Competing orders in the cuprate superconductors

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Subir Sachdev
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Ying Zhang

Talk online at
http://pantheon.yale.edu/~subir

Superconductivity in a doped Mott insulator

Review: S. Sachdev, *Science* **286**, 2479 (1999).

Hypothesis: cuprate superconductors are characterized by additional order parameters which compete with the order of BCS theory. These orders lead to new low energy excitations.

Will present experimental and theoretical support for order parameters linked to spin density waves and associated "charge" order

Describe physics by expanding away from quantum critical points separating phases associated with these competing order parameters: this provides a systematic and controlled theory of the low energy excitations (including their behavior near imperfections such as impurities and vortices and their response to applied fields) and of crossovers into "incoherent" regimes at finite temperature.

Outline

I. **Spin density waves (SDW) in LSCO**
   Tuning order and transitions by a magnetic field.

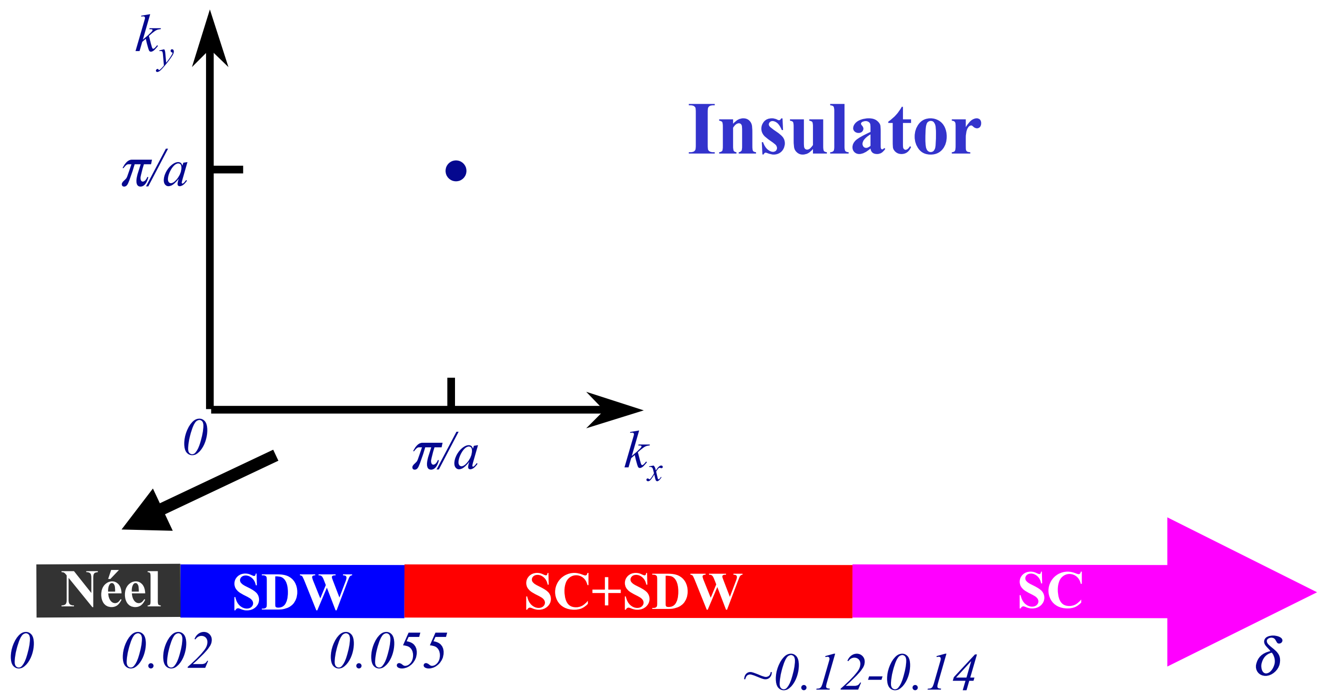
II. Connection with charge order – phenomenological theory
   STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

III. Connection with charge order – microscopic theory
   Global phase diagram: Connection to theory of magnetic ordering transitions in Mott insulators. Includes non-magnetic superconductors with bond-centered modulation of exchange and pairing energies with even periods---**bond order waves**

IV. Conclusions

I. Tuning magnetic order in LSCO by a magnetic field

T=0 phases of LSCO



(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

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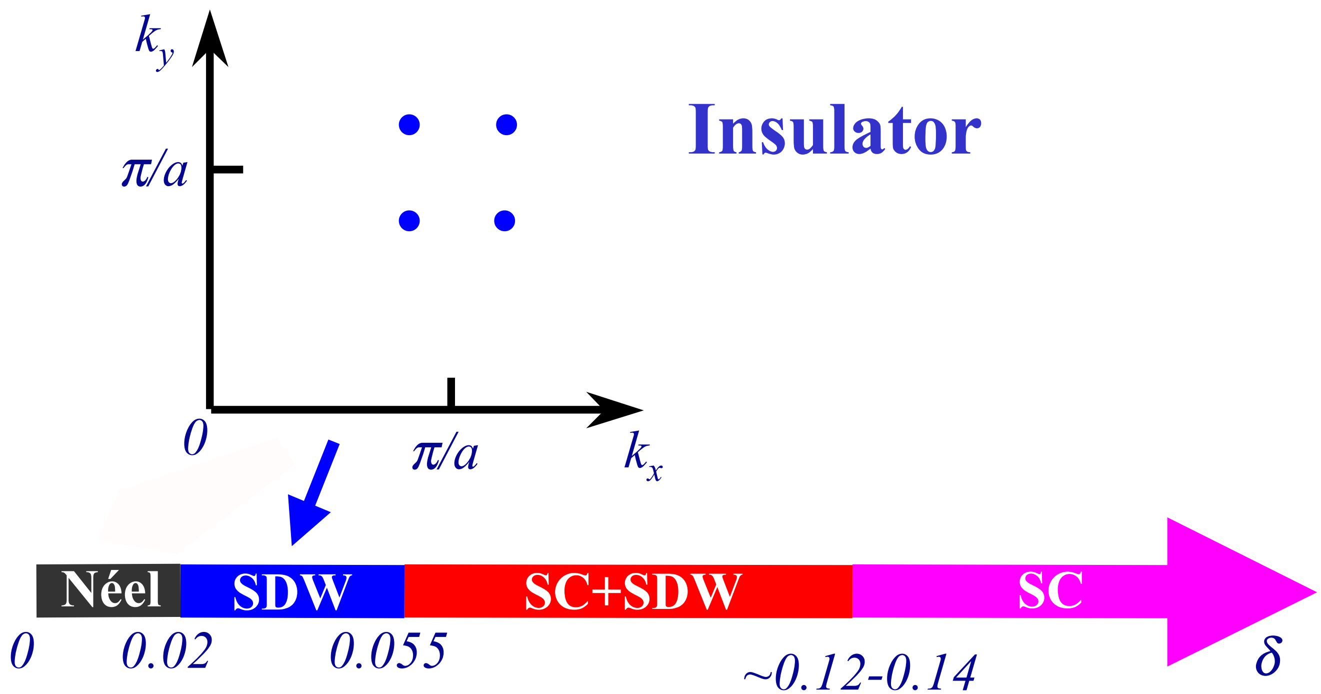

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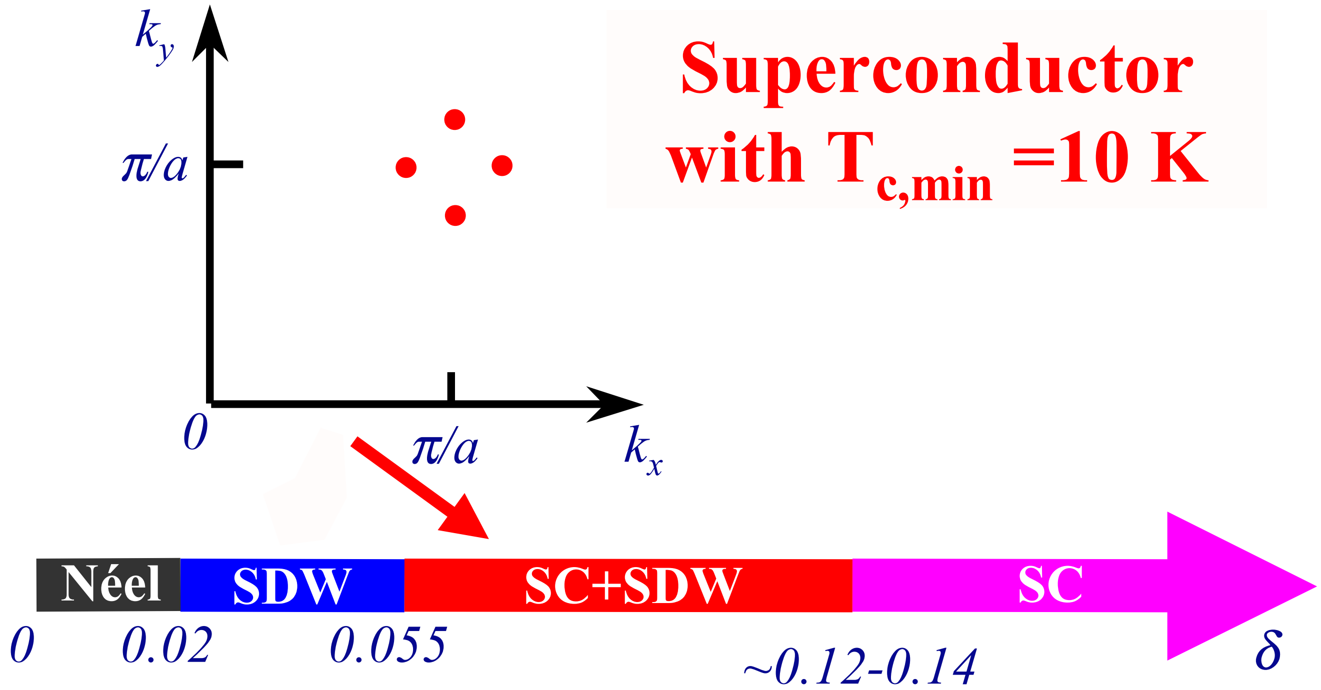

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S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

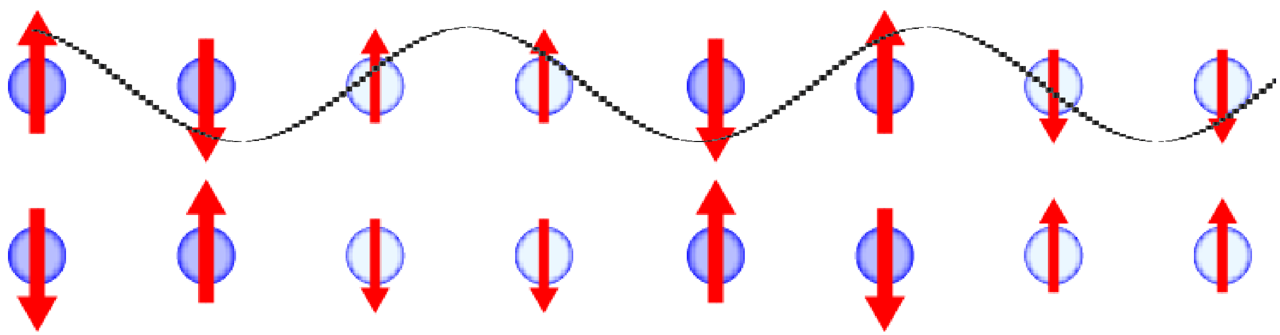
SDW order parameter for general ordering wavevector

$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

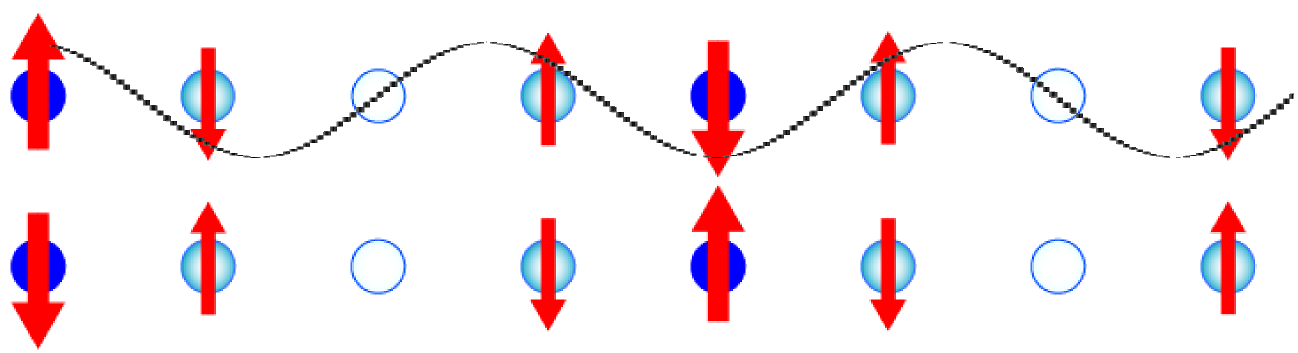
$\Phi_\alpha(\boldsymbol{r})$ is a ***complex*** field and $\boldsymbol{K}=(3\pi/4,\pi)$

$$\Phi_\alpha = e^{i\theta} n_\alpha$$

Spin density wave is ***collinear*** (and not spiral):



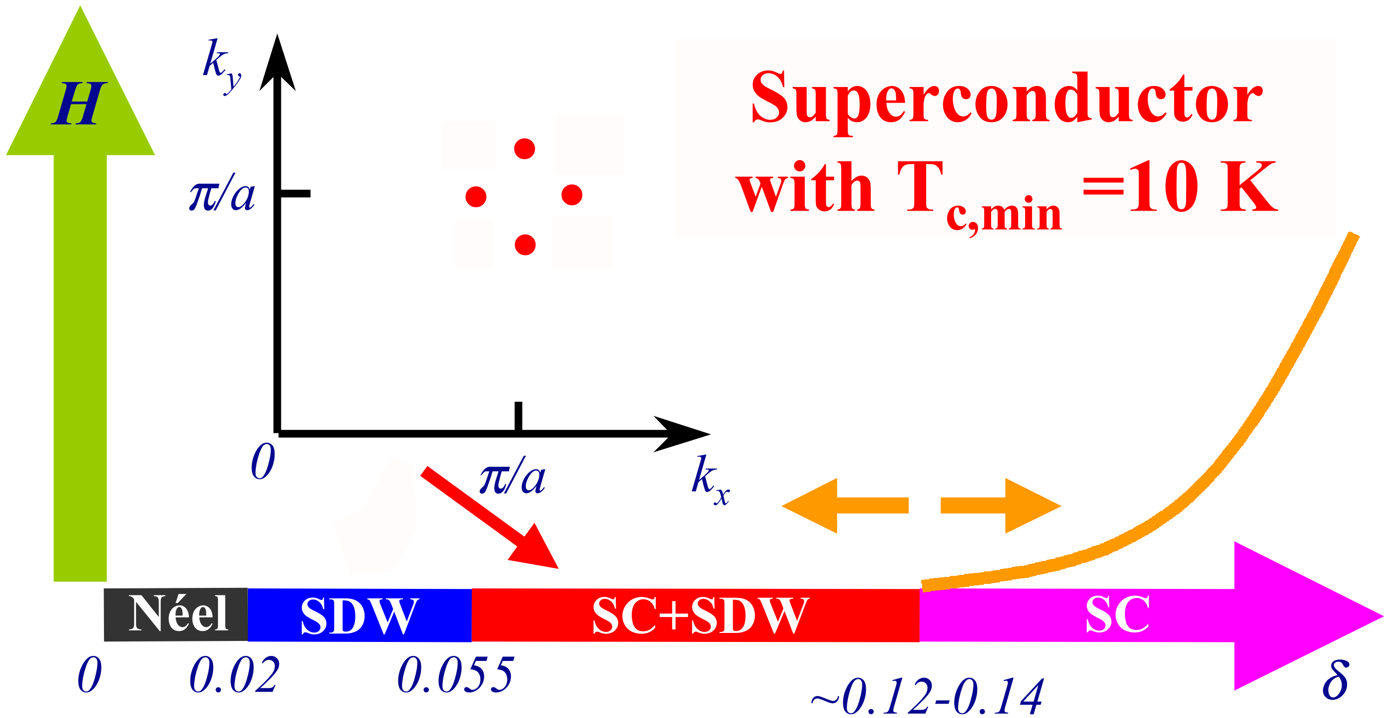
Bond-centered



Site-centered

I. Tuning magnetic order in LSCO by a magnetic field

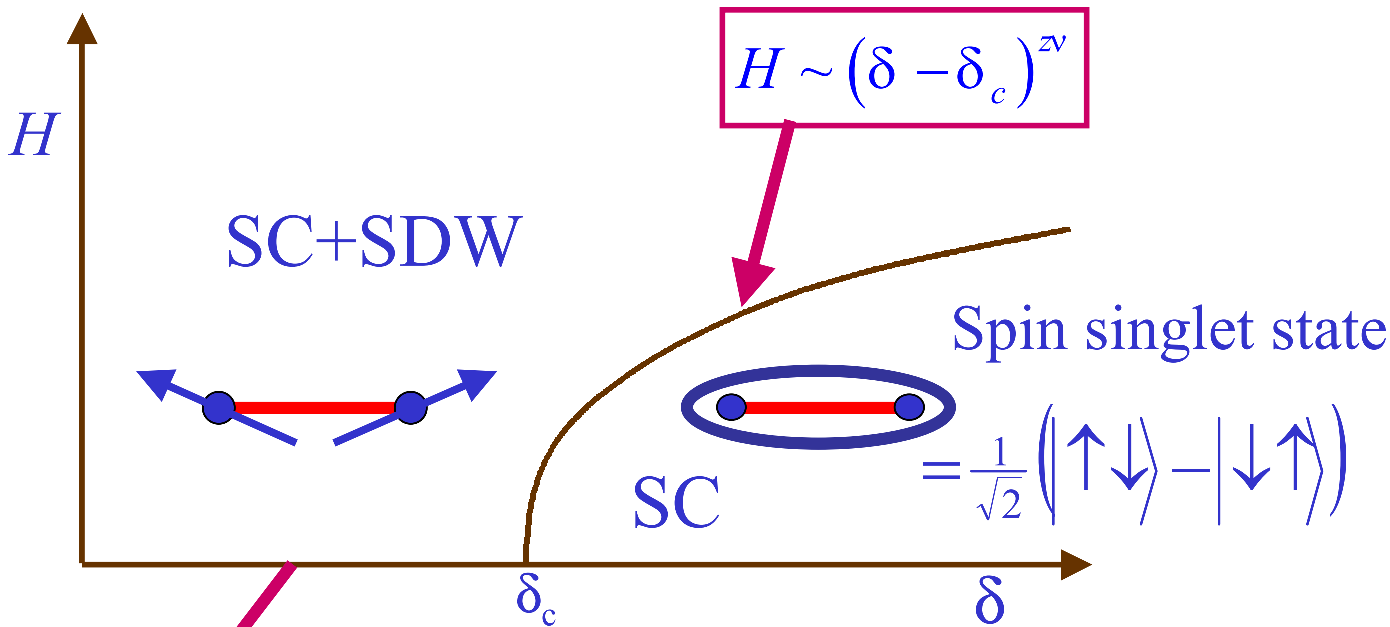
T=0 phases of LSCO



Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases: expansion away from quantum critical point provides a controlled theory for strong correlations in SC and SC+SDW phases (***even in SC phase of materials which may not have a SC+SDW phase at any*** δ)

Follow intensity of elastic Bragg spots in a magnetic field

Effect of the Zeeman term: precession of SDW order about the magnetic field



Elastic scattering intensity

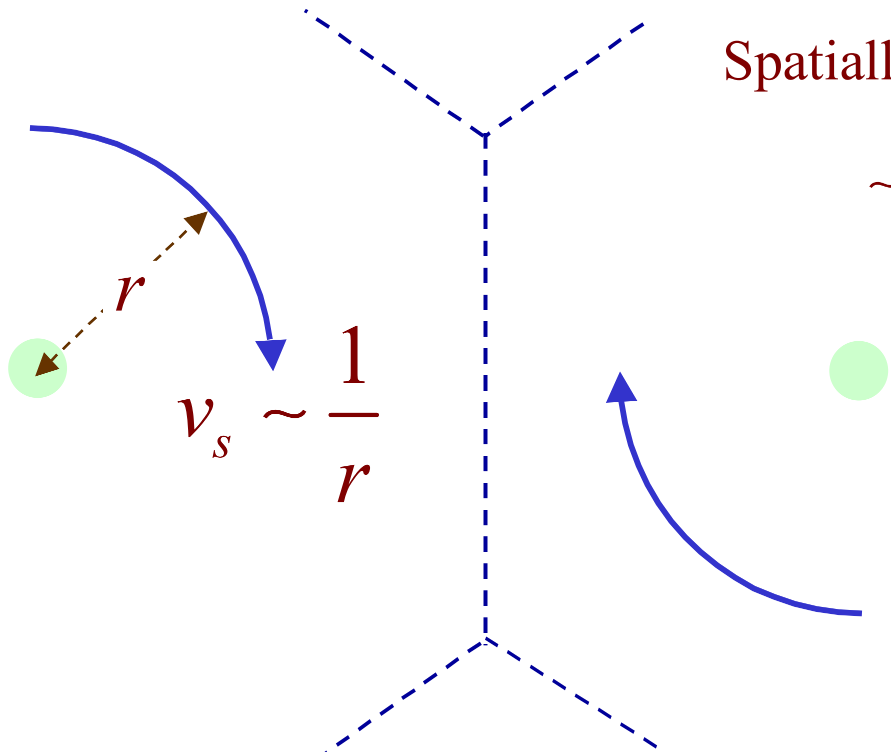
$$I(H) = I(0) + a\left(\frac{H}{J}\right)^2$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

Effect is negligible over experimental field scales

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Competing order is enhanced in a "halo" around each vortex

The suppression of SC order appears to the SDW order as an effective δ :

$$\delta_{\text{eff}}\left(H\right) = \delta - C \frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

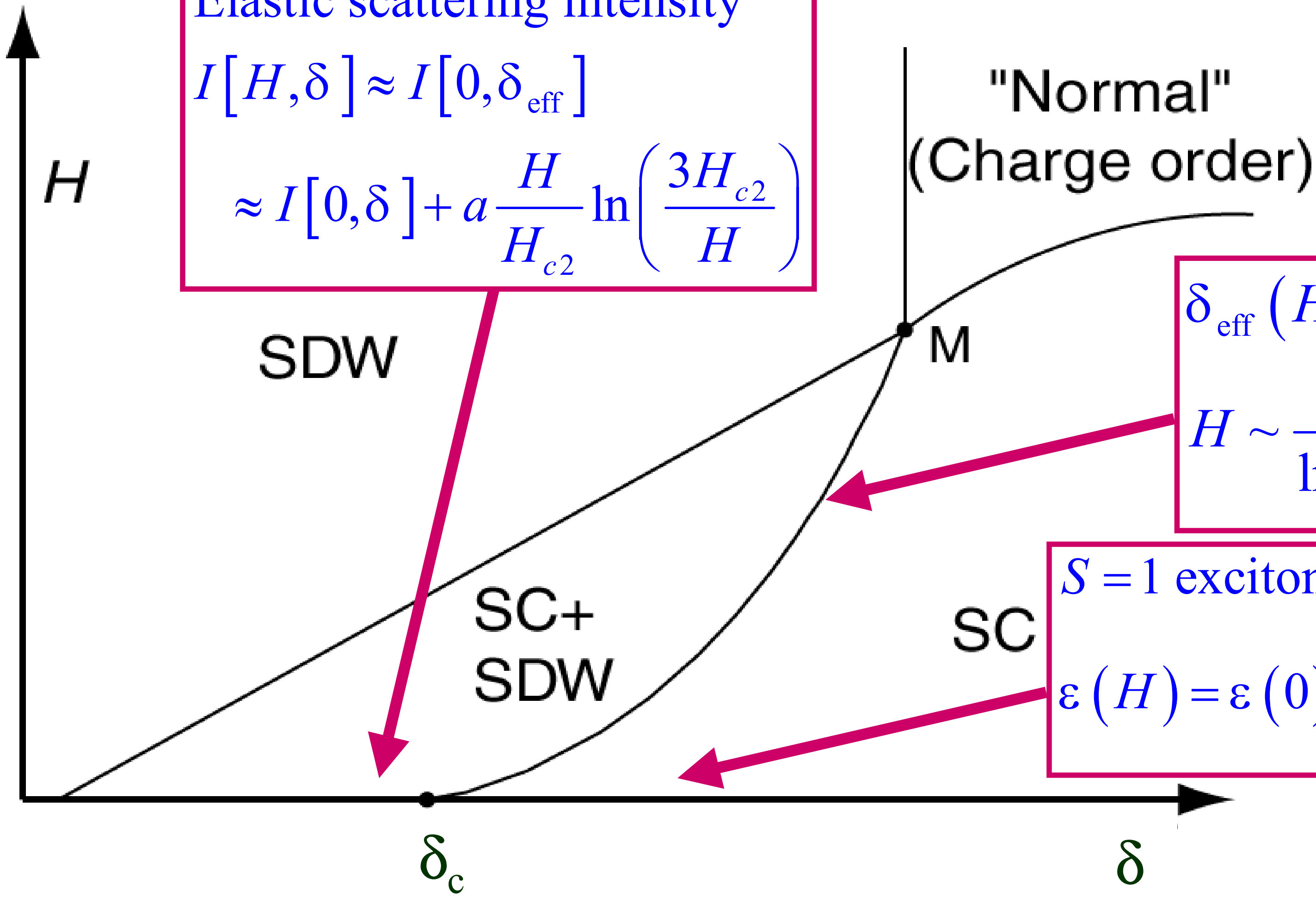
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Main results

$T=0$

Elastic scattering intensity

$$I[H,\delta] \approx I[0,\delta_{\text{eff}}]$$
$$\approx I[0,\delta] + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$



$$\delta_{\text{eff}}(H) = \delta_c \quad \Rightarrow$$
$$H \sim \frac{(\delta - \delta_c)}{\ln\left(1/(\delta - \delta_c)\right)}$$

$S = 1$ exciton energy

$$\varepsilon(H) = \varepsilon(0) - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Lowering of characteristic energy of dynamic spin fluctuations was measured earlier in LSCO by B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder, *Science* **291**, 1759 (2001).

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

Quantum theory for dynamic and critical spin fluctuations

$$\mathcal{S}_b = \int d^2r \int_0^{1/T} d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + s\left|\Phi_\alpha\right|^2 + \frac{g_1}{2}\left(\left|\Phi_\alpha\right|^2\right)^2 + \frac{g_2}{2}\left|\Phi_\alpha^2\right|^2 \right]$$

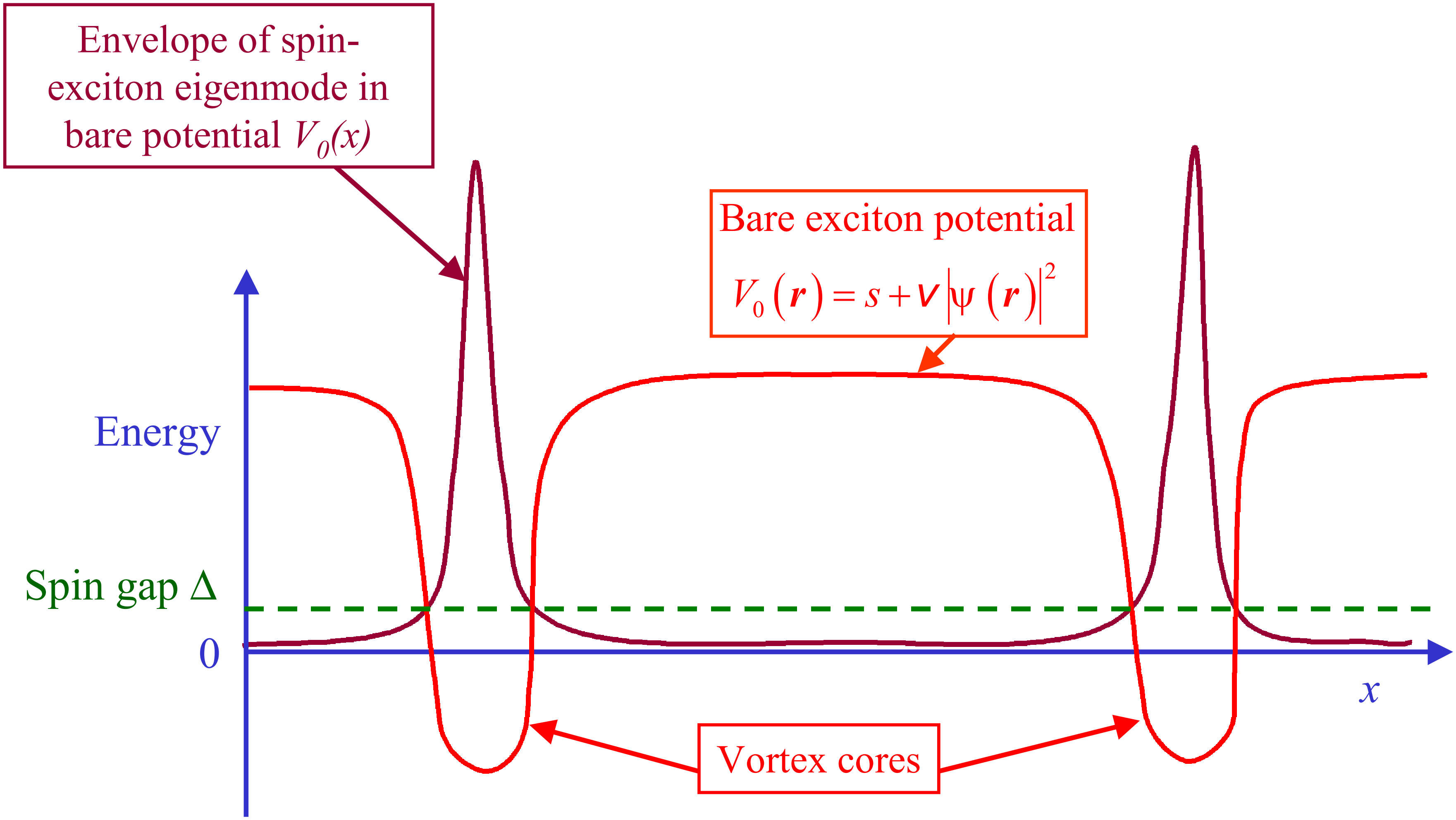
$$\mathcal{S}_c = \int d^2r d\tau \left[\frac{v}{2} \left|\Phi_\alpha\right|^2 \left|\psi\right|^2 \right]$$

$$Z\left[\psi\left(r\right)\right] = \int D\Phi\left(r,\tau\right) e^{-F_{GL} - \mathcal{S}_b - \mathcal{S}_c}$$

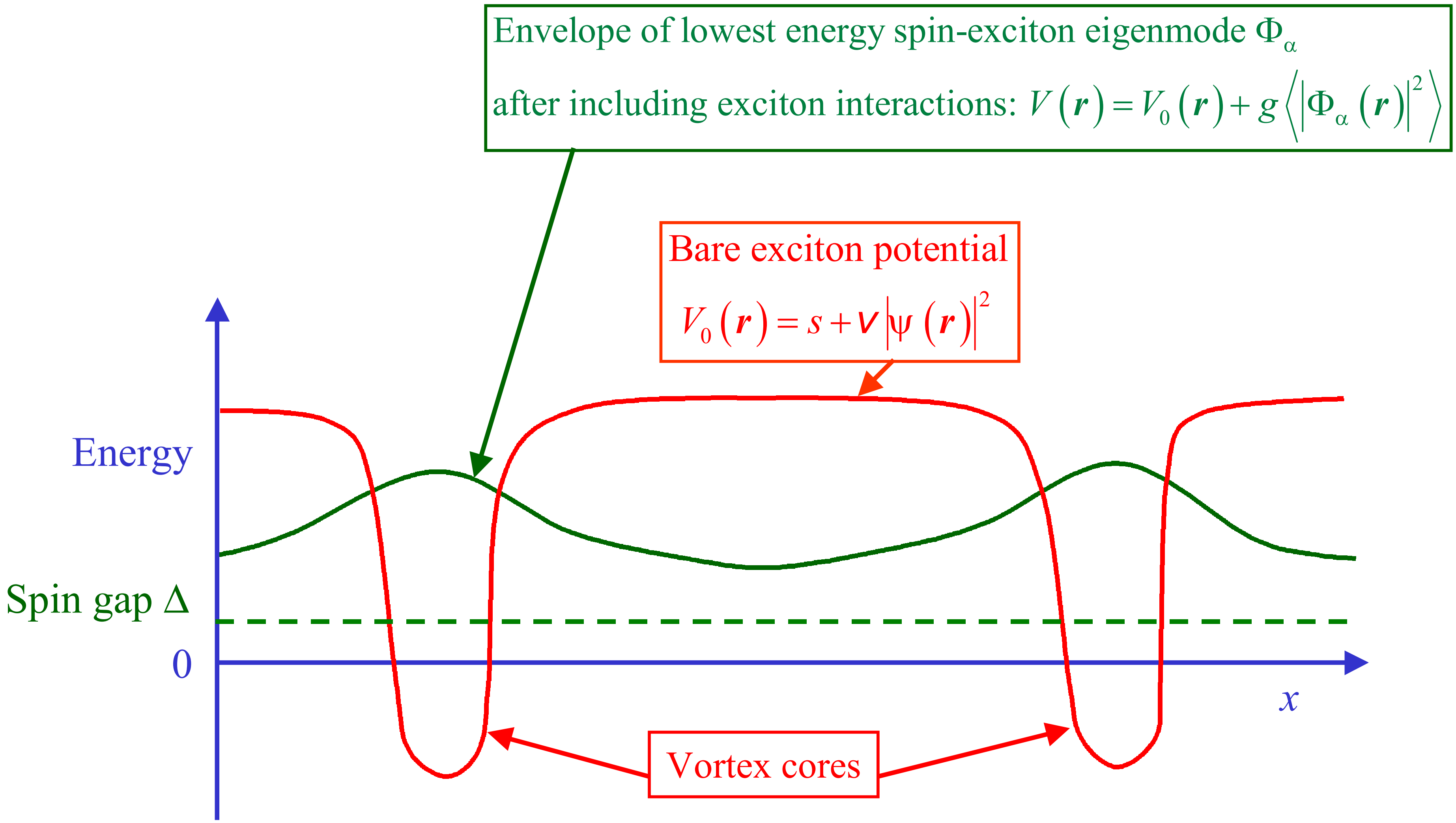
$$\frac{\delta \ln Z\left[\psi\left(r\right)\right]}{\delta\psi\left(r\right)} = 0$$

$$F_{GL} = \int d^2r \left[-\left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2} + \left|\left(\nabla_r - iA\right)\psi\right|^2 \right]$$

Static Ginzburg-Landau theory for non-critical superconductivity



D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) proposed **static** magnetism (with Δ=0) localized within vortex cores (Talk 23aC1)



Strongly relevant repulsive interactions between excitons imply that excitons must be extended as $\Delta \to 0$.

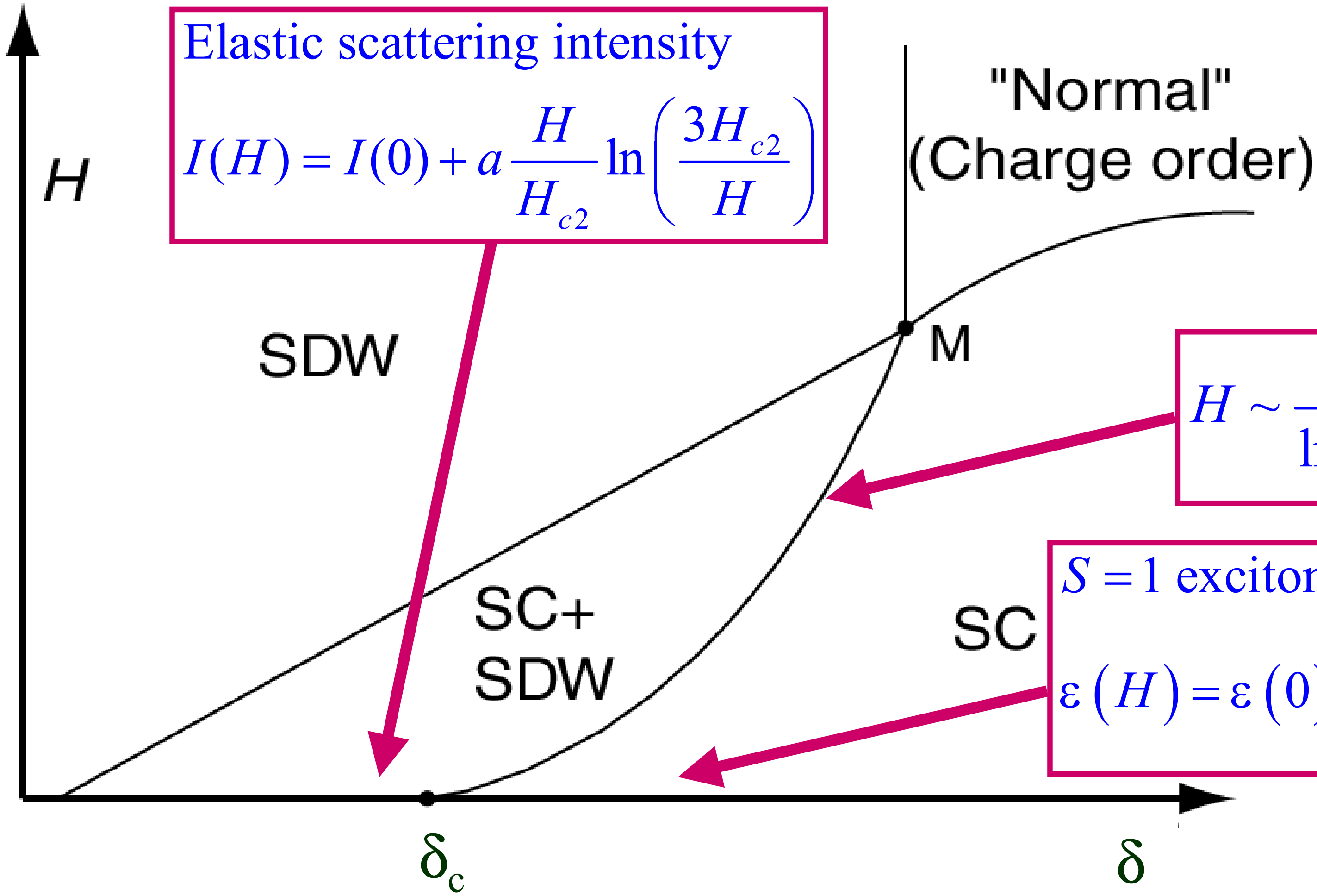
E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett*. **87**, 067202 (2001).

A.J. Bray and M.A. Moore, *J. Phys*. C **15**, L7 65 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, *Phys. Rev. Lett*. **43**, 942 (1979).

Main results

T=0



Elastic scattering intensity

$$I(H) = I(0) + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$H \sim \frac{(\delta - \delta_c)}{\ln\left(1/(\delta - \delta_c)\right)}$$

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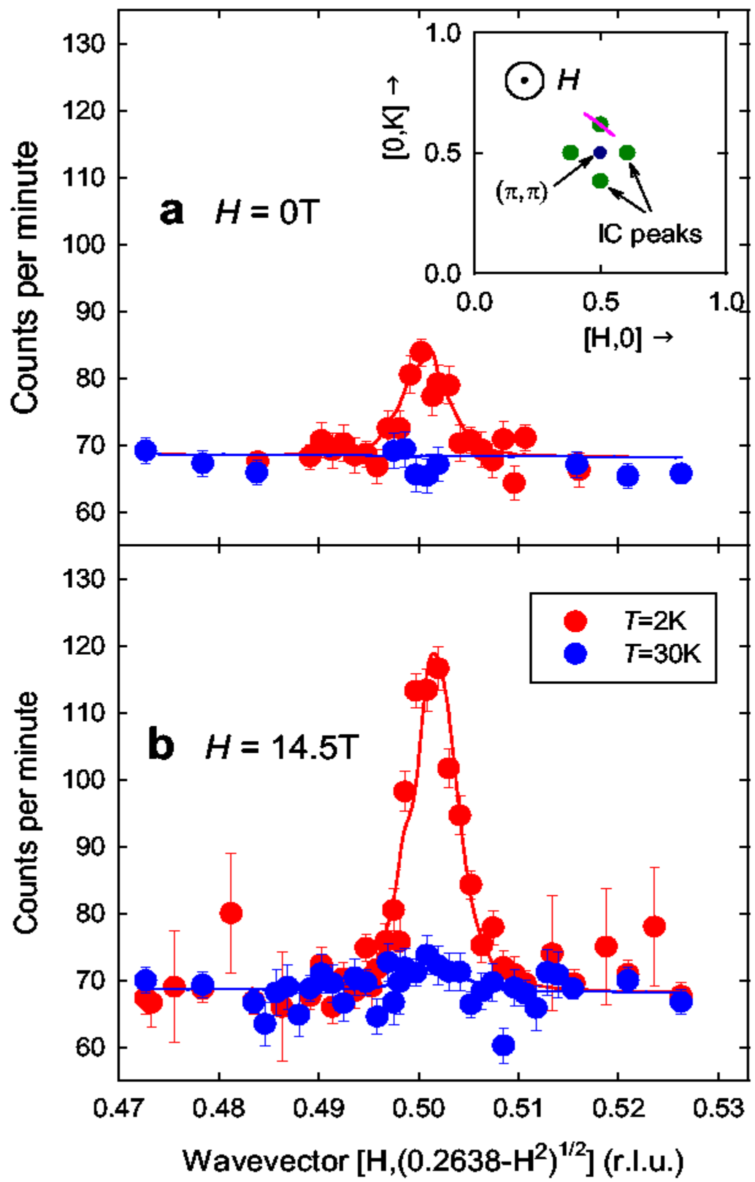
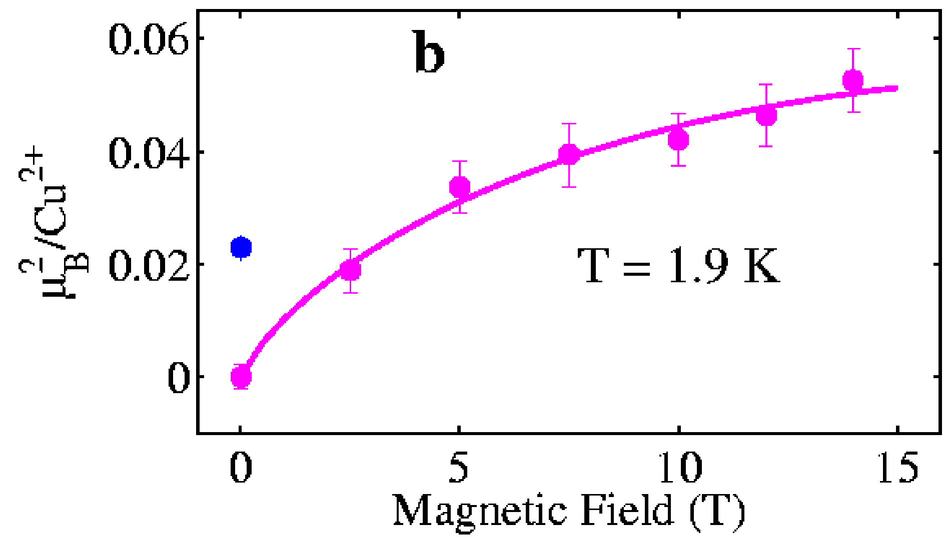
Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



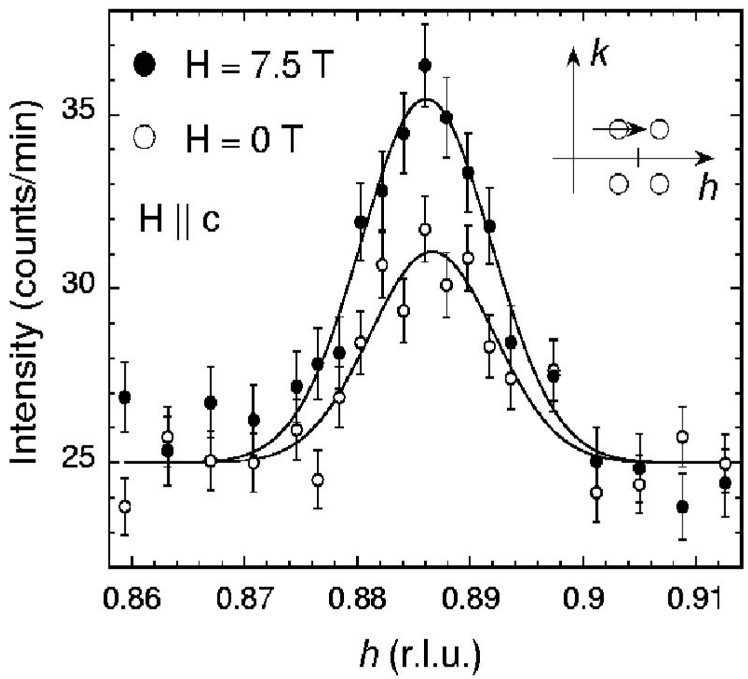
Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

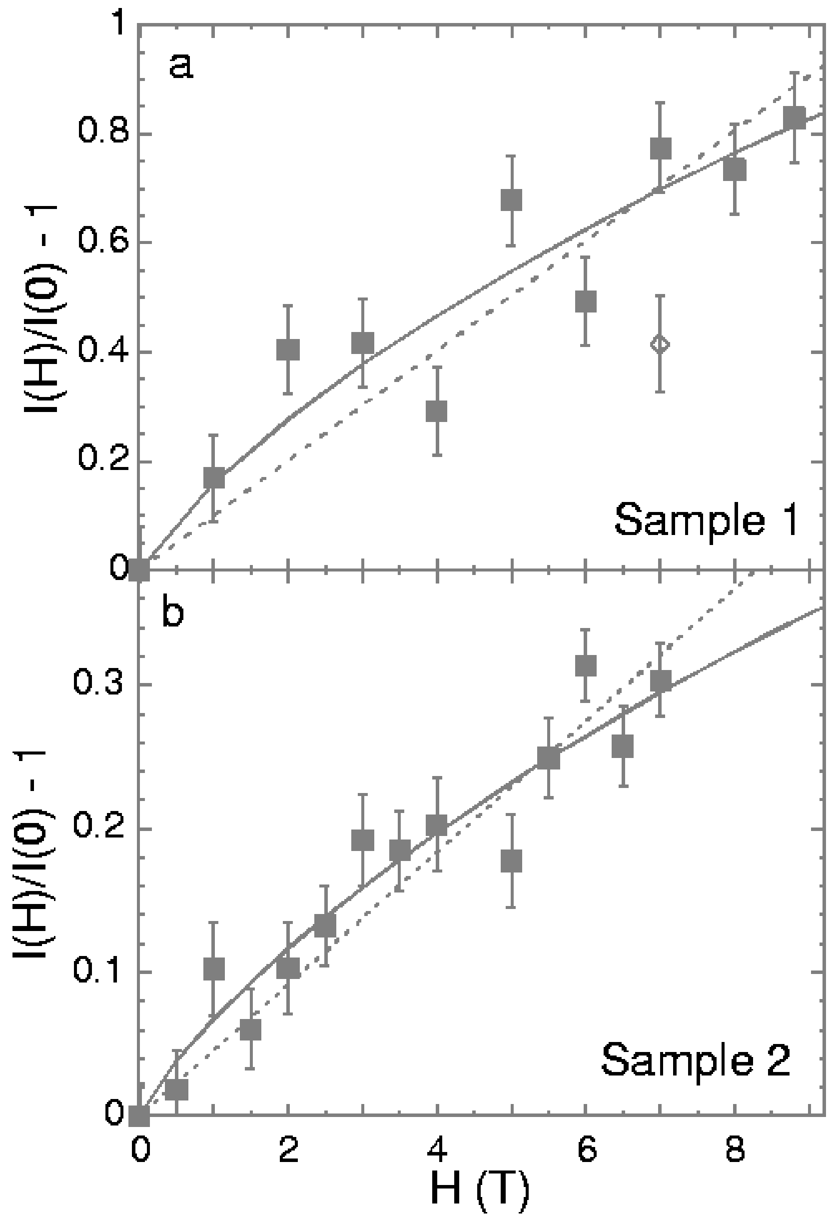
See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev.* B **62**, R14677 (2000).

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, *Phys. Rev.* B **66**, 014528 (2002).





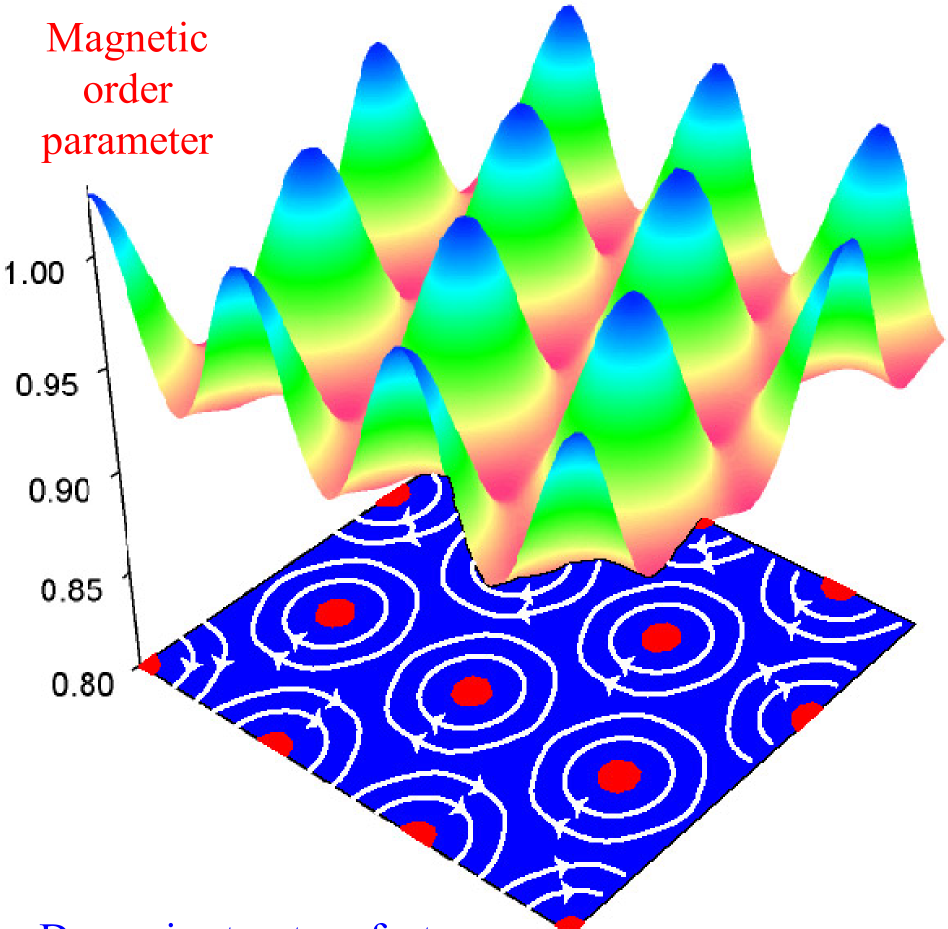
Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

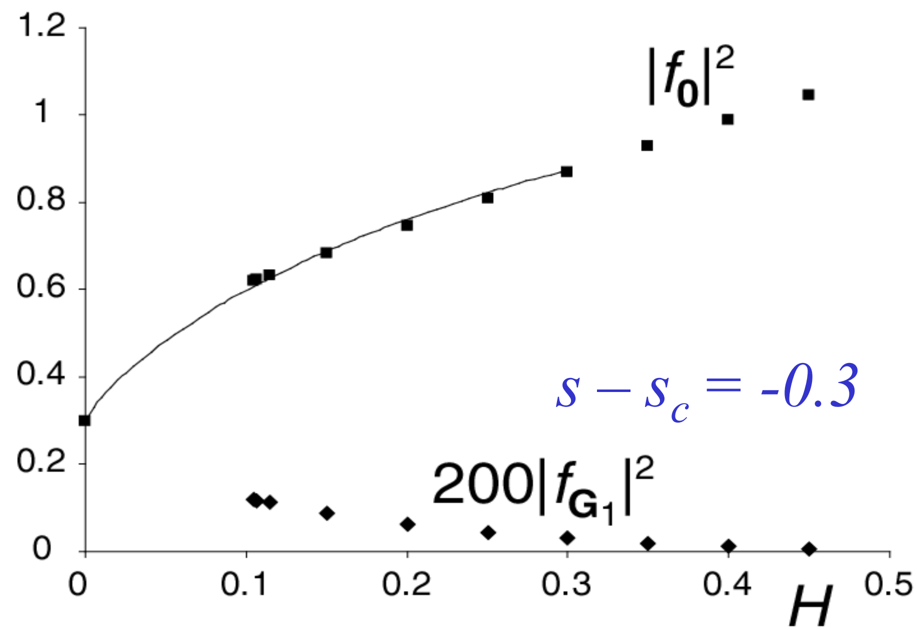
Best fit value - a = 2.4 with H_{c2} = 60 T

Structure of *long-range* SDW order in SC+SDW phase

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



$$\delta \left| f_0 \right|^2 \propto H \ln(1/H)$$



Dynamic structure factor

$$S(\boldsymbol{k},\omega) = (2\pi)^3 \delta(\omega) \sum_{\boldsymbol{G}} \left| f_{\boldsymbol{G}} \right|^2 \delta(\boldsymbol{k}-\boldsymbol{G}) + \cdots$$

$\boldsymbol{G} \rightarrow$ reciprocal lattice vectors of vortex lattice.

$\boldsymbol{k}$ measures deviation from SDW ordering wavevector $\boldsymbol{K}$

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II. Connections with charge order – phenomenological theory

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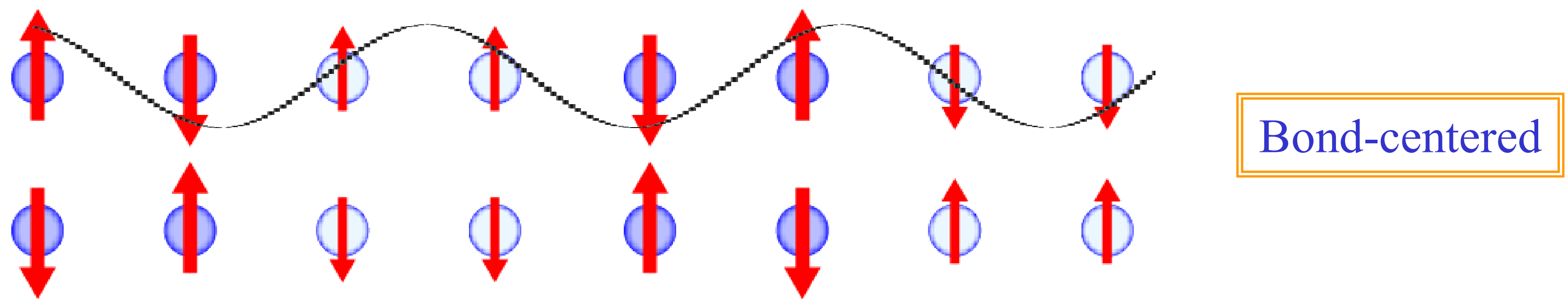
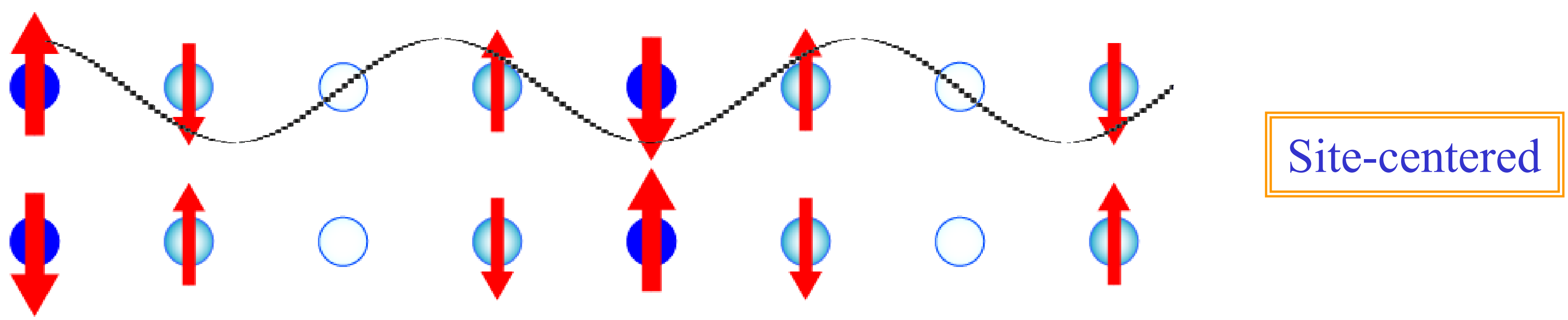
Spin density wave is ***collinear*** (and not spiral):





A collinear spin density wave necessarily has an accompanying modulation in the site charge densities, exchange and pairing energy per link etc. at half the wavelength of the SDW

"Charge" order: periodic modulation in local observables invariant under spin rotations and time-reversal.

$$\text{Order parmeter} \sim \sum_{\alpha} \Phi_{\alpha}^{2}\left(\boldsymbol{r}\right)$$

$$\delta\rho\left(\boldsymbol{r}\right) \propto S_{\alpha}^{2}\left(\boldsymbol{r}\right) = \sum_{\alpha} \Phi_{\alpha}^{2}\left(\boldsymbol{r}\right) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).
J. Zaanen and O. Gunnarsson, *Phys. Rev.* B **40**, 7391 (1989).
H. Schulz, *J. de Physique* **50**, 2833 (1989).
K. Machida, Physica **158C**, 192 (1989).

***Prediction*: Charge order should be pinned in halo around vortex core**

K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001).
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Pinning of static charge order by vortex cores in SC phase, with dynamic SDW correlations

A.Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, cond-mat/0110329
Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343

Superflow reduces energy of dynamic spin exciton, but action so far does not lead to static charge order because all terms are invariant under the "sliding" symmetry:

$$\Phi_\alpha(\boldsymbol{r}) \to \Phi_\alpha(\boldsymbol{r}) e^{i\vartheta}$$

Small vortex cores break this sliding symmetry on the lattice scale, and lead to a pinning term, which picks particular phase of the local charge order

$$\mathcal{S}_{\text{pin}} = \zeta \sum_{\text{All } \boldsymbol{r}_v \text{ where } \psi(\boldsymbol{r}_v)=0} \int_0^{1/T} d\tau \left[\sum_\alpha \Phi_\alpha^2(\boldsymbol{r}_v) e^{i\vartheta} + \text{c.c.} \right]$$

With this term, SC phase has static charge order but dynamic SDW

i.e. **there is no static spin order**

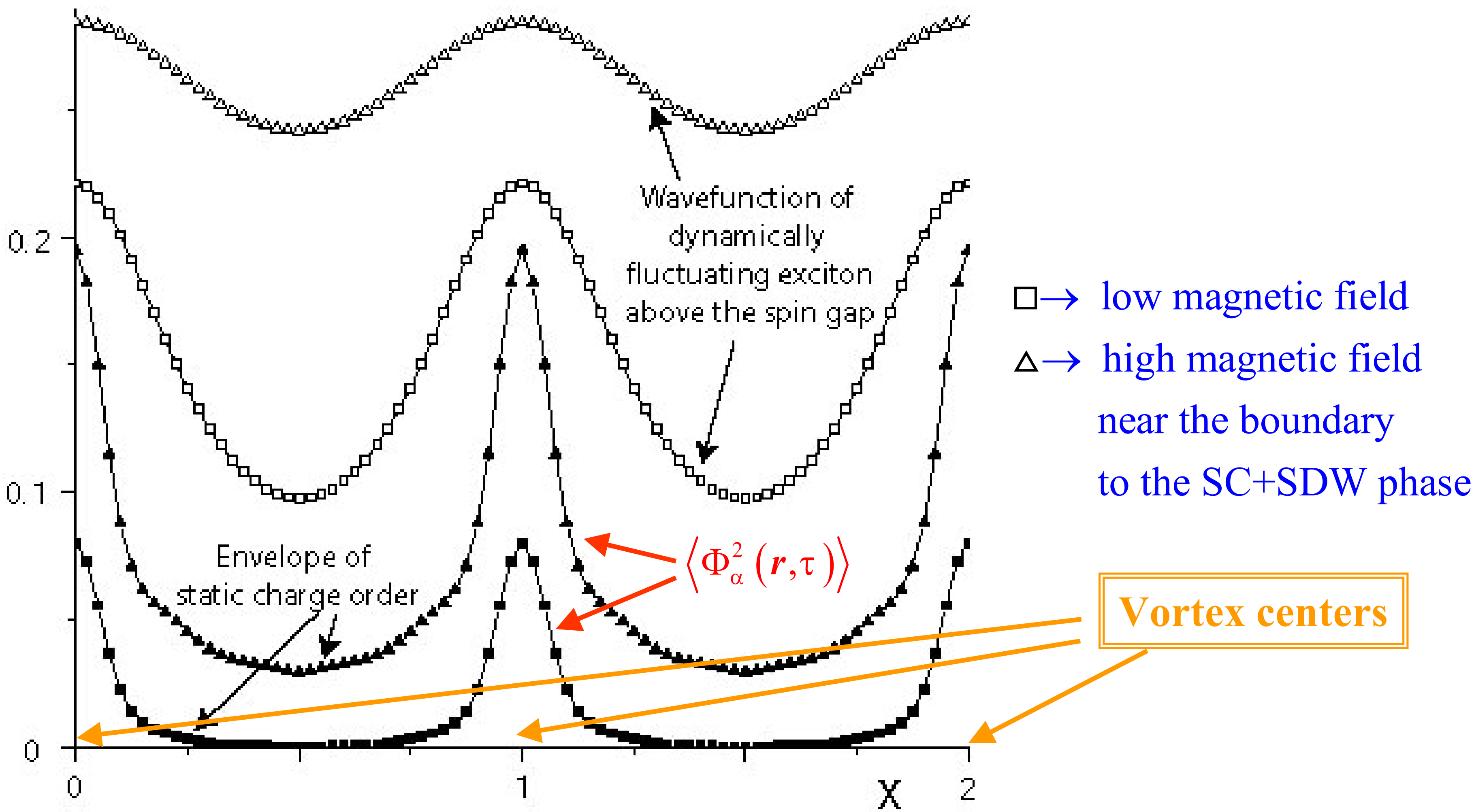
$$\left\langle \Phi_\alpha^2(\boldsymbol{r}) \right\rangle \neq 0 \quad ; \quad \left\langle \Phi_\alpha(\boldsymbol{r}) \right\rangle = 0$$

;

$$\delta\rho(\boldsymbol{r}) \propto \sum_\alpha \Phi_\alpha^2(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.} \qquad S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

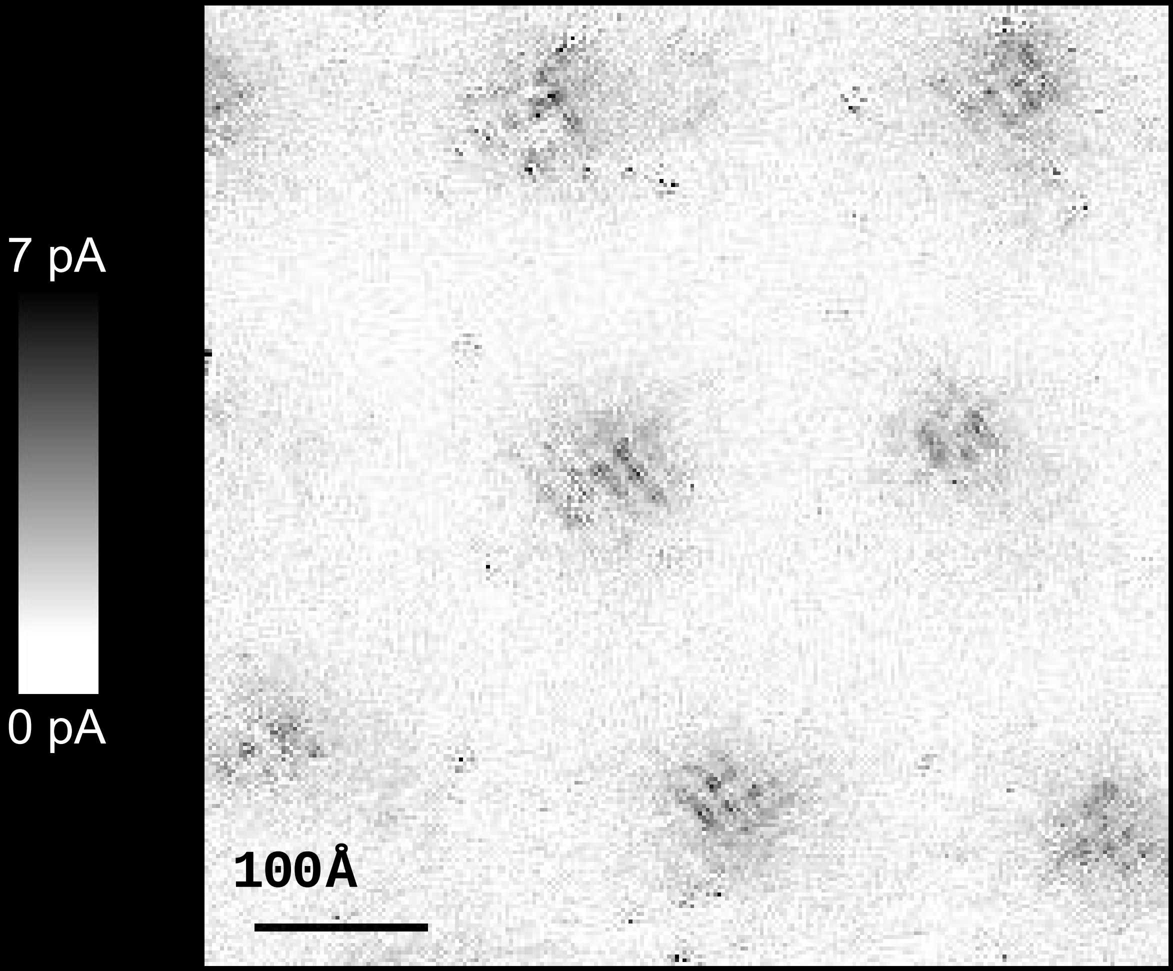
Pinning of static charge order by vortex cores in SC phase, with dynamic SDW correlations

$$\left\langle \Phi_\alpha^2(\boldsymbol{r},\tau)\right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\boldsymbol{r},\tau)\Phi_\alpha^*(\boldsymbol{r}_v,\tau_1)\right\rangle^2 \quad ; \quad \left\langle \Phi_\alpha(\boldsymbol{r},\tau)\right\rangle = 0$$



Ying Zhang, E. Demler, and S. Sachdev, *Phys. Rev.* B, Sep 1 (2002), cond-mat/0112343.

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

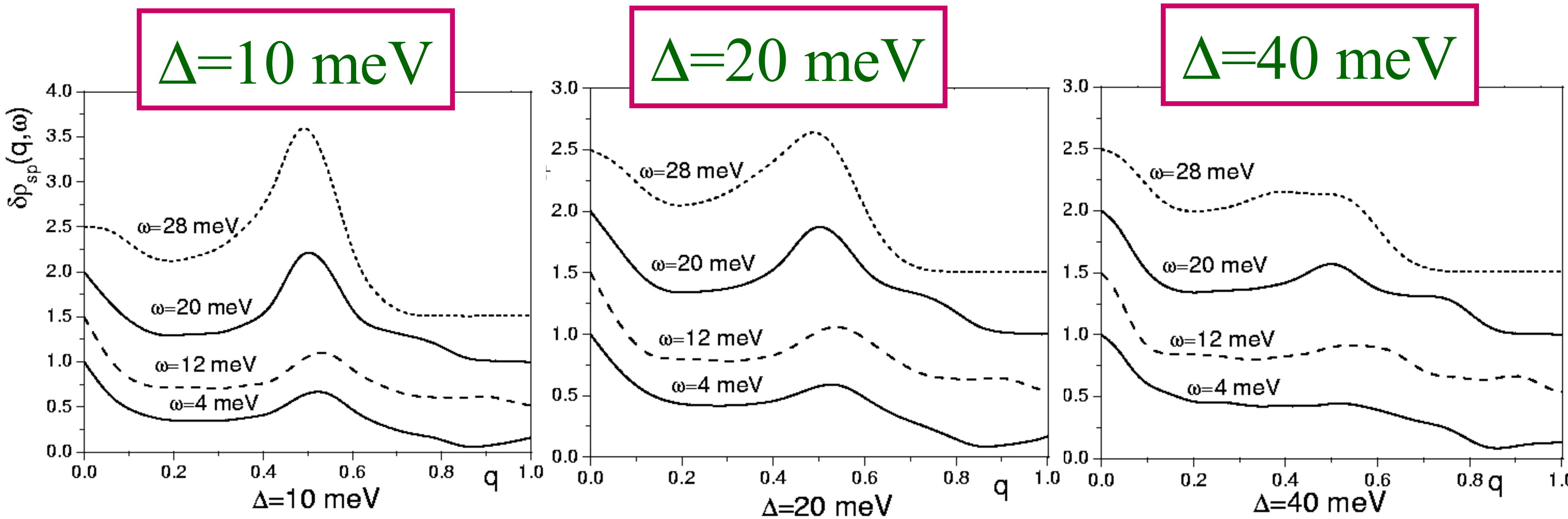
Poster 22BP66

Spectroscopy of charge order (FT-STS)

(Talk by Seamus Davis, 22V2)

A.Polkovnikov, M. Vojta, and S. Sachdev, *Phys. Rev.* B **65**, 220509 (2002); cond-mat/0208334.

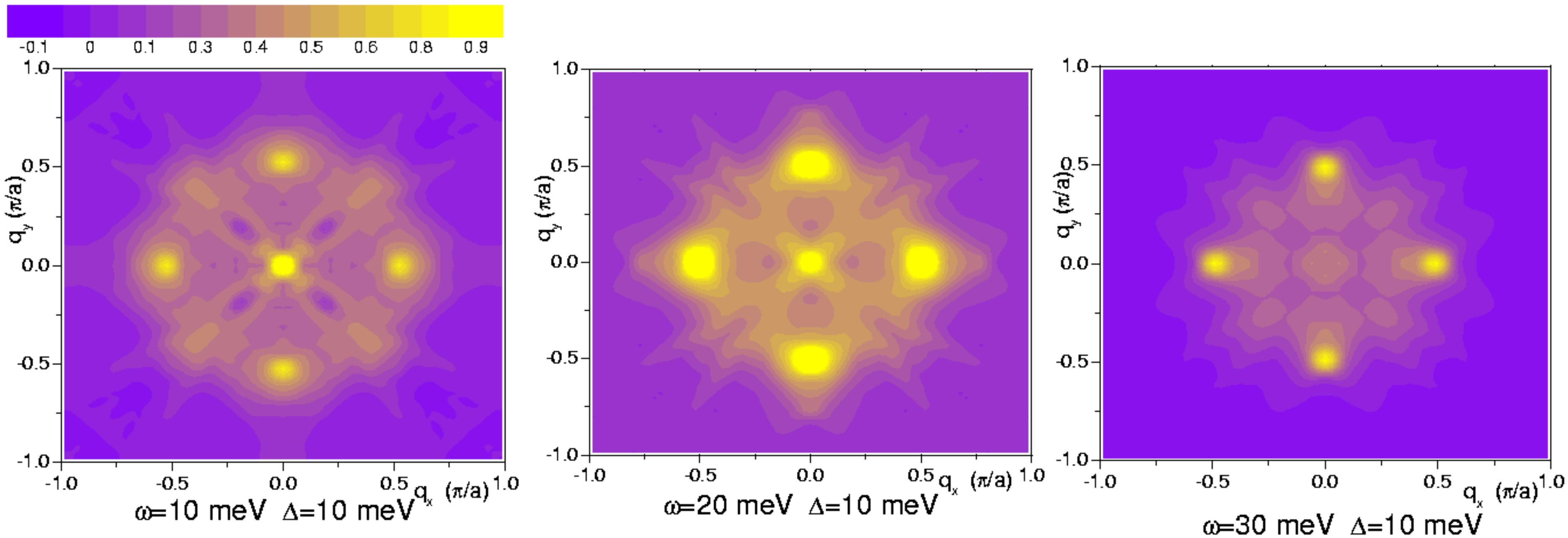
$$H = H_{BCS}\left(c_{i\sigma}\right) + \gamma \sum_i \Phi_\alpha\left(r_i\right) c_{i\sigma}^\dagger \tau_{\sigma\sigma'}^\alpha c_{i\sigma'} + h.c.$$



Spatial Fourier transform of LDOS measured at wavevector $(\pi q/a, 0)$, energy ω with spin gap Δ

Superconducting gap = 40 meV

Spin gap Δ=10 meV



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A.Polkovnikov, S. Sachdev, and M. Vojta, cond-mat/0208334

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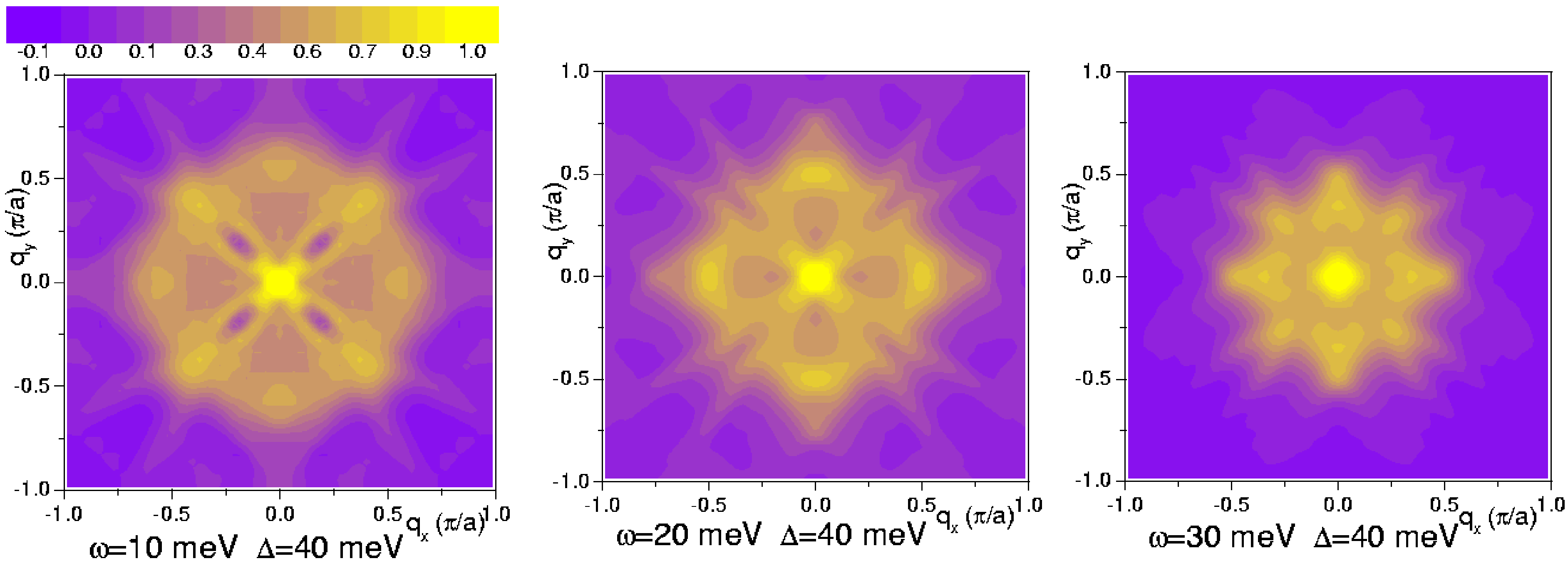

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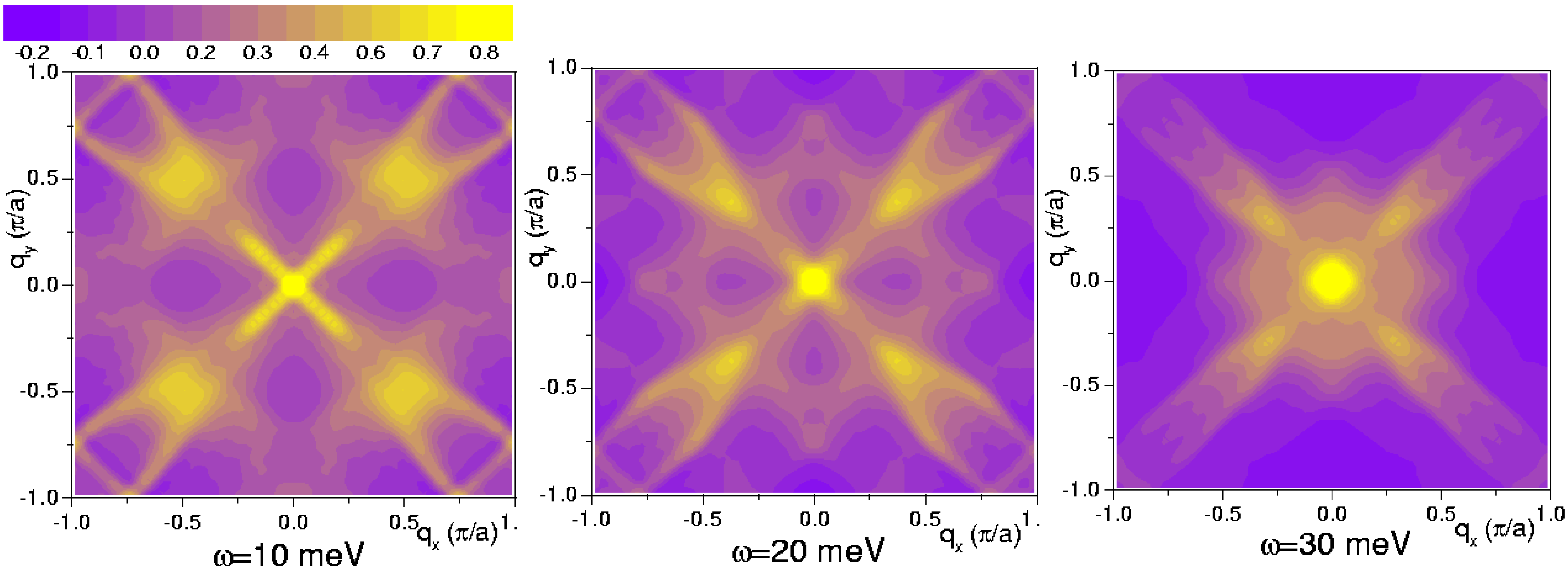
A.Polkovnikov, S. Sachdev, and M. Vojta, cond-mat/0208334

Scattering of quasiparticles off a point impurity

J.M. Byers, M.E. Flatté, and D. J. Scalapino, *Phys. Rev. Lett.* **71**, 3363 (1993).
Q.-H. Wang and D.-H. Lee, cond-mat/0205118.



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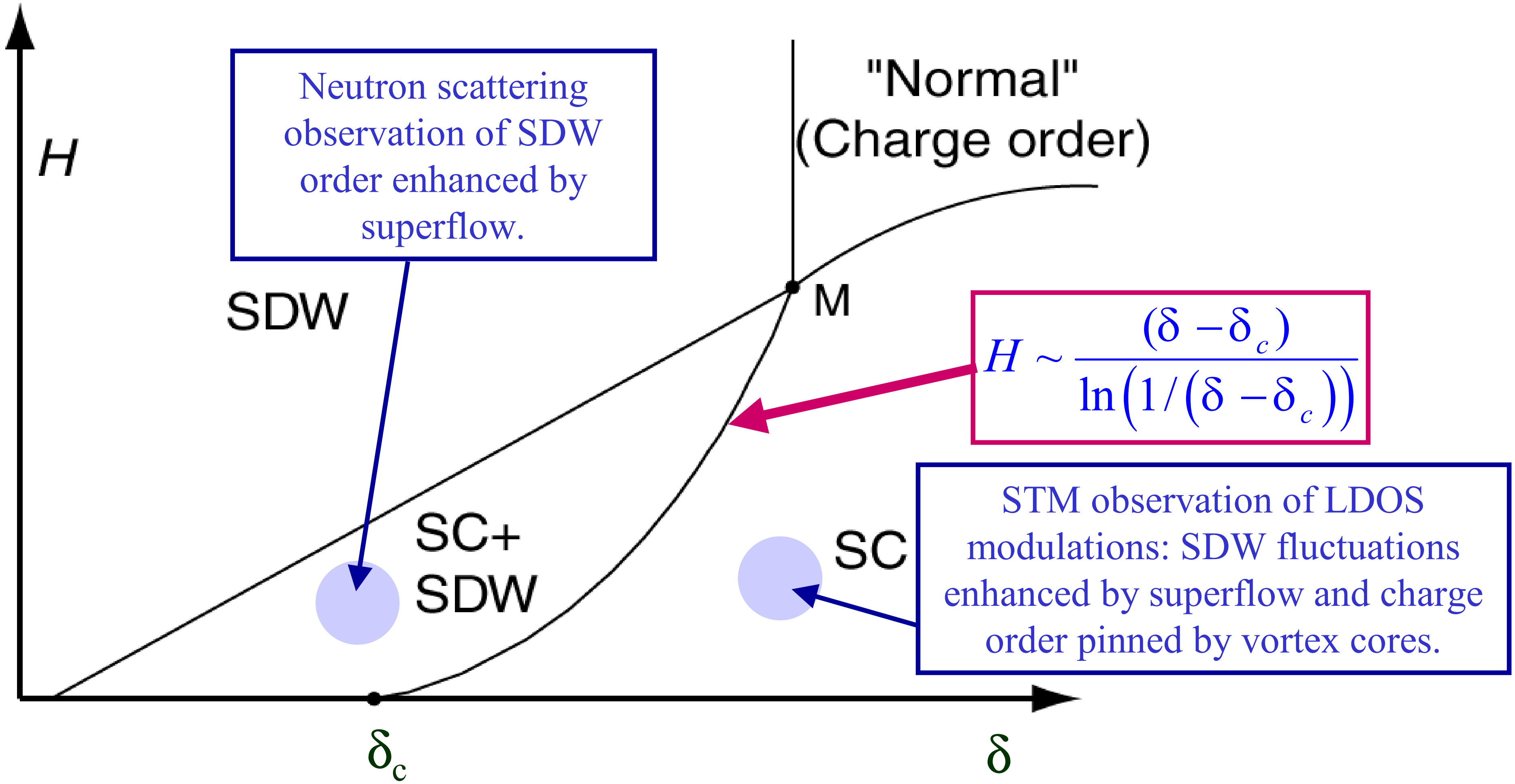
A.Polkovnikov, S. Sachdev, and M. Vojta, cond-mat/0208334

Summary of theory and experiments

(extreme Type II superconductivity)

T=0



E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Quantitative connection between the two experiments ?

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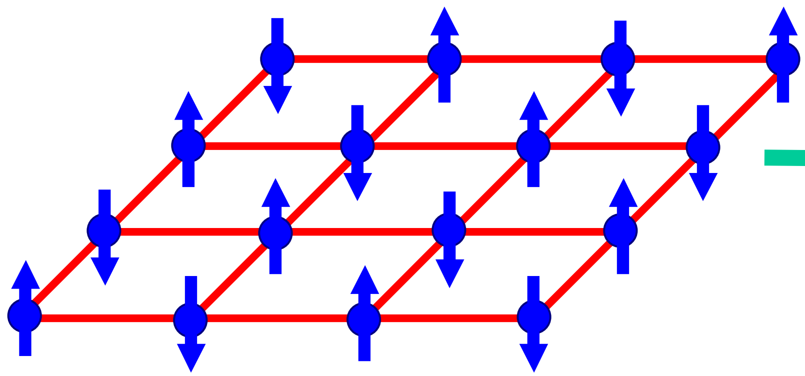
2D Antiferromagnets with an odd number of $S=1/2$ spins per unit cell

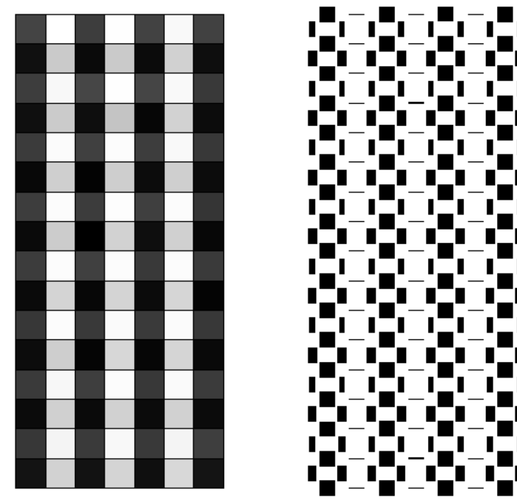
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991); S.S. and N.R. *Int. J. Mod. Phys.* B **5**, 219 (1991).

A. Collinear spins, Berry phases, and bond-order



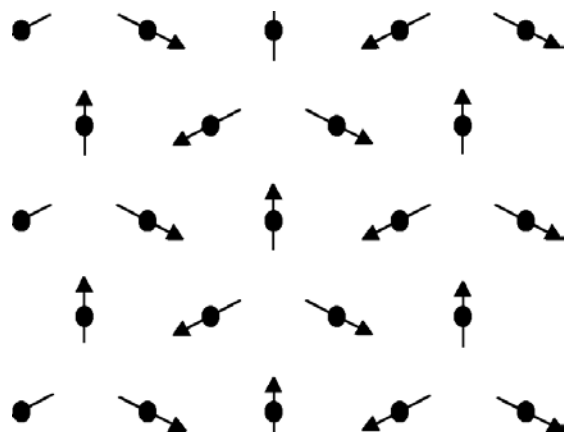
Néel ordered state

Quantum transition restoring spin rotation invariance



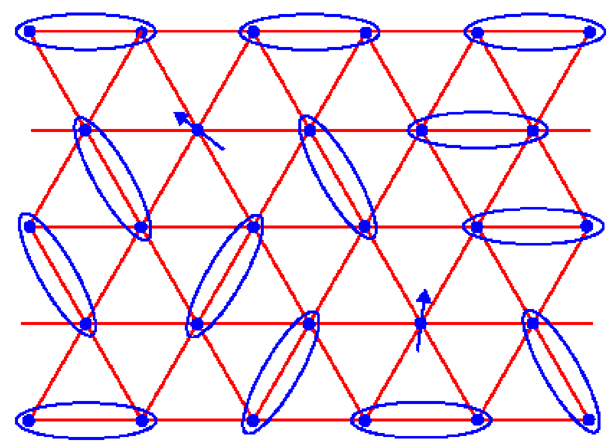
Bond-ordered state with confinement of spinons and $S=1$ spin exciton

B. Non-collinear spins and deconfined spinons.



Non-collinear ordered antiferromagnet

Quantum transition restoring spin rotation invariance

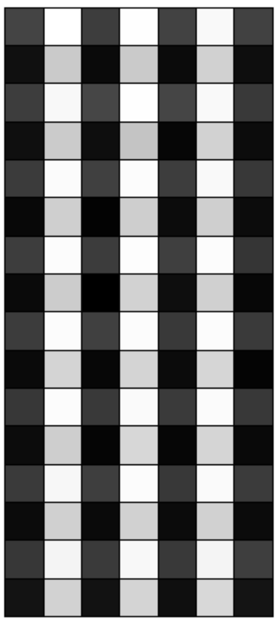


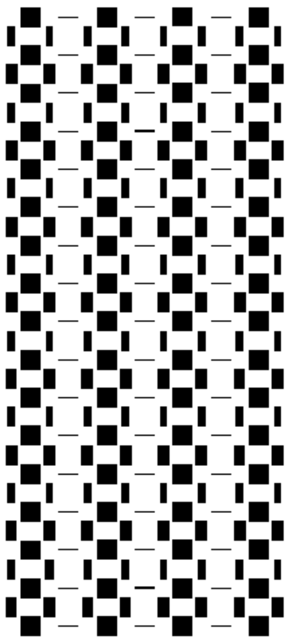
RVB state with visons, deconfined $S=1/2$ spinons, Z_2 gauge theory

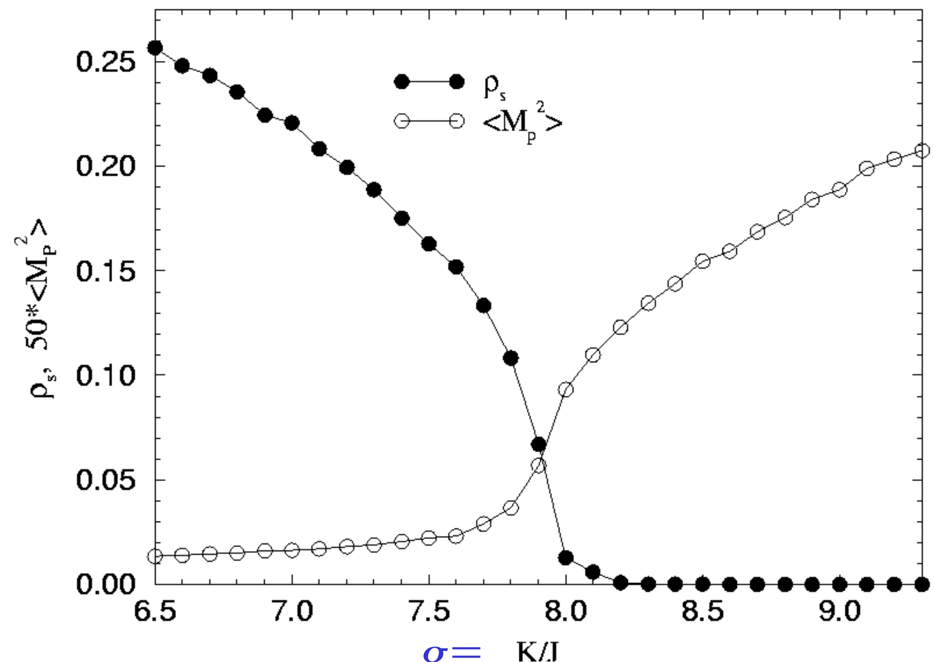
Bond order wave in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, cond-mat/0205270

First large scale numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry





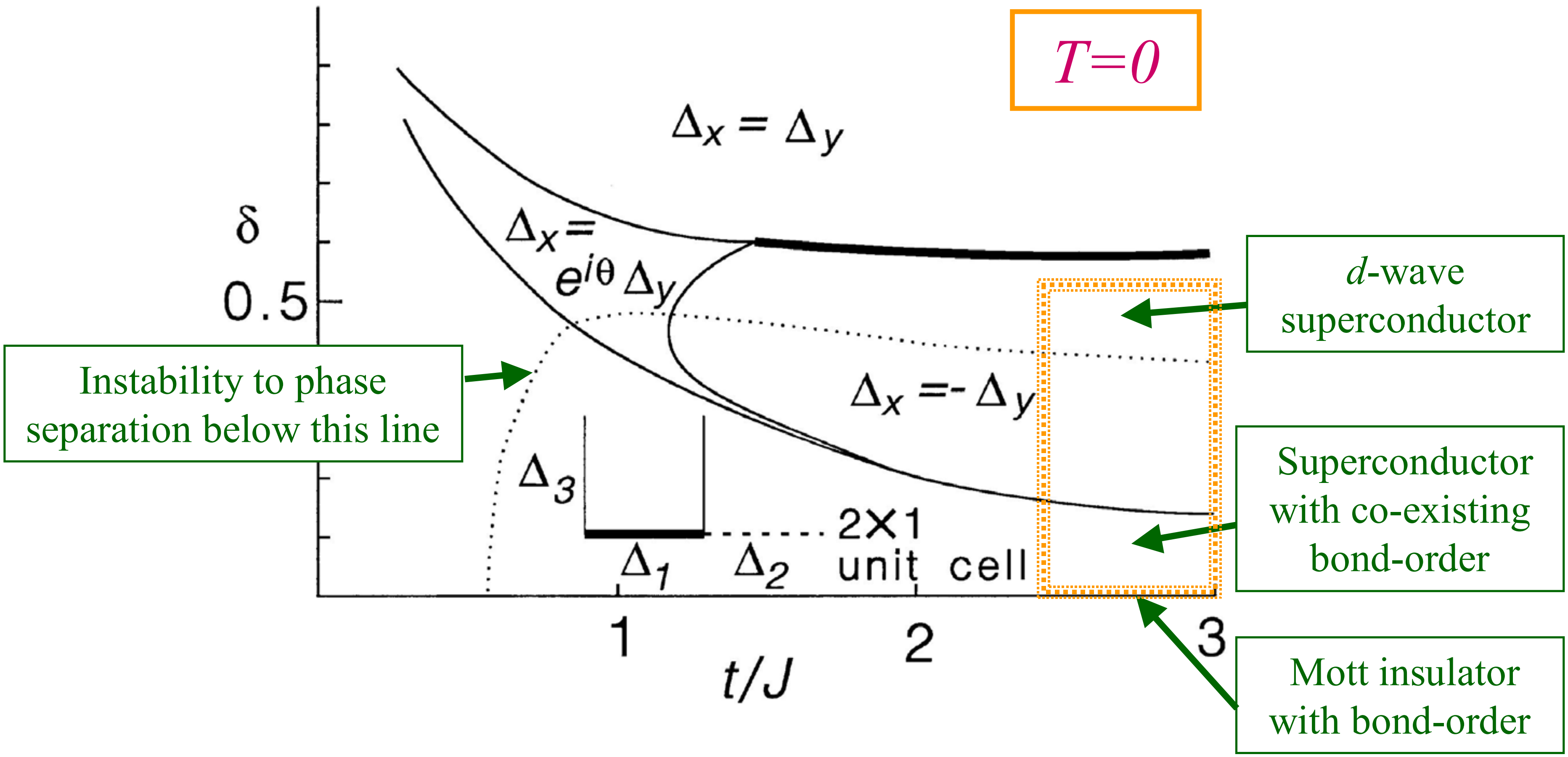


$g=$ K/J

$$H = 2J\sum_{\langle ij\rangle}\left(S_i^x S_j^x + S_i^y S_j^y\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+\right)$$

Doping a bond-ordered Mott insulator

Approach the non-magnetic *d*-wave superconductor by starting from a non-magnetic Mott insulator: allows a systematic Sp(*N*) theory of translational symmetry breaking, while preserving spin rotation invariance.

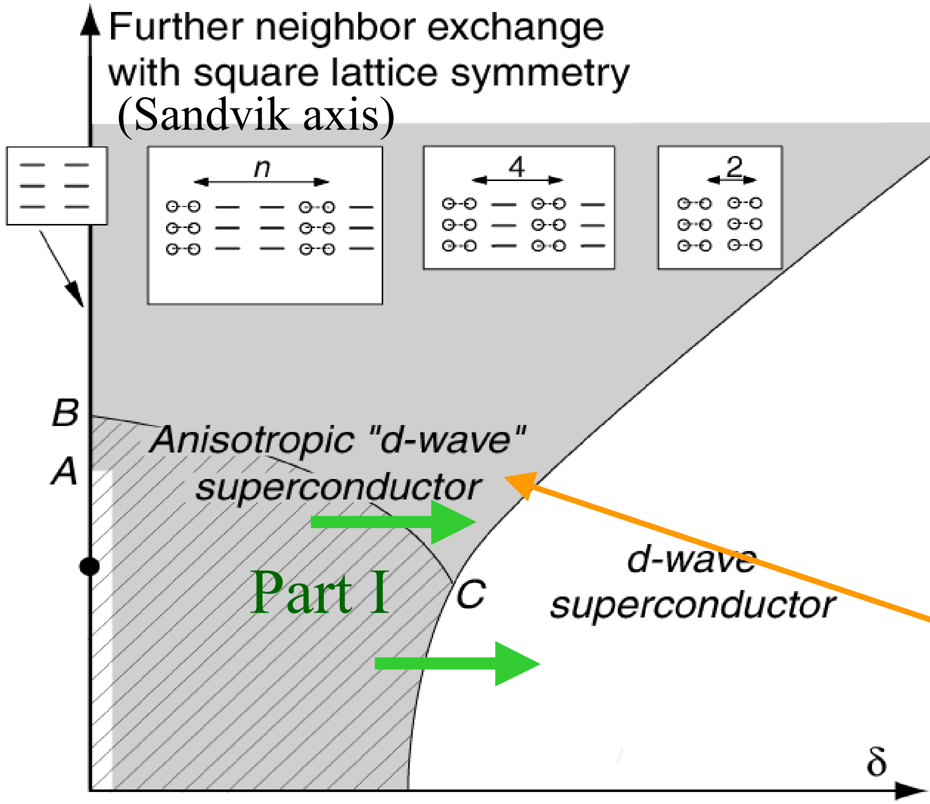


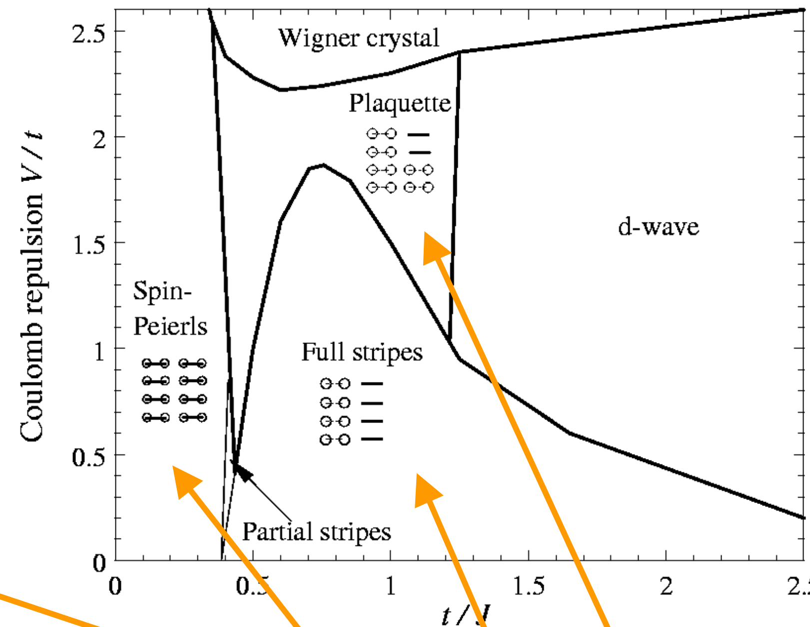
S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).

III.Global phase diagram

Include long-range Coulomb interactions: frustrated phase separation

V.J. Emery, S.A. Kivelson, and H.Q. Lin, *Phys. Rev. Lett.* **64**, 475 (1990).





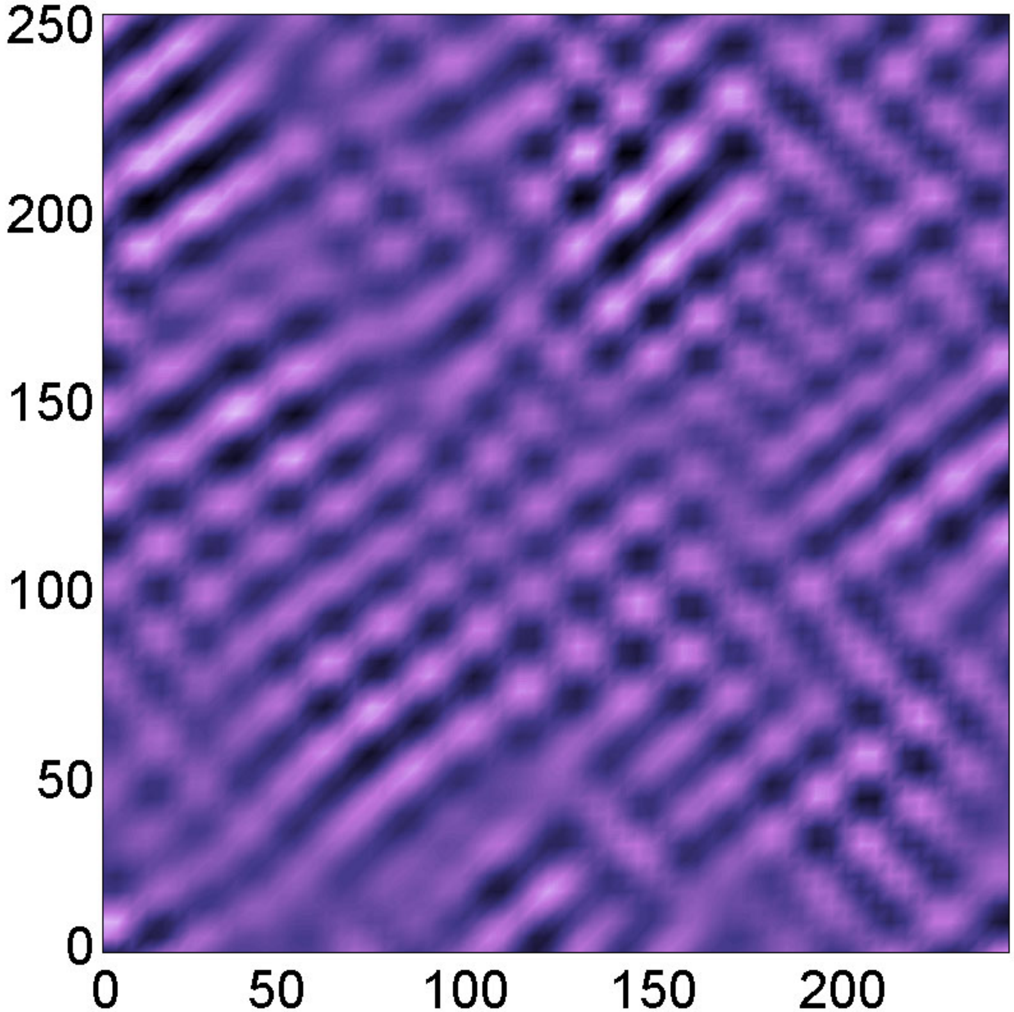
Hatched region --- static spin order
Shaded region ---- static charge order

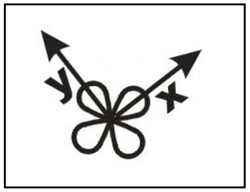
Non-magnetic “d-wave” superconductor with even period bond-order.

M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000).
M. Vojta, cond-mat/0204284.

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. White and D. Scalapino, *Phys. Rev. Lett.* **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett.* **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

III. STM image of LDOS modulations in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field

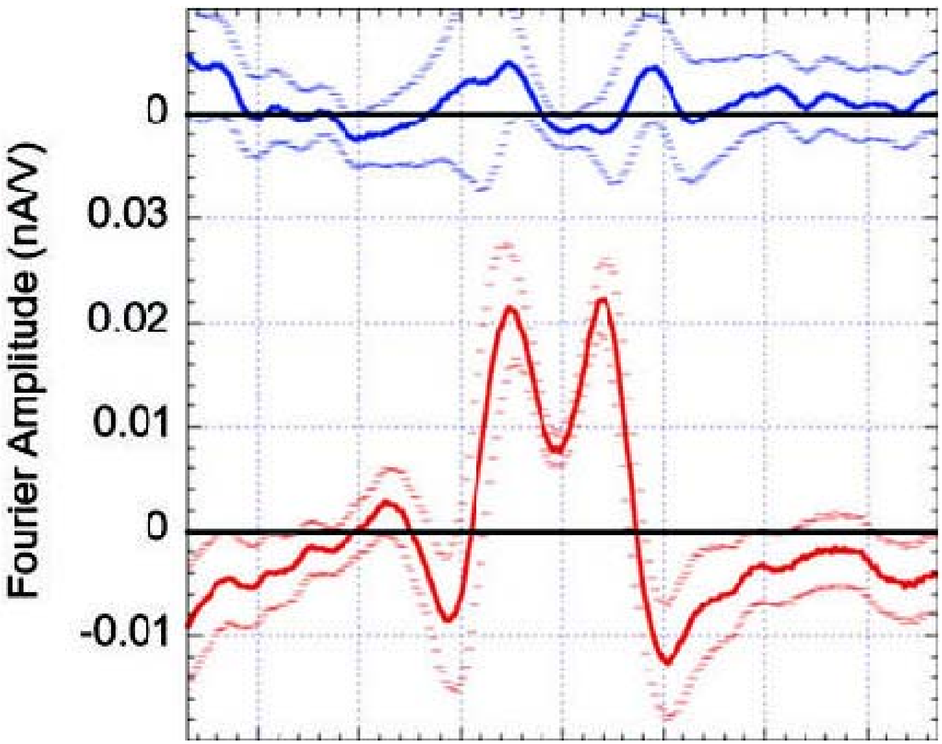




Period = 4 lattice spacings

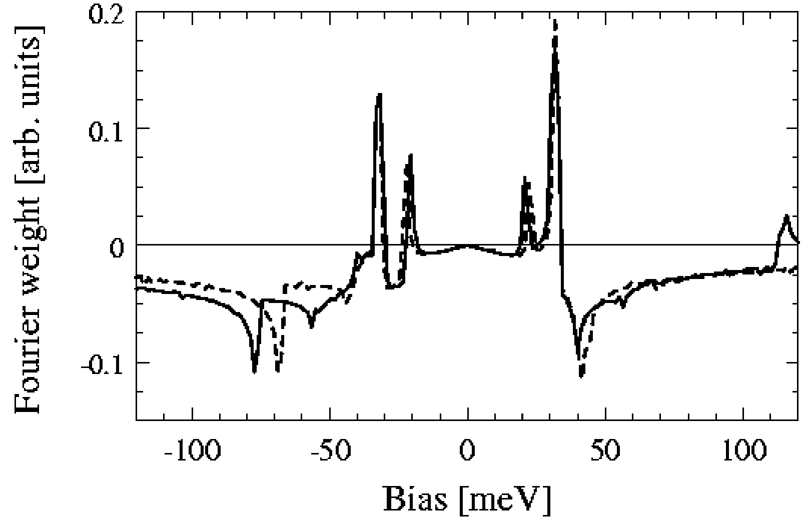
C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

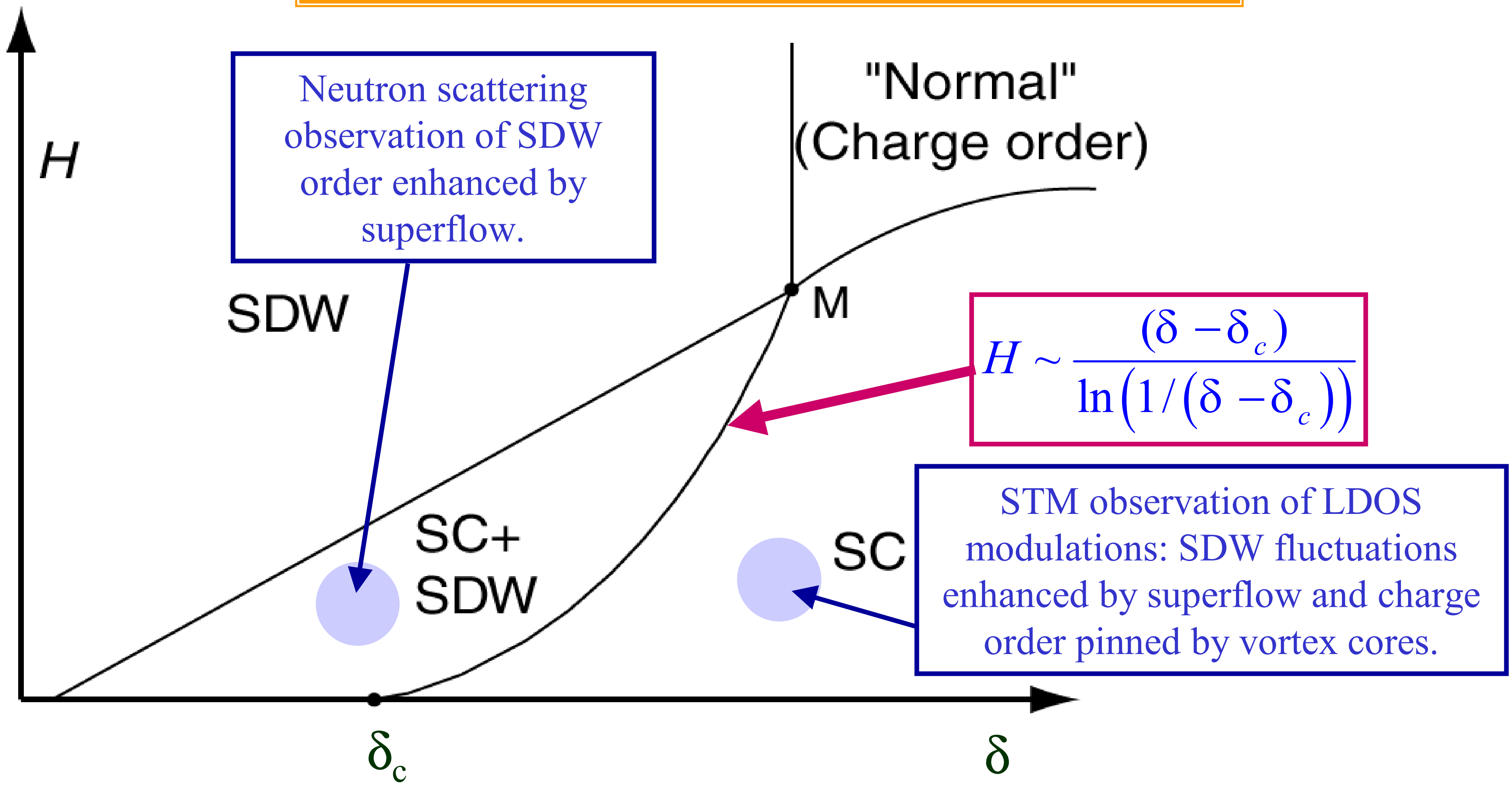


Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

M. Vojta, cond-mat/0204284.
D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, cond-mat/0204011

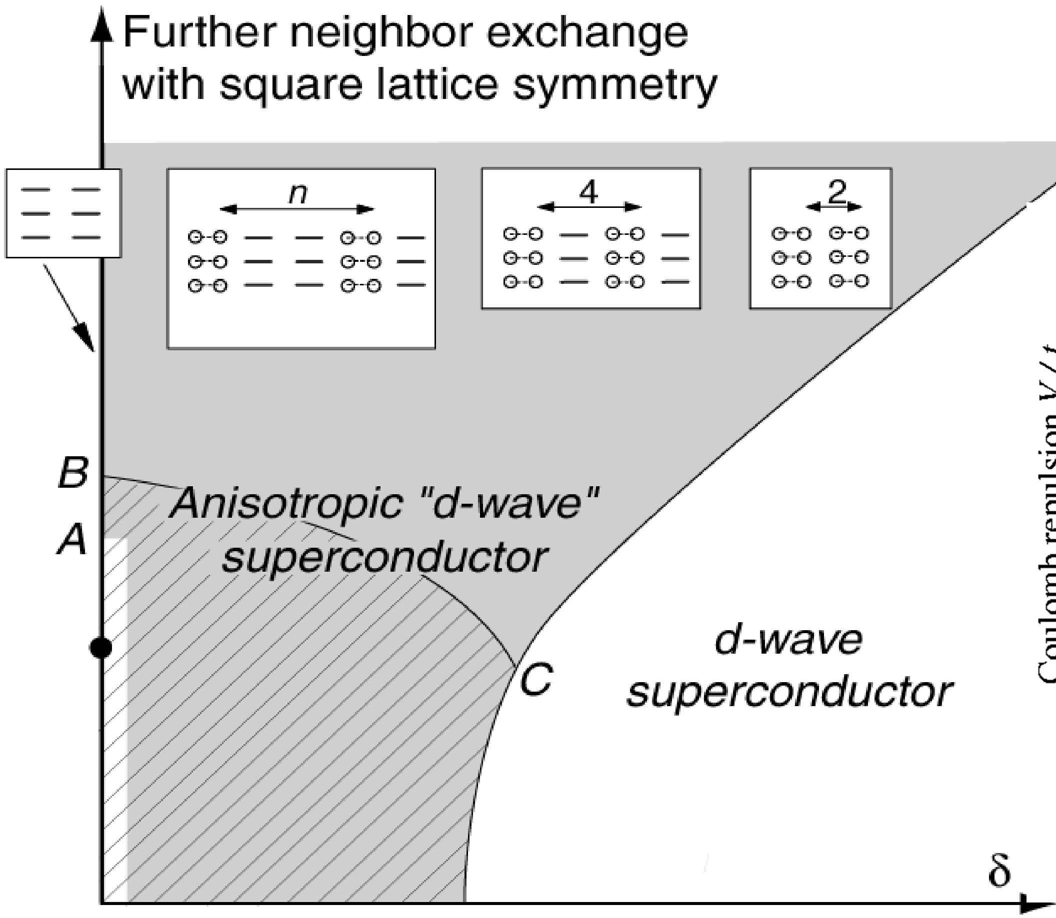
Conclusions

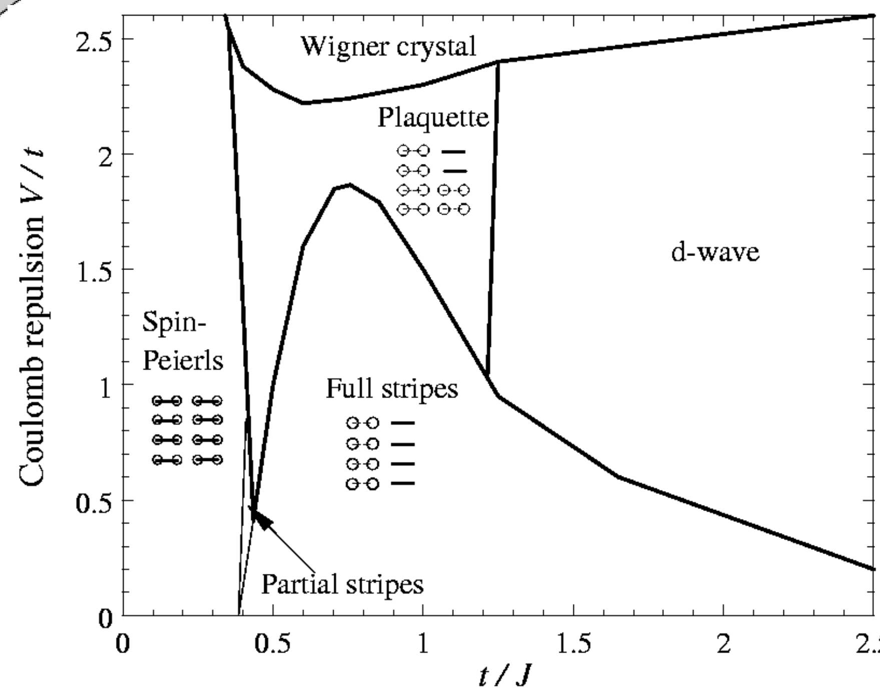
Theory of SC+SDW to SC quantum phase transition in a magnetic field



Conclusions

Global phase diagram in zero field





Hatched region --- static spin order
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See also
V.J. Emery, S.A. Kivelson, and H.Q. Lin, *Phys. Rev. Lett*. **64**, 475 (1990).
J. Zaanen, *Physica* C **217**, 317 (1999),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers. The correct paramagnetic Mott insulator has bond-order and confinement of spinons (collinear spins in magnetically ordered state).

II. The Mott insulator reveals itself vortices and near impurities. Predicted effects seen recently in STM and NMR experiments.

III. Semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also proposes a connection to STM experiments.

IV. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

V. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures ?