Multi-point correlators of conformal field theories: implications for quantum critical transport

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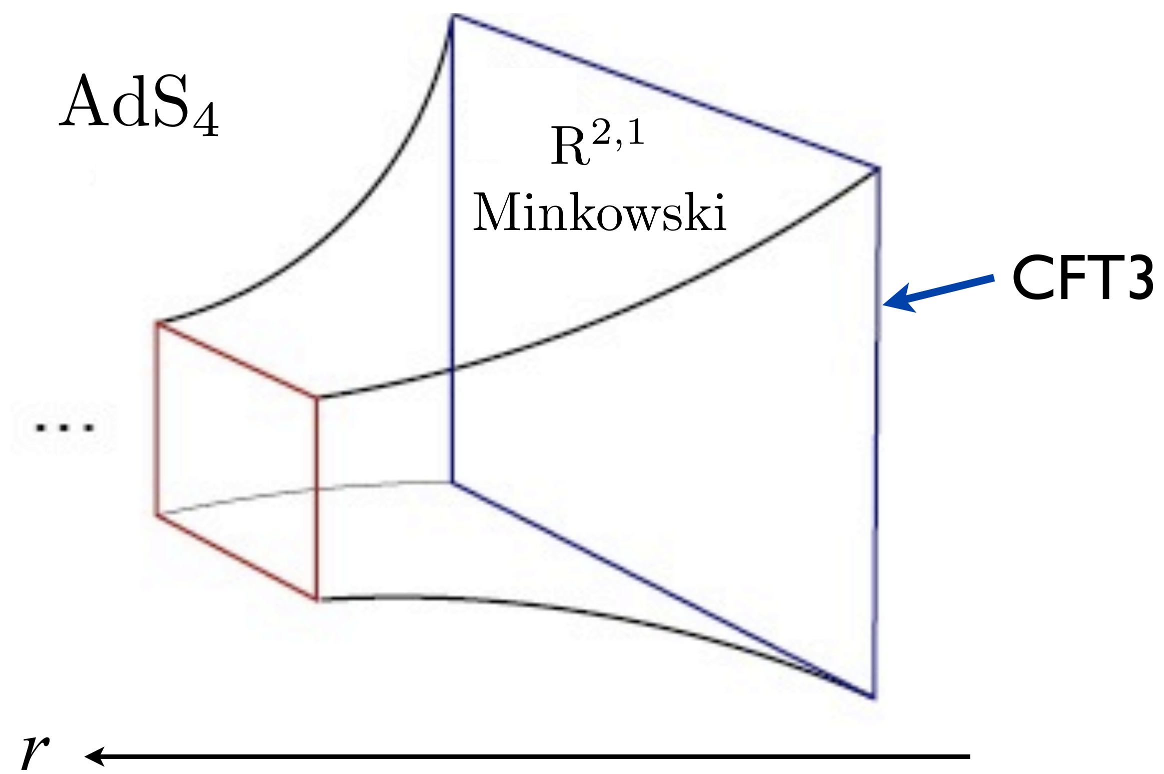
arXiv:1210.5247

PHYSICS
HARVARD

Relate boundary CFT correlators to bulk gravity correlators at boundary



$$\int \mathcal{D}\phi \exp\left(-\mathcal{S}_{\rm bulk}\right)\Big|_{\rm bdy} = \left\langle \exp\left(\int d^D x\, \phi_0(\boldsymbol{x}) O(\boldsymbol{x})\right)\right\rangle_{\rm CFT}$$

boundary condition is

$$\lim_{r\to 0} \phi(\boldsymbol{x}, r) = r^{D-\Delta}\phi_0(\boldsymbol{x}).$$

$$\langle O(\boldsymbol{x}_1)\ldots O(\boldsymbol{x}_n)\rangle_{\rm CFT} = Z^n \lim_{r\to 0} r_1^{-\Delta}\ldots r_n^{-\Delta}\,\langle \phi(\boldsymbol{x}_1, r_1)\ldots\phi(\boldsymbol{x}_n, r_n)\rangle_{\rm bulk}$$

Correlators of a conserved U(1) current

$$\langle J_\mu(\boldsymbol{k}) J_\nu(\boldsymbol{k})\rangle_{\mathrm{CFT}} = \mathcal{K}\, k \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

Associated with this current is a bulk U(1) gauge field

$$\mathcal{S}_M = \frac{1}{4g_M^2}\int d^{D+1}x\,\sqrt{g}F_{ab}F^{ab}$$

Bulk-boundary correspondence:

$$\langle J_\mu(\boldsymbol{x}_1)\dots J_\nu(\boldsymbol{x}_n)\rangle_{\mathrm{CFT}} \;=\; (Zg_M^{-2})^n \lim_{r\to 0} r_1^{2-D}\dots r_n^{2-D}\,\langle A_\mu(\boldsymbol{x}_1, r_1)\dots A_\nu(\boldsymbol{x}_n, r_n)\rangle_{\mathrm{bulk}}$$

$$\mathcal{K} = \frac{1}{g_M^2}.$$

Correlators of the stress-energy tensor

$$\langle T_{\mu\nu}(\boldsymbol{k}) T_{\rho\sigma}(-\boldsymbol{k})\rangle_{\rm CFT} = \mathcal{C}_T |k|^3 \left(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\nu\rho}\delta_{\mu\sigma} - \delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\nu}\frac{k_\rho k_\sigma}{k^2} + \delta_{\rho\sigma}\frac{k_\mu k_\nu}{k^2} \right.$$
$$\left. -\delta_{\mu\rho}\frac{k_\nu k_\sigma}{k^2} - \delta_{\nu\rho}\frac{k_\mu k_\sigma}{k^2} - \delta_{\mu\sigma}\frac{k_\nu k_\rho}{k^2} - \delta_{\nu\sigma}\frac{k_\mu k_\rho}{k^2} + \frac{k_\mu k_\nu k_\rho k_\sigma}{k^4} \right)$$

Associated are the fluctuations of the bulk metric

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Bulk-boundary correspondence:

$$\mathcal{C}_T \propto \frac{L^2}{2\kappa^2}.$$

Fix coefficients from CFT_3 at *zero* temperature

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right]$$

Coupling	Correlator
G_N	$\dfrac{\langle TJJ\rangle}{\sqrt{\langle TT\rangle\langle JJ\rangle}}$
g_4^2	$\langle JJ\rangle$
γ	$\dfrac{\langle TJJ\rangle^{---}}{\langle TJJ\rangle^{--+}}$

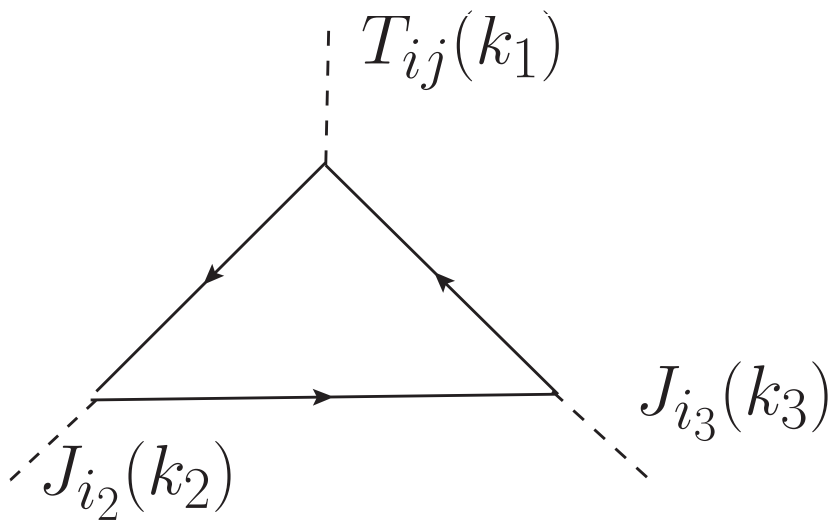
Current, stress tensor fermions:

$$J^{\ell}_{f,i} = \overline{\psi}_\alpha\, T^{\ell}_{\alpha\beta}\gamma_i\,\psi_\beta$$

$$T_{f,ij} = \frac{i}{4}\sum_{\alpha=1}^{N_f}\Big(\overline{\psi}_\alpha\gamma_i\left(D_j\psi_\alpha\right) + \overline{\psi}_\alpha\gamma_j\left(D_i\psi_\alpha\right) - \left(D_i^*\overline{\psi}_\alpha\right)\gamma_j\psi_\alpha - \left(D_j^*\overline{\psi}_\alpha\right)\gamma_i\psi_\alpha\Big)$$

$$\mathcal{L} = \sum_{\alpha=1}^{N_f} i\overline{\psi}_\alpha\gamma^i D_i\psi_\alpha + \sum_{a=1}^{N_s}\left(|D_i z_a|^2 + s|z_a|^2 + \frac{u}{2}\left(|z_a|^2\right)^2\right)$$

Free CFT3s saturate exact CFT3 bound, $|\gamma| \leq 1/12$.

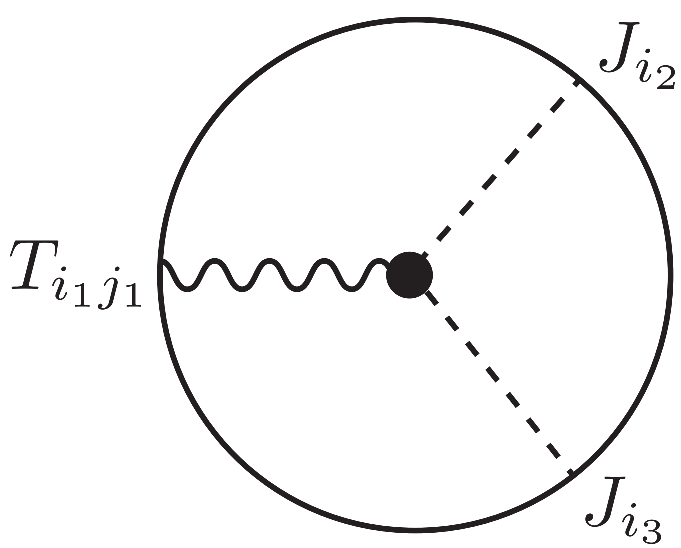


- 1 Stress-tensor vertex, 2 current vertices
- Tensor-valued momentum integrals

$$\langle J_\mu(\boldsymbol{k}_1) J_\nu(\boldsymbol{k}_2) T_{\rho\sigma}(-\boldsymbol{k}_1 - \boldsymbol{k}_2)\rangle \sim \frac{k_1 k_2}{(k_1 + k_2)^5} k_{1\mu} k_{1\nu} k_{1\rho} k_{1\sigma}$$

+ 185 terms

- Helicity contractions

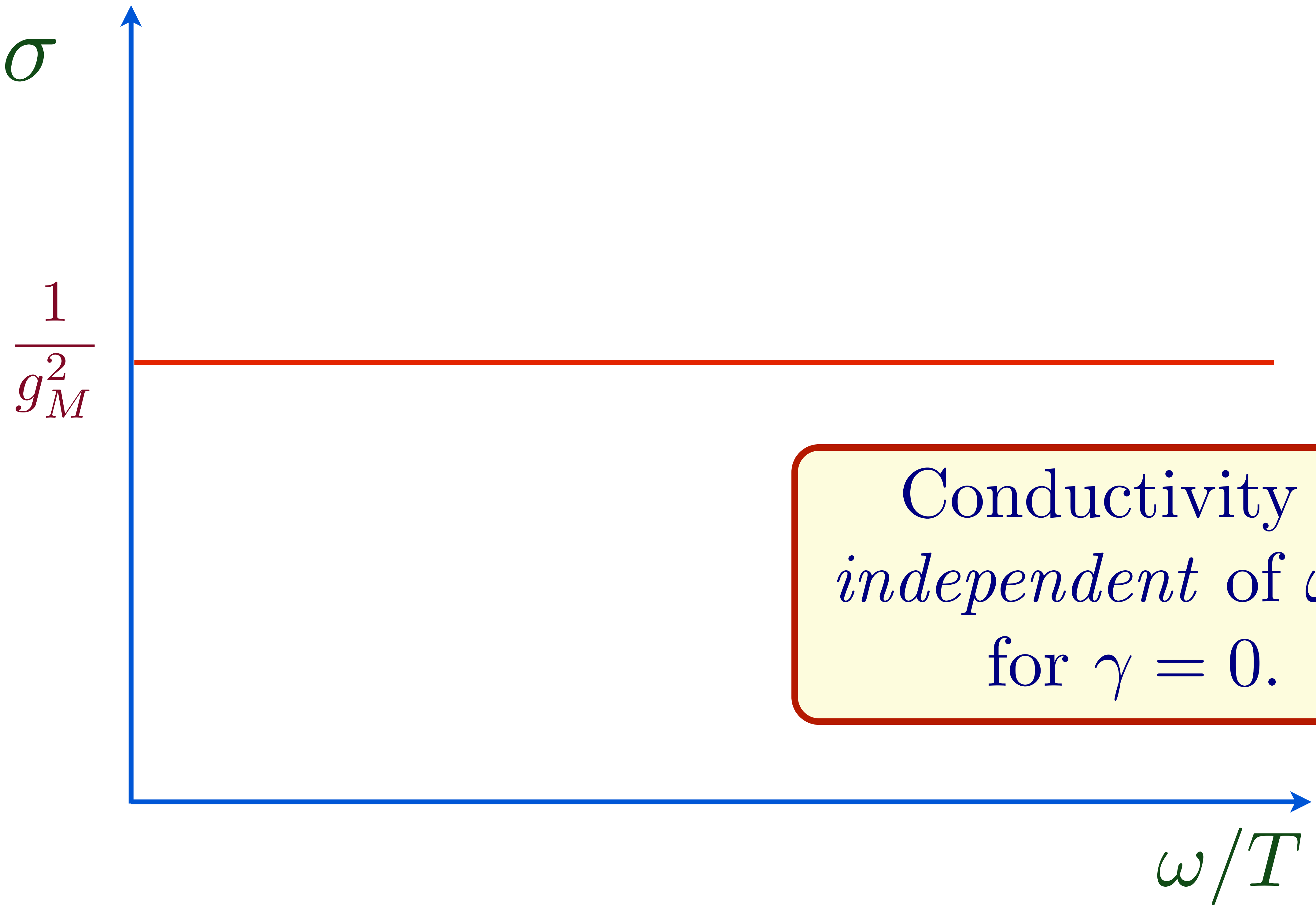


- Expand metric: $\eta_{\mu\nu} + z^2 h_{\mu\nu}$
- Derive non-linear graviton-gauge field vertex
- Evaluate tree-level Witten diagram
- Helicity contractions

Suvrat Raju

- Fermions: $\gamma_f = \frac{1}{12} + \mathcal{O}(1/N_F)$
- Scalars: $\gamma_s = -\frac{1}{12} + \mathcal{O}(1/N_F)$

AdS4 theory of electrical transport in a strongly interacting CFT3 for *T > 0*



Conductivity is *independent* of ω/T for $\gamma = 0$.

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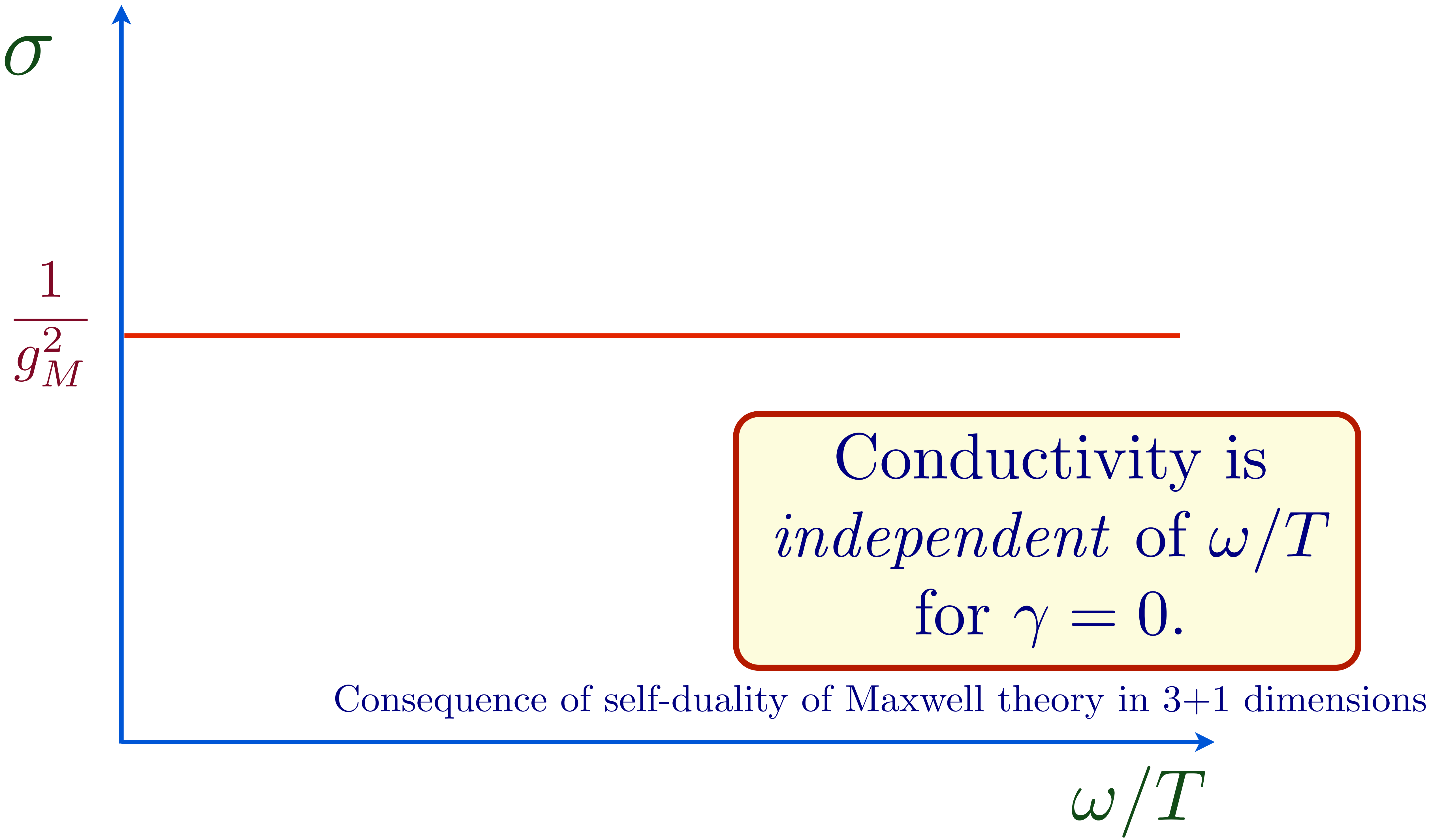
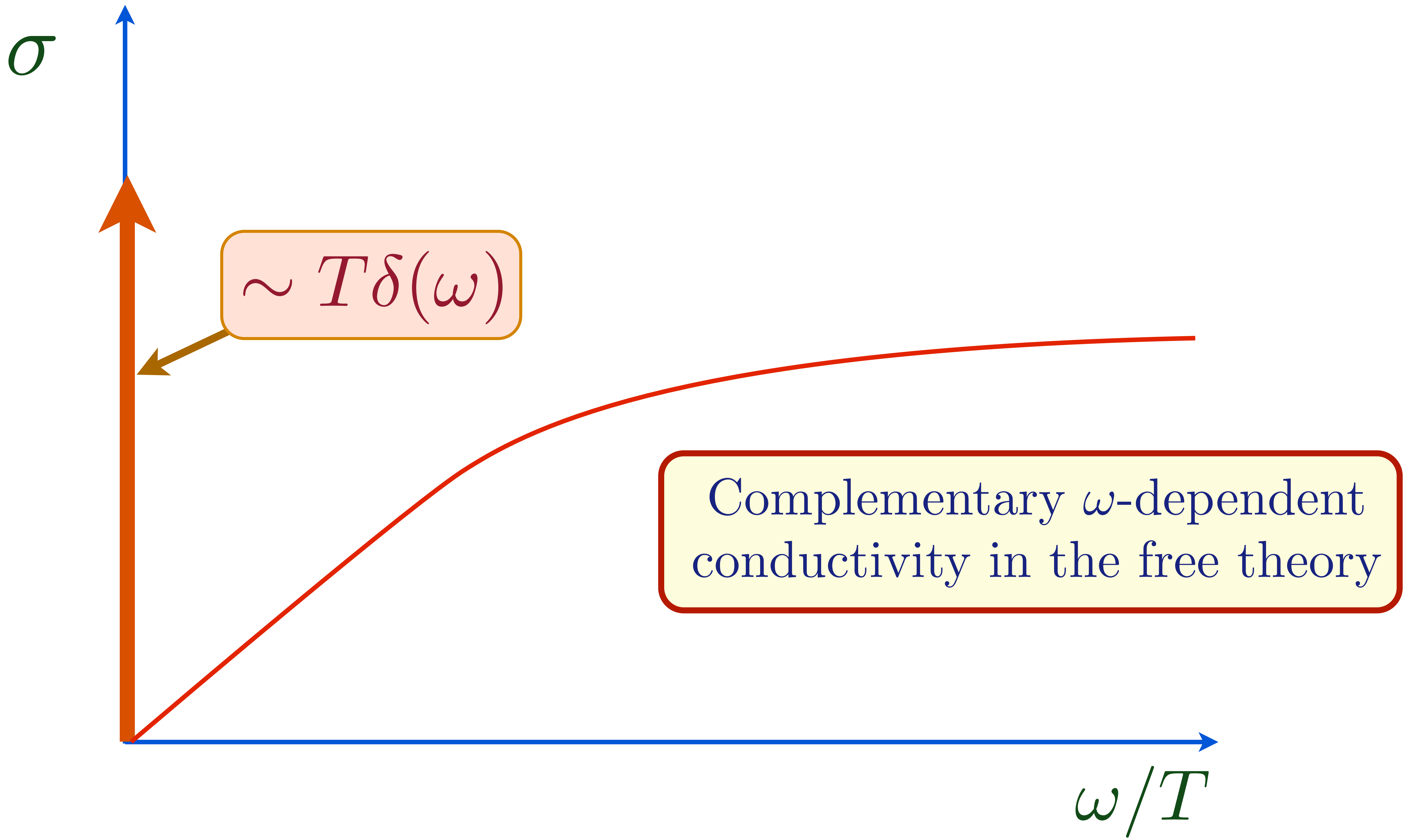
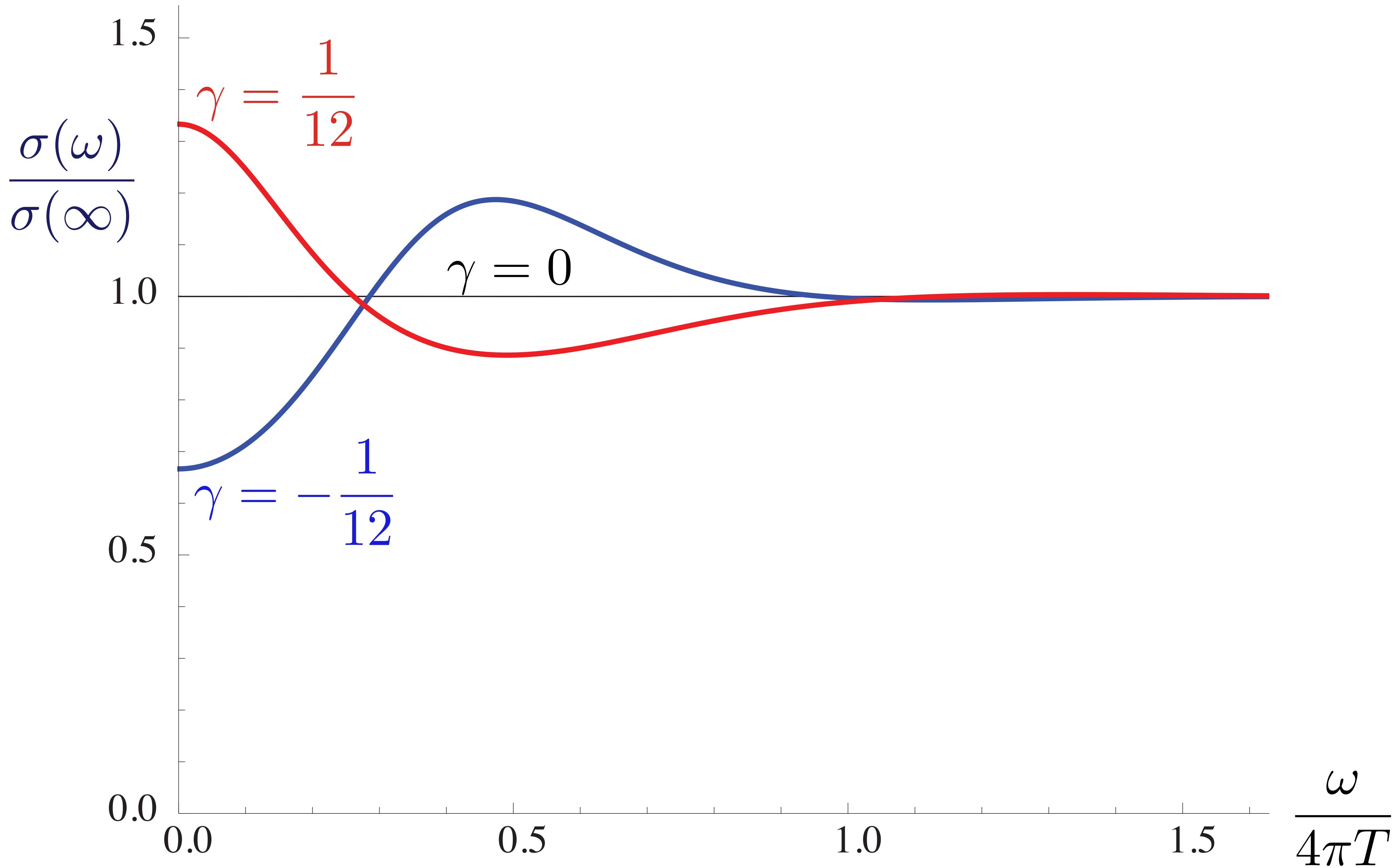


C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

Electrical transport in a free CFT3 for *T* > *0*



AdS$_4$ theory of "nearly perfect fluids"



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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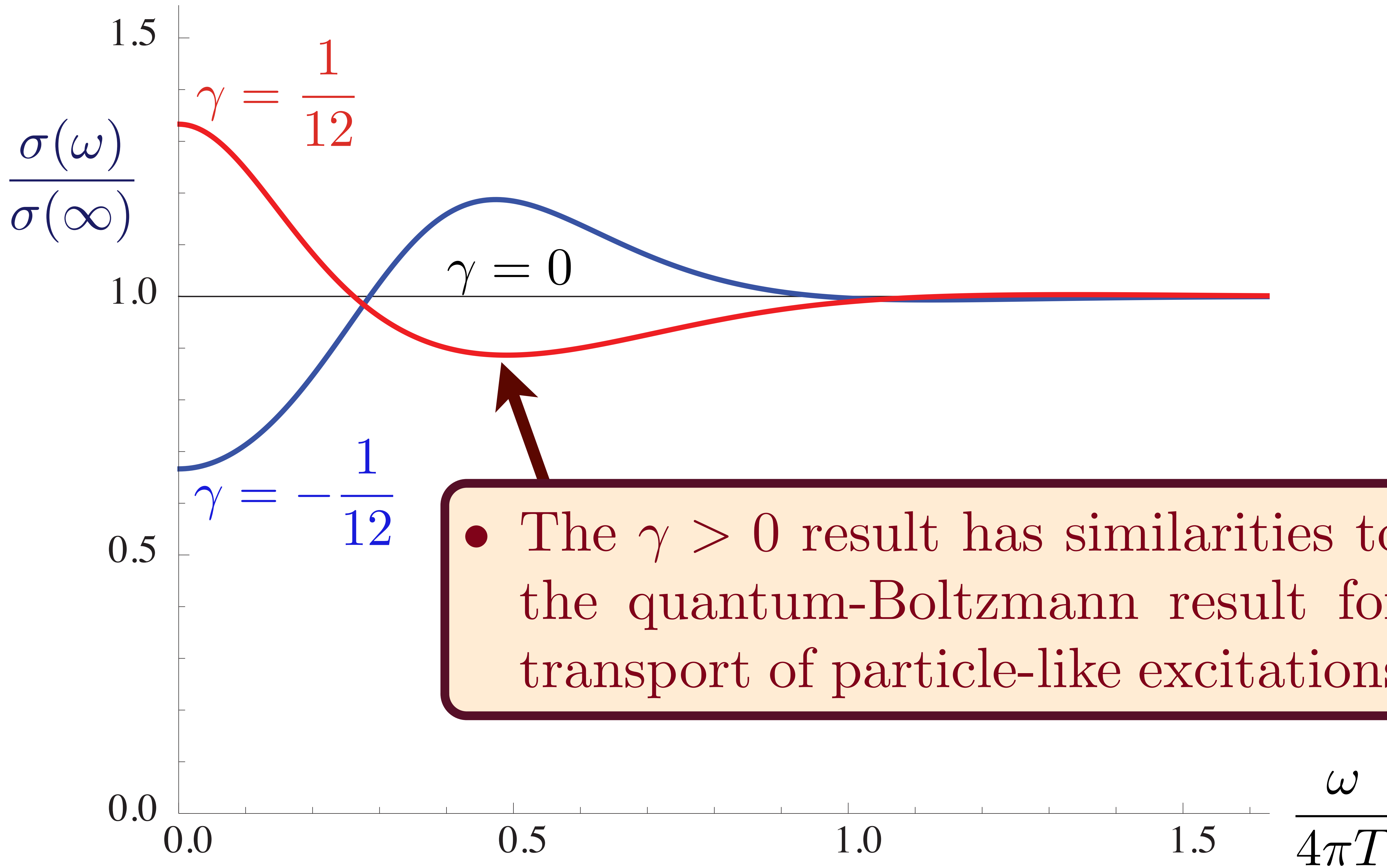

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

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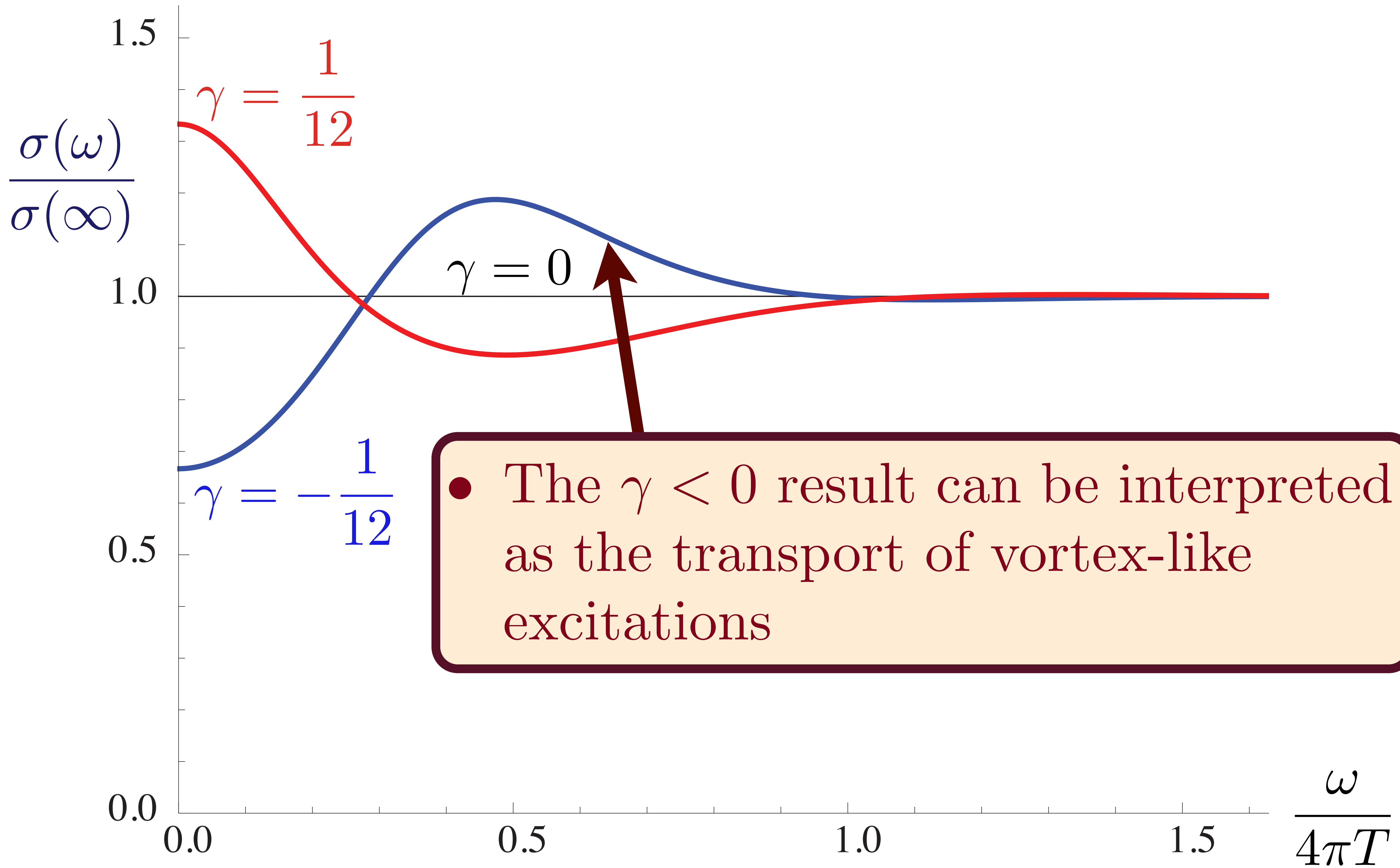

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

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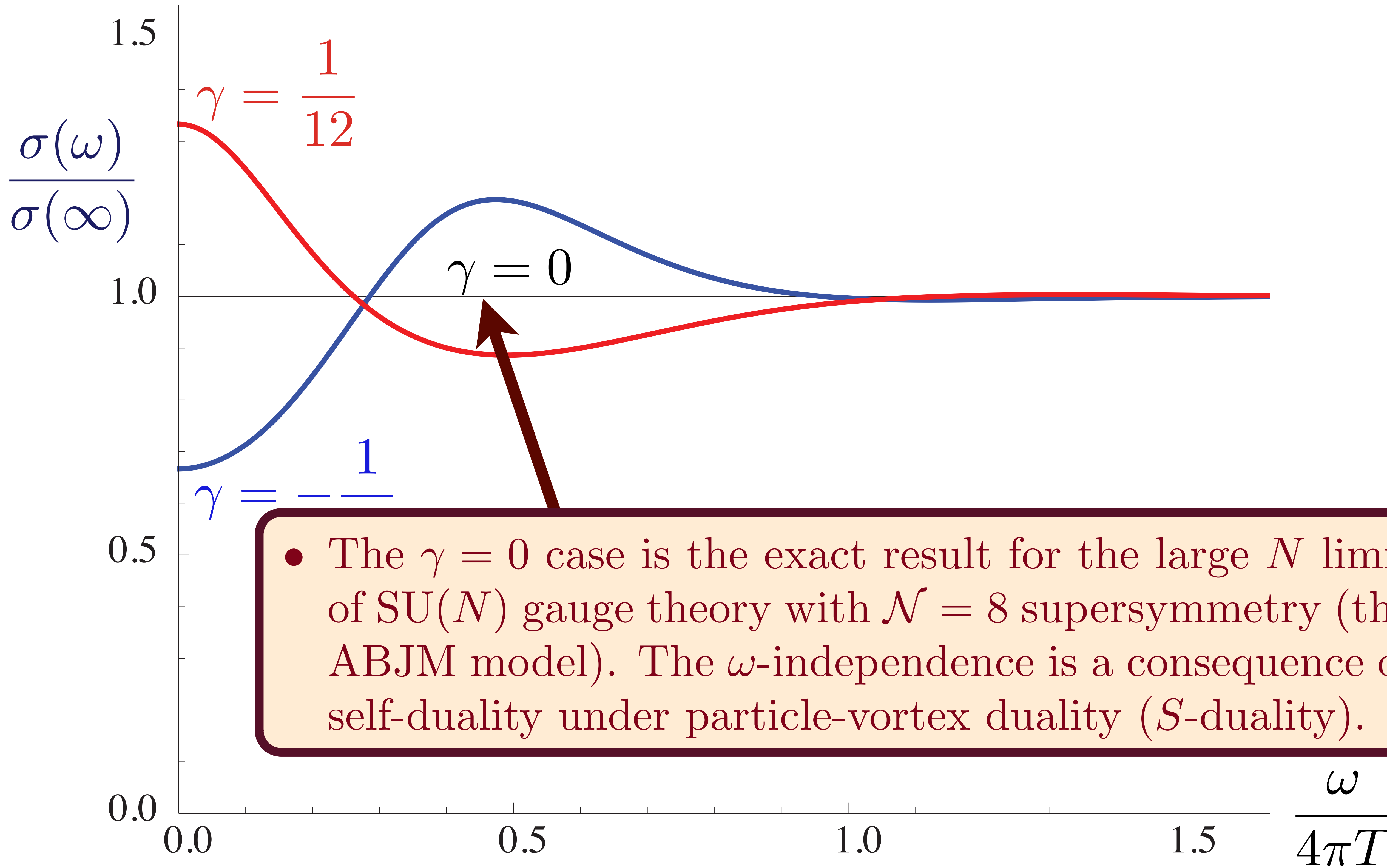

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

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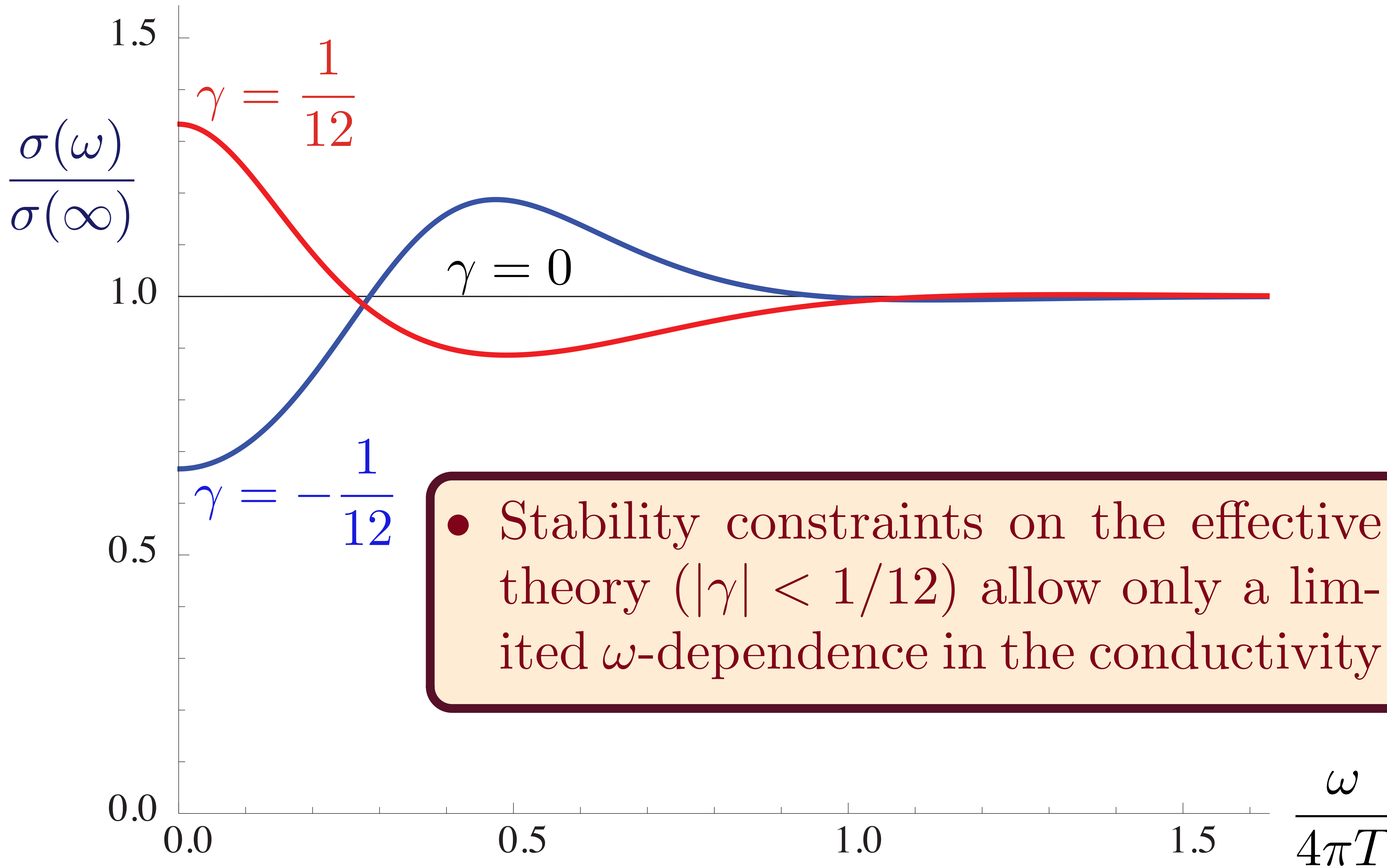

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Traditional CMT

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- Start with strongly interacting CFT without particle- or wave-like excitations

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- Solve Einsten-Maxwell-... equations, allowing for a horizon at non-zero temperatures.