Quantum phase transitions of insulators, superconductors and metals in two dimensions

Talk online: sachdev.physics.harvard.edu

PHYSICS

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Outline

1. Phenomenology of the cuprate superconductors
   (and other compounds)

2. QPT of antiferromagnetic insulators
   (and bosons at rational filling)

3. QPT of d-wave superconductors:
   Fermi points of massless Dirac fermions

4. QPT of Fermi surfaces:
   A. Finite wavevector ordering (SDW/CDW):
      "Hot spots" on Fermi surfaces
   B. Zero wavevector ordering (Nematic):
      "Hot Fermi surfaces"

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Max Metlitski

Strategy

1. Write down local field theory for order parameter and fermions

2. Apply renormalization group to field theory

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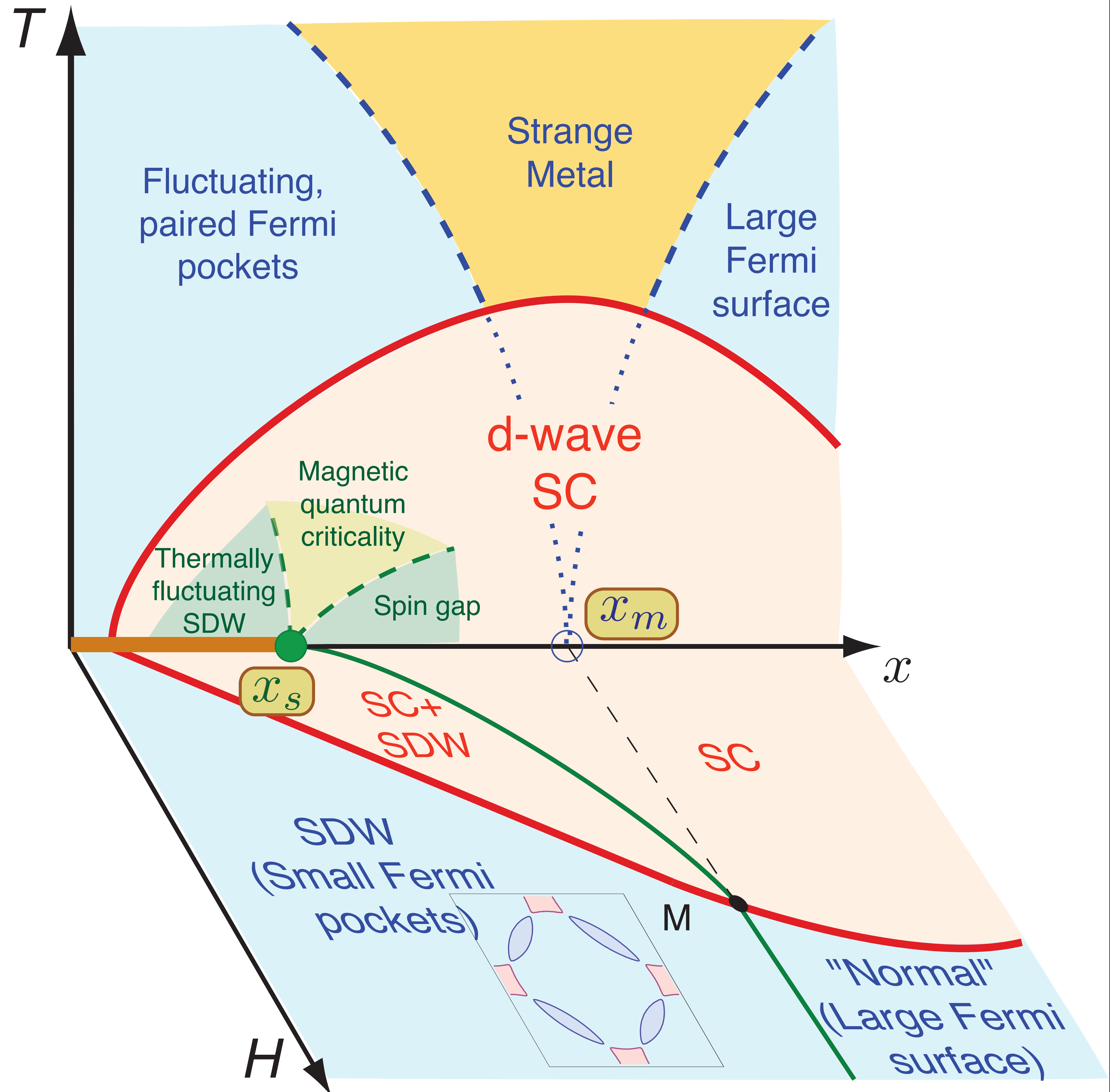
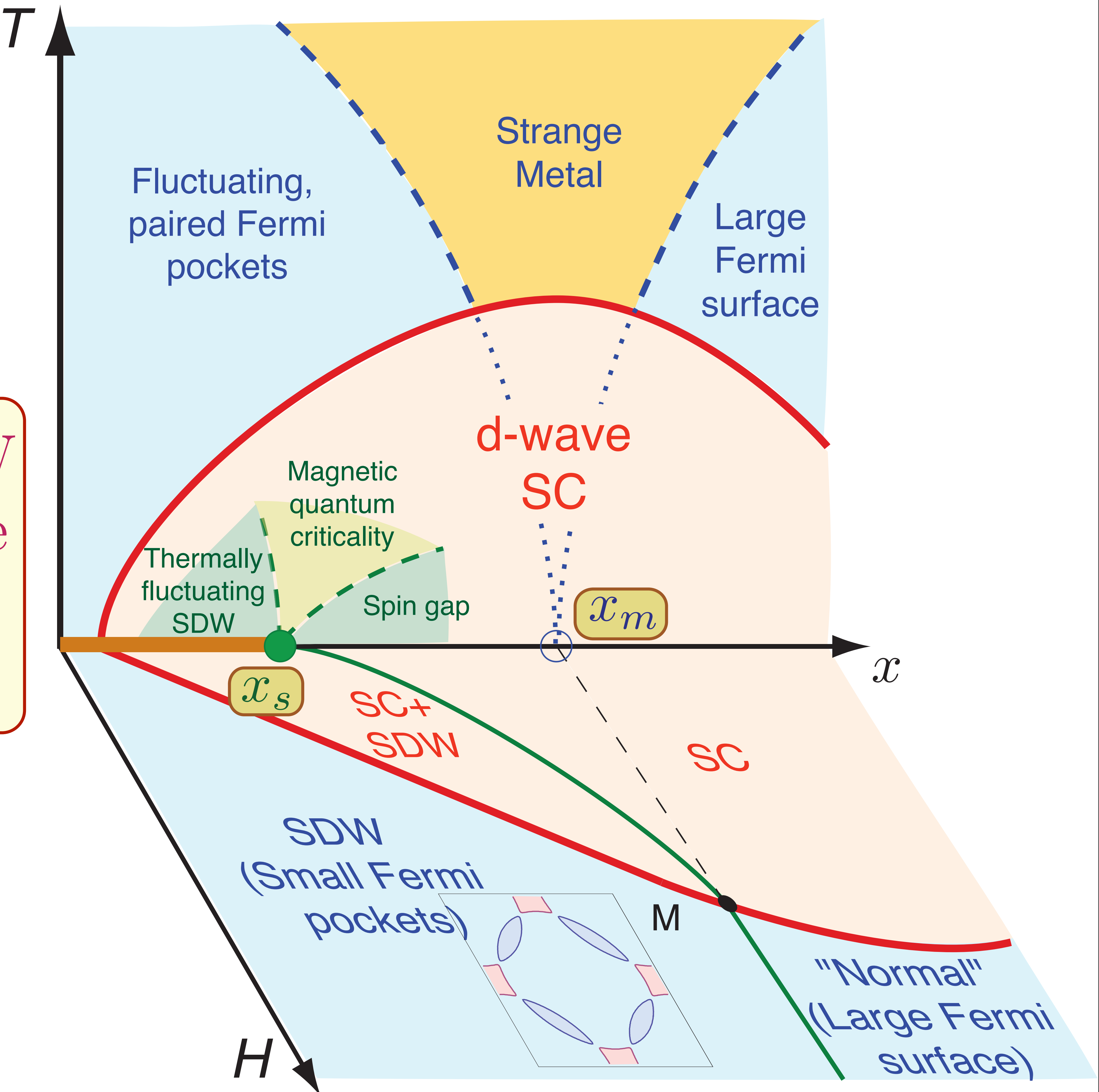
Order parameter at a non-zero wavevector: "Hot spots" on the Fermi surface.



Friday, April 13, 2012

Theory of SDW quantum phase transition in metal



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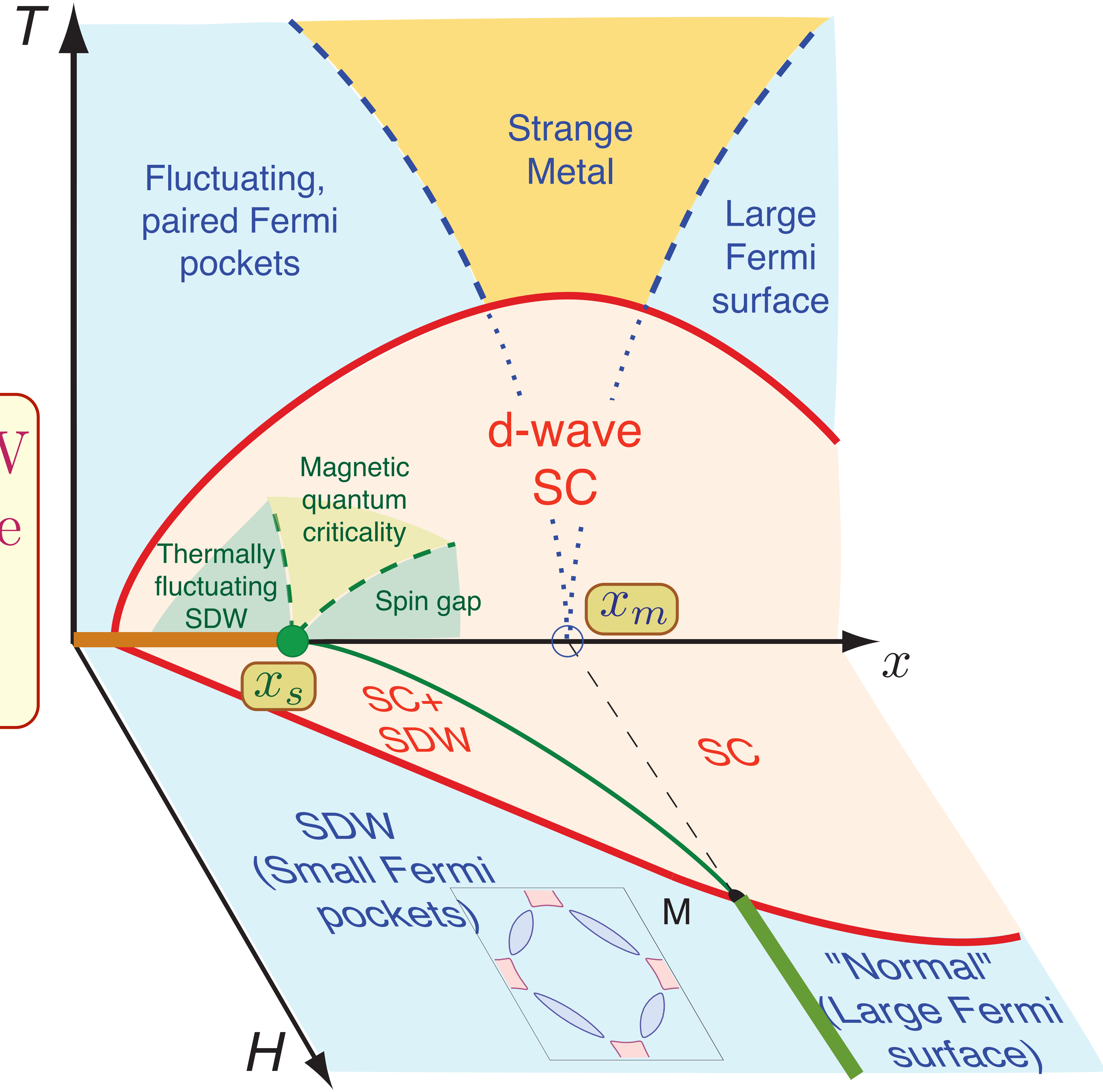
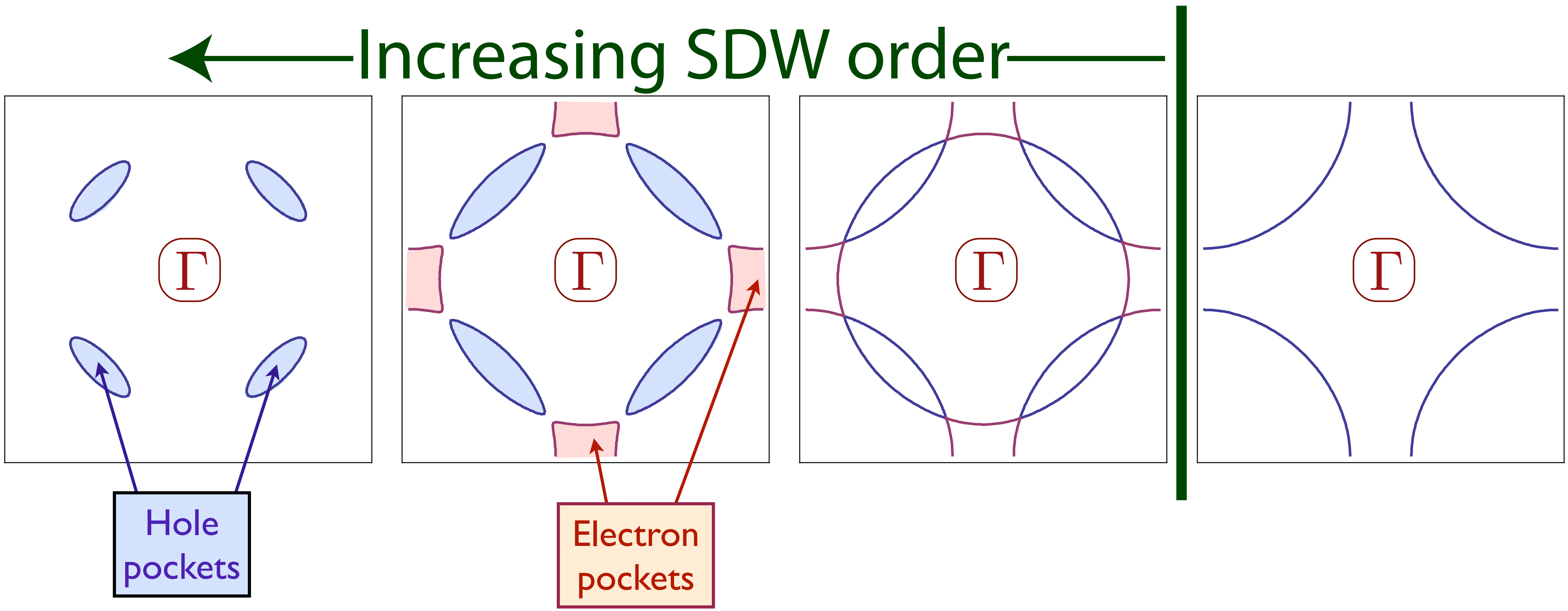


Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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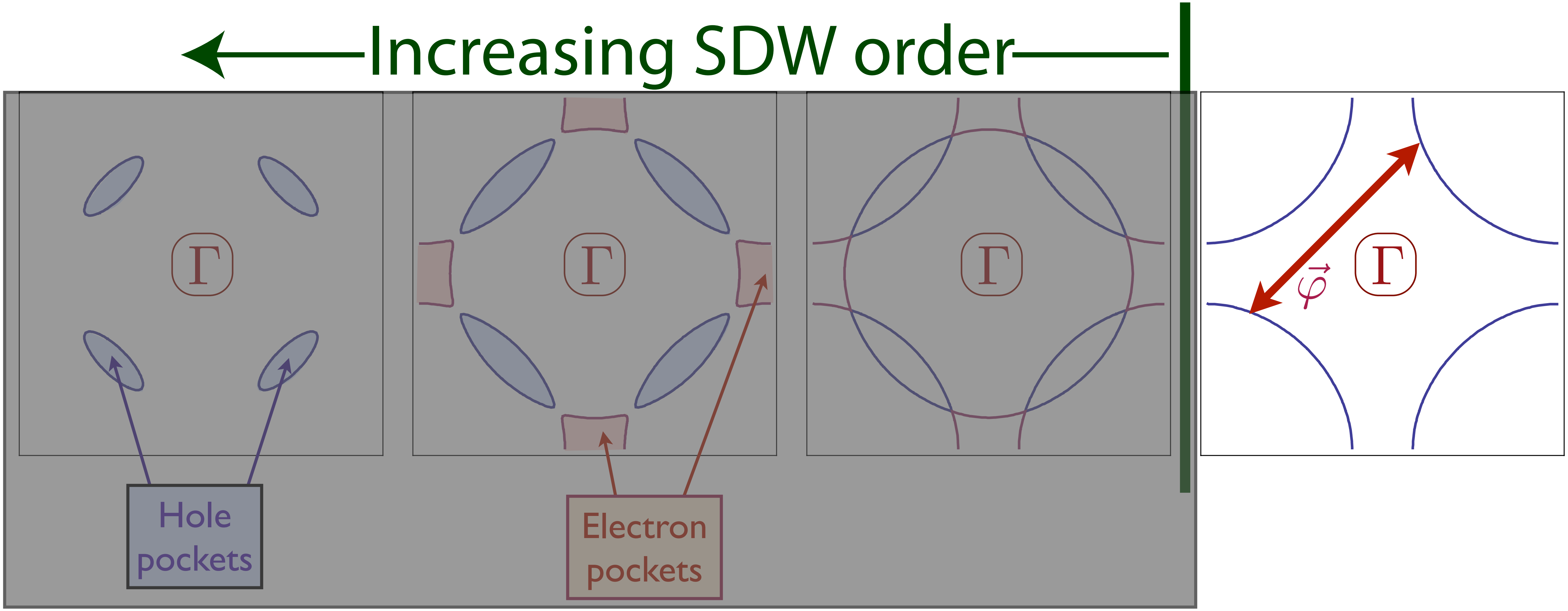


$\vec{\varphi}$ fluctuations act on the large Fermi surface

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

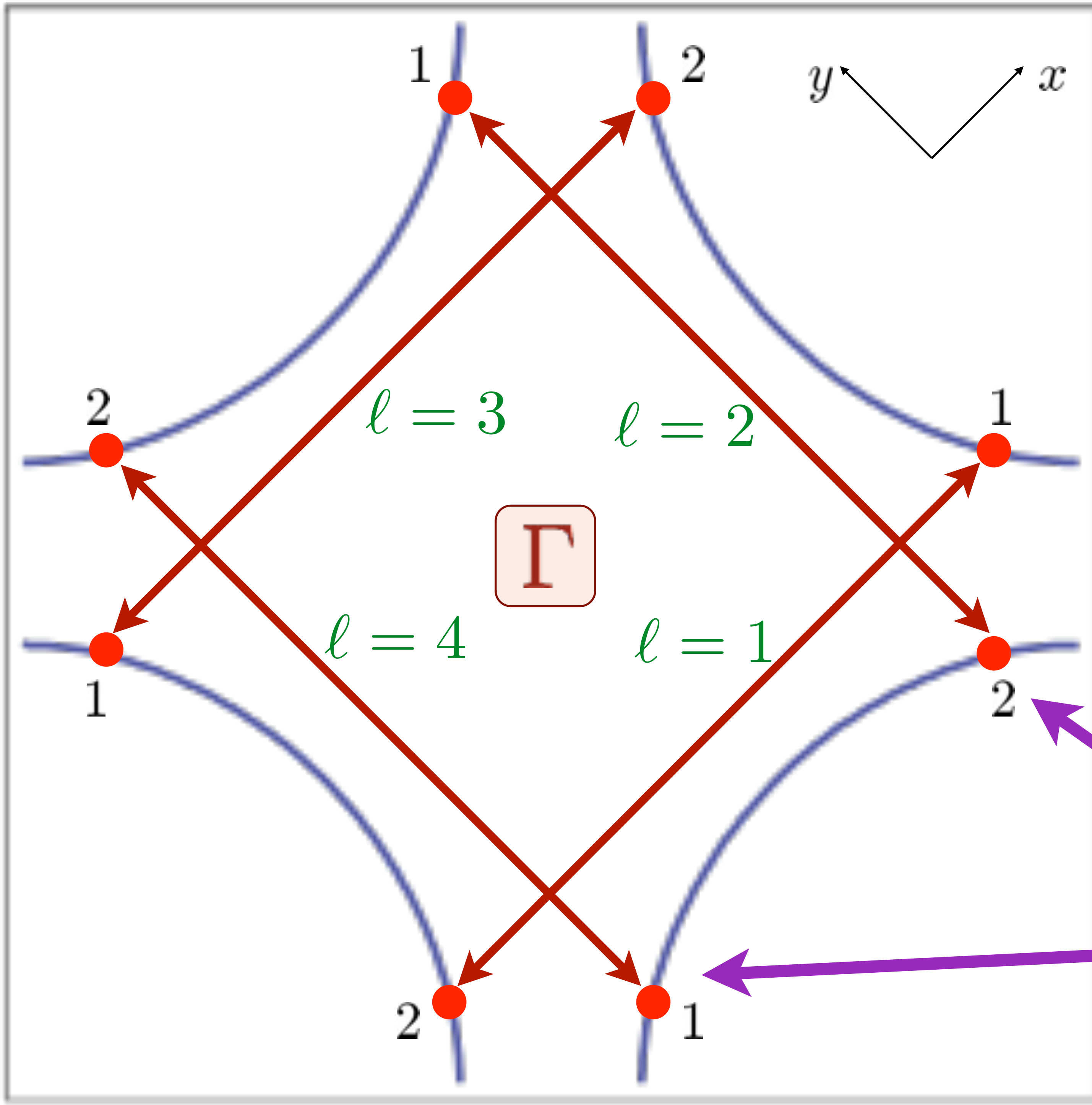
Start from the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{\widetilde{\zeta}}{2} \left(\partial_\tau \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]$$

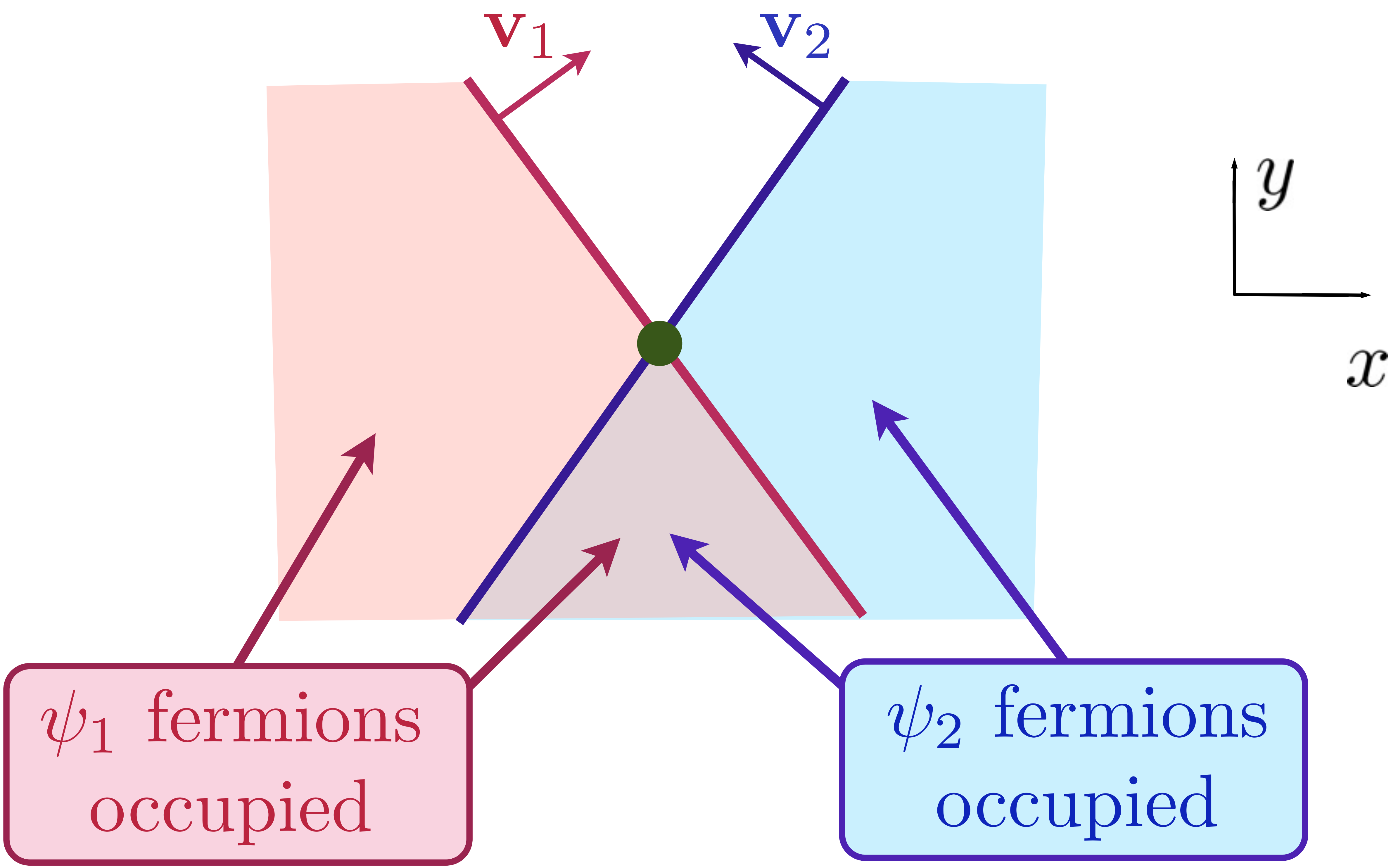


Low energy fermions
$\psi^{\ell}_{1\alpha}$, $\psi^{\ell}_{2\alpha}$
$\ell = 1, \ldots, 4$

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha} \left(\zeta \partial_\tau - i \mathbf{v}^{\ell}_1 \cdot \boldsymbol{\nabla}_r \right) \psi^{\ell}_{1\alpha} + \psi^{\ell\dagger}_{2\alpha} \left(\zeta \partial_\tau - i \mathbf{v}^{\ell}_2 \cdot \boldsymbol{\nabla}_r \right) \psi^{\ell}_{2\alpha}$$

$$\mathbf{v}^{\ell=1}_1 = (v_x, v_y), \; \mathbf{v}^{\ell=1}_2 = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$



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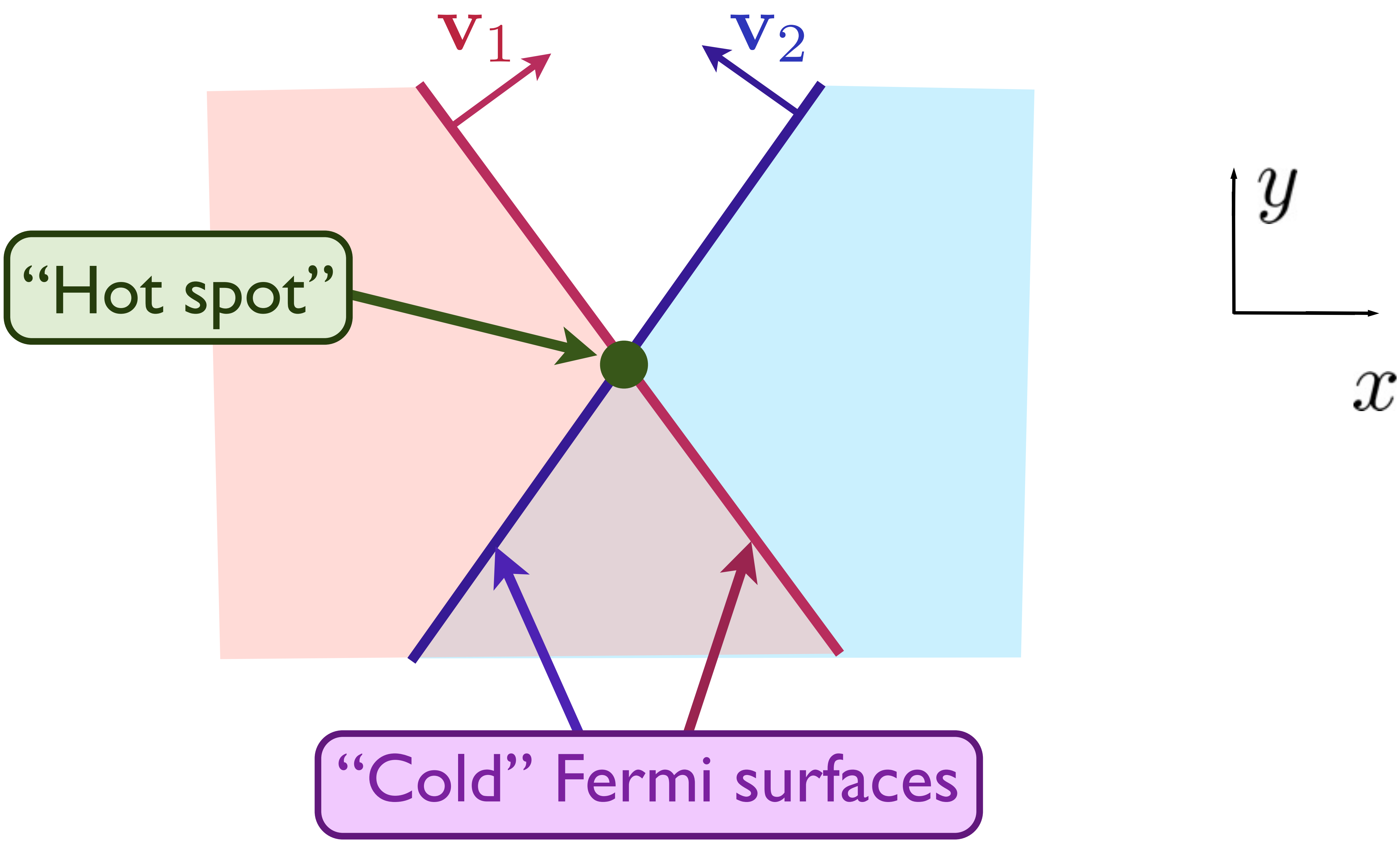

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Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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"Yukawa" coupling: $$\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi^{\ell\dagger}_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{2\beta} + \psi^{\ell\dagger}_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{1\beta}\right)$$

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Hertz-Moriya-Millis (HMM) theory

Integrate out fermions and obtain non-local corrections to $\mathcal{L}_\varphi$

$$\mathcal{L}_\varphi = \frac{1}{2}\vec{\varphi}^2\left[\mathbf{q}^2 + \gamma|\omega|\right]/2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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Exponent $z = 2$ and mean-field criticality (upto logarithms)

But, higher order terms contain an infinite number of marginal couplings ......

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} \left(\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r \right) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} \left(\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r \right) \psi_{2\alpha}^\ell$$

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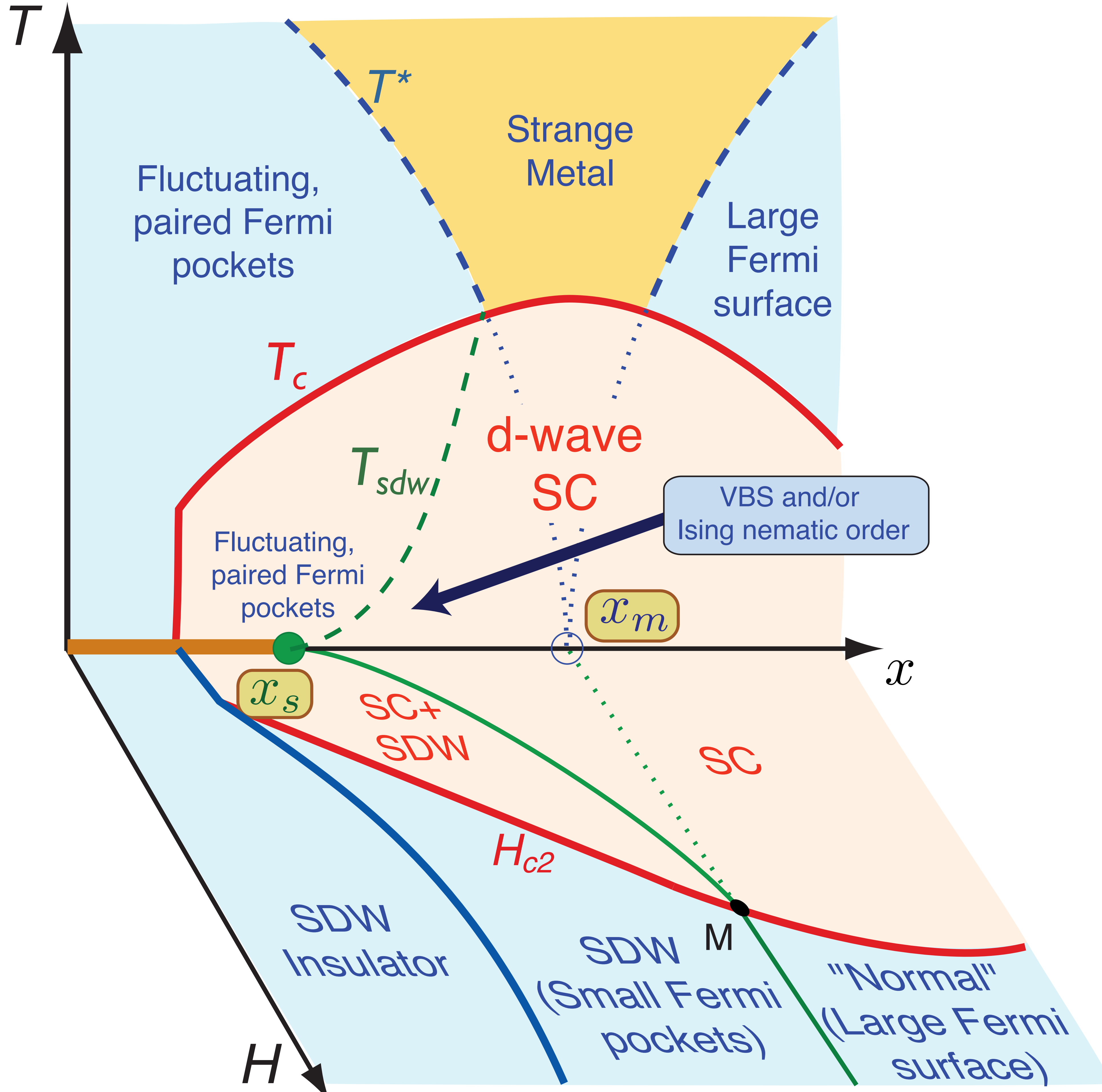
Perform RG on both fermions and $\vec{\varphi}$,
using a *local* field theory.

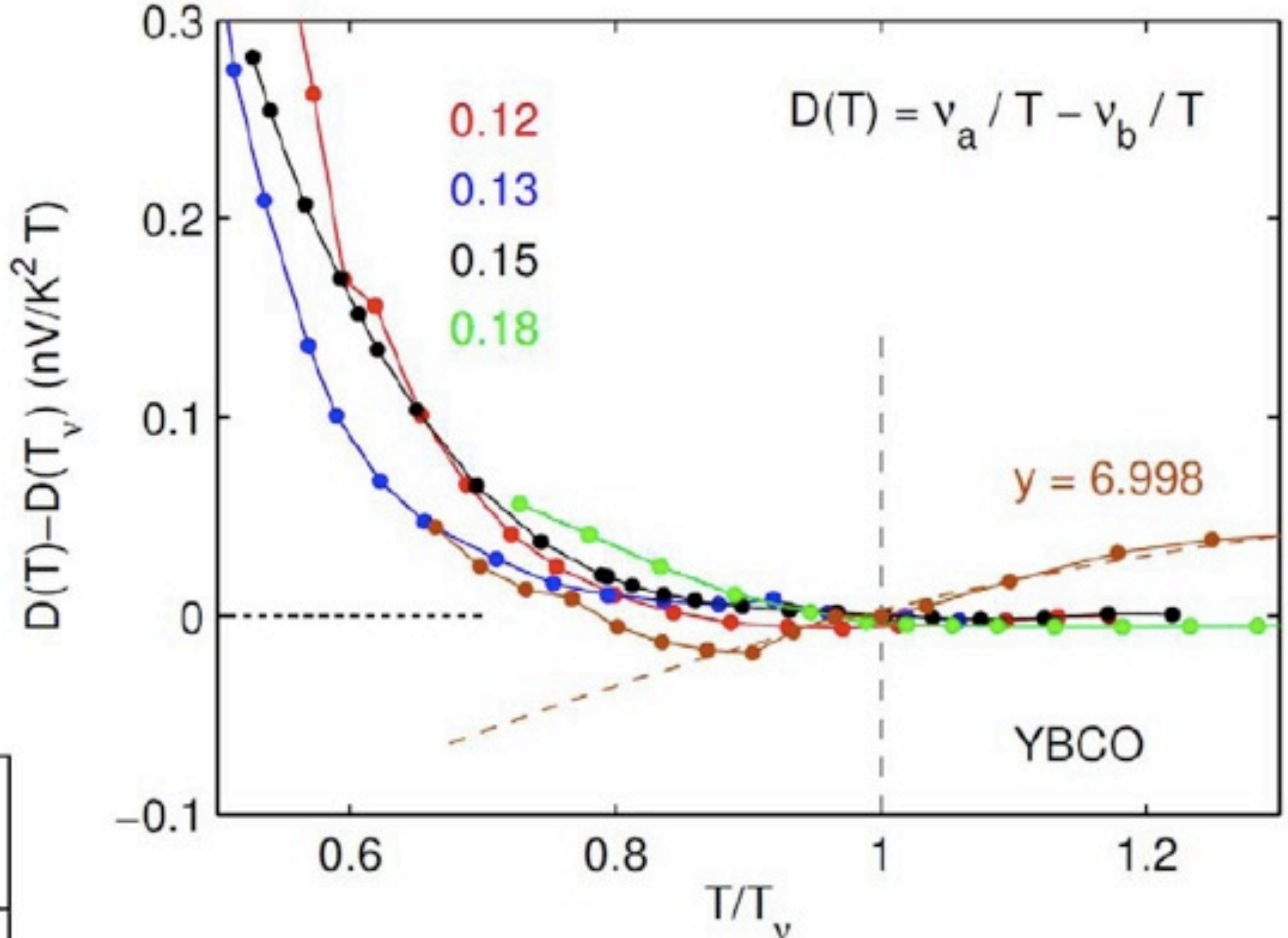
Order parameter at zero wavevector:
"Hot Fermi surface".

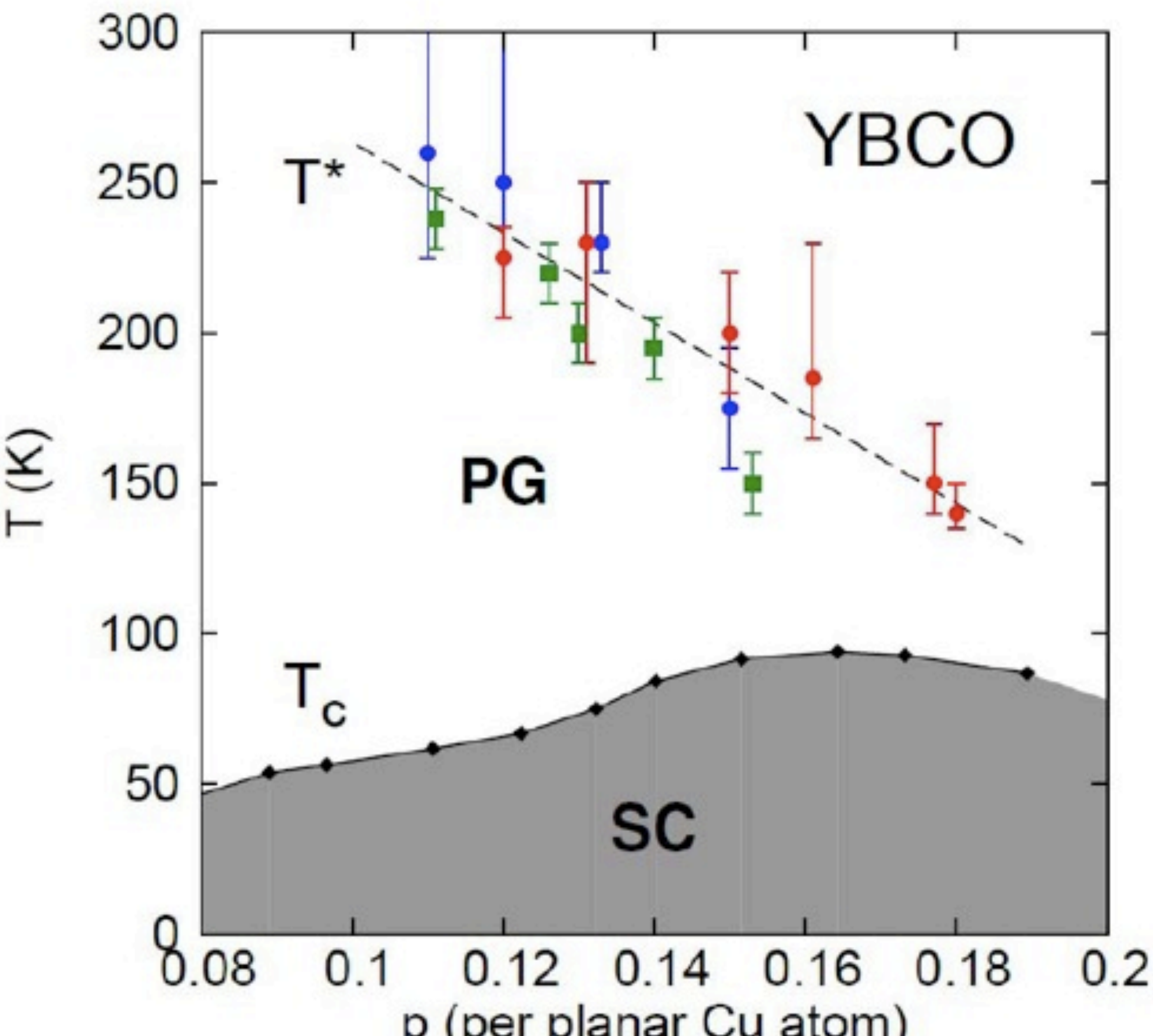


T
T*
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
T_c
T_{sdw}
d-wave SC
VBS and/or Ising nematic order
Fluctuating, paired Fermi pockets
x_m
x
x_s
SC+ SDW
SC
H_{c2}
SDW Insulator
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

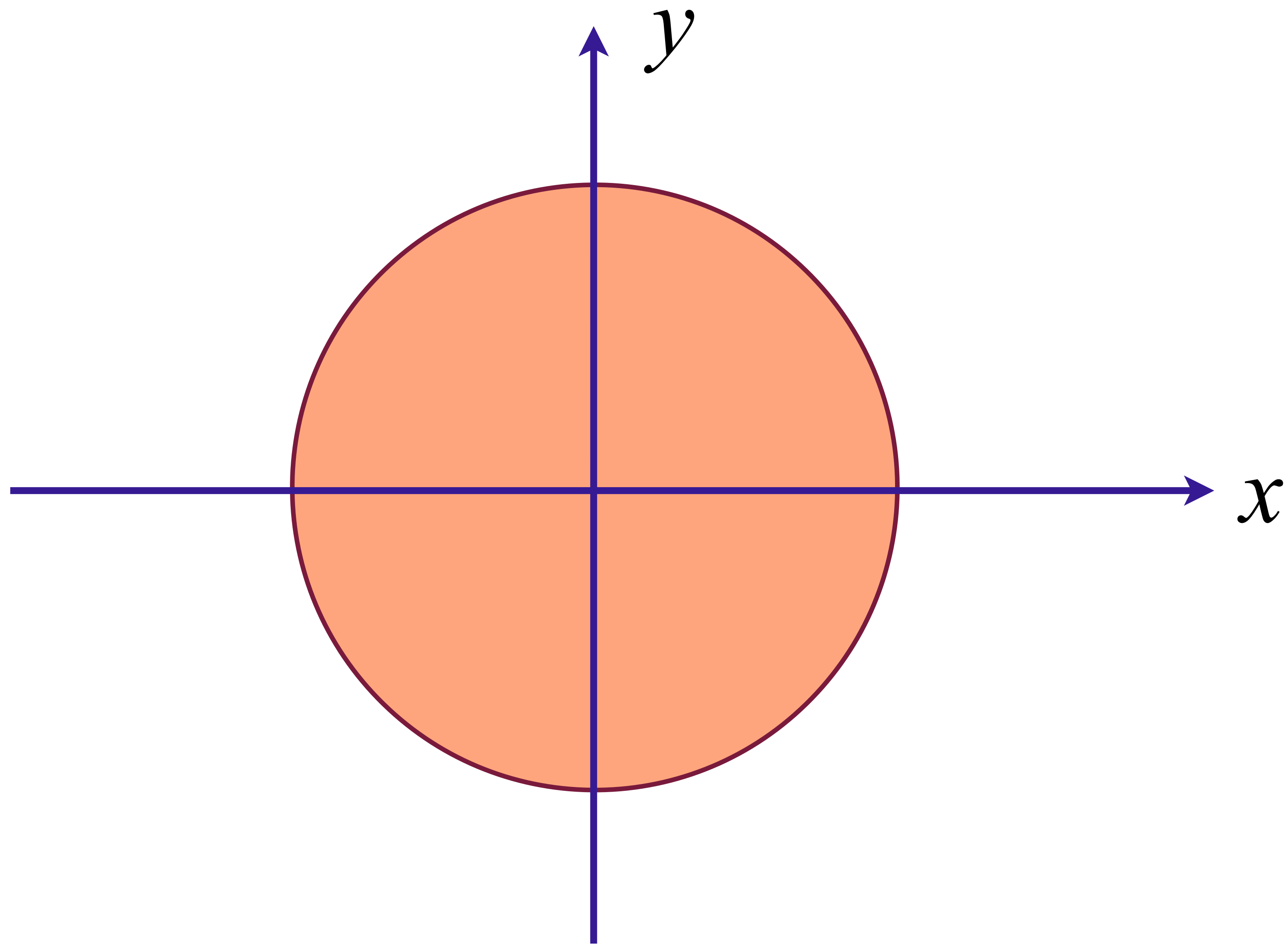
Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430, Nature, in press



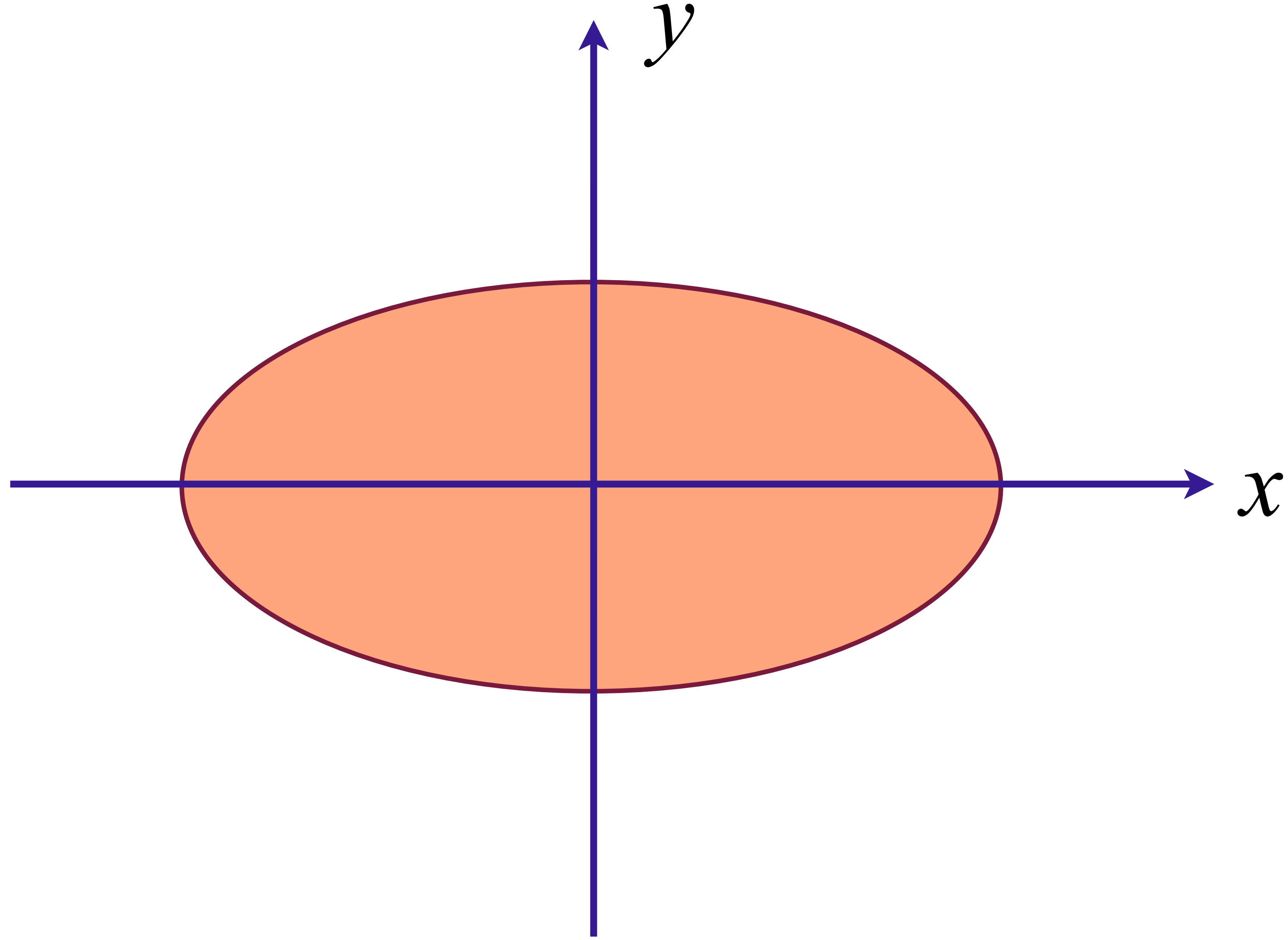


Quantum criticality of Pomeranchuk instability



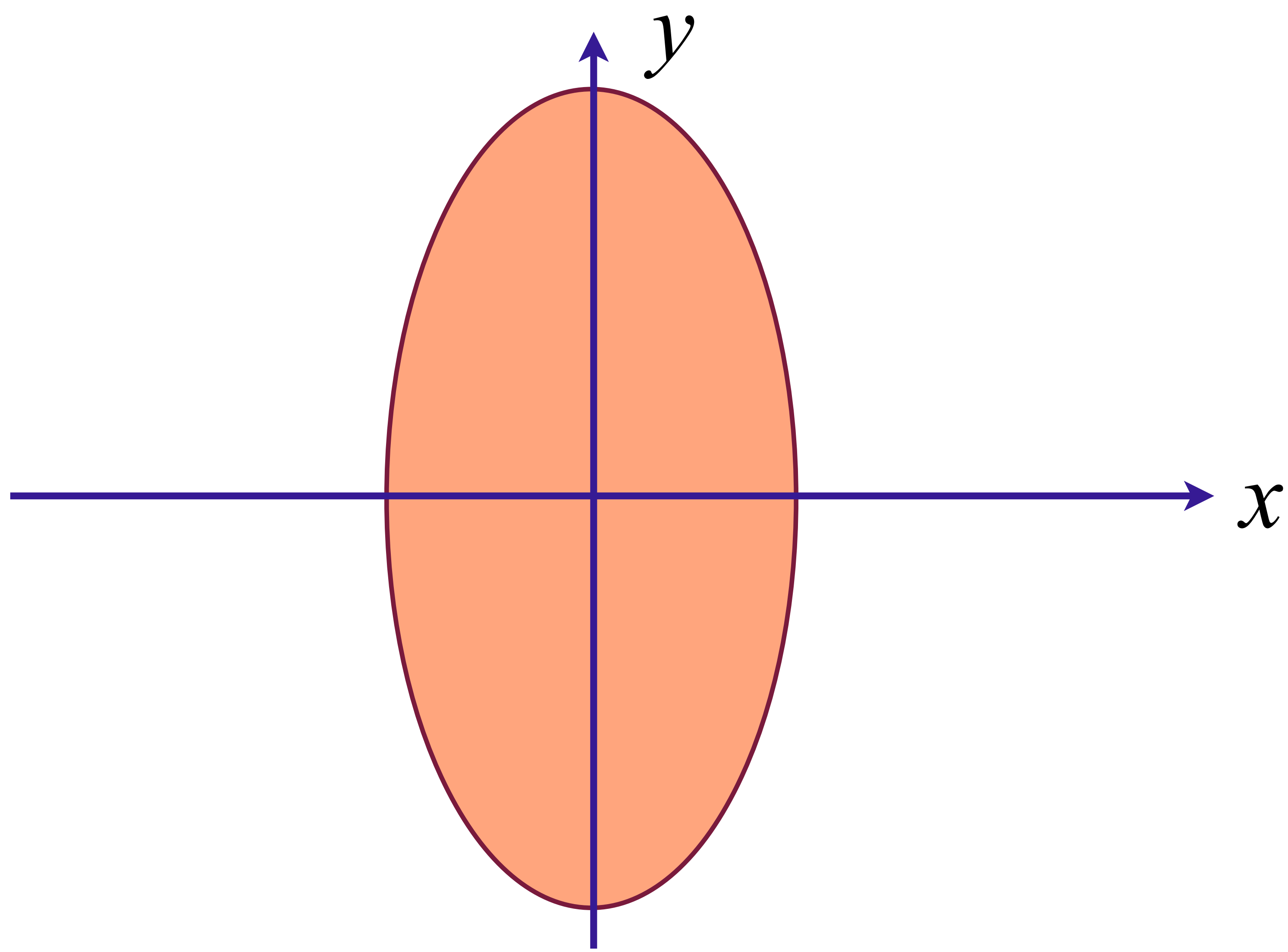
Fermi surface with full square lattice symmetry

Quantum criticality of Pomeranchuk instability



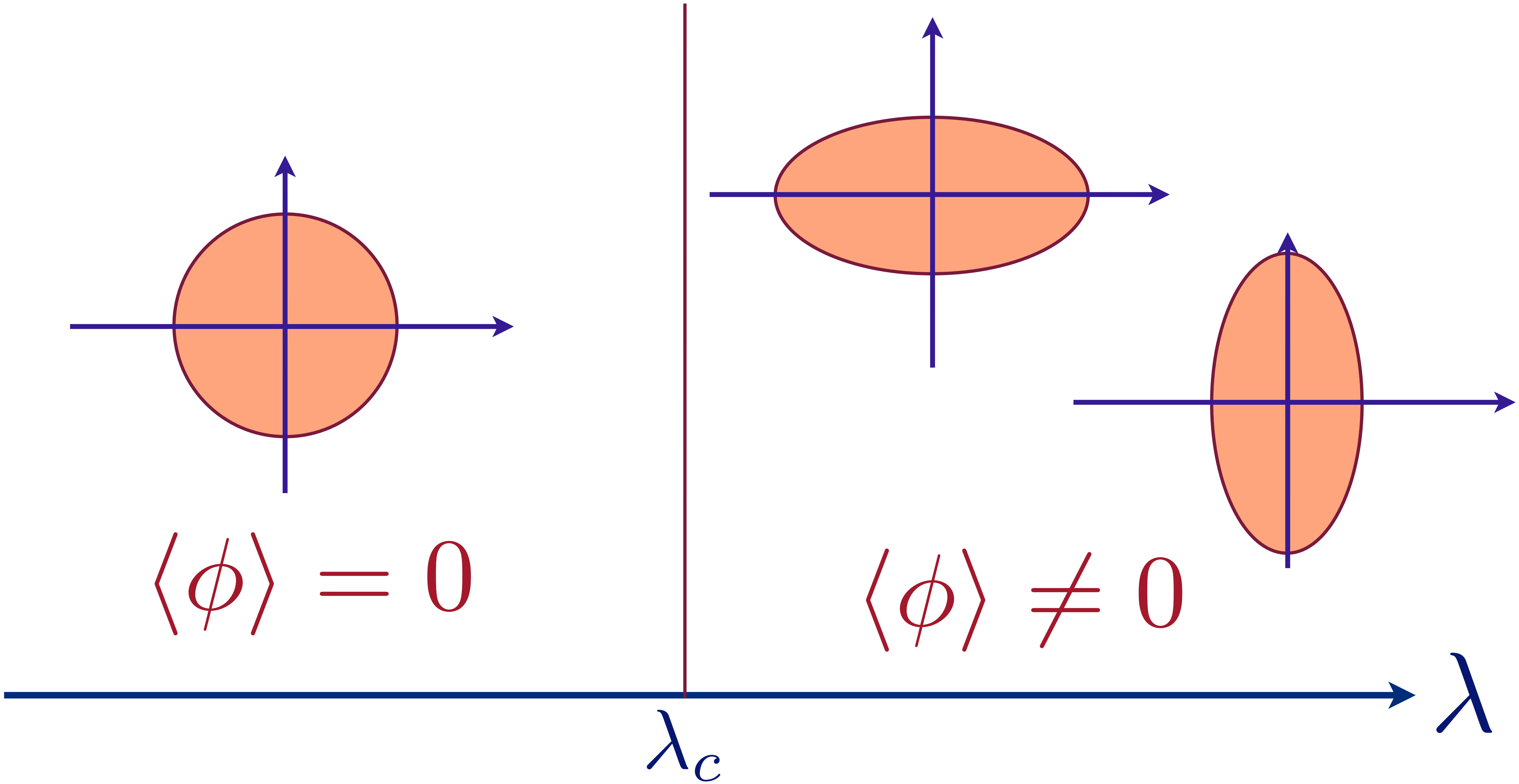
Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Pomeranchuk instability



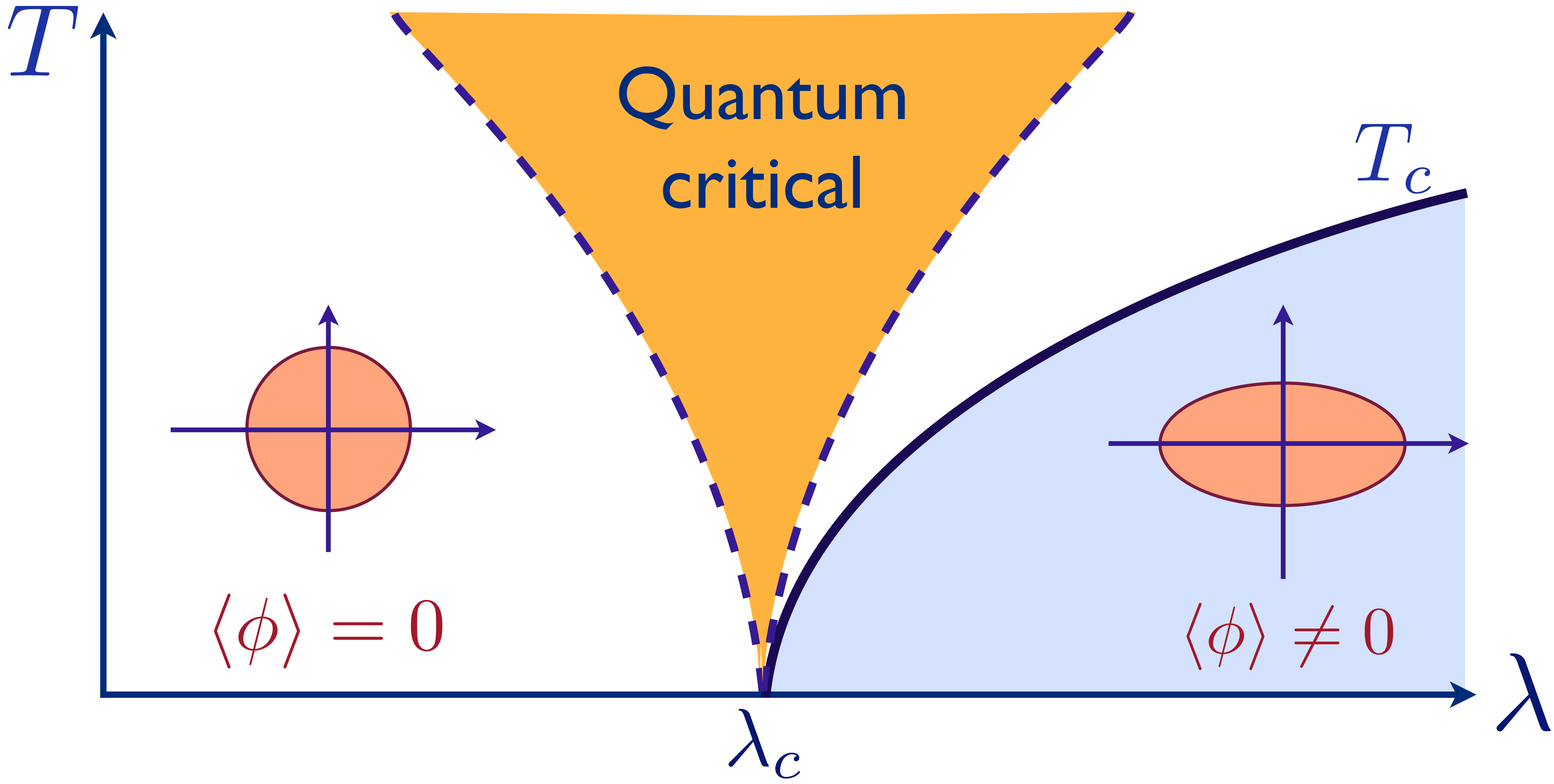
Spontaneous elongation along y direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Pomeranchuk instability



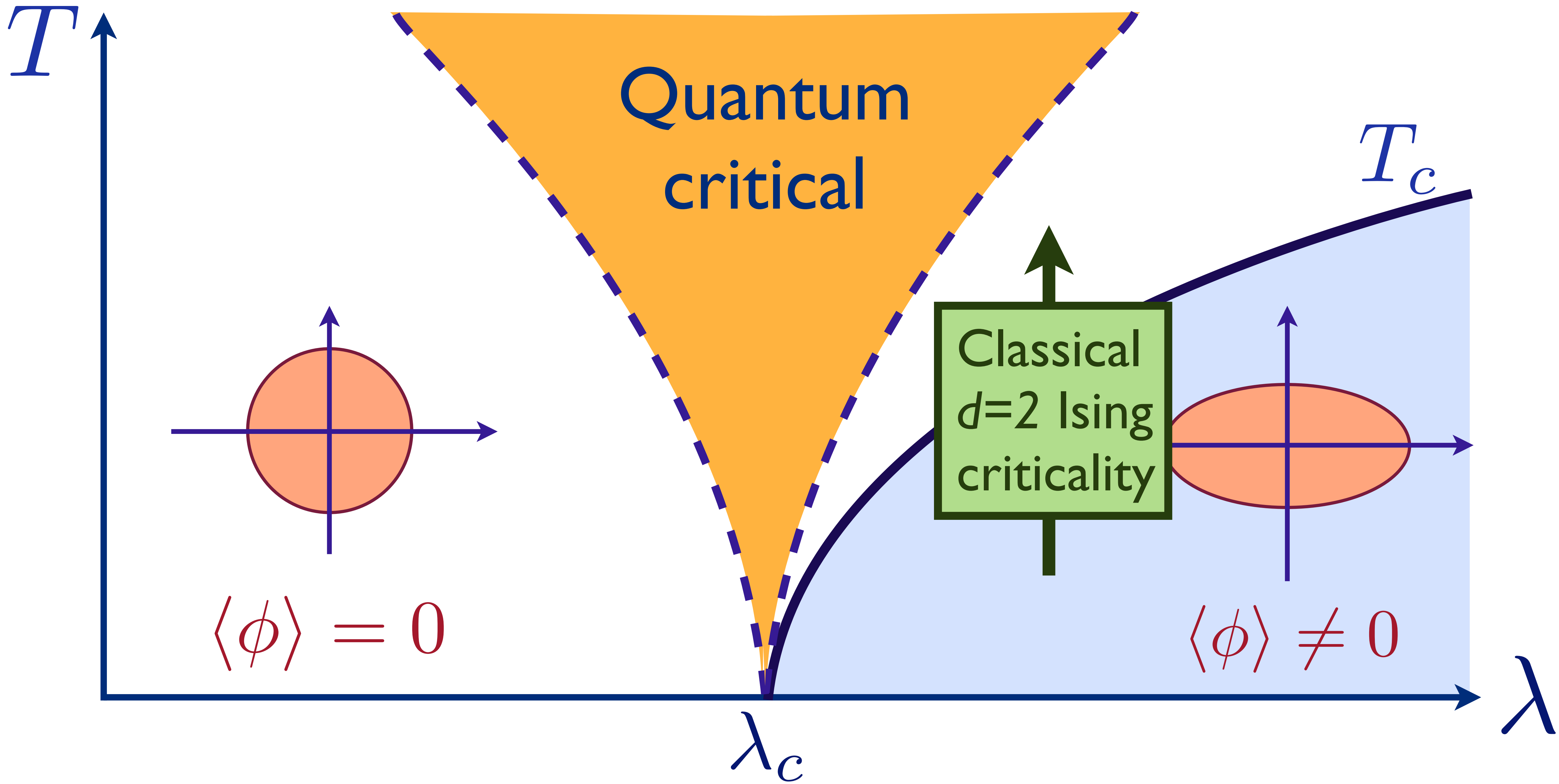
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

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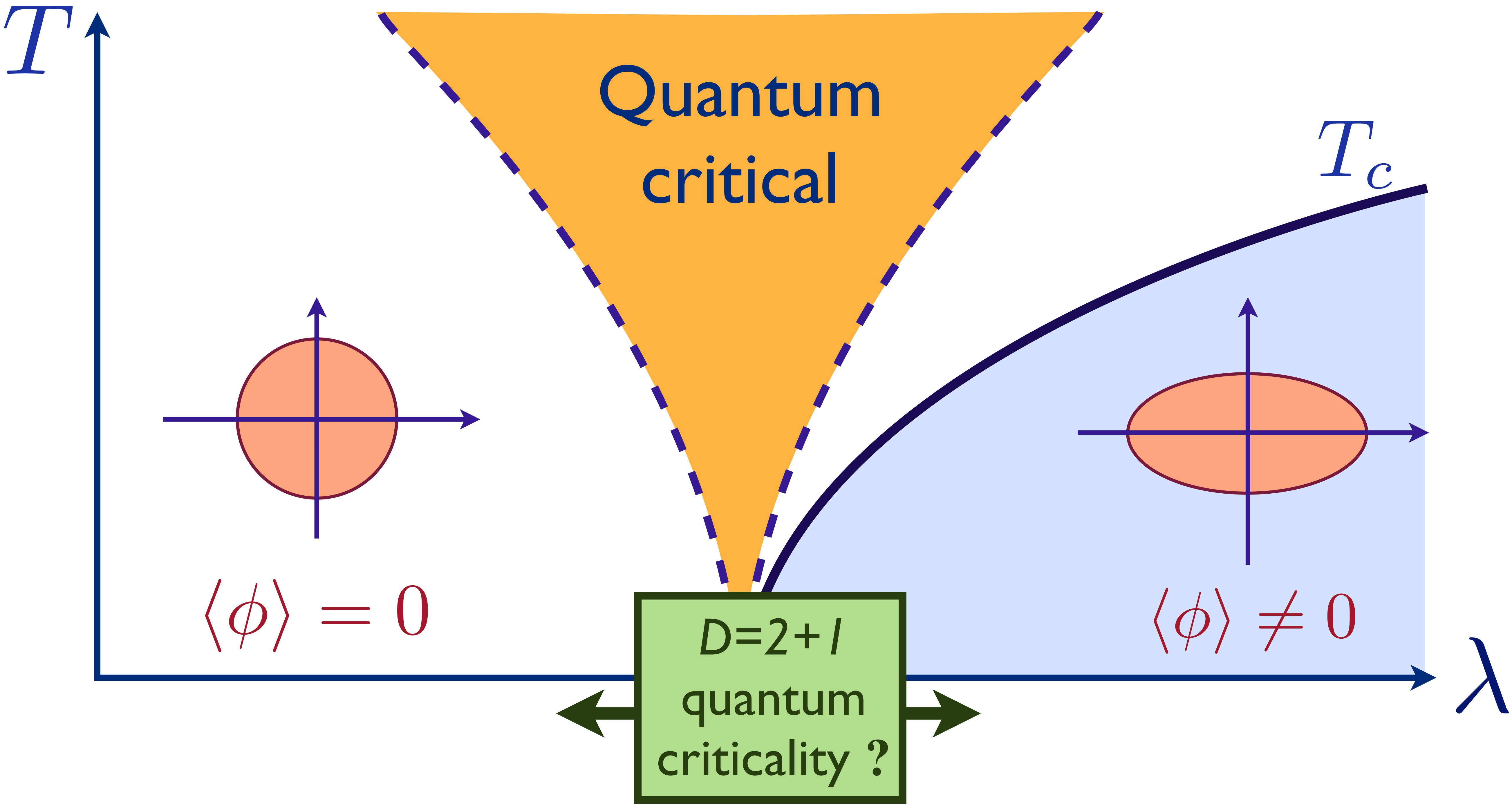

Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Pomeranchuk instability

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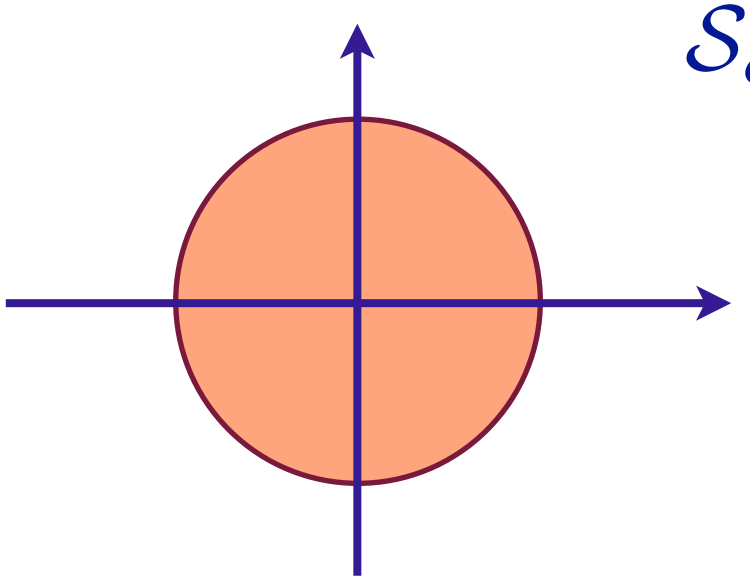
Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

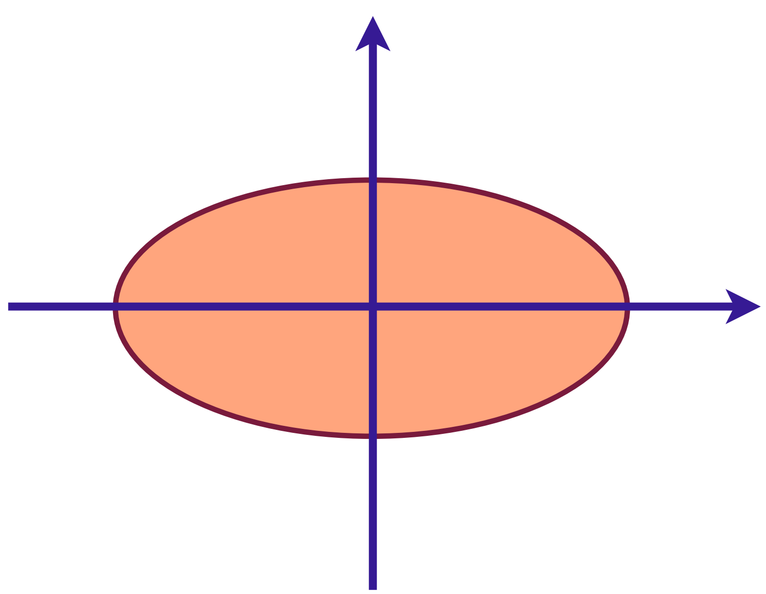
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Pomeranchuk instability

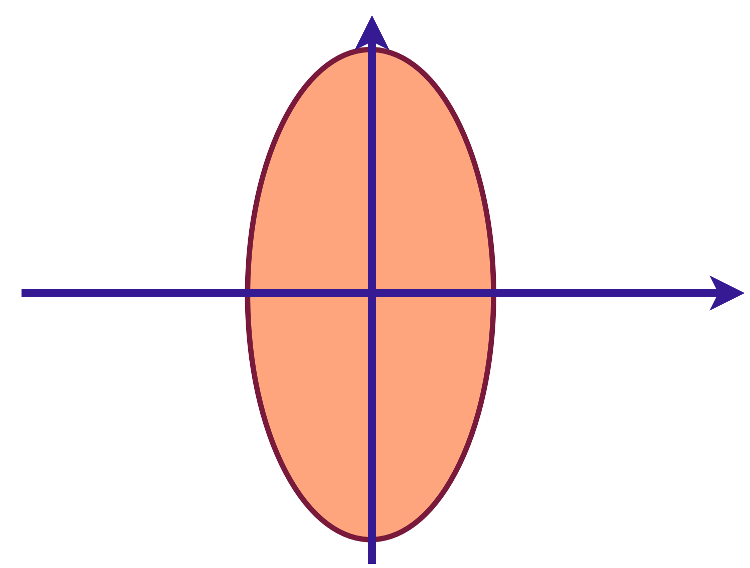
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \, \phi \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

for spatially independent ϕ



$\langle \phi \rangle > 0$



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Quantum criticality of Pomeranchuk instability

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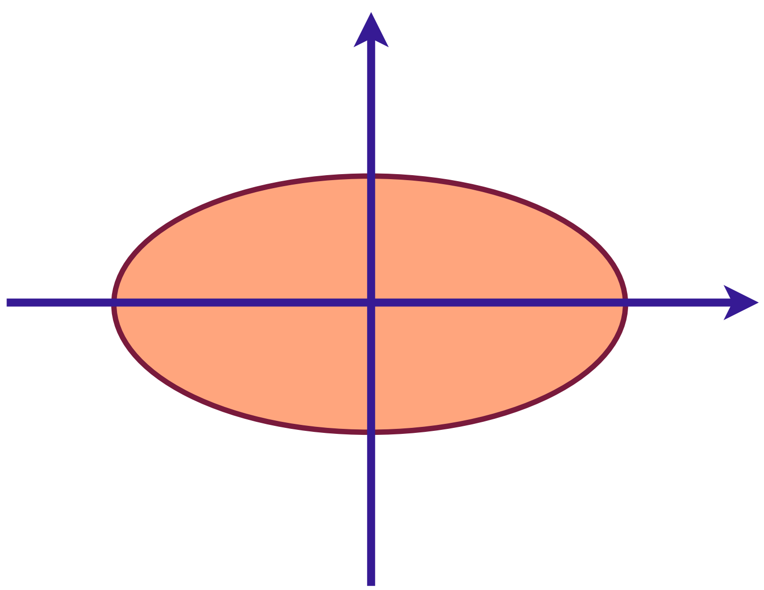
$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



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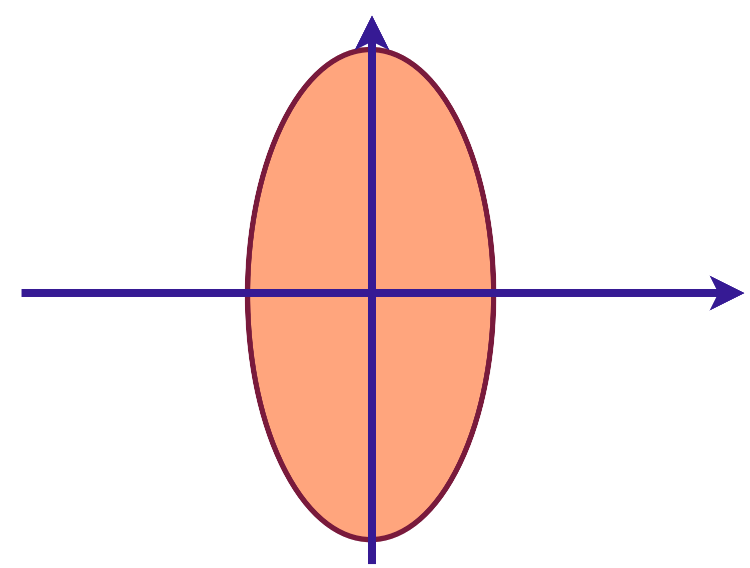

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Quantum criticality of Pomeranchuk instability

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

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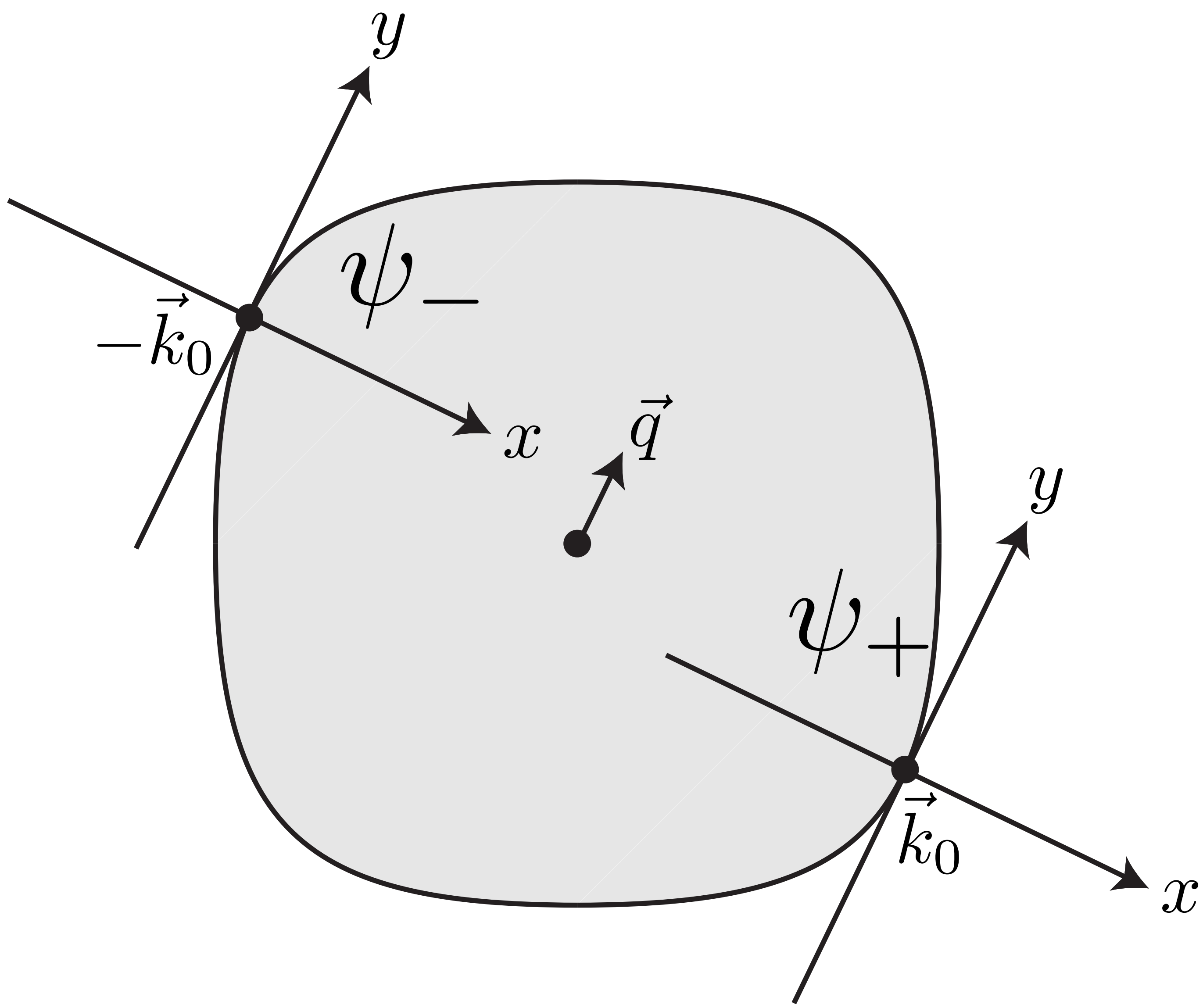
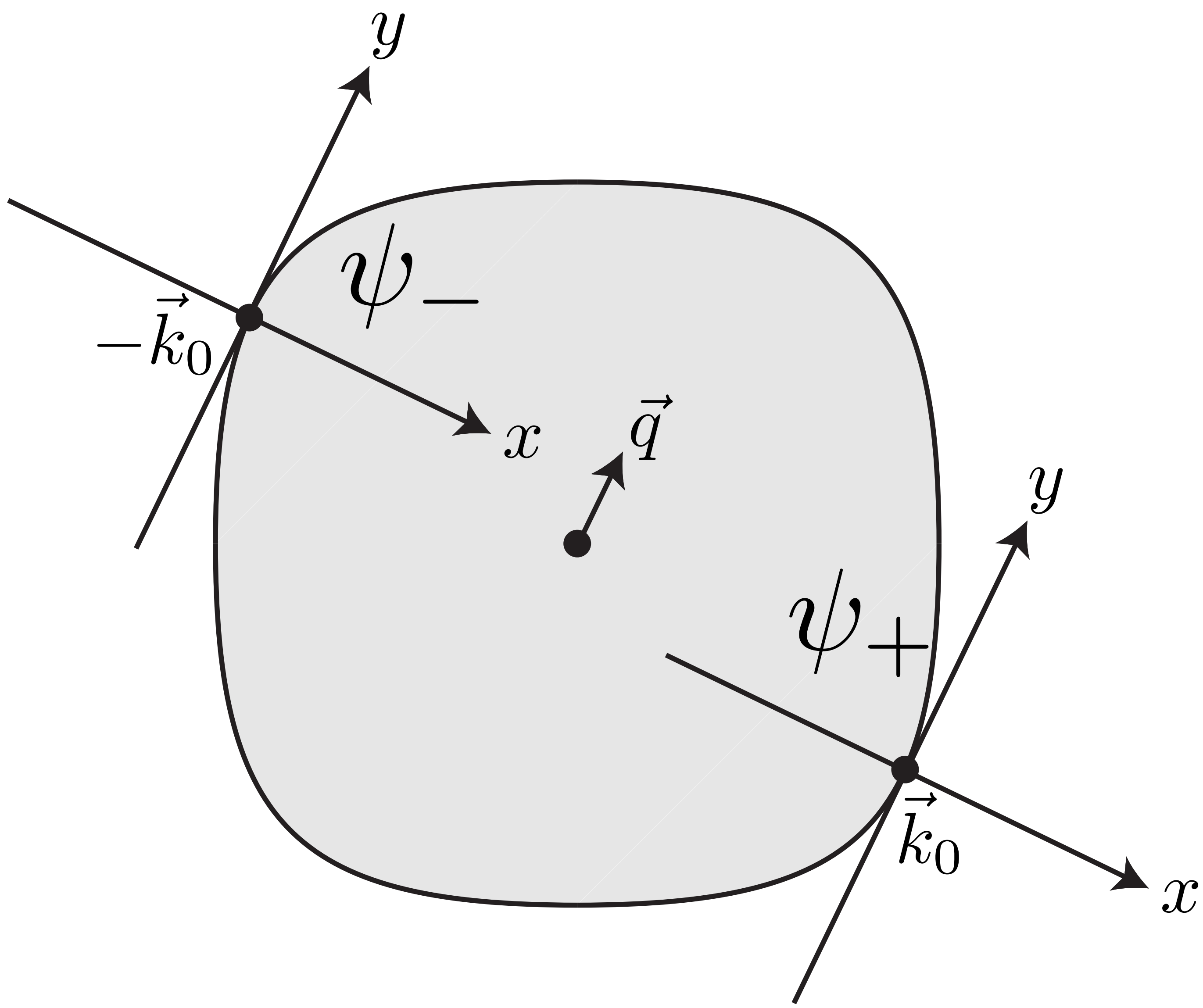
$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

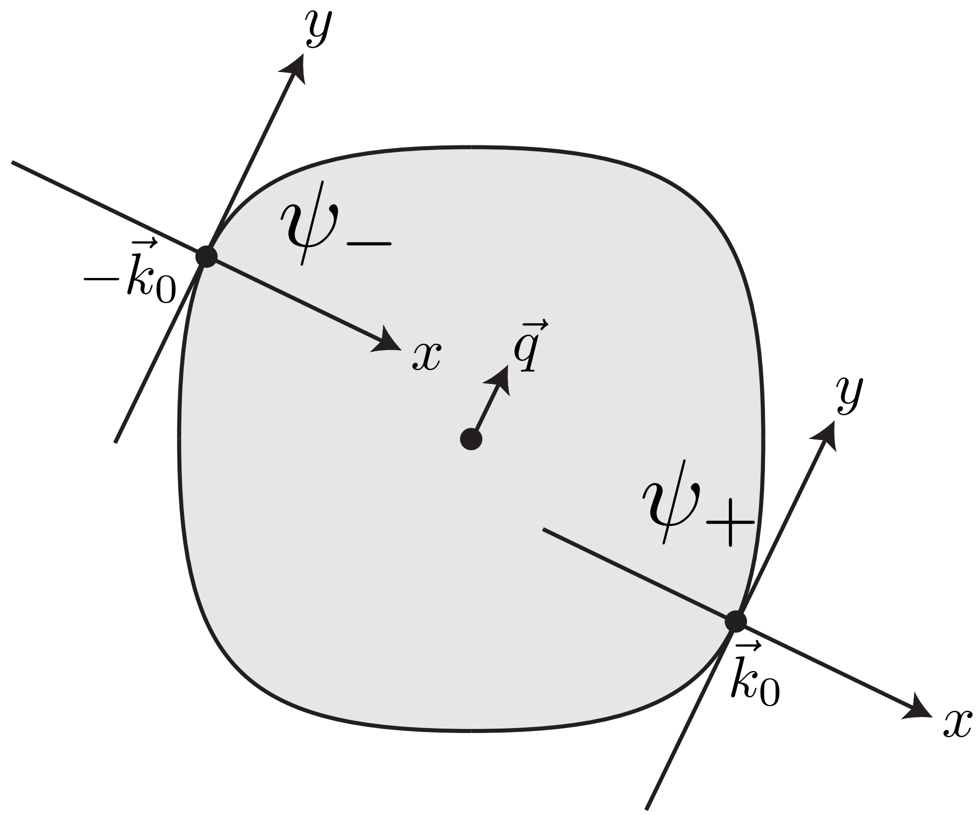


A ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Expand fermion kinetic energy at wavevectors about $\vec{k}_0$



$$
\begin{aligned}
\mathcal{L} \;=\;& \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\
& - \lambda\phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2
\end{aligned}
$$

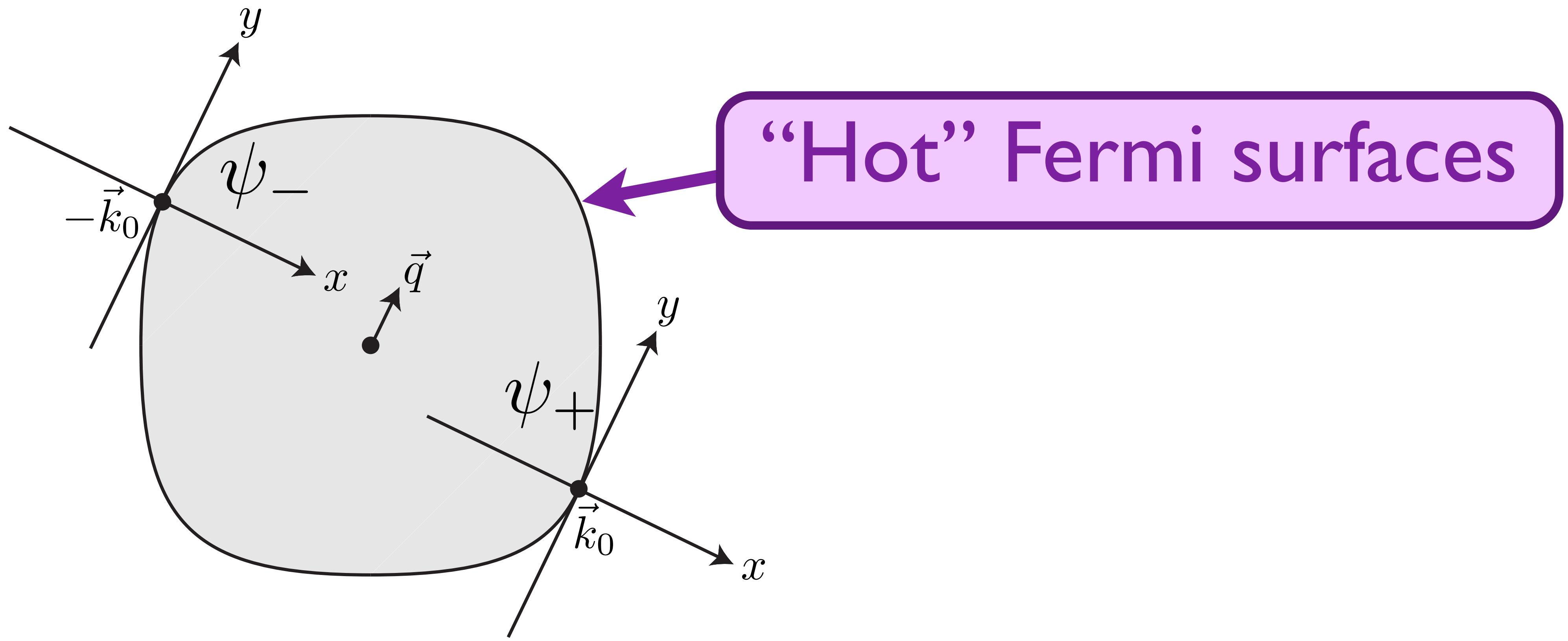


Emergent "Galilean invariance" at low energy ($s = \pm$):

$$\phi(x, y) \to \phi(x, y + \theta x), \quad \psi_s(x, y) \to e^{-is(\frac{\theta}{2} y + \frac{\theta^2}{4} x)} \psi_s(x, y + \theta x)$$

which implies for the fermion Green's function

$$G(q_x, q_y) = G(s q_x + q_y^2).$$



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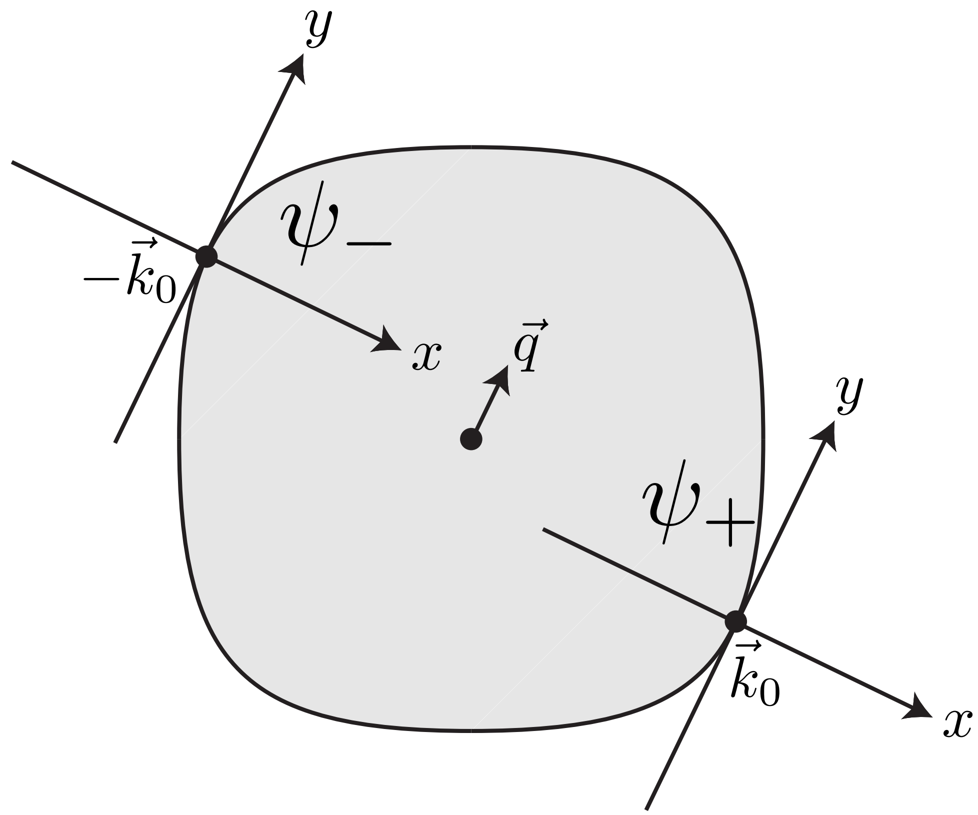
which implies for the fermion Green’s function

$$G(q_x, q_y) = G(sq_x + q_y^2).$$

Every point on the Fermi surface $sq_x + q_y^2 = 0$ has the same singularity: “Hot Fermi surface”.



Hertz-Moriya-Millis (HMM) theory

Integrate out fermions and obtain effective action for ϕ

$$\mathcal{L}_\phi = \frac{1}{2}\phi^2 \left[\frac{q_y^2}{g} + \frac{|\omega|}{4\pi|q_y|} \right]$$

Exponent $z = 3$ and mean-field criticality ?

Strategy

1. Write down local field theory for order parameter and fermions

2. Apply renormalization group to field theory

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Order parameter at a non-zero wavevector: "Hot spots" on the Fermi surface.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

"Yukawa" coupling: $\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$

Under the rescaling $x' = xe^{-\ell}$, $\tau' = \tau e^{-z\ell}$, the spatial gradients are fixed if the fields transform as

$$\vec{\varphi}' = e^{(d+z-2)\ell/2}\vec{\varphi} \quad ;" \ \psi' = e^{(d+z-1)\ell/2}\psi.$$

Then the Yukawa coupling transforms as

$$\lambda' = e^{(4-d-z)\ell/2}\lambda$$

For $d = 2$, with $z = 2$ the Yukawa coupling is invariant, and the bare time-derivative terms ζ, $\widetilde{\zeta}$ are irrelevant.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

"Yukawa" coupling: $$\mathcal{L}_c = -\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$$

With $z = 2$ scaling, ζ is irrelevant.
So we take $\zeta \to 0$
(⚠ watch for dangerous irrelevancy).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Set $\vec{\varphi}$ wavefunction renormalization by keeping co-efficient of $(\boldsymbol{\nabla}_r\vec{\varphi})^2$ fixed (as usual).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Set fermion wavefunction renormalization by keeping Yukawa coupling fixed.

Y. Huh and S. Sachdev, *Phys. Rev. B* **78**, 064512 (2008).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

"Yukawa" coupling: $\mathcal{L}_c = -\vec{\varphi}\cdot\left(\psi^{\ell\dagger}_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{2\beta} + \psi^{\ell\dagger}_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{1\beta}\right)$

We find consistent two-loop RG factors, as $\zeta \to 0$, for the velocities v_x, v_y, and the wavefunction renormalizations.

Consistency check: the expression for the boson damping constant, $\gamma = \dfrac{2}{\pi v_x v_y}$, is preserved under RG.

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}$$

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The velocities flow as

$$\frac{1}{v_x}\frac{dv_x}{d\ell} = \frac{\mathcal{A}(\alpha)+\mathcal{B}(\alpha)}{2} \quad ; \quad \frac{1}{v_y}\frac{dv_y}{d\ell} = \frac{-\mathcal{A}(\alpha)+\mathcal{B}(\alpha)}{2}$$

$$\mathcal{A}(\alpha) \equiv \frac{3}{\pi N}\frac{\alpha}{1+\alpha^2}$$

$$\mathcal{B}(\alpha) \equiv \frac{1}{2\pi N}\left(\frac{1}{\alpha}-\alpha\right)\left(1+\left(\frac{1}{\alpha}-\alpha\right)\tan^{-1}\frac{1}{\alpha}\right)$$

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The anomalous dimensions of $\vec{\varphi}$ and ψ are

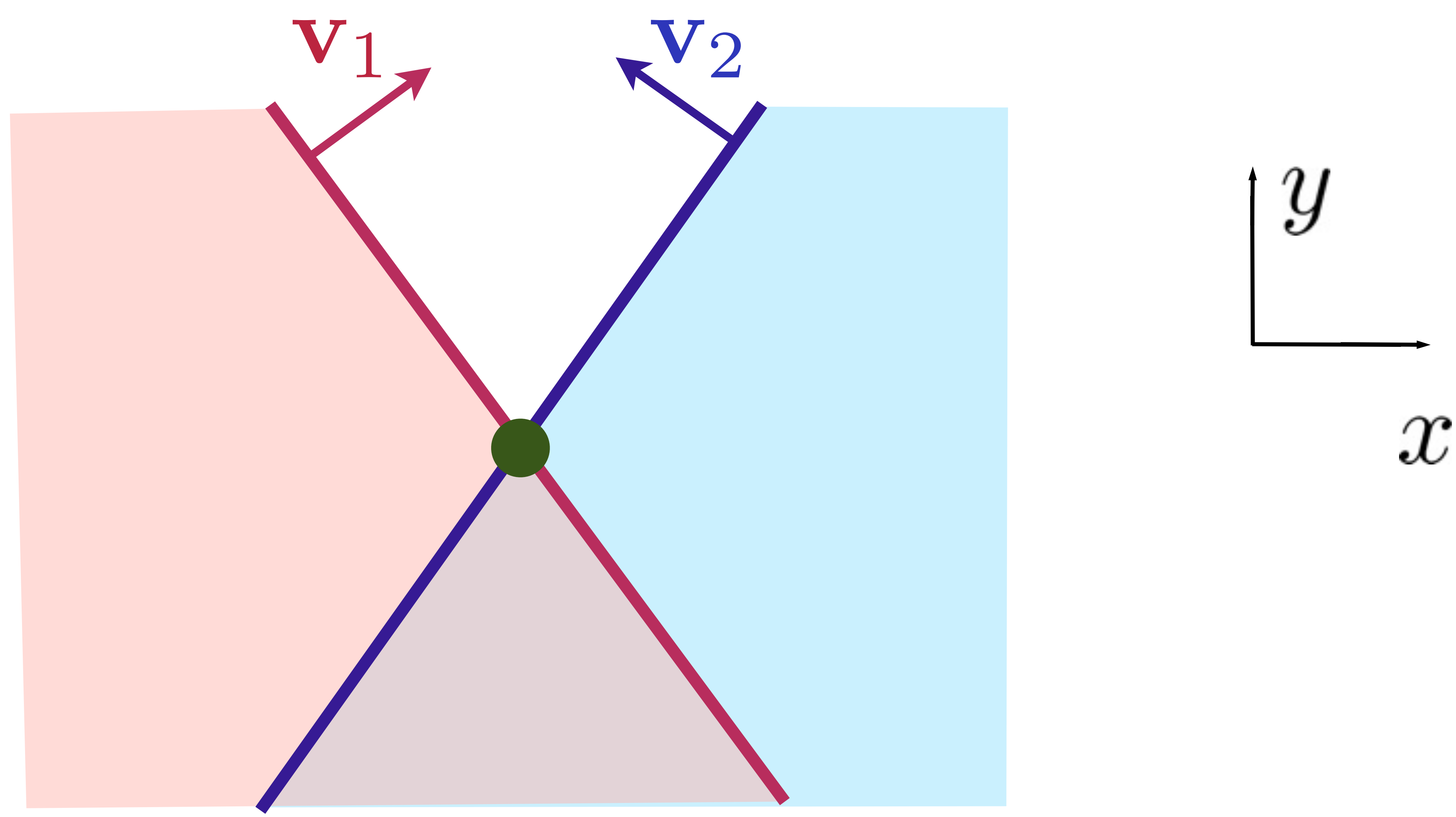
$$\eta_\varphi = \frac{1}{2\pi N}\left(\frac{1}{\alpha} - \alpha + \left(\frac{1}{\alpha^2} + \alpha^2\right)\tan^{-1}\frac{1}{\alpha}\right)$$

$$\eta_\psi = -\frac{1}{4\pi N}\left(\frac{1}{\alpha} - \alpha\right)\left(1 + \left(\frac{1}{\alpha} - \alpha\right)\tan^{-1}\frac{1}{\alpha}\right)$$

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \rightarrow 0$ logarithmically in the infrared.

Dynamical Nesting

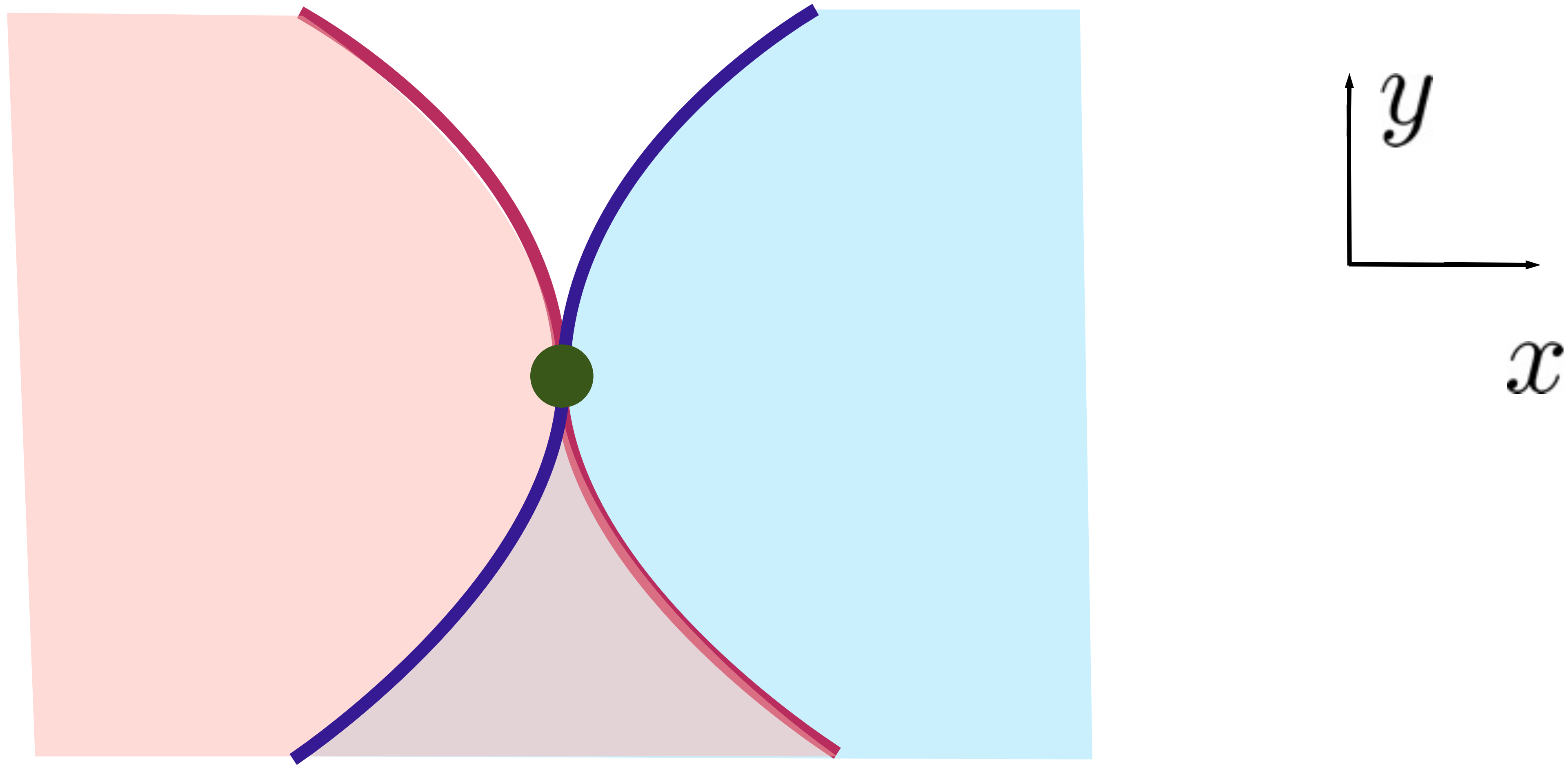


Bare Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting



Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

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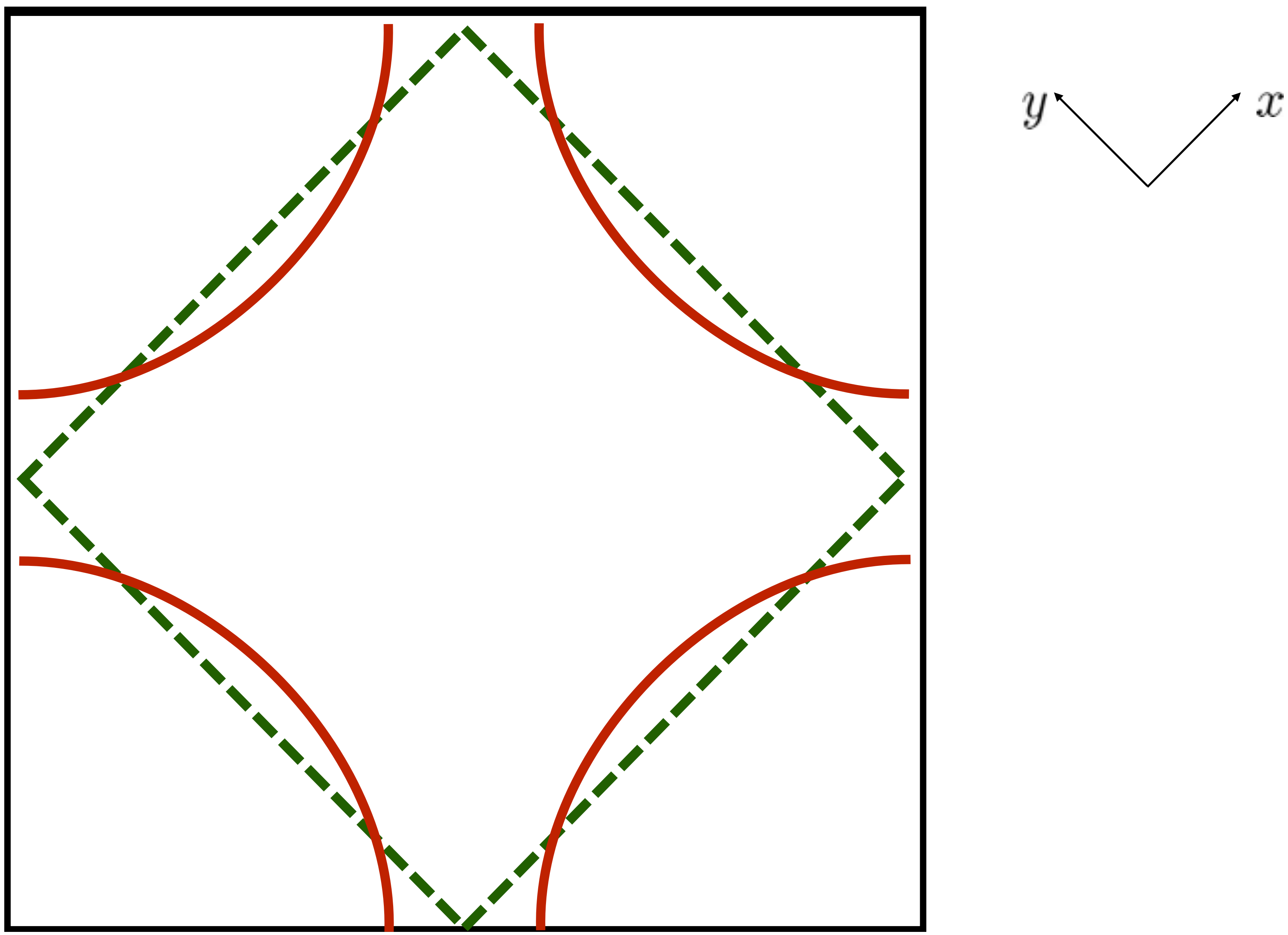
Dynamical Nesting



Bare Fermi surface

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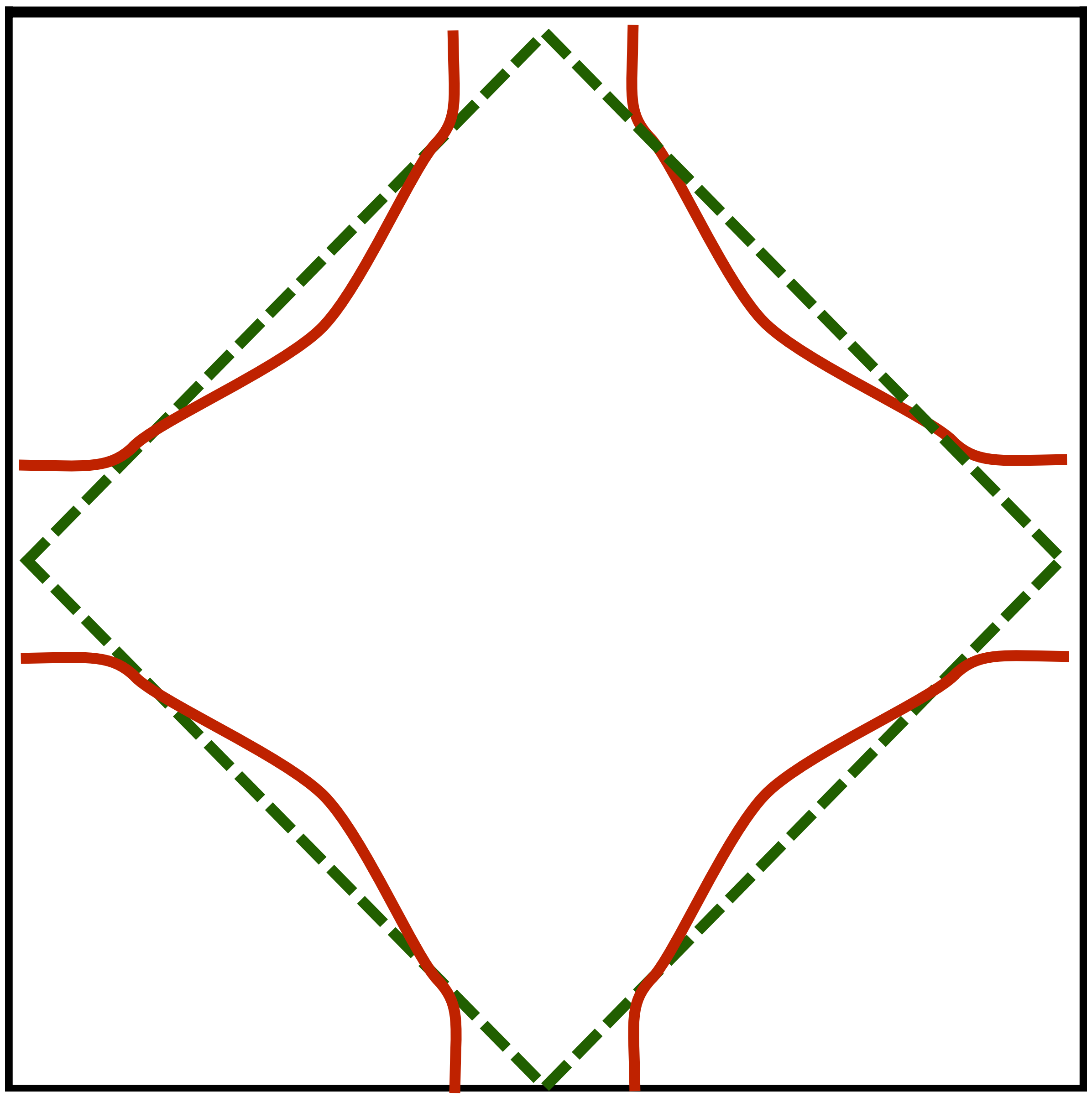
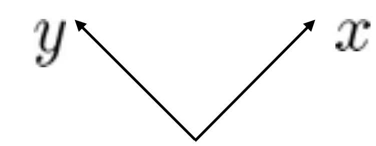
Dynamical Nesting





Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

In $\vec{\varphi}$ SDW fluctuations, characteristic q and ω scale as

$$q \sim \omega^{1/2} \exp\left(-\frac{3}{64\pi^2}\left(\frac{\ln(1/\omega)}{N}\right)^3\right).$$

However, $1/N$ expansion cannot be trusted in the asymptotic regime.

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

$$\frac{1}{N}\frac{1}{(q^2+\gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v}\cdot\mathbf{q}+i\zeta\omega+i\frac{1}{N\sqrt{\gamma}v}\sqrt{\omega}F\left(\frac{v^2q^2}{\omega}\right)}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

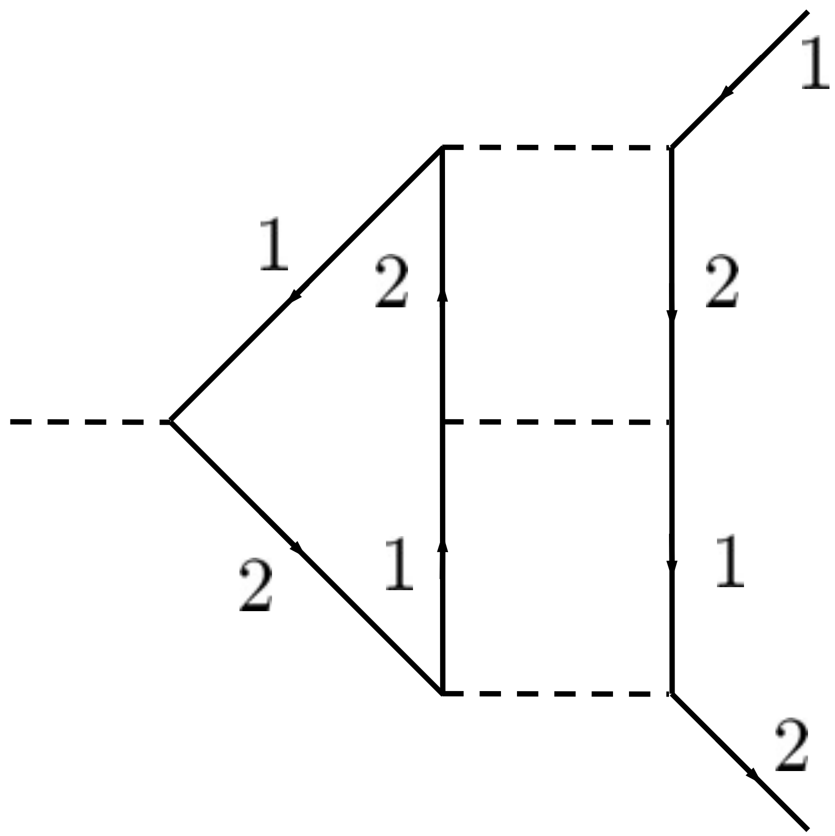
$$\frac{1}{N} \frac{1}{(q^2 + \gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v} \cdot \mathbf{q} + i\zeta\omega + i \frac{1}{N\sqrt{\gamma}v} \sqrt{\omega} F\left(\frac{v^2 q^2}{\omega}\right)}$$

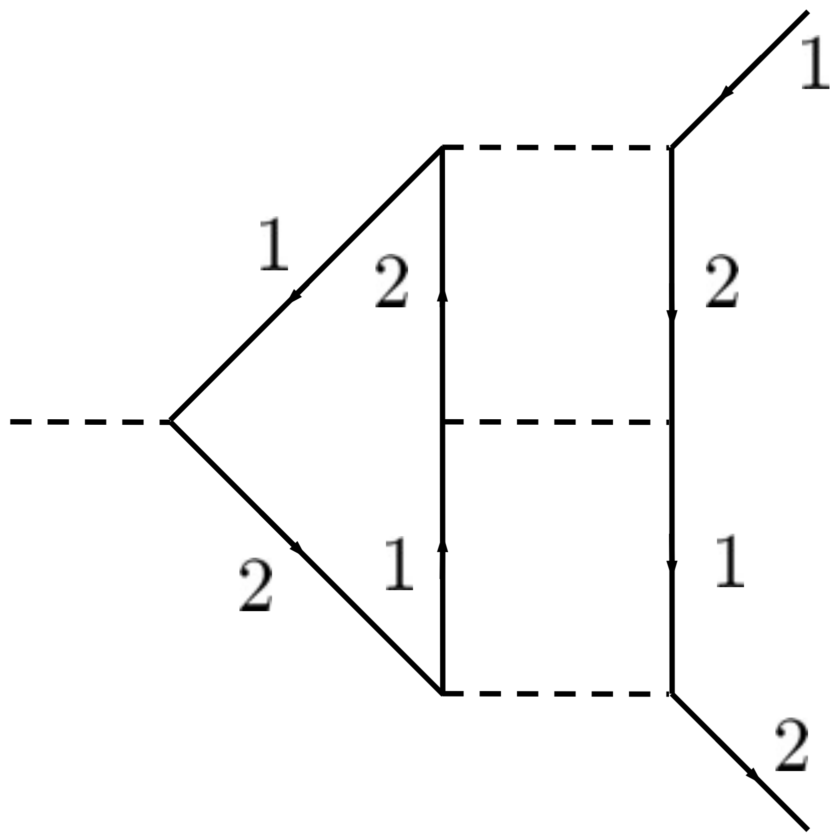
Dangerous

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \dfrac{1}{N^2} \times \dfrac{1}{\zeta^2} \times \dfrac{1}{\omega}$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \dfrac{1}{N^2} \times \dfrac{1}{\zeta^2} \times \dfrac{1}{\omega}$

Actual order $\sim \dfrac{1}{N^0}$

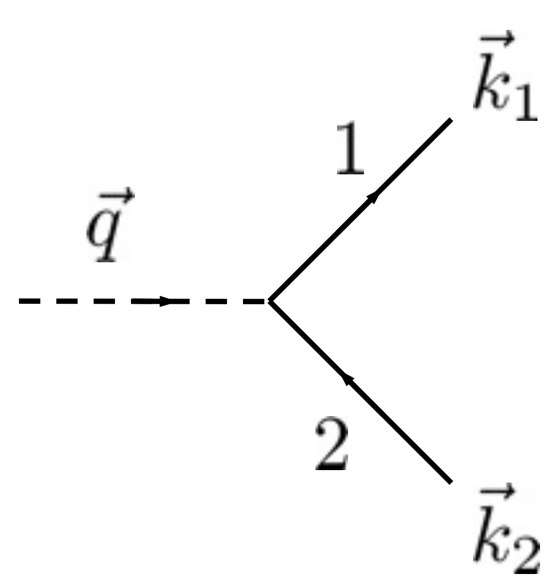
Double line representation

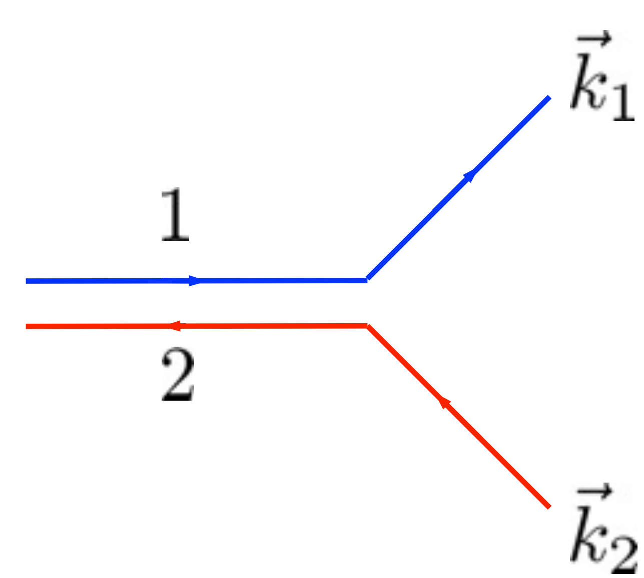
- A way to compute the order of a diagram.
- Extra powers of N come from the Fermi-surface

$$G(\omega, \vec{k}) = \frac{1}{-\Sigma_1(\omega, \vec{k}) - \vec{v} \cdot \vec{k}} \qquad \Sigma_1 \sim \frac{1}{N}$$

- What are the conditions for all propagators to be on the Fermi surface?
- Concentrate on diagrams involving a single pair of hot-spots
- Any bosonic momentum may be (uniquely) written as

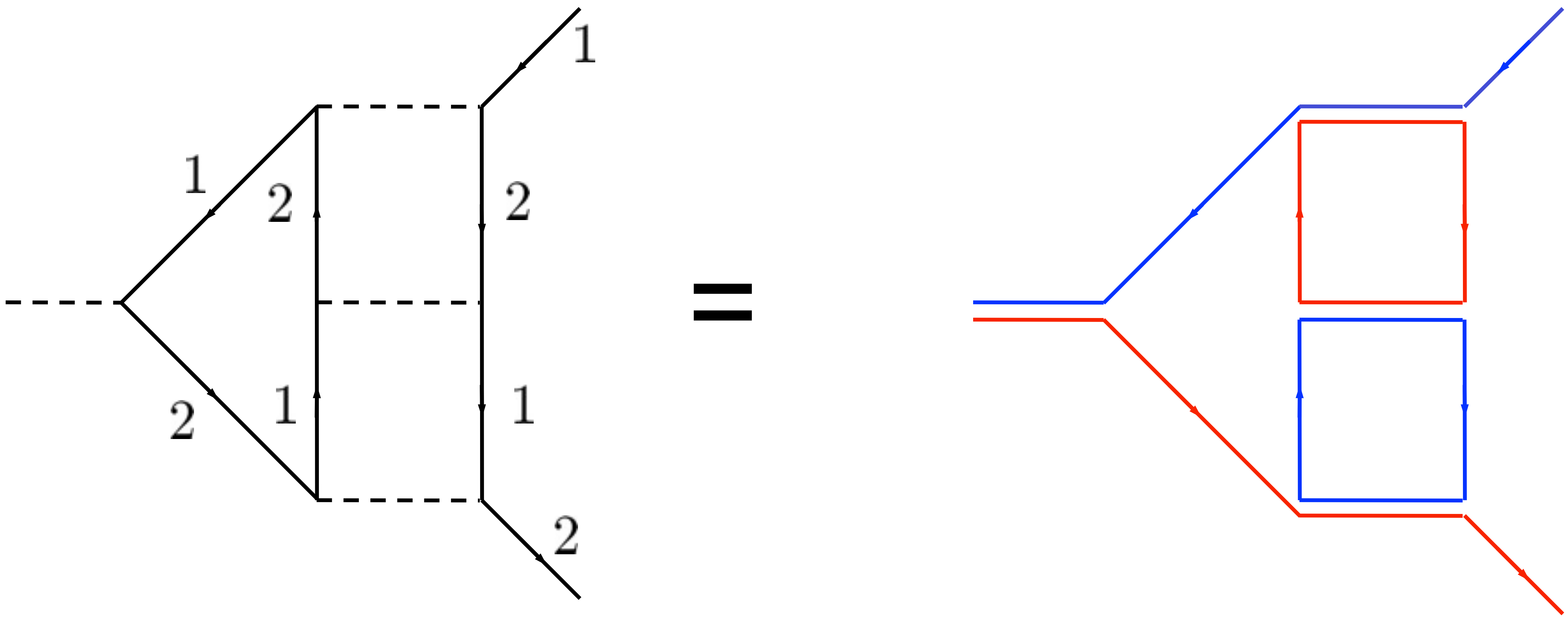
$$\vec{q} = \vec{k}_1 - \vec{k}_2 \qquad \vec{k}_1 \in \text{FS of } \psi_1 \qquad \vec{k}_2 \in \text{FS of } \psi_2$$





R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
S. W. Tsai, A. H. Castro Neto, R. Shankar, and D. K. Campbell, Phys. Rev. B **72**, 054531 (2005).

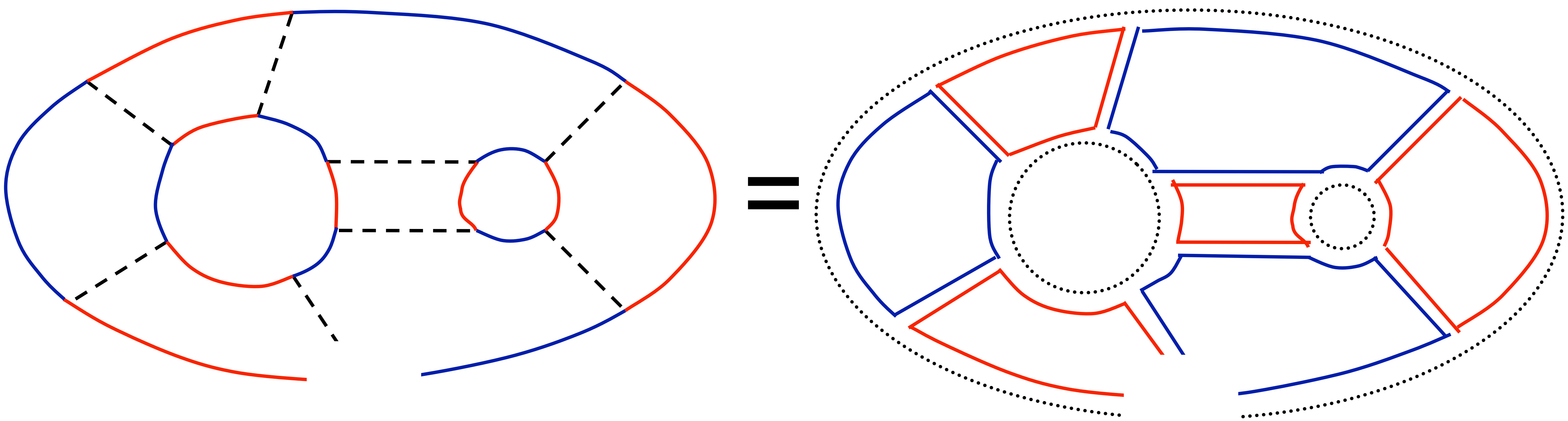
New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Singularities as $\zeta \to 0$ appear when fermions in closed blue and red line loops are exactly on the Fermi surface

$$\text{Actual order} \sim \frac{1}{N^0}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

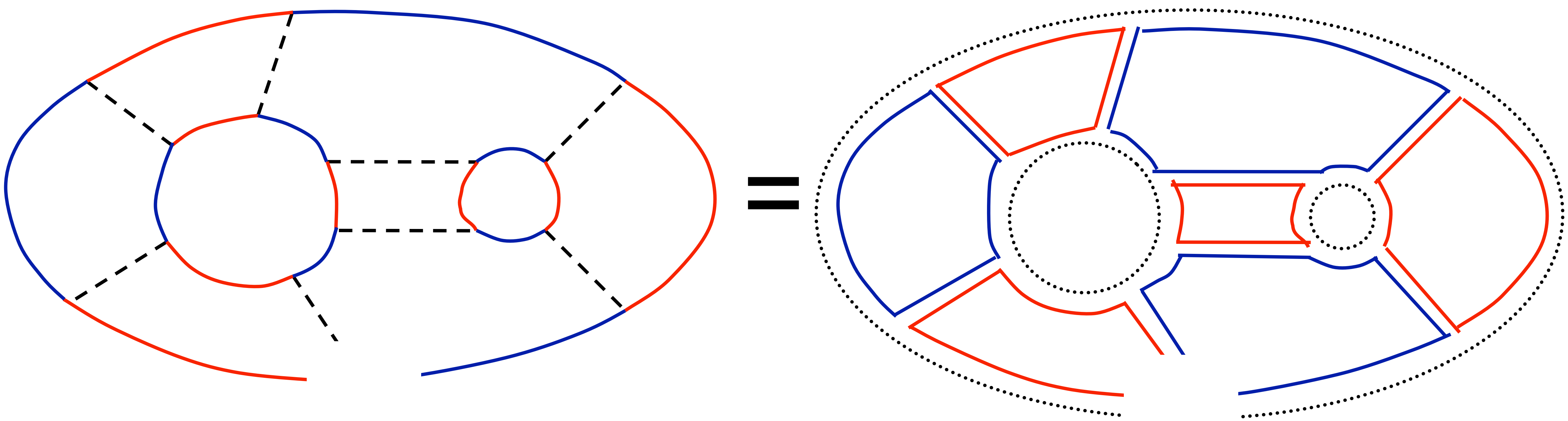


$$\text{Actual order} \sim \frac{1}{N^0}$$

Graph is **planar** after turning fermion propagators also into double lines by drawing additional dotted single line loops for each fermion loop

Sung-Sik Lee, arXiv:0905.4532

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



$$\text{Actual order} \sim \frac{1}{N^0}$$

A consistent analysis requires resummation of all planar graphs

Theory for the onset of spin density wave order in metals is *strongly* coupled in two dimensions