Strange metals: entanglement, field theory, and holography

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PHYSICS
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HARVARD

1. Entanglement, holography, and CFTs
2. Field theory of a non-Fermi liquid
3. Generalized holography beyond CFTs
4. Holography of strange metals

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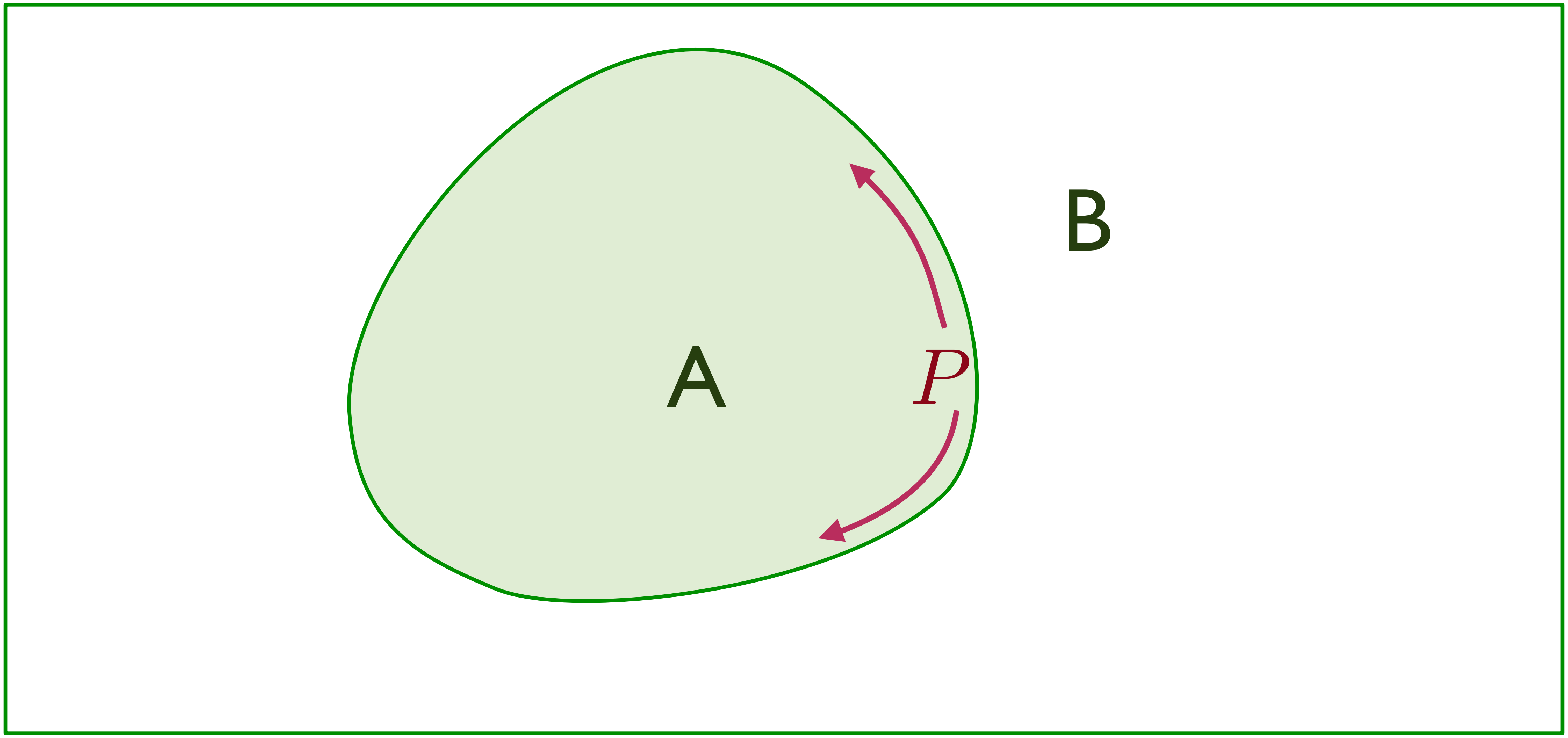
Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy

$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

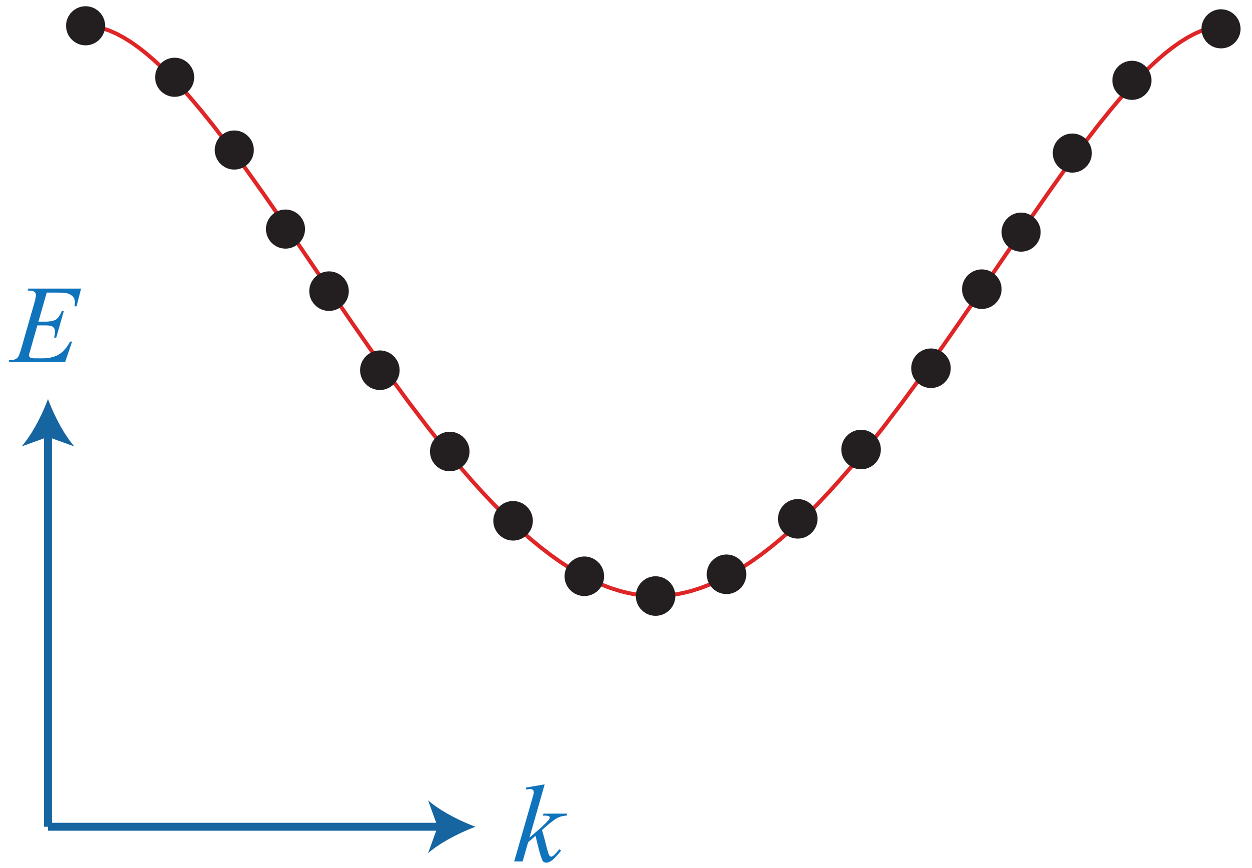
$$\text{Take } |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right)$$

$$\text{Then } \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$
$$= \frac{1}{2}\left(|\uparrow\rangle_A \langle\uparrow|_A + |\downarrow\rangle_A \langle\downarrow|_A\right)$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$
$$= \ln 2$$

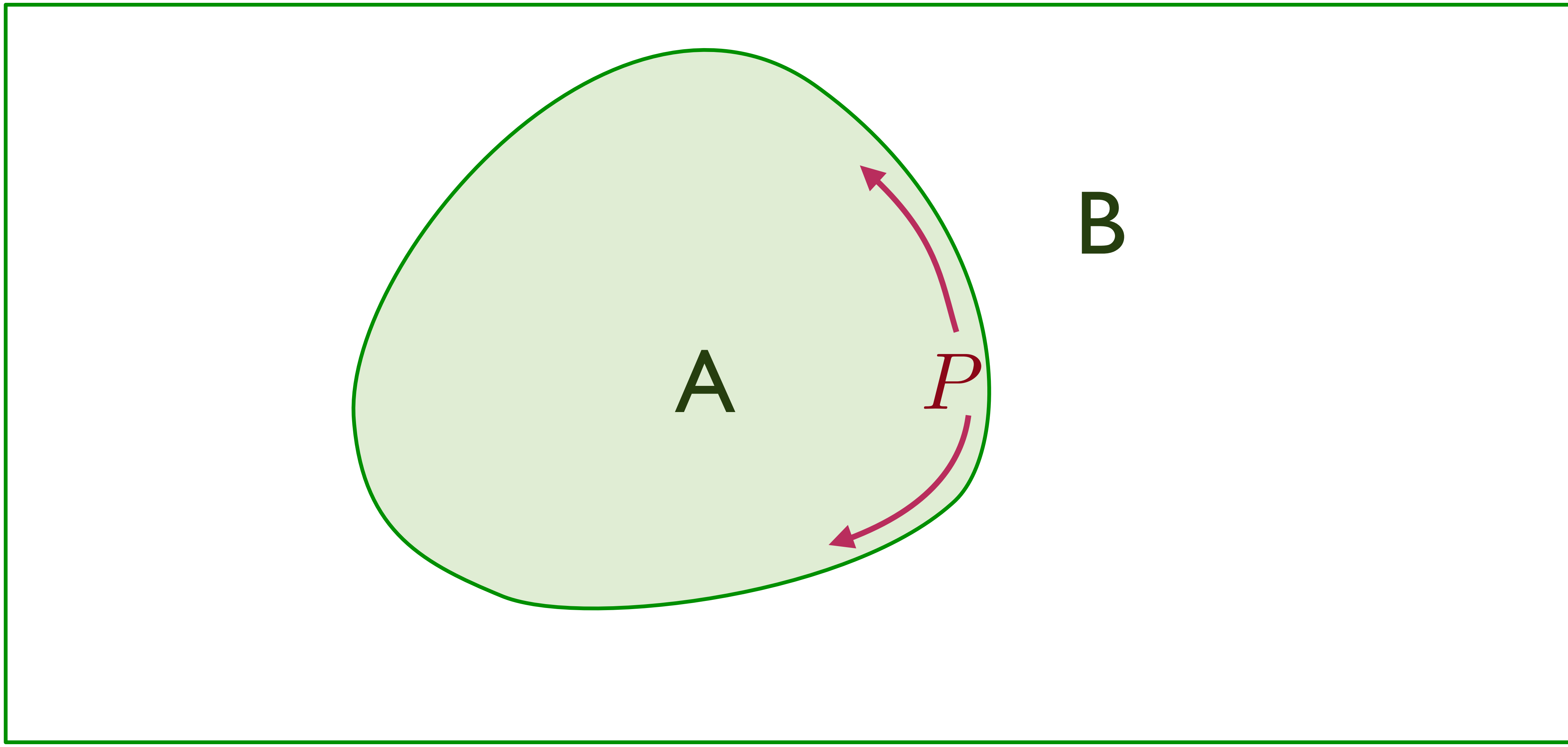
Entanglement entropy of a band insulator

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator



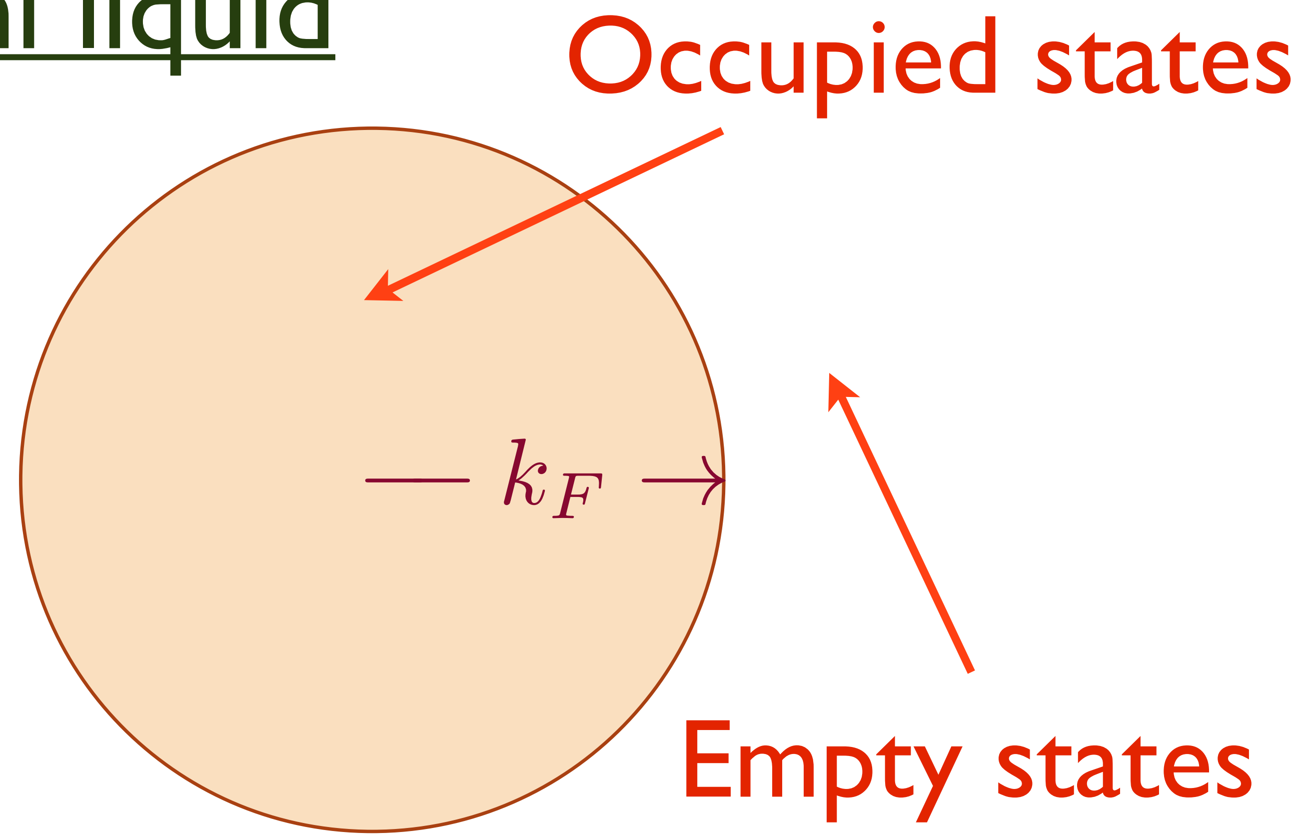
$$S_E = aP - b\exp(-cP)$$

where P is the surface area (perimeter) of the boundary between A and B.

The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

$$+ \, 4 \text{ Fermi terms}$$



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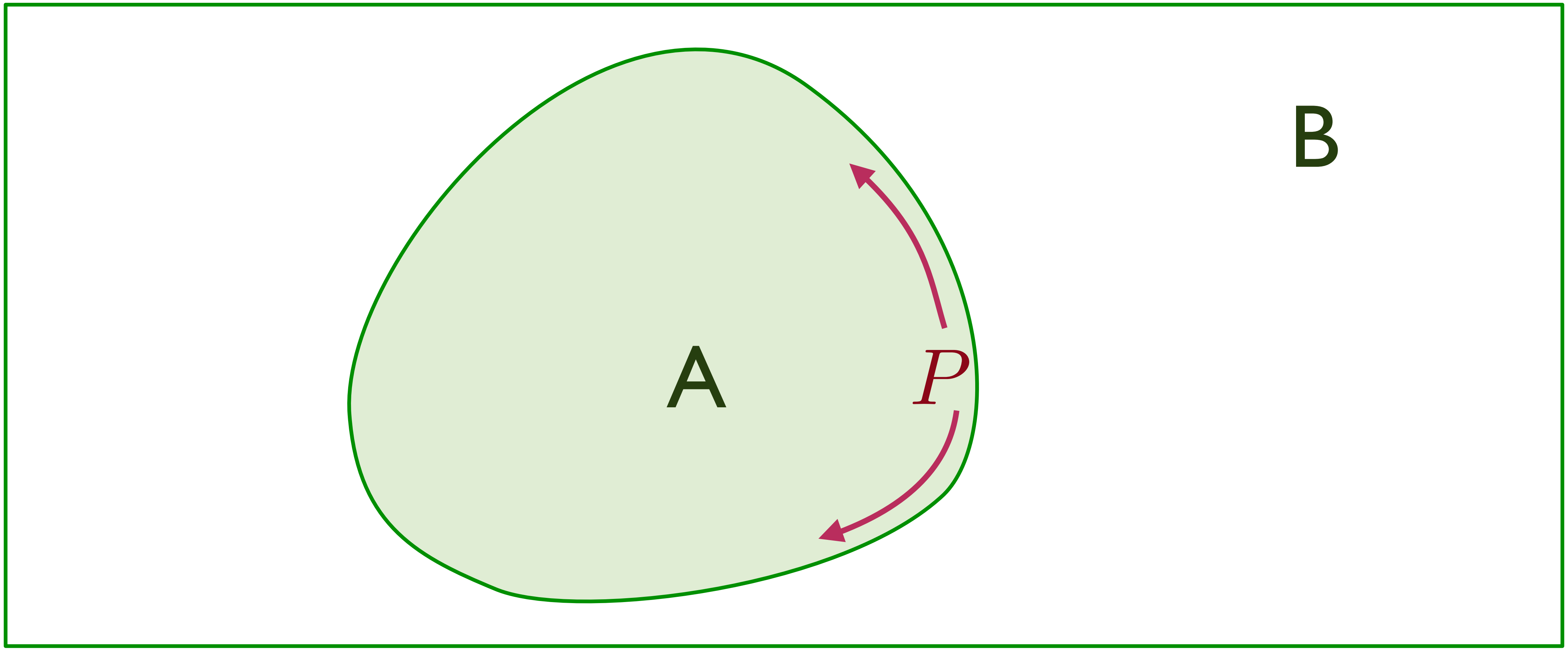
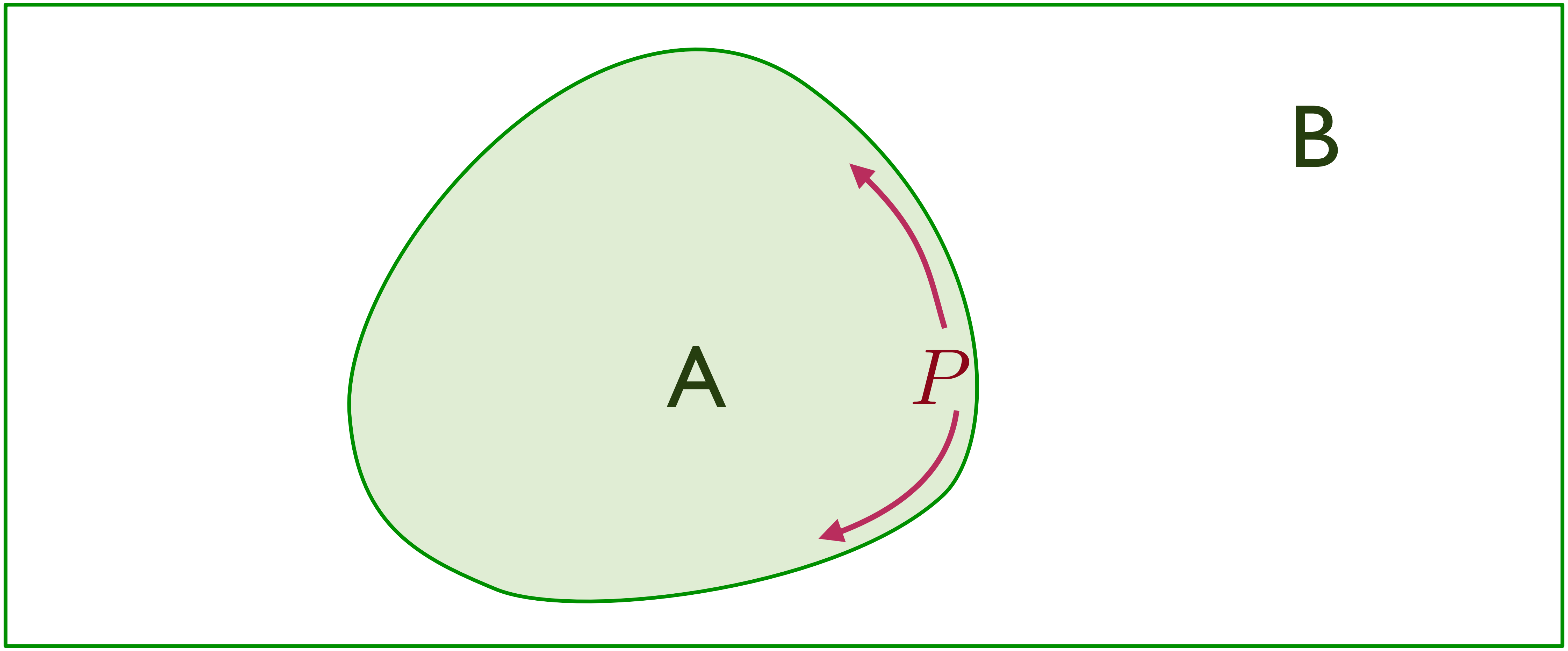


Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of the Fermi liquid



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The prefactor $1/12$ is *universal*: it is independent of the shape of the entangling region, and of the strength of the interactions.

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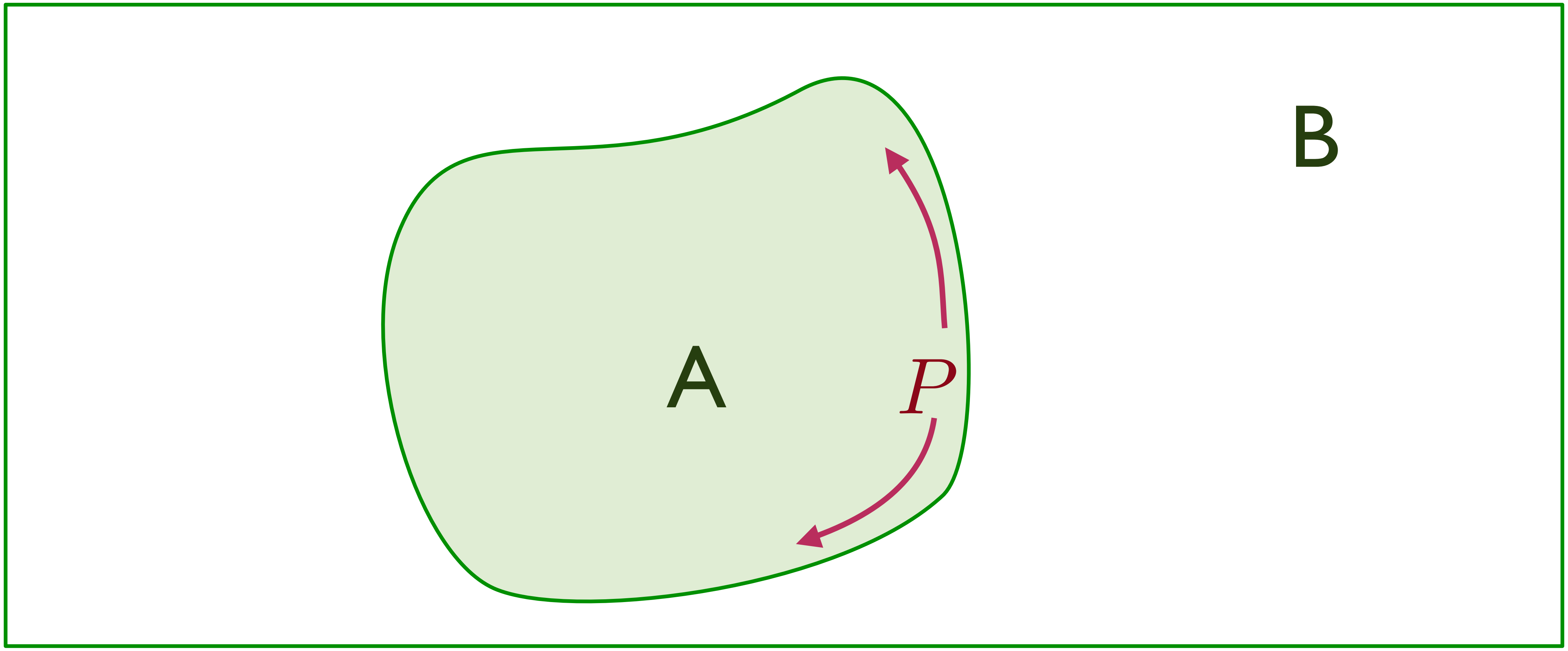

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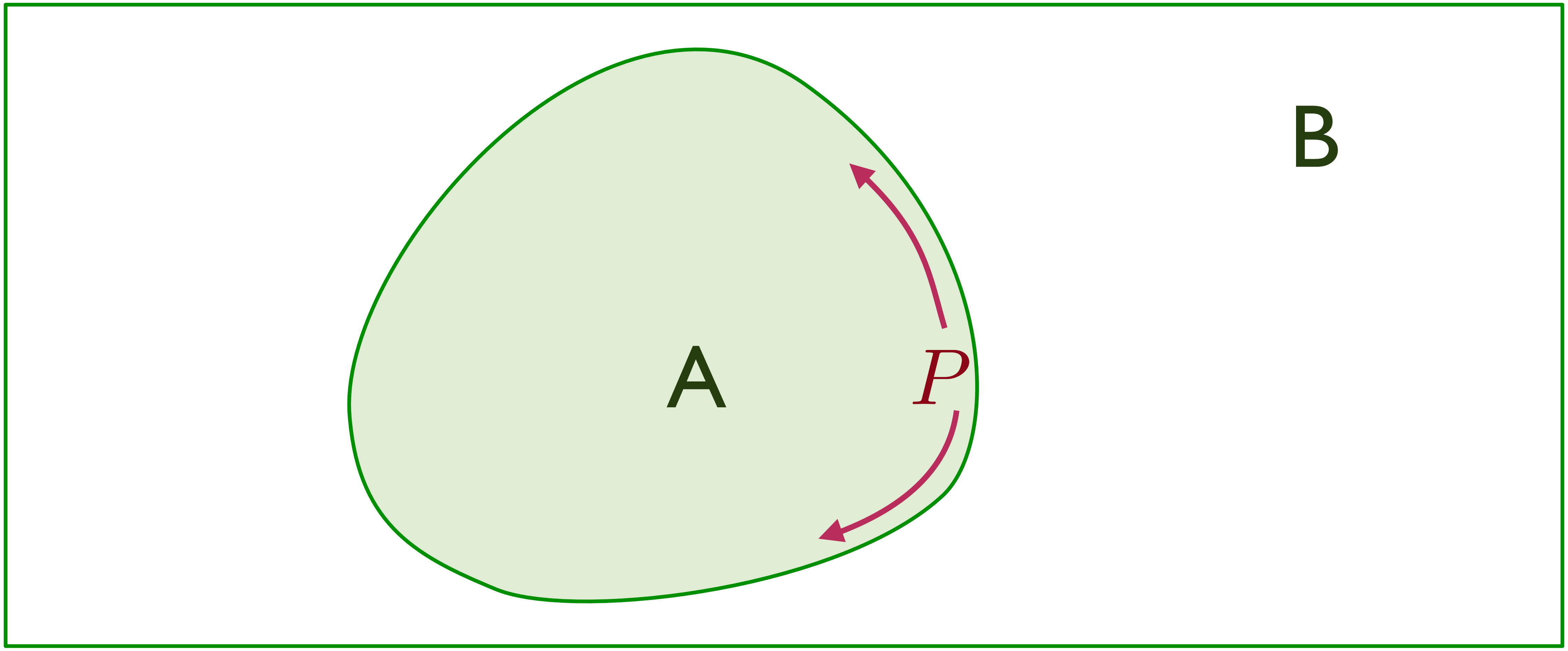

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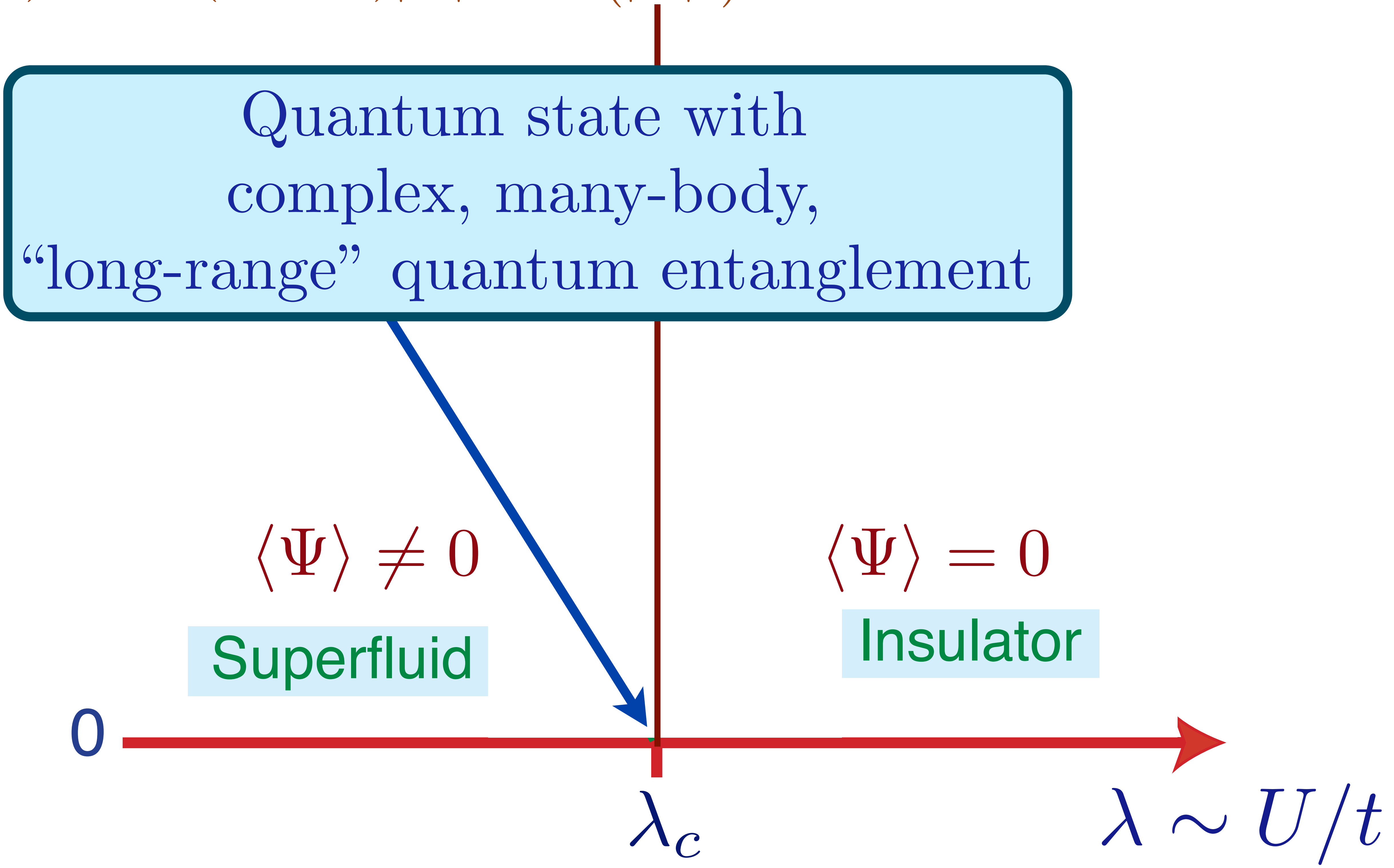
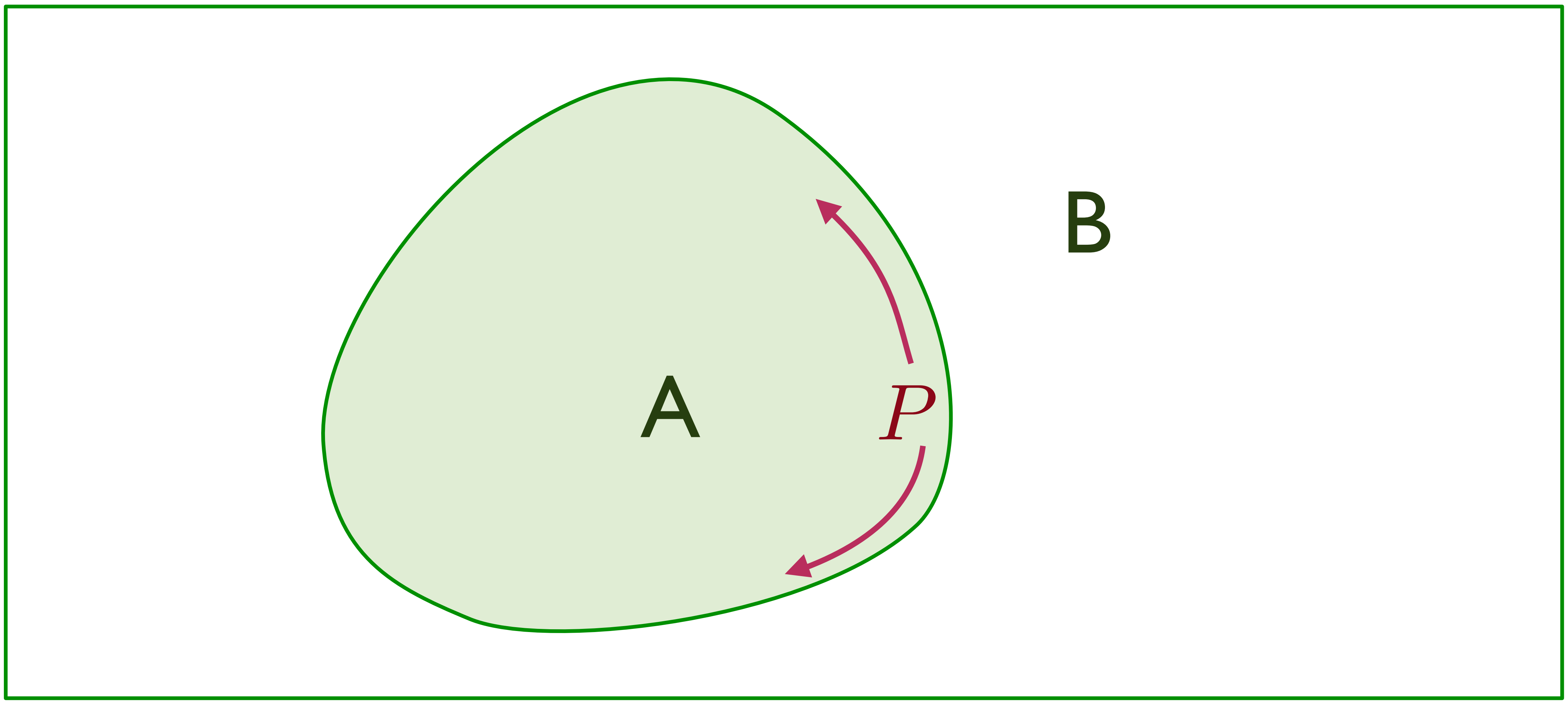
$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



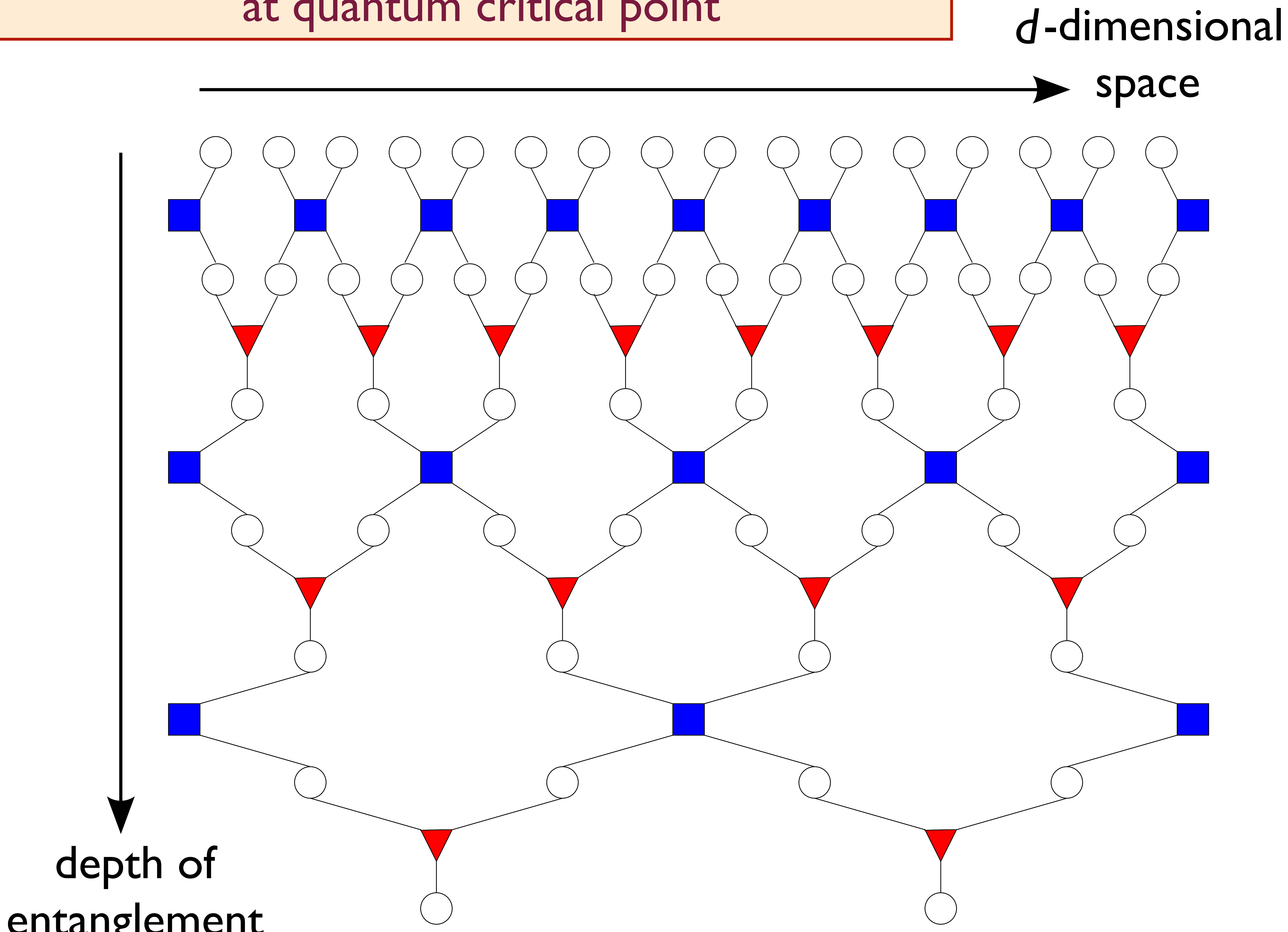
Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009); H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011); I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Tensor network representation of entanglement at quantum critical point



G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)

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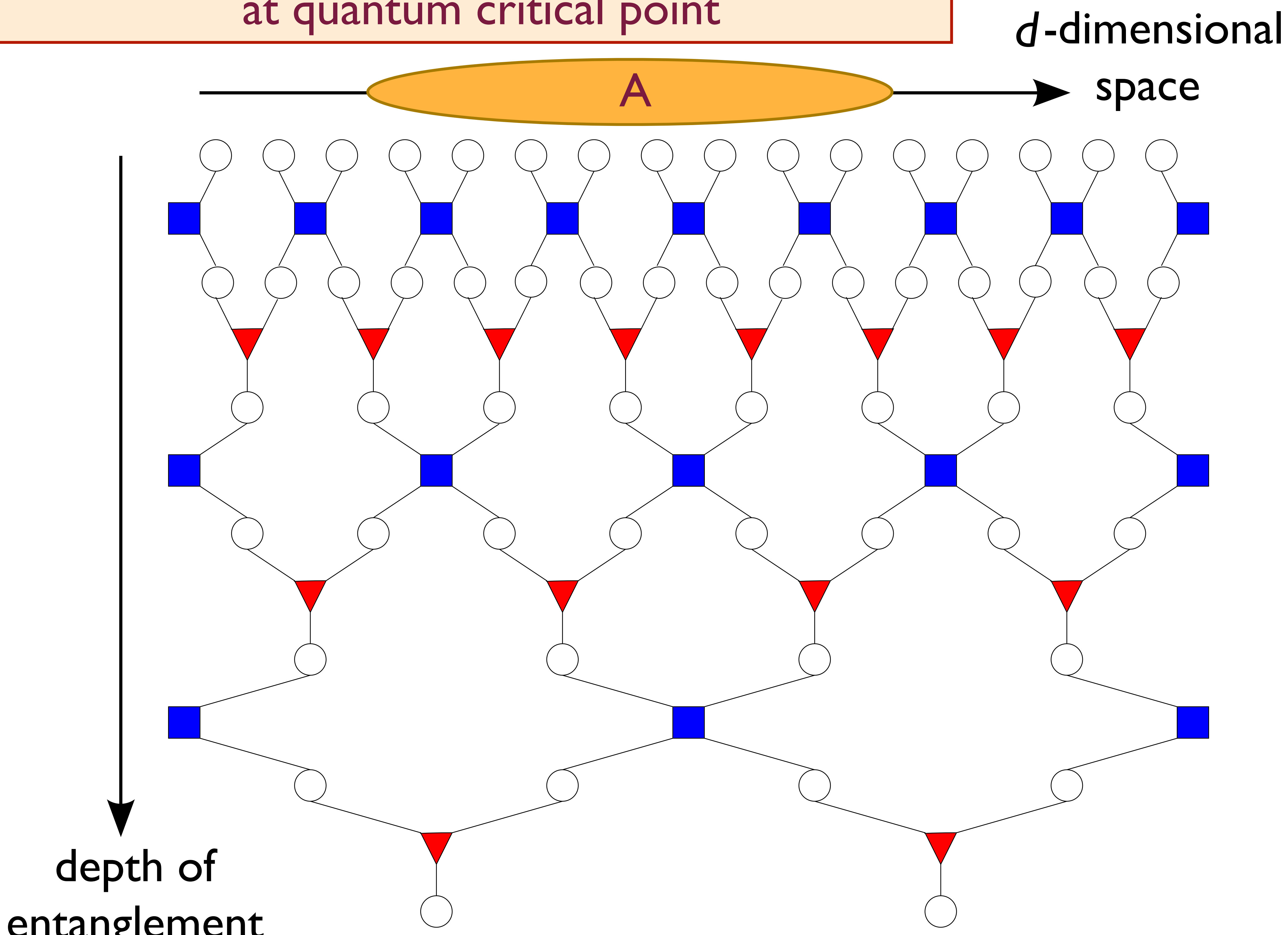

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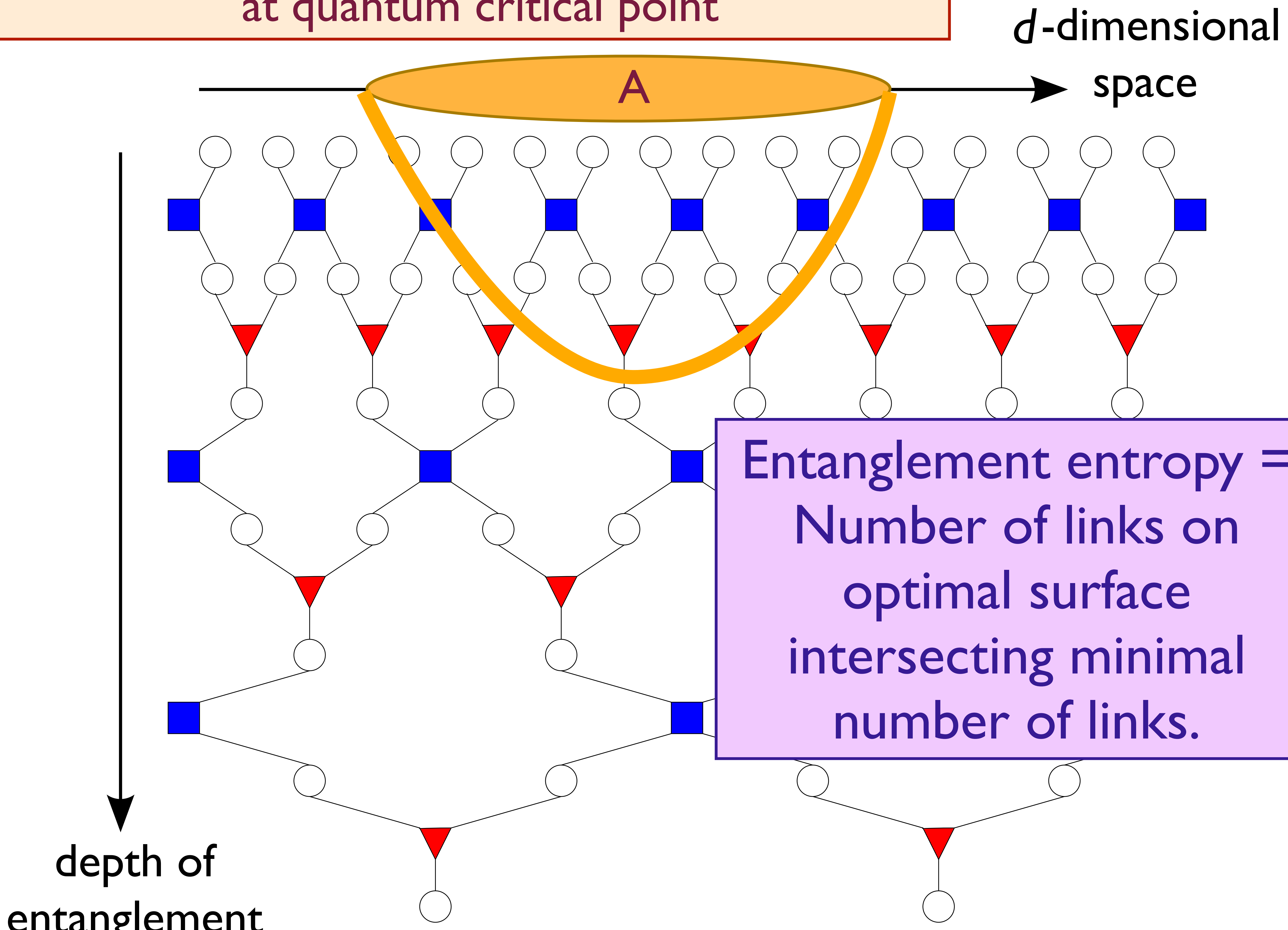

Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Brian Swingle, arXiv:0905.1317

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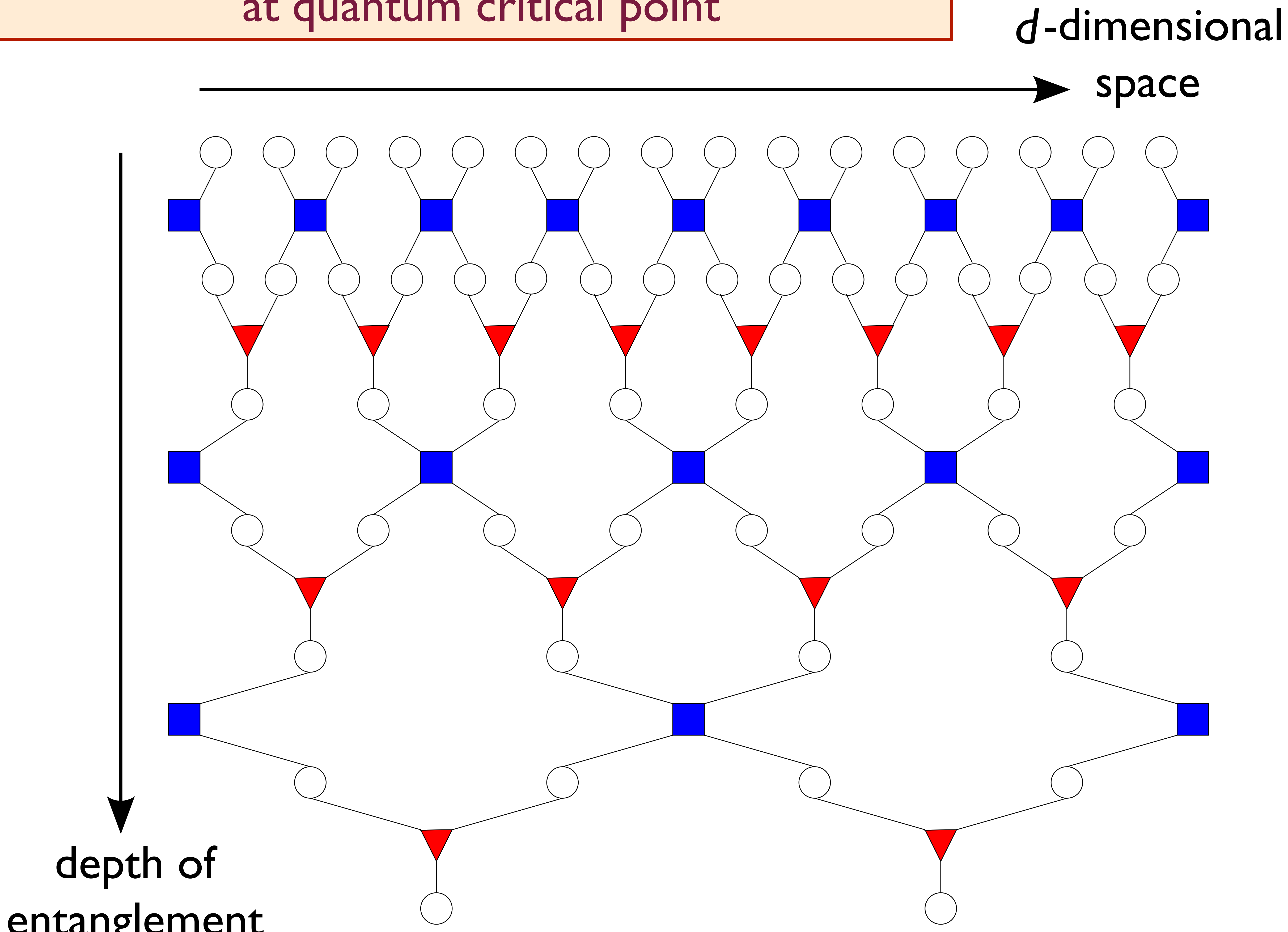
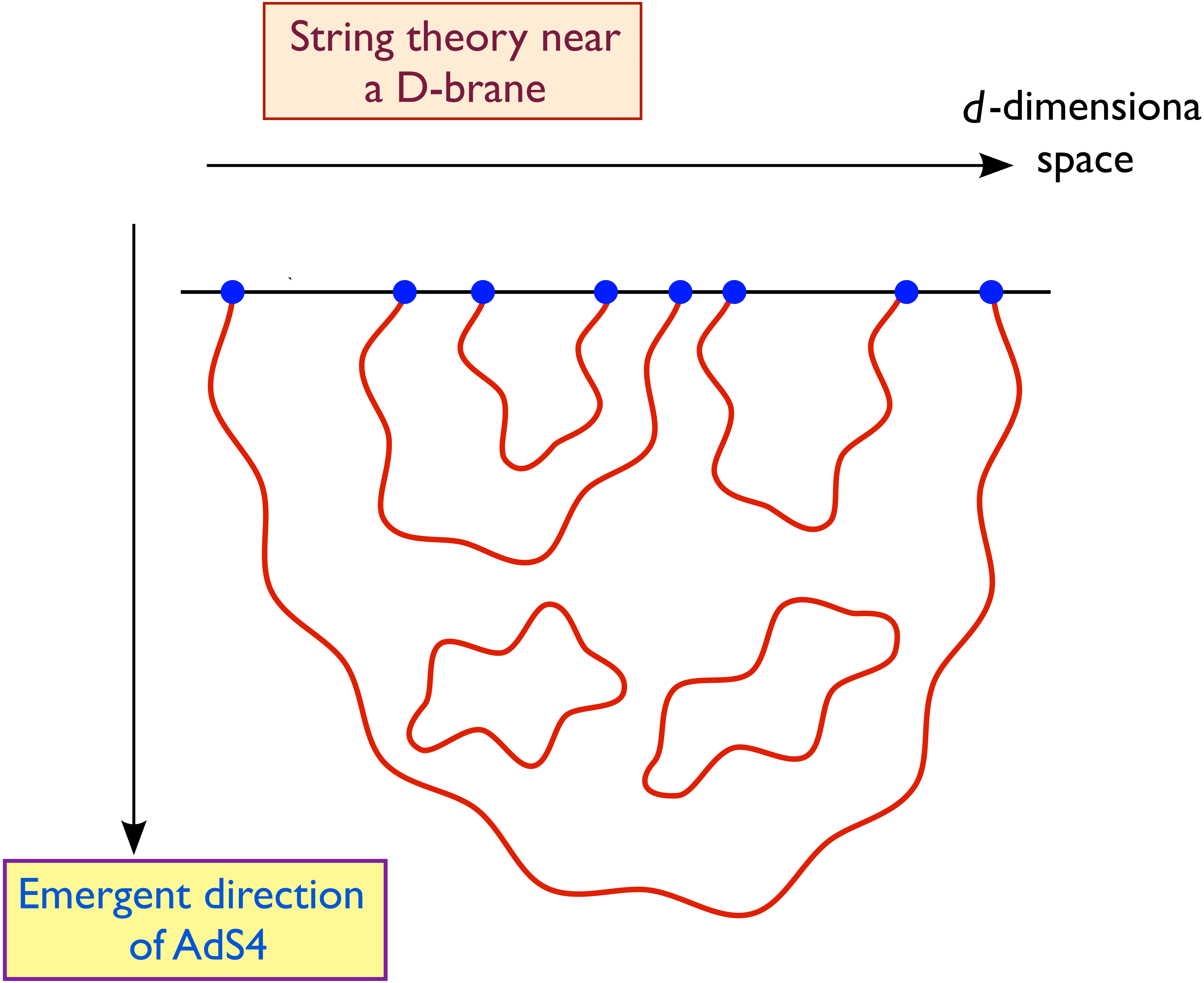
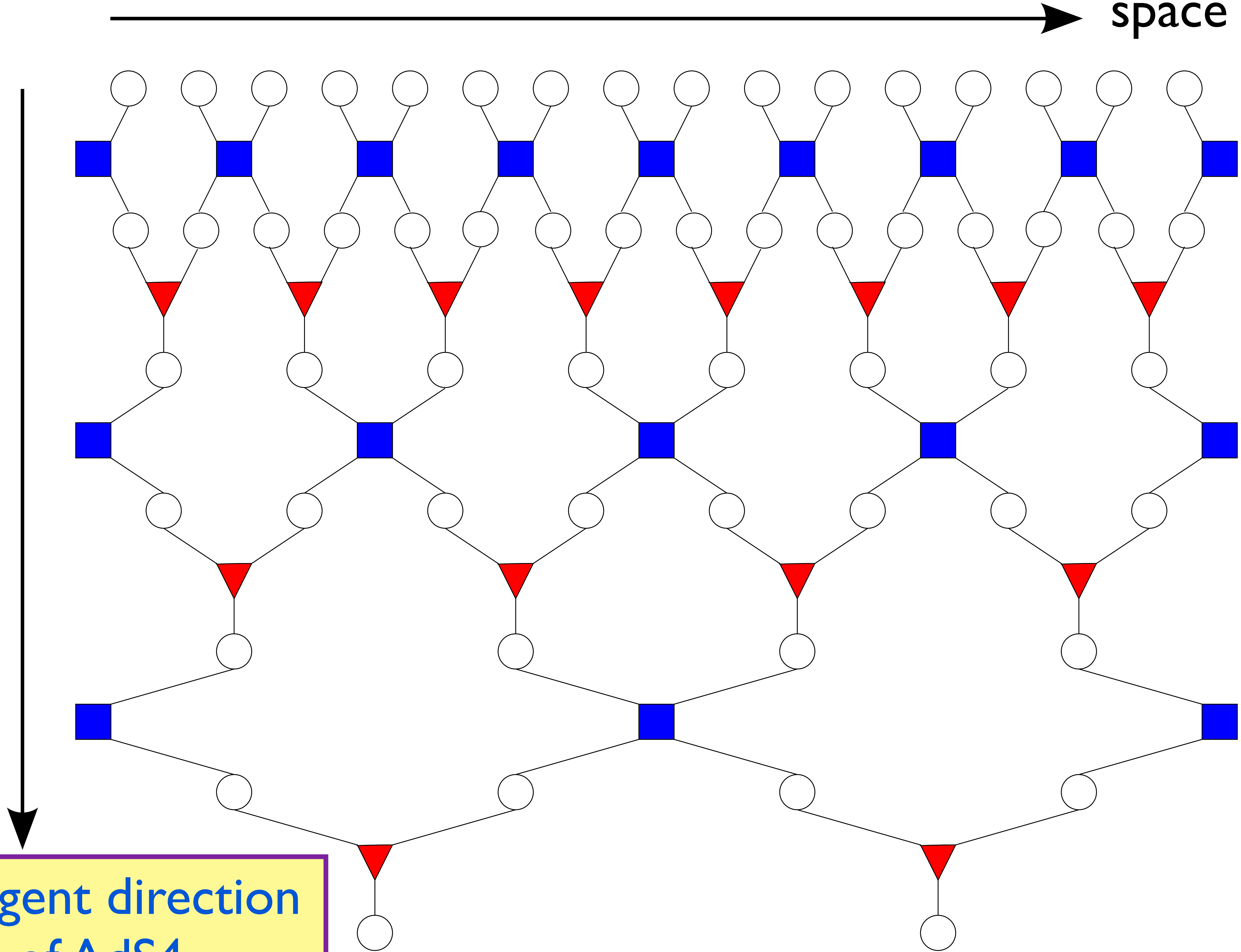



String theory near a D-brane
d-dimensional space
Emergent direction of AdS4

Tensor network representation of entanglement at quantum critical point



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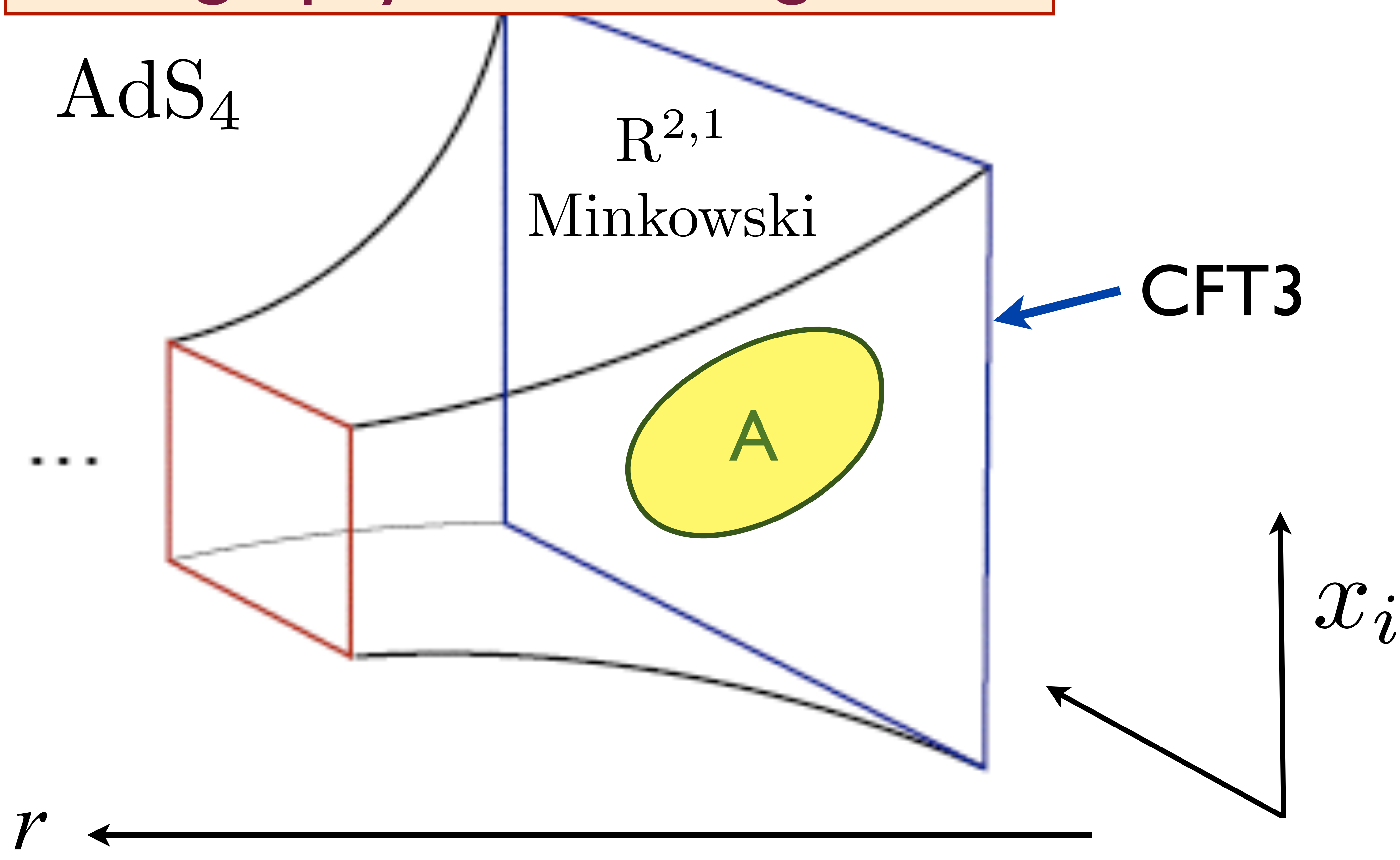
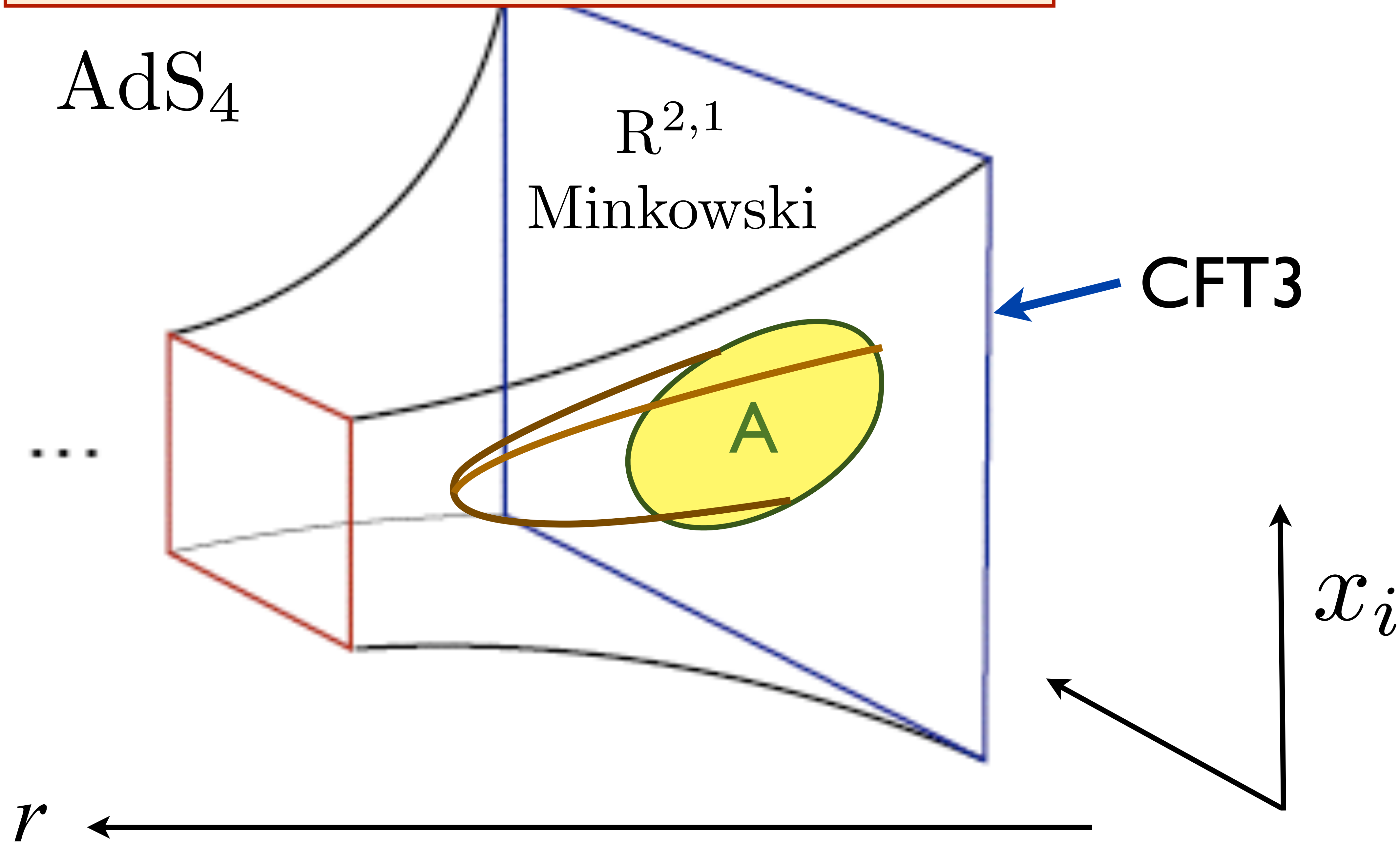
Holography and Entanglement



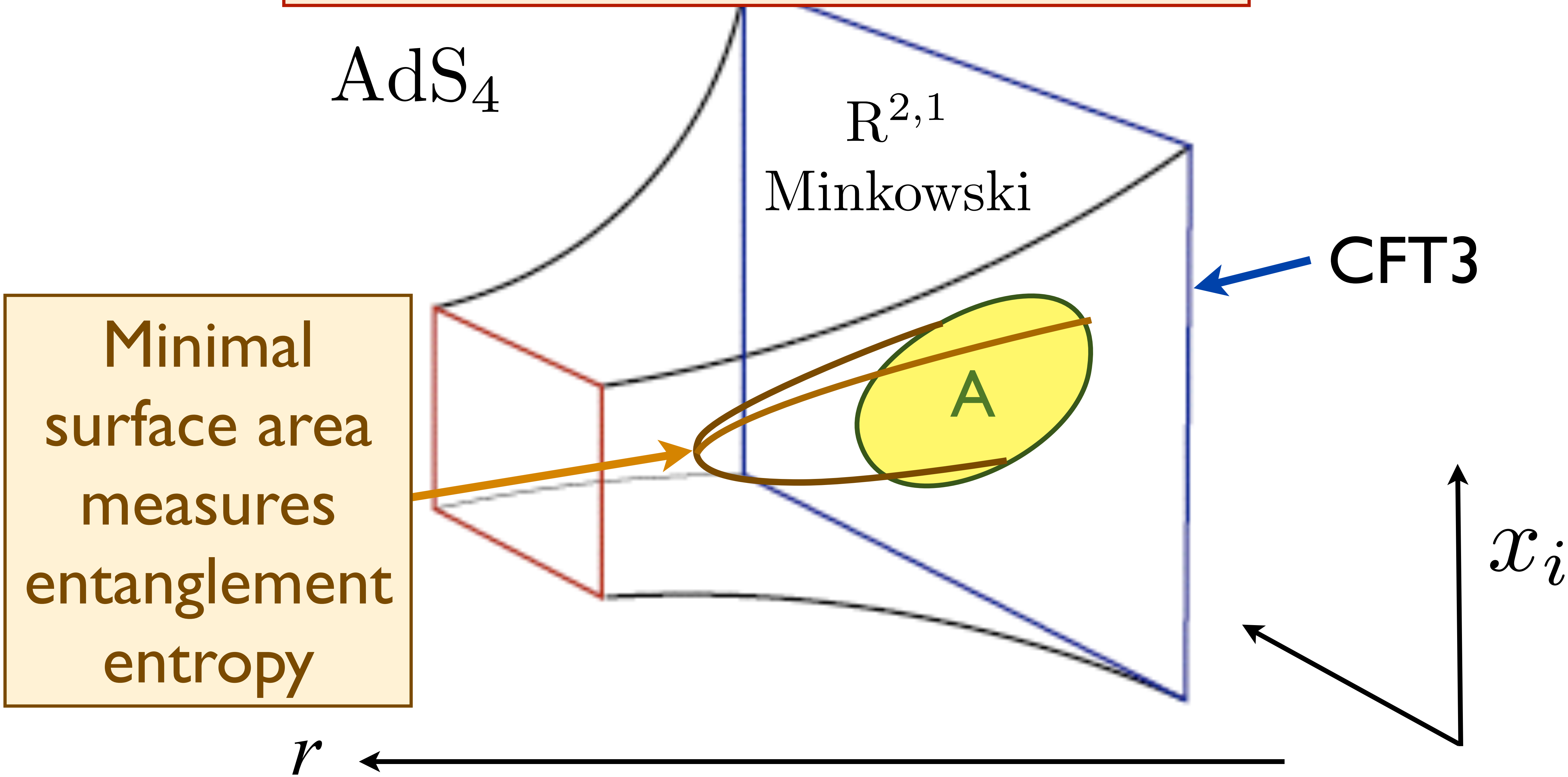
Holography and Entanglement



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Holography and Entanglement



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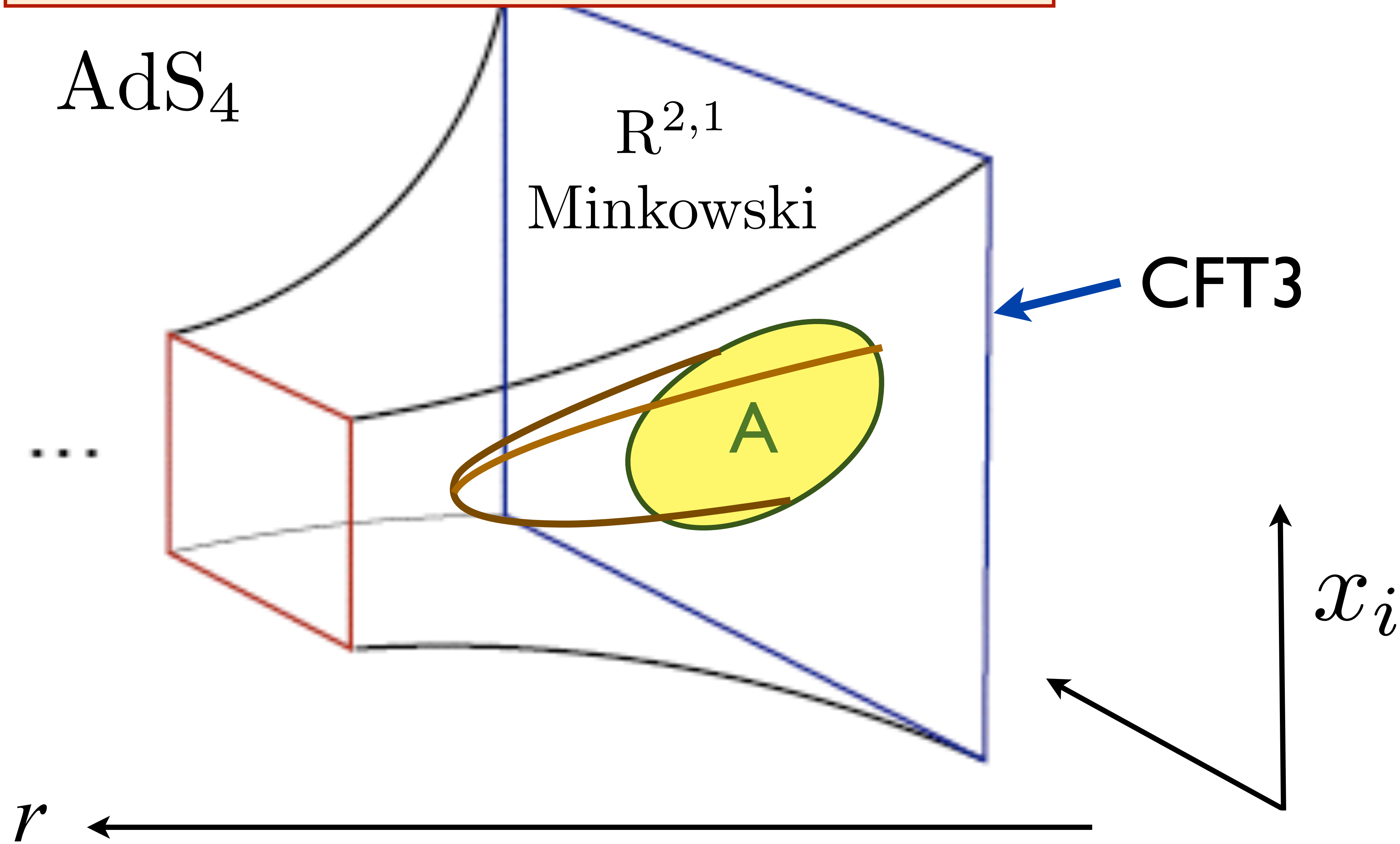

- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

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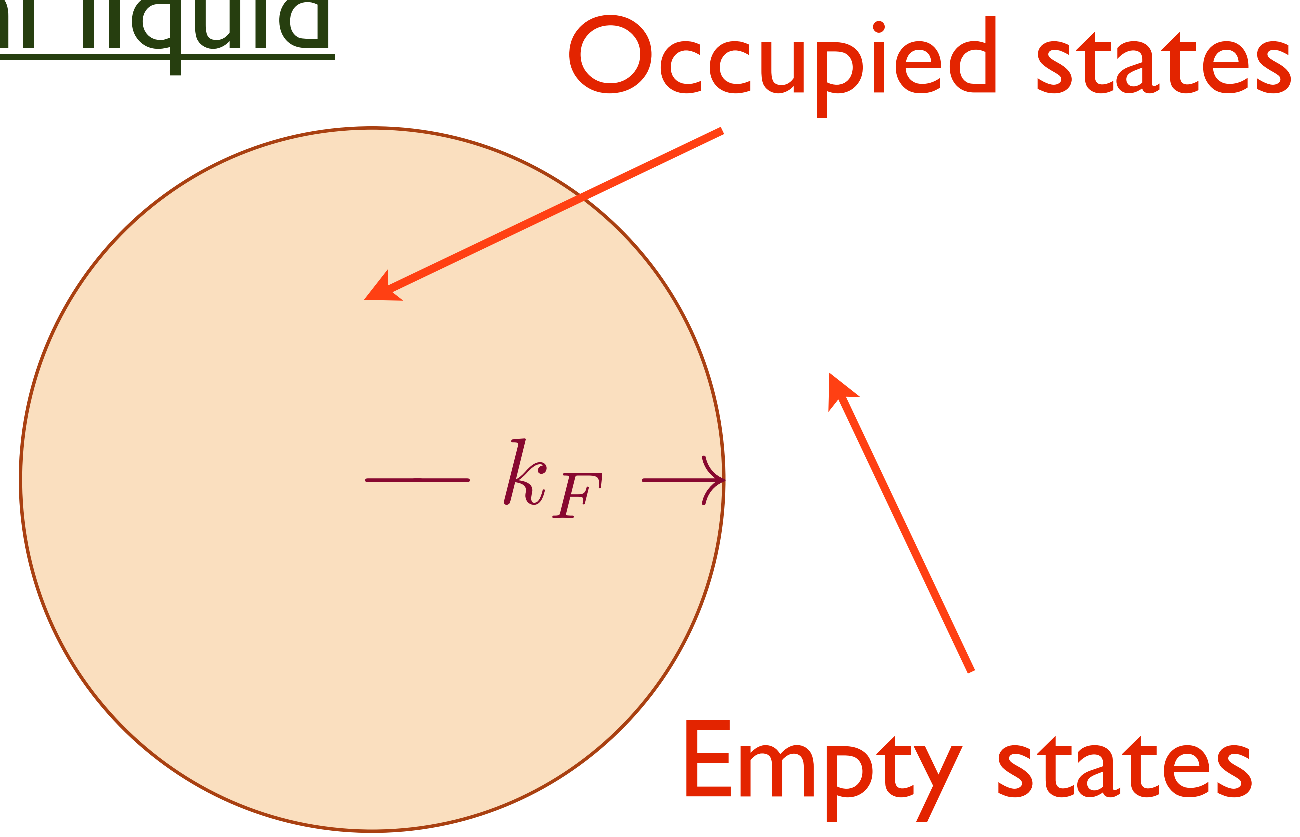
The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$
$$+ \, 4 \text{ Fermi terms}$$



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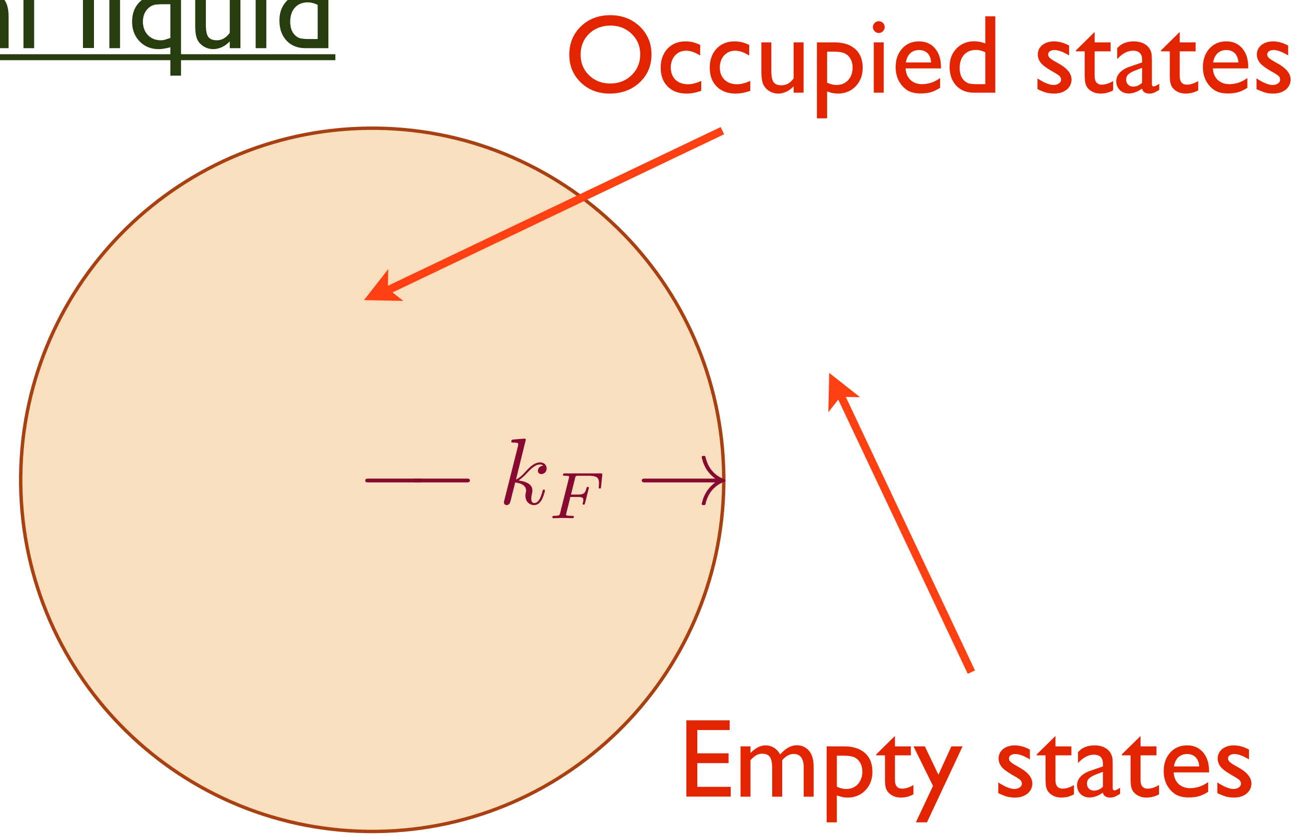
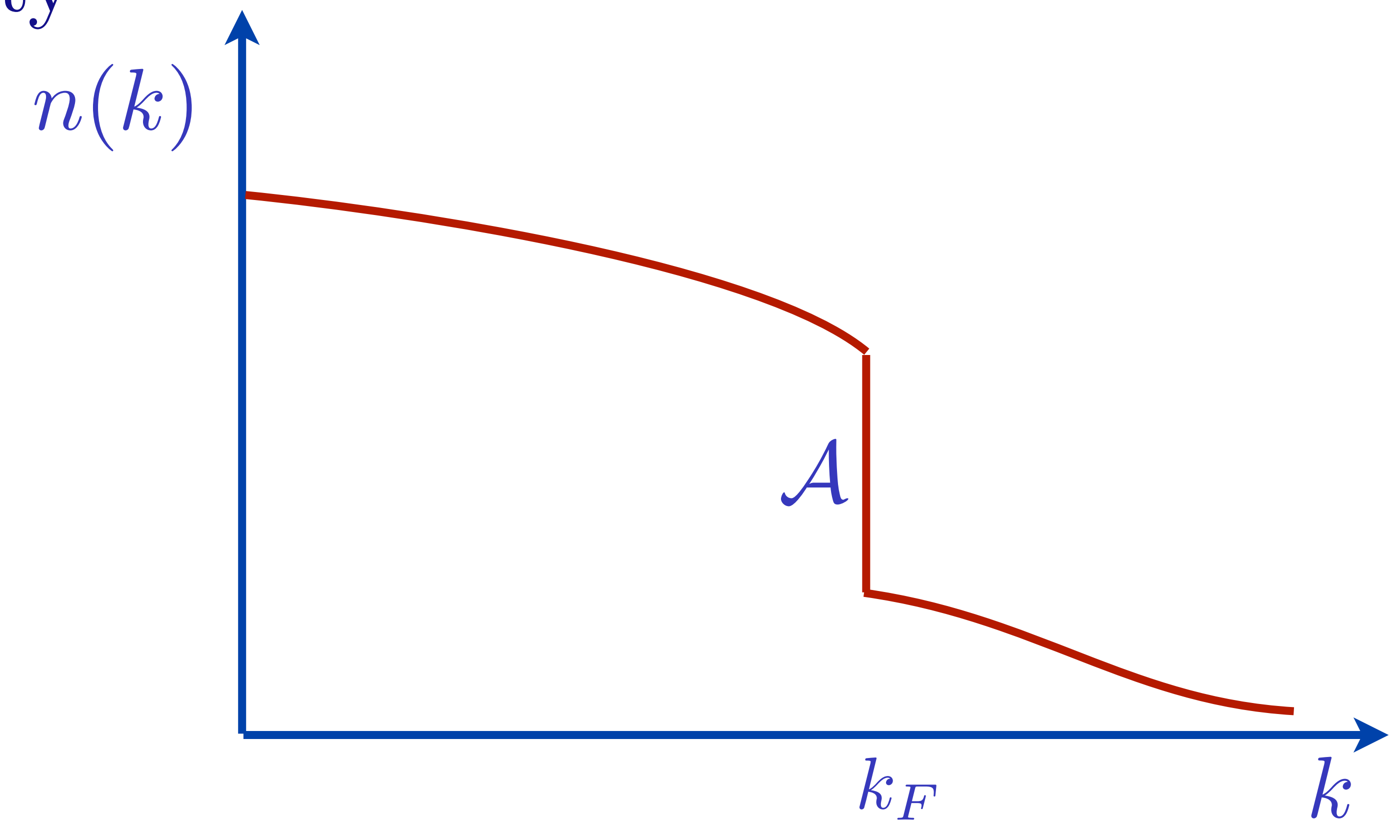


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density



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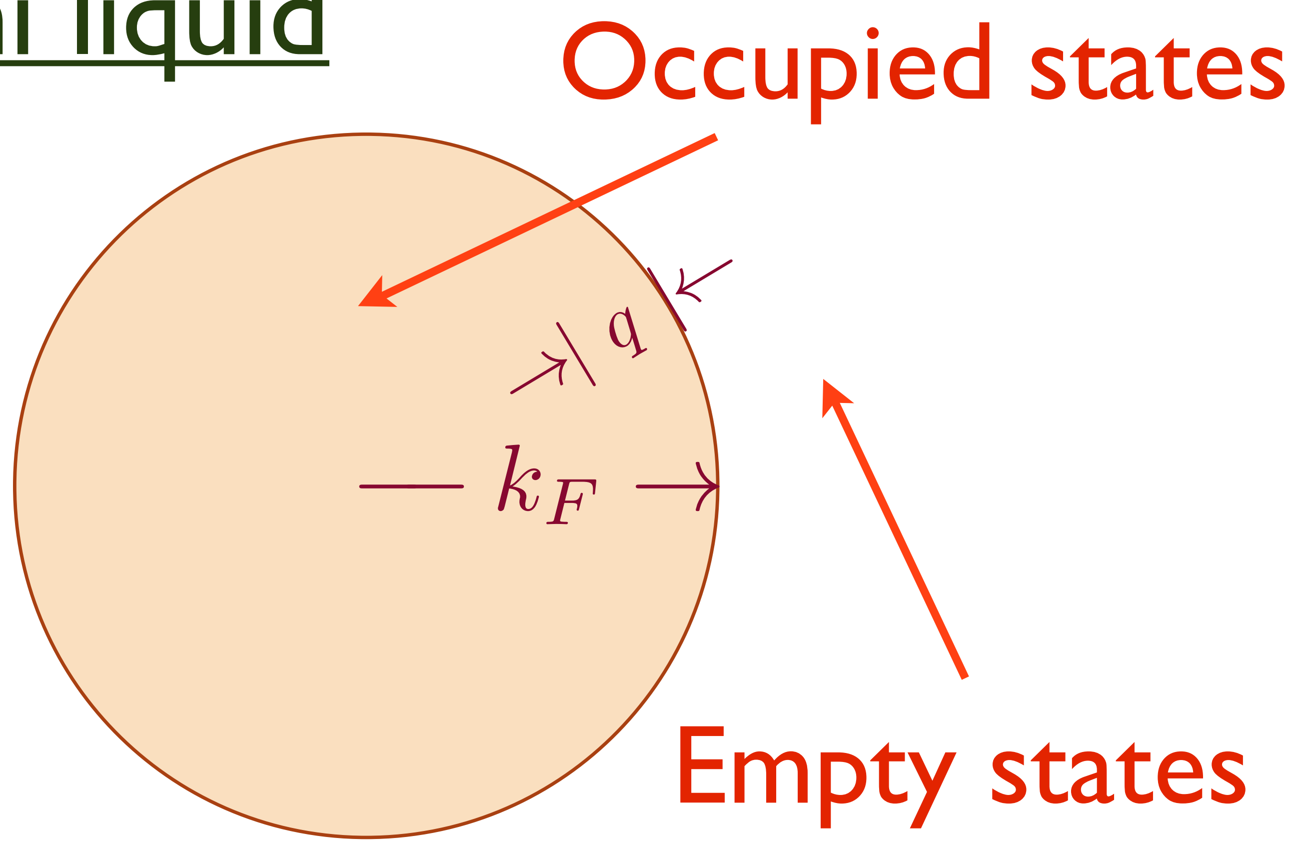

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- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.

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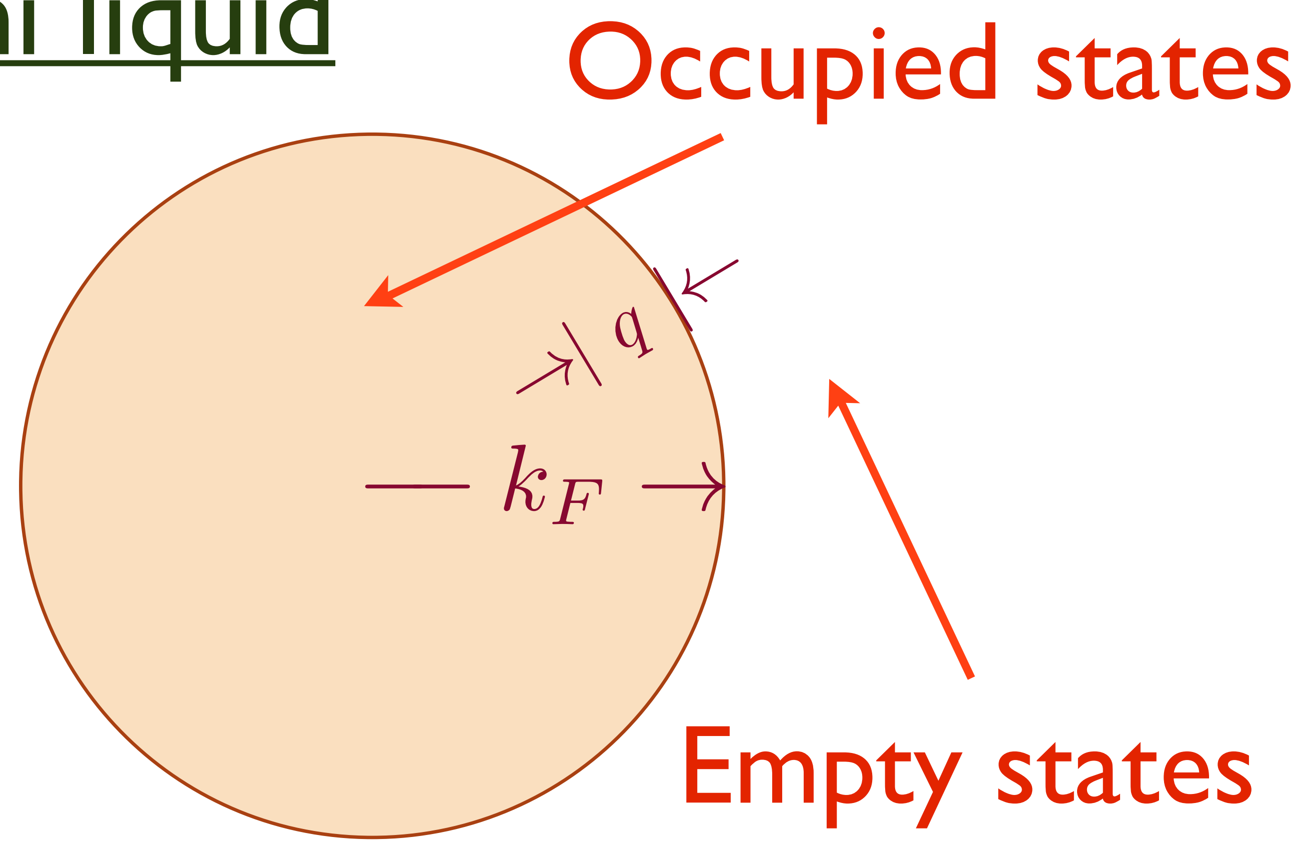
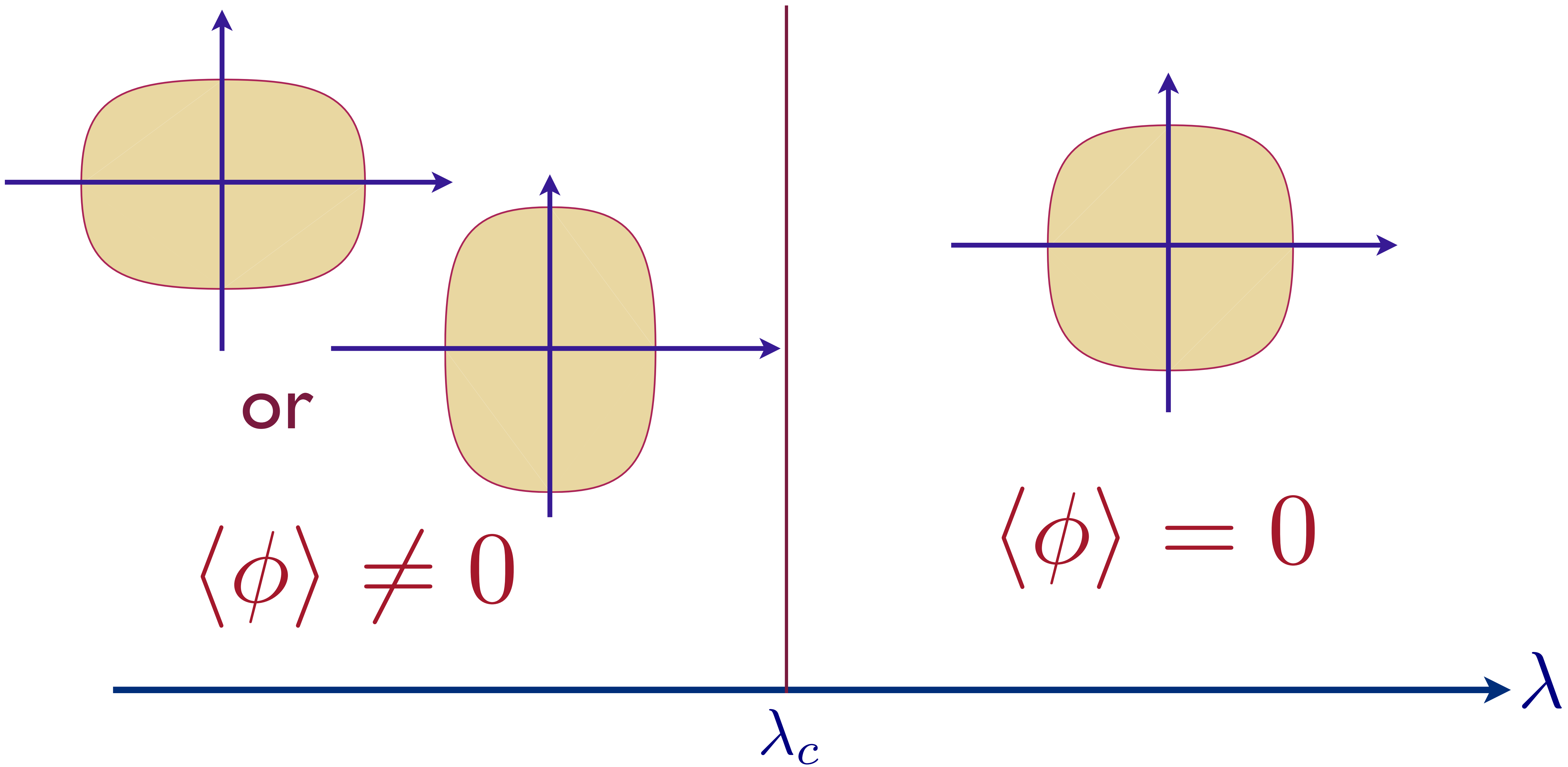


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T$. It is useful to write this is as $S \sim T^{(d-\theta)/z}$, with violation of hyperscaling exponent $\theta = d - 1$.

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

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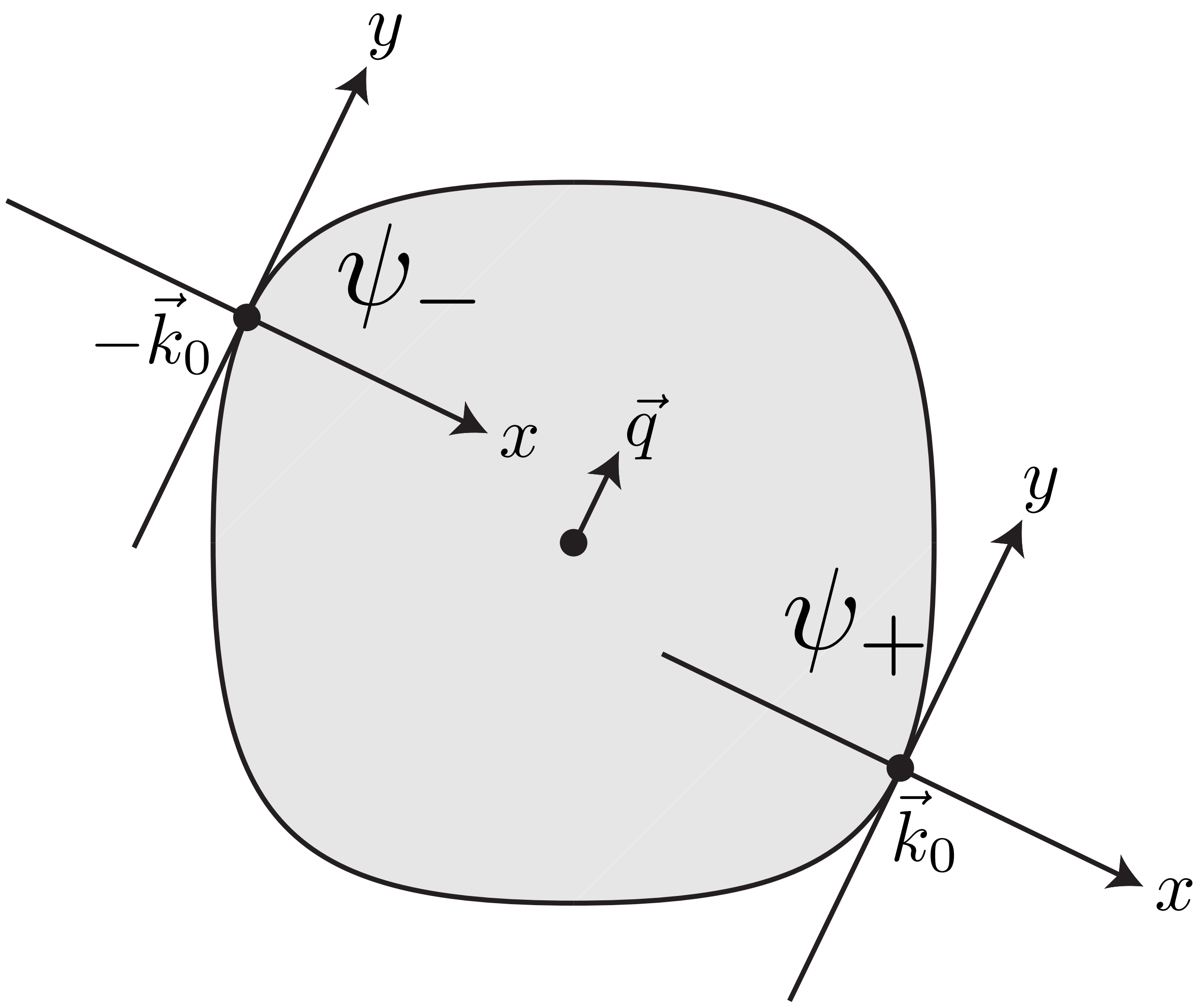

- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

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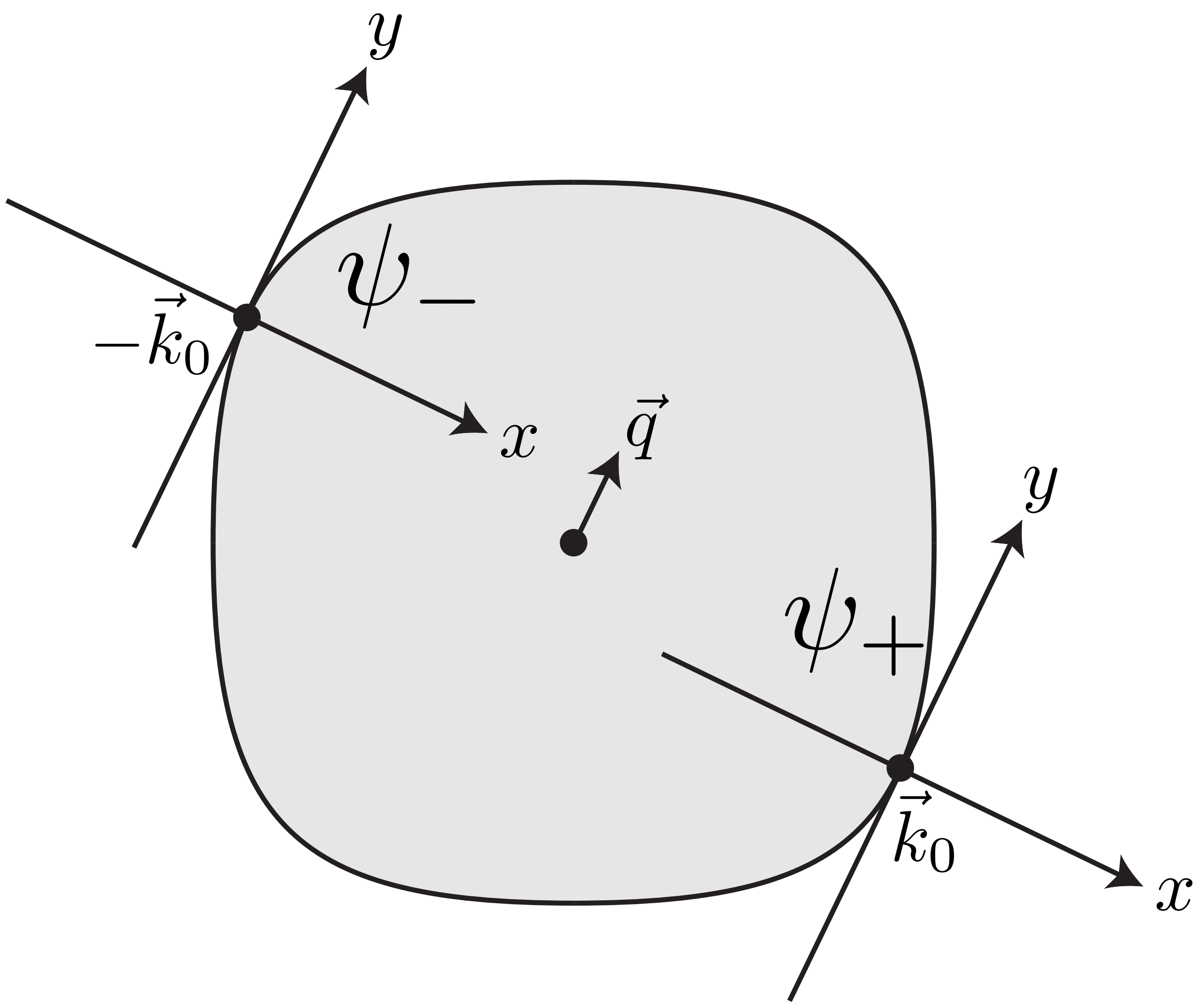
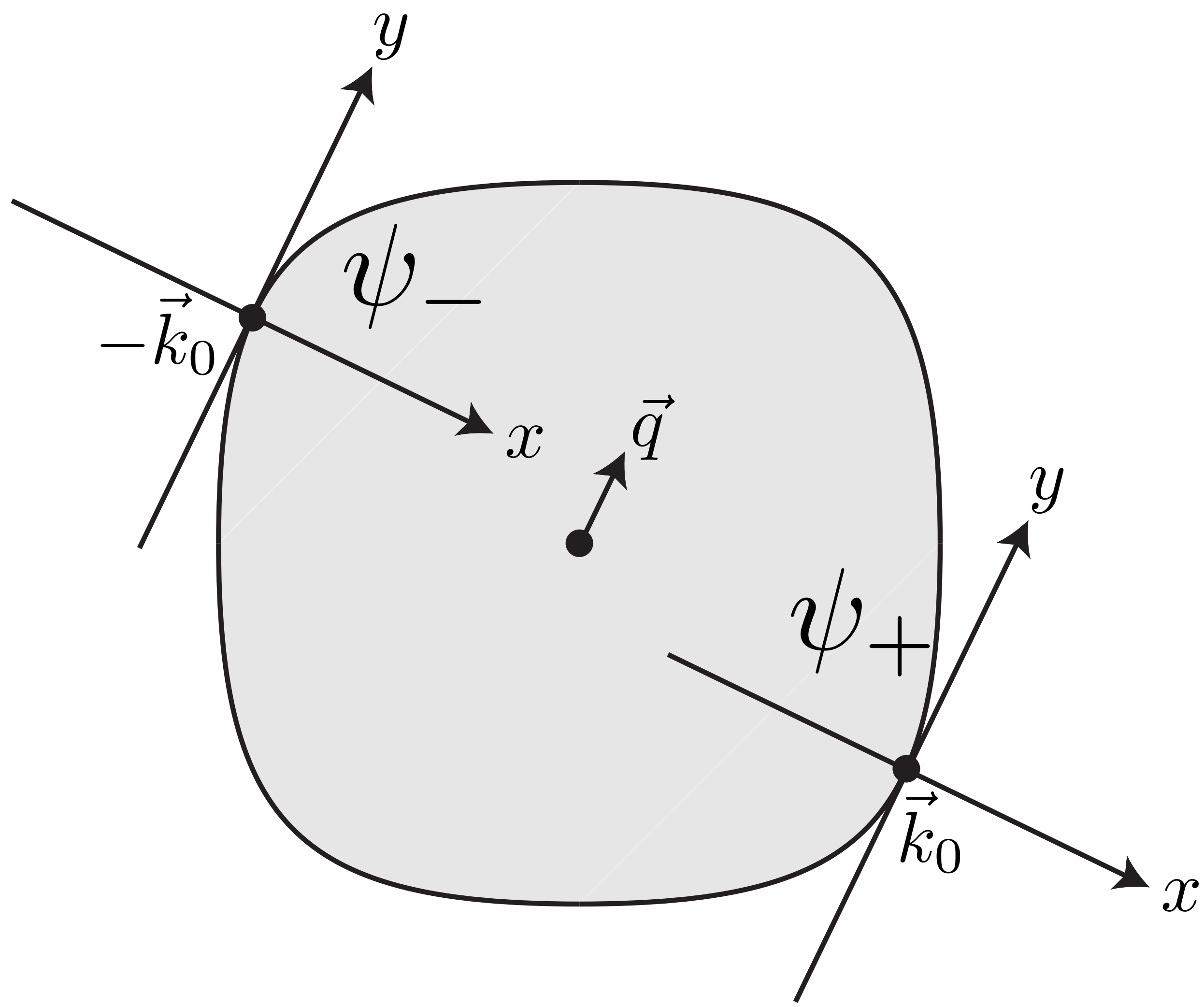


- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering in a metal



$$\mathcal{L}[\psi_\pm, \phi] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering in a metal

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- \\ &\quad - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2\end{aligned}$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering in a metal

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& - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2
\end{aligned}
$$

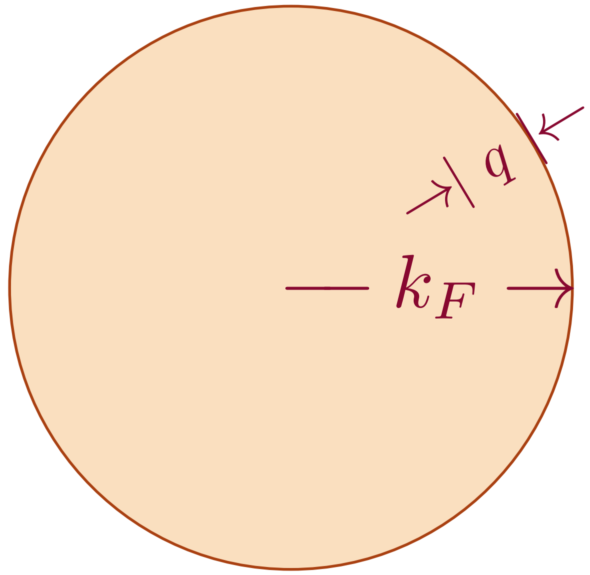
Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$
\begin{aligned}
\phi &\to \phi\, s \\
\psi &\to \psi\, s^{(2z+1)/4} \\
g &\to g\, s^{(3-2z)/4}
\end{aligned}
$$

So the action is invariant provided $z = 3/2$.

FL Fermi liquid



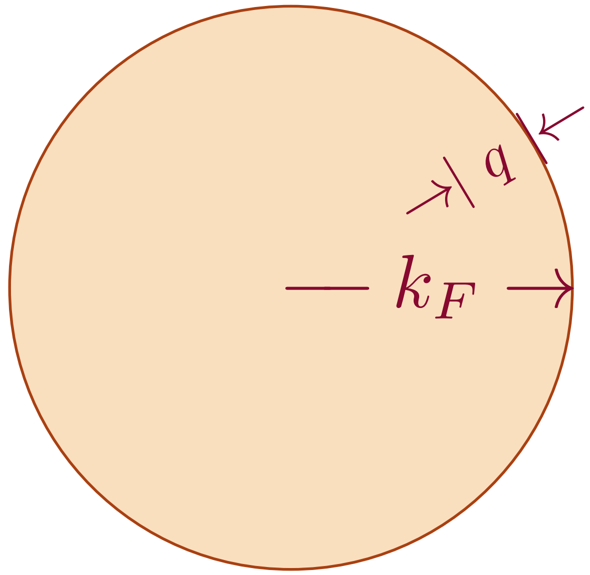
- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

FL Fermi liquid



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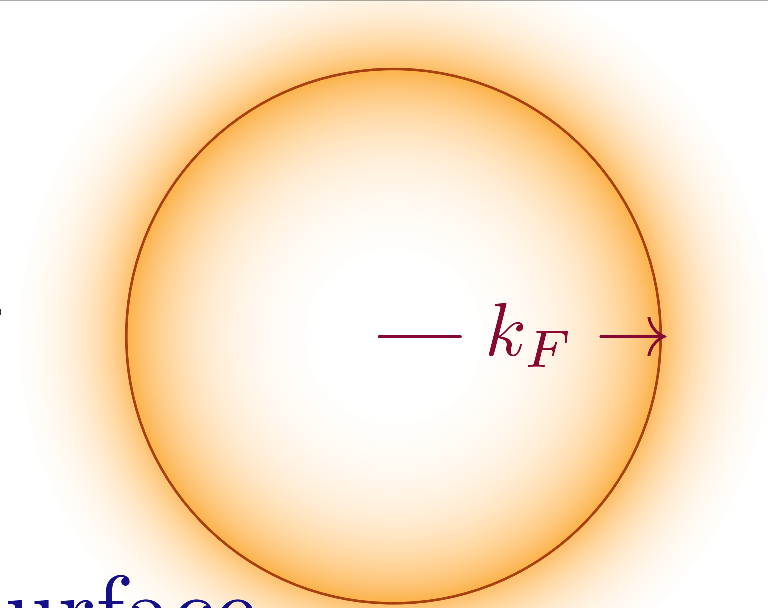
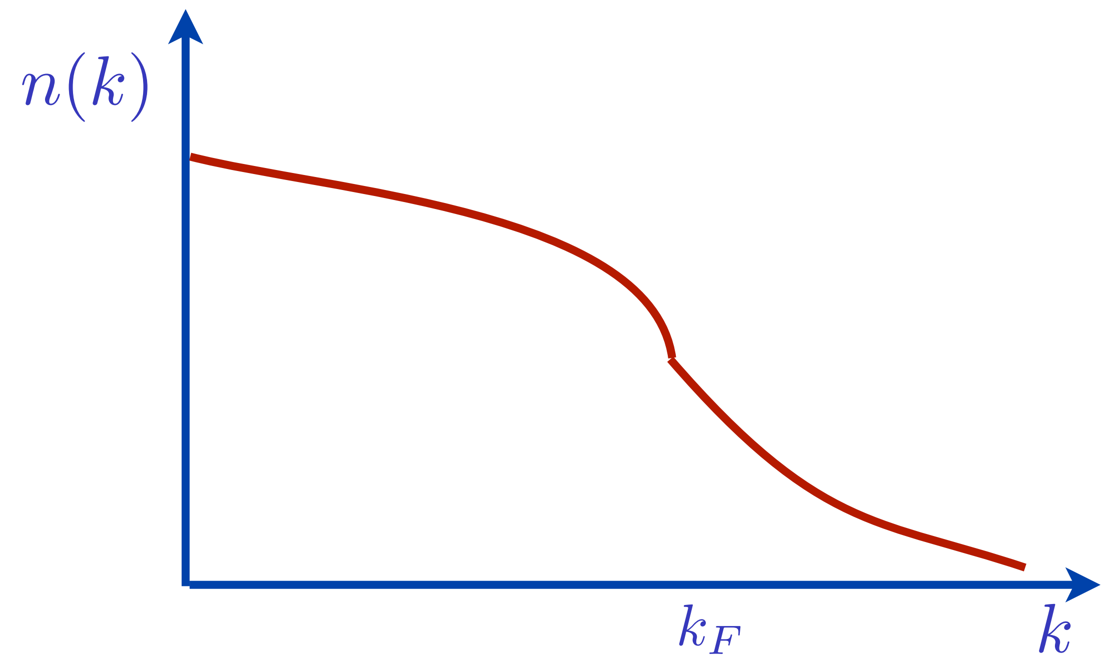
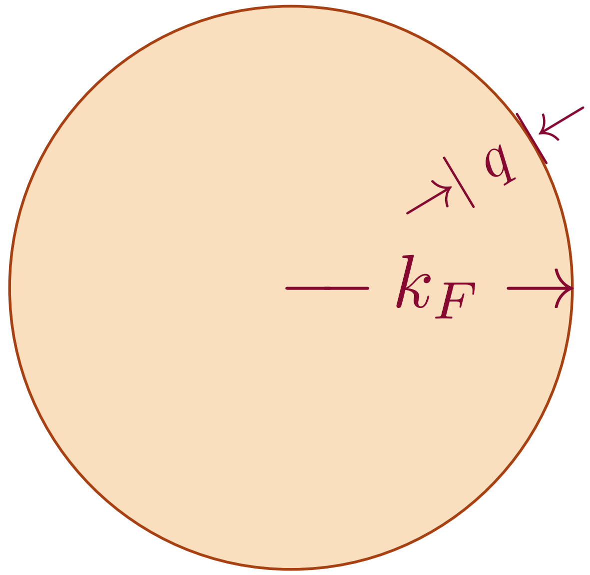
NFL Nematic QCP



- Fermi surface with $k_F^d \sim \mathcal{Q}$.



FL Fermi liquid



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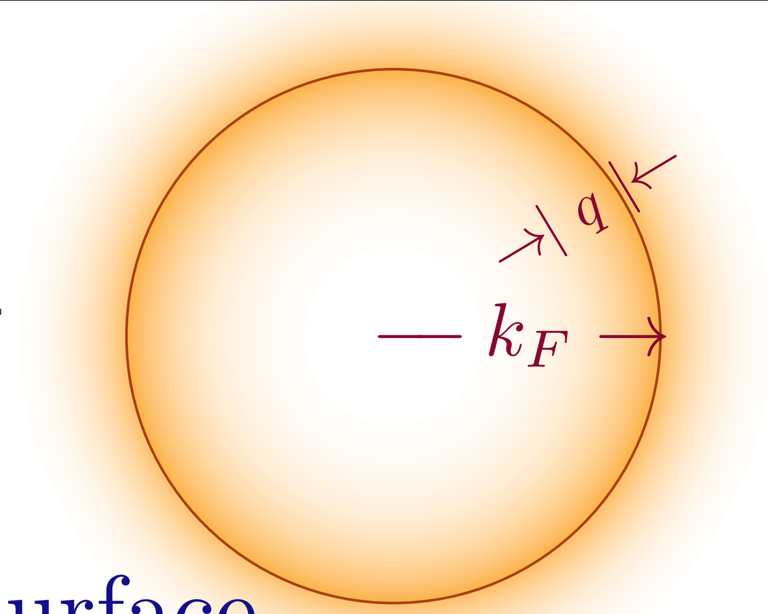
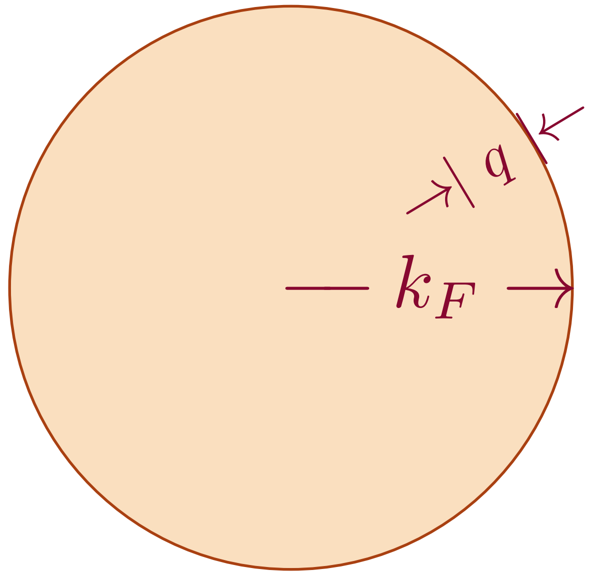
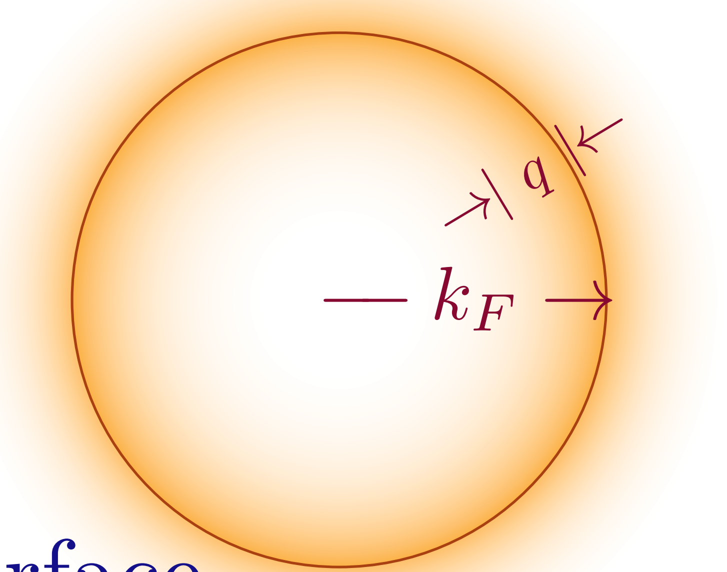
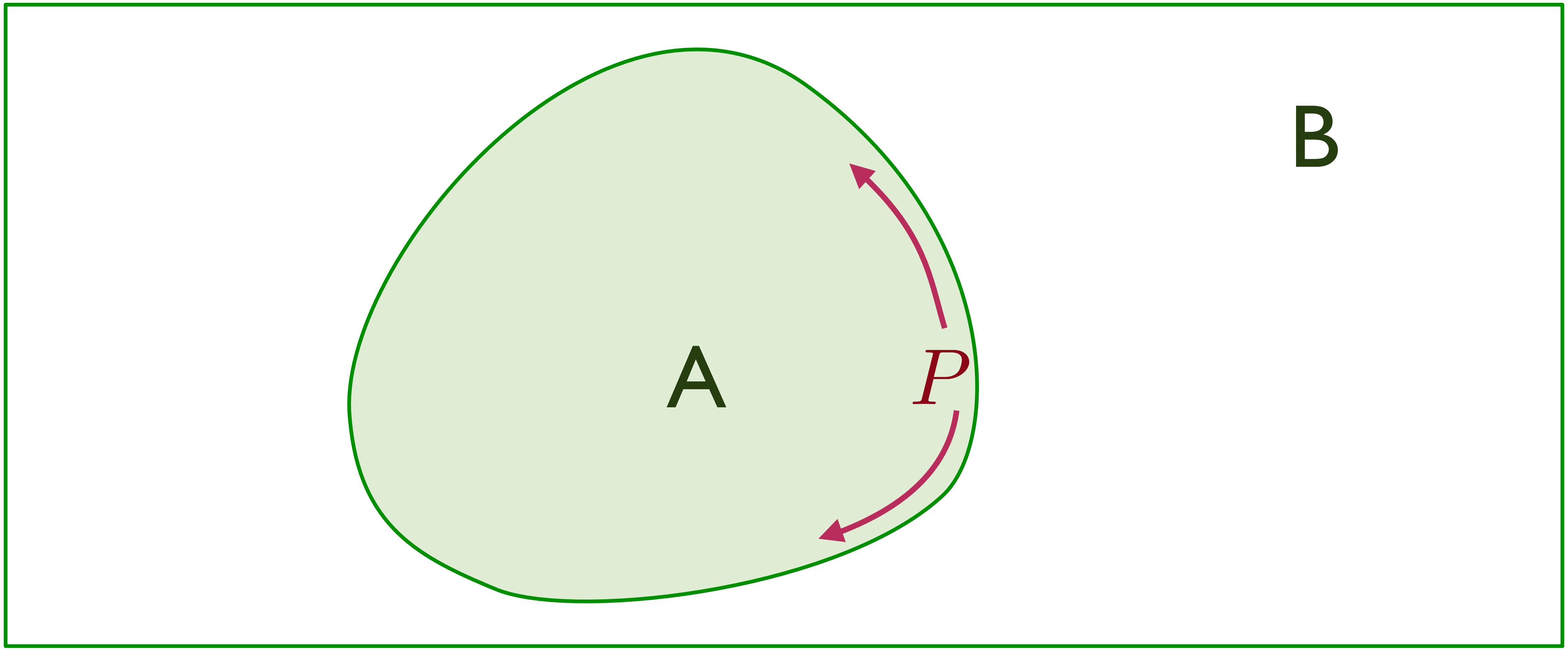


- Fermi surface with $k_F^d \sim \mathcal{Q}$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

FL Fermi liquid	NFL Nematic QCP
	
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \lvert q \rvert^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	

Entanglement entropy of the non-Fermi liquid

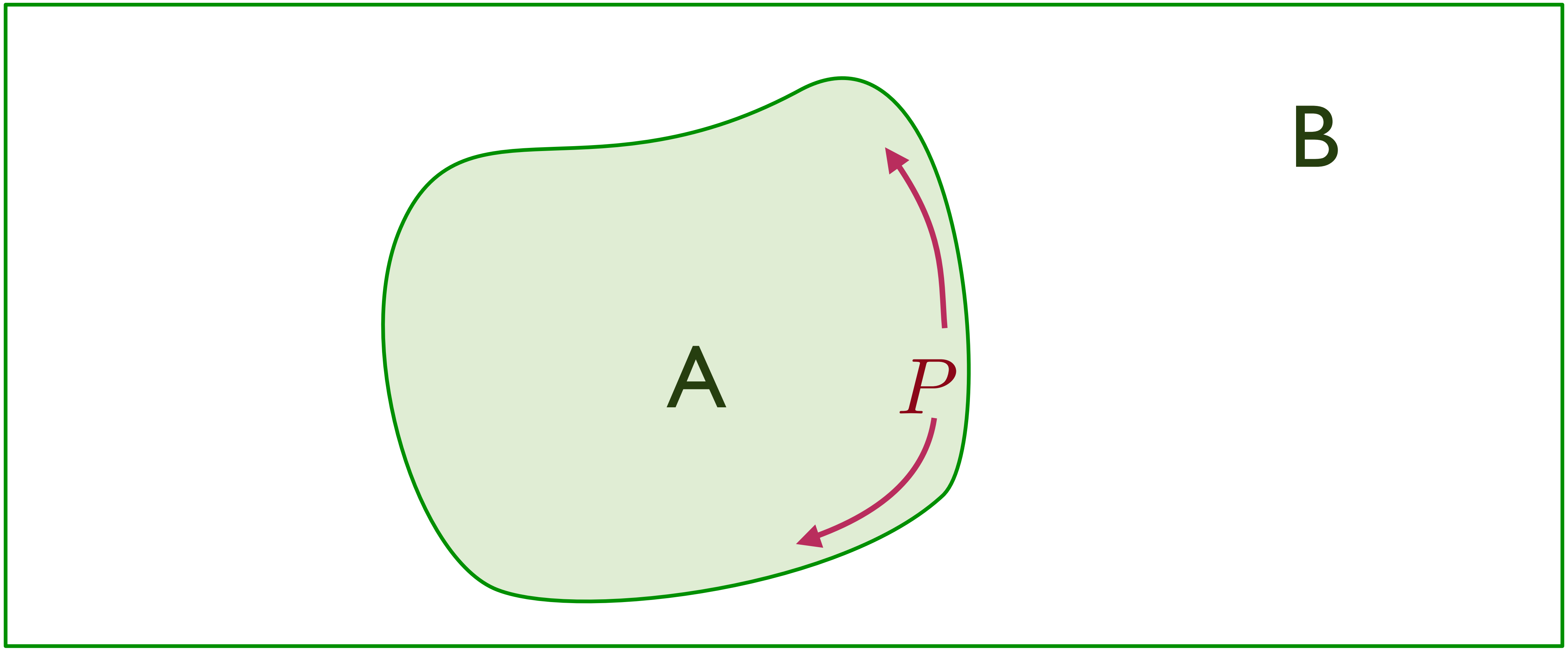


Logarithmic violation of “area law”: $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $\mathcal{C}_E$ is expected to be universal but $\neq 1/12$:
independent of the shape of the entangling region, and dependent
only on IR features of the theory.

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Entanglement entropy of the non-Fermi liquid



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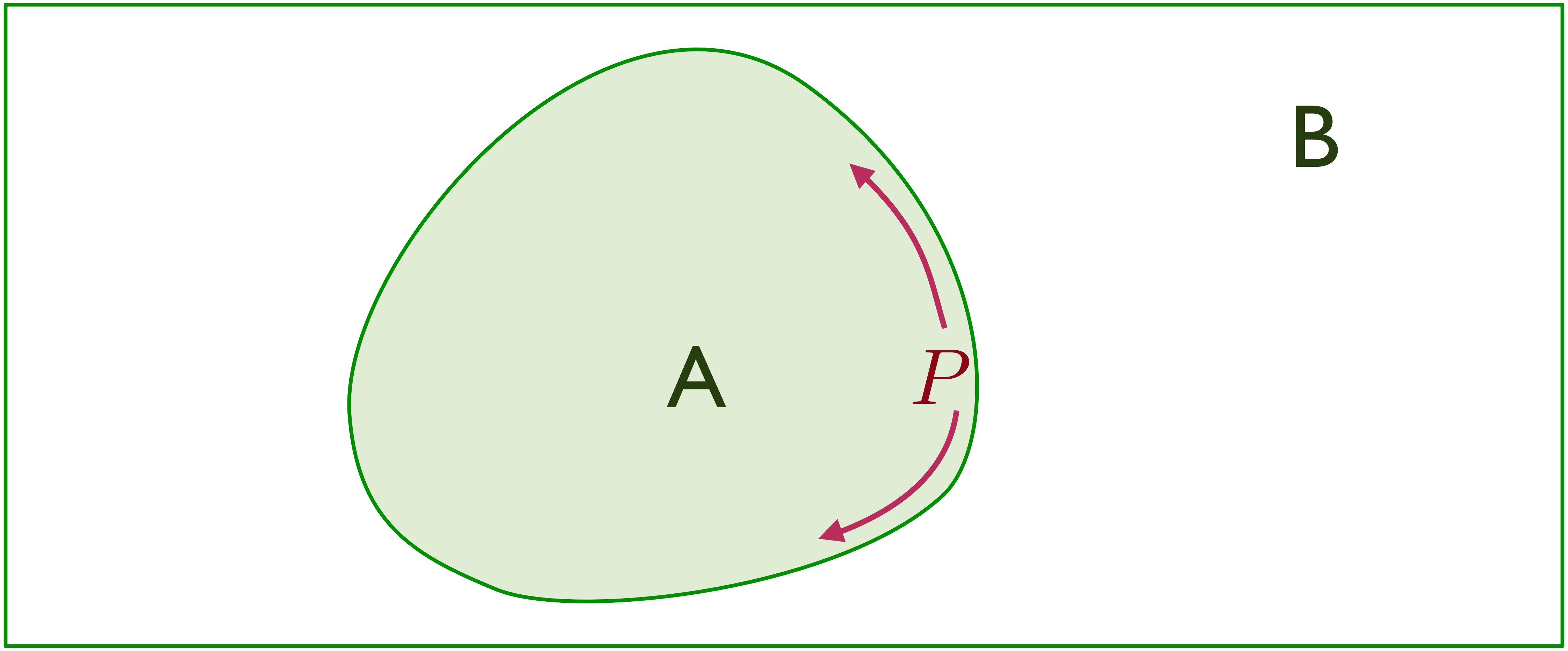

Logarithmic violation of "area law": $S_E = \mathcal{C}_E\, k_F P \ln(k_F P)$

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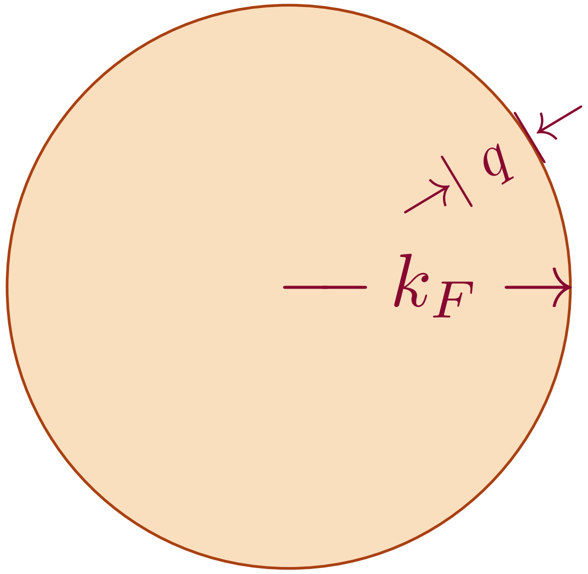
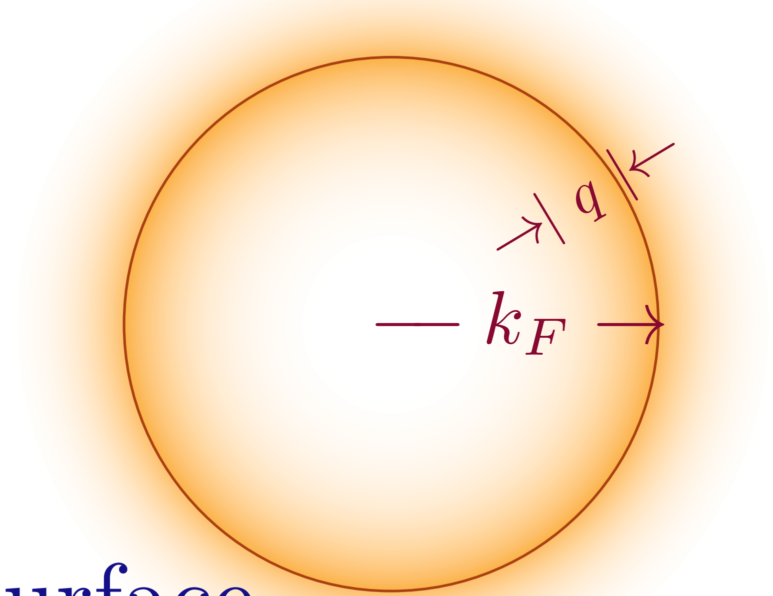
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

FL Fermi liquid	NFL Nematic QCP
 • $k_F^d \sim \mathcal{Q}$, the fermion density	 • Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \|q\|^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	• $S_E \sim k_F^{d-1} P \ln P$.

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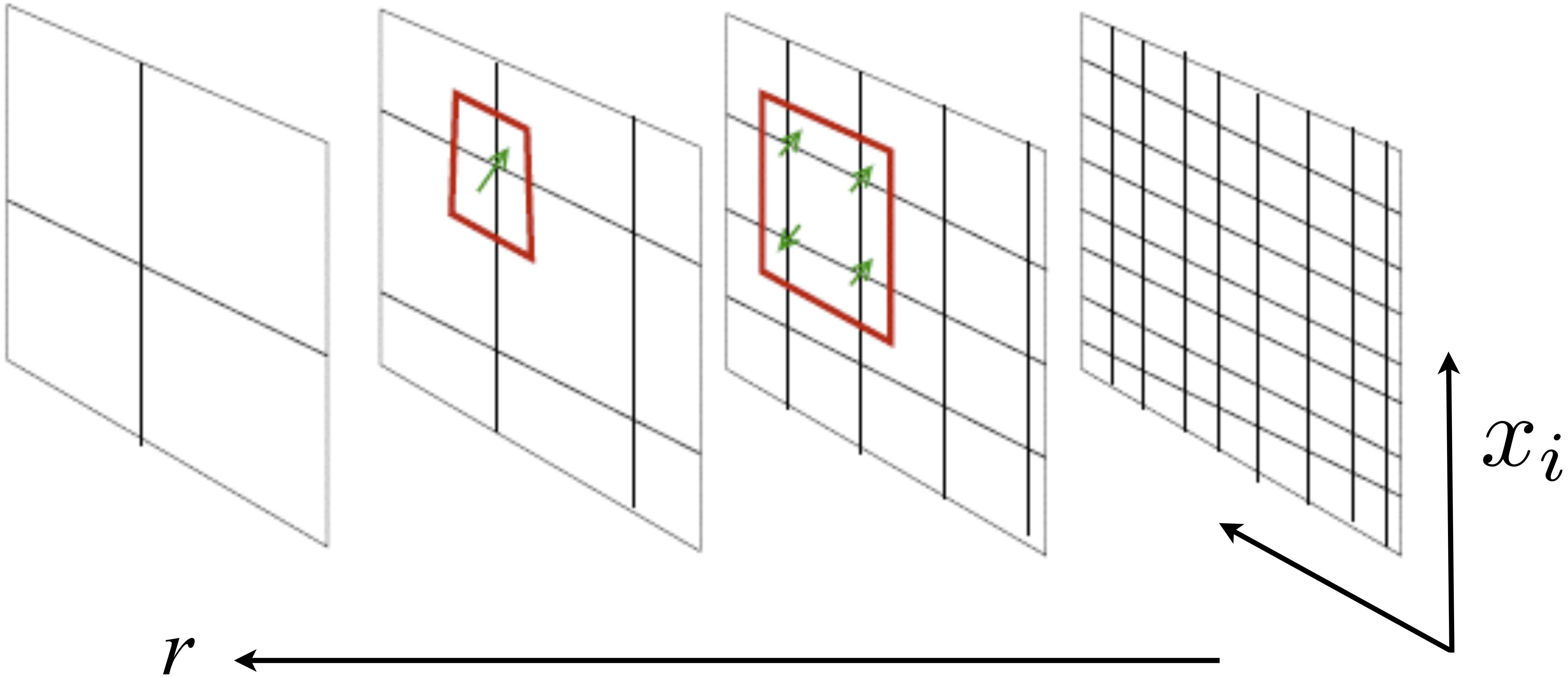
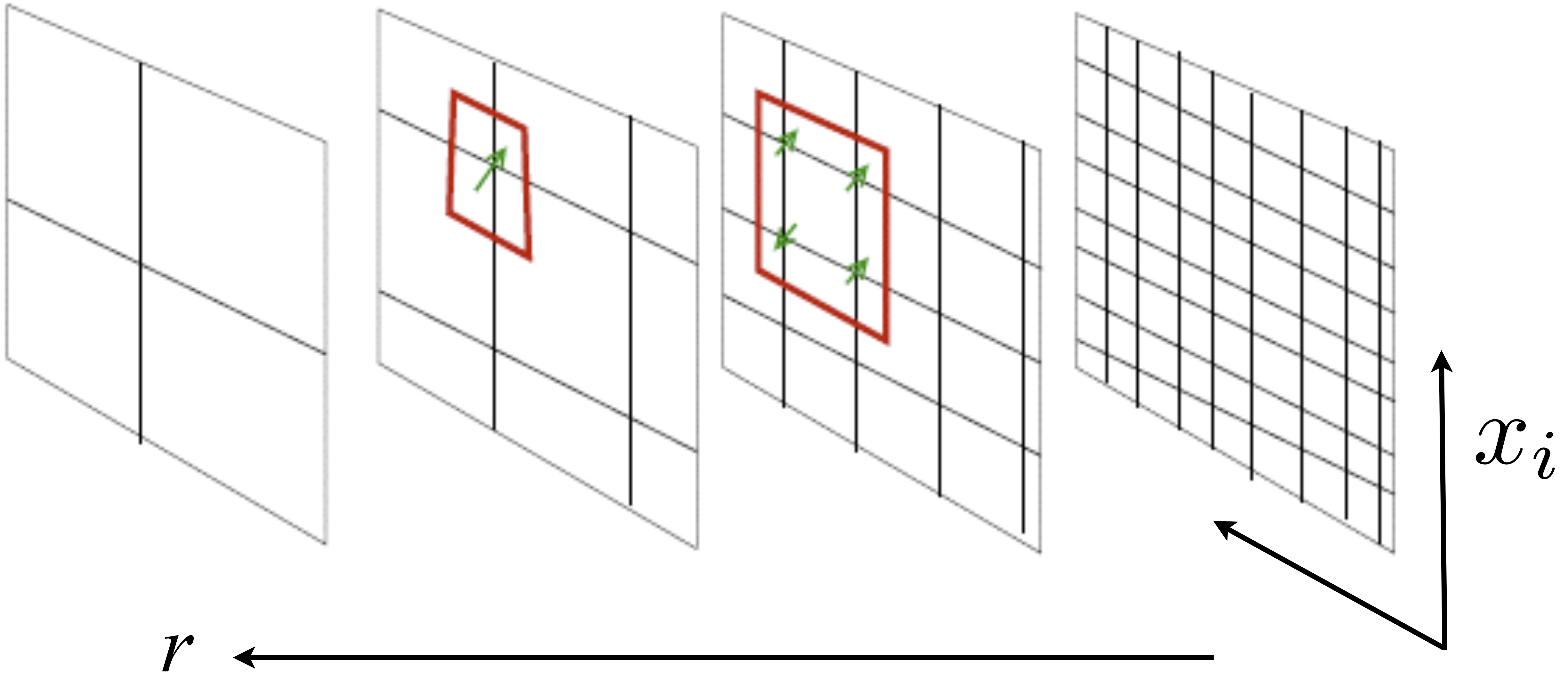
Generalized holography



Generalized holography

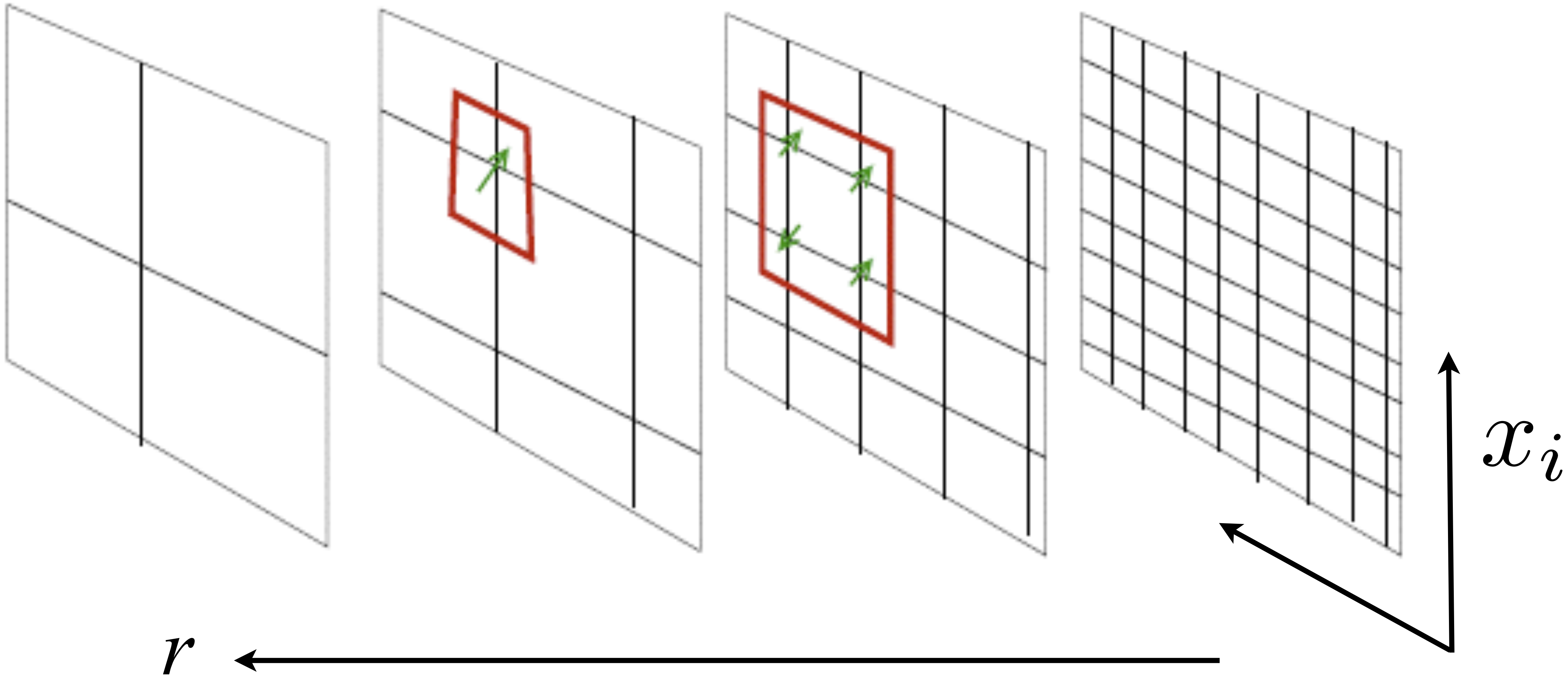


Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Generalized holography



The most general such metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

Generalized holography

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

This is the most general metric which is invariant under the scale transformation

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points). We will see shortly that θ is the violation of hyperscaling exponent.
We have used reparametrization invariance in r to define it so that it scales as

$$r \rightarrow \zeta^{(d-\theta)/d} r\,.$$

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

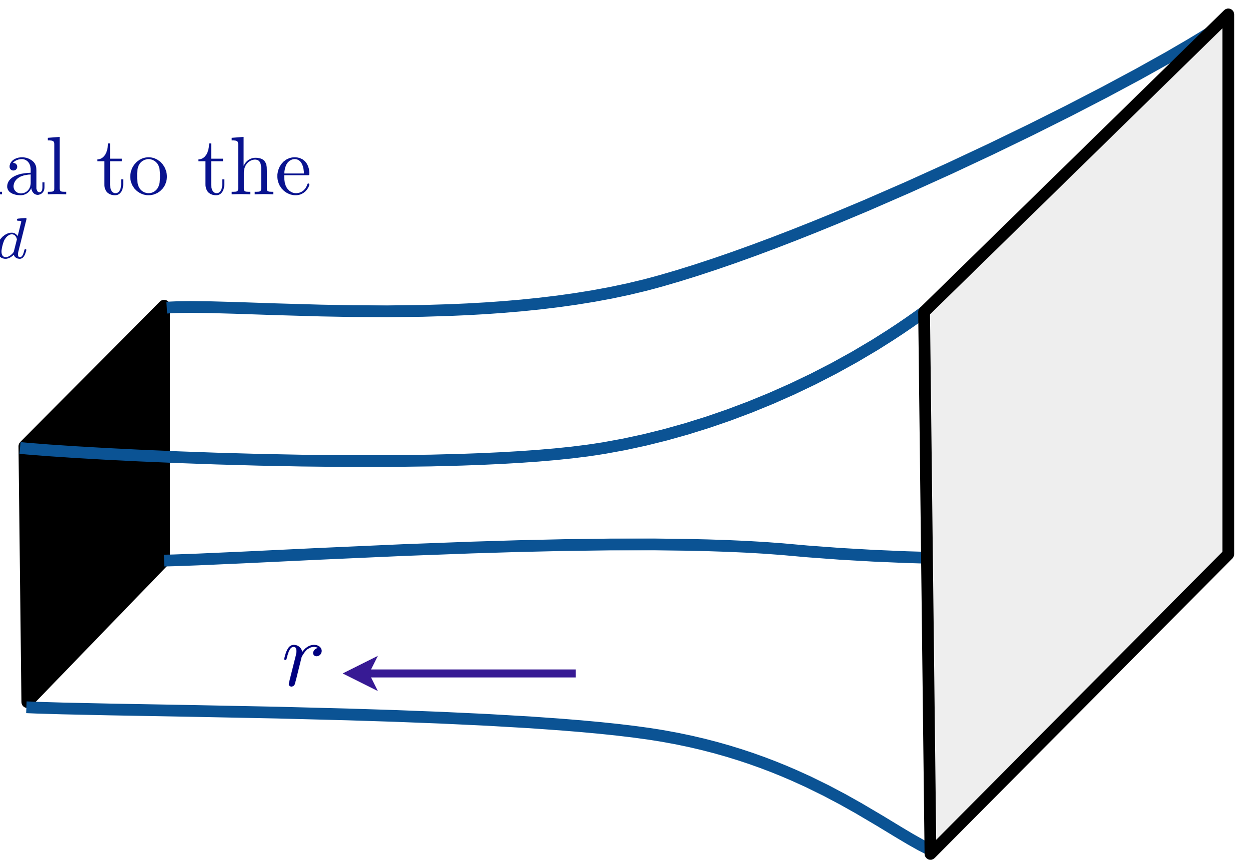
Generalized holography

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

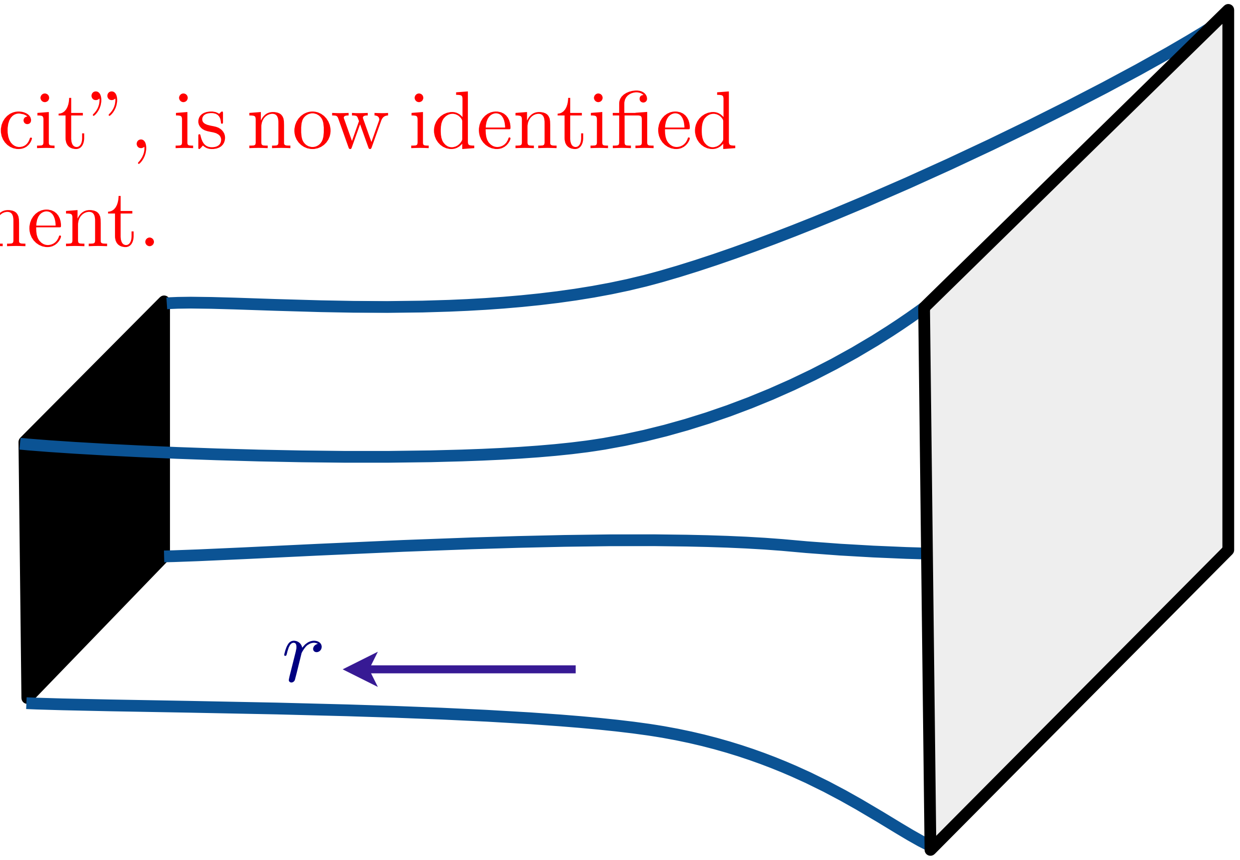
Generalized holography

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$, the "dimension deficit", is now identified as the violation of hyperscaling exponent.



L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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The Fermi liquid has $\theta = d - 1$ and $z = 1$: so the Fermi liquid does *not* have such a gravity dual.
The non-Fermi liquid in $d = 2$ has $\theta = d-1$, and this implies $z \geq 3/2$. So the lower bound is precisely the value obtained for the non-Fermi liquid!

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Generalized holography

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Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim \begin{cases} P & , \quad \text{for } \theta < d-1 \\ P \ln P & , \quad \text{for } \theta = d-1 \\ P^{\theta/(d-1)} & , \quad \text{for } \theta > d-1 \end{cases} .$$

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The non-Fermi liquid has log-violation of "area law", and this appears precisely at the correct value $\theta = d-1$!
Moreover, the co-efficient of $P\ln P$ computed holographically is independent of the shape of the entangling region just as expected for a circular Fermi surface!!

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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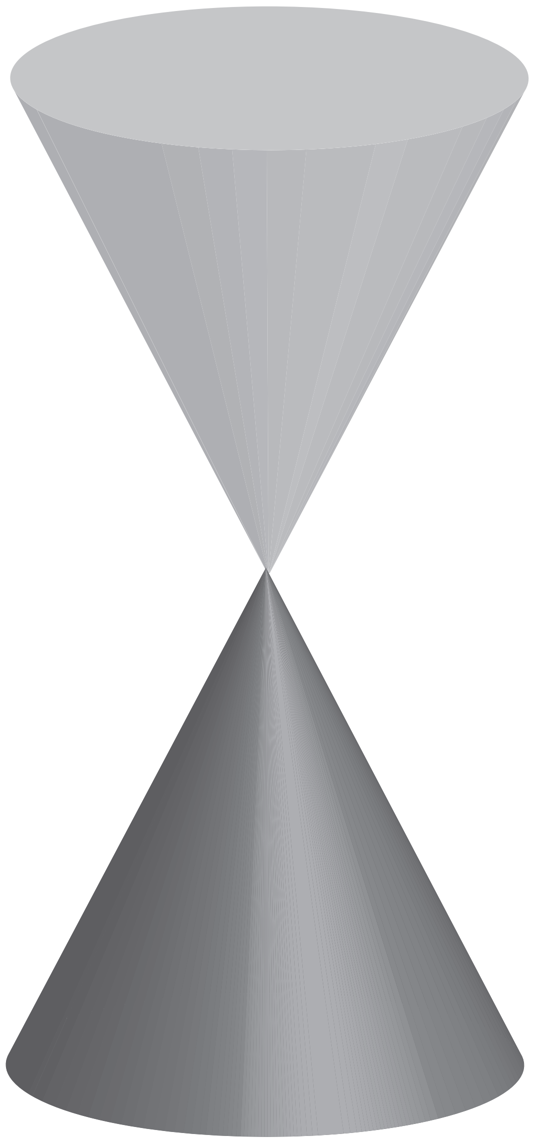
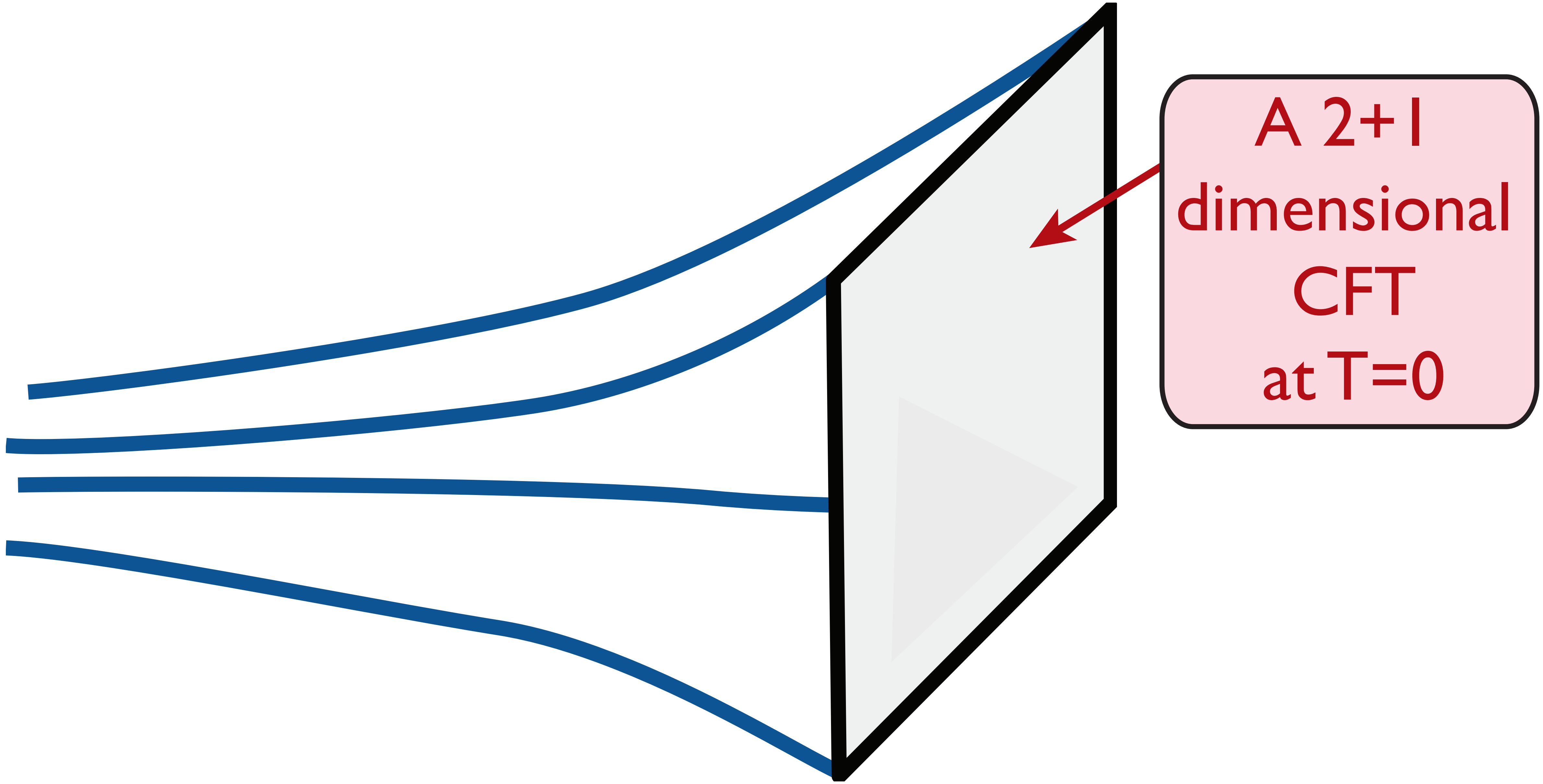
Begin with a CFT

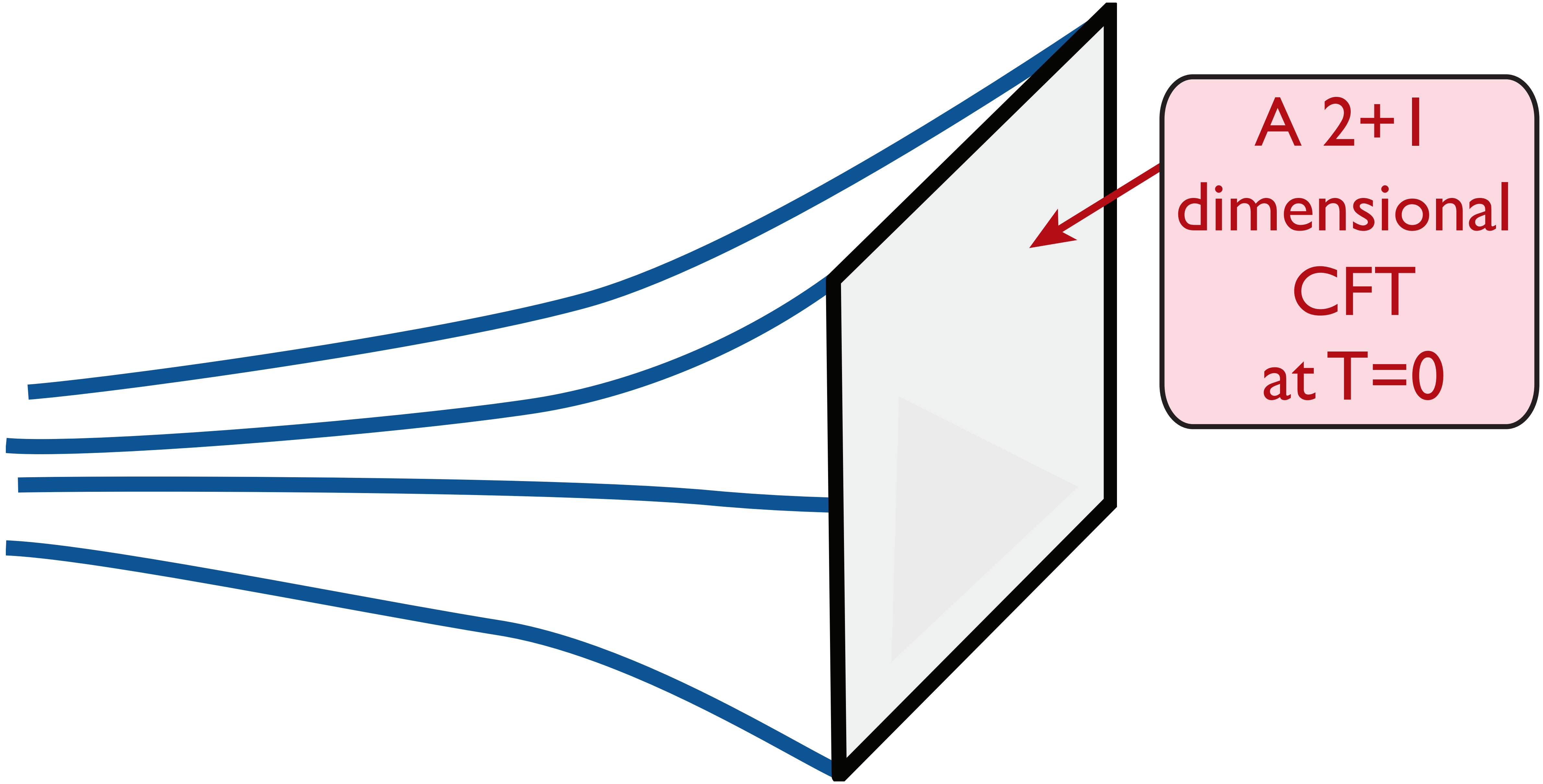


Holographic representation: AdS_4



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

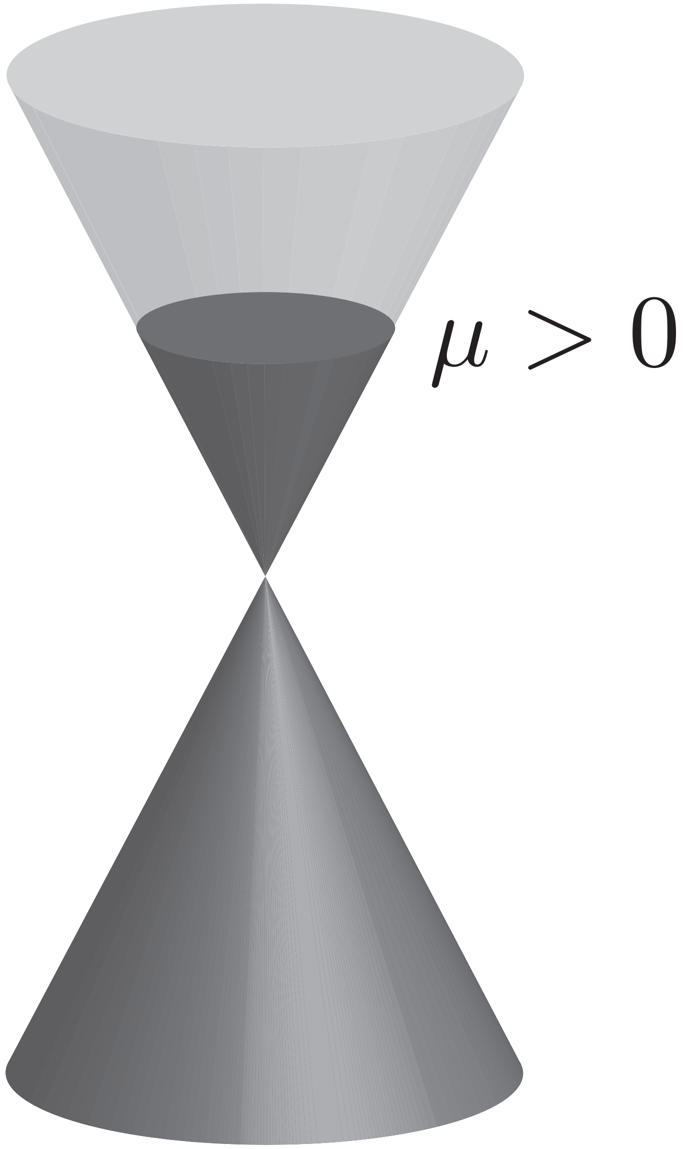
Holographic representation: AdS$_4$



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1$

Apply a chemical potential



AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right] .$$

This is to be solved subject to the constraint

$$A_\mu(r \to 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where $\mathcal{A}_\mu$ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

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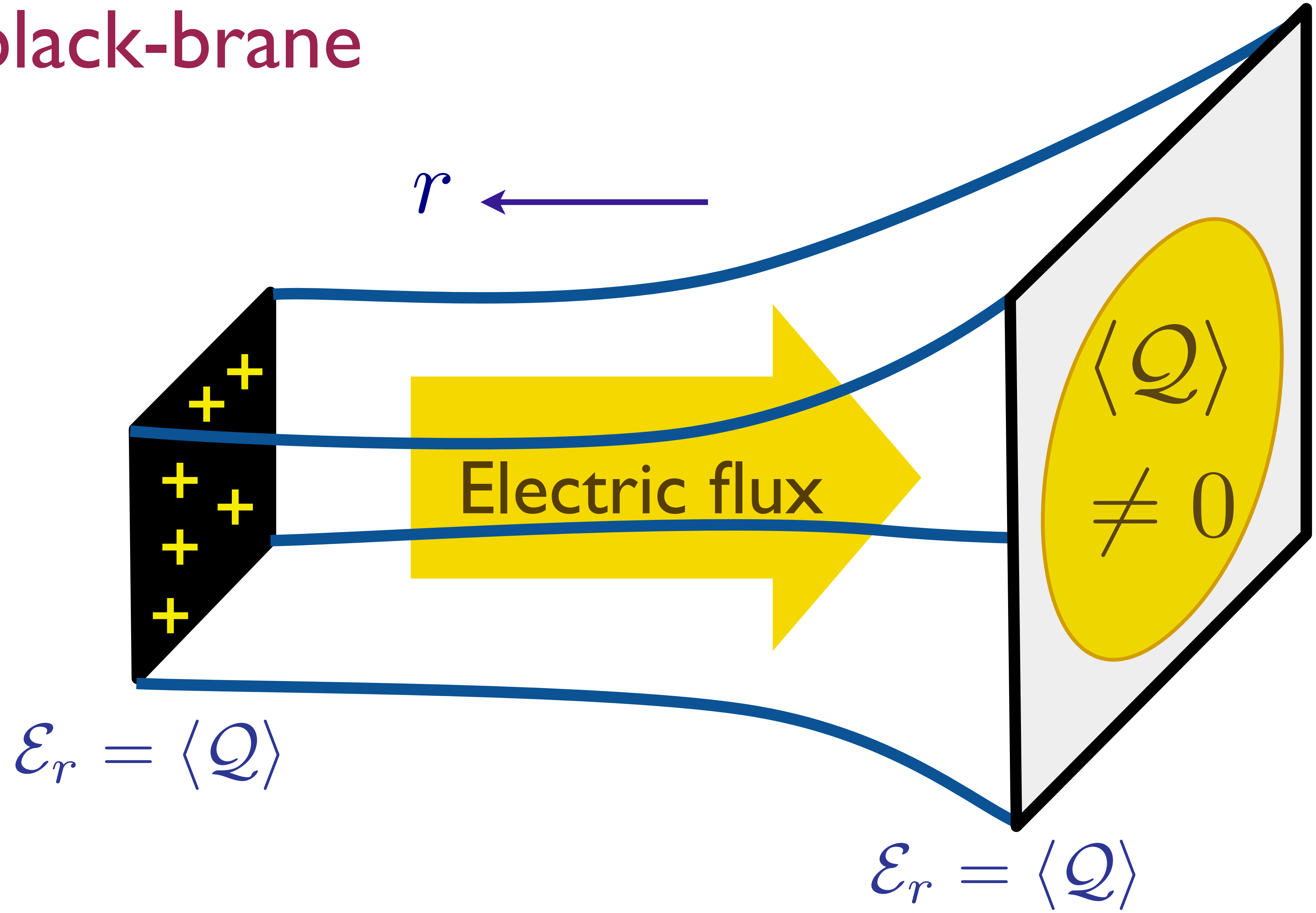
$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

At non-zero chemical potential we simply require $\mathcal{A}_\tau = \mu$.

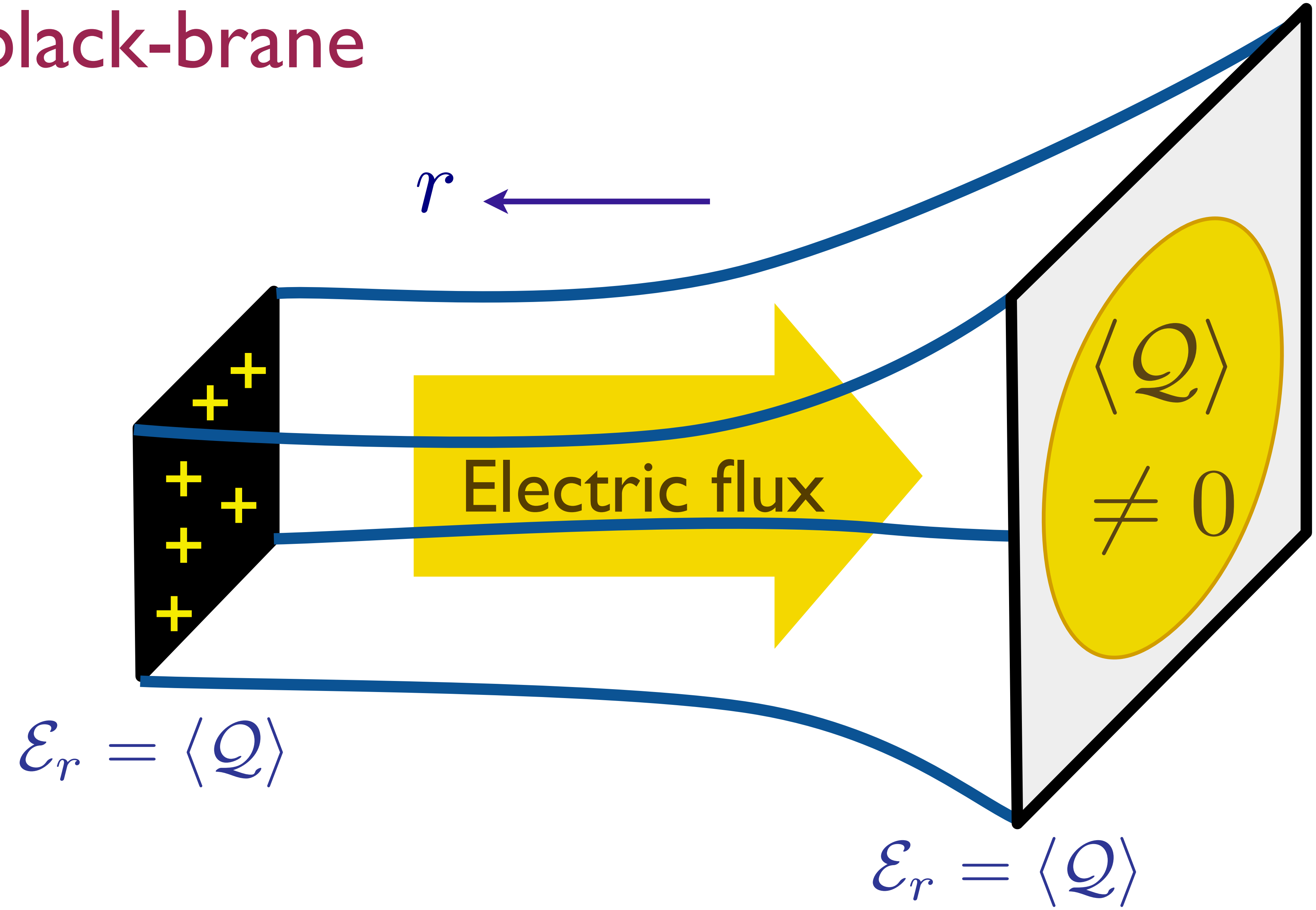
The Maxwell-Einstein theory of the applied chemical potential yields a AdS4-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4g_4^2}F_{ab}F^{ab}\right]$$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

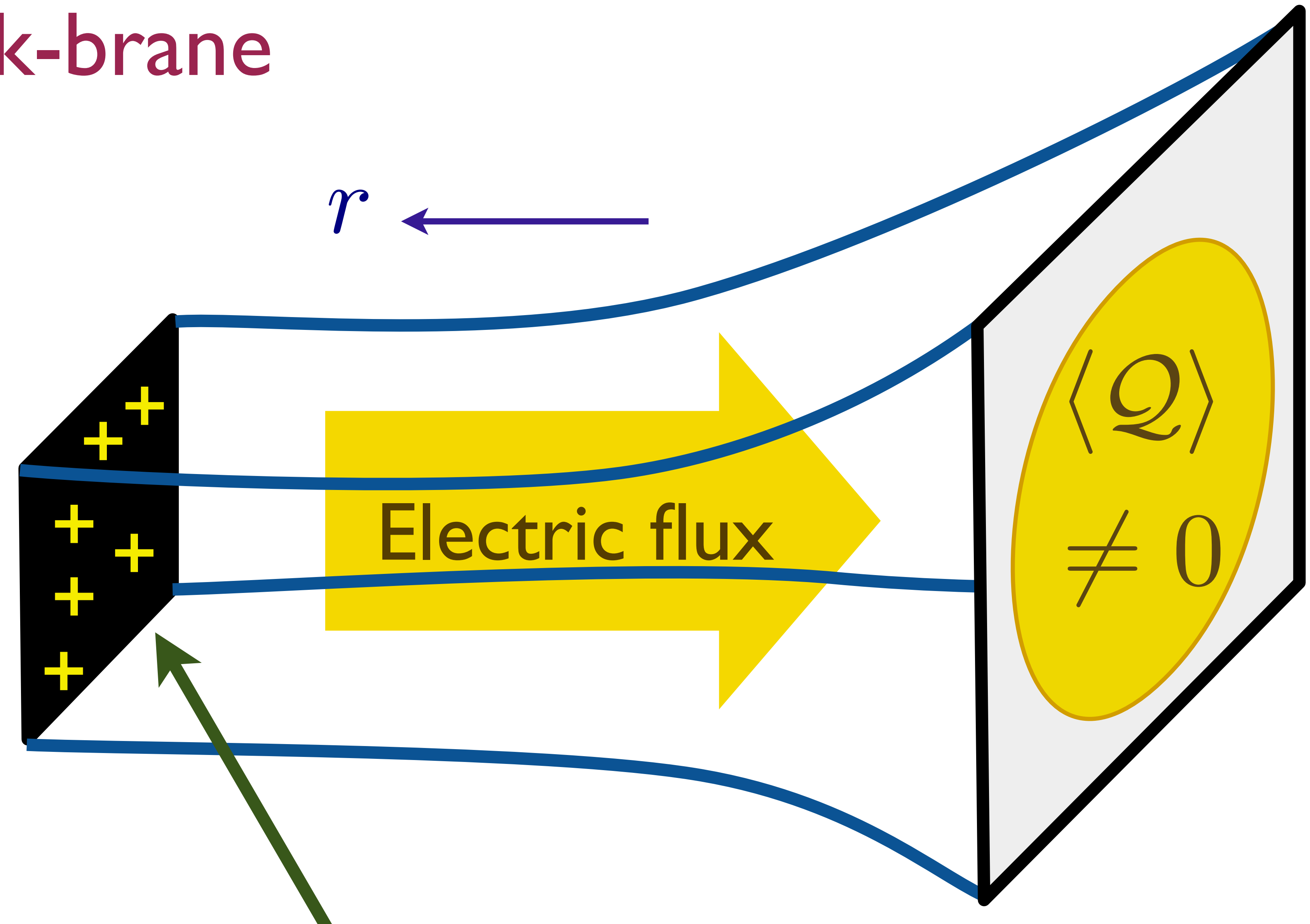
The Maxwell-Einstein theory of the applied chemical potential yields a AdS4-Reissner-Nordtröm black-brane



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right)$ and $R = \frac{\sqrt{6}Lg_4}{\kappa\mu}$, and $A_\tau = \mu\left(1 - \frac{r}{R}\right)$

The Maxwell-Einstein theory of the applied chemical potential yields a AdS4-Reissner-Nordtröm black-brane



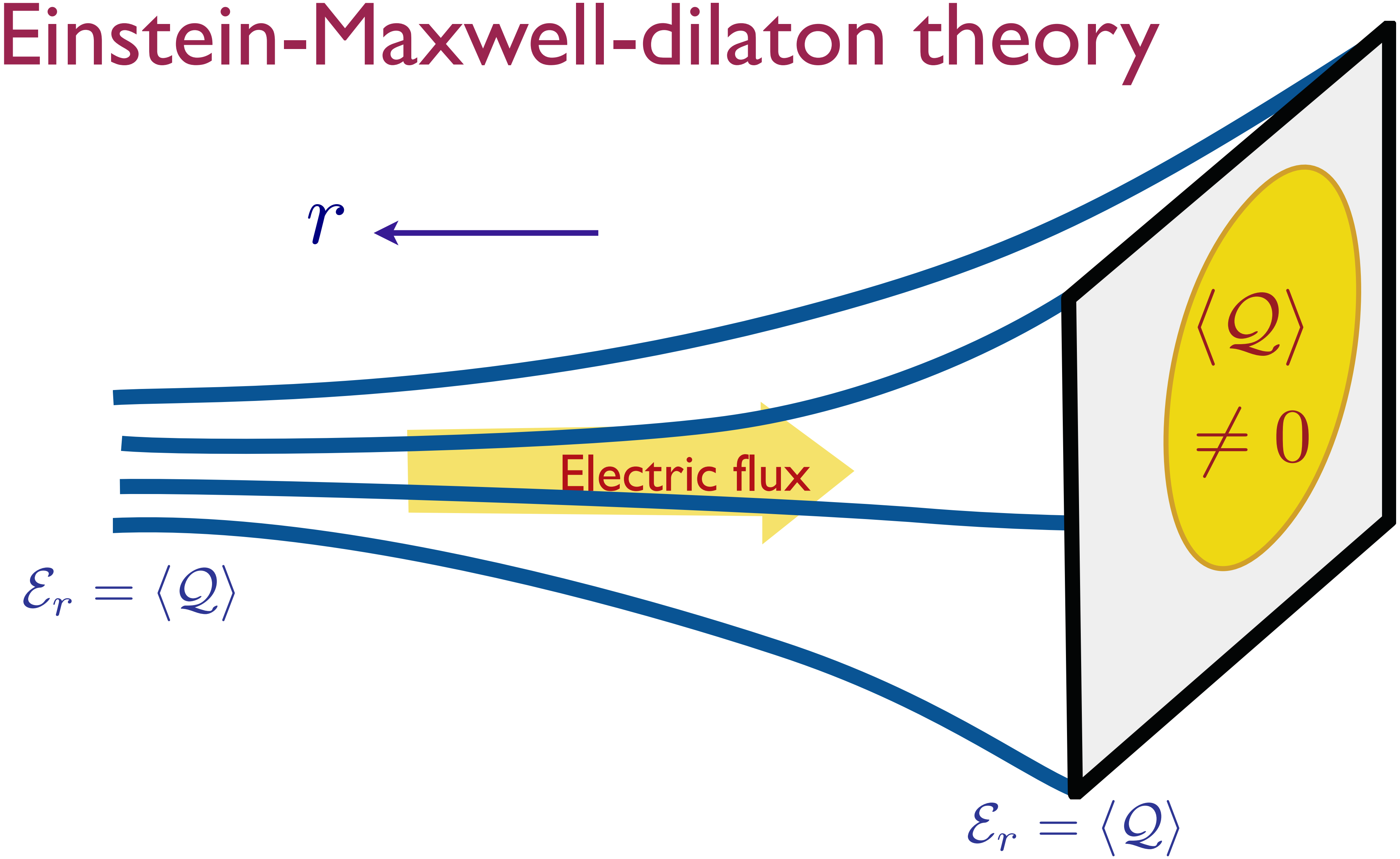
At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\mathrm{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Holography of a non-Fermi liquid

Einstein-Maxwell-dilaton theory



$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab} F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The $r \to \infty$ limit of the metric of the Einstein-Maxwell-dilaton (EMD) theory has the most general form with

$$\begin{aligned}
\theta &= \frac{d^2\beta}{\alpha + (d-1)\beta} \\
z &= 1 + \frac{\theta}{d} + \frac{8(d(d-\theta)+\theta)^2}{d^2(d-\theta)\alpha^2}.
\end{aligned}$$

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

Computation of the entanglement entropy in the EMD theory via the Ryu-Takayanagi formula for $\theta = d - 1$ yields

$$\mathcal{S}_E = \mathcal{C}_E \mathcal{Q}^{(d-1)/d} P \ln P$$

where $\mathcal{C}_E$ is independent of UV details.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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This is precisely as expected for a Fermi surface, when combined with the Luttinger theorem!

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of a non-Fermi liquid

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To relax momentum, add a 'random-field' coupling to the field operator $\mathcal{O}$:

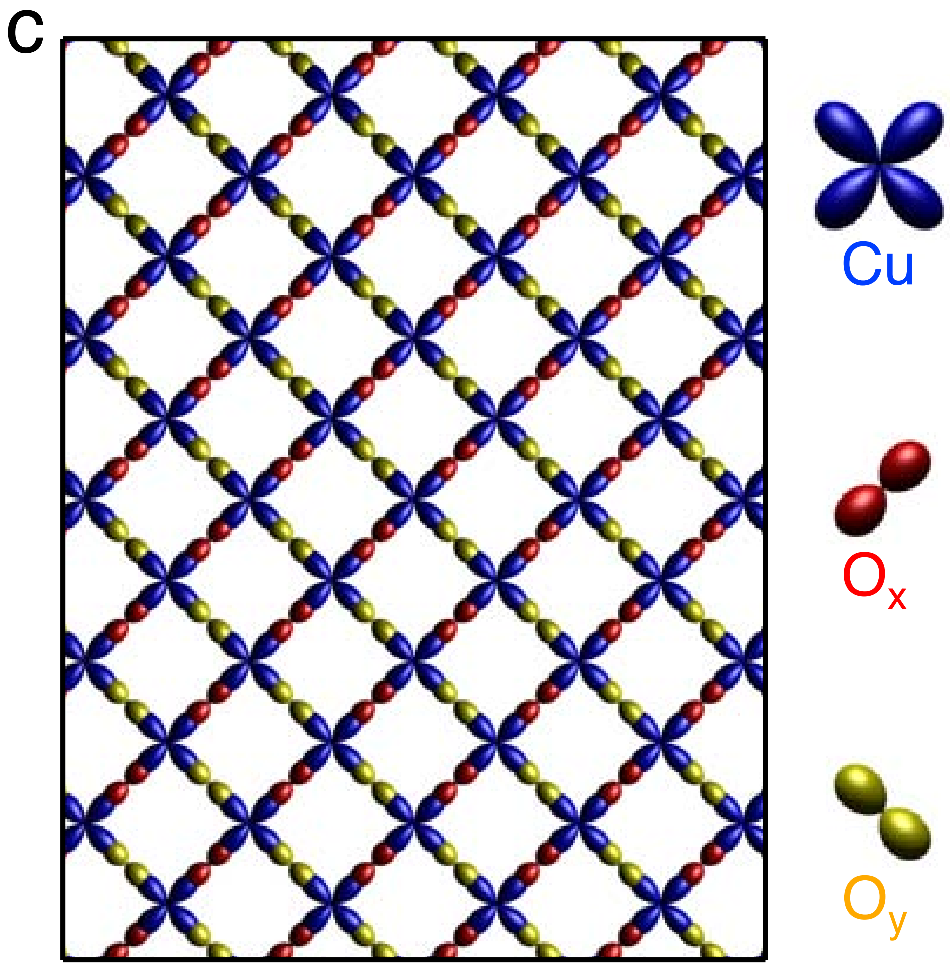
$$\mathcal{S} \to \mathcal{S} + \int d^d r d\tau h(r)\mathcal{O}(r,\tau) \quad \text{with } \overline{h(r)} = 0 \text{ and } \overline{h(r)h(r')} = h_0^2 \delta^d(r - r')$$

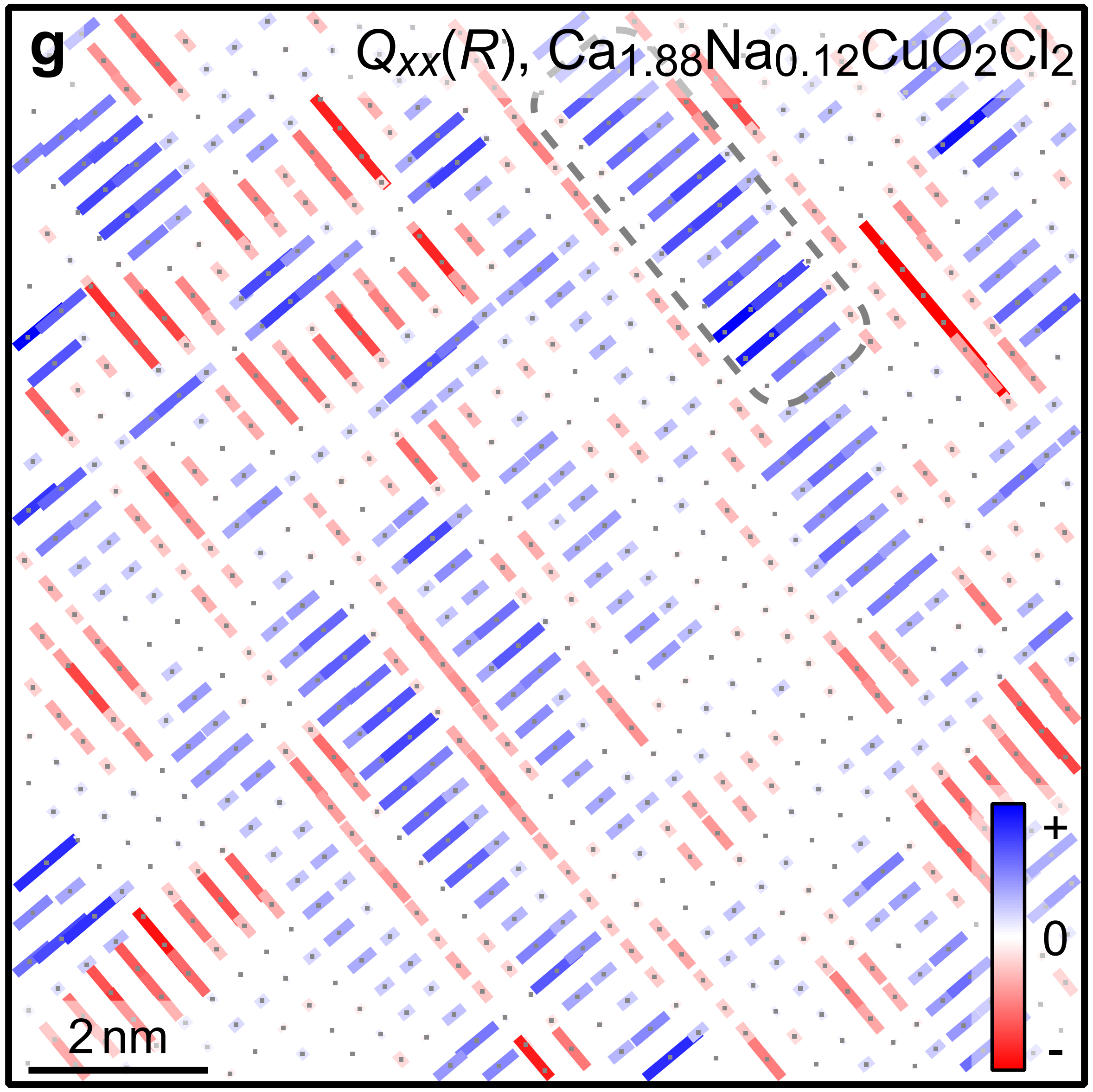
S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`
A.Lucas, S. Sachdev, and K. Schalm `arXiv:1401.7933`

Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi

Nature Physics, **8**, 534 (2012).





Evidence for "nematic" order
(*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

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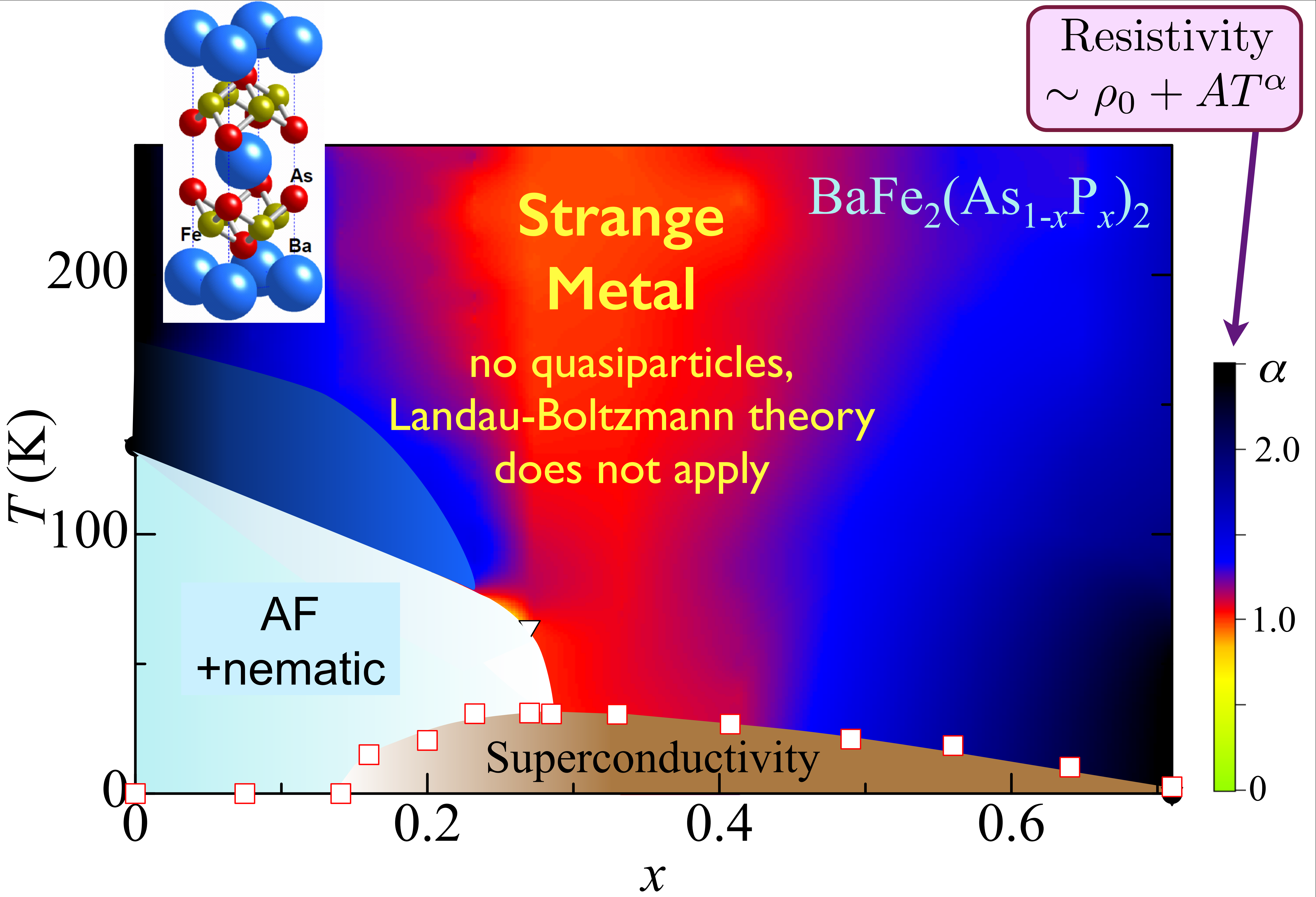
Solution of Einstein-Maxwell equations for small h_0 yields the resistivity

$$\rho(T) \sim h_0^2\, T^{(d-z+\eta)/z}\,,$$

where $\dim[\mathcal{O}] = (d+z-2+\eta)/2$. This agrees with the *memory function* computation of the bosonic contribution of the "standard model" field theory. The crossover at higher energies to the Wilson-Fisher CFT (with $z=1$, $\eta \approx 0$) yields $\rho(T) \sim T$.

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`
A.Lucas, S. Sachdev, and K. Schalm `arXiv:1401.7933`



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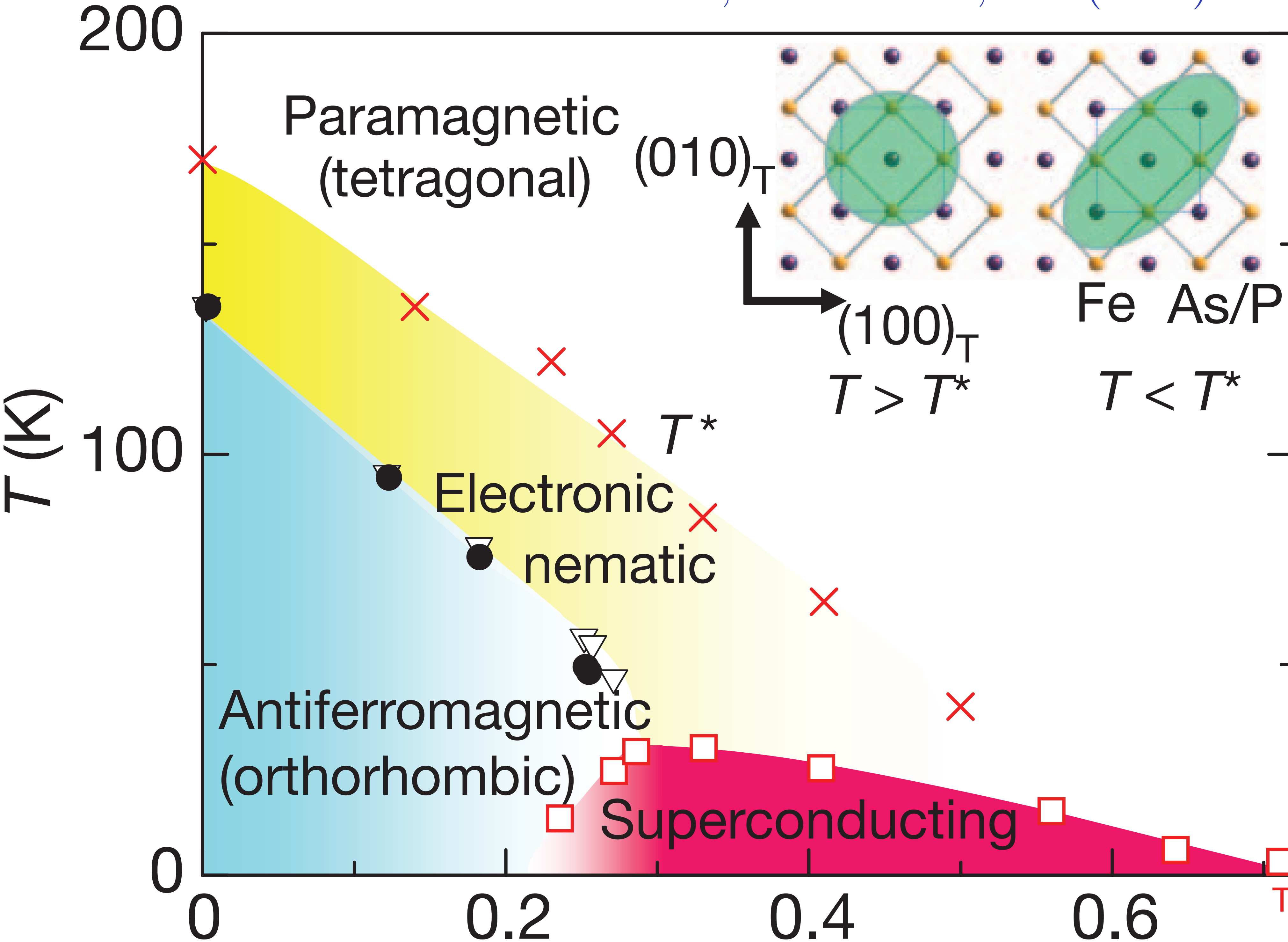

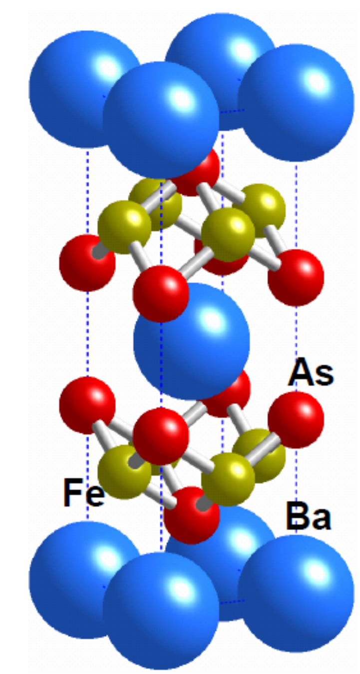

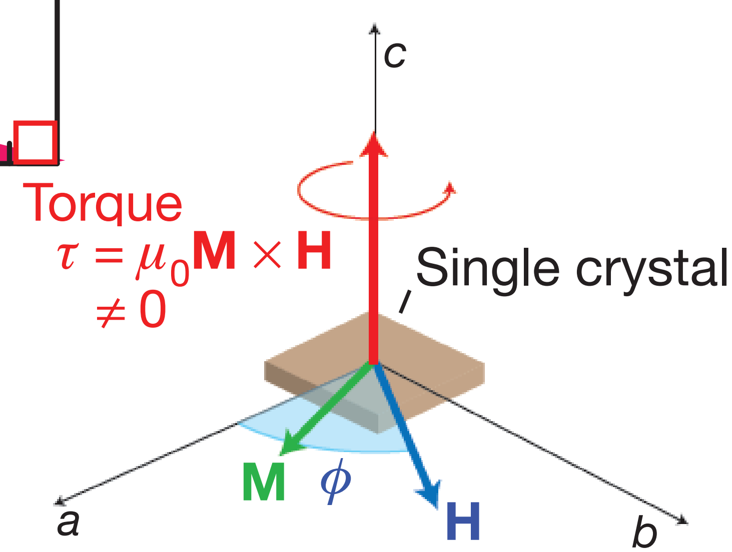

$\mathrm{BaFe_2(As_{1-x}P_x)_2}$

- Strongly-coupled quantum criticality leads to a novel regime of quantum dynamics without quasiparticles.

- The simplest examples are conformal field theories in 2+1 dimensions, realized by ultracold atoms in optical lattices.

- Holographic theories provide an excellent quantitative description of quantum Monte Carlo studies of quantum-critical boson models

- Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography