Strange metals and gauge-gravity duality

Loughborough University, May 17, 2012

Subir Sachdev

Lecture at the 100th anniversary Solvay conference,
Theory of the Quantum World, chair D.J. Gross.
arXiv:1203.4565

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Liza Huijse

Max Metlitski

Brian Swingle

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
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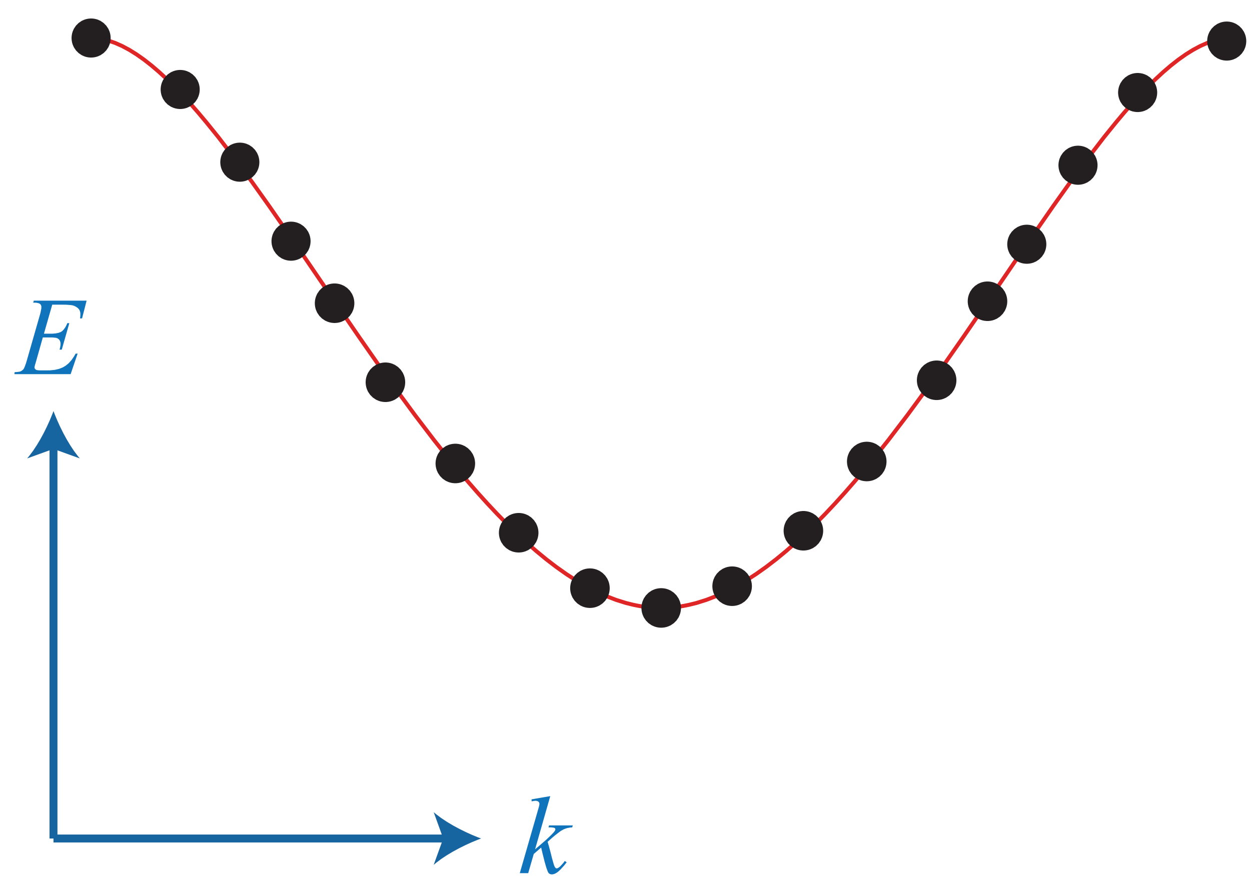
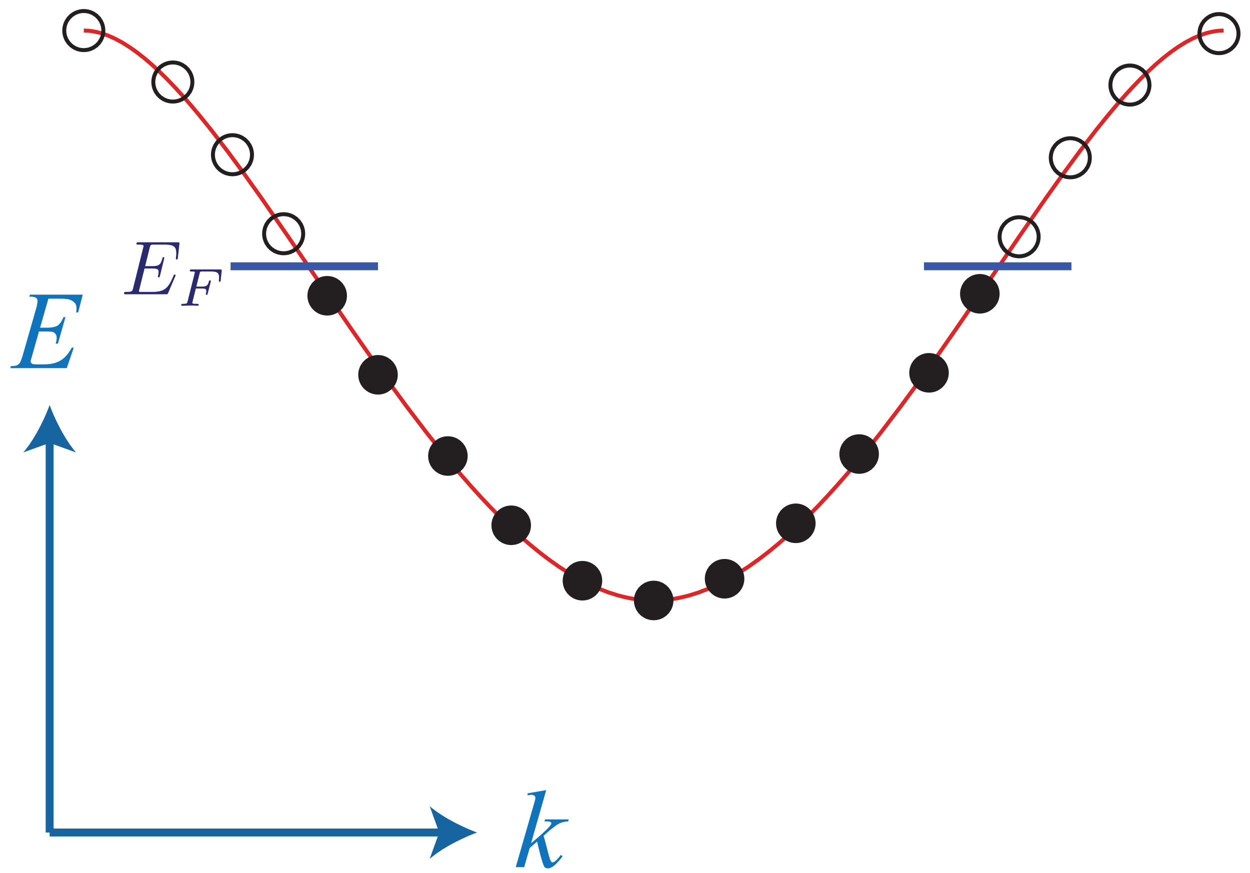
Band insulators



An even number of electrons per unit cell

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically connected to independent electron states

Metals



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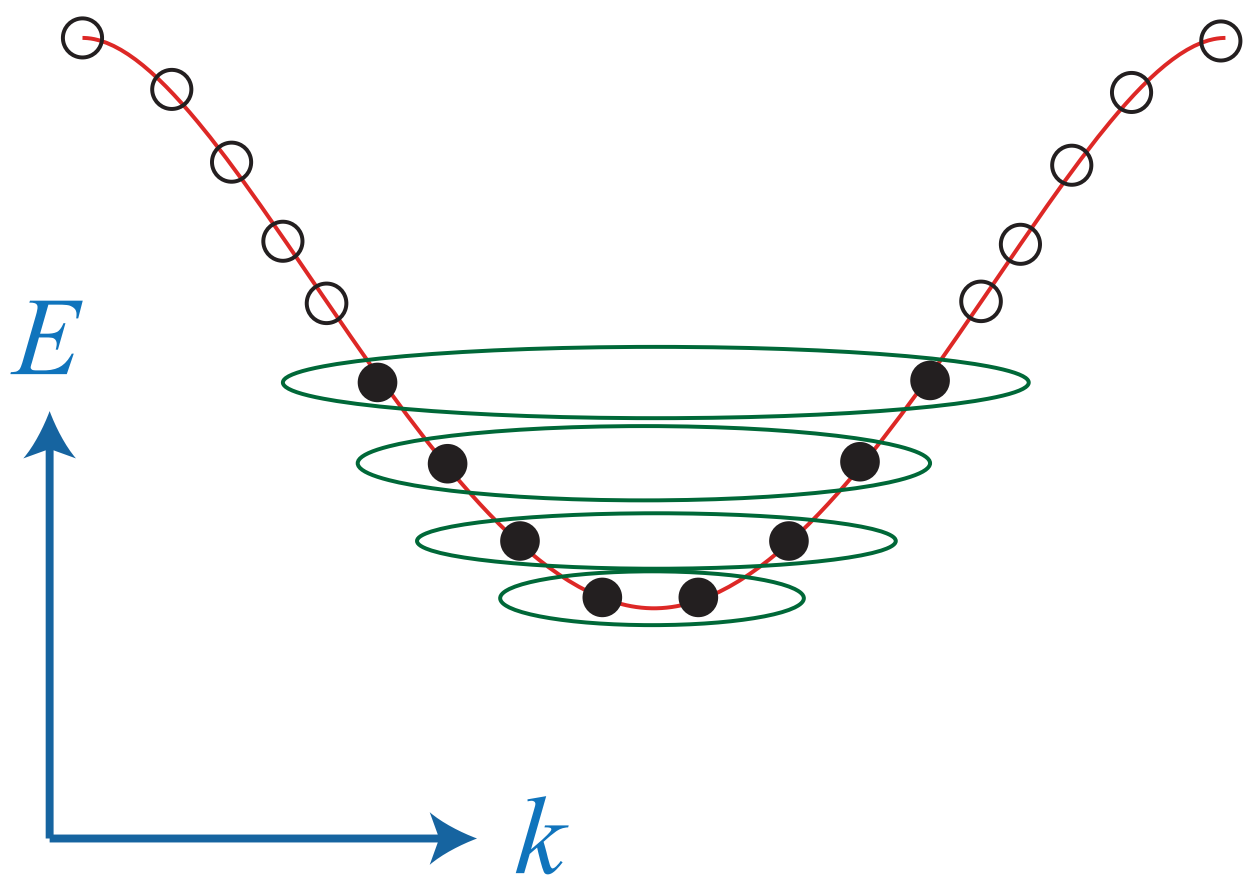
Superconductors



Modern phases of quantum matter

Not adiabatically connected

to independent electron states:

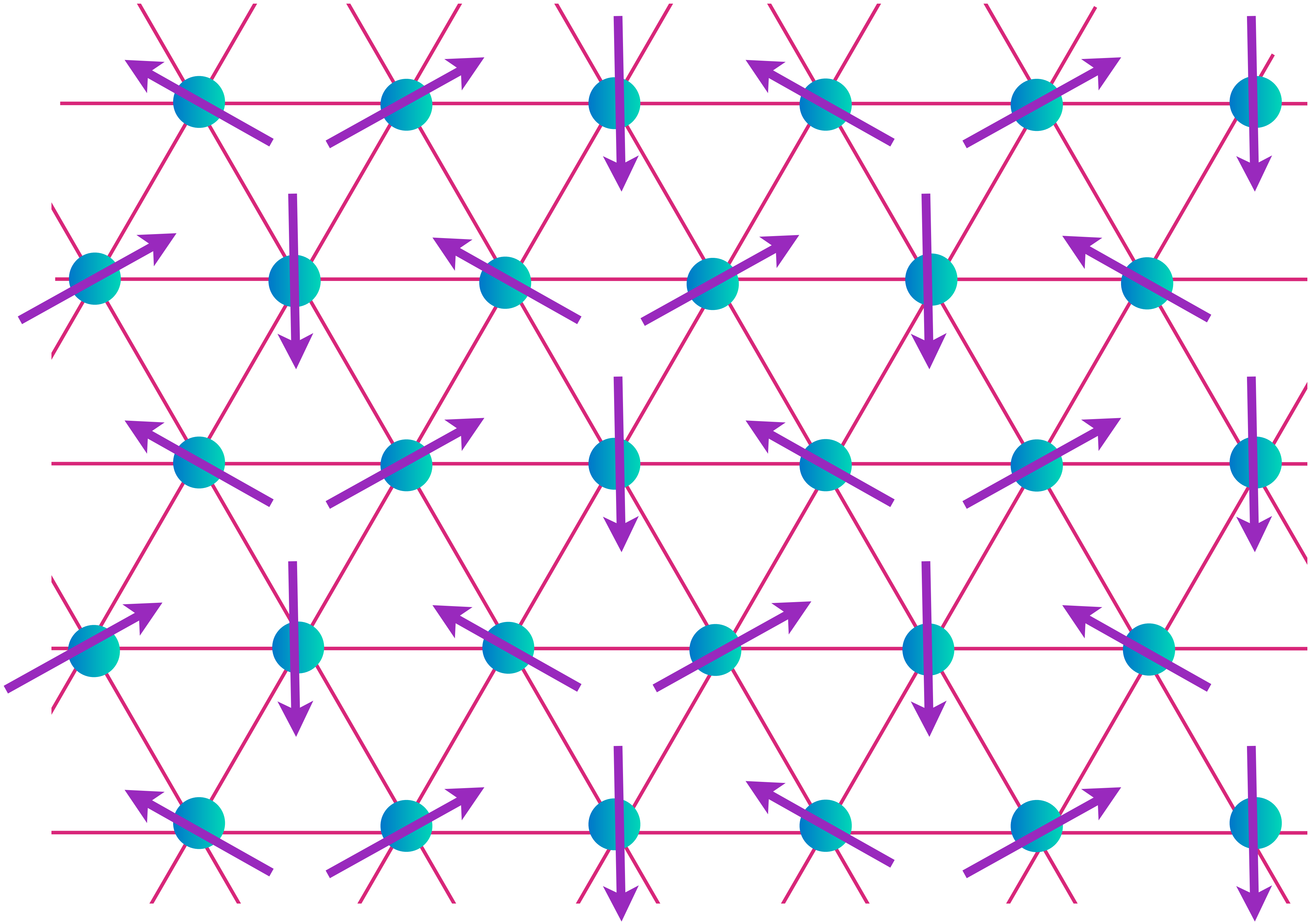
Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle, long-range
quantum entanglement

Mott insulator: Triangular lattice antiferromagnet

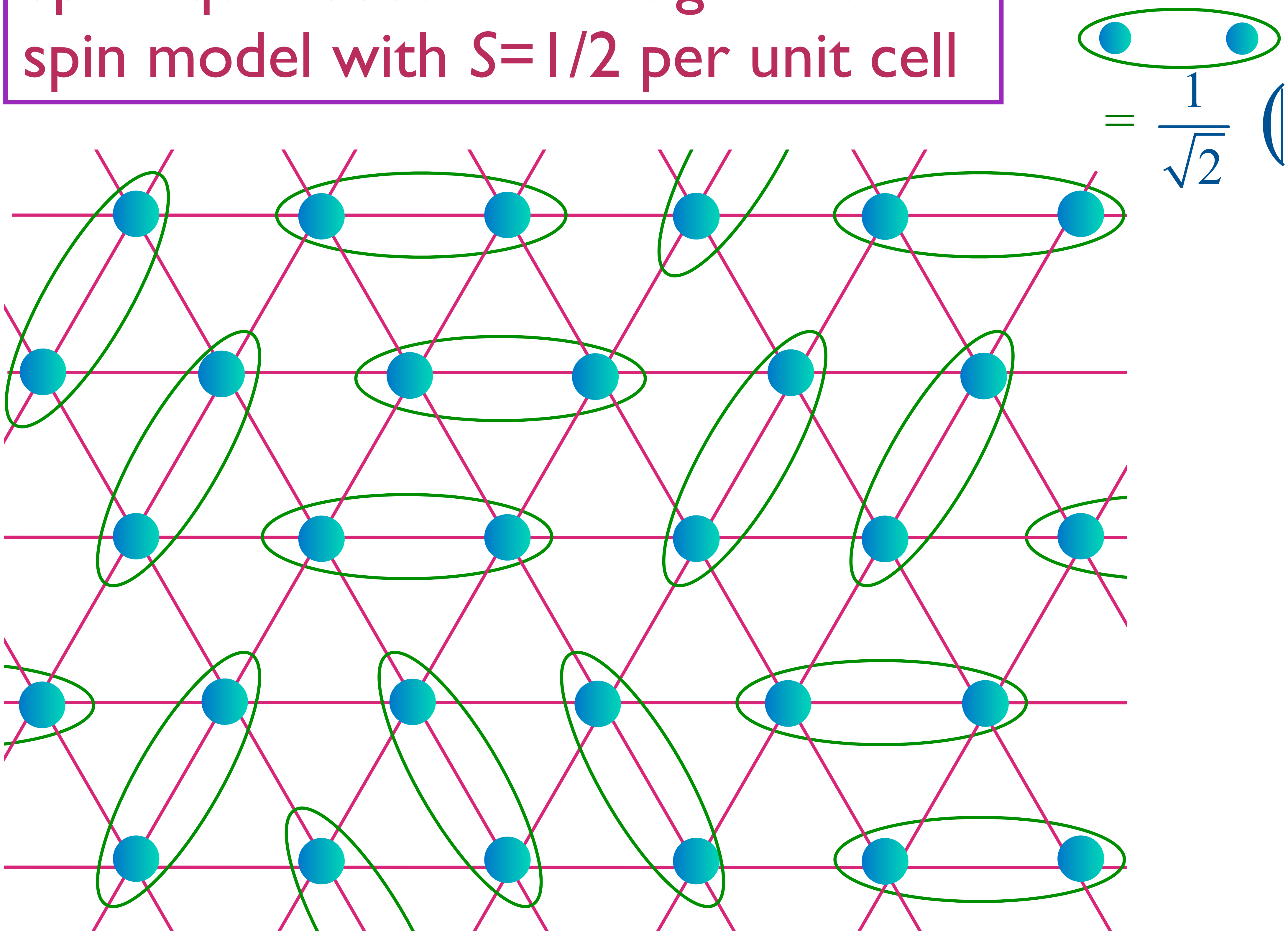
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Mott insulator: Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



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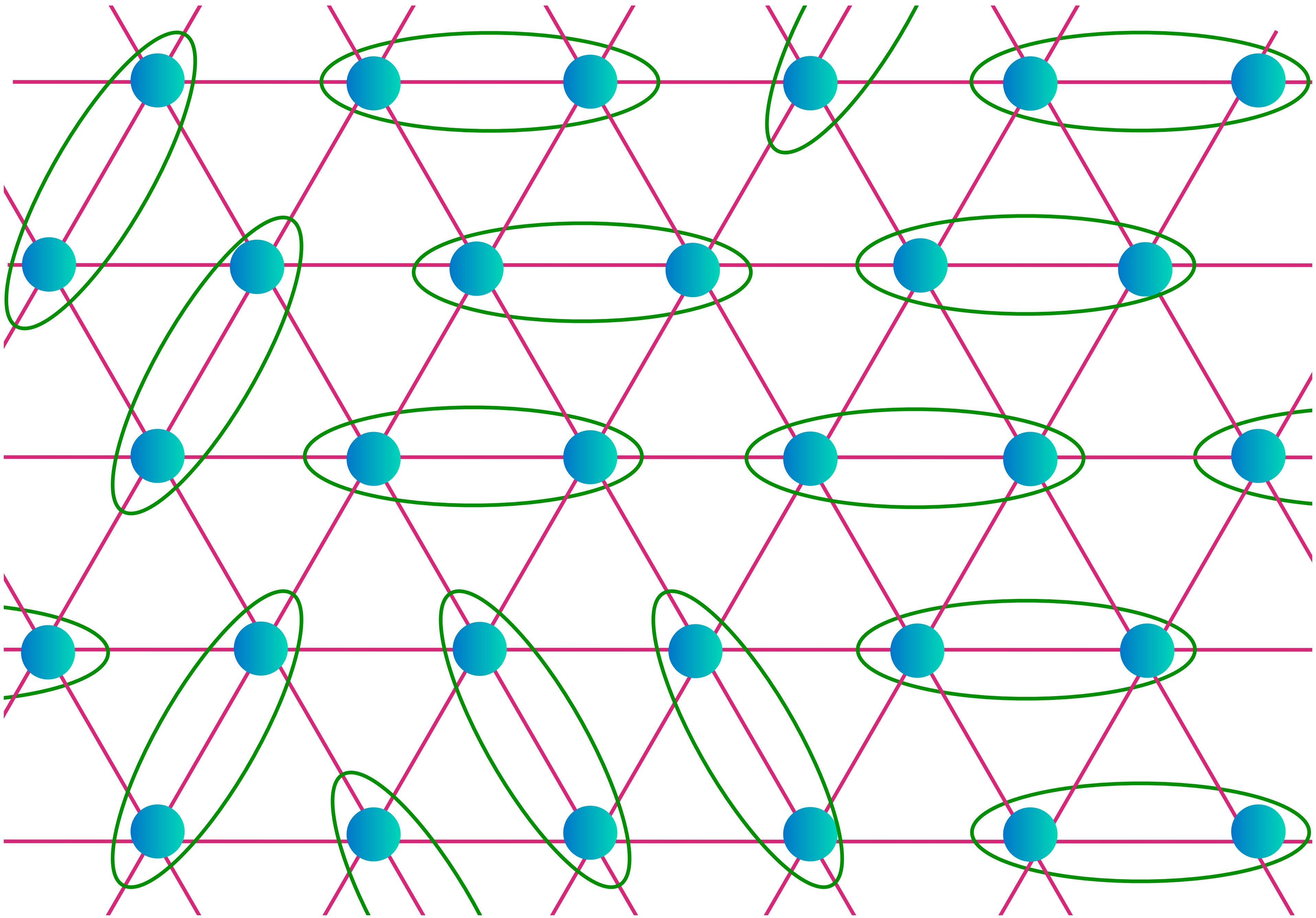

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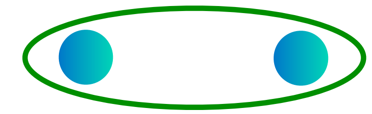

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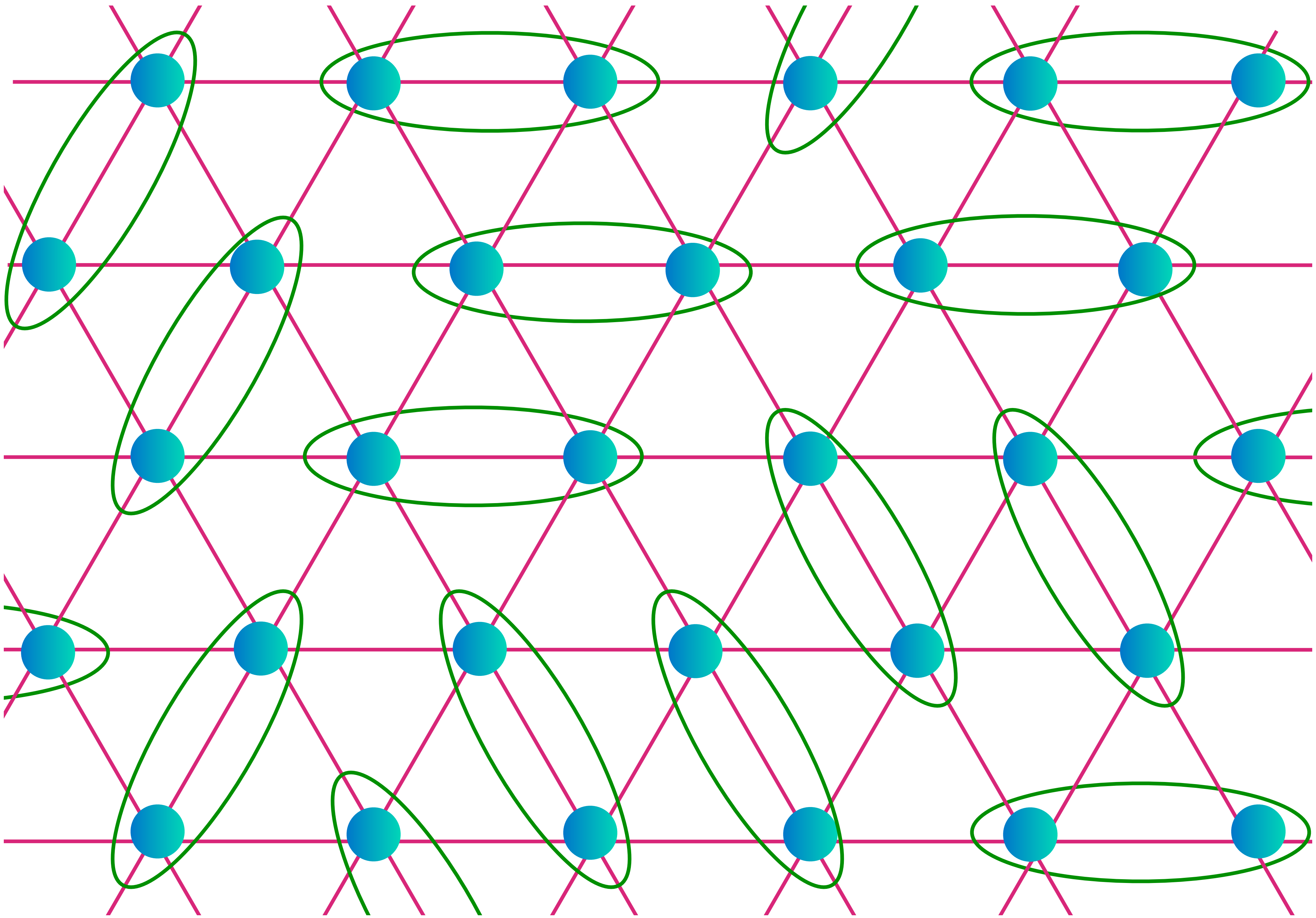

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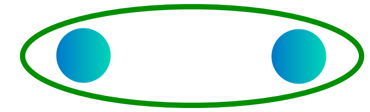

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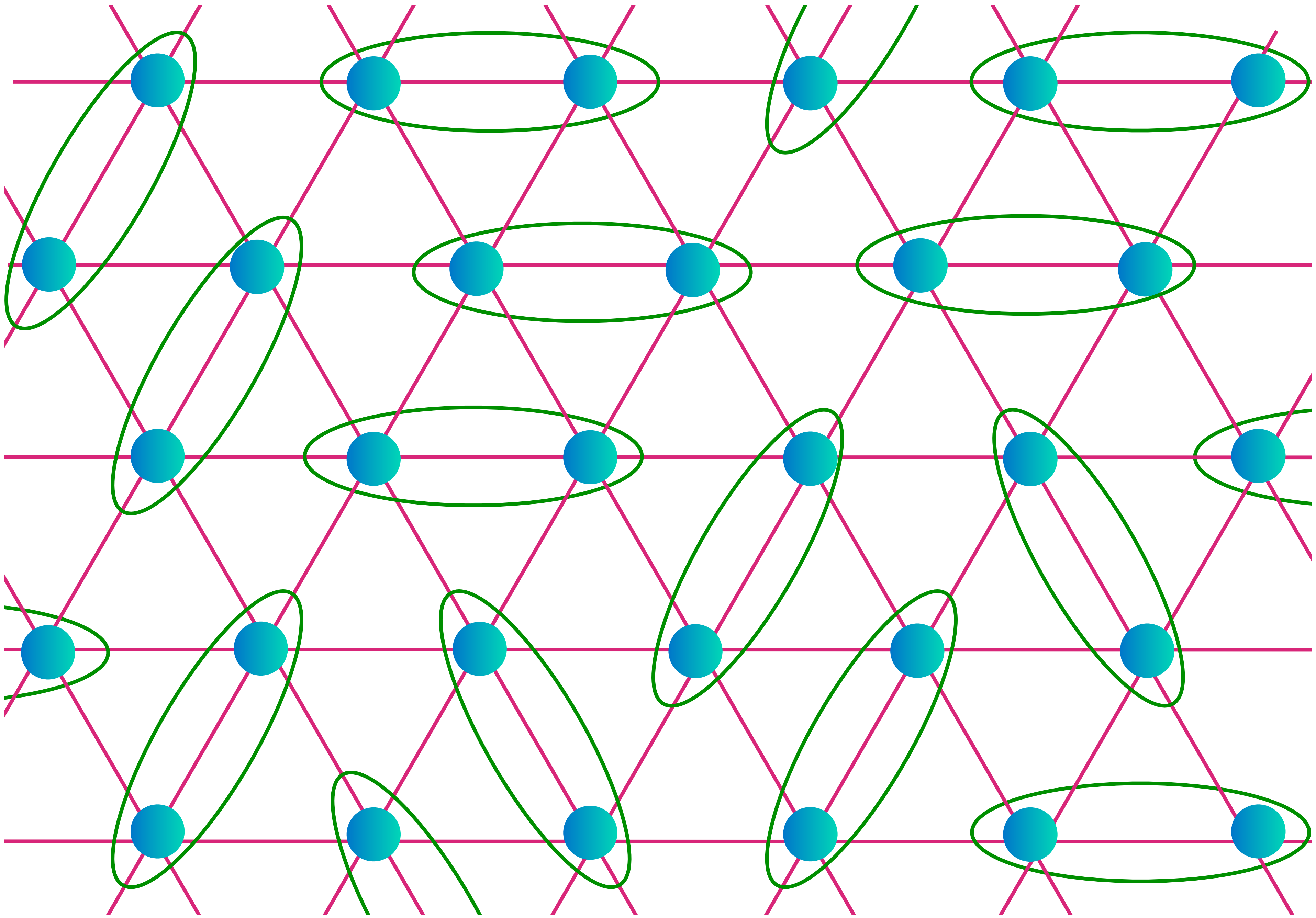

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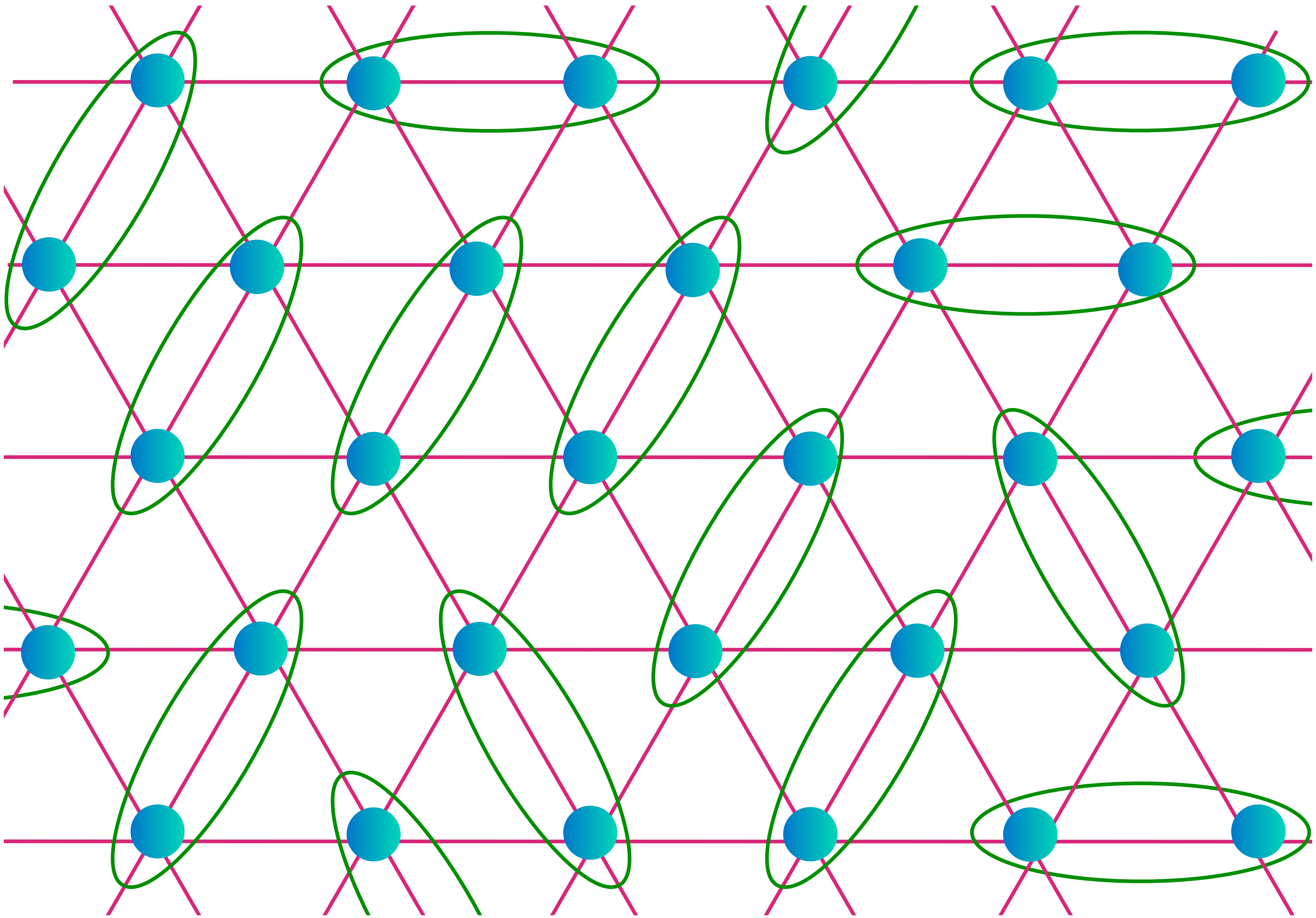

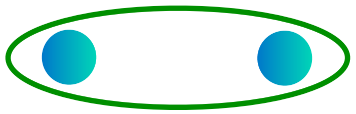

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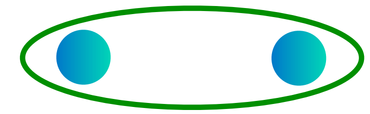

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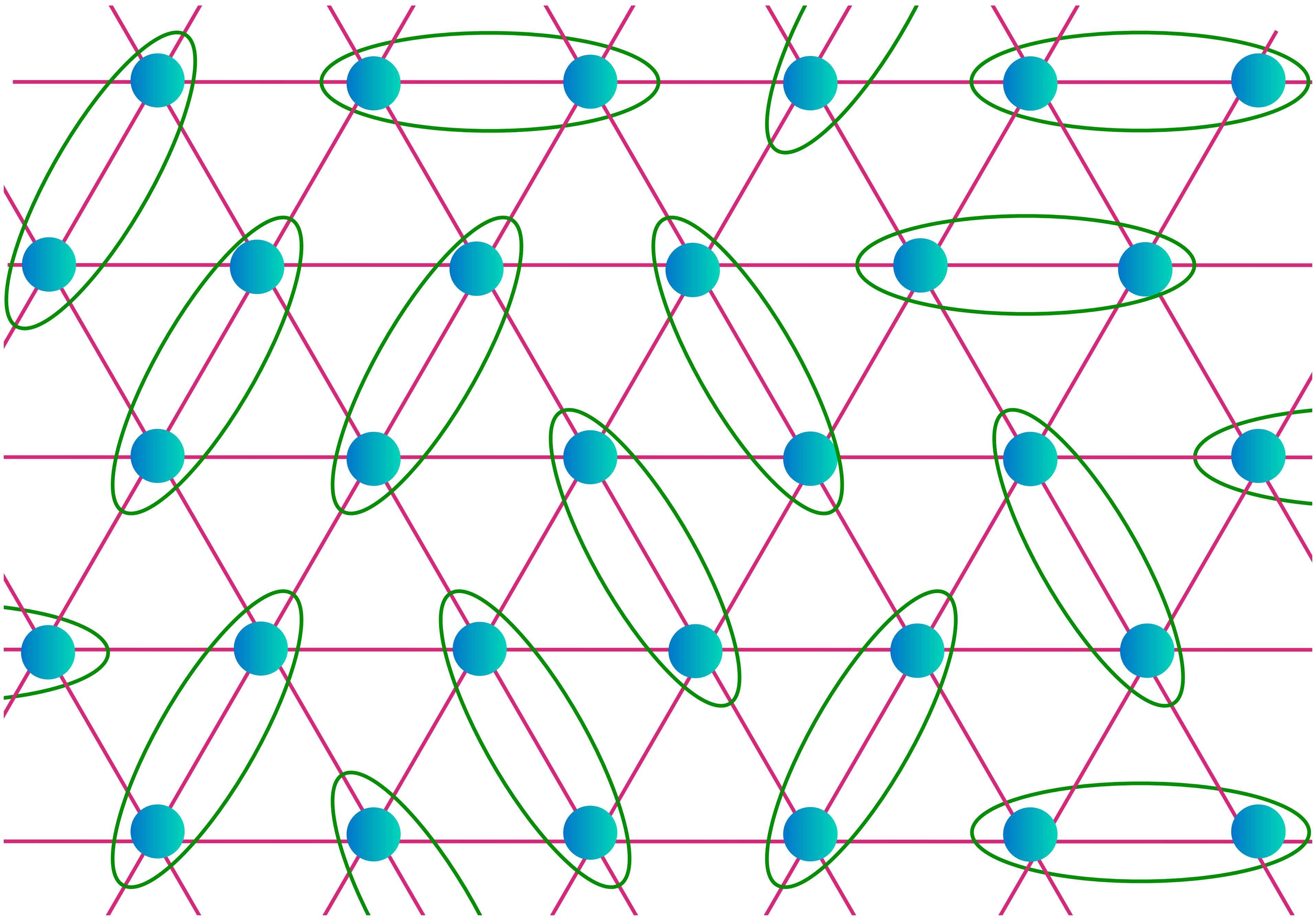

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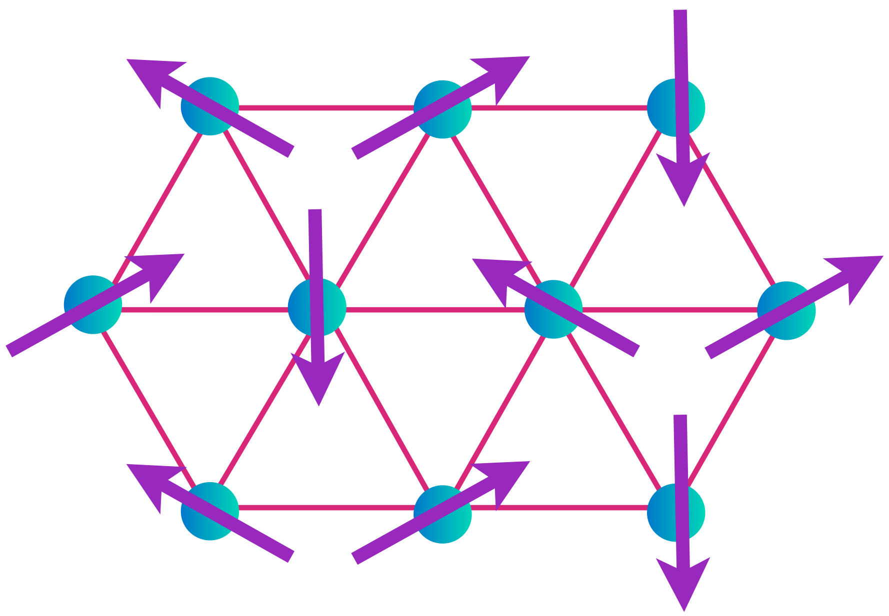
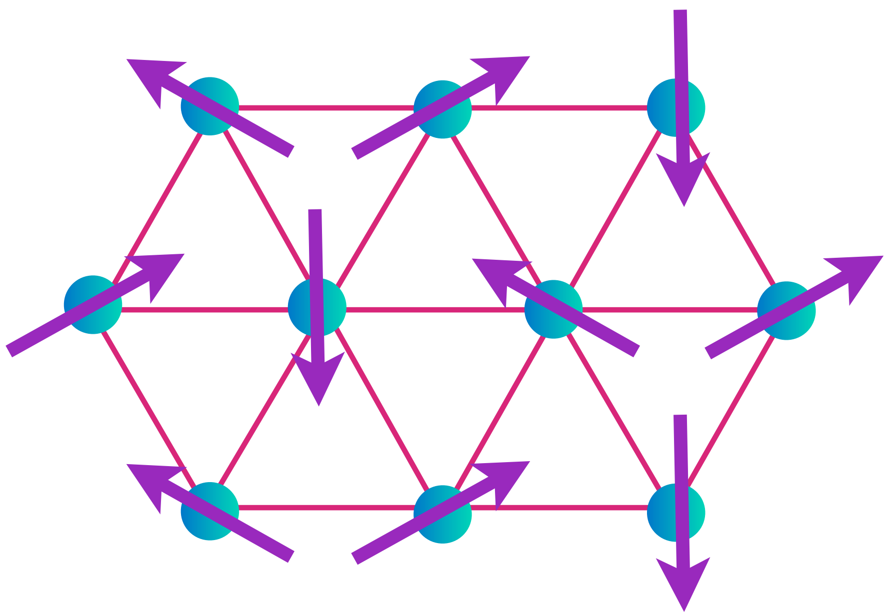


Quantum “disordered” state with exponentially decaying spin correlations.

non-collinear Néel state

s_c $\qquad$ s

Mott insulator: Triangular lattice antiferromagnet



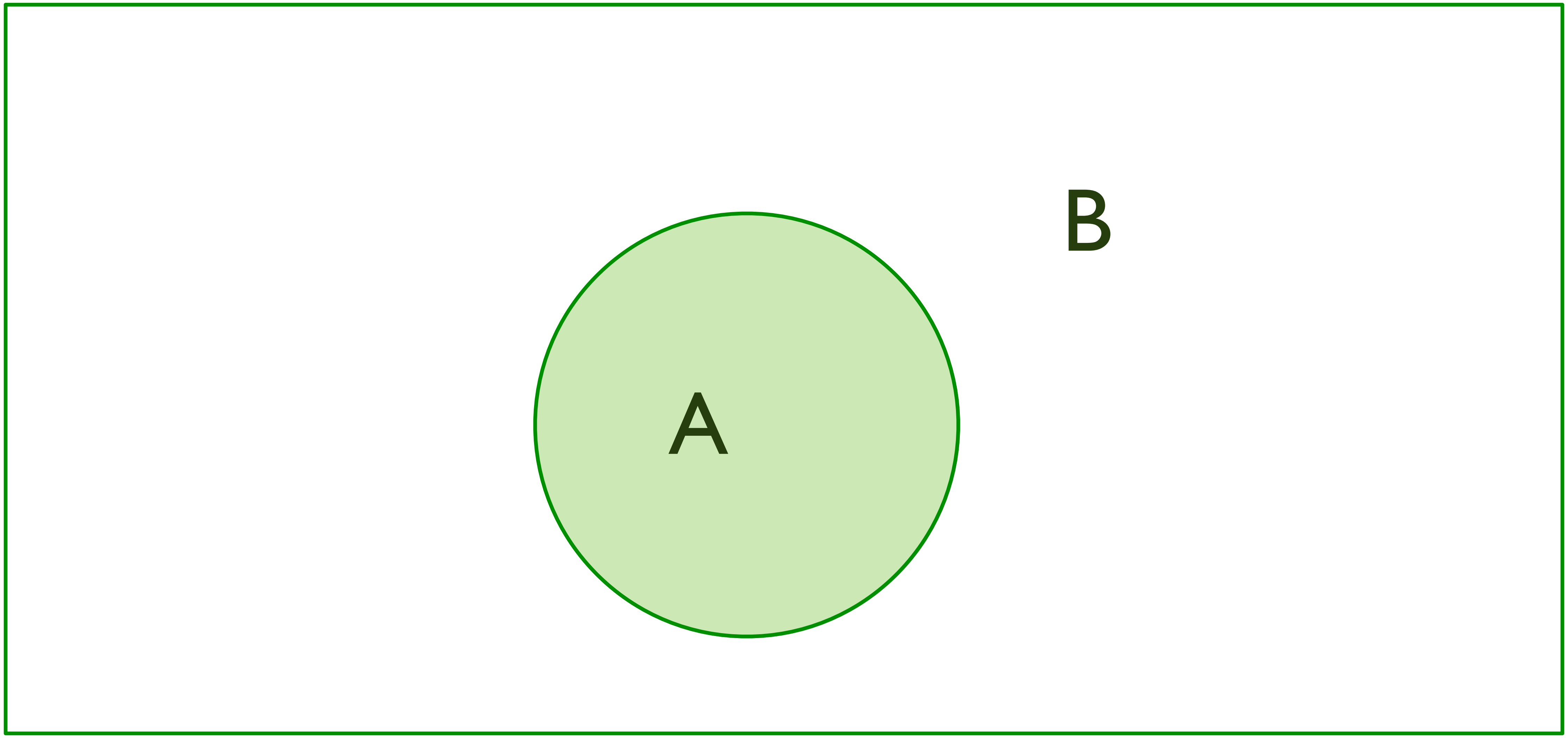
non-collinear Néel state

Quantum "disordered" state with exponentially decaying spin correlations.

Z_2 spin liquid with *long-range* entanglement.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Topological order in the Z_2 spin liquid ground state

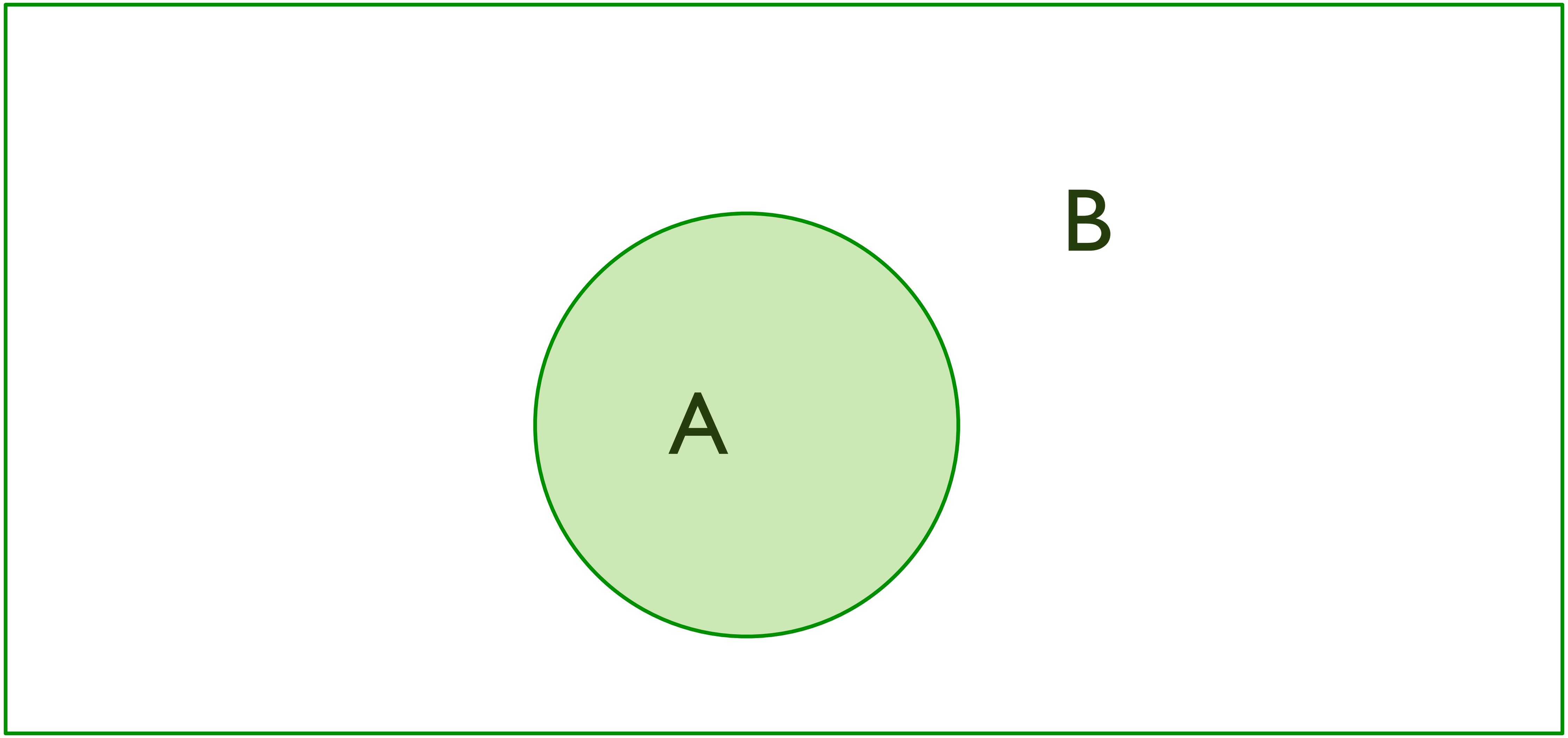


$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

Topological order in the Z_2 spin liquid ground state



Entanglement entropy of a band insulator:

$$S_{EE} = aP - \exp(-bP)$$

where P is the surface area (perimeter) of the boundary between A and B.

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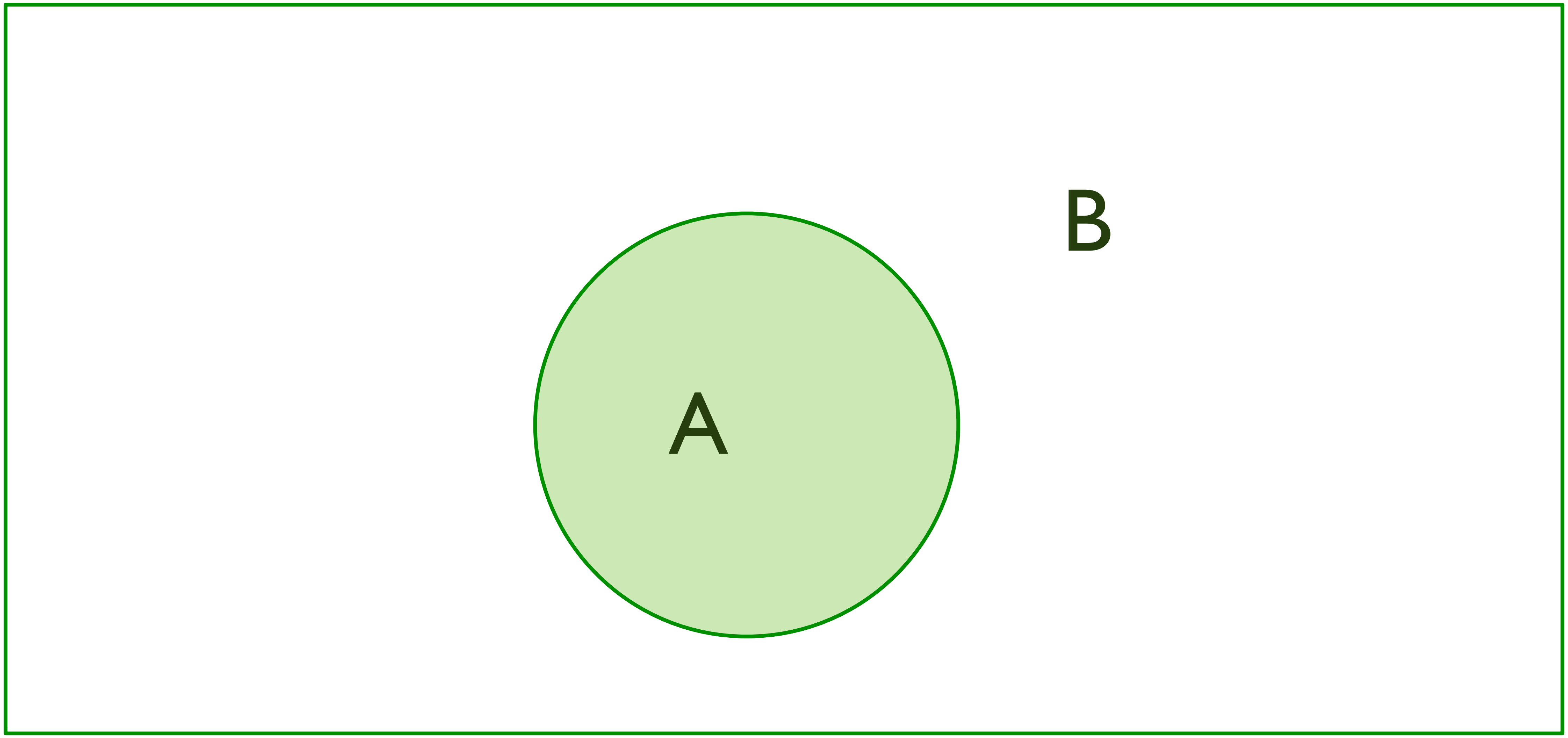


Entanglement entropy of a Z_2 spin liquid:

$$S_{EE} = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

The $\ln(2)$ is a universal characteristic of the Z_2 spin liquid, and implies *long-range* quantum entanglement.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

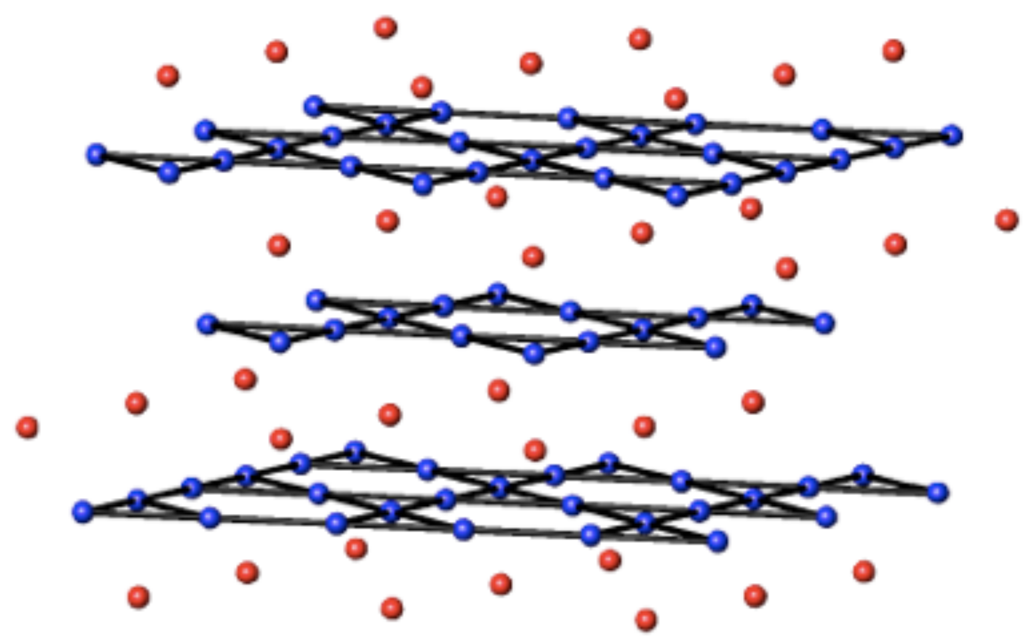
Promising candidate: the kagome antiferromagnet

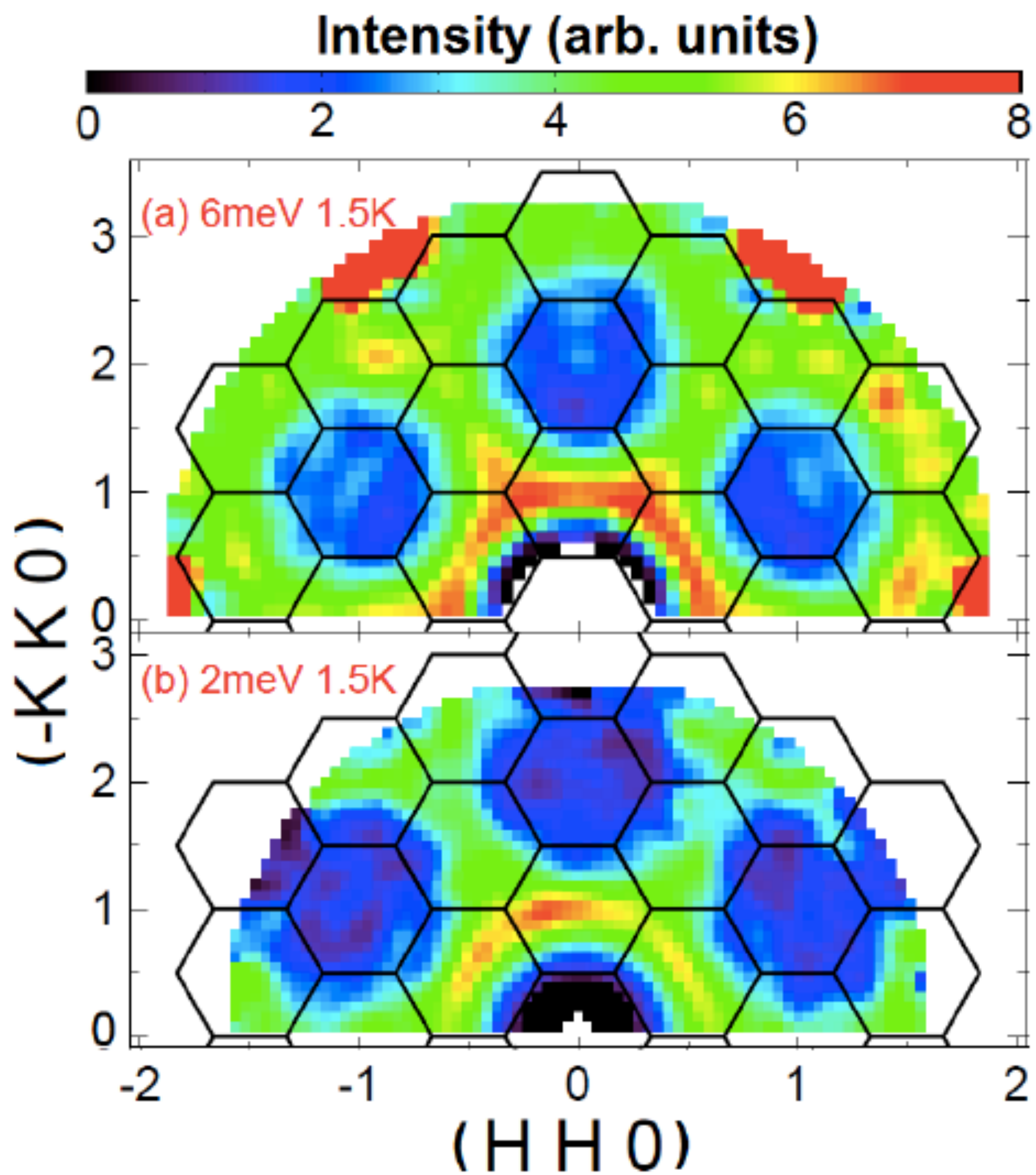
Numerical evidence for a gapped spin liquid:

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Young Lee,
APS meeting, March 2012

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





Quantum superposition and entanglement

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String theory

Quantum critical points of electrons in crystals

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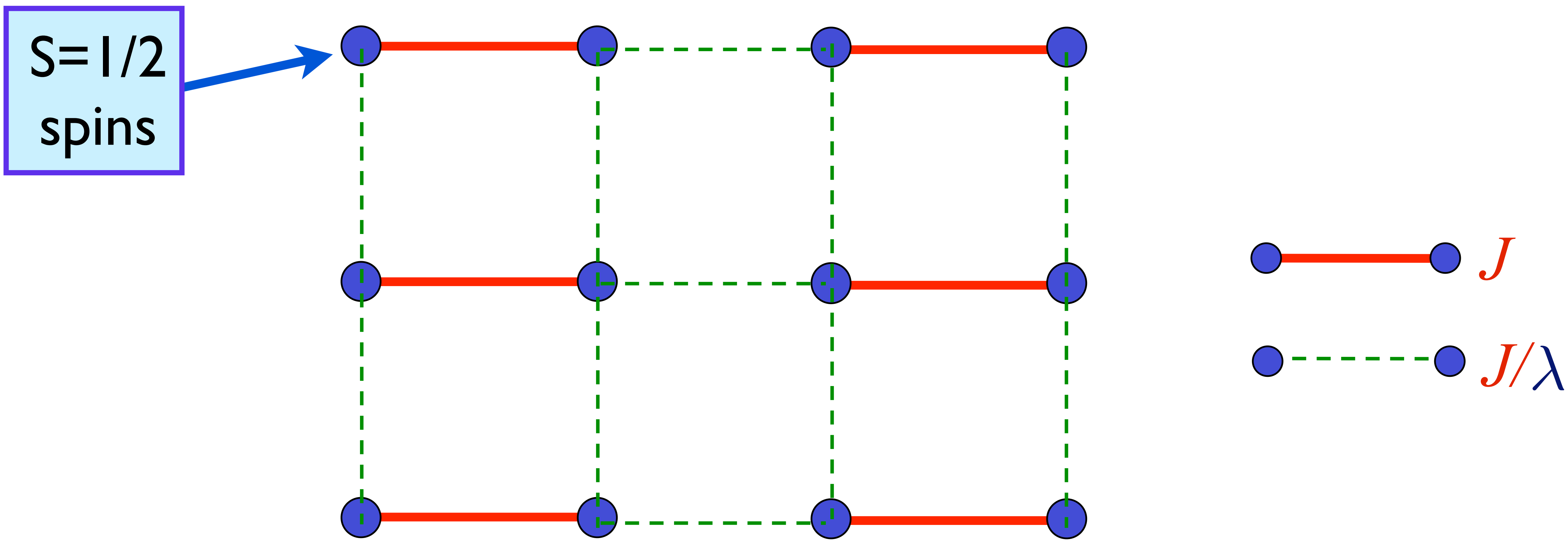
Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

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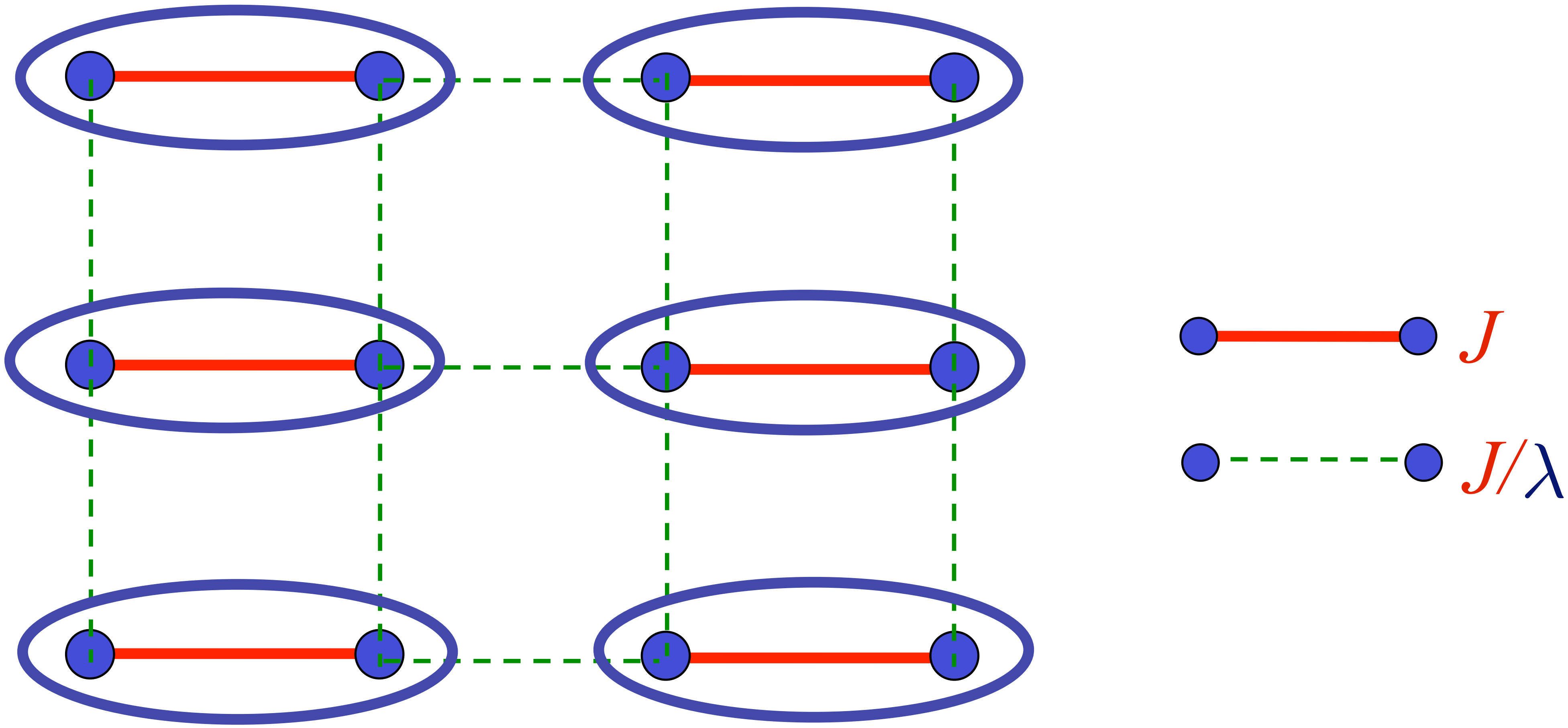

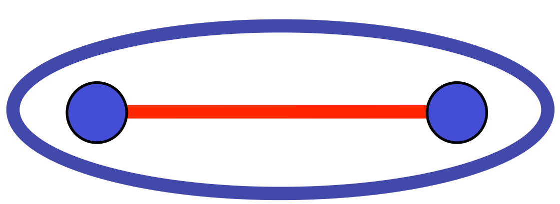

$$= \frac{1}{\sqrt{2}} \Big(\big| \uparrow\downarrow \big\rangle - \big| \downarrow\uparrow \big\rangle \Big)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

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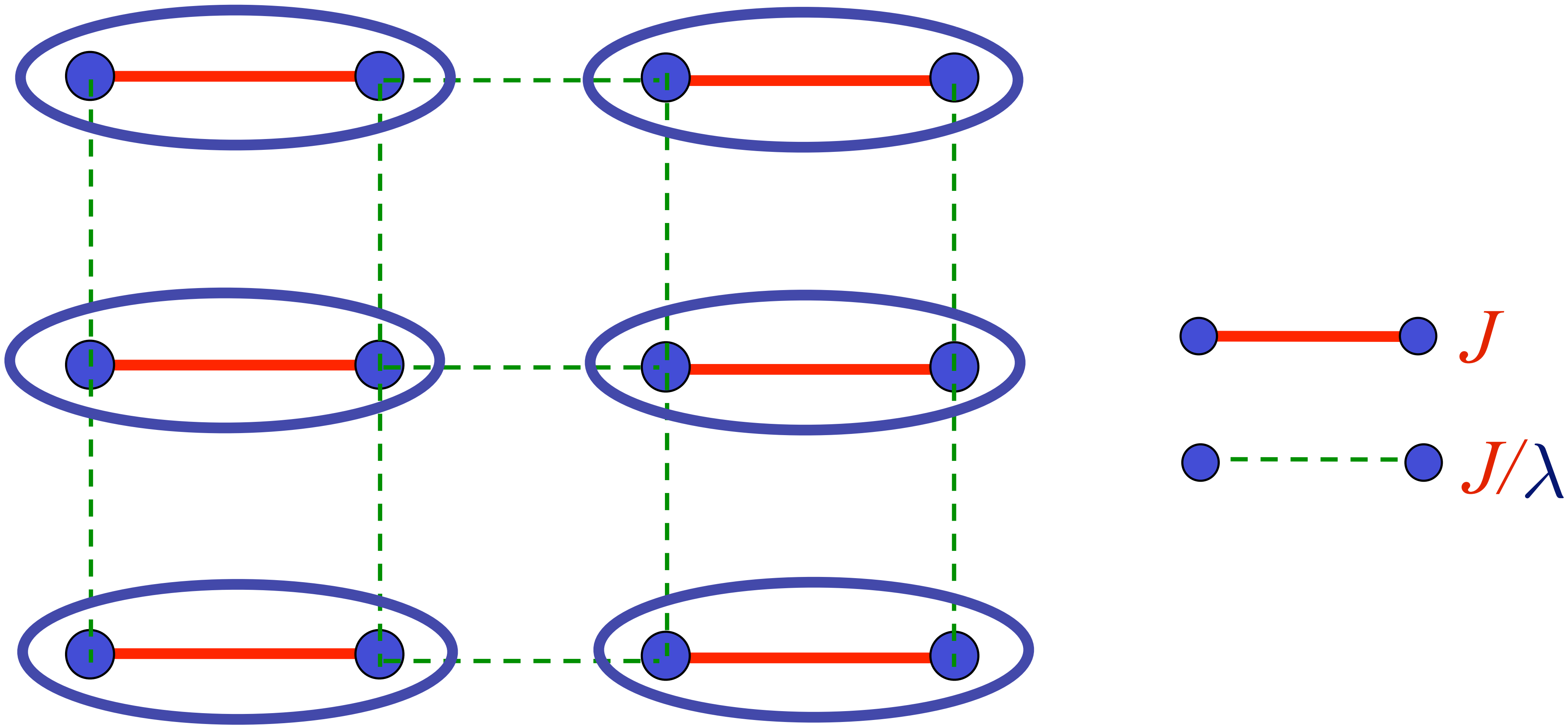

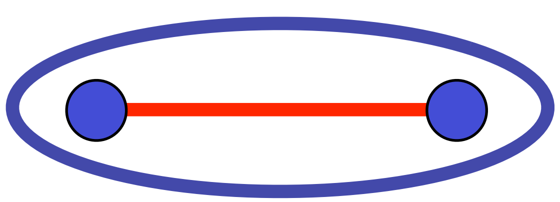

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Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

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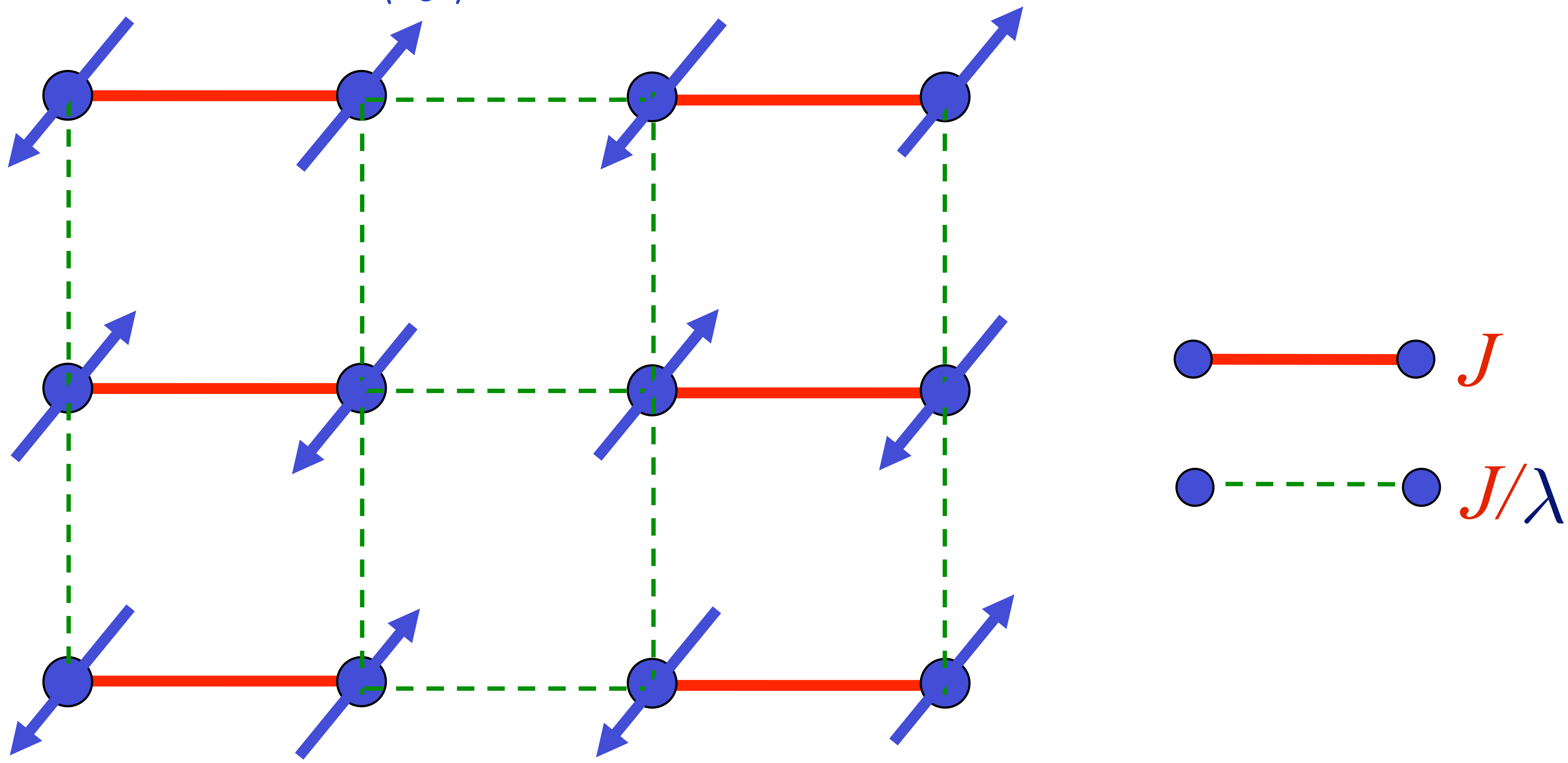

For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

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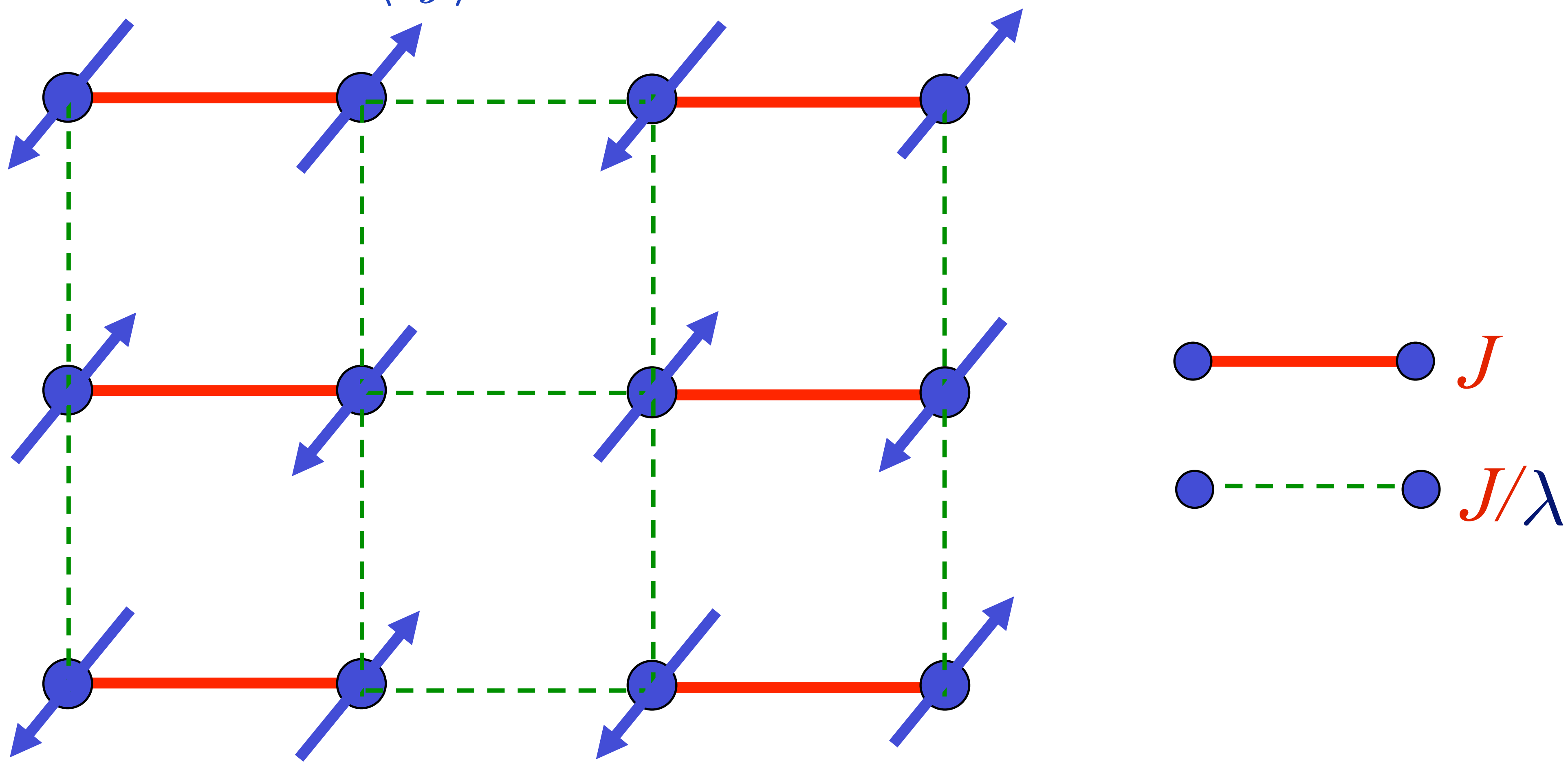

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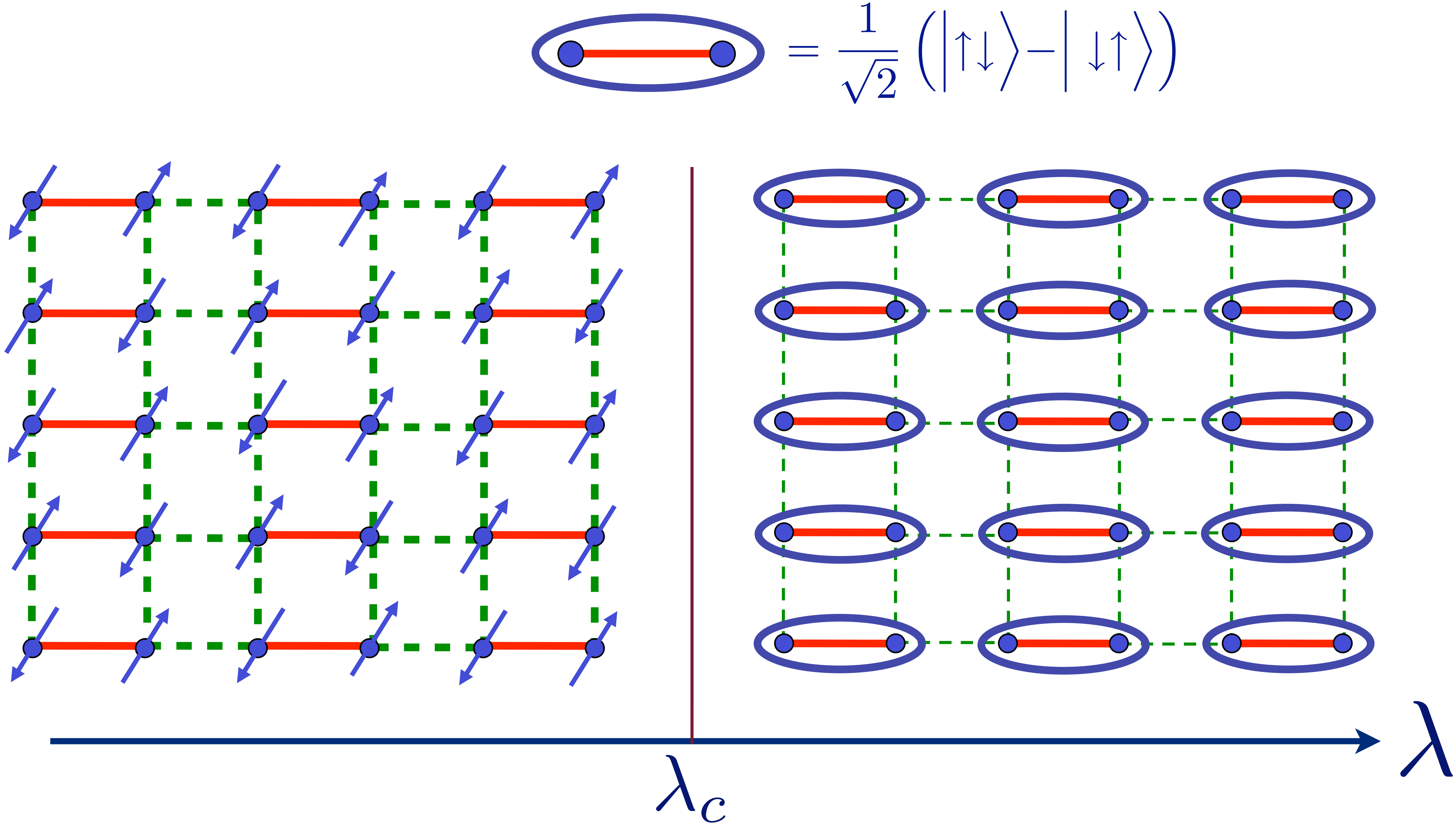
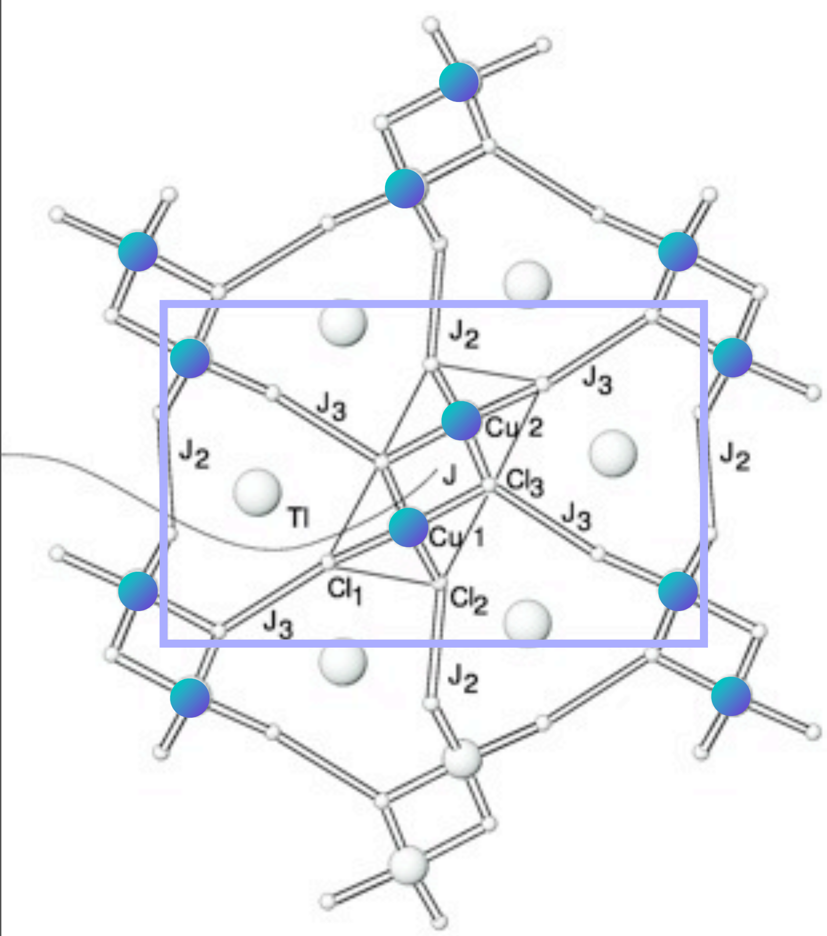
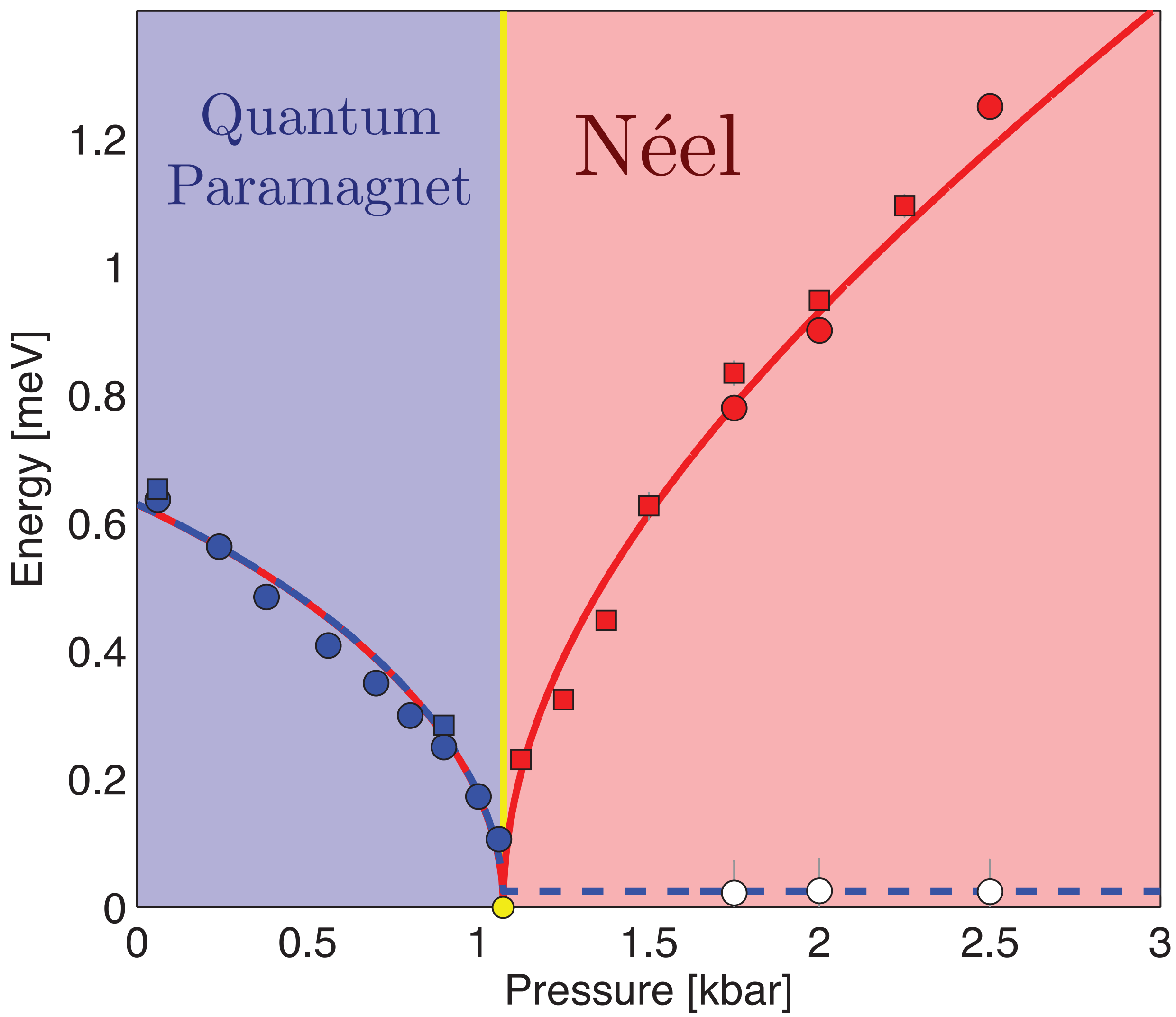
No EPR pairs


$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ

Excitations of $TlCuCl_3$ with varying pressure





Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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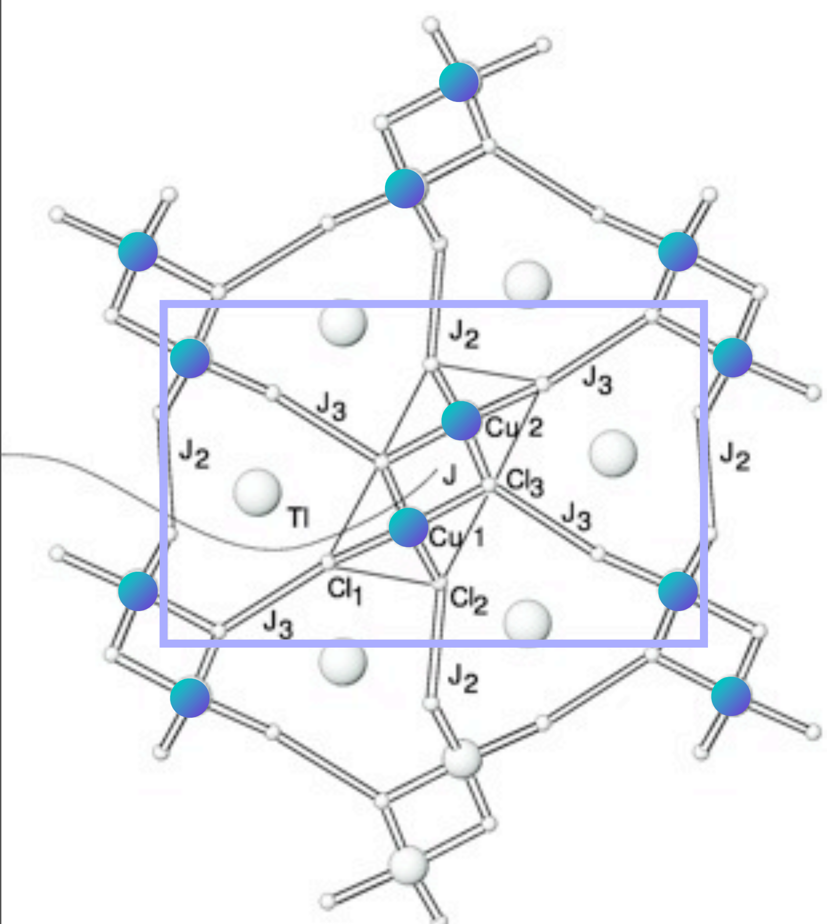

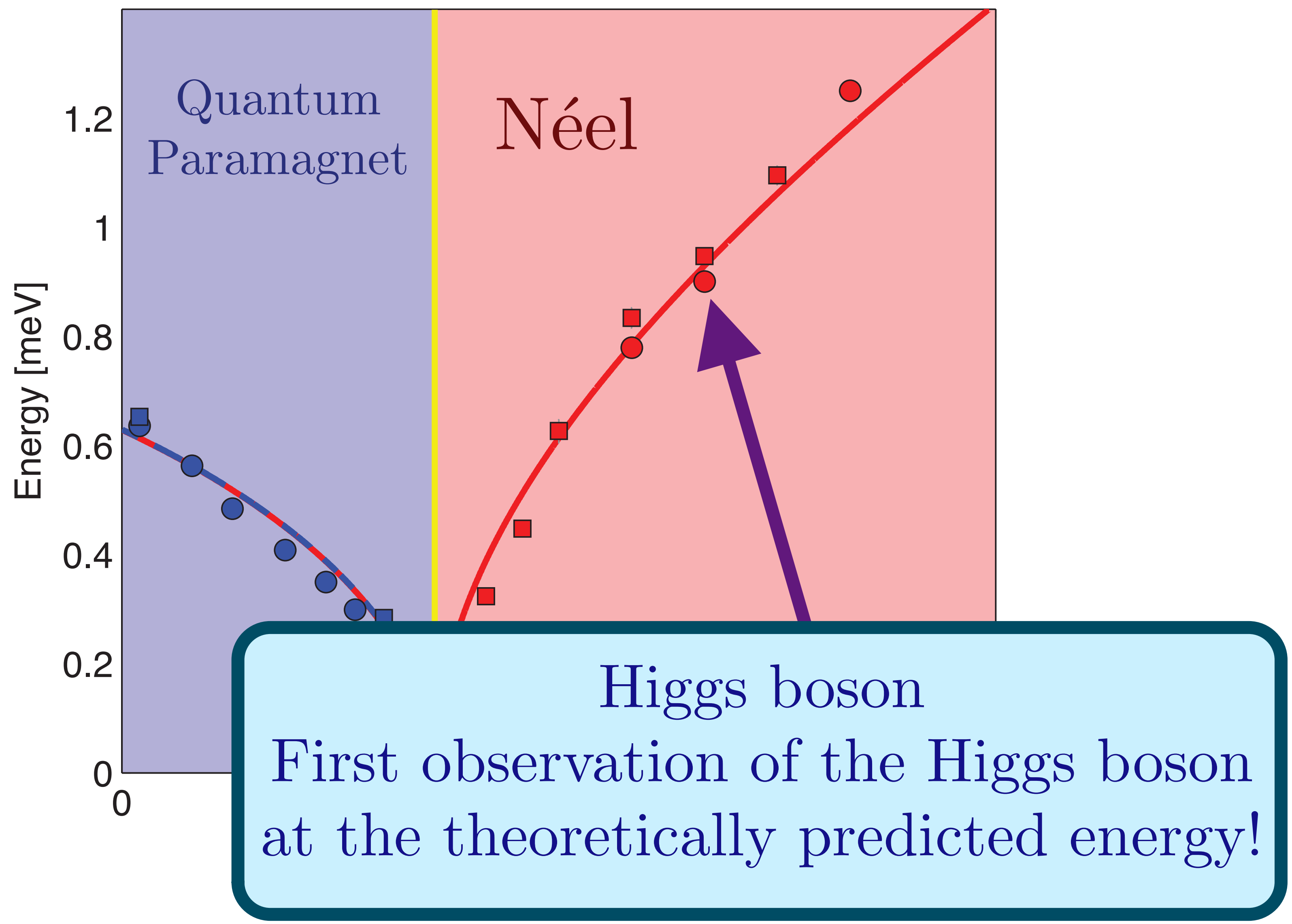

S. Sachdev, arXiv:0901.4103

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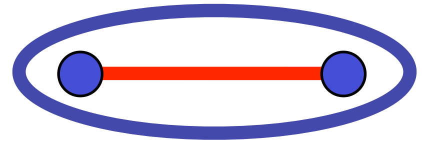

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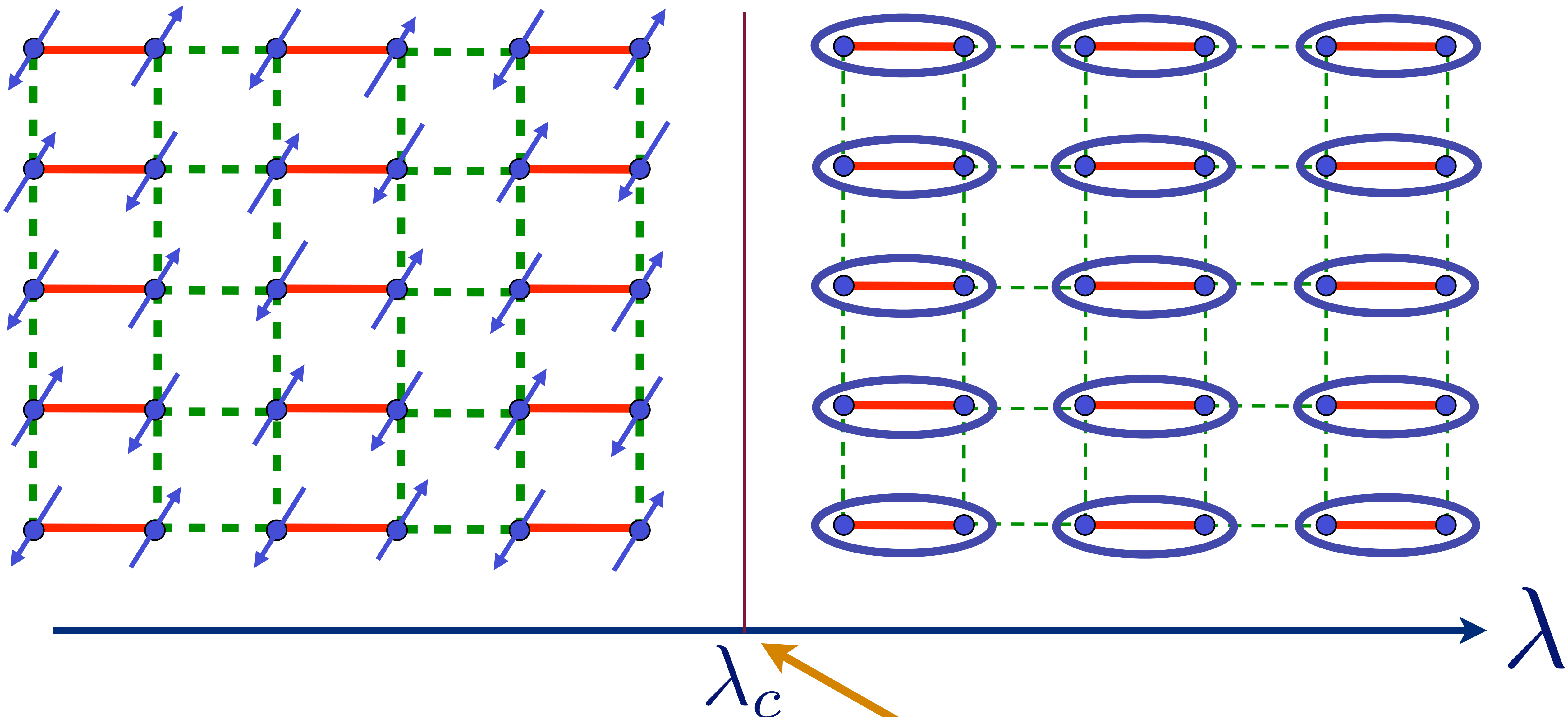
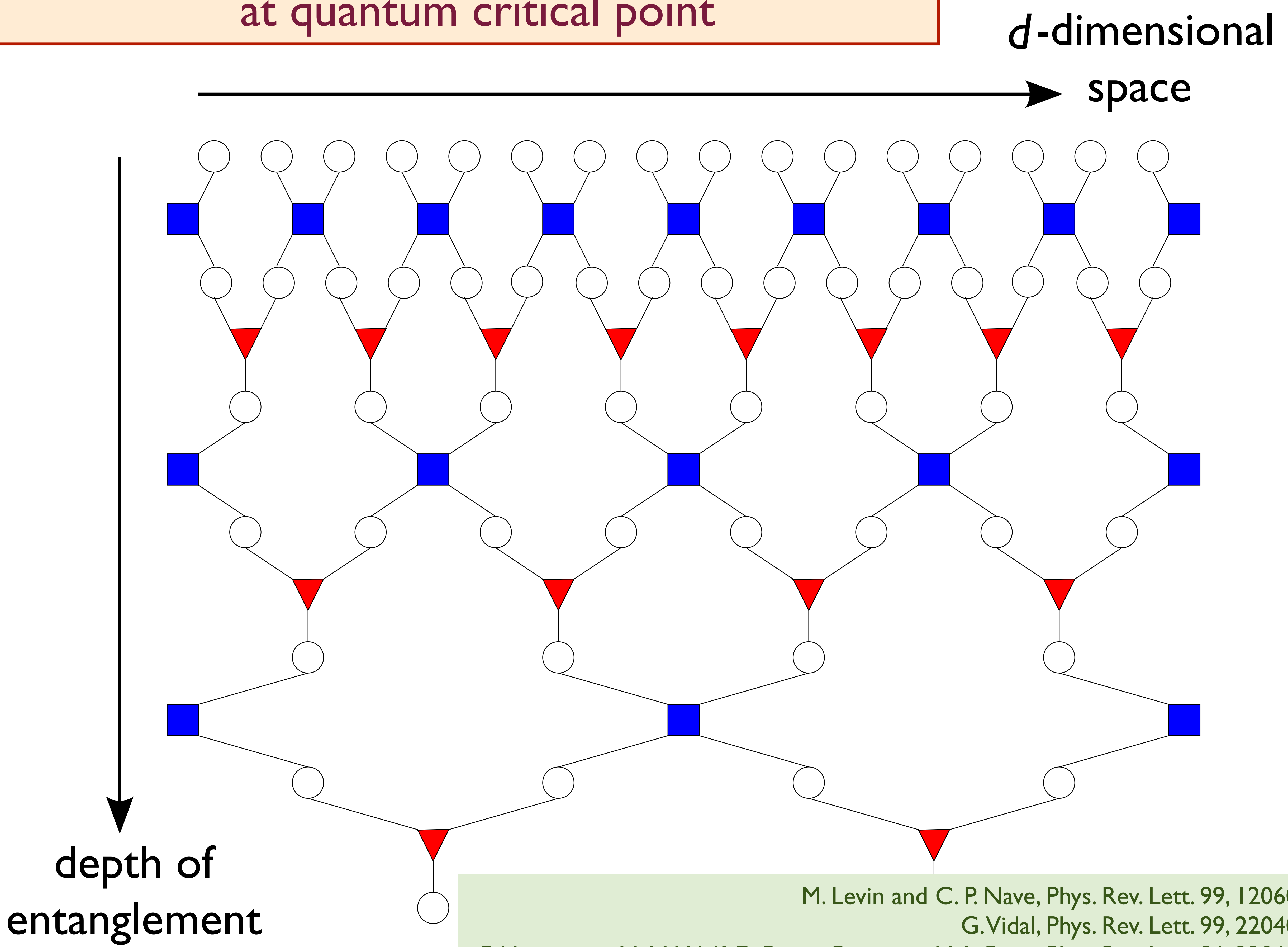


Quantum critical point with non-local entanglement in spin wavefunction

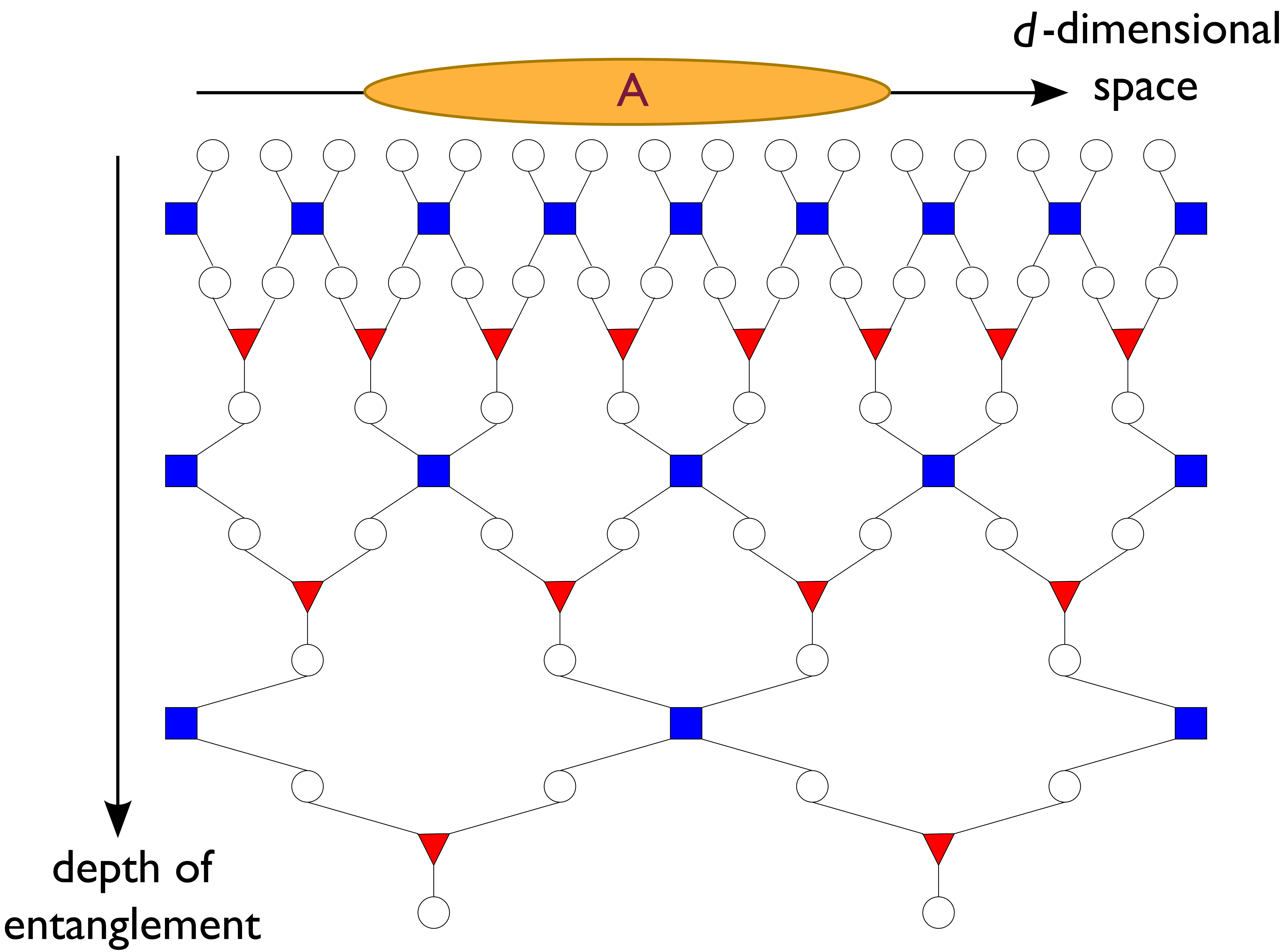
A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).

Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

Entanglement entropy



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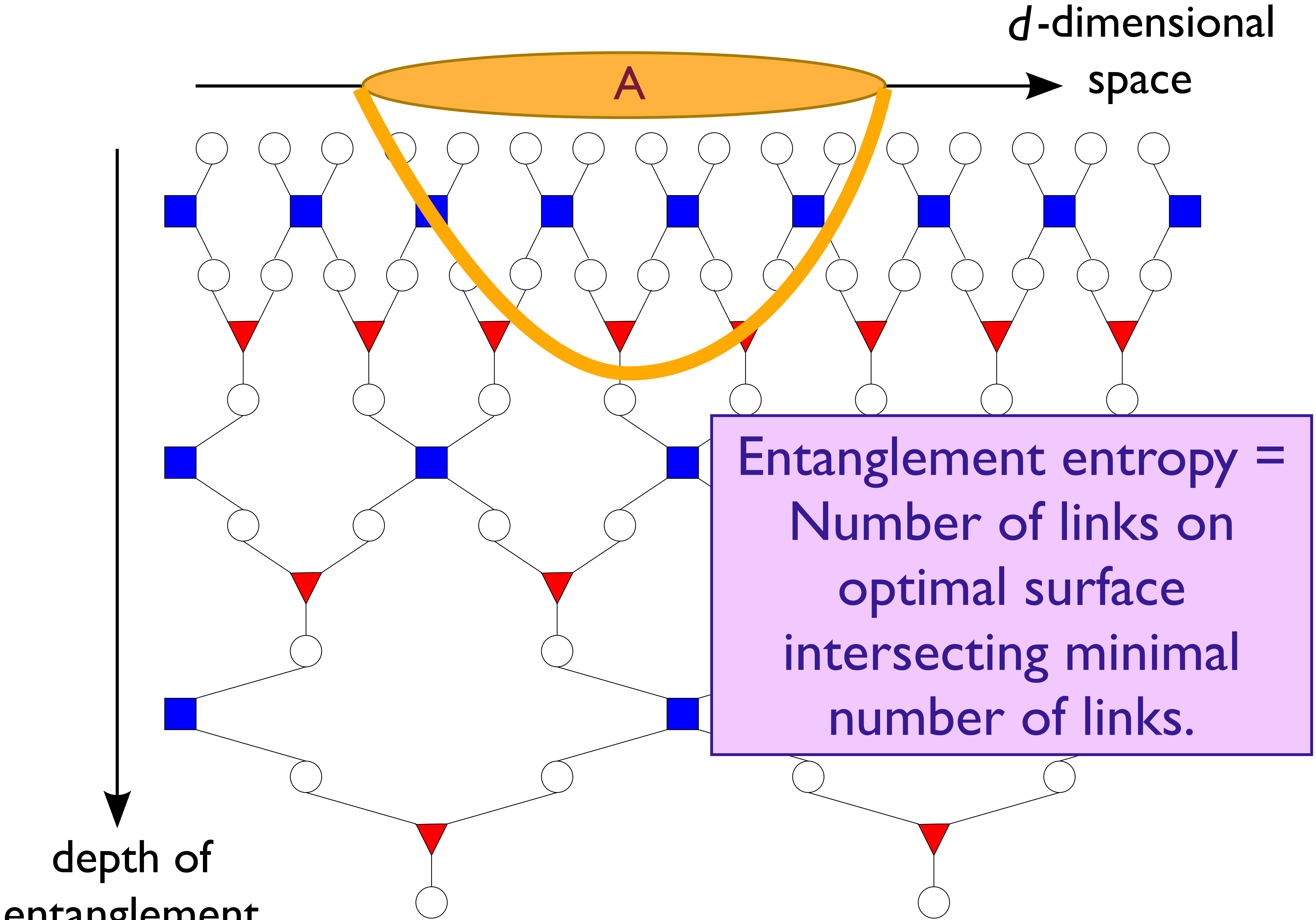
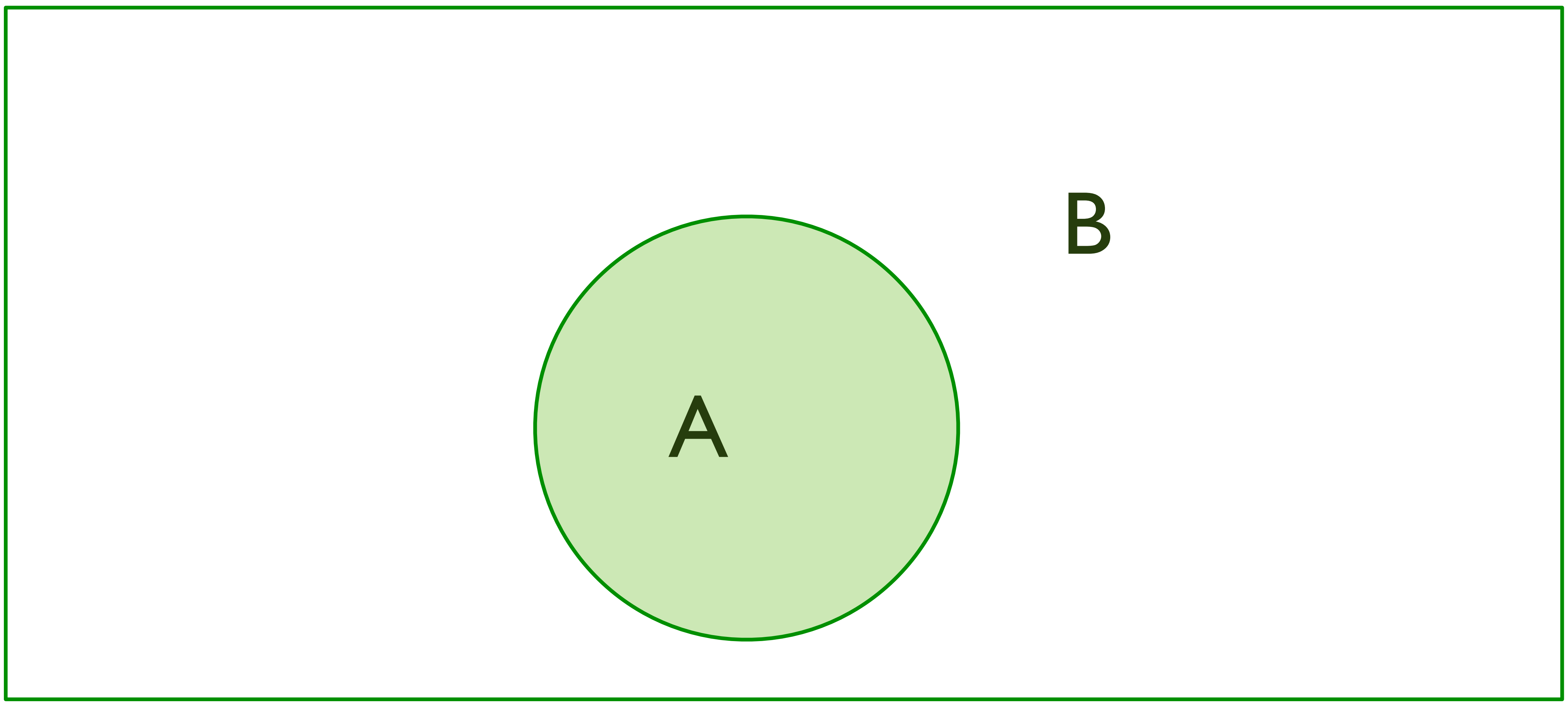


Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Brian Swingle, arXiv:0905.1317

Long-range entanglement at the quantum critical point

- Long-range entanglement: entanglement entropy obeys $S_{EE} = aL - \gamma$, where γ is a universal number associated with the quantum critical point.



M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Characteristics of quantum critical point

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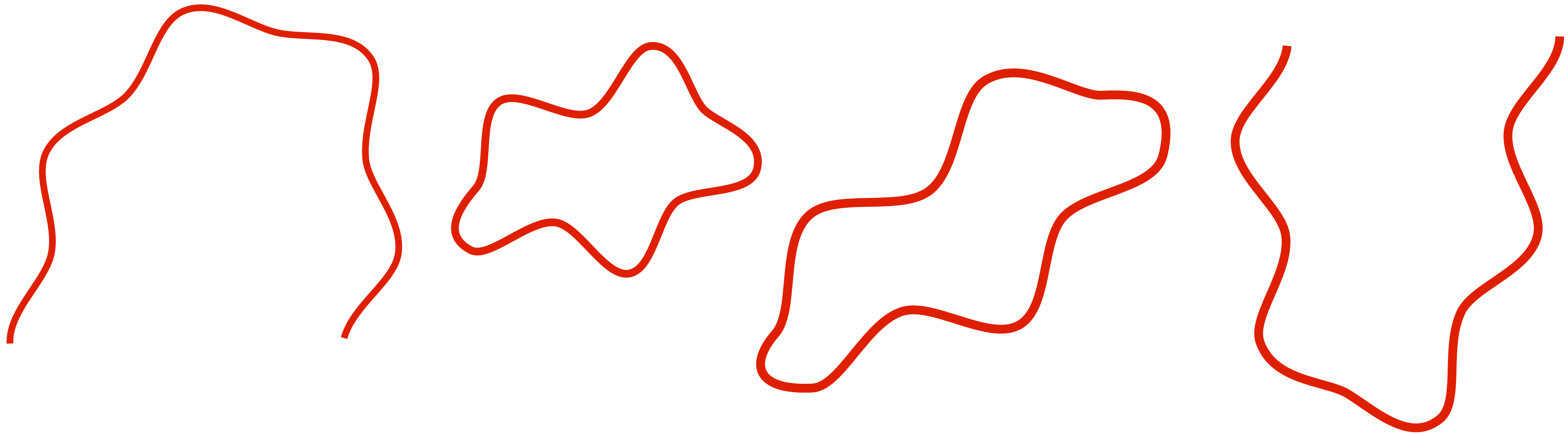
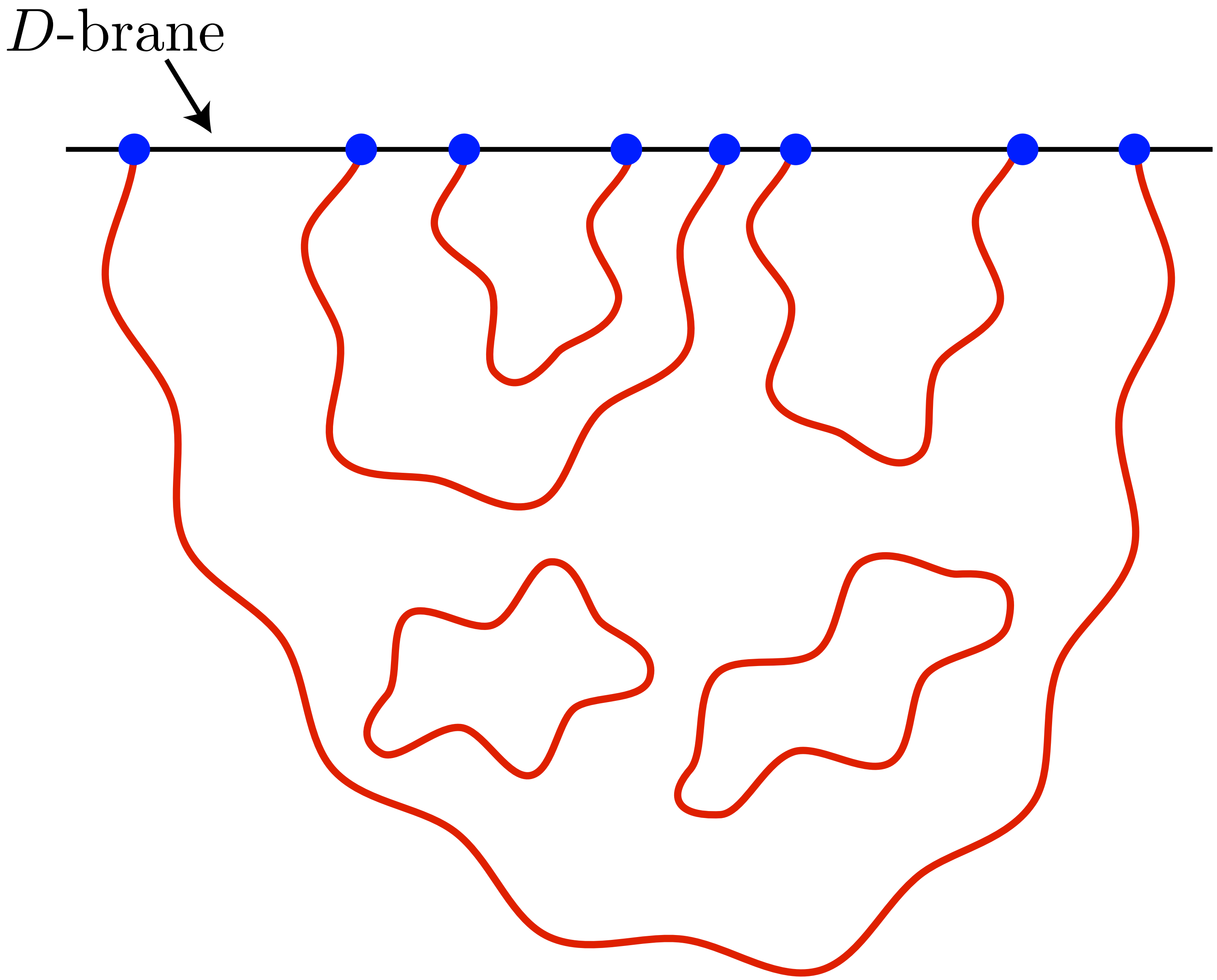
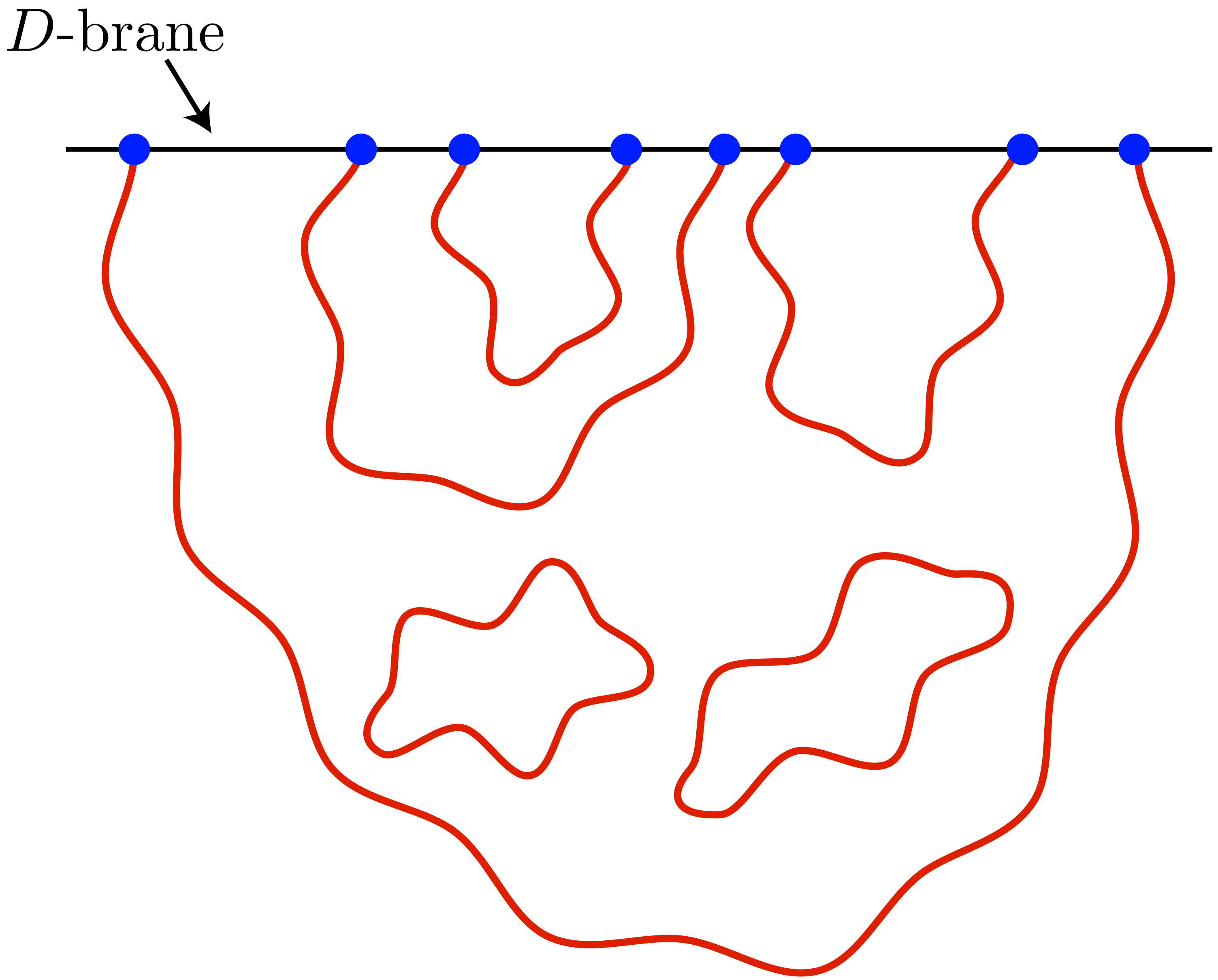


- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



- A D-brane is a d-dimensional surface on which strings can end.



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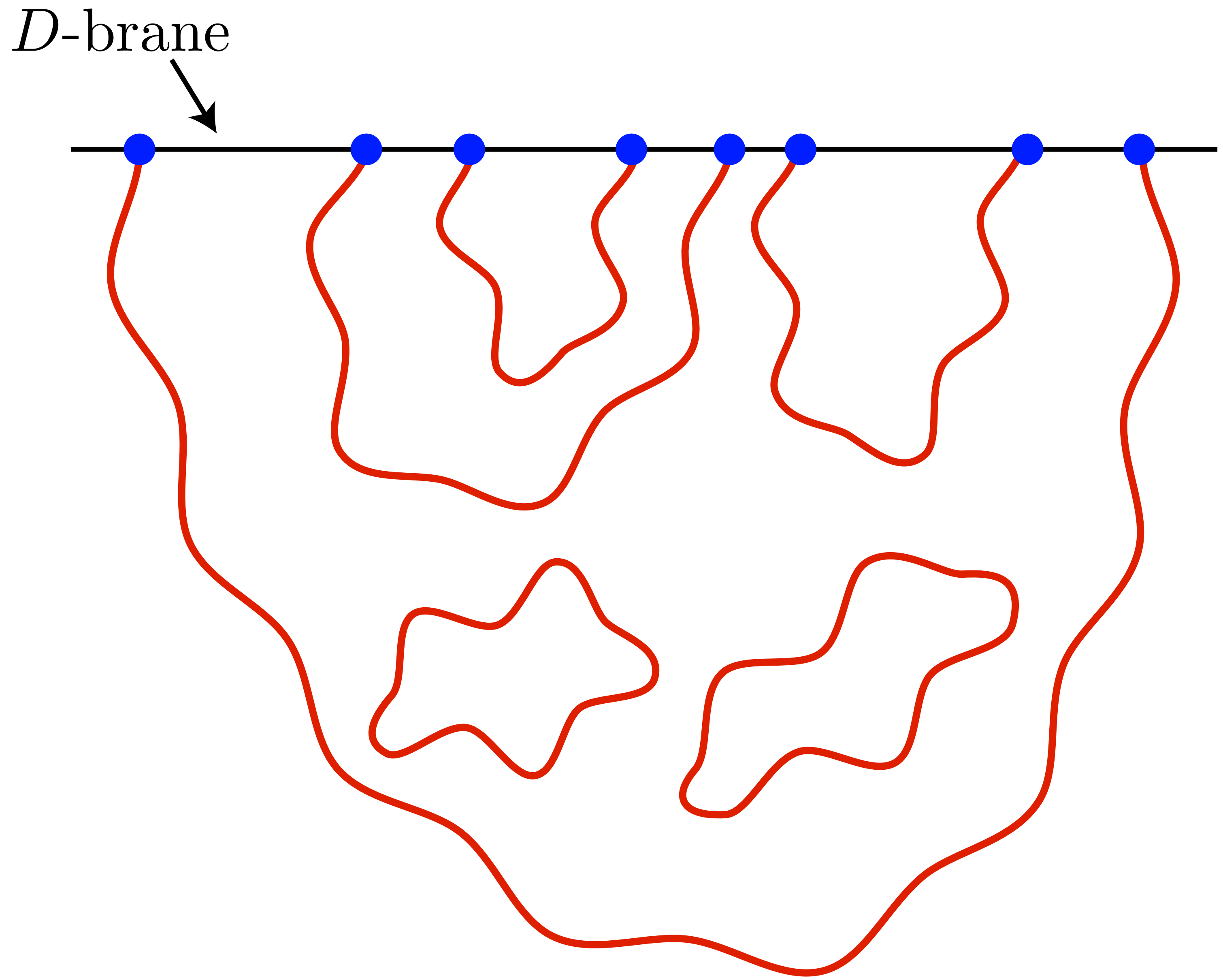
- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us.



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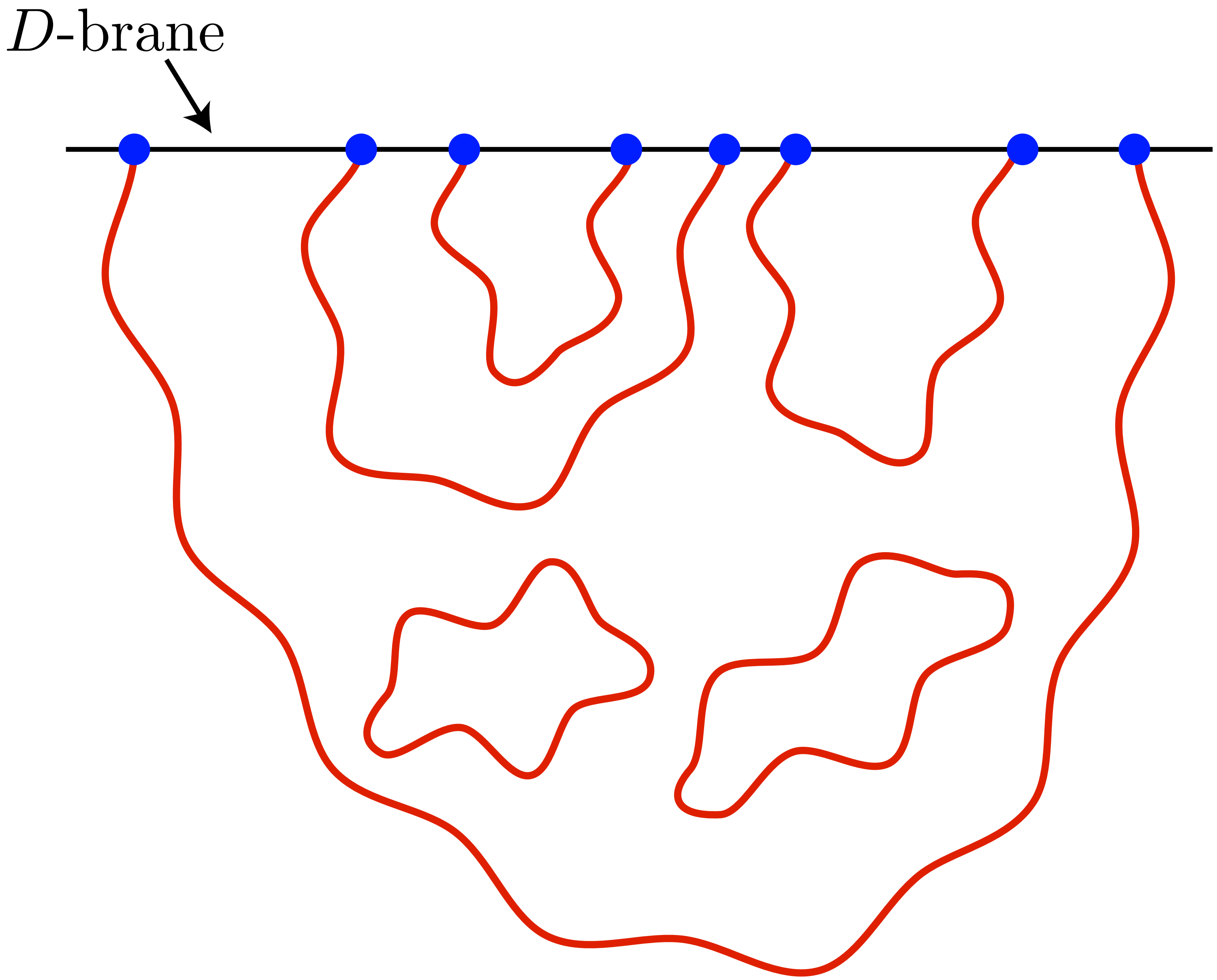

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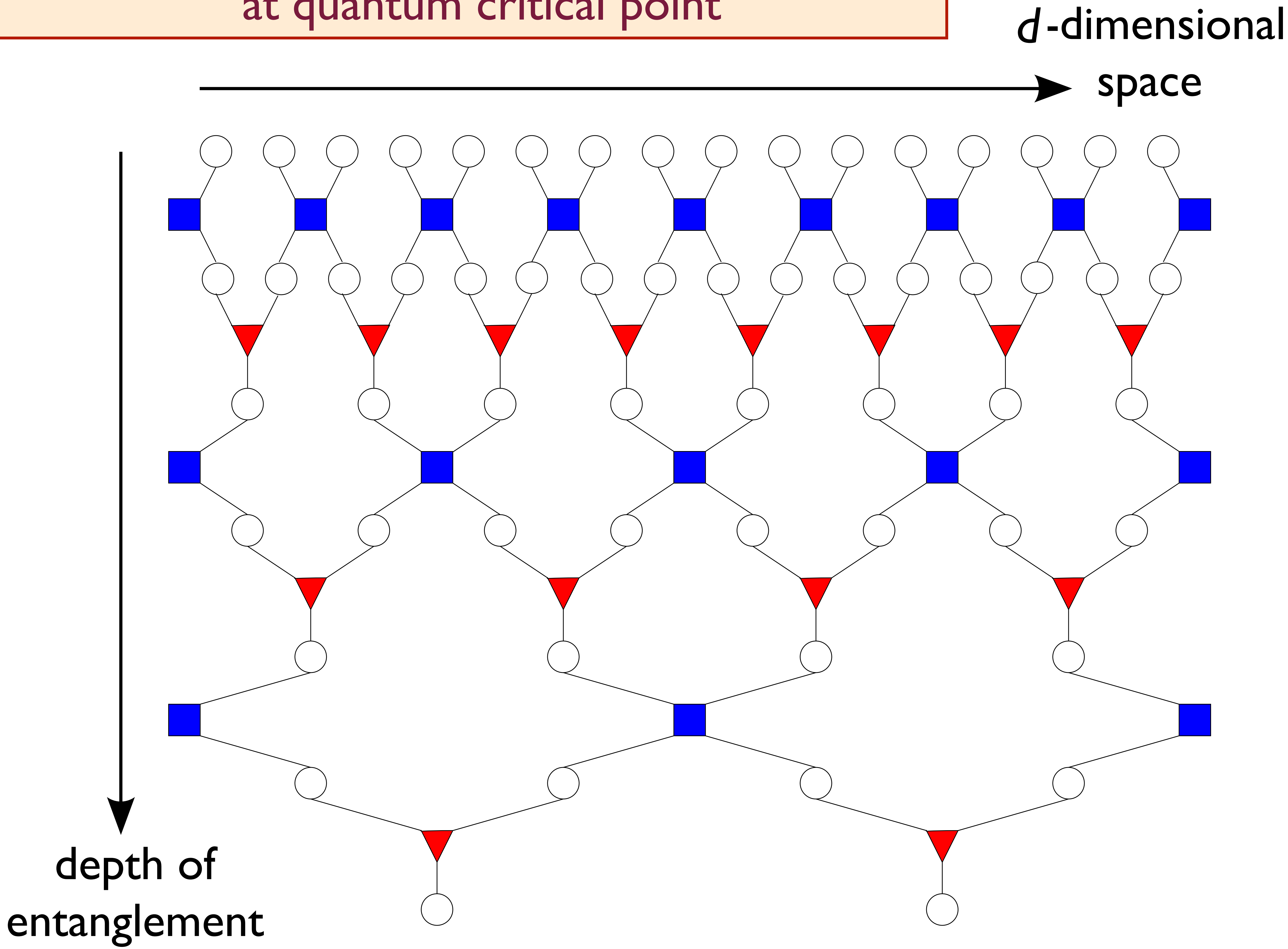
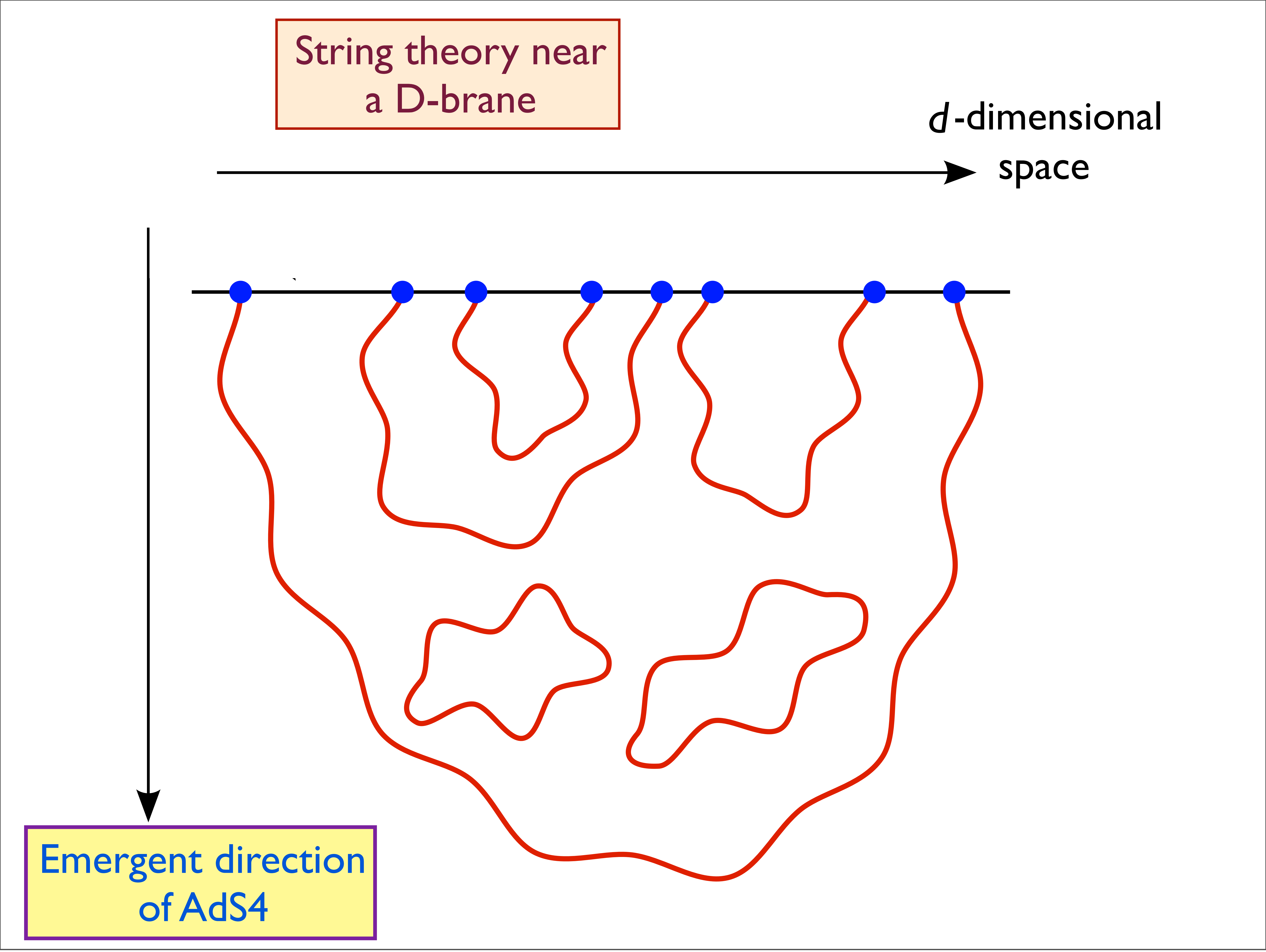
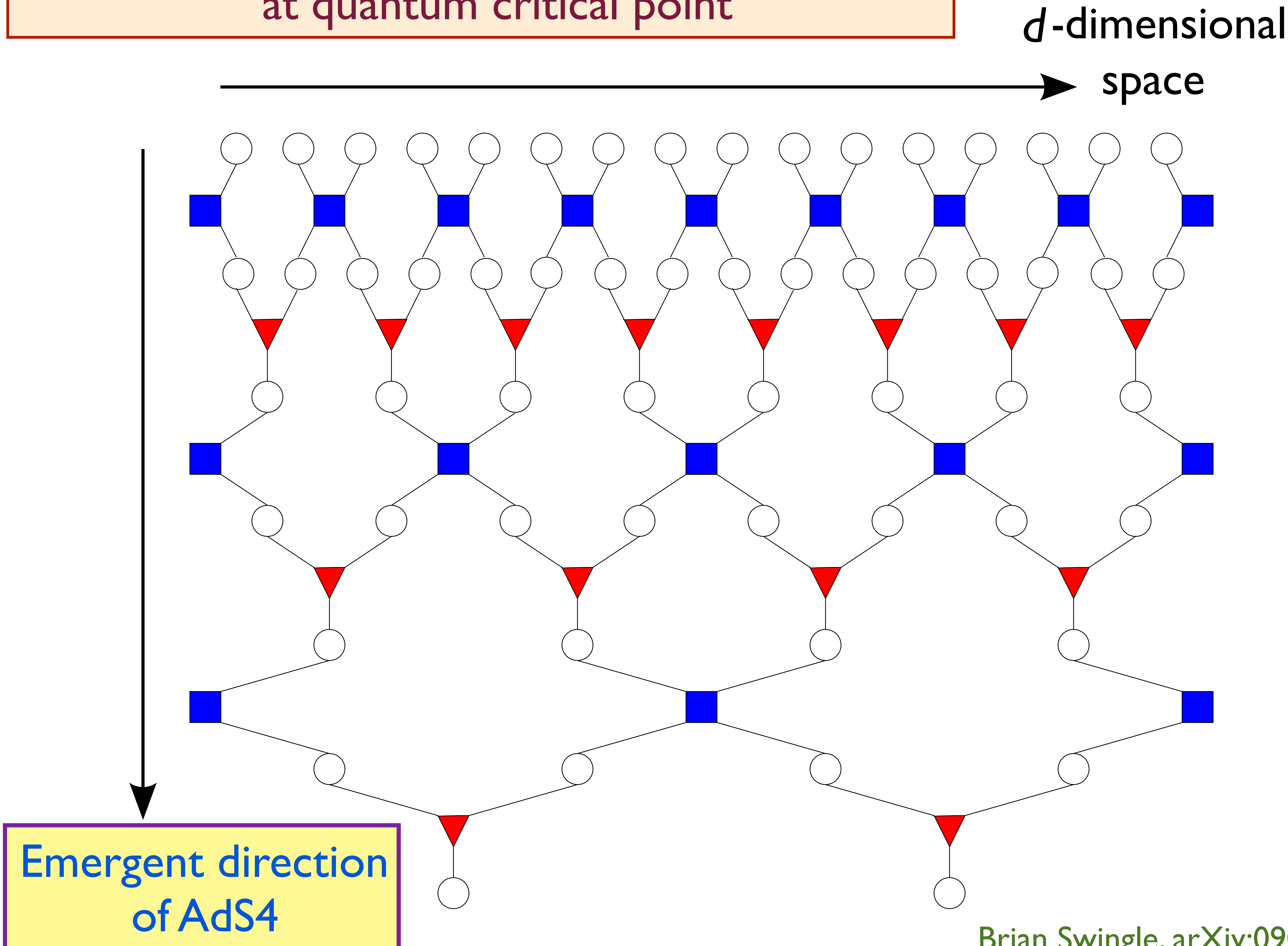


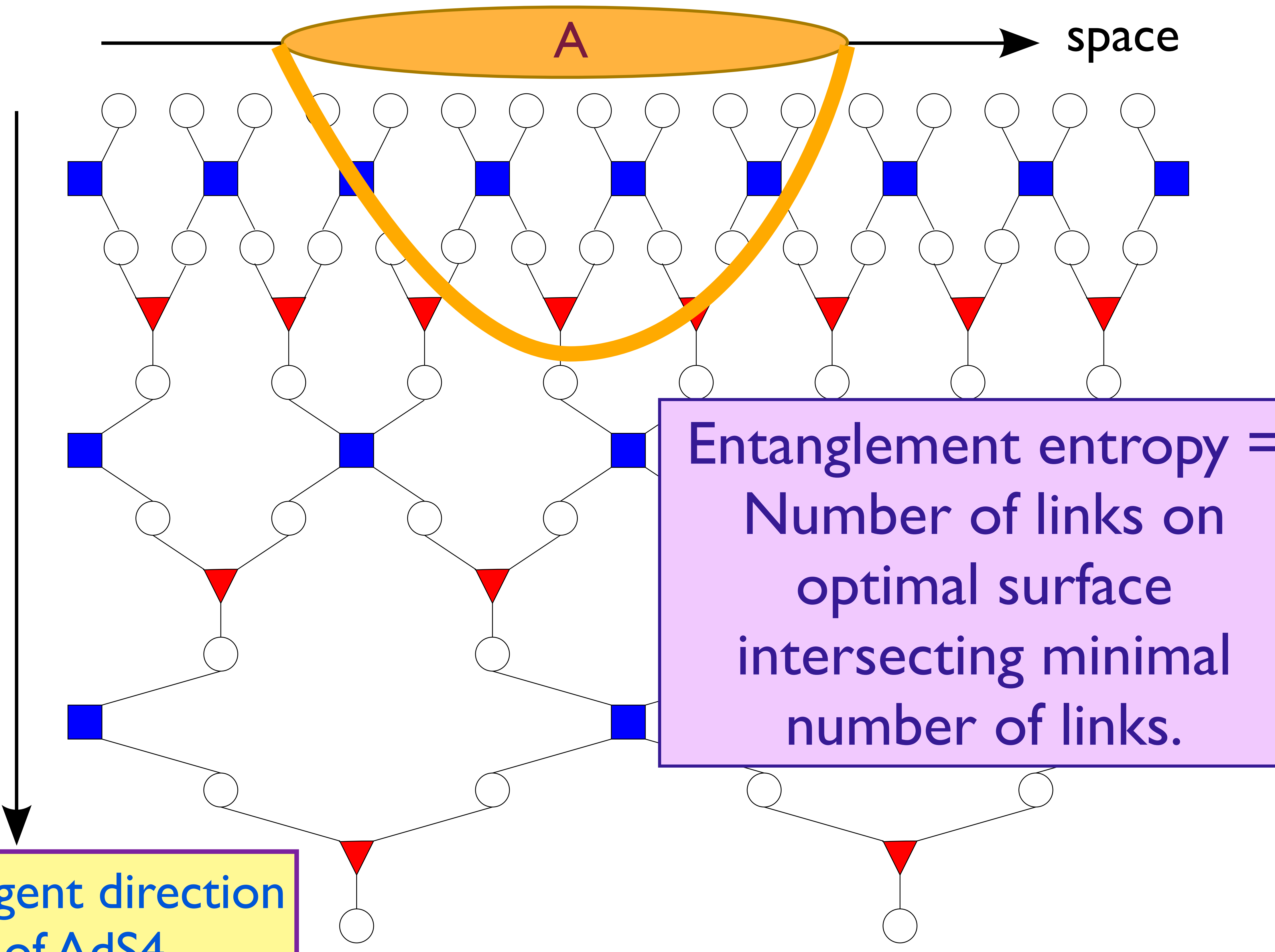

String theory near a D-brane
d-dimensional space
Emergent direction of AdS4

Tensor network representation of entanglement at quantum critical point



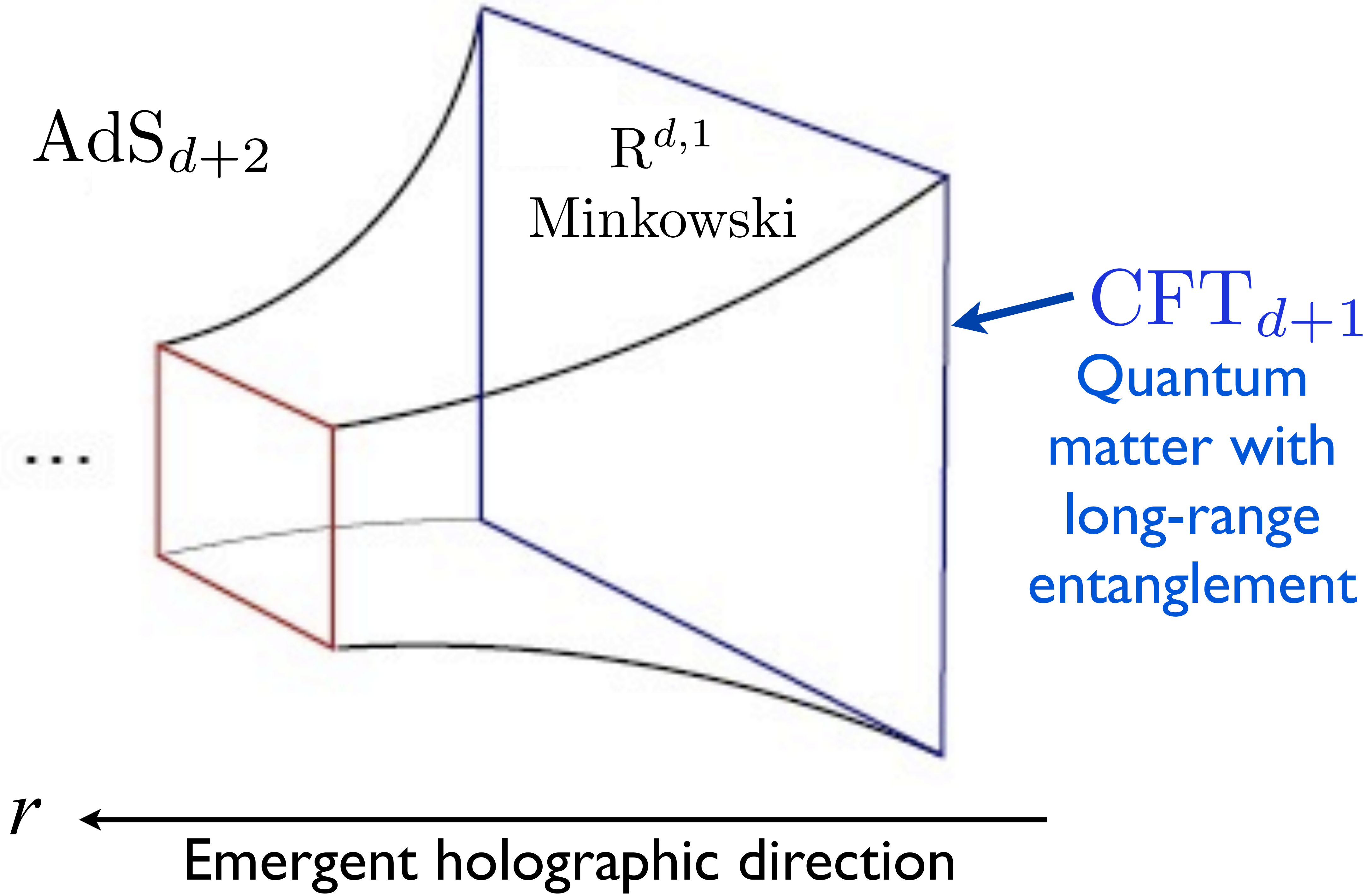
Brian Swingle, arXiv:0905.1317

Entanglement entropy

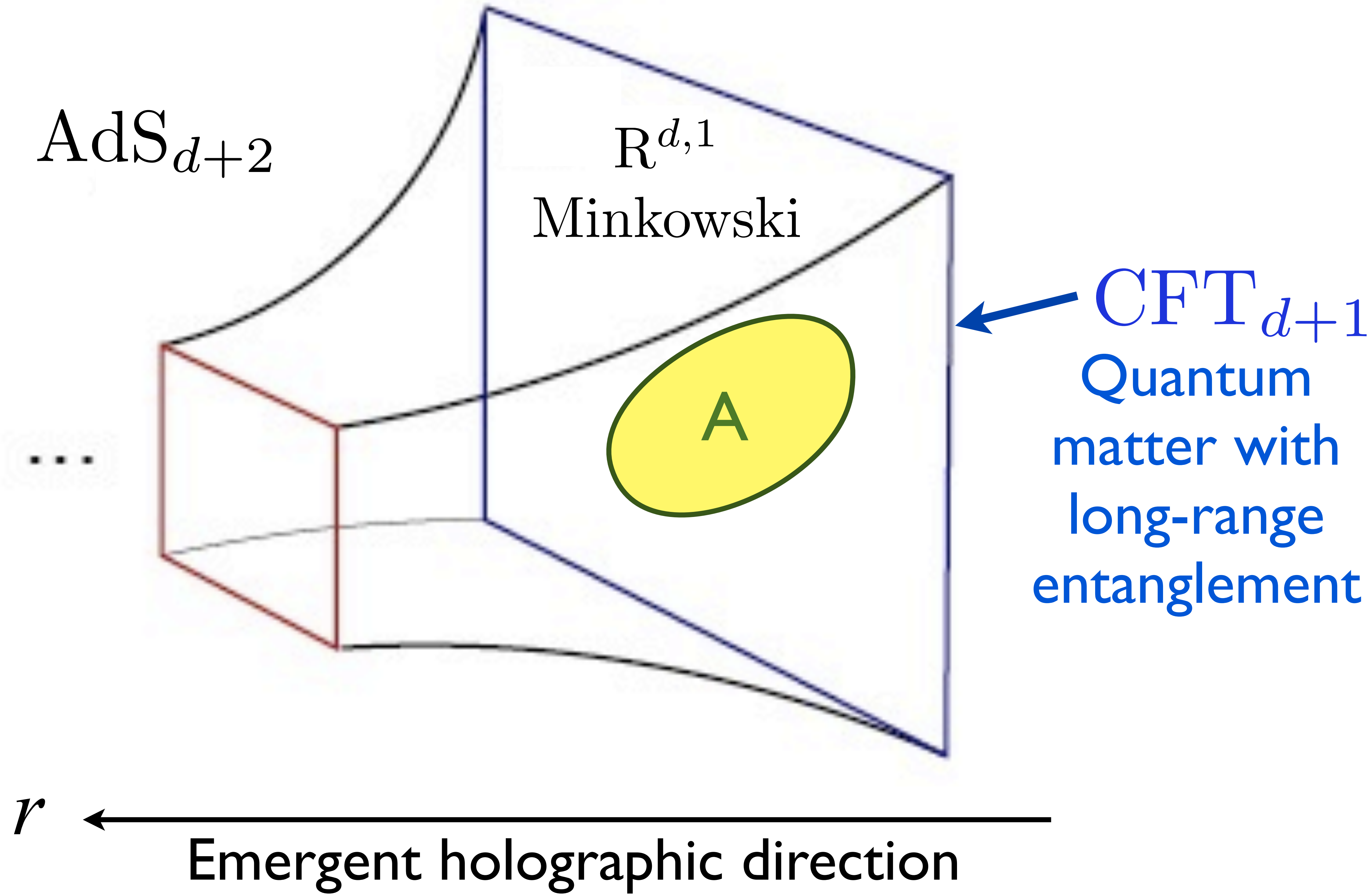


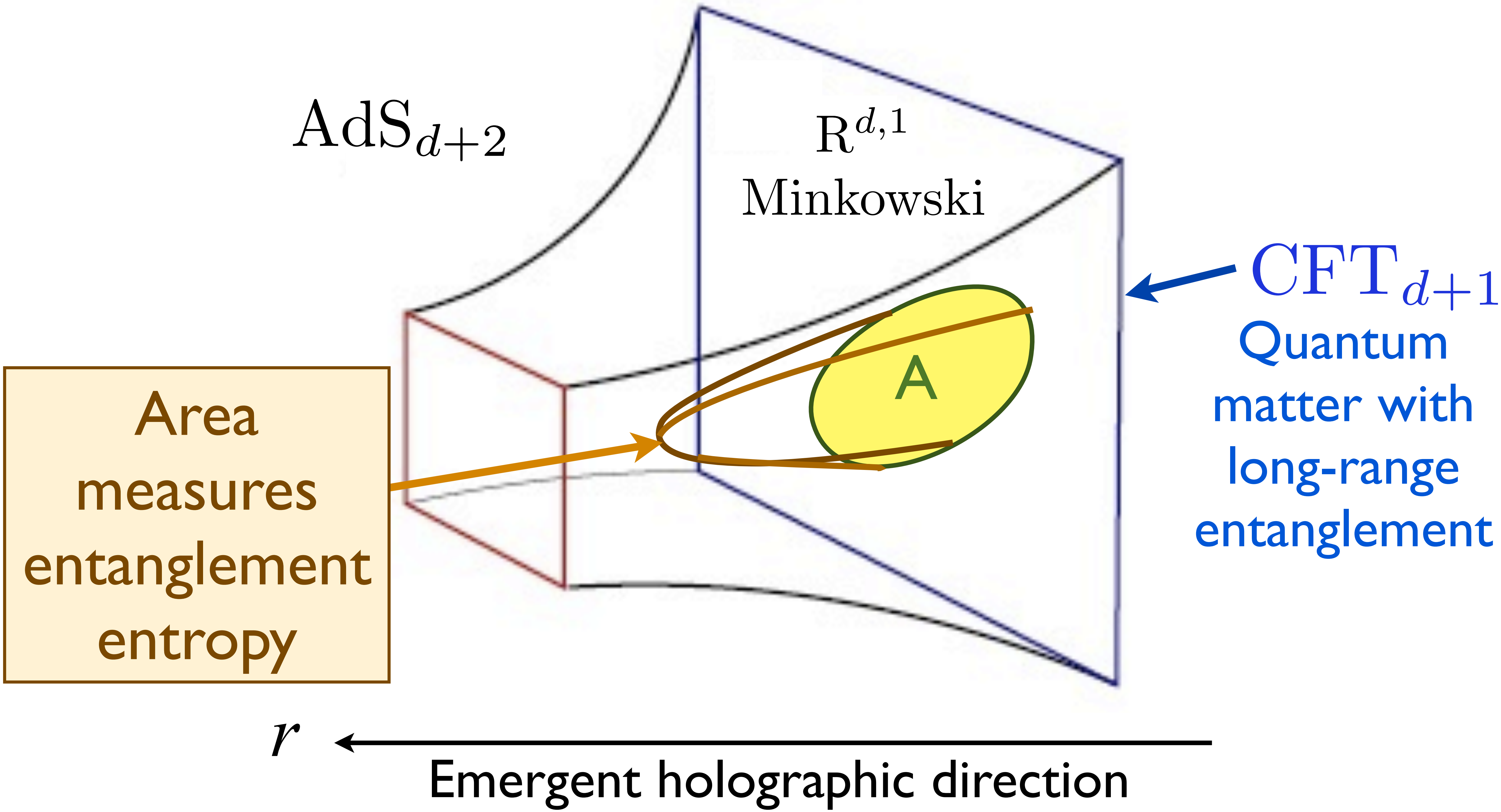
Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Emergent direction of AdS4



J. McGreevy, arXiv0909.0518


AdS_{d+2}
$\mathrm{R}^{d,1}$
Minkowski
CFT_{d+1}
Quantum matter with long-range entanglement
A
...
r
Emergent holographic direction

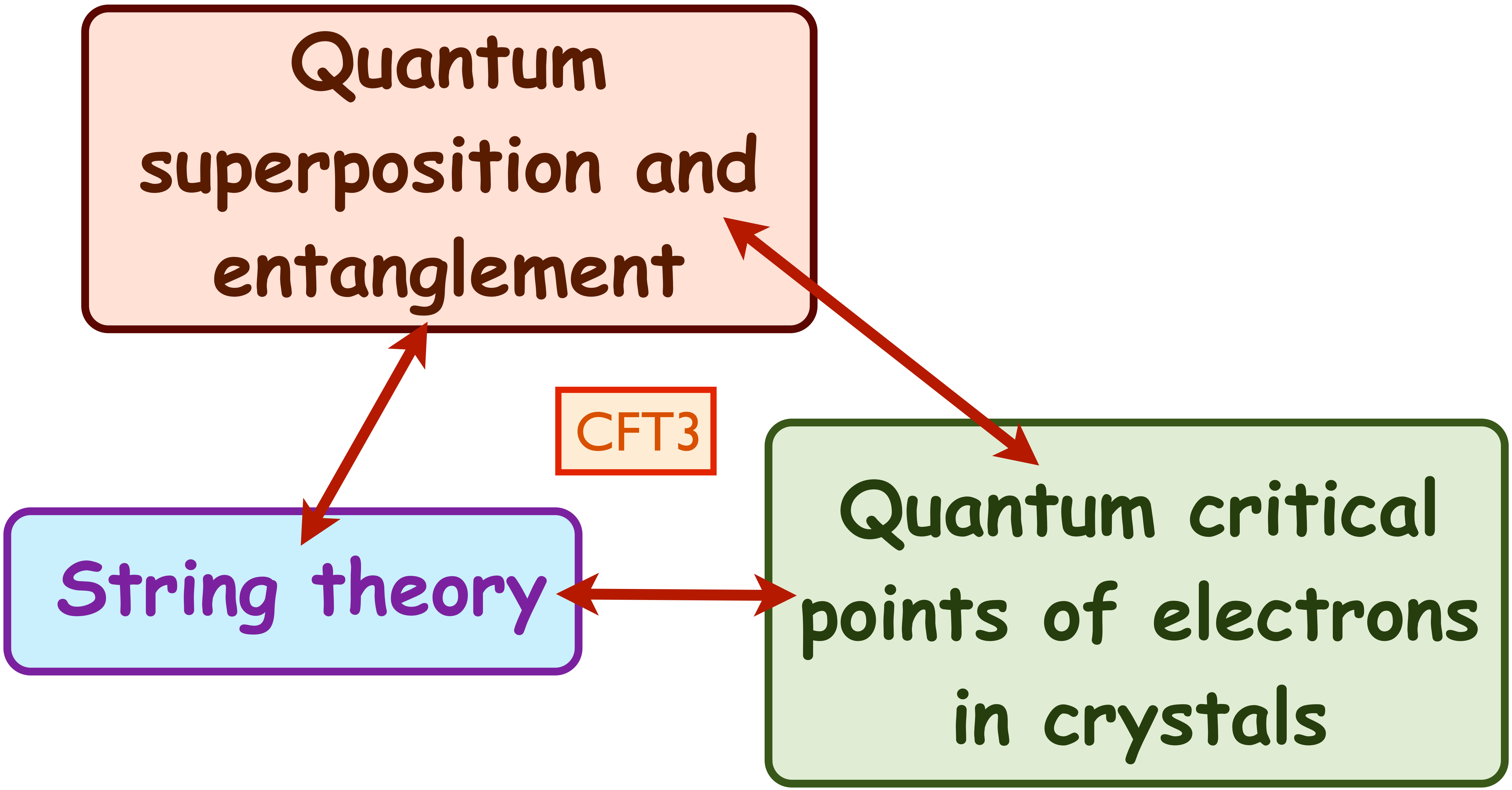


S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

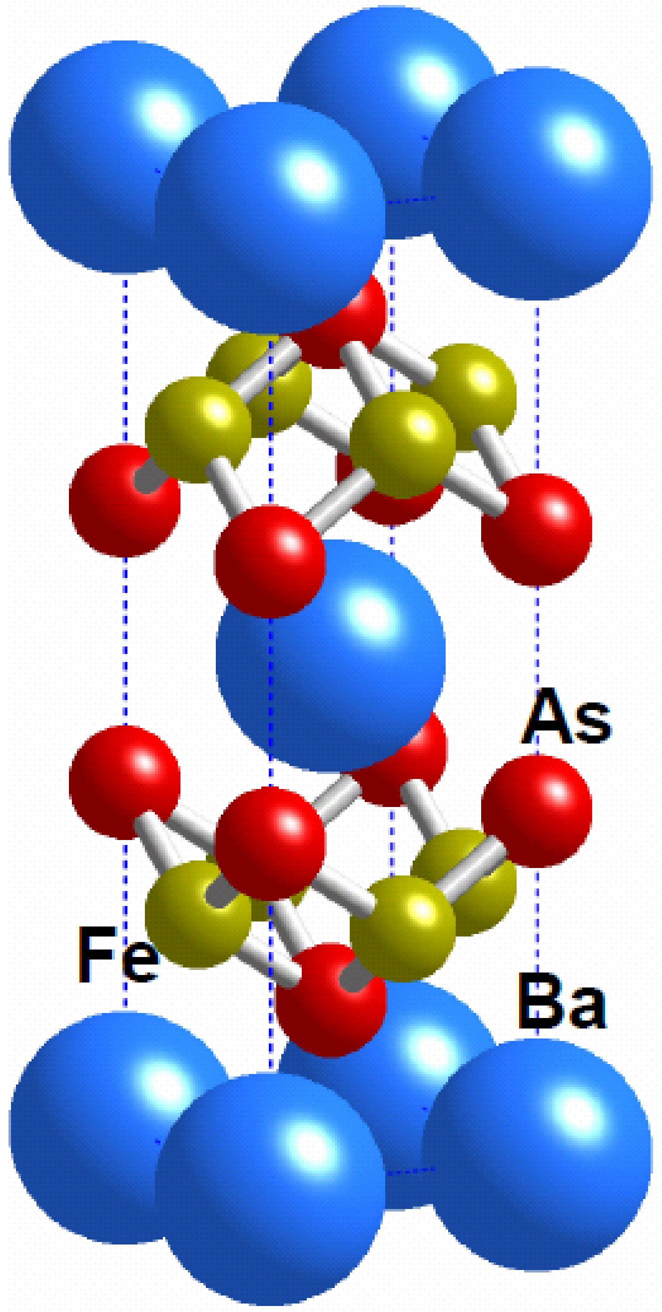

Quantum superposition and entanglement
CFT3
String theory
Quantum critical points of electrons in crystals

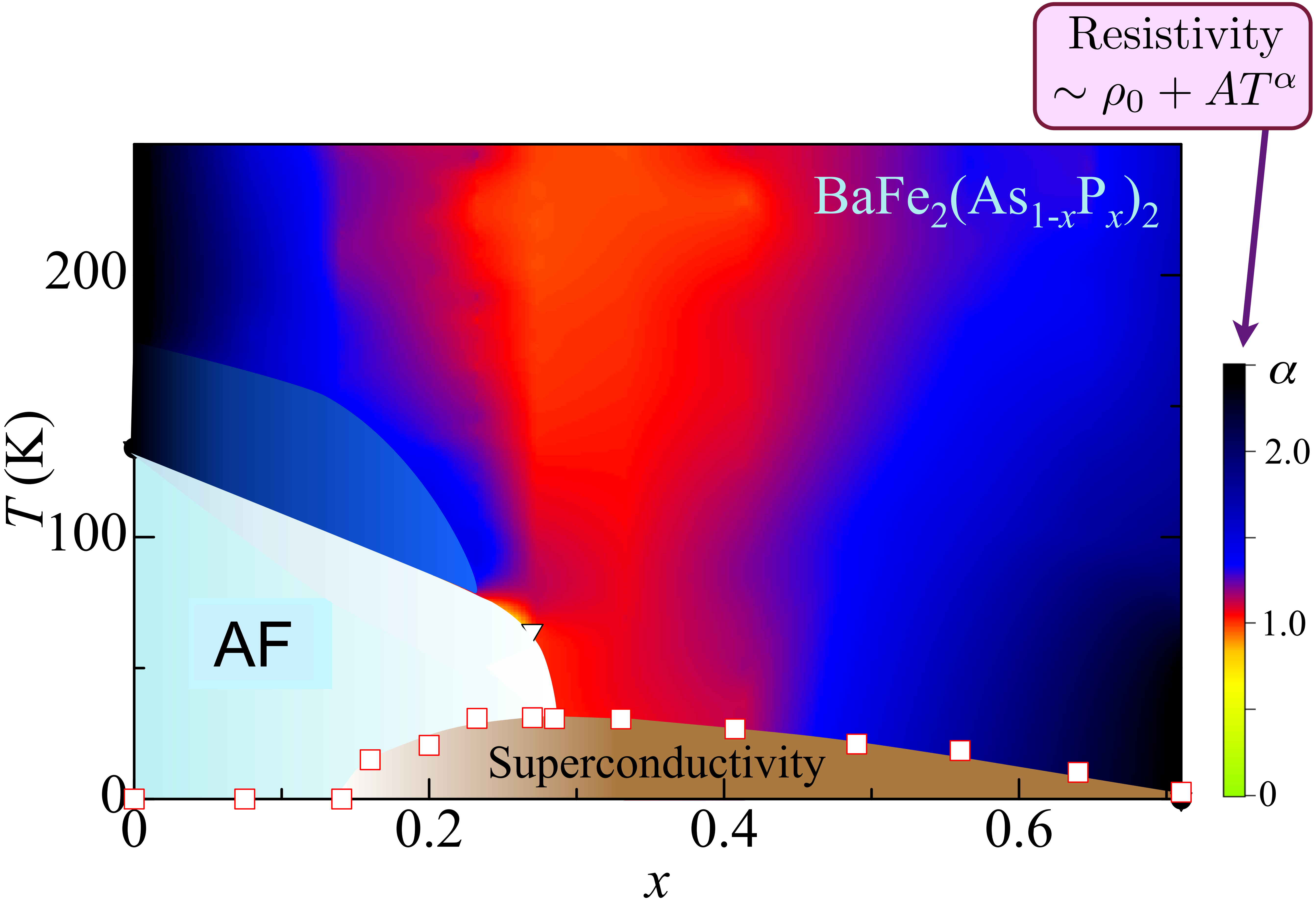
Metals, "strange metals", and high temperature superconductors

Insights from gravitational "duals"

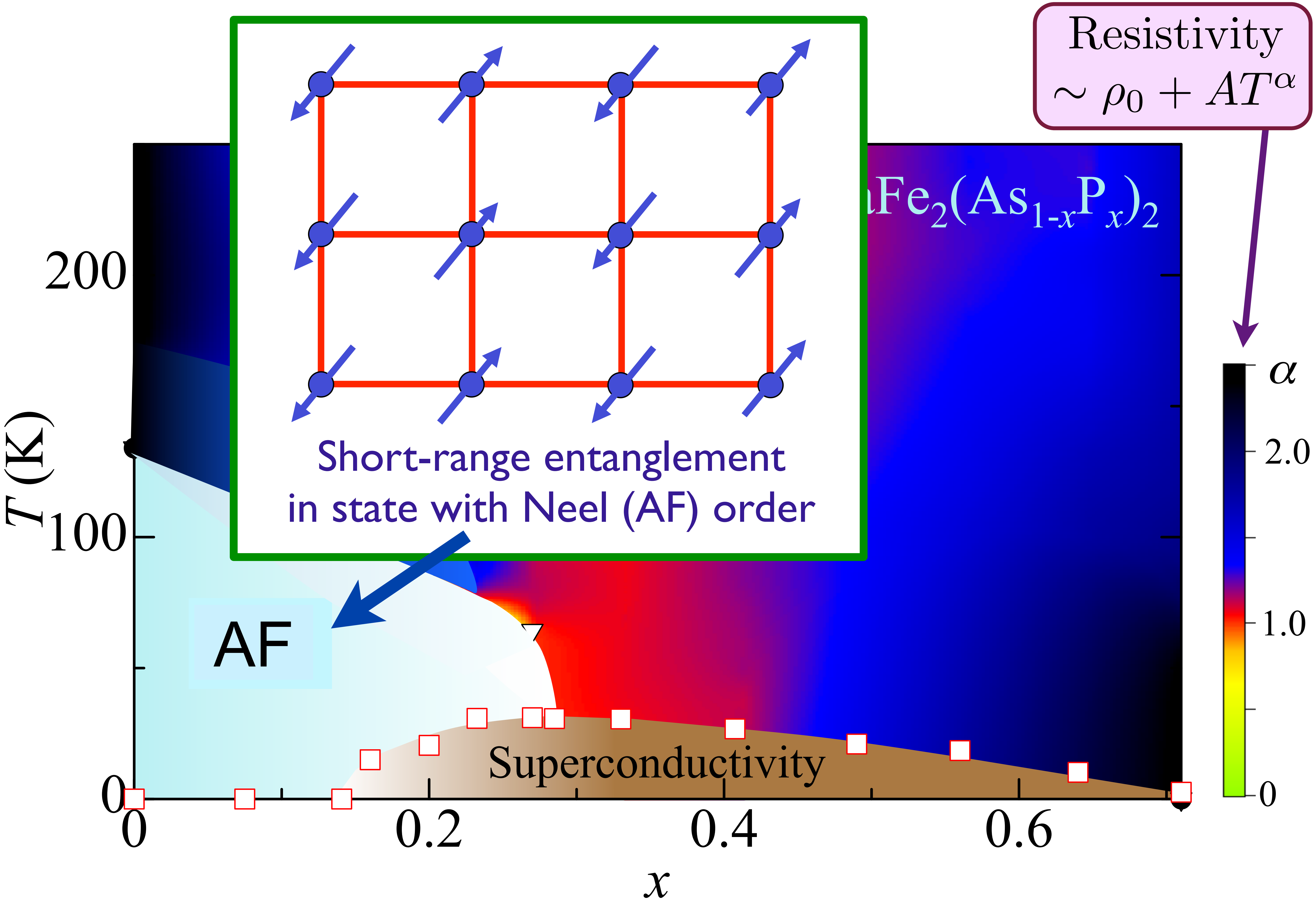
Iron pnictides:

a new class of high temperature superconductors

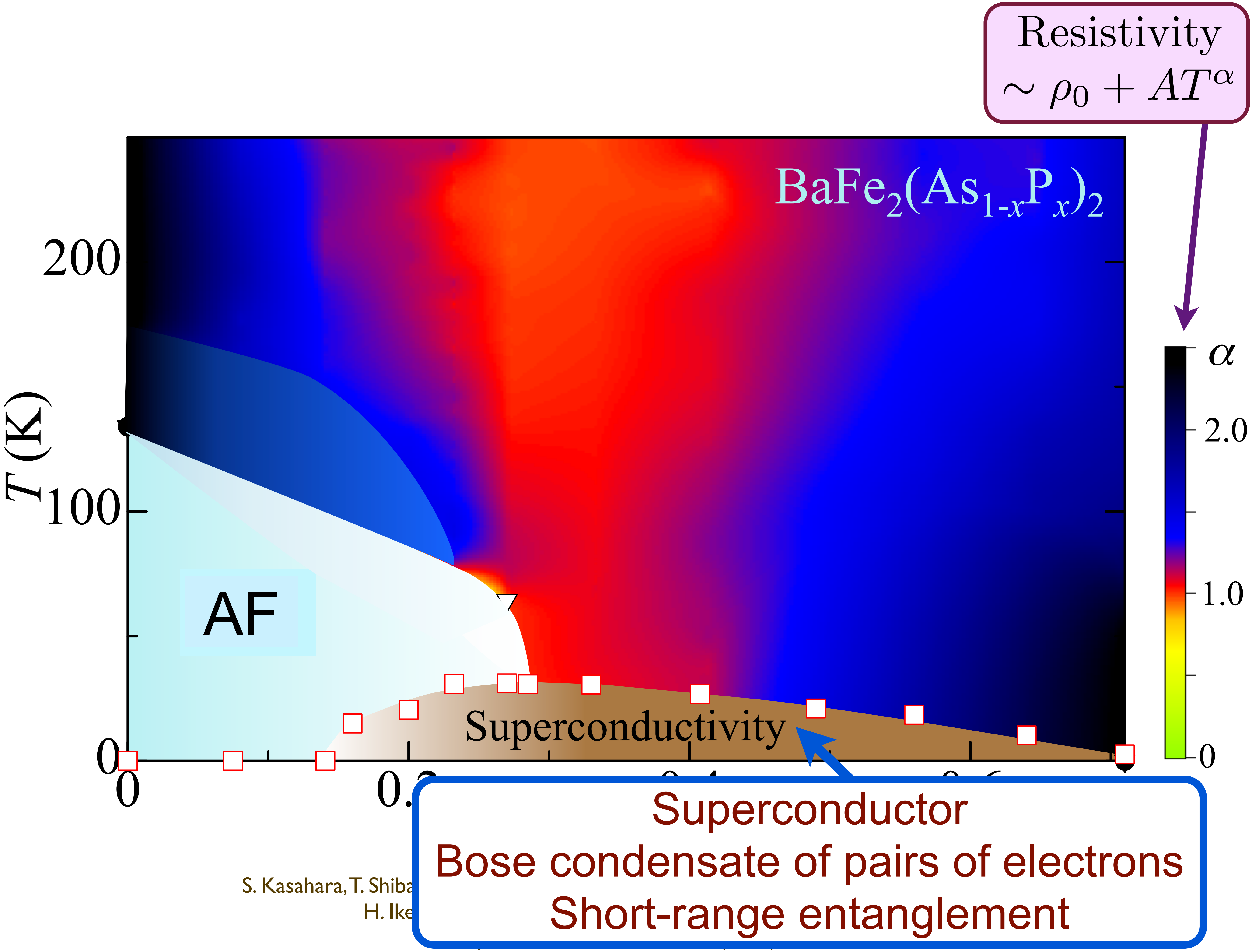


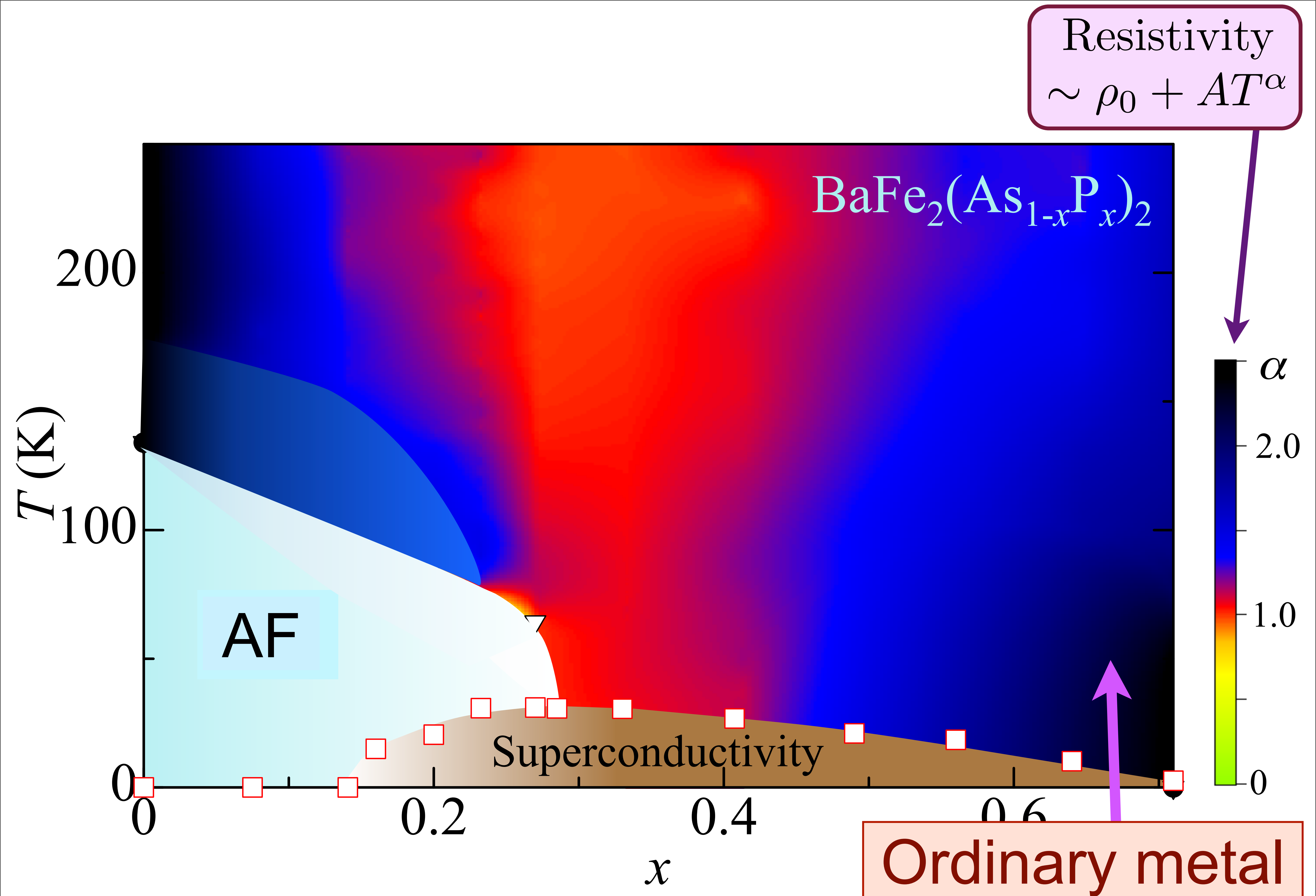


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



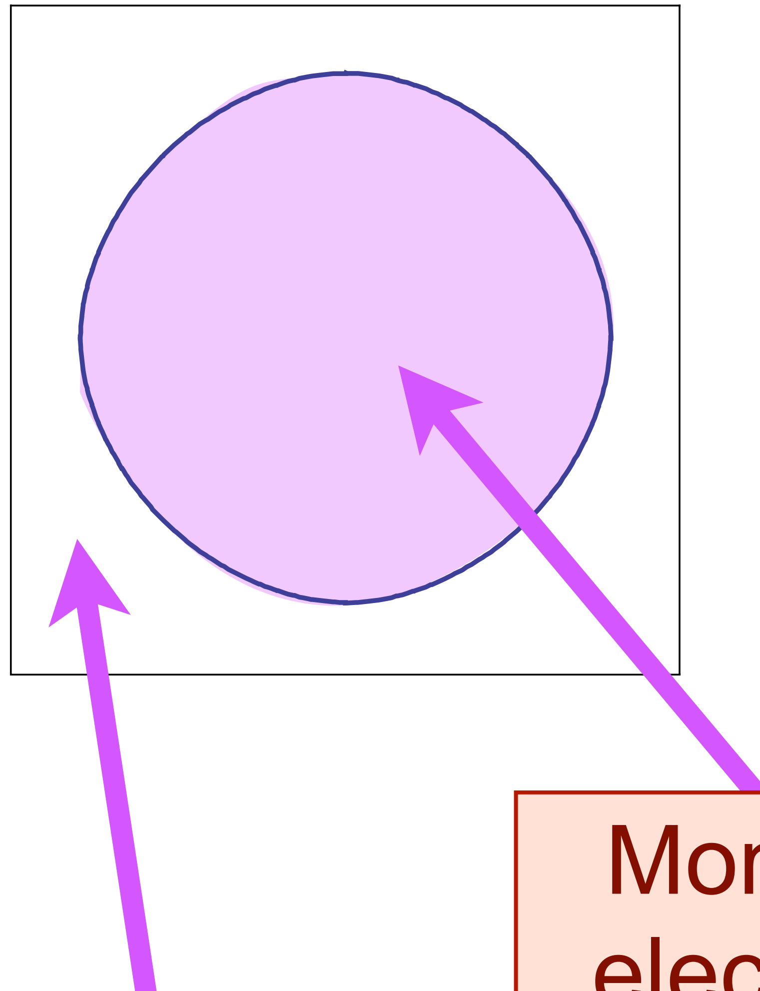
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)


Resistivity
$\sim \rho_0 + AT^\alpha$
$BaFe_2(As_{1-x}P_x)_2$
200
100
0
T (K)
α
2.0
1.0
0
AF
Superconductivity
0
0.2
Superconductor
Bose condensate of pairs of electrons
Short-range entanglement
S. Kasahara, T. Shiba
H. Ike



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Ma
Physical Review B **81**, 184519 (2010)

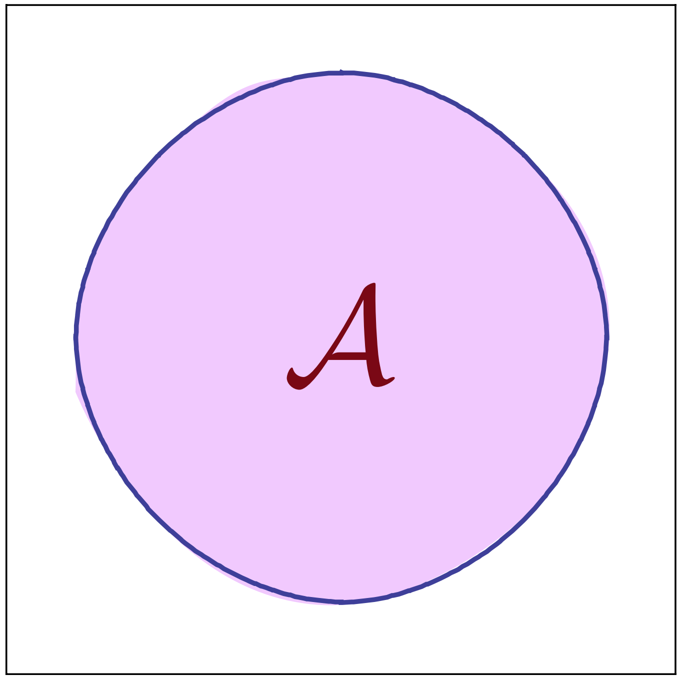
Sommerfeld-Bloch theory of ordinary metals



Momenta with electron states empty

Momenta with electron states occupied

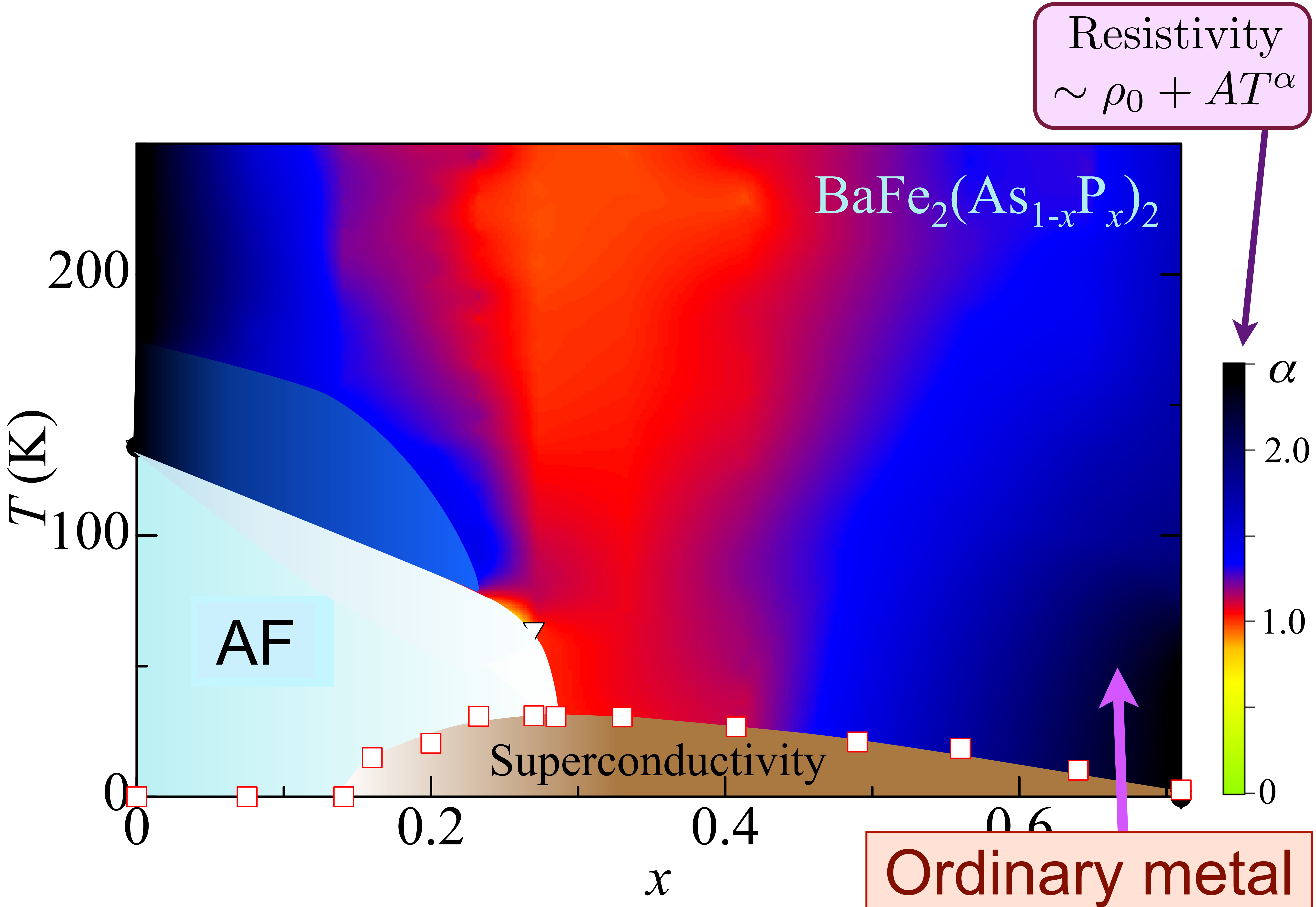
Sommerfeld-Bloch theory of ordinary metals



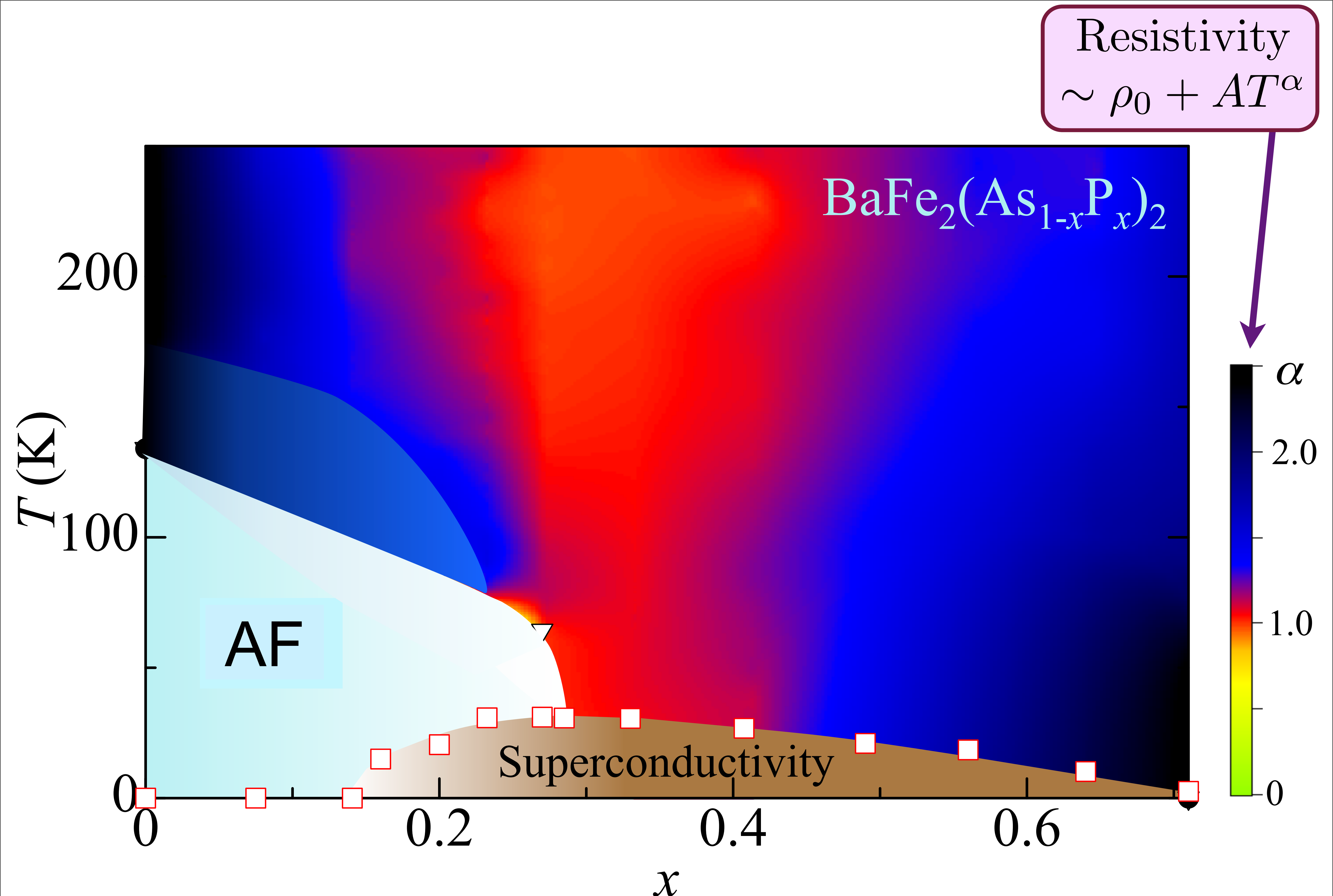
Key feature of the theory:
the **Fermi surface**

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the electron density

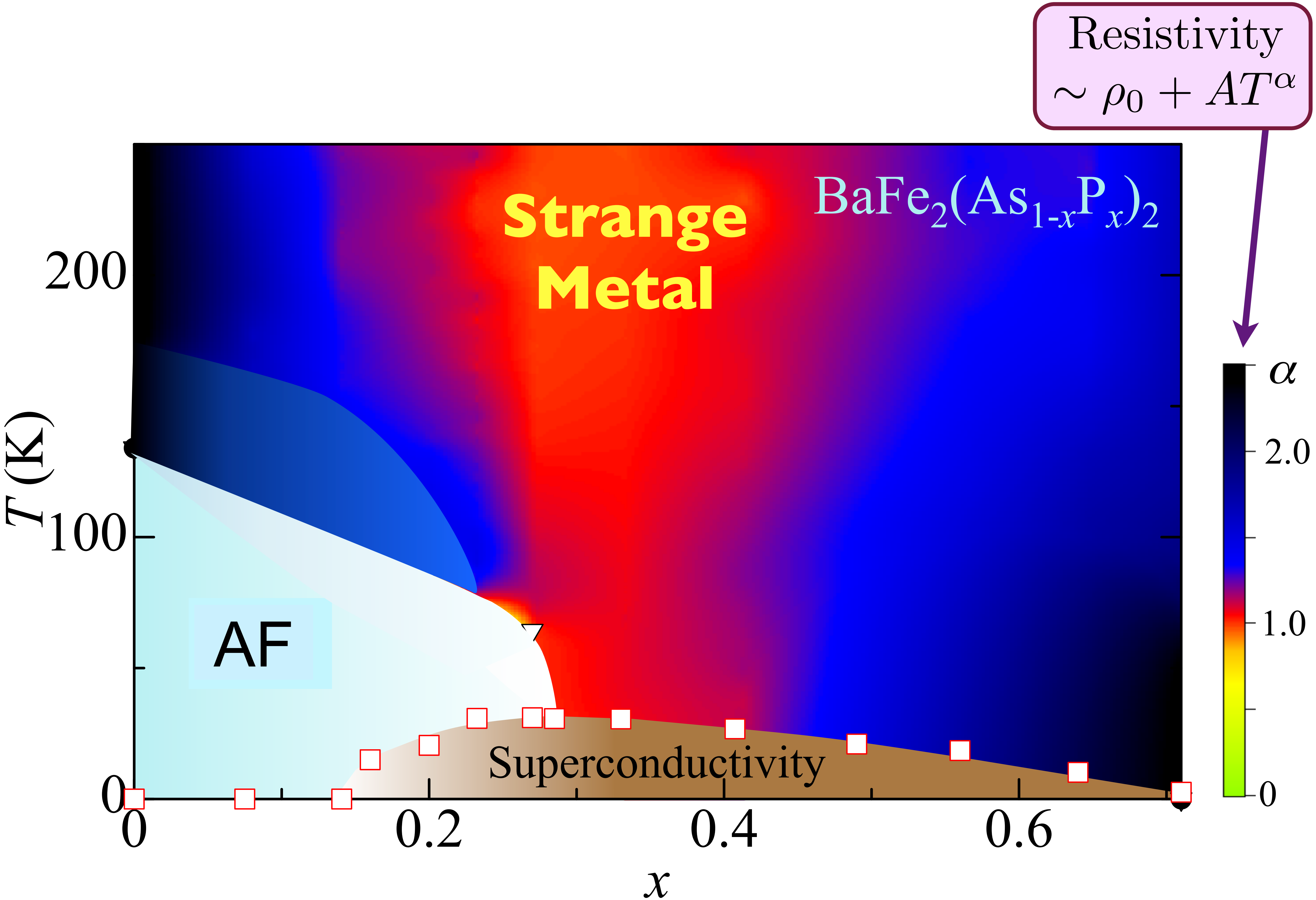
- Excitations near the Fermi surface are responsible for the familiar properties of ordinary metals, such as resistivity $\sim T^2$.



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Mat
Physical Review B **81**, 184519 (2010)

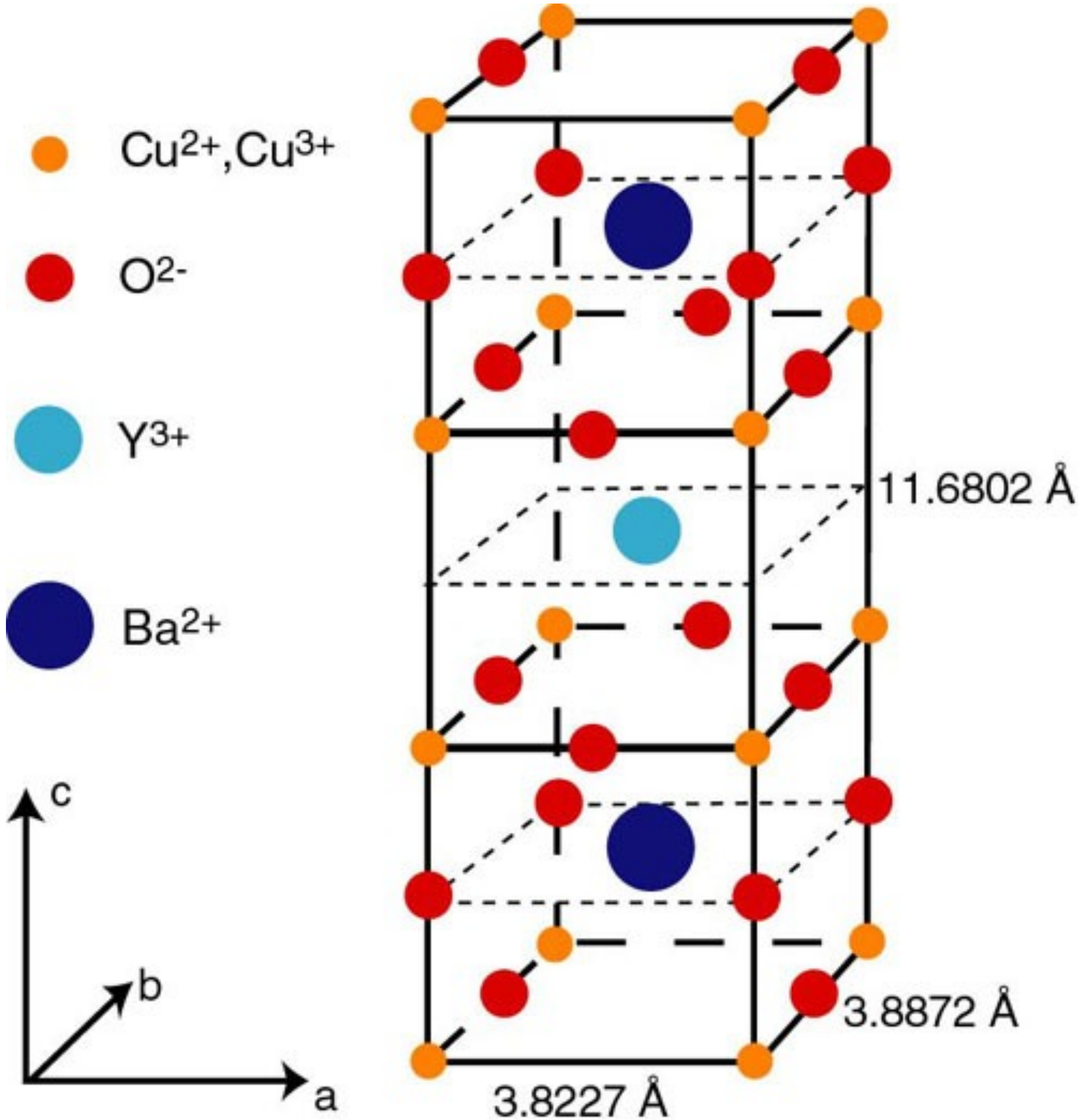


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



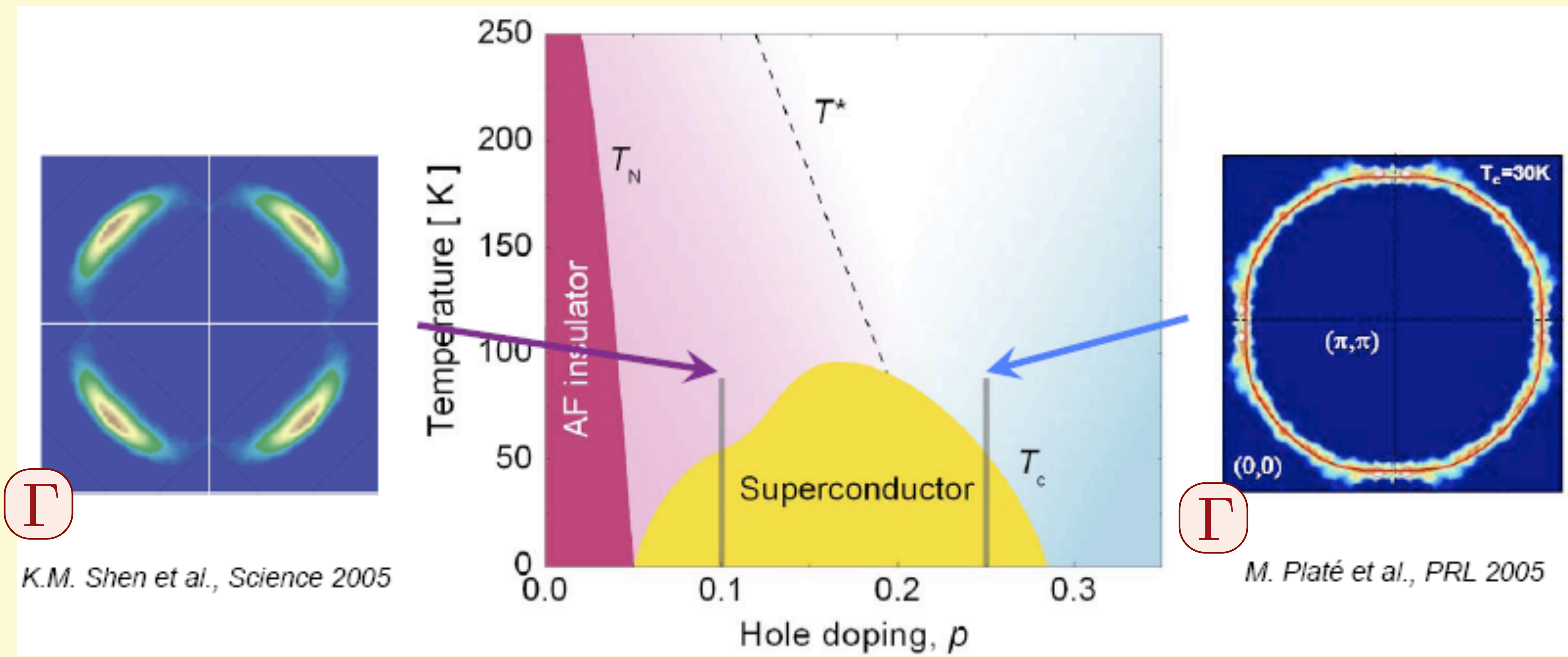
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$

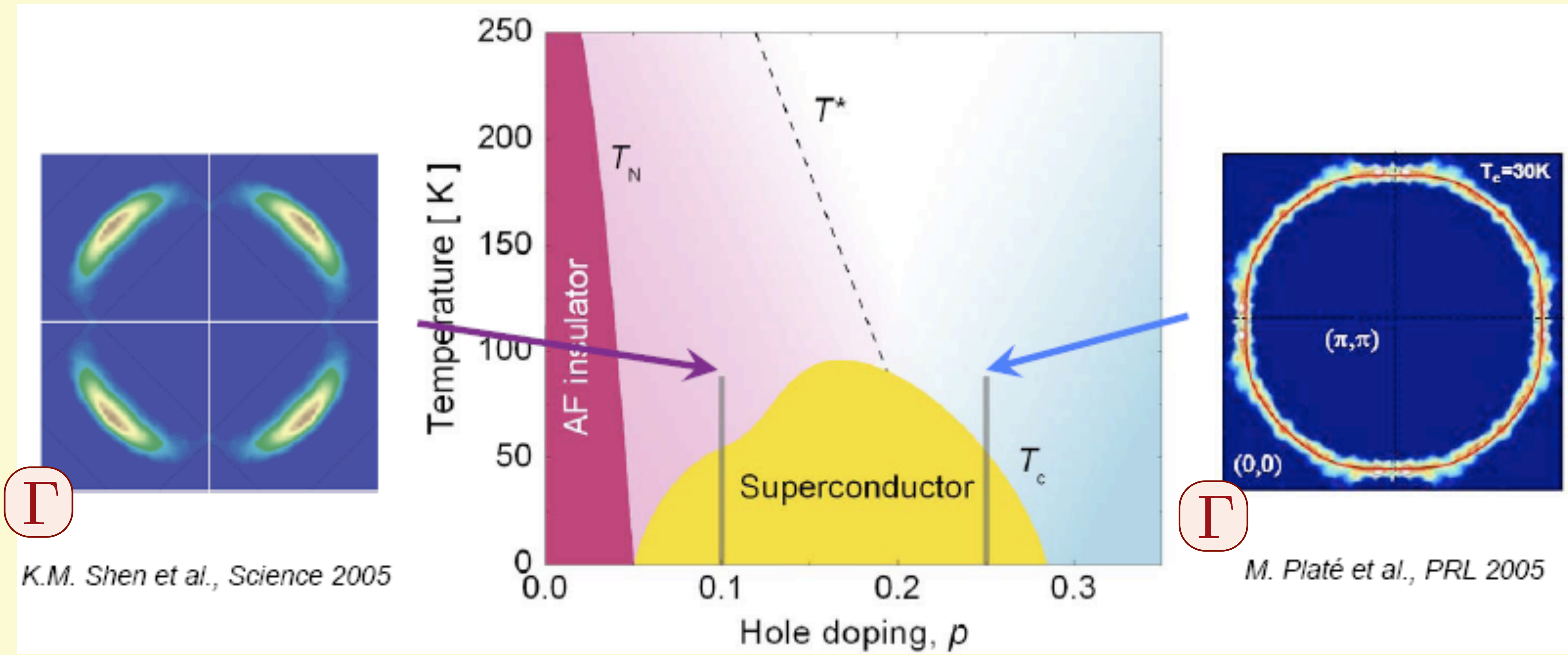
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole Fermi-pockets

Large hole Fermi surface

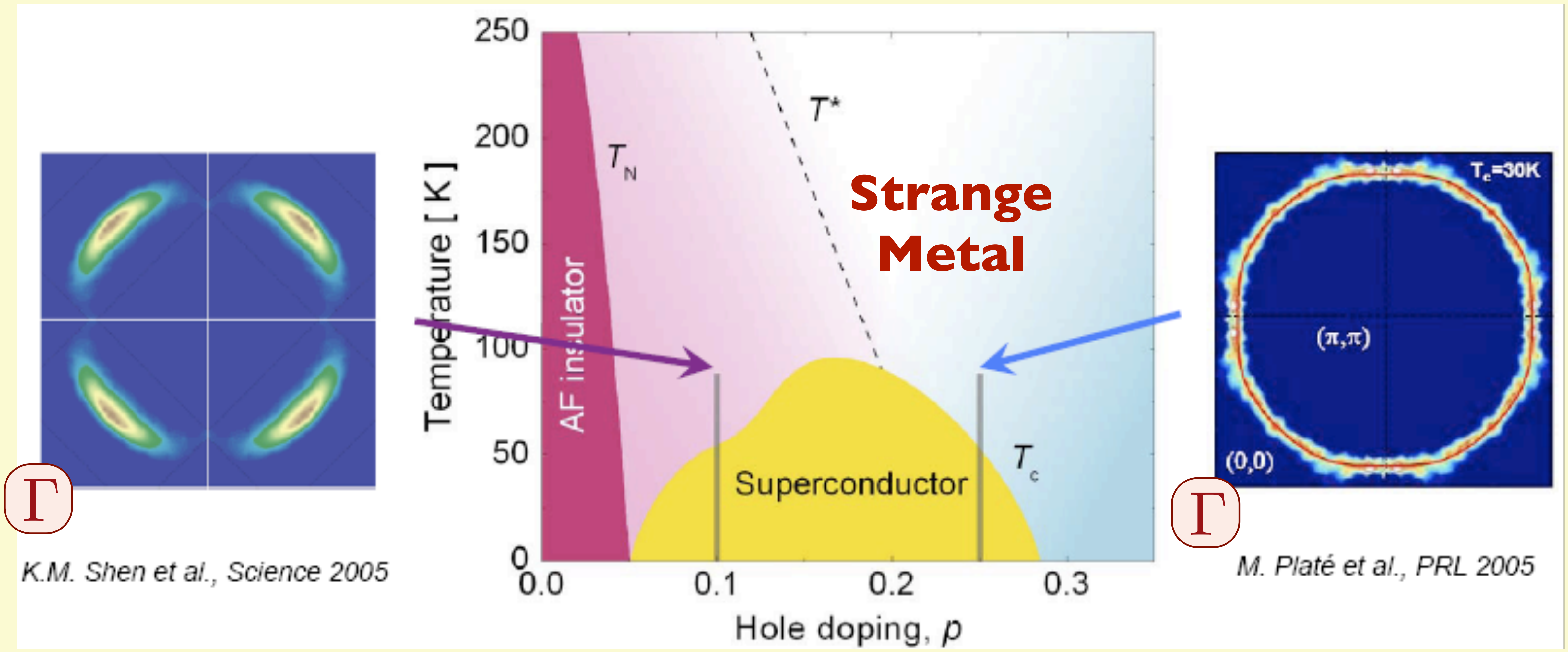
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole Fermi-pockets

Large hole Fermi surface

Evolution of the (ARPES) Fermi surface on the cuprate phase diagram

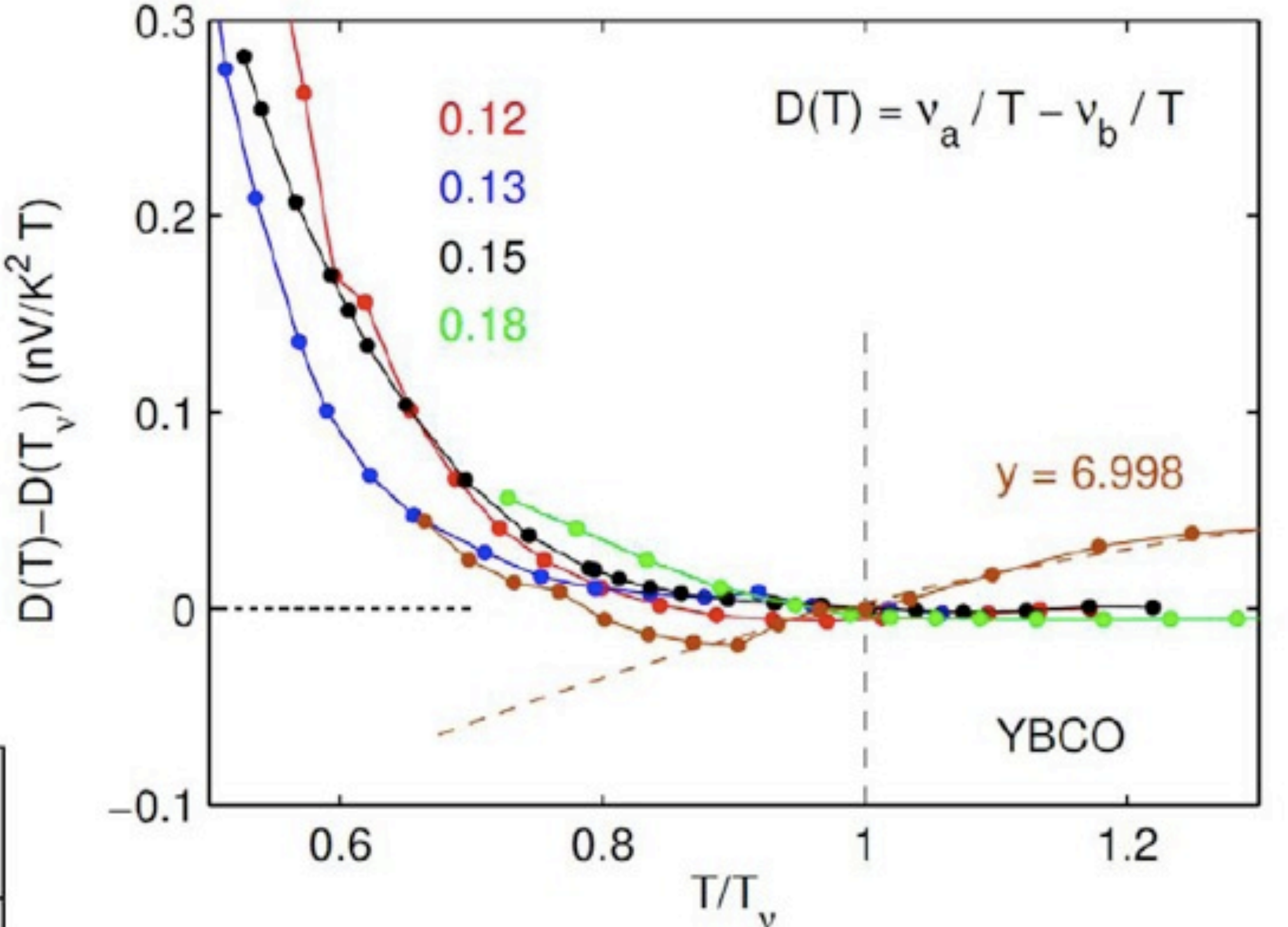


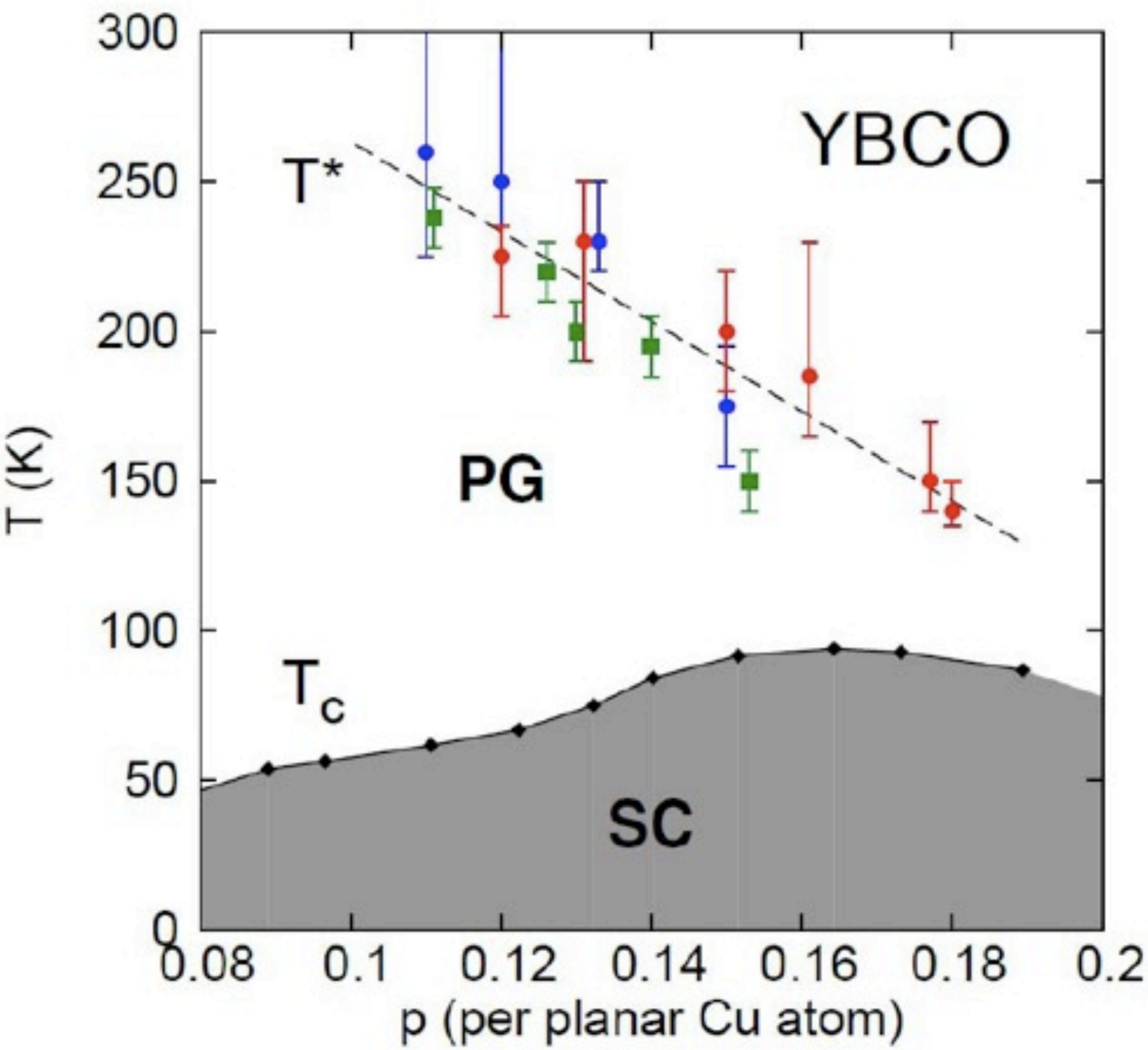
Smaller hole Fermi-pockets

Large hole Fermi surface

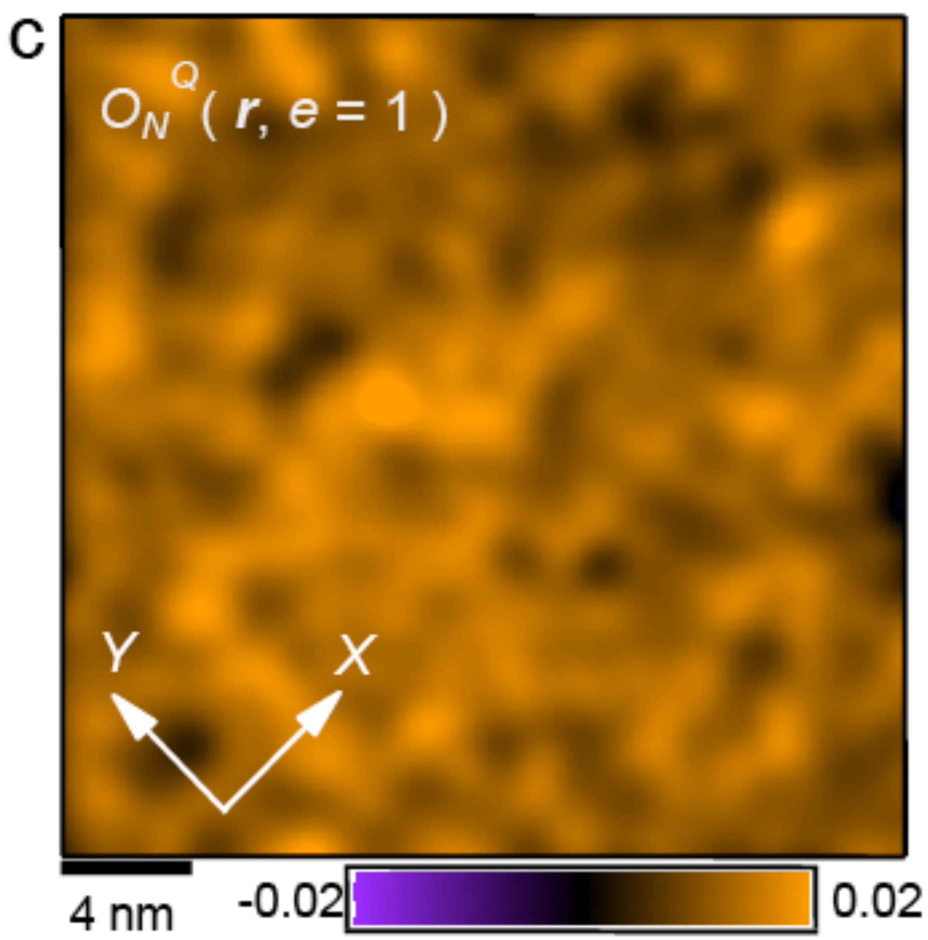
Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

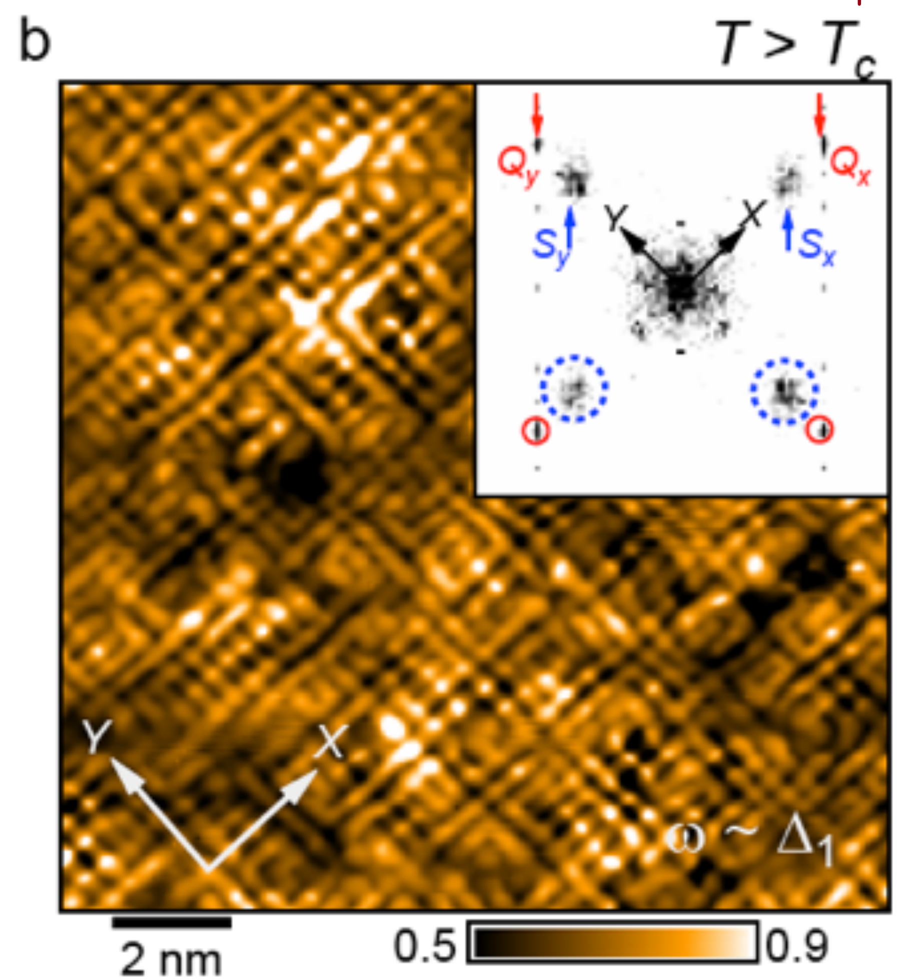
R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
Nature, **463**, 519 (2010).



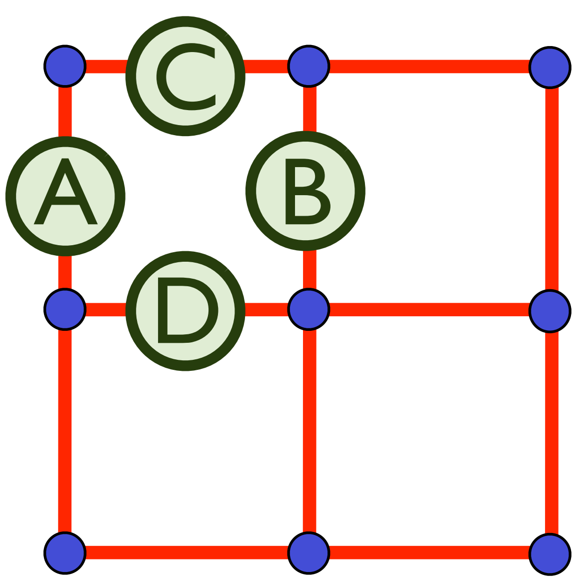


STM measurements of $Z(r)$, the energy asymmetry in density of states in $Bi_2Sr_2CaCu_2O_{8+\delta}$.





M. J. Lawler, K. Fujita, Jhinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, Nature **466**, 347 (2010)



$$O_N = Z_A + Z_B - Z_C - Z_D$$

Strong anisotropy of electronic states between *x* and *y* directions: Electronic "Ising-nematic" order

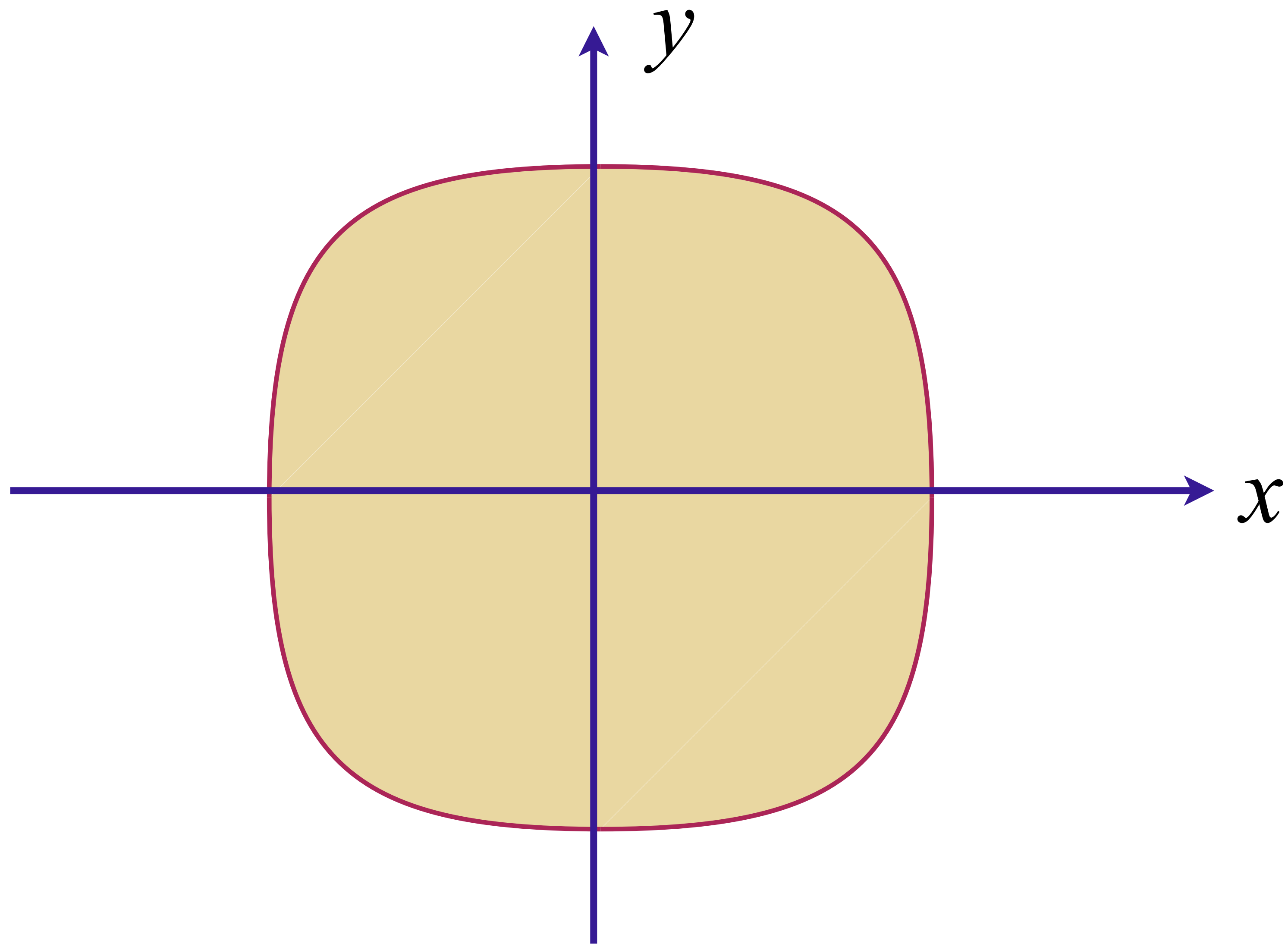
A. Field theory

B. Gauge-gravity duality

A. Field theory

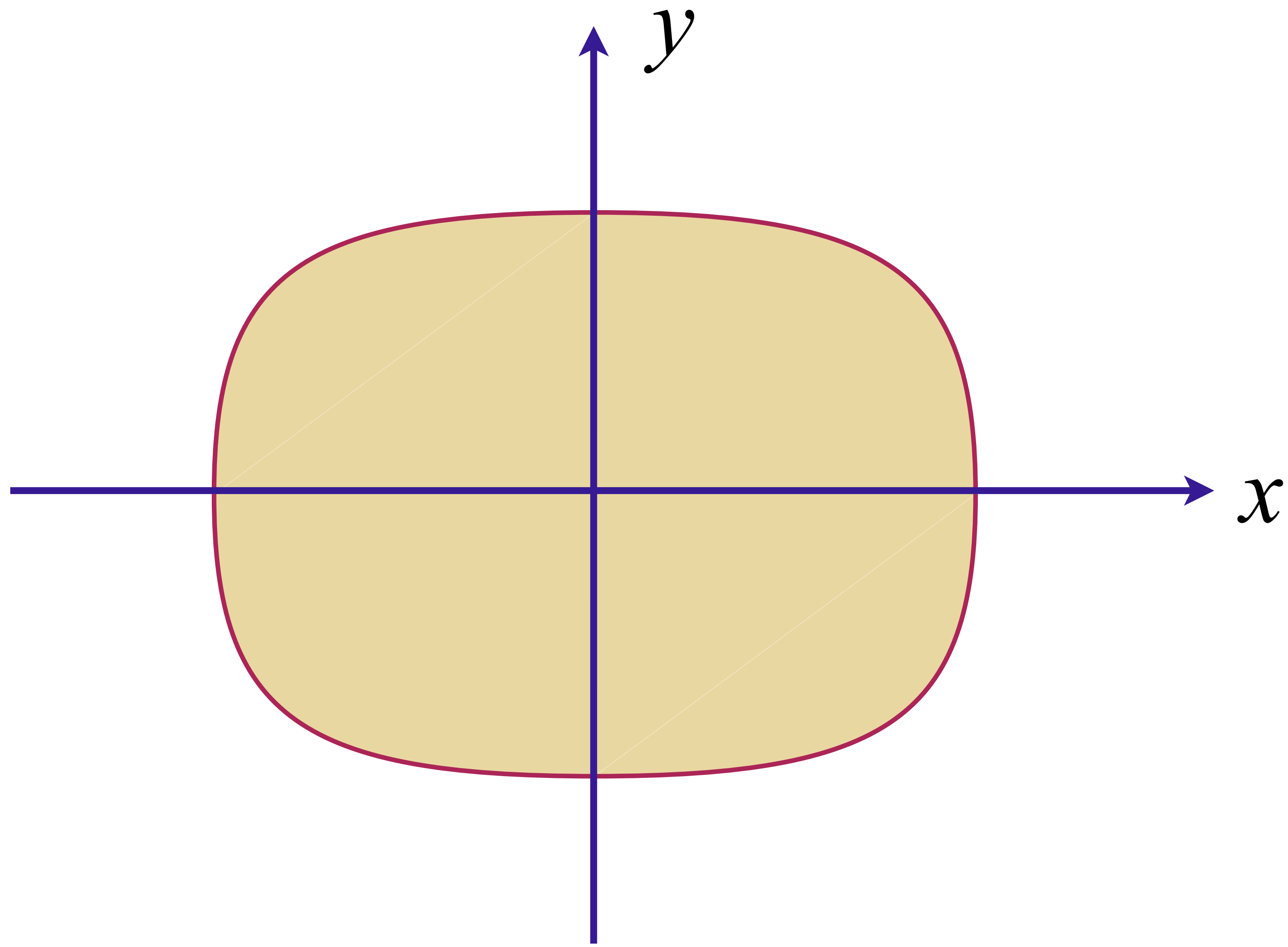
B. Gauge-gravity duality

Quantum criticality of Ising-nematic ordering



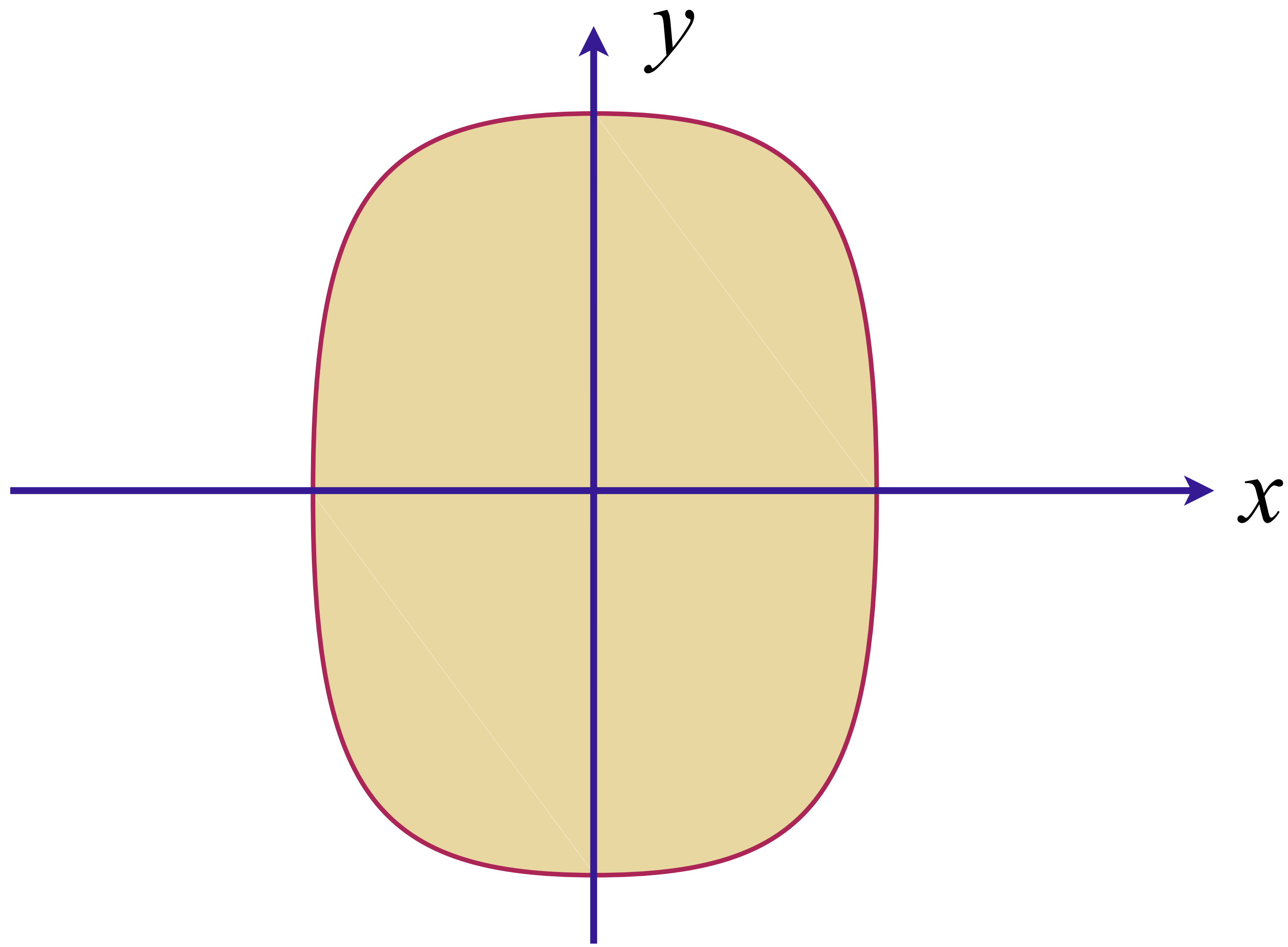
Fermi surface with full square lattice symmetry

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:

Quantum criticality of Ising-nematic ordering



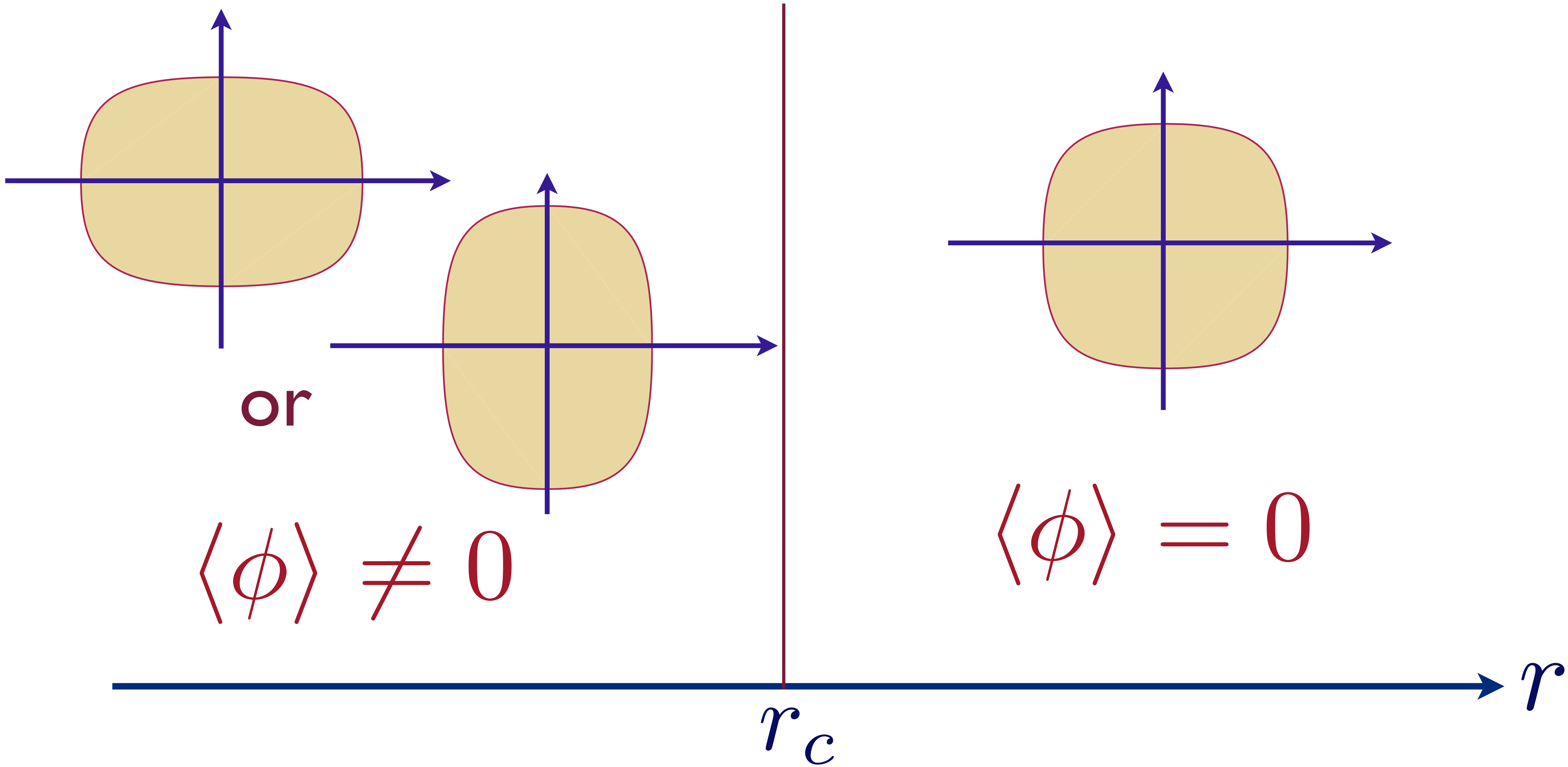
Spontaneous elongation along y direction:

Ising-nematic order parameter

$$\phi \sim \int d^2k \, (\cos k_x - \cos k_y) \, c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$$

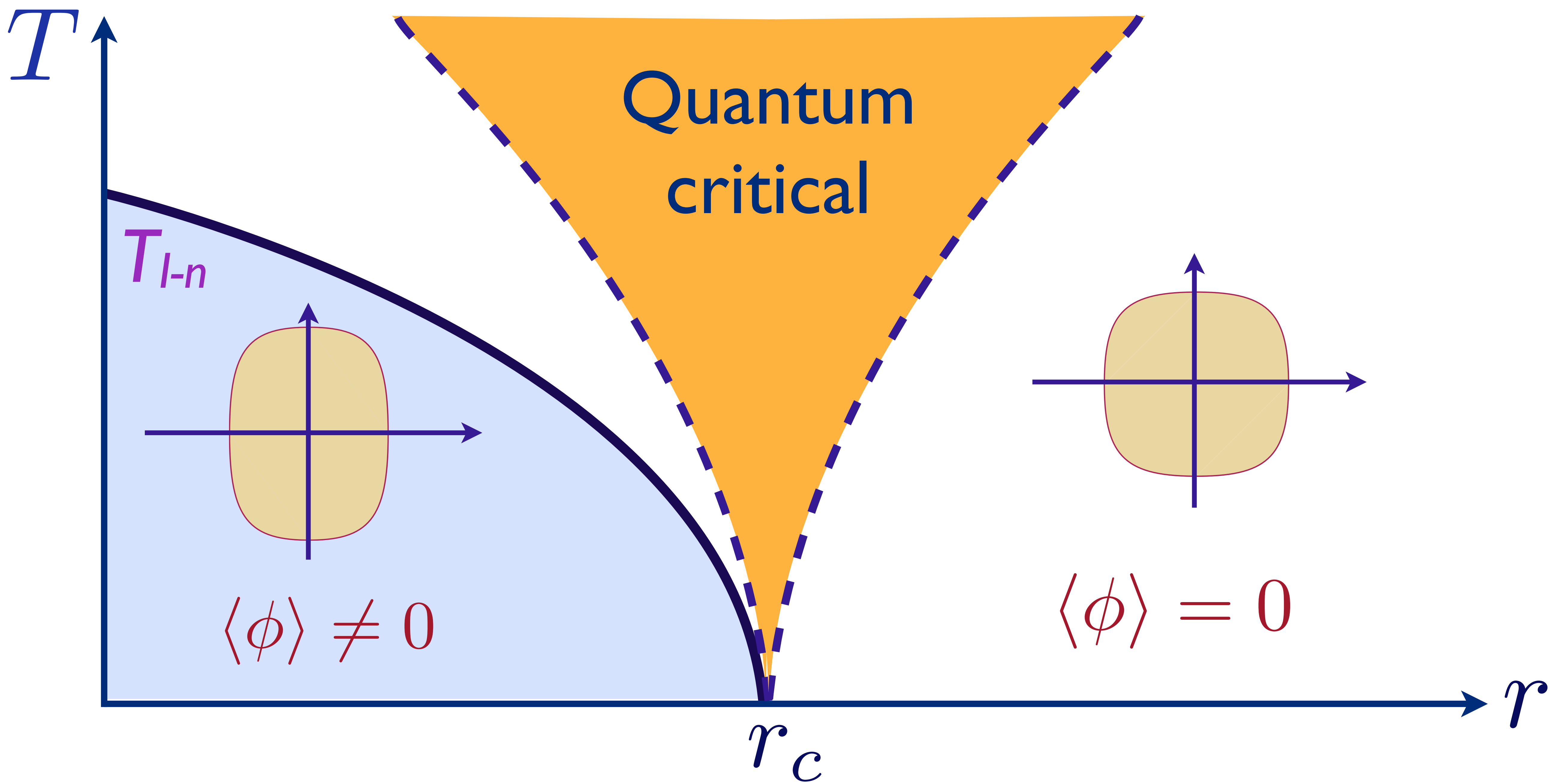
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering



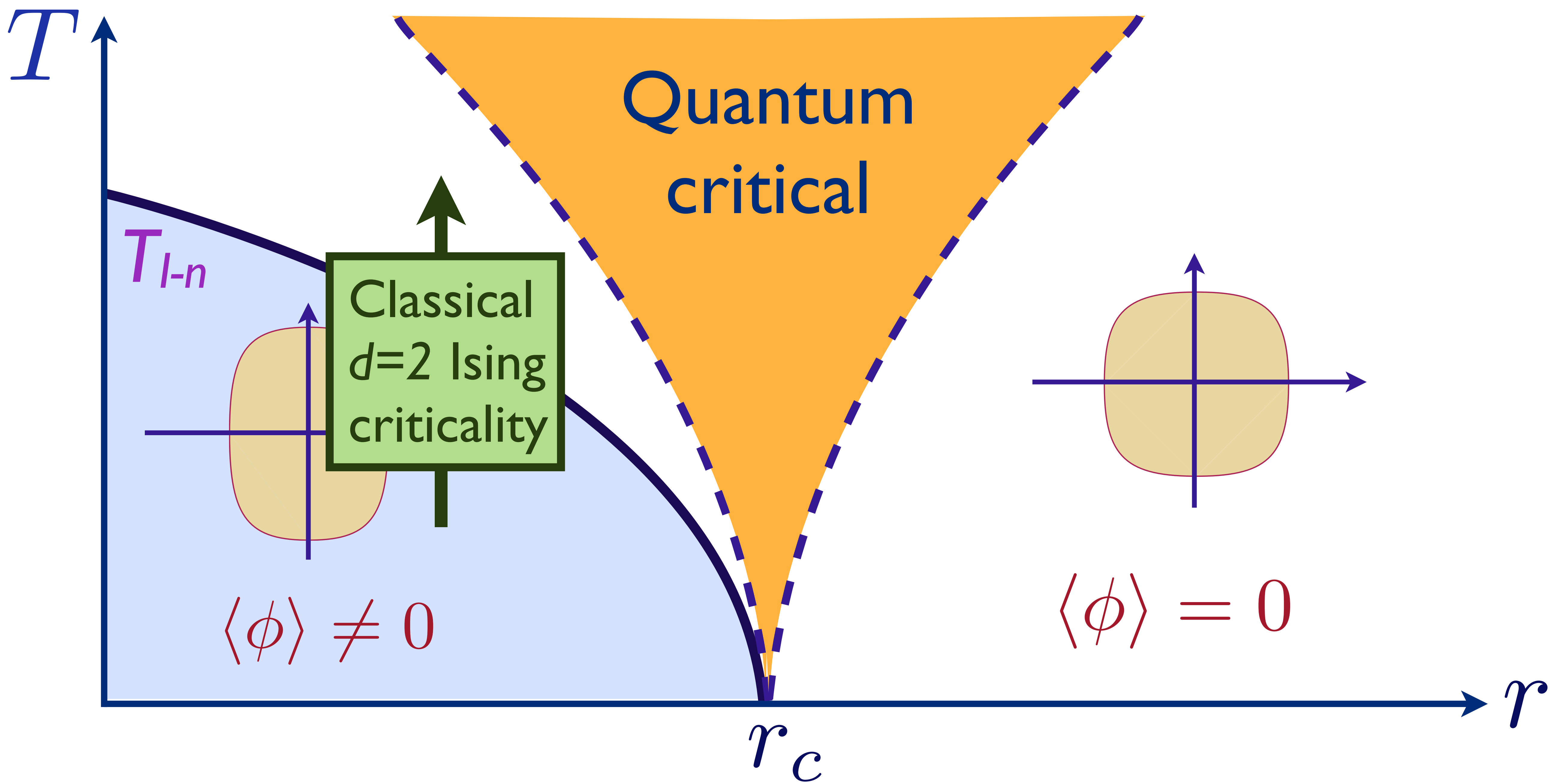
Pomeranchuk instability as a function of coupling r

Quantum criticality of Ising-nematic ordering



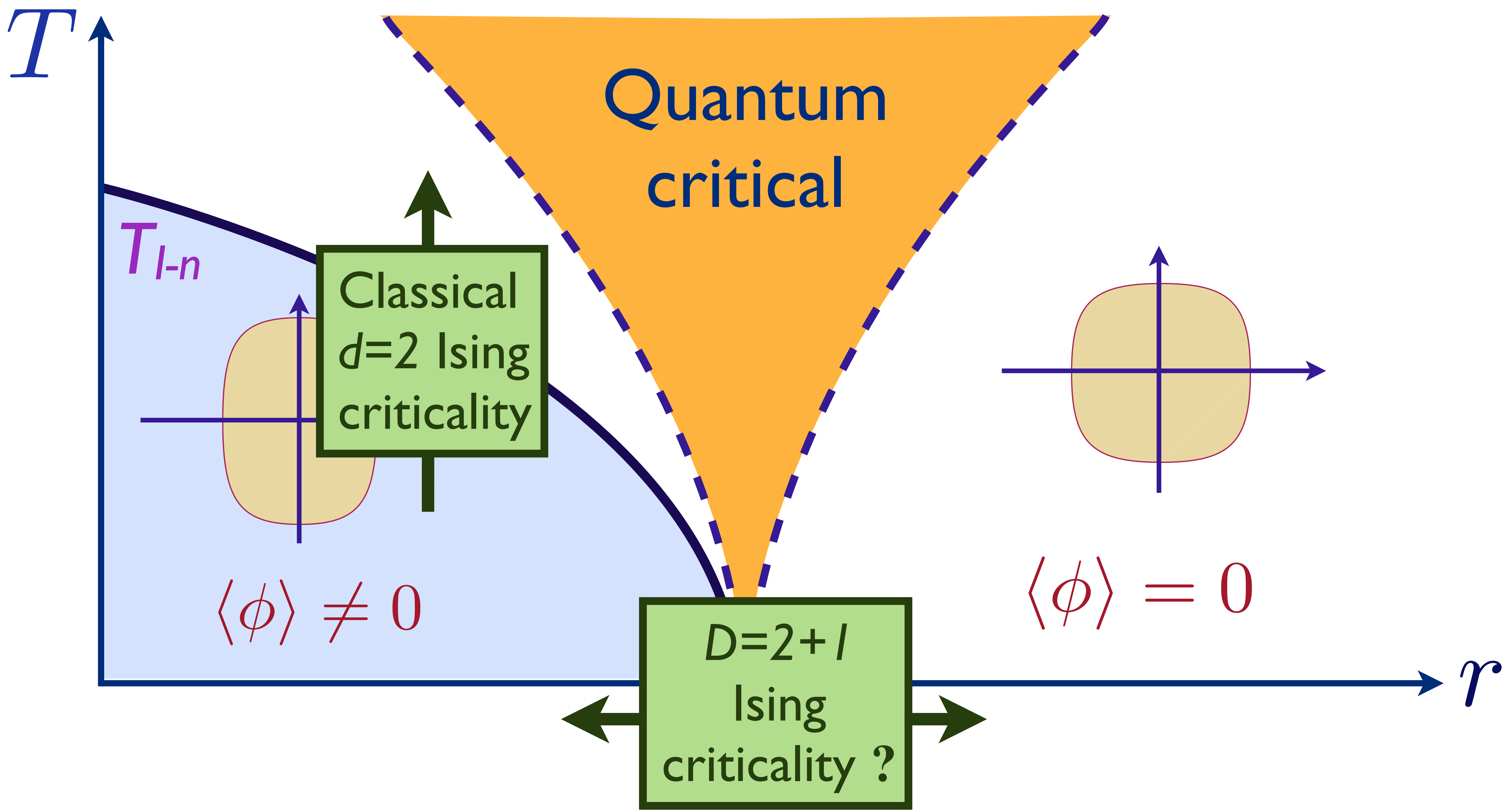
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



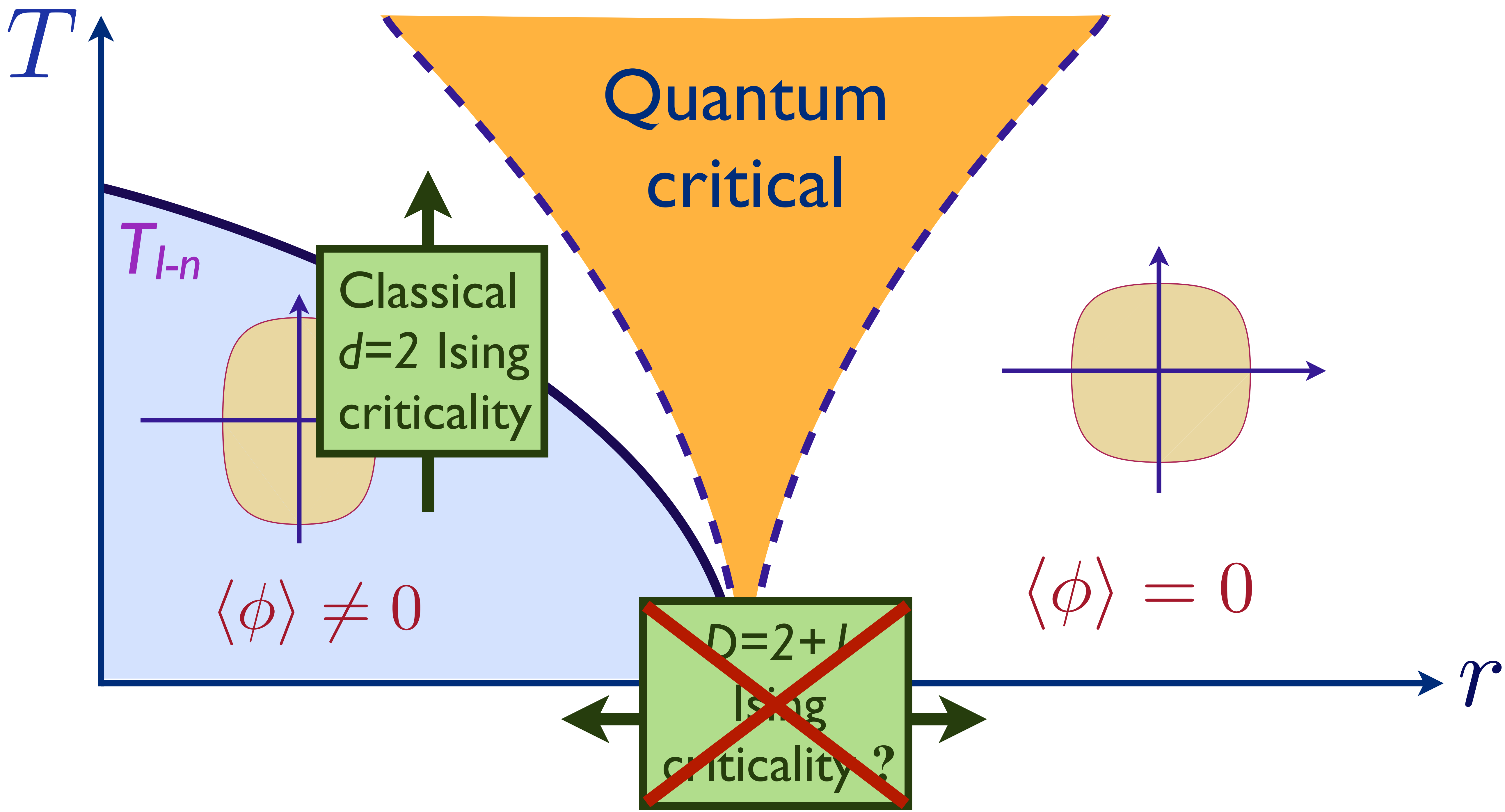
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



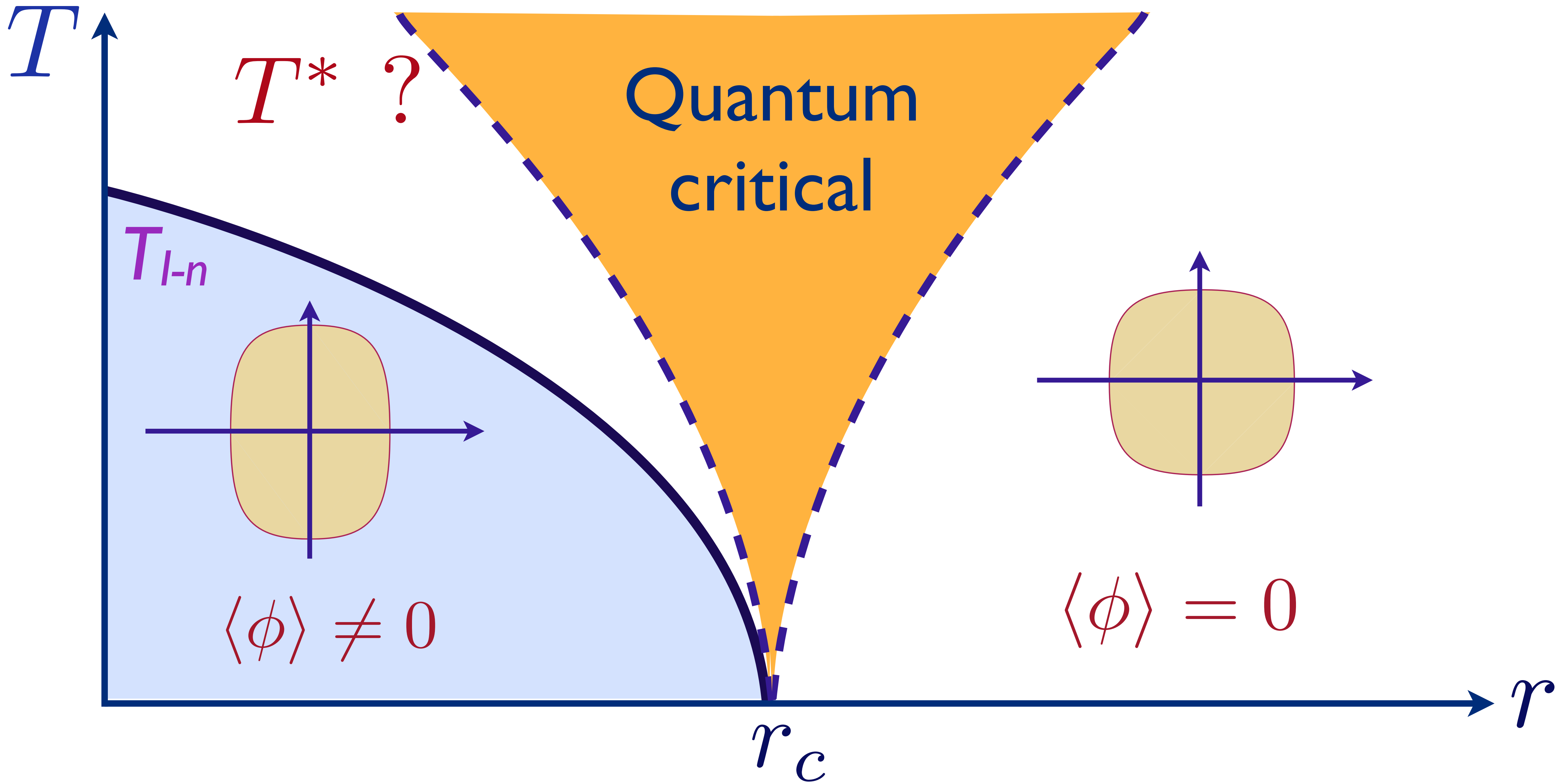
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



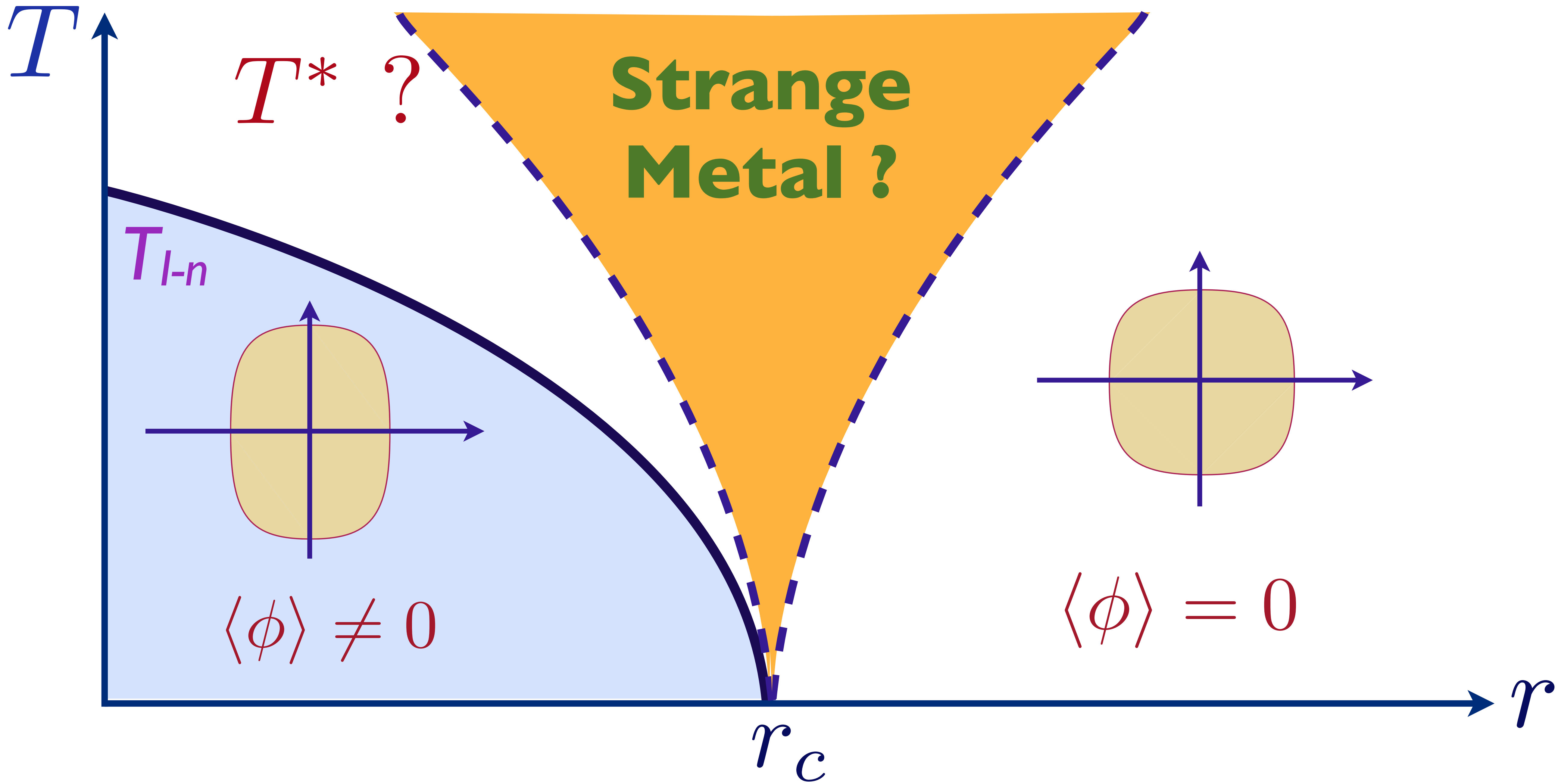
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

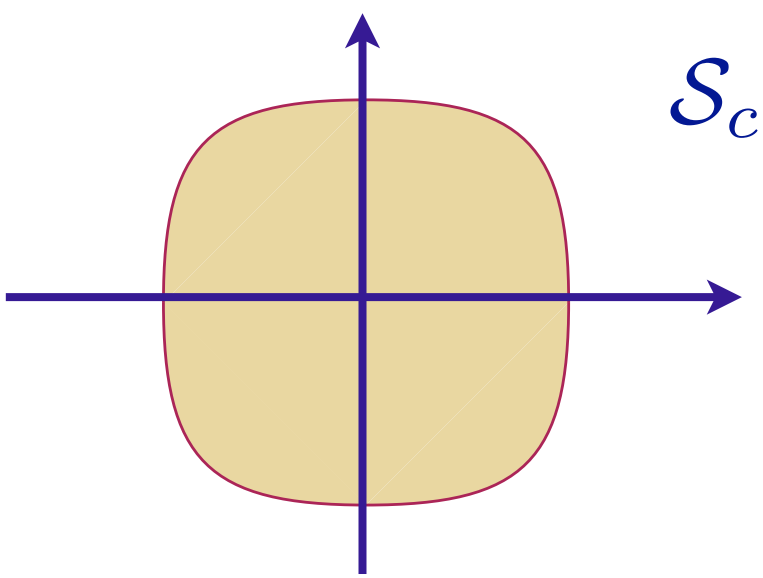
$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right]$$

Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

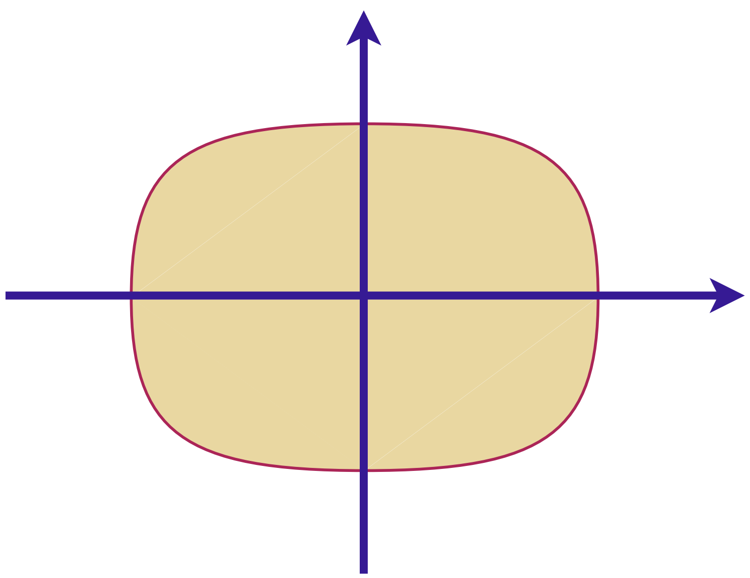
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Ising-nematic ordering

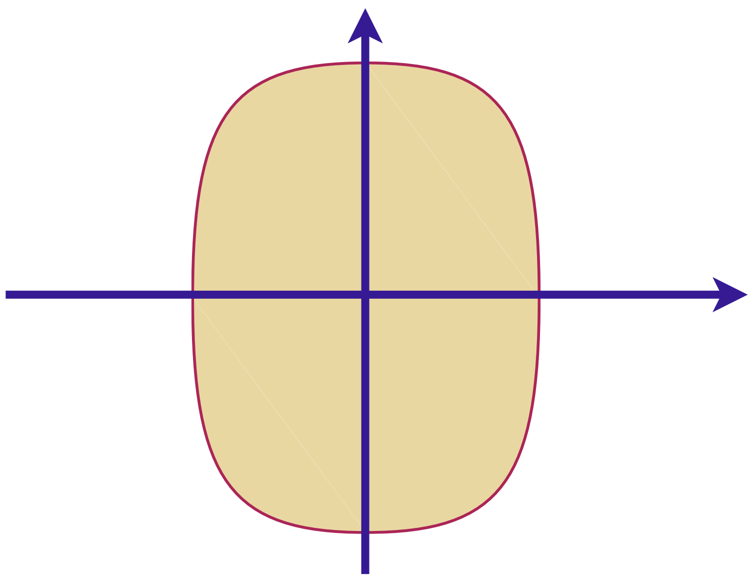
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$\langle \phi \rangle > 0$



$\langle \phi \rangle < 0$

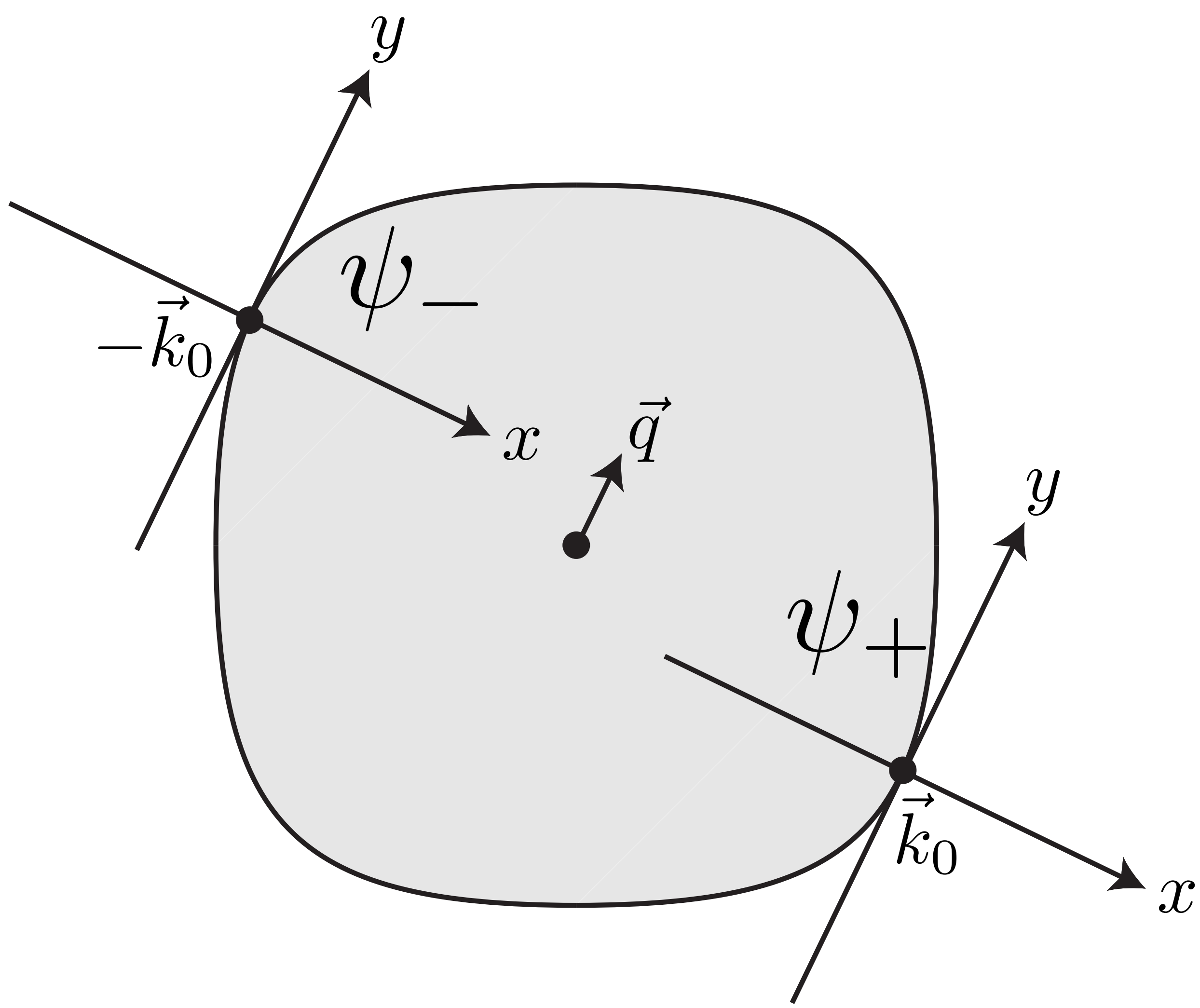
Quantum criticality of Ising-nematic ordering

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

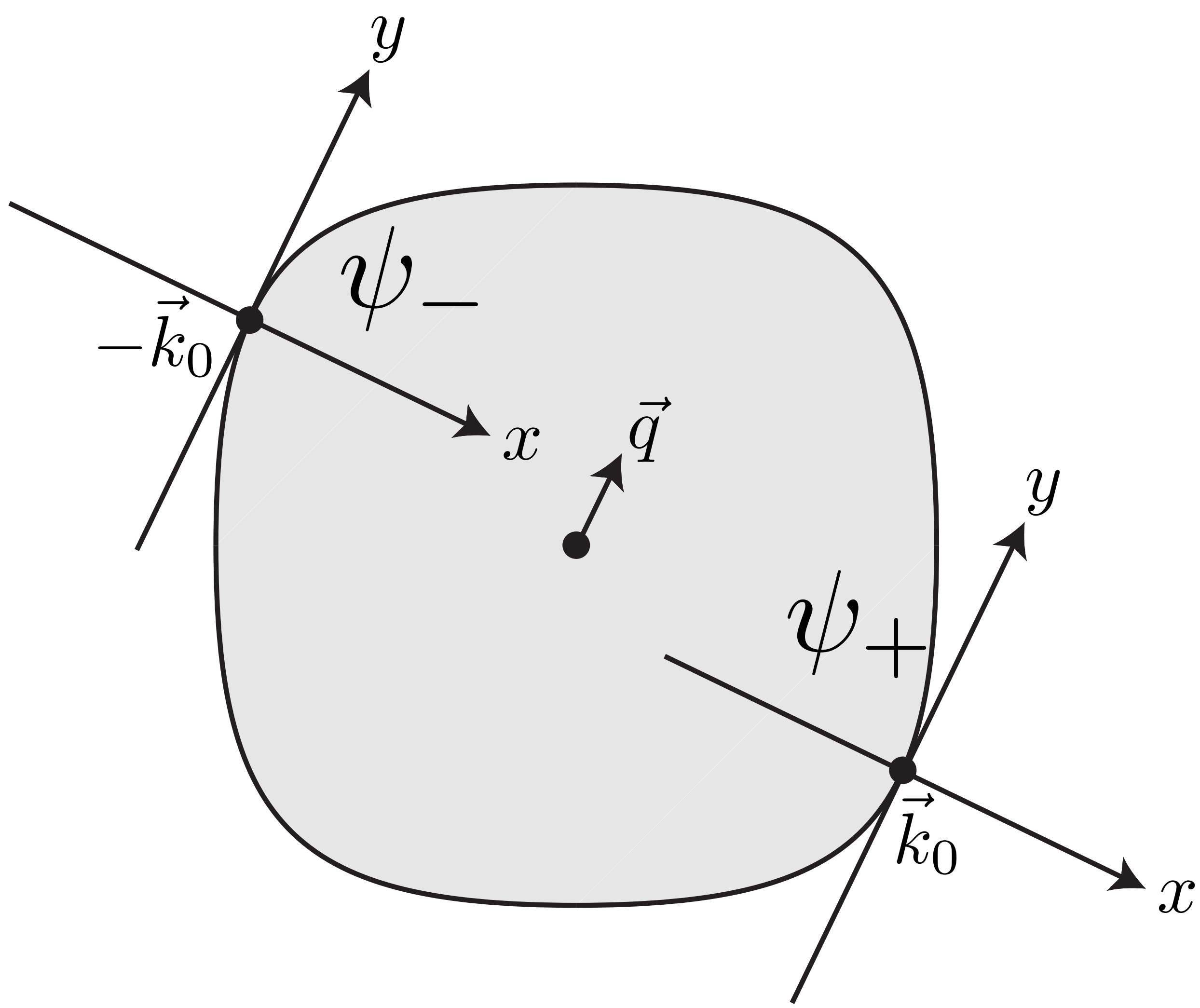
$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum criticality of Ising-nematic ordering



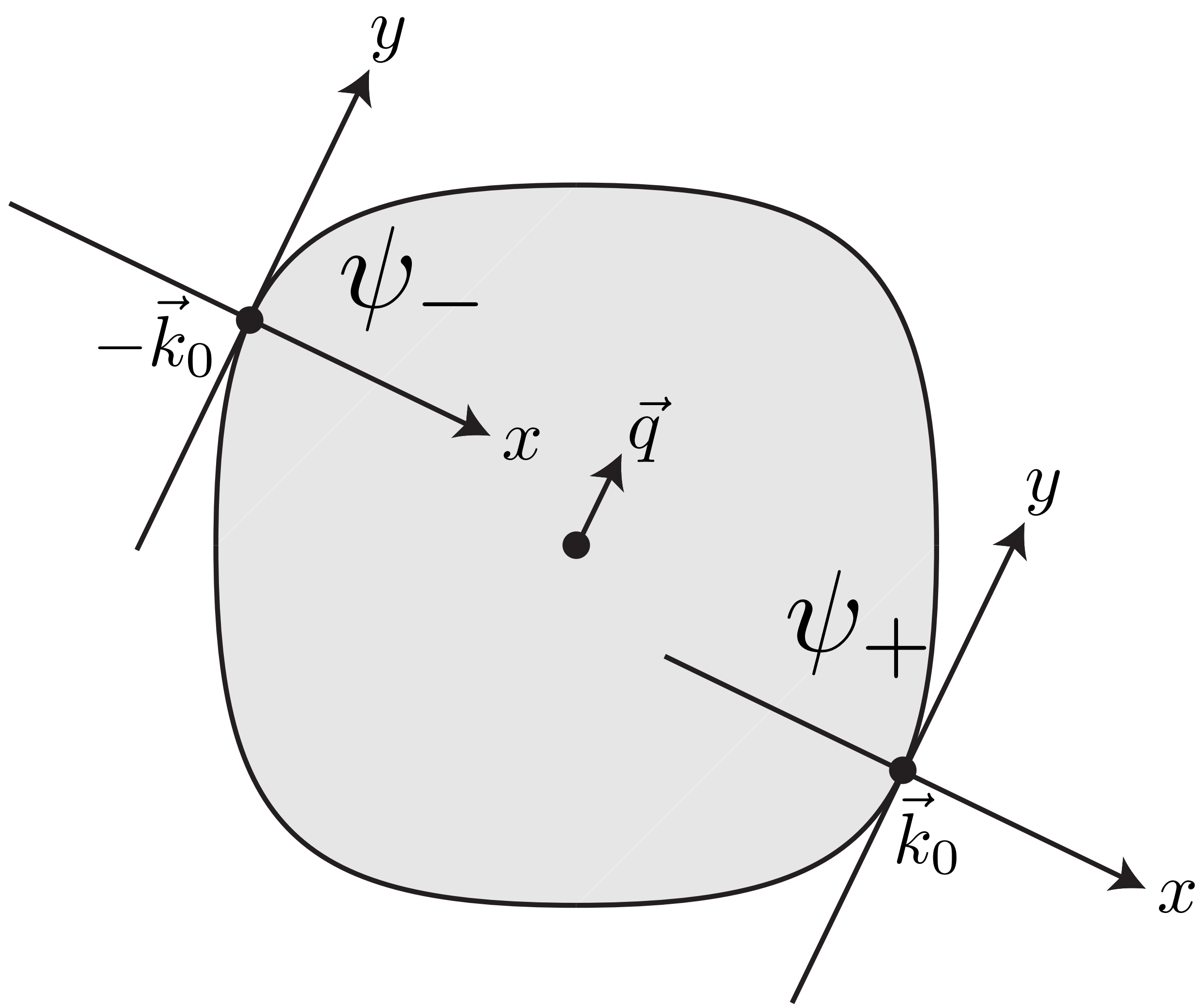
- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Quantum criticality of Ising-nematic ordering



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering



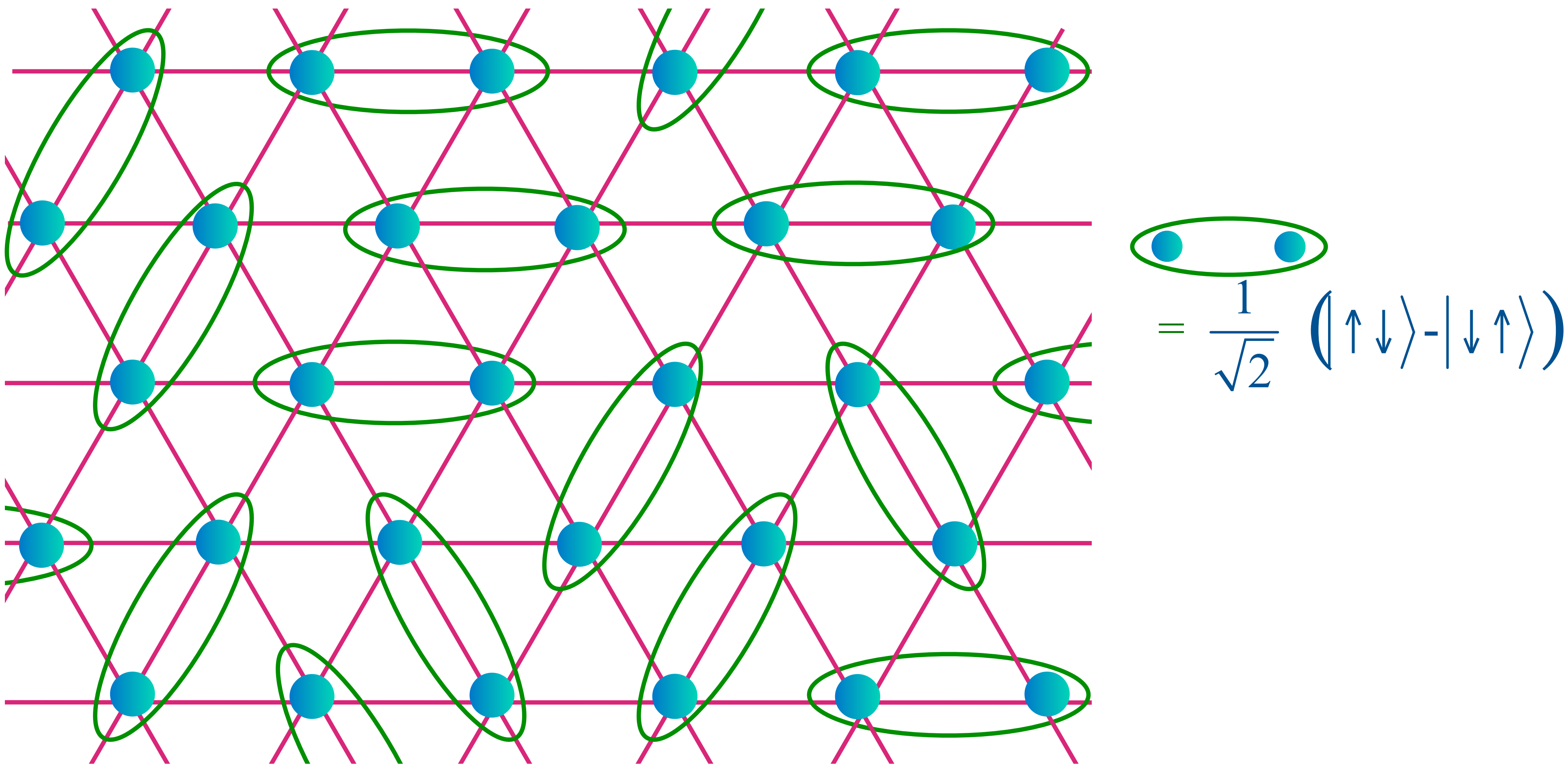
$$\mathcal{L}[\psi_\pm, \phi] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

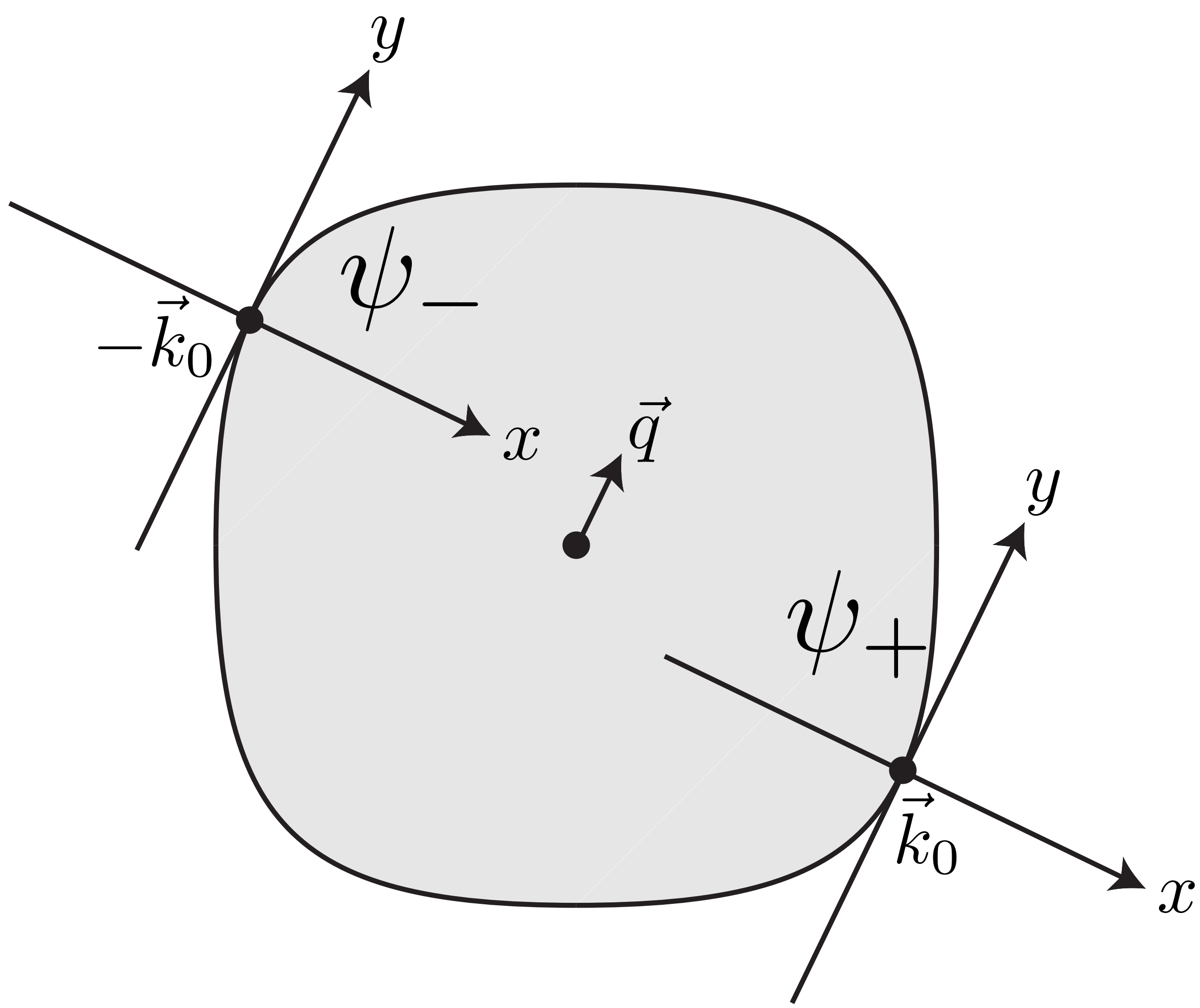
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

Quantum criticality of Ising-nematic ordering



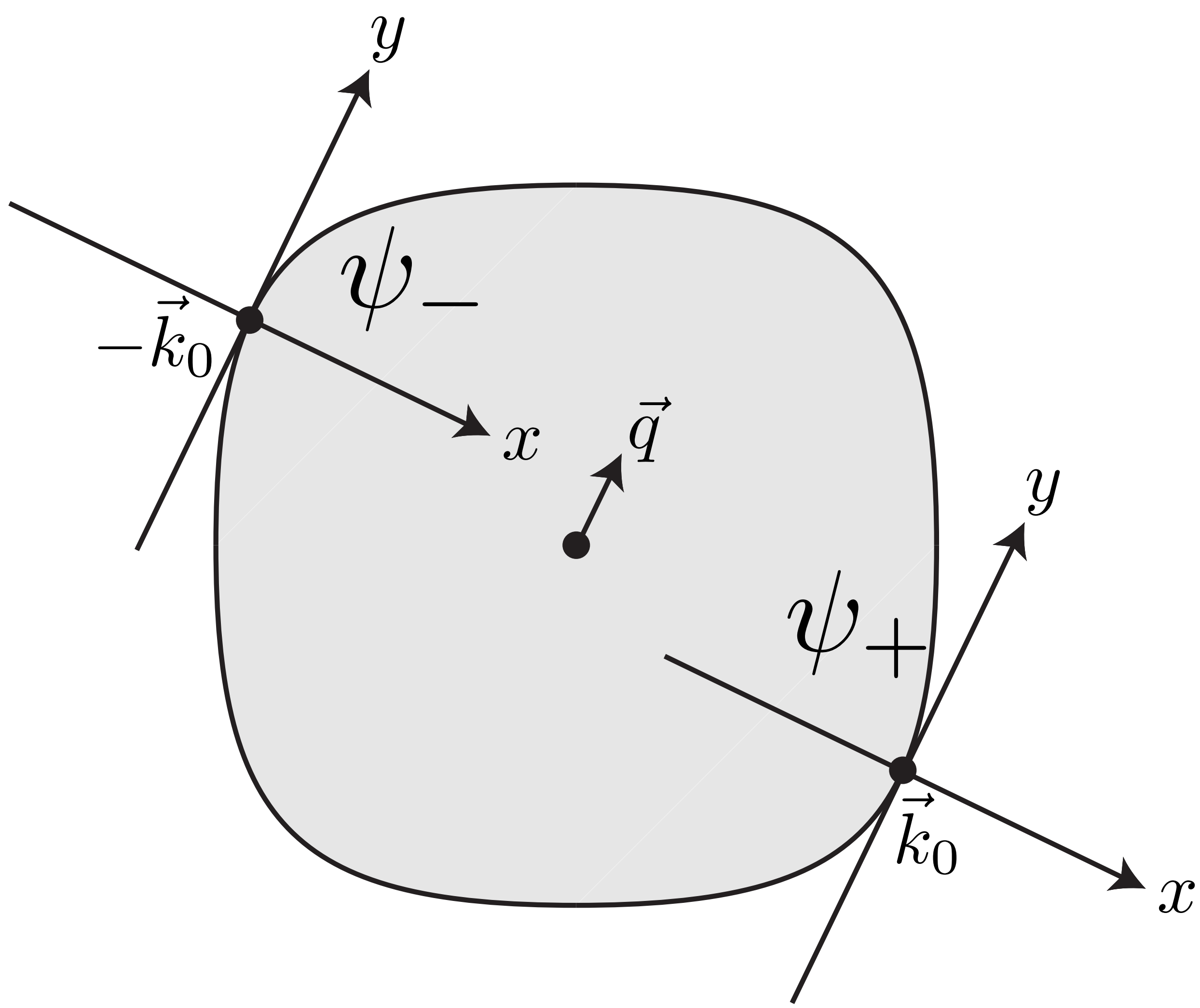
$$\mathcal{L}[\psi_\pm, \phi] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-\phi\left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Field theory of U(1) spin liquid



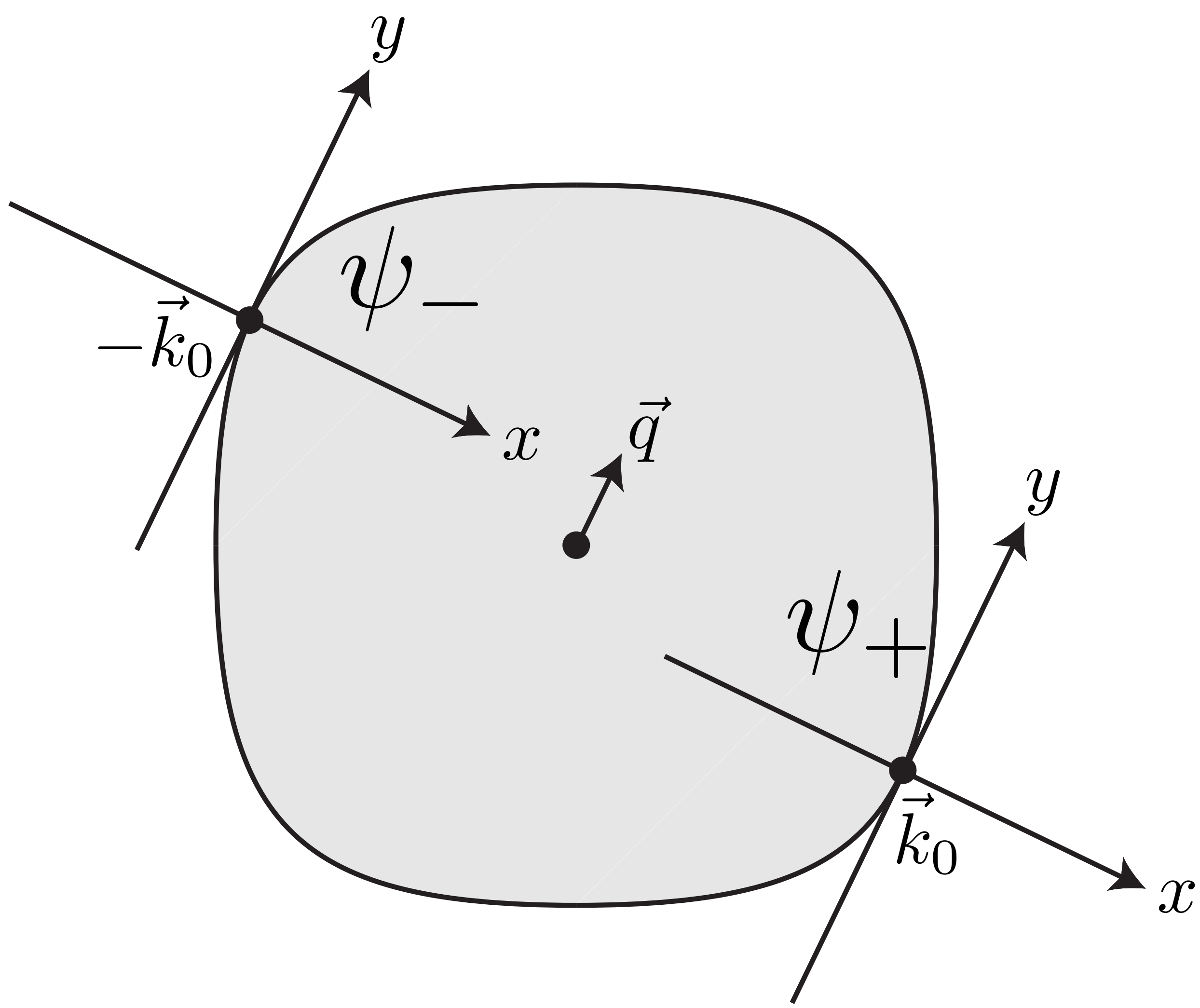
$$\mathcal{L}[\psi_\pm, a] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-a\left(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering



$$\mathcal{L}[\psi_\pm, \phi] =$$

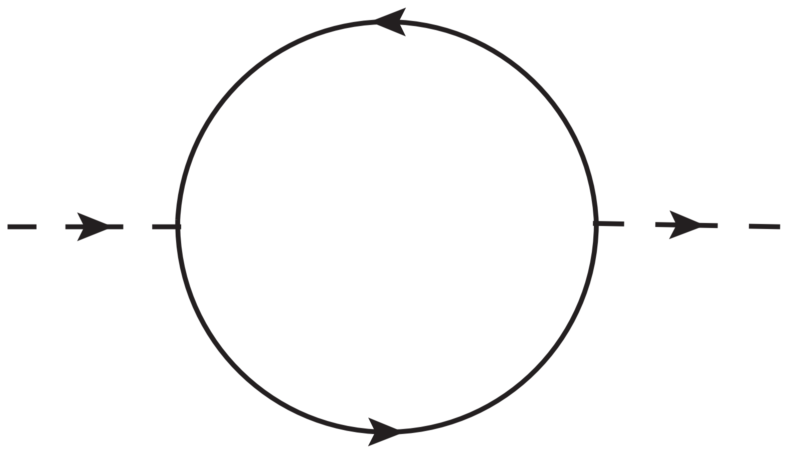
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$



One loop ϕ self-energy with N_f fermion flavors:

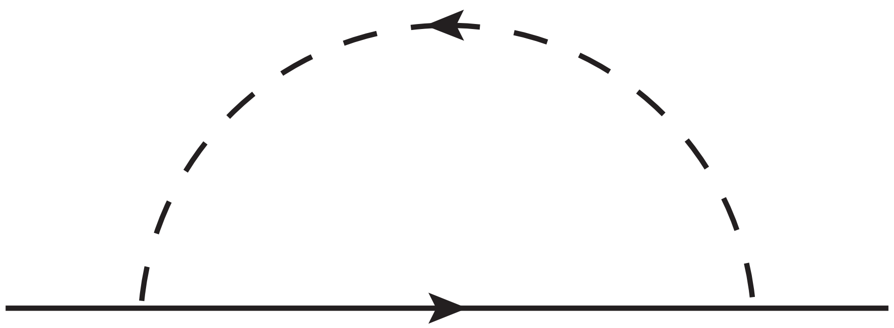
$$D(\vec{q},\omega) = N_f \int \frac{d^2k}{4\pi^2}\frac{d\Omega}{2\pi}\frac{1}{\left[-i(\Omega+\omega)+k_x+q_x+(k_y+q_y)^2\right]\left[-i\Omega-k_x+k_y^2\right]}$$

$$= \frac{N_f}{4\pi}\frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- \\ &\quad - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2\end{aligned}$$



Electron self-energy at order $1/N_f$:

$$\begin{aligned}\Sigma(\vec{k}, \Omega) &= -\frac{1}{N_f} \int \frac{d^2 q}{4\pi^2} \frac{d\omega}{2\pi} \frac{1}{\left[-i(\omega + \Omega) + k_x + q_x + (k_y + q_y)^2\right] \left[\dfrac{q_y^2}{g^2} + \dfrac{|\omega|}{|q_y|}\right]} \\ &= -i \frac{2}{\sqrt{3} N_f} \left(\frac{g^2}{4\pi}\right)^{2/3} \operatorname{sgn}(\Omega) |\Omega|^{2/3}\end{aligned}$$

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$

Schematic form of ϕ and fermion Green's functions

$$D(\vec{q},\omega) = \frac{1/N_f}{q_y^2 + \dfrac{|\omega|}{|q_y|}} \quad , \quad G_f(\vec{q},\omega) = \frac{1}{q_x + q_y^2 - i\,\mathrm{sgn}(\omega)|\omega|^{2/3}/N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L}_{\rm scaling} = \psi_+^\dagger \left(-i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(+i\partial_x - \partial_y^2\right) \psi_- - g\,\phi \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \left(\partial_y \phi\right)^2$$

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Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} a &\to a\, s^{(2z+1)/4} \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$
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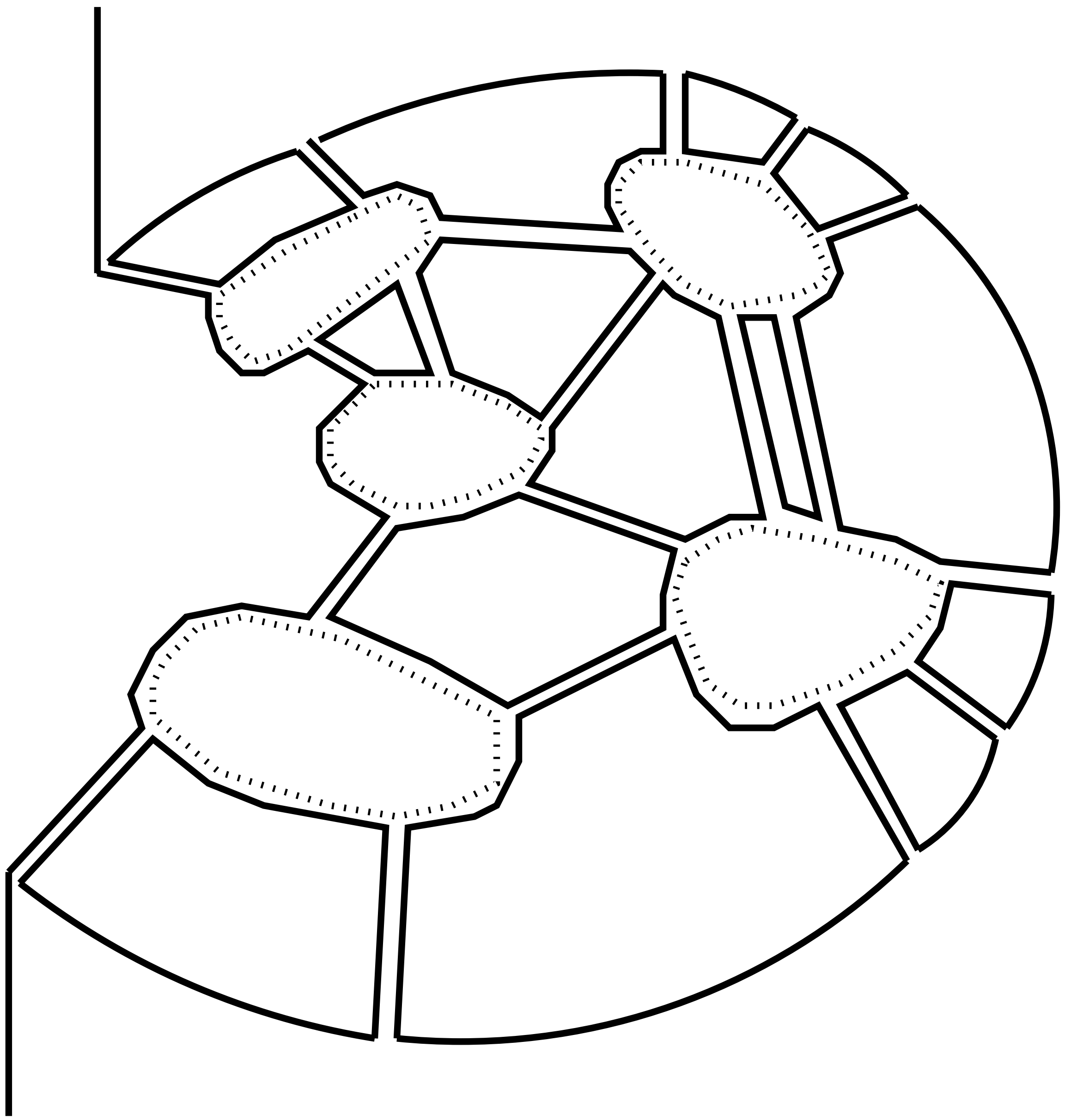
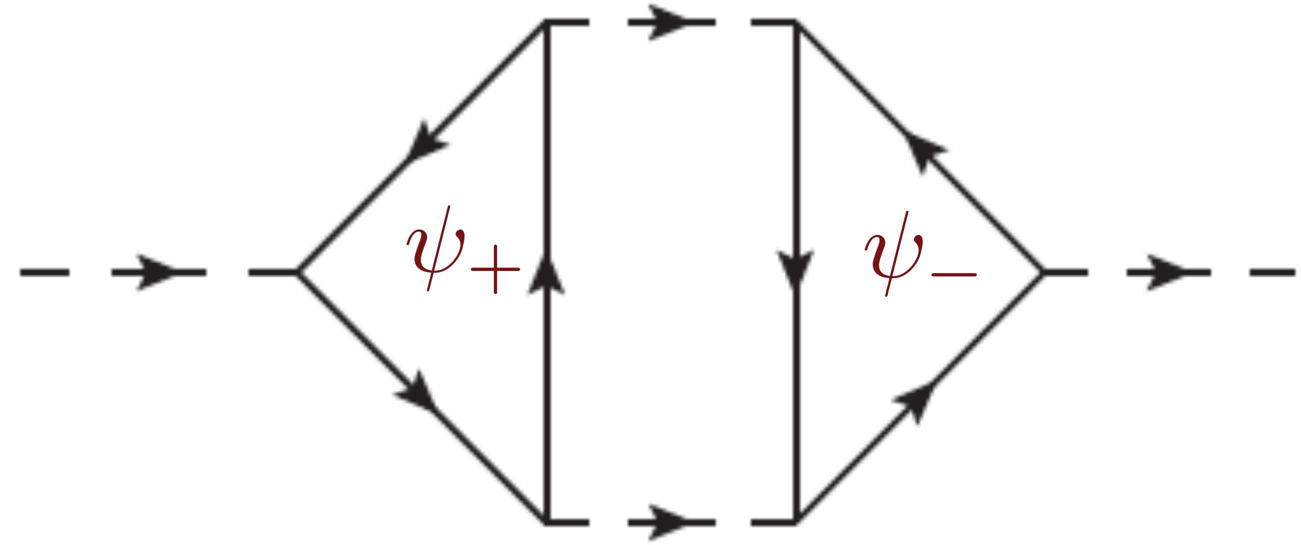
The $1/N_f$ expansion is *not* determined by counting fermion loops, because of infrared singularities created by the Fermi surface. The $|\omega|^{2/3}/N_f$ fermion self-energy leads to additional powers of N_f, and a breakdown in the loop expansion.

Computations in the 1/N expansion



All planar graphs of ψ_+ alone are as important as the leading term

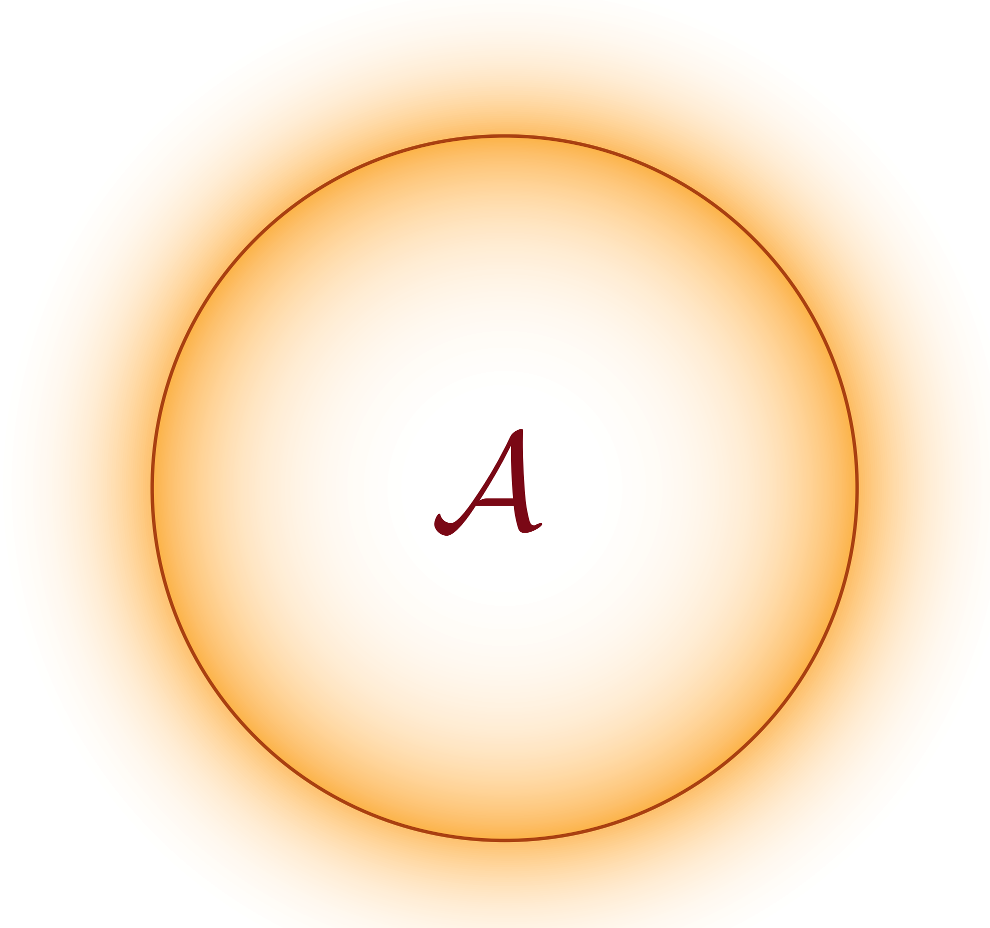


Graph mixing ψ_+ and ψ_- is $\mathcal{O}\left(N^{3/2}\right)$ (instead of $\mathcal{O}\left(N\right)$), violating genus expansion

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Properties of the strange metal at the Ising-nematic critical point



- There is a sharp Fermi surface defined by the fermion Green's function: $G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
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D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

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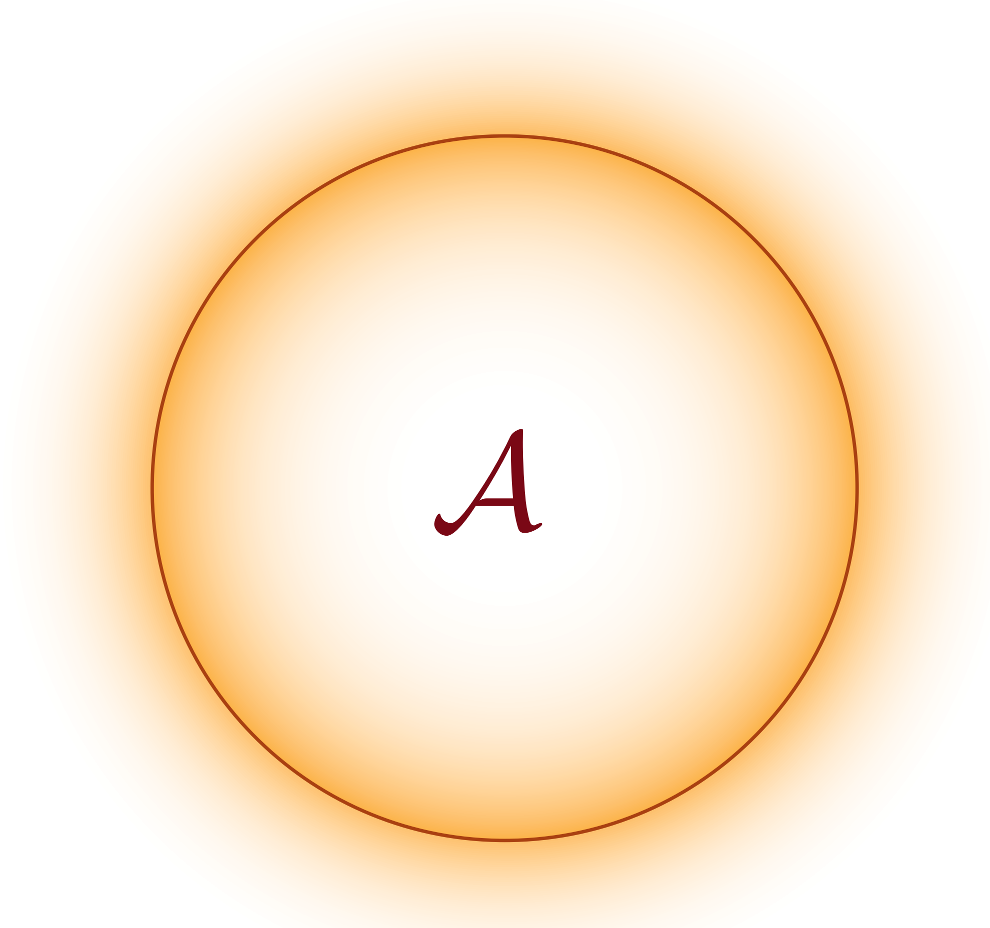

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- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

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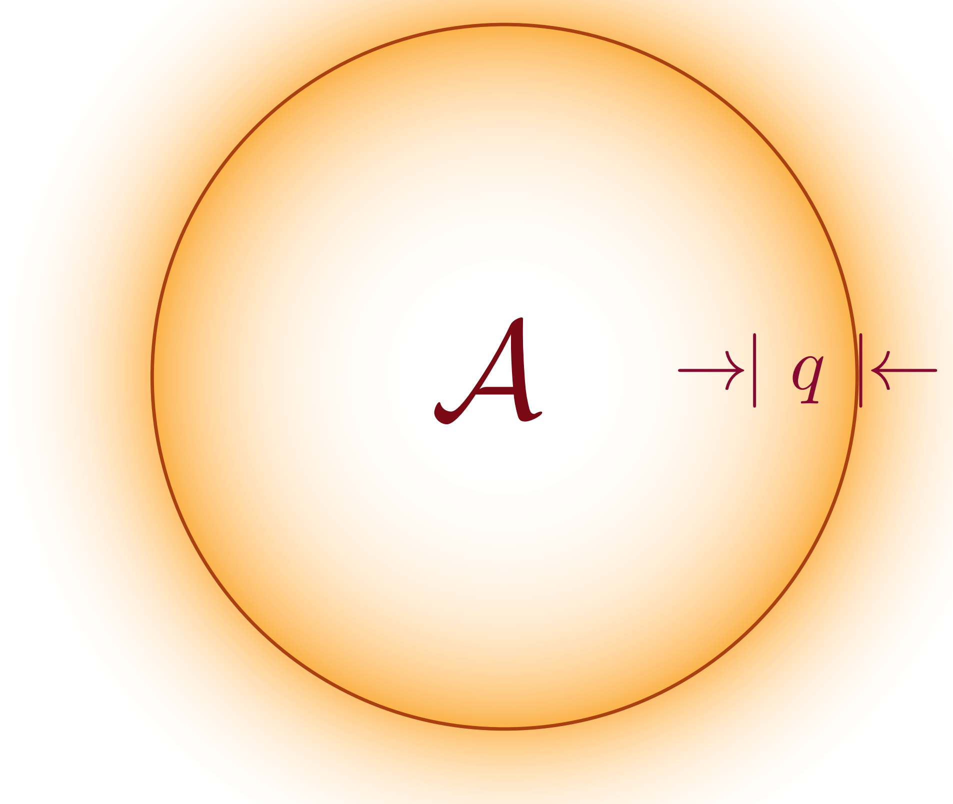

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- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density
- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

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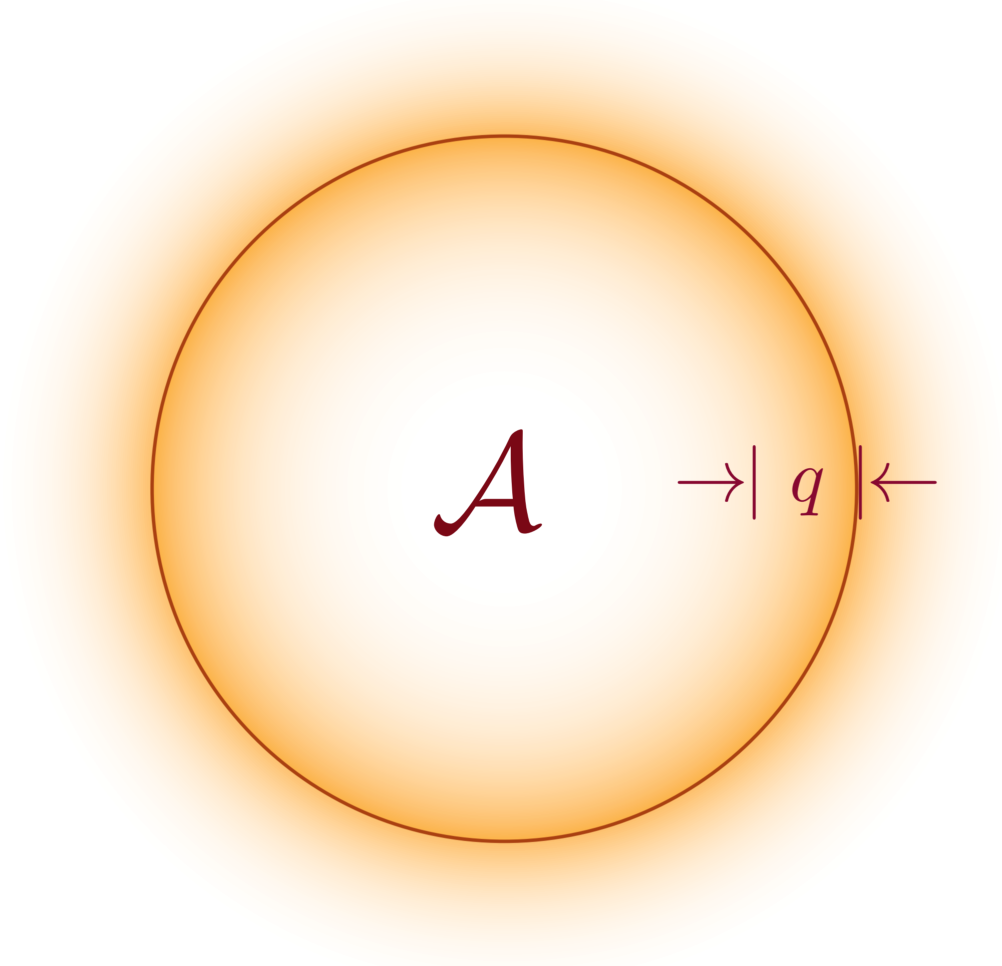

- Fermion Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
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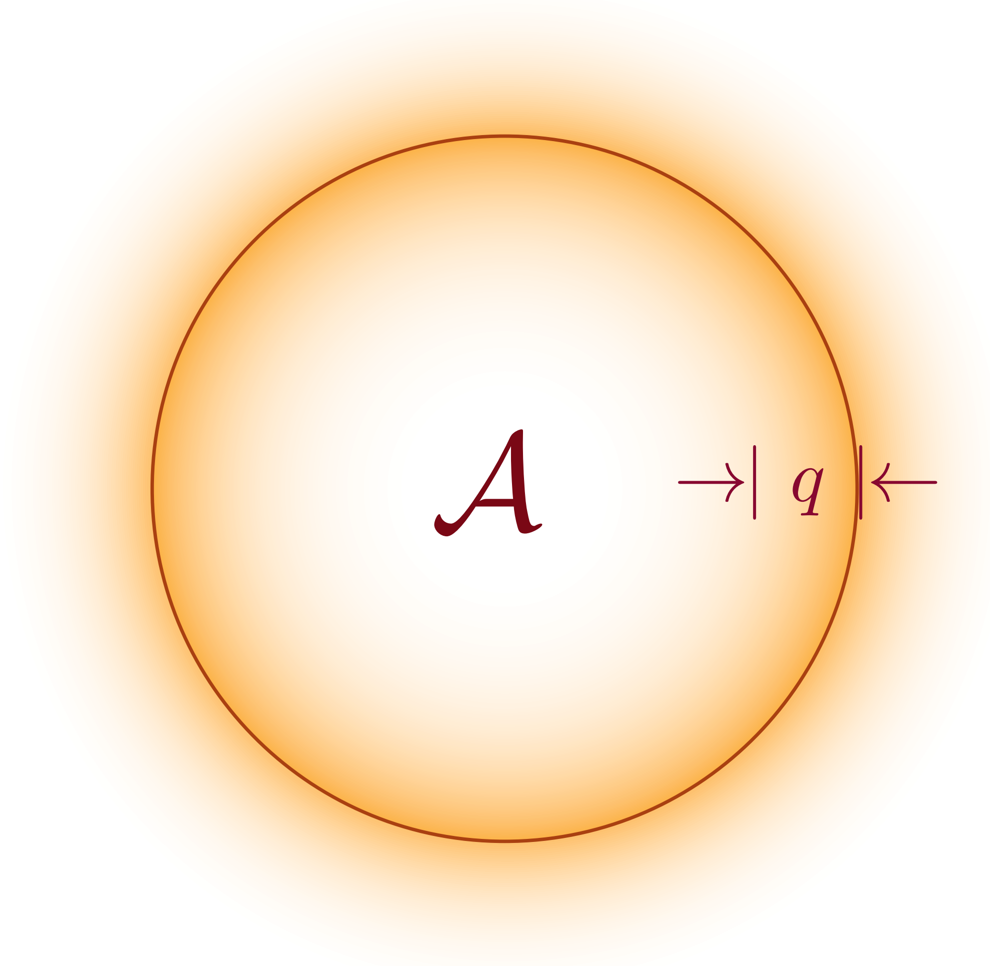
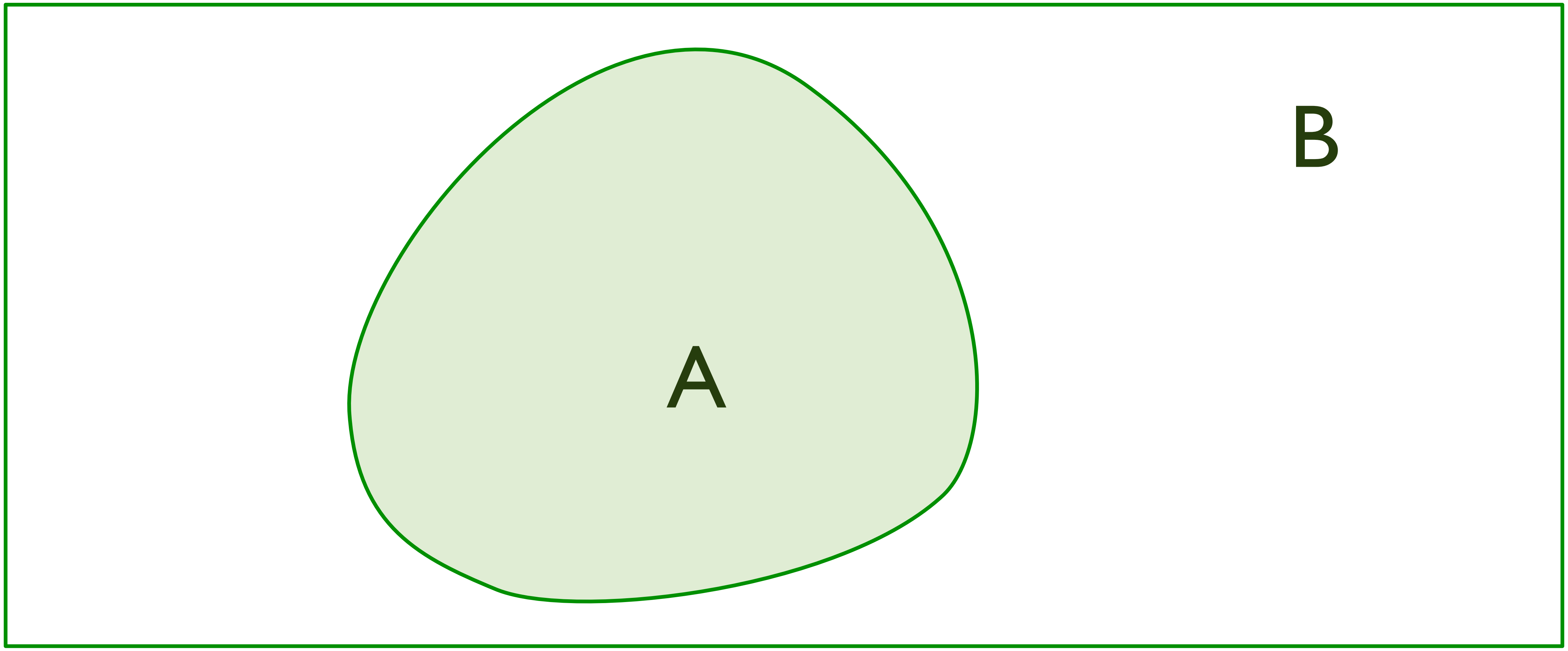


- Fermion Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\rm eff}/z}$ with $d_{\rm eff} = 1$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

Entanglement entropy

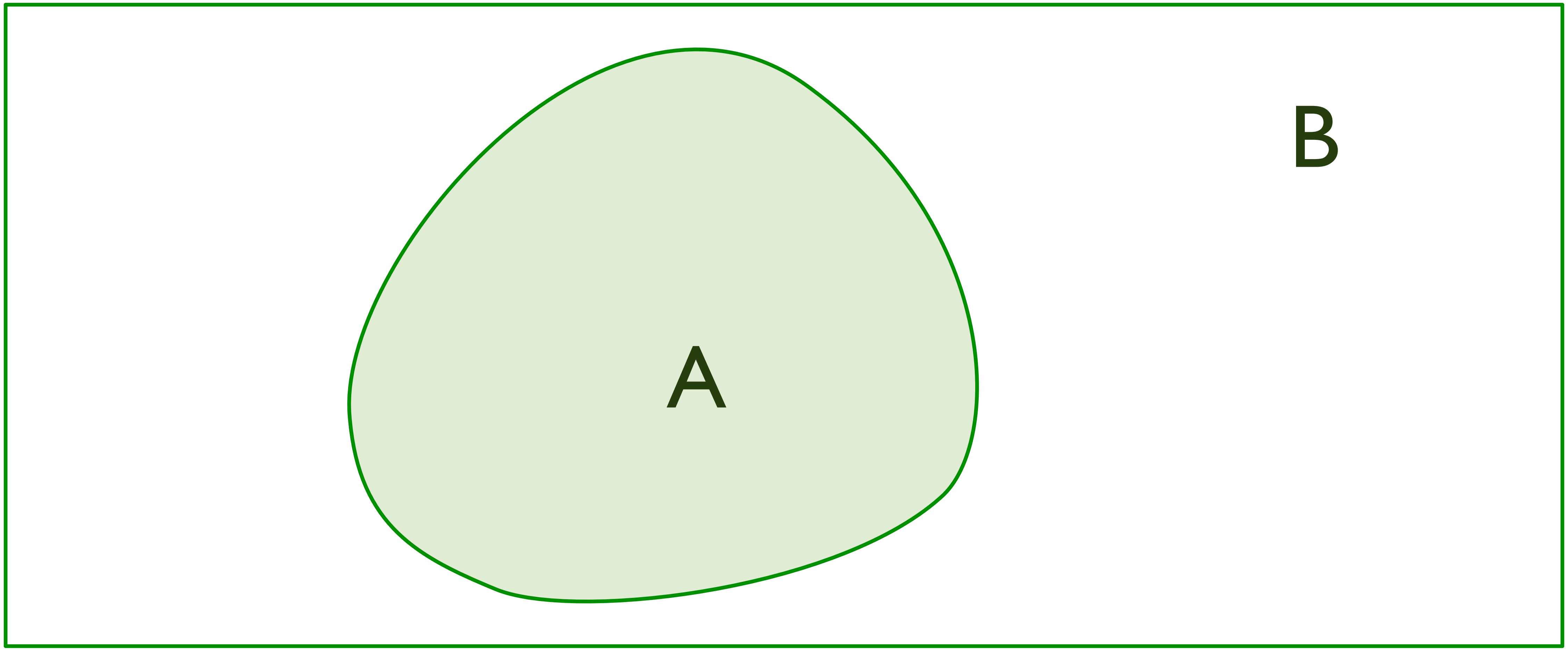


Measure strength of quantum entanglement of region A with region B.

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

Entanglement entropy $S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

Entanglement entropy of Fermi surfaces

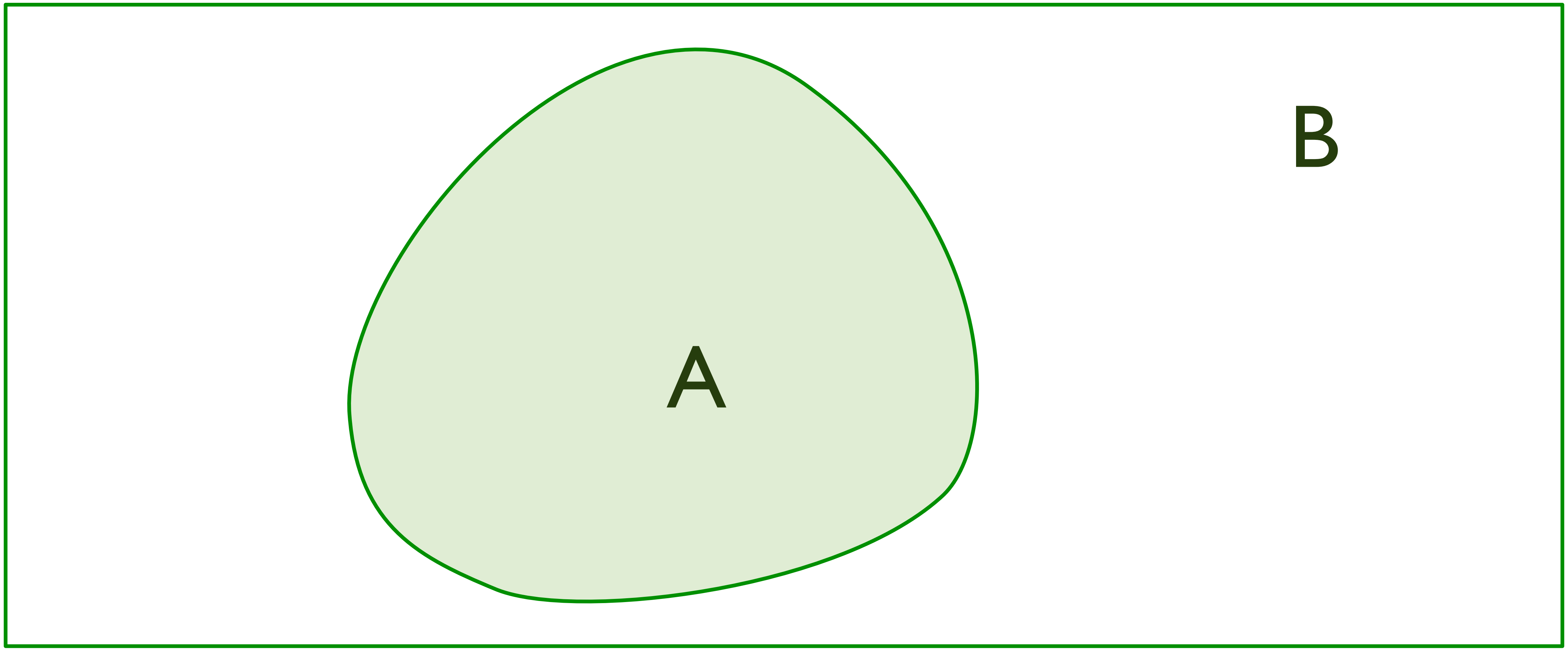


Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of Fermi surfaces



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Non-Fermi liquids have, at most, the "1/12" prefactor modified.

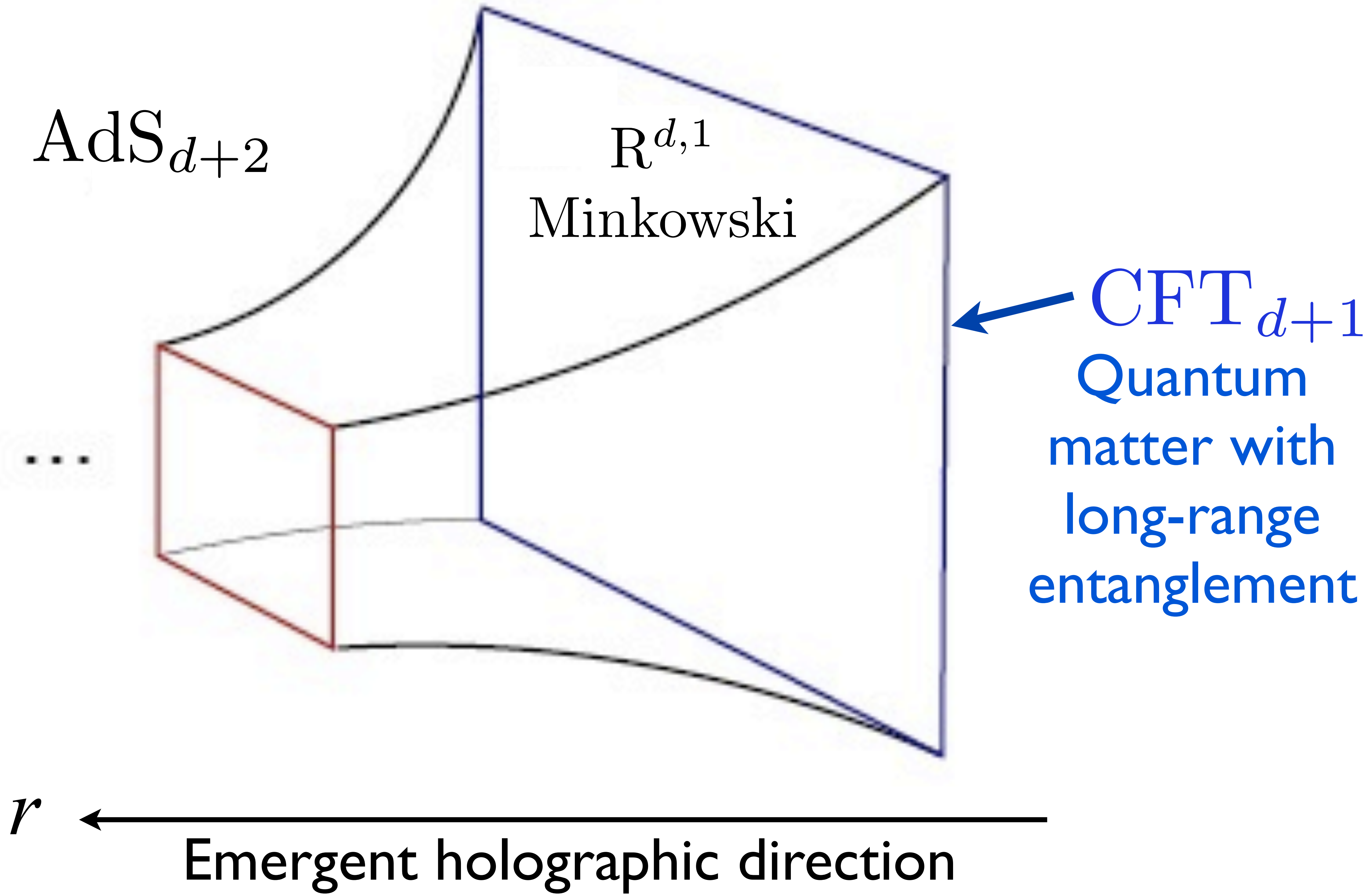
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

A. Field theory

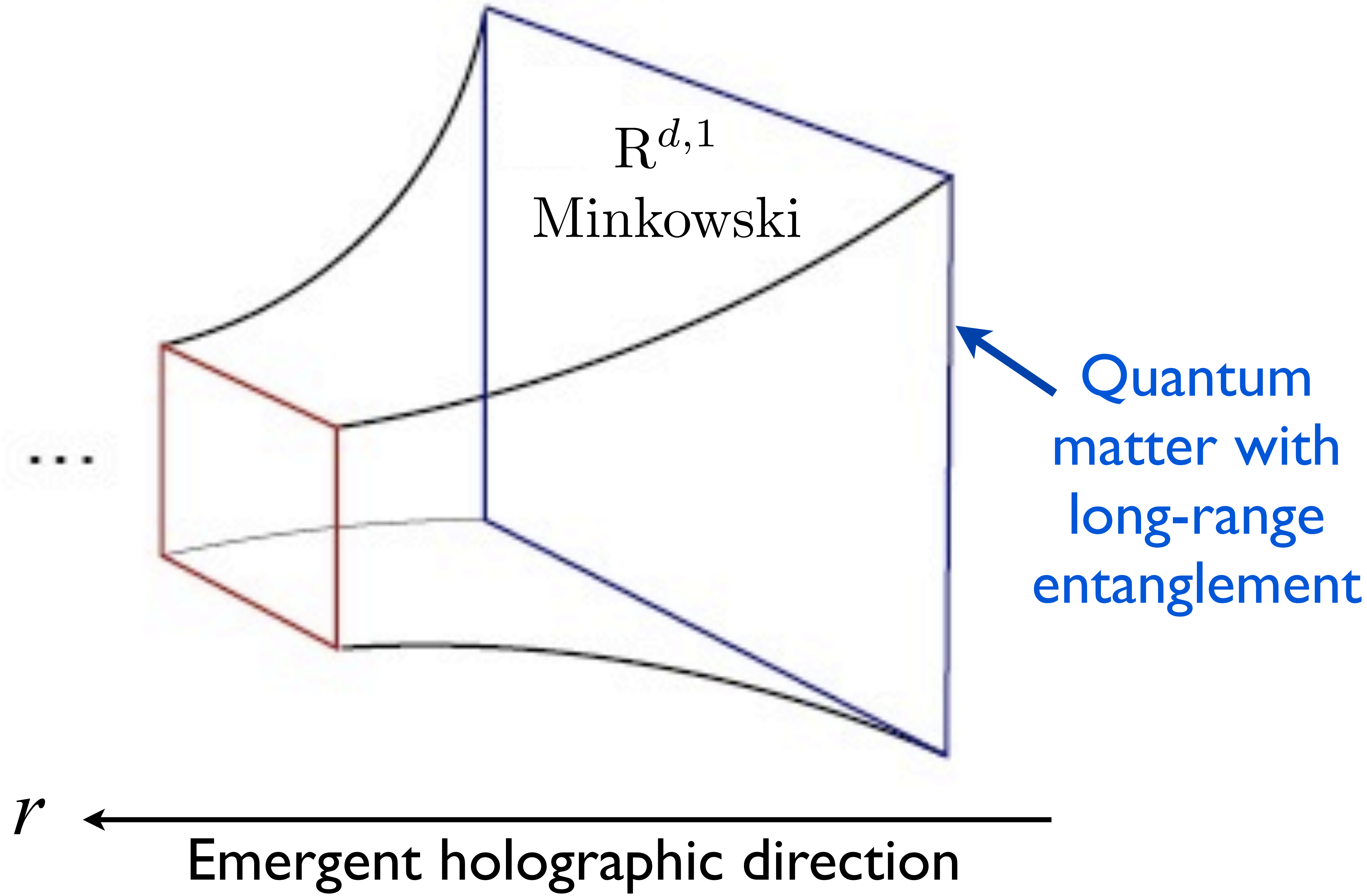
B. Gauge-gravity duality

A. Field theory

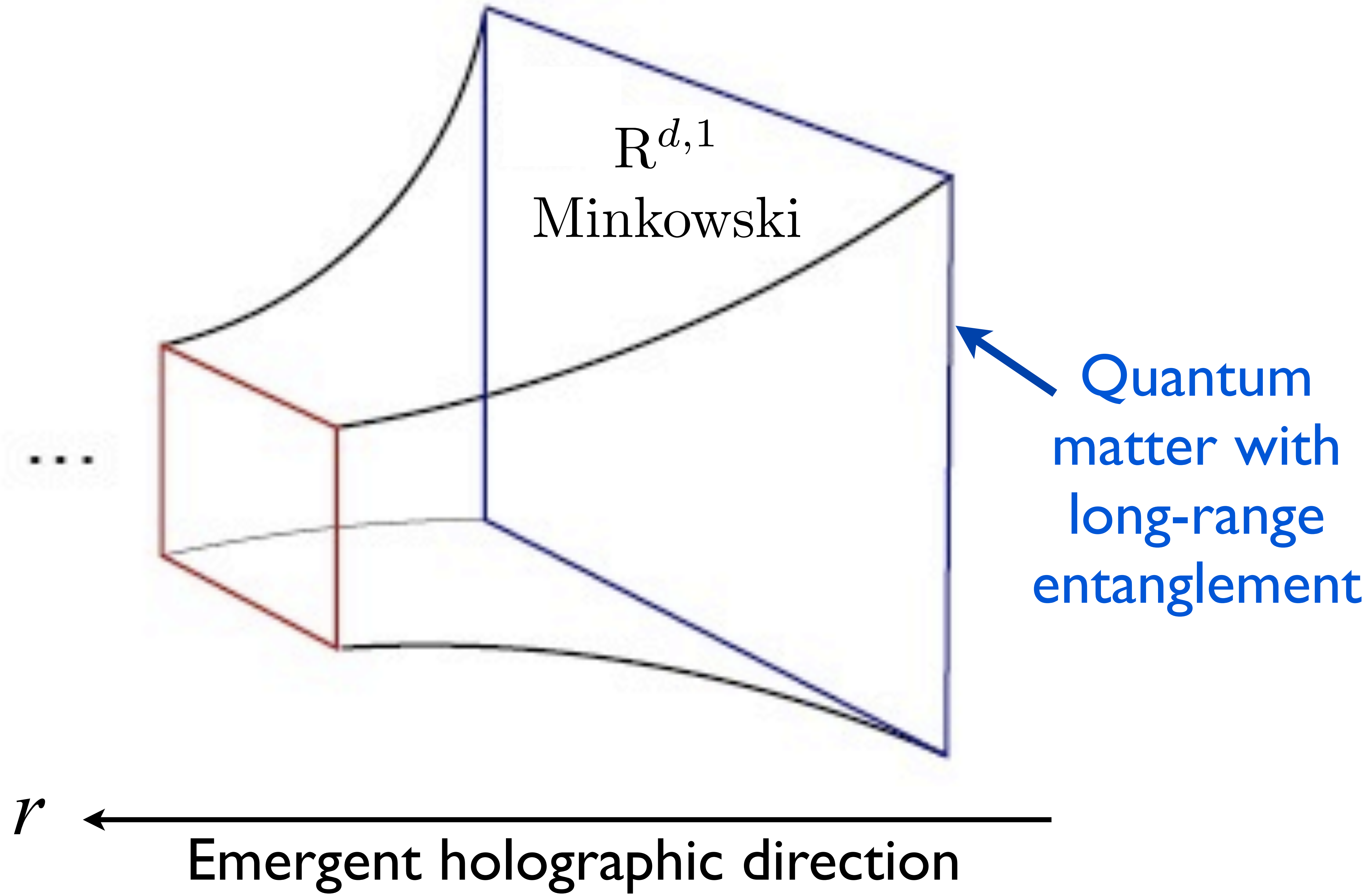
B. Gauge-gravity duality



J. McGreevy, arXiv0909.0518



$\mathrm{R}^{d,1}$
Minkowski
...
Quantum matter with long-range entanglement
r
Emergent holographic direction



Abandon conformal invariance, and only require scale invariance at long lengths and times.....

Most general metric has 2 independent exponents z and θ, where z is the dynamic critical exponent:

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

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$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

Such a theory has:

- Thermal entropy $S \sim T^{(d-\theta)/z}$.

- Entanglement entropy

$$S_E \sim \begin{cases} P & , \quad \text{for } \theta < d-1 \\ P \ln P & , \quad \text{for } \theta = d-1 \\ P^{\theta/(d-1)} & , \quad \text{for } \theta > d-1 \end{cases}$$

- The null energy condition implies $z \geq 1 + \frac{\theta}{d}$.

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d.
  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of non-Fermi liquids

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  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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$$\theta = d - 1$$

- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1202**, 137 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!
- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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$$\theta = d - 1$$

- This metric can be realized as the solution Einstein-Maxwell-dilaton holographic theory, corresponding to a boundary theory with charge $\mathcal{Q}$. The entanglement entropy then has the form

$$S_E = \Xi\, \mathcal{Q}^{(d-1)/d} P \ln\left(\mathcal{Q}^{(d-1)/d} P\right).$$

where P is surface area ('perimeter') of the entangling region, and Ξ is a dimensionless constant which is independent of all UV details, of $\mathcal{Q}$, and of any property of the entangling region. Note $\mathcal{Q}^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Conclusions

Compressible quantum matter

- Field theory of Ising-nematic ordering in a Fermi liquid ("Pomeranchuk" transition) is strongly coupled in two spatial dimensions

Conclusions

Compressible quantum matter

- Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

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After fixing $\theta = d-1$ to obtain thermal entropy density $S \sim T^{1/z}$, we found

- Log violation of the area law in entanglement entropy, S_E.

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