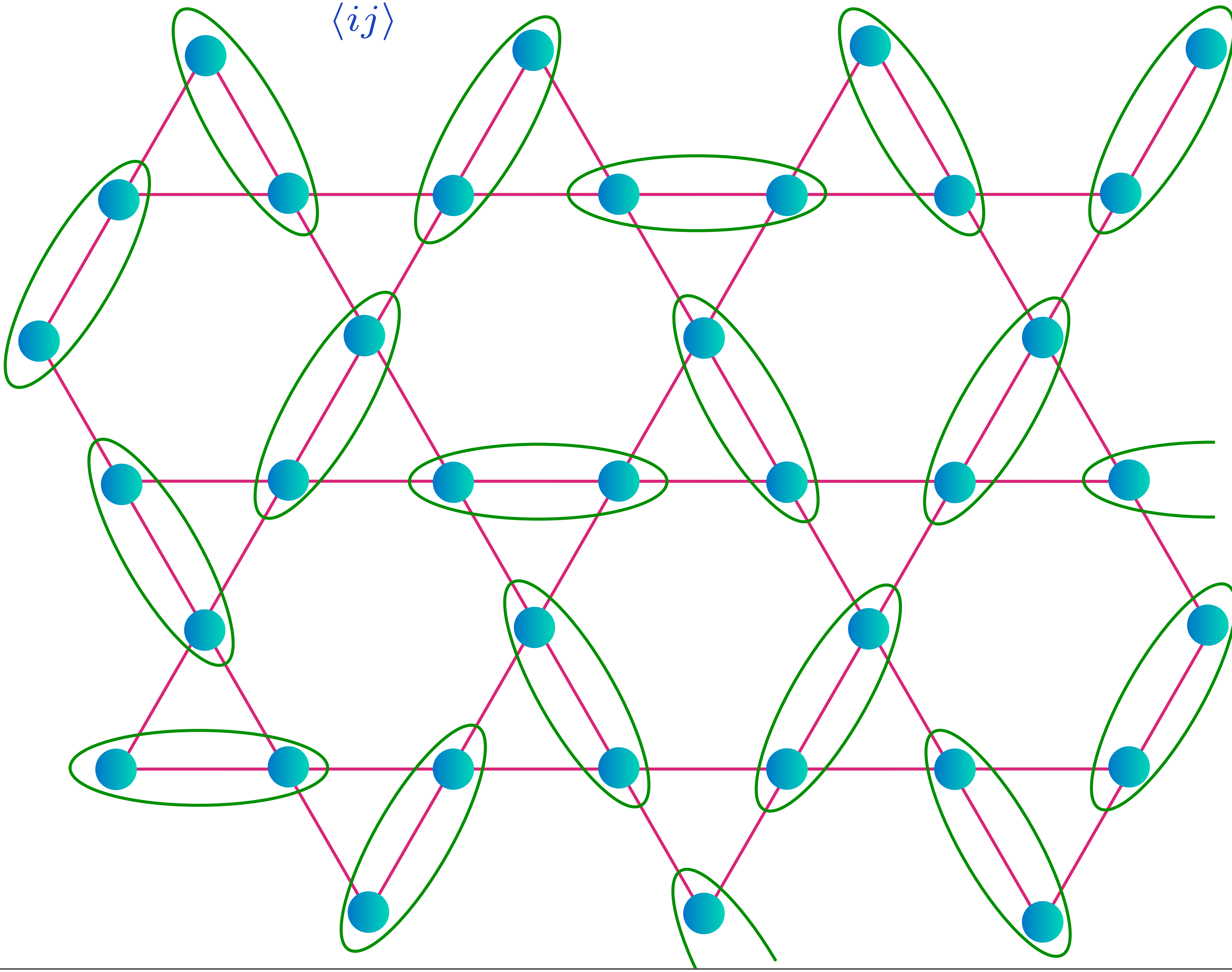

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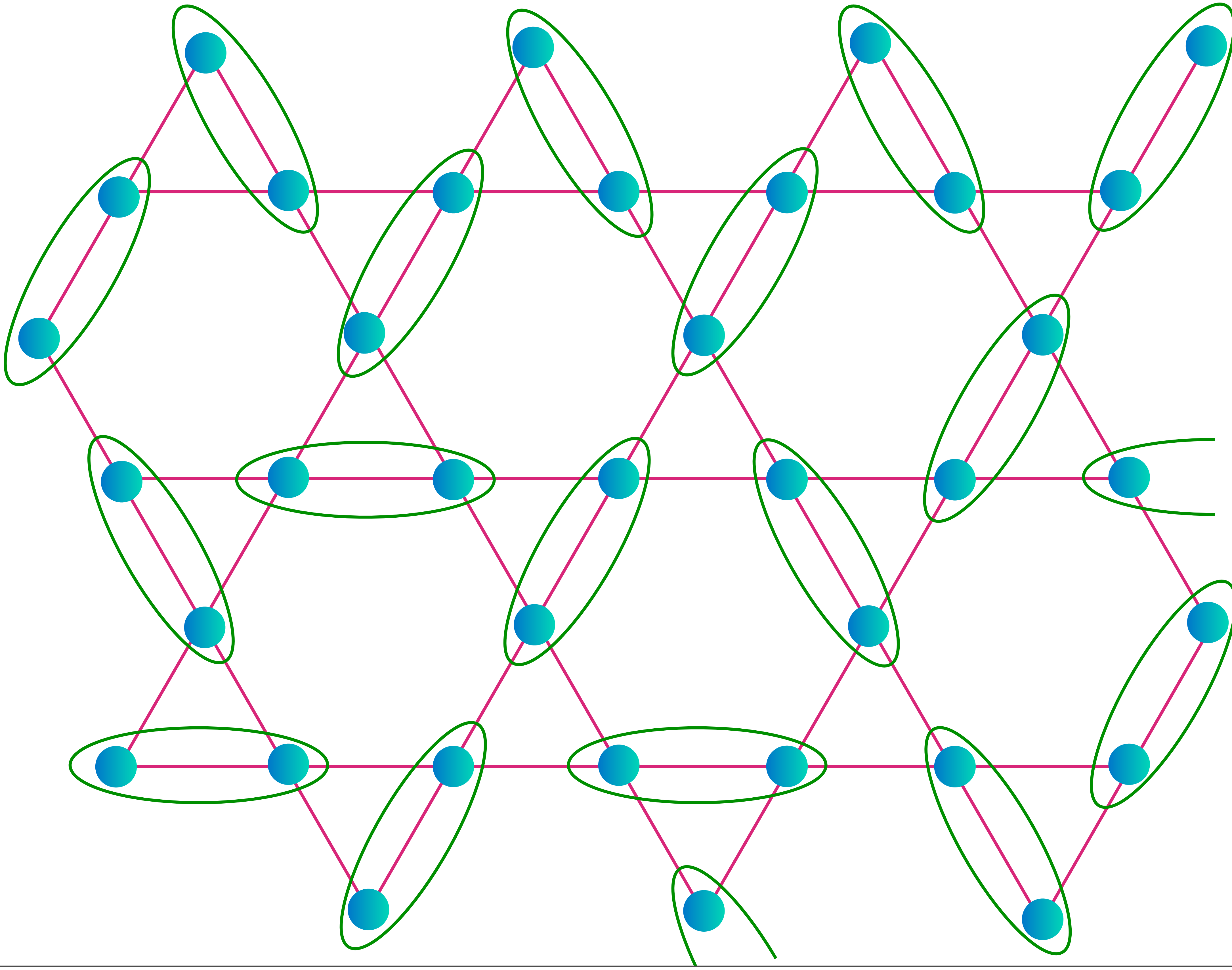
Kagome antiferromagnet

Alternative view

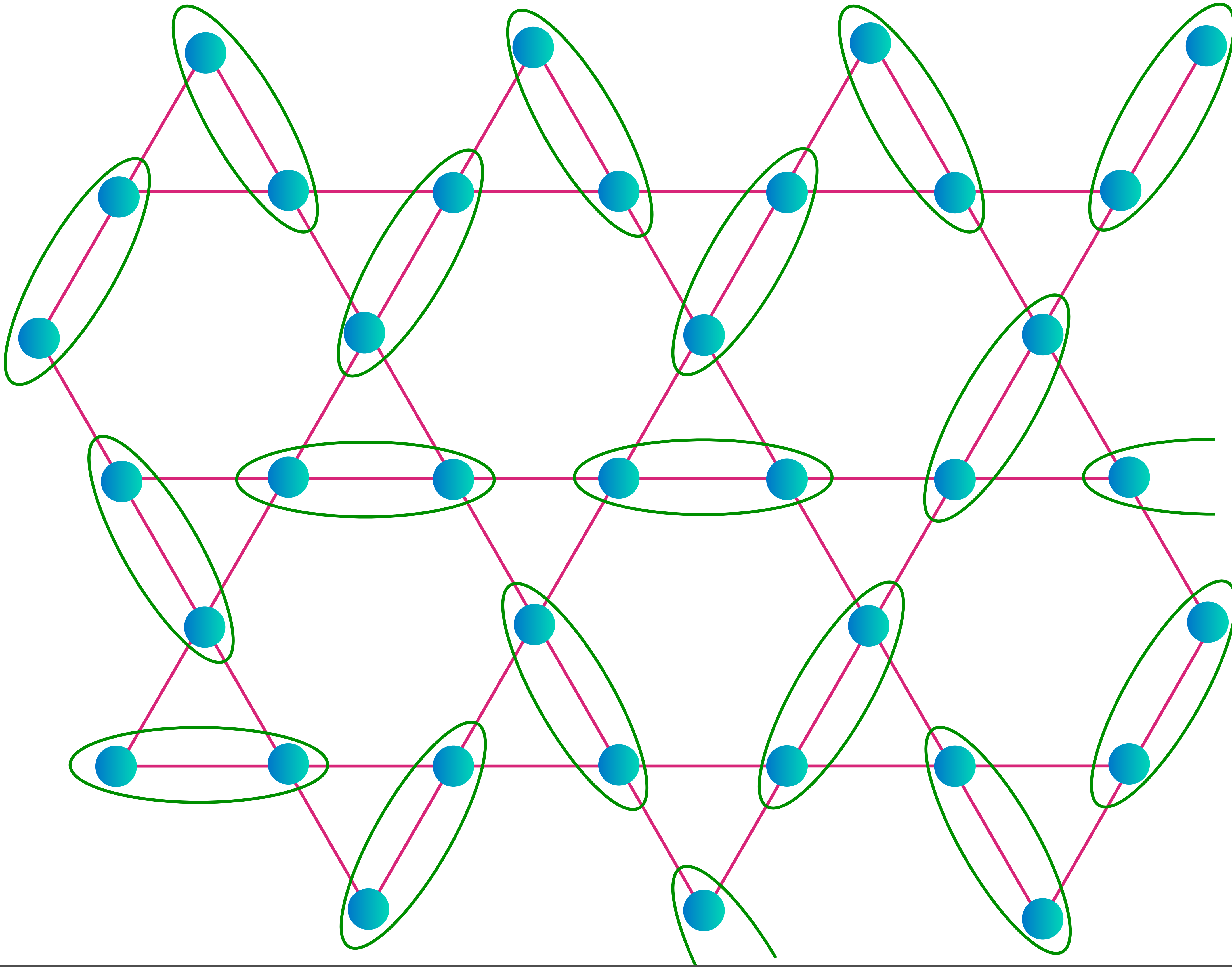
Pick a reference configuration



Kagome antiferromagnet

Alternative view

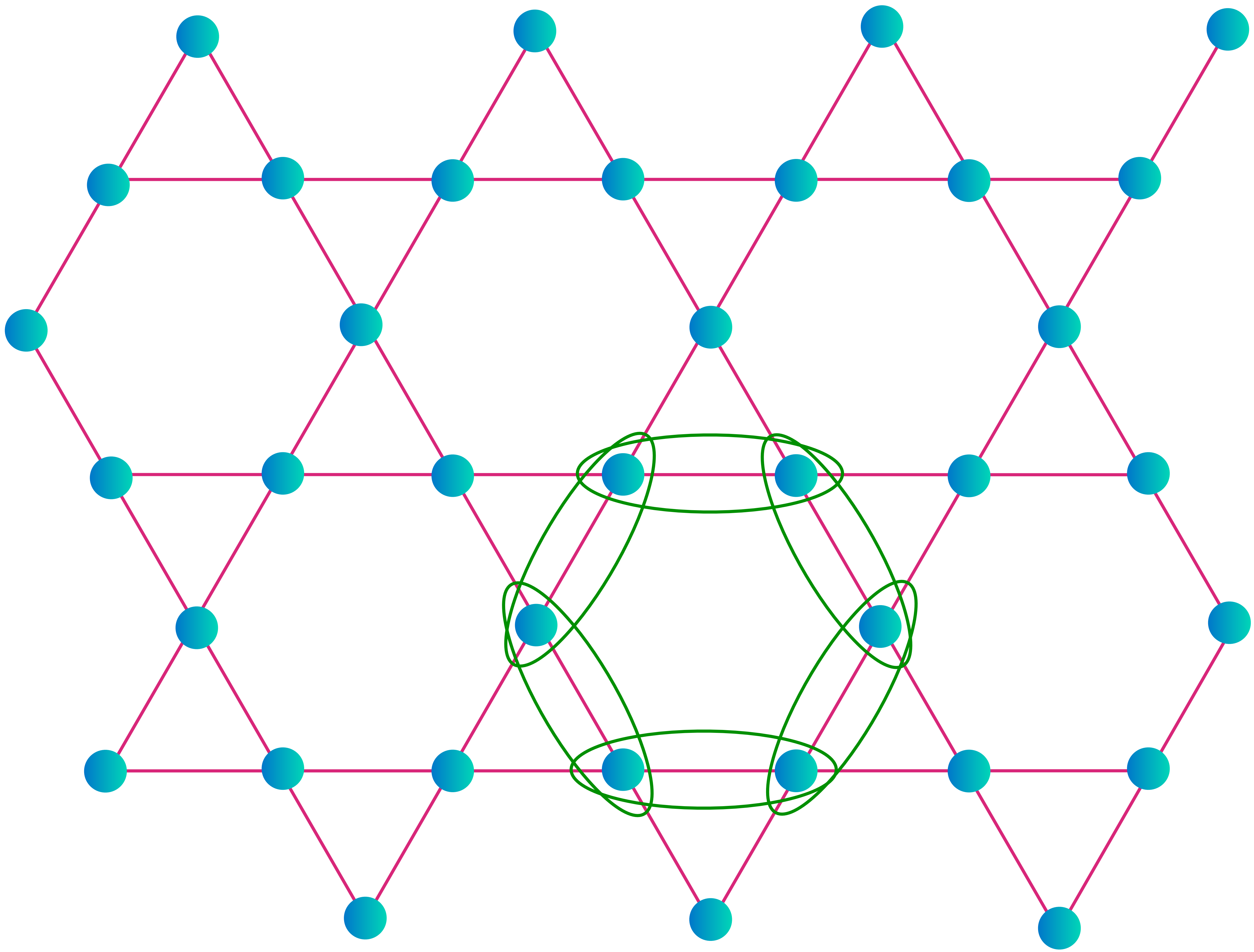
A nearby configuration



Kagome antiferromagnet

Alternative view

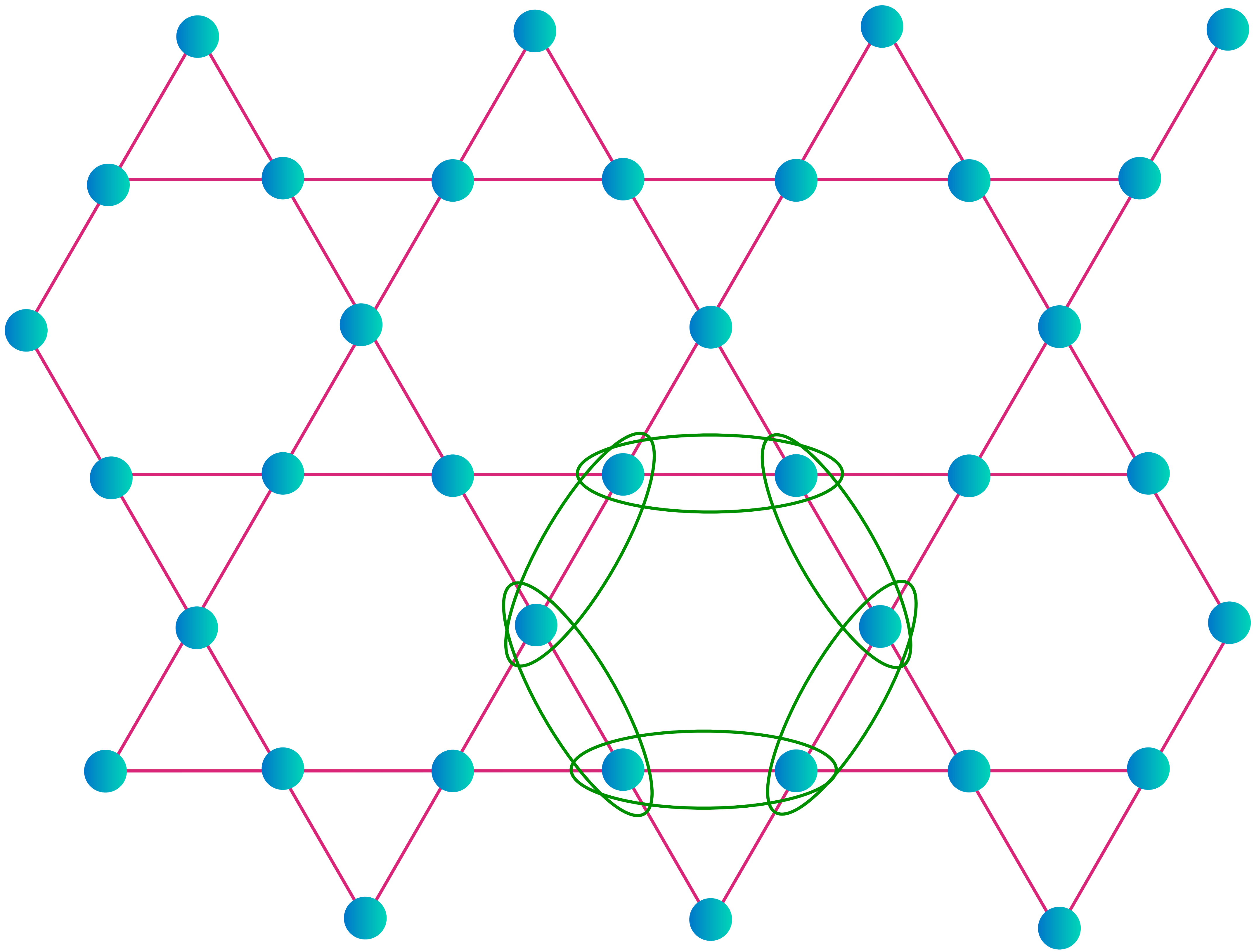
Difference: a closed loop



Kagome antiferromagnet

Alternative view

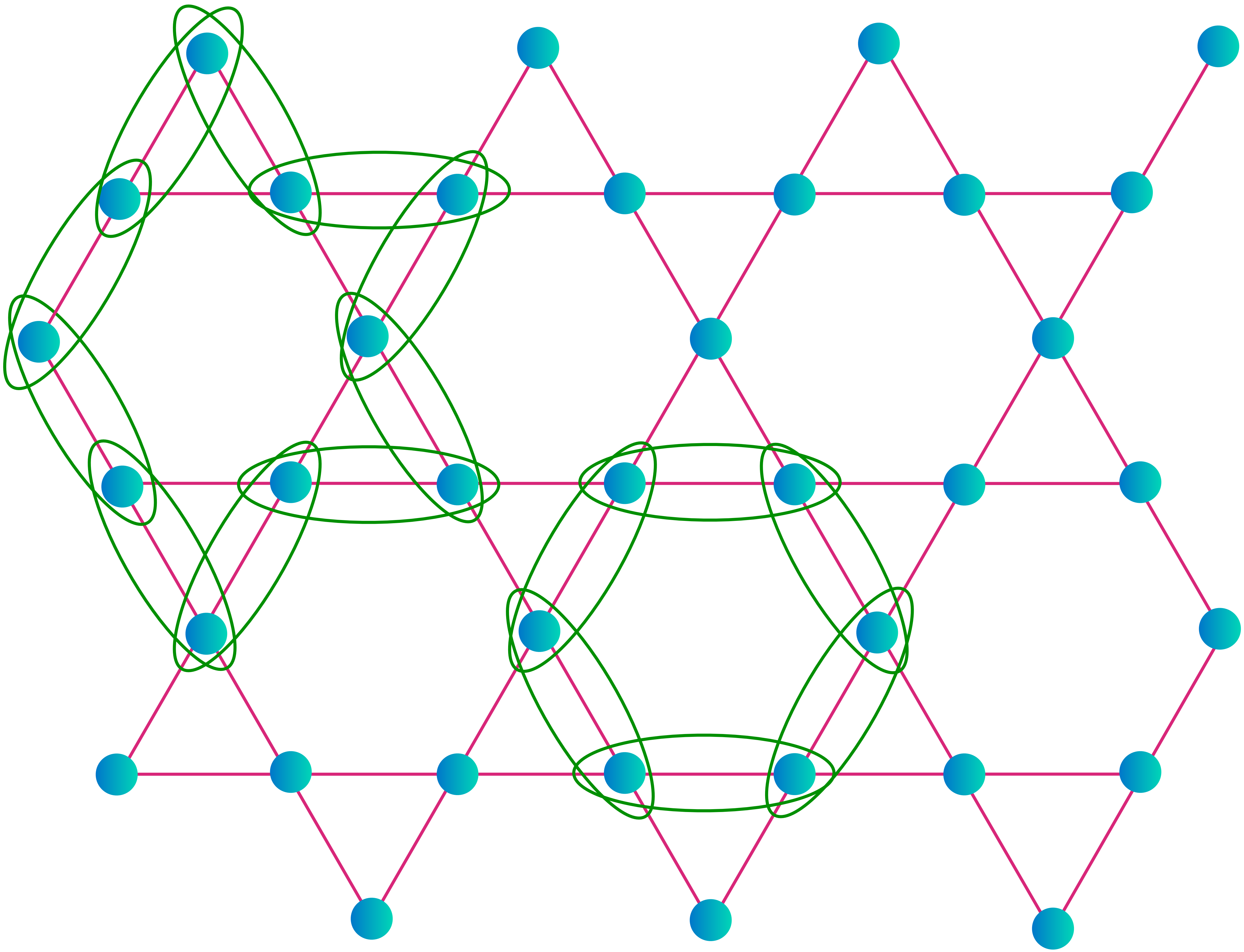
Ground state: sum over closed loops



Kagome antiferromagnet

Alternative view

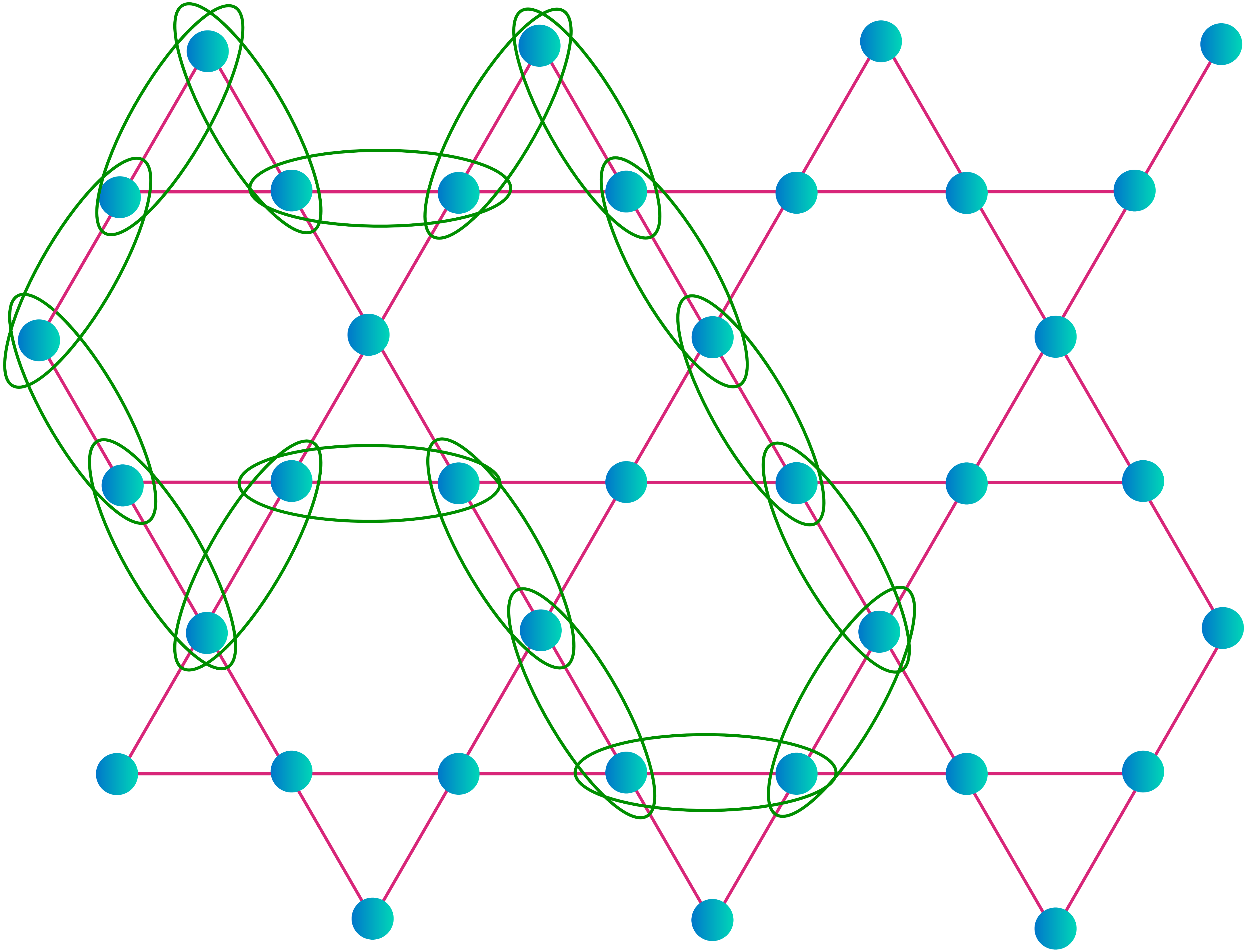
Ground state: sum over closed loops



Mott insulator: Kagome antiferromagnet

Alternative view

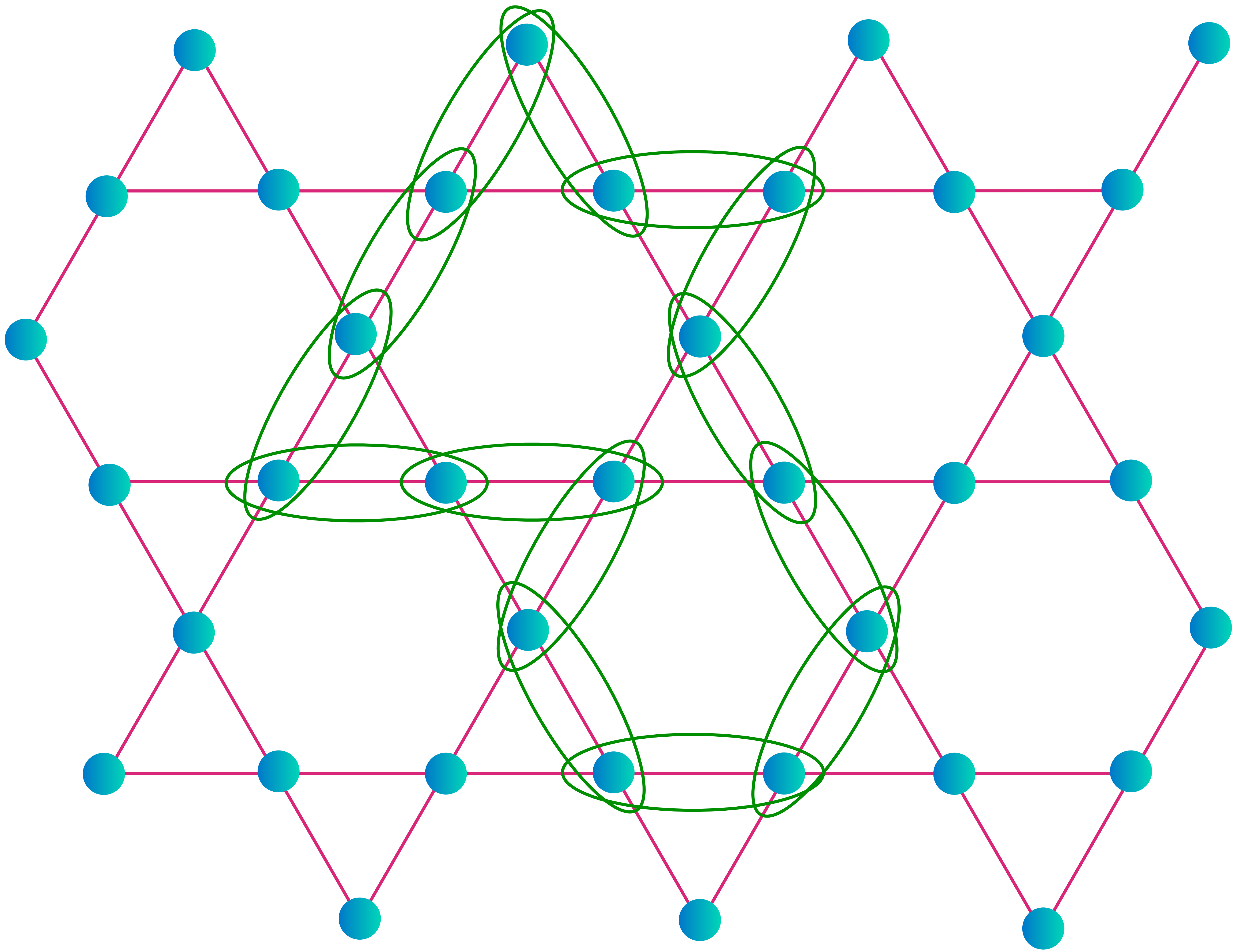
Ground state: sum over closed loops



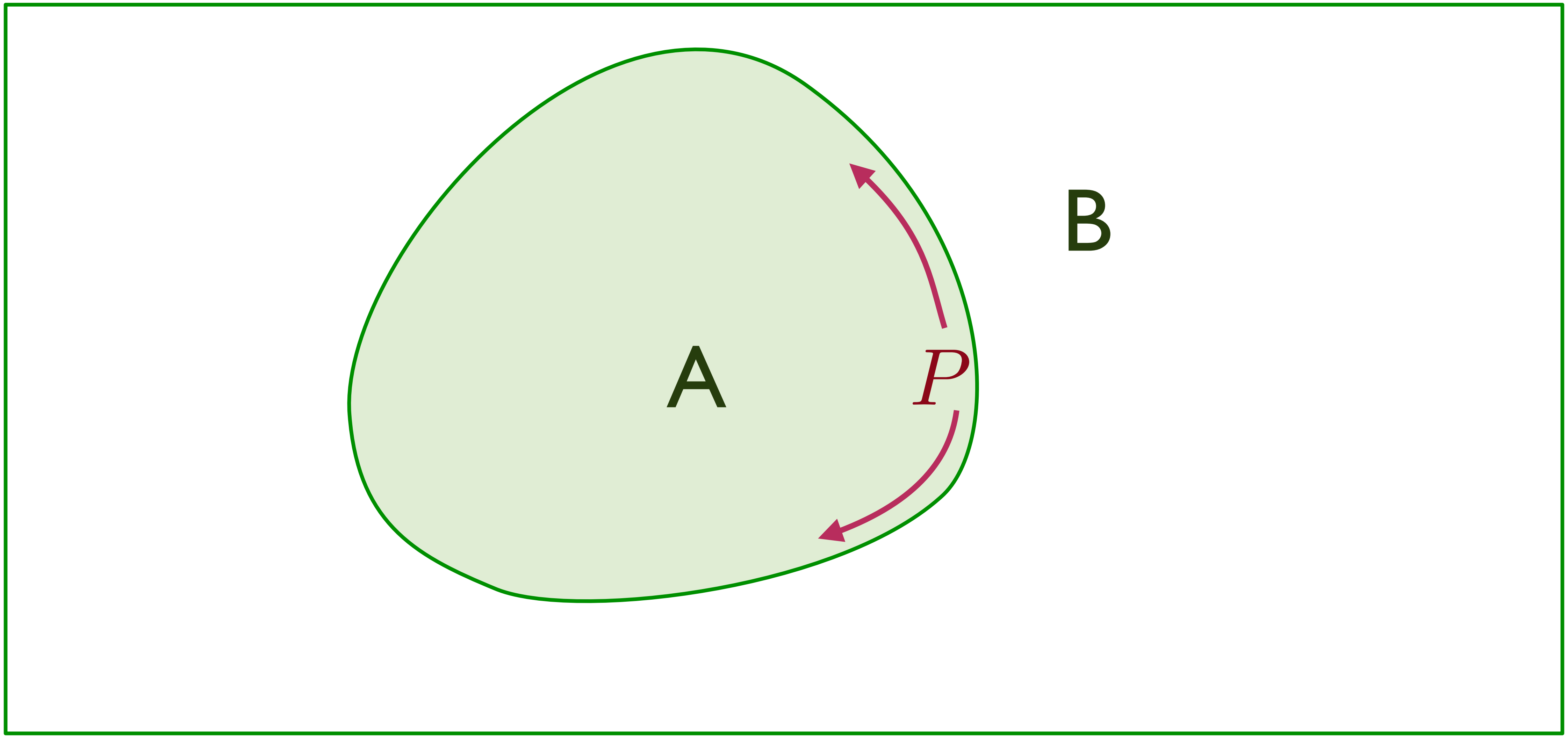
Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops



Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy

$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

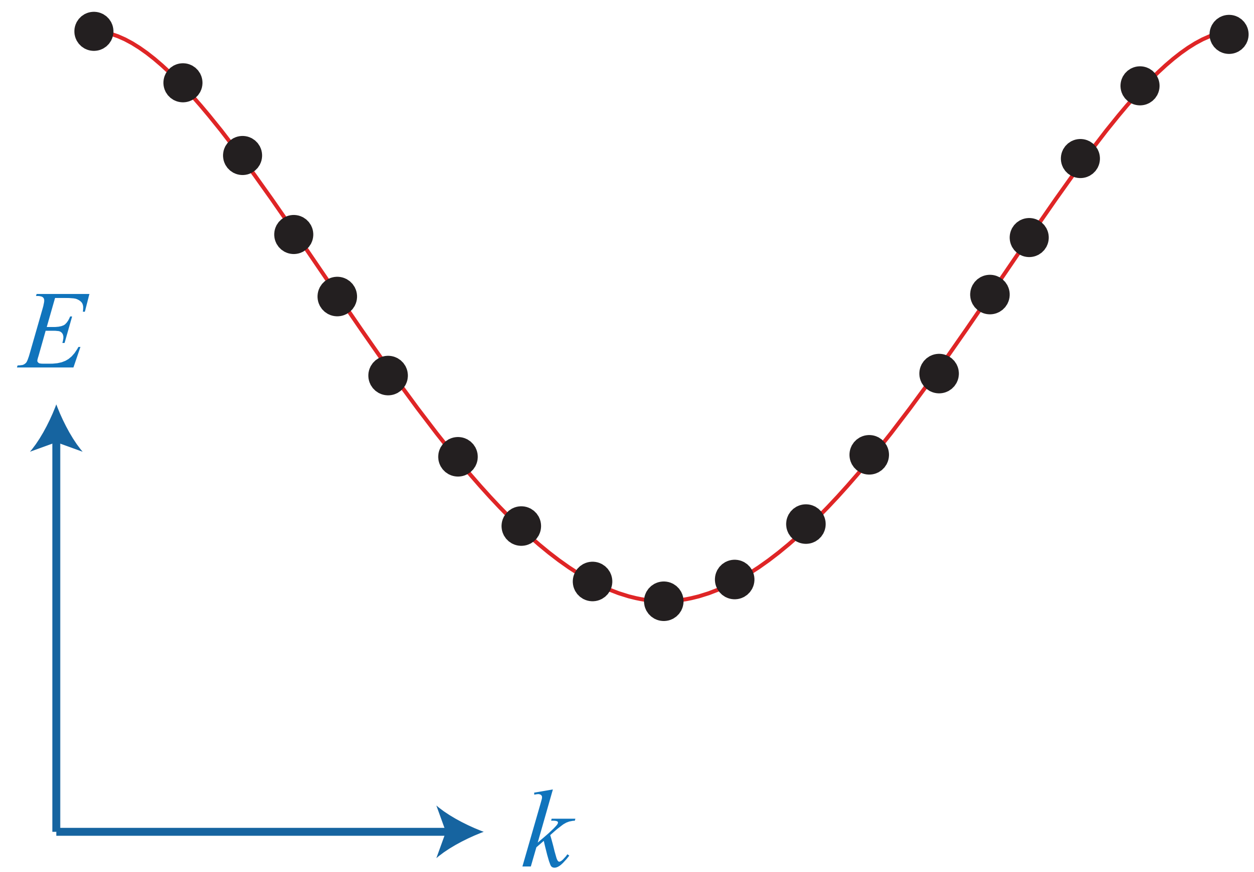
$$\text{Take } |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right)$$

$$\text{Then } \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$
$$= \frac{1}{2}\left(|\uparrow\rangle_A \langle\uparrow|_A + |\downarrow\rangle_A \langle\downarrow|_A\right)$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$
$$= \ln 2$$

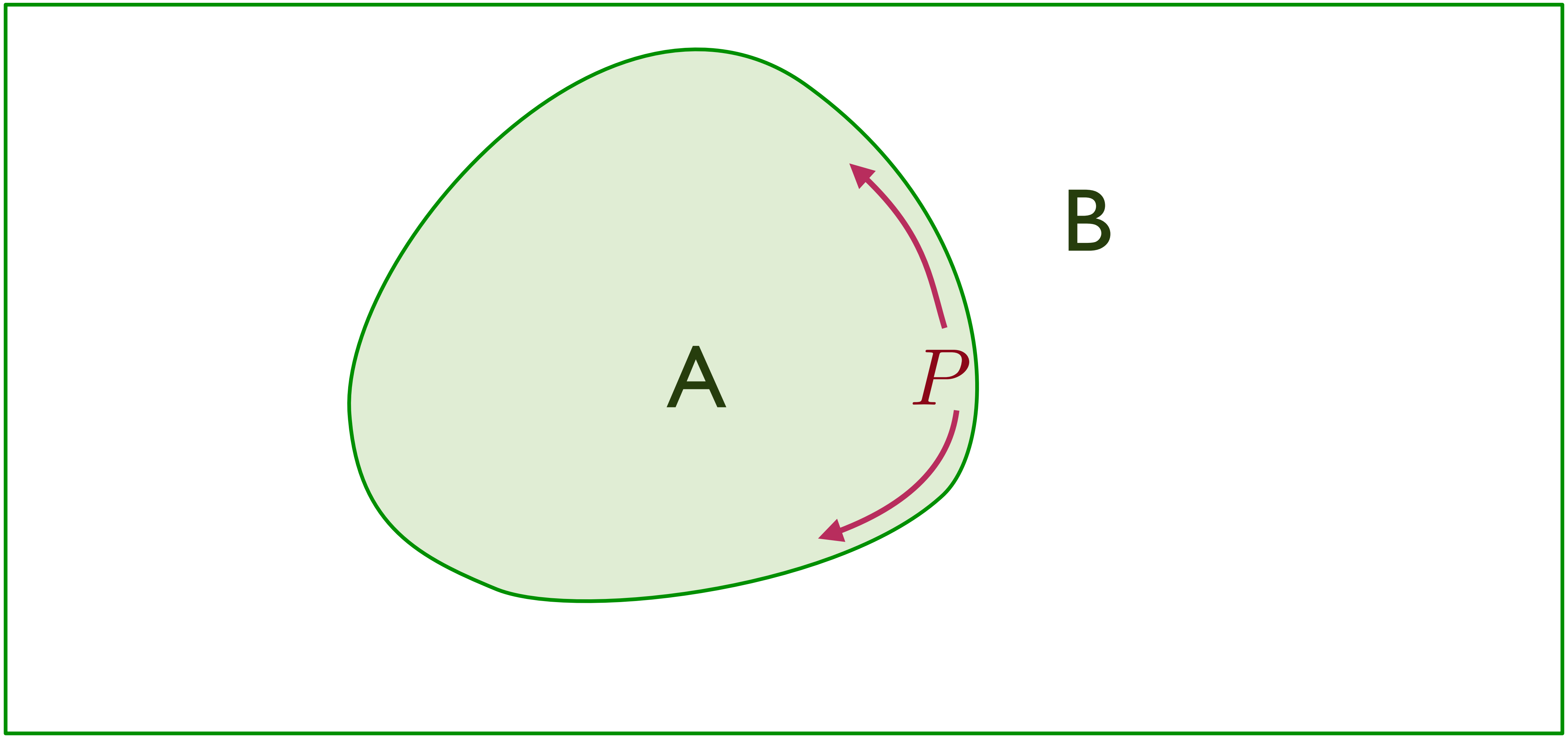
Entanglement entropy of a band insulator

Band insulators



An even number of electrons per unit cell

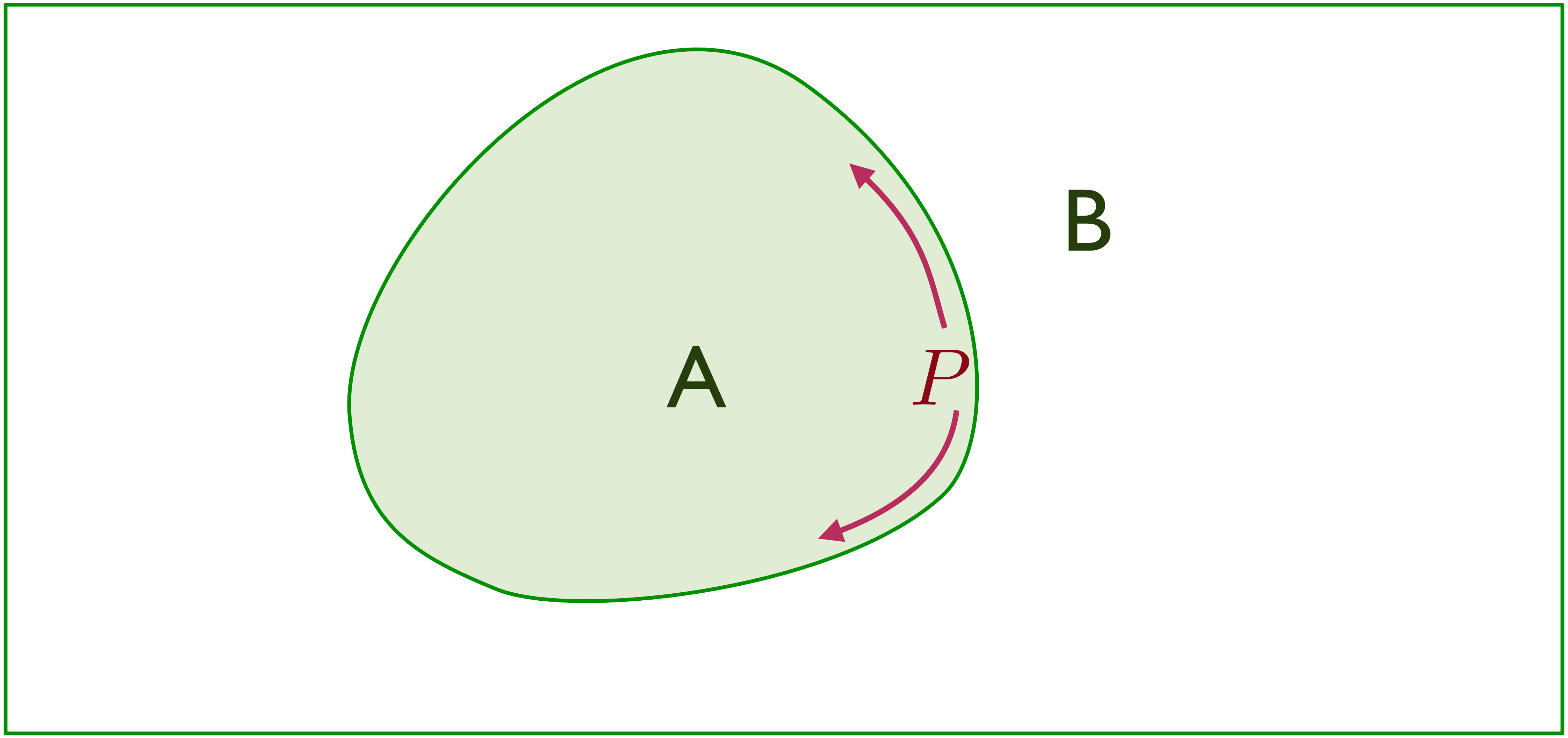
Entanglement entropy of a band insulator



$$S_E = aP - b\exp(-cP)$$

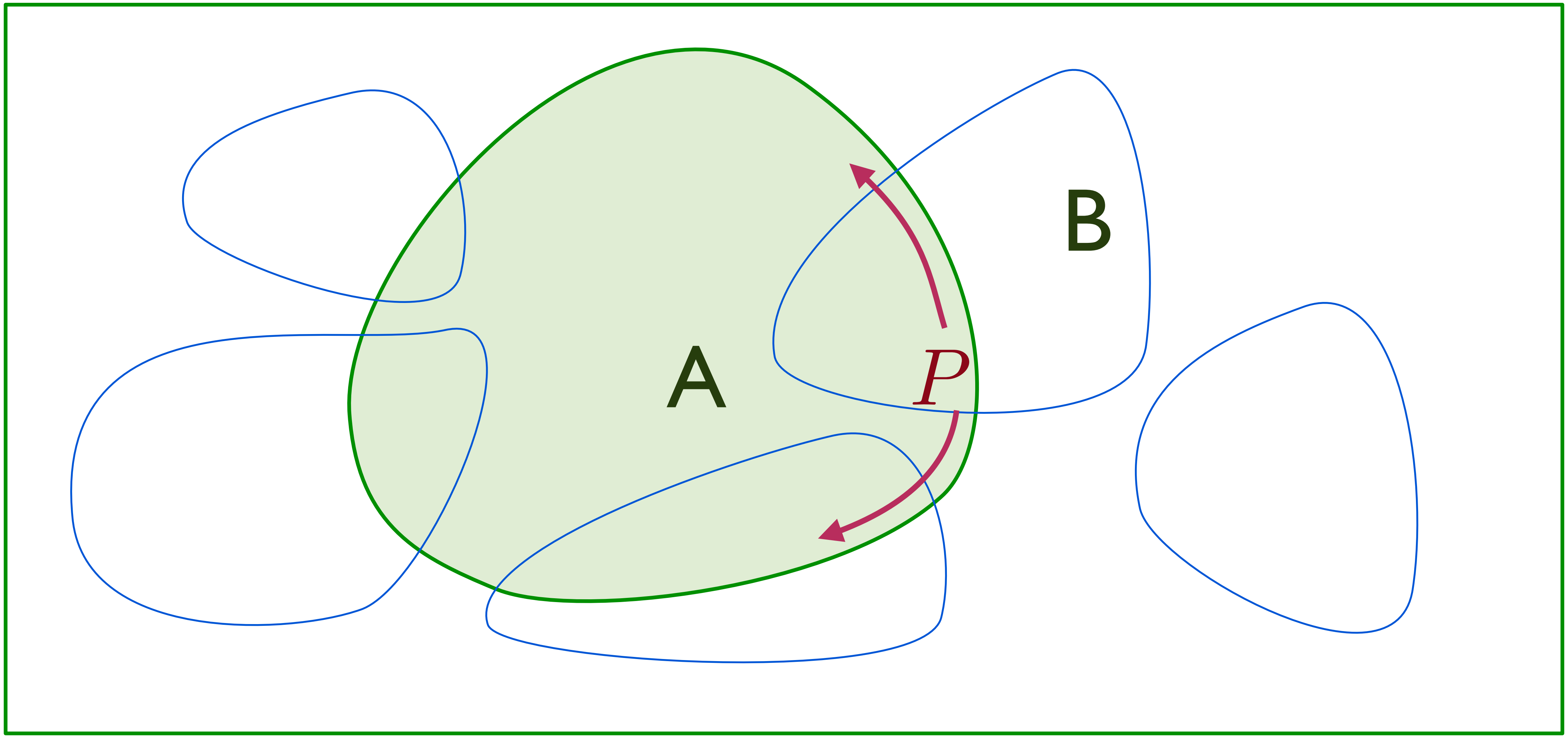
where P is the surface area (perimeter) of the boundary between A and B.

Entanglement in the Z_2 spin liquid



The sum over closed loops is characteristic of the Z_2 spin liquid, introduced in
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991),
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

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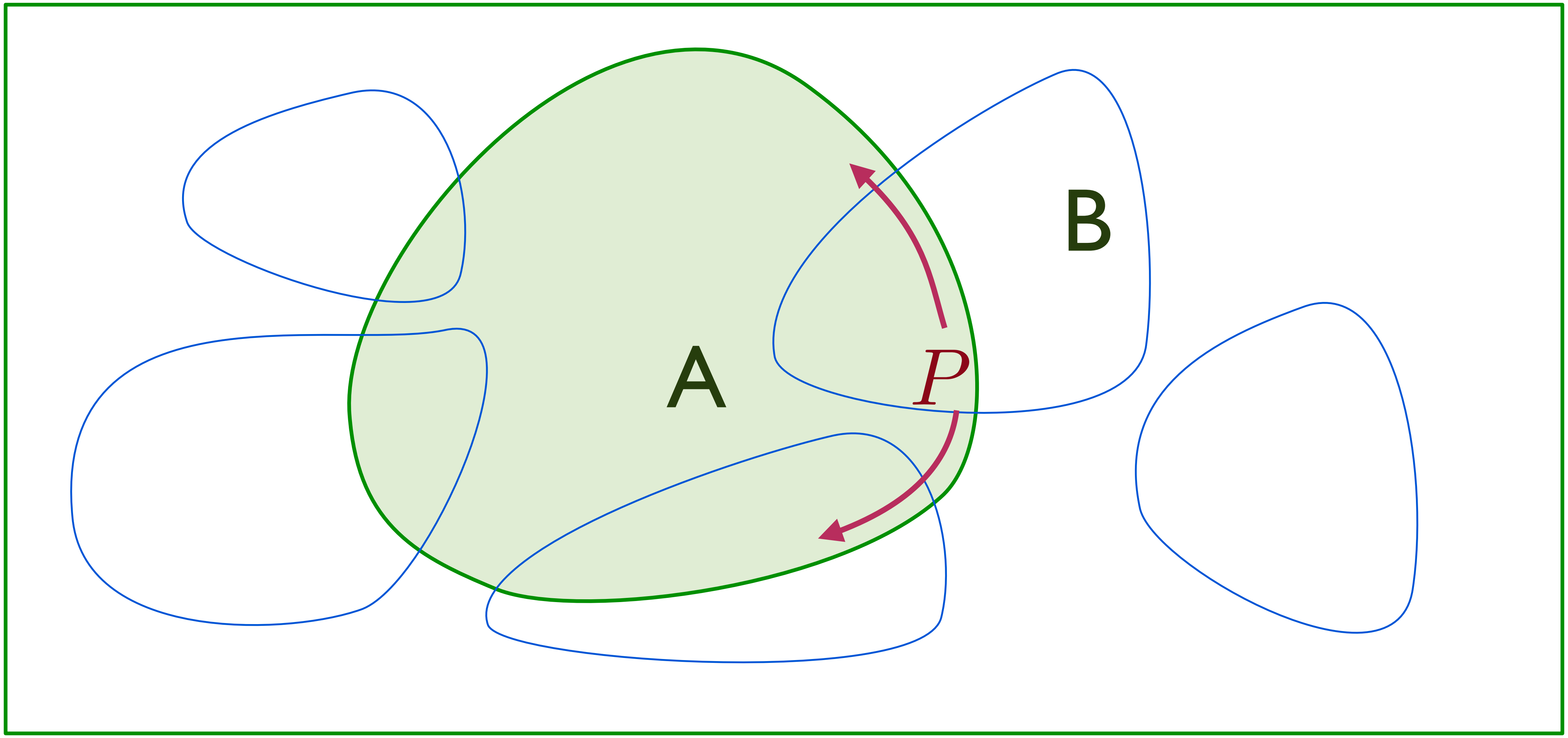
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Entanglement in the Z_2 spin liquid

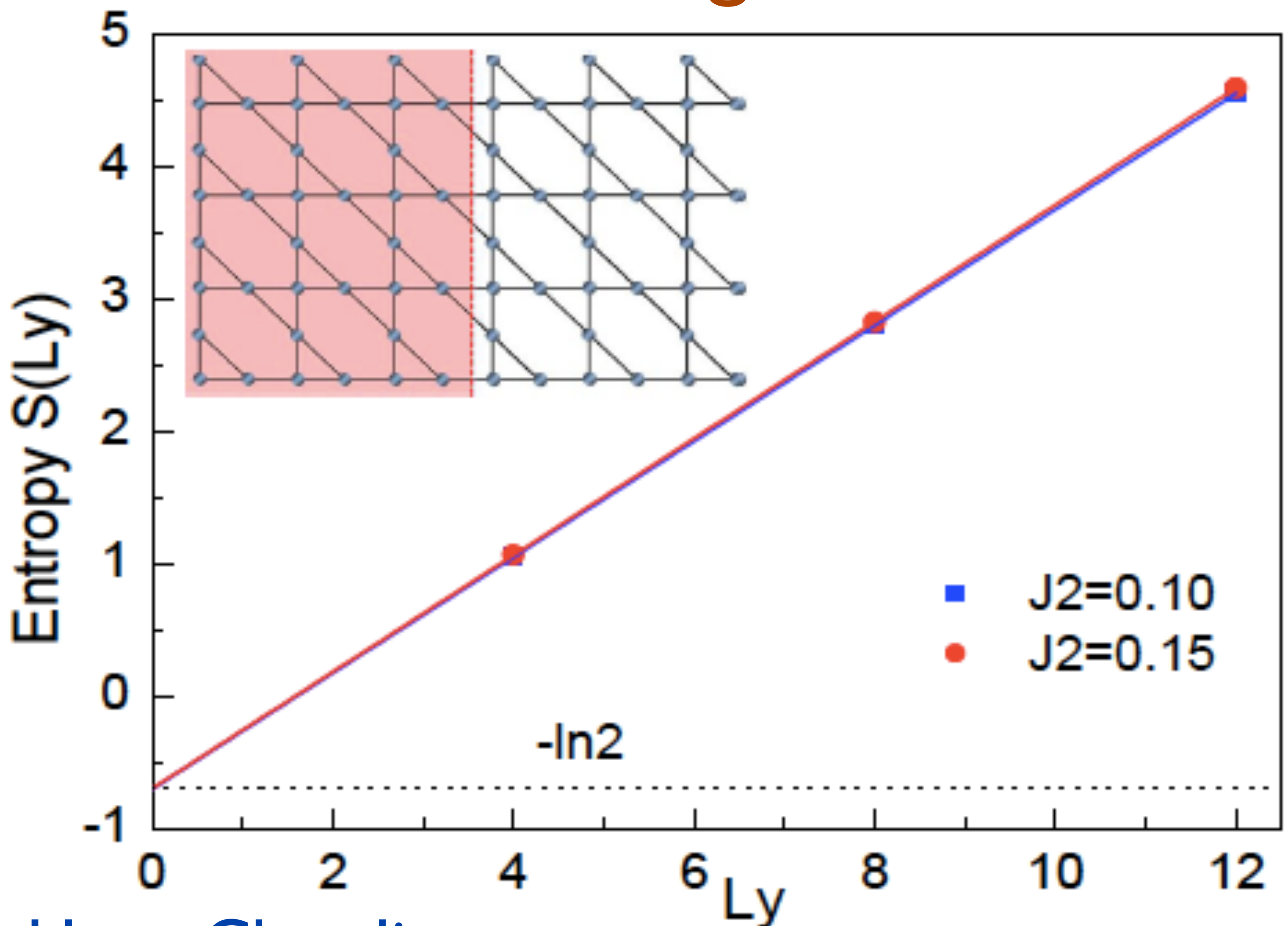


$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

A. Hamma, R. Ionicioiu, and P. Zanardi, Phys. Rev. A **71**, 022315 (2005)
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006); A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006)
Y. Zhang, T. Grover, and A. Vishwanath, Phys. Rev. B **84**, 075128 (2011)

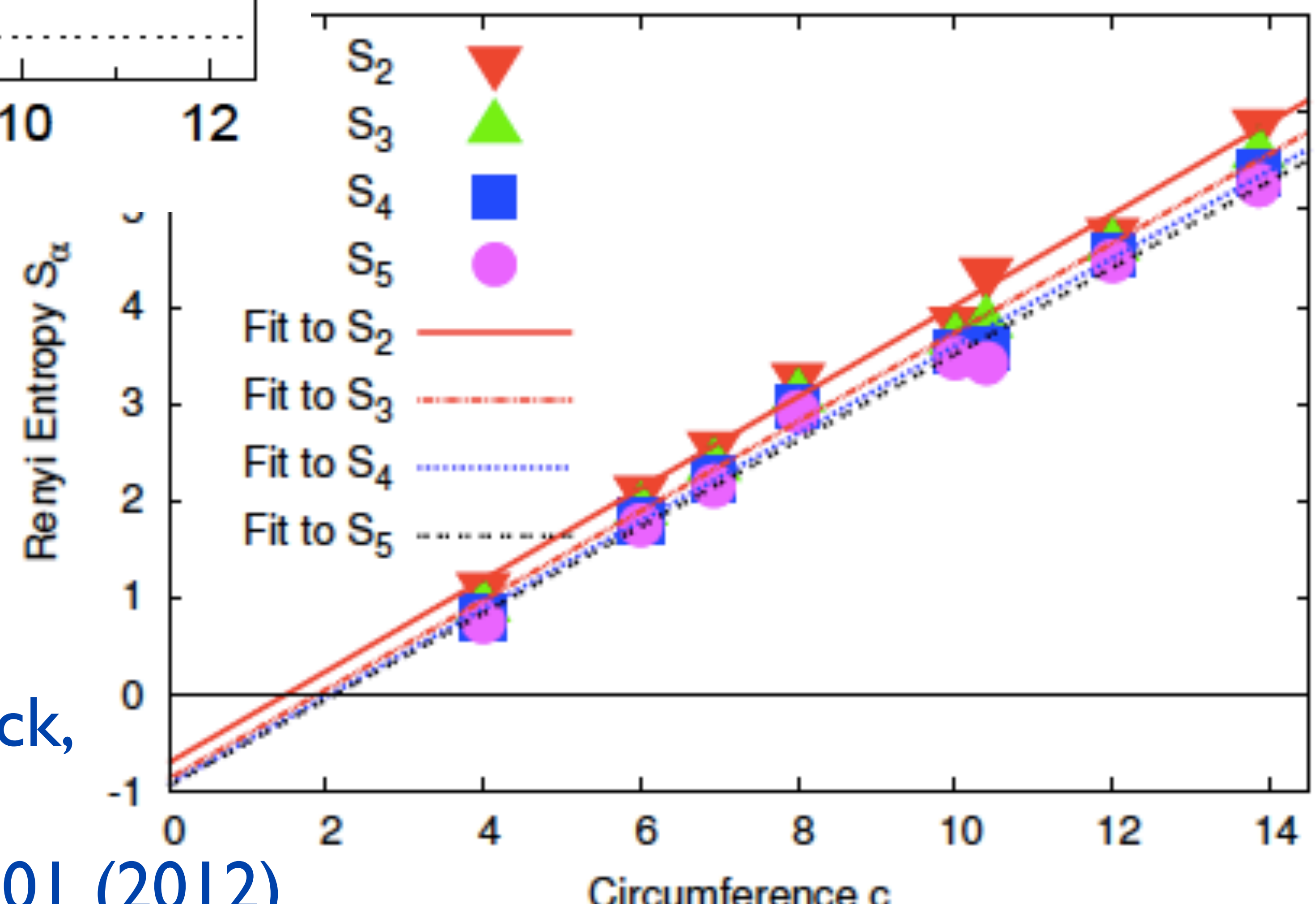
Mott insulator: Kagome antiferromagnet



Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, *Nature Physics* **8**, 902 (2012)



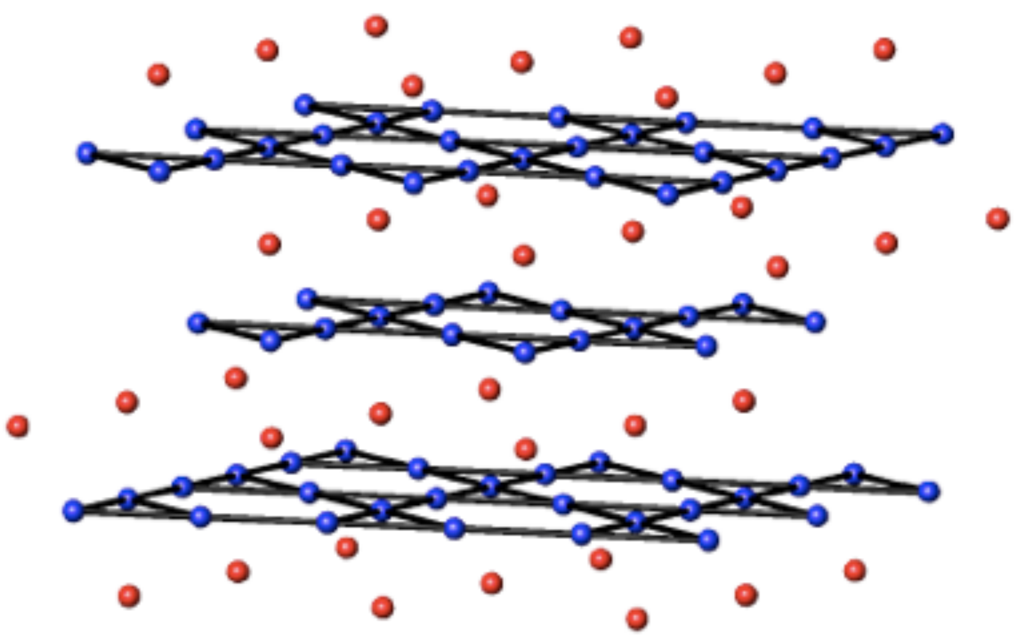
S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, *Physical Review Letters* **109**, 067201 (2012)

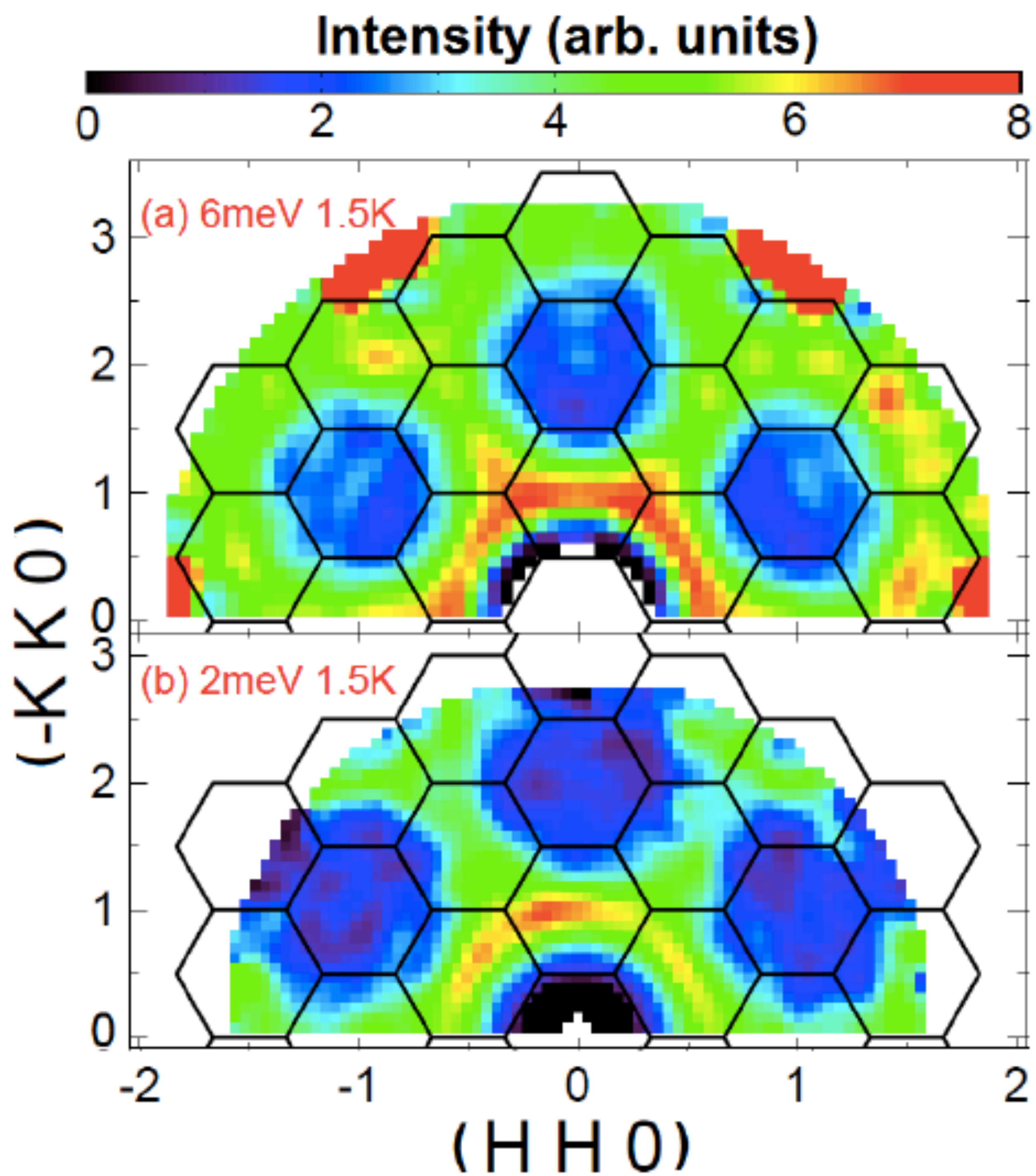
Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han[1], Joel S. Helton[2], Shaoyan Chu[3], Daniel G. Nocera[4], Jose A. Rodriguez-Rivera[2,5], Collin Broholm[2,6] & Young S. Lee[1]

Nature **492**, 406 (2012)

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





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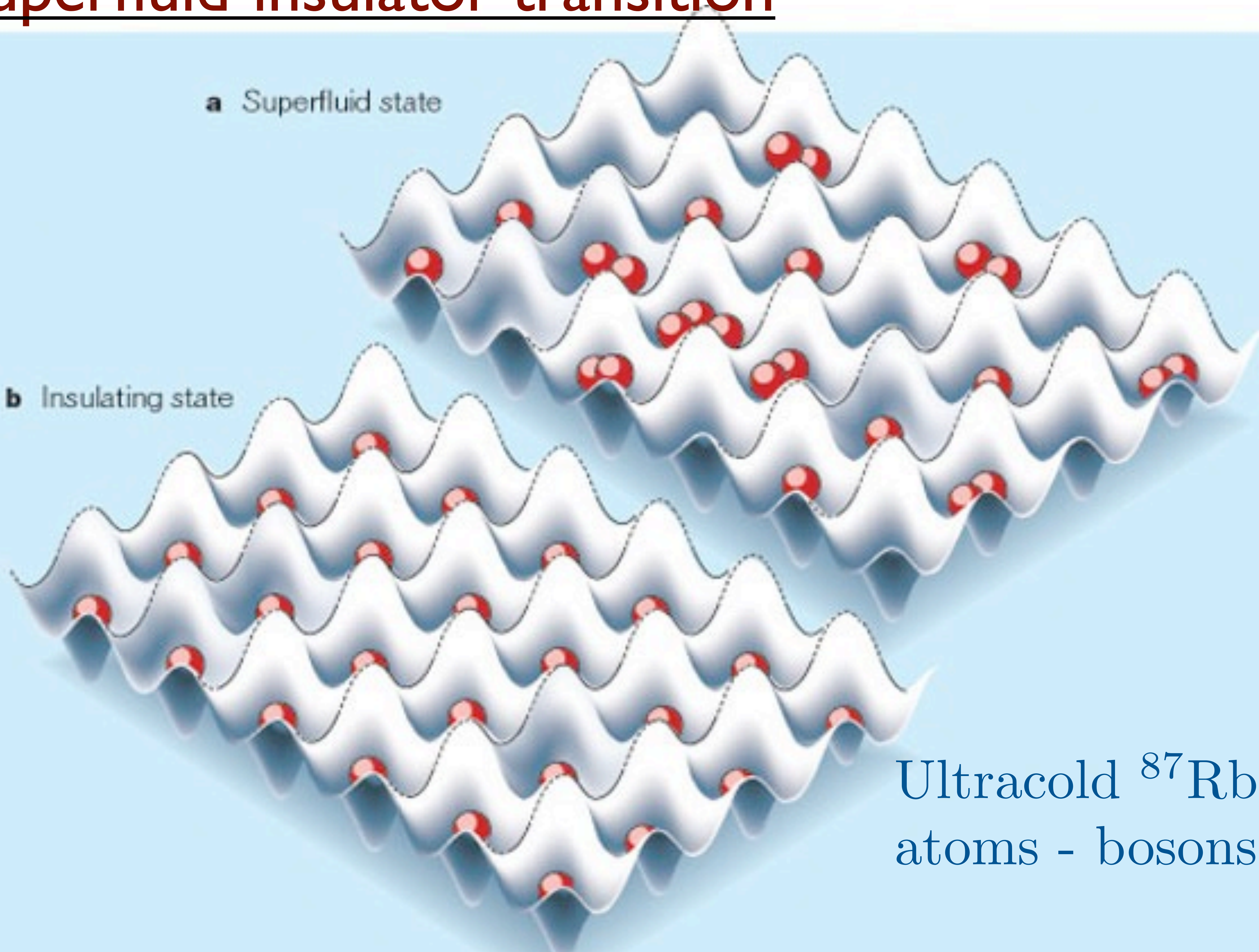
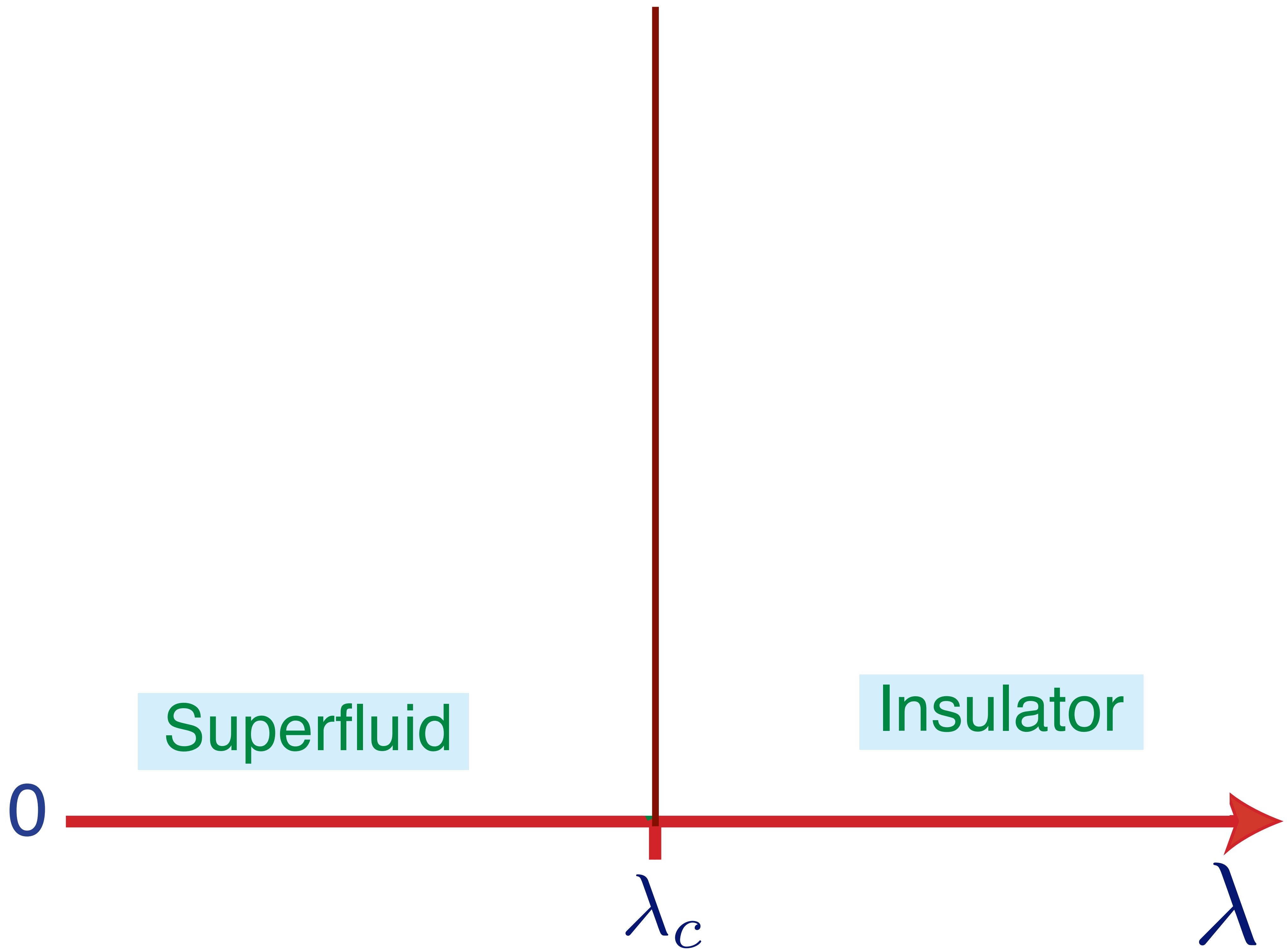
Superfluid-insulator transition

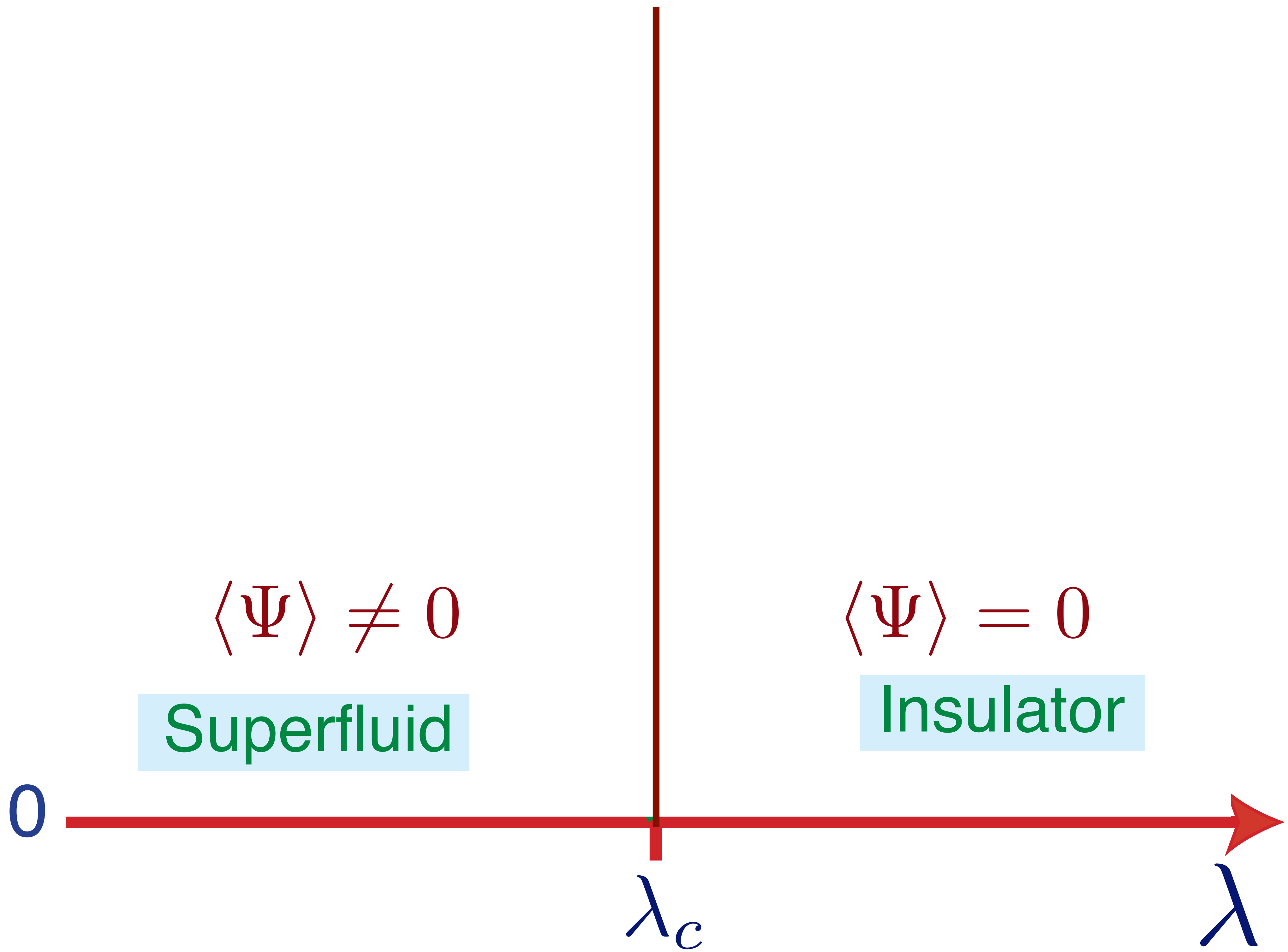


Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

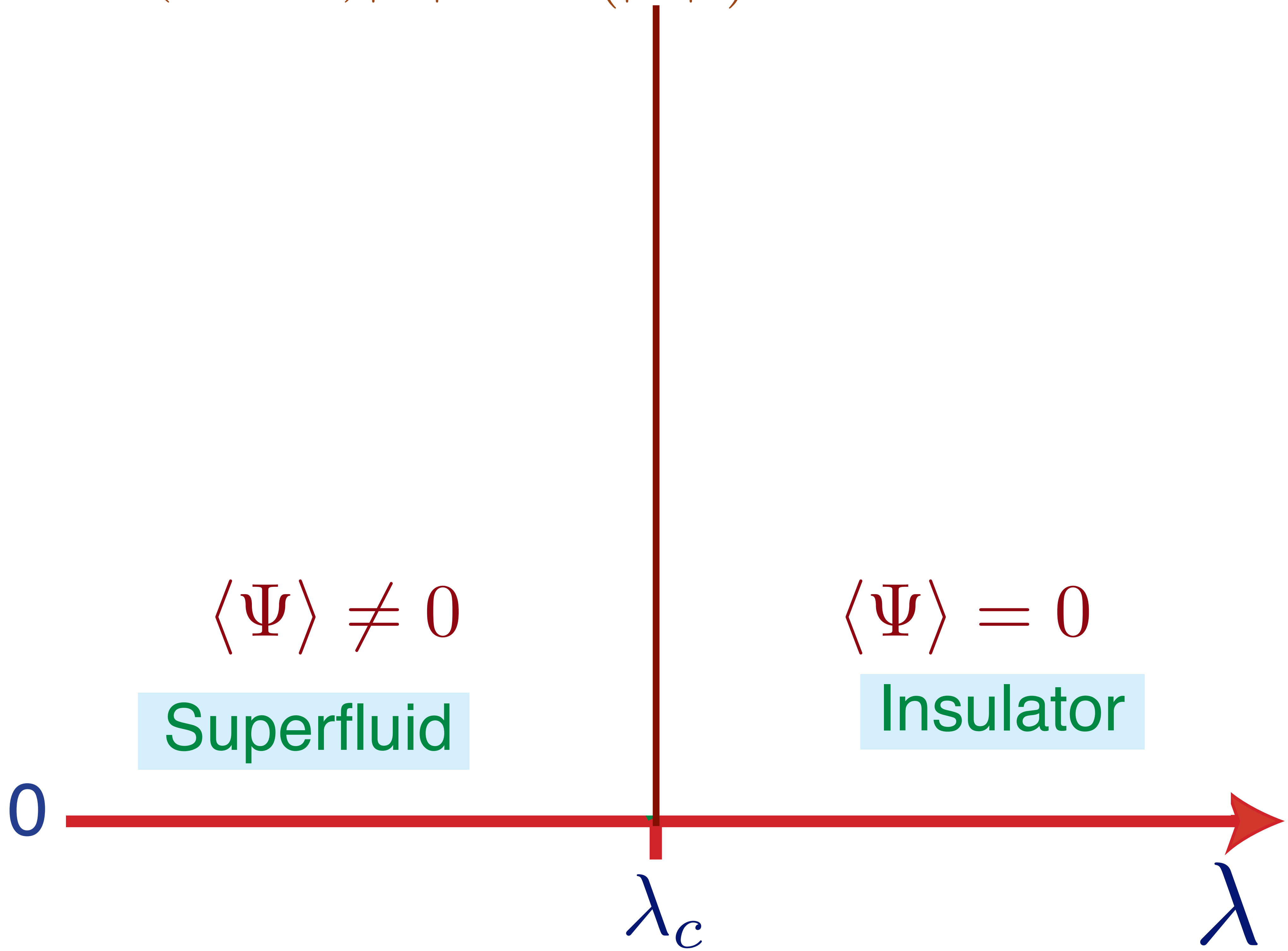

Superfluid
Insulator
0
λ_c
λ

$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



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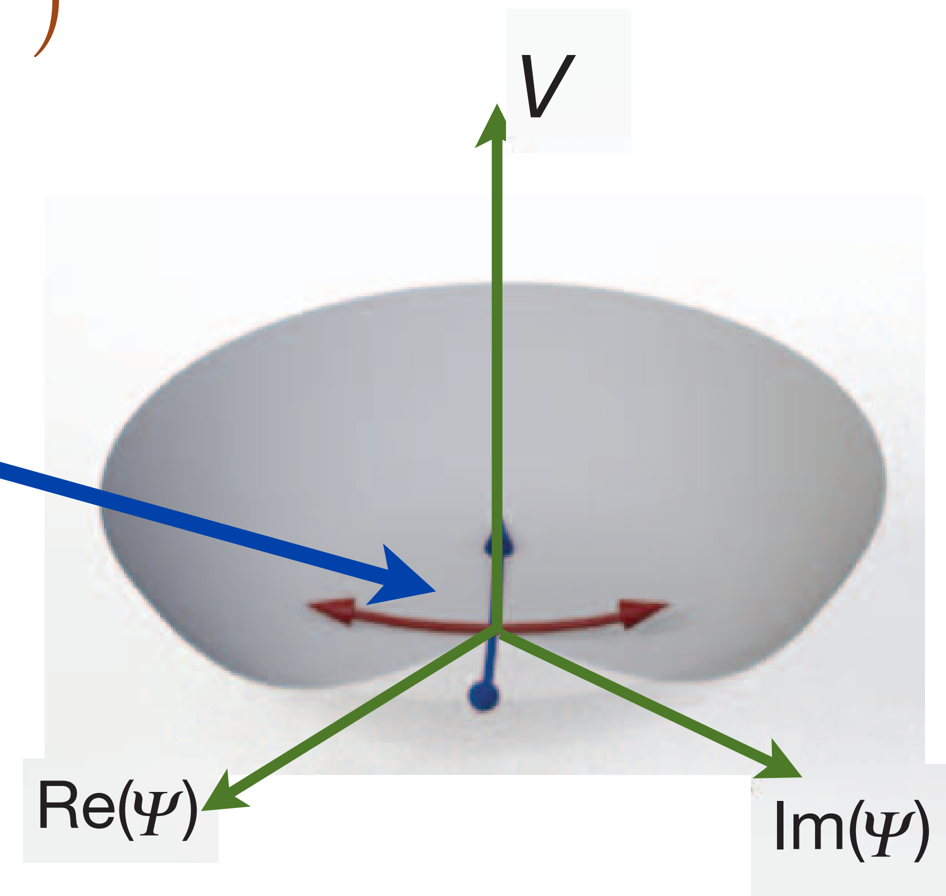
Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



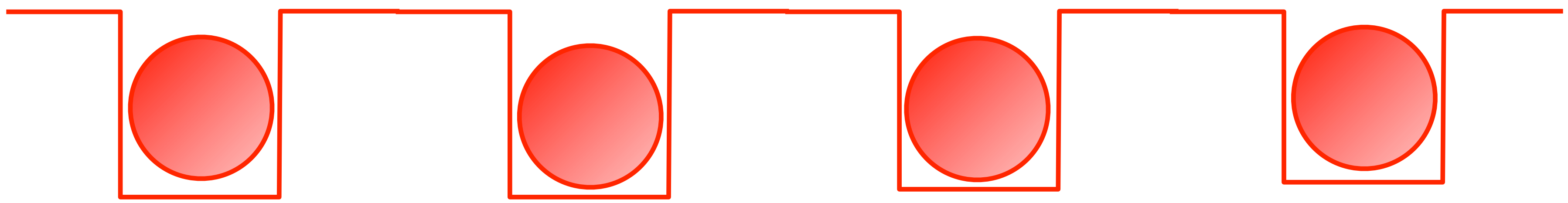
$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

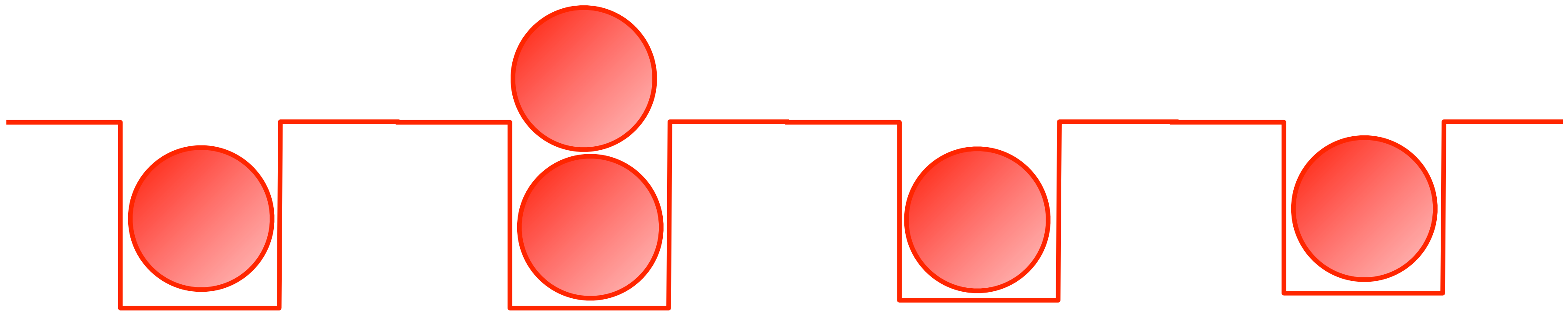
Insulator

0 λ_c λ



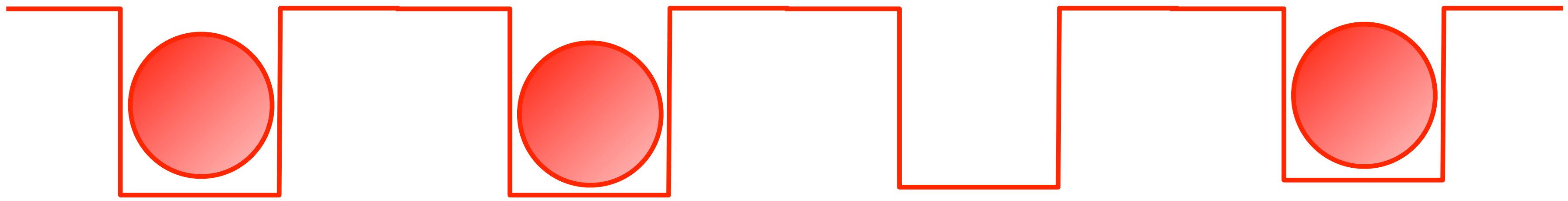
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



$$\text{Particles} \sim \Psi^\dagger$$

Excitations of the insulator:



Holes $\sim \Psi$

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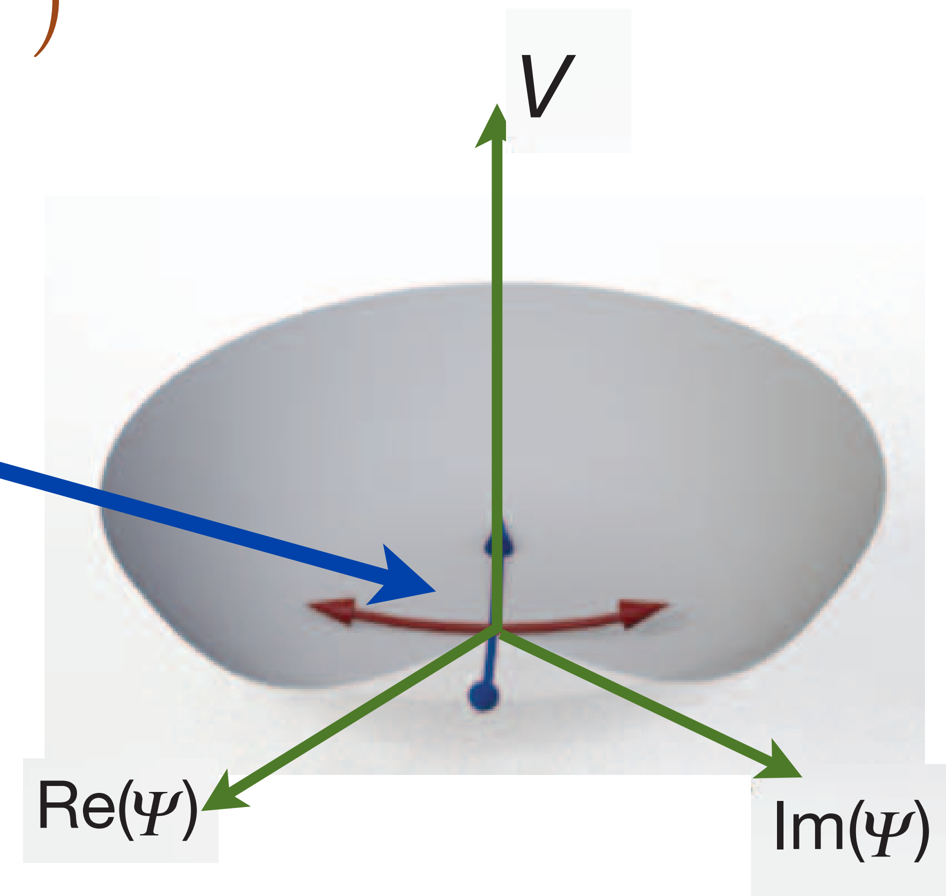

$\langle \Psi \rangle \neq 0$ Superfluid

$\langle \Psi \rangle = 0$ Insulator

0 λ_c λ

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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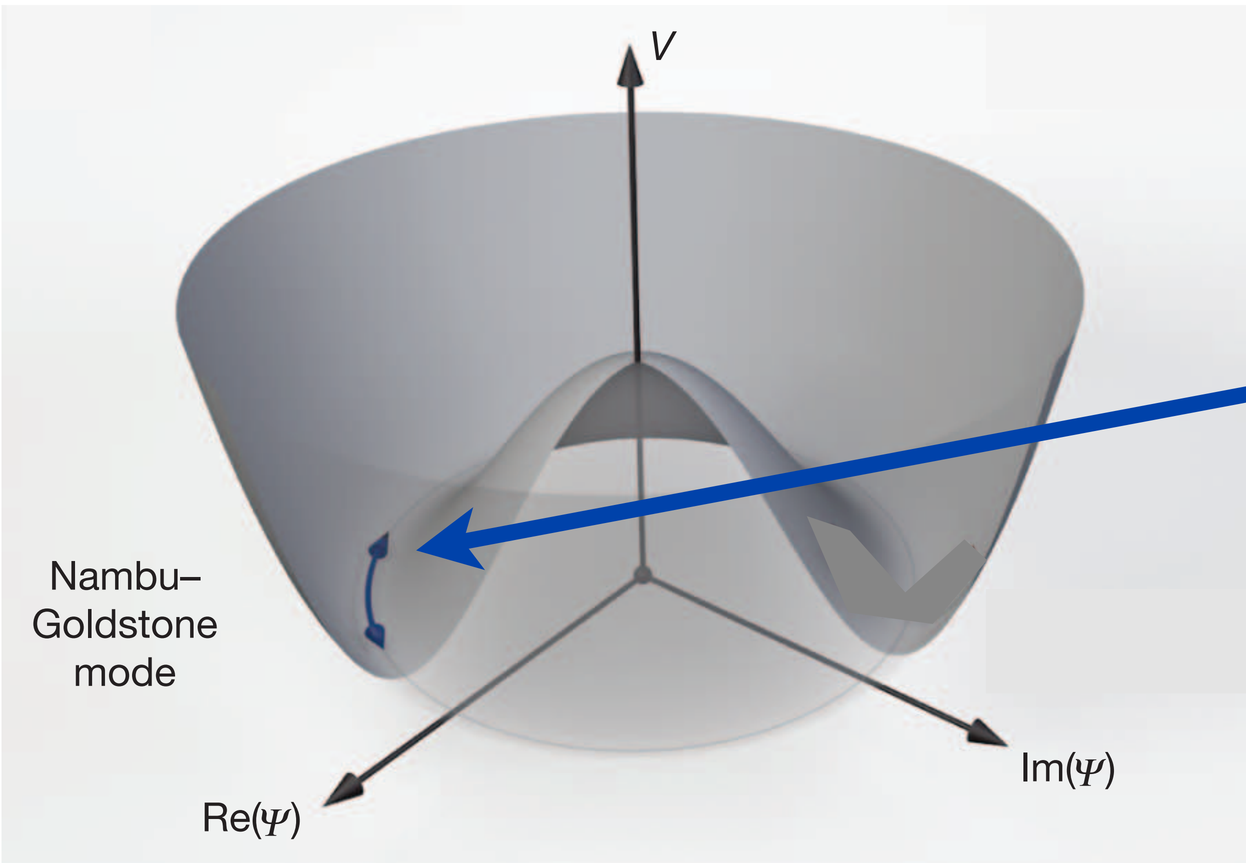


Nambu-Goldstone mode is the oscillation in the phase Ψ at a constant non-zero $|\Psi|$.

$\langle \Psi \rangle \neq 0$

Superfluid

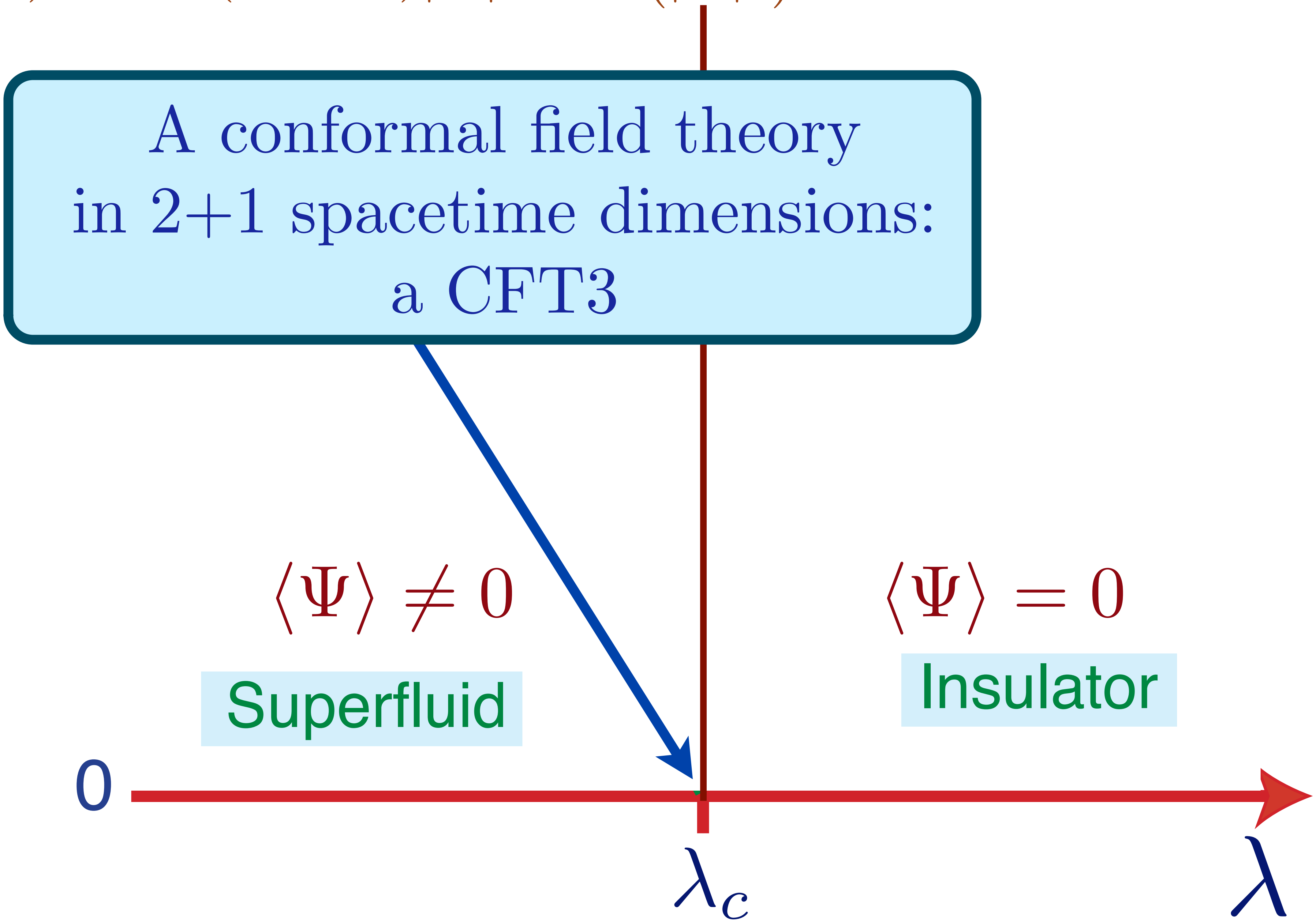
$\langle \Psi \rangle = 0$

Insulator

0 — λ_c — λ

$$
\mathcal{S} = \int d^2 r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]
$$

$$
V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2
$$



$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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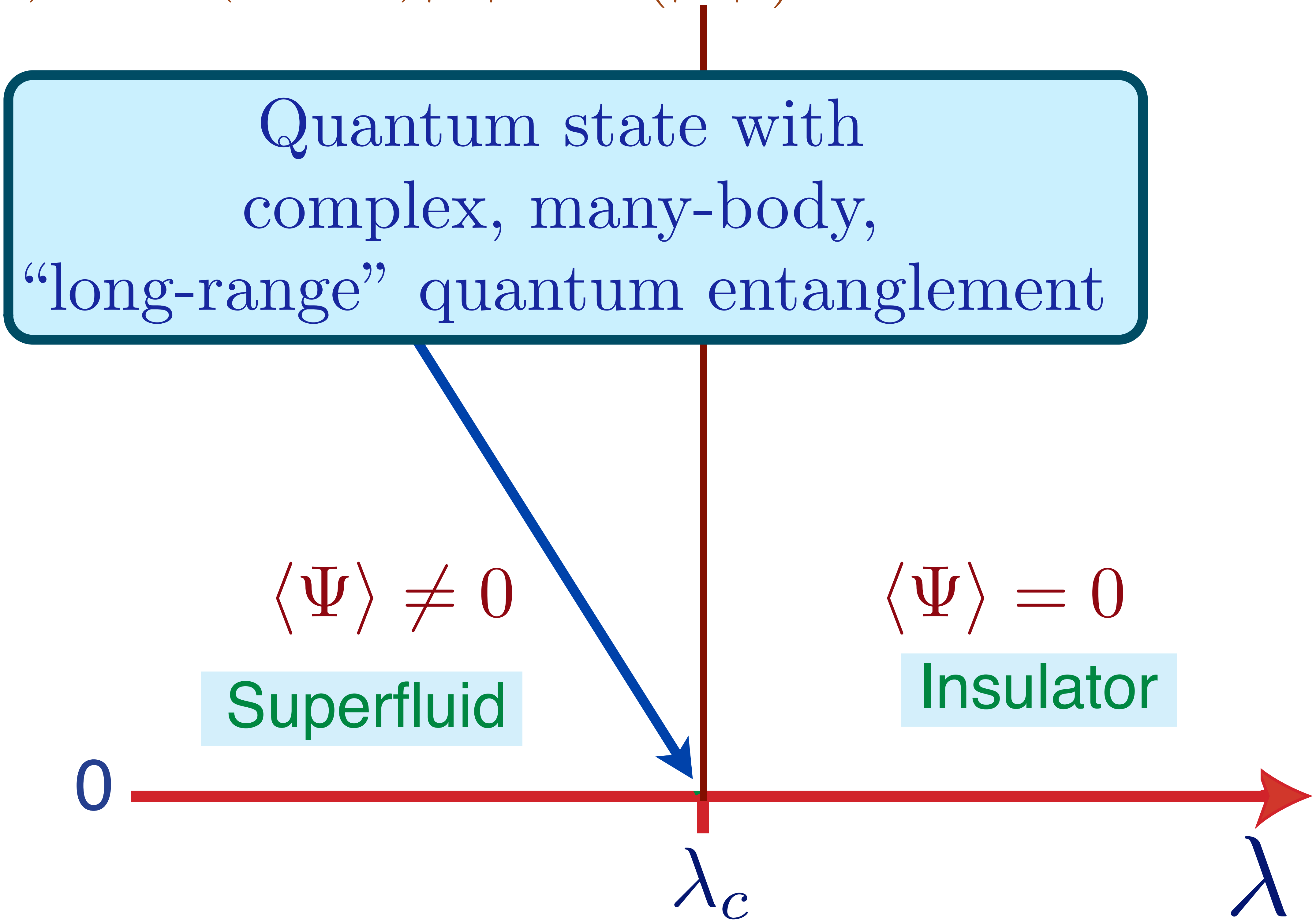

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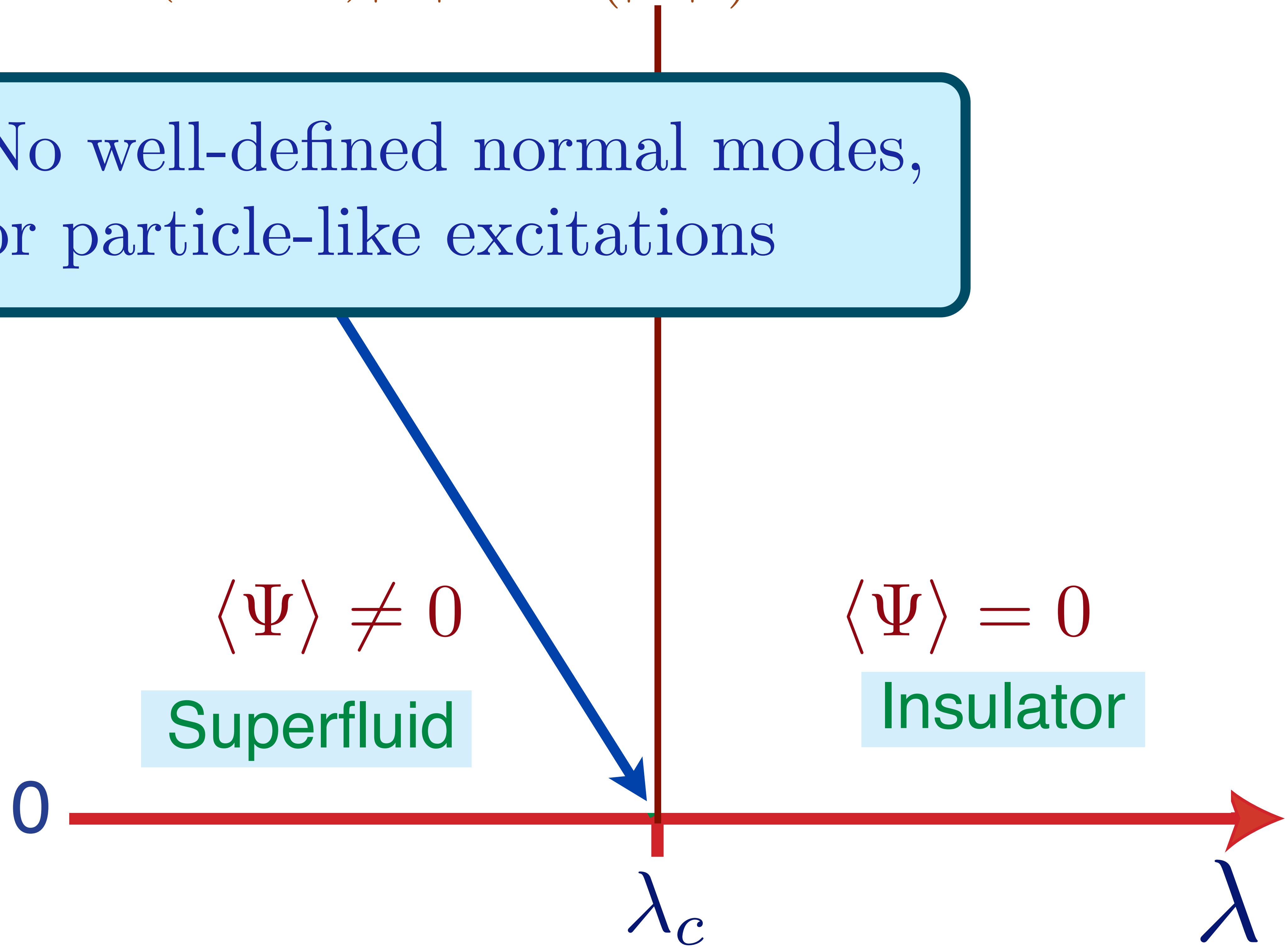

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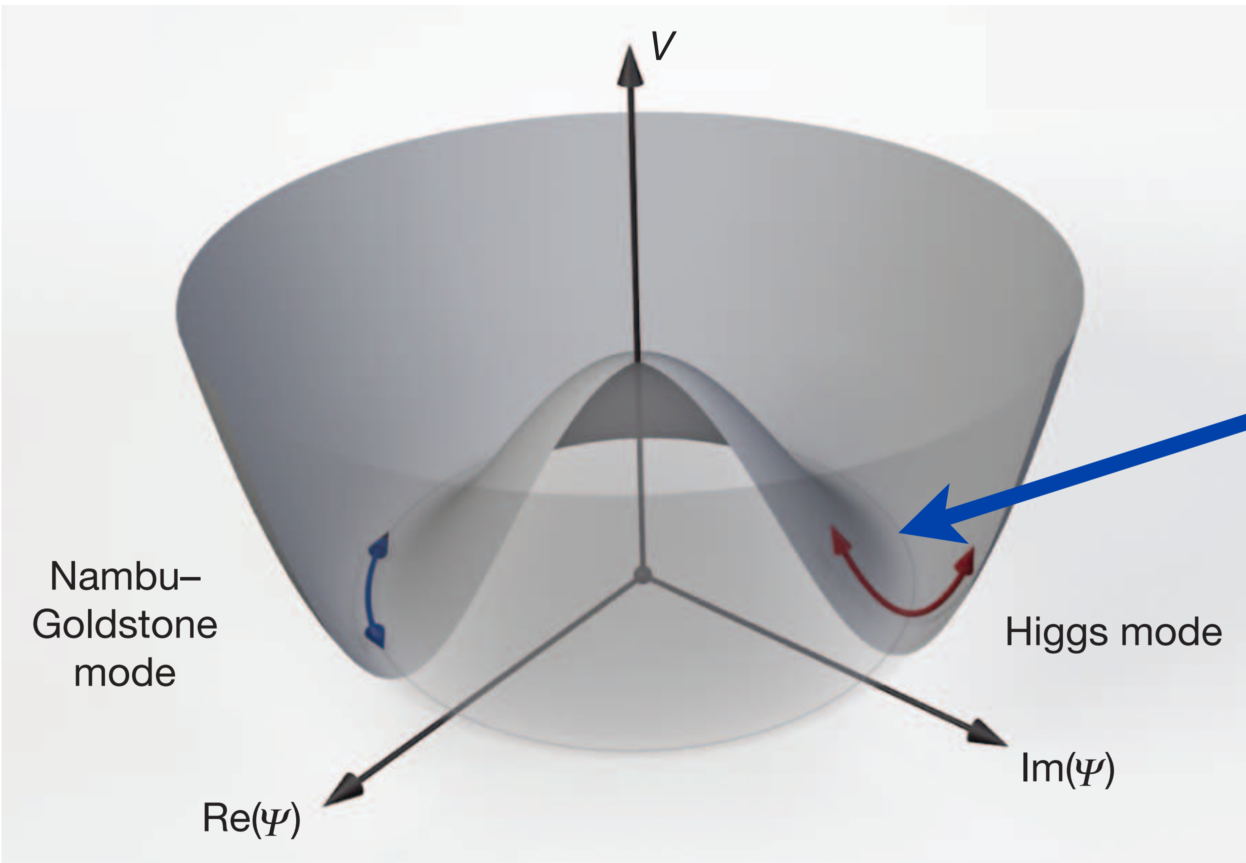

Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes.

$\langle \Psi \rangle \neq 0$ — Superfluid

$\langle \Psi \rangle = 0$ — Insulator

0 — λ_c — λ

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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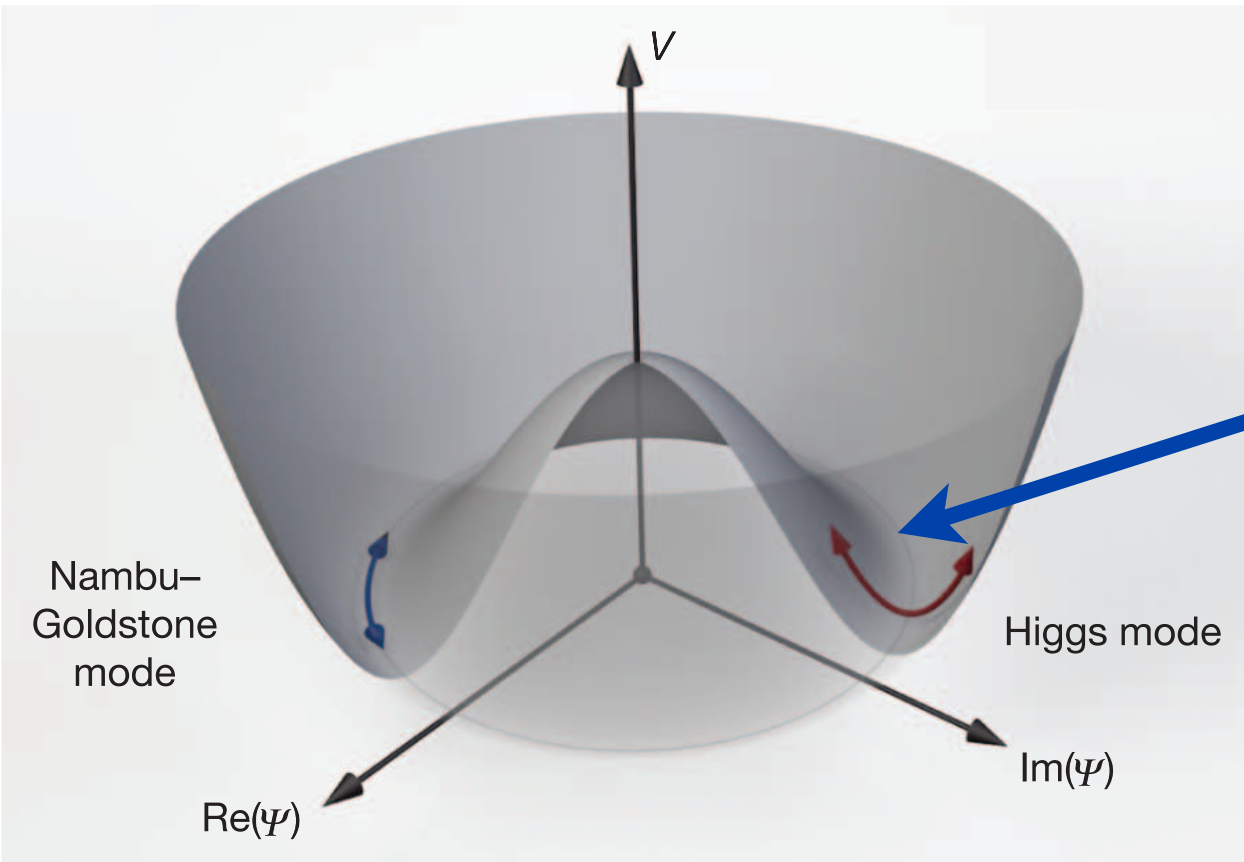

Despite rapid decay, there is a well-defined Higgs "quasi-normal mode". This is associated with a pole in the lower-half of the complex frequency plane.

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0 λ_c λ

A. V. Chubukov, S. Sachdev, and J. Ye, PRB **49**, 11919 (1994). S. Sachdev, PRB **59**, 14054 (1999).
W. Zwerger, PRL **92**, 027203 (2004).
D. Podolsky, A. Auerbach, and D. P. Arovas, PRB **84**, 174522 (2011).
D. Podolsky and S. Sachdev, PRB **86**, 054508 (2012). L. Pollet and N. Prokof'ev, PRL **109**, 010401 (2012).

$$\mathcal{S} = \int d^2 r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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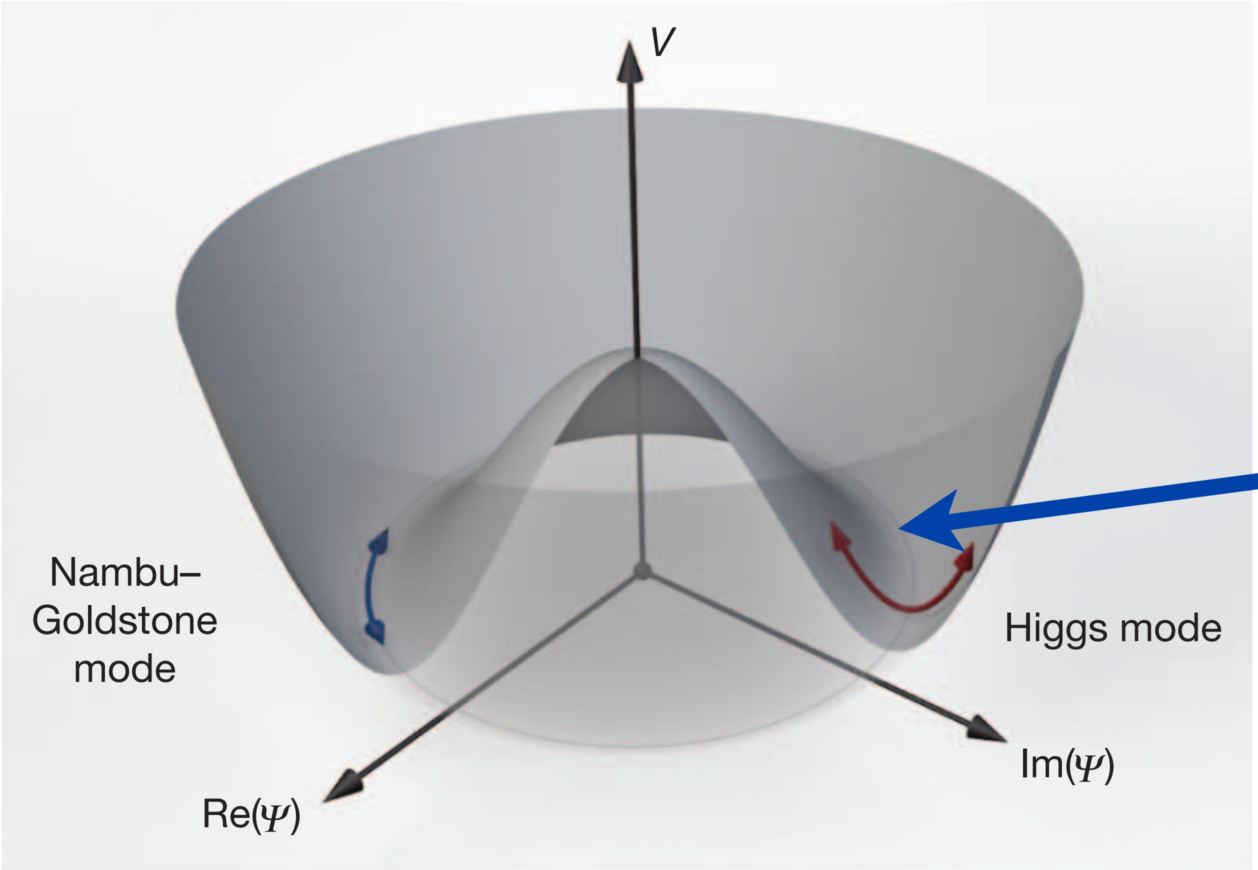
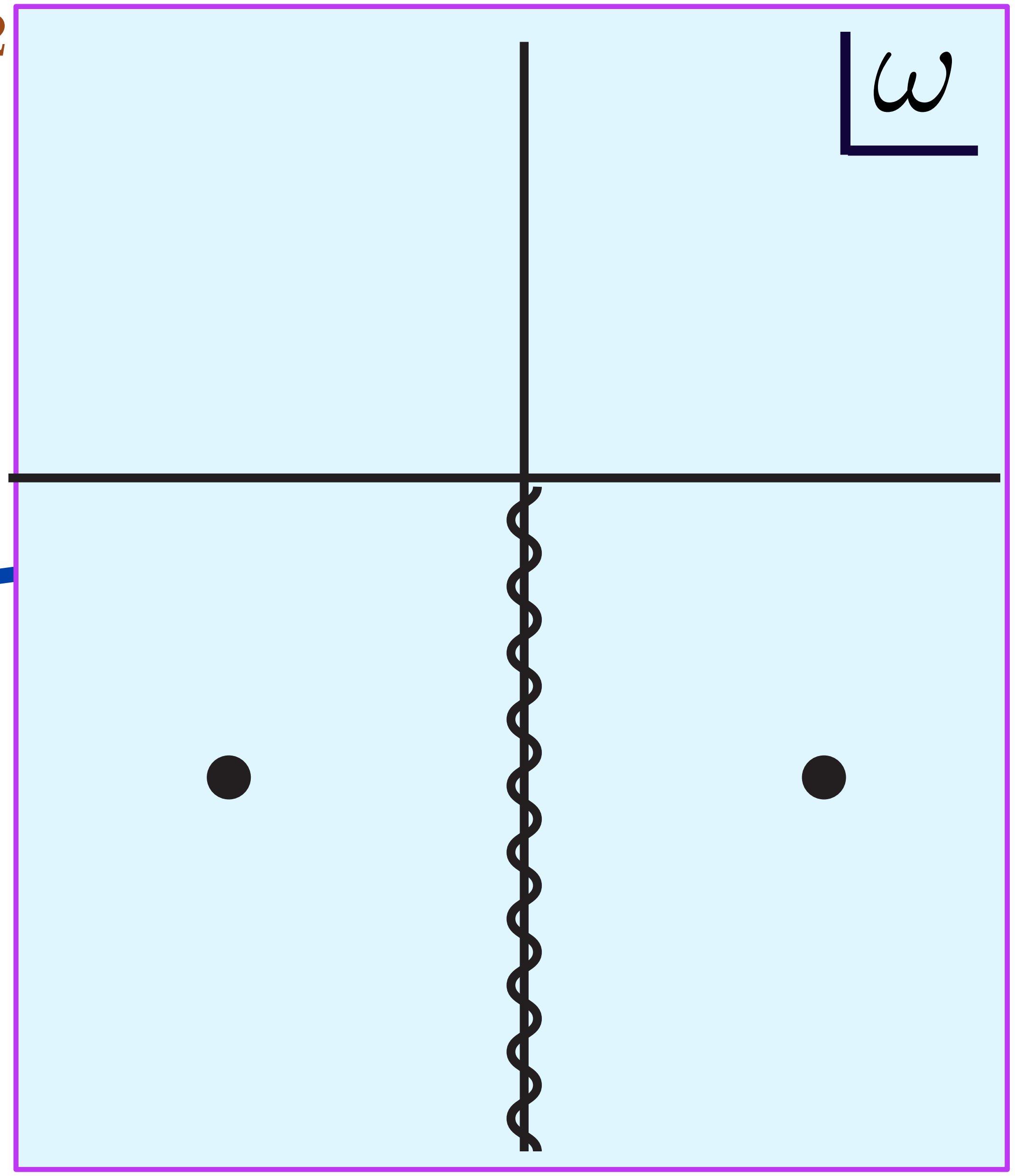


D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).



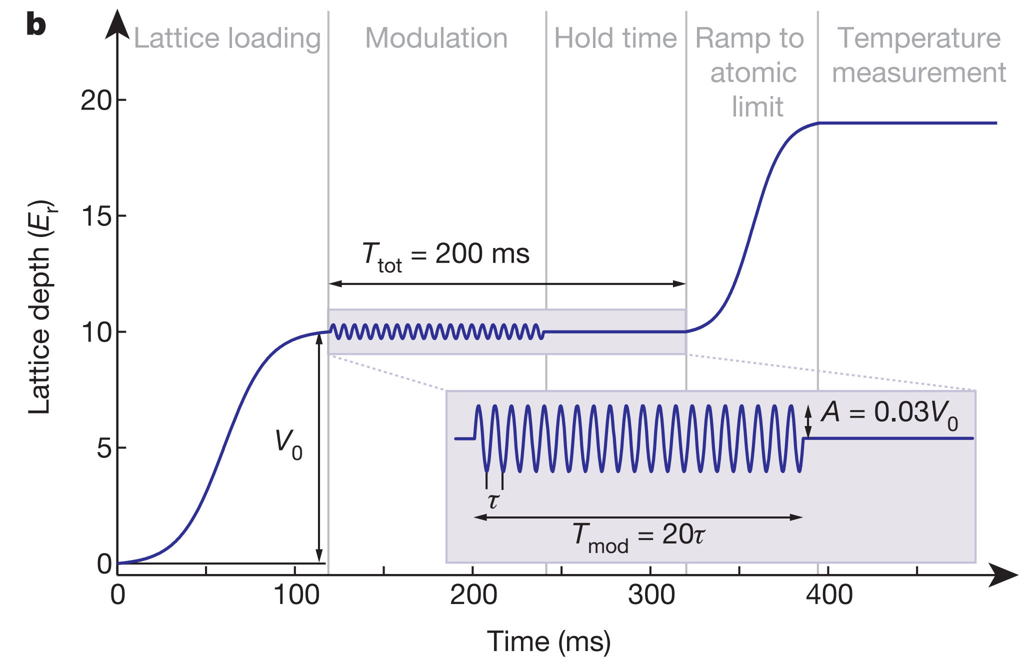
The Higgs quasi-normal mode is at the frequency

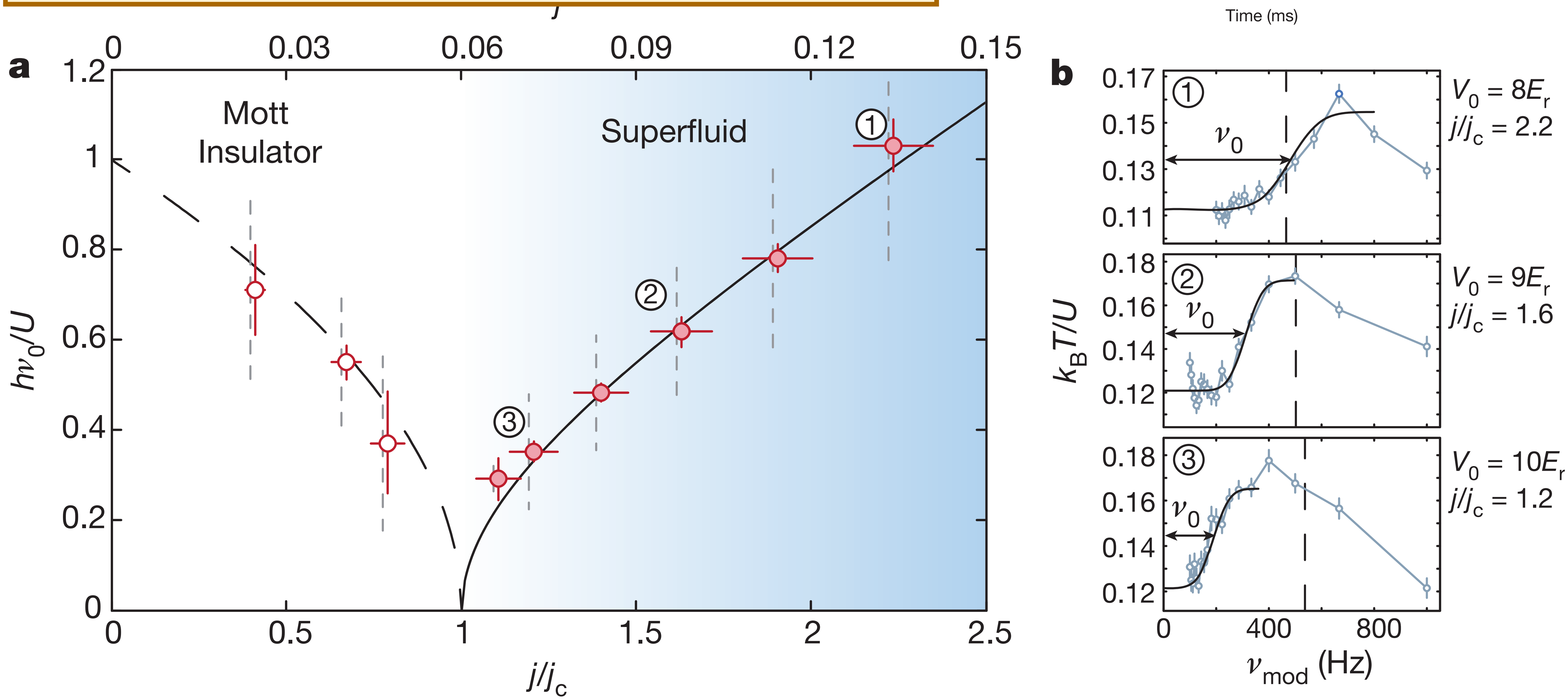
$$\frac{\omega_{\text{pole}}}{\Delta} = -i\,\frac{4}{\pi} + \frac{1}{N}\left(\frac{16\left(4 + \sqrt{2}\log\left(3 - 2\sqrt{2}\right)\right)}{\pi^2} + 2.46531203396\, i \right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

where Δ is the particle gap at the complementary point in the "paramagnetic" state with the same value of $|\lambda - \lambda_c|$, and $N = 2$ is the number of vector components of Ψ. The universal answer is a consequence of the strong interactions in the CFT3

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole





Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice: Response to modulation of lattice depth scales as expected from the LHP pole

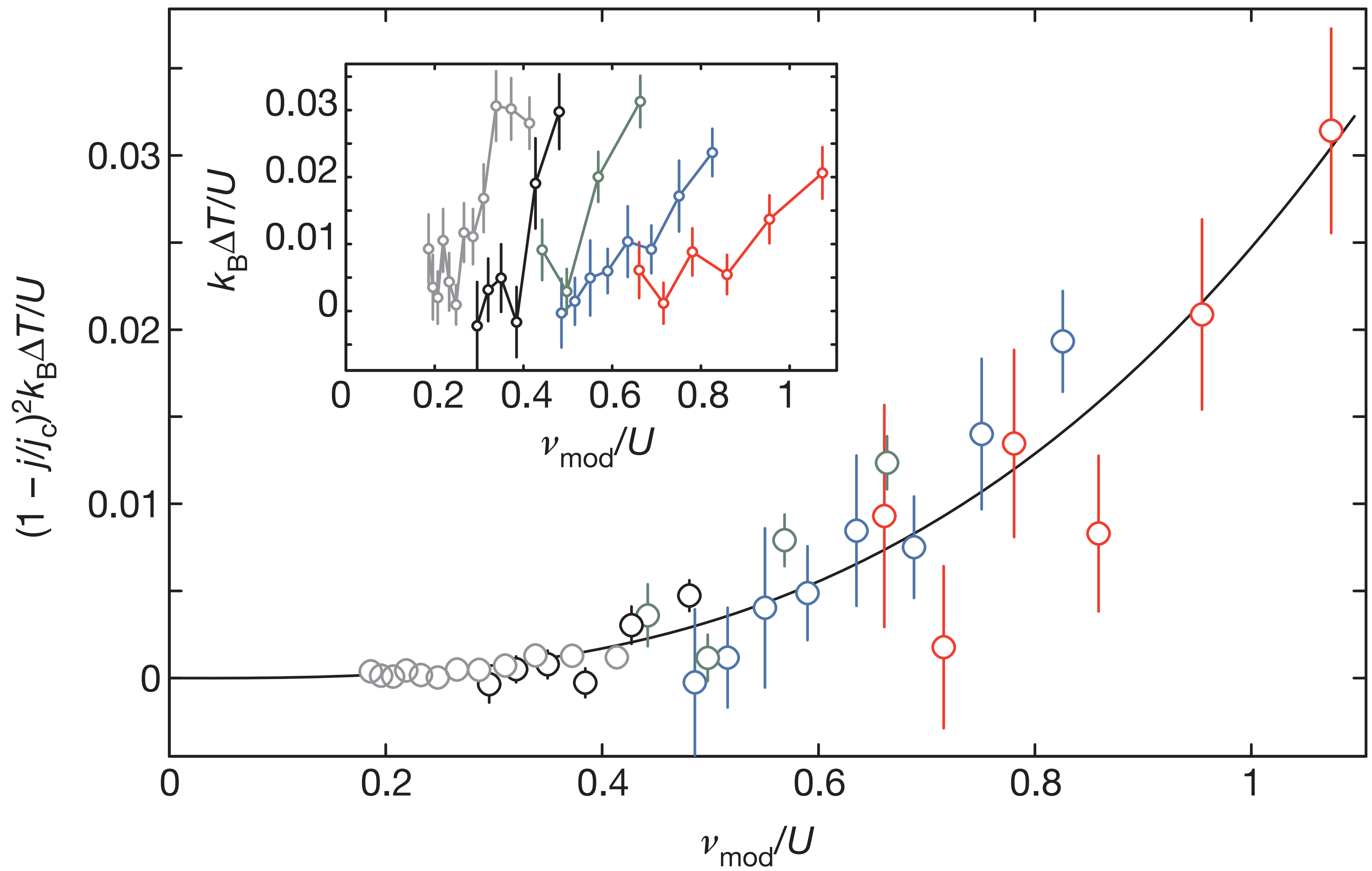


Figure 4 | Scaling of the low-frequency response. The low-frequency response in the superfluid regime shows a scaling compatible with the prediction $(1 - j/j_c)^{-2}\nu^3$ (Methods). Shown is the temperature response rescaled with $(1 - j/j_c)^2$ for $V_0 = 10E_r$ (grey), $9.5E_r$ (black), $9E_r$ (green), $8.5E_r$ (blue) and $8E_r$ (red) as a function of the modulation frequency. The black line is a fit of the form $a\nu^b$ with a fitted exponent $b = 2.9(5)$. The inset shows the same data points without rescaling, for comparison. Error bars, s.e.m.

D. Podolsky and S. Sachdev, Phy. Rev. B **86**, 054508 (2012).

Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

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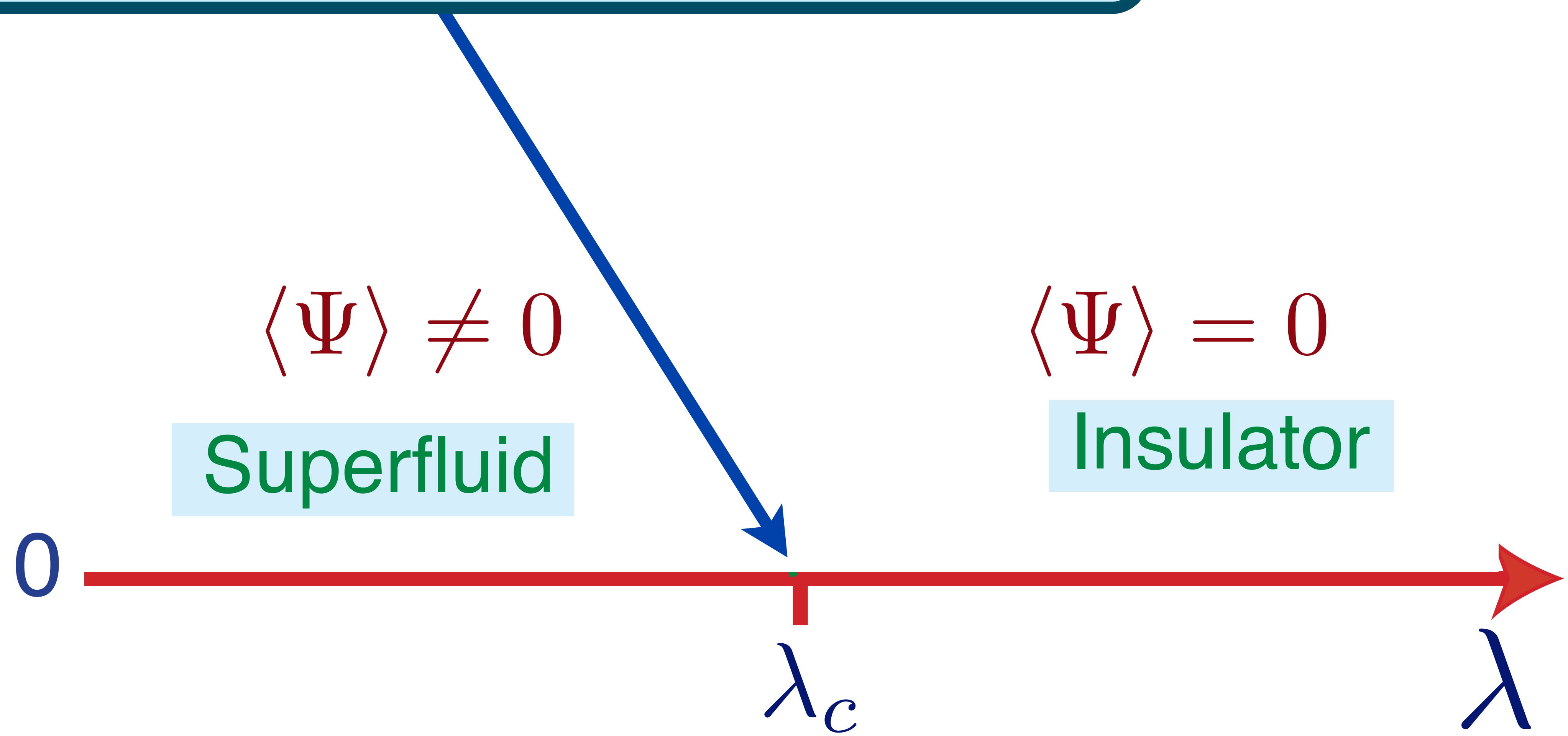
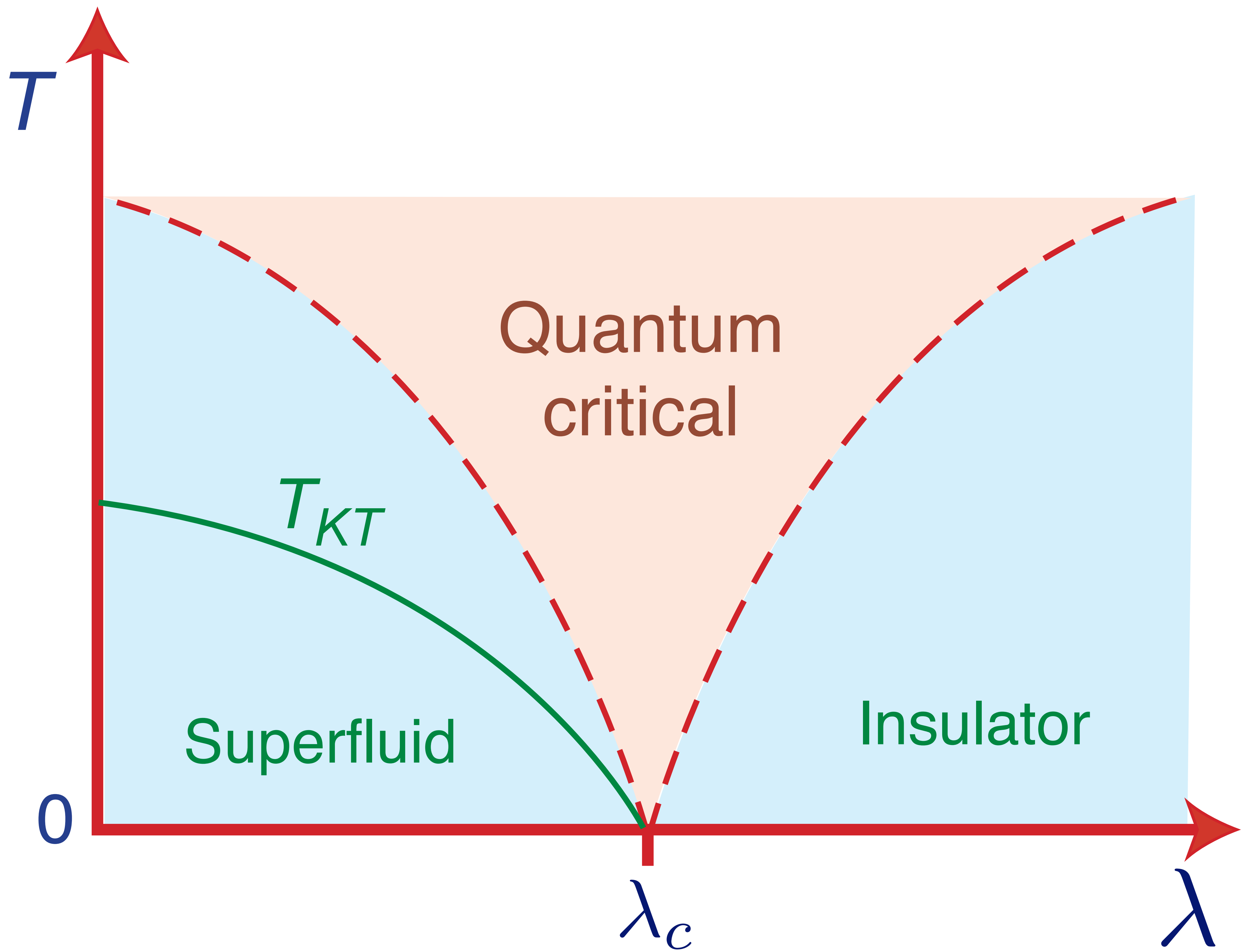
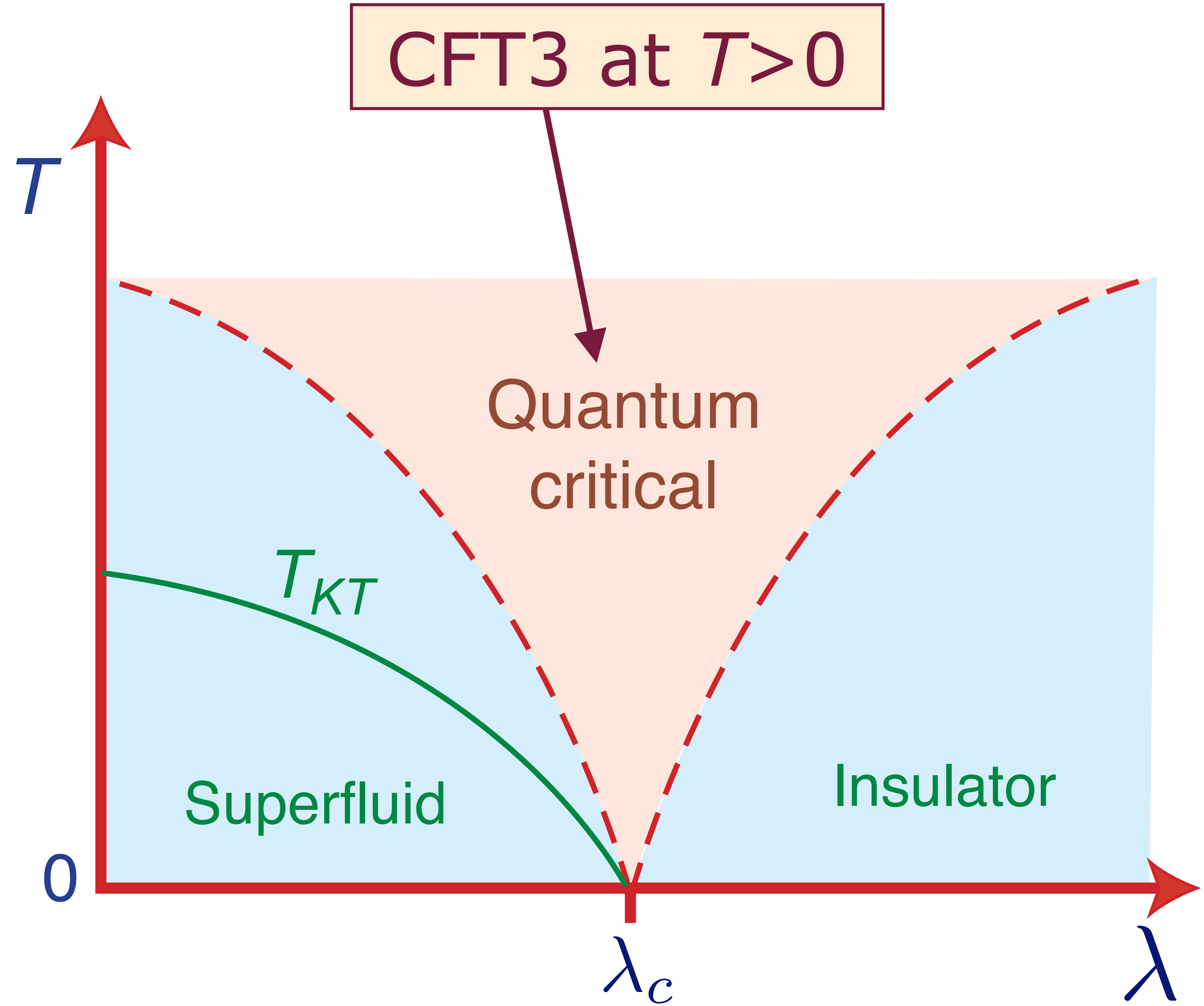
$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$

A conformal field theory
in 2+1 spacetime dimensions:
a CFT3





T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
λ_c
λ

Quantum critical dynamics

Quantum *"nearly perfect fluid"*
with shortest possible *local* equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C} \frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant.

Response functions are characterized by poles in LHP
with $\omega \sim k_B T/\hbar$.
These poles (quasi-normal modes) appear naturally in
the holographic theory.
(Analogs of Higgs quasi-normal mode.)

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical dynamics

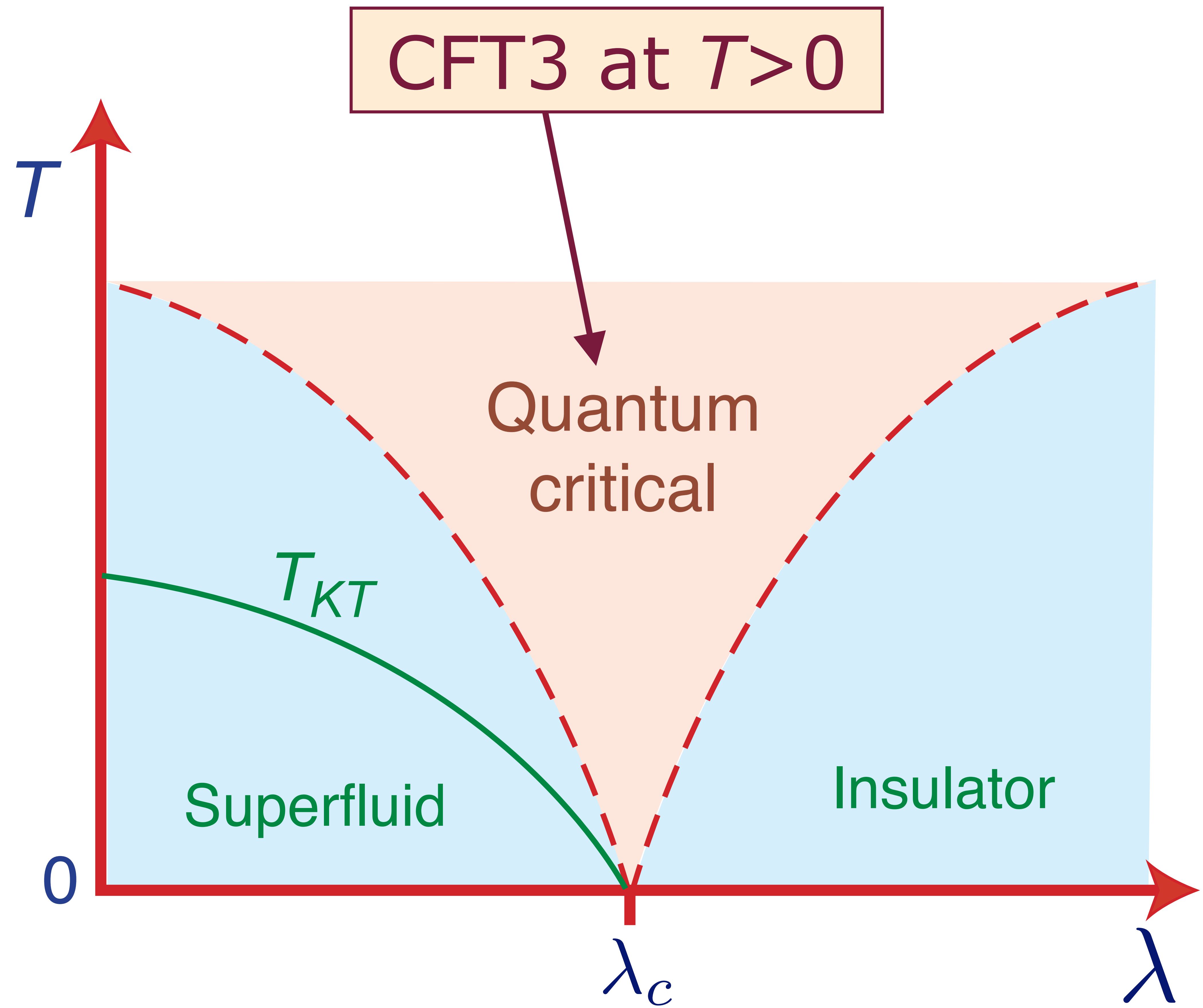
Transport co-oefficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

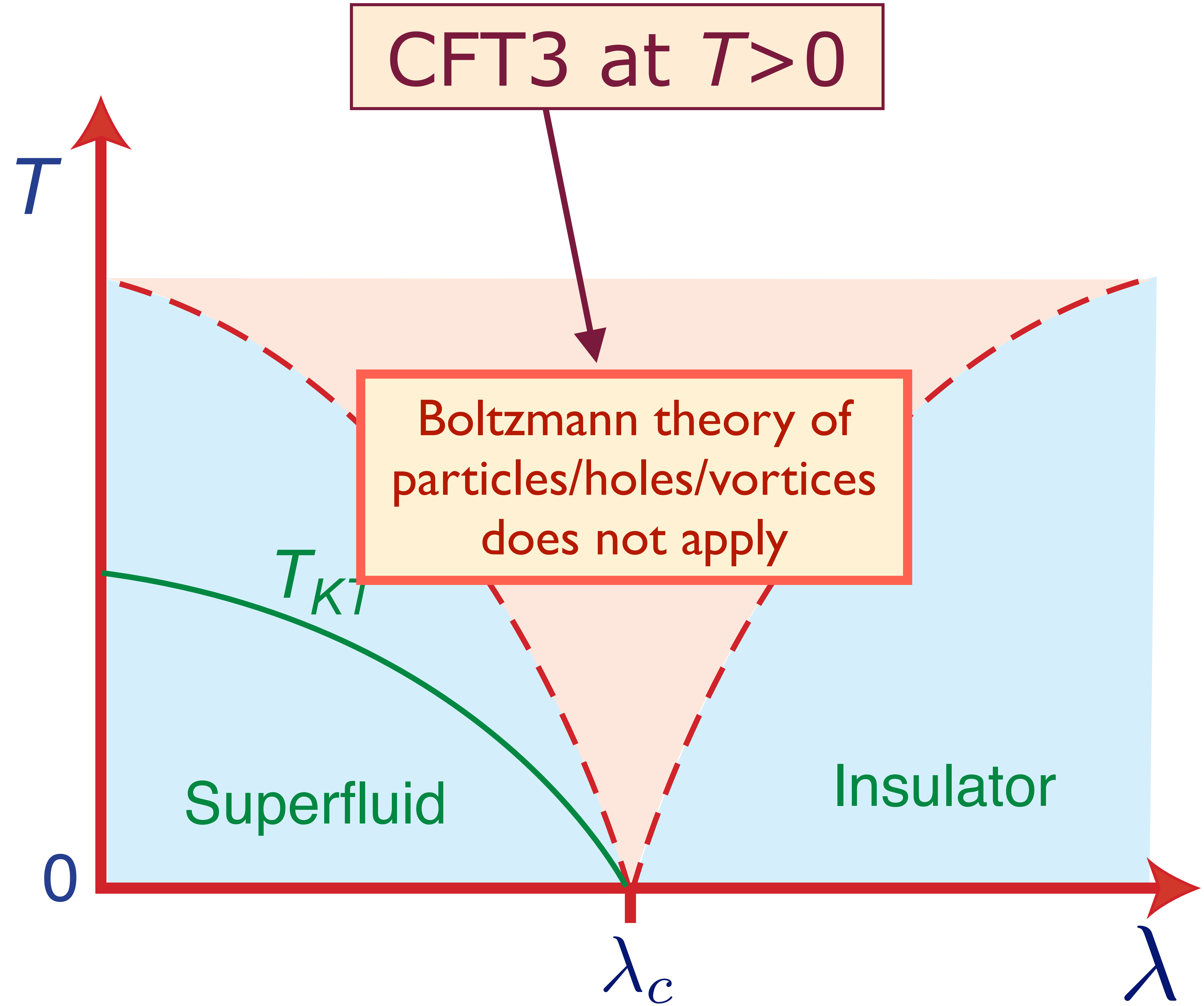
Conductivity

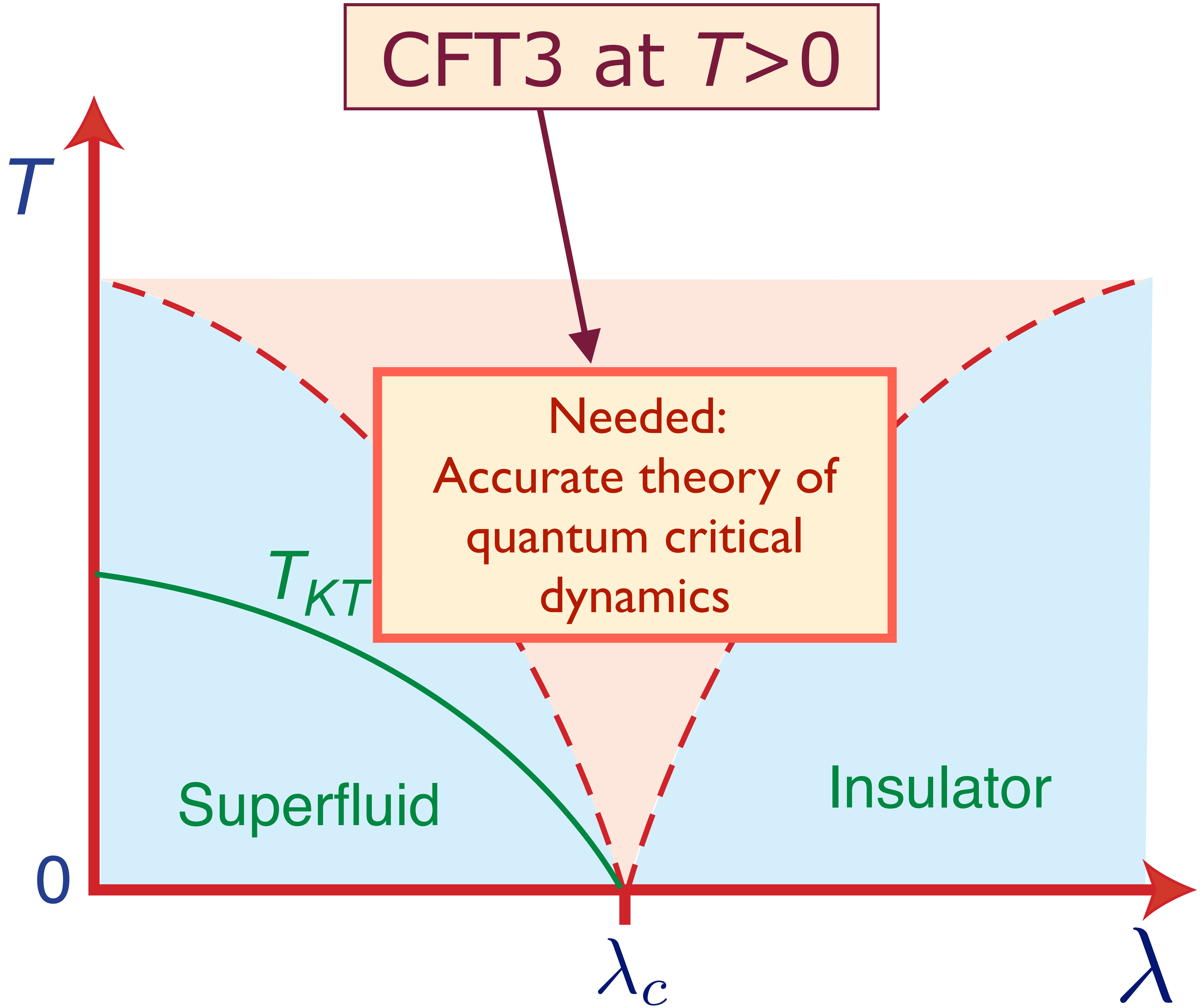
$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
λ_c
λ


CFT3 at T>0
T
Boltzmann theory of
particles/holes/vortices
does not apply
T_KT
Superfluid
Insulator
0
λ_c
λ



CFT3 at T>0
T
Needed:
Accurate theory of
quantum critical
dynamics
T_KT
Superfluid
Insulator
0
λ_c
λ

Outline

1. Z_2 Spin liquid in the kagome antiferromagnet

2. Superfluid-insulator transition of ultracold atoms in optical lattices:
   Quantum criticality and conformal field theories

3. Holography and the quasi-normal modes of black-hole horizons

4. Strange metals:
   What lies beyond the horizon ?

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1. Z_2 Spin liquid in the kagome antiferromagnet

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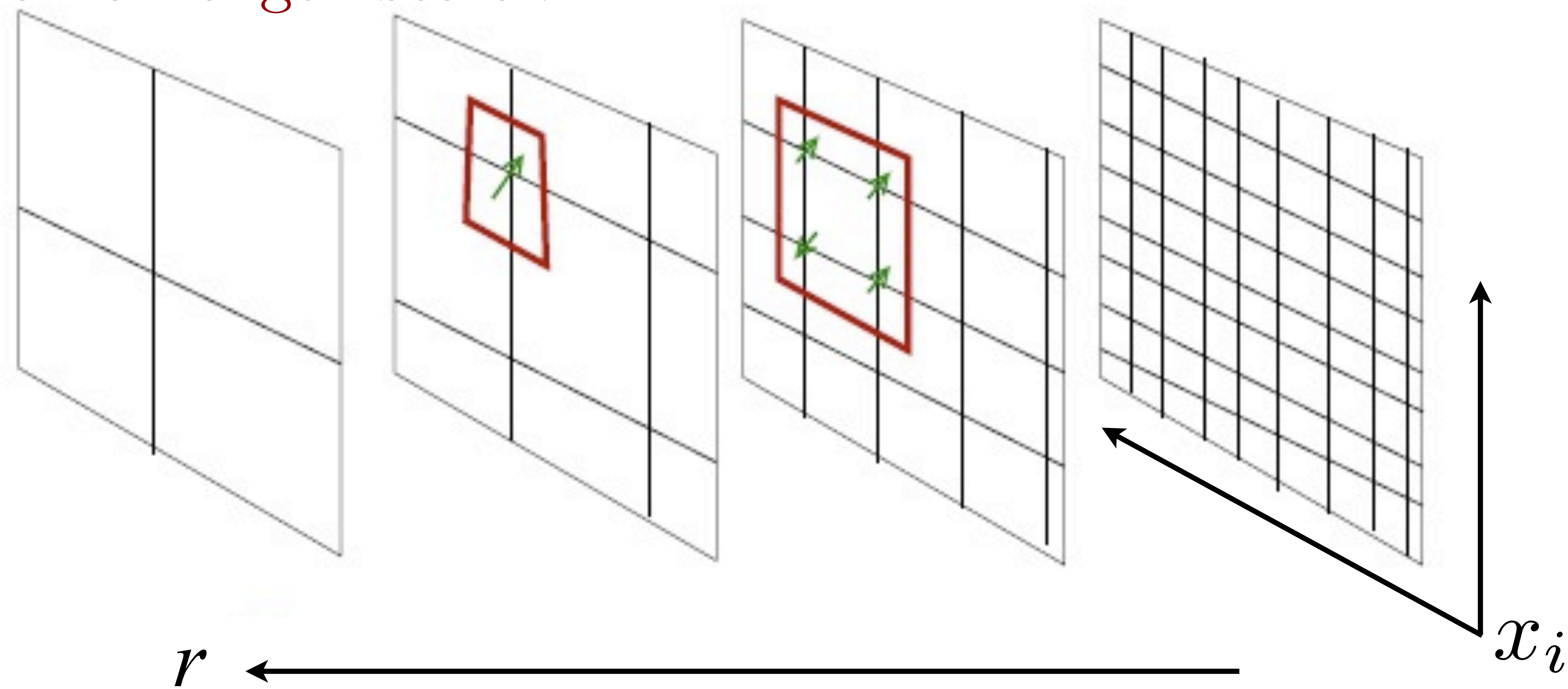
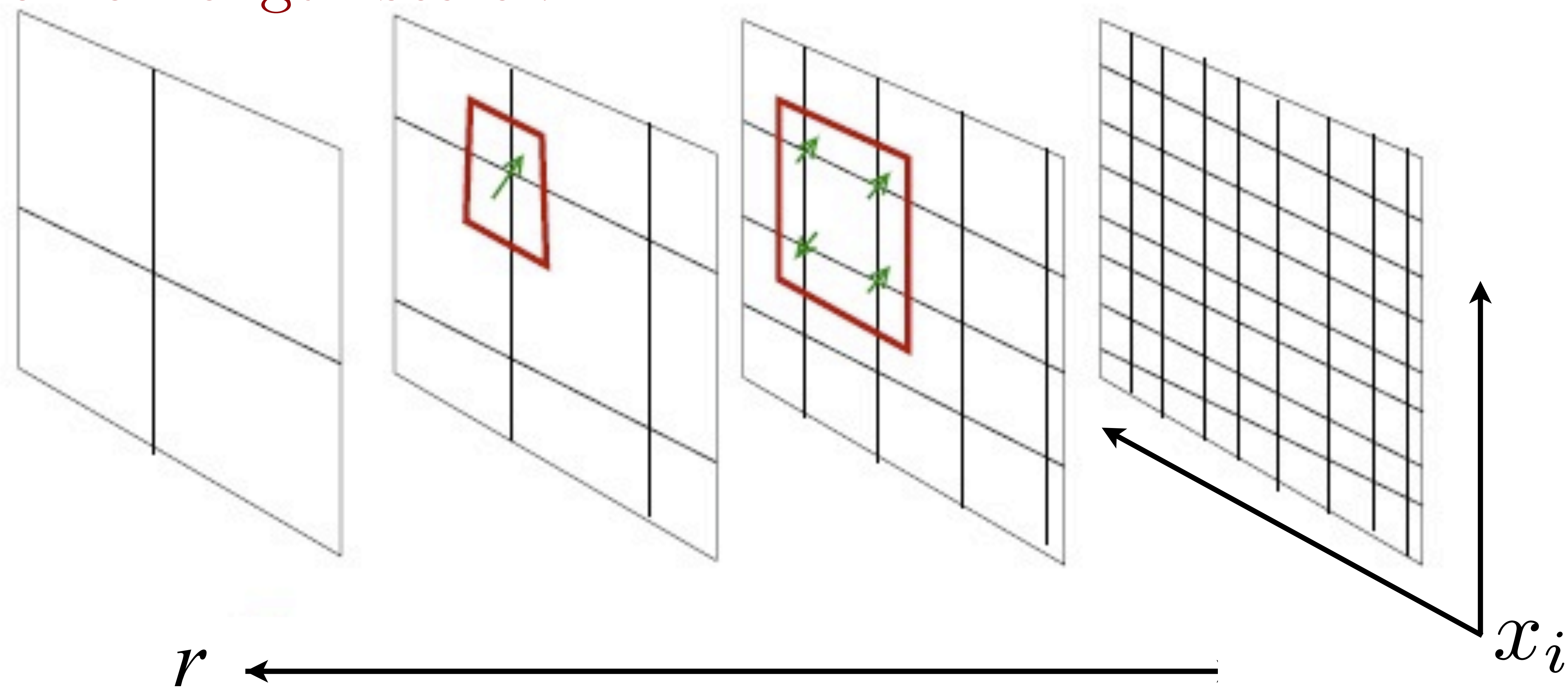
   What lies beyond the horizon ?

Renormalization group: $\Rightarrow$ Follow coupling constants of quantum many body theory as a function of length scale r



J. McGreevy, arXiv0909.0518

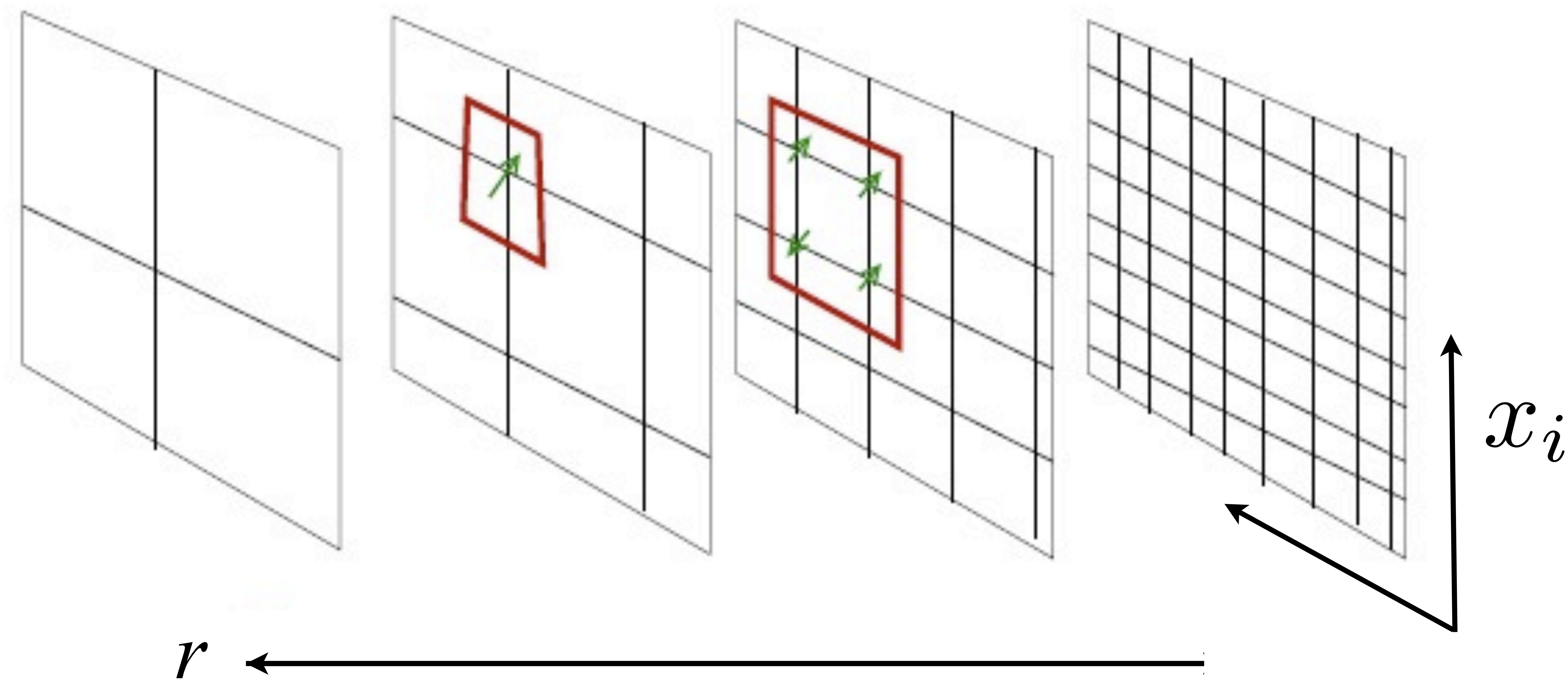
Renormalization group: $\Rightarrow$ Follow coupling constants of quantum many body theory as a function of length scale r



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

J. McGreevy, arXiv0909.0518

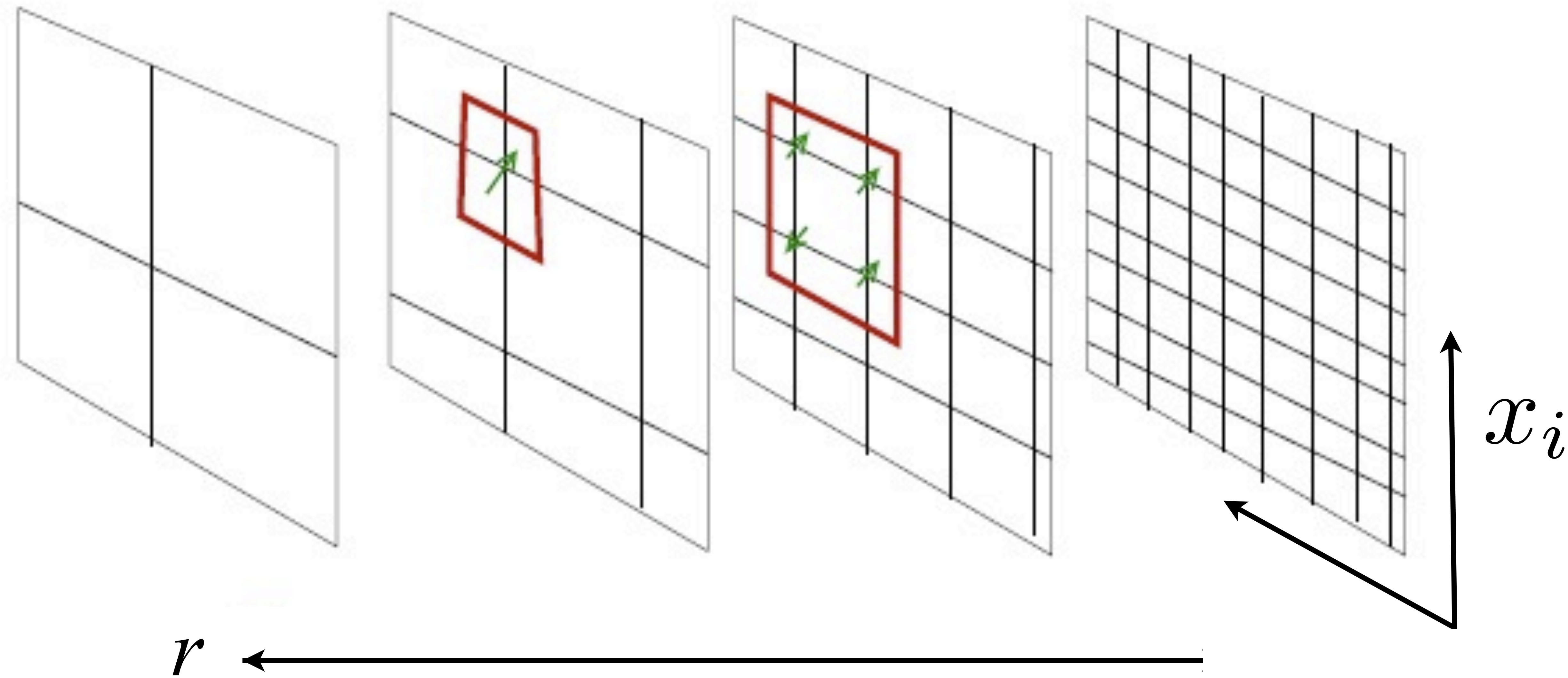
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography

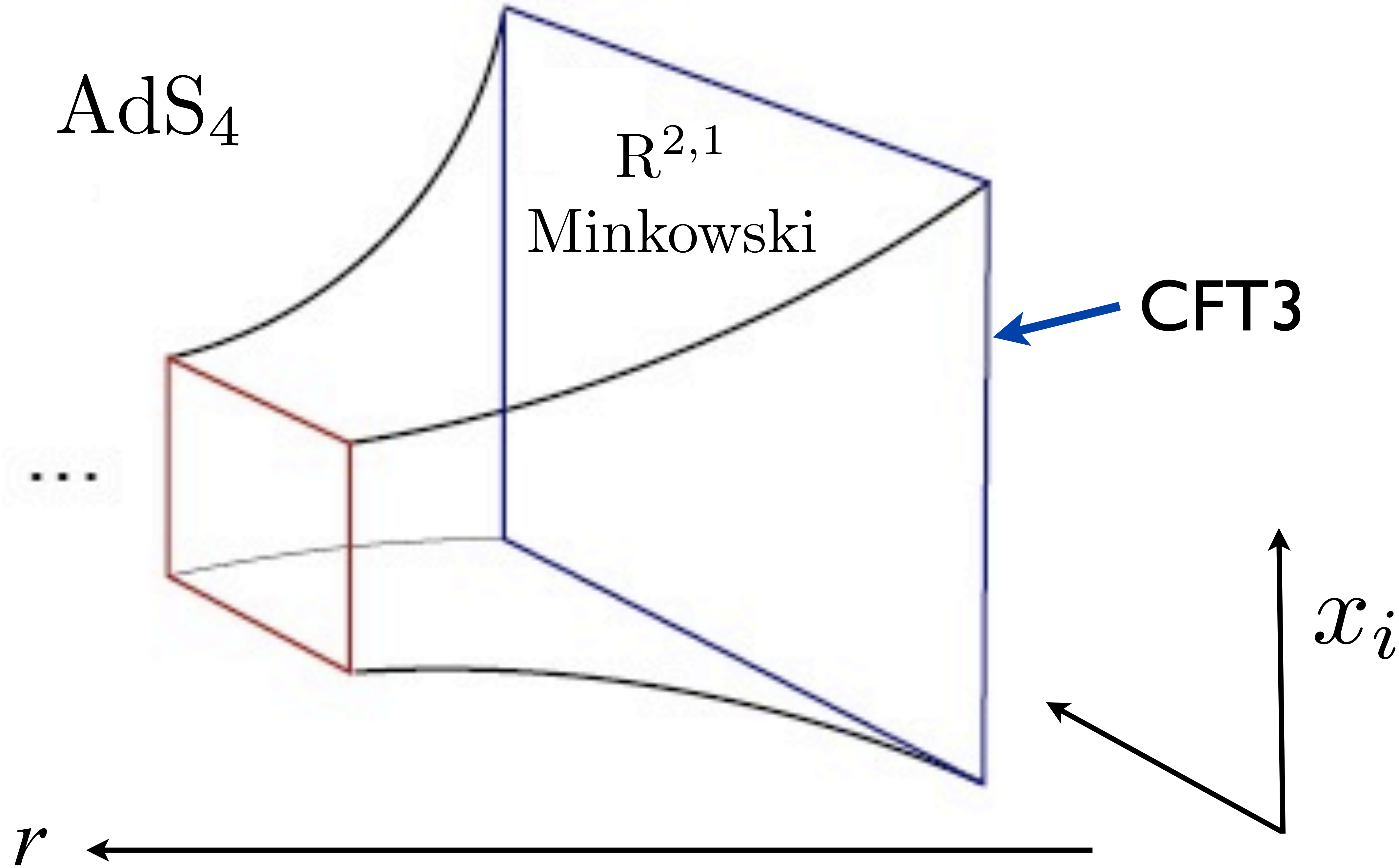


This gives the unique metric

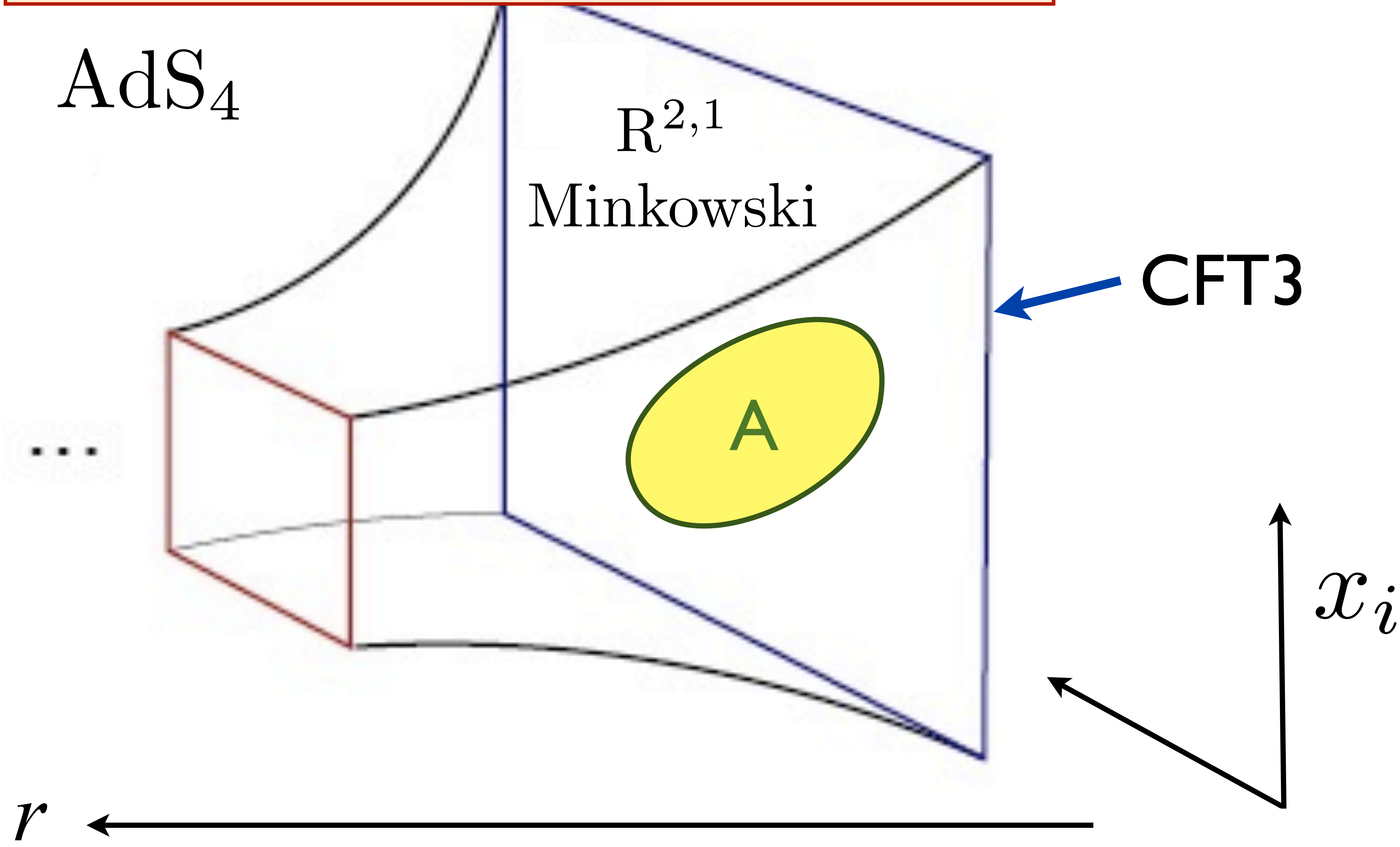
$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS_{d+2}.

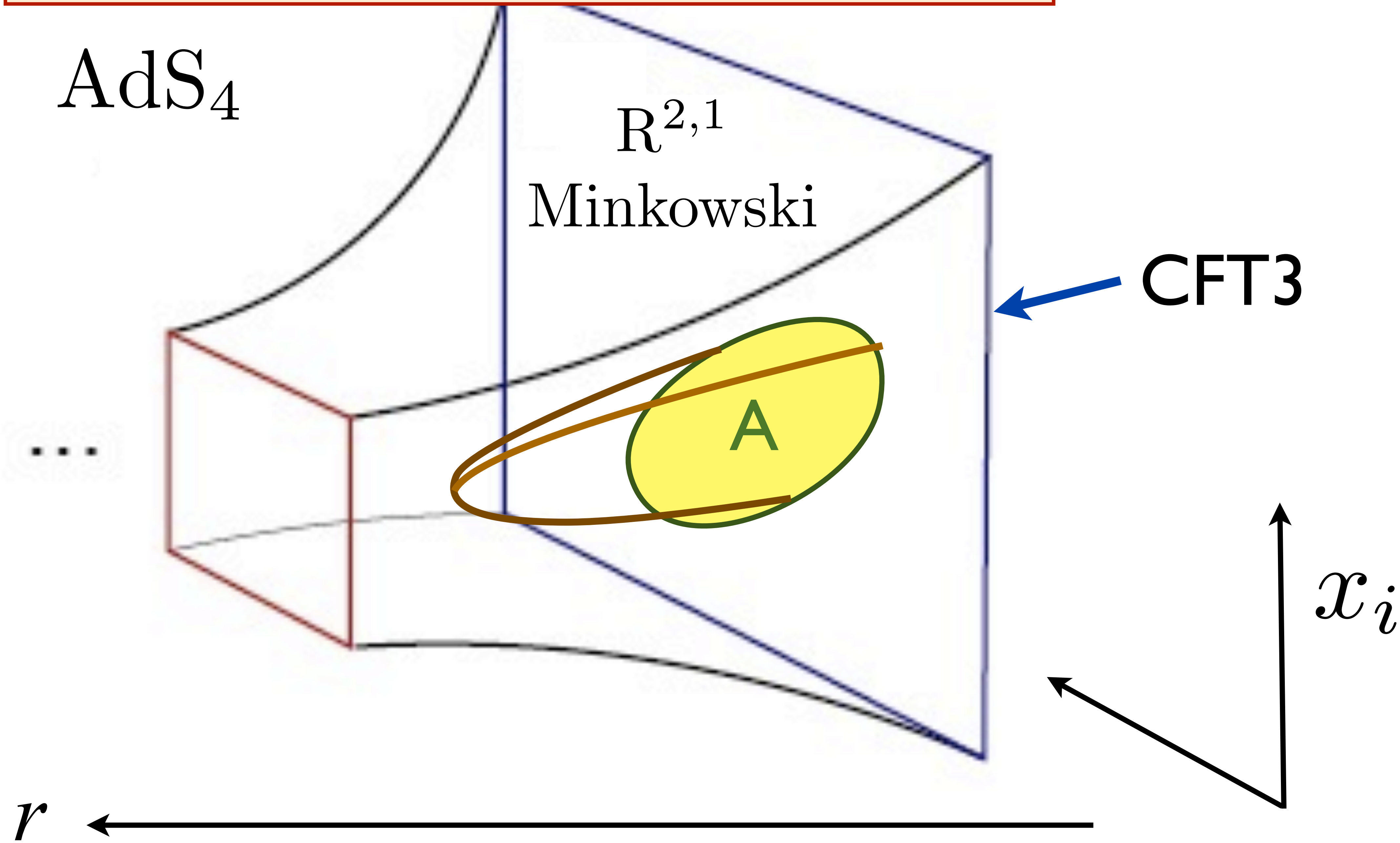
AdS/CFT correspondence



Holography and Entanglement



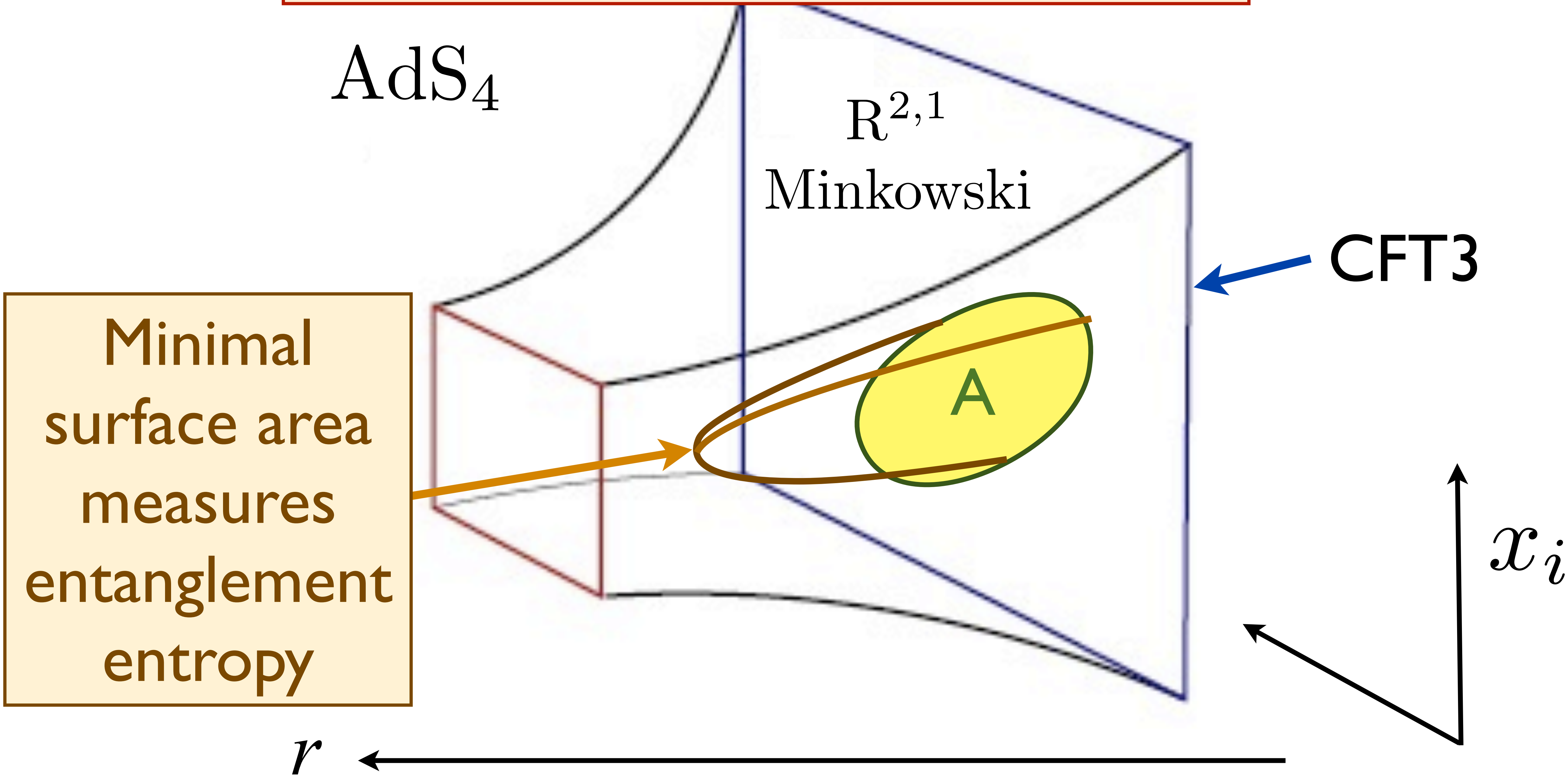
Holography and Entanglement



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

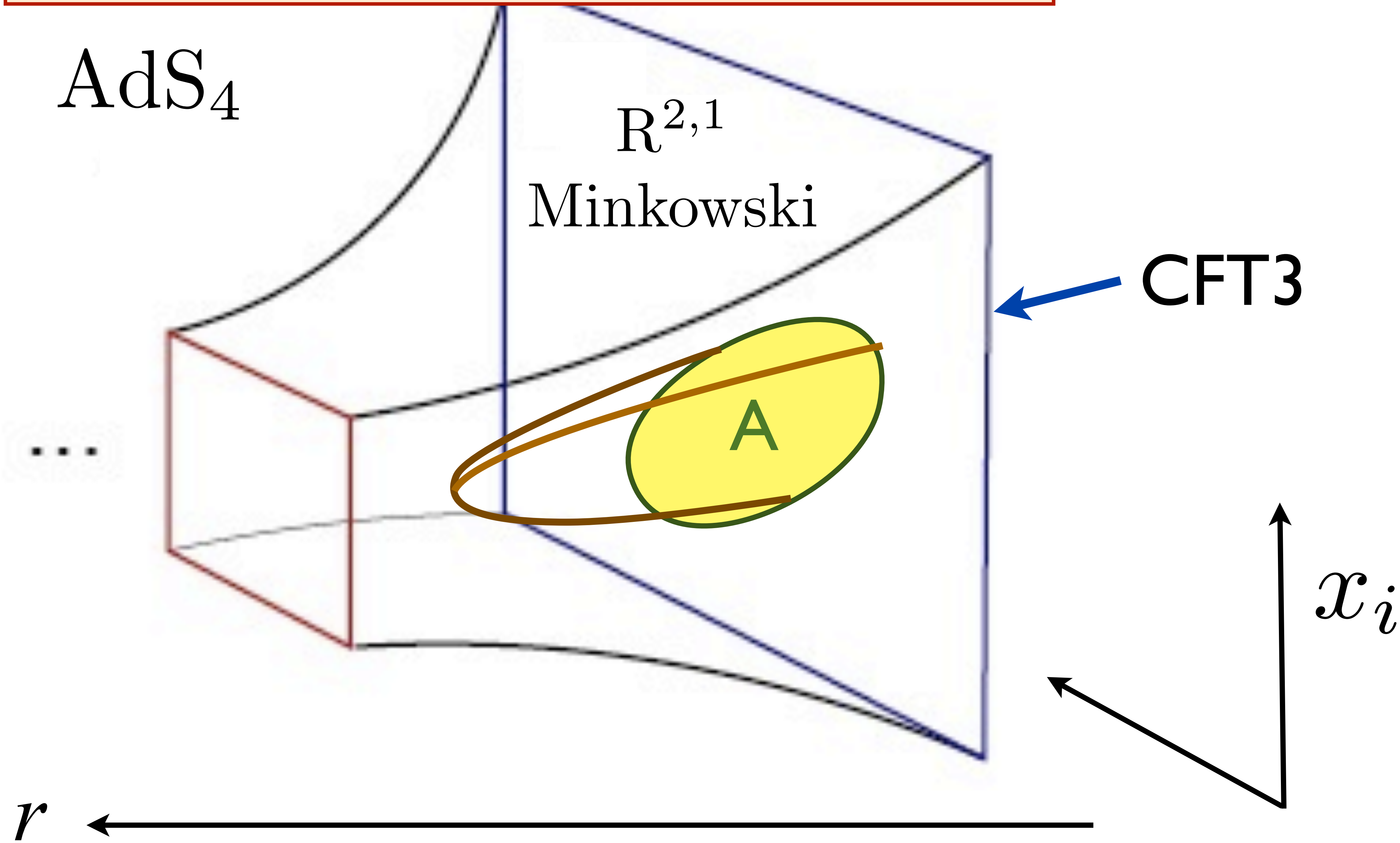
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Holography and Entanglement



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Holography and Entanglement



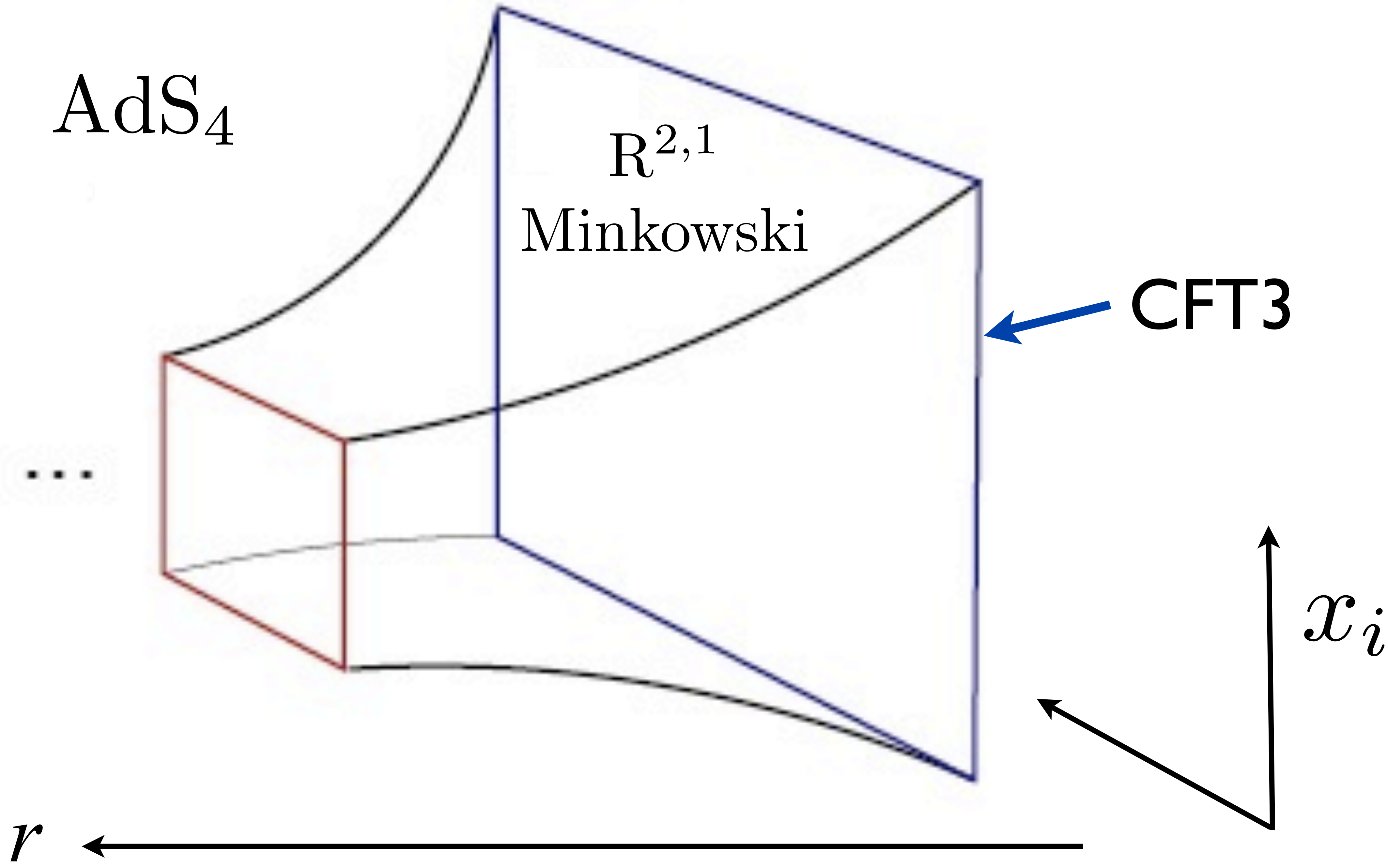
- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009); H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011); I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

AdS/CFT correspondence

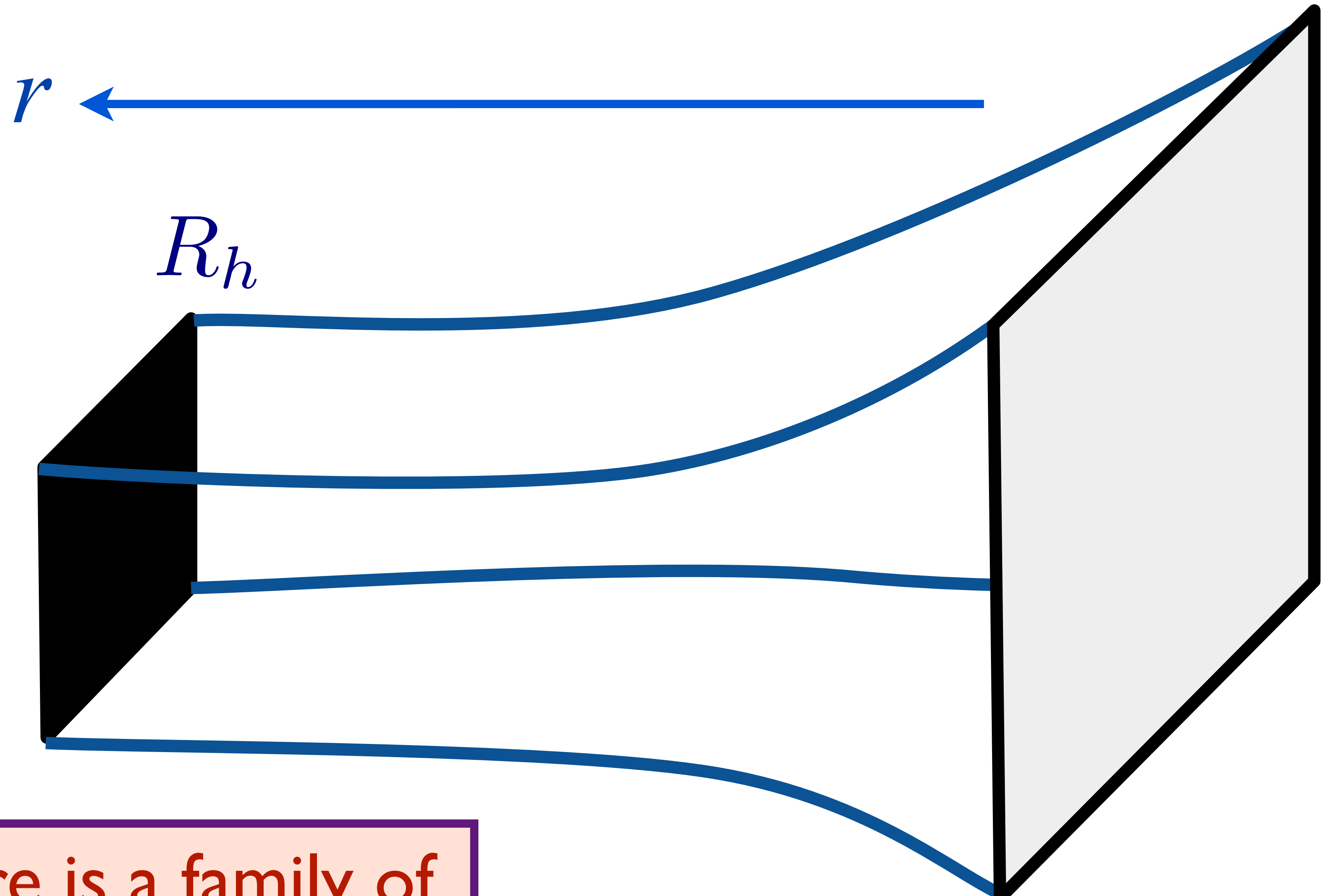


This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

AdS/CFT correspondence at non-zero temperatures

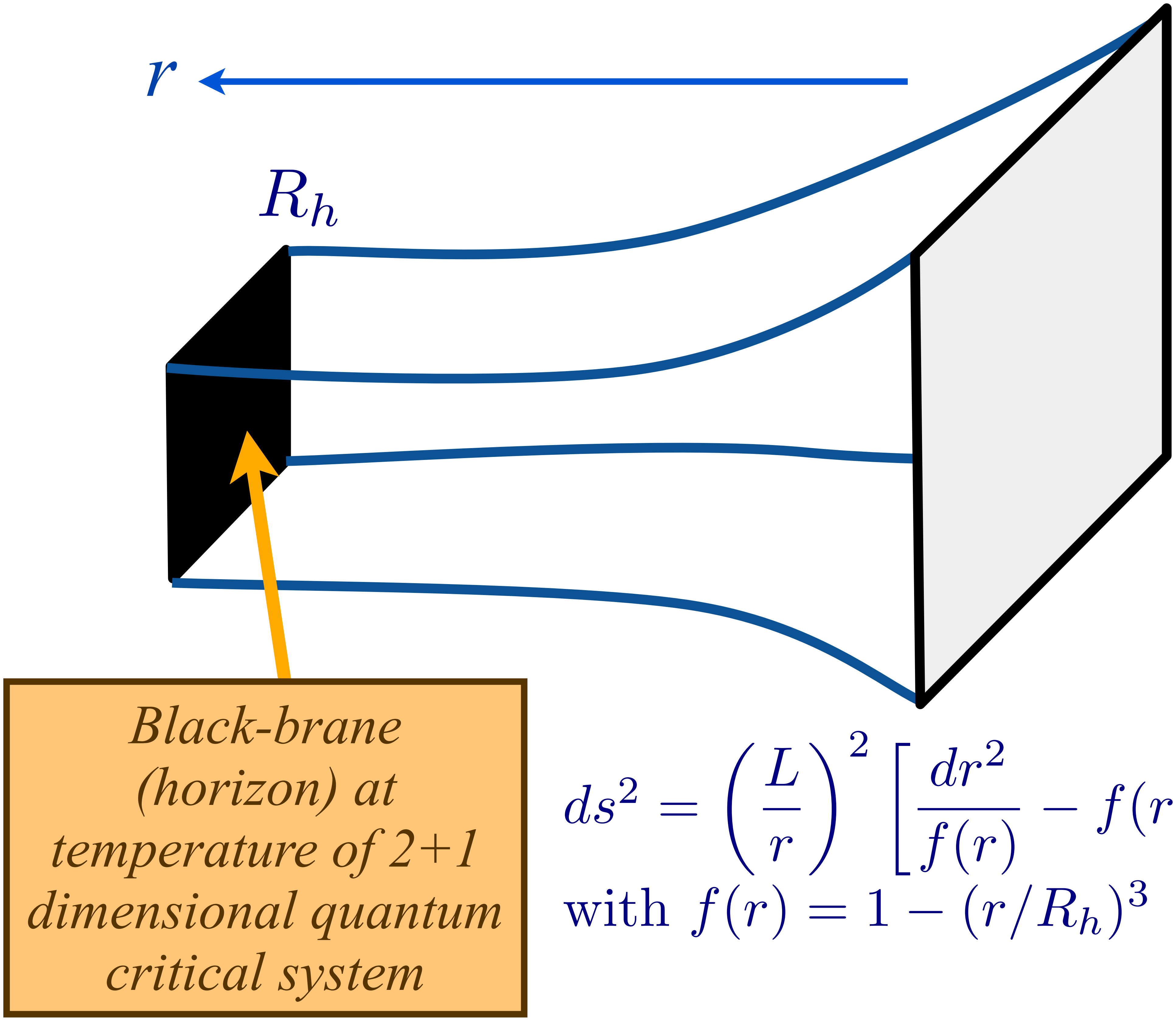
AdS_4-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS_4-Schwarzschild black-brane

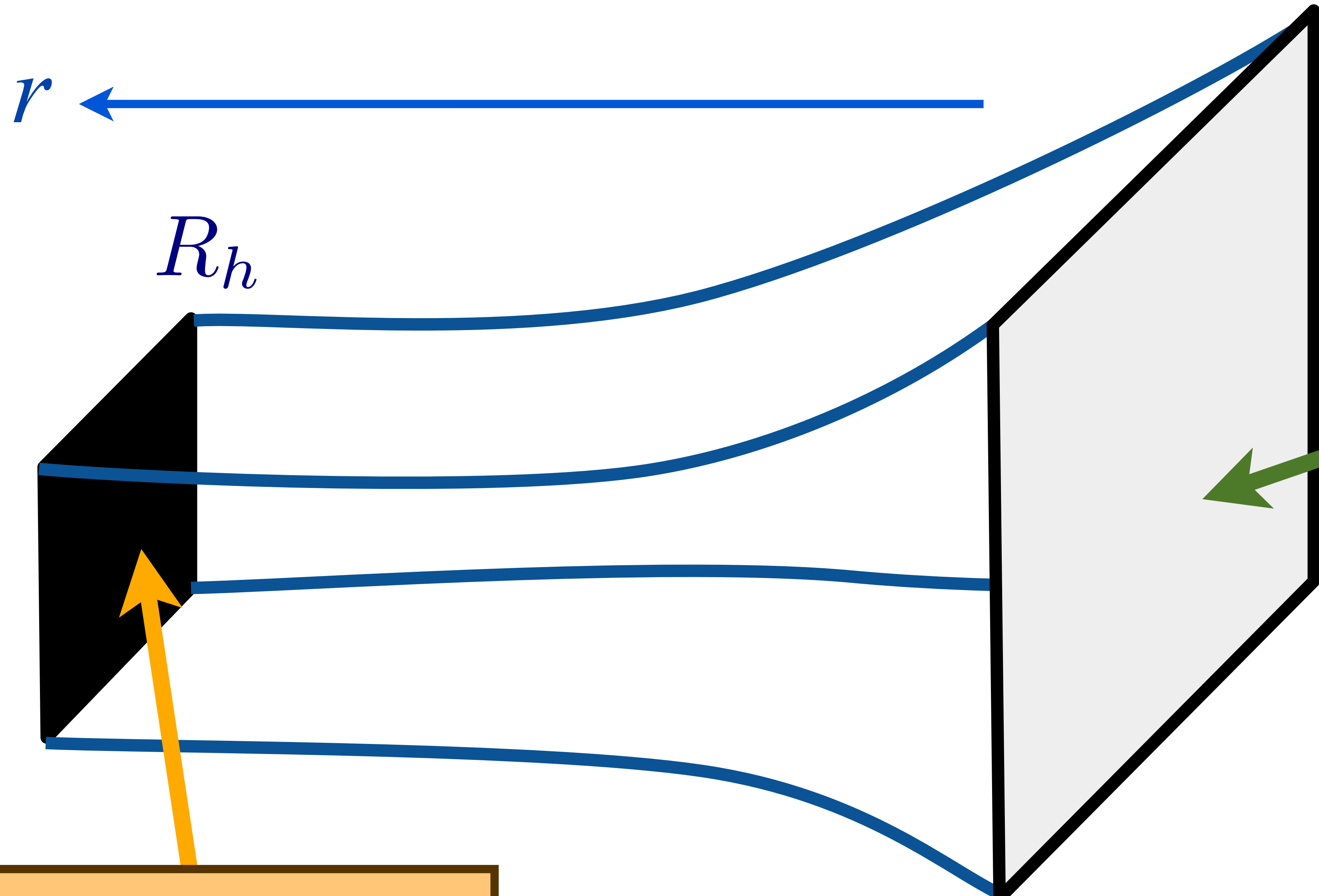


$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R_h)^3$

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point:

$$k_B T = \frac{3\hbar}{4\pi R_h}.$$

Black-brane (horizon) at temperature of 2+1 dimensional quantum critical system

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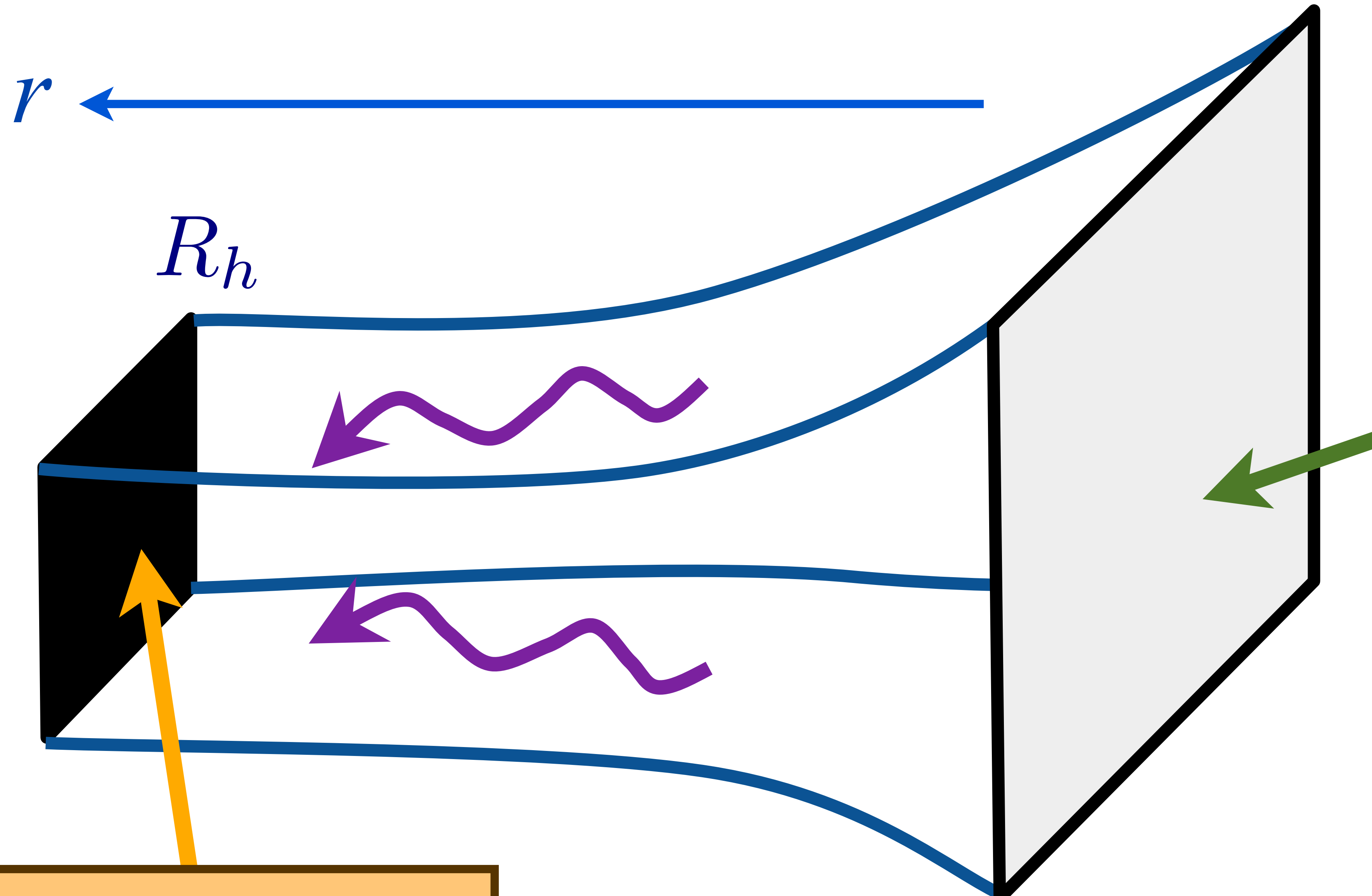

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$k_B T = \frac{3\hbar}{4\pi R_h}$.

Black-brane (horizon) at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling past the horizon

D. T. Son

AdS/CFT correspondence at non-zero temperatures

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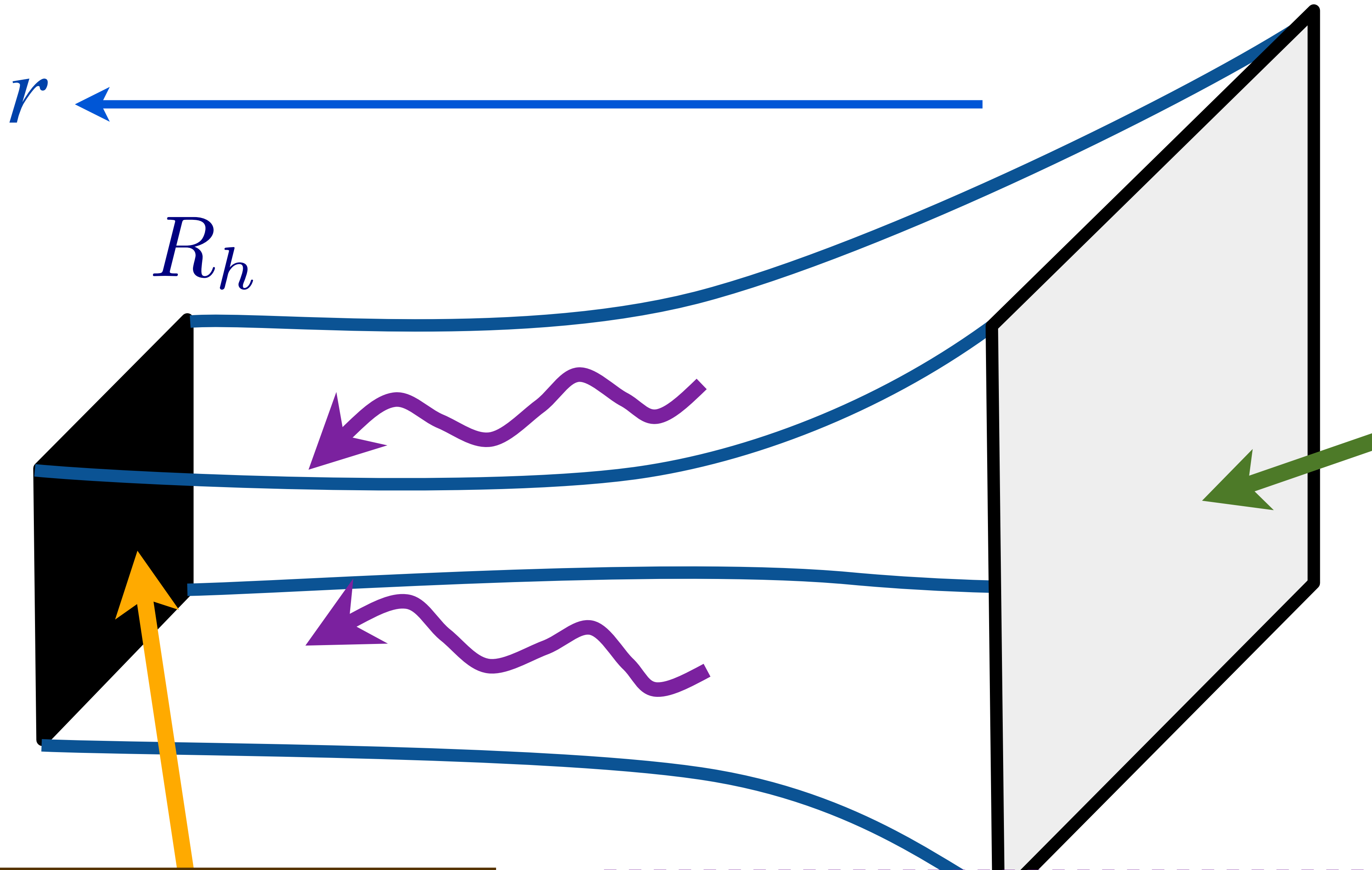

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Black-brane (horizon) at temperature of 2+1 dimensional quantum critical system

Quasi-normal modes of waves near horizon -- quasi-normal modes of quantum criticality (and Higgs)

AdS$_4$ theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],$$

Here F_{ab} is a 4-dimensional gauge field strength, which is "dual" to a conserved U(1) current of the CFT. C_{abcd} is the Weyl tensor.

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

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This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator.

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)

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Boundary and bulk methods both show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

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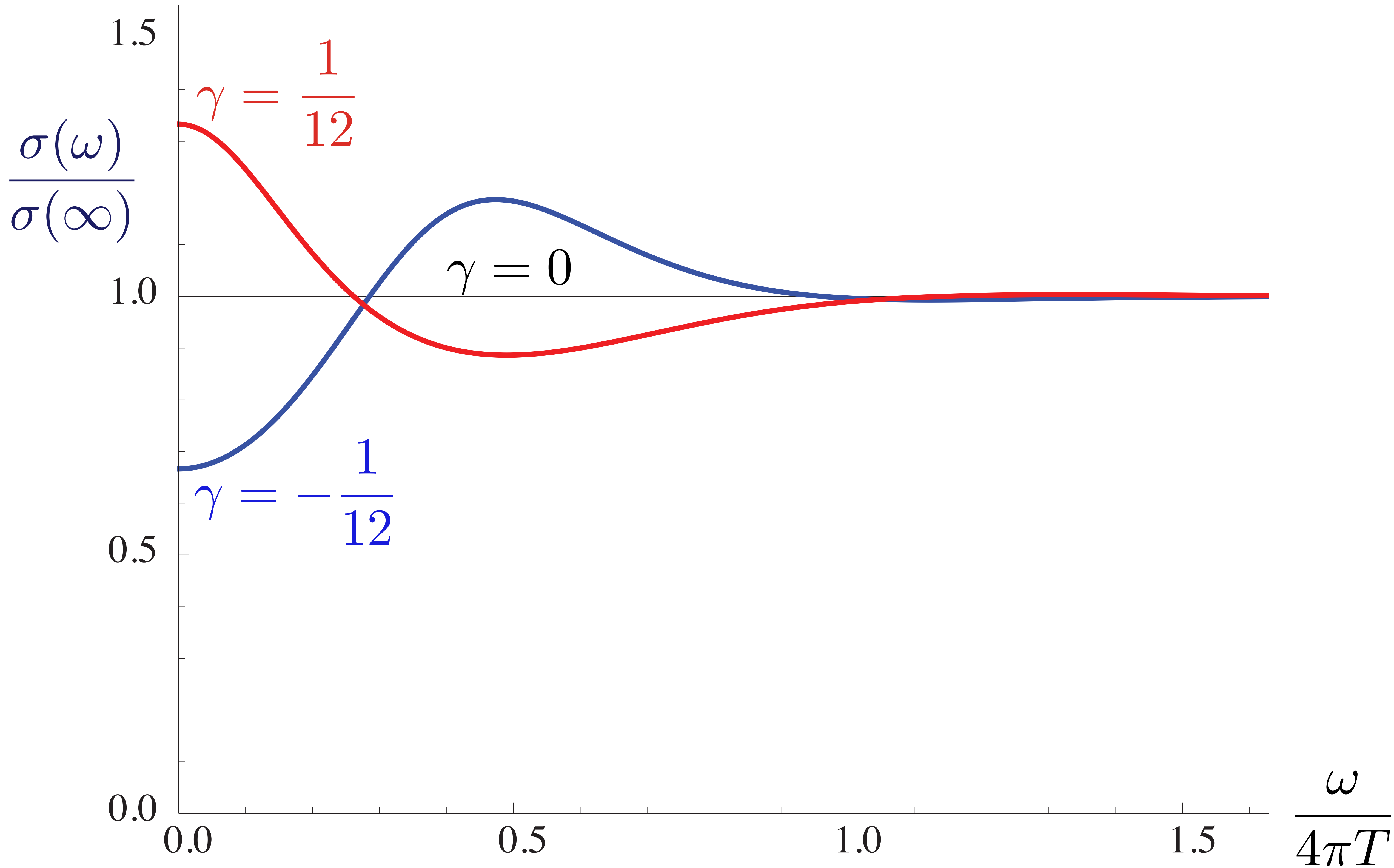

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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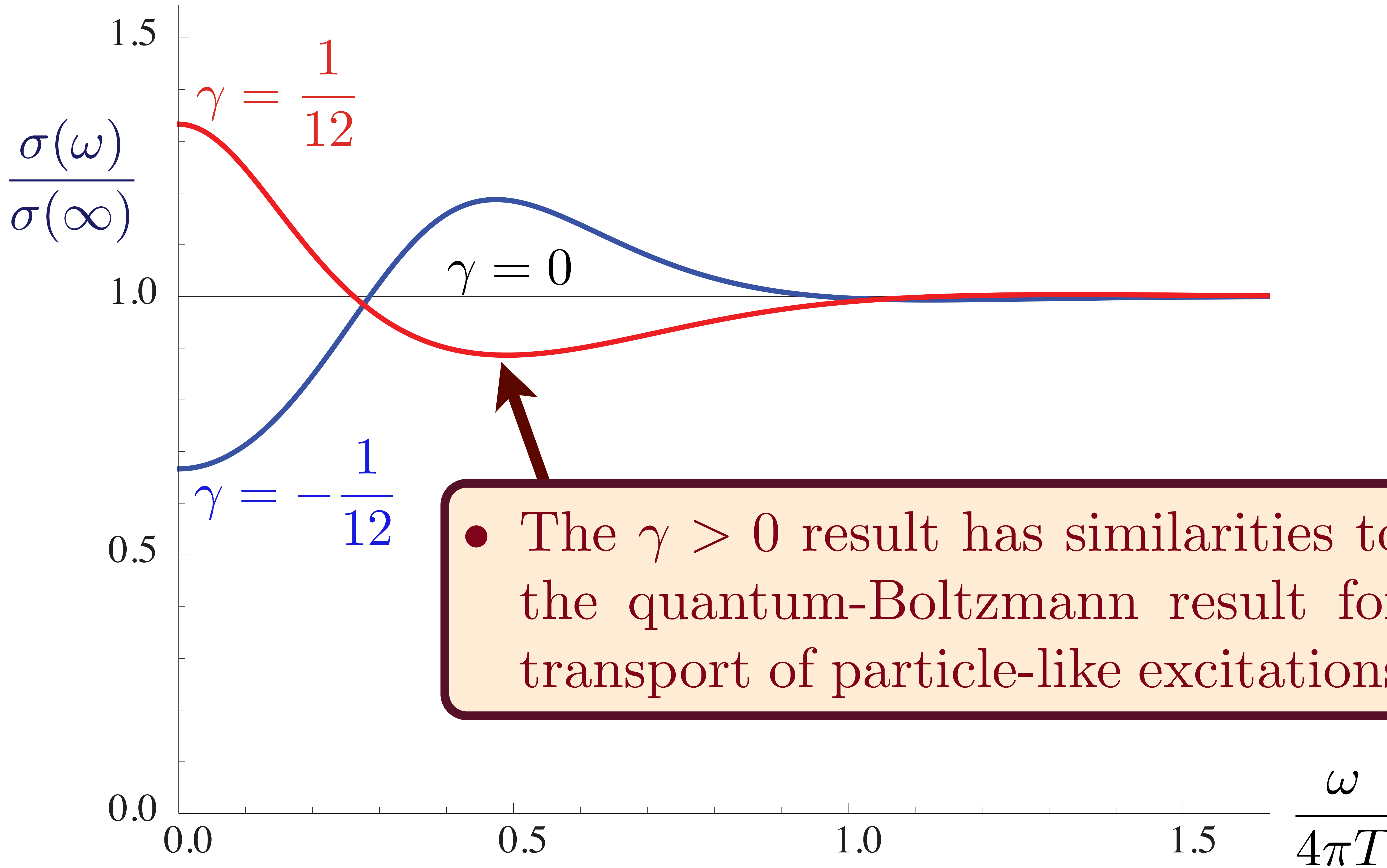
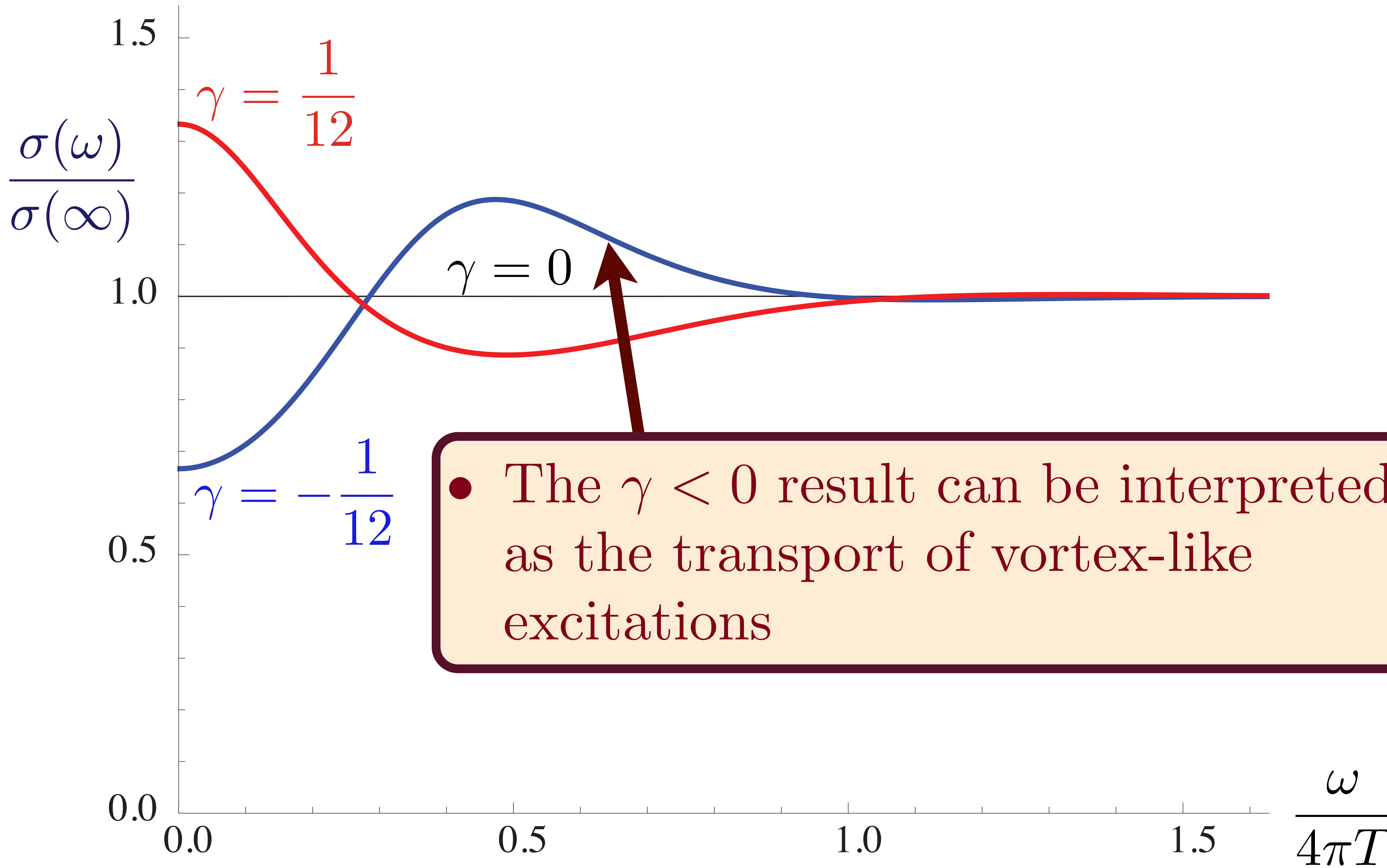


- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

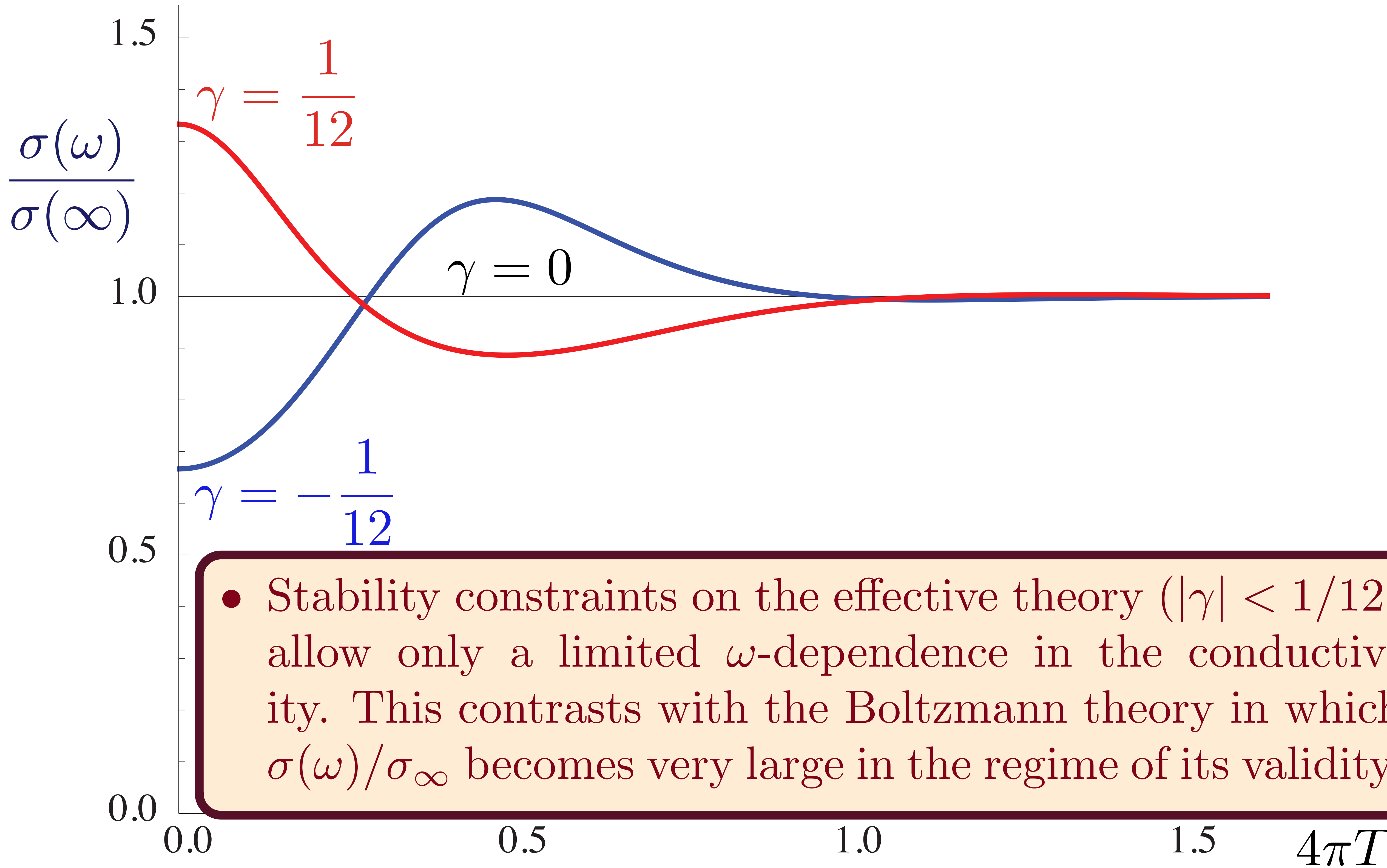
AdS_4 theory of quantum criticality



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS_4 theory of quantum criticality

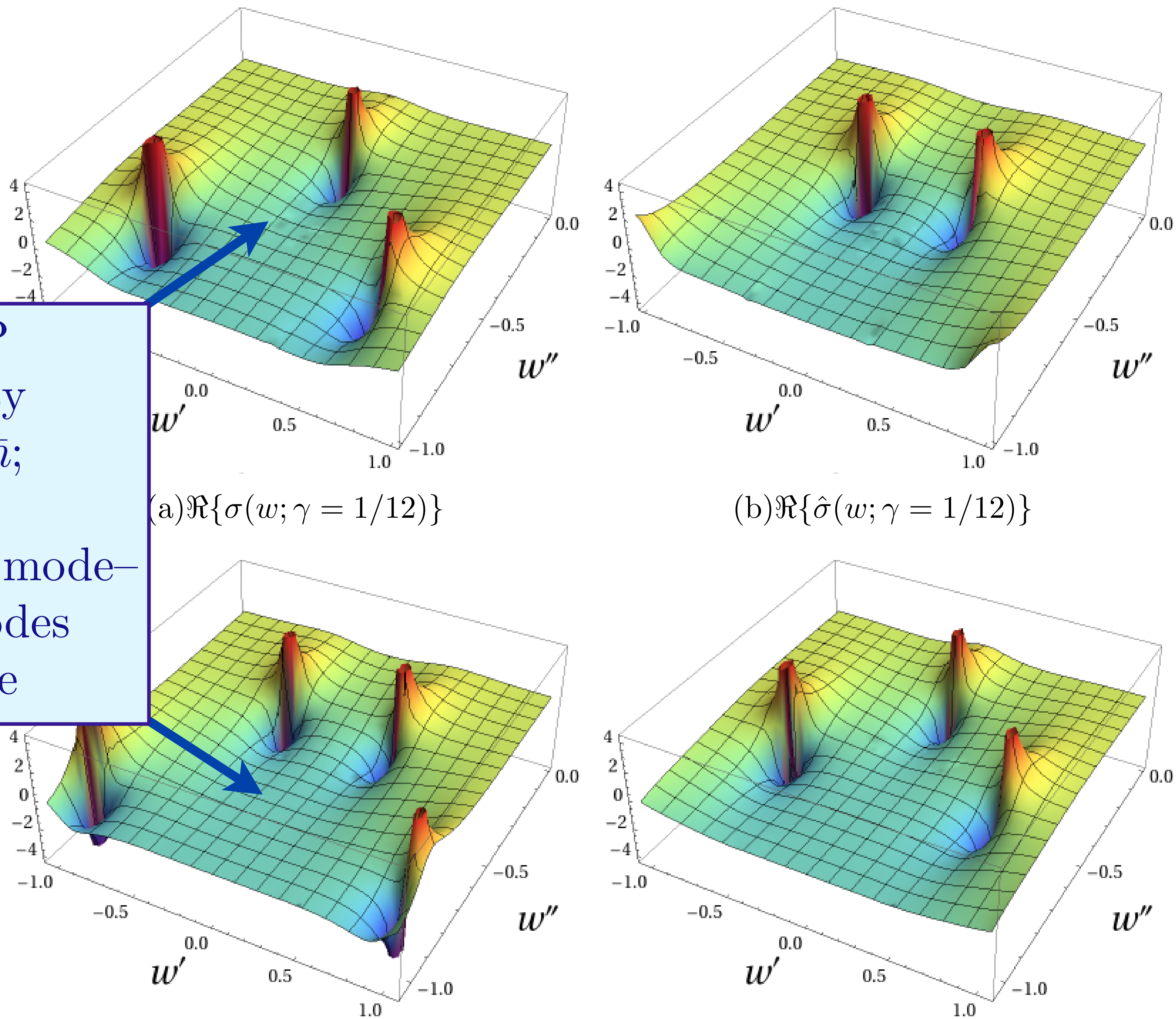


- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity. This contrasts with the Boltzmann theory in which $\sigma(\omega)/\sigma_\infty$ becomes very large in the regime of its validity.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of quantum criticality

Poles in LHP
of conductivity
at $\omega \sim k_B T/\hbar$;
analog of
Higgs quasinormal mode–
quasinormal modes
of black brane



(a) $\Re\{\sigma(w; \gamma = 1/12)\}$

(b) $\Re\{\hat{\sigma}(w; \gamma = 1/12)\}$

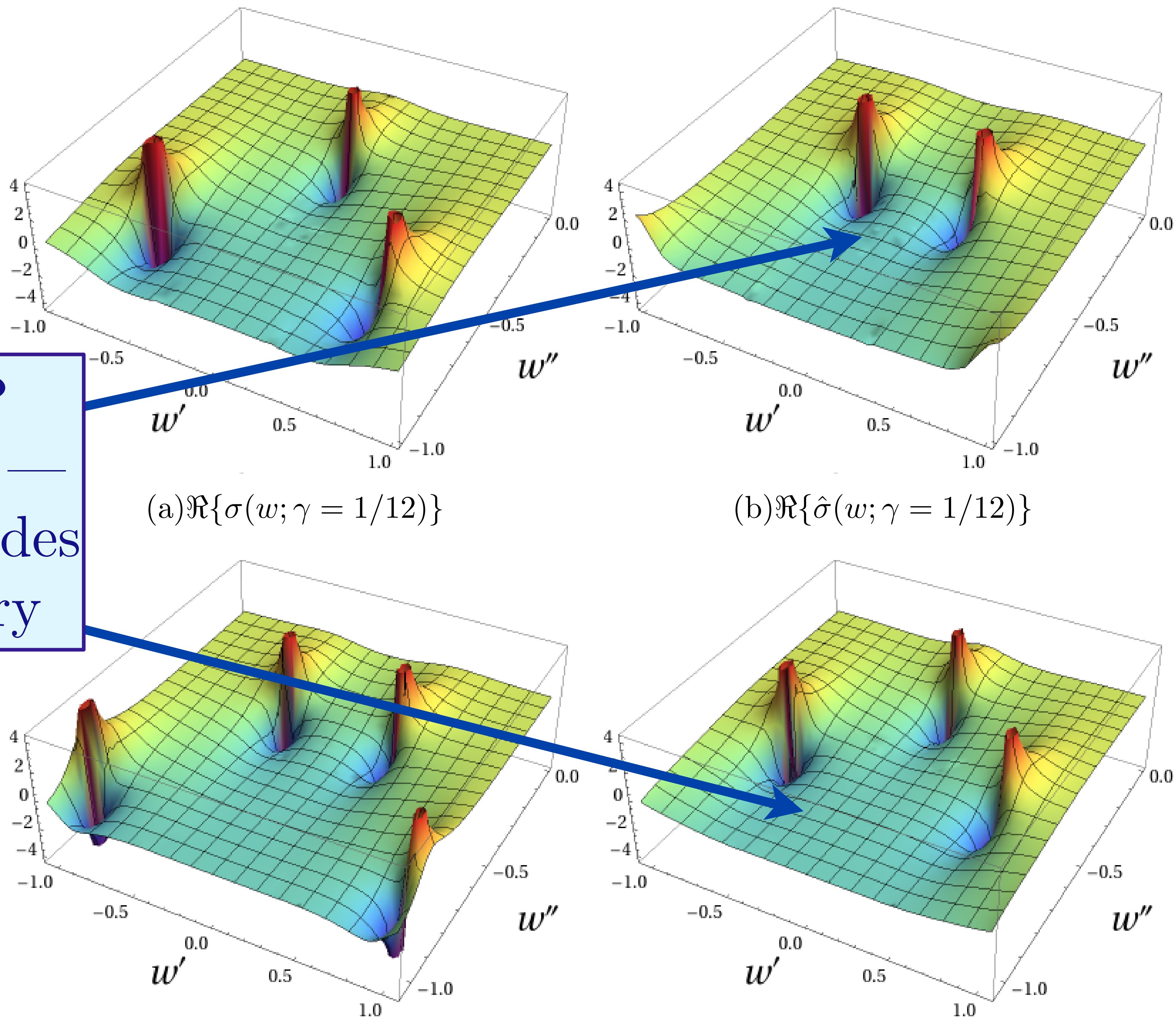
(c) $\Re\{\sigma(w; \gamma = -1/12)\}$

(d) $\Re\{\hat{\sigma}(w; \gamma = -1/12)\}$

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of quantum criticality

Zeros in LHP
of conductivity —
quasinormal modes
of S-dual theory



(a) $\Re\{\sigma(w;\gamma=1/12)\}$ (b) $\Re\{\hat{\sigma}(w;\gamma=1/12)\}$

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W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of quantum criticality

It can be shown that the conductivity of *any* CFT3 must satisfy two sum rules

$$\begin{aligned} \int_0^\infty d\omega \mathrm{Re}\left[\sigma(\omega) - \sigma(\infty)\right] &= 0 \\ \int_0^\infty d\omega \mathrm{Re}\left[\frac{1}{\sigma(\omega)} - \frac{1}{\sigma(\infty)}\right] &= 0 \end{aligned}$$

- The AdS$_4$ theory satisfies *both* sum rules exactly.
- The Boltzmann theory must make a choice between the "particle" or "vortex" basis, and so satisfies only *one* of the sum rules.

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

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- Start with strongly interacting CFT without particle- or wave-like excitations

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- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

AdS$_4$ theory of quantum criticality

PRL **95,** 180603 (2005) PHYSICAL REVIEW LETTERS week ending 28 OCTOBER 2005

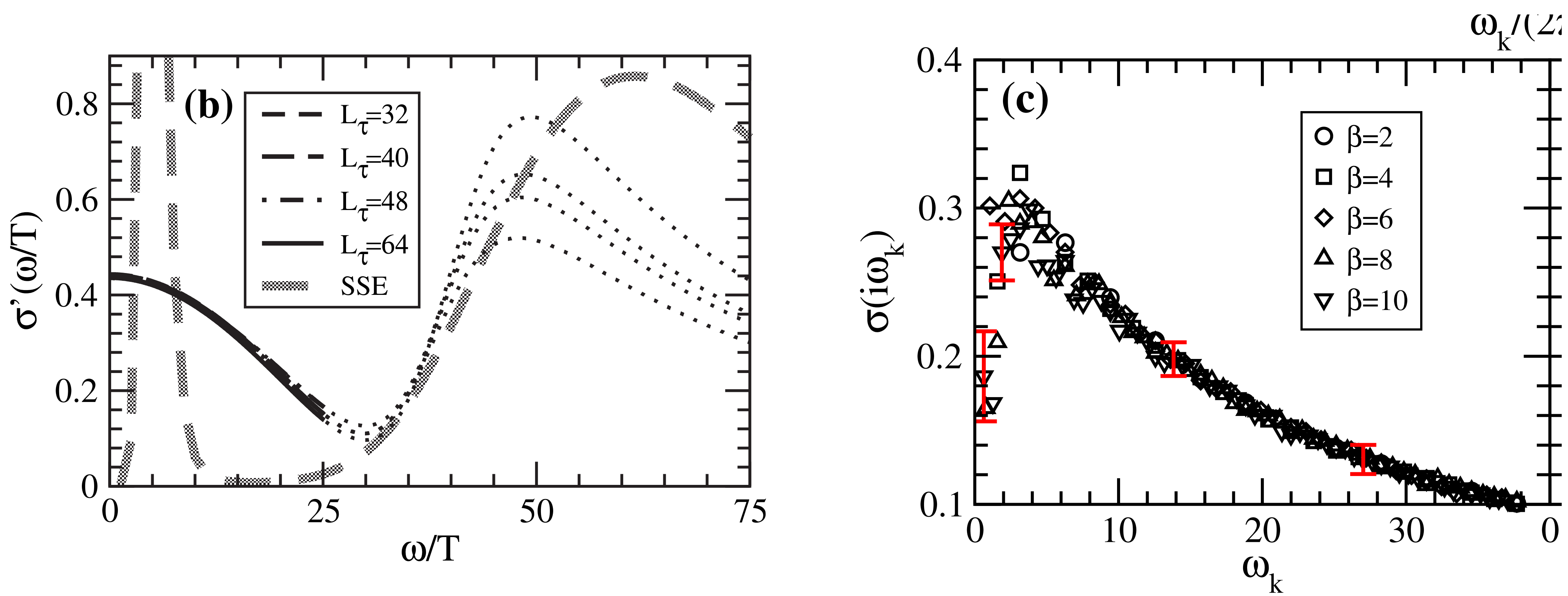
Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada

(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature T. We find clear evidence for *deviations* from ω_k scaling of the conductivity towards ω_k/T scaling at low Matsubara frequencies ω_k. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with ω/T at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.



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QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

W. Witzack-Krempa and S. Sachdev, arXiv:1302.0847

Outline

1. Z_2 Spin liquid in the kagome antiferromagnet

2. Superfluid-insulator transition of ultracold atoms in optical lattices:
   Quantum criticality and conformal field theories

3. Holography and the quasi-normal modes of black-hole horizons

4. Strange metals:
   What lies beyond the horizon ?

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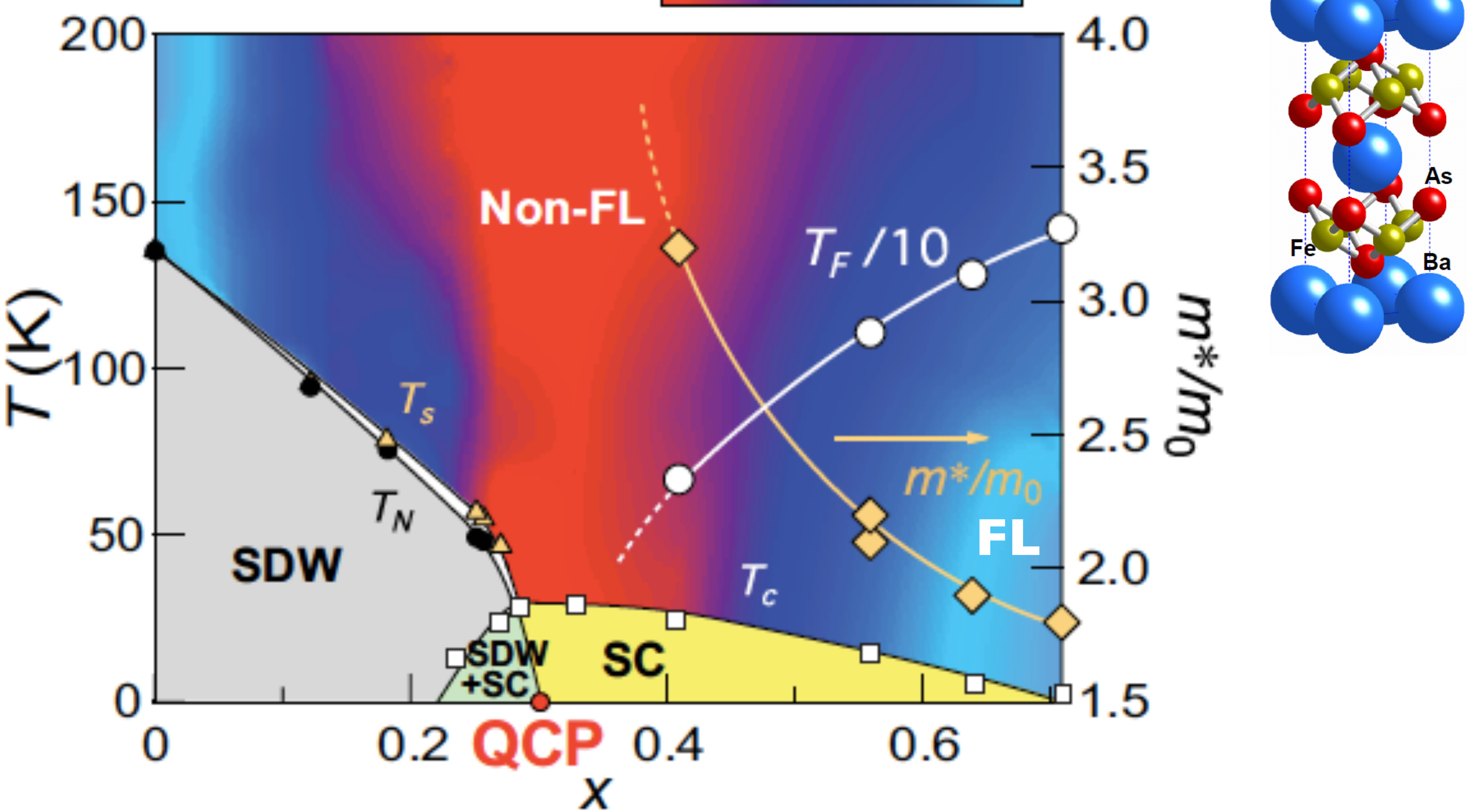
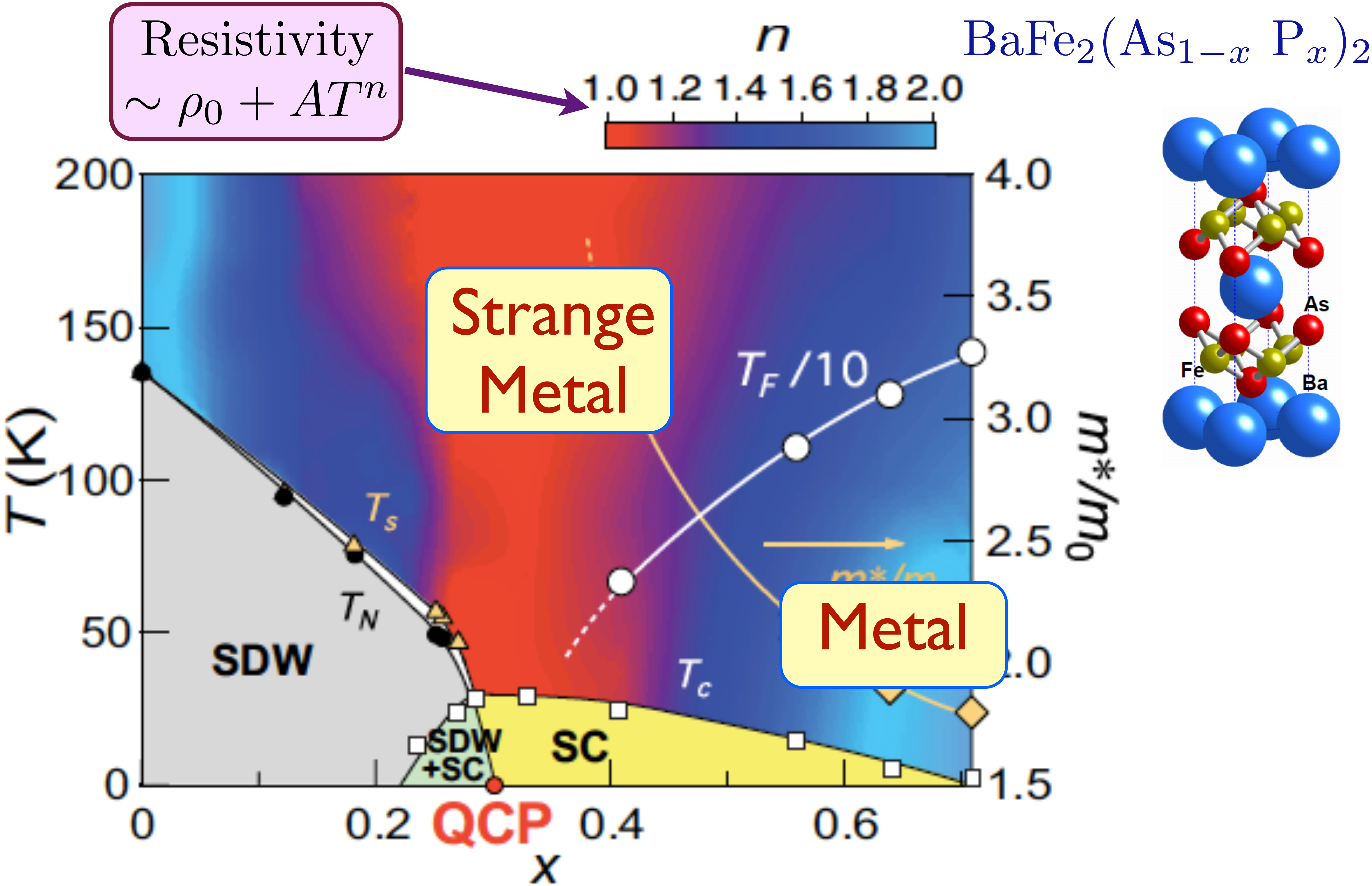
$BaFe_2(As_{1-x}\ P_x)_2$

Resistivity $\sim \rho_0 + AT^n$



K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).



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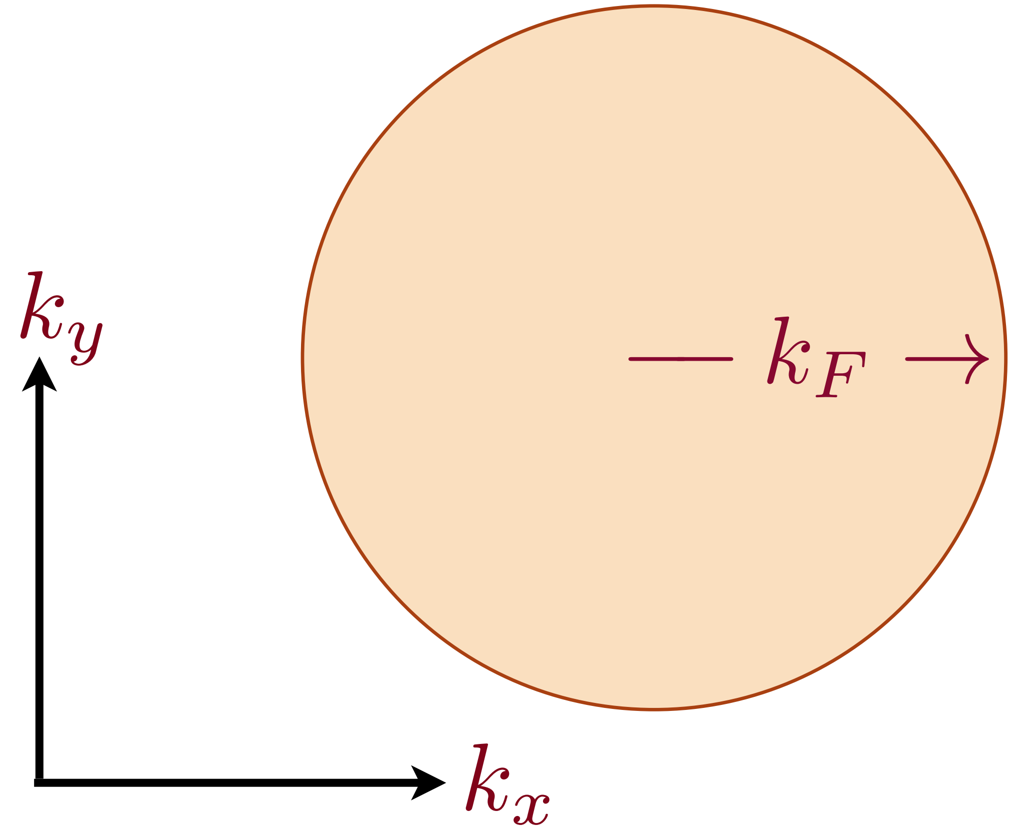
The Metal



Electrons (fermions) occupy states inside a Fermi "surface" (circle) of radius k_F which is determined by the density of electrons, $\mathcal{Q}$.

A Strange Metal

Can bosons form a metal ?

A Strange Metal

O. I. Motrunich and M. P. A. Fisher, *Phys. Rev.* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Phys. Rev.* D **84**, 026001 (2011)
S. Sachdev, arXiv:1209.1637

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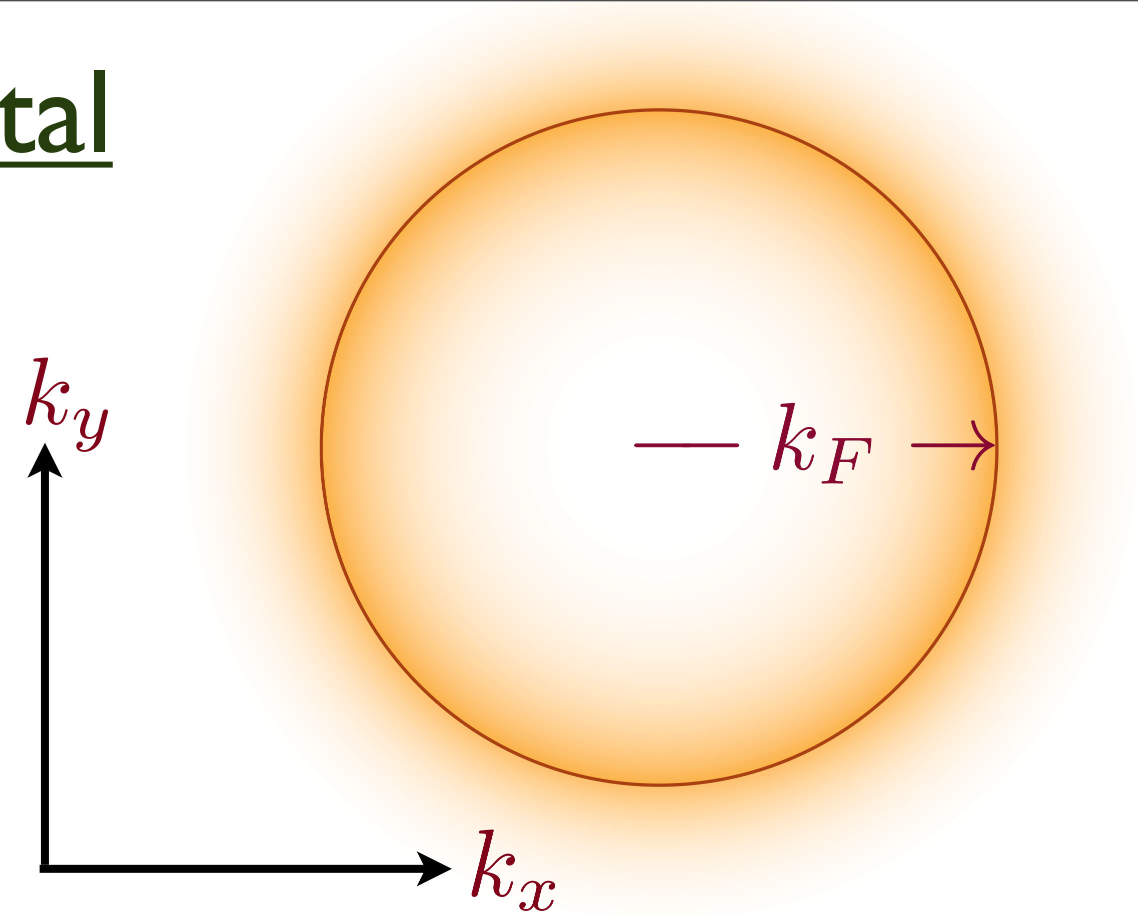

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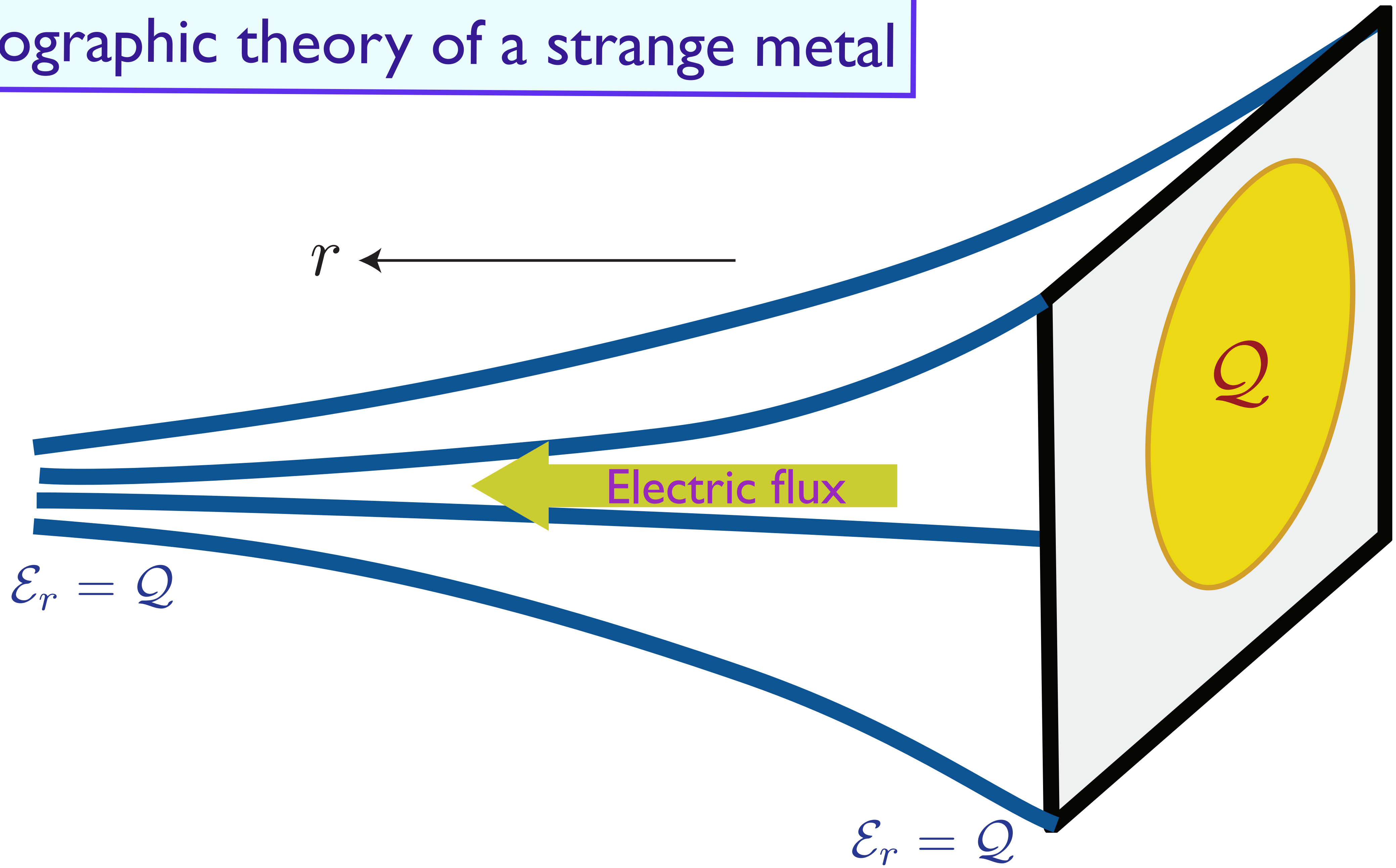
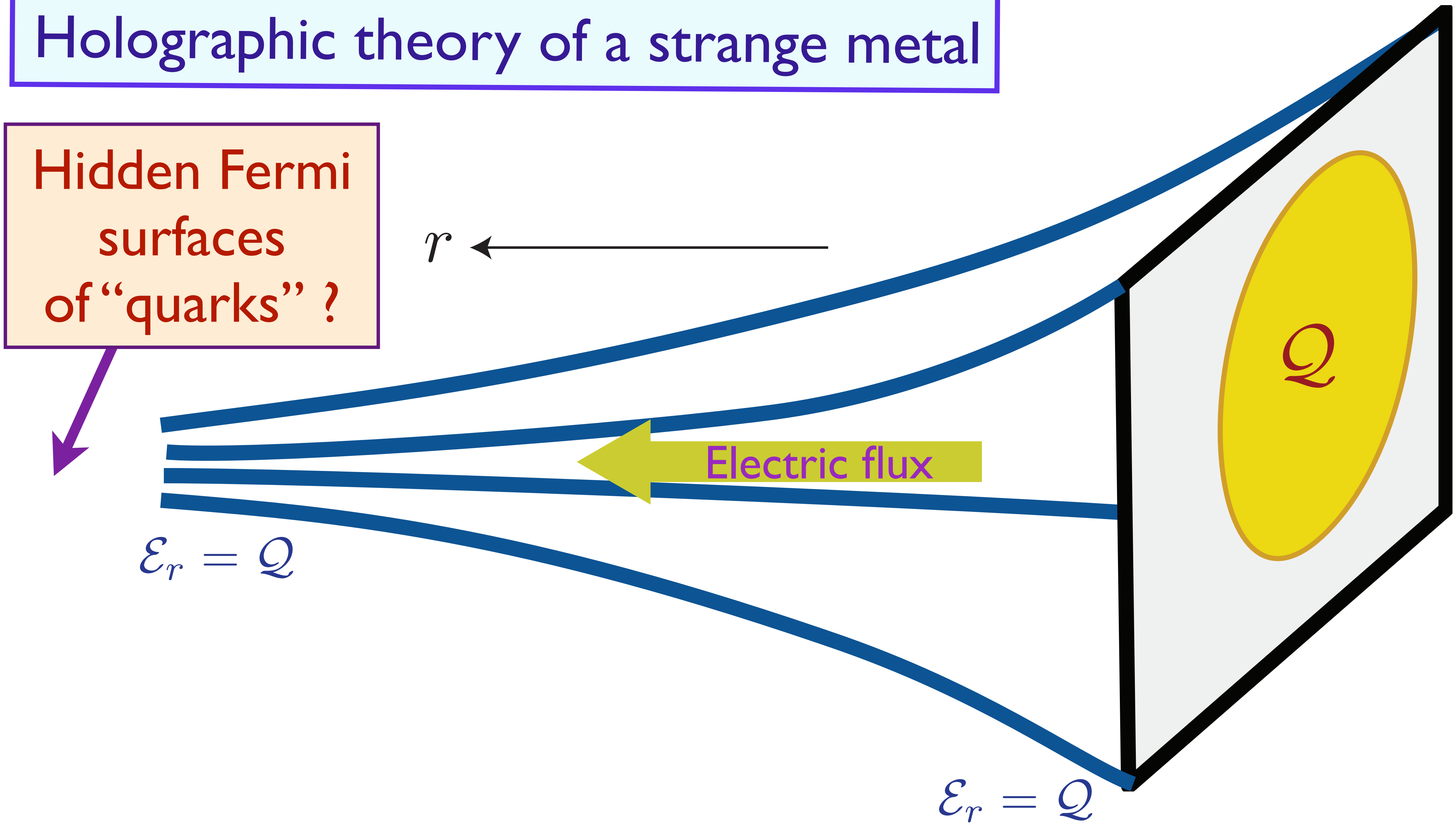
- Each quark is charged under an *emergent* gauge force, which encapsulates the entanglement in the ground state.
- The quarks have "hidden" Fermi surfaces of radius k_F.

Holographic theory of a strange metal



The density of particles $\mathcal{Q}$ creates an electric flux $\mathcal{E}_r$ which modifies the metric of the emergent spacetime.

Holographic theory of a strange metal



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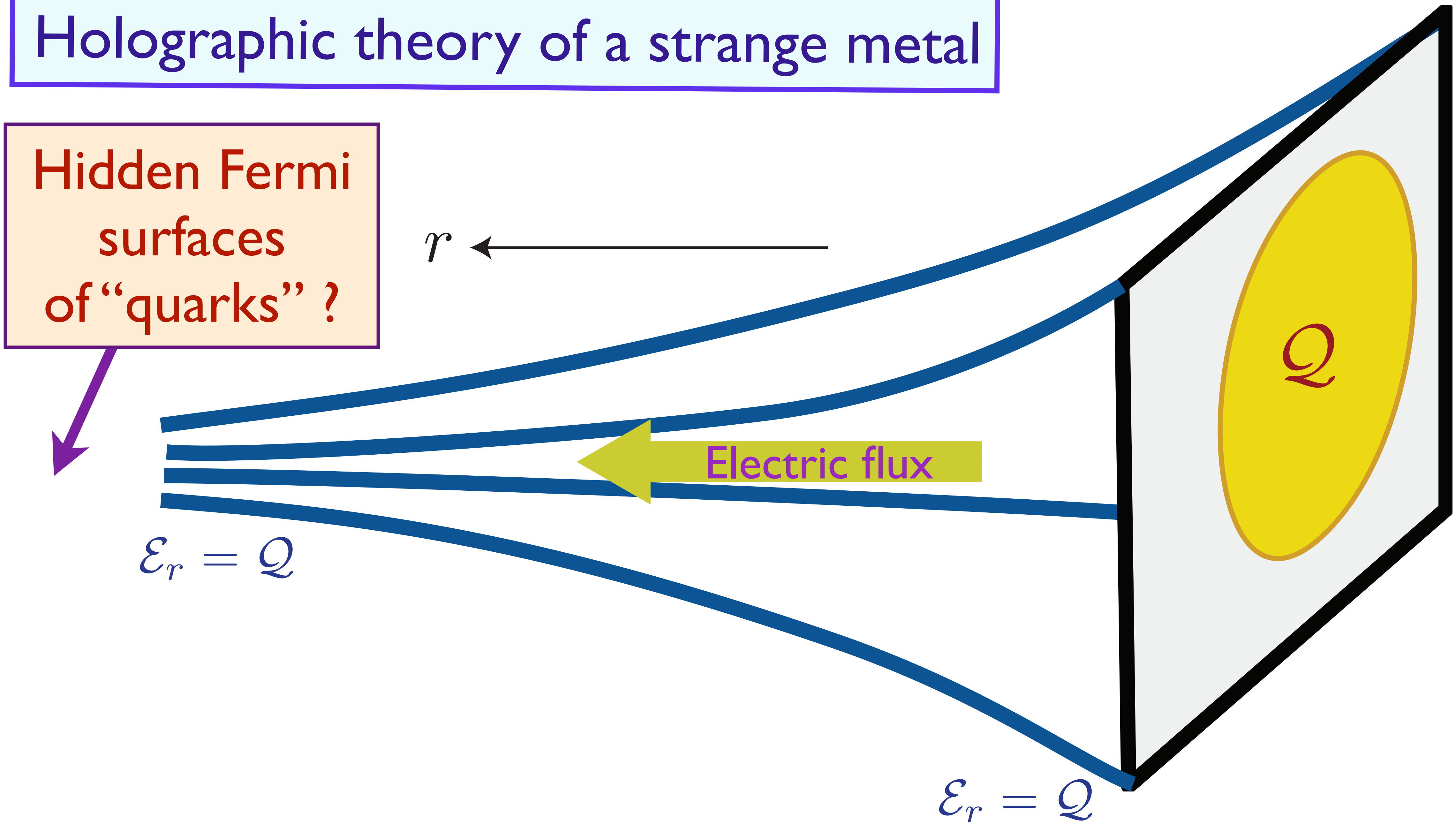
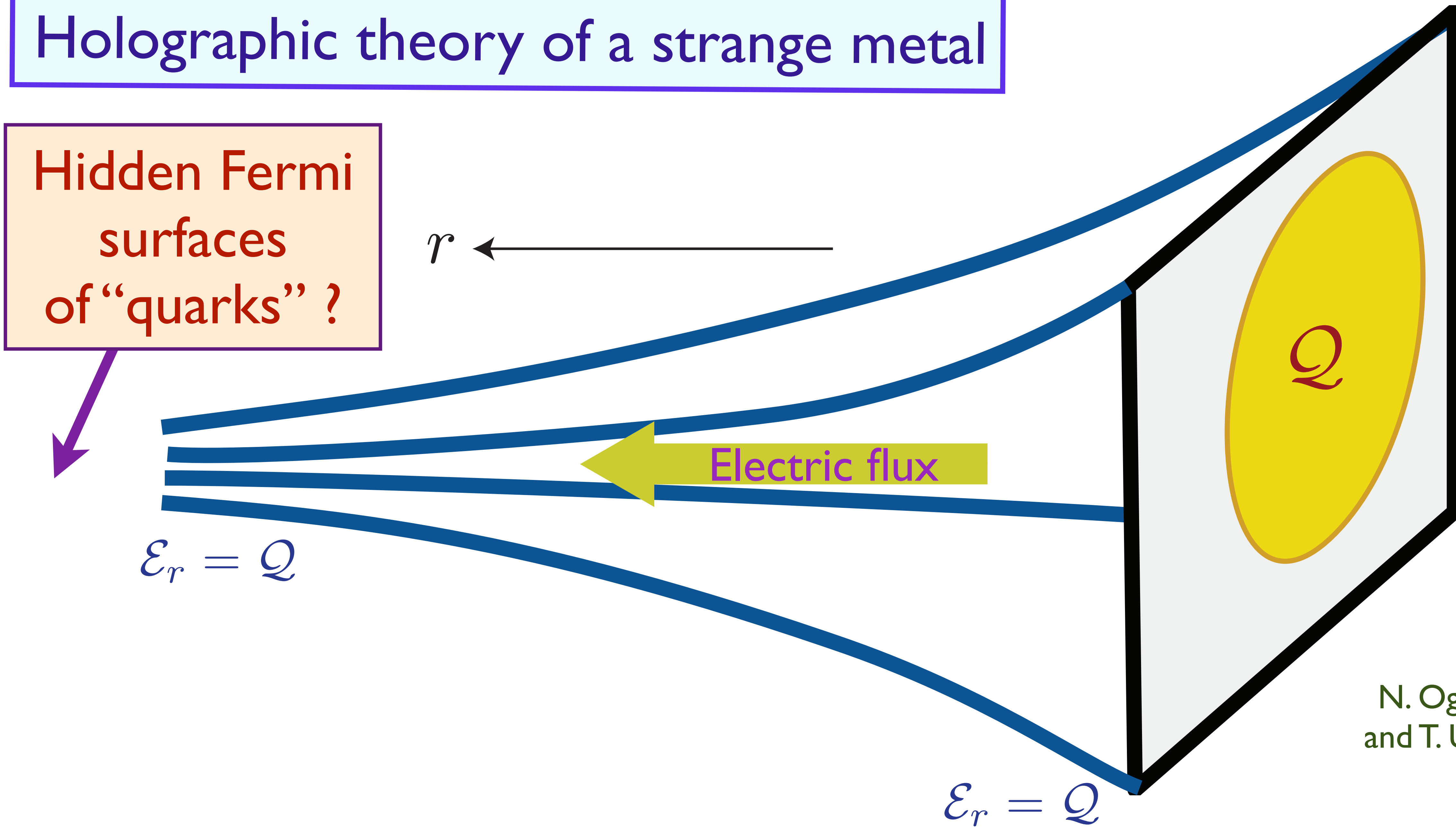


The general metric transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Holographic theory of a strange metal



N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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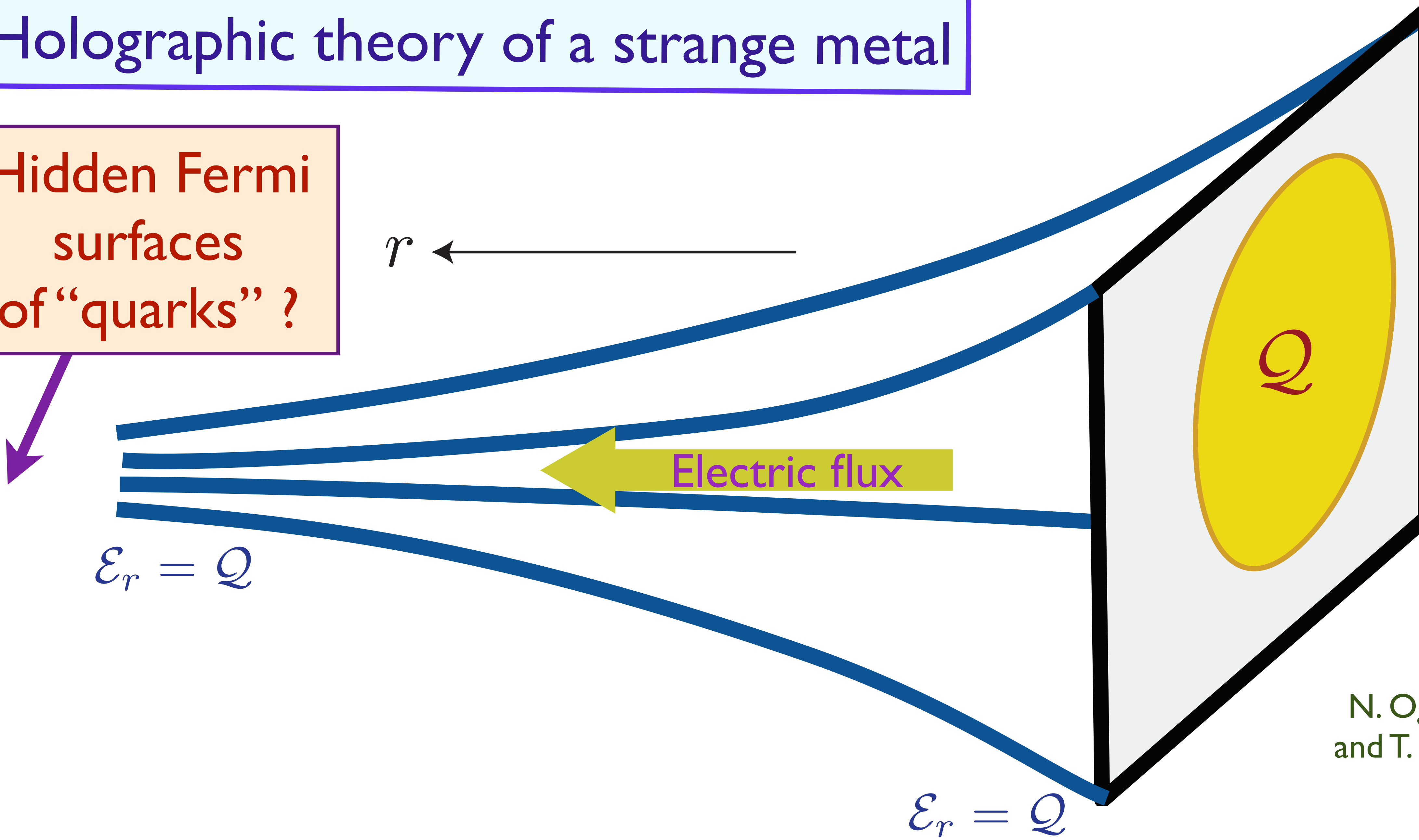
The value $\theta = d - 1$ reproduces *all* the essential characteristics of the entropy and entanglement entropy of a strange metal.

Holographic theory of a strange metal

Hidden Fermi surfaces of “quarks” ?



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The null-energy condition of gravity yields $z \geq 1 + \theta/d$. In $d = 2$, this leads to $z \geq 3/2$. Field theory on strange metal yields $z = 3/2$ to 3 loops!

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Conclusions

Realizations of many-particle entanglement:
Z_2 spin liquids and
conformal quantum critical points

Conclusions

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with many-particle quantum entanglement.

Much recent progress offers hope of a holographic description of "strange metals"