Quantum phase transitions of insulators, superconductors and metals in two dimensions

Talk online: sachdev.physics.harvard.edu

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Outline

1. Phenomenology of the cuprate superconductors
   (and other compounds)

2. QPT of antiferromagnetic insulators
   (and bosons at rational filling)

3. QPT of d-wave superconductors:
   Fermi points of massless Dirac fermions

4. QPT of Fermi surfaces:
   A. Finite wavevector ordering (SDW/CDW):
      "Hot spots" on Fermi surfaces
   B. Zero wavevector ordering (Nematic):
      "Hot Fermi surfaces"

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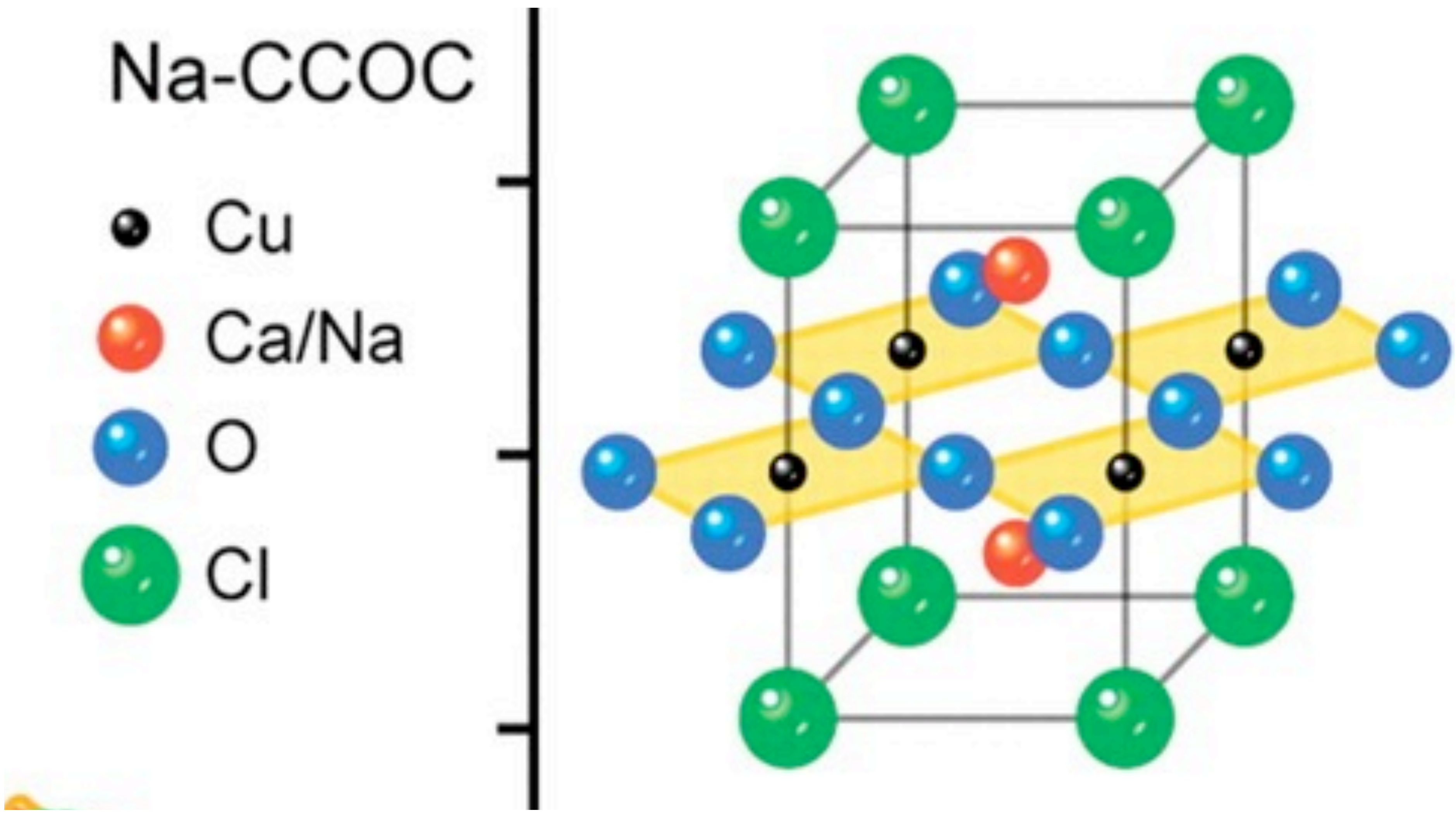
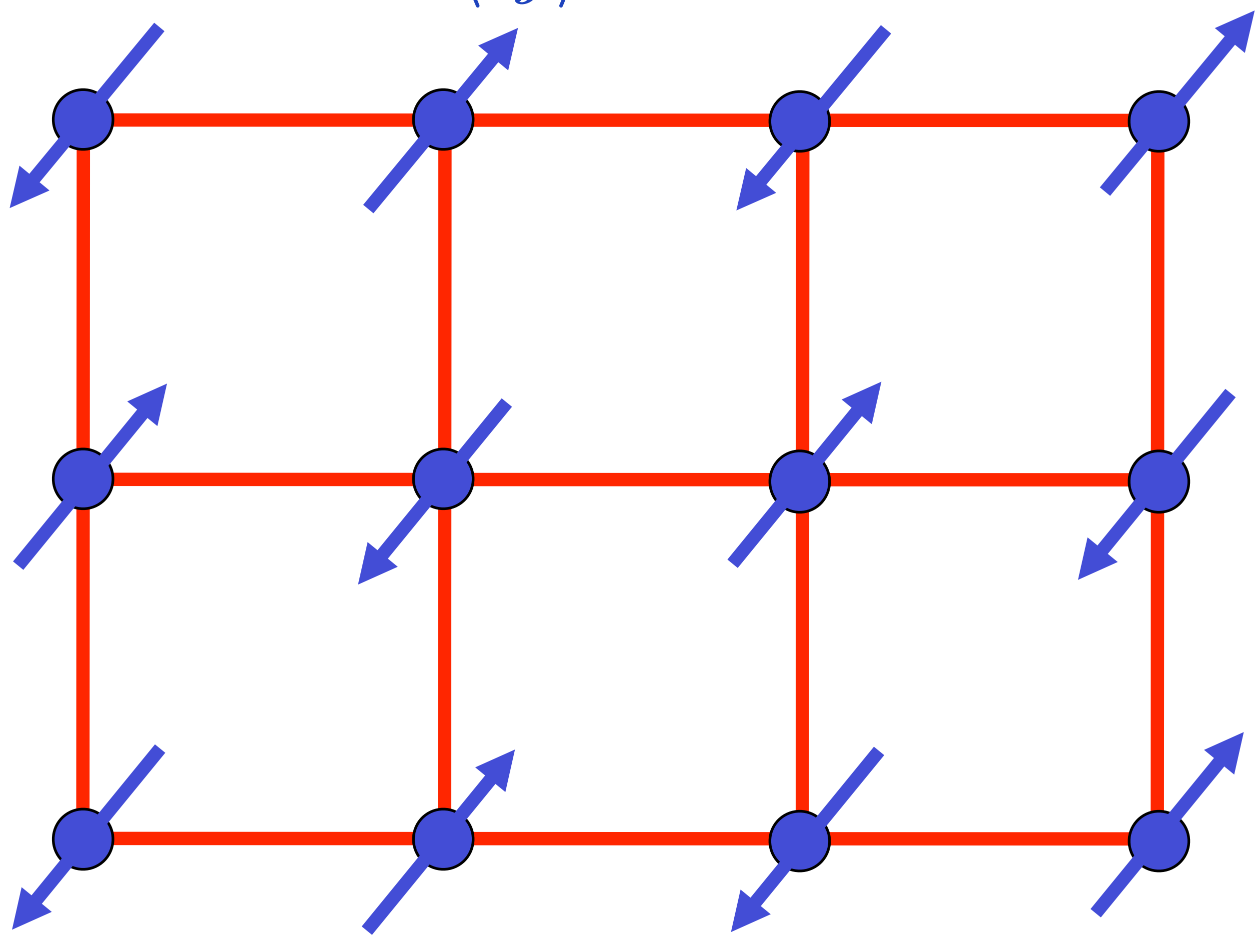
The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



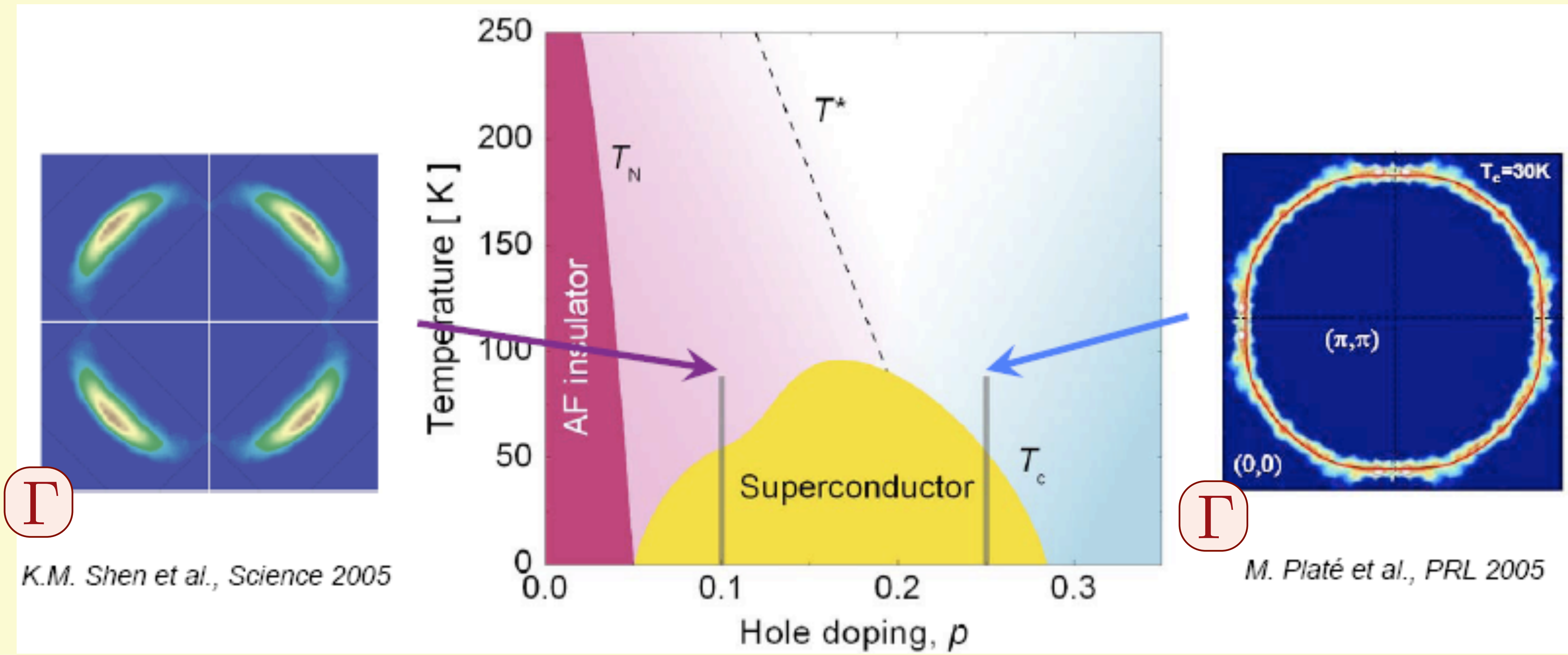
Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Smaller hole Fermi-pockets

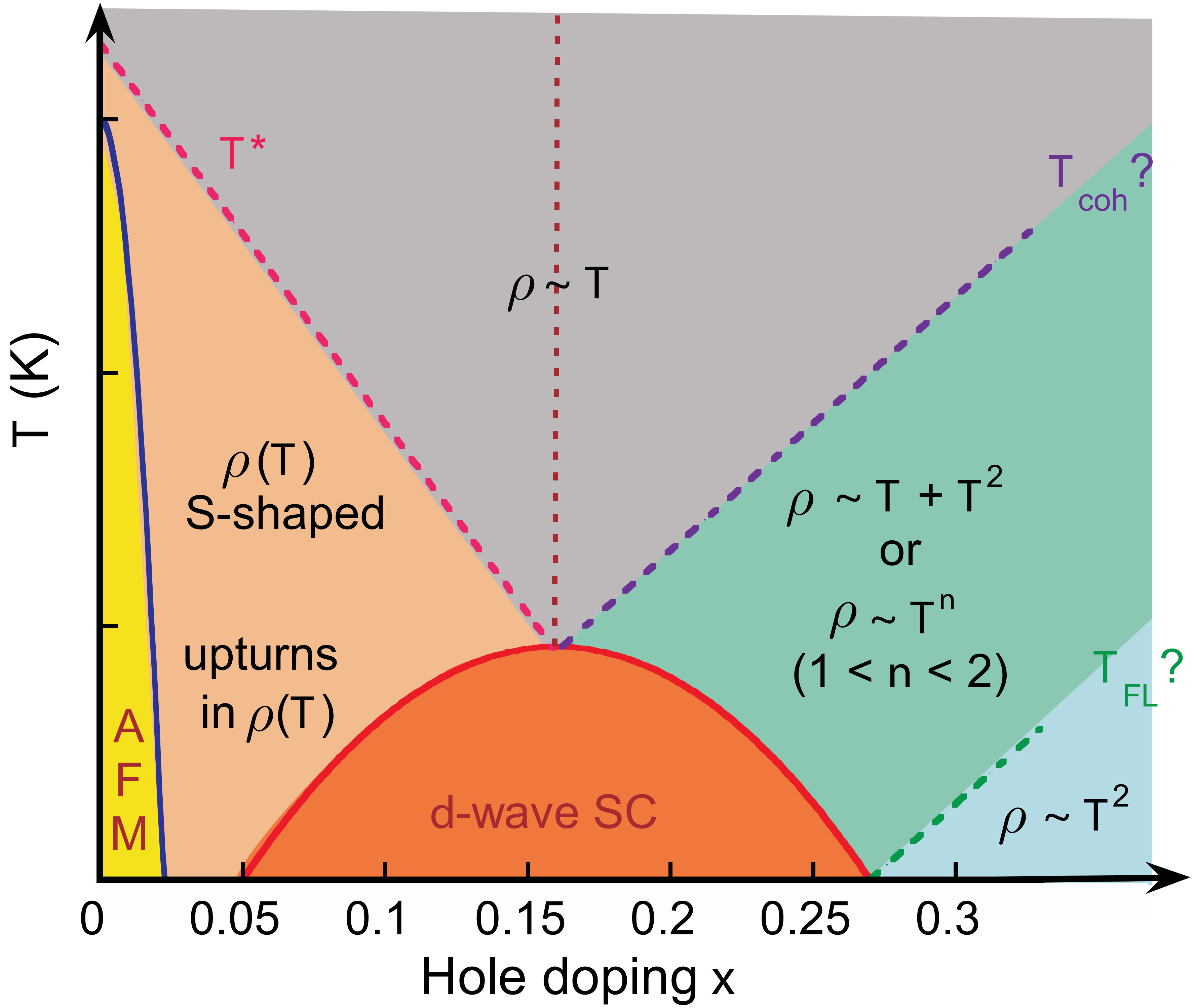
Large hole Fermi surface

Antiferro-
magnetism

d-wave
supercon-
ductivity

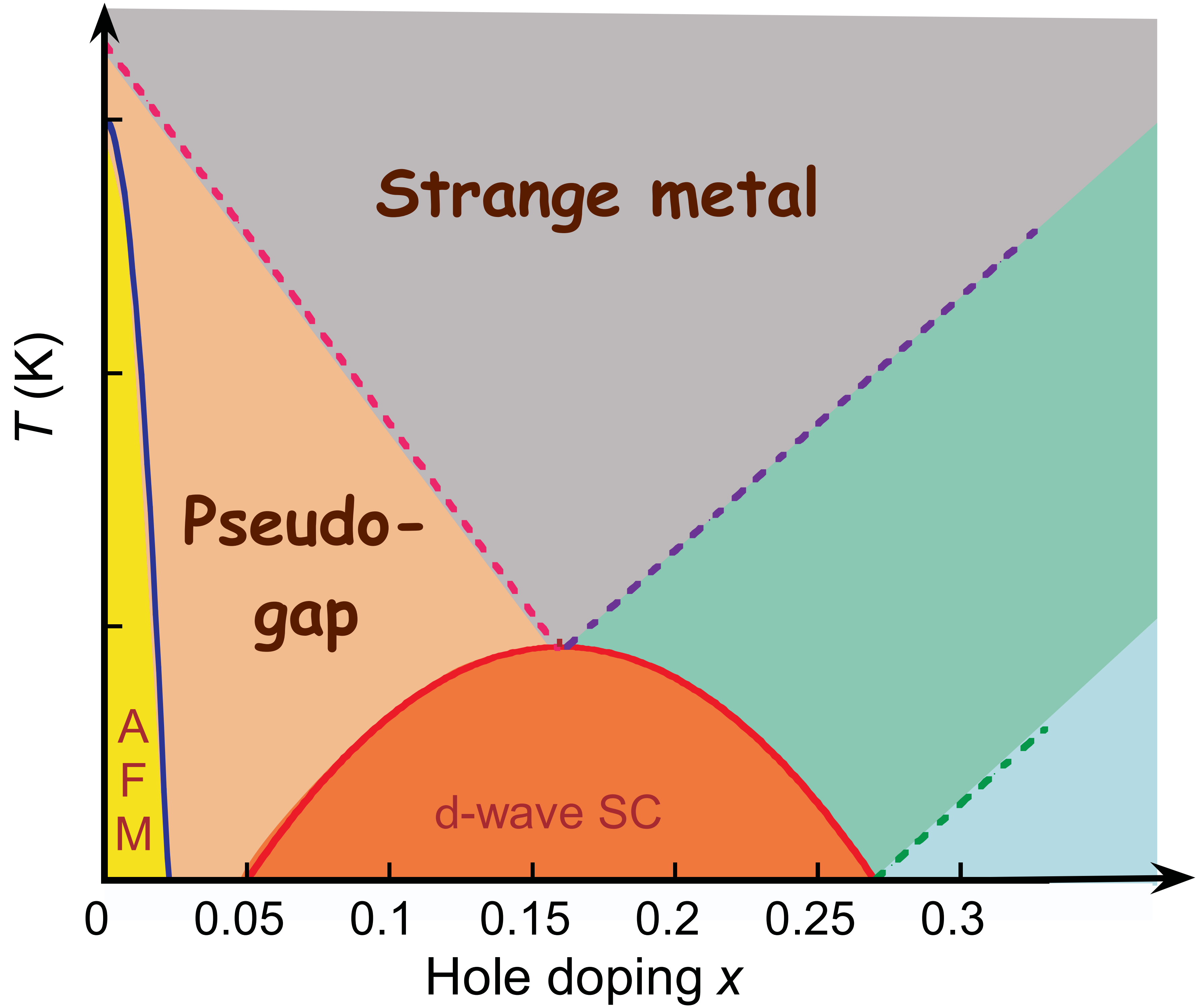
Fermi
surface

Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



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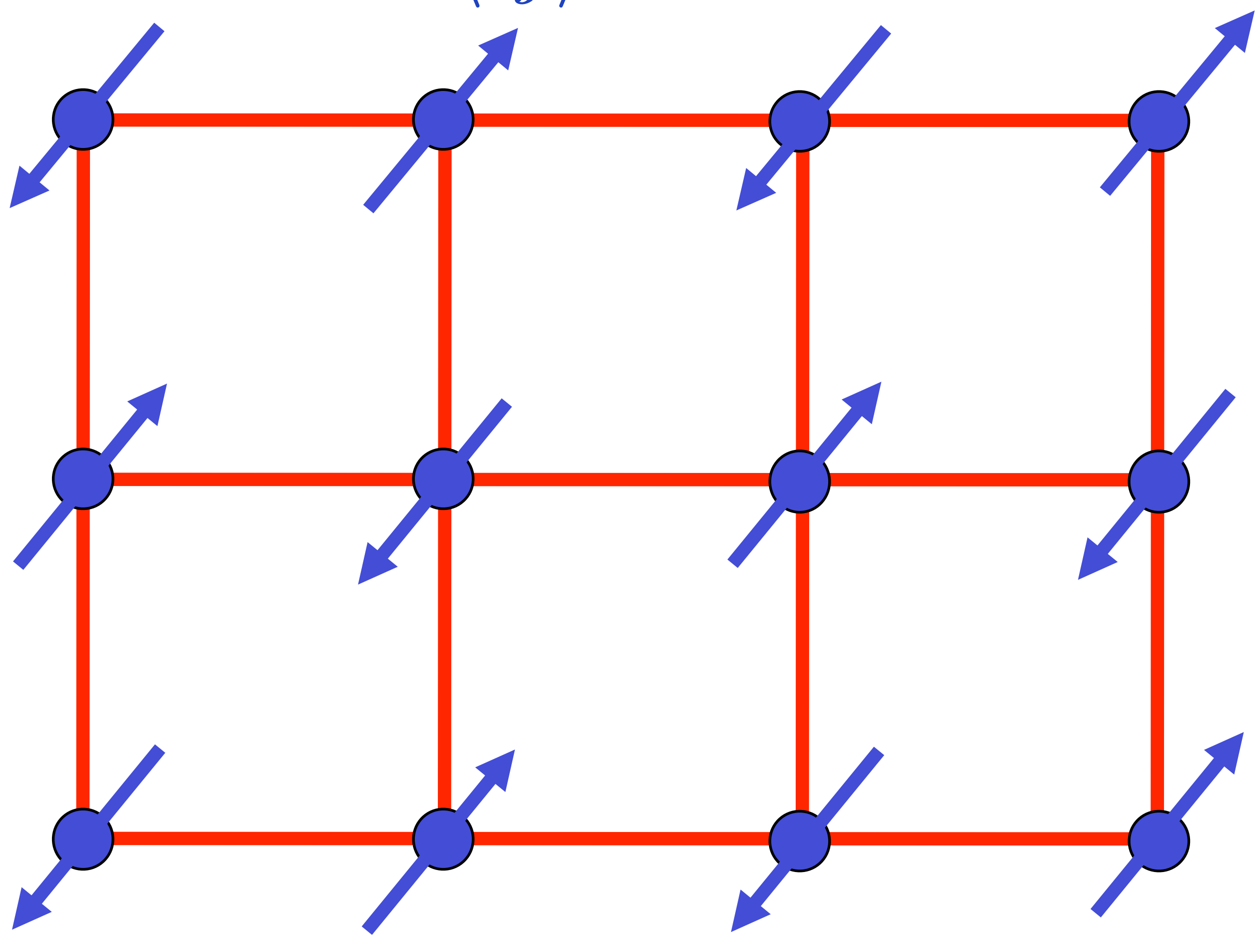

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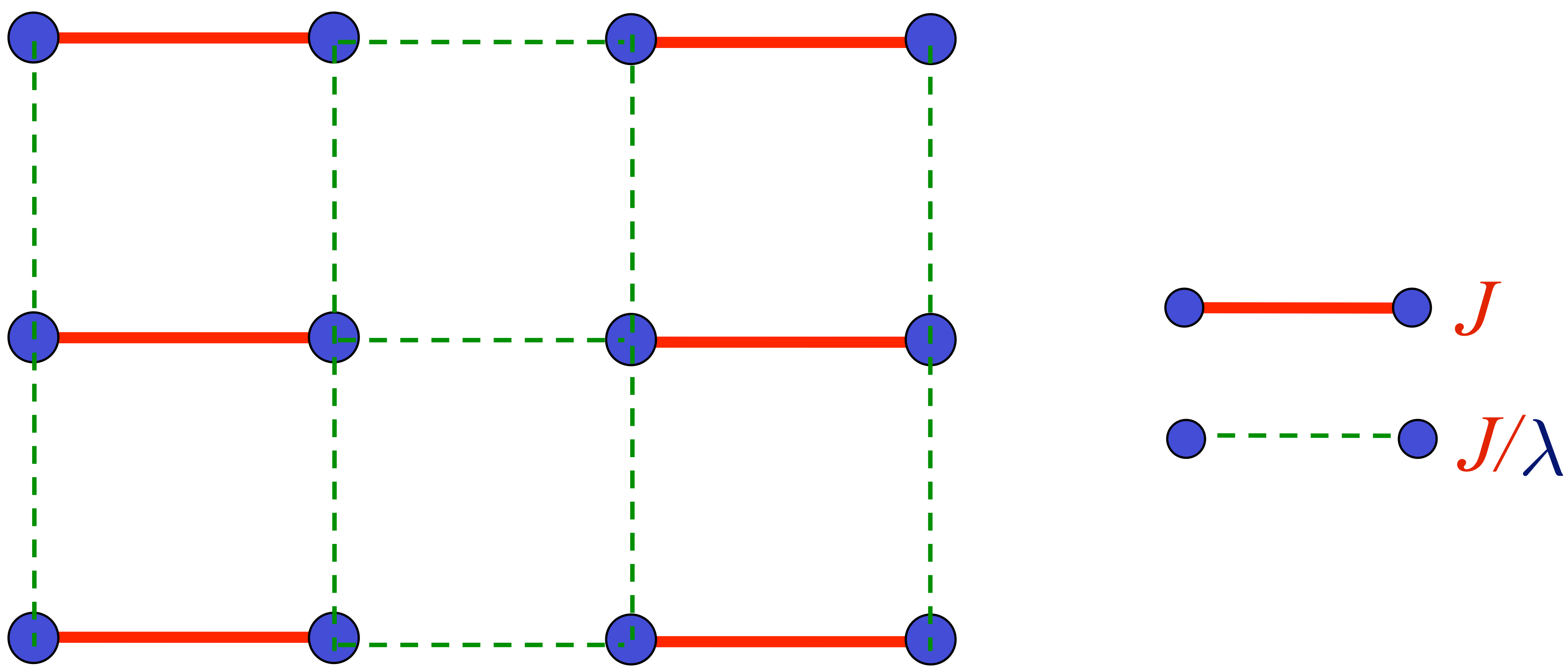
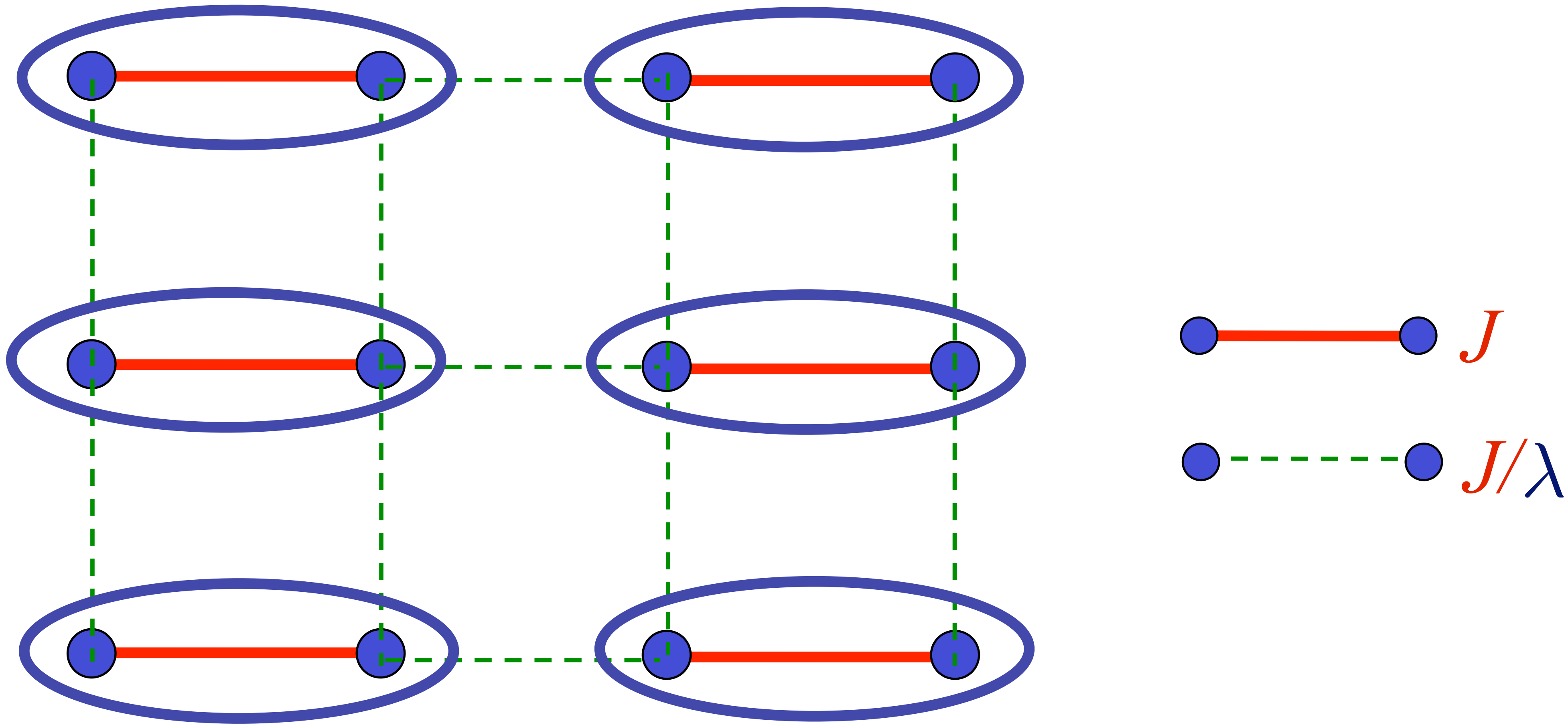


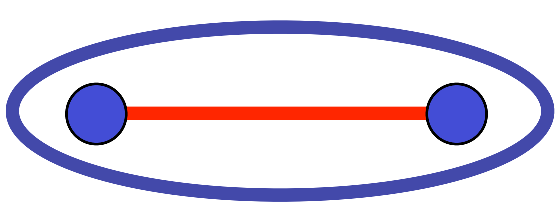
Weaken some bonds to induce spin entanglement in a new quantum phase

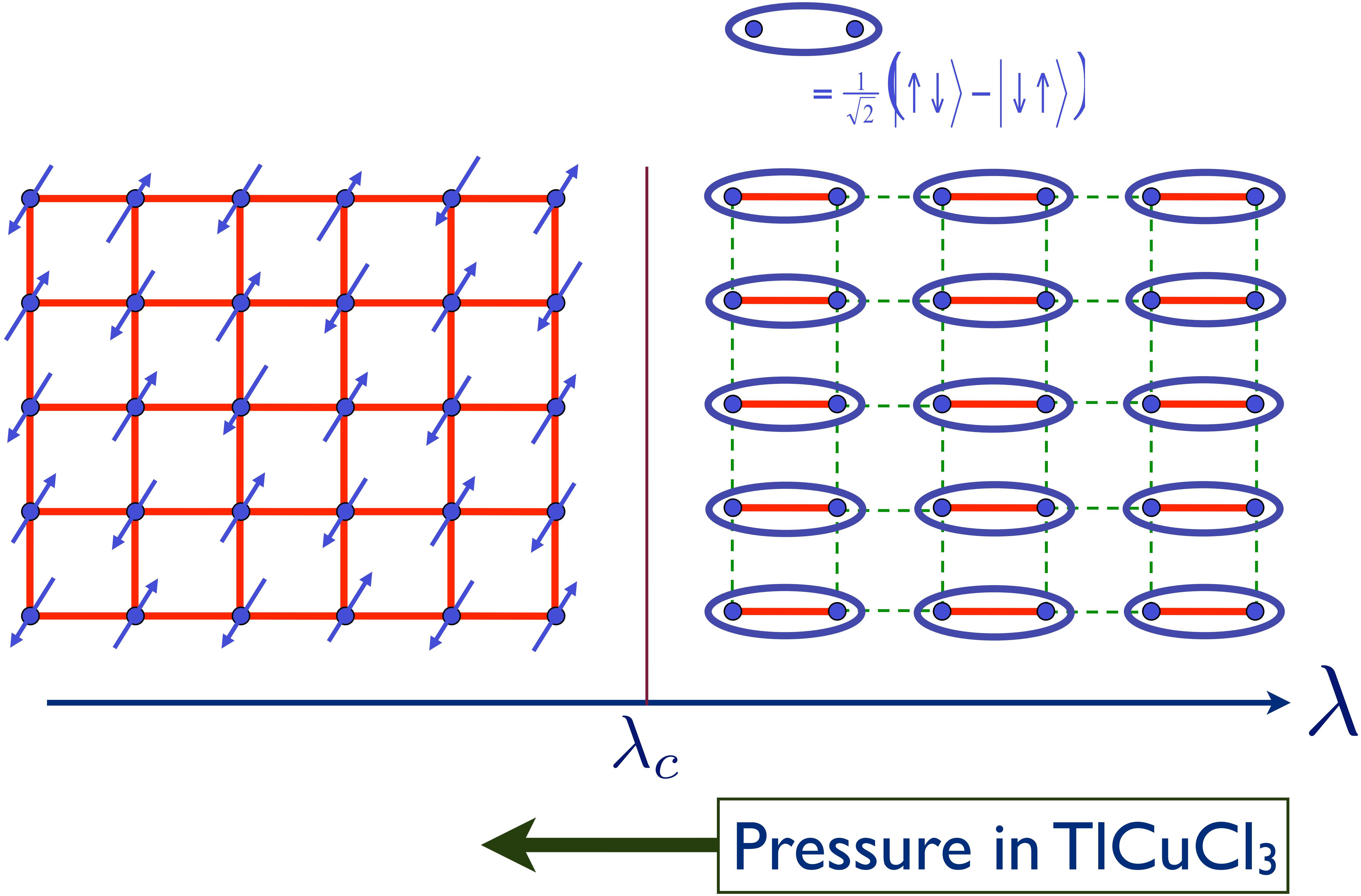
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



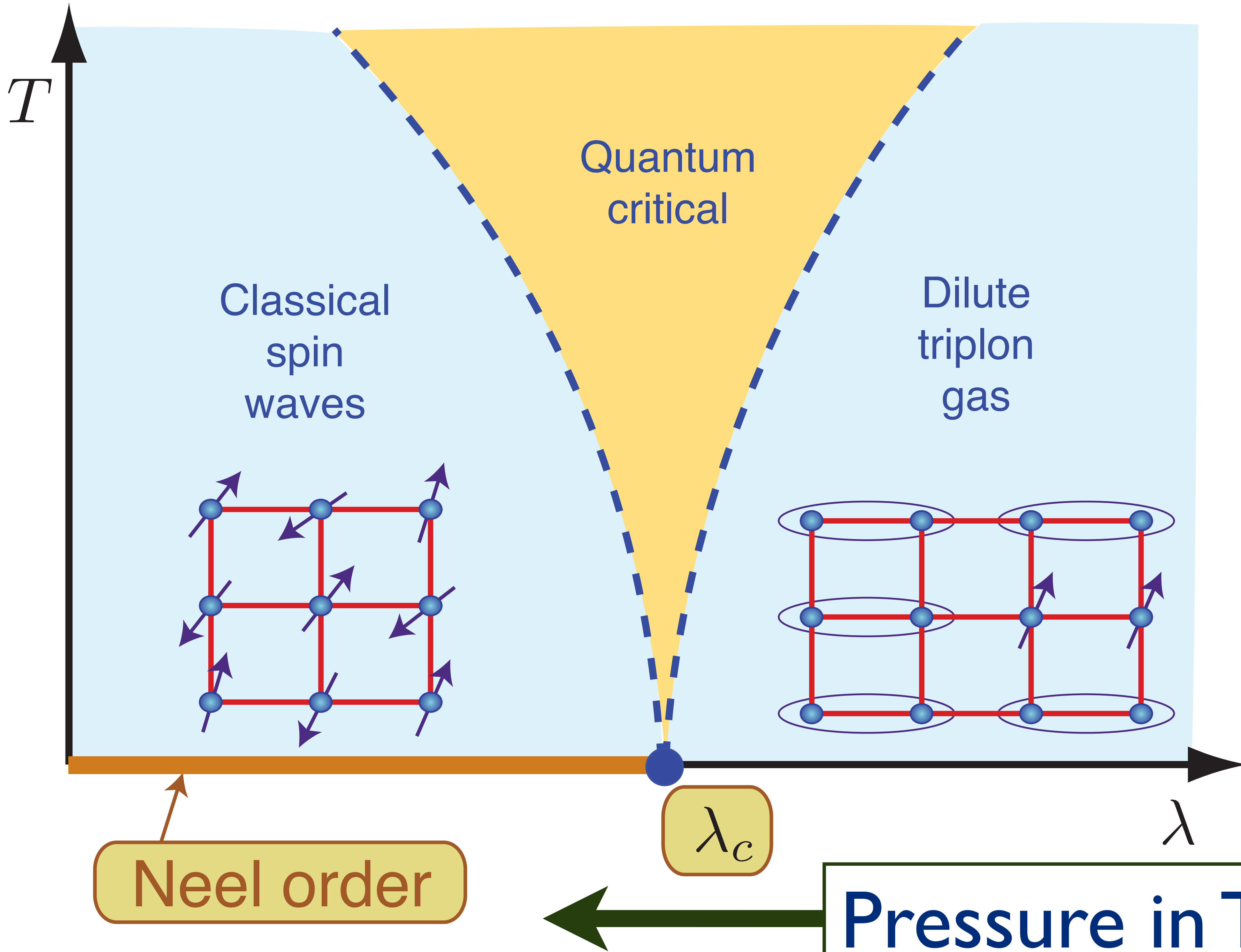
Ground state is a "quantum paramagnet" with spins locked in valence bond singlets

 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

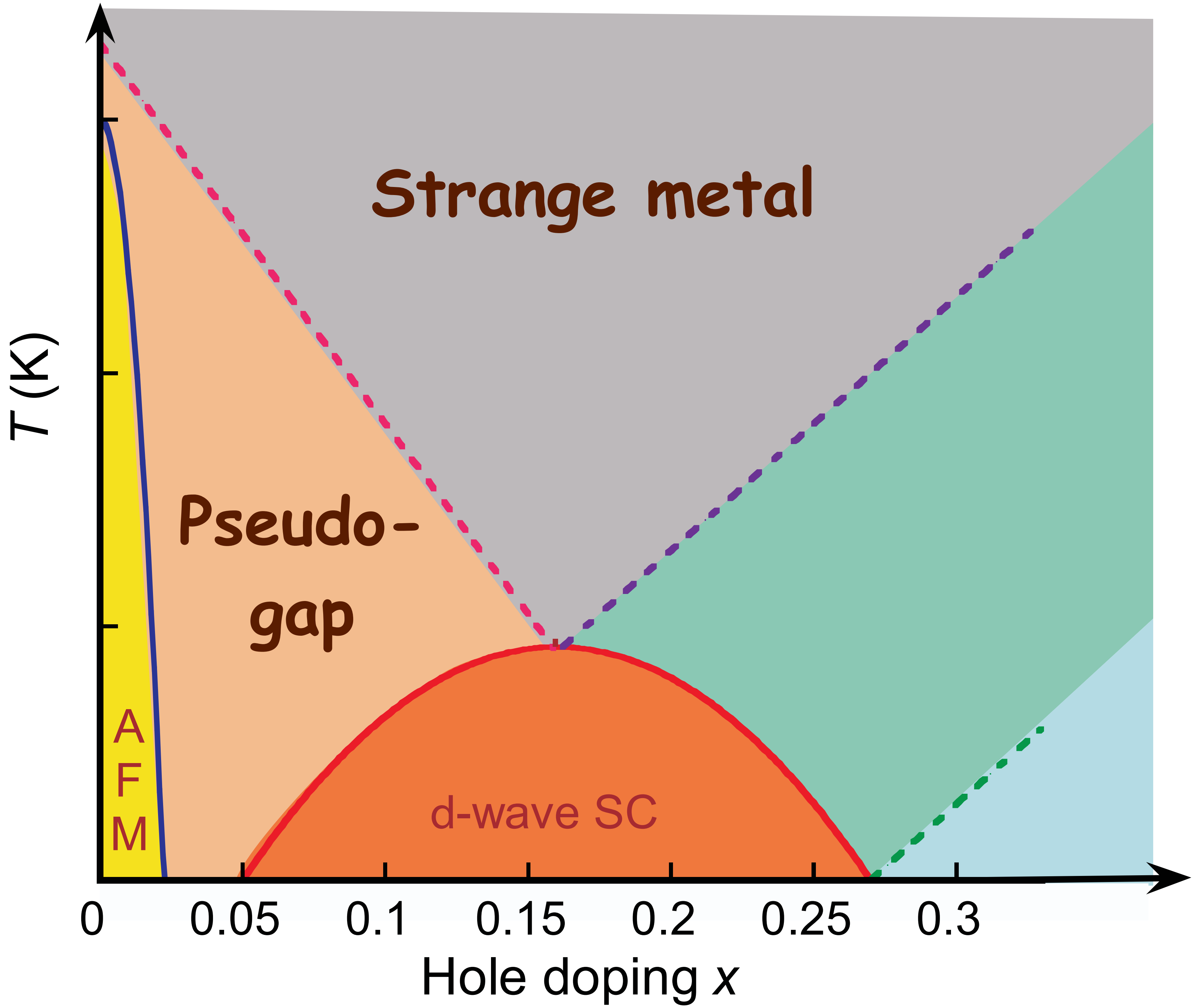
Canonical quantum critical phase diagram of coupled-dimer antiferromagnet



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

Christian Ruegg et al. , *Phys. Rev. Lett.* **100**, 205701 (2008)

Crossovers in transport properties of hole-doped cuprates



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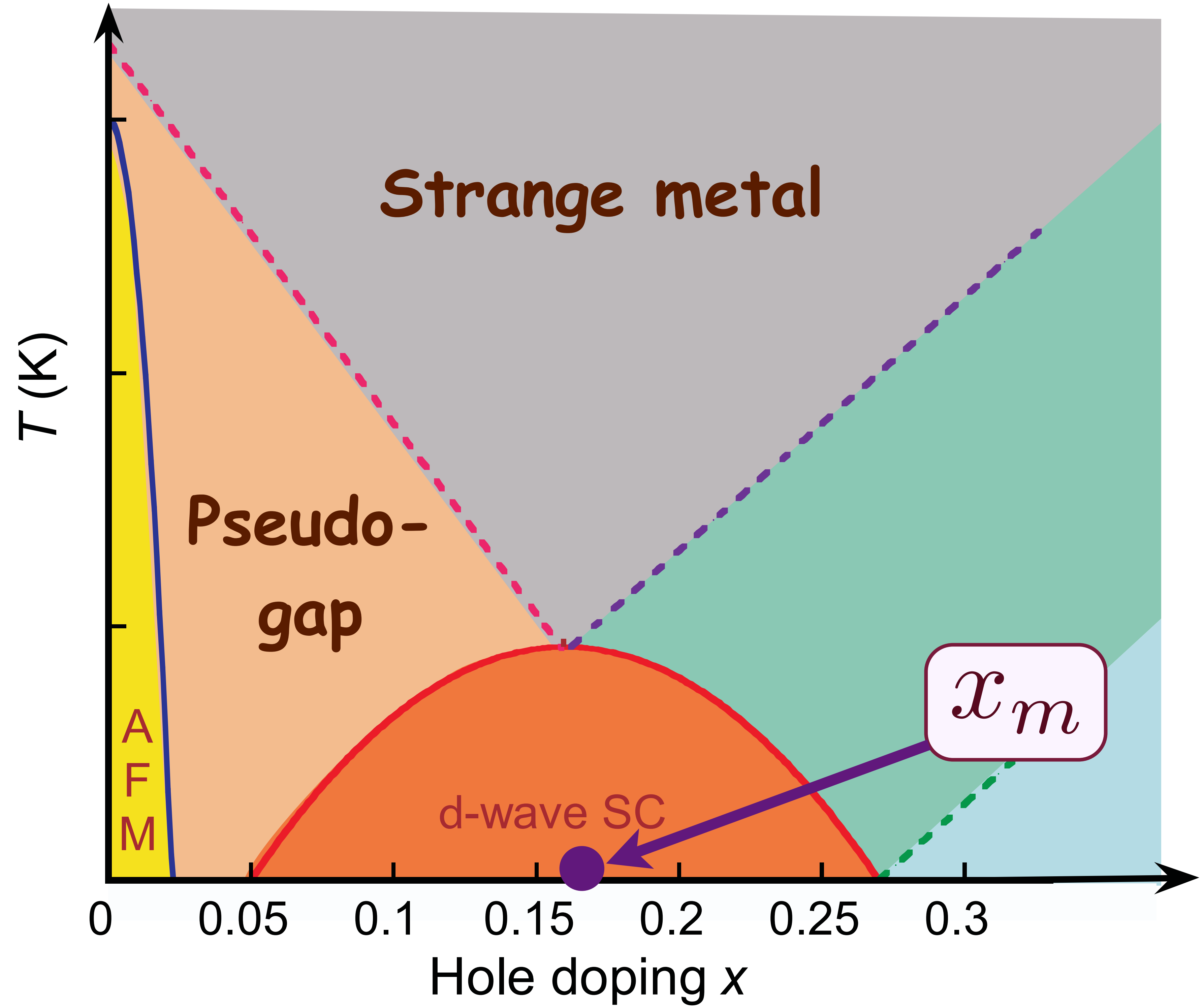


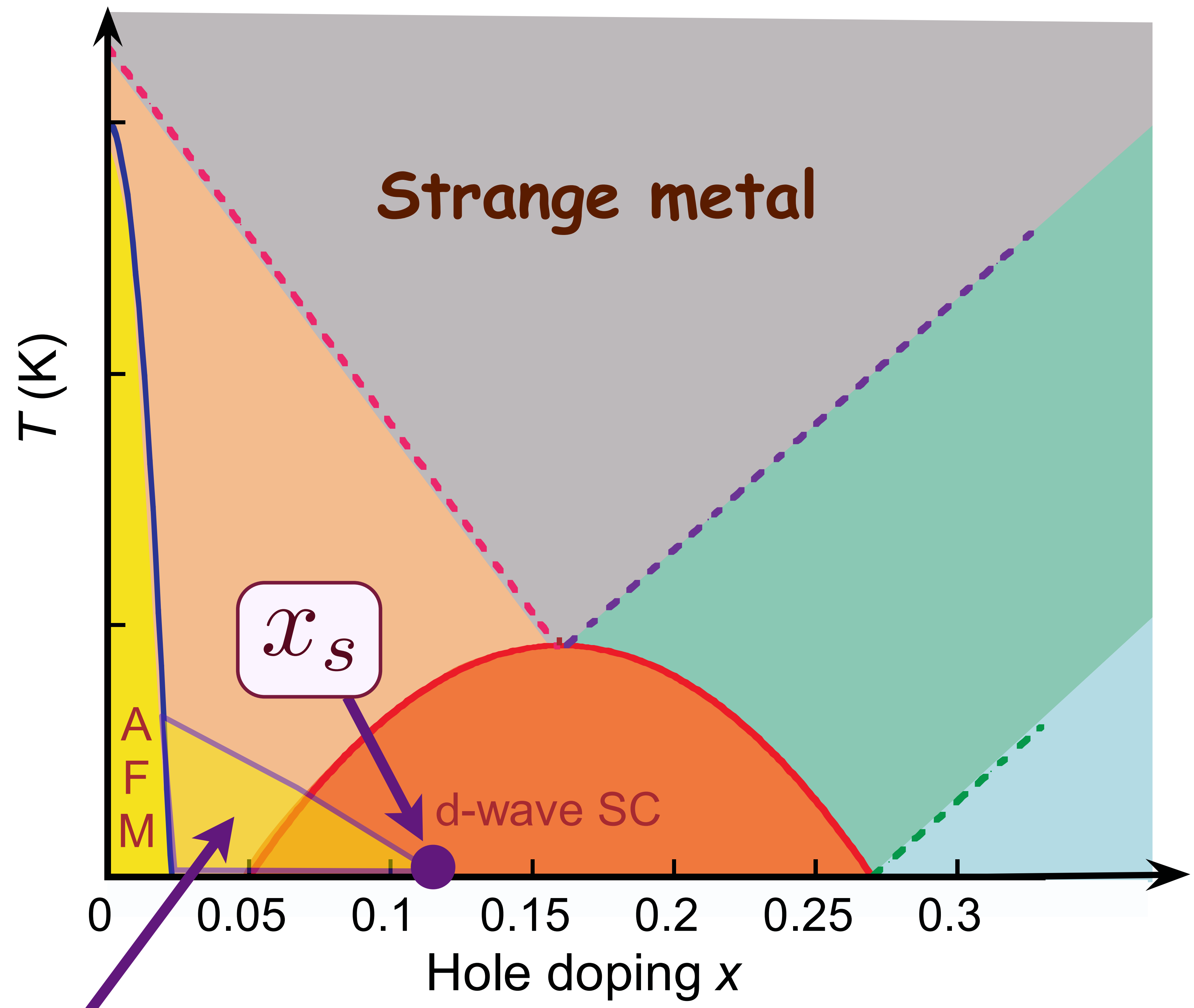
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

A. J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T

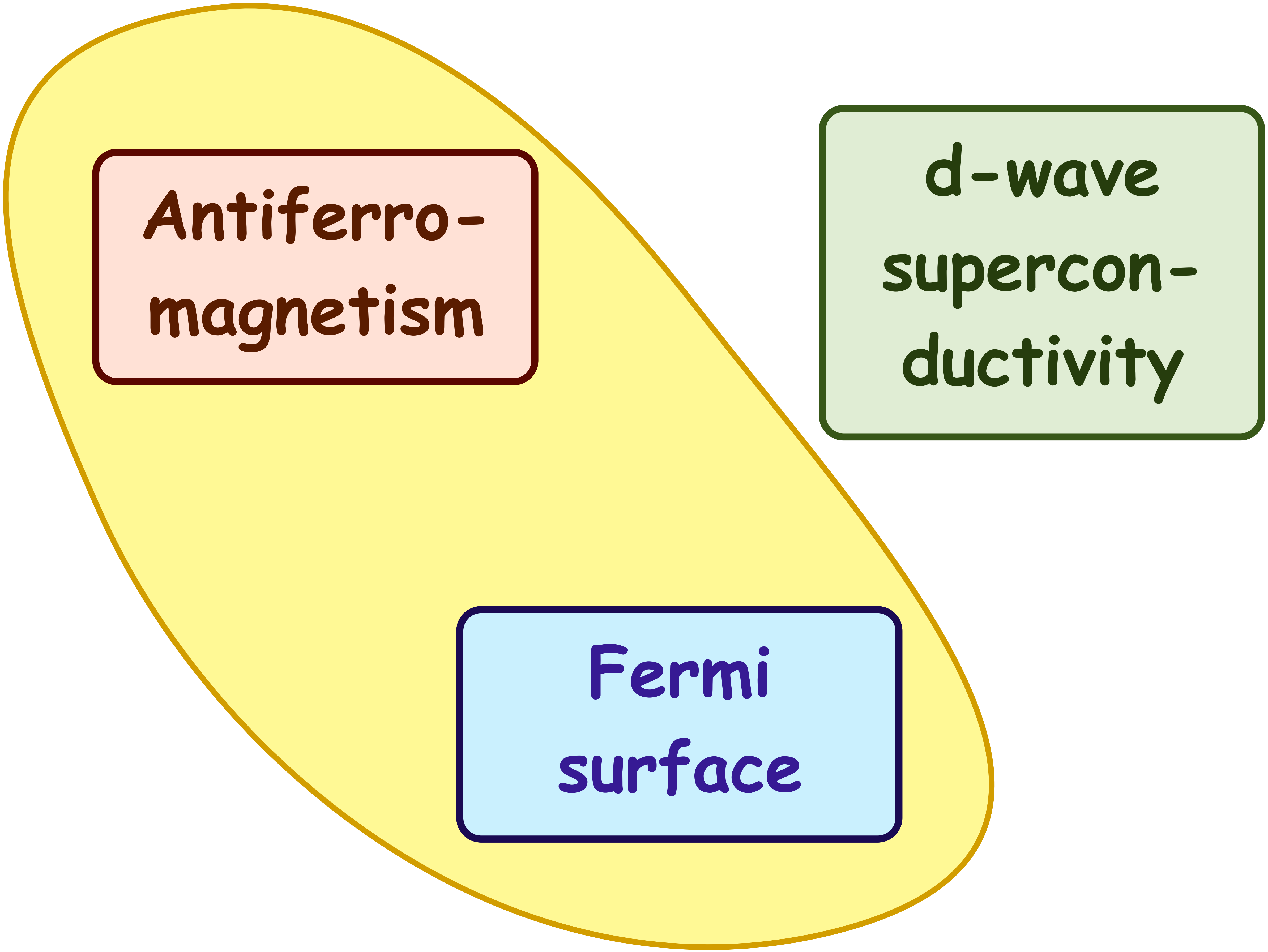


Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Antiferro-
magnetism

d-wave
supercon-
ductivity

Fermi
surface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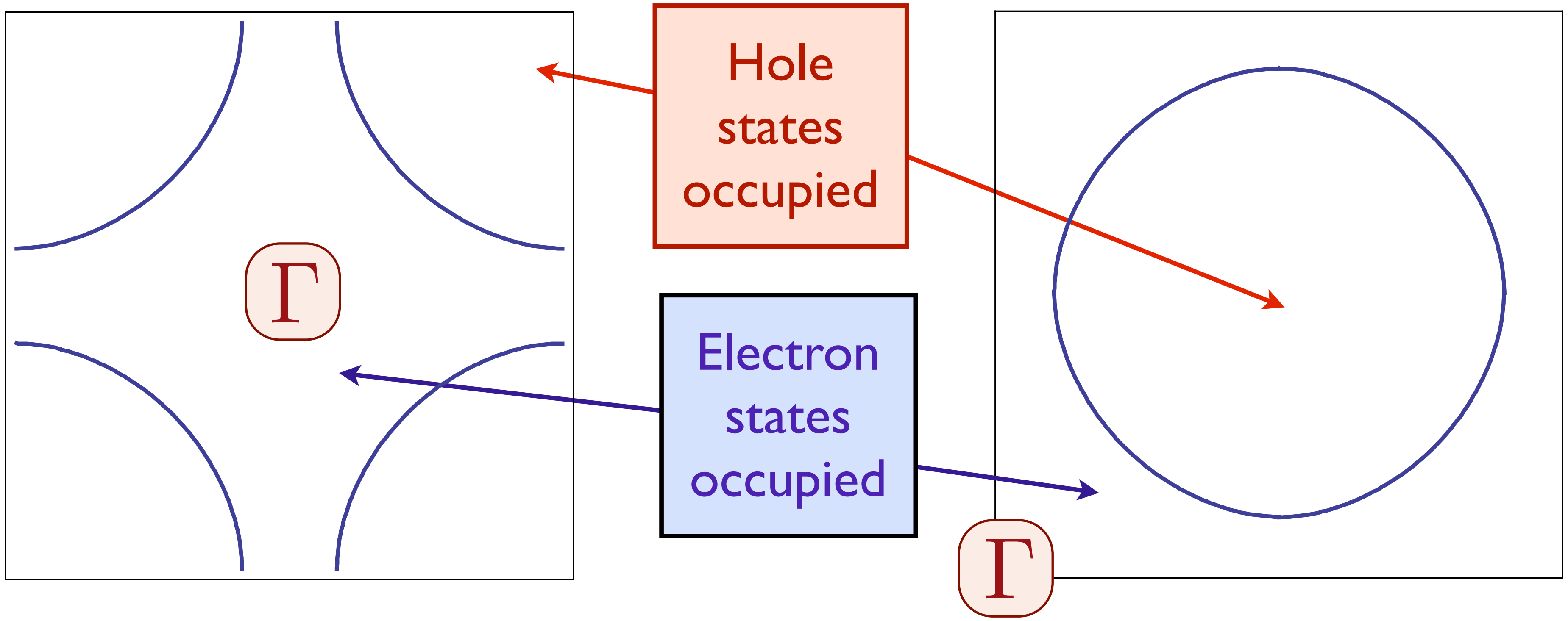
"Large" Fermi surfaces in cuprates



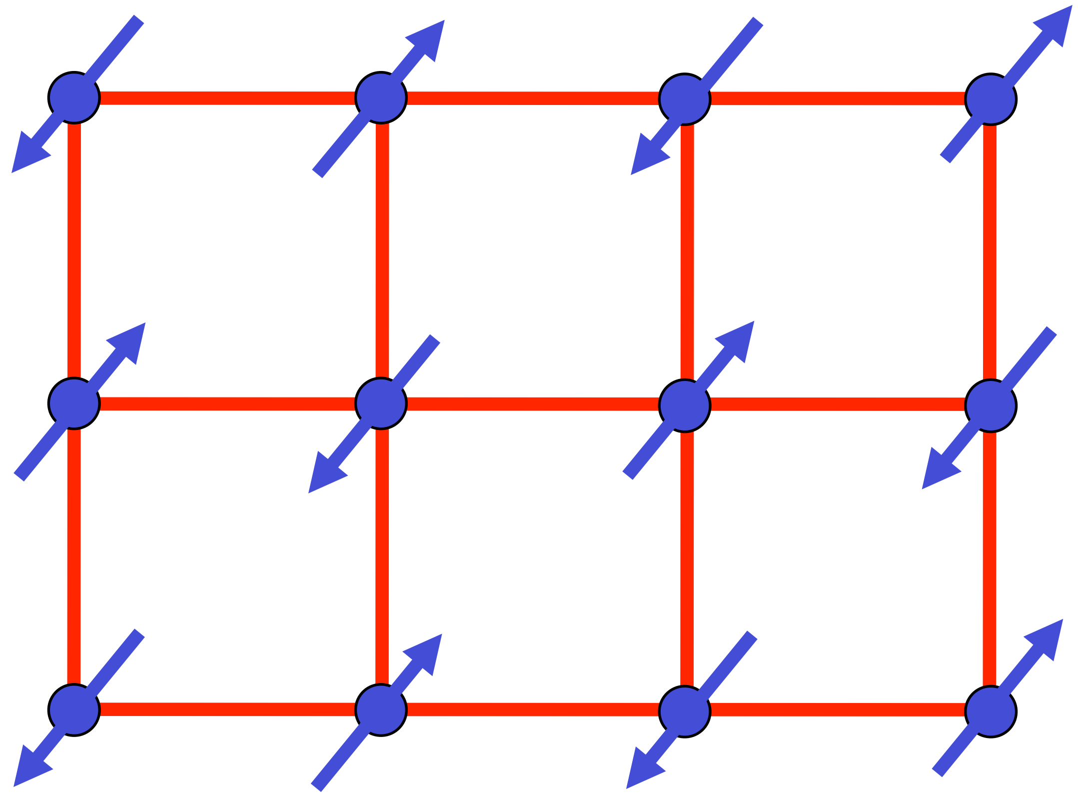
$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

The area of the occupied electron/hole states:

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

$$\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$$

Spin density wave theory

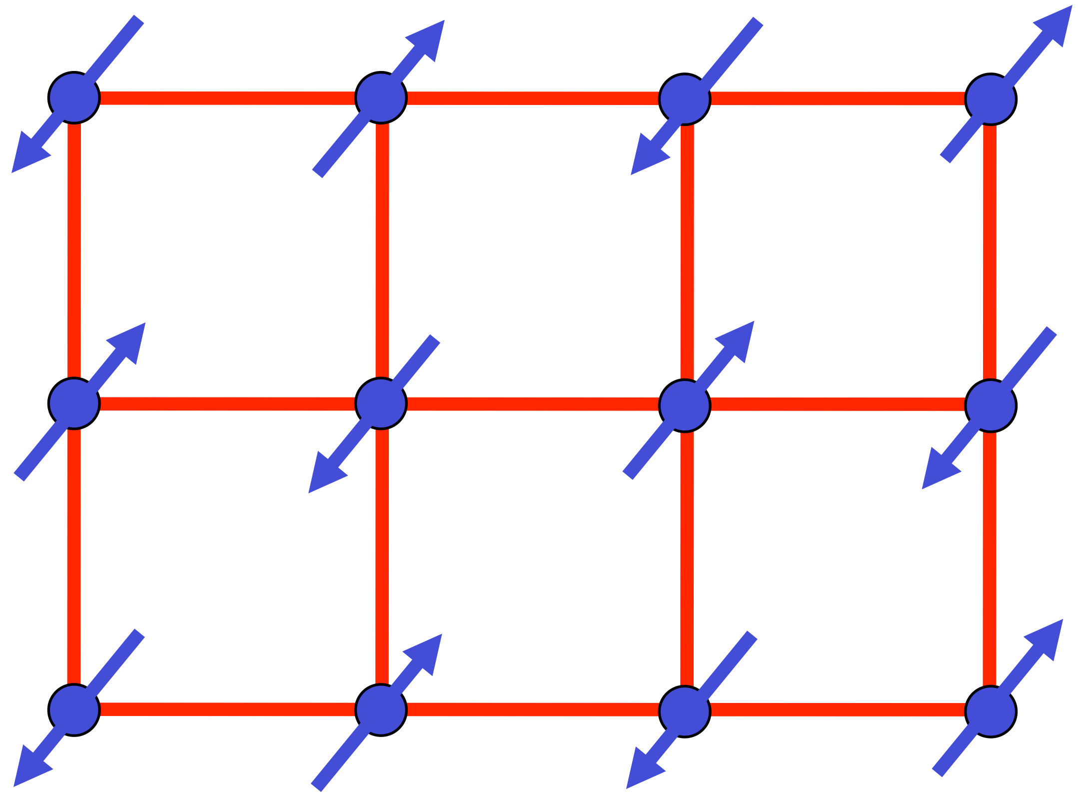


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau)e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the spin density wave (SDW) order parameter, and $\mathbf{K}$ is the ordering wavevector. For simplicity, we consider $\mathbf{K} = (\pi, \pi)$.

Spin density wave theory



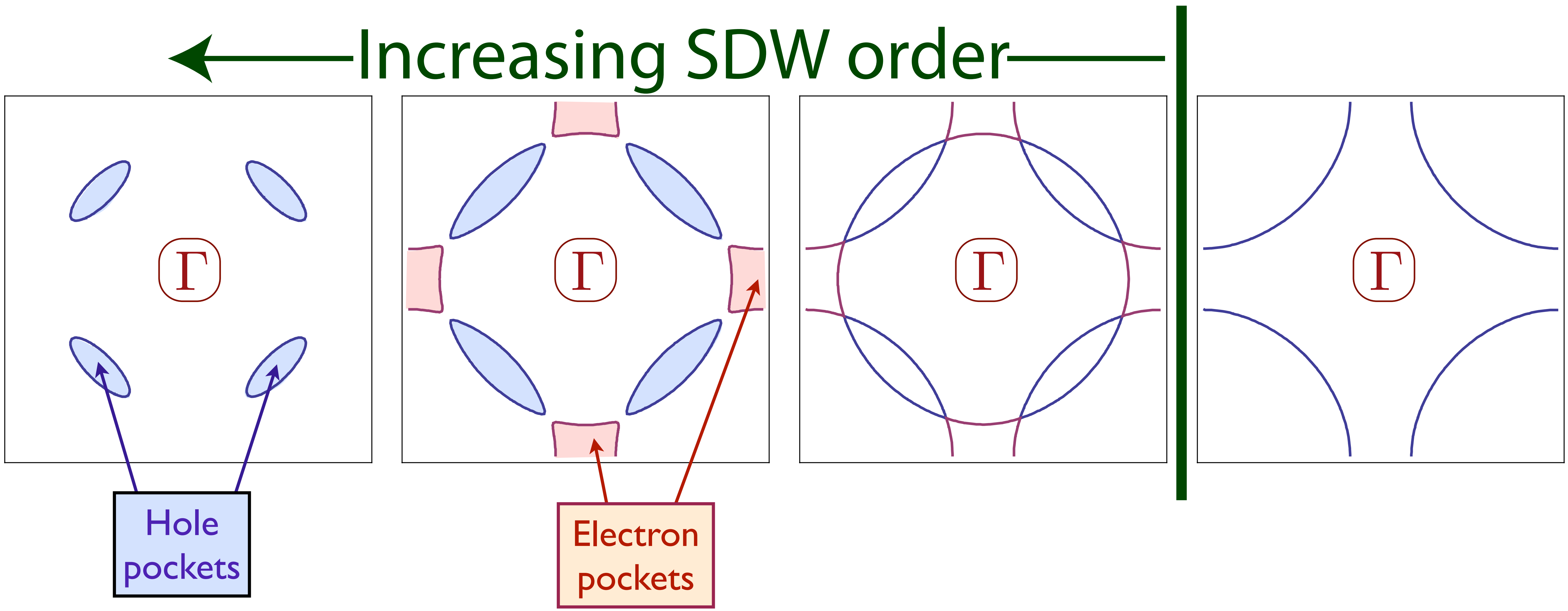
Spin density wave Hamiltonian

$$H_{\text{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

Diagonalize $H_0 + H_{\text{sdw}}$ for $\vec{\varphi} = (0, 0, \varphi)$

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

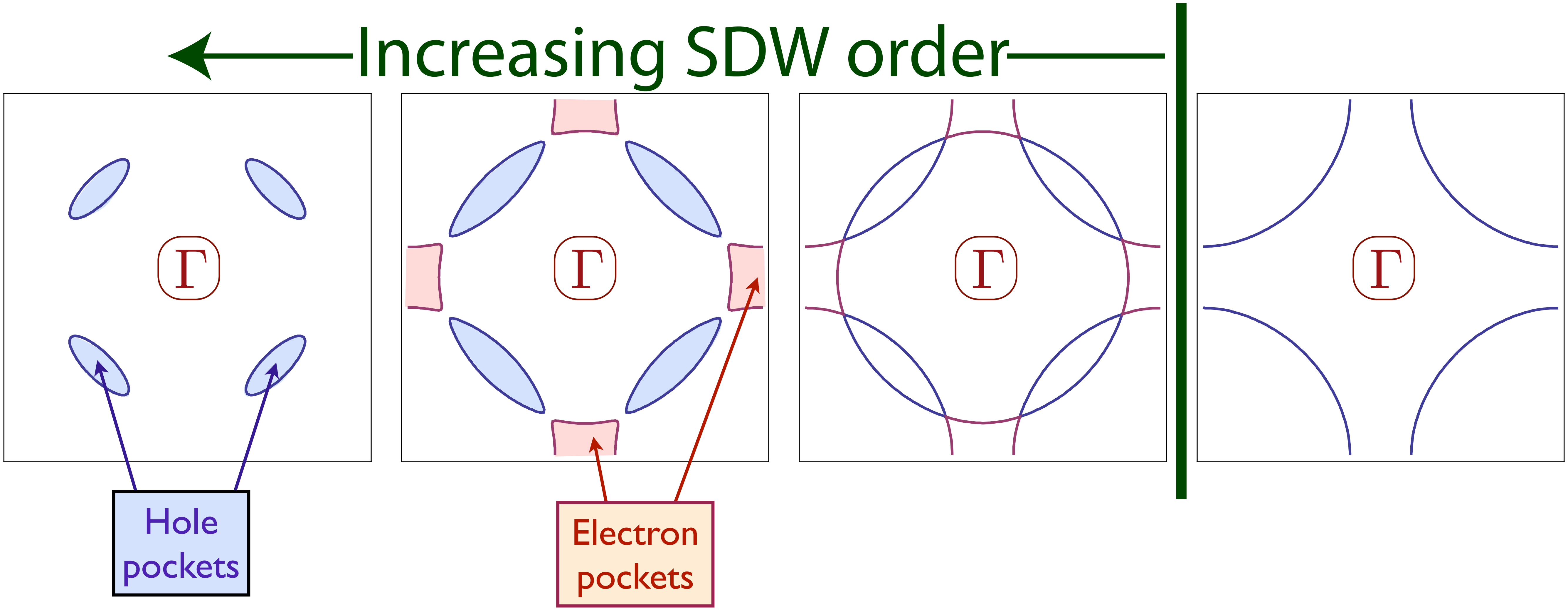
Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



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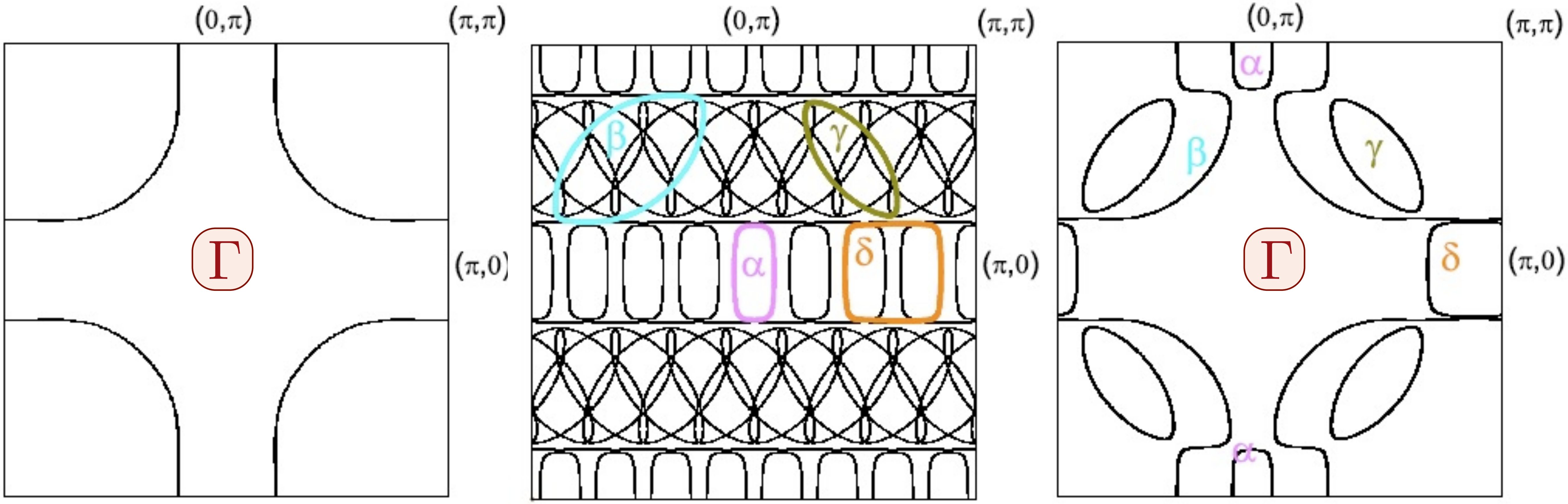
Spin density wave theory in hole-doped cuprates



Incommensurate order in $YBa_2Cu_3O_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Spin density wave theory in hole-doped cuprates

For incommensurate ordering, the SDW order parameter consists of 2 complex 3-component vectors $\vec{\Phi}_x$, $\vec{\Phi}_y$:

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\Phi}_x(\mathbf{r},\tau) e^{i\mathbf{K}_x \cdot \mathbf{r}} + \vec{\Phi}_y(\mathbf{r},\tau) e^{i\mathbf{K}_y \cdot \mathbf{r}} + \text{c.c.}$$

where $\mathbf{K}_x = (\pi(1-\vartheta), \pi)$ and $\mathbf{K}_y = (\pi, \pi(1-\vartheta))$, with $\vartheta = 1/4$ near $1/8$ doping.

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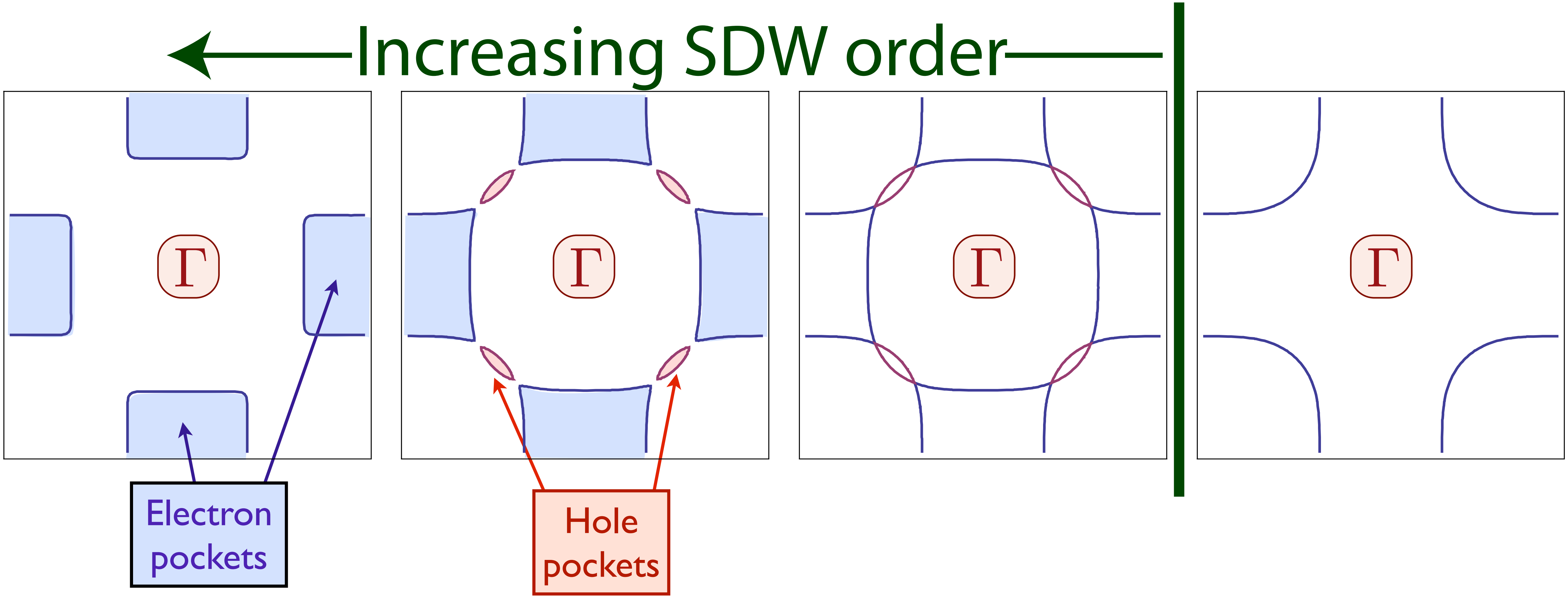
We can also define additional 'composite order parameters' which may be present even though SDW is not long-range:

- Charge density wave orders $\propto \vec{\Phi}_x^2$, $\vec{\Phi}_y^2$

$$\langle \delta\rho(\mathbf{r},\tau) \rangle = \vec{\Phi}_x^2(\mathbf{r},\tau) e^{i2\mathbf{K}_x\cdot\mathbf{r}} + \vec{\Phi}_y^2(\mathbf{r},\tau) e^{i2\mathbf{K}_y\cdot\mathbf{r}} + \text{c.c.}$$

- Nematic order $\propto |\vec{\Phi}_x|^2 - |\vec{\Phi}_y|^2$. This ordering breaks the $x \leftrightarrow y$ symmetry of the lattice, but need not have SDW or CDW ordering.

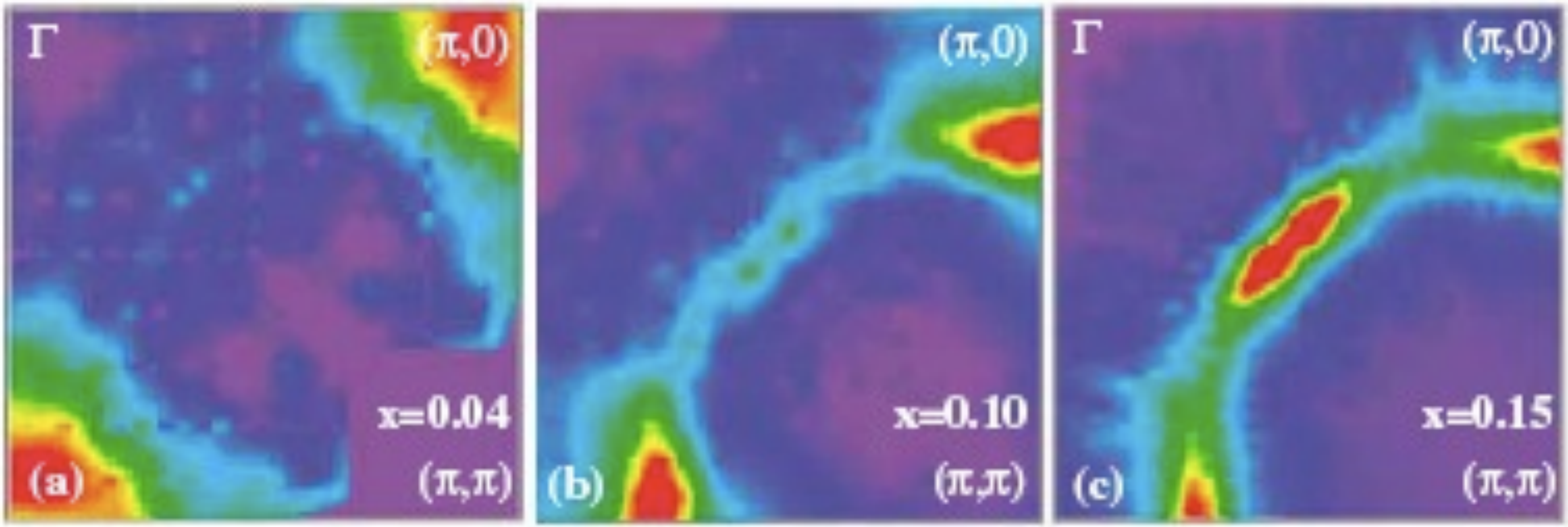
Electron-doped cuprates



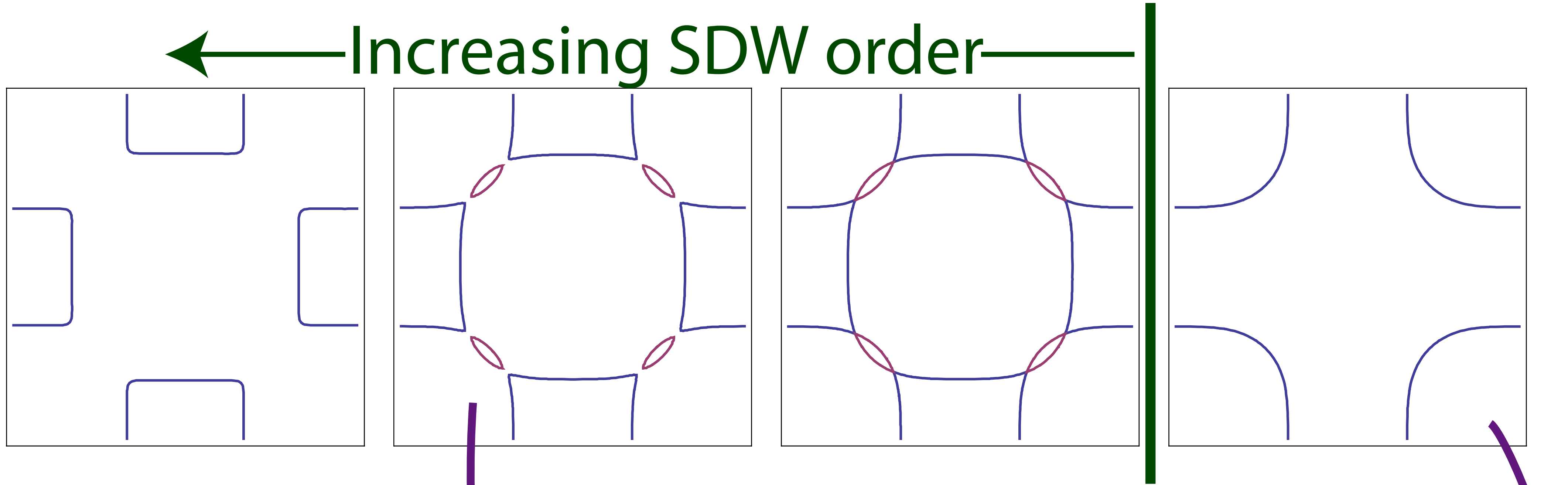
Large Fermi surface breaks up into electron and hole pockets

D. Senechal and A.-M. S. Tremblay, *Physical Review Letters* **92**, 126401 (2004)
J. Lin, and A. J. Millis, *Physical Review B* **72**, 214506 (2005).

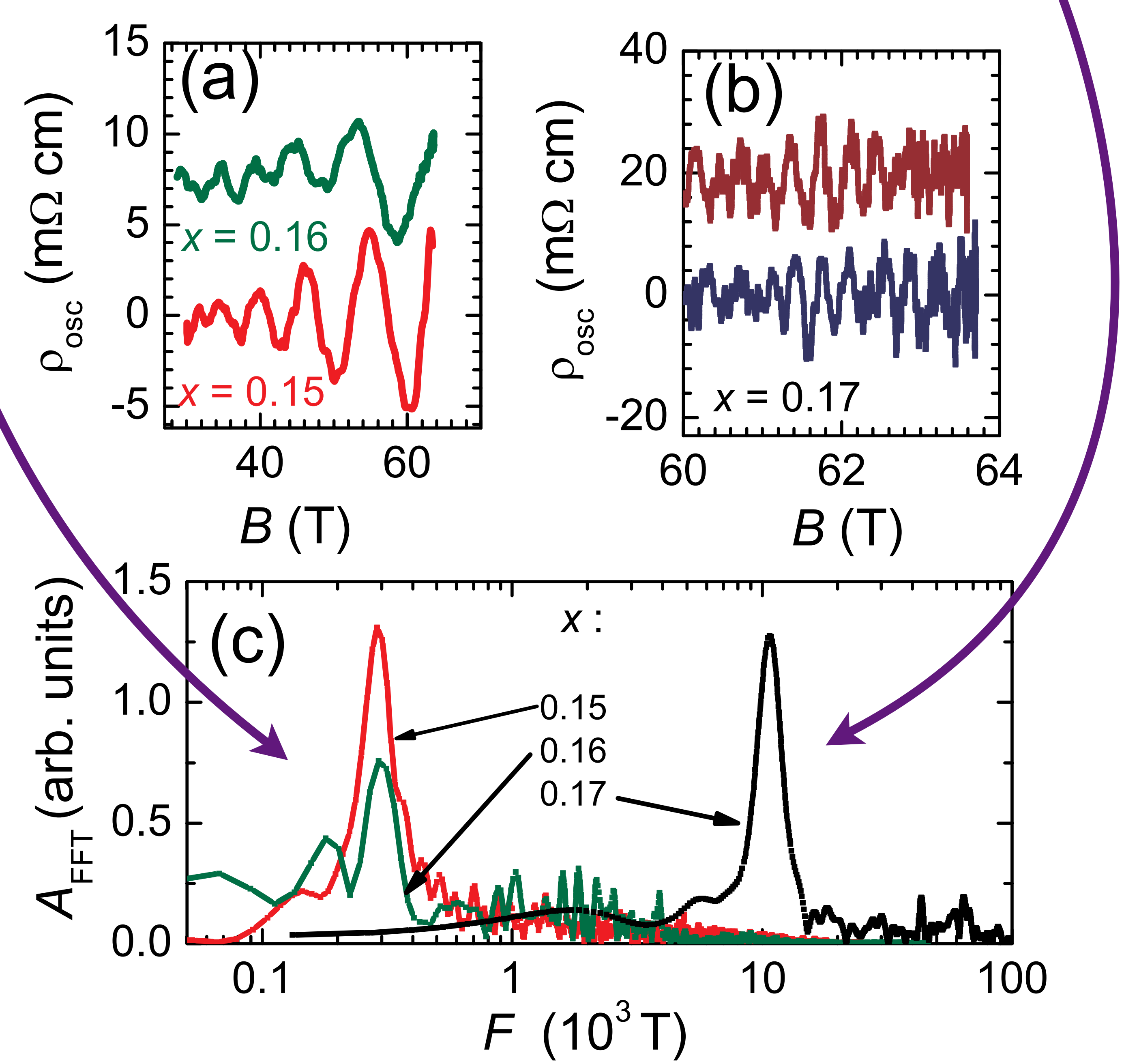
Photoemission in NCCO



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).



Quantum oscillations



$Nd_{2-x}Ce_xCuO_4$

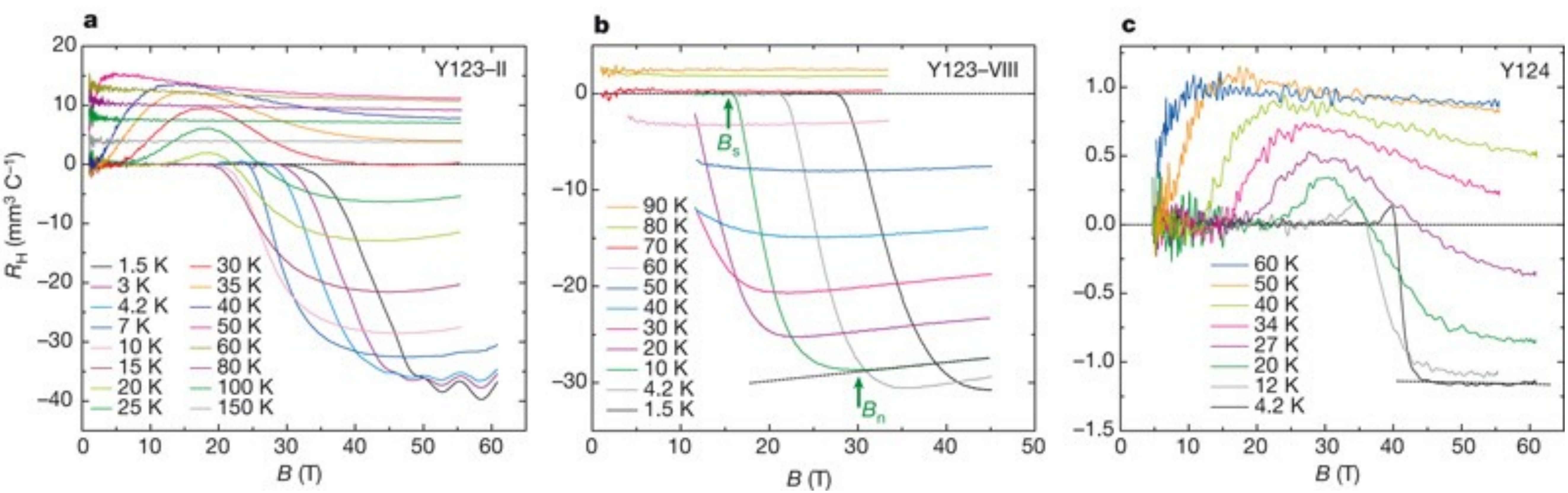
T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



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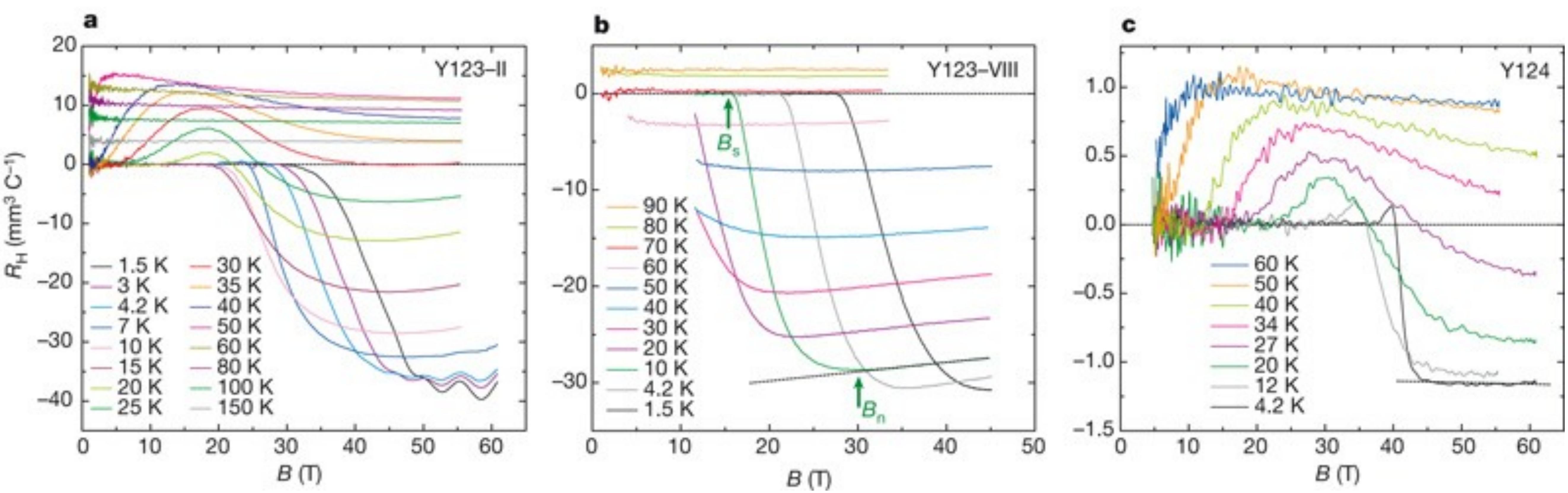


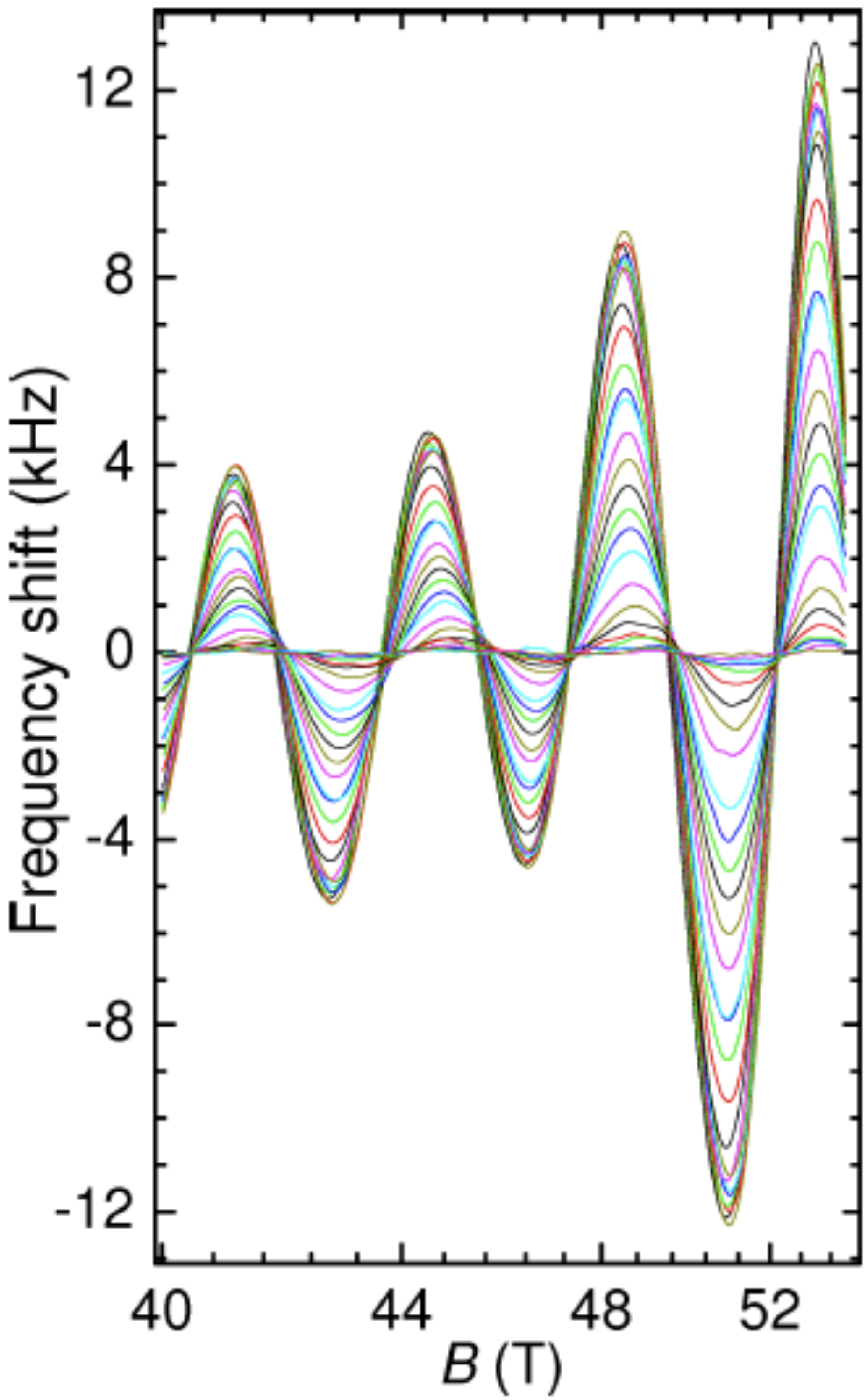


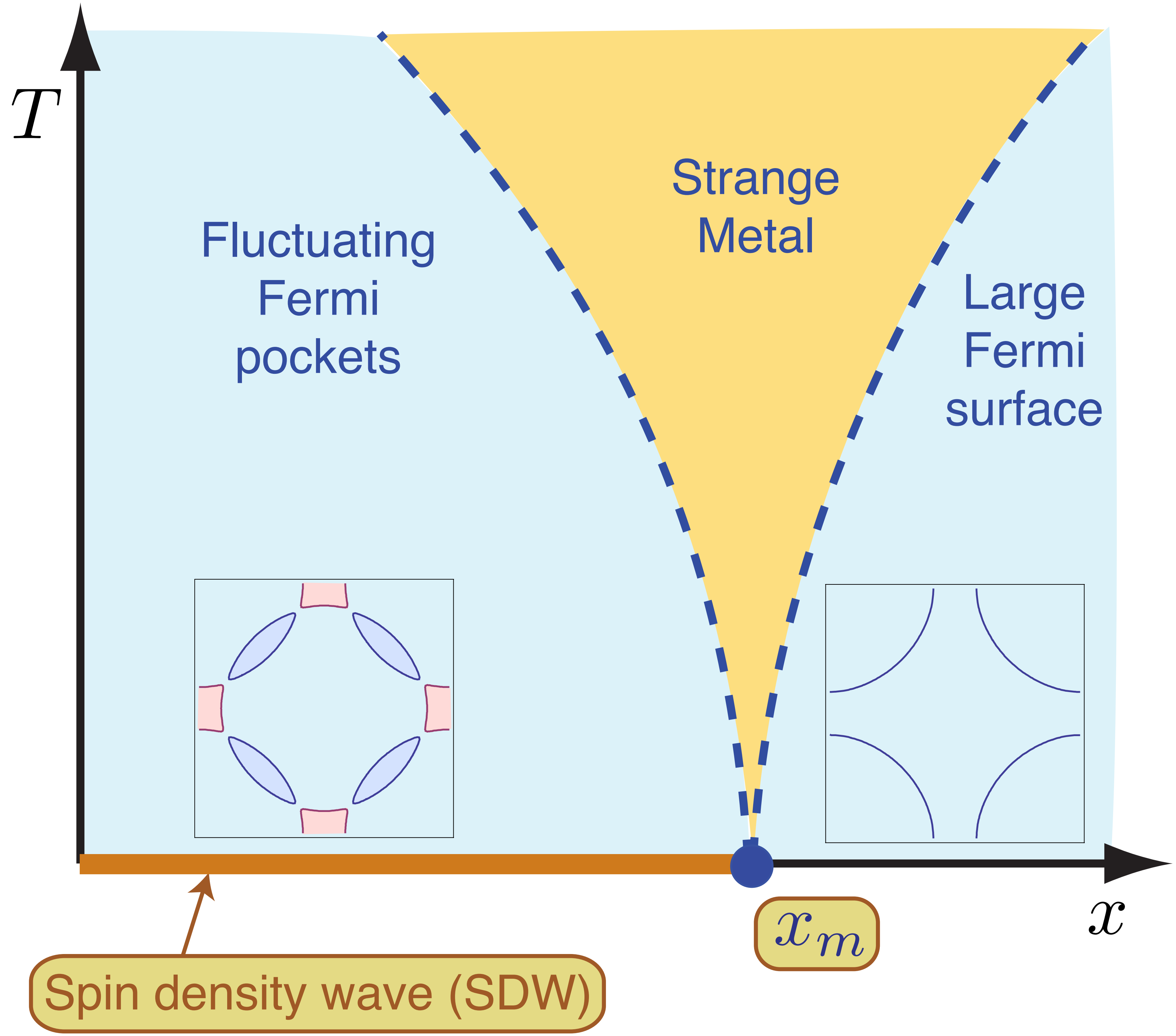
FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of ~ 50 dB between $T = 1$ and 18 K. The actual T values are provided in Fig. 3.

Fermi liquid behaviour in an underdoped high Tc superconductor

Suchitra E. Sebastian, N. Harrison, M. M. Altarawneh, Ruixing Liang, D. A. Bonn, W. N. Hardy, and G. G. Lonzarich

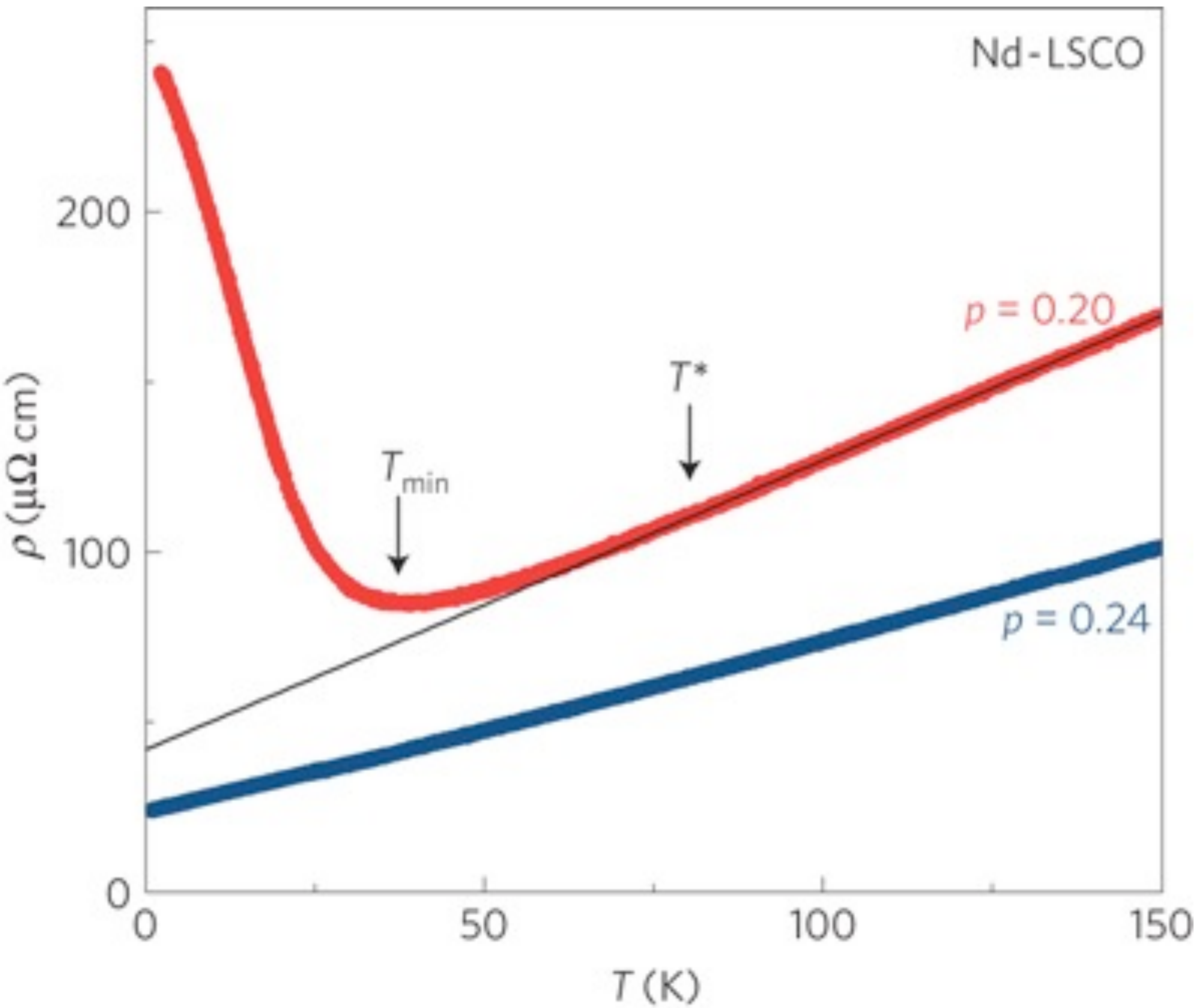
arXiv:0912.3022

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c

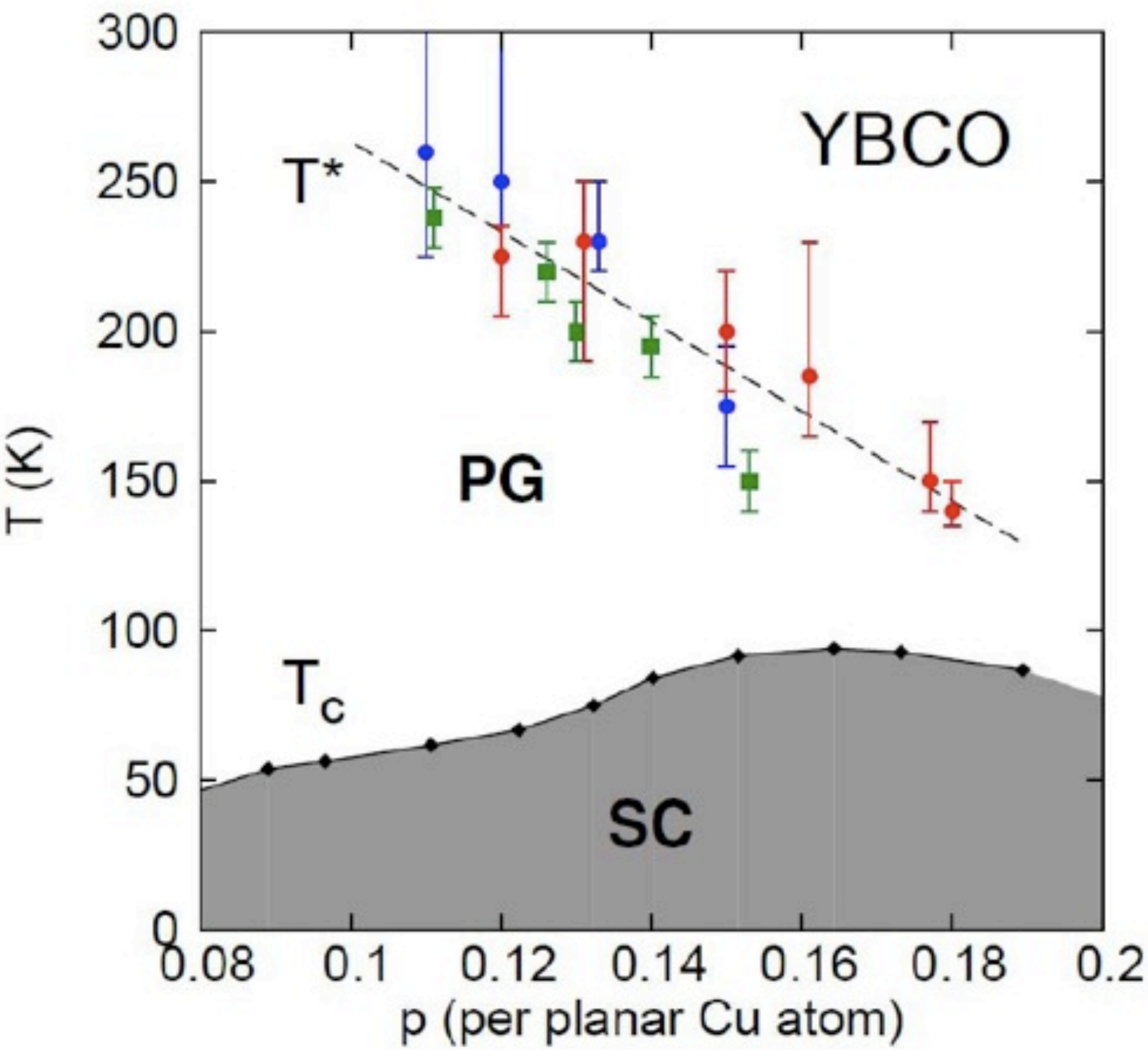


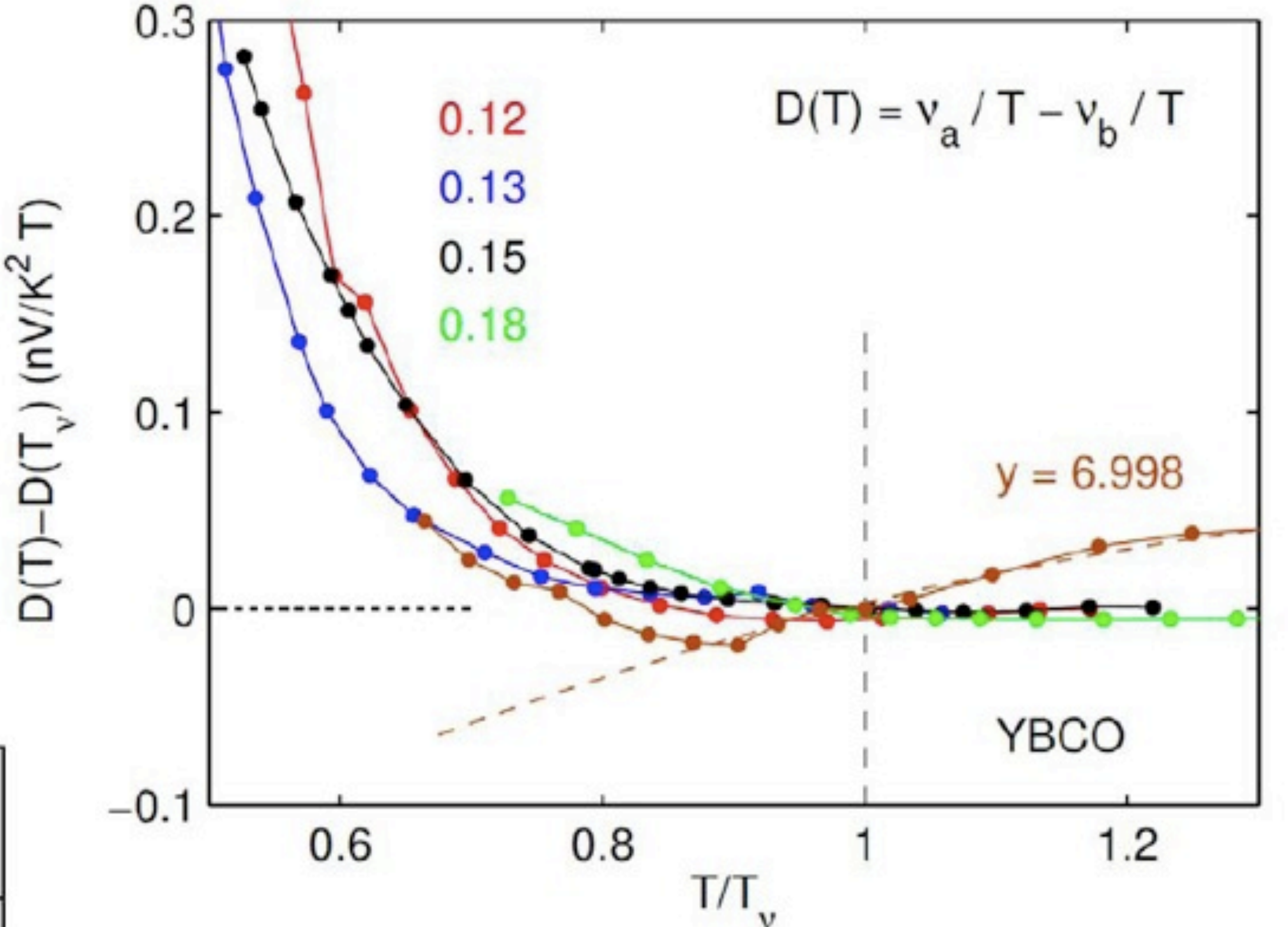
- Magnetic field of upto 35 T used to suppress superconductivity
- Identifies $x_m \approx 0.24$

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-T_c superconductor
R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)

Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430, Nature, in press





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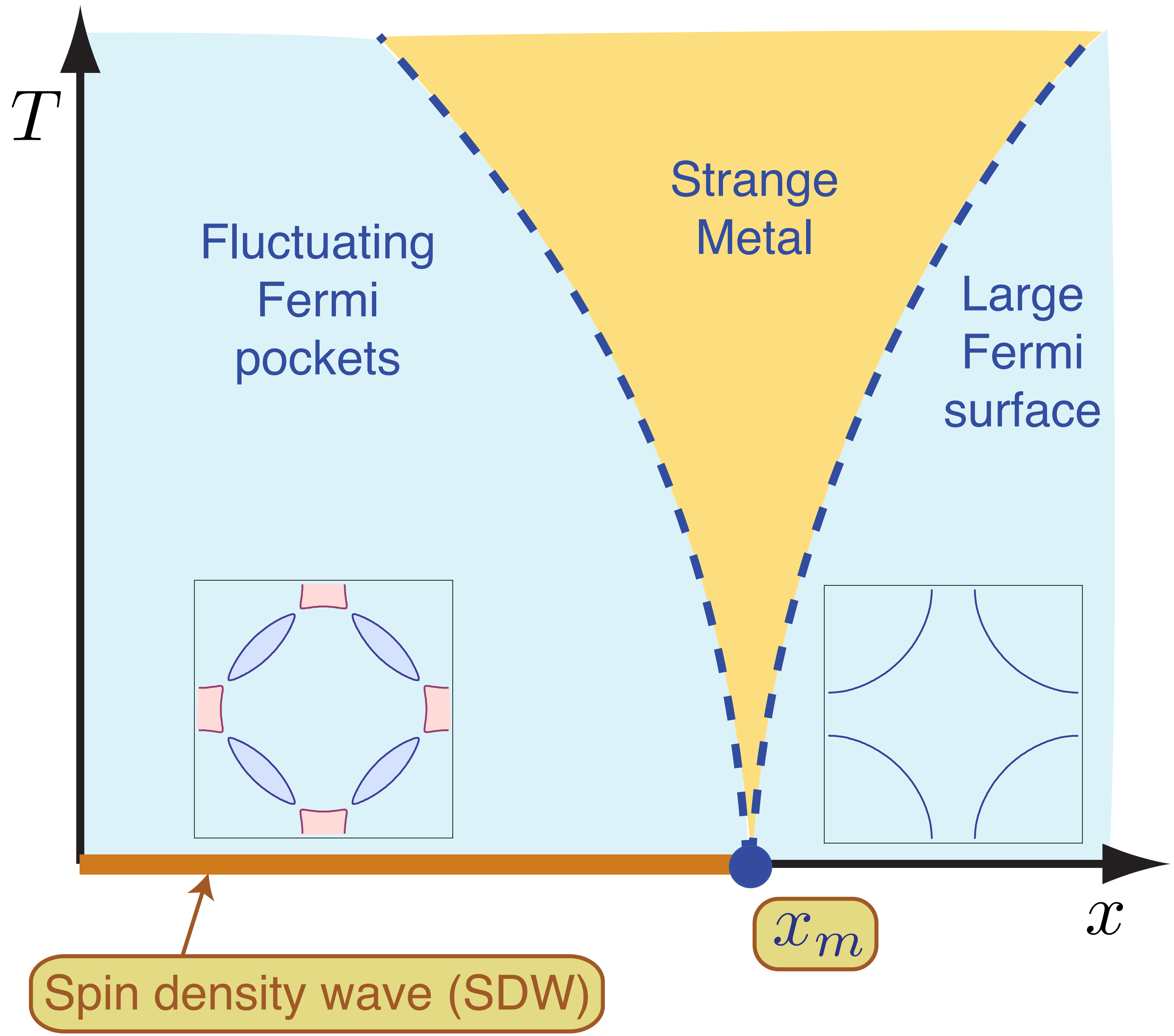

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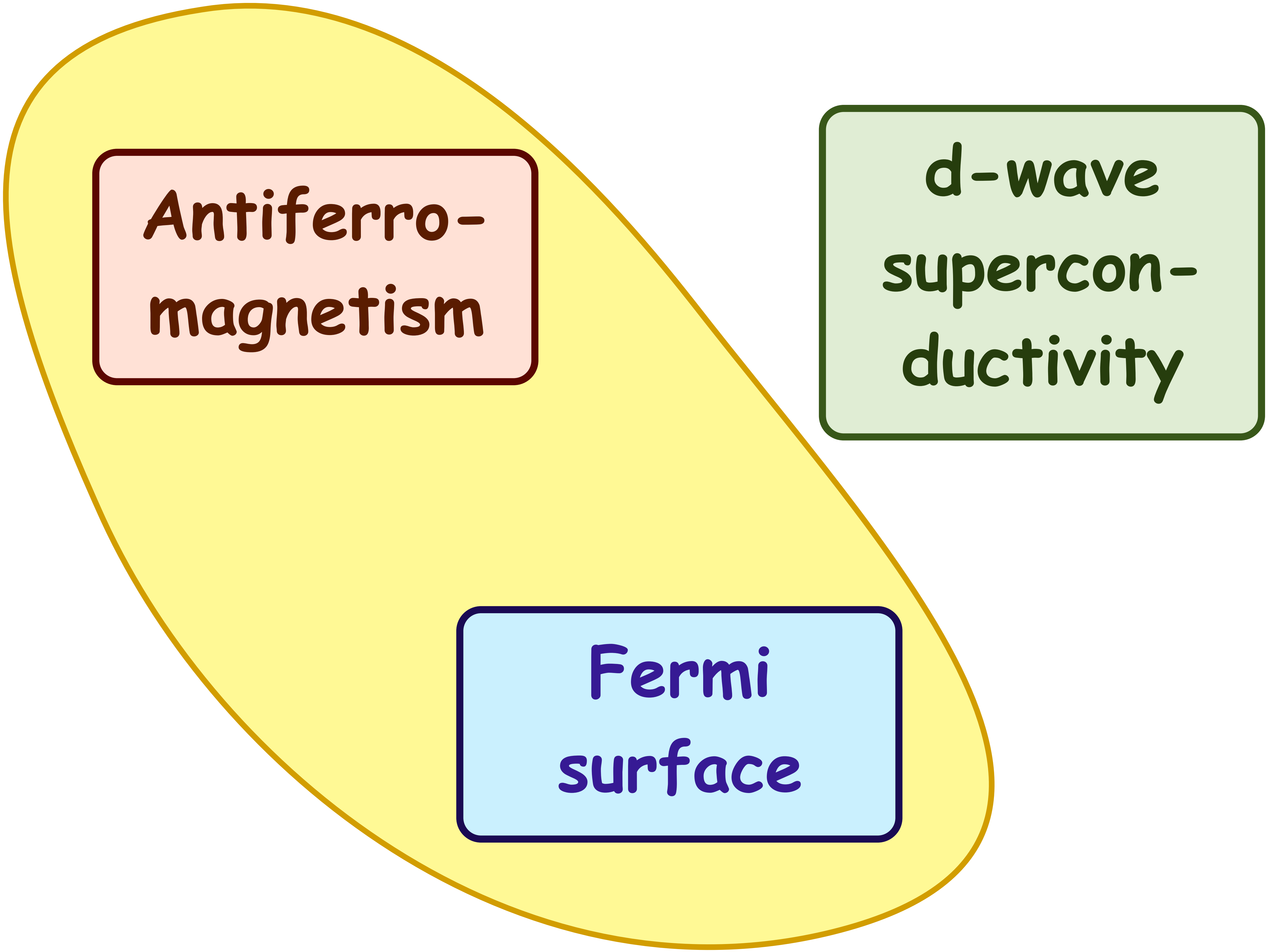
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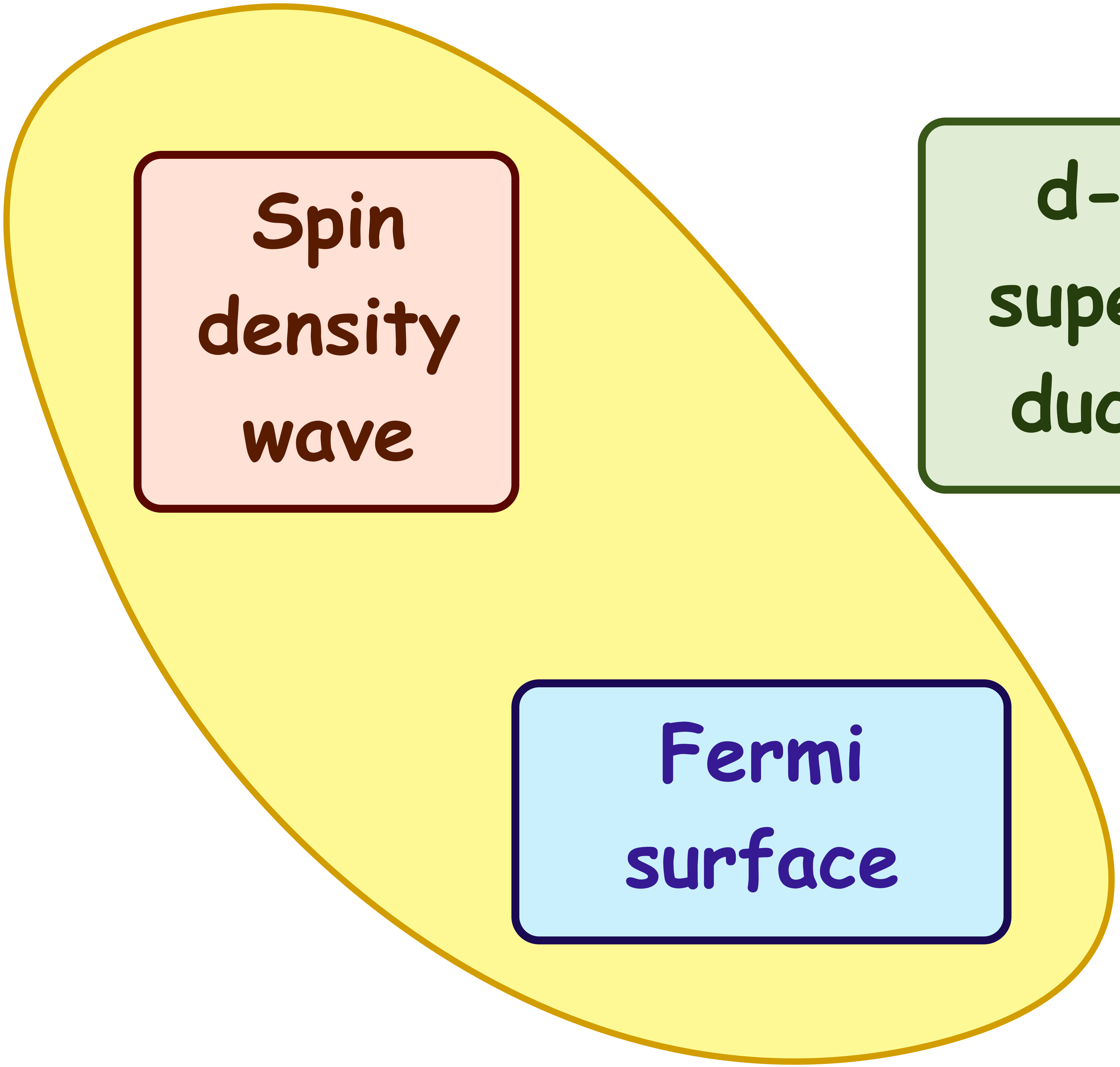
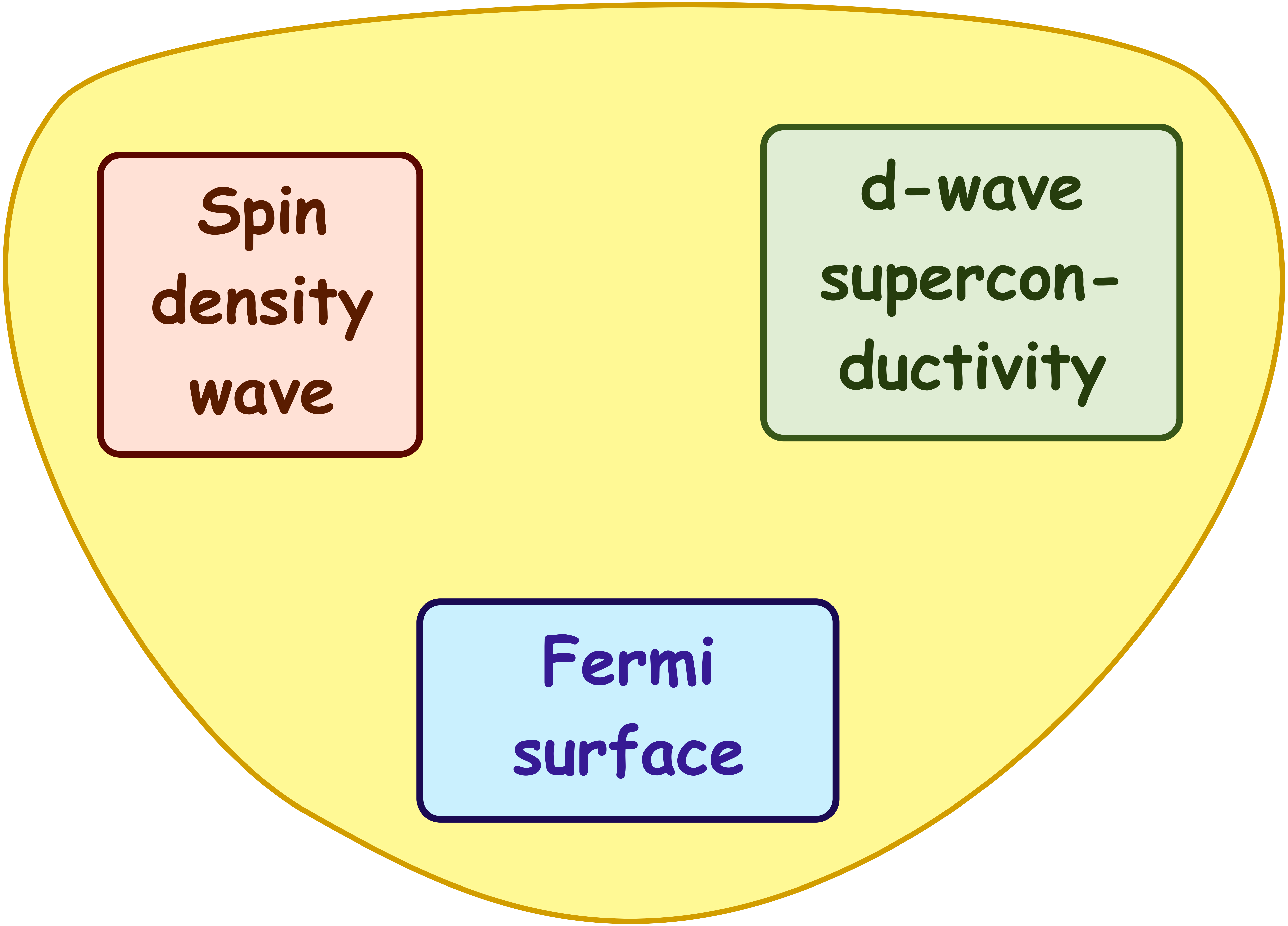

Spin density wave
d-wave supercon-ductivity
Fermi surface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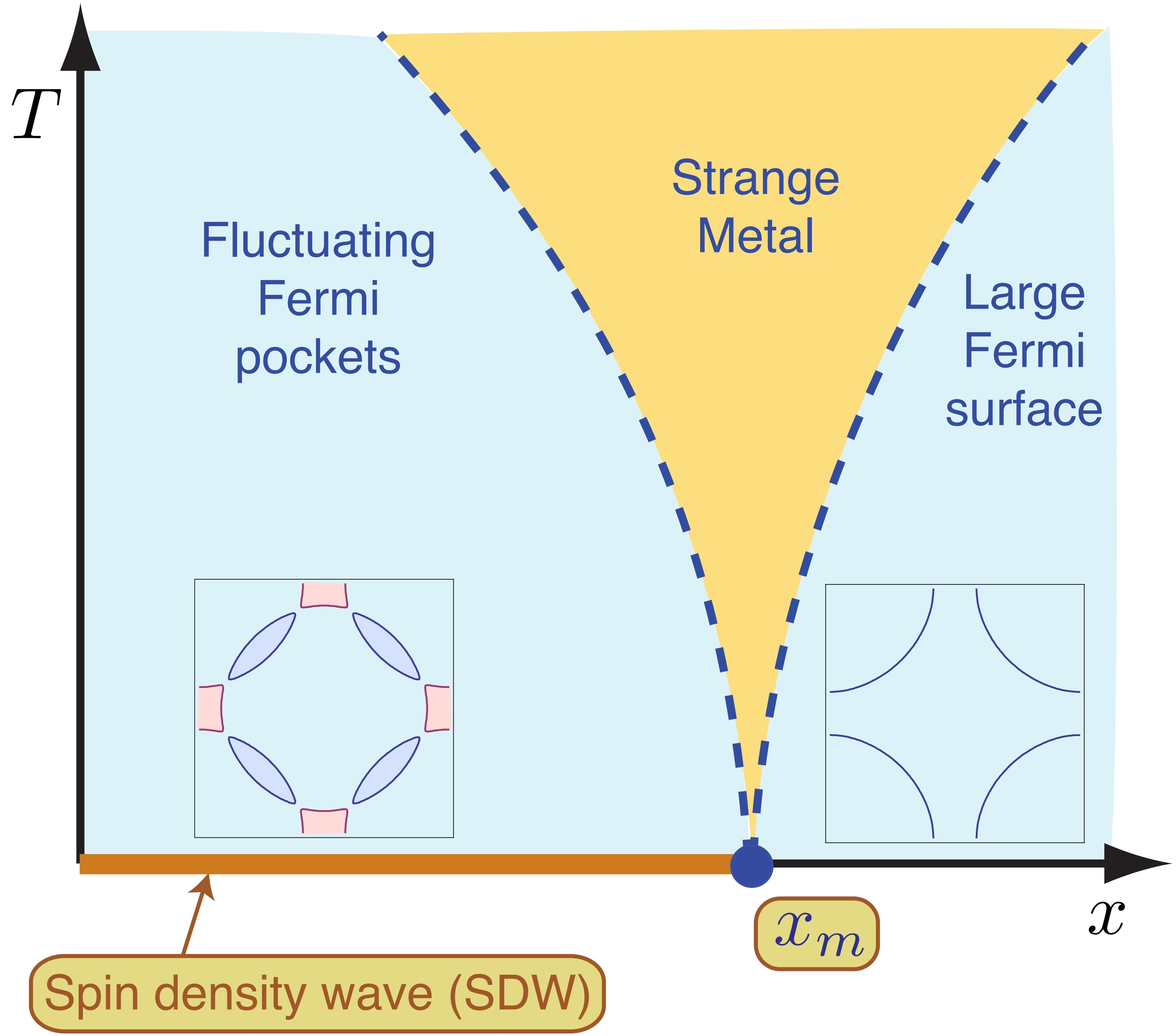
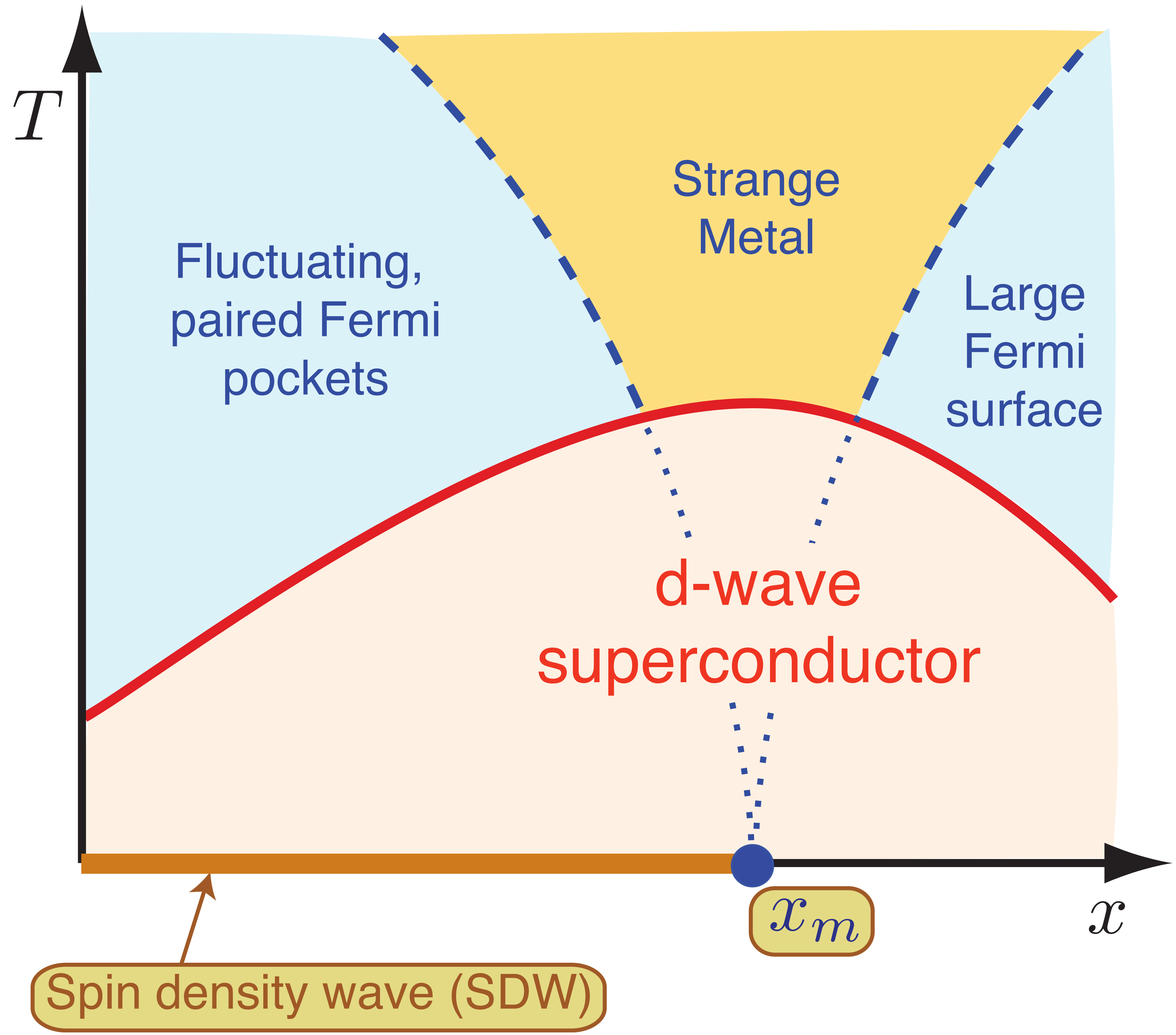


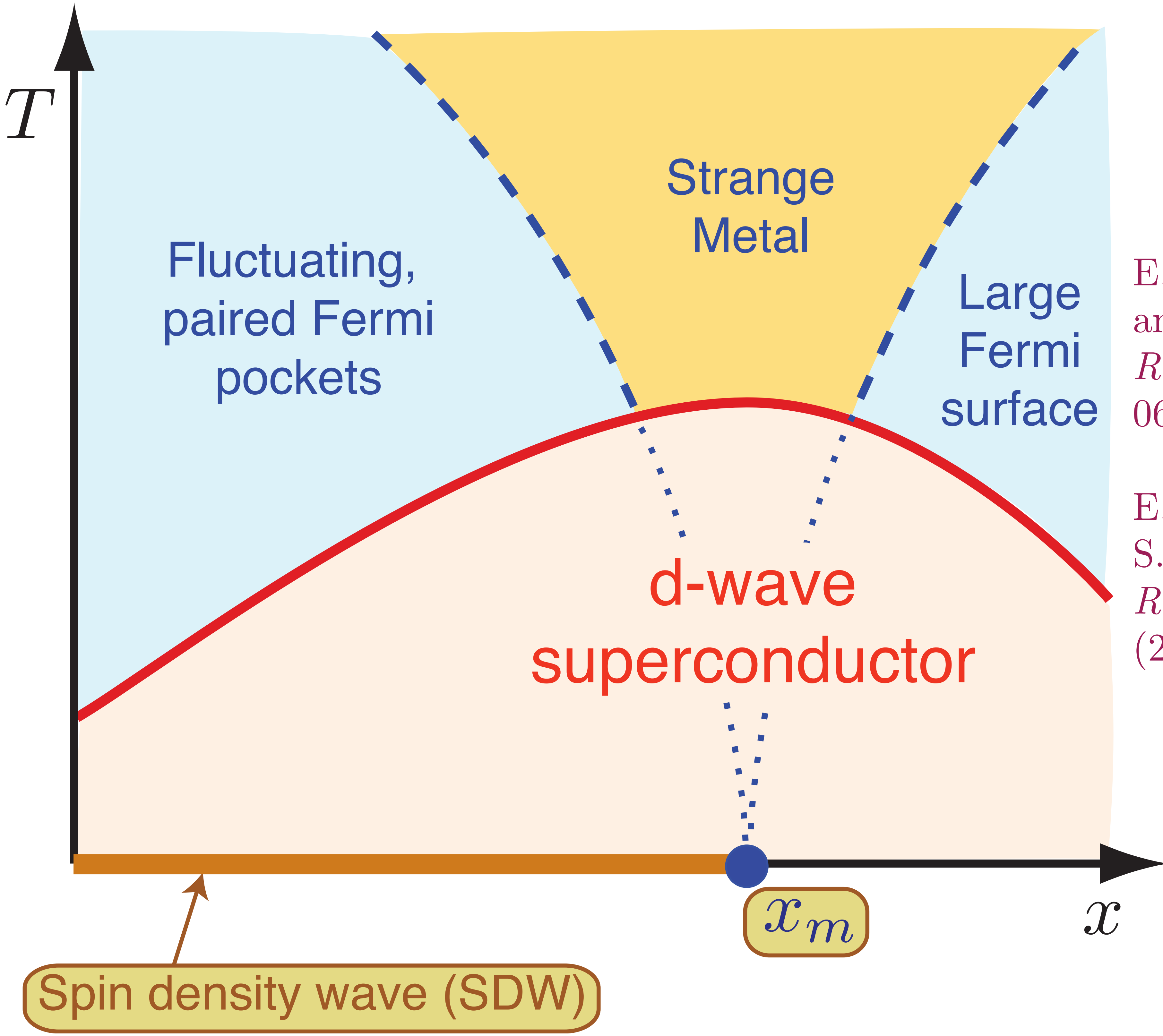
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates

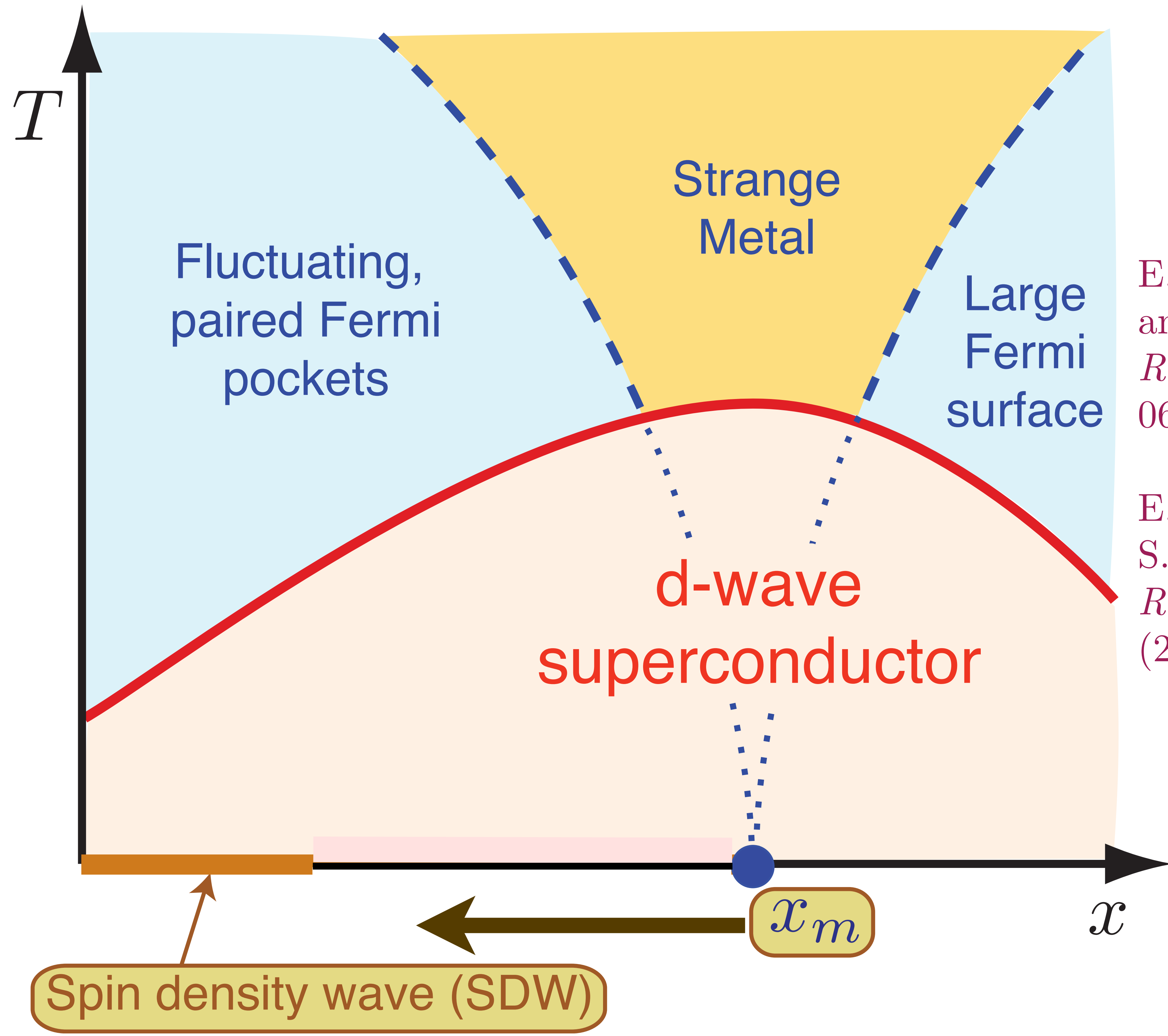


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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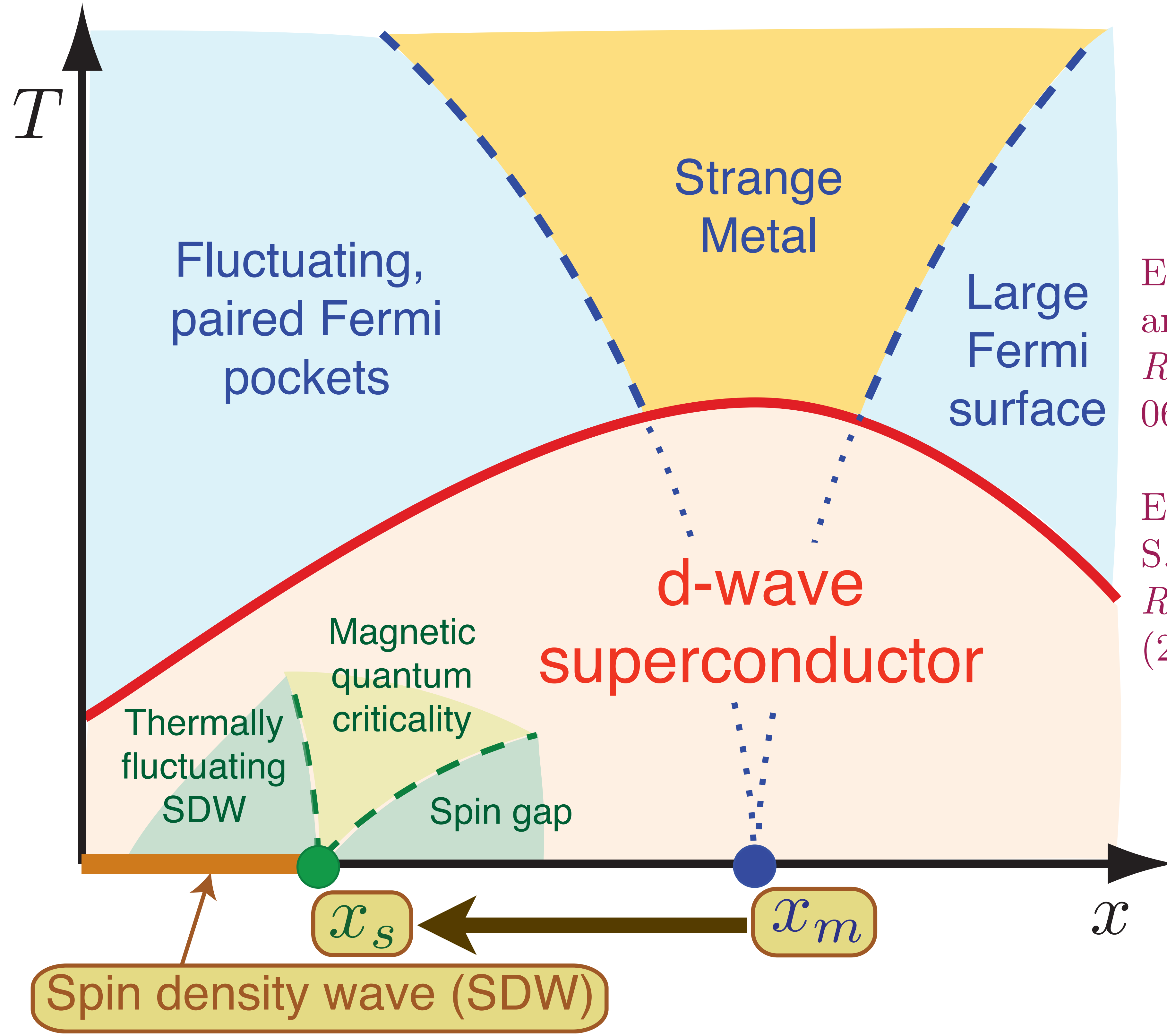
Theory of quantum criticality in the cuprates



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

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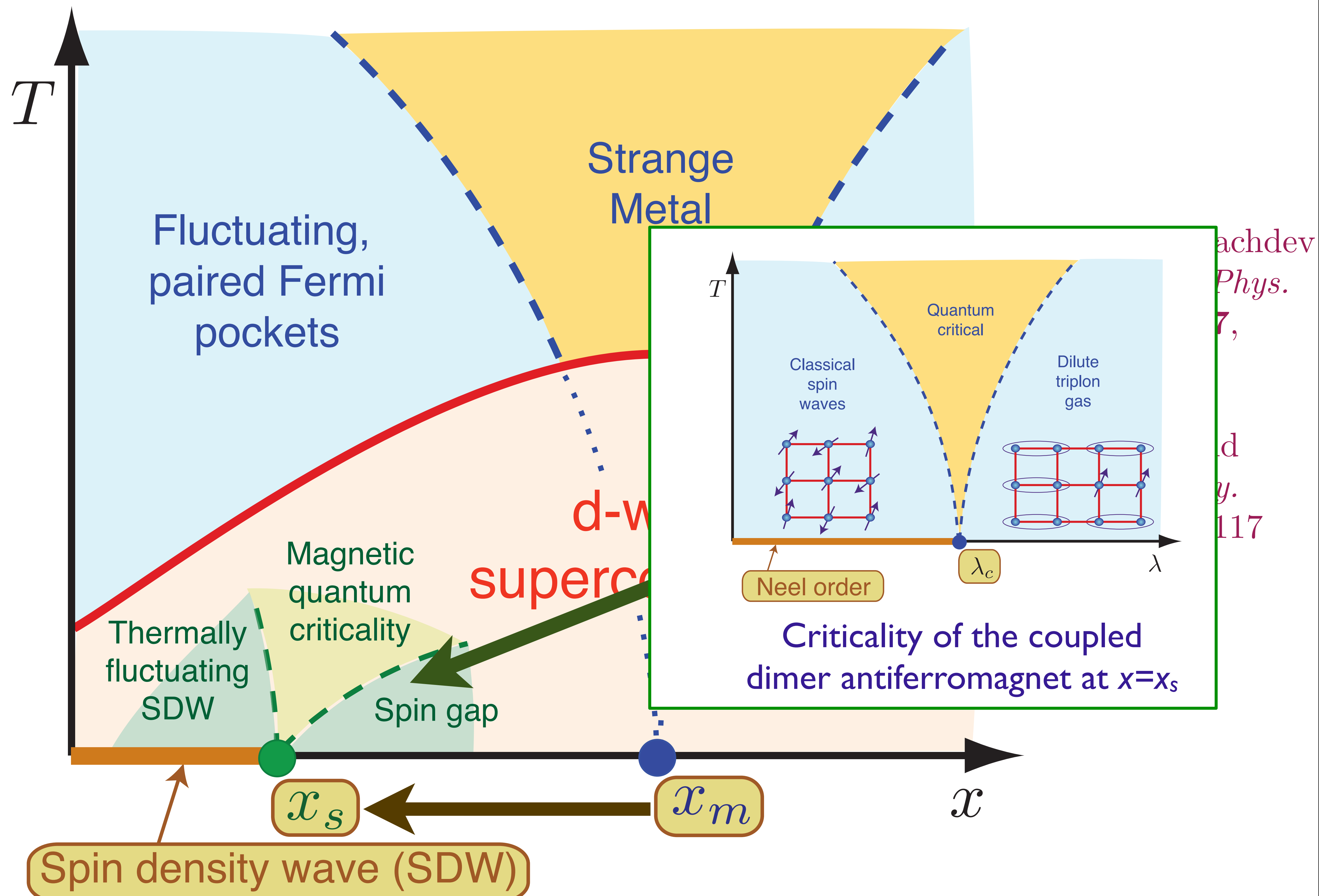

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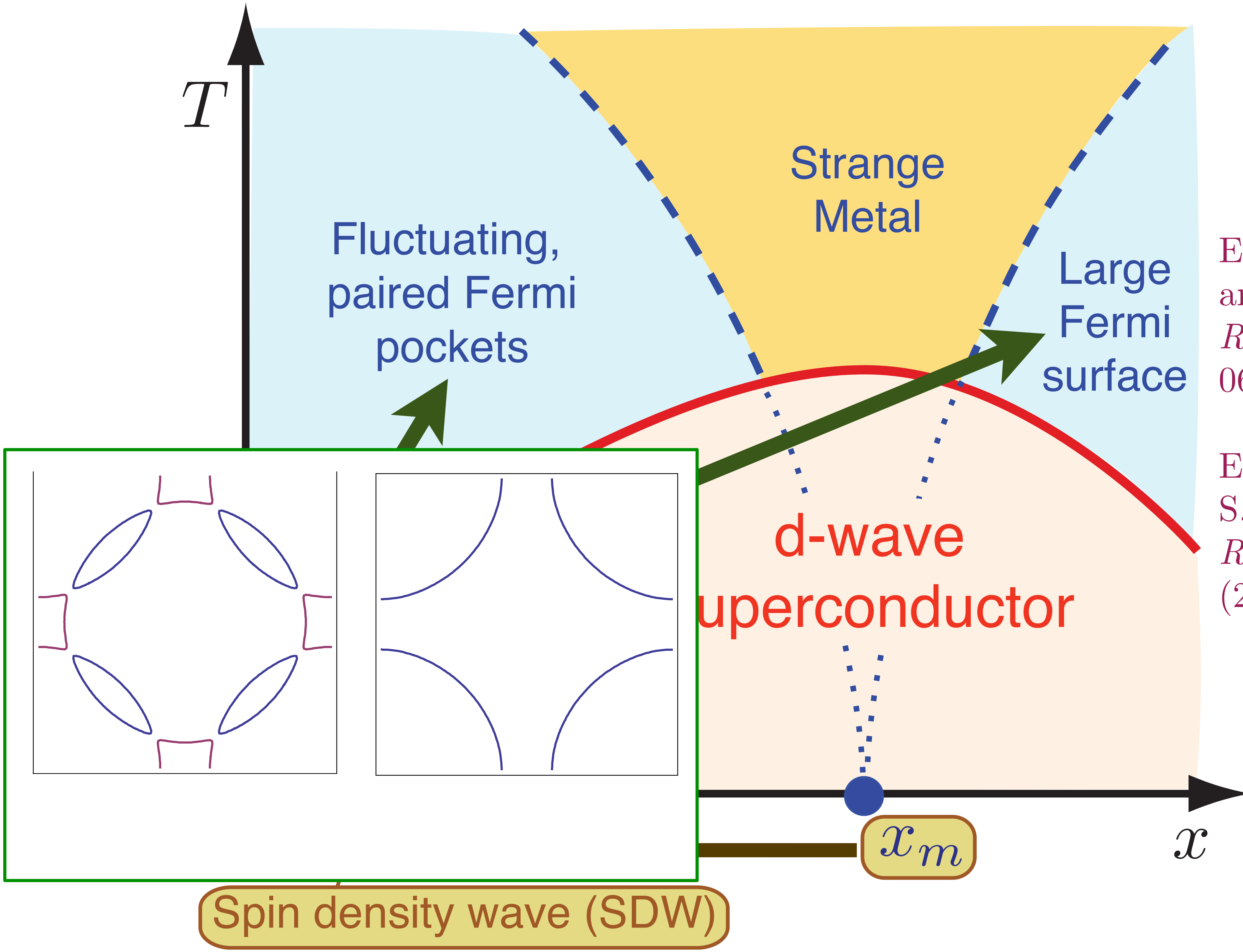
Theory of quantum criticality in the cuprates



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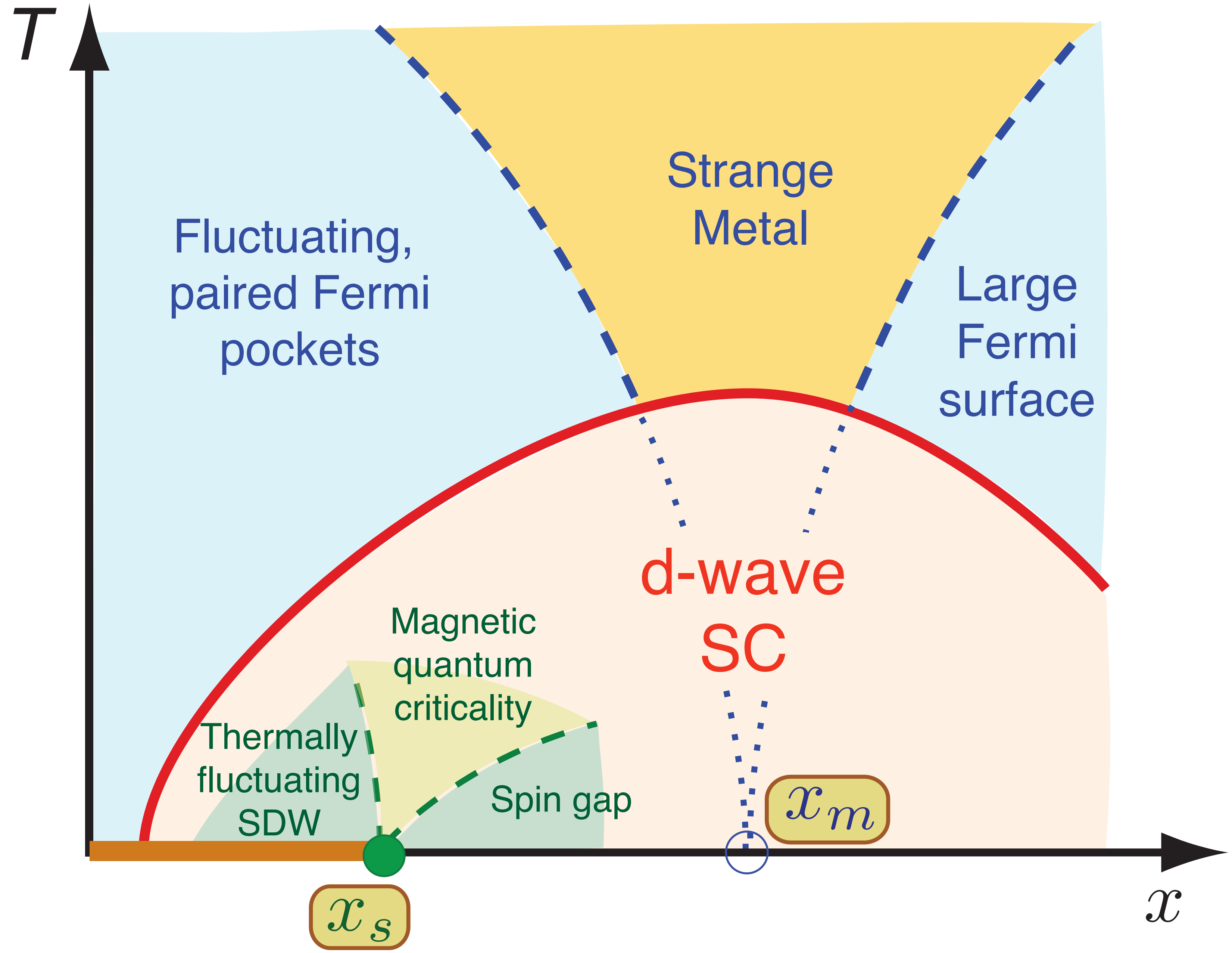
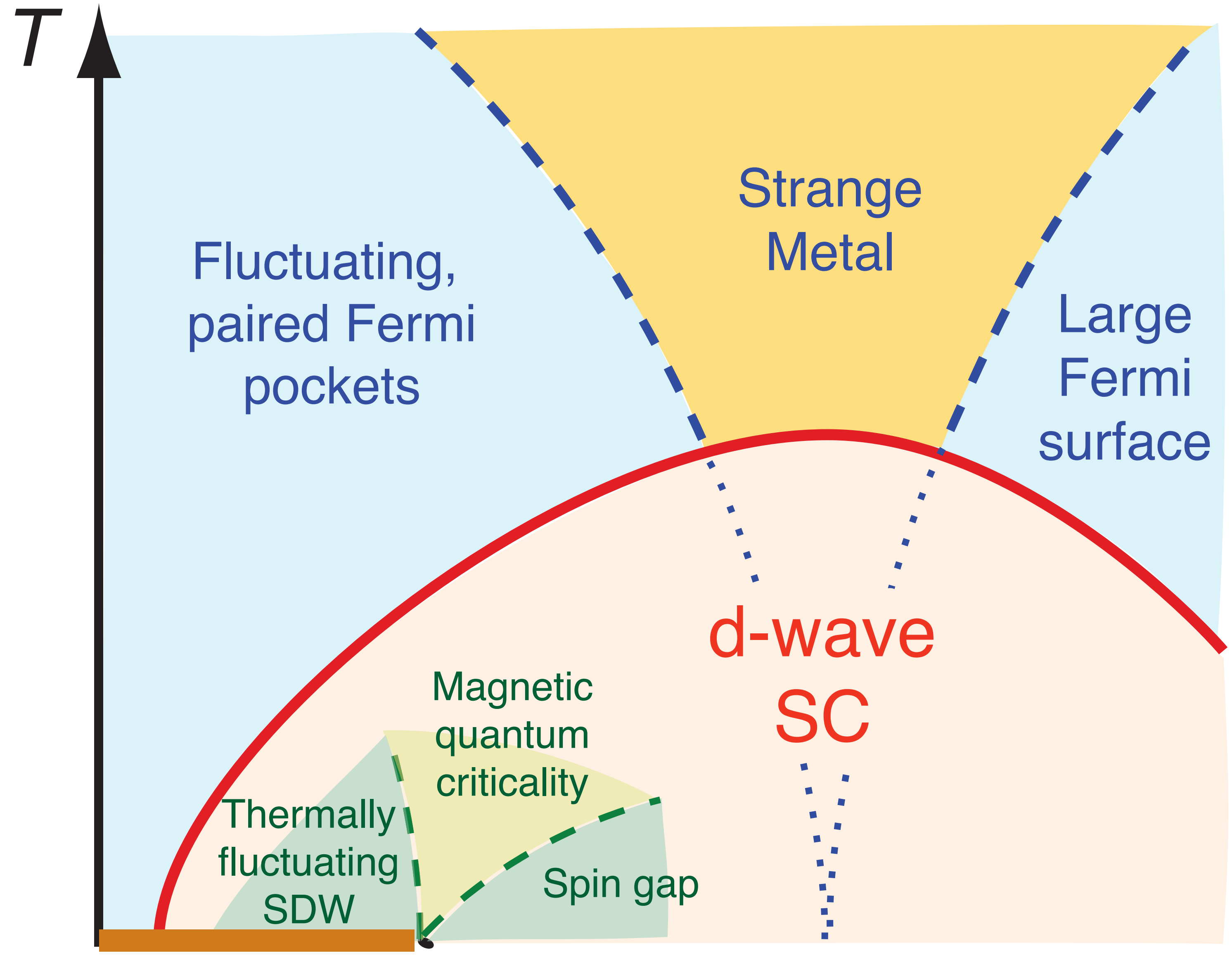


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x



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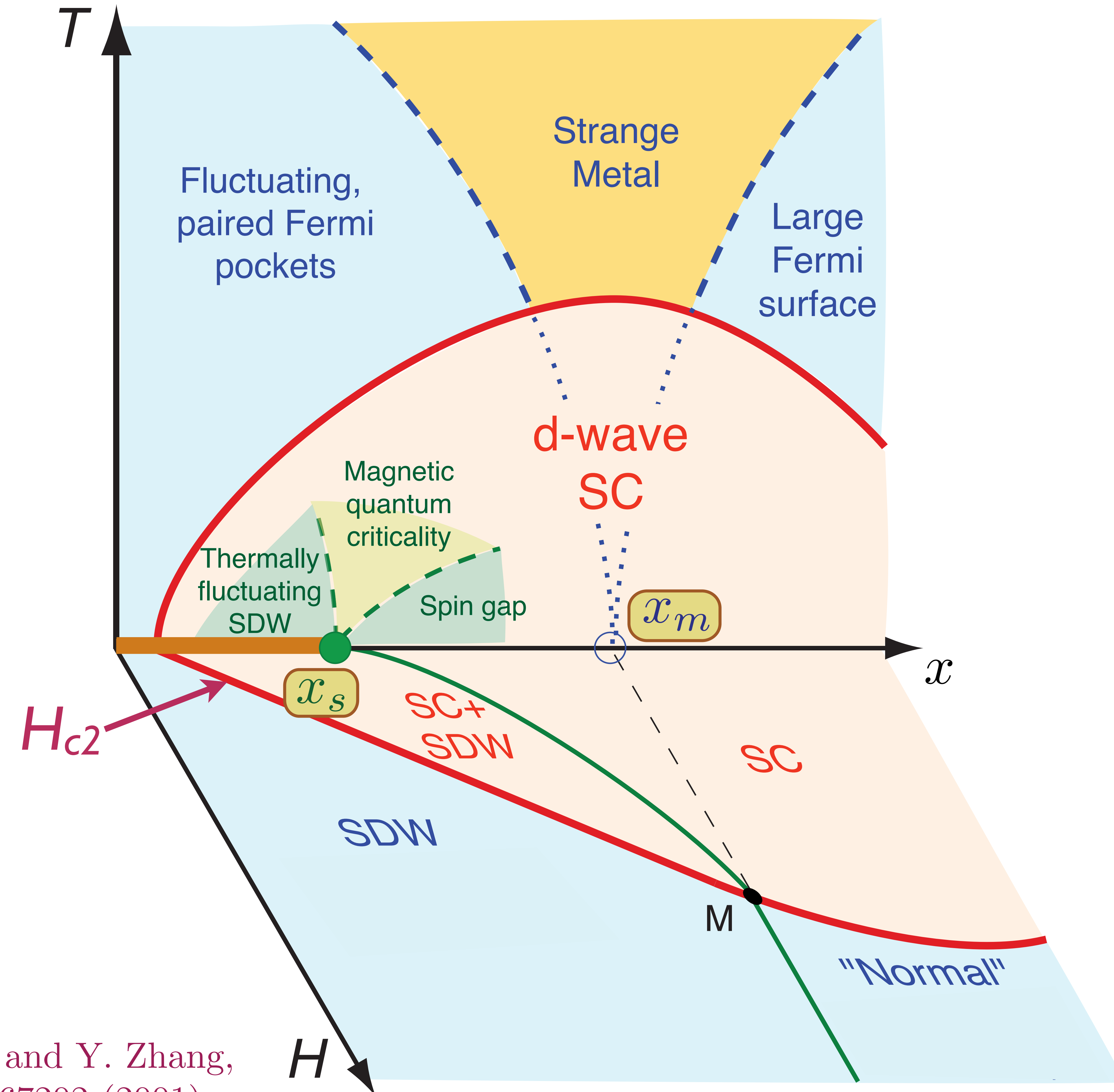

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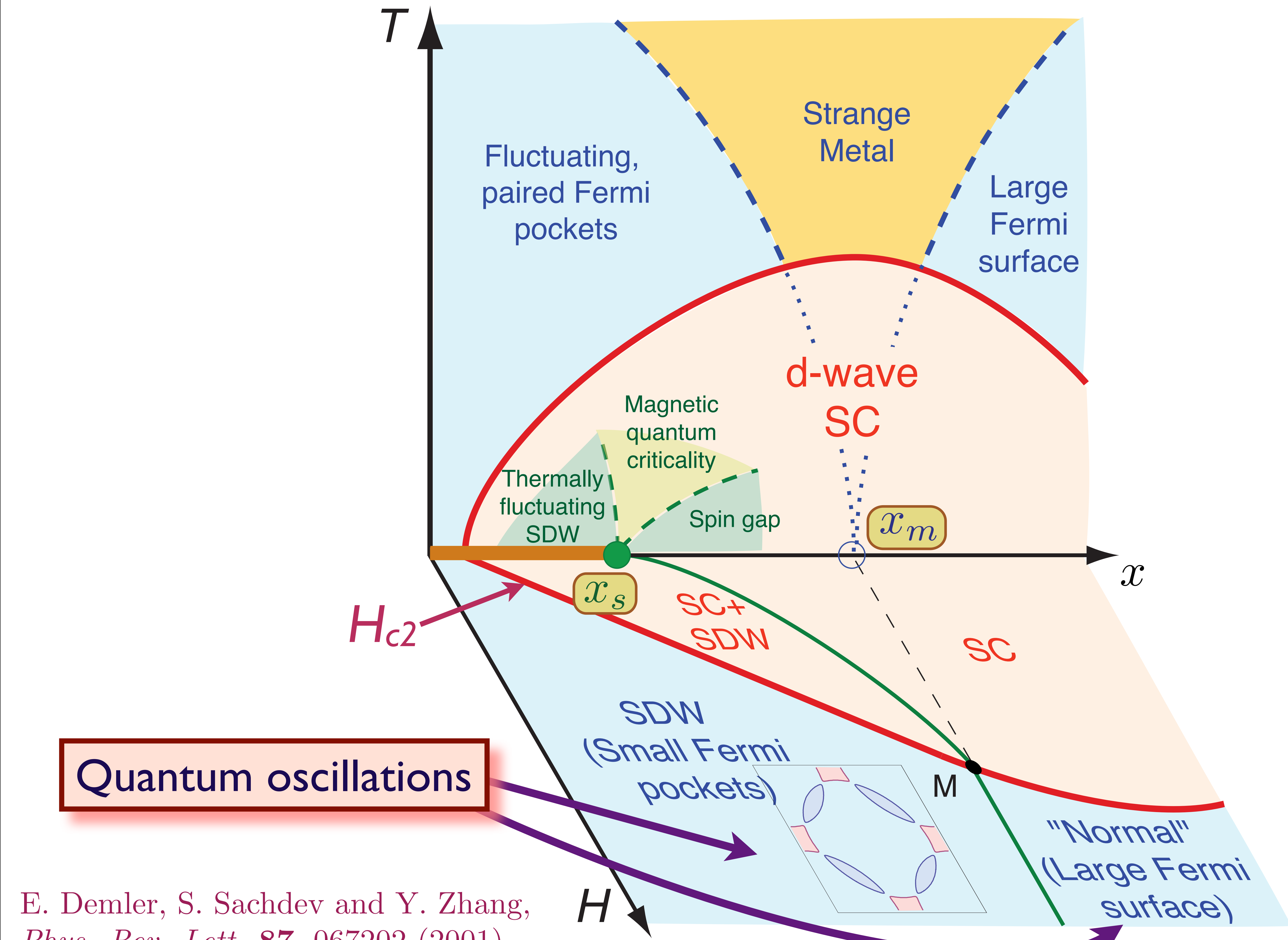

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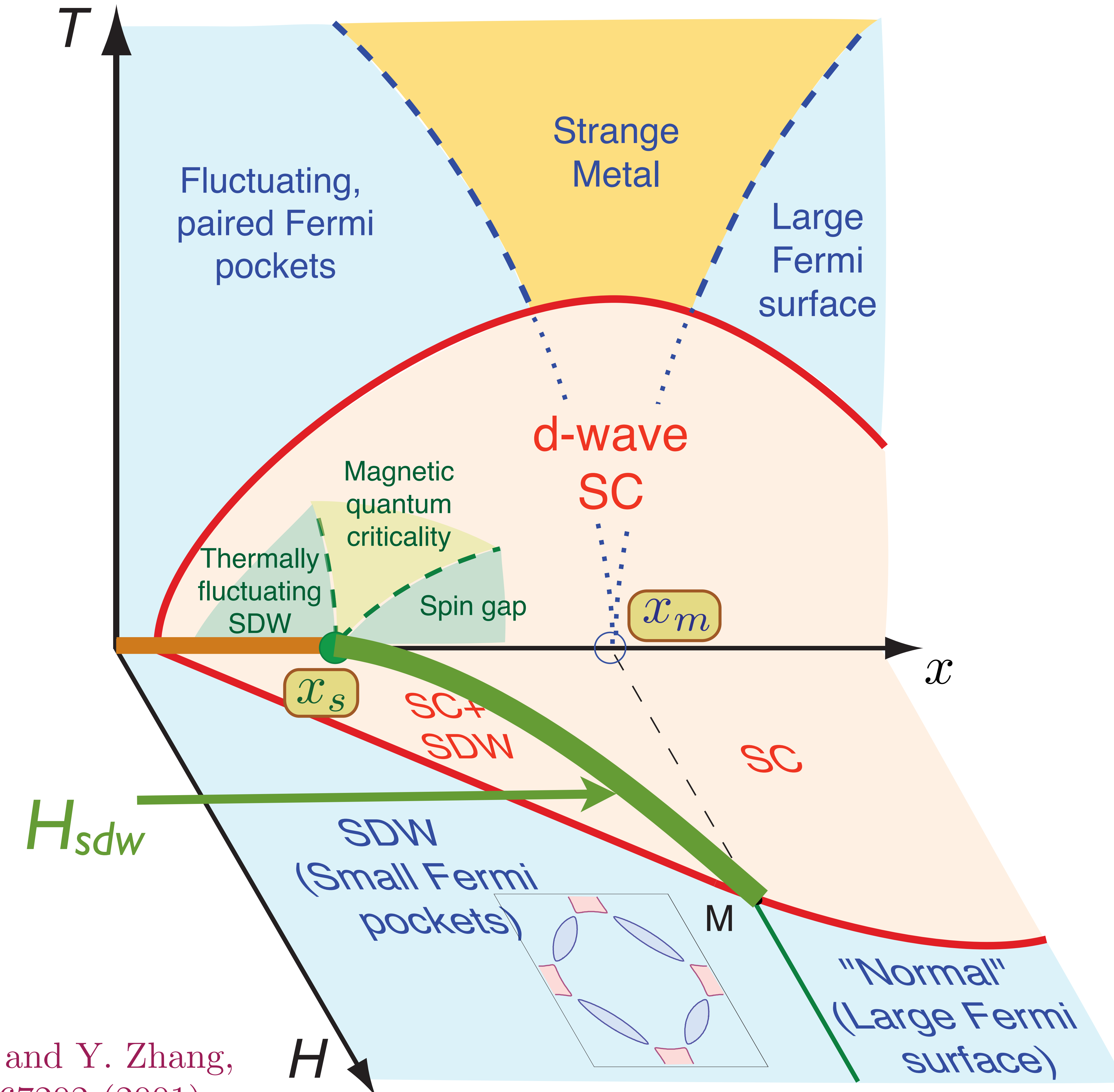

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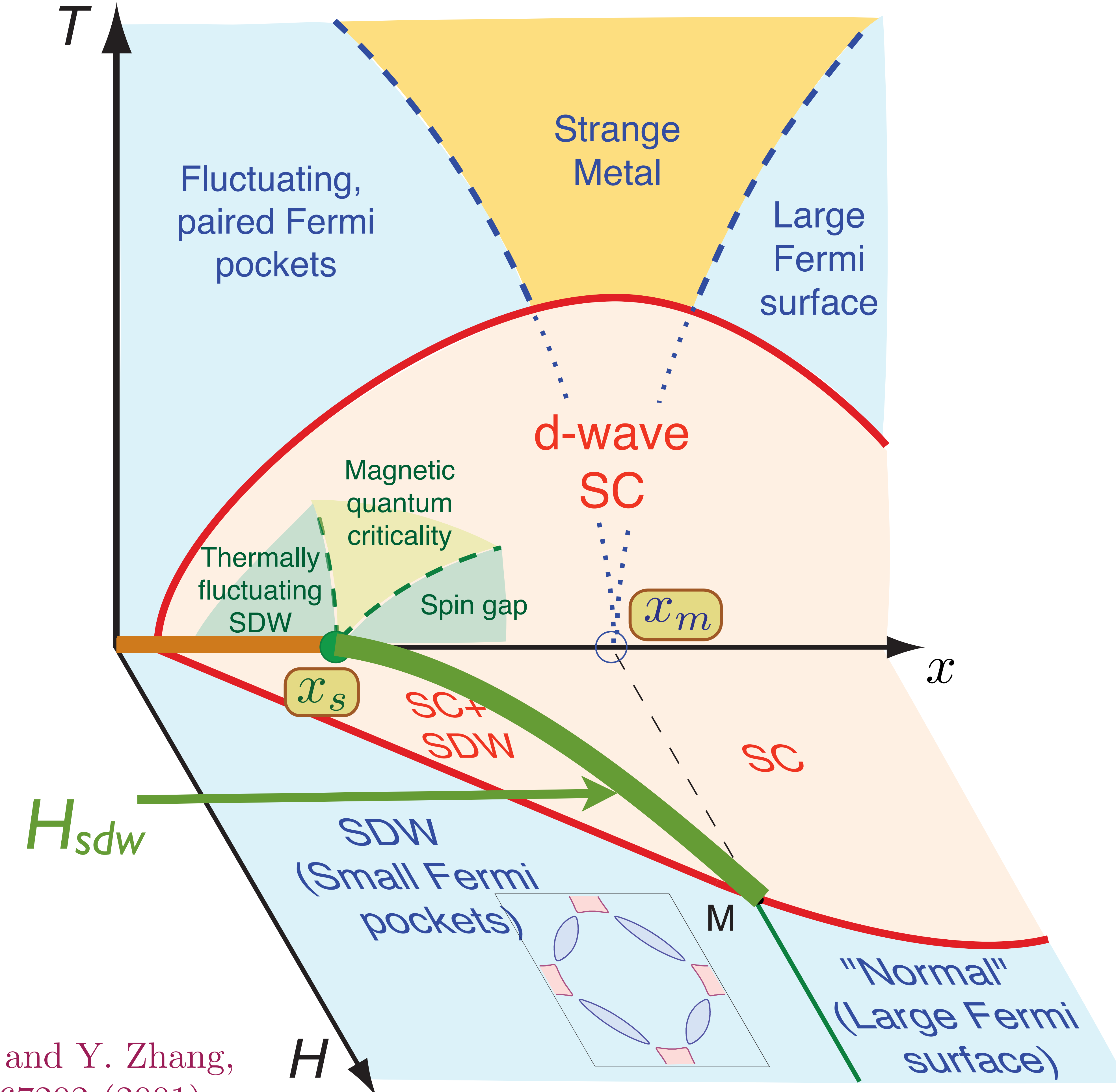

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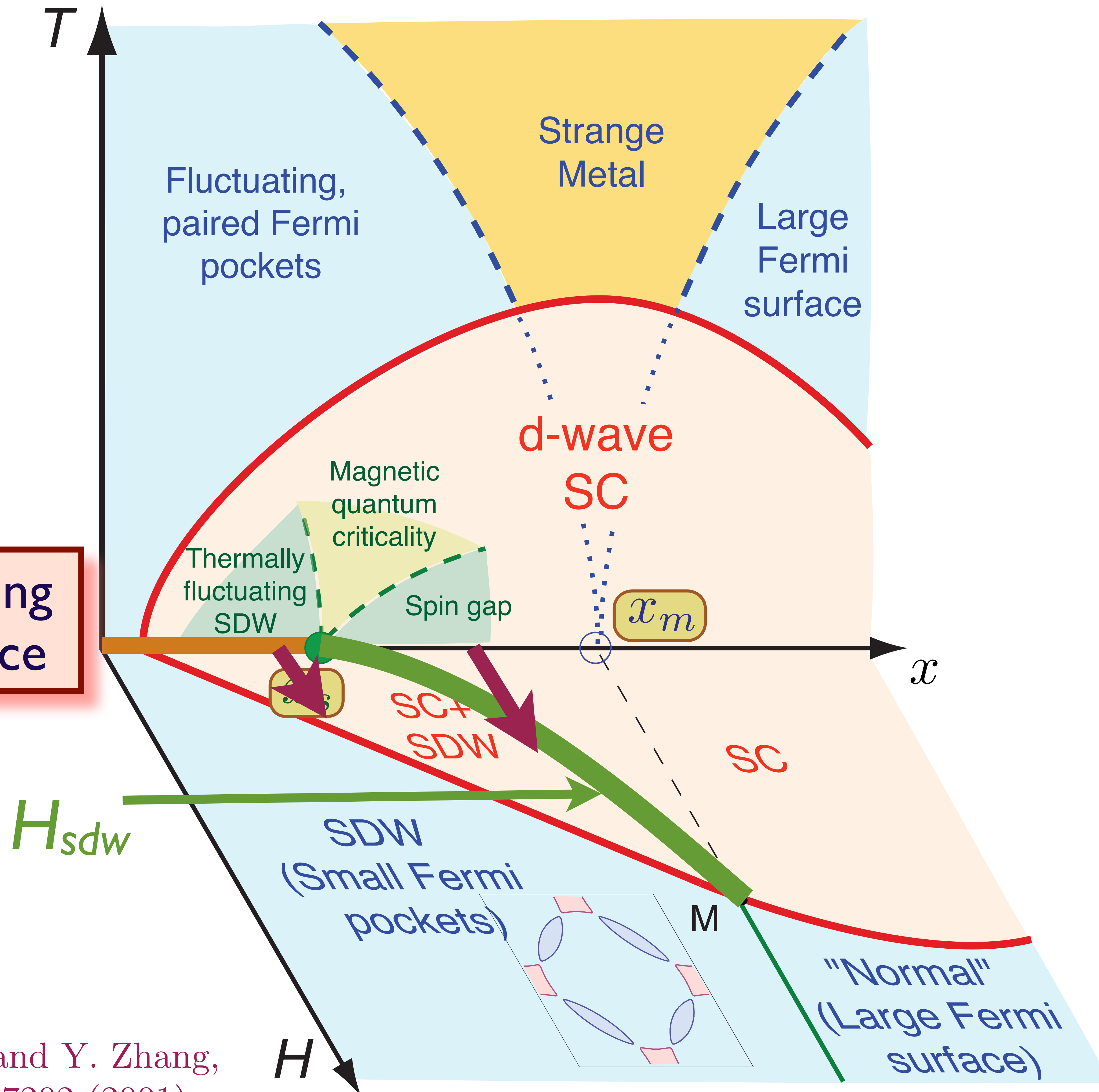
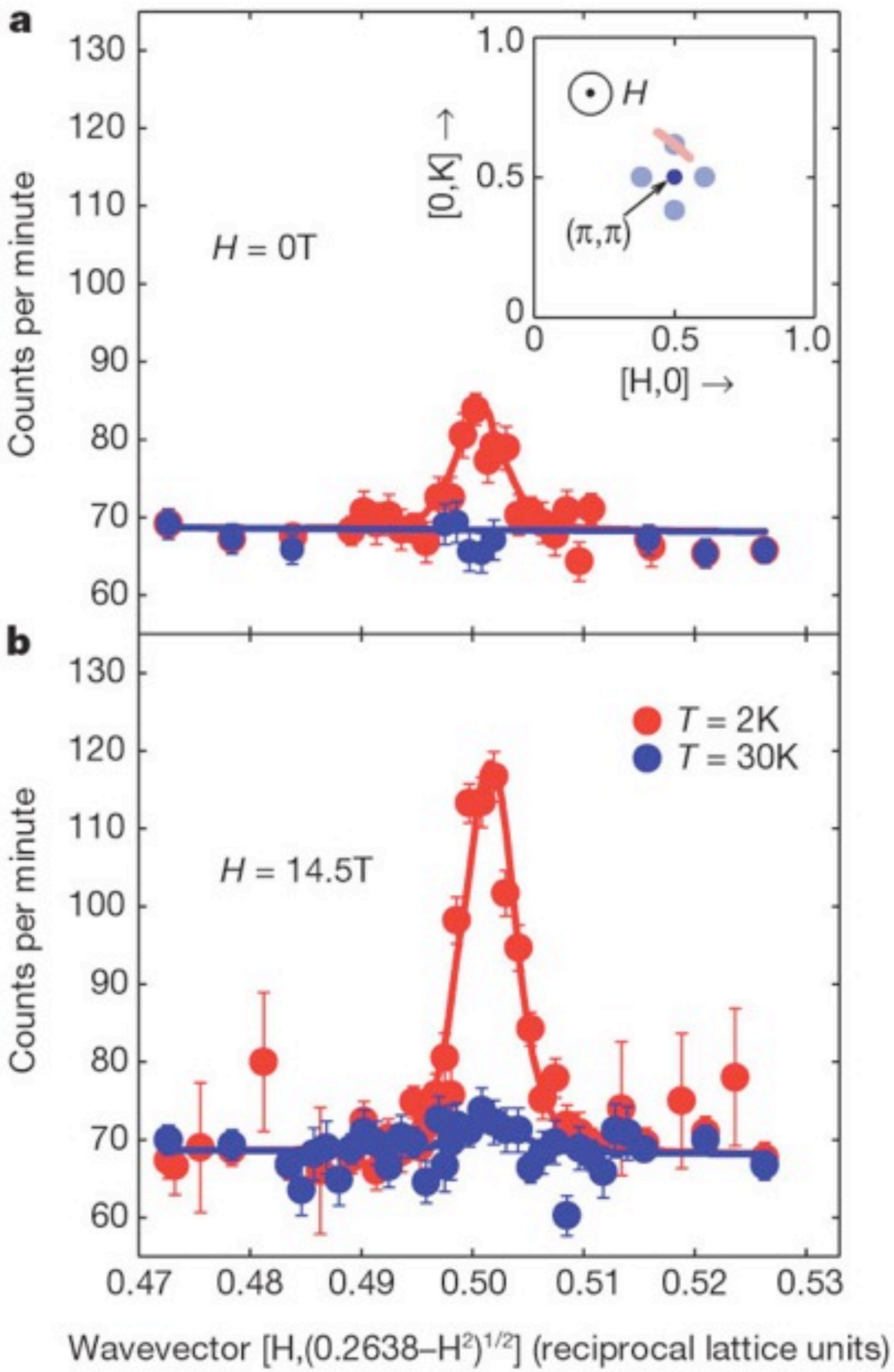
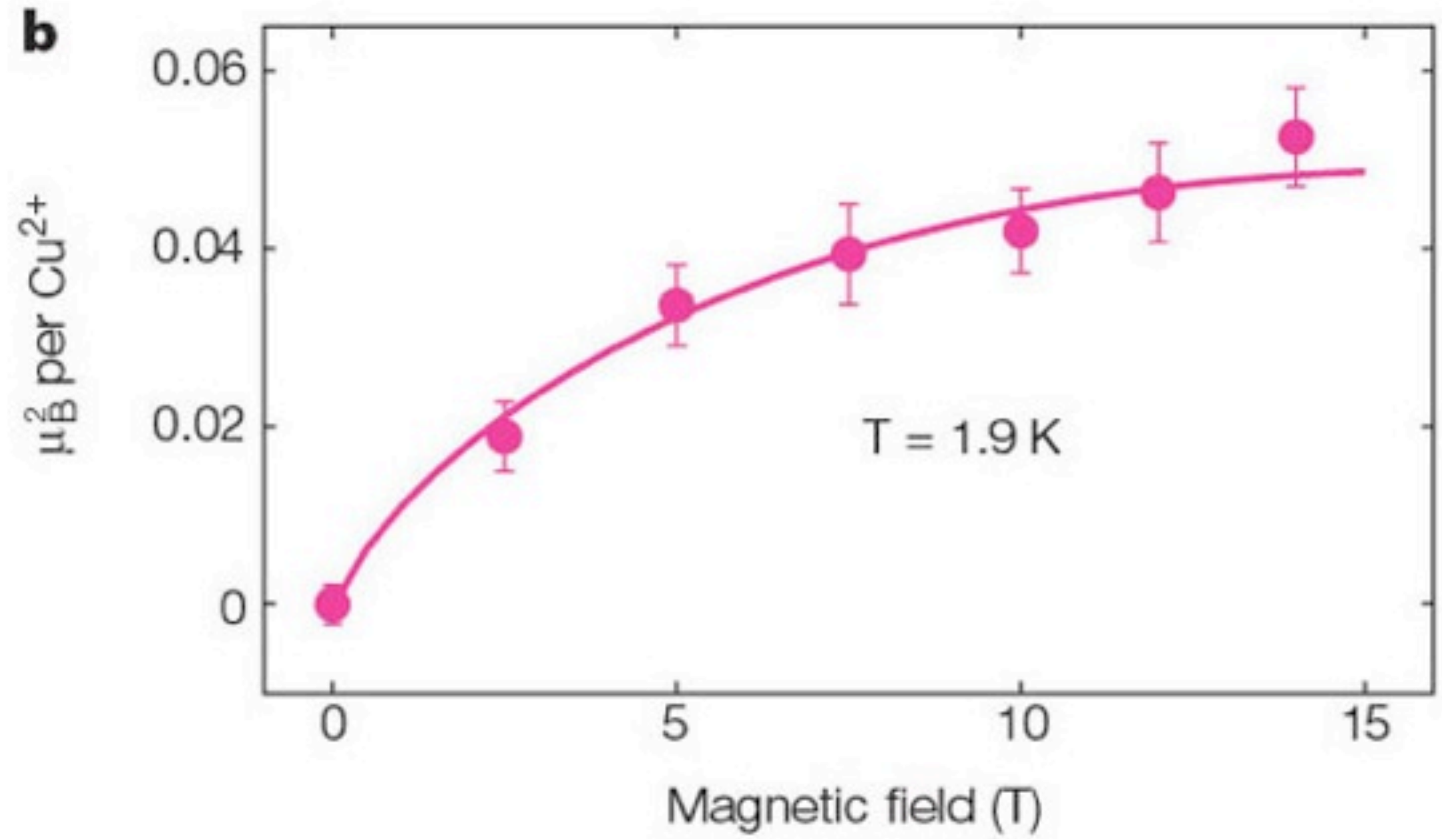


Neutron scattering & muon resonance

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).





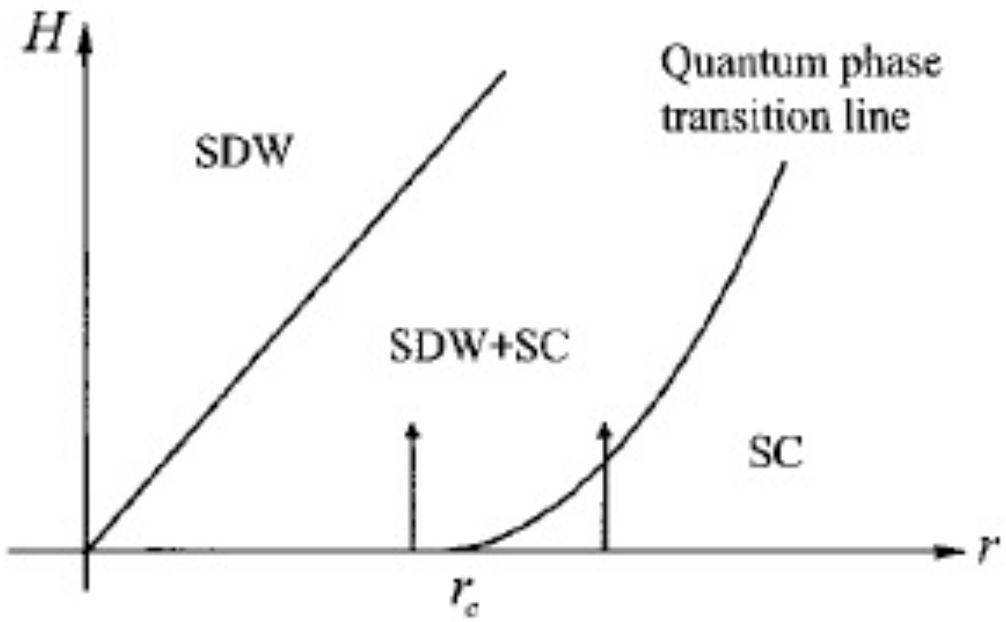
B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, and T. E. Mason, Nature **415**, 299 (2002)

B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder *Science* **291**, 1759 (2001).

PHYSICAL REVIEW B **71**, 220508(R) (2005)

Field-induced transition between magnetically disordered and ordered phases in underdoped $La_{2-x}Sr_xCuO_4$

B. Khaykovich,[1] S. Wakimoto,[2] R. J. Birgeneau,[3] M. A. Kastner,[1] Y. S. Lee,[1] P. Smeibidl,[4] P. Vorderwisch,[4] and K. Yamada[5]



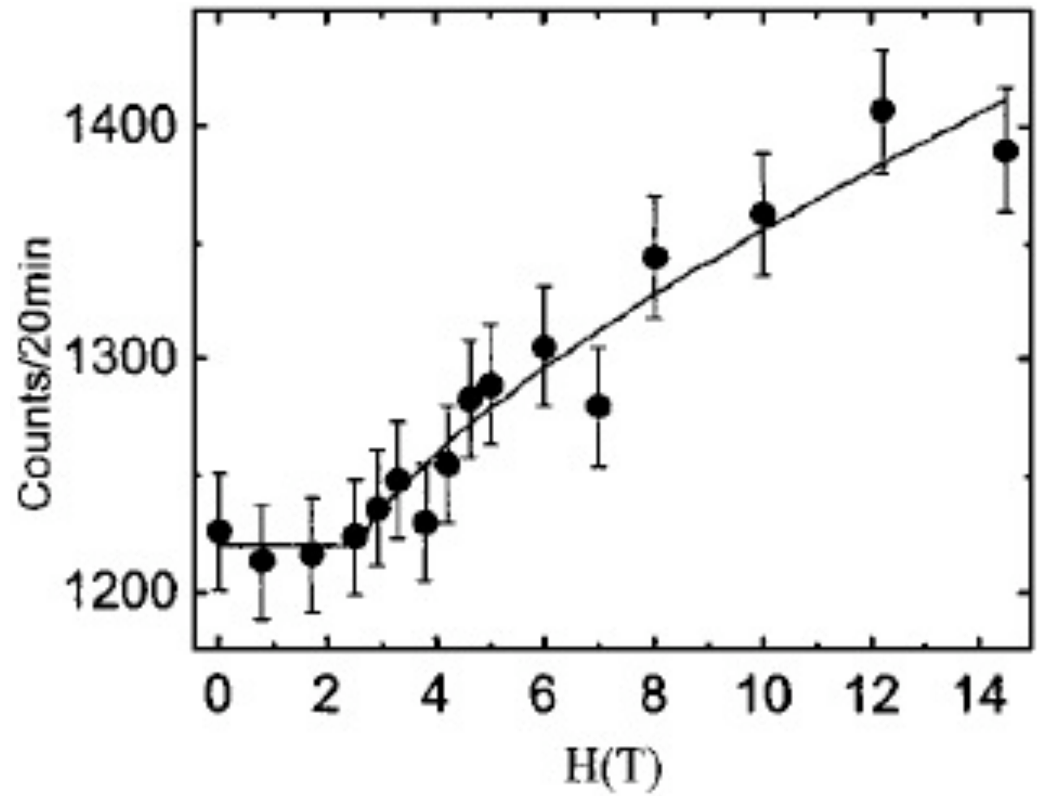
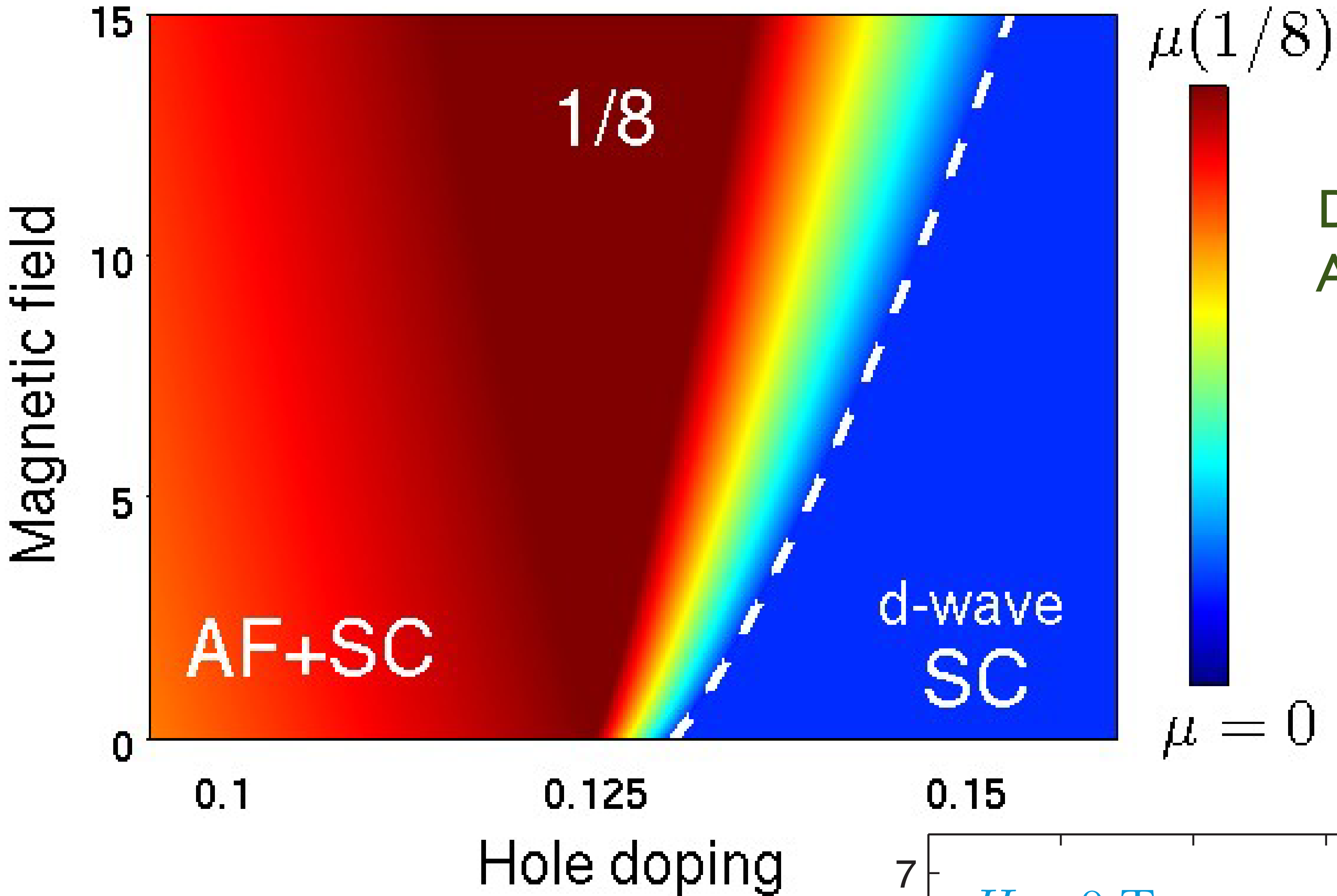
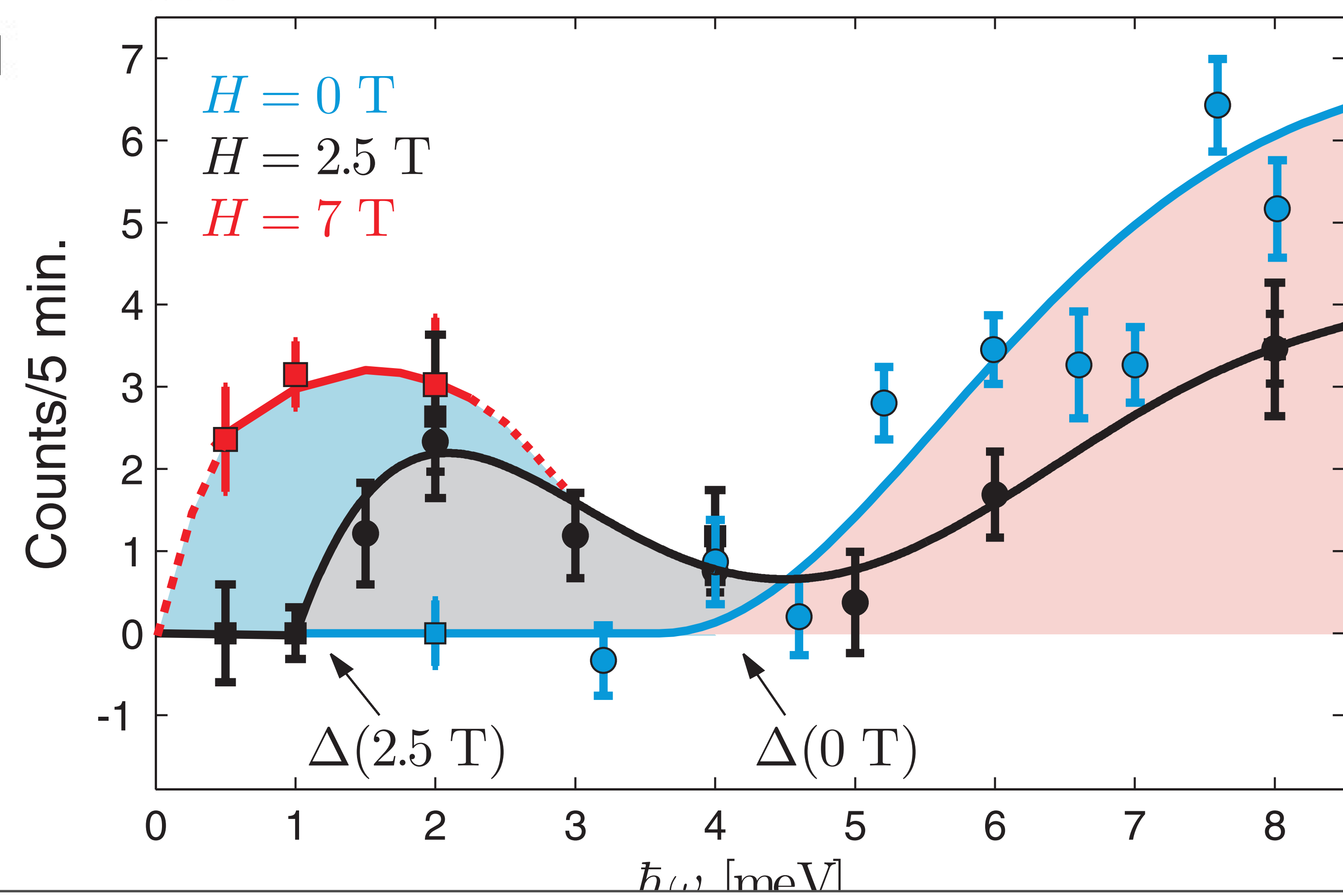


FIG. 1. (a) A fragment of the theoretical phase diagram, adopted from Refs. 4 and 20. The vertical axis is the magnetic field and the horizontal axis is the coupling strength between superconductivity and magnetic order. (b) Field dependence of the magnetic Bragg peak corresponding to the incommensurate SDW peak at $Q=(1.125,0.125,0)$. Every point is measured after field cooling at $T=1.5$ K. The data are fitted to $I=I_0+A|H-H_c|^{2\beta}$ above H_c as explained in the text. Spectrometer configuration: 45-60-Be—S—Be-60-open; cold Be filters were used before and after the sample to eliminate contamination from high-energy neutrons; $E=4$ meV.

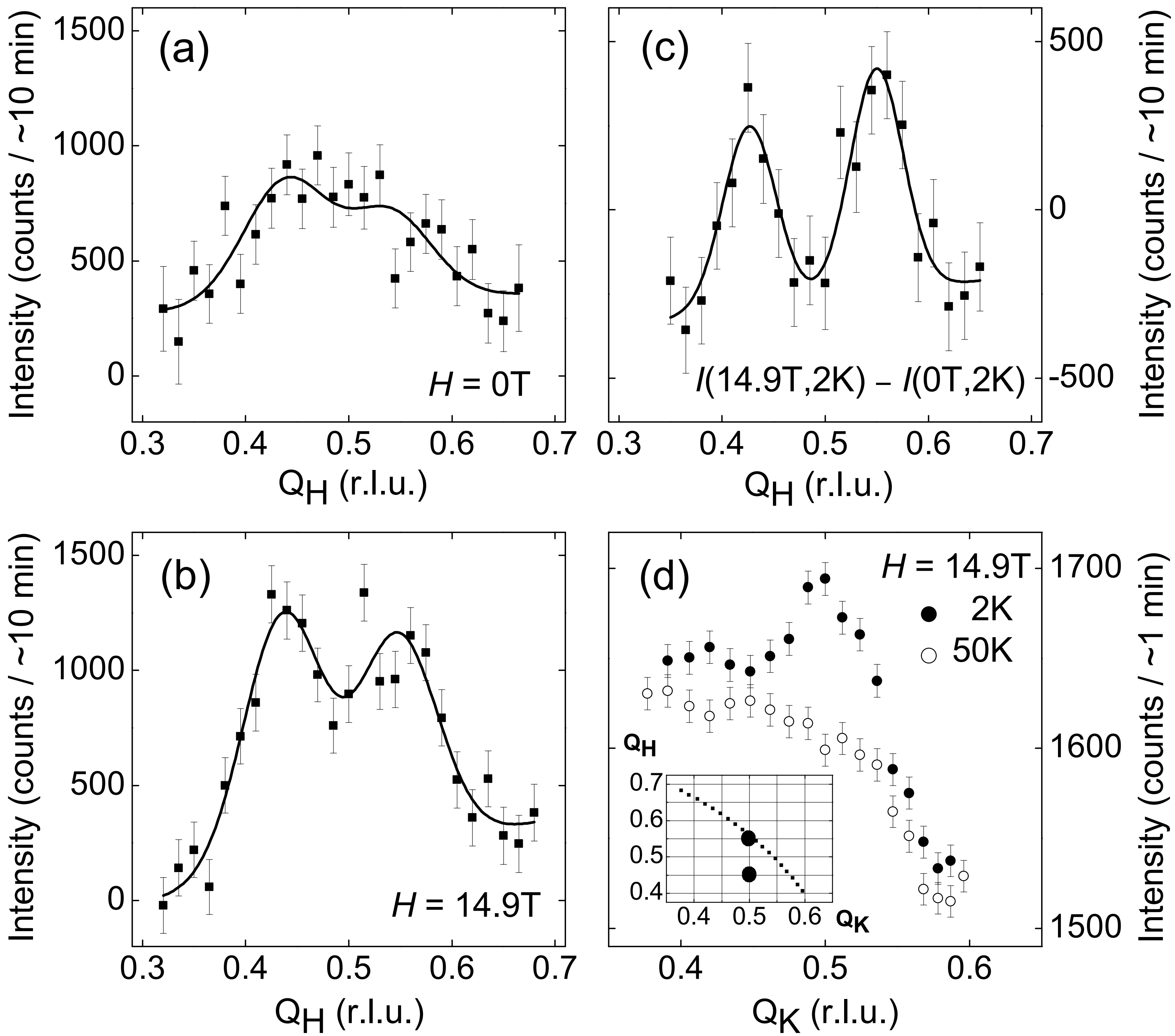
Friday, December 18, 2009

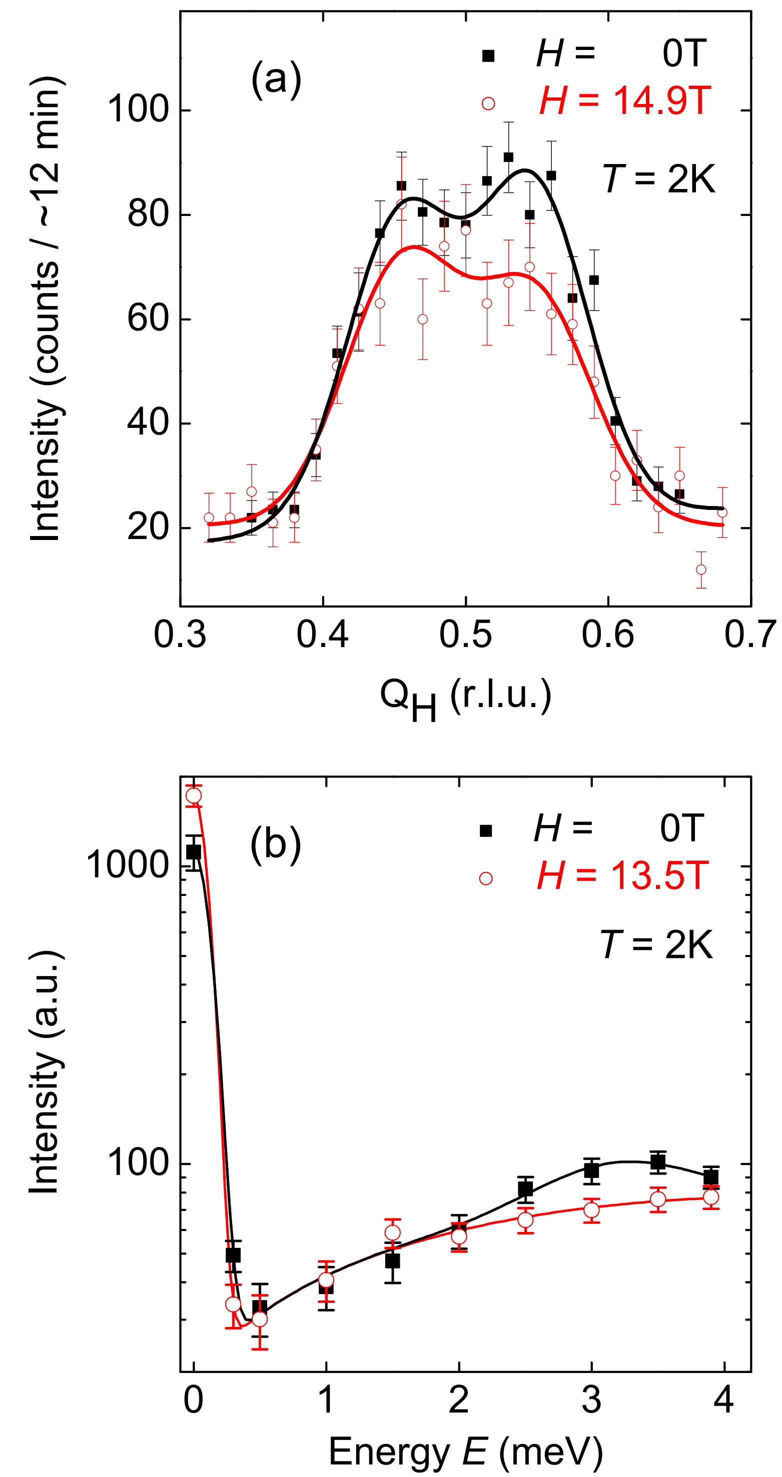


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).

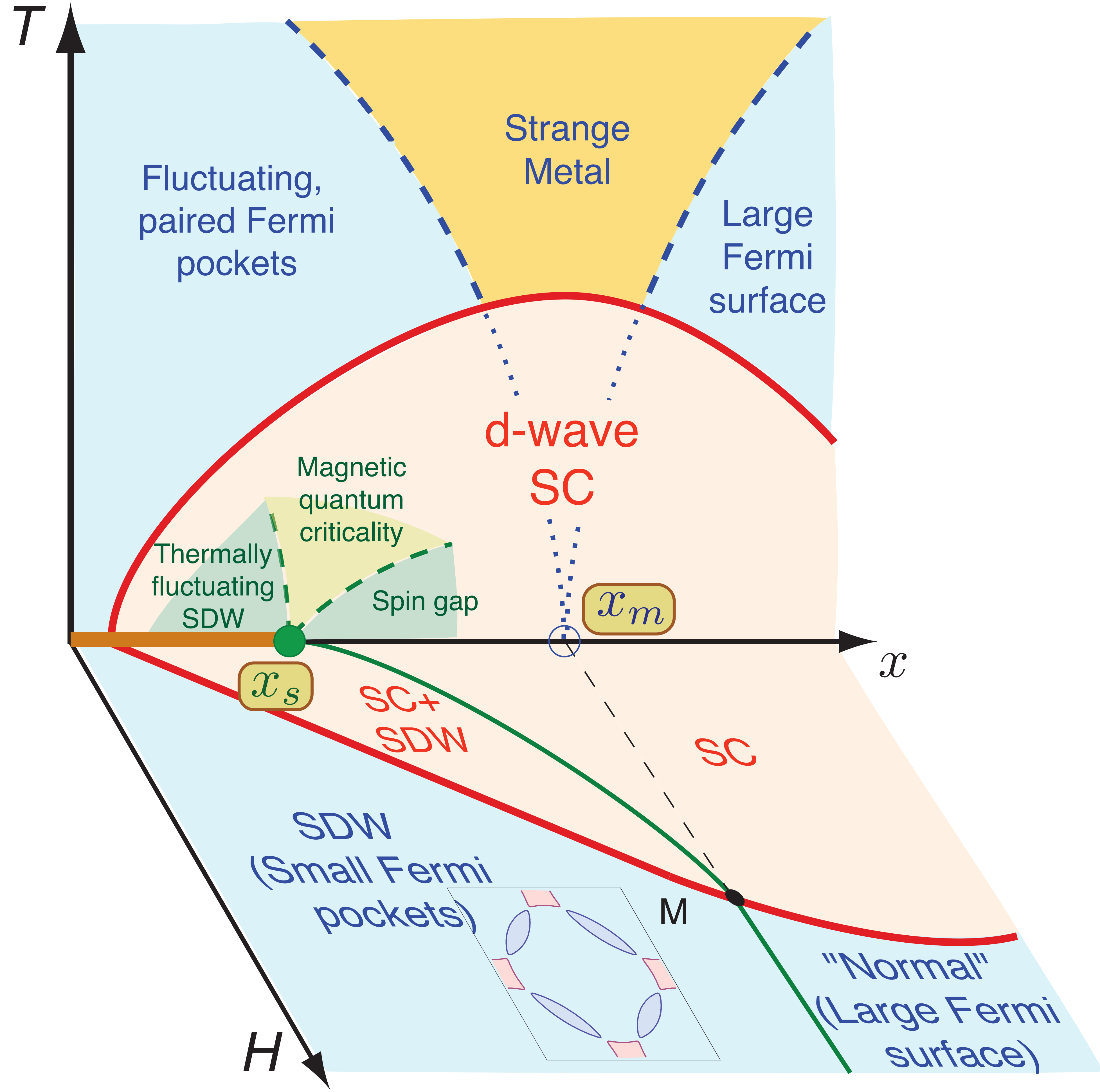


J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).





D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)


T
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Large
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surface
d-wave
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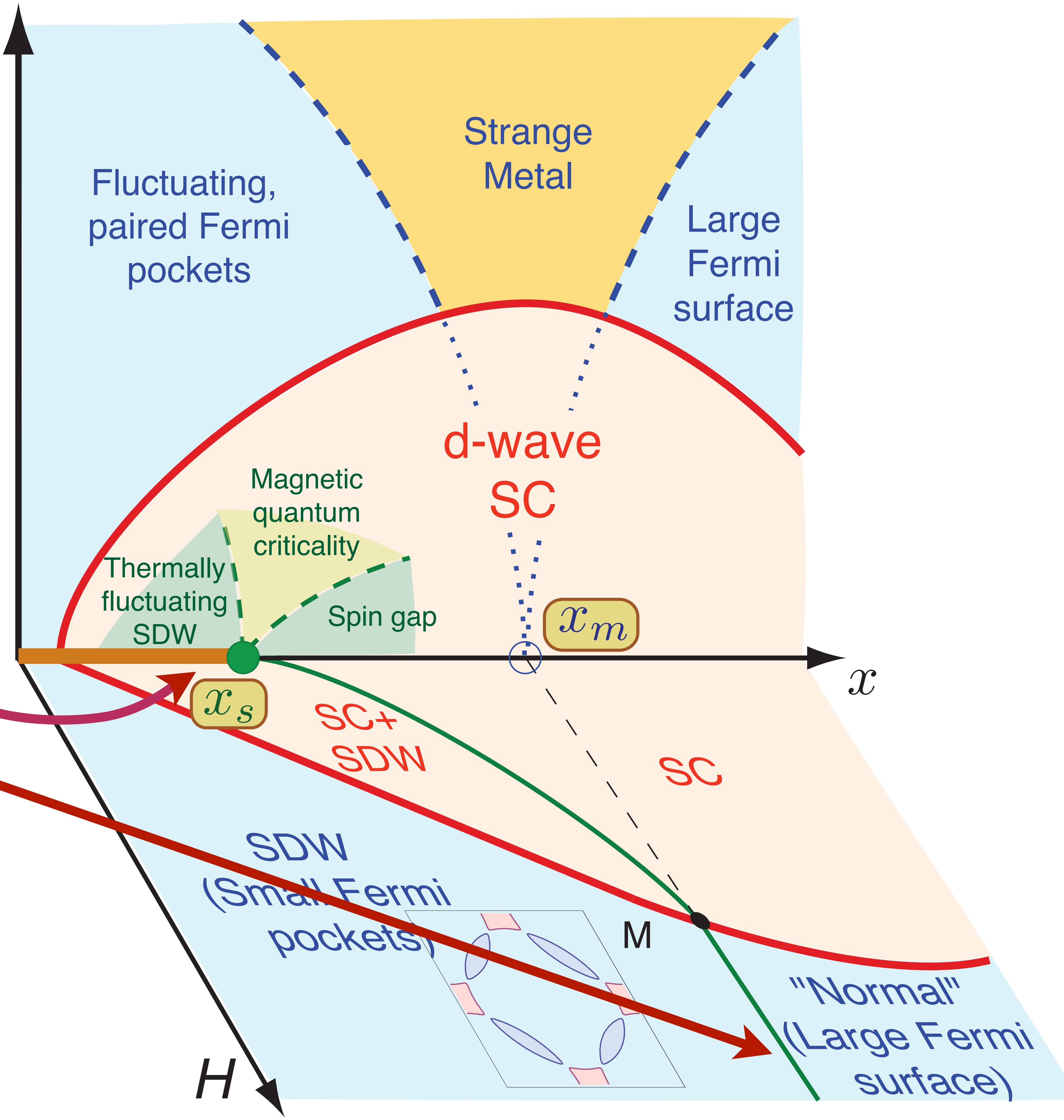
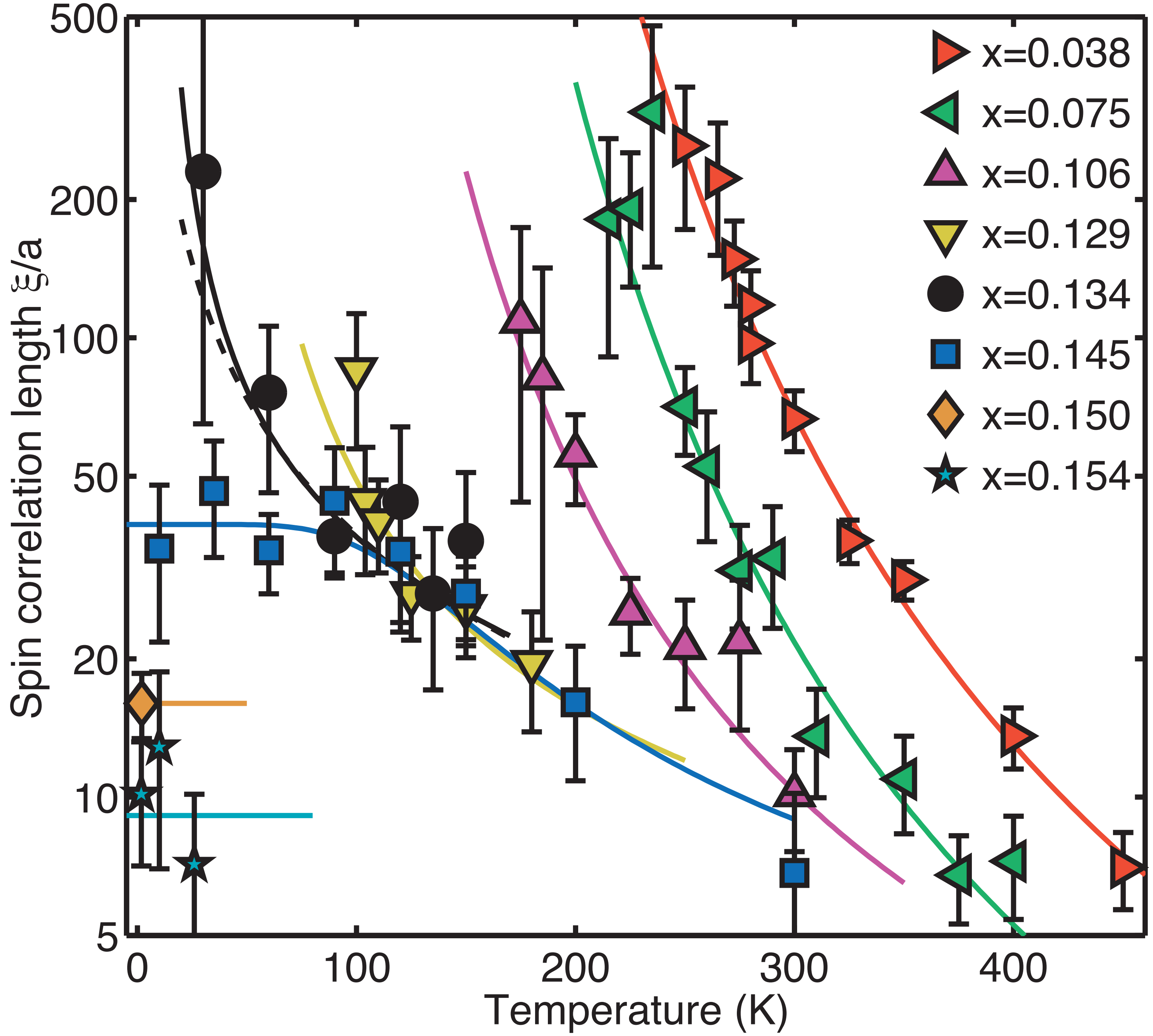
Thermally
fluctuating
SDW
Spin gap
x_m
x_s
x
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

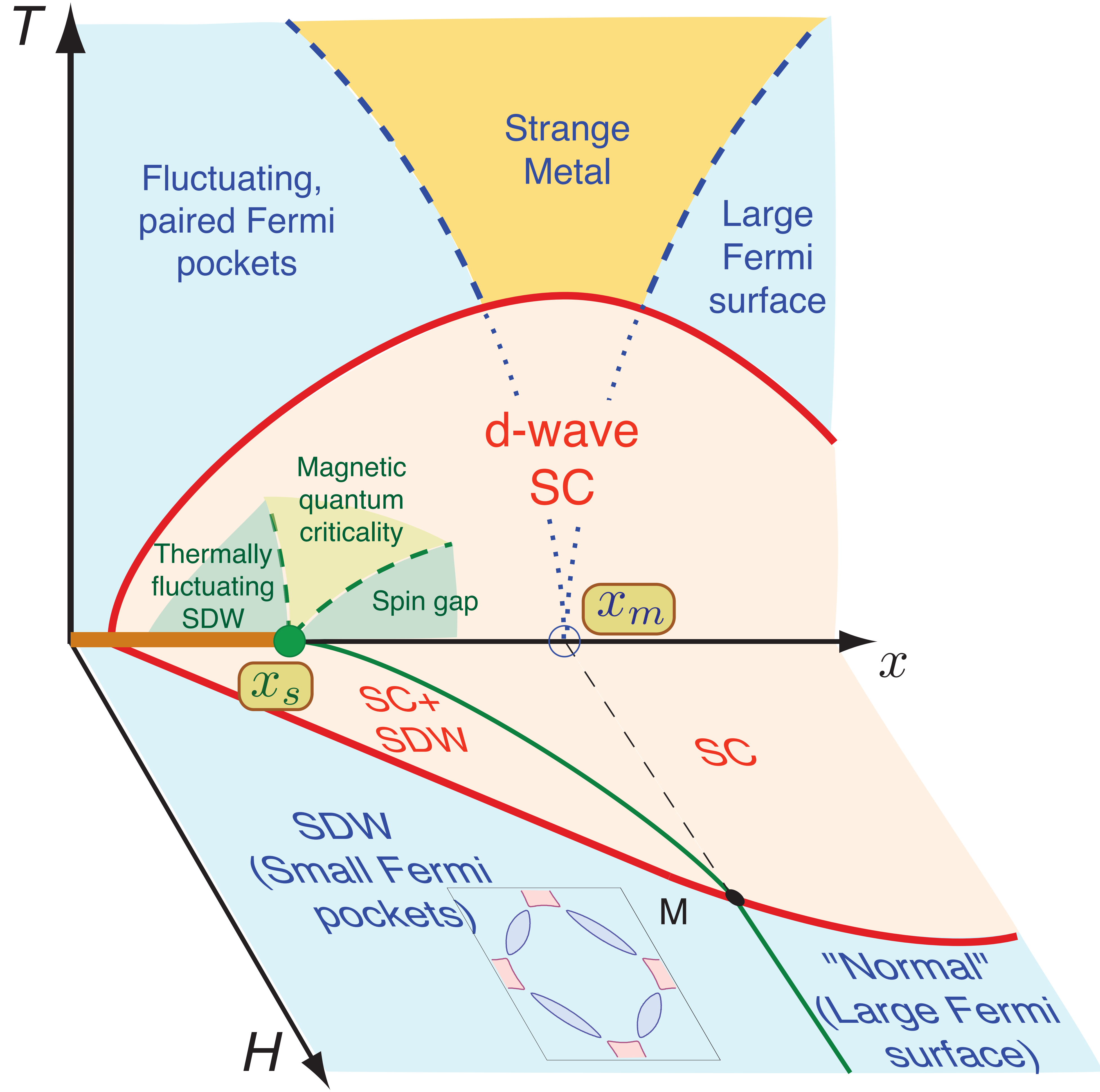
Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.



$Nd_{2-x}Ce_xCuO_4$



E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).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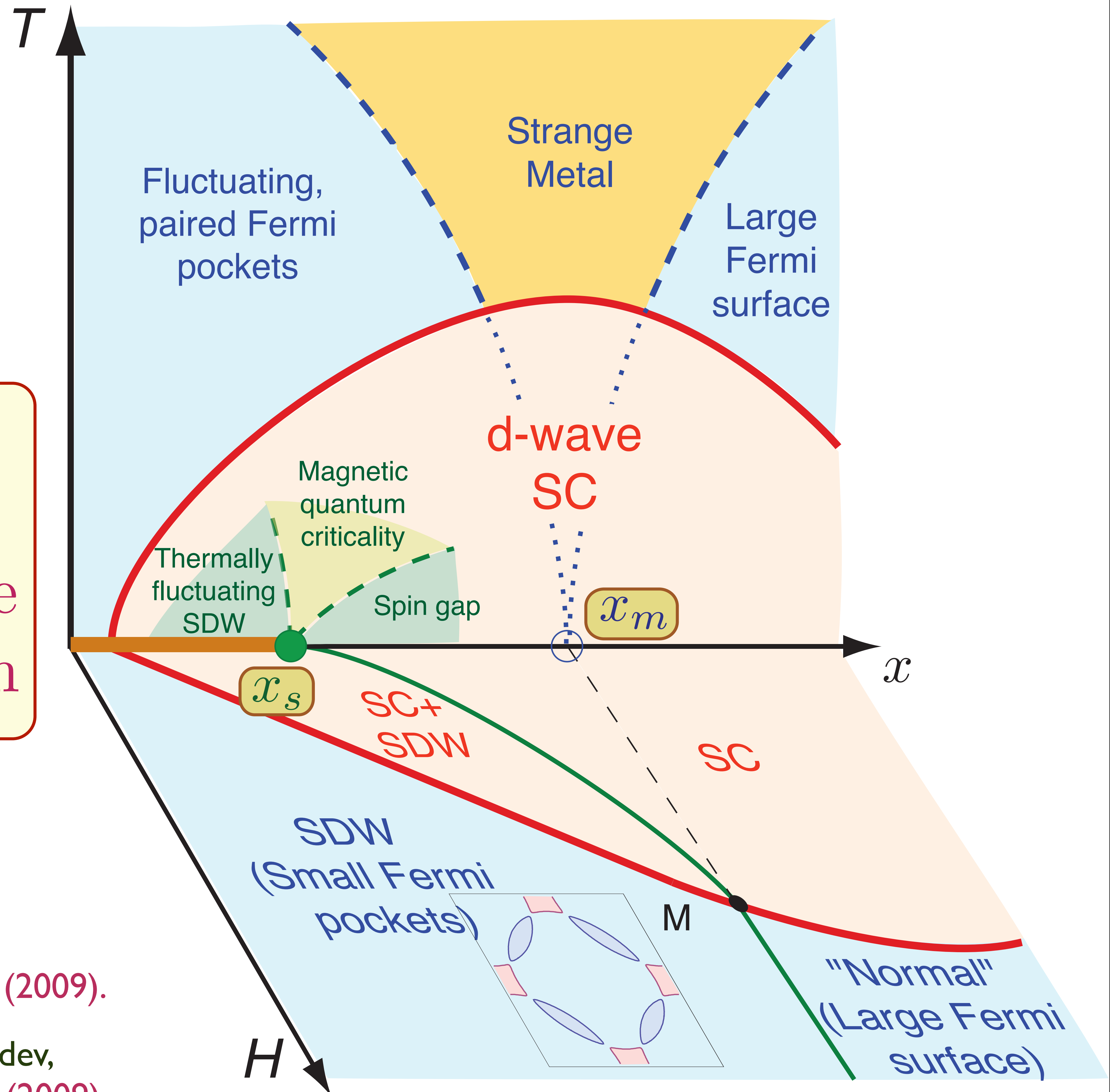
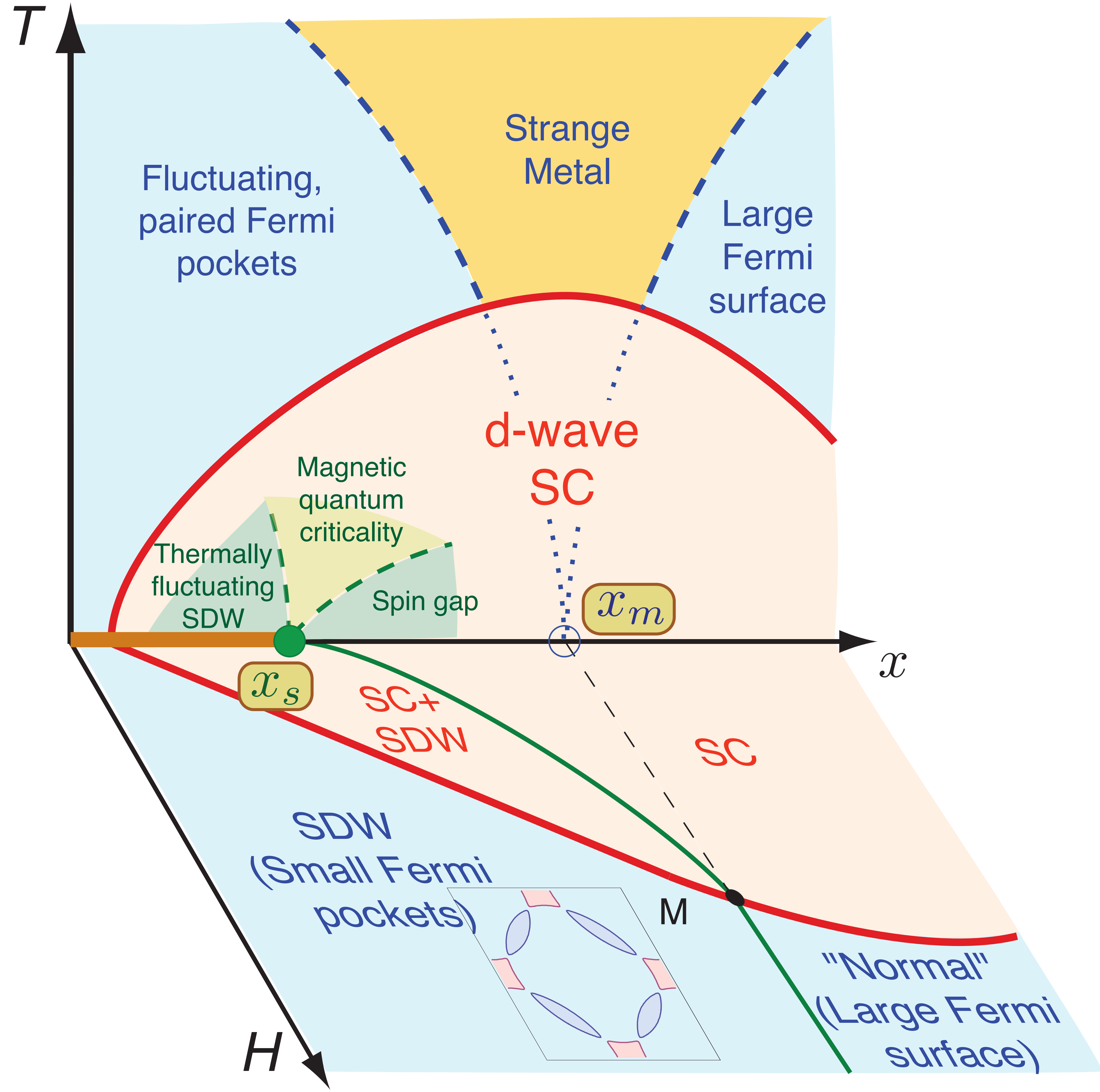
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Theory of qualitative features of the phase diagram



V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

Eun Gook Moon and S. Sachdev, *Physical Review B* **80**, 035117 (2009).



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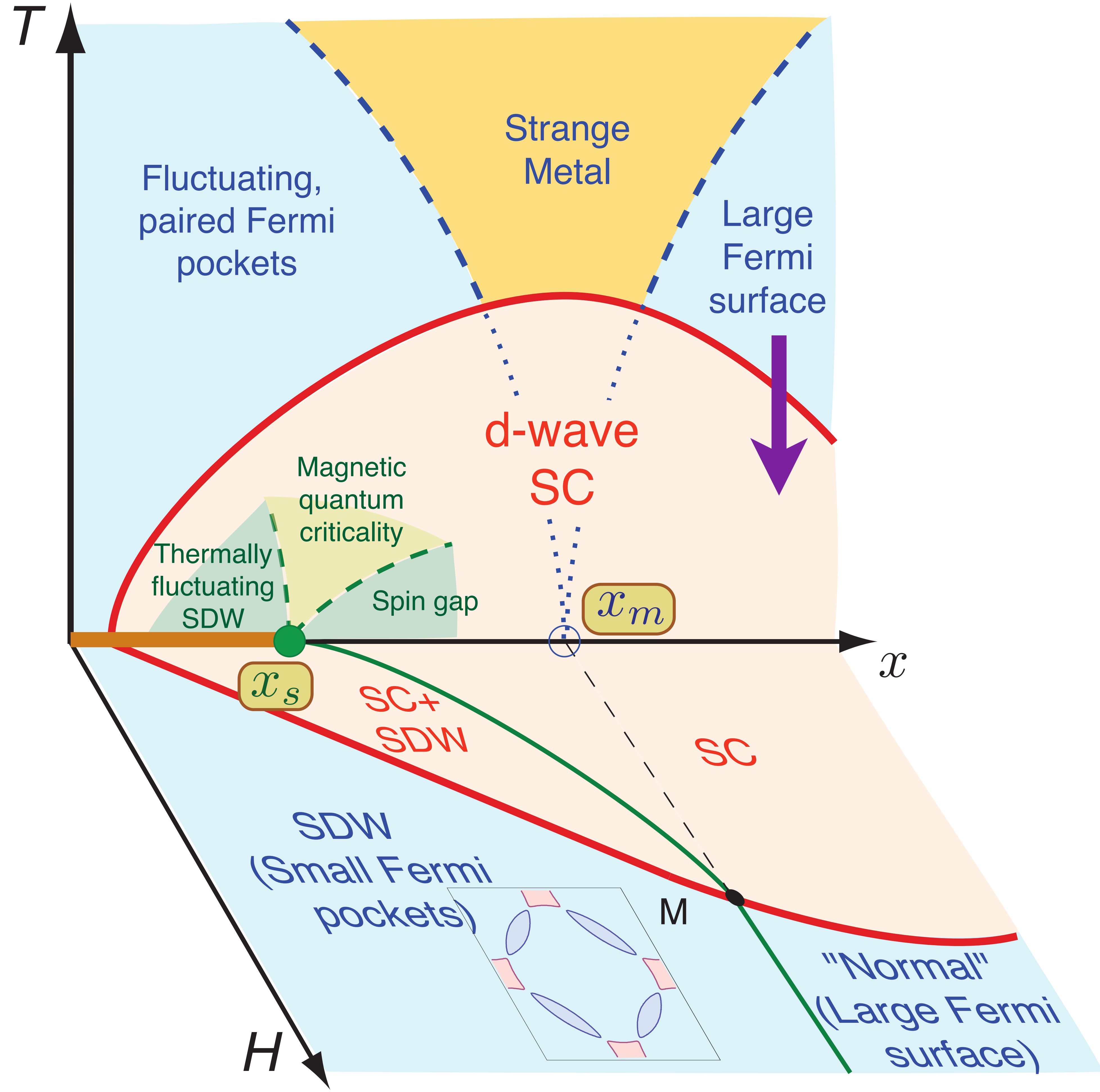
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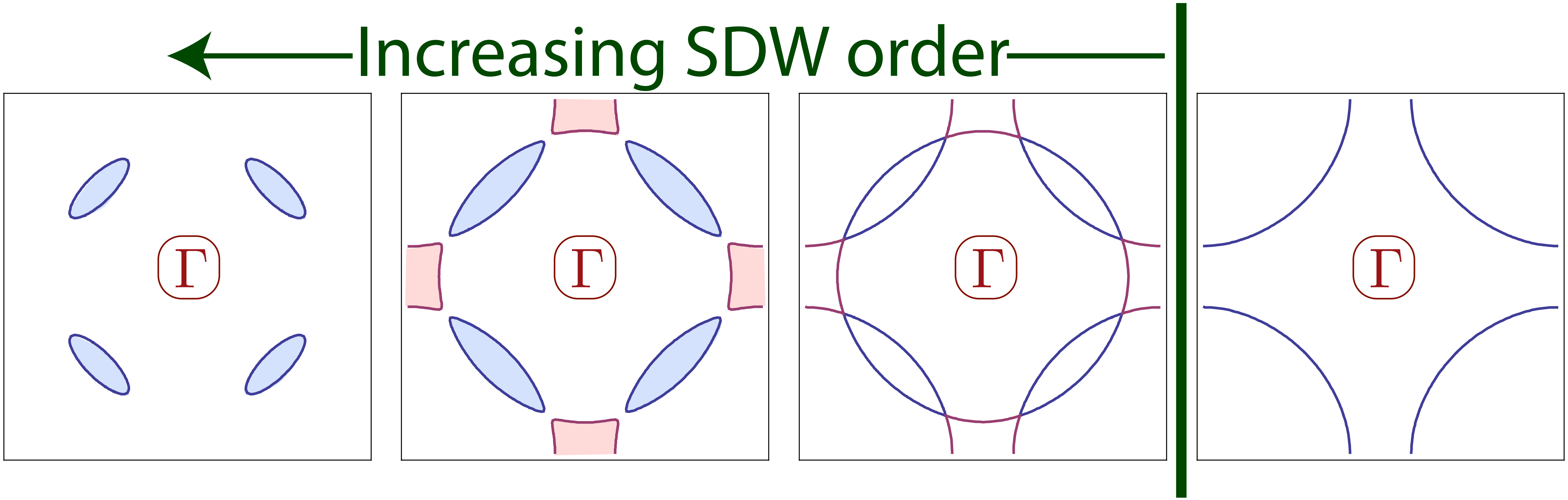
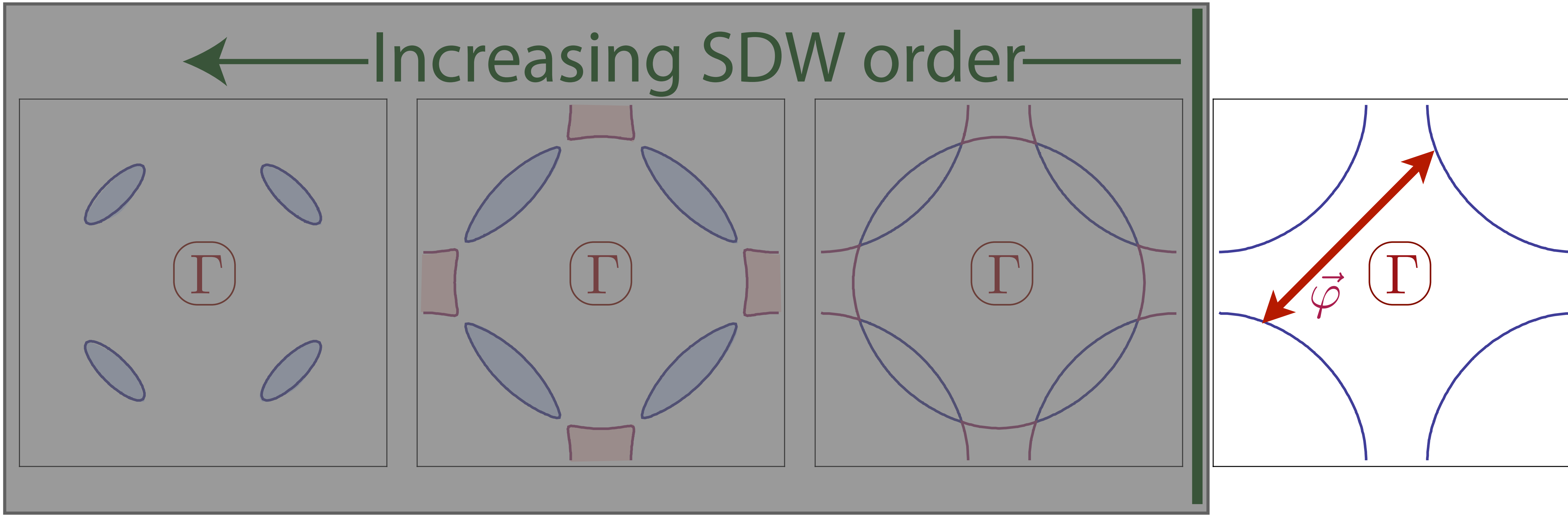
Spin density wave theory in hole-doped cuprates



Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

David Pines, Douglas Scalapino

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^{\dagger}_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q} - \mathbf{K})^2},$$

with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

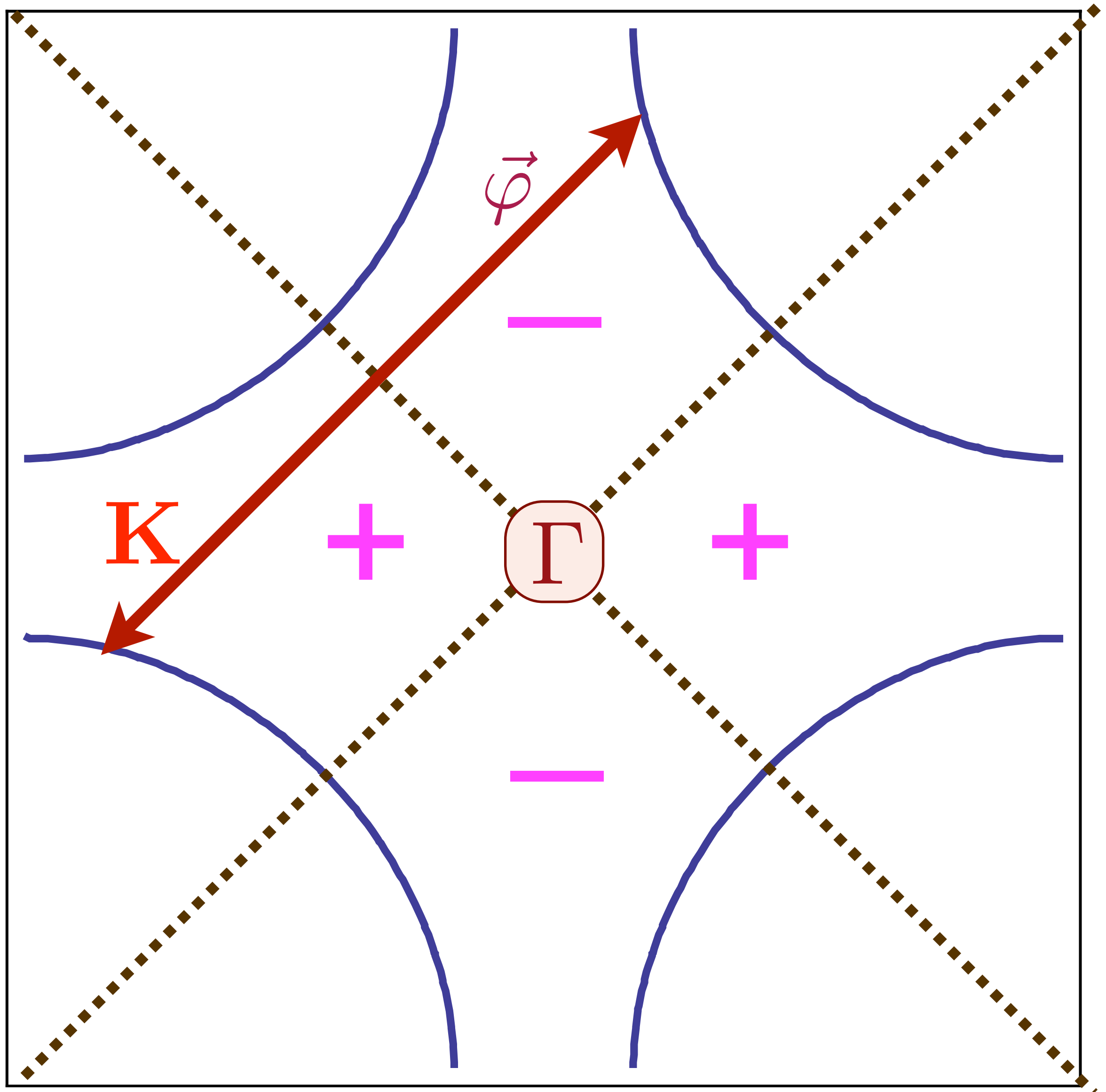
BCS Gap equation

In BCS theory, this interaction leads to the 'gap equation' for the pairing gap $\Delta_{\mathbf{k}} \propto \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$.

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{p}} \left(\frac{3\chi_0}{\xi^{-2} + (\mathbf{p} - \mathbf{k} - \mathbf{K})^2} \right) \frac{\Delta_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_{\mathbf{p}}^2}}$$

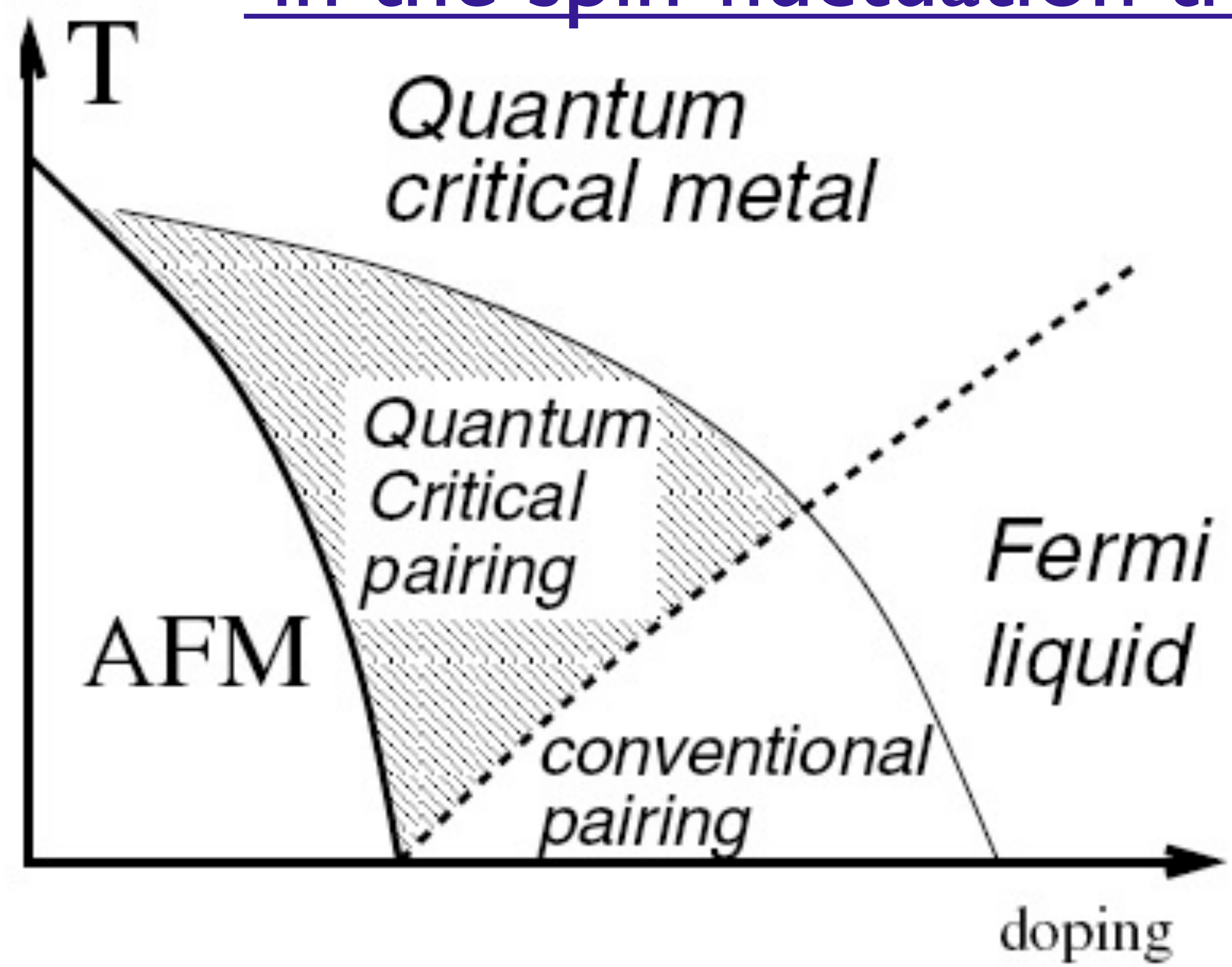
Non-zero solutions of this equation require that $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{p}}$ have opposite signs when $\mathbf{p} - \mathbf{k} \approx \mathbf{K}$.

d-wave pairing of the large Fermi surface



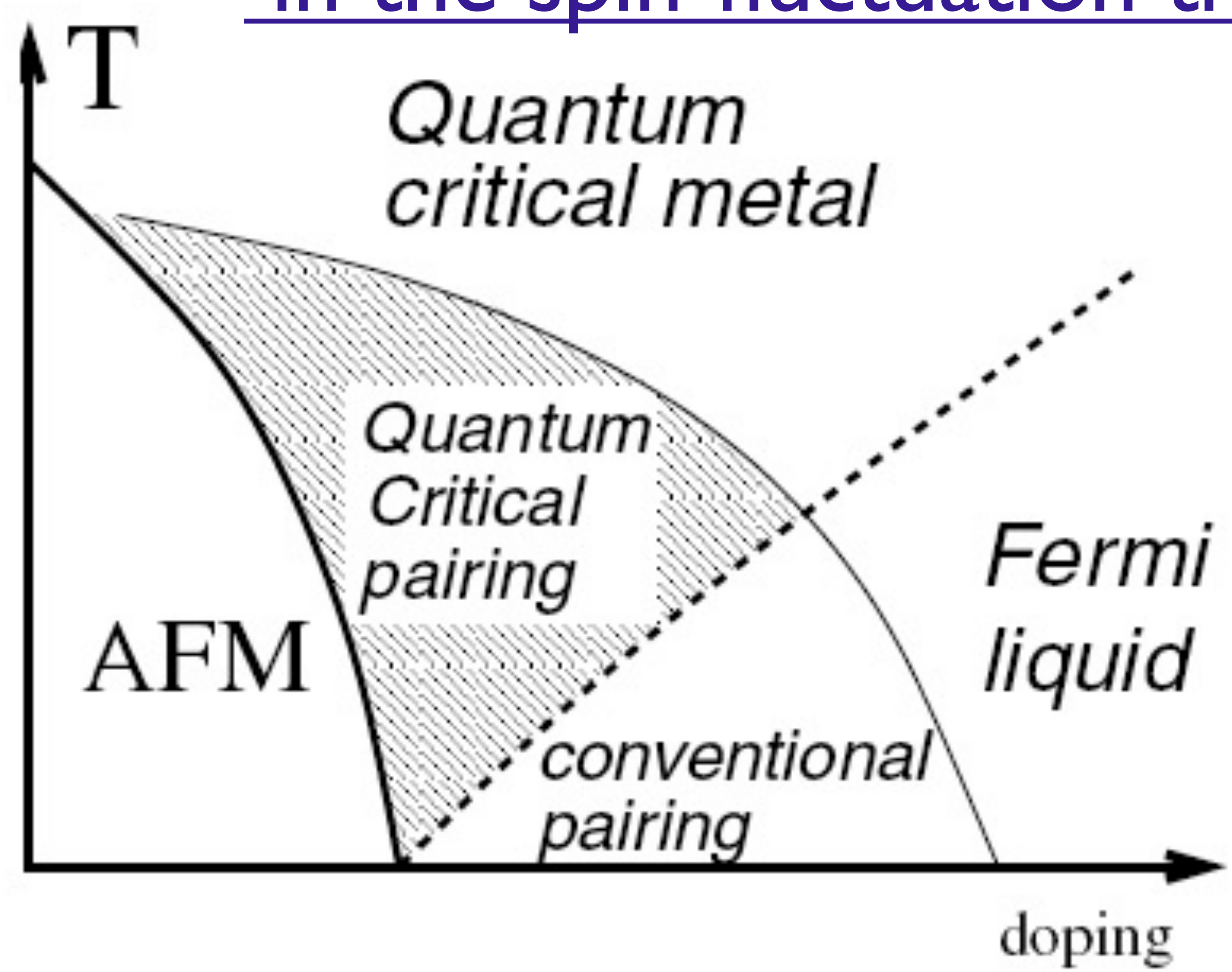
$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



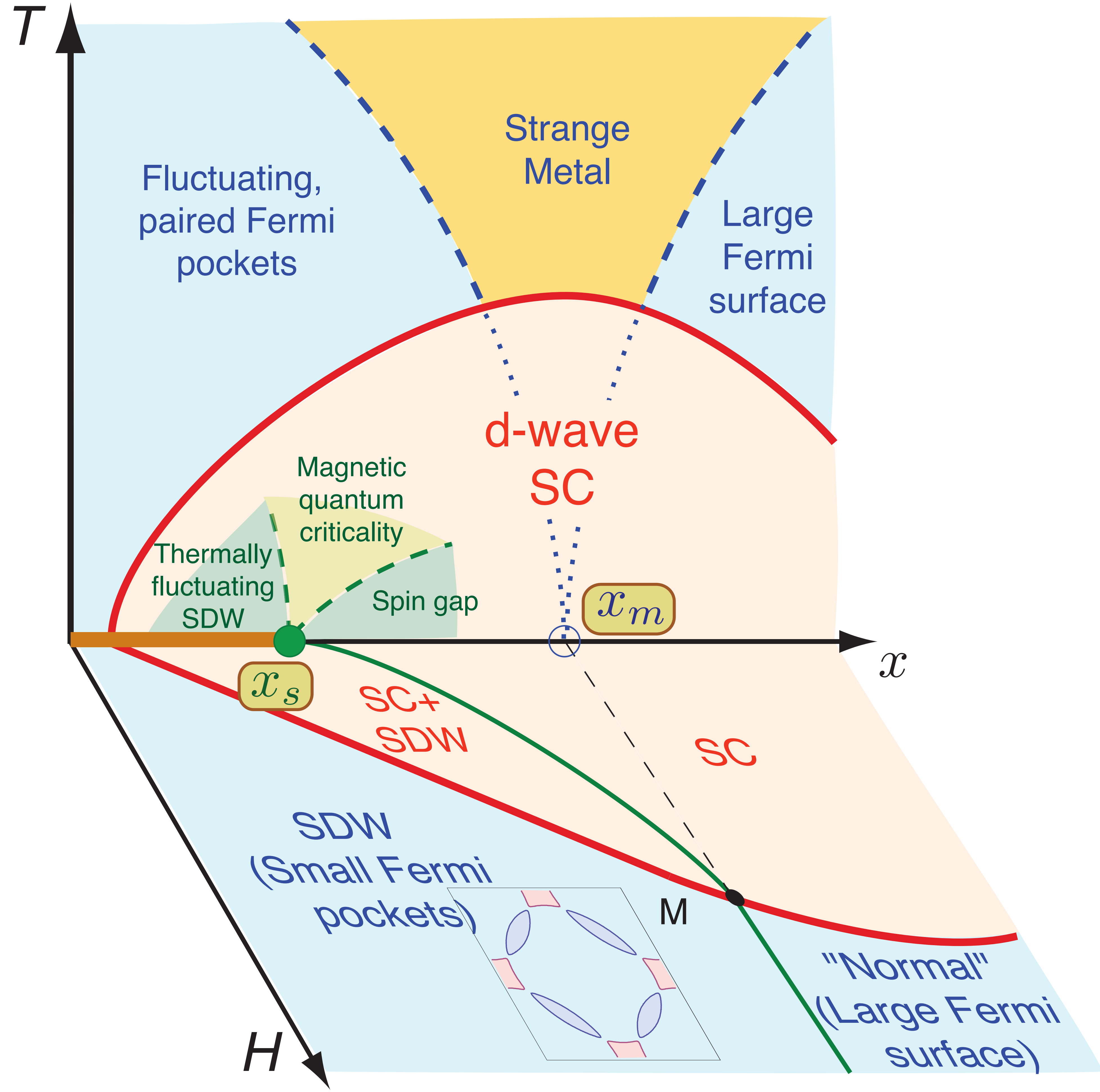
Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



- T_c *increases* upon approaching the SDW transition.
  SDW and SC orders do not compete, but attract each other.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).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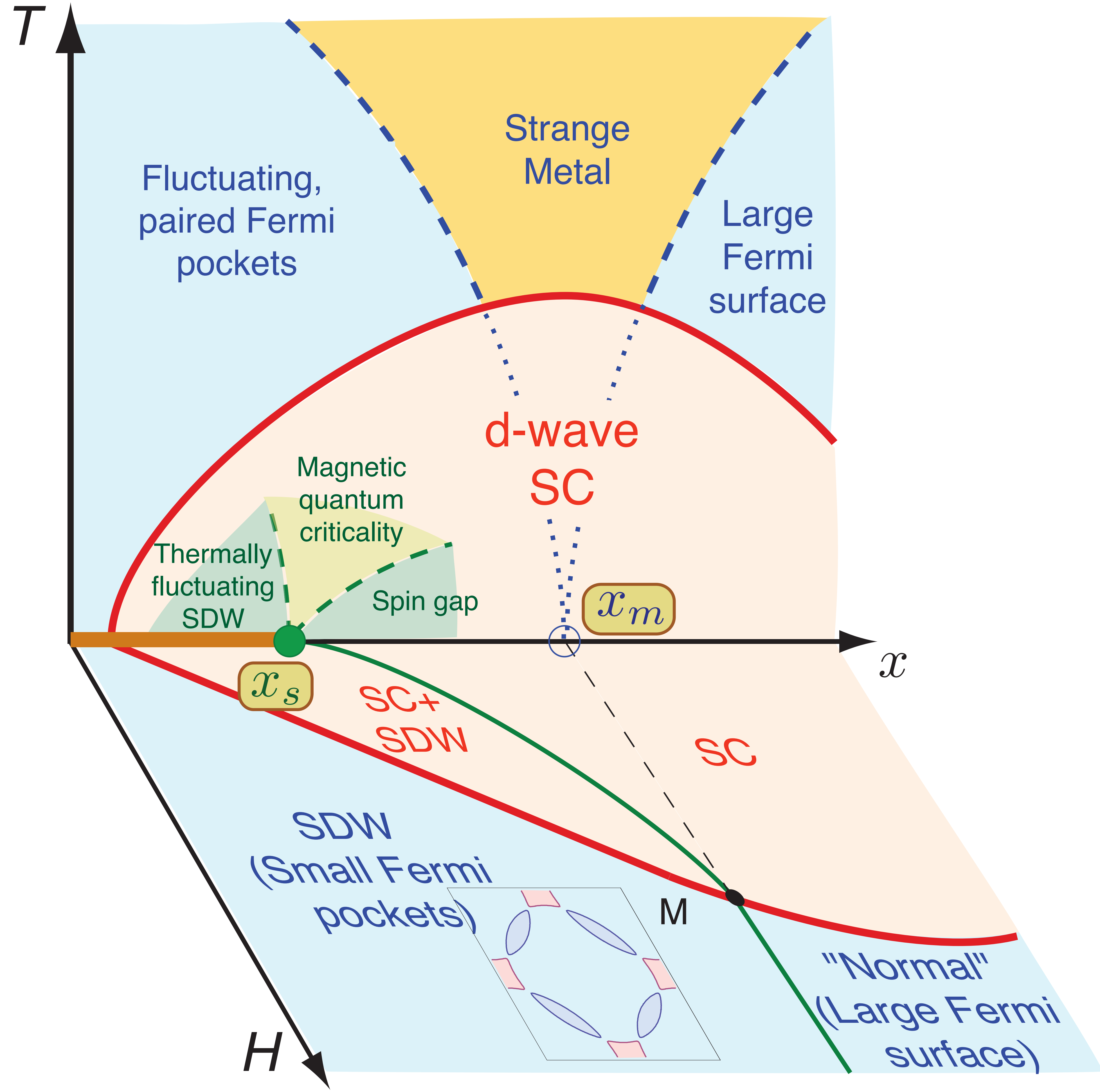
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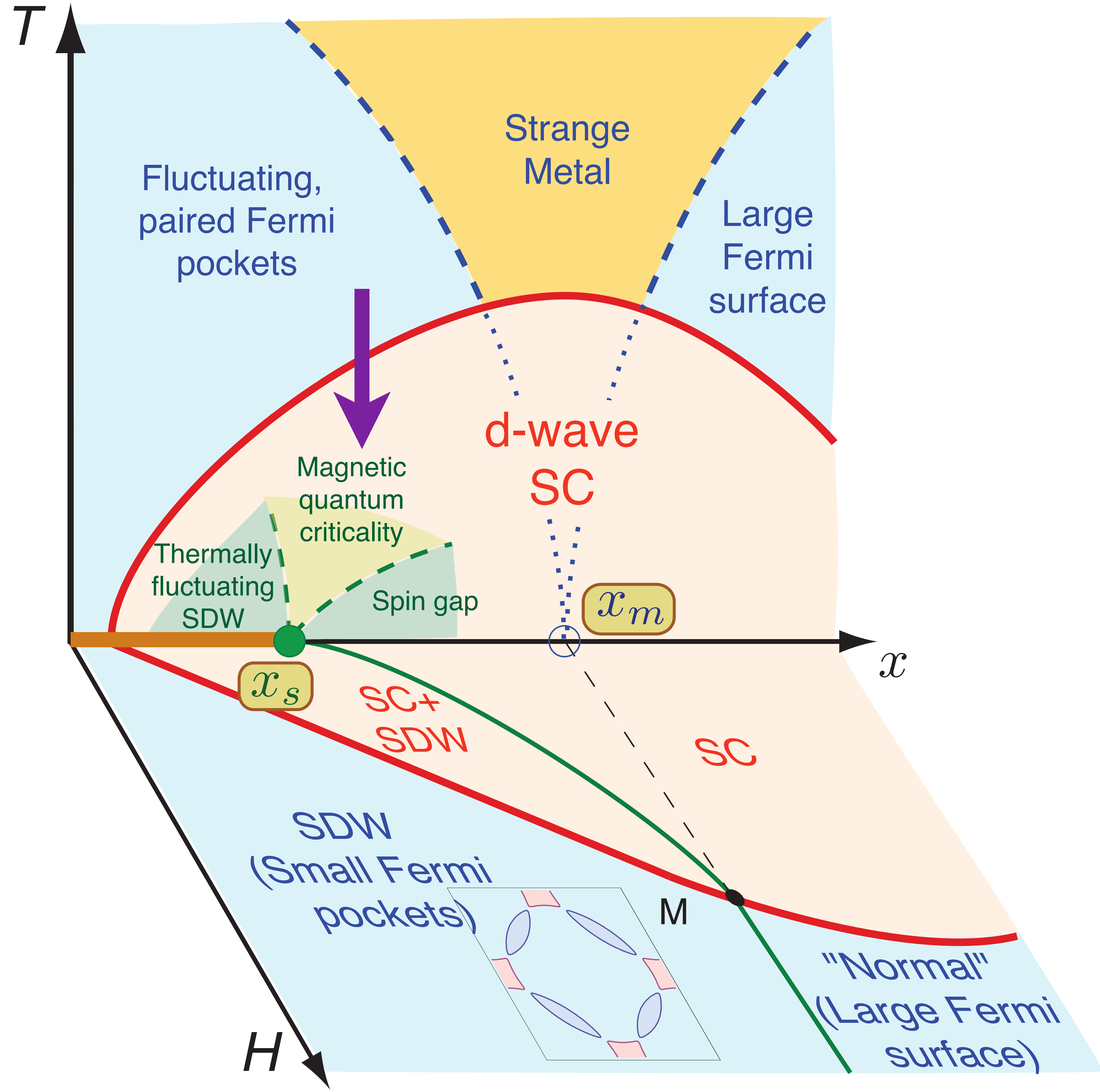

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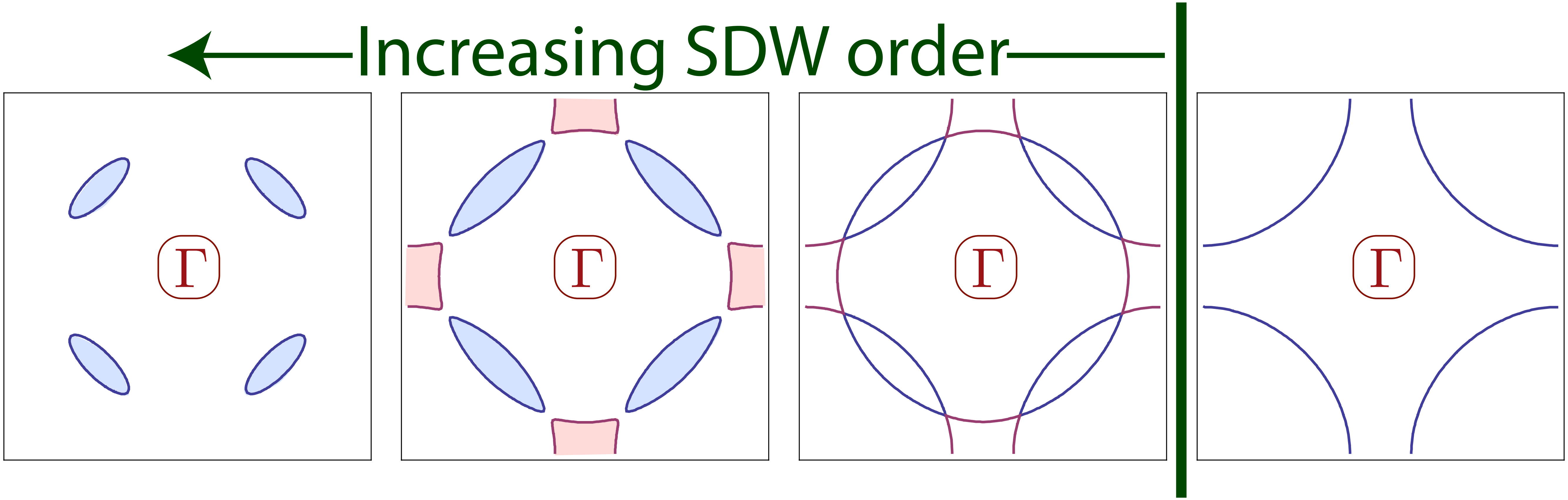
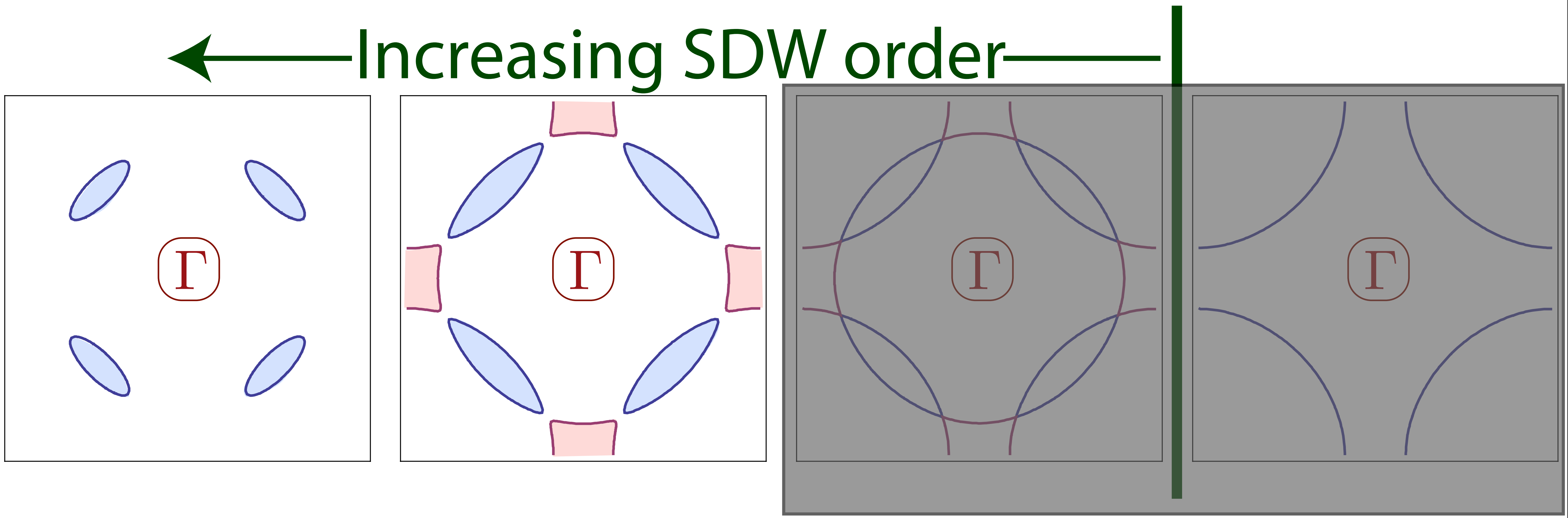
Spin density wave theory in hole-doped cuprates



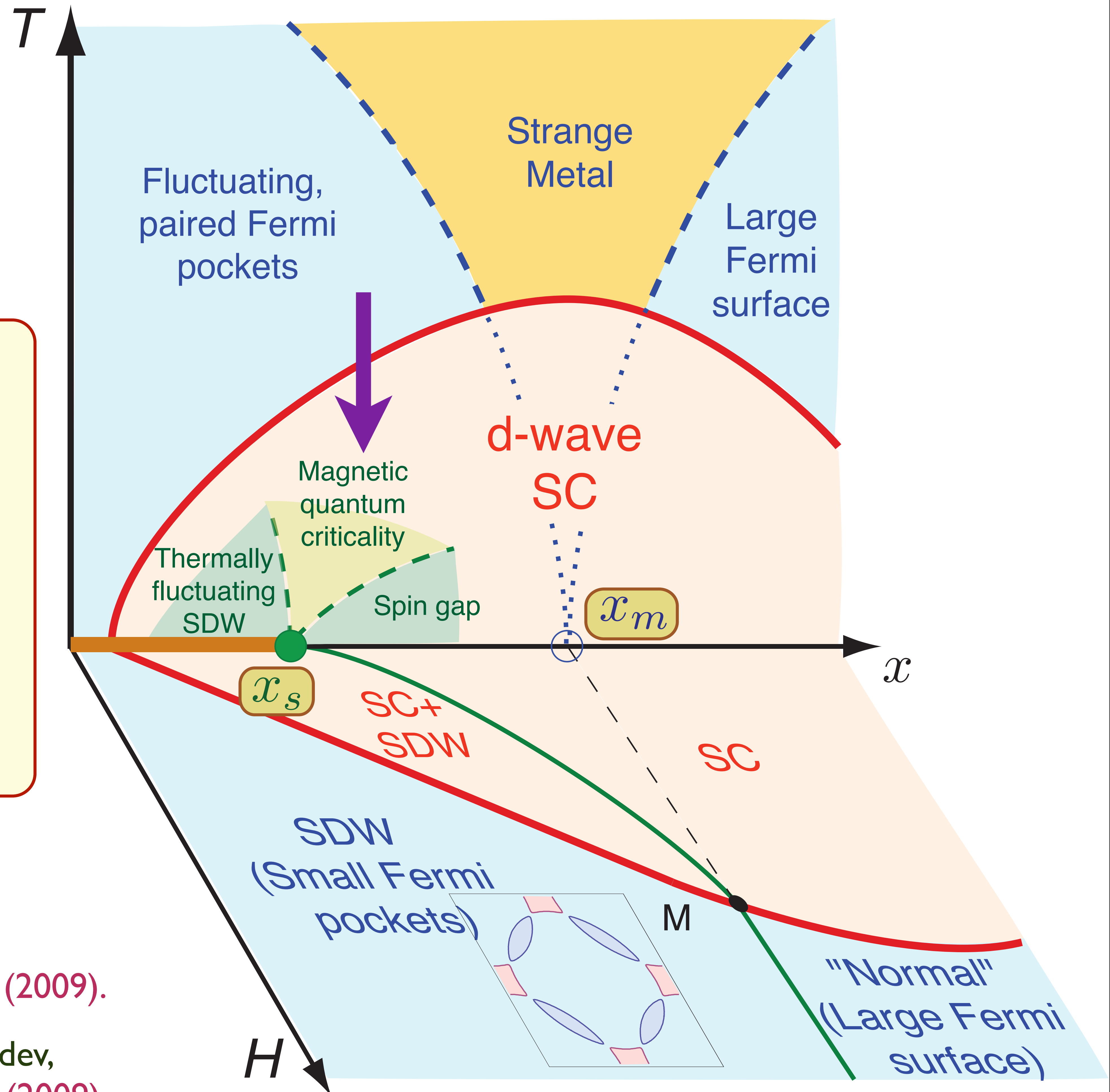
d-wave superconductivity in the underdoped cuprates



Pairing of pocket Fermi surfaces leads to decreasing T_c

V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Eun Gook Moon and S. Sachdev,
Physical Review B **80**, 035117 (2009).

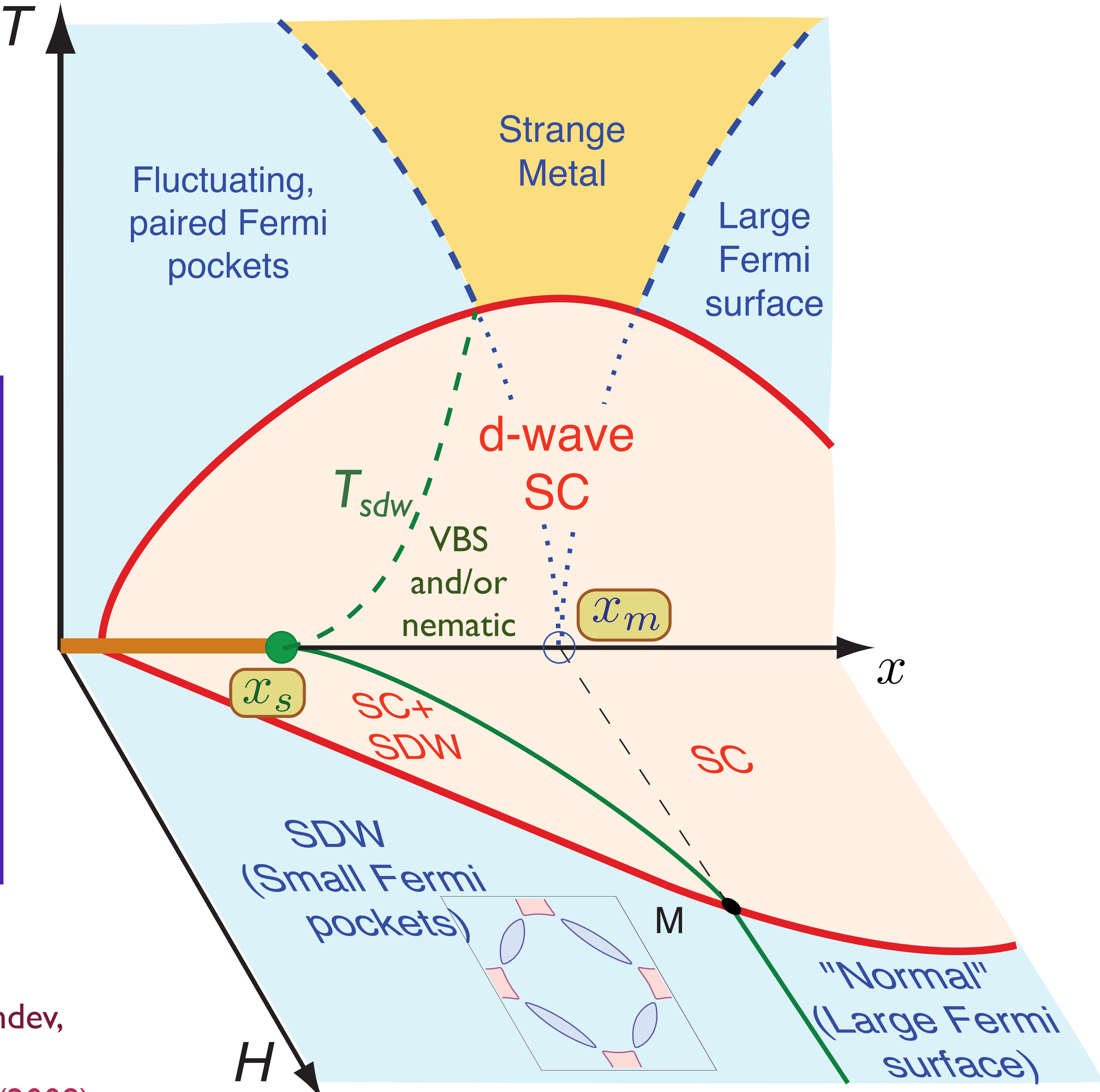


Physics of competition: d-wave SC and SDW "eat up" same pieces of the large Fermi surface.

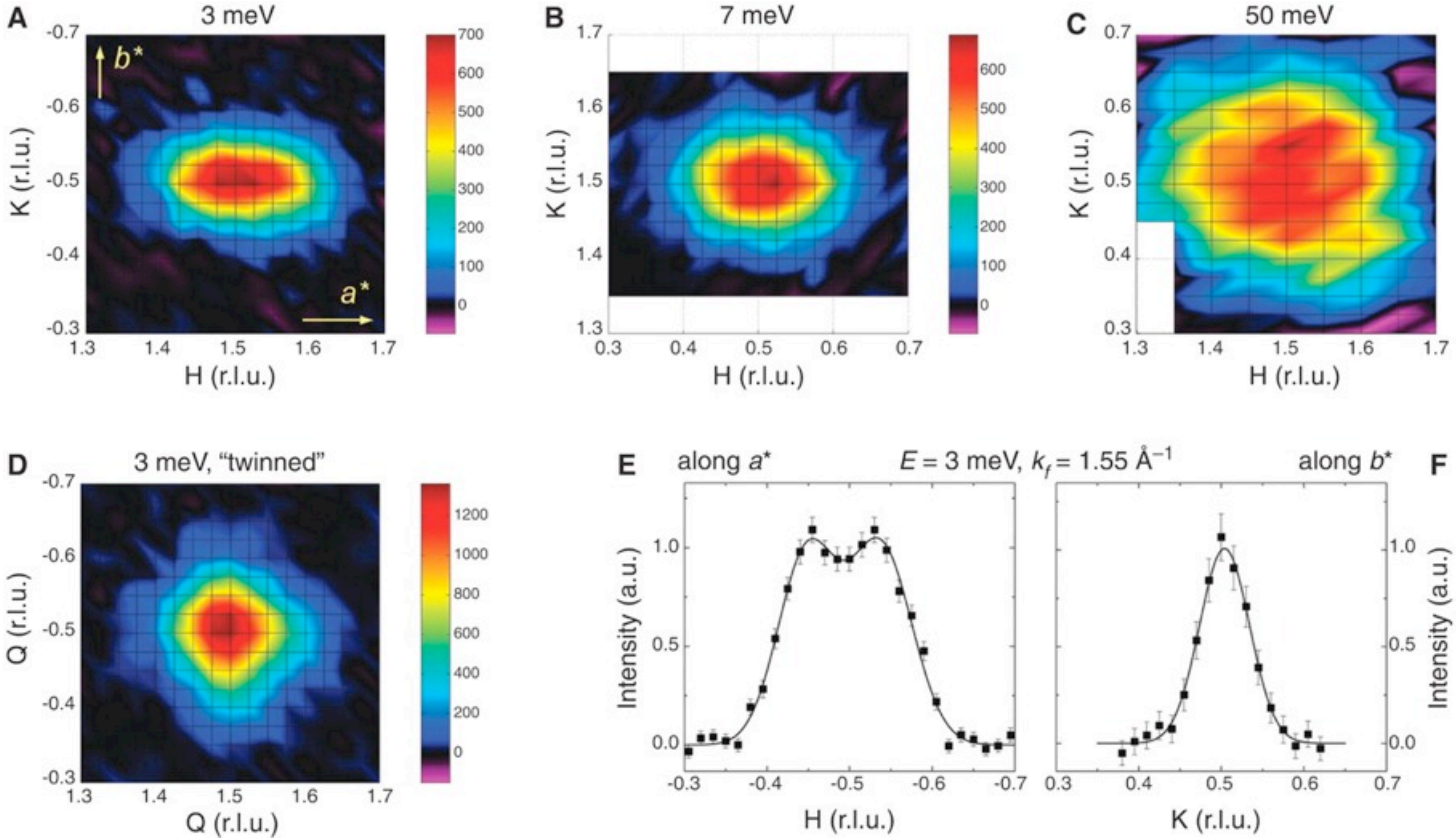
V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

Eun Gook Moon and S. Sachdev, *Physical Review B* **80**, 035117 (2009).

Onset of superconductivity disrupts SDW order, but associated CDW/VBS/nematic ordering can survive



R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).



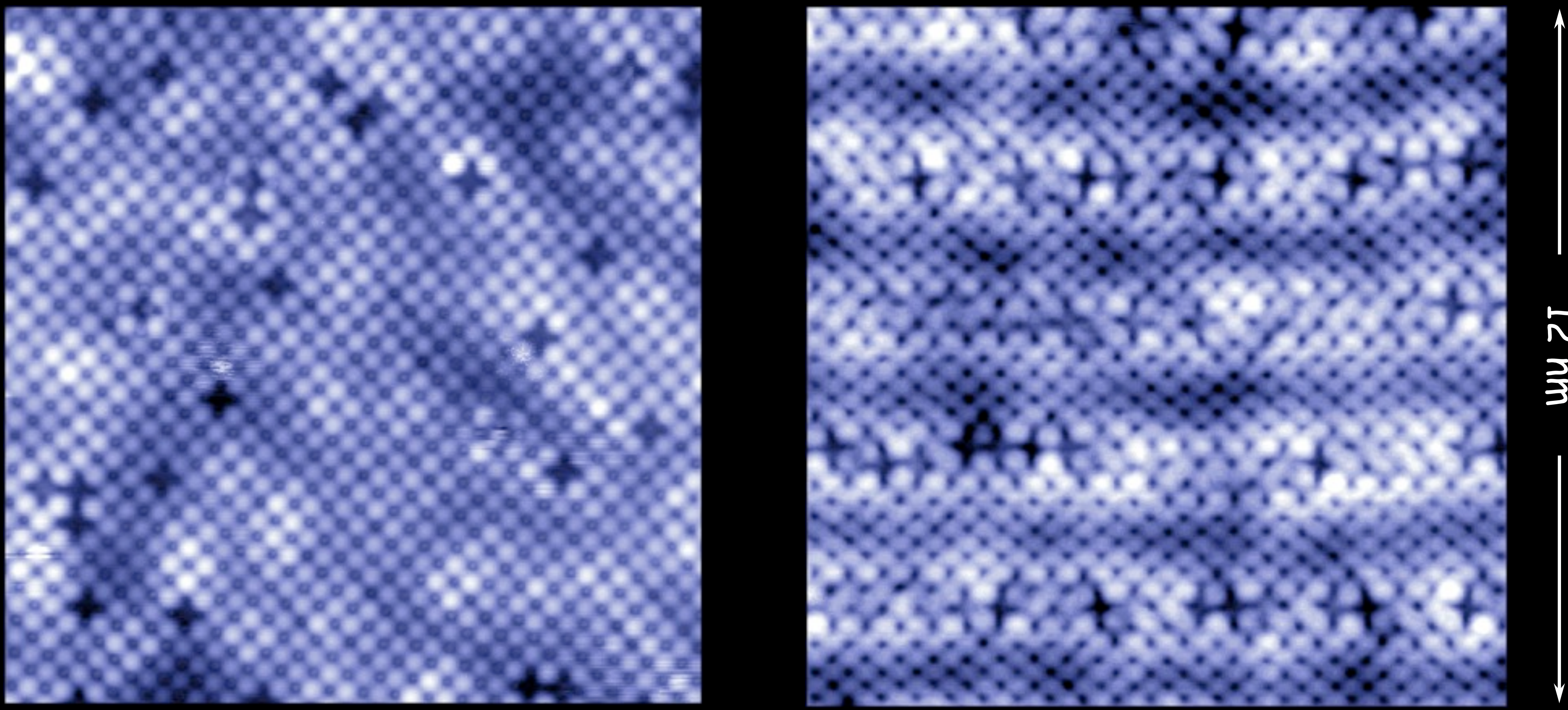
S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).

Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

R-map at E=150meV

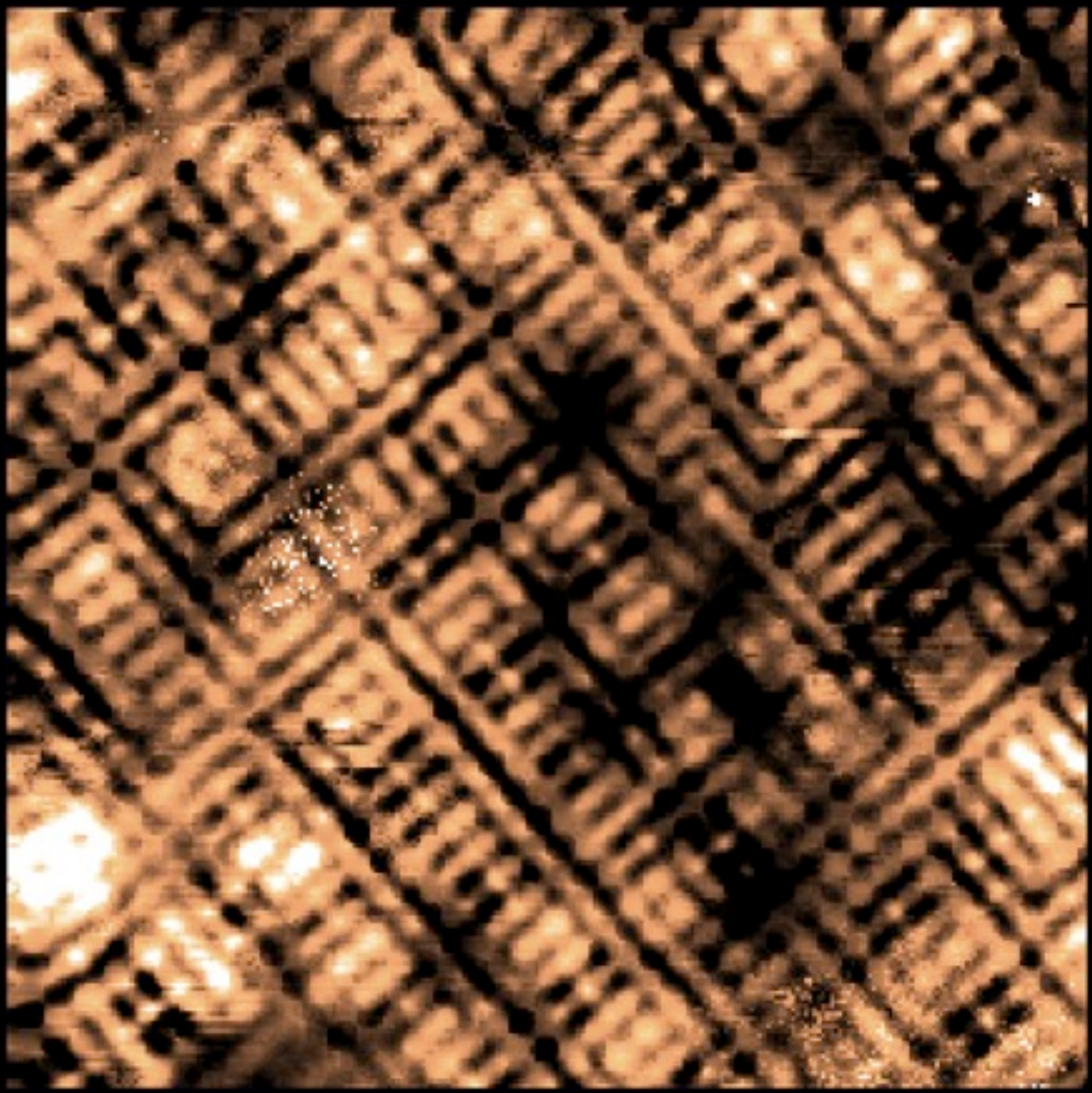
$Ca_{1.90}Na_{0.10}CuO_2Cl_2$ $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Y. Kohsaka et al. Science 315, 1380 (2007)

R-map at E=150meV

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



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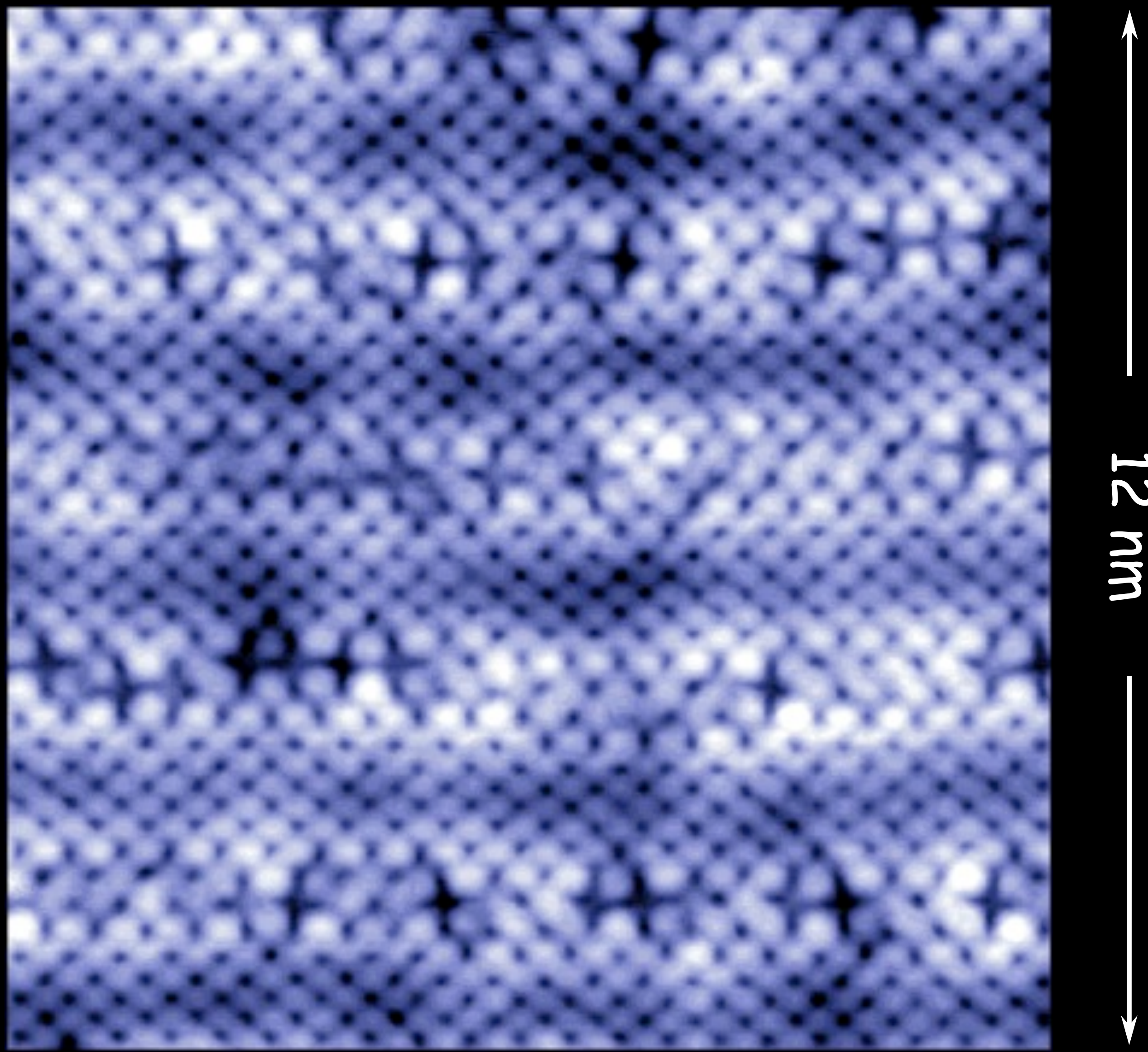

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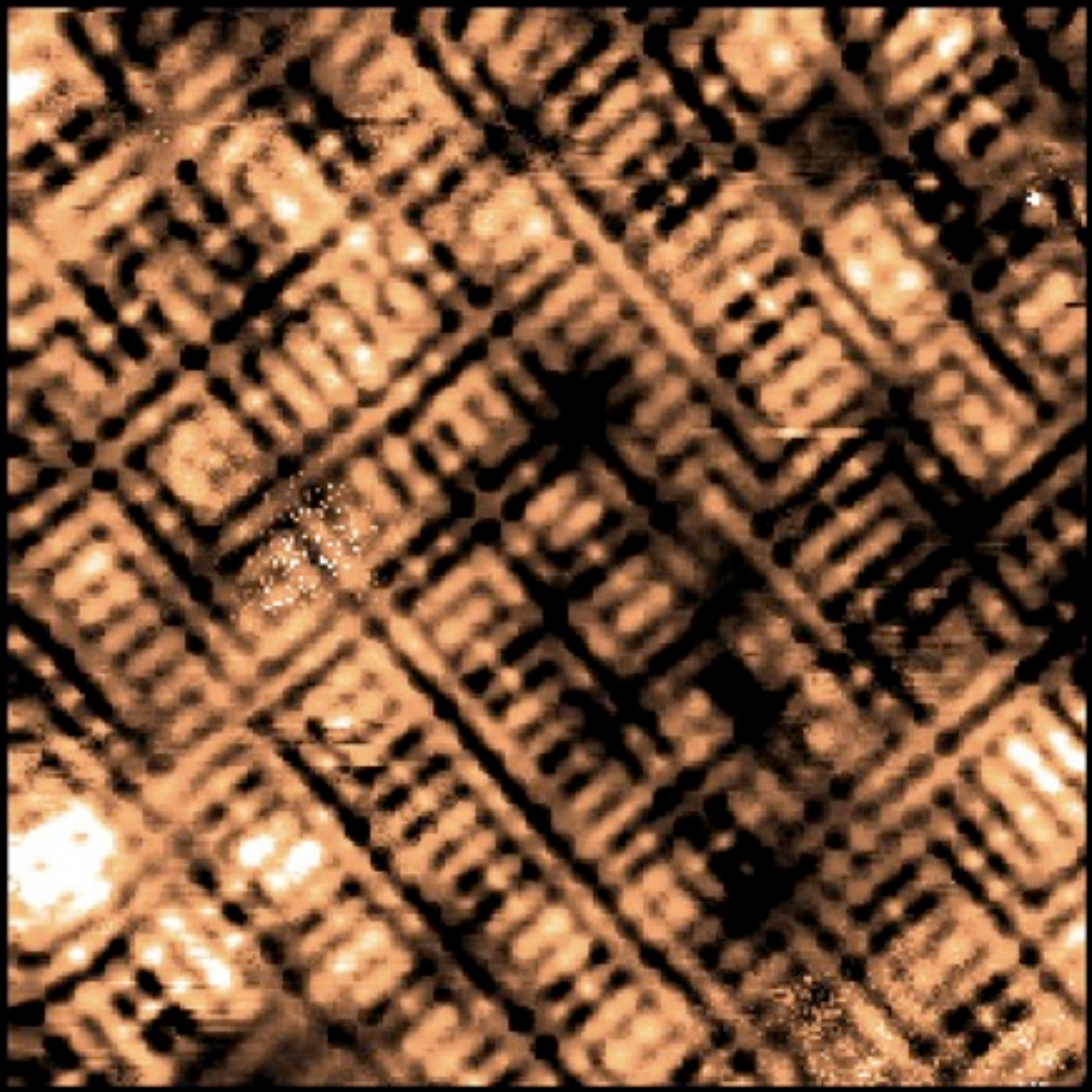

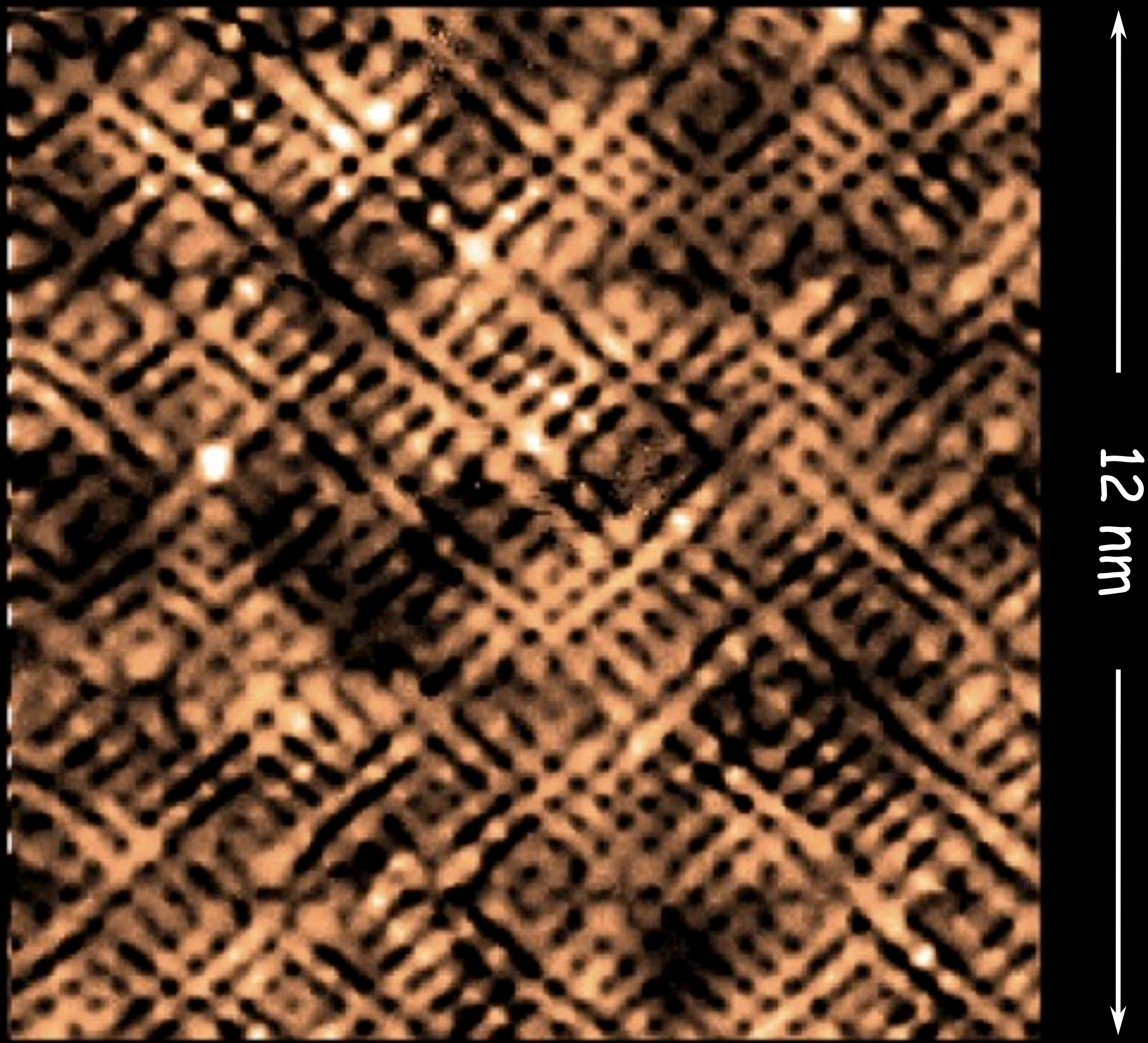

Y. Kohsaka et al. Science 315, 1380 (2007)

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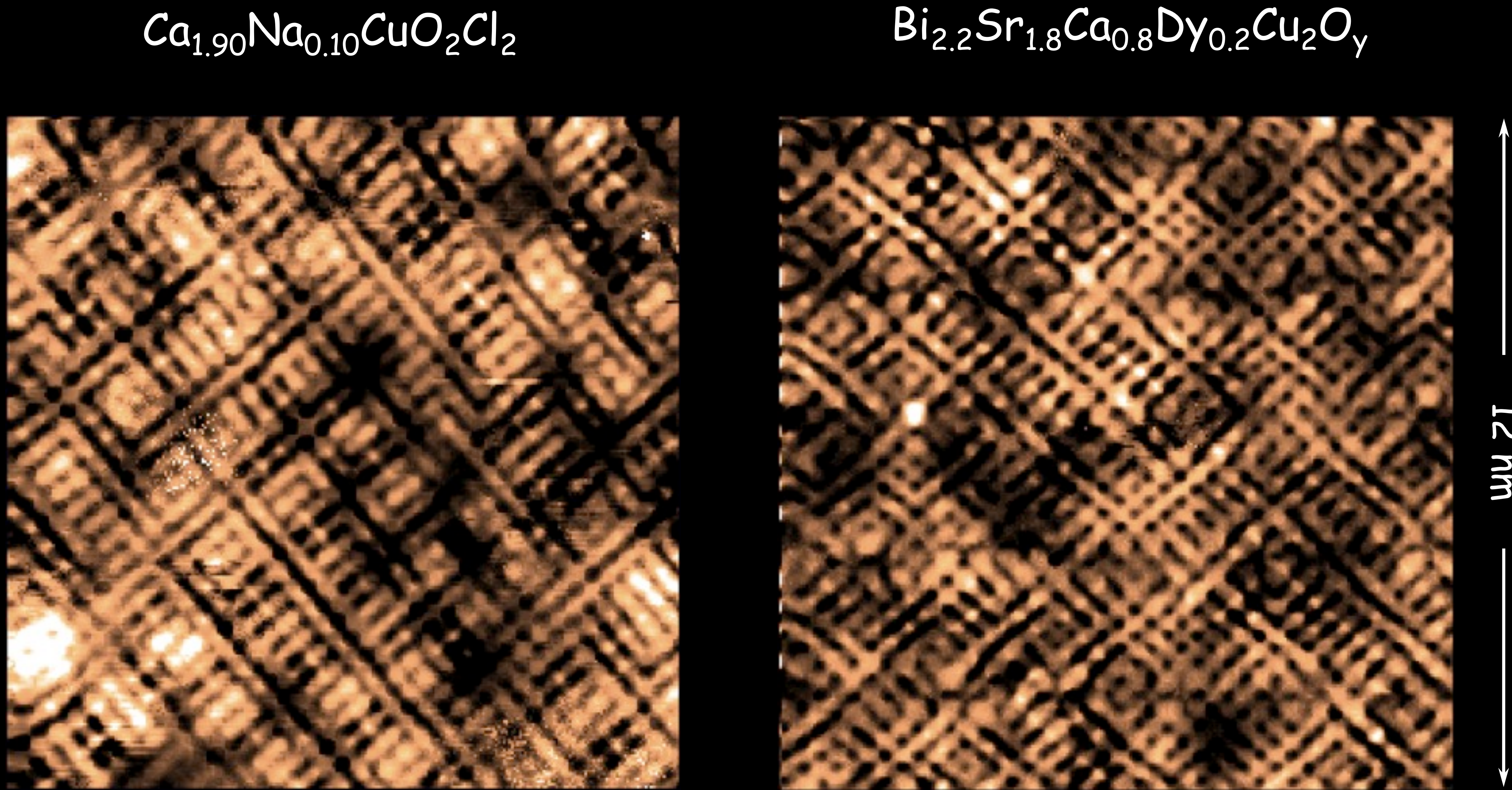


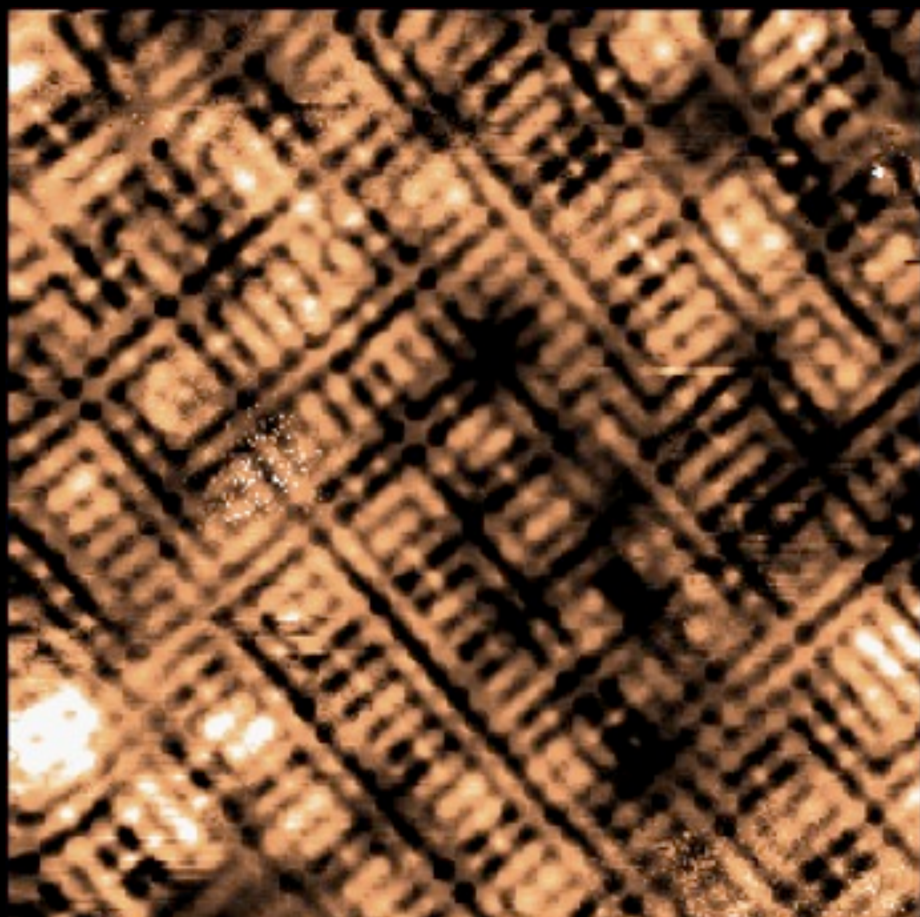
Indistinguishable bond-centered TA contrast

with disperse $4a_0$-wide nanodomains

Y. Kohsaka et al. Science 315, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

R map (150 mV)

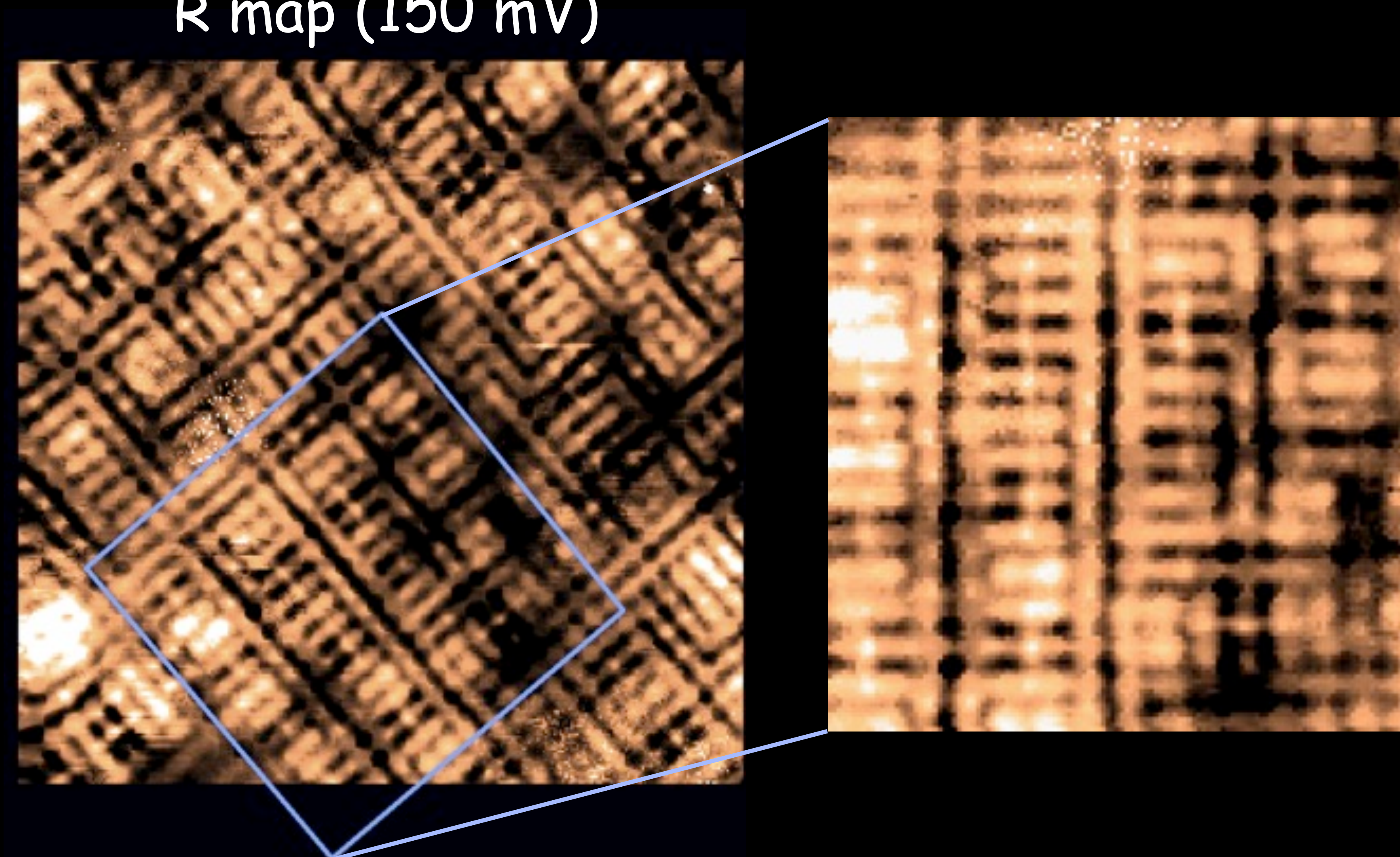


12 nm

$Ca_{1.88}Na_{0.12}CuO_2Cl_2$, 4 K

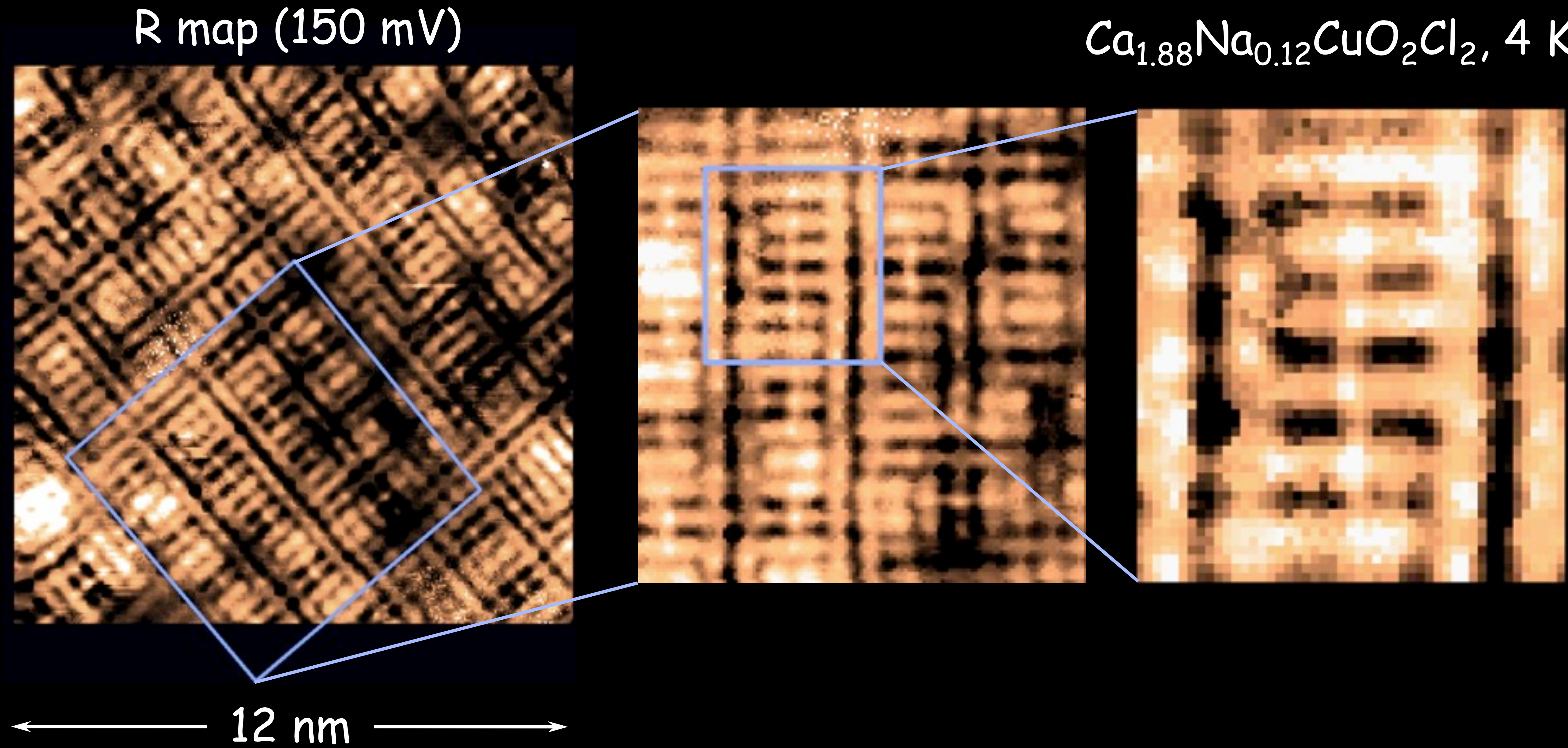
Y. Kohsaka et al. Science 315, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



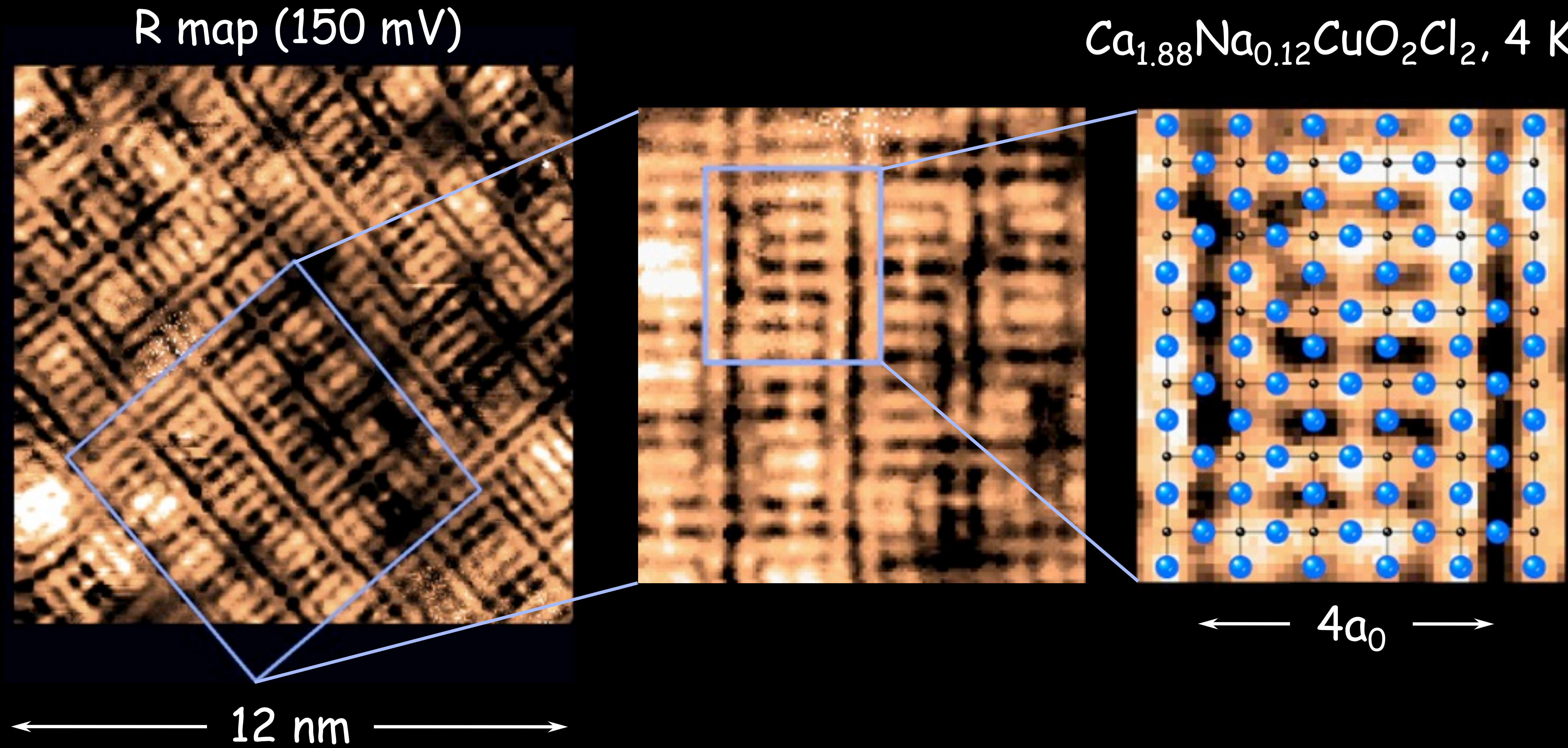
Y. Kohsaka et al. **Science 315**, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



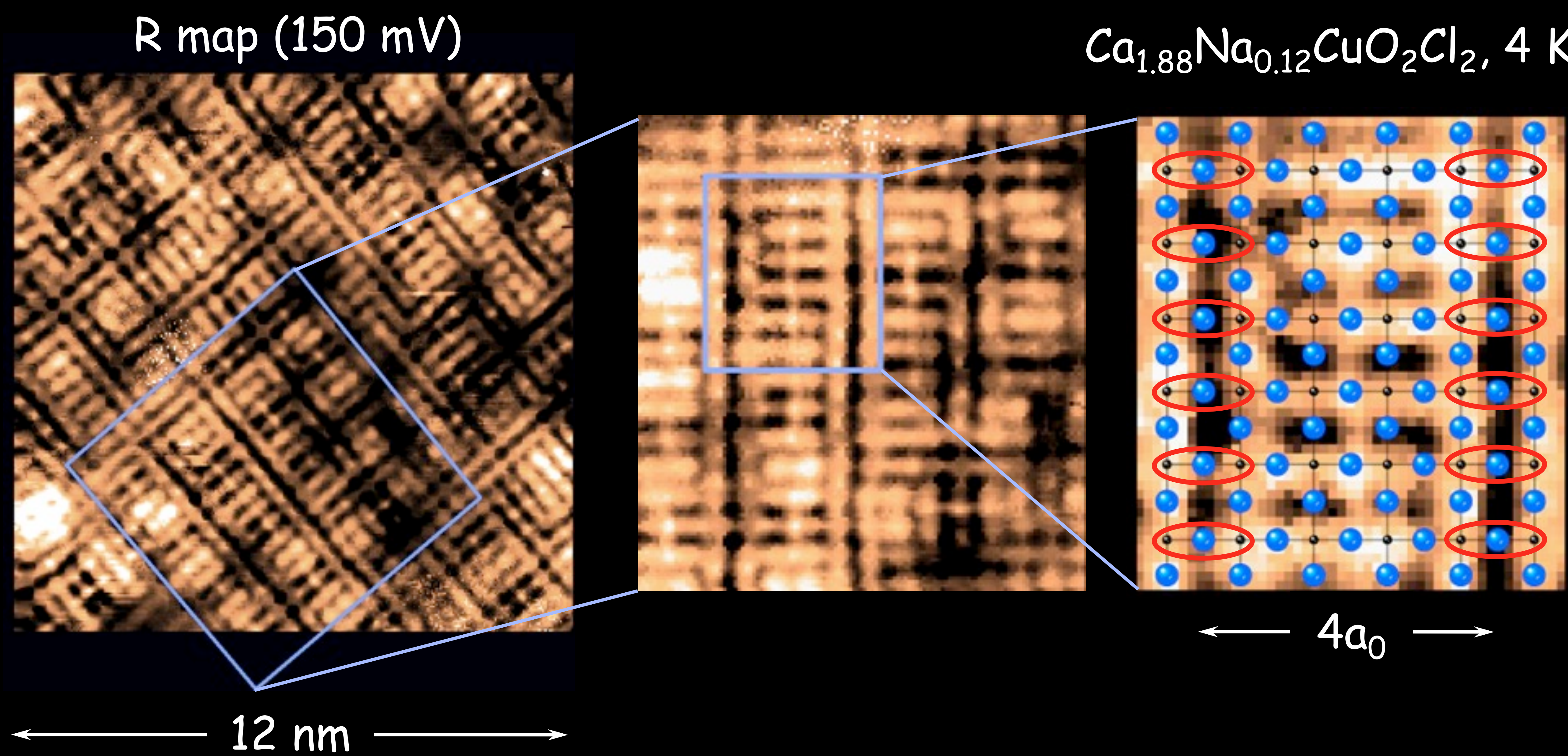
Y. Kohsaka et al. Science 315, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

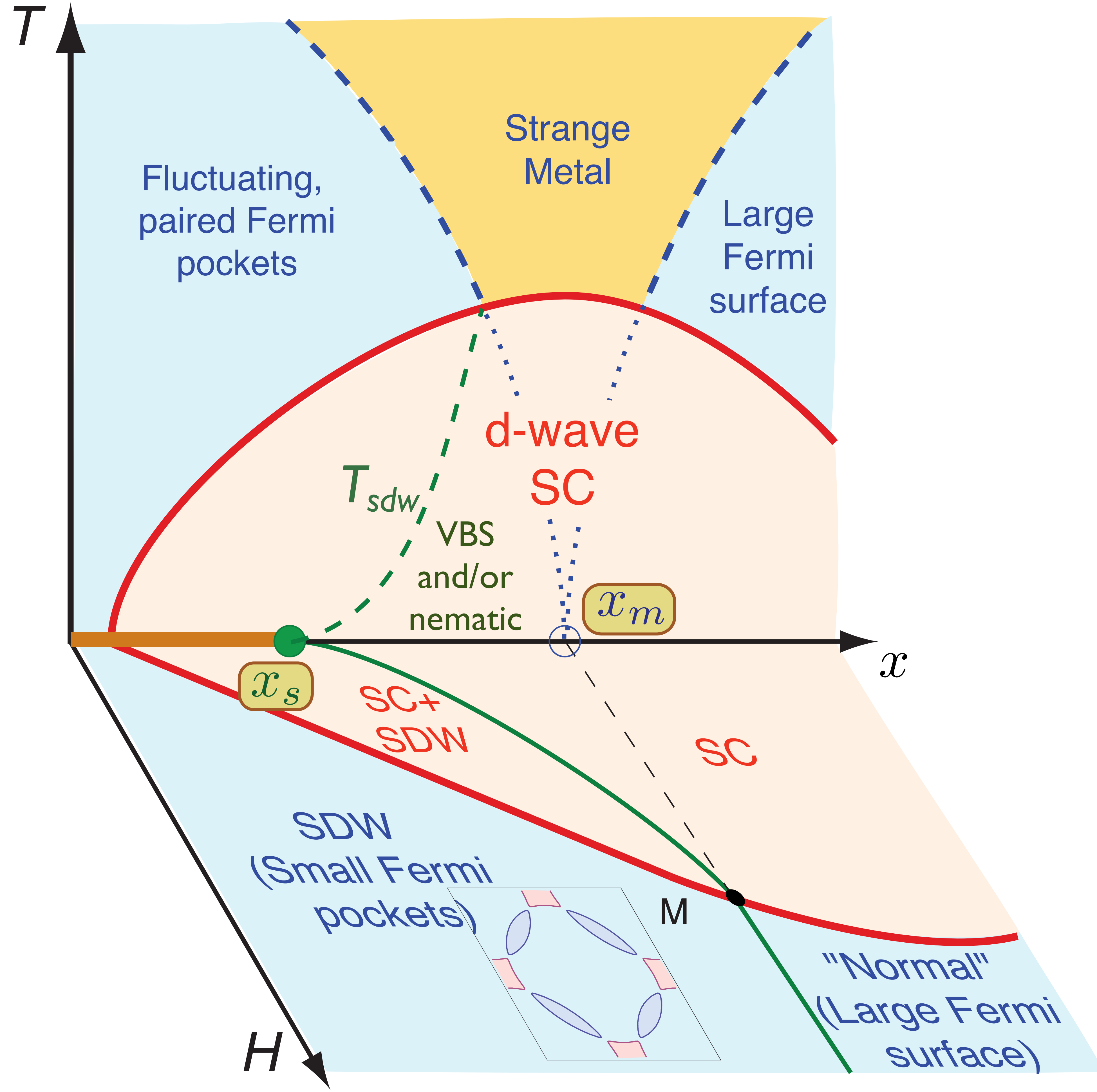


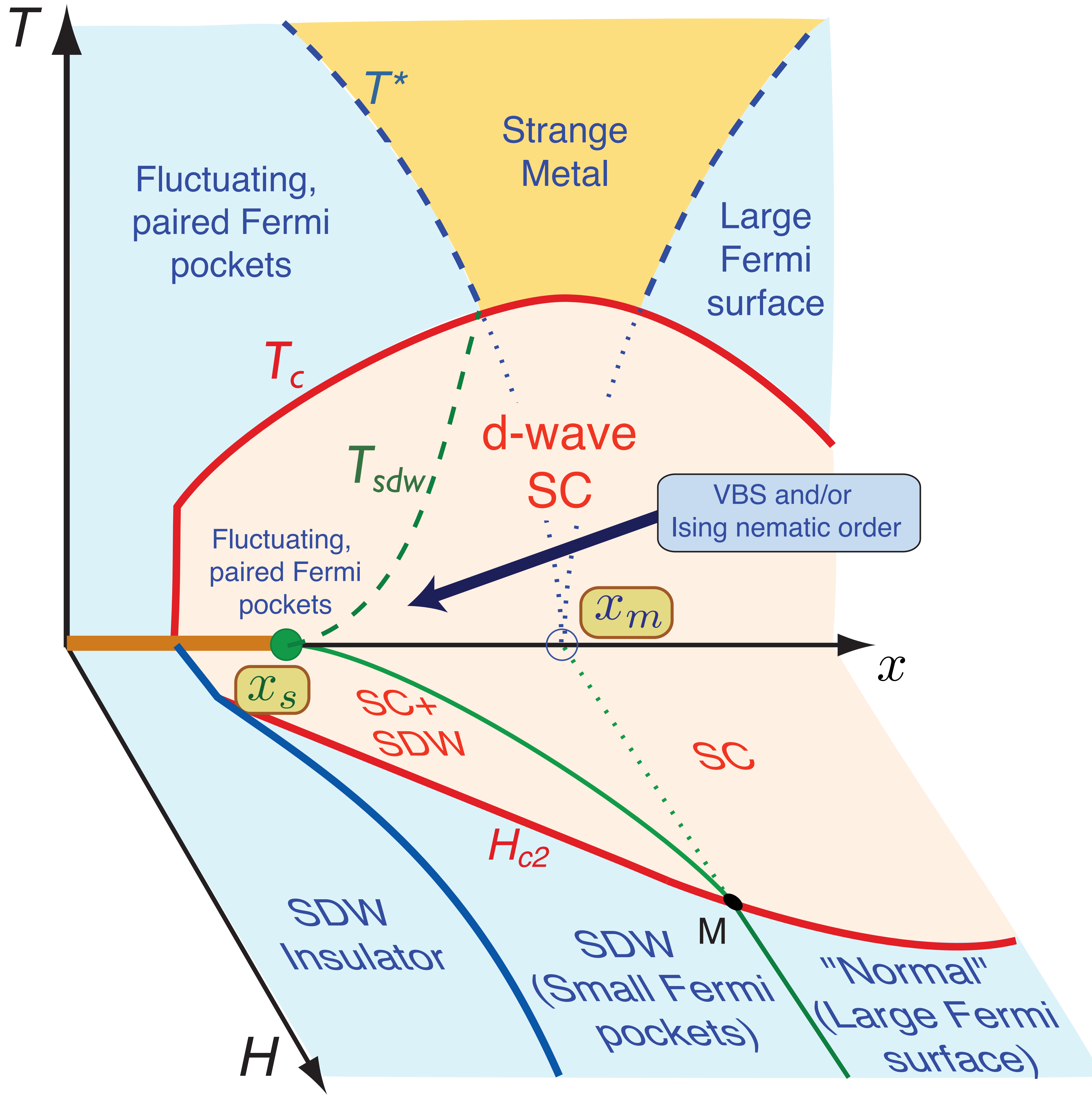
Y. Kohsaka et al. **Science 315**, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



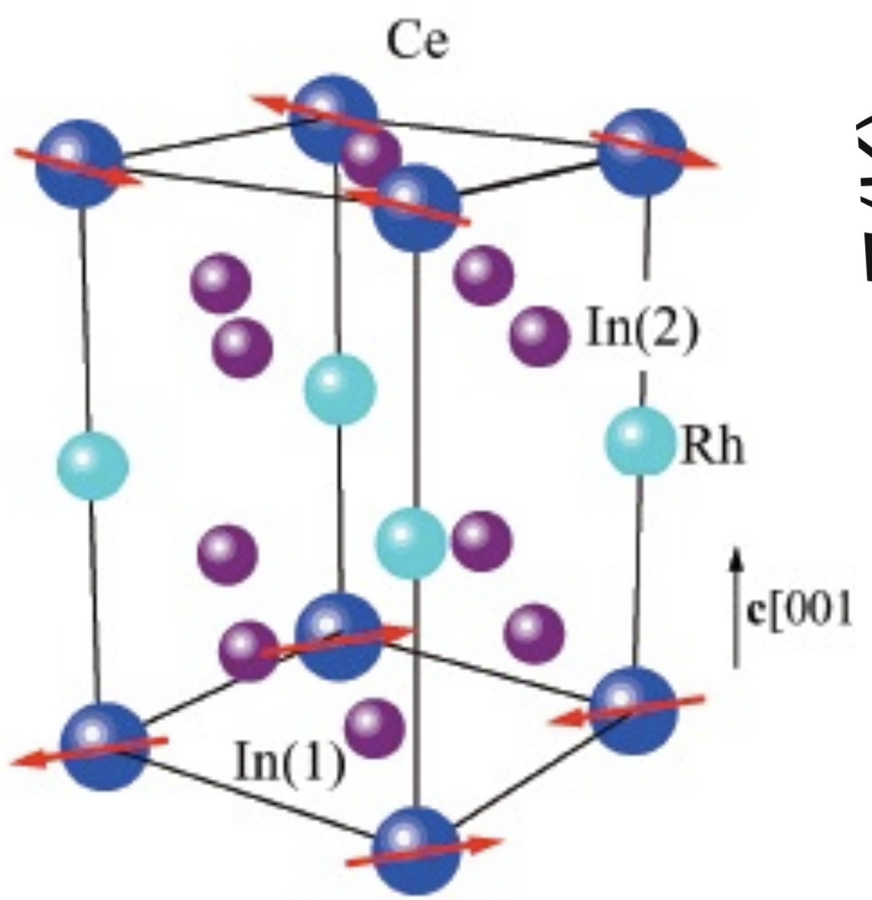
Y. Kohsaka et al. **Science 315**, 1380 (2007)

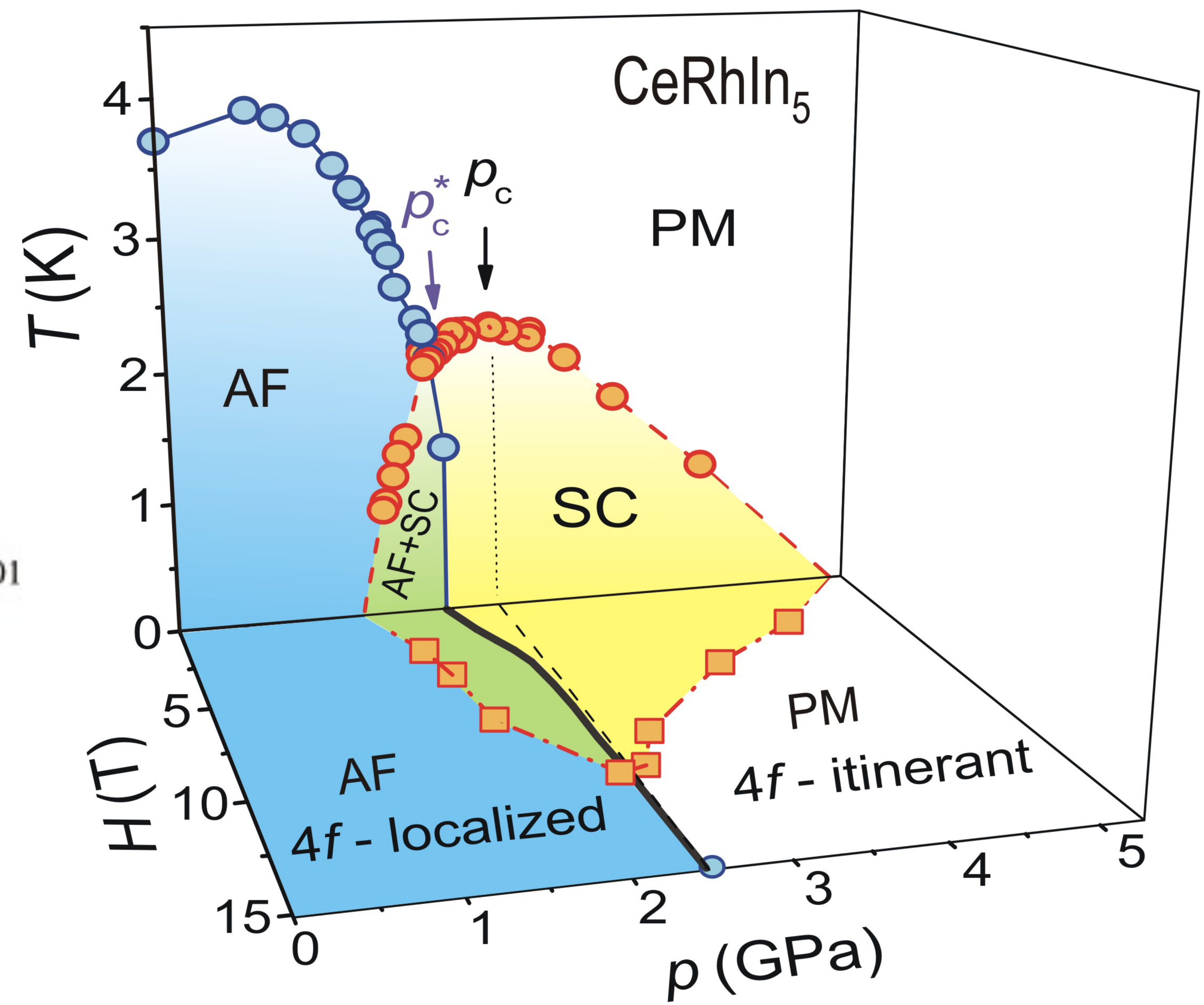

T
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
d-wave
SC
T_{sdw}
VBS
and/or
nematic
x_m
x
x_s
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H



T
T*
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
T_c
T_{sdw}
d-wave SC
VBS and/or Ising nematic order
Fluctuating, paired Fermi pockets
x_m
x
x_s
SC+ SDW
SC
H_{c2}
M
SDW Insulator
SDW (Small Fermi pockets)
"Normal" (Large Fermi surface)
H

Similar phase diagram for $CeRhIn_5$





G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity

Conclusions

Needed:
Theory for the onset of spin density wave order in metals in two dimensions