Quantum matter without quasiparticles: SYK models, strange metals, and black holes

University of Maryland, College Park, December 13, 2016

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PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Talk online: sachdev.physics.harvard.edu

PHYSICS
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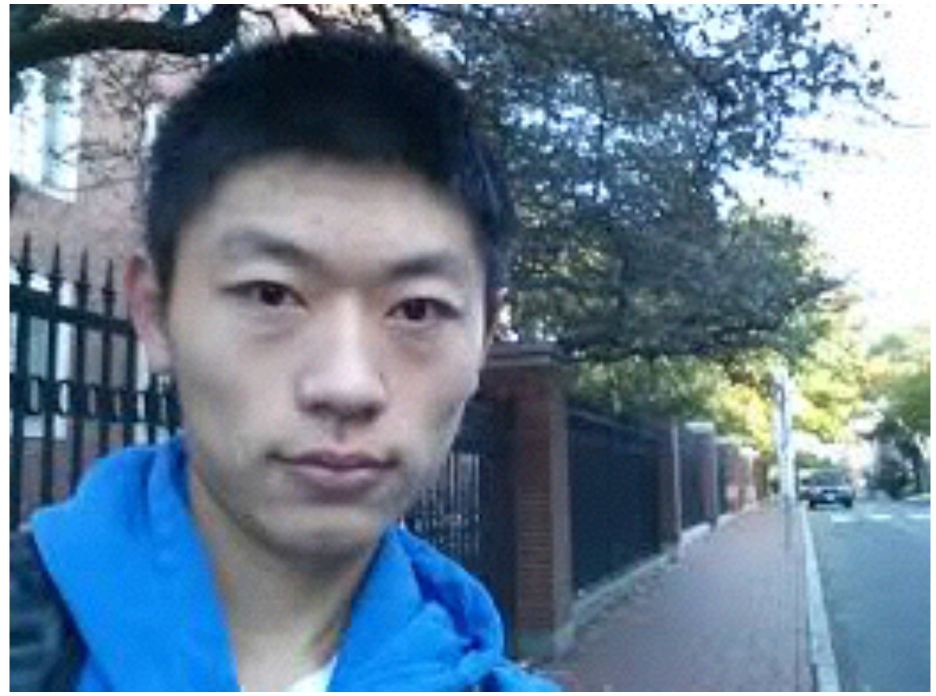
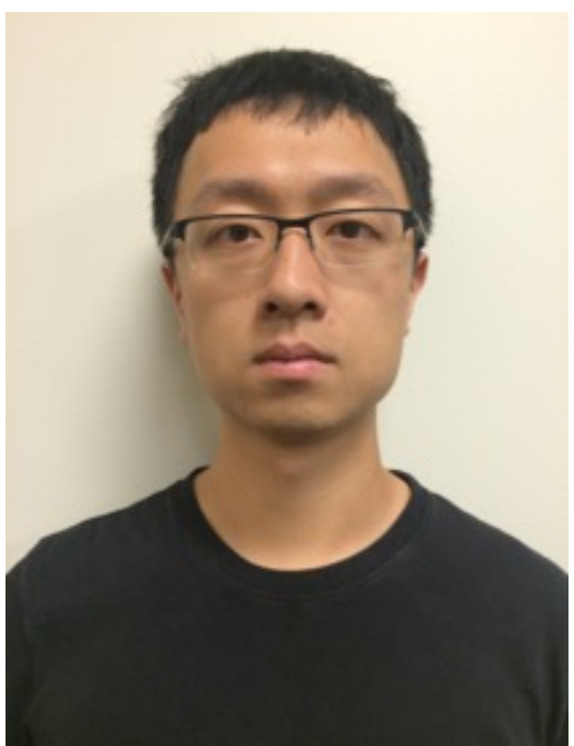
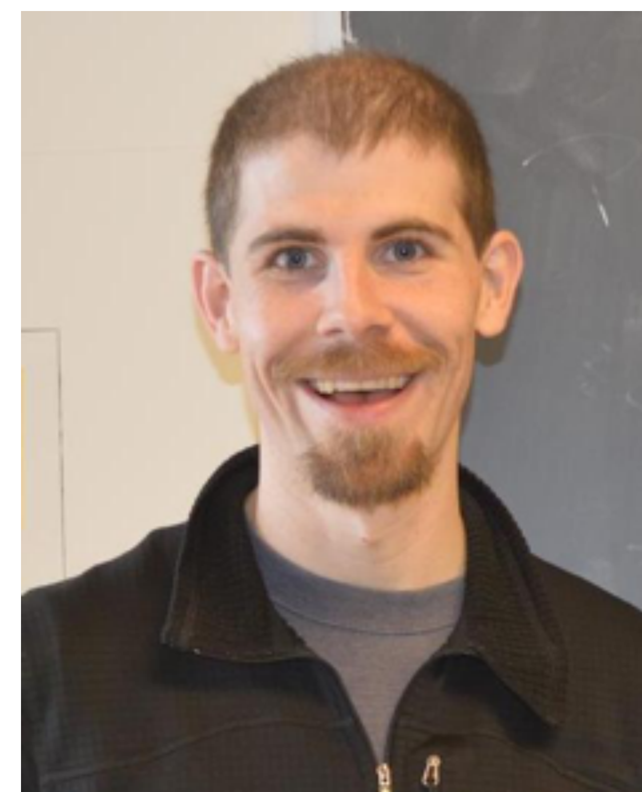
Aavishkar Patel, Harvard


Wenbo Fu, Harvard


Antoine Georges, Paris

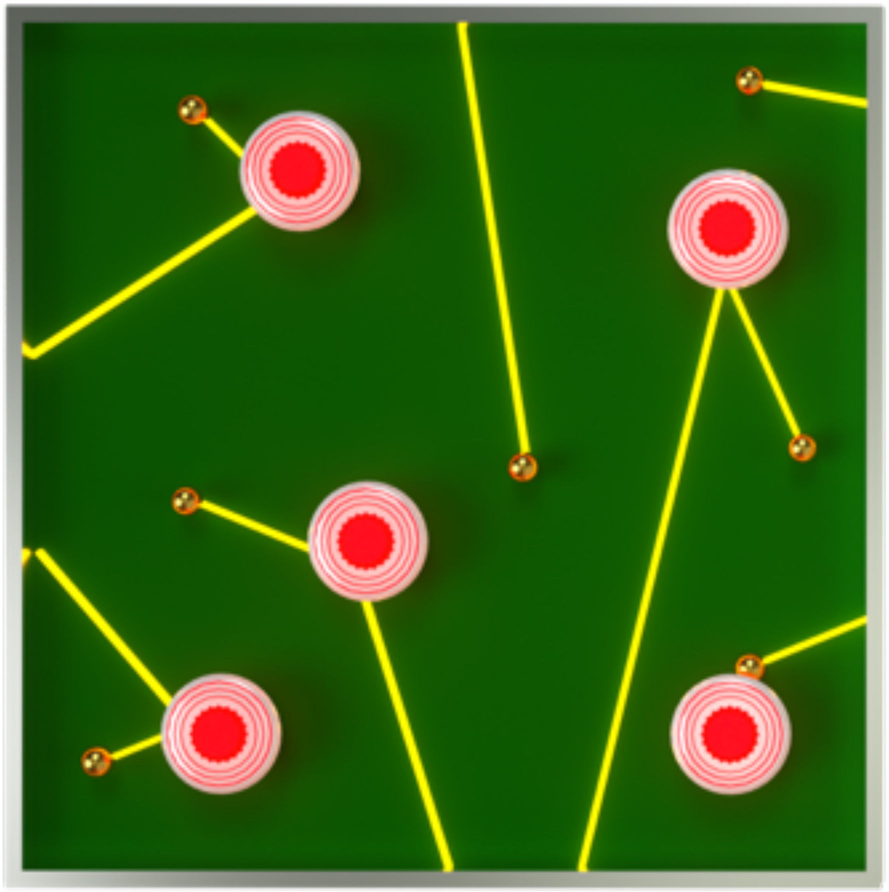

Yingfei Gu, Stanford


Richard Davison, Harvard


Kristan Jensen, SFSU

Quantum matter with quasiparticles:

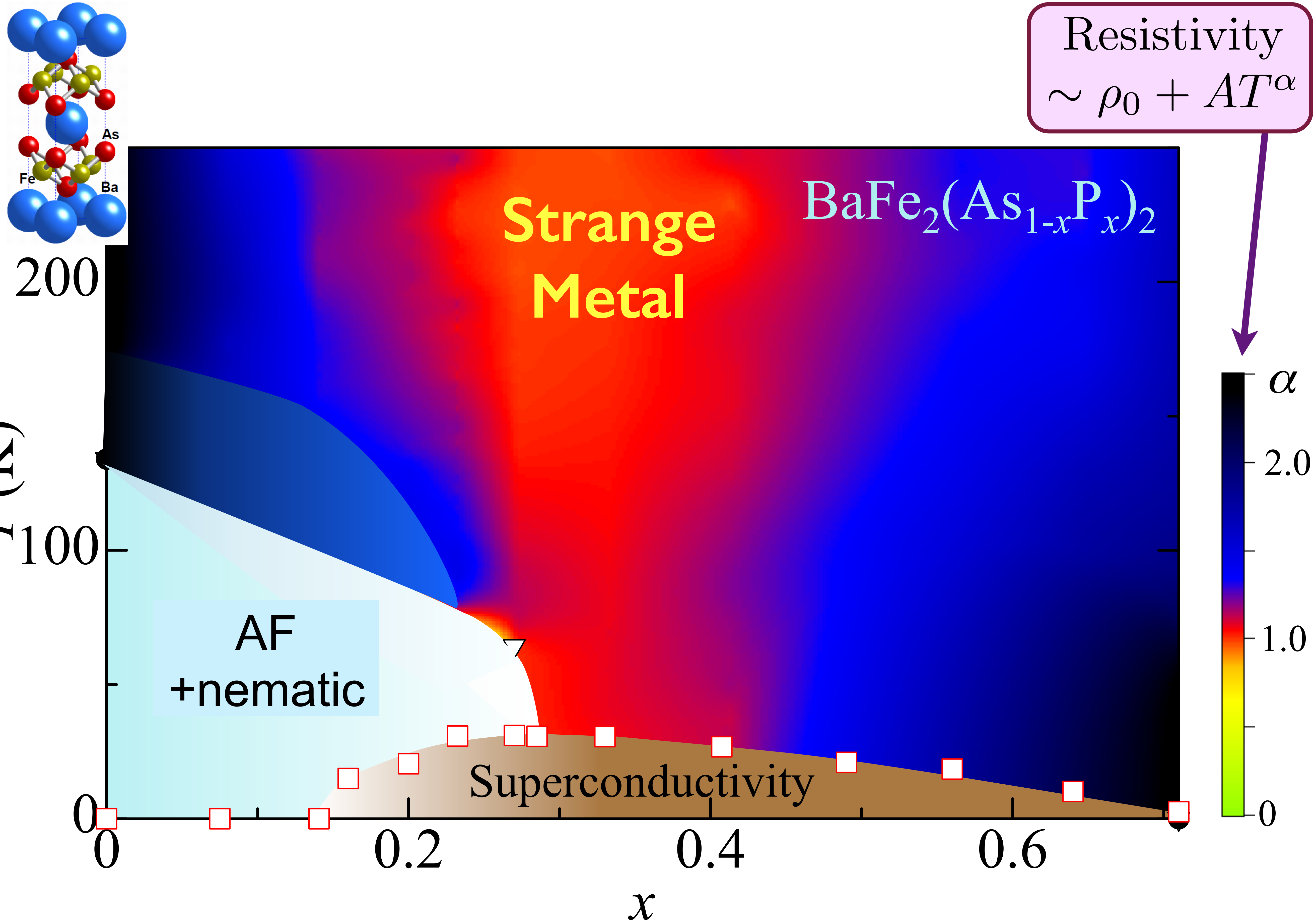
- *Landau quasi-particles & holes*
- *Phonon*
- *Magnon*
- *Roton*
- *Plasmon*
- *Polaron*
- *Exciton*
- *Laughlin quasiparticle*
- *Bogoliubovon*
- *Anderson-Higgs mode*
- *Massless Dirac Fermions*
- *Weyl fermions*
- *….*



Quantum matter with quasiparticles:

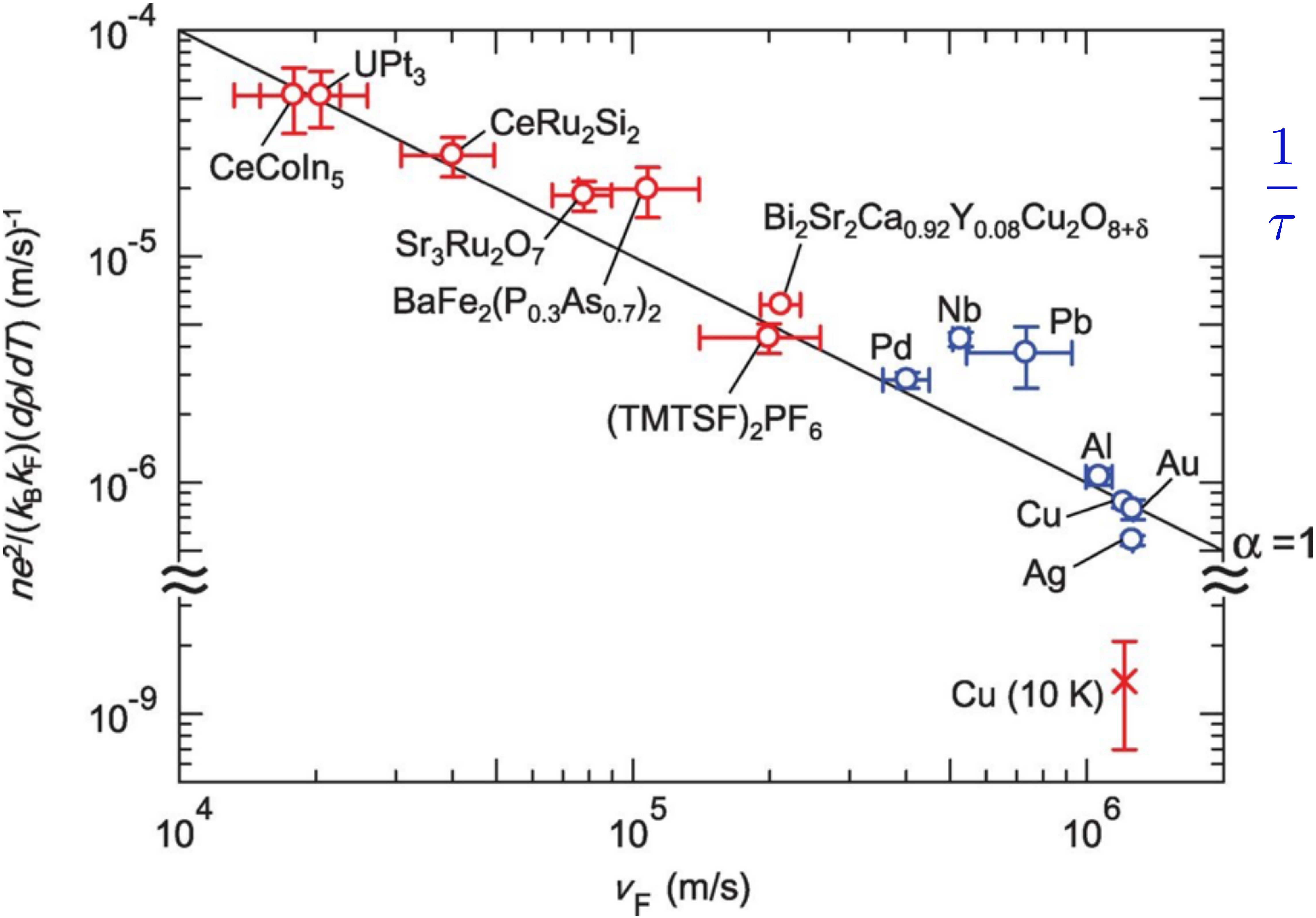
Most generally, a quasiparticle is an "additive" excitation:

Quasiparticles can be combined to yield additional excitations, with energy determined by the energies and densities of the constituents. Such a procedure yields all the low-lying excitations. Then we can apply the Boltzmann-Landau theory to make predictions for dynamics.



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Strange metals



$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie, *Science*. **339,** 804 (2013)

Quantum matter <u>without</u> quasiparticles:

No quasiparticle structure to excitations.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some exotic quasiparticles inaccessible to current experiments……..

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Consider how rapidly the system loses "phase coherence", reaches local thermal equilibrium, or becomes "chaotic"

Local thermal equilibration or phase coherence time, τ_φ:

- There is an *lower bound* on τ_φ in all many-body quantum systems as $T \to 0$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

  where C is a T-independent constant. Systems *without* quasiparticles have $\tau_\varphi \sim \hbar/(k_B T)$.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T;
  e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
  and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A bound on quantum chaos:

- In classical chaos, we measure the sensitivity of the position at time t, $q(t)$, to variations in the initial position, $q(0)$, *i.e.* we measure

$$\left(\frac{\partial q(t)}{\partial q(0)}\right)^2 = \left(\{q(t), p(0)\}_{\text{P.B.}}\right)^2$$

- By analogy, we define τ_L as the LYAPUNOV TIME over which the wave-function of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$\left\langle \left| [\hat{A}(t), \hat{B}(0)] \right|^2 \right\rangle \sim e^{t/\tau_L}$$

This quantum time was argued to obey lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}.$$

There is no analogous bound in classical mechanics.

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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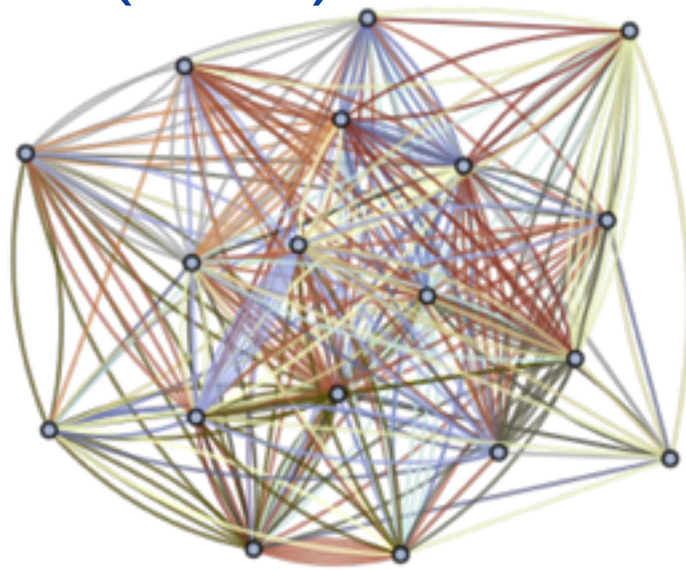
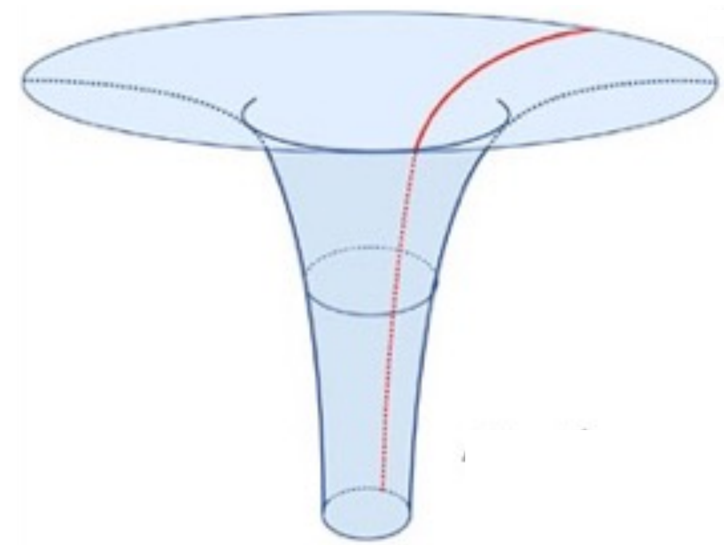
Quantum matter without quasiparticles
≈ fastest possible many-body quantum chaos

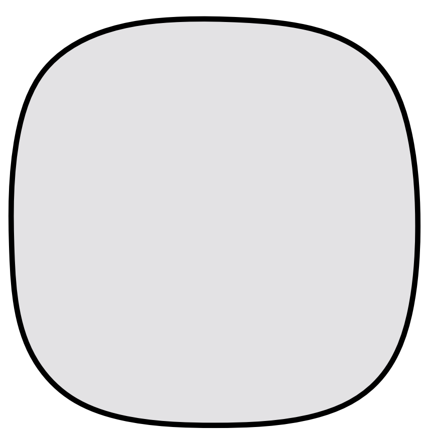
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons



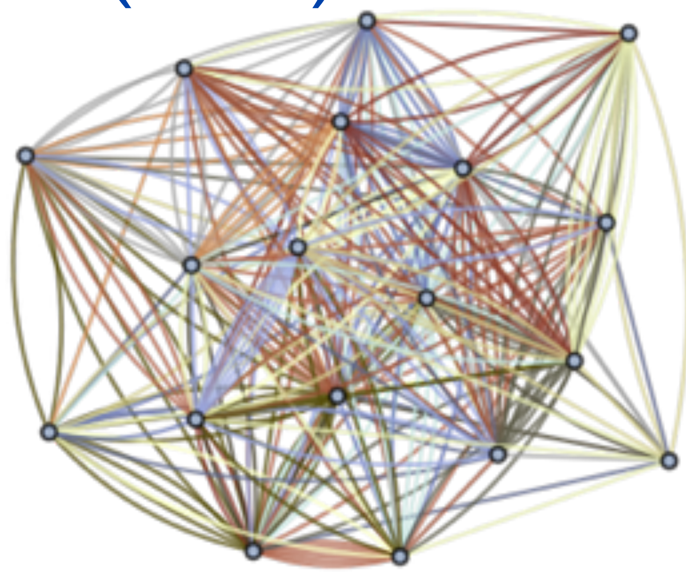


Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

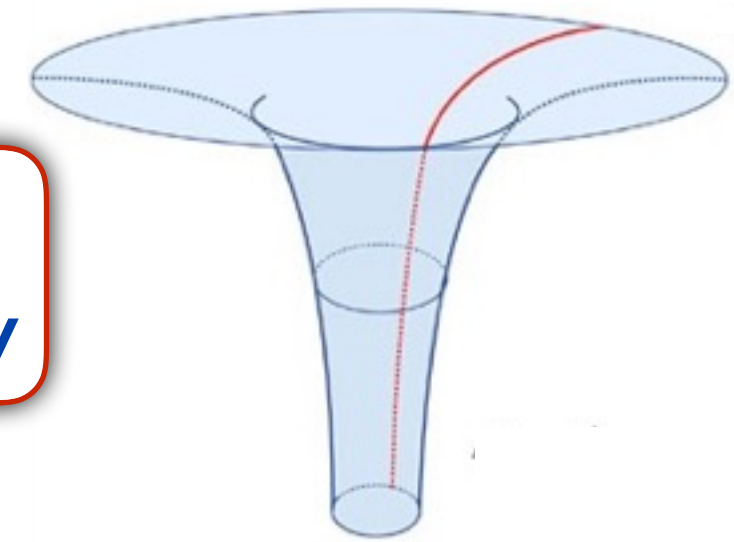
Quantum matter without quasiparticles:

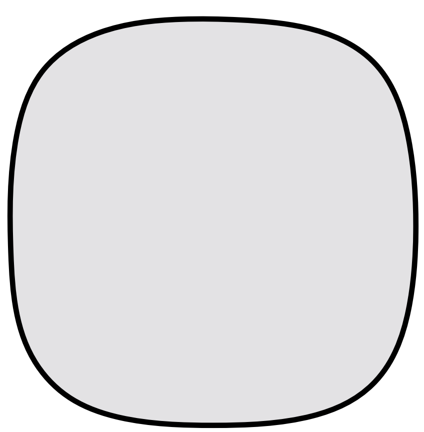
The Sachdev-Ye-Kitaev (SYK) models



Same low energy theory

Black holes with AdS_2 horizons





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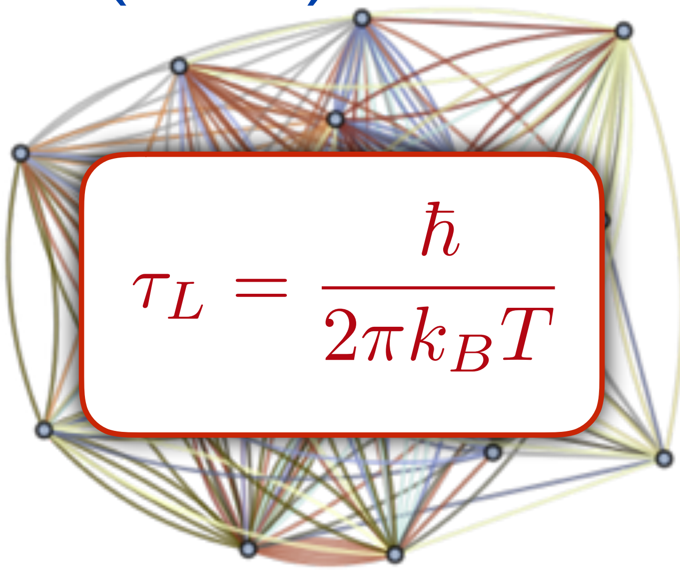
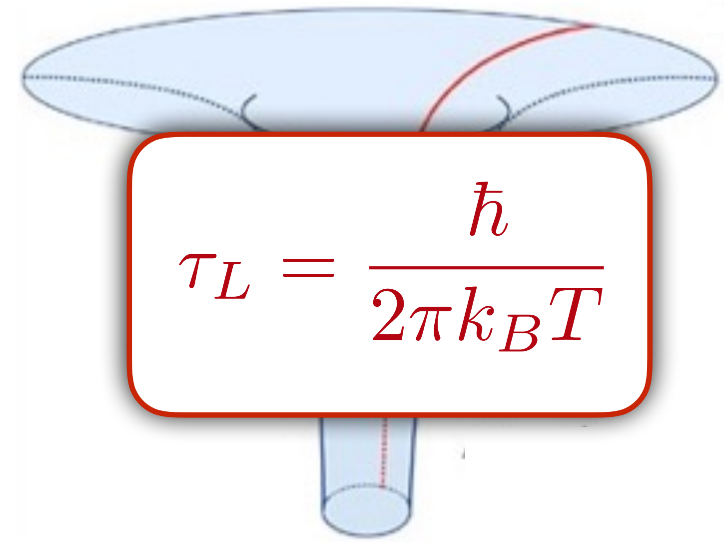


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Black holes with AdS_2 horizons



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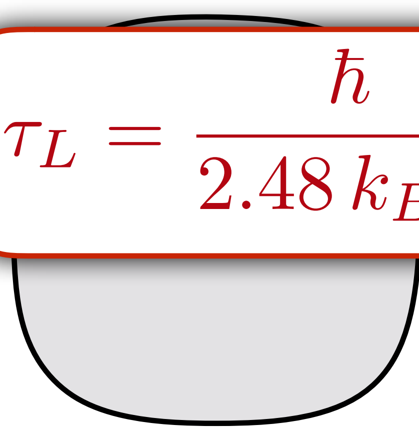


$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS_2
- Fastest possible quantum chaos with $\tau_L = \dfrac{\hbar}{2\pi k_B T}$

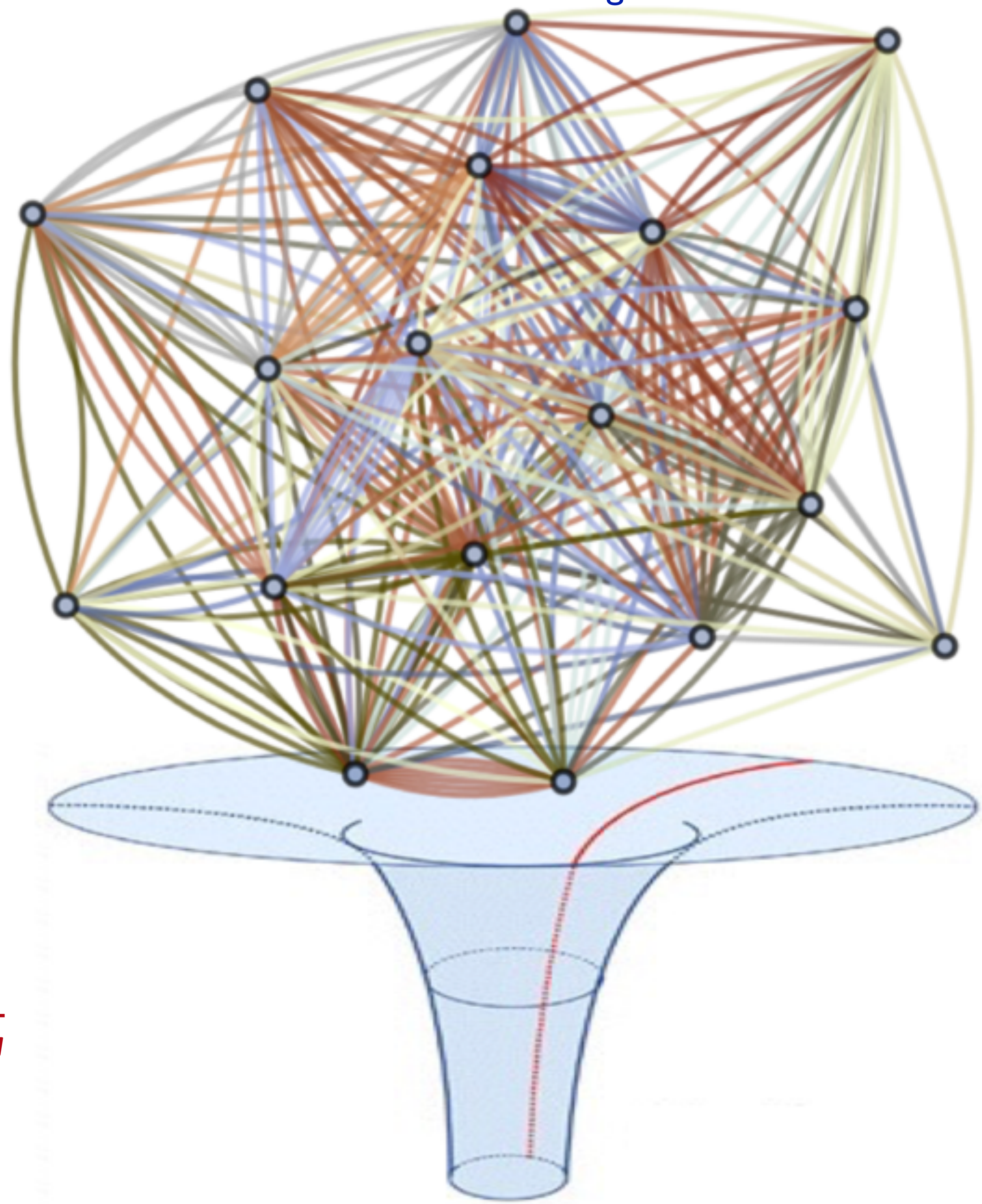


Figure credit: L. Balents

Infinite-range model with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

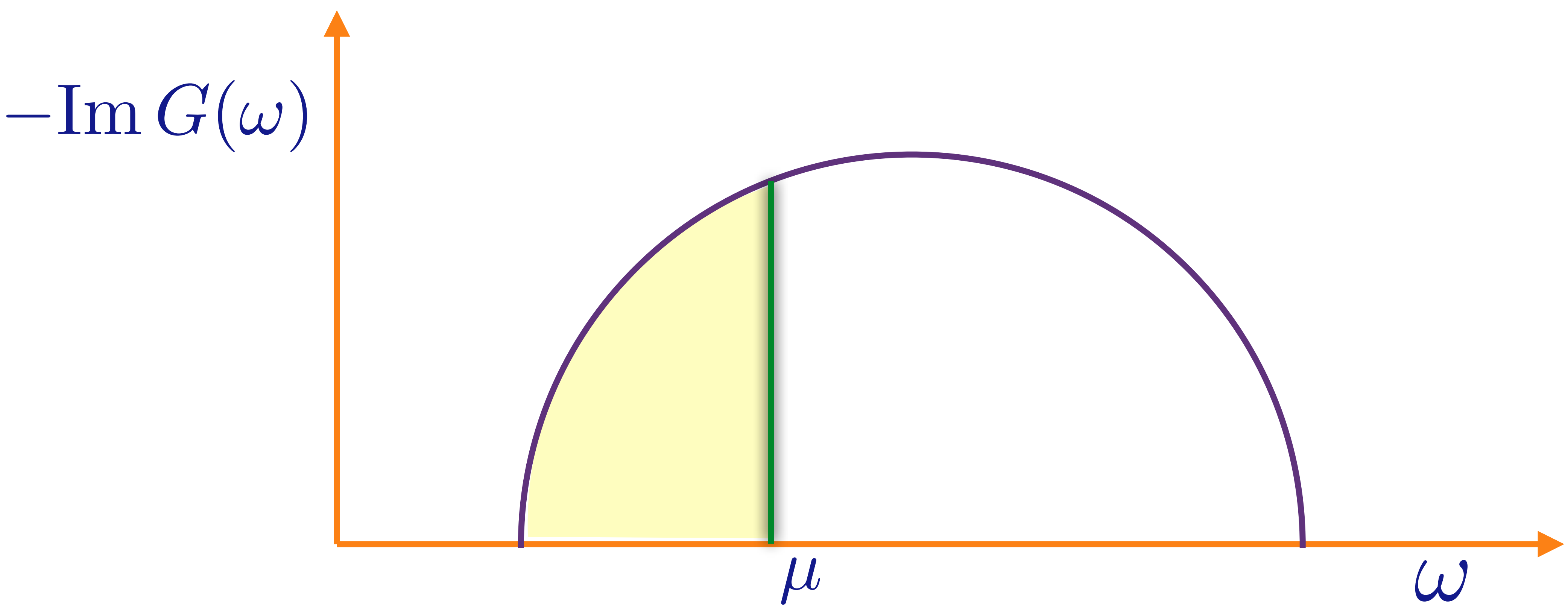
Fermions occupying the eigenstates of a *N* x *N* random matrix

Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy E_α. By Fermi's Golden rule, for E_α at the Fermi energy

$$\begin{aligned}\frac{1}{\tau_\alpha} &= \pi J^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta) \\ &= \frac{\pi^3 J^2 \rho_0^2}{4} T^2\end{aligned}$$

where ρ_0 is the density of states at the Fermi energy.

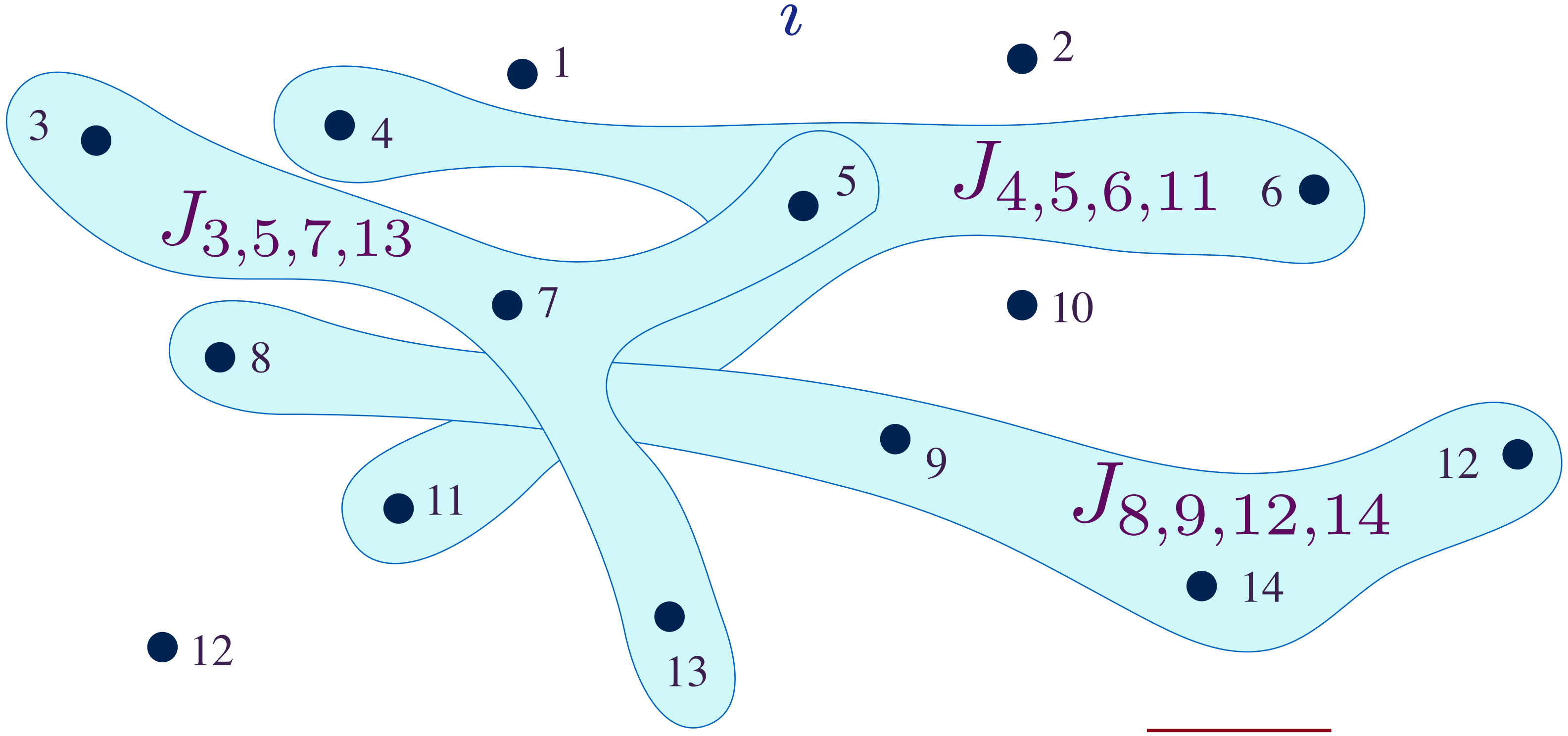
Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK and AdS_2

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = NS_0 + \ldots$
Not a ground state degeneracy: due to an exponentially small (in N) many-body level spacing at all energies down to the ground state energy.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

SYK and AdS_2

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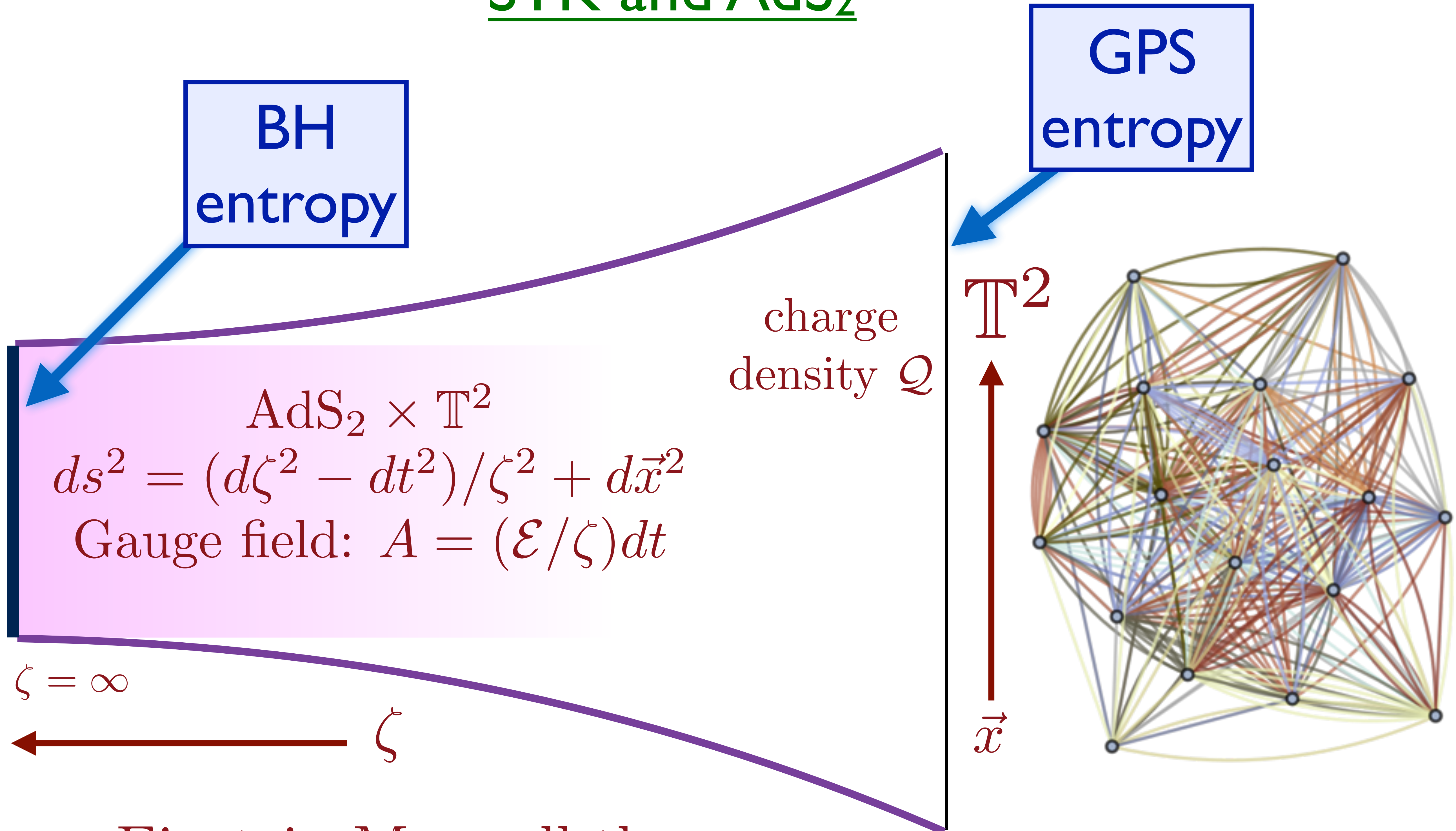
A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

- This entropy, and other dynamic correlators of the SYK models, imply that the SYK model is holographically dual to black holes with an AdS_2 horizon. The Bekenstein-Hawking entropy of the black hole equals NS_0:

$$\mathbf{GPS} = \mathbf{BH}.$$

S. Sachdev, PRL **105**, 151602 (2010)

SYK and AdS_2



Einstein-Maxwell theory
+ cosmological constant

S. Sachdev, PRL **105**, 151602 (2010)

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

SYK and AdS_2

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

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SYK and AdS$_2$

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$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished
S. Sachdev, PRX **5**, 041025 (2015)

SYK and AdS_2

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2}$$
$$\Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

A. Kitaev, unpublished

SYK and AdS_2

Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism

- SL(2,R) is the isometry group of AdS_2.

SYK and AdS$_2$

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1)\mathcal{D}\Sigma(\tau_1, \tau_2)e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4}G_s(f(\tau_1) - f(\tau_2))e^{i\phi(\tau_1)-i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)e^{-NS_{\text{eff}}[f,\phi]}.$$

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

SYK and AdS_2

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)e^{-NS_{\rm eff}[f,\phi]}.$$

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\rm eff}[f,\phi] = \frac{K}{2}\int_0^{1/T} d\tau(\partial_\tau\phi + i(2\pi\mathcal{E}T)\partial_\tau\epsilon)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \,\{\tan(\pi T(\tau+\epsilon(\tau),\tau\},$$

where $f(\tau) \equiv \tau + \epsilon(\tau)$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g,\tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 .$$

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
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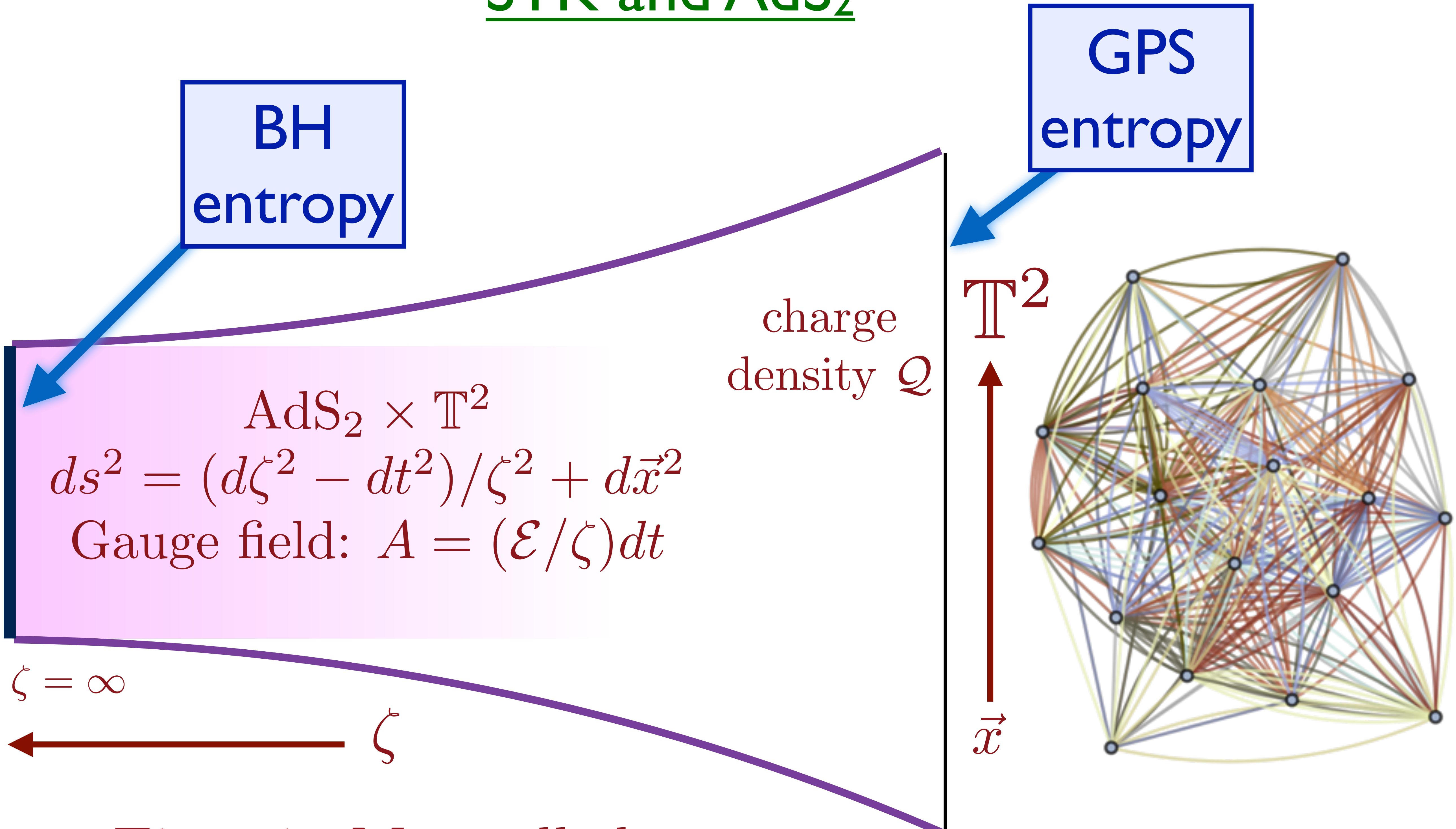
SYK and AdS_2

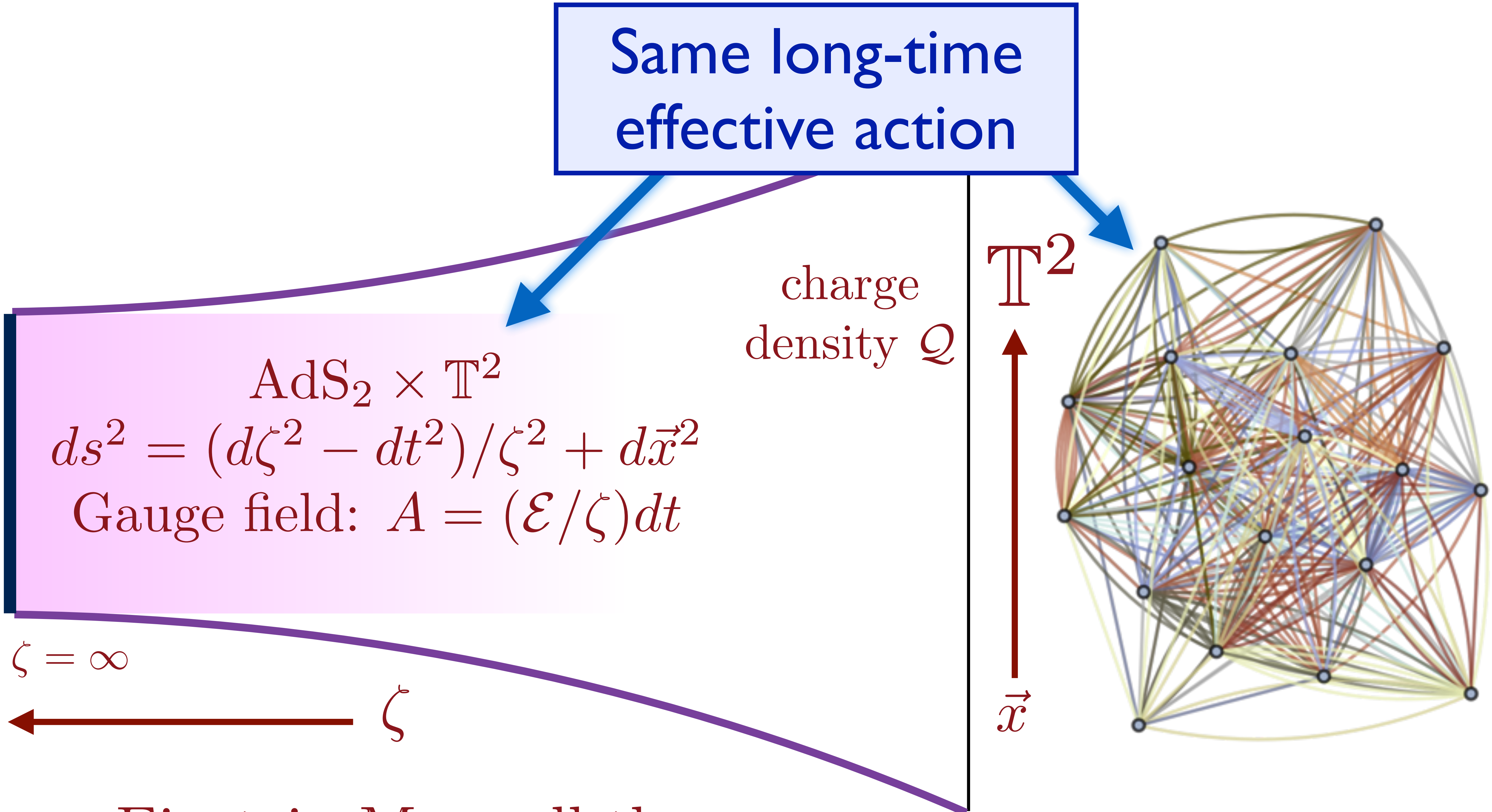


Einstein-Maxwell theory
+ cosmological constant

S. Sachdev, PRL **105**, 151602 (2010)

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

SYK and AdS_2



Einstein-Maxwell theory
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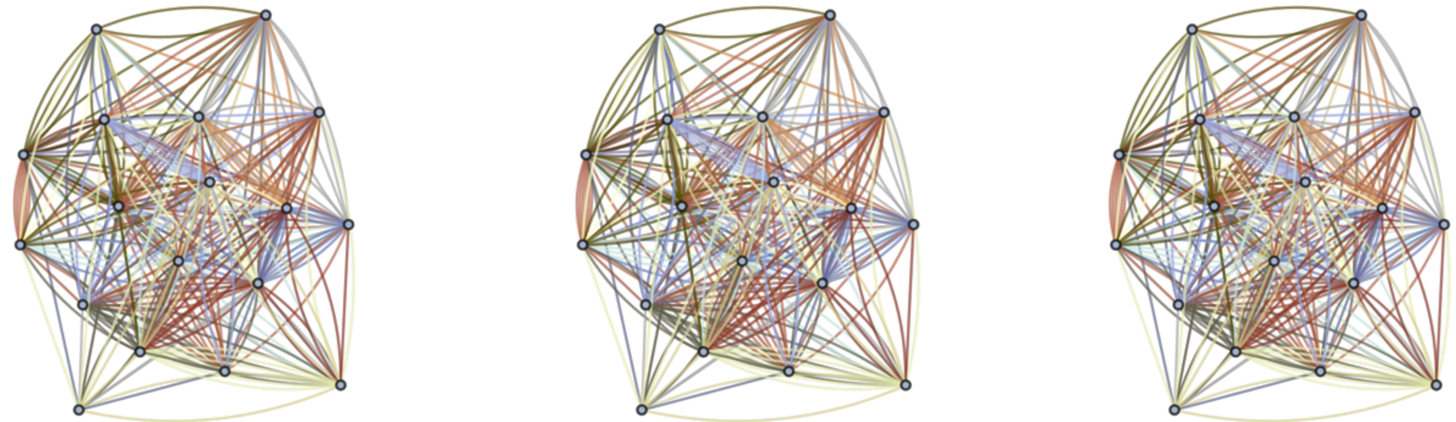
Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

Coupled SYK models



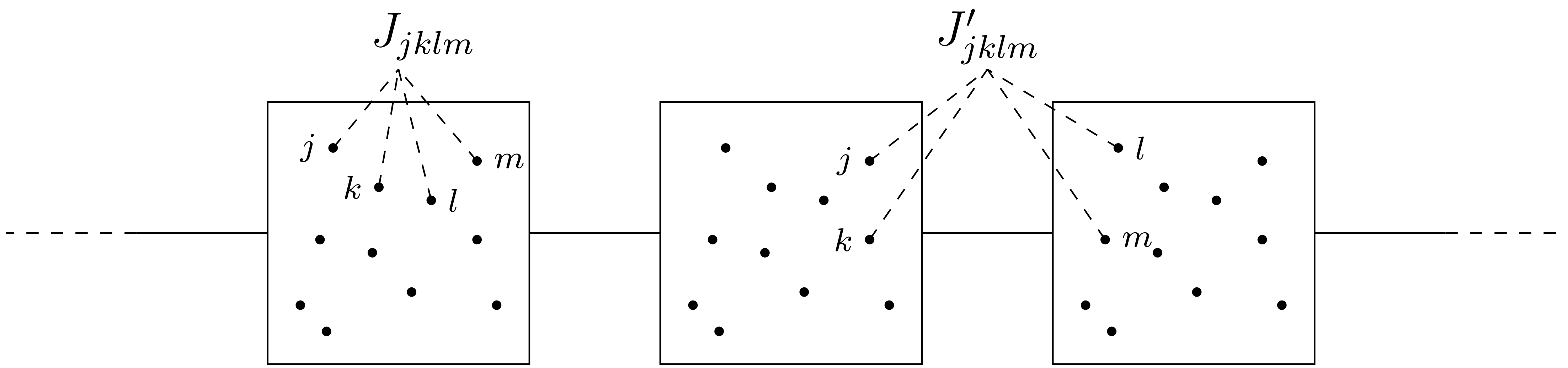
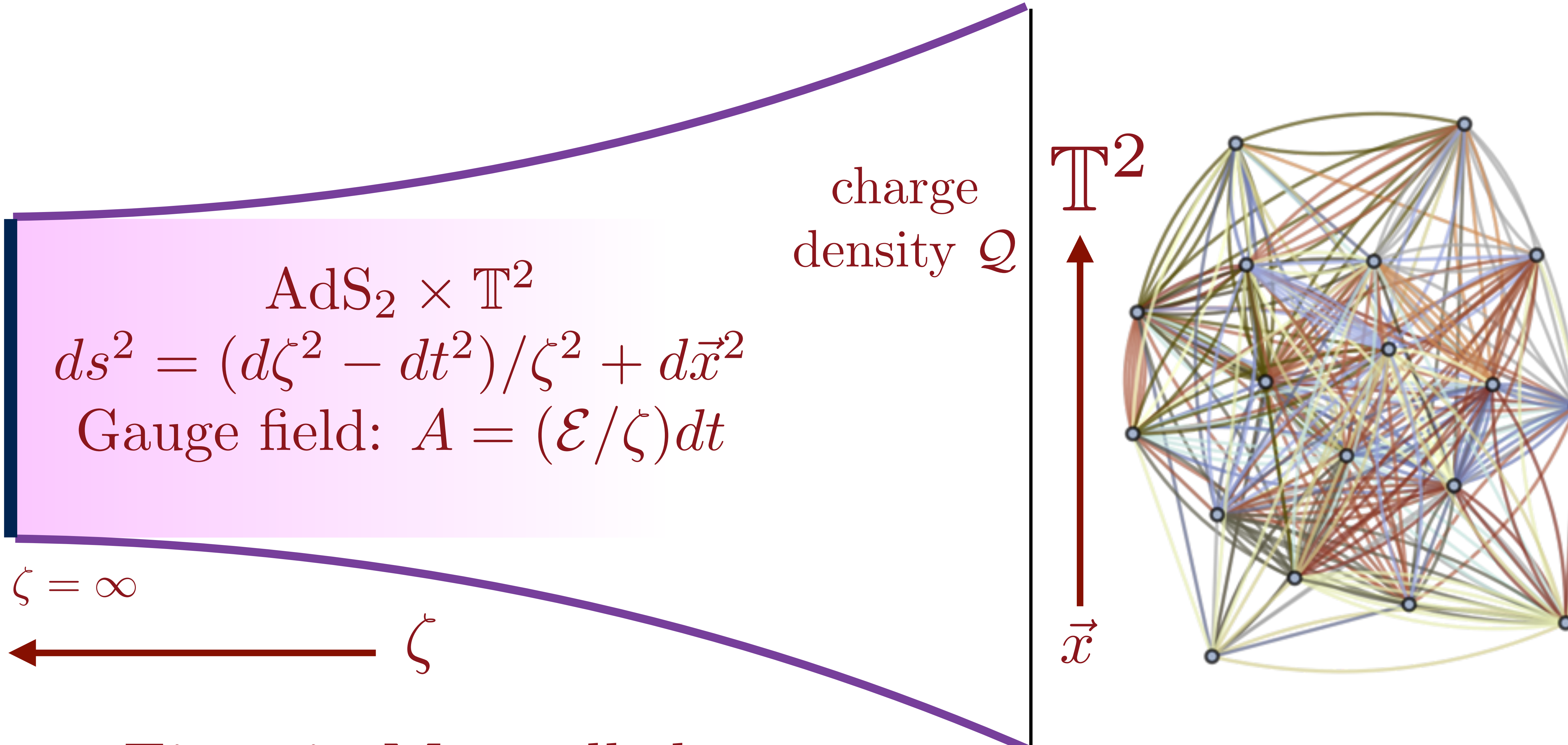


Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

SYK and AdS_2



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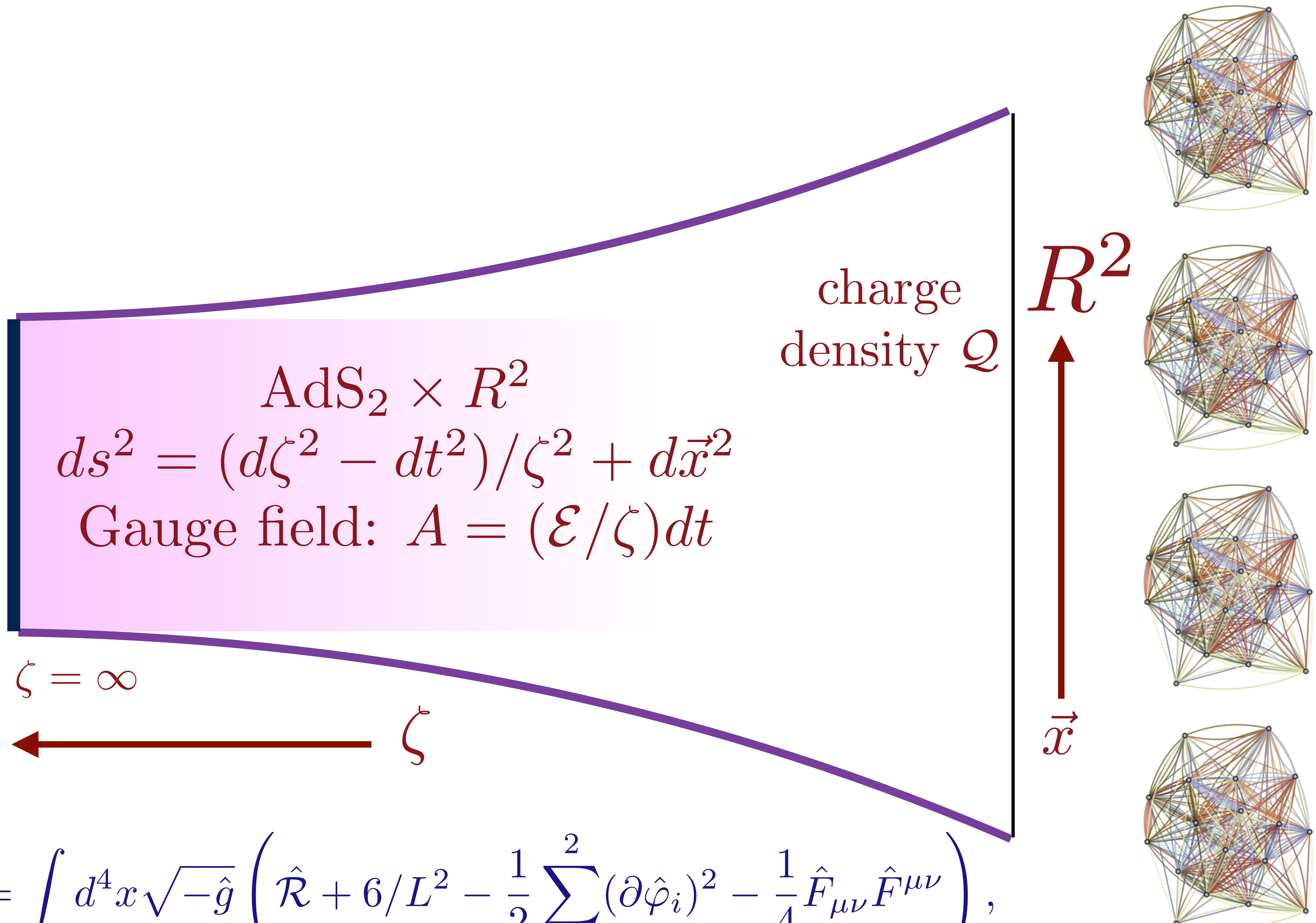
Coupled SYK and AdS_4



$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$ leading to momentum disspation

Coupled SYK and AdS$_4$

The response functions of the density, $\mathcal{Q}$, and the energy, E exhibit diffusion

$$\begin{pmatrix} \langle \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} / T \\ \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; E - \mu \mathcal{Q} \rangle_{k,\omega} / T \end{pmatrix} = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

$$D = \begin{pmatrix} \sigma & \alpha \\ \alpha T & \overline{\kappa} \end{pmatrix} \chi_s^{-1}.$$

The Seebeck co-efficient (thermopower), α/σ, is given exactly by a thermodynamic derivative

$$\frac{\alpha}{\sigma} = \frac{\partial S_0}{\partial \mathcal{Q}}$$

The coupled-SYK and AdS$_4$ models realize
a disordered metal with no quasiparticle excitations.
(a "strange metal")

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

Quantum chaos:

- In both the SYK and holographic models, the growth of chaos is characterized by

$$\left\langle \left| \{c(x,t), c^\dagger(0,0)\} \right|^2 \right\rangle \sim \exp\left(\frac{1}{\tau_L} \left(t - \frac{|x|}{v_B} \right) \right)$$

where the Lyapunov time saturates the lower bound $\hbar/(2\pi k_B T)$ and the BUTTERFLY VELOCITY $v_B \sim T^{1/2}$.

- The thermal diffusivity, D_E is given exactly by

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There is no universal relationship between the charge diffusivity, D_c, and v_B.

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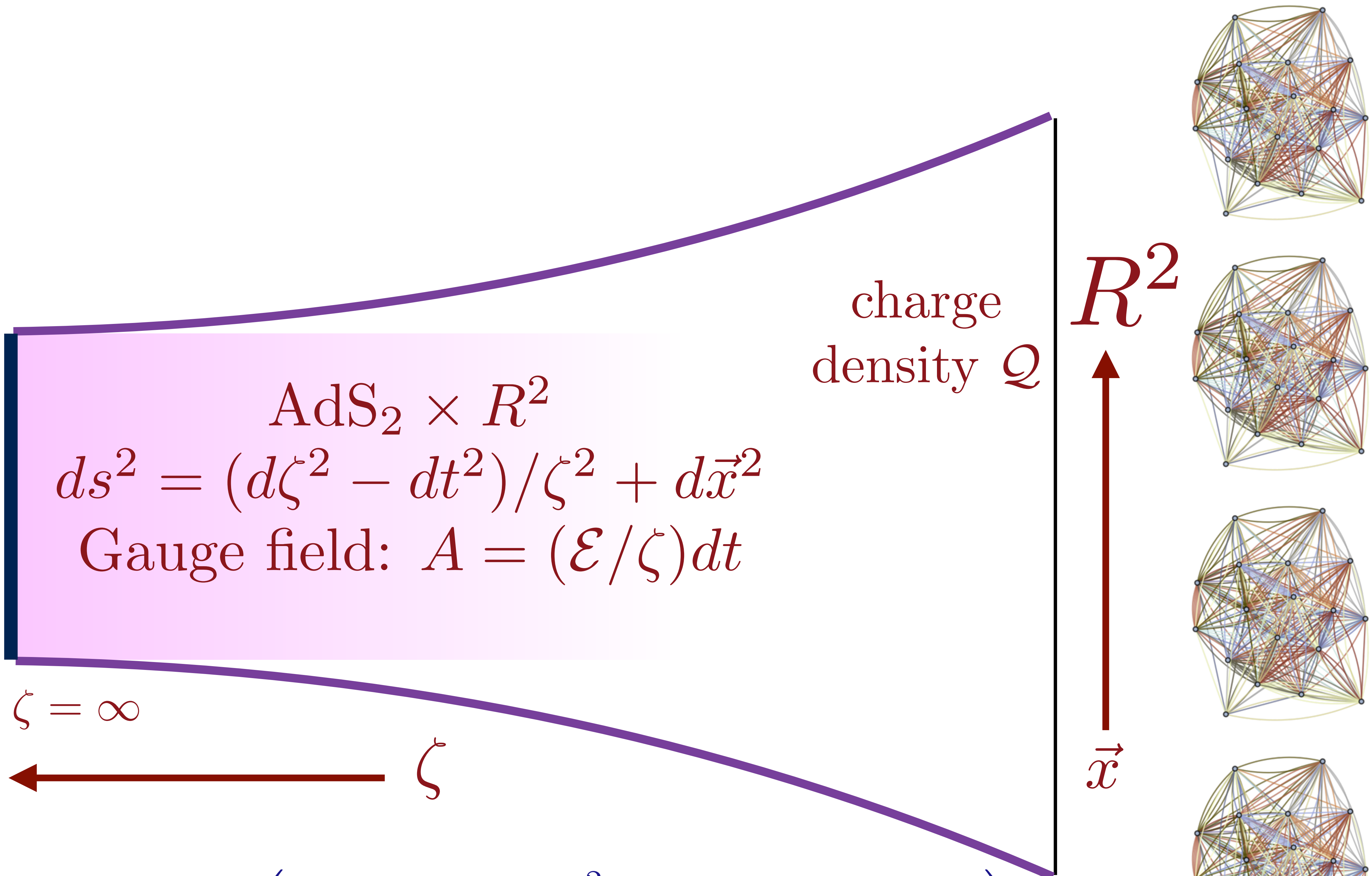
- Quantum chaos is intimately linked to the loss of phase coherence from electron-electron interactions. As the time derivative of the local phase is determined by the local energy, phase fluctuations and chaos are linked to interaction-induced energy fluctuations, and hence thermal diffusivity.

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

Coupled SYK and AdS_4

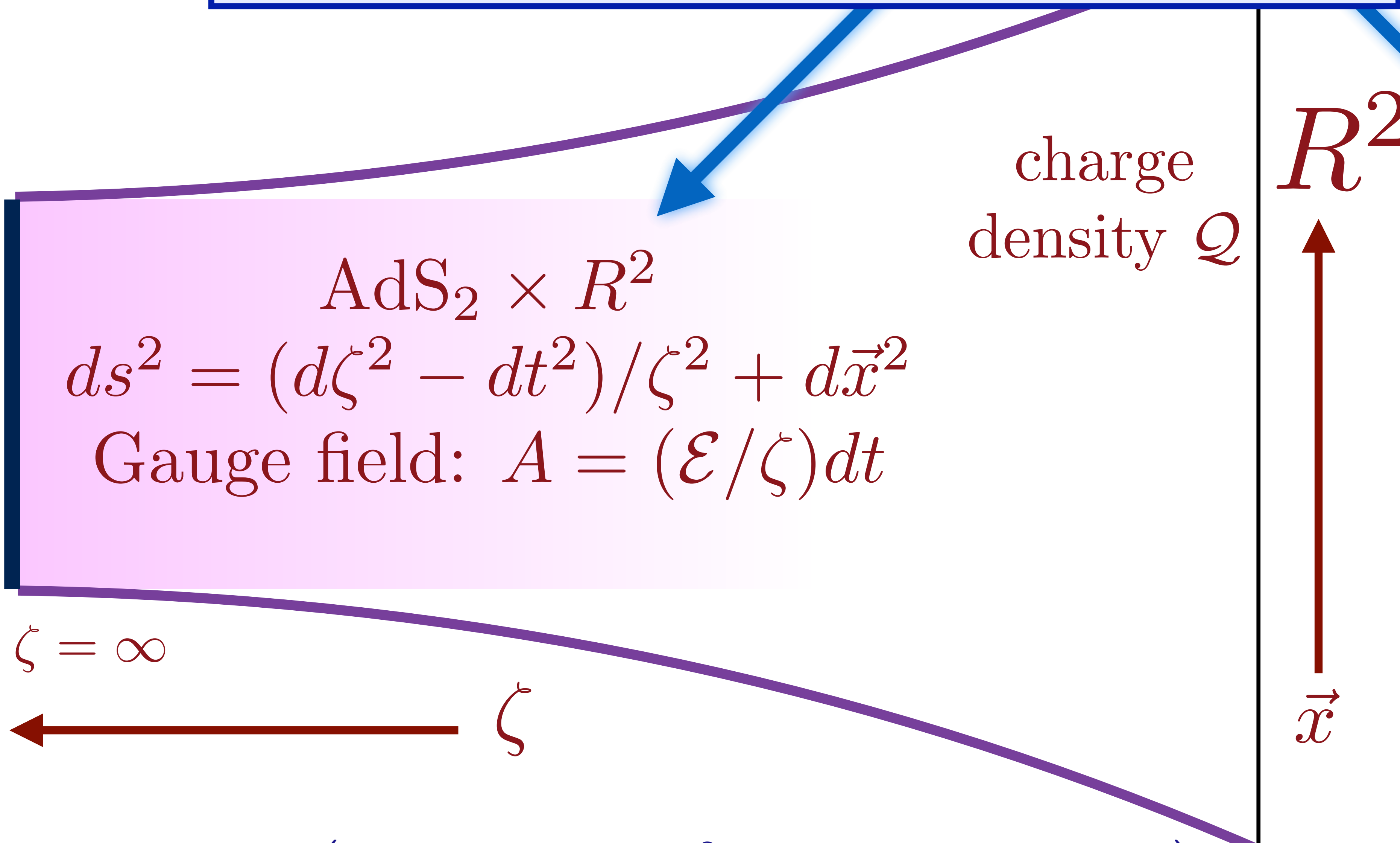


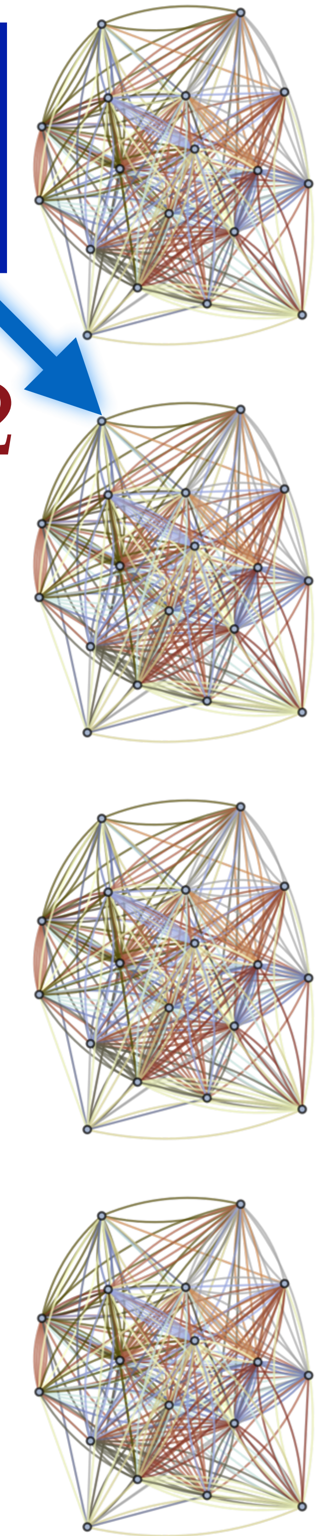
$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$ leading to momentum disspation

Coupled SYK and AdS_4

Matching correlators for thermoelectric diffusion, and quantum chaos





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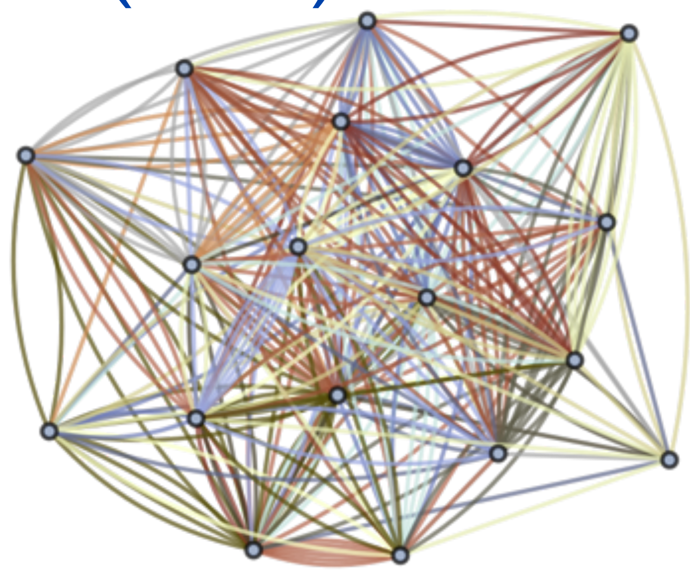
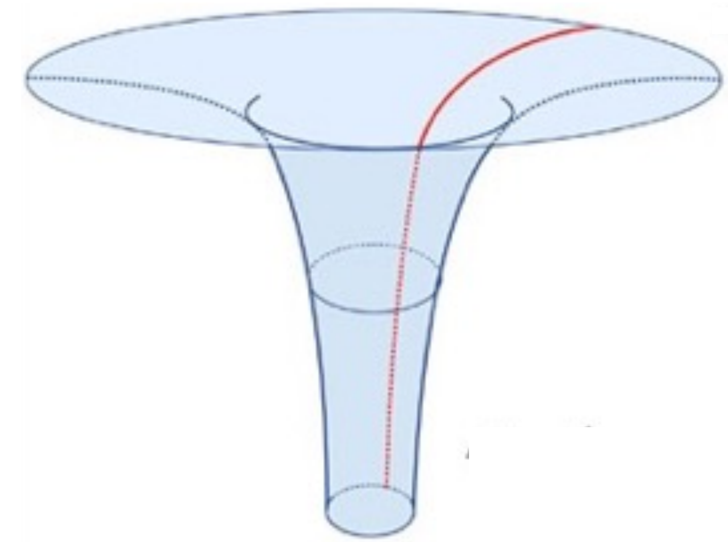
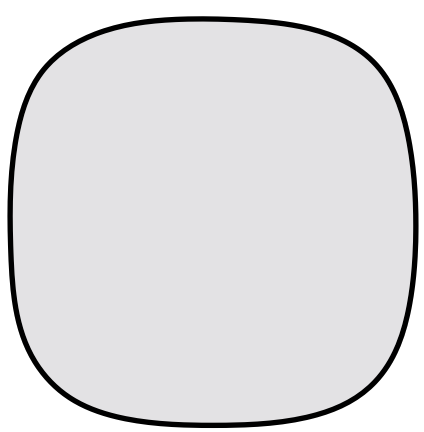
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons





Fermi surface coupled to a gauge field

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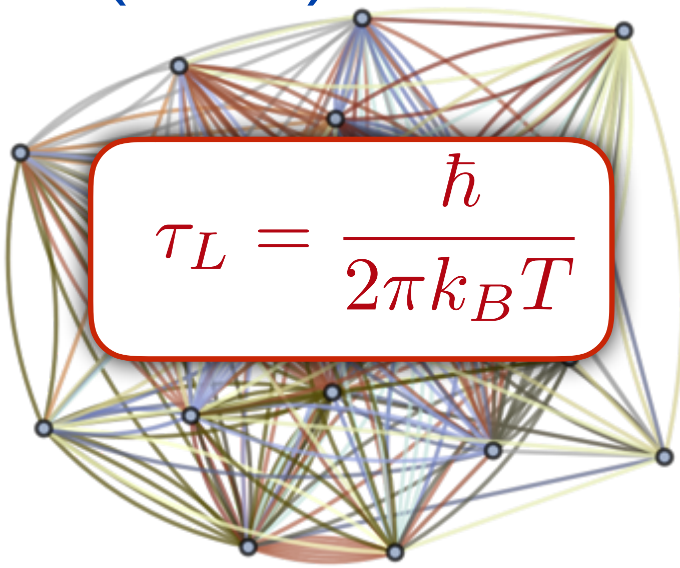

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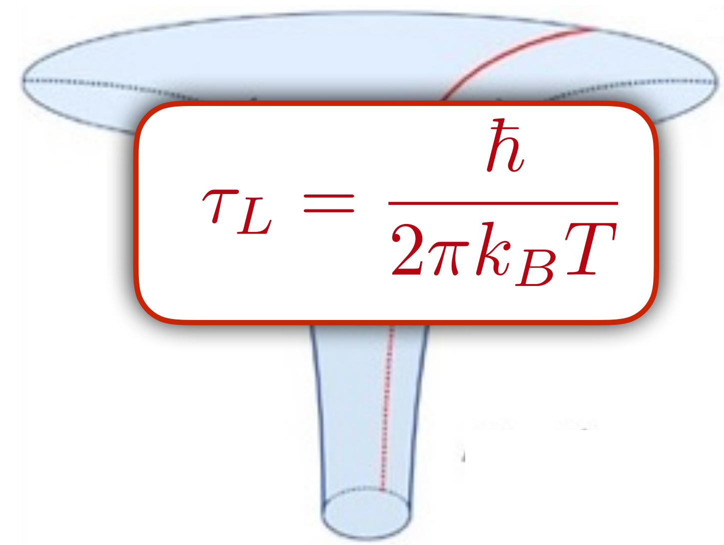

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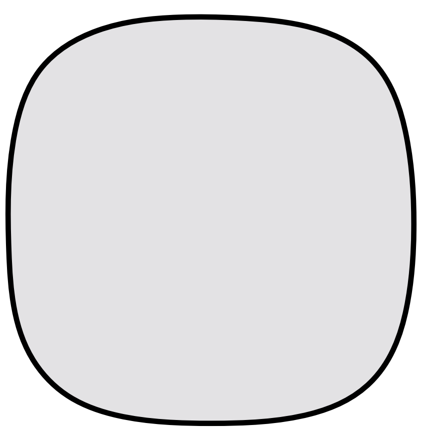

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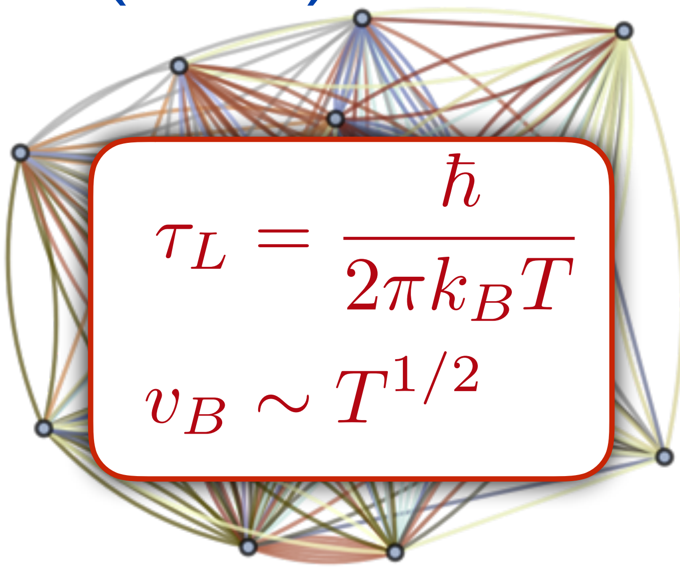

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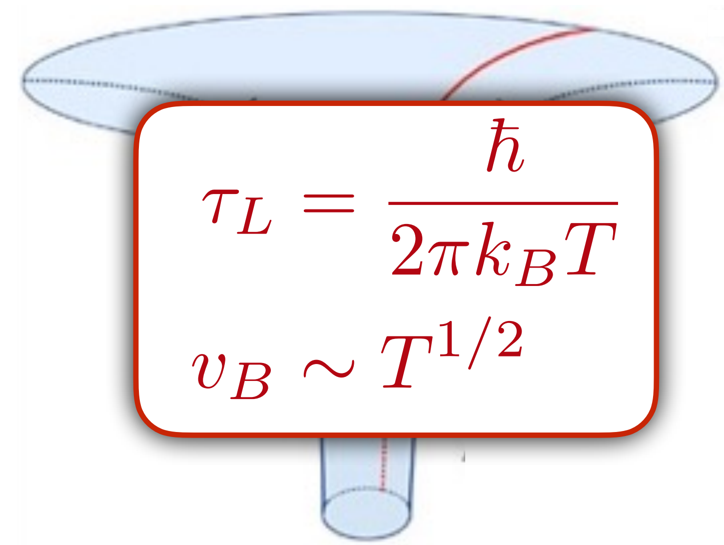

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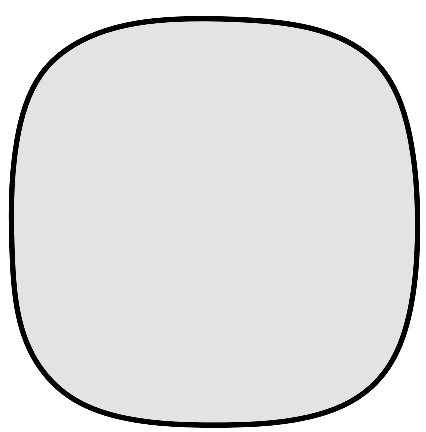

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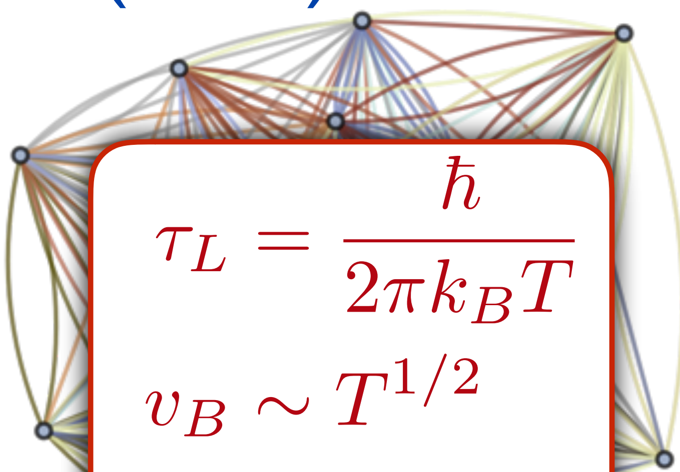

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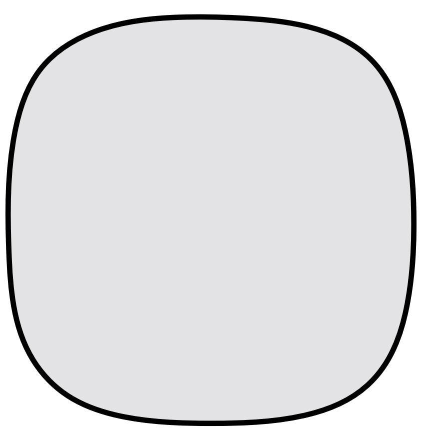

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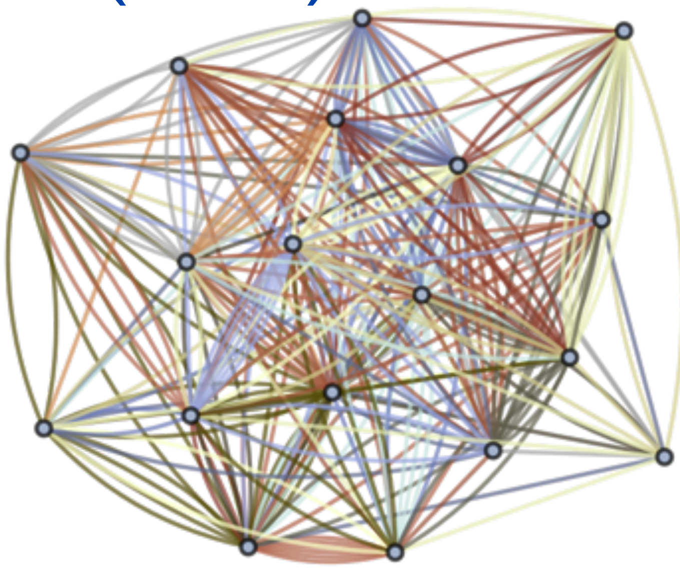

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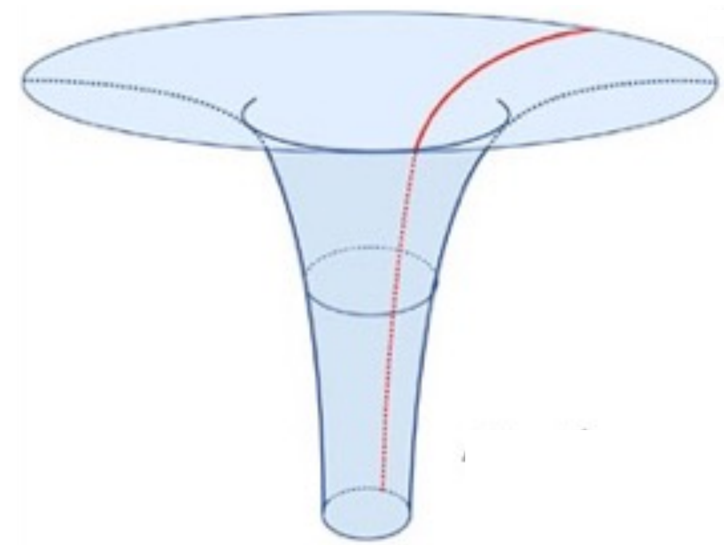

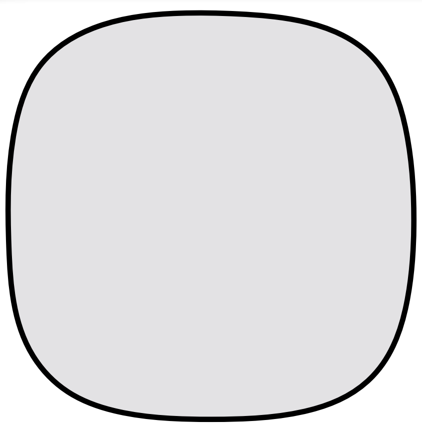

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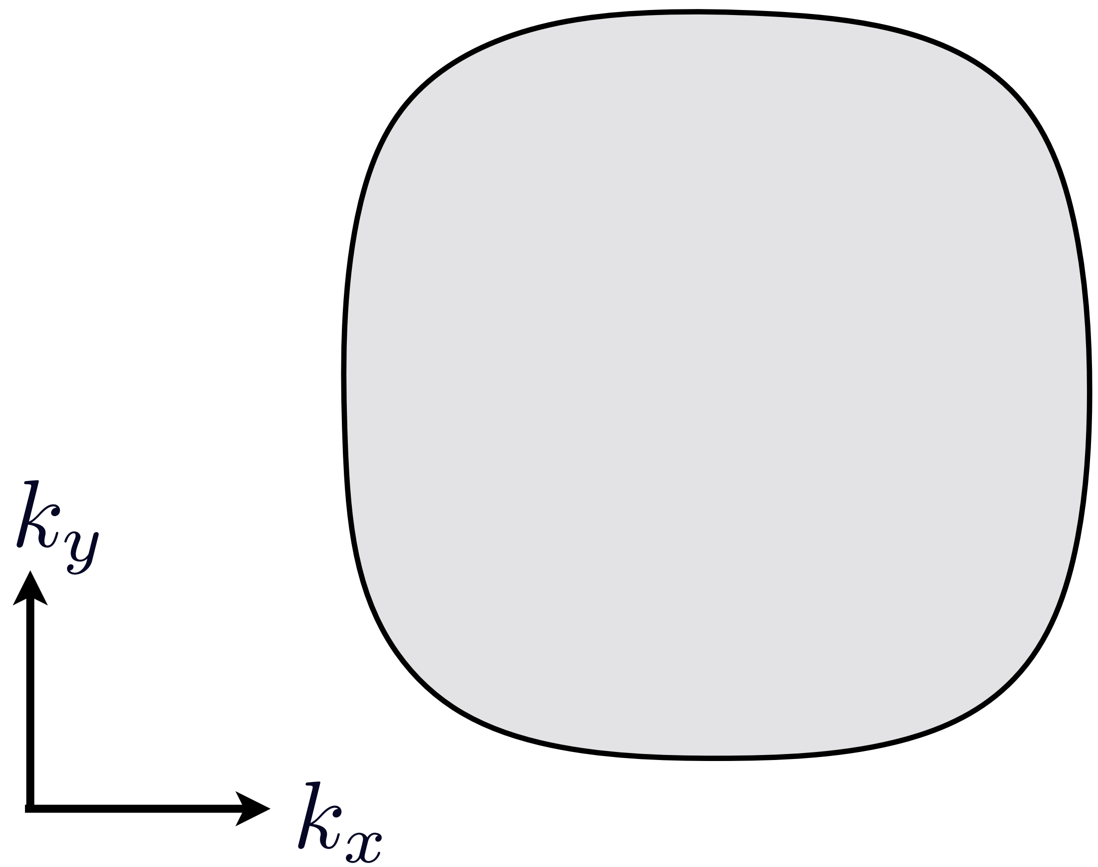

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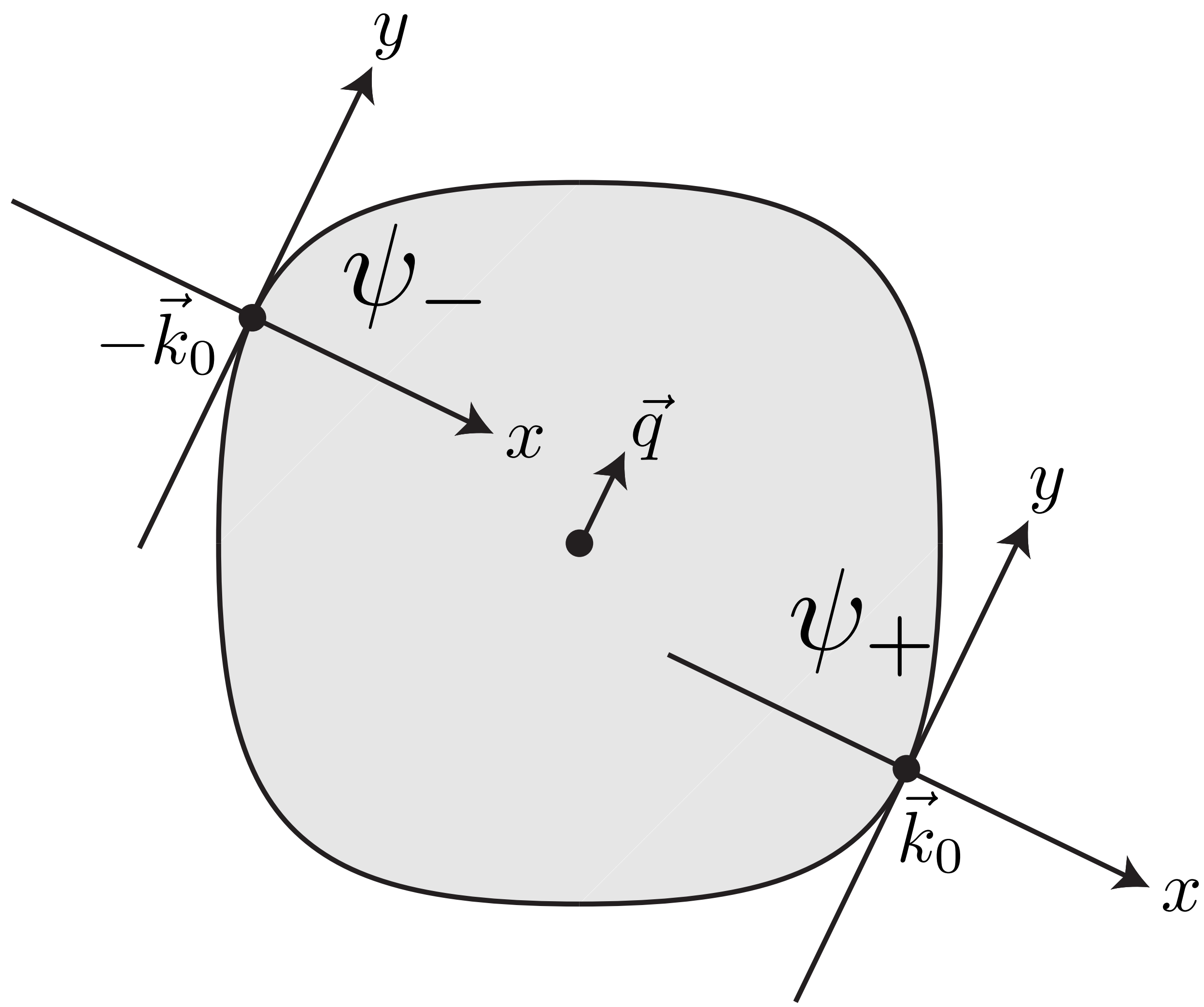


$$\mathcal{L}[\psi_\pm, a] =$$

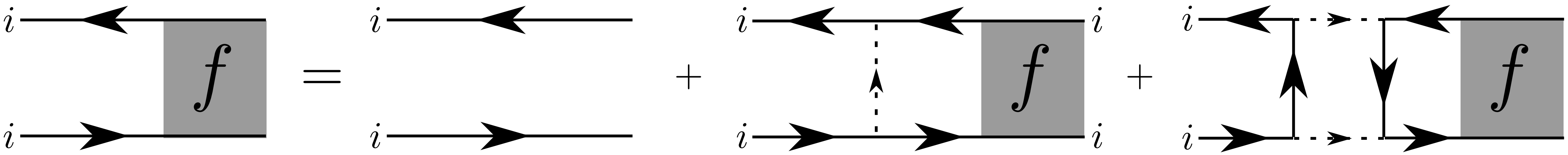
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos



$$f(t) = \frac{1}{N^2}\theta(t) \sum_{i,j=1}^{N} \int d^2x \; \mathrm{Tr}\Big[e^{-\beta H/2}\{\psi_i(x,t), \psi_j^\dagger(0)\} \times e^{-\beta H/2}\{\psi_i(x,t), \psi_j^\dagger(0)\}^\dagger \Big]$$

$$\sim \exp\Big((t - x/v_B)/\tau_L\Big)$$

A.A. Patel and S. Sachdev, arXiv:1611.00003

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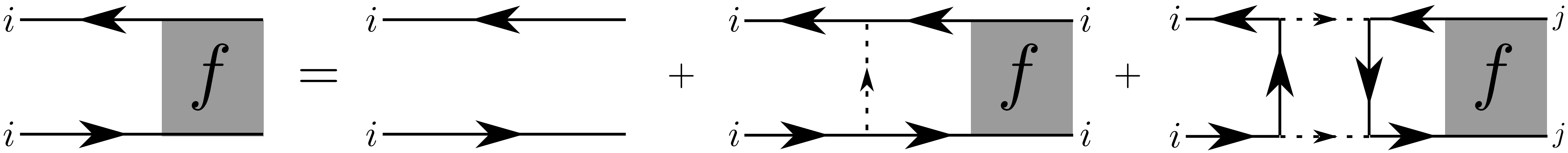

Strongly-coupled theory with no quasiparticles and fast scrambling:

$$\tau_L \approx \frac{\hbar}{2.48\, k_B T}$$

$$v_B \approx 4.1\, \frac{N T^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}}$$

$$D_E \approx 0.42\, v_B^2 \tau_L$$

N is the number of fermion flavors, v_F is the Fermi velocity, γ is the Fermi surface curvature, e is the gauge coupling constant.

A.A. Patel and S. Sachdev, arXiv:1611.00003

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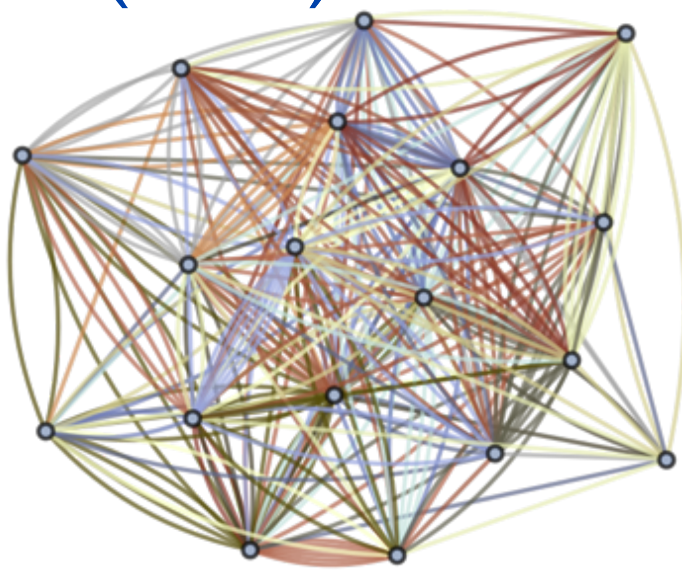

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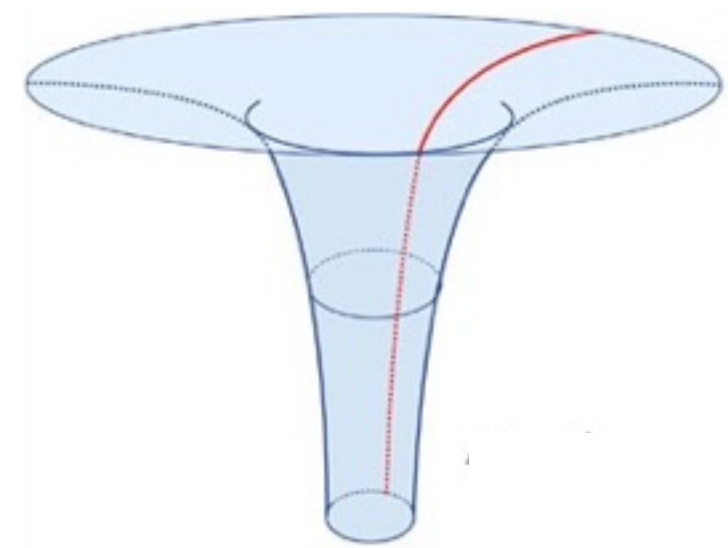

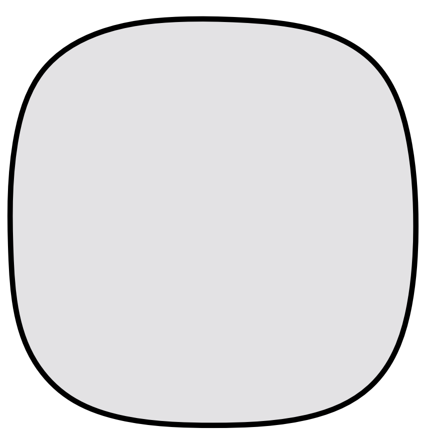

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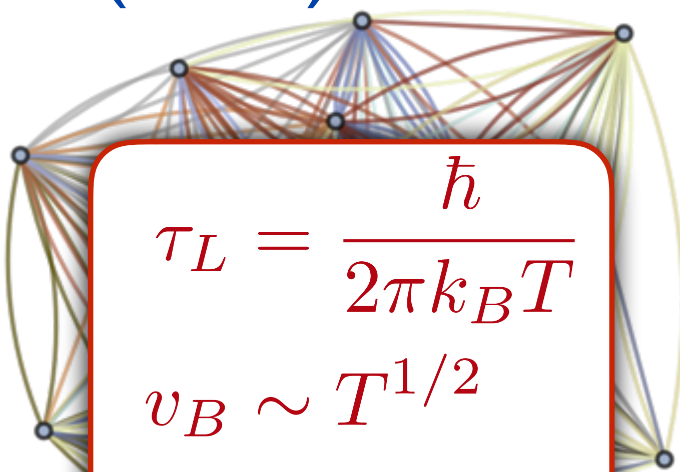

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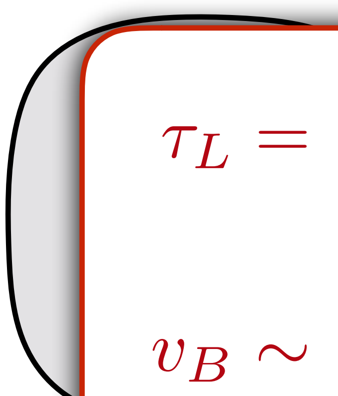

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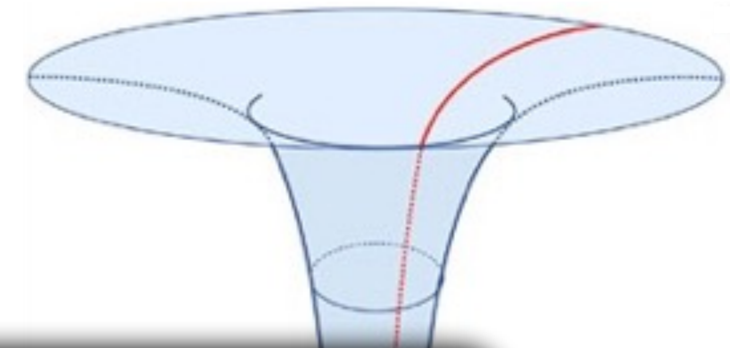
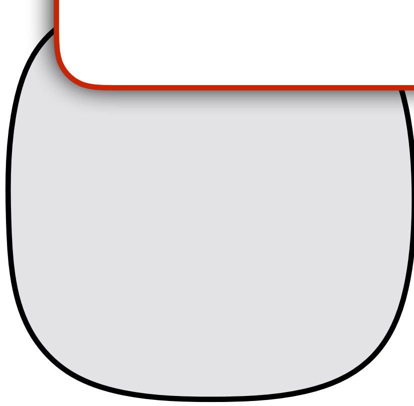


Thermal diffusivity, D_E:

$D_E =$ (universal number) $\times v_B^2 \tau_L$

in all three models



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