Strange metals: field theory vs. holography

McGill University
February 16, 2012

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PHYSICS
VE RI TAS
HARVARD

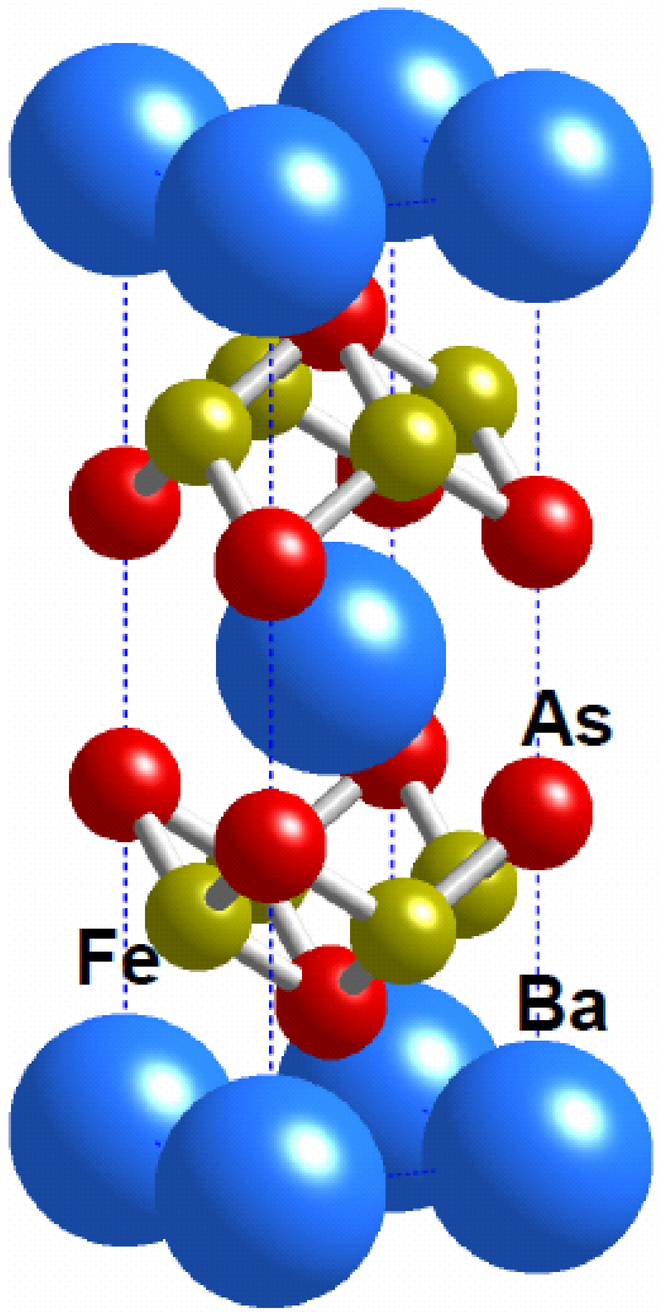
Liza Huijse

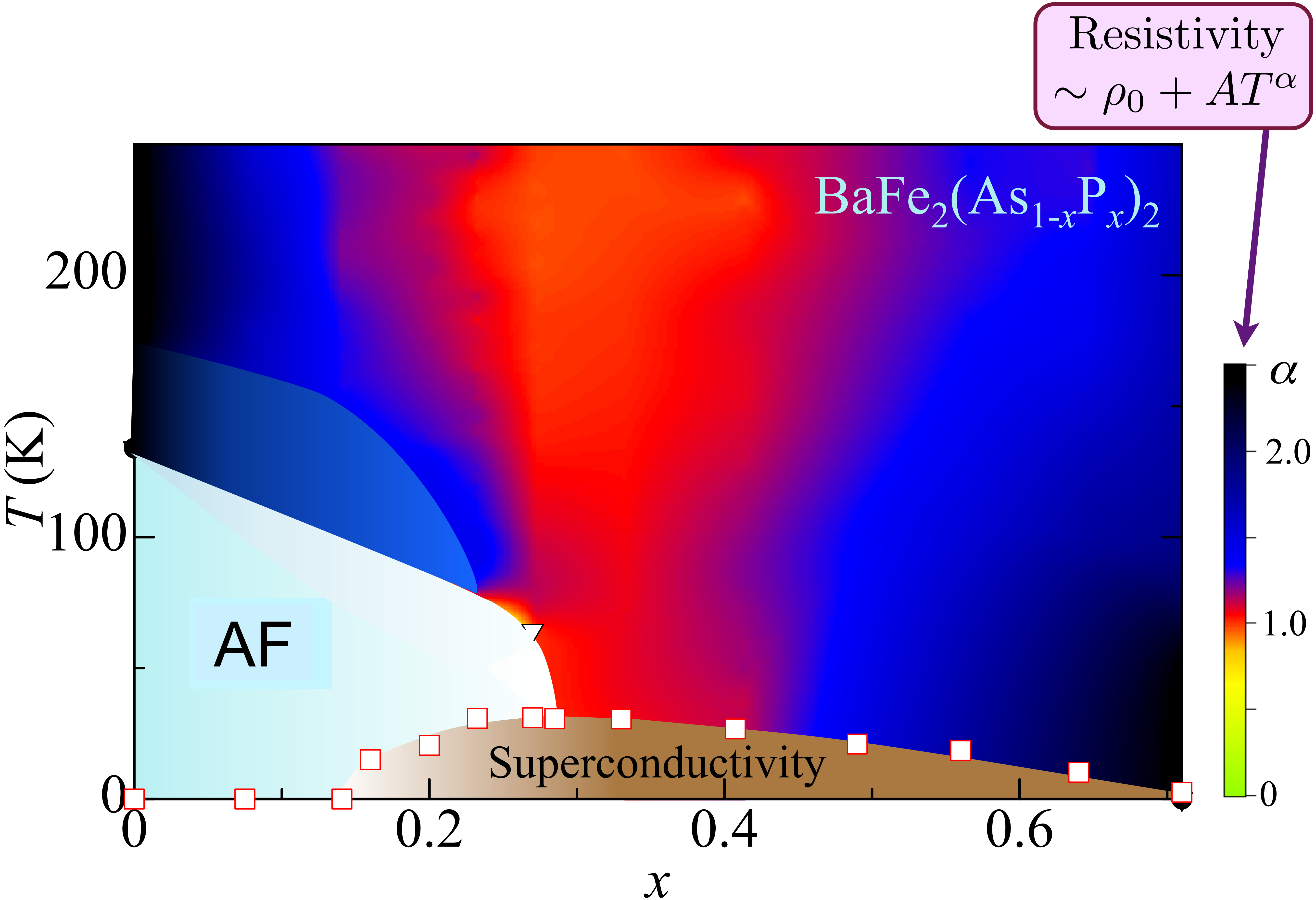
Max Metlitski

Brian Swingle

Iron pnictides:

a new class of high temperature superconductors





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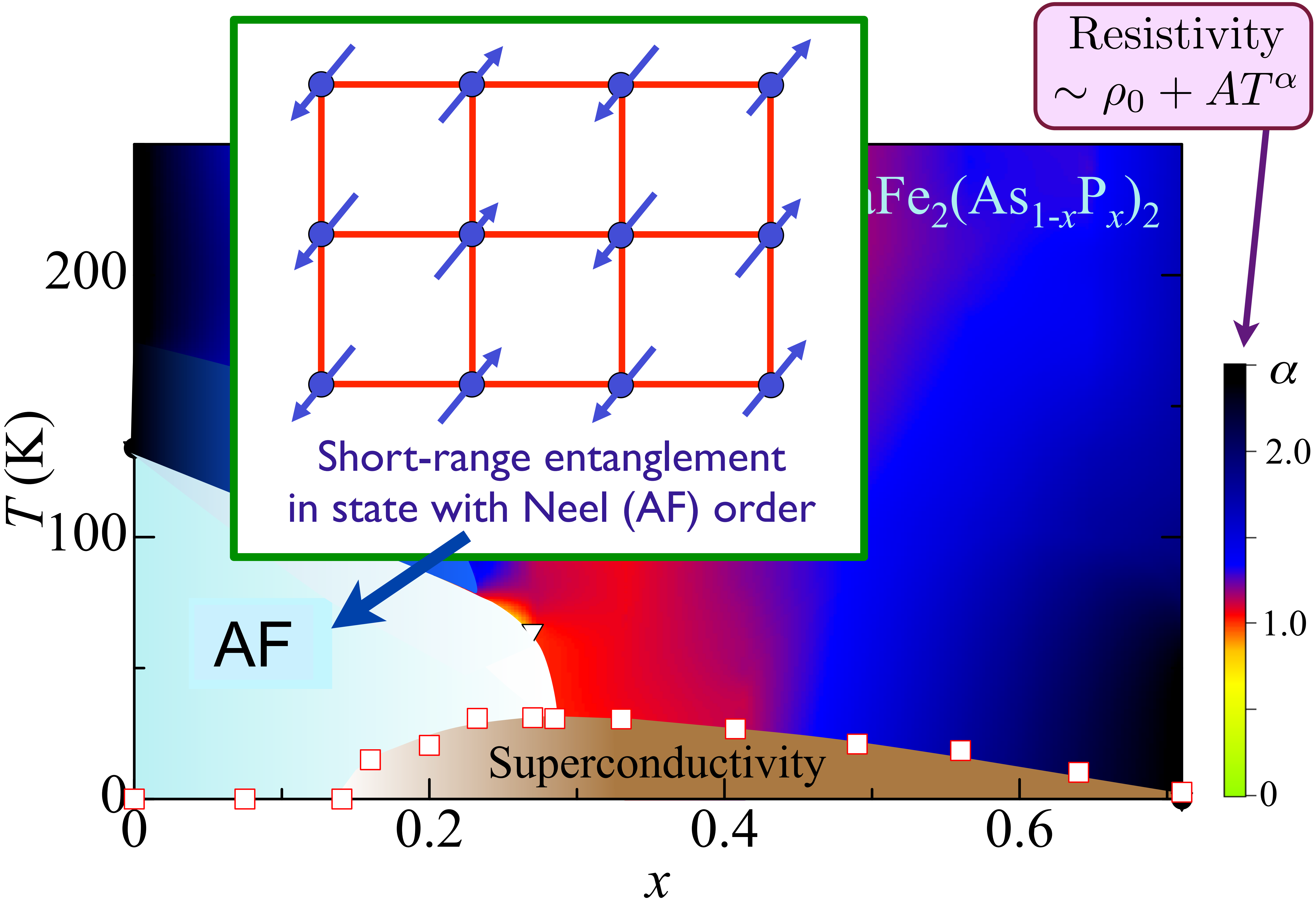

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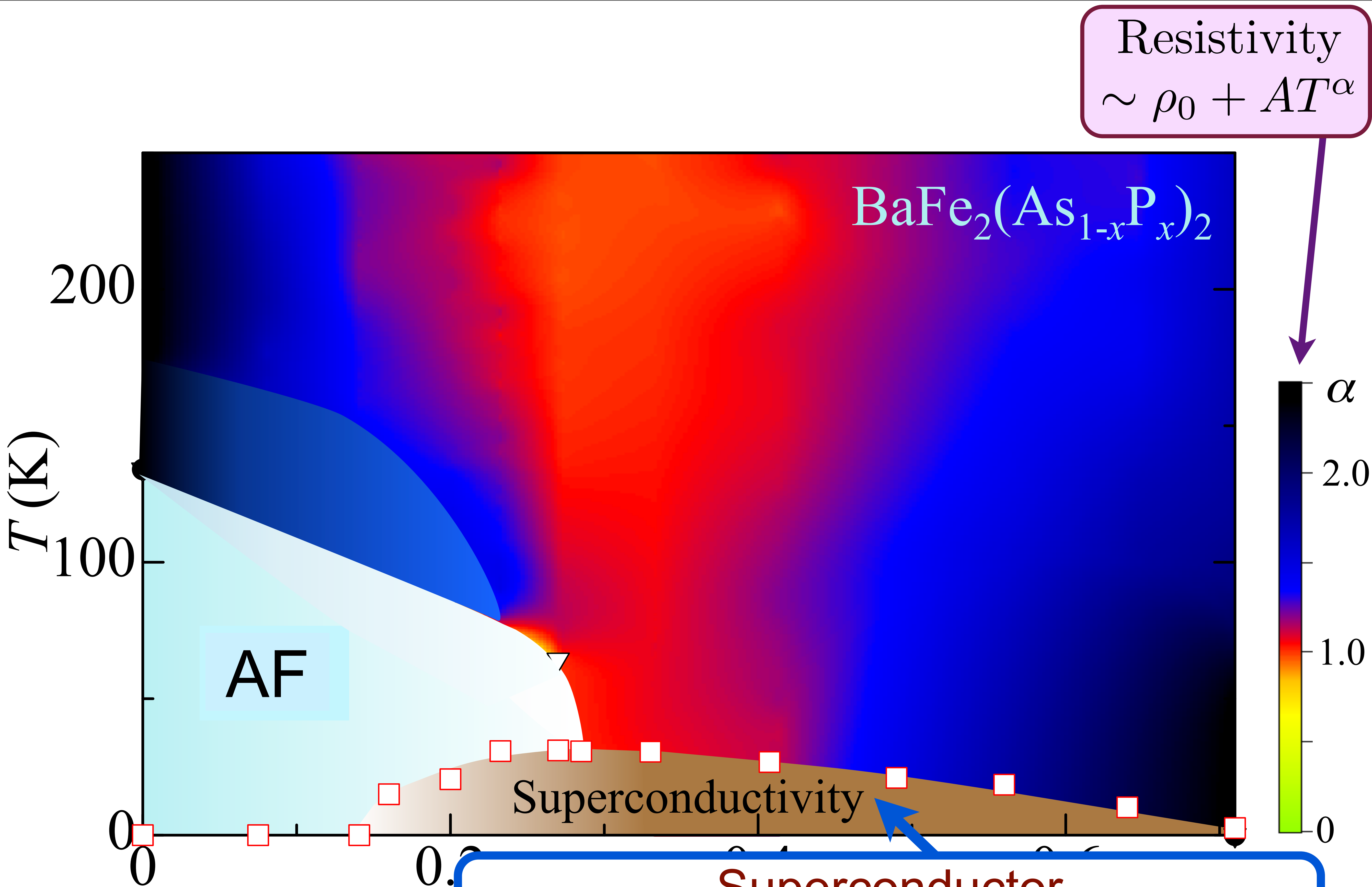
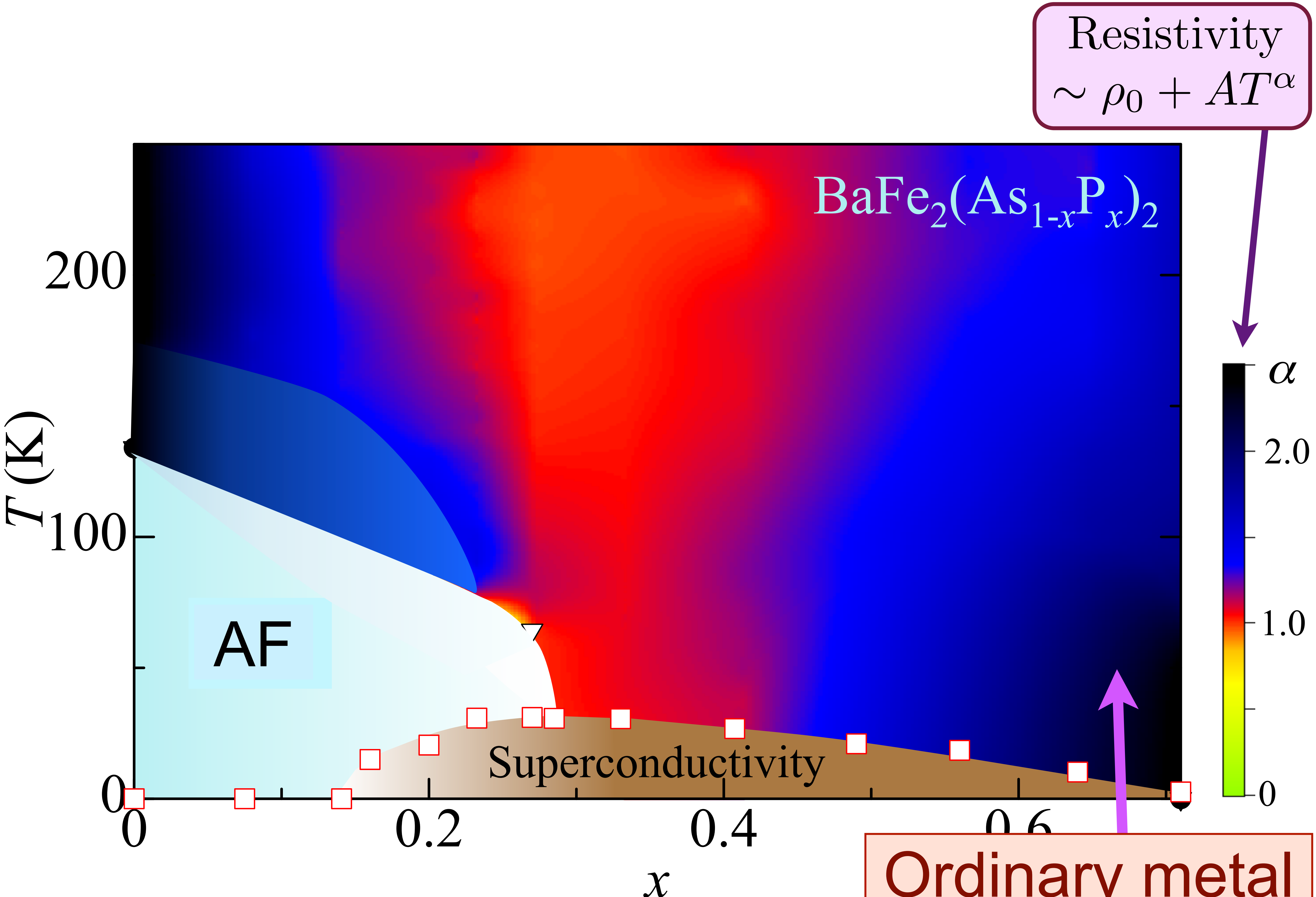


Superconductor
Bose condensate of pairs of electrons
Short-range entanglement

S. Kasahara, T. Shiba
H. Ike

Friday, February 17, 2012



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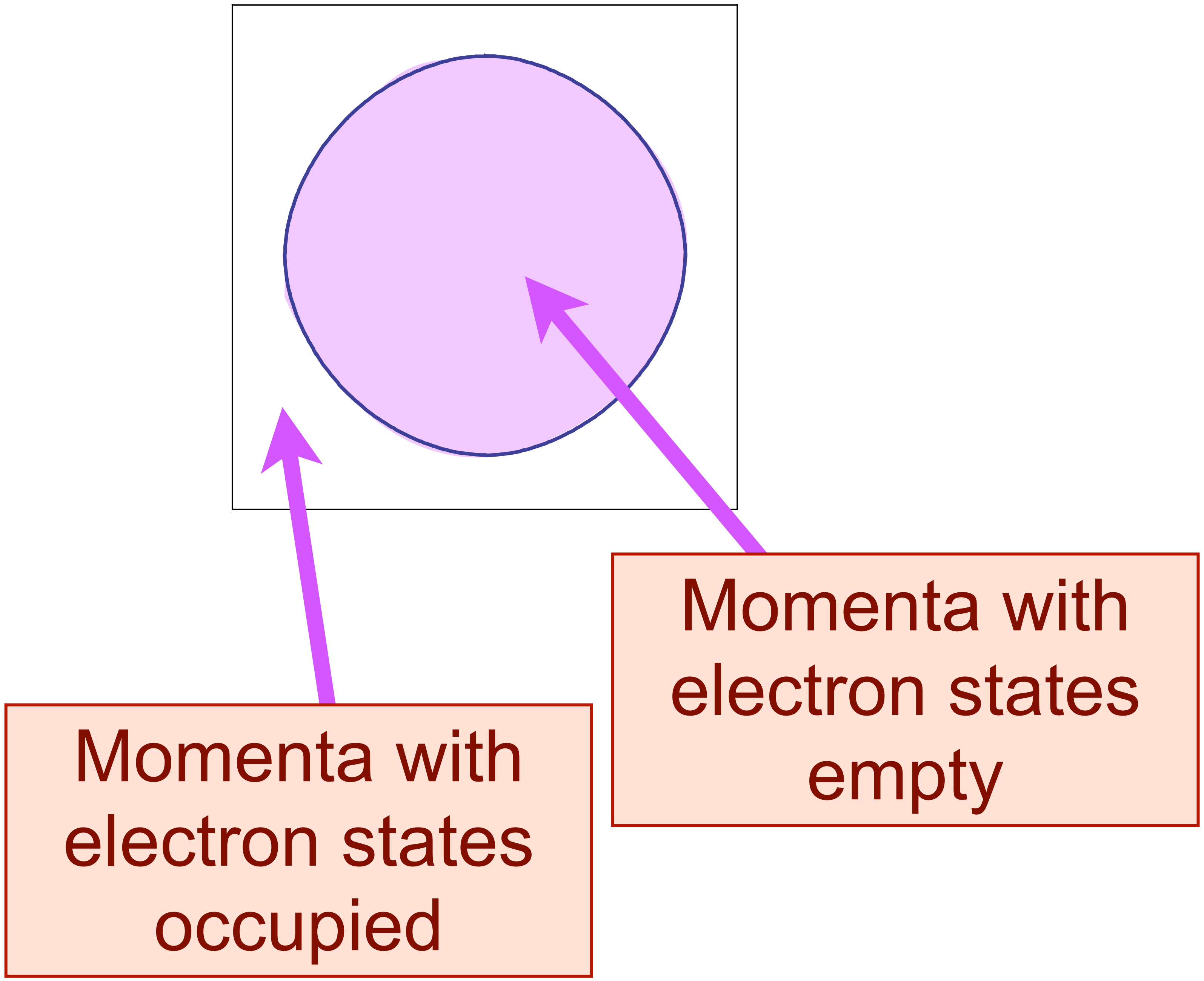
Sommerfeld-Bloch-Landau theory of ordinary metals



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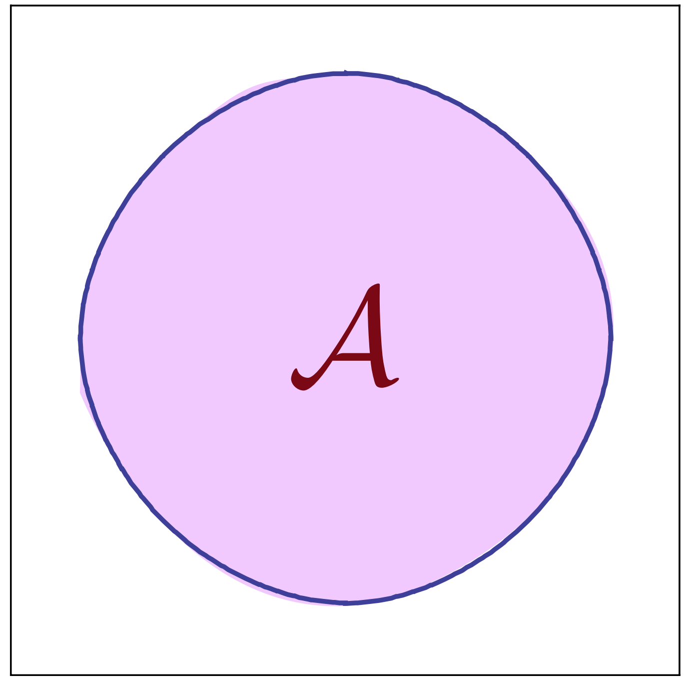
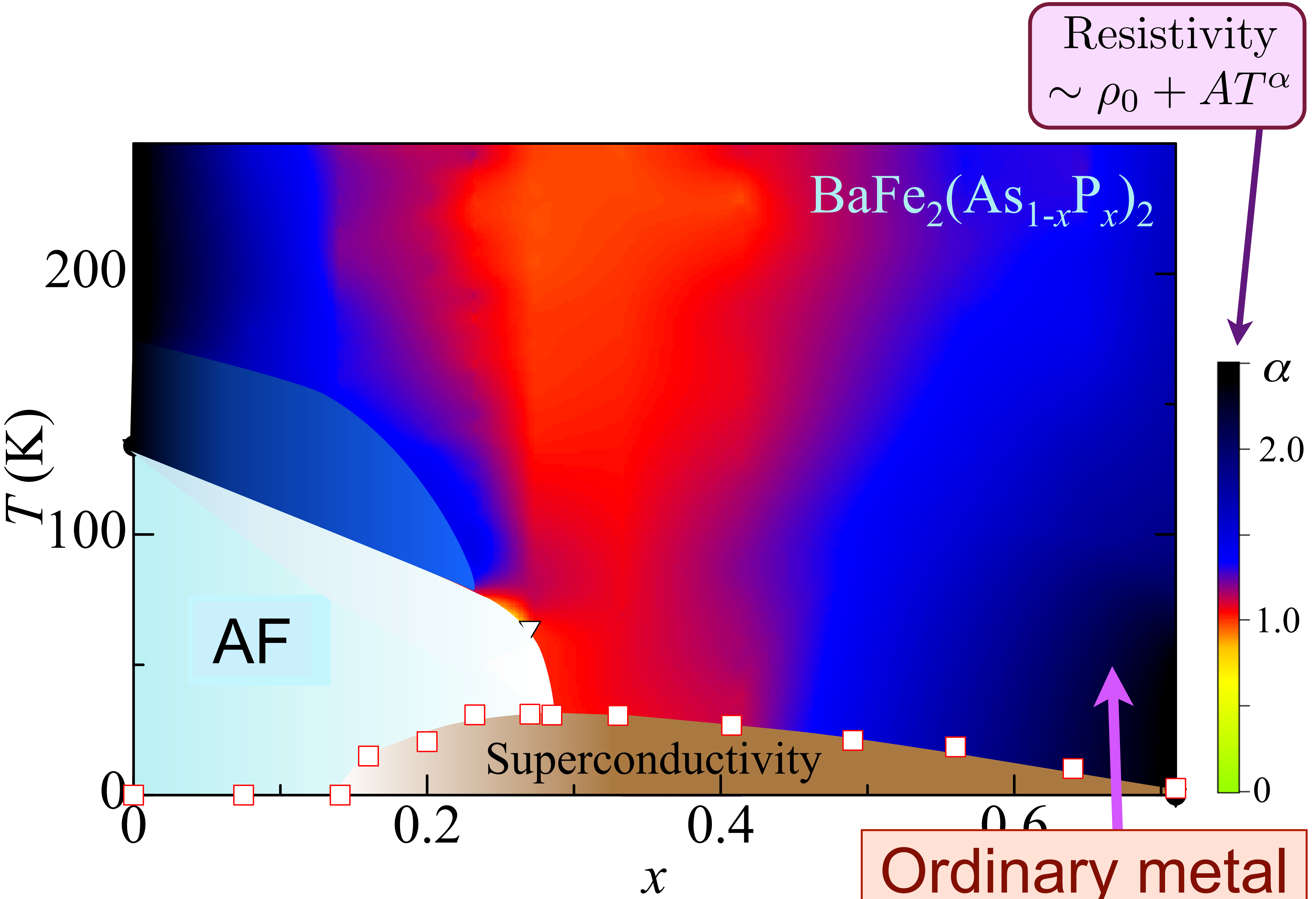


Key feature of the theory:
the **Fermi surface**

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the electron density
- Excitations near the Fermi surface are responsible for the familiar properties of ordinary metals, such as resistivity $\sim T^2$.



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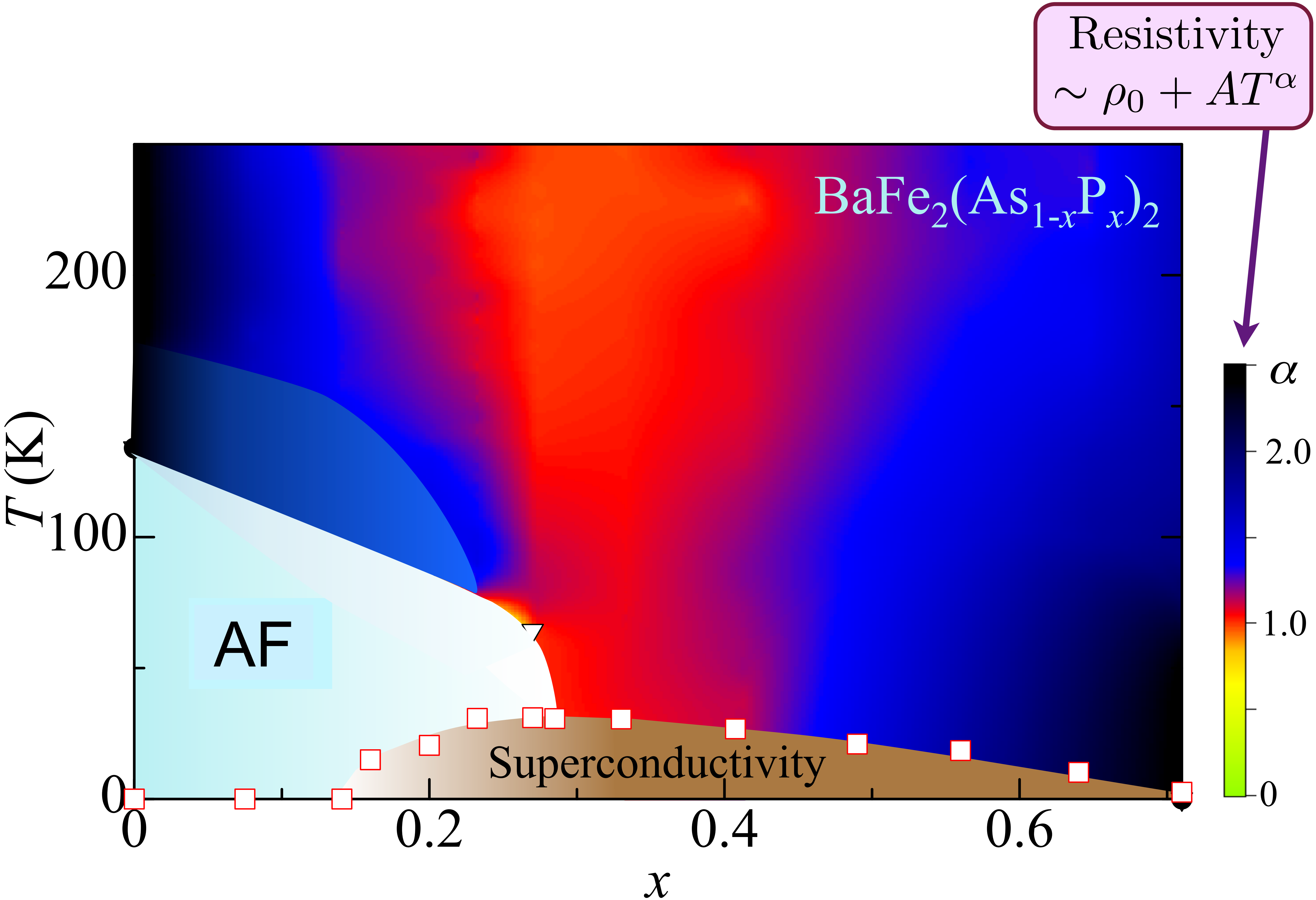

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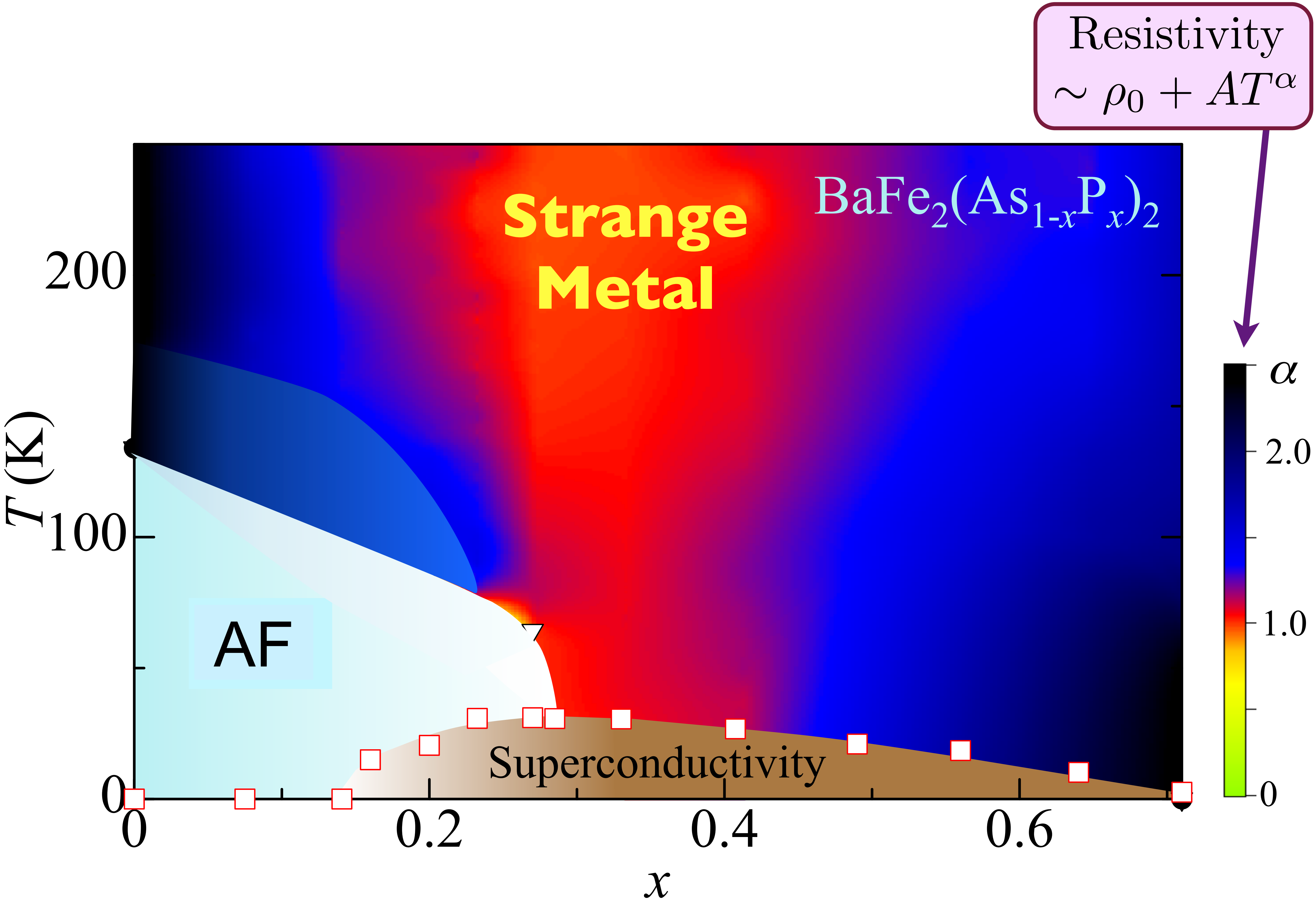

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

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- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe *zero temperature* phases where $d\langle \mathcal{Q} \rangle / d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu \mathcal{Q}$.

The only compressible phase of traditional condensed matter physics which does not break the translational or $\mathrm{U}(1)$ symmetries is the Landau Fermi liquid

Strange metals

A. Field theory

B. Holography

Strange metals

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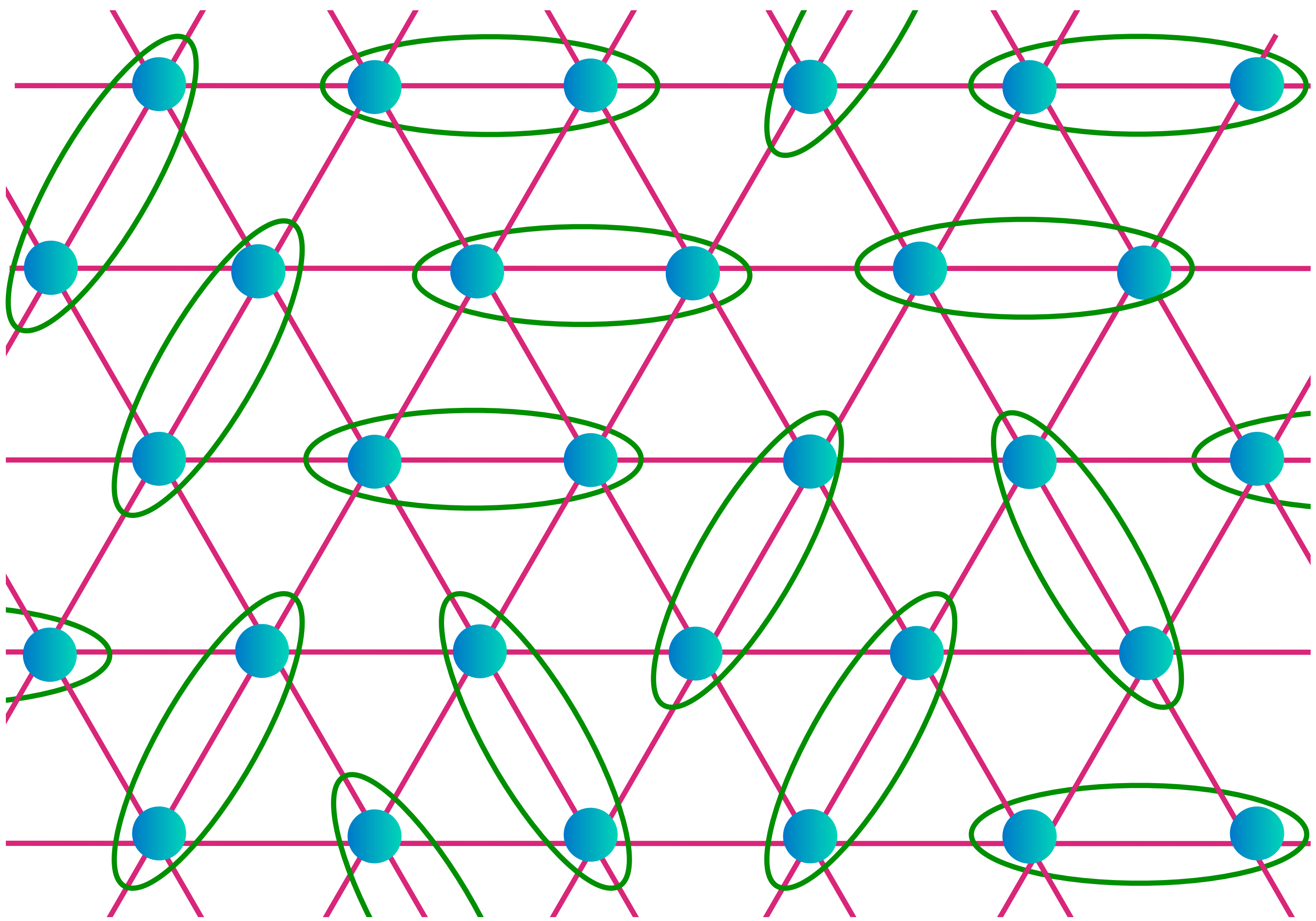
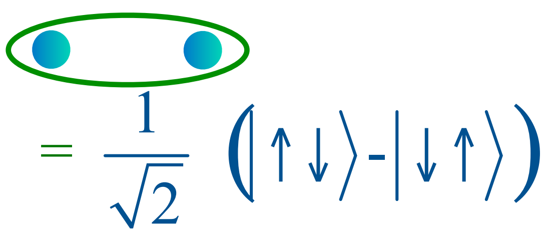
The Non-Fermi Liquid (NFL)

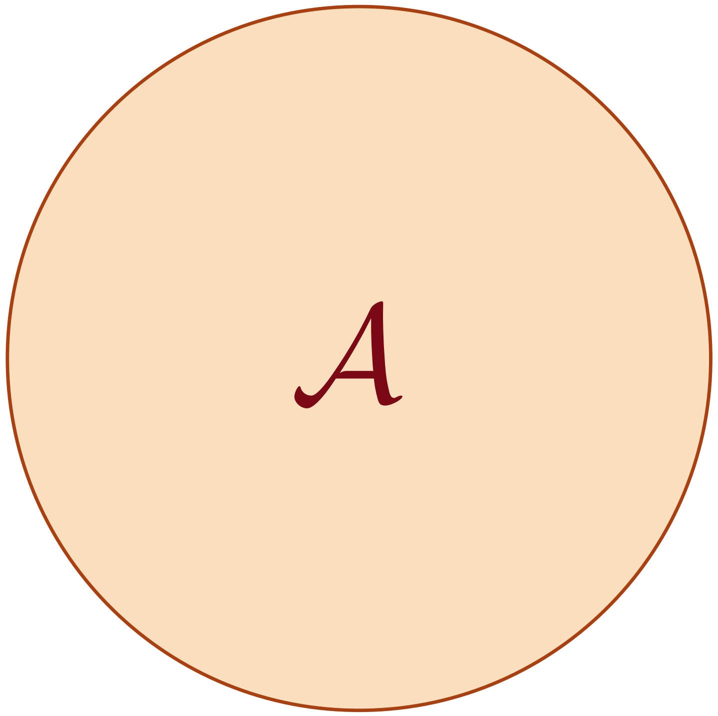
- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.





$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

Fermi surface of an ordinary metal



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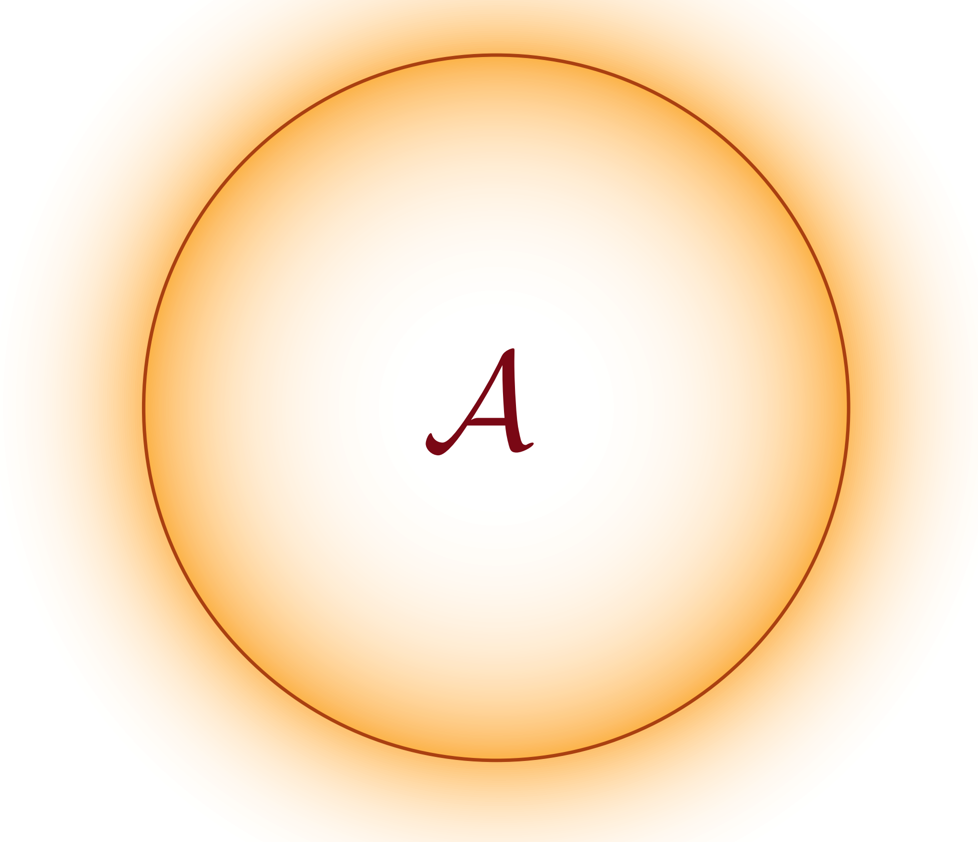
Fermions coupled to a gauge field



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S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
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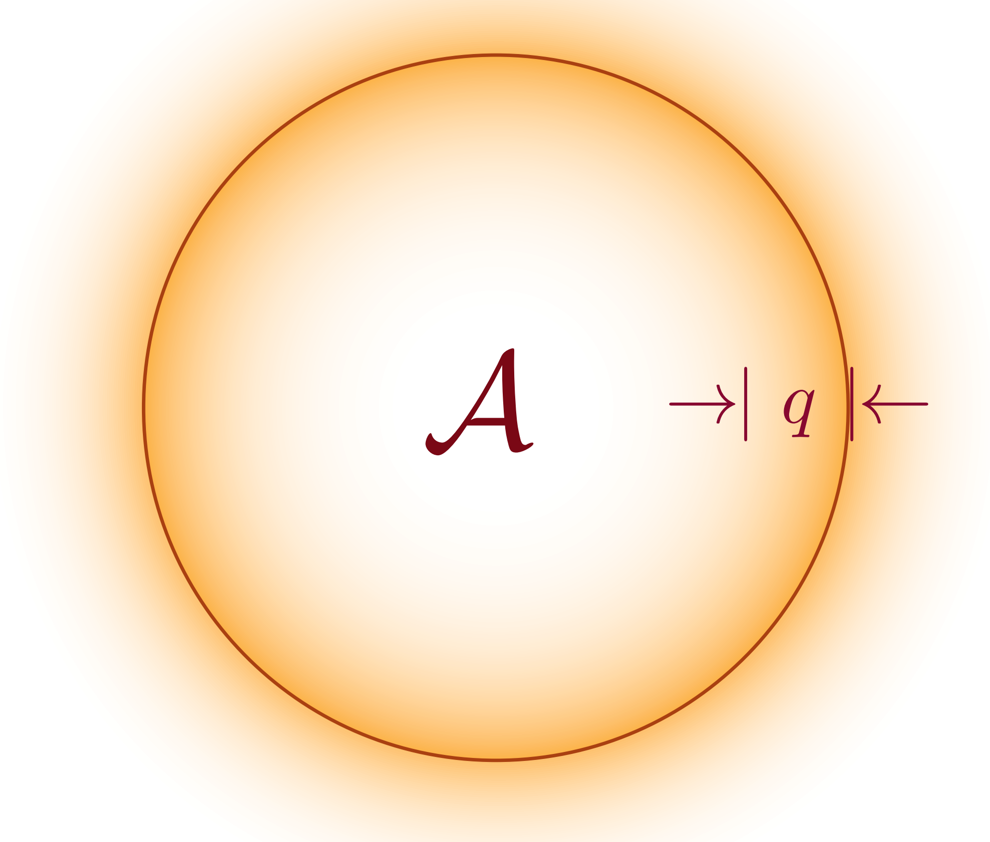

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- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
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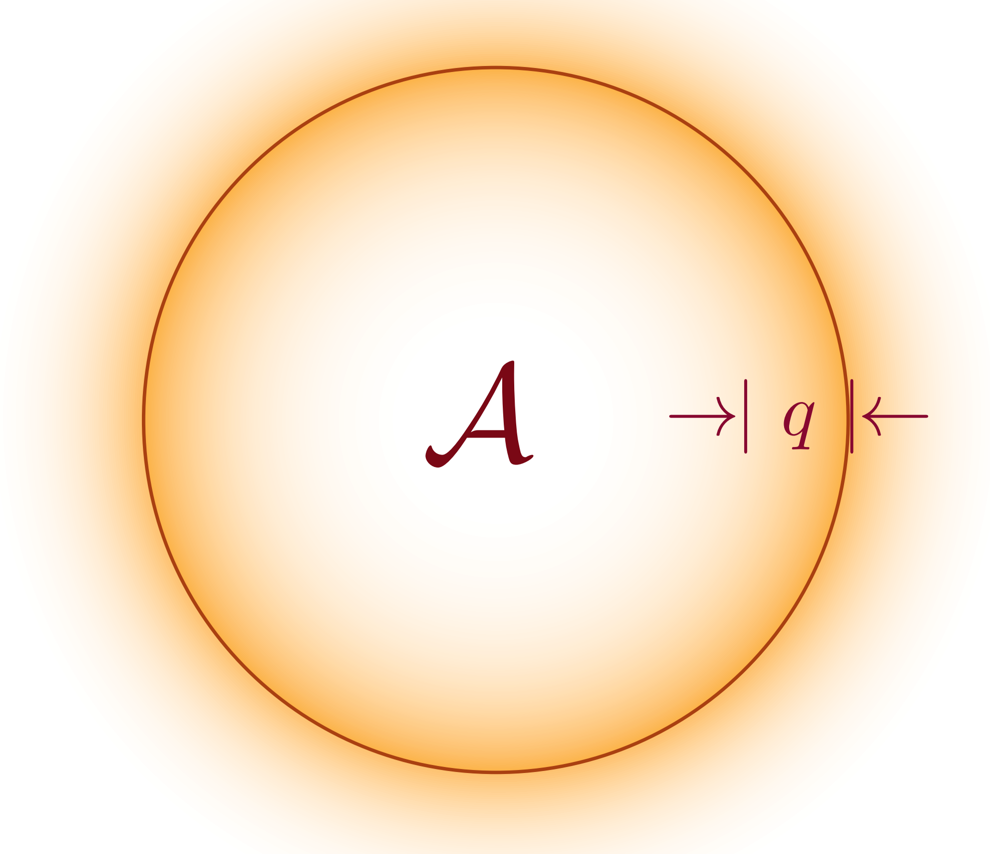

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- Gauge-dependent Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.

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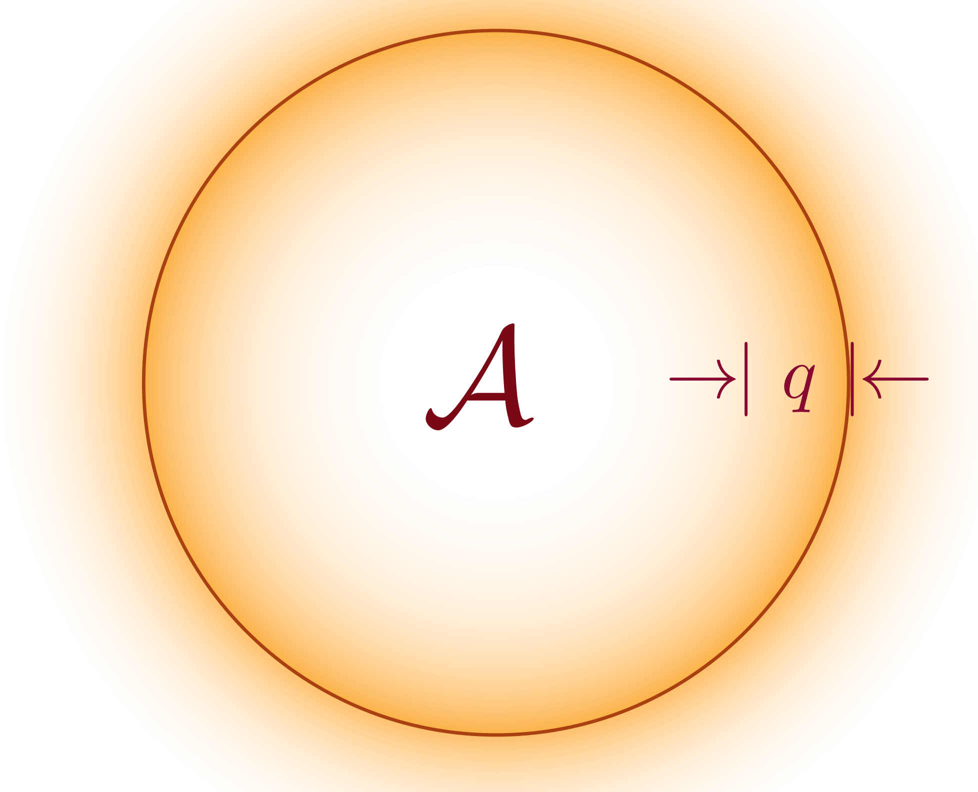

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- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\rm eff}/z}$ with $d_{\rm eff} = 1$.

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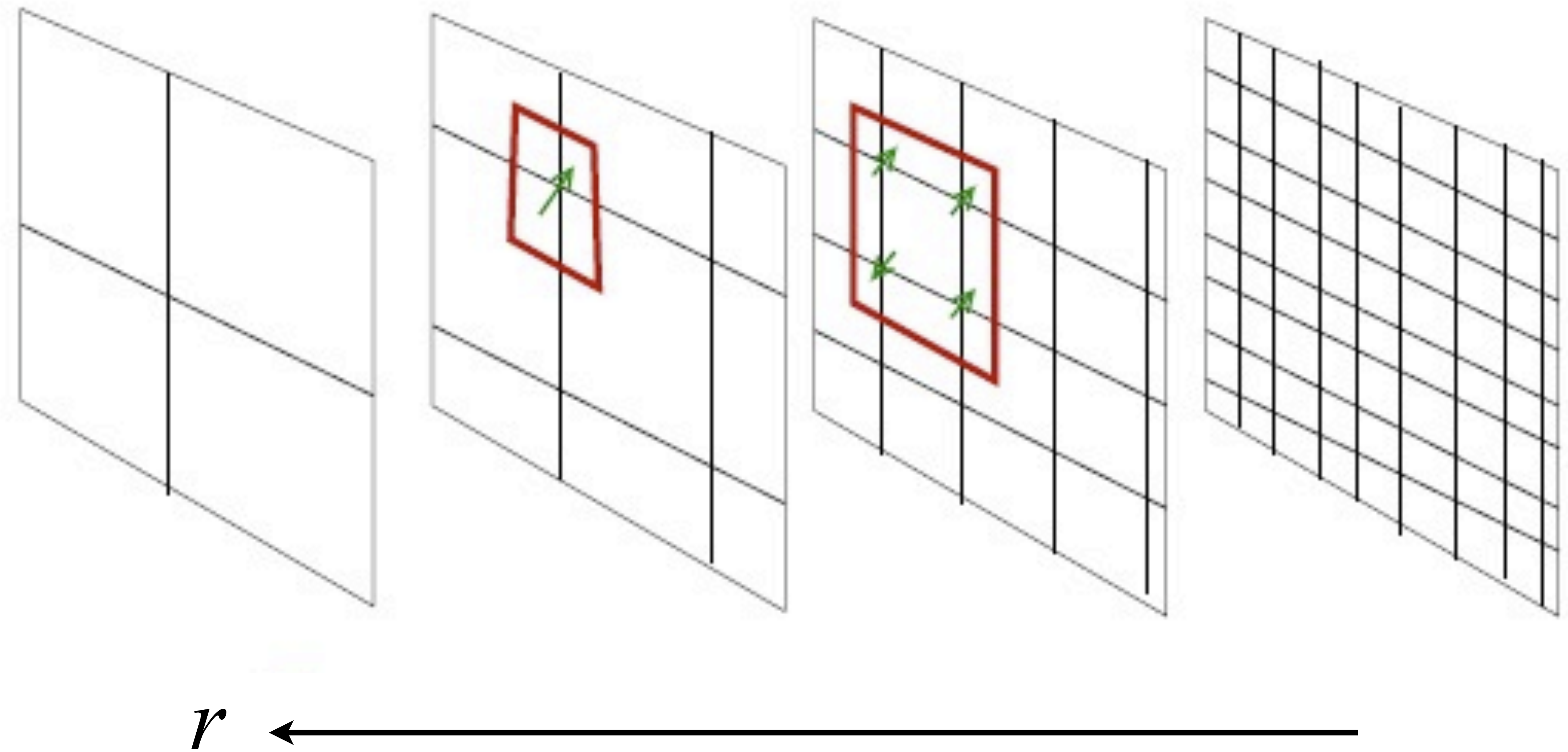
- We will now present a conjectured gravity dual of this theory.



J. McGreevy, arXiv0909.0518

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

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This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

Consider the following (most) general metric for the holographic theory

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

This metric transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

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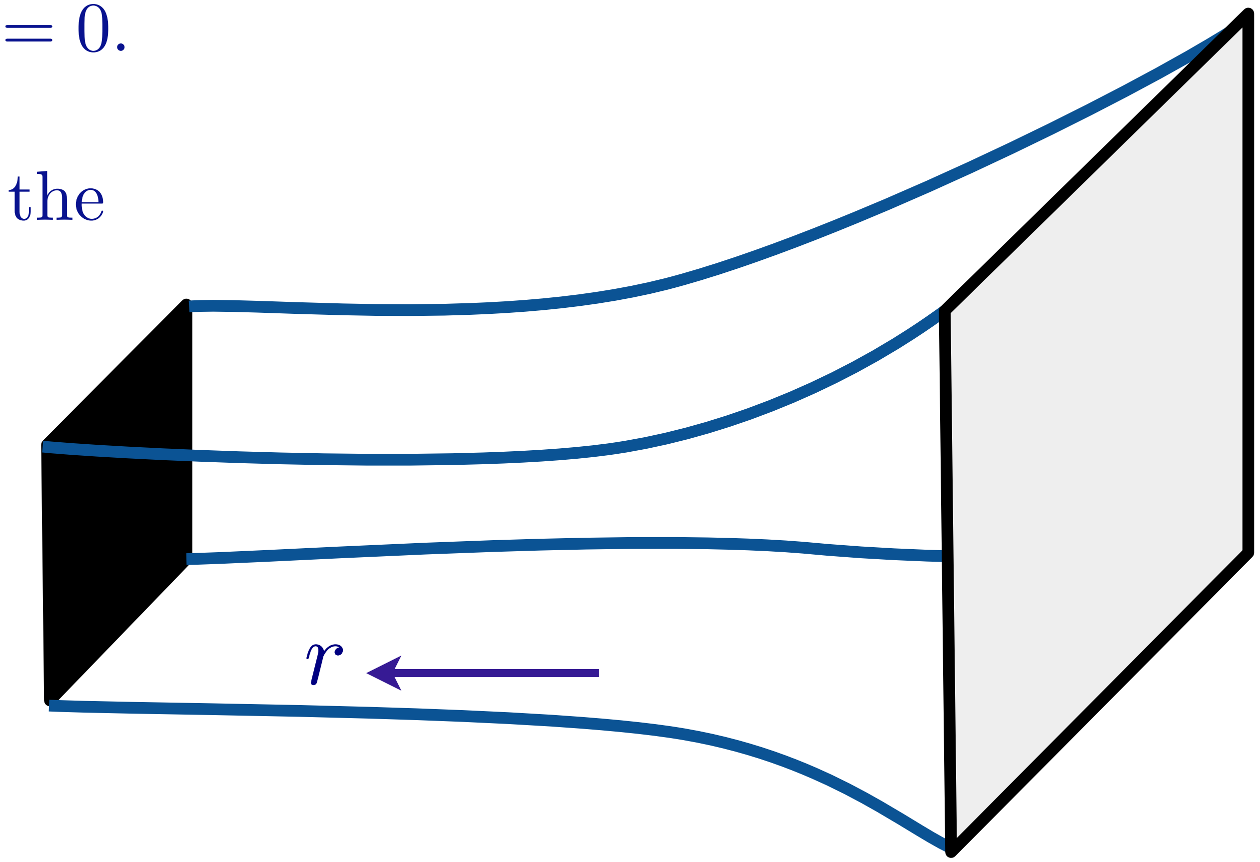
What is θ ? ($\theta = 0$ for "relativistic" quantum critical points).

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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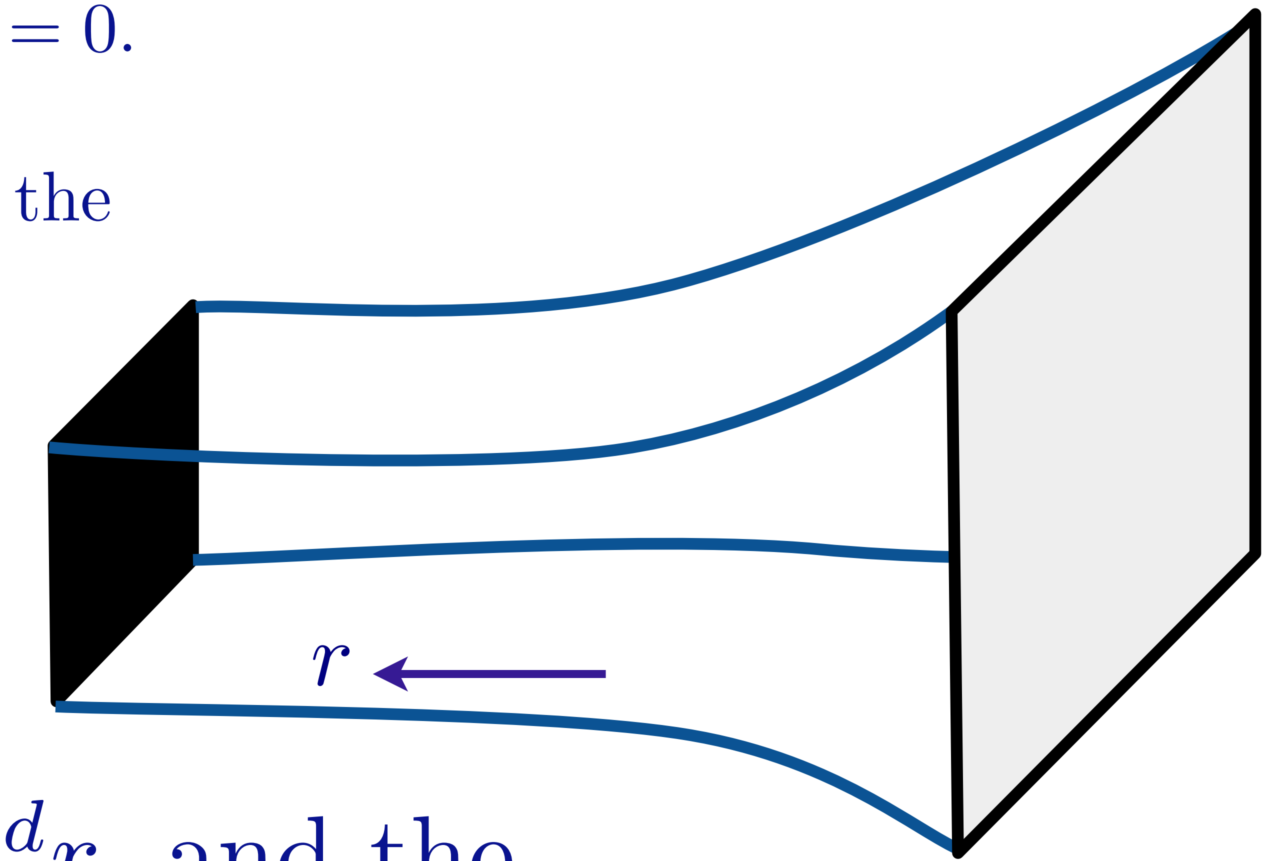


Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\rm eff}/z}$$

where $\theta = d - d_{\rm eff}$ measures "dimension deficit" in the phase space of low energy degrees of a freedom.

Holography of non-Fermi liquids

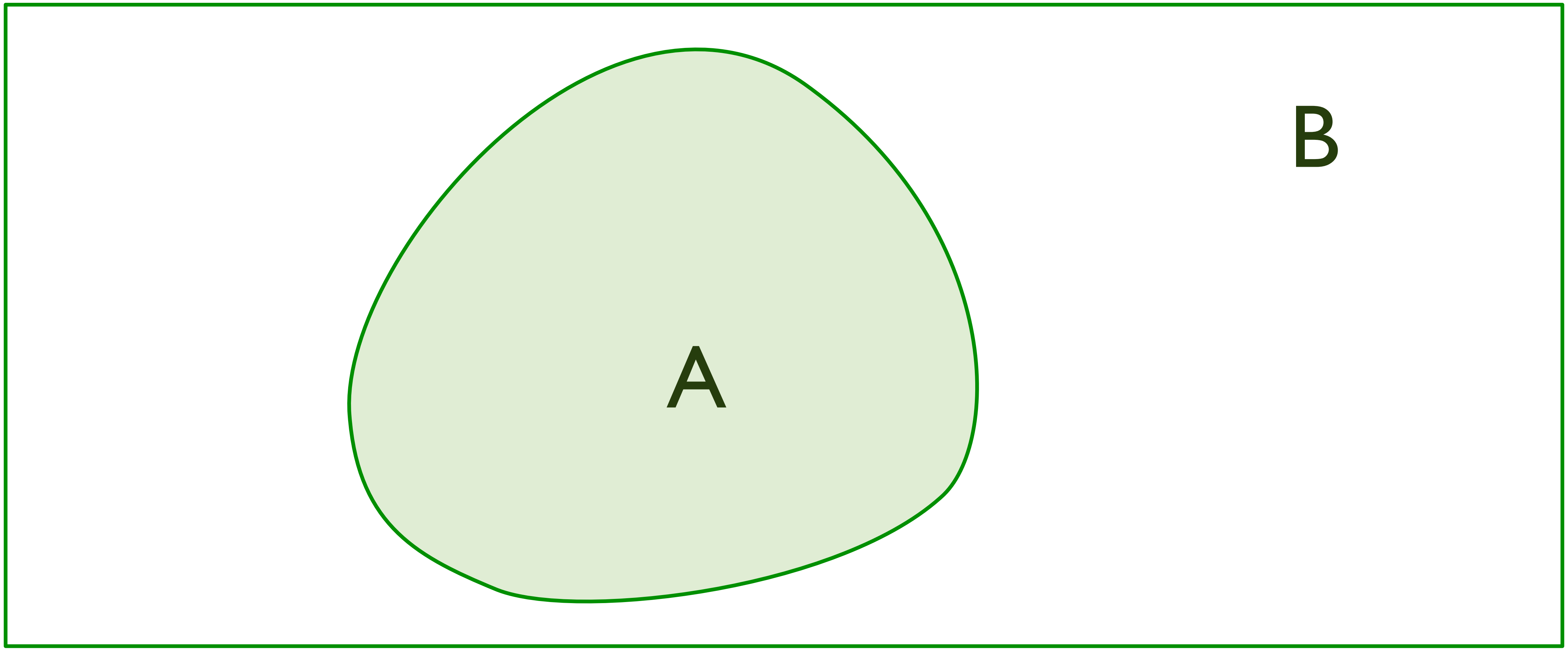
$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

A non-Fermi liquid has gapless fermionic excitations on the Fermi surface, which disperse in the single transverse direction with dynamic critical exponent z, with entropy density $\sim T^{1/z}$. So we expect compressible quantum states to have $d_{\text{eff}} = 1$, or

$$\theta = d - 1$$

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Entanglement entropy

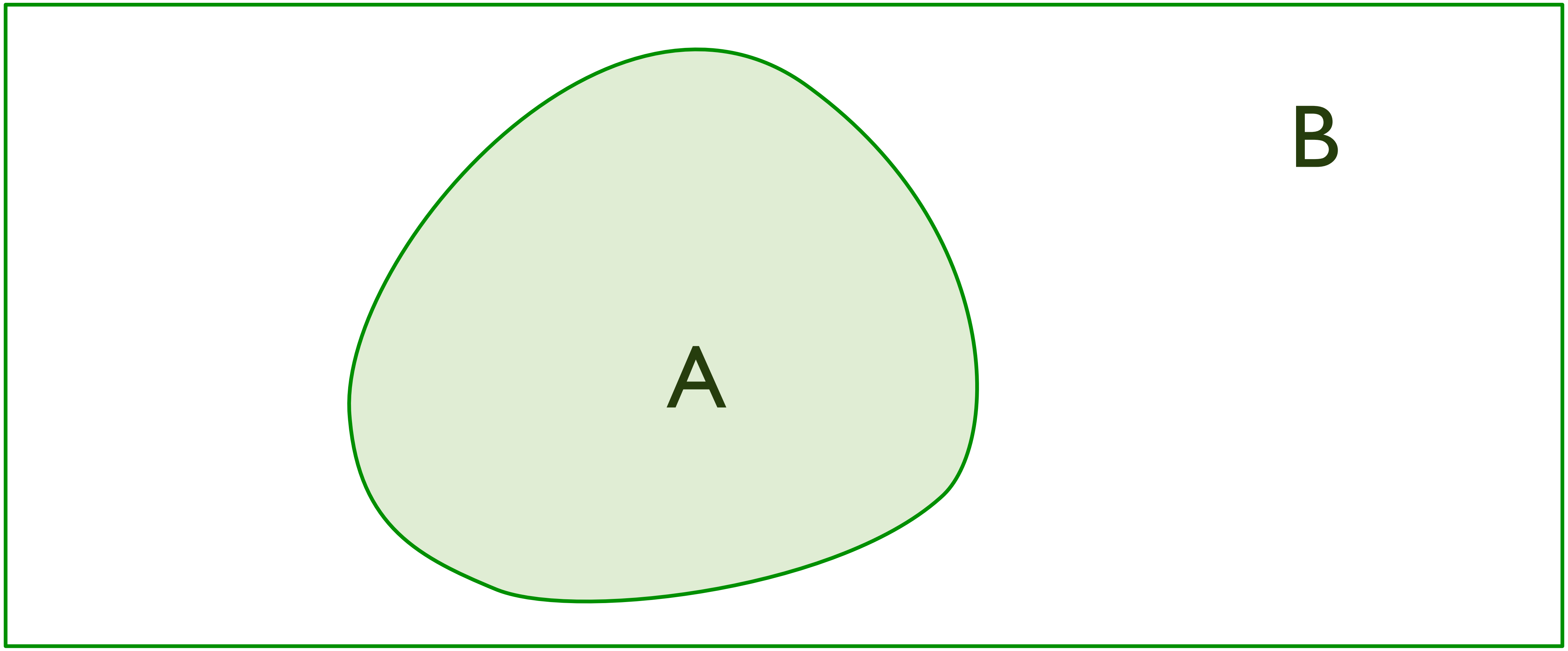


Measure strength of quantum entanglement of region A with region B.

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

Entanglement entropy $S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

Entanglement entropy of Fermi surfaces

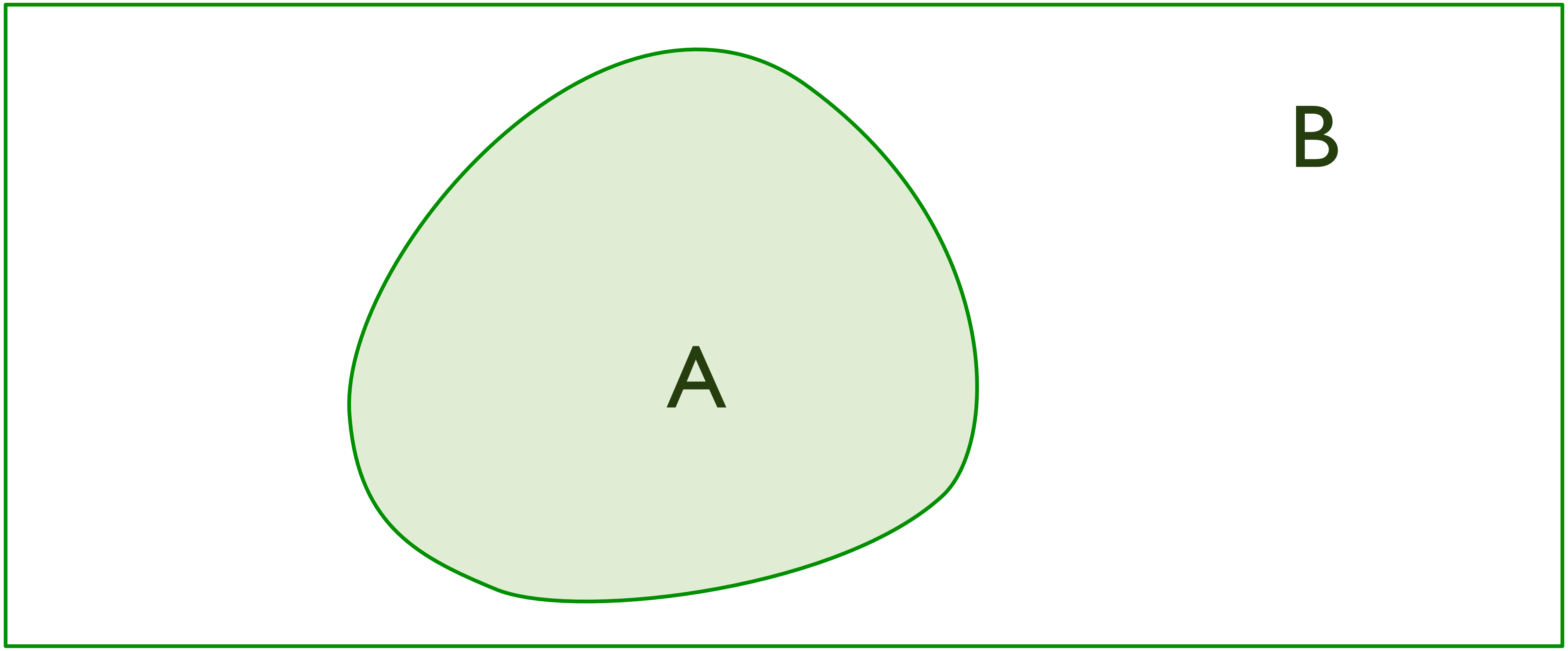


Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of Fermi surfaces



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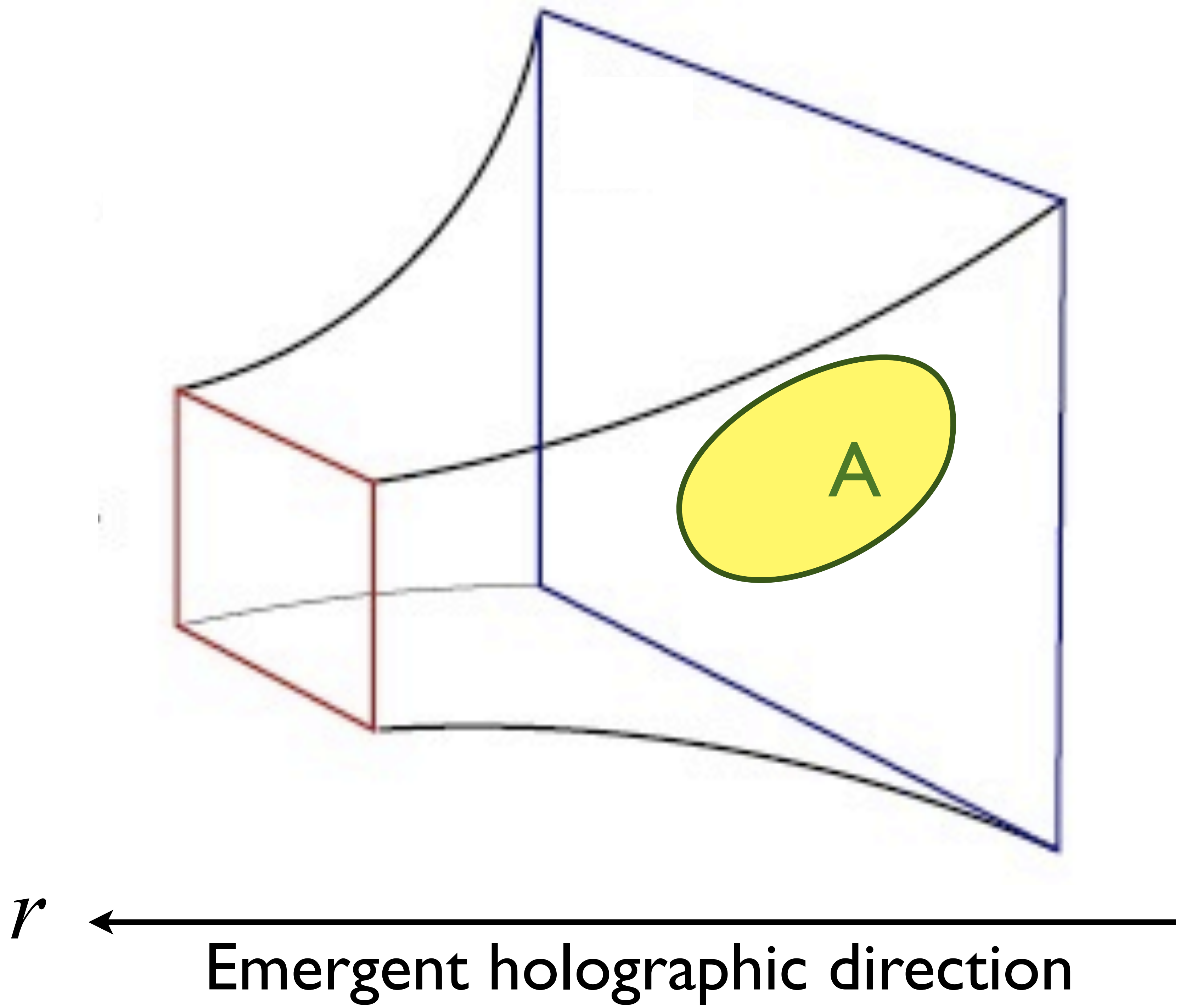
Non-Fermi liquids have, at most, the "1/12" prefactor modified.

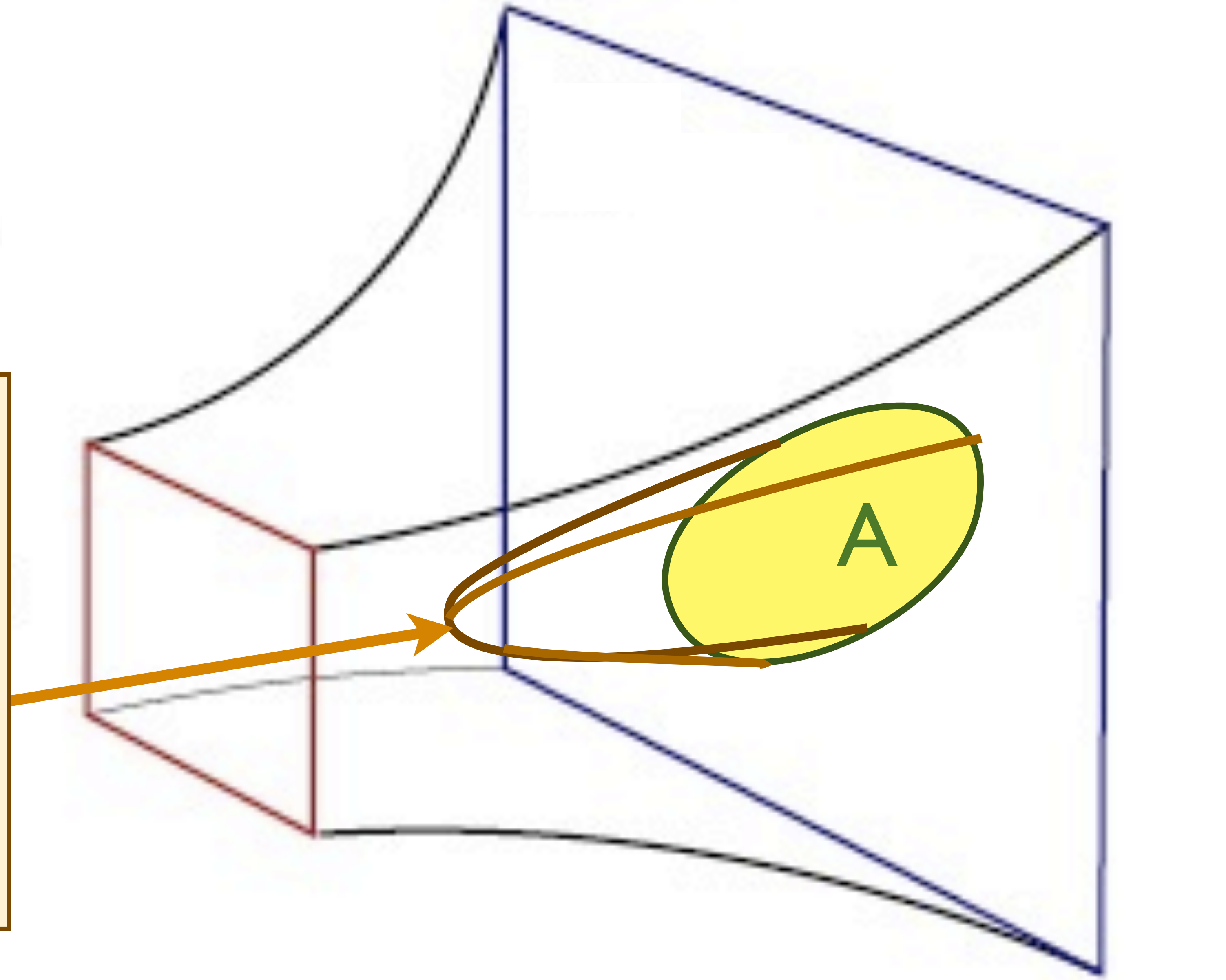
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Holographic entanglement entropy



Holographic entanglement entropy

Area of minimal surface equals entanglement entropy



r ← Emergent holographic direction

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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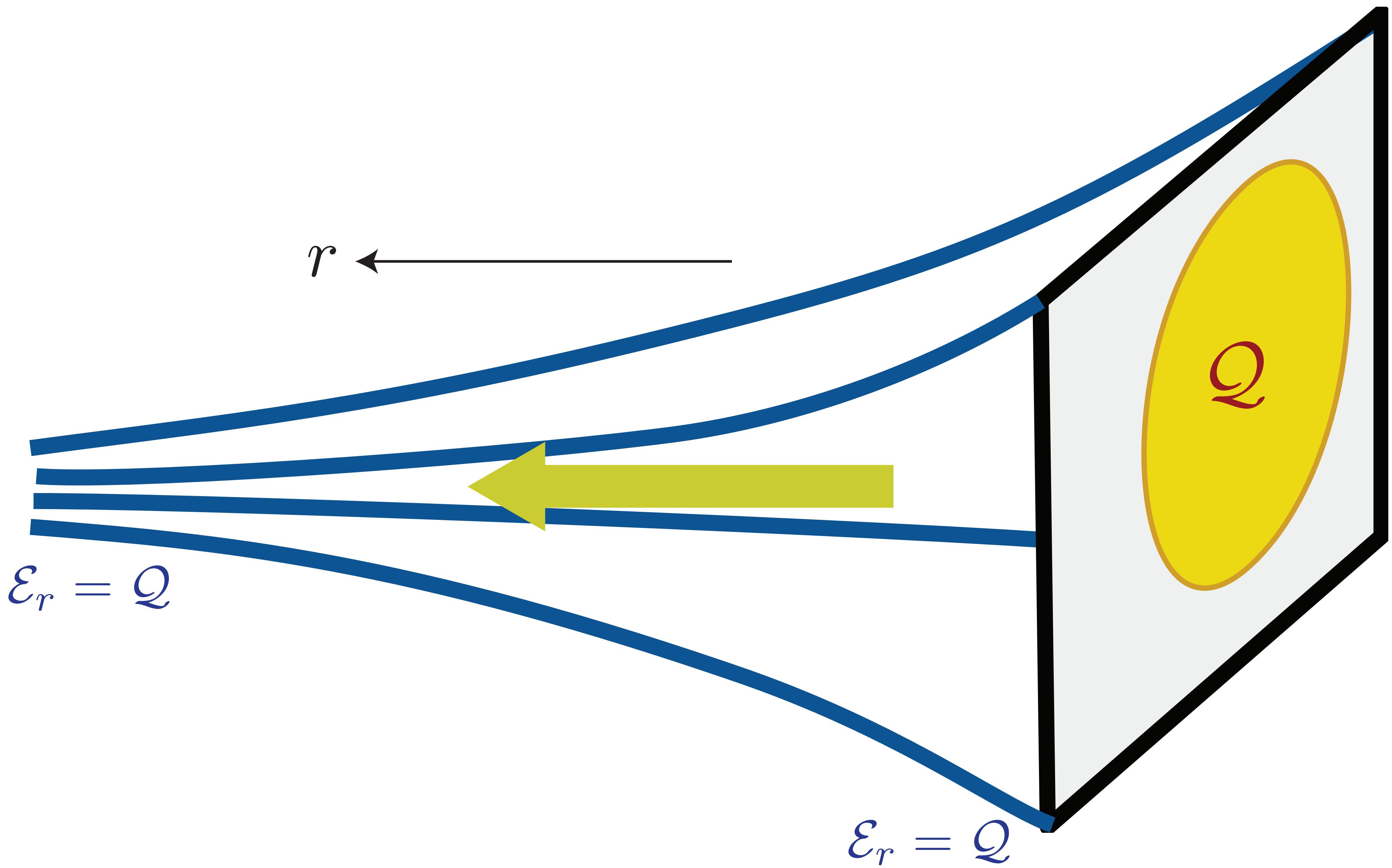
- The metric can be realized as the solution of a Einstein-Maxwell-Dilaton theory with no explicit fermions. The density of the "hidden Fermi surfaces" of the boundary gauge-charged fermions can be deduced from the electric flux leaking to $r \to \infty$.

K. Goldstein, S. Kachru, S. Prakash, and S. P. Trivedi JHEP **1008**, 078 (2010)

Holographic theory of a non-Fermi liquid (NFL)



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L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The co-efficient of the logarithmic term in the entanglement entropy is insensitive to all short-distance details, and depends only upon the fermion density.
- The two methods of deducing with fermion density, from the electric flux as $r \to \infty$ and from the entanglement entropy, are consistent with the Luttinger relation !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Inequalities

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The area law of entanglement entropy is obeyed for

$$\theta \leq d - 1.$$

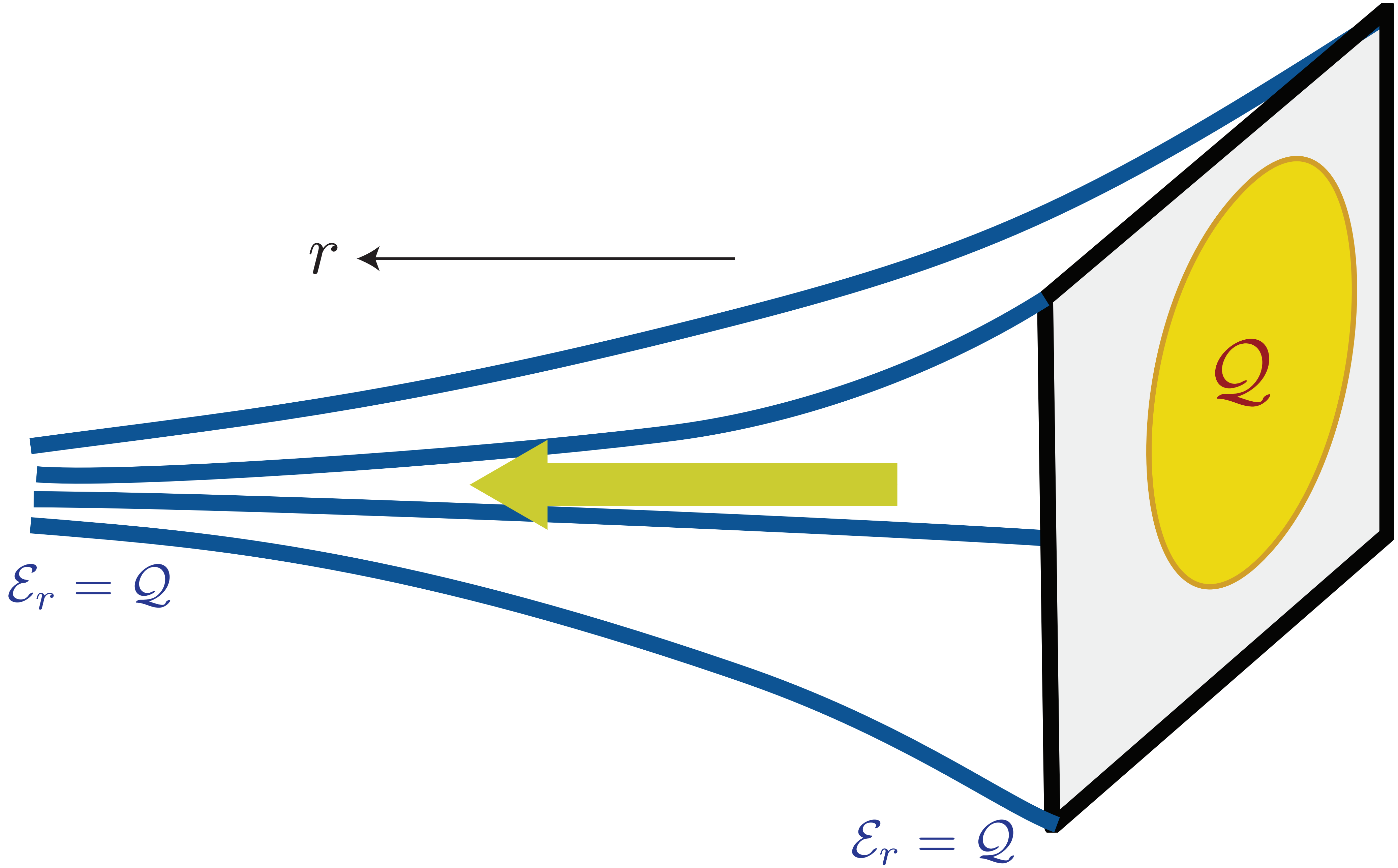
The "null energy condition" of the gravity theory yields

$$z \geq 1 + \frac{\theta}{d}.$$

Remarkably, for $d = 2$, $\theta = d - 1$ and $z = 1 + \theta/d$, we obtain $z = 3/2$, the same value associated with the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

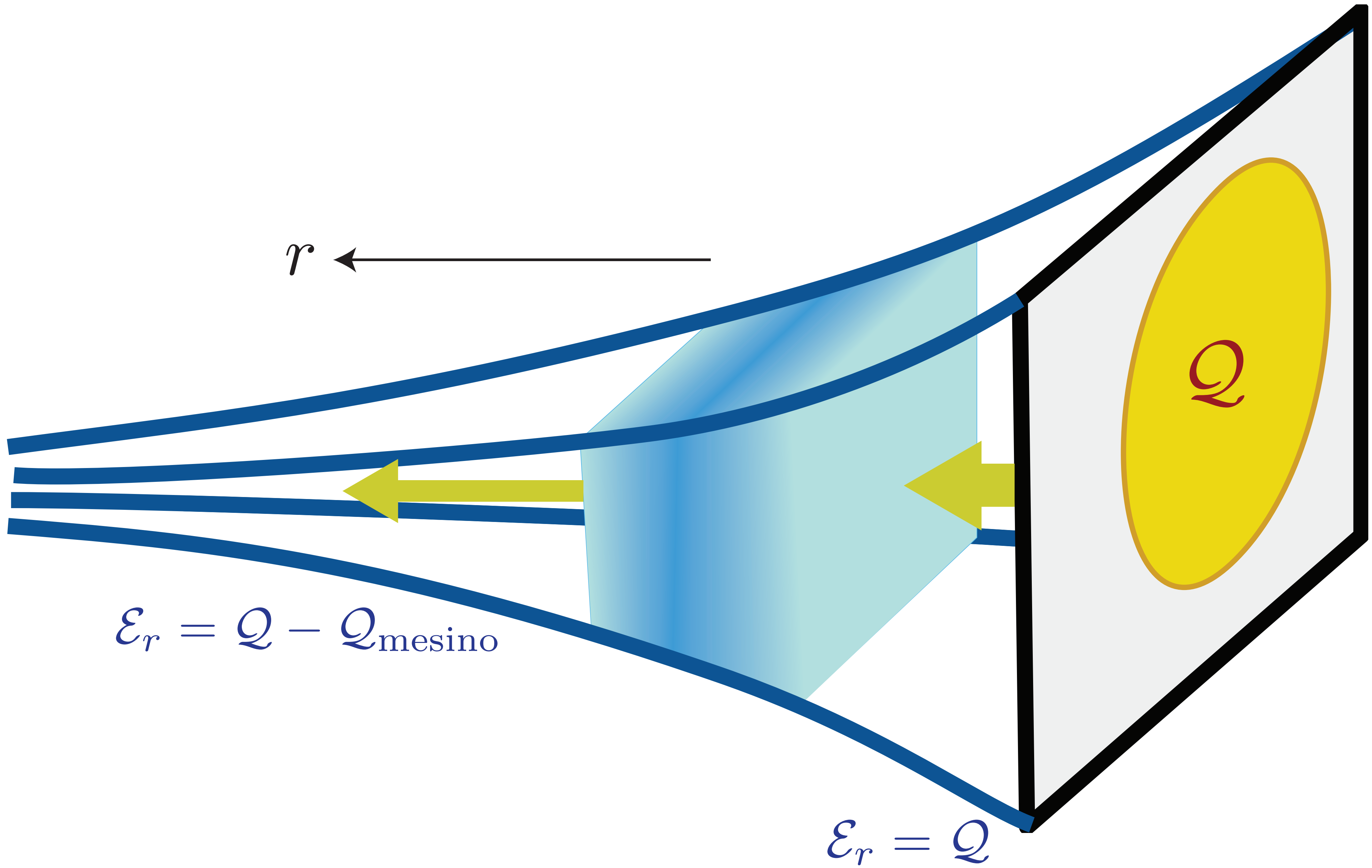
Holographic theory of a non-Fermi liquid (NFL)



Gauss Law in the bulk

$\Leftrightarrow$ Luttinger theorem on the boundary

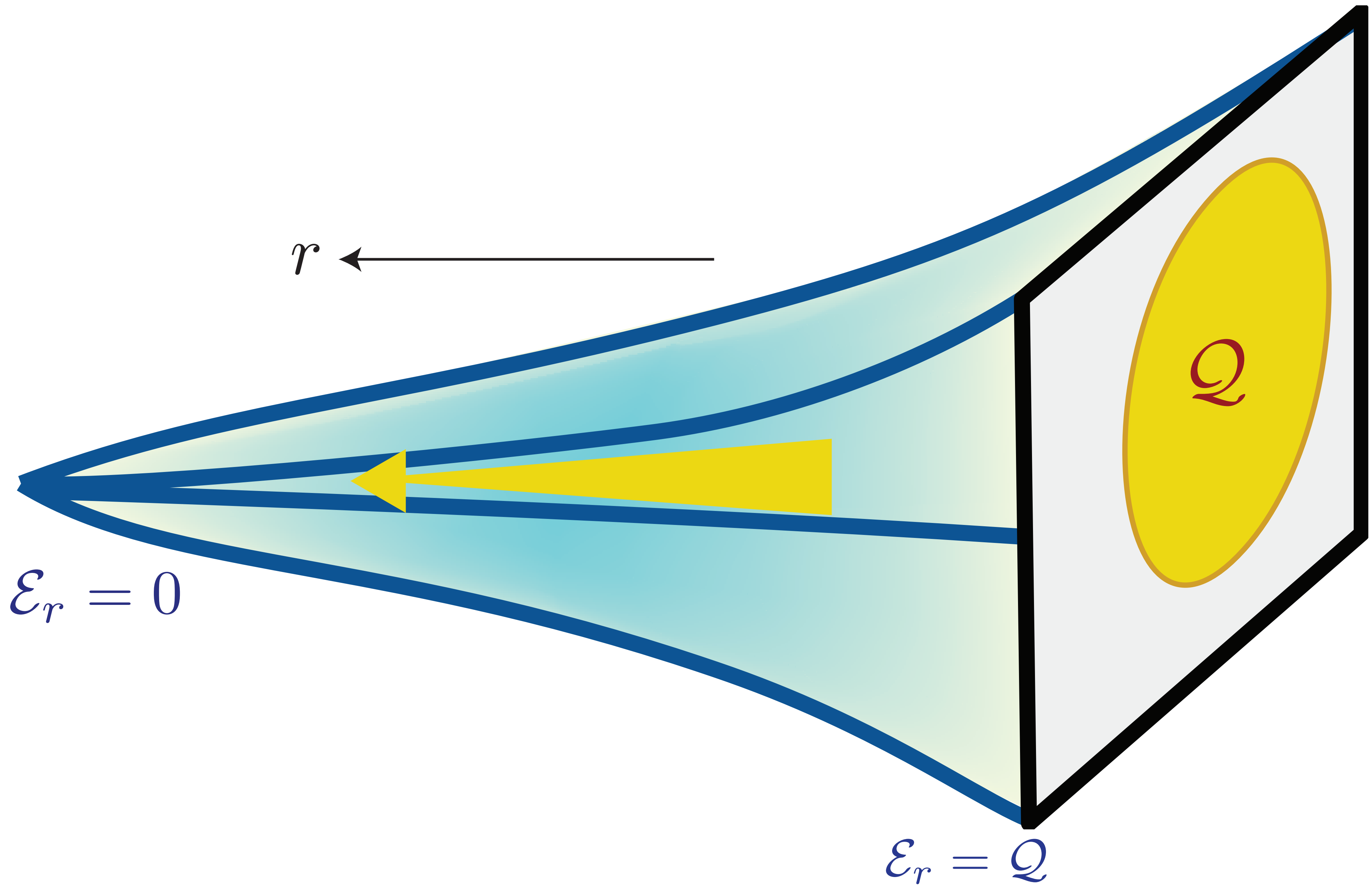
Holographic theory of a fractionalized-Fermi liquid (FL*)



Gauss Law in the bulk

$\Leftrightarrow$ Luttinger theorem on the boundary

Holographic theory of a Fermi liquid (FL)



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Theory of a non-Fermi liquid (NFL)

Field theory	Holography
A gauge-dependent Fermi surface of overdamped gapless fermions.	Fermi surface is hidden.

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Logarithmic violation of area law of entanglement entropy, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.	Logarithmic violation of area law of entanglement entropy for $\theta = d - 1$, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.

Theory of a non-Fermi liquid (NFL)

Field theory	Holography
Three-loop analysis shows $z = 3/2$ in $d = 2$.	Existence of gravity dual implies $z \geq 1 + \theta/d$; leads to $z \geq 3/2$ for $\theta = d-1$ in $d = 2$.

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Gauge neutral 'mesinos' reduce the volume enclosed by Fermi surfaces of gauge-charged fermions to $\mathcal{Q} - \mathcal{Q}_{\text{mesino}}$.	Gauge neutral 'mesinos' reduce the volume enclosed by hidden Fermi surfaces to $\mathcal{Q} - \mathcal{Q}_{\text{mesino}}$.