Quantum phase transitions of insulators, superconductors and metals in two dimensions

Talk online: sachdev.physics.harvard.edu

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Outline

1. Phenomenology of the cuprate superconductors
   (and other compounds)

2. QPT of antiferromagnetic insulators
   (and bosons at rational filling)

3. QPT of d-wave superconductors:
   Fermi points of massless Dirac fermions

4. QPT of Fermi surfaces:
   A. Finite wavevector ordering (SDW/CDW):
      "Hot spots" on Fermi surfaces
   B. Zero wavevector ordering (Nematic):
      "Hot Fermi surfaces"

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   B. Zero wavevector ordering (Nematic): "Hot Fermi surfaces"

Yejin Huh, Harvard

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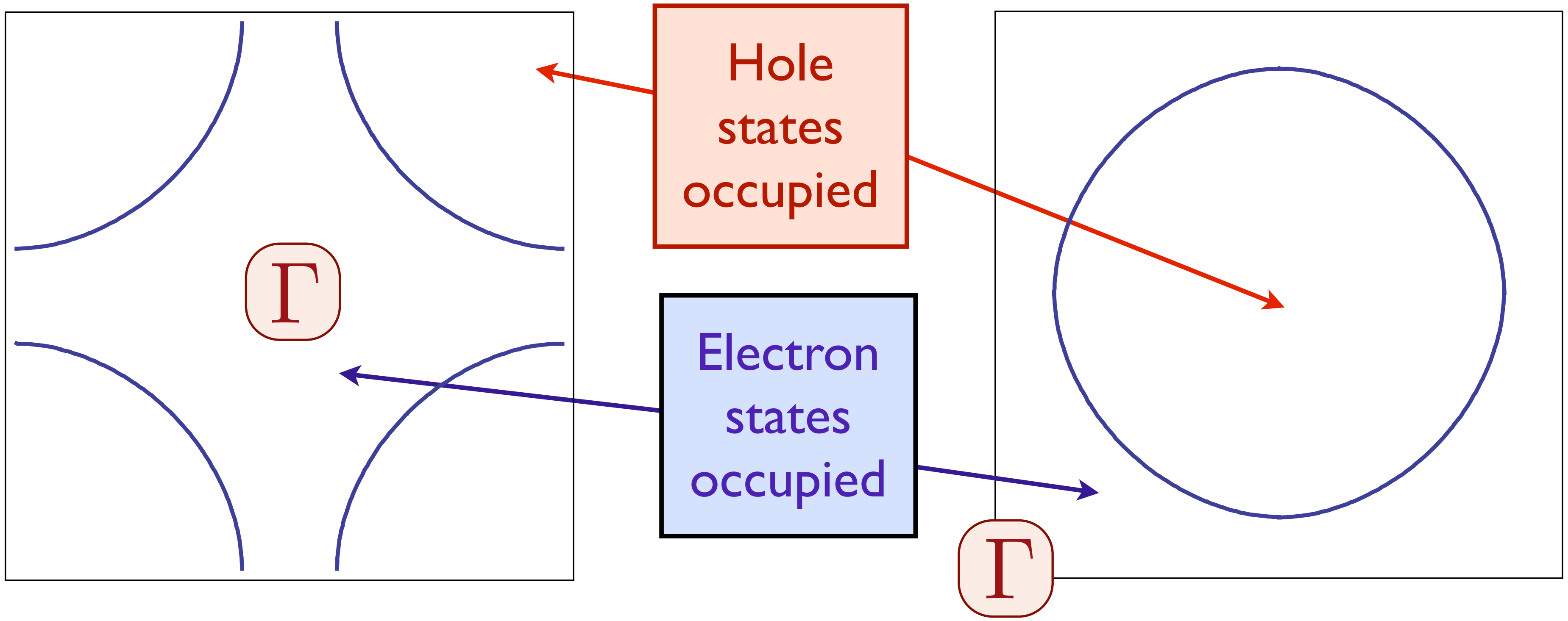
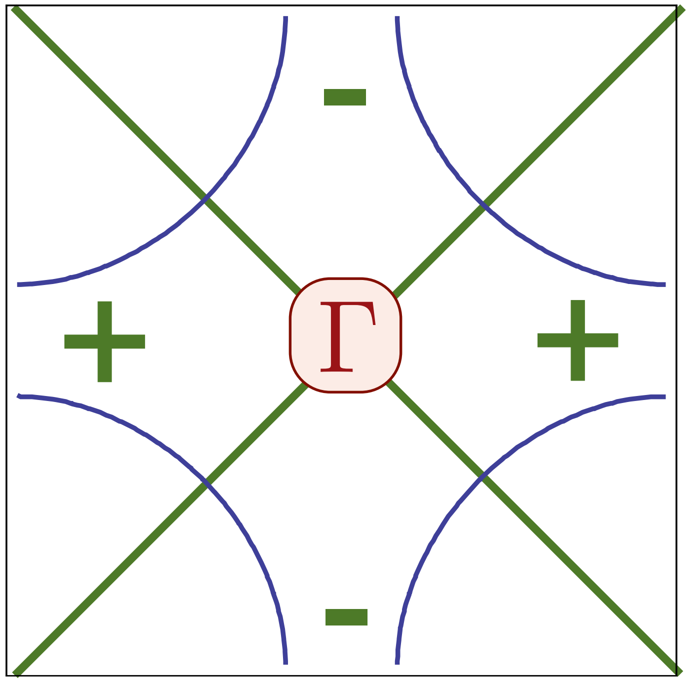
d-wave superconductivity in cuprates

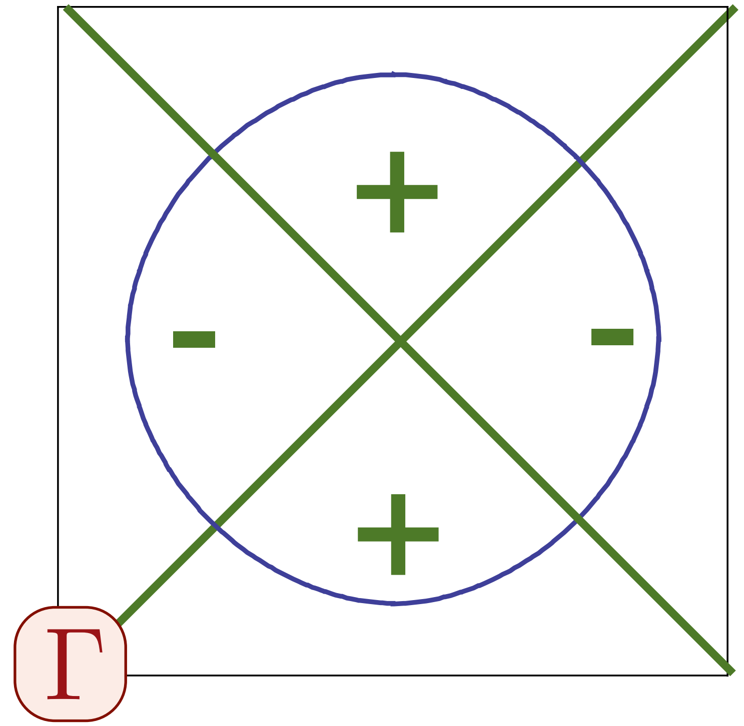


$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

- Begin with free electrons.

d-wave superconductivity in cuprates

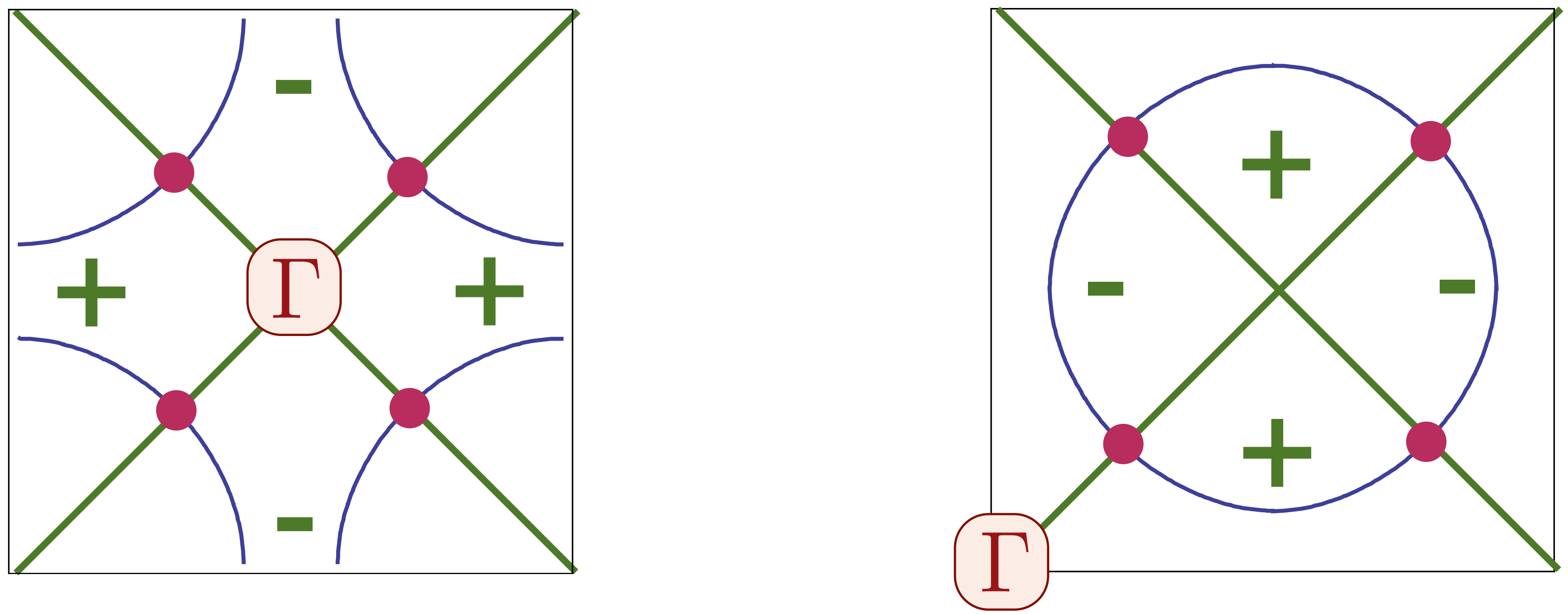




$$H = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

- Begin with free electrons.
- Add d-wave pairing interaction $\Delta_k \sim \cos k_x - \cos k_y$ which vanishes along diagonals

d-wave superconductivity in cuprates



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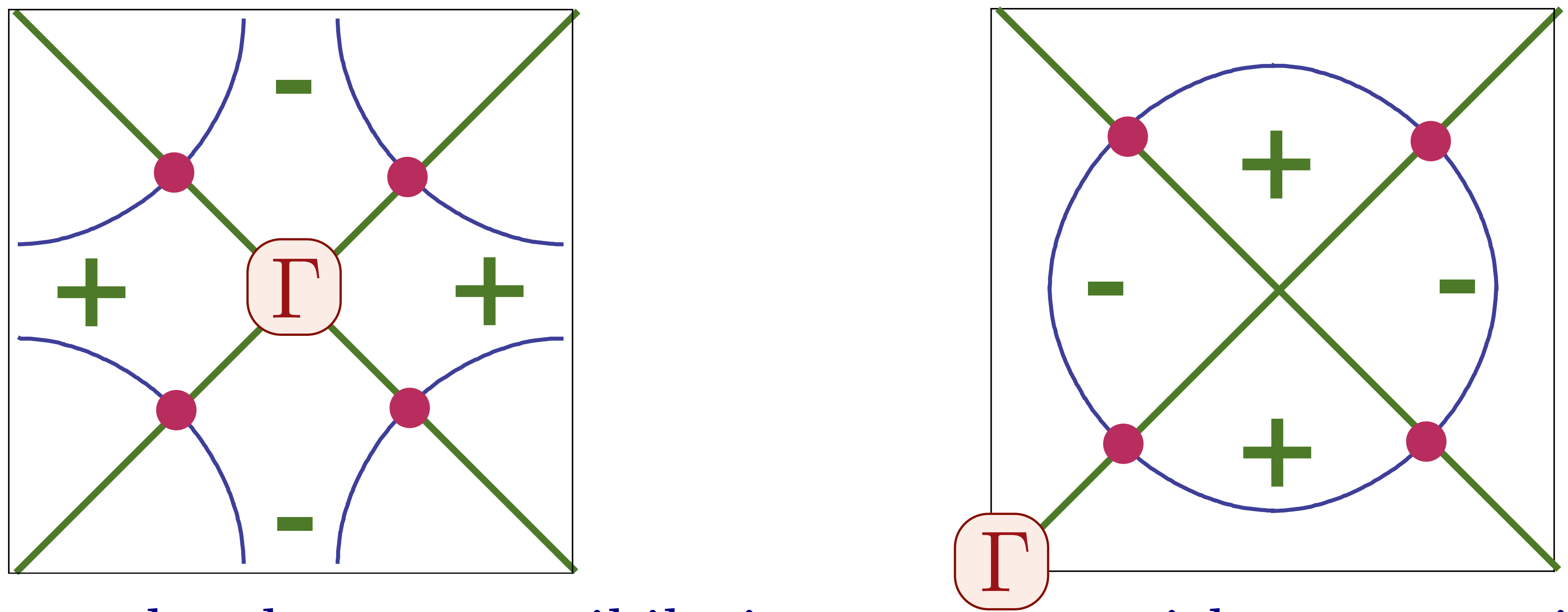
- Begin with free electrons.
- Add d-wave pairing interaction Δ_k which vanishes along diagonals
- Obtain Bogoliubov quasiparticles with dispersion $\sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$

d-wave superconductivity in cuprates



- Denote the electron annihilation operator with momenta in the vicinity of the nodes as (Q,Q), $(-Q,Q)$, $(-Q,-Q)$, and $(Q,-Q)$ by f_{1a}, f_{2a}, f_a, and f_{4a} respectively; here $a = \uparrow, \downarrow$ is an electron spin index.

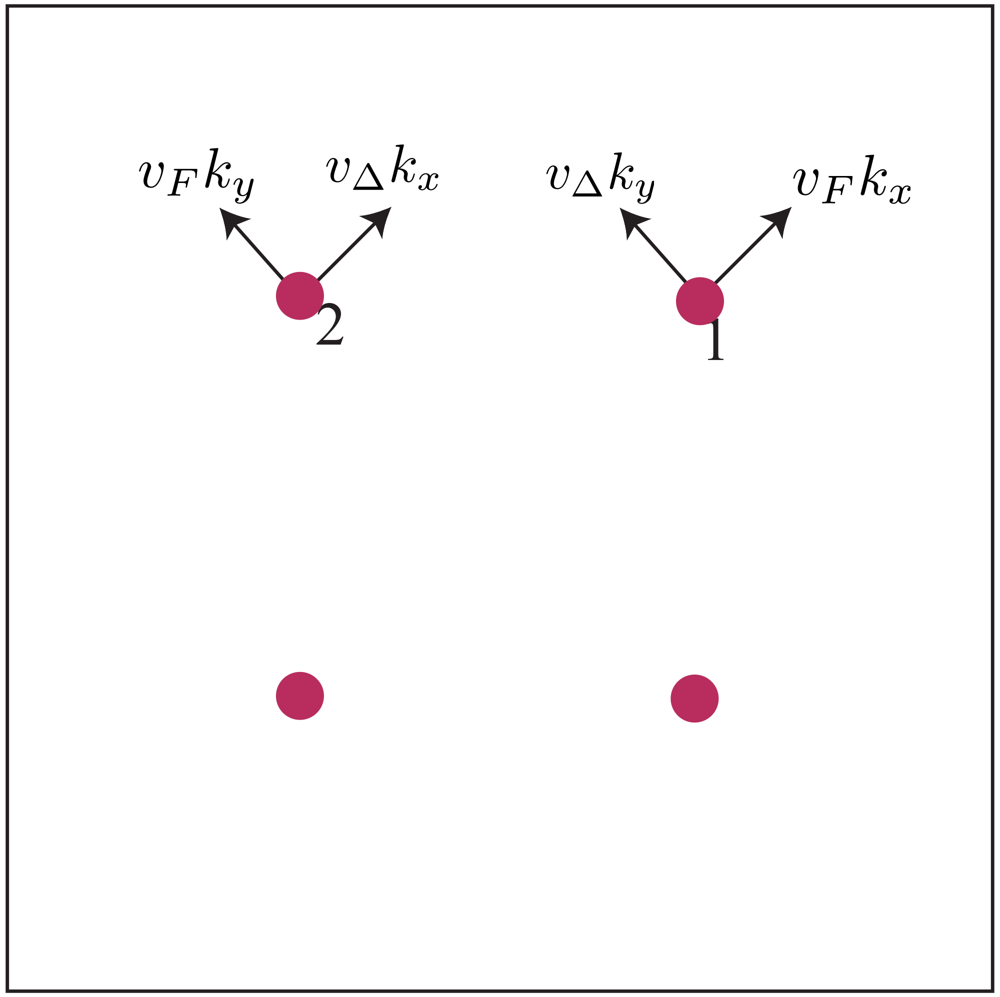
- Introduce the 4 2-component Nambu spinors

$$\Psi_{1a} = \begin{pmatrix} f_{1a} \\ \varepsilon_{ab} f_{3b}^{\dagger} \end{pmatrix} \quad , \quad \Psi_{2a} = \begin{pmatrix} f_{2a} \\ \varepsilon_{ab} f_{4b}^{\dagger} \end{pmatrix}$$

where $\varepsilon_{ab} = -\varepsilon_{ba}$ and $\varepsilon_{ab} = 1$.

We will use Pauli matrices τ^i which act on the Nambu particle-hole space.

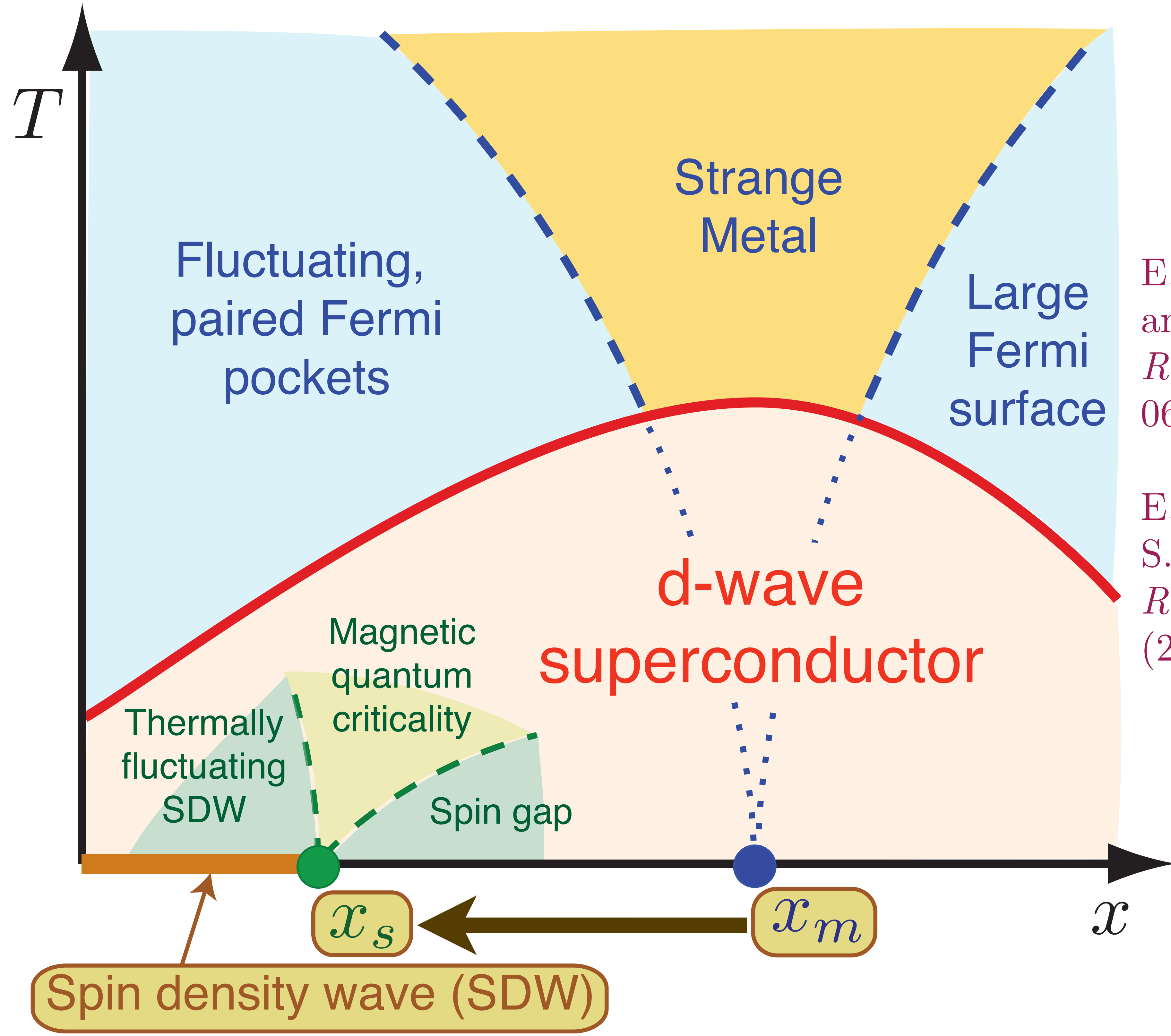
d-wave superconductivity in cuprates



4 two-component Dirac fermions

$$
\begin{aligned}
S_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{1a} \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_{1a} \\
&+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{2a} \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_{2a}.
\end{aligned}
$$

Theory of quantum criticality in the cuprates

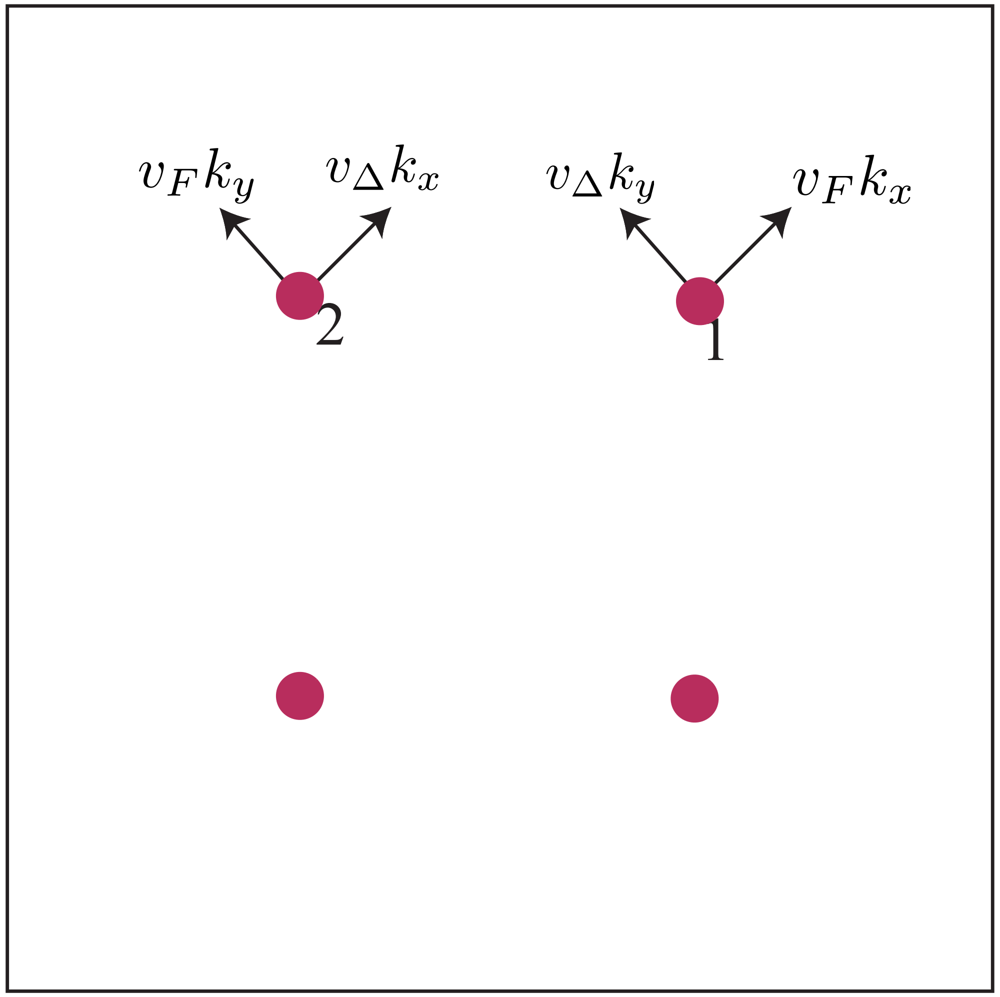


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

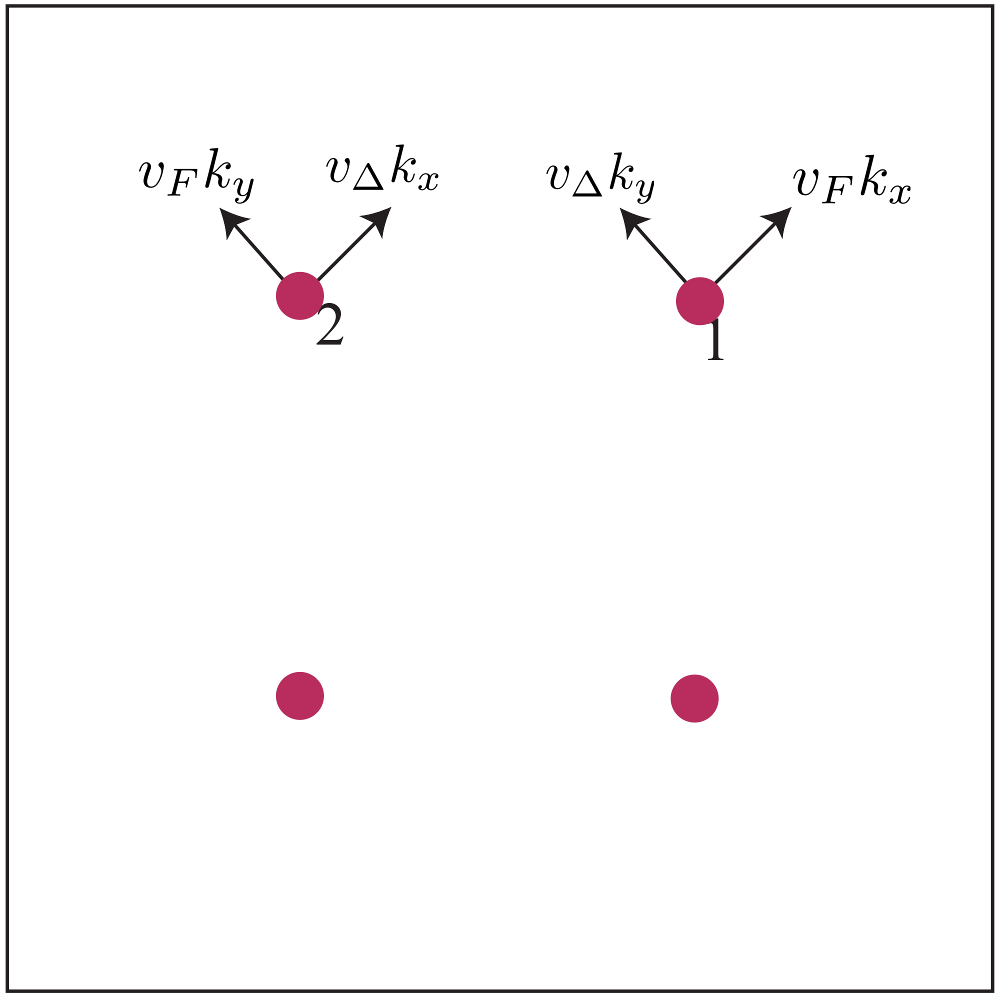
Onset of SDW order in a d-wave superconductor



4 two-component Dirac fermions

$$\begin{aligned} S_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{1a} \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_{1a} \\ &+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{2a} \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_{2a}. \end{aligned}$$

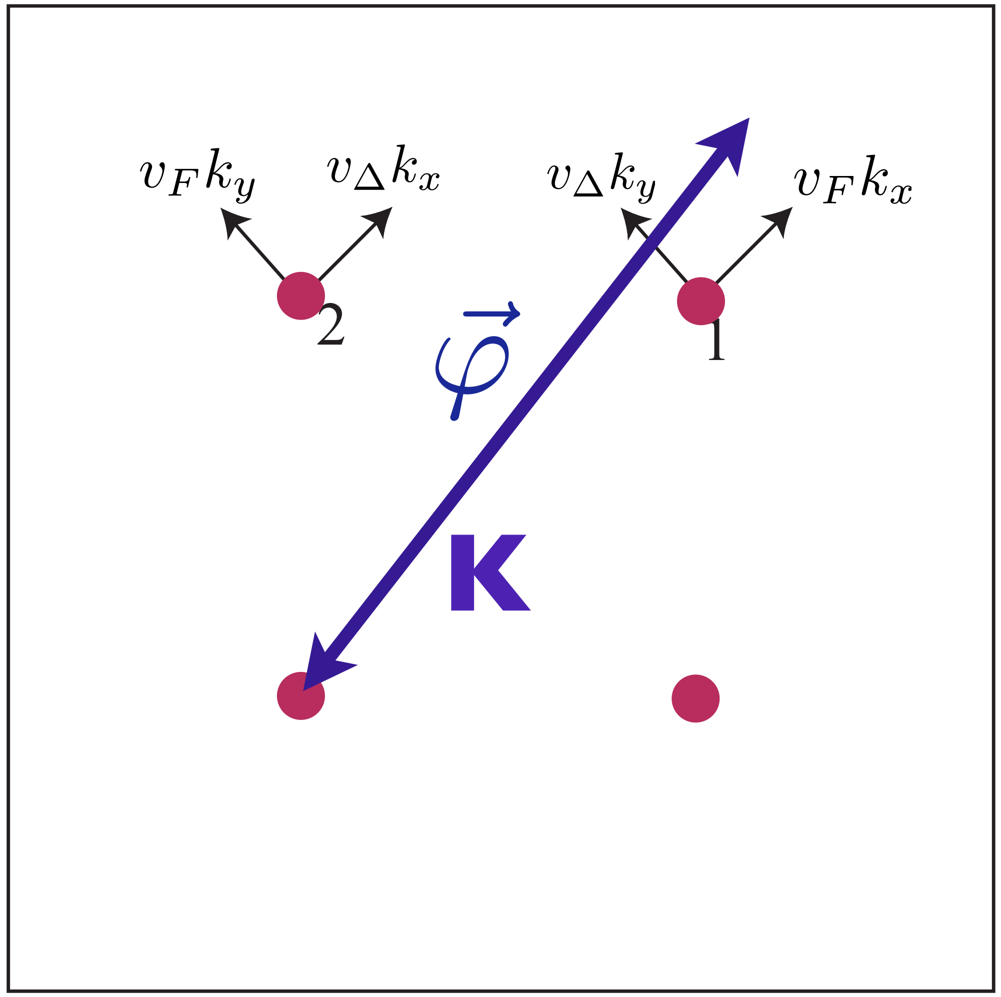
Onset of SDW order in a d-wave superconductor



Landau-Ginzburg field theory for SDW order

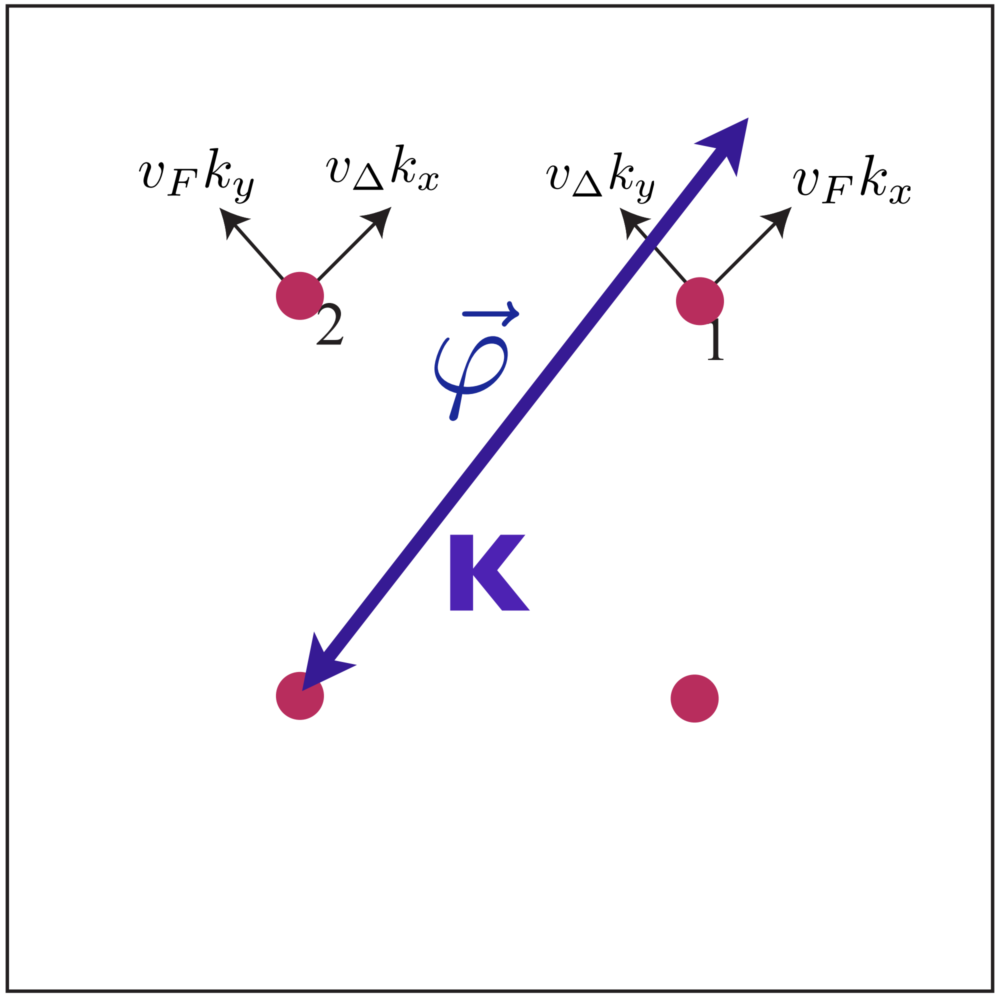
$$\mathcal{S}_{\vec{\varphi}} = \int d^2x d\tau \left[\frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2} (\nabla \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \frac{u_0}{24} \vec{\varphi}^4 \right];$$

Onset of SDW order in a d-wave superconductor

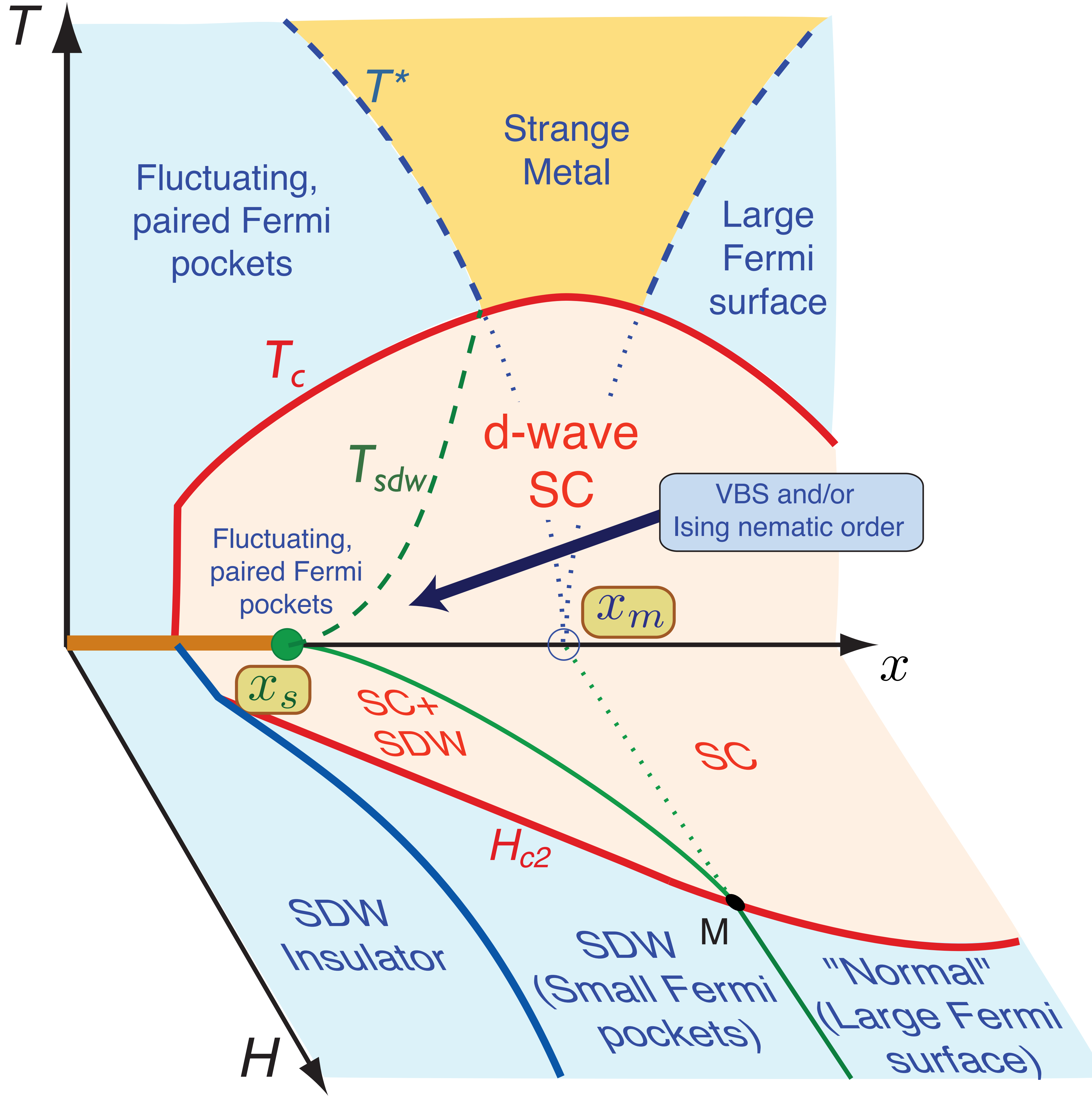


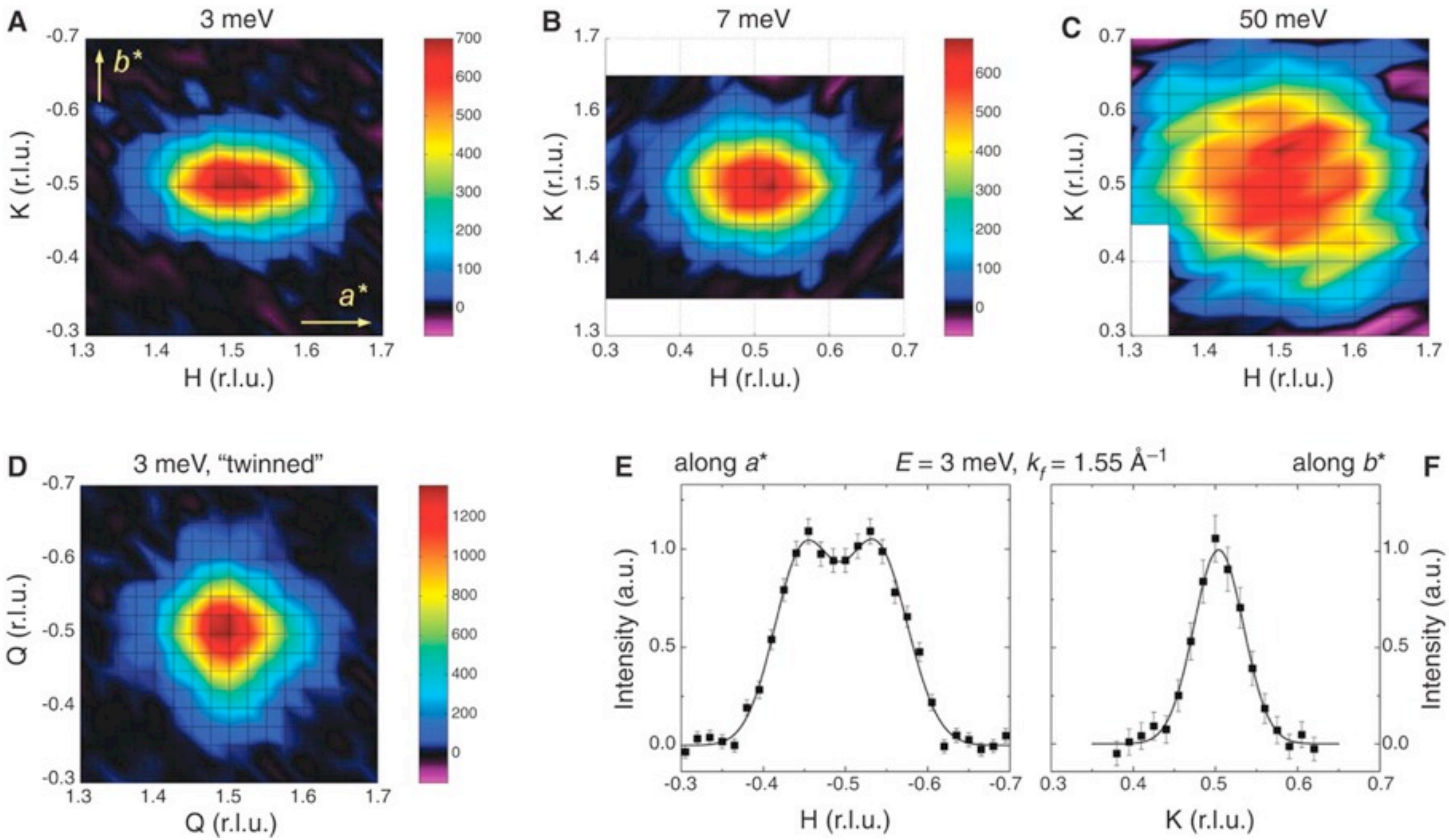
- No coupling between SDW order, $\vec{\varphi}$, and Dirac fermions, $\Psi_{1,2}$, which is linear in $\vec{\varphi}$.

Onset of SDW order in a d-wave superconductor



- No coupling between SDW order, $\vec{\varphi}$, and Dirac fermions, $\Psi_{1,2}$, which is linear in $\vec{\varphi}$.
- Universality class of SDW ordering transition is the same as that in the coupled-dimer antiferromagnet. Corrections to scaling arise from coupling of $|\vec{\varphi}|^2$ (and of nematic order) to the Dirac fermions.


T
T*
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
T_c
T_{sdw}
d-wave SC
VBS and/or Ising nematic order
Fluctuating, paired Fermi pockets
x_m
x_s
x
SC+ SDW
SC
H_{c2}
M
SDW Insulator
SDW (Small Fermi pockets)
"Normal" (Large Fermi surface)
H

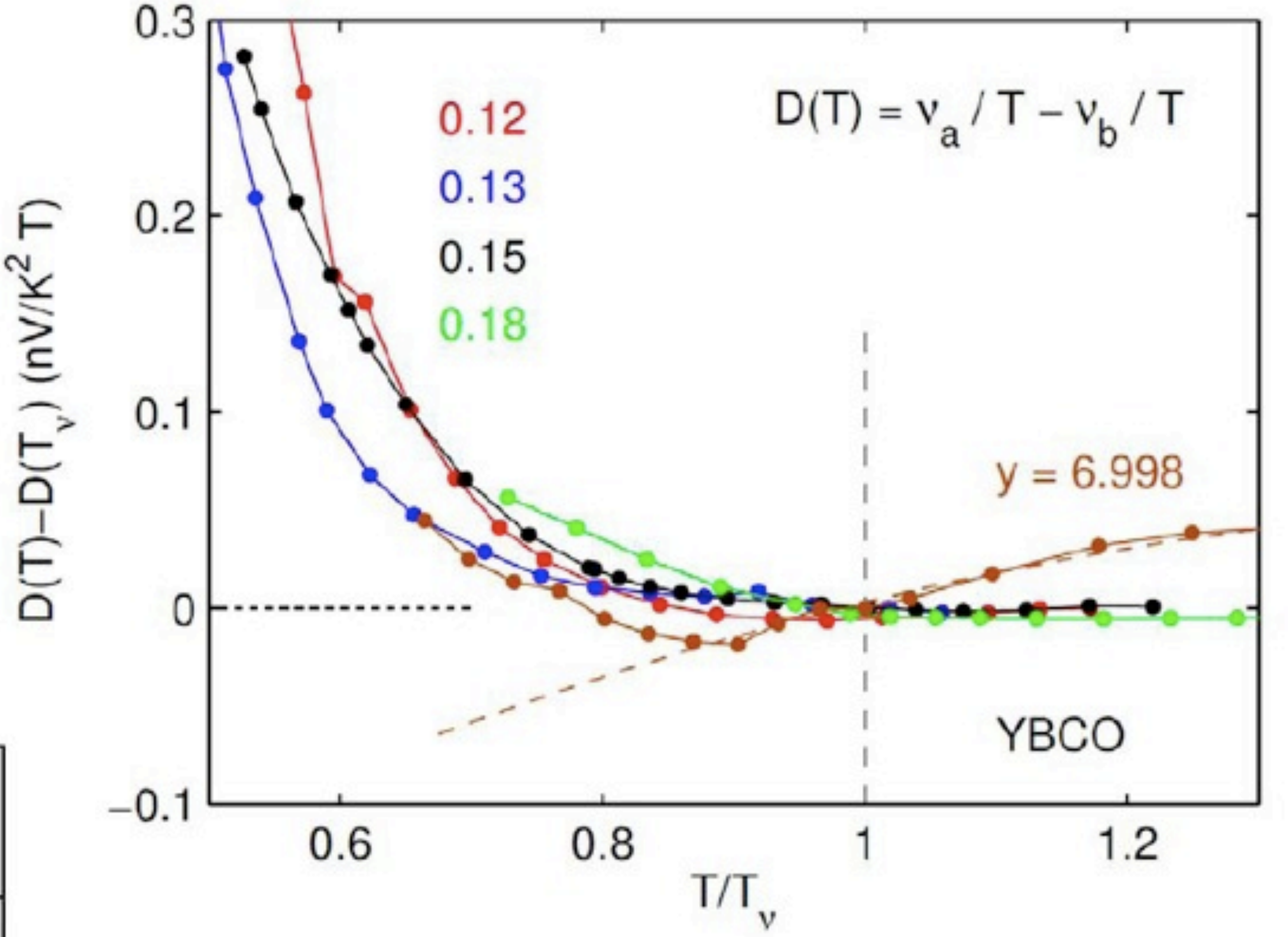


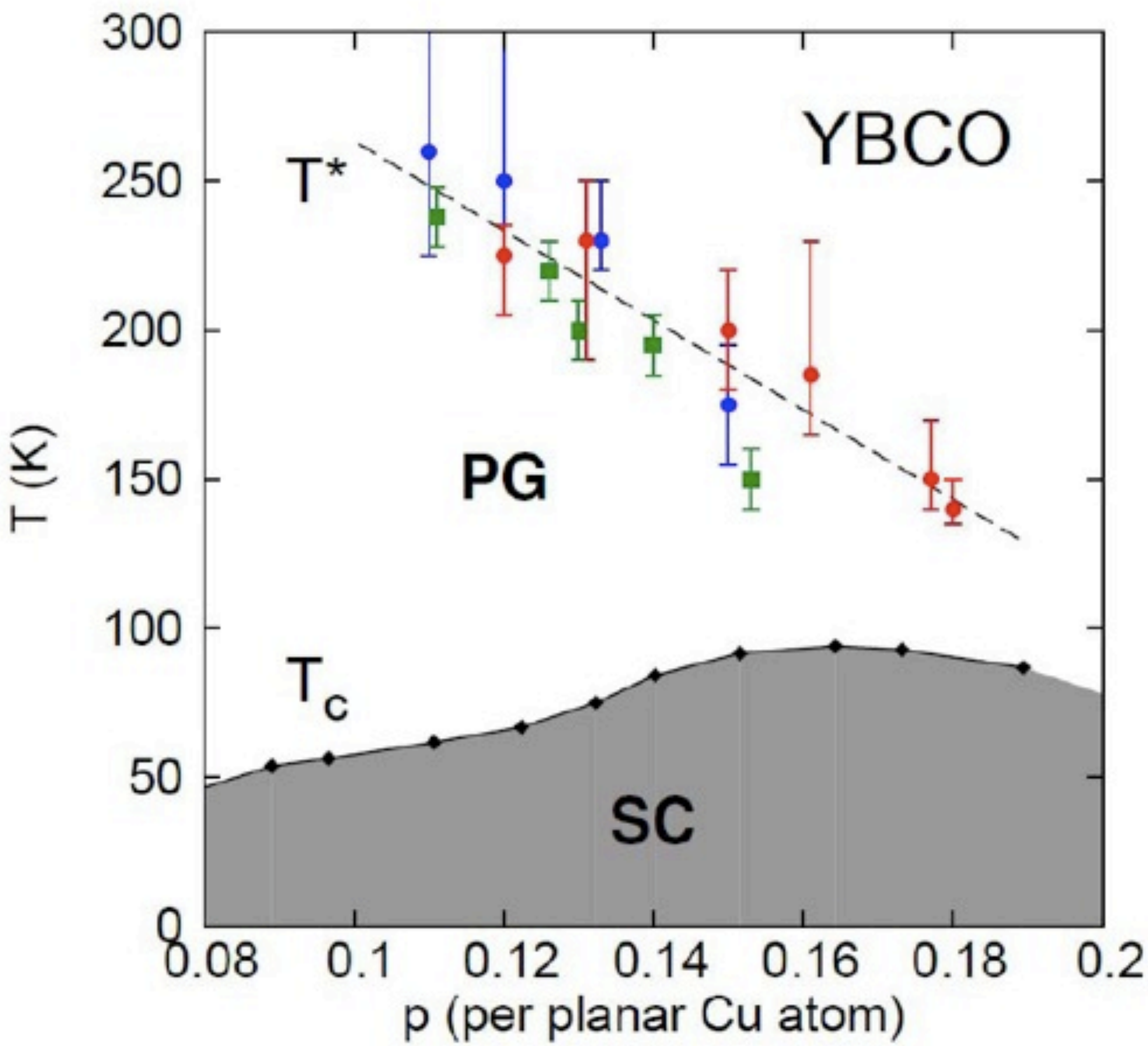
Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430





S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).

Discrete symmetry breaking in d-wave superconductors

Transformation of Dirac fermions under square lattice space group and time-reversal

	T_x	T_y	R	I	$\mathcal{T}$
$\Psi_{1\alpha}$	$e^{iQ}\Psi_{1\alpha}$	$e^{iQ}\Psi_{1\alpha}$	$i\tau^z\Psi_{2\alpha}$	$\Psi_{2\alpha}$	$-\tau^y\Psi_{1\alpha}$
$\Psi_{2\alpha}$	$e^{-iQ}\Psi_{2\alpha}$	$e^{iQ}\Psi_{2\alpha}$	$-i\varepsilon_{\alpha\beta}\left[\Psi^\dagger_{1\beta}\tau^x\right]^T$	$\Psi_{1\alpha}$	$-\tau^y\Psi_{2\alpha}$

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Nematic order parameter is $\phi \sim \Psi^\dagger_{1a}\tau^x\Psi_{1a} + \Psi^\dagger_{2a}\tau^x\Psi_{2a}$ which is odd under R and even under I.
This order parameter is s-wave pairing $\sim \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow}$

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This order parameter is s-wave pairing $\sim \sum_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger$

Time-reversal order parameter is $\phi \sim \Psi_{1a}^\dagger\tau^y\Psi_{1a} + \Psi_{2a}^\dagger\tau^y\Psi_{2a}$ which is odd under $\mathcal{T}$ and even under all other operations.
This order parameter is id_{xy}-wave pairing $\sim i\sum_{\mathbf{k}} \sin k_x \sin k_y c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger$

Discrete symmetry breaking in d-wave superconductors

Now consider a discrete spontaneous symmetry breaking, with Ising symmetry, described by a real scalar field ϕ.
Two cases of experimental interest are:

- Break 4-fold lattice rotation symmetry to 2-fold lattice rotations: leads to a superconductor with **nematic** order: equivalent to $d_{x^2-y^2} + s$ pairing.

$$H = H_\phi + \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger + \text{c.c.} \right)$$

$$H_\phi = \phi \sum_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger + \text{c.c.}$$

Discrete symmetry breaking in d-wave superconductors

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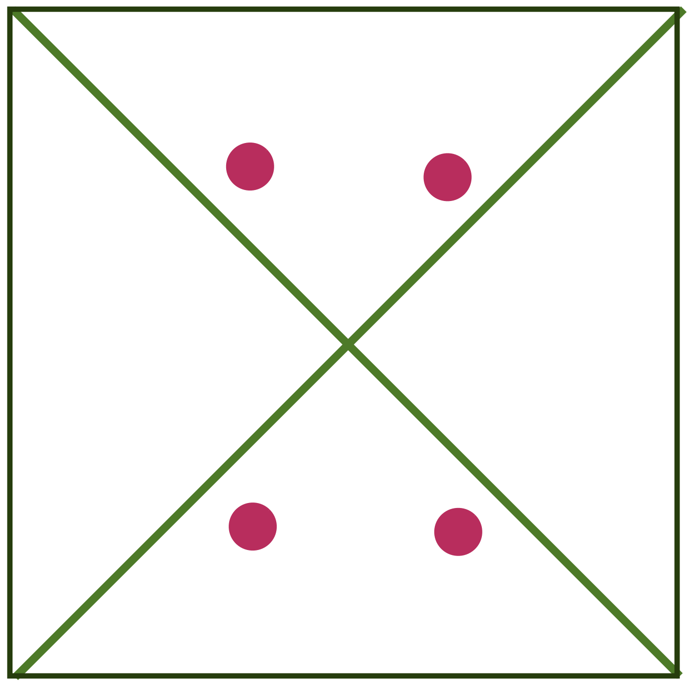
Two cases of experimental interest are:

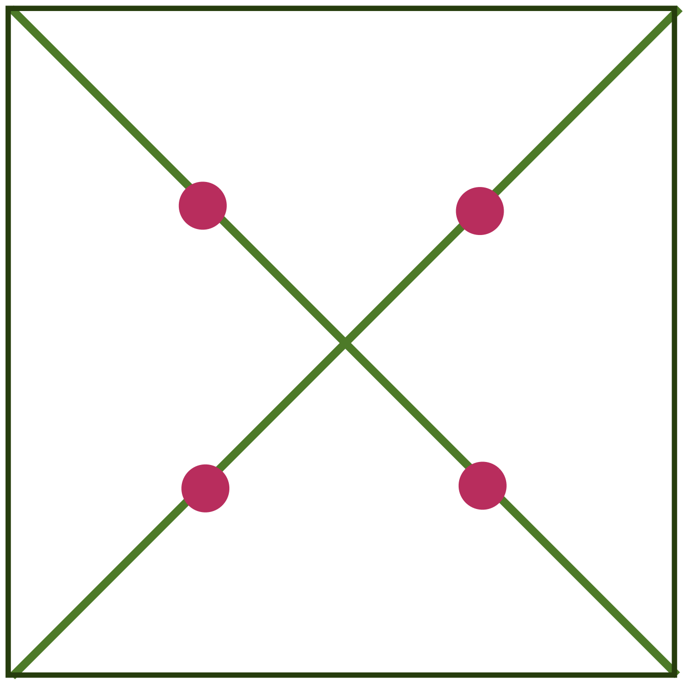
- Break 4-fold lattice rotation symmetry to 2-fold lattice rotations: leads to a superconductor with **nematic** order: equivalent to $d_{x^2-y^2} + s$ pairing.
- Time-reversal symmetry breaking: leads to a $d_{x^2-y^2} + id_{xy}$ superconductor, in which the Dirac fermions are massive

$$H = H_\phi + \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

$$H_\phi = i\phi \sum_{\mathbf{k}} \sin k_x \sin k_y c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \text{c.c.}$$

Lattice rotation symmetry breaking



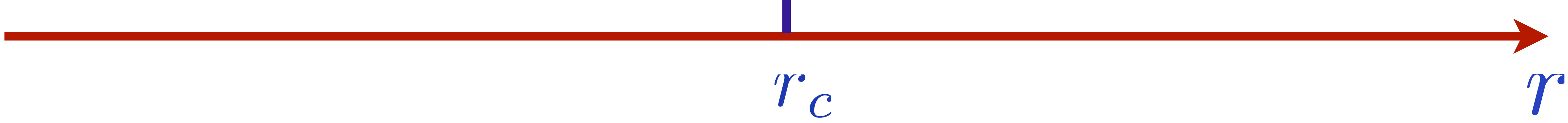


$d_{x^2-y^2}$ superconductor
+ nematic order

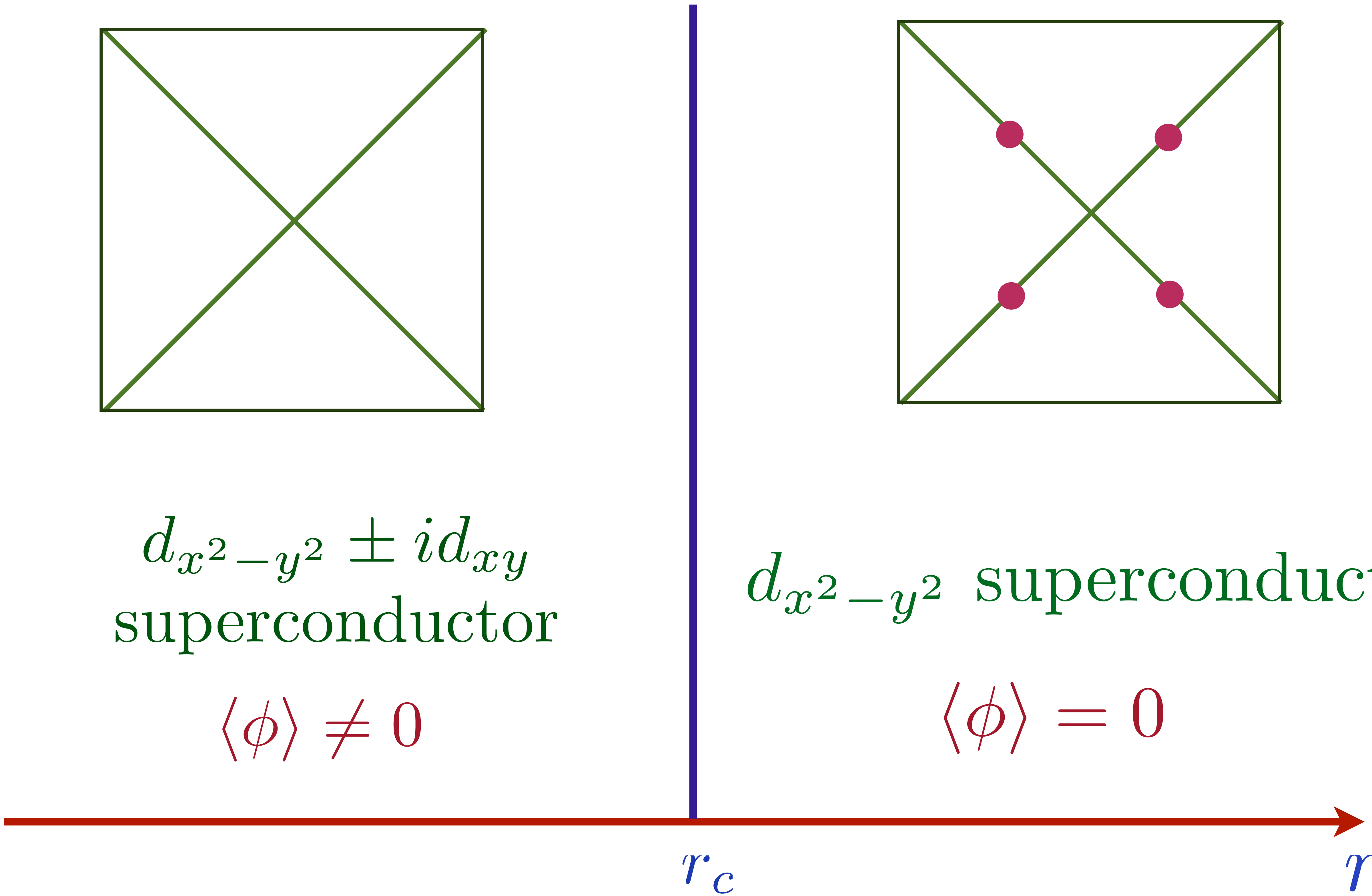
$\langle \phi \rangle \neq 0$

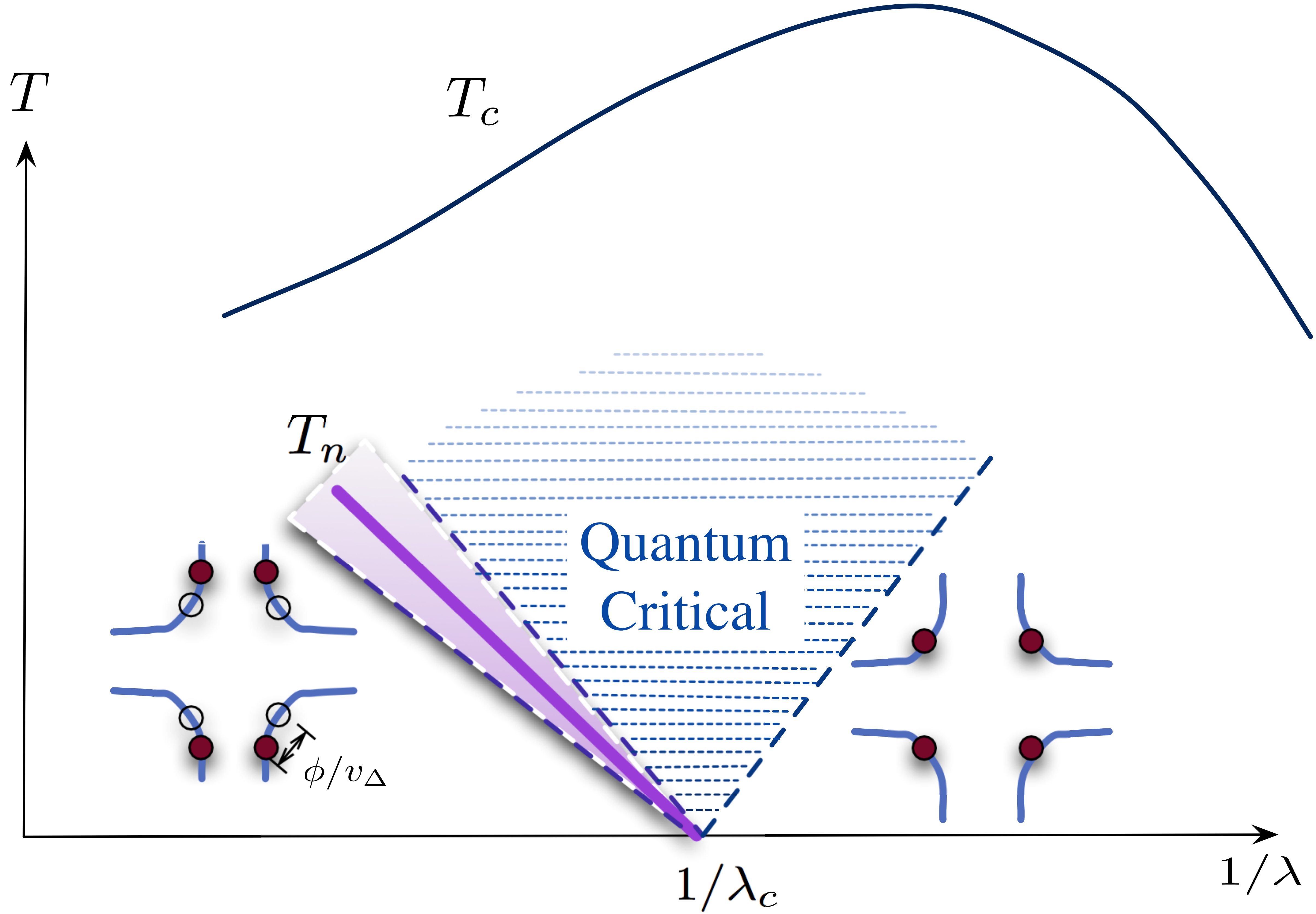
$d_{x^2-y^2}$ superconductor

$\langle \phi \rangle = 0$



Time-reversal symmetry breaking





M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000)
E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, Phys. Rev. B **77**, 184514 (2008).

Discrete symmetry breaking in d-wave superconductors

Field theory for transition with Ising order described by a real scalar field ϕ:

$$\mathcal{S} = \mathcal{S}_\Psi + \mathcal{S}_\phi + \mathcal{S}_{\Psi\phi}$$

4 two-component Dirac fermions

$$\begin{aligned}
\mathcal{S}_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{1a}^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_{1a} \\
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\end{aligned}$$

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Ising field theory

$$\mathcal{S}_\phi = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{u_0}{24}\phi^4\right];$$

Ising order and Dirac fermions couple via a "Yukawa" term.

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^\dagger \tau^x \Psi_{1a} + \Psi_{2a}^\dagger \tau^x \Psi_{2a} \right) \right],$$

Nematic ordering

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^\dagger \tau^y \Psi_{1a} + \Psi_{2a}^\dagger \tau^y \Psi_{2a} \right) \right]$$

Time reversal symmetry breaking

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

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Time reversal symmetry breaking

For the latter case *only*, with $v_F = v_\Delta = c$, theory reduces to relativistic Gross-Neveu model

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the scalar order parameter

$$S_\phi = \frac{N_f}{v_\Delta v_F}\Gamma\left[\lambda_0\phi(x,\tau);\frac{v_\Delta}{v_F}\right] + \frac{N_f}{2}\int d^2x d\tau \Big(r\phi^2(x,\tau)\Big) + \text{irrelevant terms}$$

where Γ is a non-local and non-analytic functional of ϕ.

The theory has only 2 couplings constants: r and v_Δ/v_F.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

Friday, December 18, 2009

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the nematic order parameter

$$S_\phi = \frac{N_f}{2}\int_{k,\omega} |\phi(k,\omega)|^2 \Bigg[r + \frac{\lambda_0^2}{8 v_F v_\Delta}\left(\frac{\omega^2 + v_F^2 k_x^2}{\sqrt{\omega^2 + v_F^2 k_x^2 + v_\Delta^2 k_y^2}} + (x \leftrightarrow y)\right)\Bigg]$$

$$+\text{higher order terms which cannot be neglected}$$

E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, Phys. Rev. B **77**, 184514 (2008).

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the T-breaking order parameter

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E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson,
Phys. Rev. B **77**, 184514 (2008).

Expansion in number of fermion spin components N_f

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$$
\begin{aligned}
S_\phi &= \frac{N_f}{v_\Delta v_F} \Gamma \left[\lambda_0 \phi(x,\tau); \frac{v_\Delta}{v_F} \right] \\
&\quad + \frac{N_f}{2} \int d^2 x d\tau \Big(r \phi^2(x,\tau) \Big) \\
&\quad + \text{irrelevant terms}
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where Γ is a non-local and non-analytic functional of ϕ.

The theory has only 2 couplings constants: r and v_Δ / v_F.

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where Γ is a non-local and non-analytic functional of ϕ.

There is a systematic expansion in powers of $1/N_f$ for renormalization group equations and all critical properties.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the nematic order parameter

Because the order parameter couples to a fermion current, a constant ϕ can be gauged away, and the effective potential is *unrenormalized*

$$V(\phi) = \frac{r}{2}\phi^2 + \frac{u}{4}\phi^4 + \dots \quad (1)$$

The order parameter critical exponent $\gamma = 1$.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

Renormalization group analysis

Couplings are local in the fermion action, so perform RG on fermion self energy

- The fermion self energy determines the wavefunction renormalization of the the fermions (η_f) and the renormalization of the velocities v_F and v_Δ.

- The wavefunction renormalization of ϕ (η_b) is set by the requirement that the Yukawa coupling $\phi\Psi^\dagger\tau^x\Psi$ have unit magnitude.

- The non-renormalization of the effective potential yields the correlation length exponent $\nu = 1/(2-\eta_f)$.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

Renormalization group analysis

Couplings are local in the fermion action, so perform RG on fermion self energy

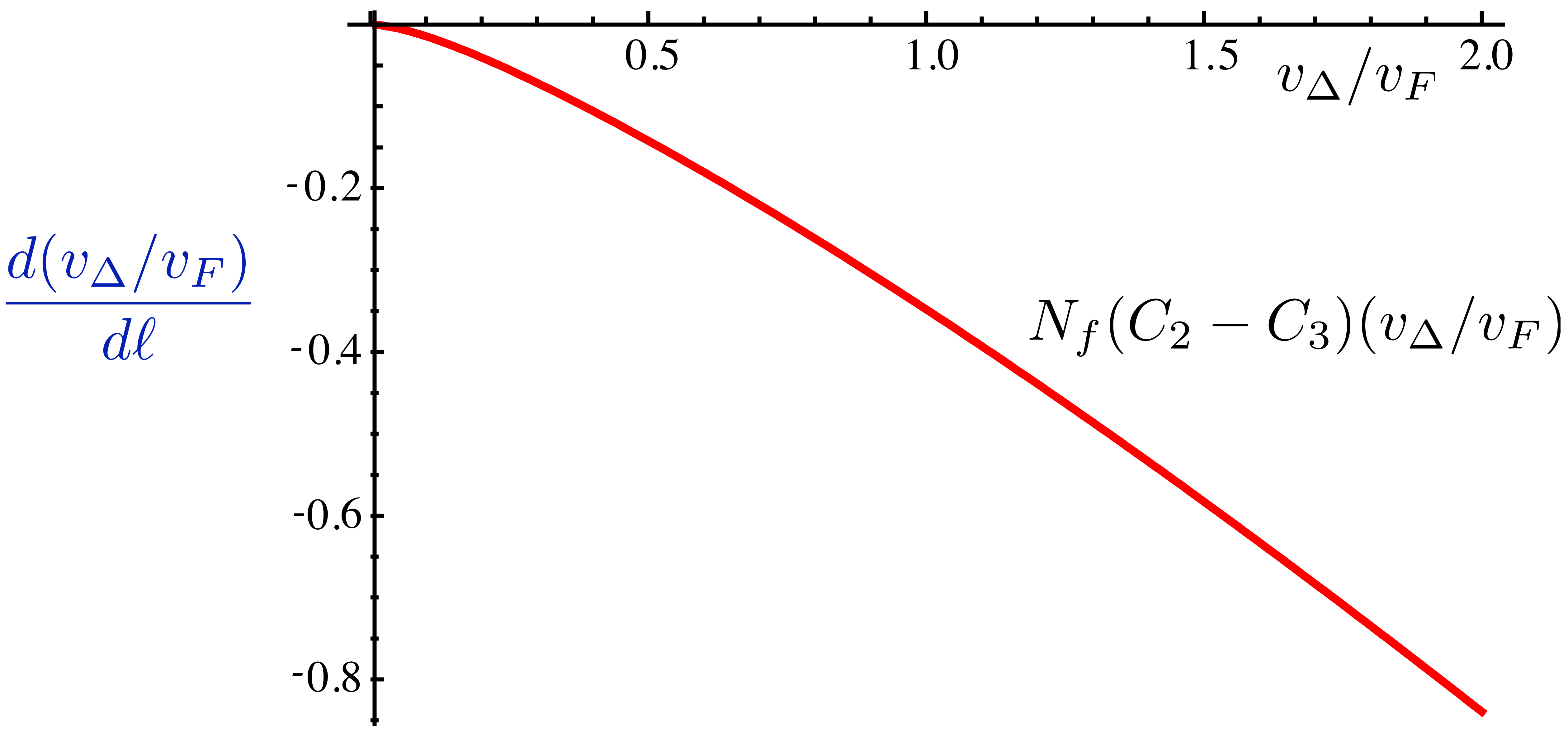
The $1/N_f$ expansion has only one coupling constant at criticality: v_Δ/v_F.

The RG has the structure:

$$\begin{aligned}
\text{dynamic critical exponent : } z &= 1 + \frac{1}{N_f} F_1(v_\Delta/v_F) \\
\text{fermion anomalous dimension : } \eta_f &= \frac{1}{N_f} F_2(v_\Delta/v_F) \\
\text{RG flow equation : } \frac{d(v_\Delta/v_F)}{d\ell} &= \frac{1}{N_f} F_3(v_\Delta/v_F)
\end{aligned}$$

where we have computed the functions $F_{1,2,3}(v_\Delta/v_F)$.

Renormalization group analysis



Renormalization group analysis

The RG flow is to $v_\Delta/v_F \to 0$ with

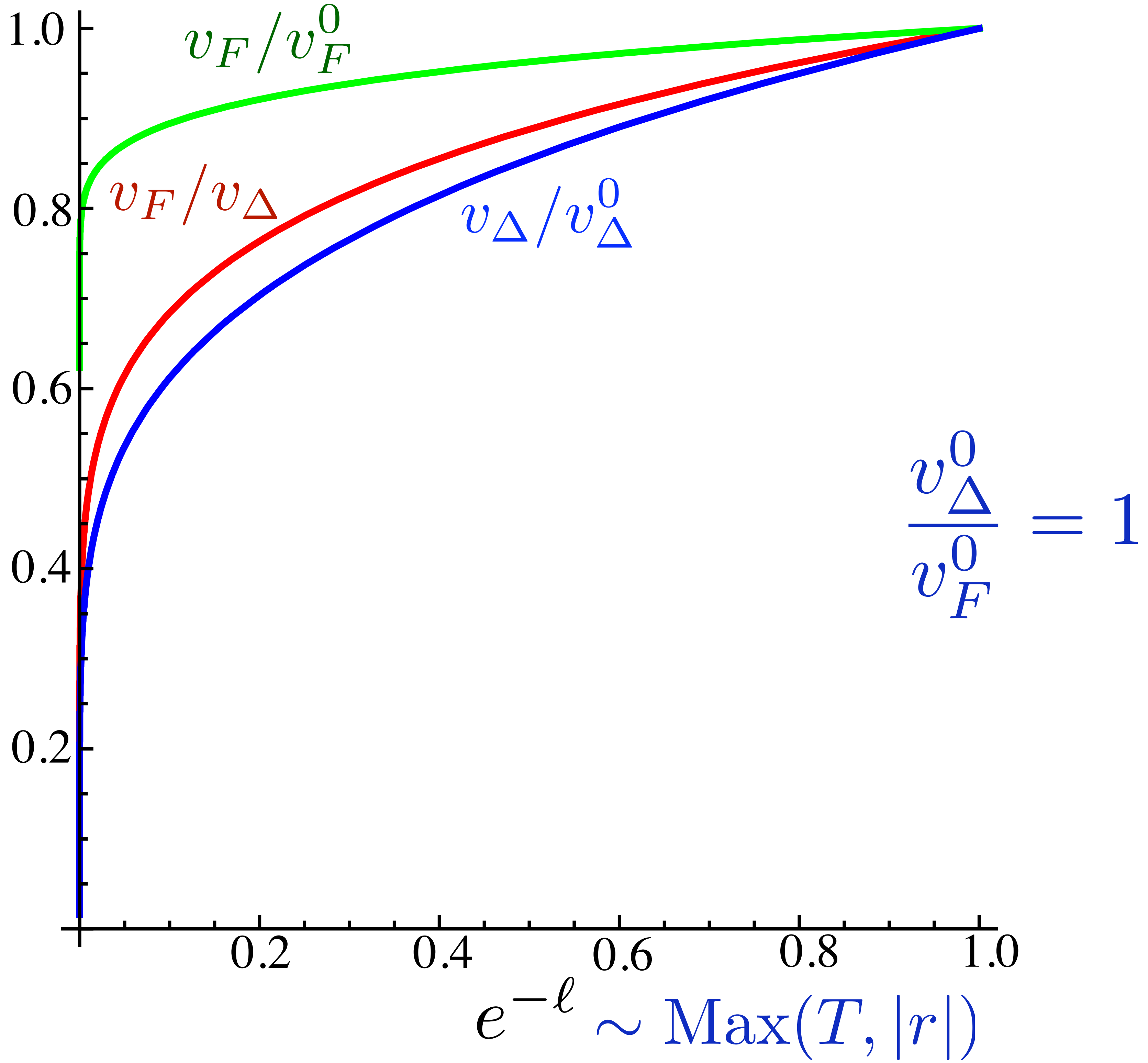
$$\frac{d(v_\Delta/v_F)}{d\ell} = -\frac{8}{\pi^2 N_f}(v_\Delta/v_F)^2 \ln\left(\frac{0.4699}{(v_\Delta/v_F)}\right)$$

This implies that as we approach the critical point, $r \to 0$, $T \to 0$,

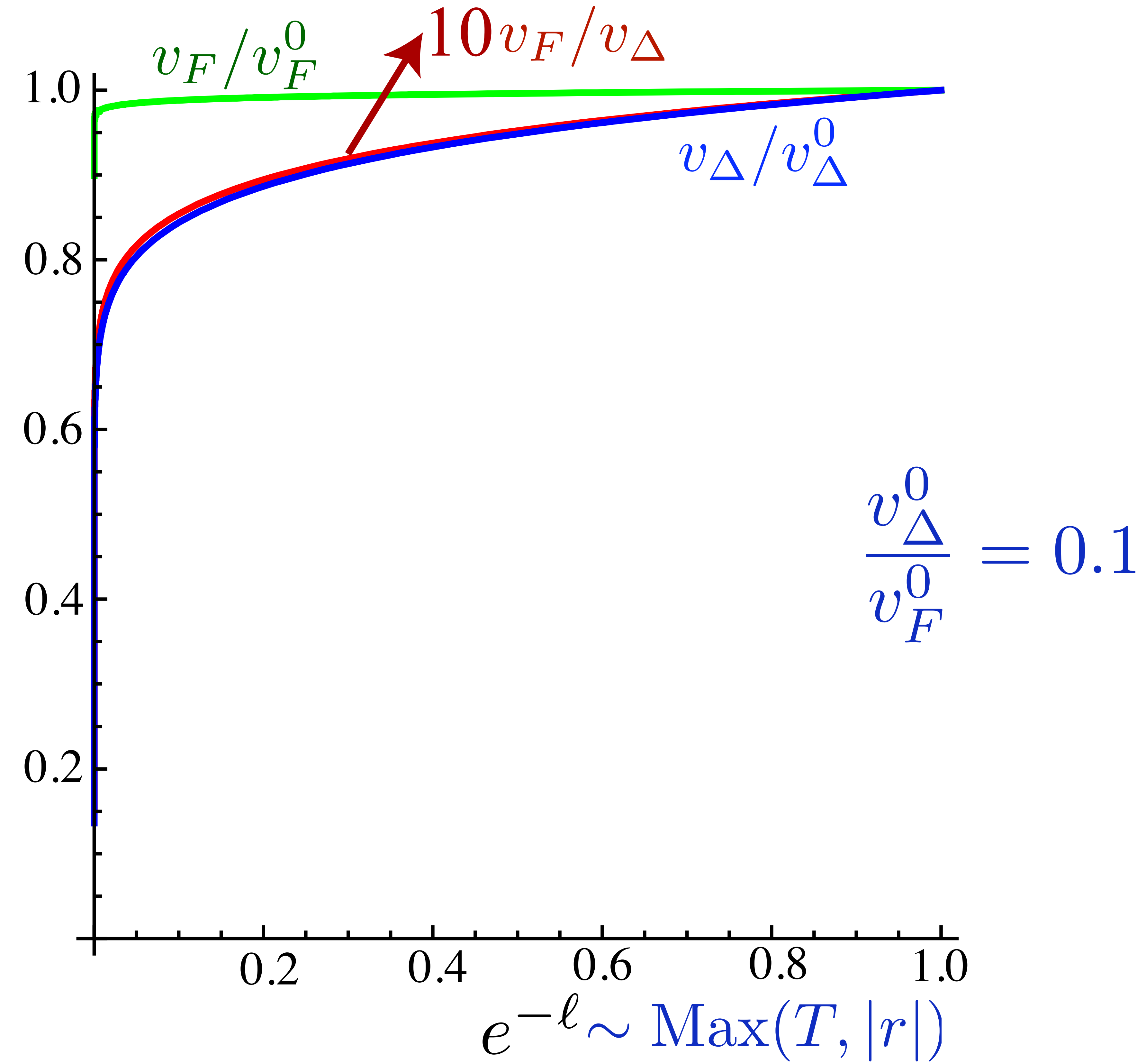
$$\frac{v_\Delta}{v_F} = \frac{\pi^2 N_f}{8} \frac{1}{\ln\left(\frac{\Lambda}{\mathrm{Max}(|r|,T)}\right) \ln\left[\frac{0.3809}{N_f} \ln\left(\frac{\Lambda}{\mathrm{Max}(|r|,T)}\right)\right]}$$

So v_Δ/v_F has a minimum as a function of r at the quantum critical point. More precise results are obtained by a numerical integration of the RG equation.

Renormalization group analysis

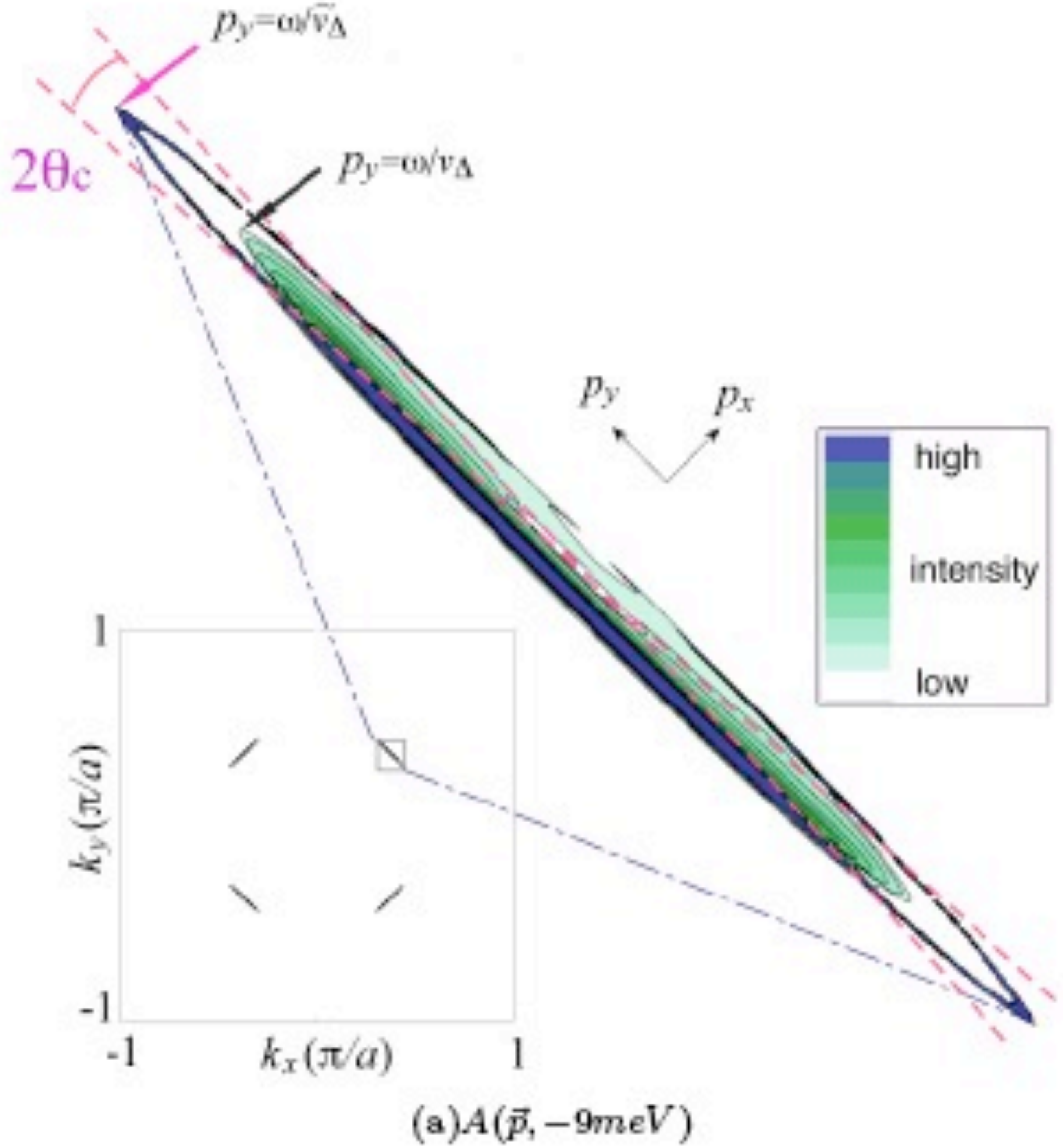


Renormalization group analysis



Fermion spectral functions

ϕ fluctuations broaden the fermion spectral functions except in a wedge near the nodal points



E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, Phys. Rev. B **77**, 184514 (2008).