Quantum critical transport in graphene

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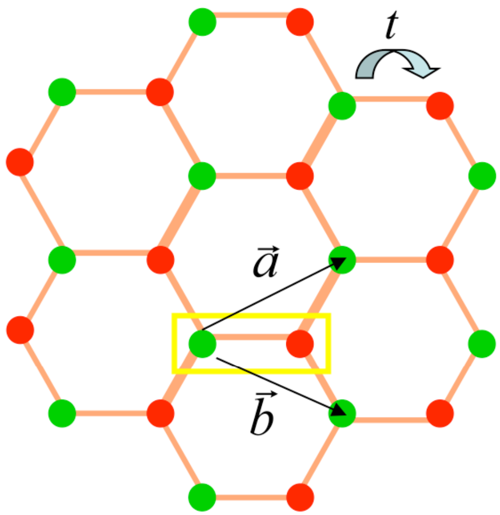
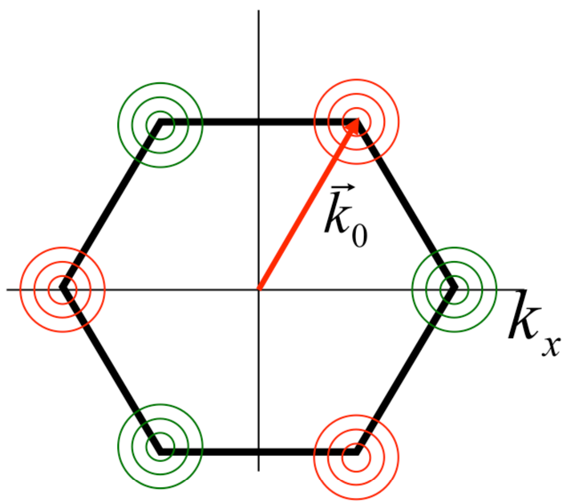
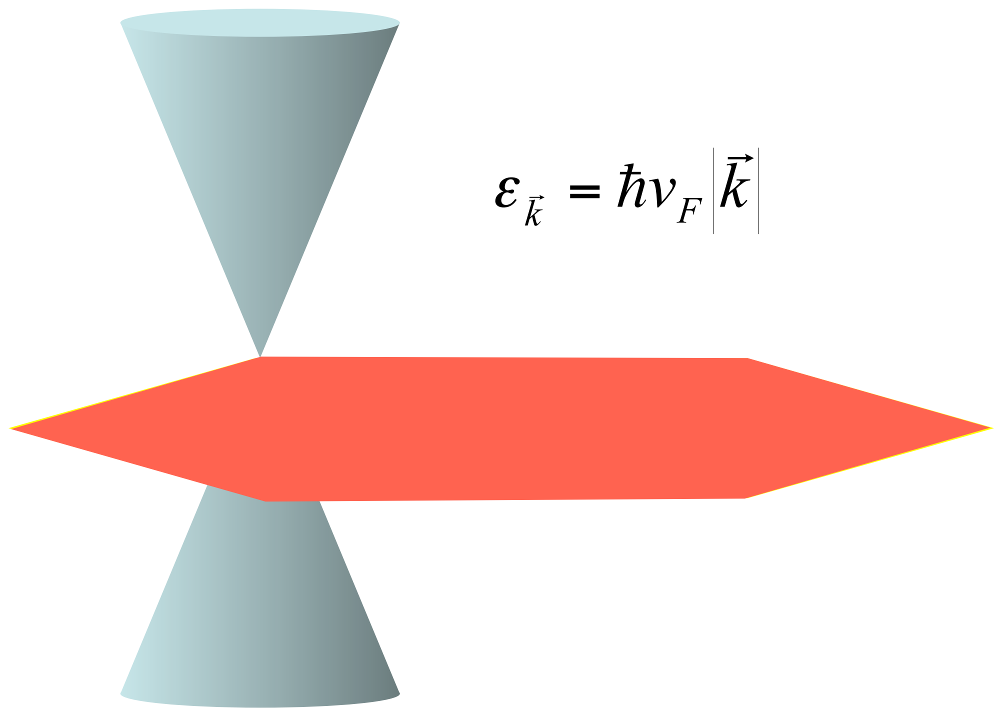
arXiv:0801.2970
arXiv:0802.4289

Graphene







$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

Graphene

Low energy theory has 4 two-component Dirac fermions, ψ_σ, $\sigma = 1 \dots 4$, interacting with a $1/r$ Coulomb interaction

$$\begin{aligned} \mathcal{S} &= \int d^2 r d\tau \psi_\sigma^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\ &+ \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r - r'|} \psi_{\sigma'}^\dagger \psi_{\sigma'}(r') \end{aligned}$$

Dimensionless "fine-structure" constant $\alpha = e^2/(\hbar v_F)$.
RG flow of α:

$$\frac{d\alpha}{d\ell} = -\alpha^2 + \dots$$

Behavior is similar to a conformal field theory (CFT) in 2+1 dimensions with $\alpha \sim 1/\ln(\text{scale})$

Conductivity is finite without impurities and with particle-hole symmetry

	Particles	**Holes**
Momentum	→	←
Electrical current	→	→

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k, \omega) = \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k,\omega)$

However, for *all* CFT3s, at $\hbar\omega \ll k_B T$, we have "phase" randomizing collisions and relaxation to local thermodynamic equilibrium. This leads to the hydrodynamic behavior

$$\chi(k,\omega) = 4e^2 \chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D \chi_c$$

where χ_c is the **compressibility** and D is the **diffusion constant**.

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Collisionless-hydrodynamic crossover in graphene

$$\sigma_Q(\omega) = \begin{cases} \dfrac{e^2}{h}\left[\dfrac{\pi}{2} + \mathcal{O}\left(\dfrac{1}{\ln(\Lambda/\omega)}\right)\right] & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left[0.760 + \mathcal{O}\left(\dfrac{1}{|\ln(\alpha(T))|}\right)\right] & , \quad \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

I. Herbut, V. Juricic, and O. Vafek, *Phys. Rev. Lett.* **100**, 046403 (2008).

where $\alpha(T)$ is the T-dependent fine structure constant which obeys

$$\alpha(T) = \frac{\alpha}{1 + (\alpha/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

L. Fritz, M. Mueller, J. Schmalian and S. Sachdev, arXiv:0802.4289

See also A. Kashuba, arXiv:0802.2216

Generalization

In the hydrodynamic regime, we include

- A bias voltage, leading to particle-hole asymmetry
- Dilute concentration of impurities
- A weak magnetic field

Transport properties can be computed from the equations of a relativistic fluid in an electromagnetic field

The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^{μ}, the current,
- ρ, the **difference** in density from undoped graphene.
- ε, the local energy
- P, the local pressure,
- u^{μ}, the local velocity, and
- σ_Q, a universal conductivity, which is the **single transport co-efficient**.

The dependence of ε, P, σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \end{aligned}$$

Conservation laws/equations of motion

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\ T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\ J^\mu &= \rho u^\mu \end{aligned}$$

Constitutive relations which follow from Lorentz transformation to moving frame

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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Momentum relaxation from impurities

$$\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu + \frac{1}{\tau_{\rm imp}} \left(\delta^\mu_\nu + u^\mu u_\nu\right) T^{\nu\gamma} u_\gamma \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q \left(g^{\mu\nu} + u^\mu u^\nu\right) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda\right) + \mu \frac{\partial_\mu T}{T}\right]
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\end{aligned}$$

Solve initial value problem and relate results to response functions (Kadanoff+Martin)

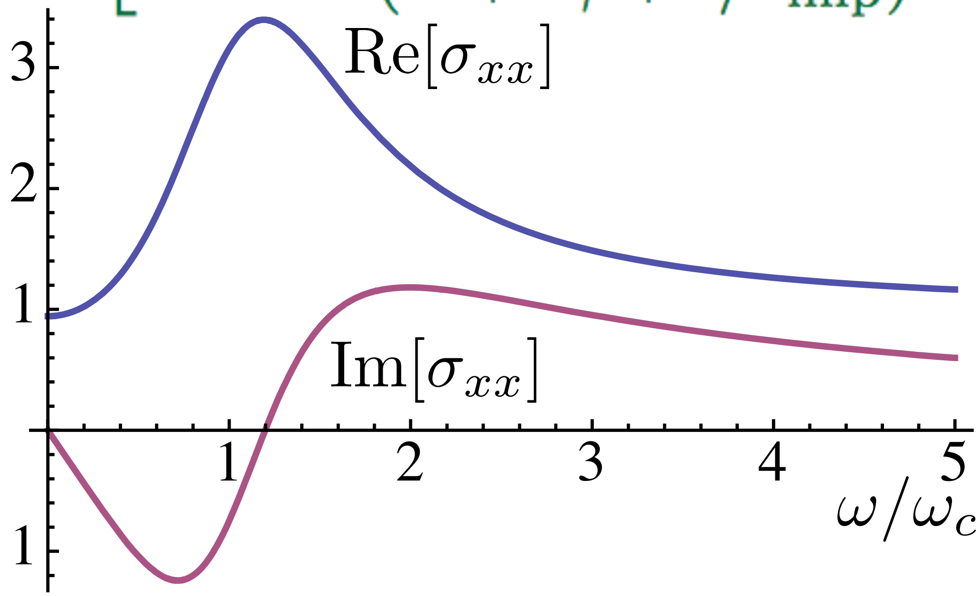
S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\rm imp})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\rm imp})}{(\omega + i\gamma + i/\tau_{\rm imp})^2 - \omega_c^2} \right] .$$



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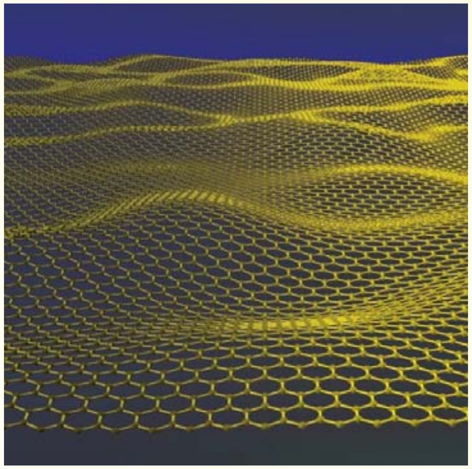
$$= \sigma_Q + \frac{4e^2\rho^2 v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} \quad \text{as } B \to 0$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

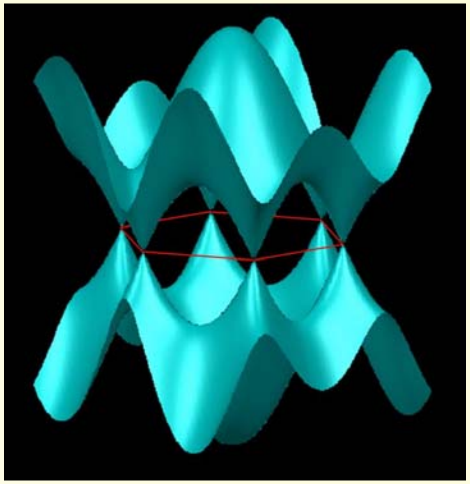
Cyclotron resonance in graphene

Markus Mueller and S. Sachdev, arXiv:0801.2970.



$$\omega = \pm\omega_c^{rel} - i\gamma - i/\tau$$

$$v = 1.1 \times 10^6 \, m/s$$
$$\approx c/300$$



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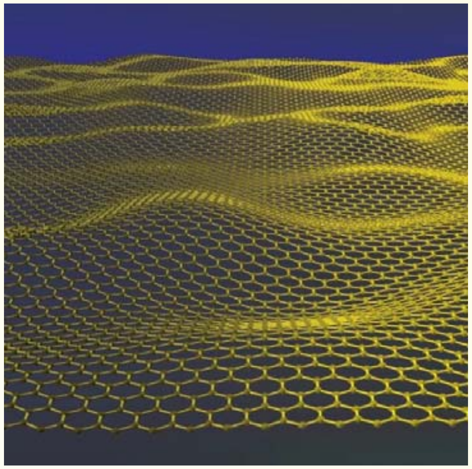

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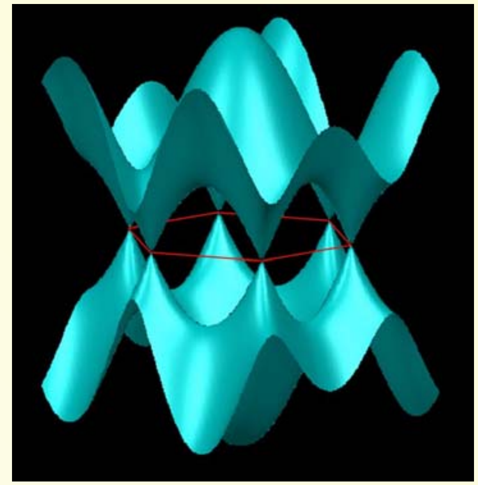

Conditions to observe resonance

Negligible Landau quantization $\quad E_{LL} = \hbar v\sqrt{\frac{2eB}{\hbar c}} << k_B T$

Hydrodynamic,
collison-dominated regime $\quad \hbar\omega_c^{rel} << k_B T$

Negligible broadening $\quad \gamma, \tau^{-1} < \omega_c^{rel}$

Relativistic, quantum critical regime $\quad \rho \le \rho_{th} = \frac{(k_B T)^2}{(\hbar v)^2}$

$T \approx 300K$

$B \approx 0.1T$

$\rho \approx 10^{11} cm^{-2}$

$\omega_c \approx 10^{13} s^{-1}$

Conclusions

- Universal quantum critical conductivity of pure graphene

$$\sigma_Q(\omega) = \begin{cases} \dfrac{e^2}{h}\left[\dfrac{\pi}{2} + \mathcal{O}\left(\dfrac{1}{\ln(\Lambda/\omega)}\right)\right] & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left[0.760 + \mathcal{O}\left(\dfrac{1}{|\ln(\alpha(T))|}\right)\right] & , \quad \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

$$\alpha(T) = \frac{\alpha}{1 + (\alpha/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

- Hydrodynamic theory for thermo-magneto-electric response functions

- Room temperature hydrodynamic cyclotron resonance.