

States of quantum matter with long-range entanglement in d spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

States of quantum matter with long-range entanglement in d spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids



Liza Huijse



Max Metlitski



Brian Swingle

Compressible quantum matter

A. Fermi liquids: graphene

*B. Holography: Reissner - Nördstrom
solution*

*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*

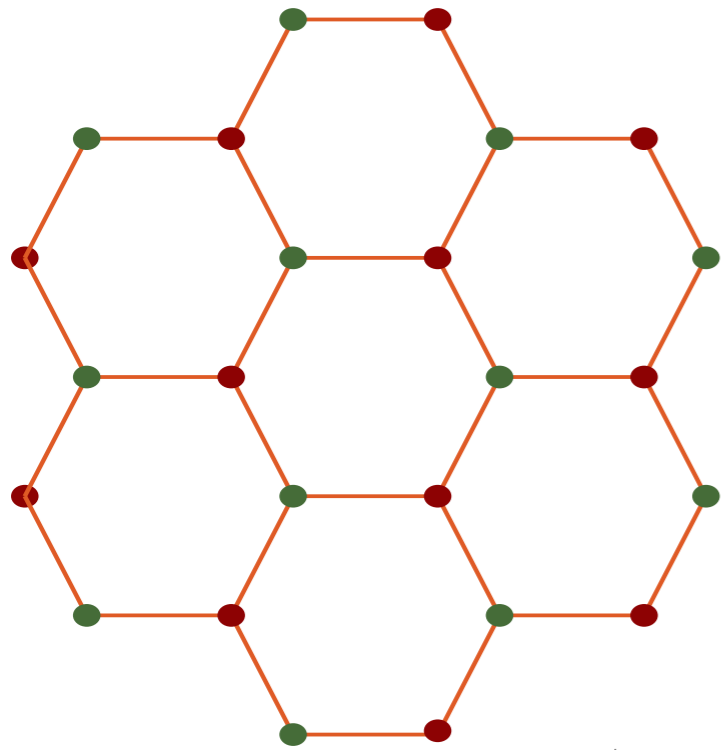
Compressible quantum matter

A. Fermi liquids: graphene

B. Holography: Reissner - Nördstrom solution

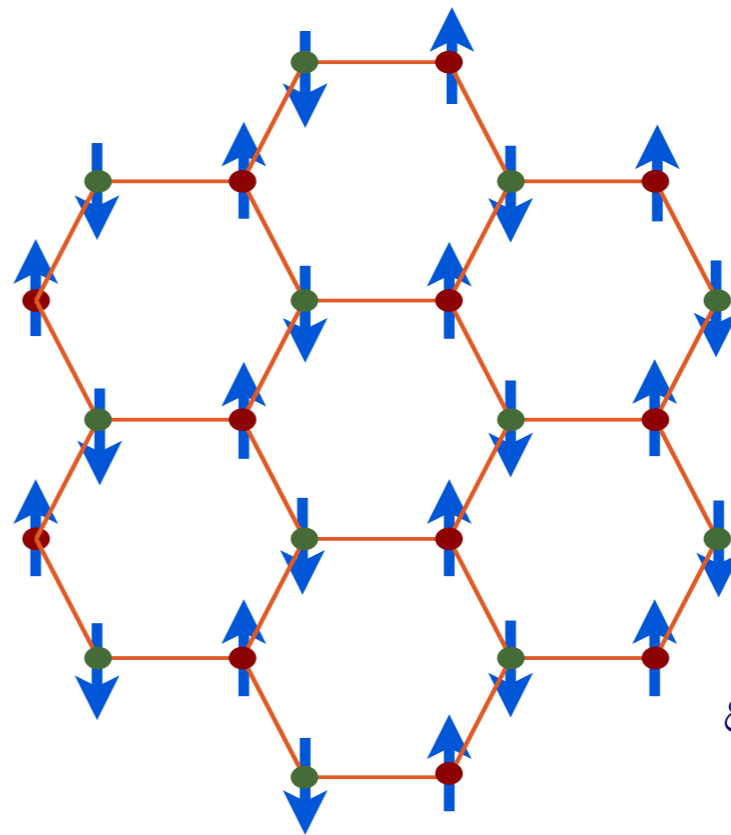
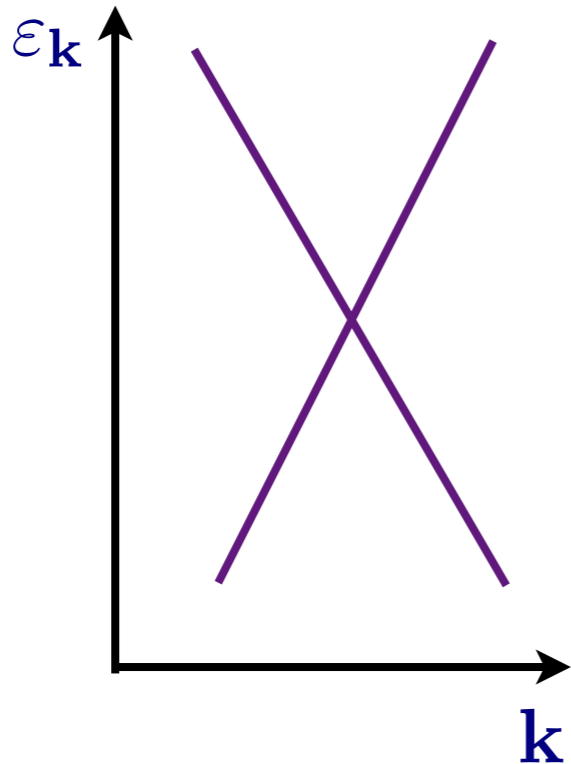
*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*



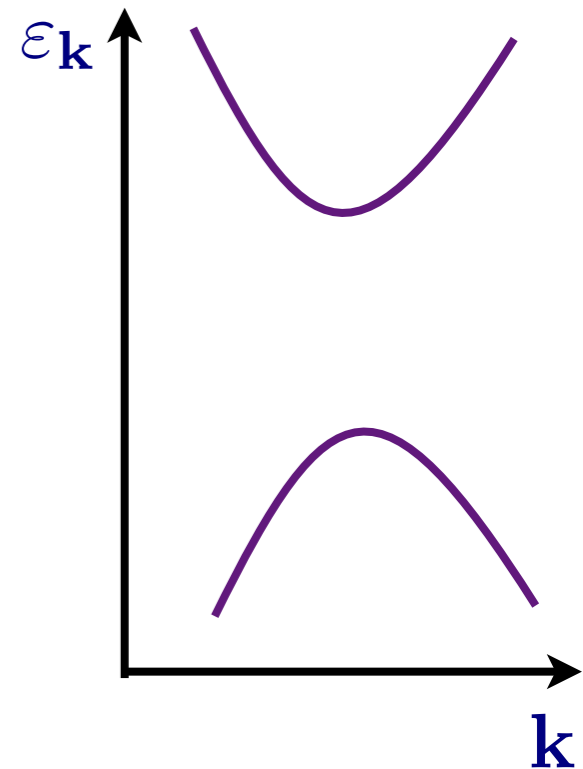
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

$$\langle \varphi^a \rangle \neq 0$$

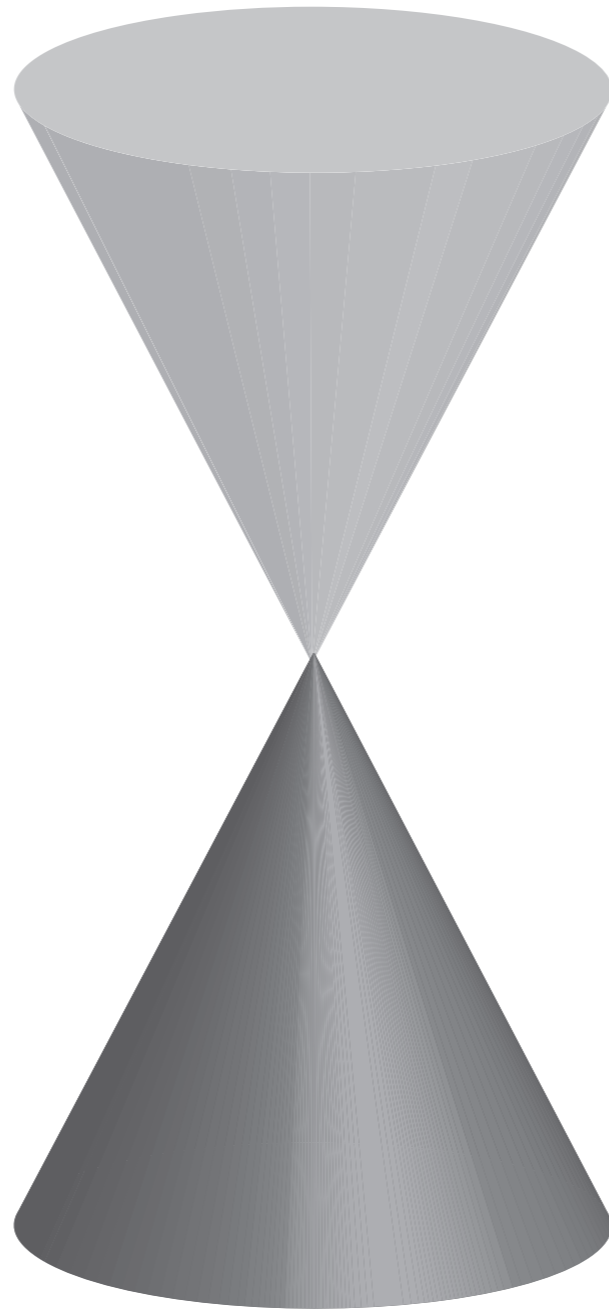


S

Free CFT3

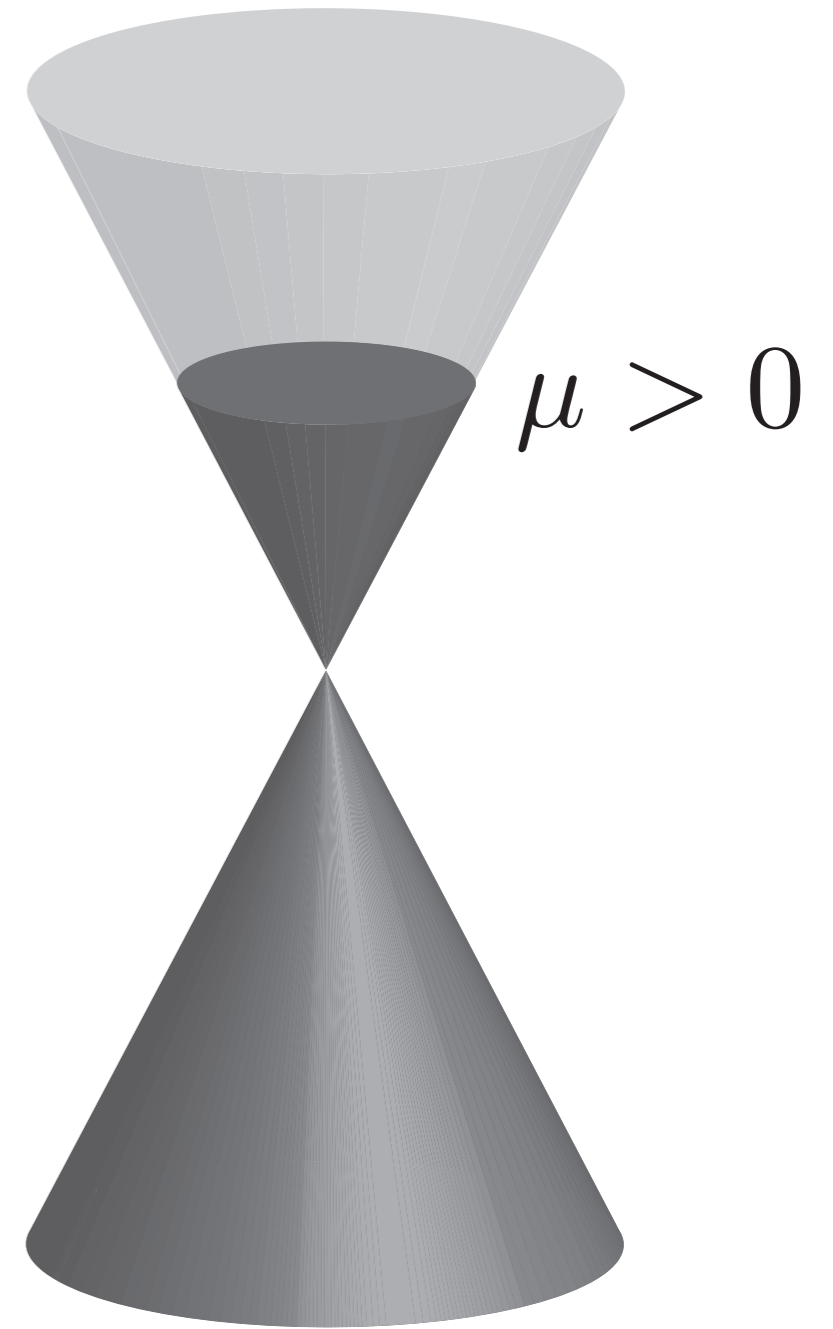
Interacting CFT3
with long-range entanglement

Quantum phase transition in graphene tuned by a chemical potential (gate voltage)



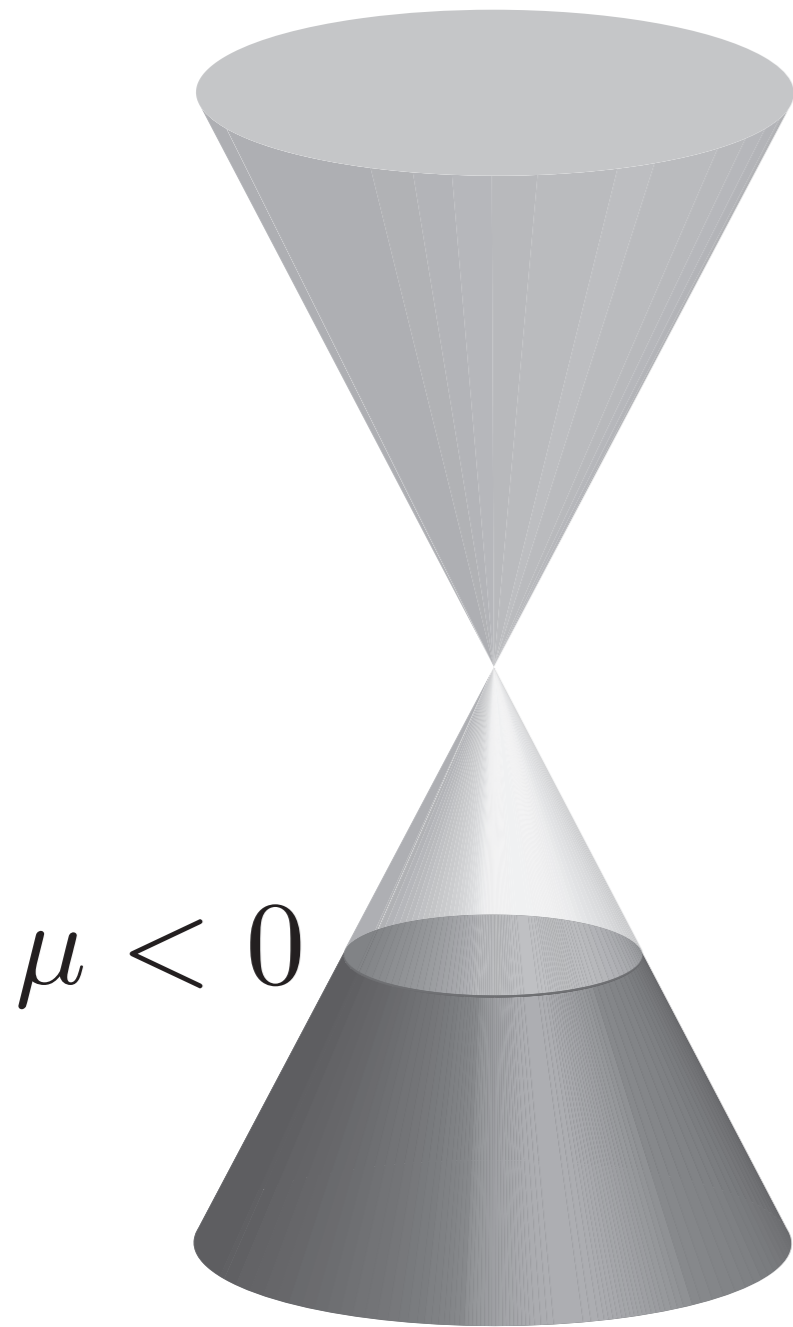
**Dirac
semi-metal**

Quantum phase transition in graphene tuned by a chemical potential (gate voltage)



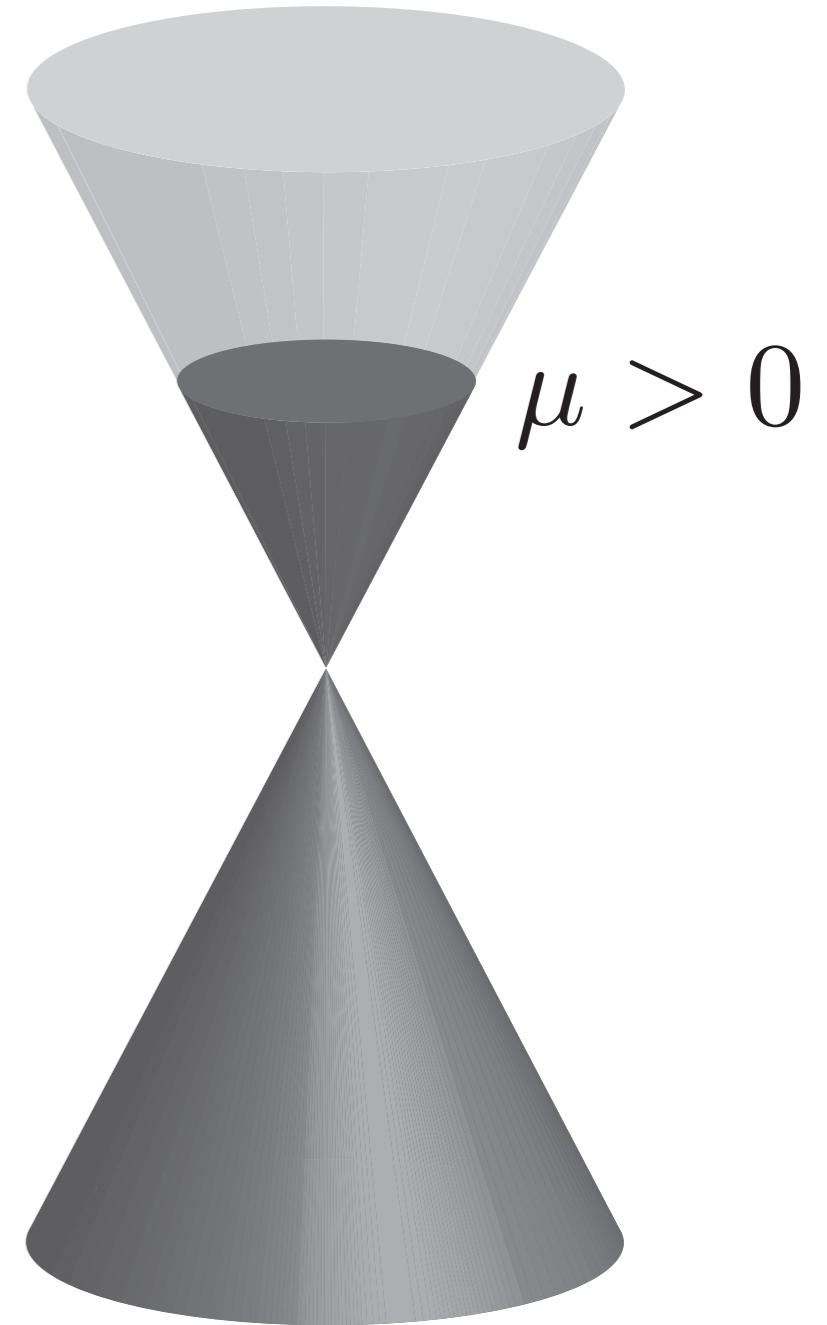
**Electron
Fermi surface**

Quantum phase transition in graphene tuned by a chemical potential (gate voltage)



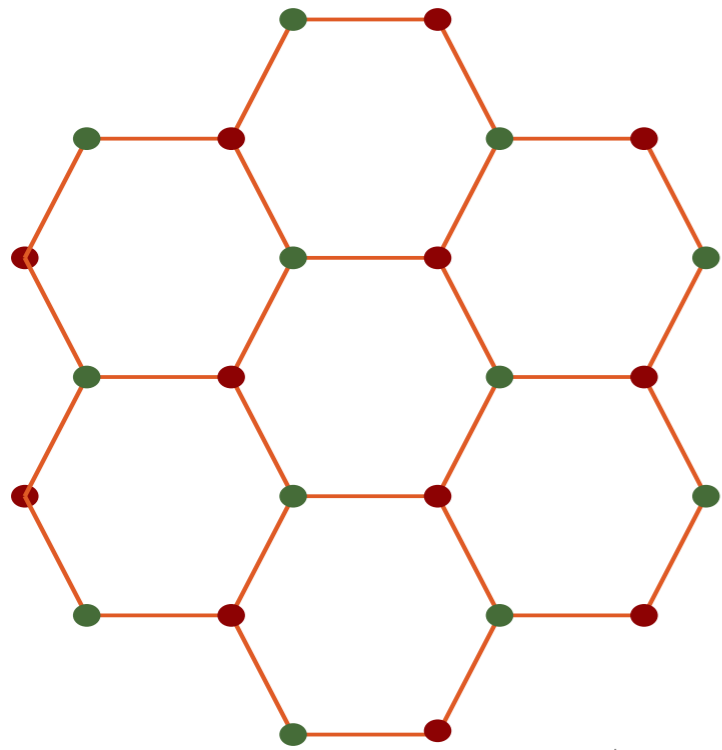
$$\mu < 0$$

**Hole
Fermi surface**



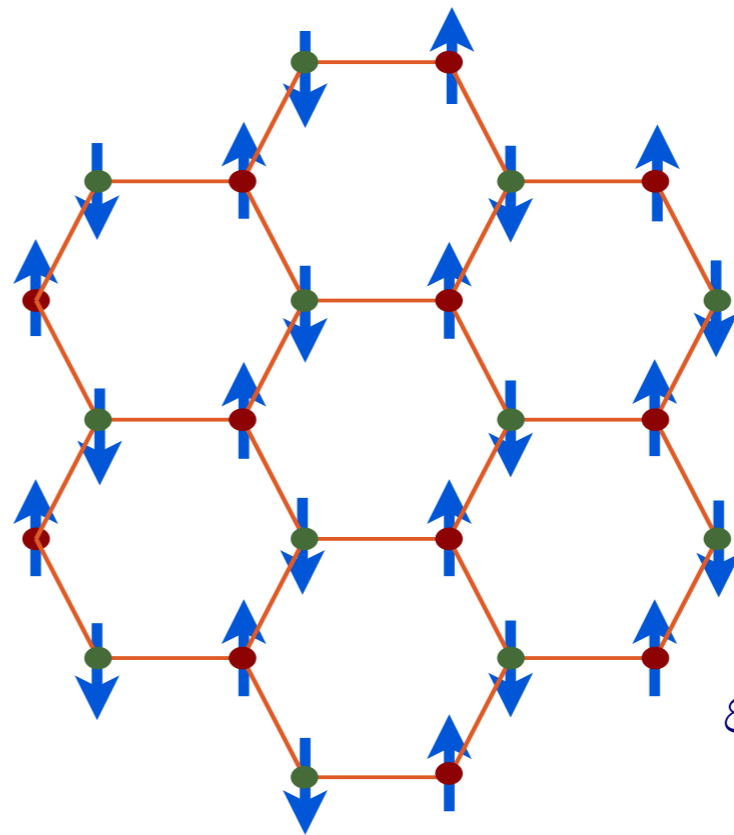
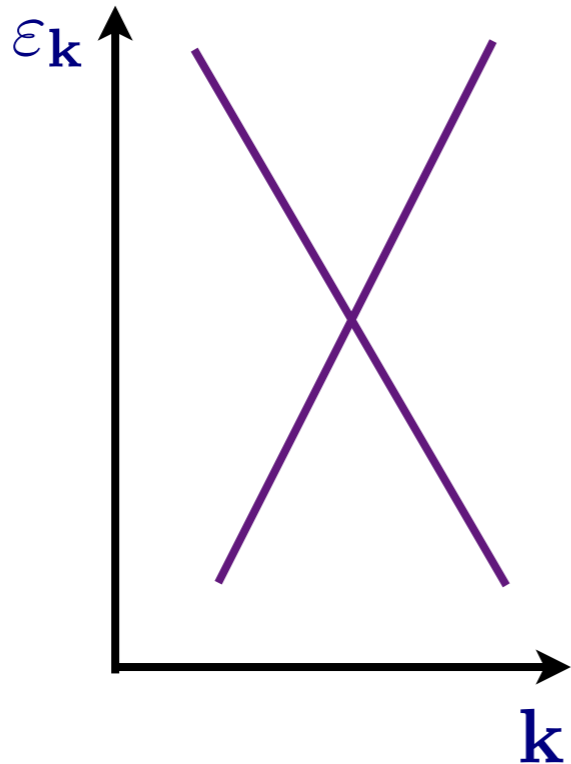
$$\mu > 0$$

**Electron
Fermi surface**



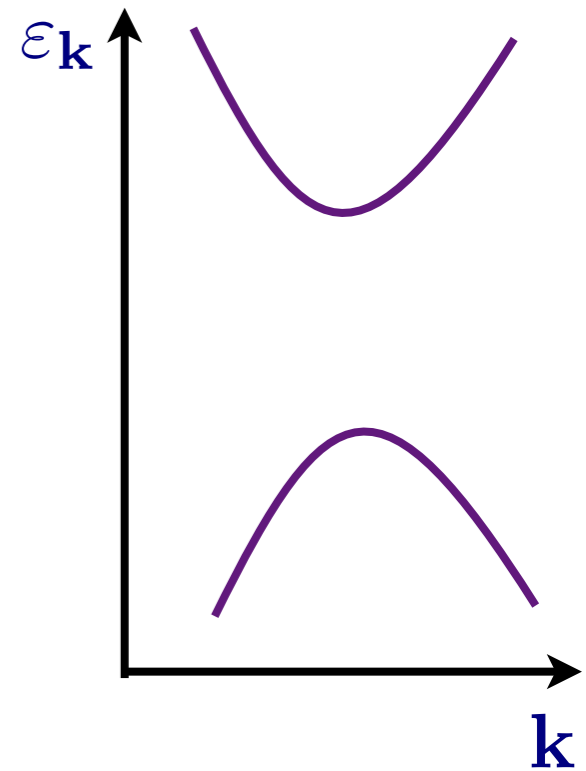
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

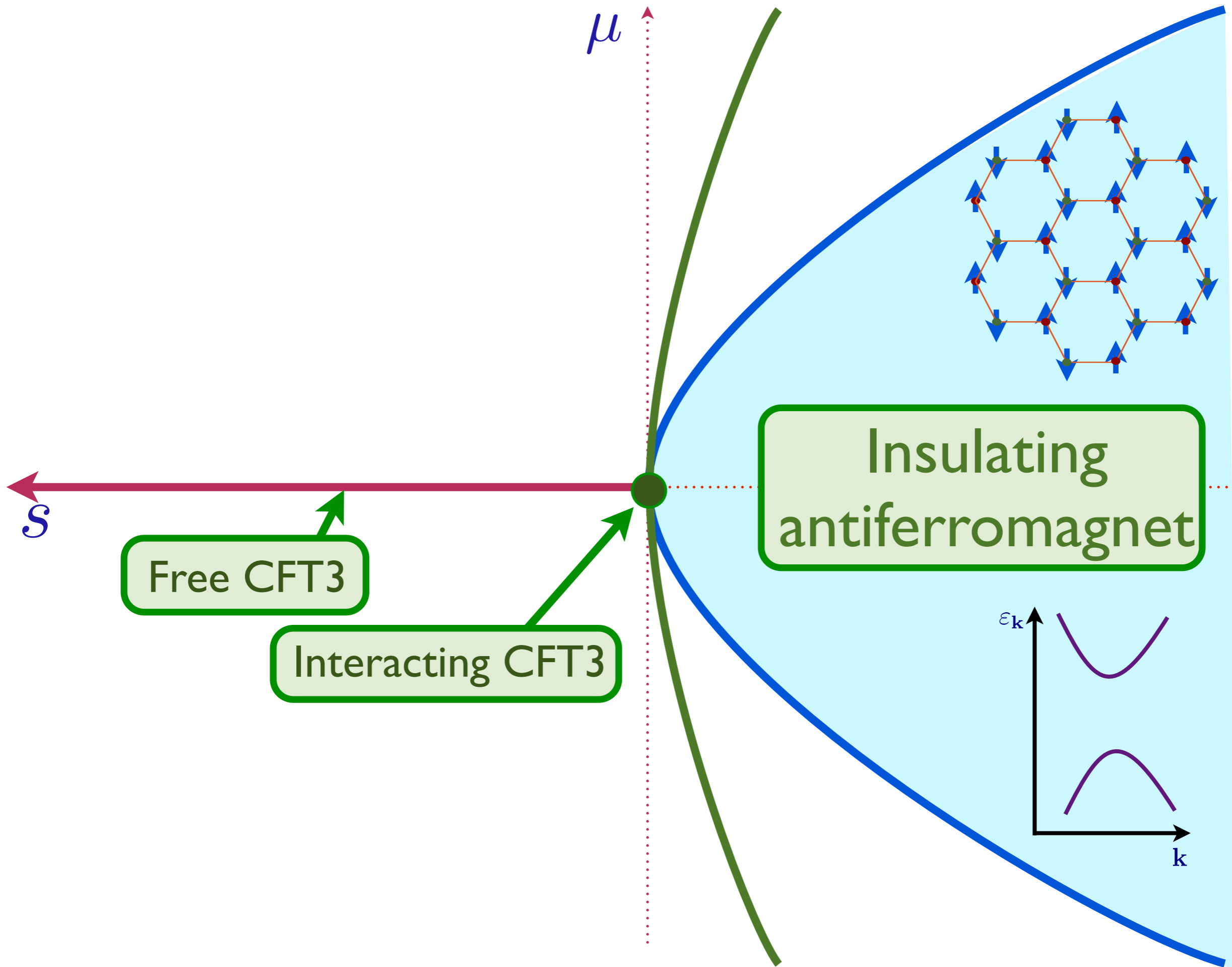
$$\langle \varphi^a \rangle \neq 0$$

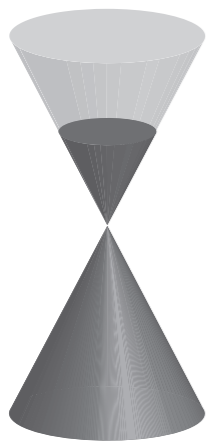


S

Free CFT3

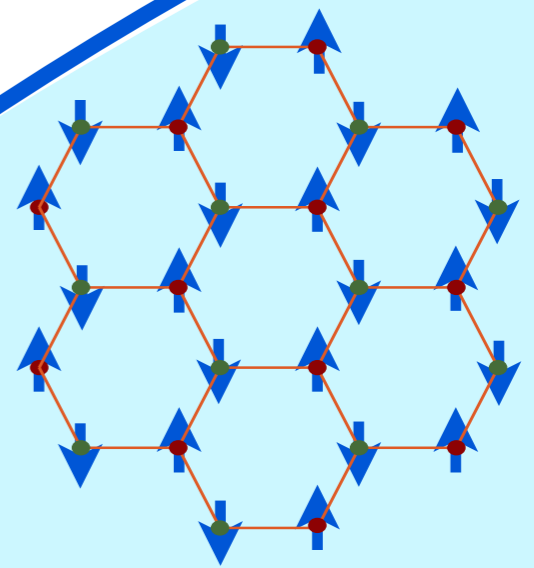
Interacting CFT3
with long-range entanglement





Electron metal

μ



Insulating antiferromagnet

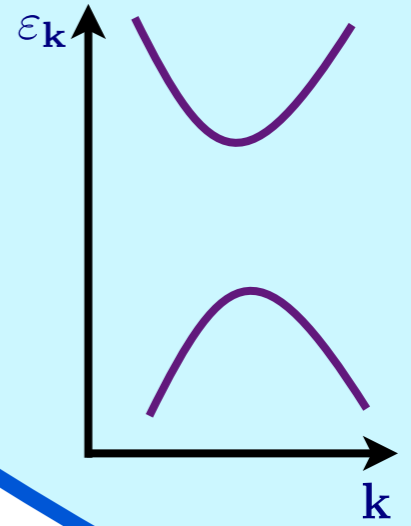
S

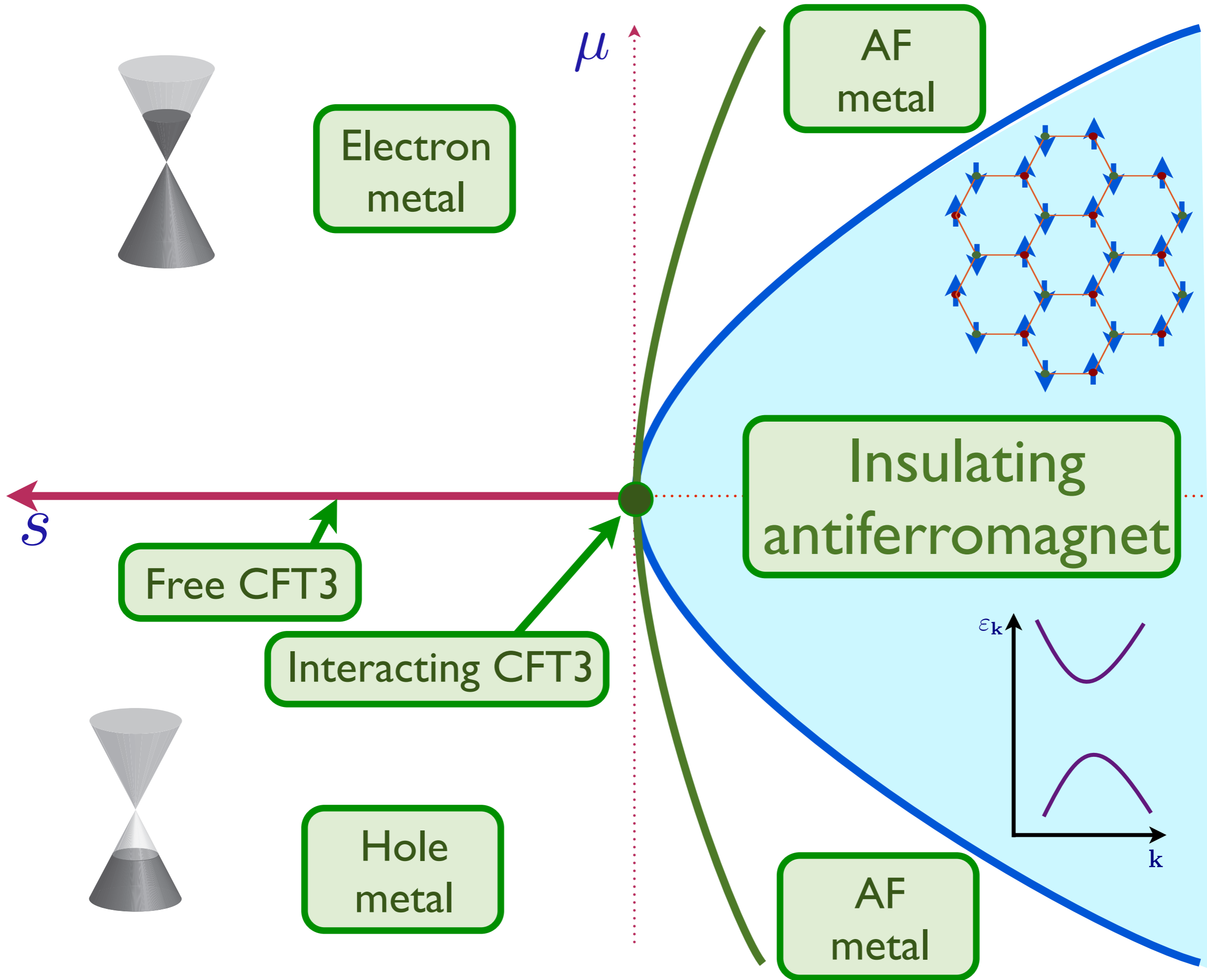
Free CFT3

Interacting CFT3



Hole metal





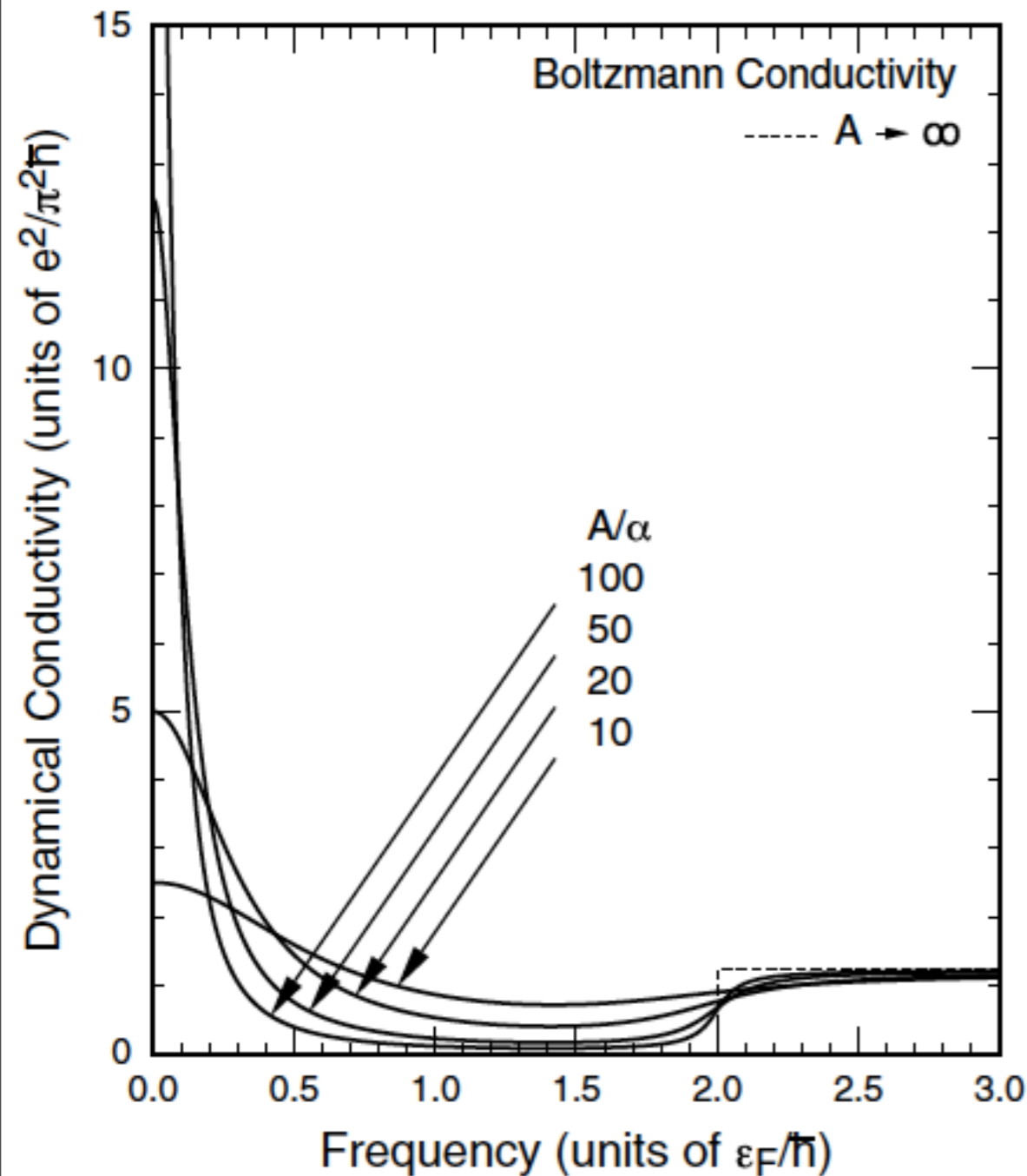
Transport in graphene at non-zero μ

From the Kubo formula

$$\sigma(\omega) = 2 (ev_F)^2 \frac{\hbar}{i} \sum_{ss'} \int \frac{d^2k}{4\pi^2} \frac{f(\varepsilon_s(\mathbf{k})) - f(\varepsilon_{s'}(\mathbf{k}))}{(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}))(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}) + \hbar\omega + i\eta)}$$

where $\varepsilon_s(\mathbf{k}) = s\hbar v_F |\mathbf{k}|$ and $s, s' = \pm 1$ for the valence and conduction bands.

Transport in graphene at non-zero μ

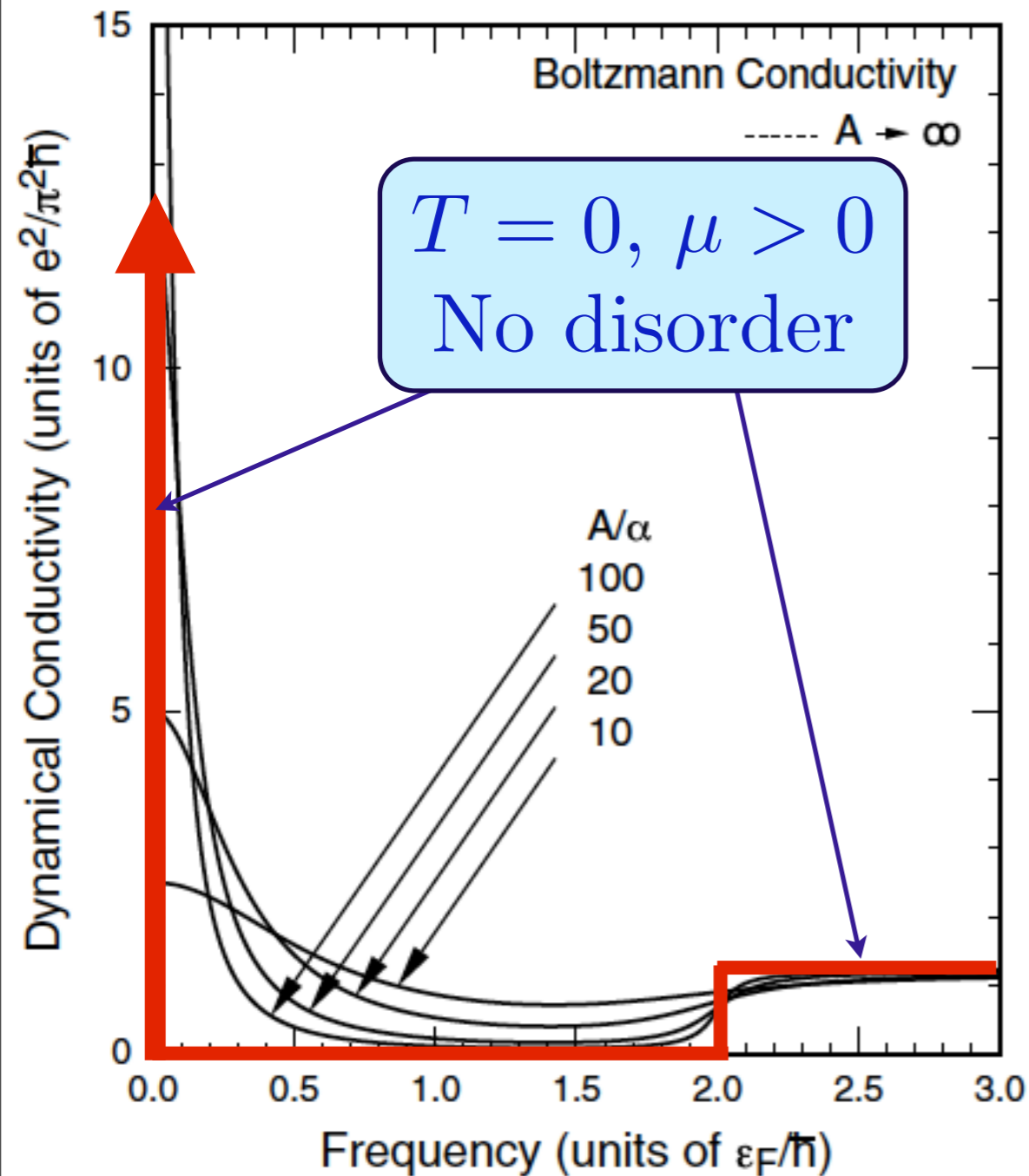


A is inversely proportional to disorder. In the clean limit $A \rightarrow \infty$, at $T = 0$

$$\text{Re}[\sigma(\omega)] = \frac{e^2}{\hbar} \left[\frac{\epsilon_F}{\hbar} \delta(\omega) + \frac{1}{4} \theta(|\omega| - 2\epsilon_F) \right]$$

Notice delta function is present even at $T = 0$ at non-zero density: this is a generic consequence of the conservation of momentum in any clean interacting Fermi liquid. Only “umklapp” scattering can broaden this delta function.

Transport in graphene at non-zero μ



A is inversely proportional to disorder.
In the clean limit $A \rightarrow \infty$, at $T = 0$

$$\text{Re}[\sigma(\omega)] = \frac{e^2}{\hbar} \left[\frac{\varepsilon_F}{\hbar} \delta(\omega) + \frac{1}{4} \theta(|\omega| - 2\varepsilon_F) \right]$$

Notice delta function is present even at $T = 0$ at non-zero density: this is a generic consequence of the conservation of momentum in any clean interacting Fermi liquid. Only “umklapp” scattering can broaden this delta function.

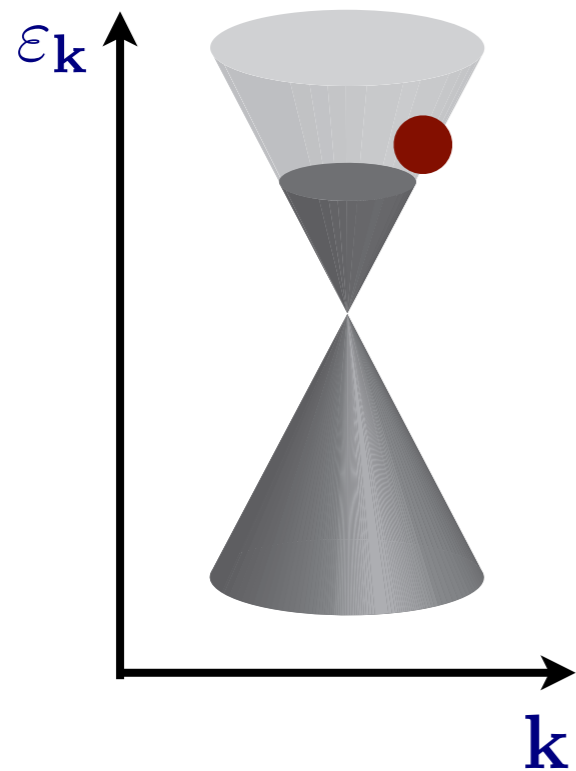
Particles



Momentum



Current



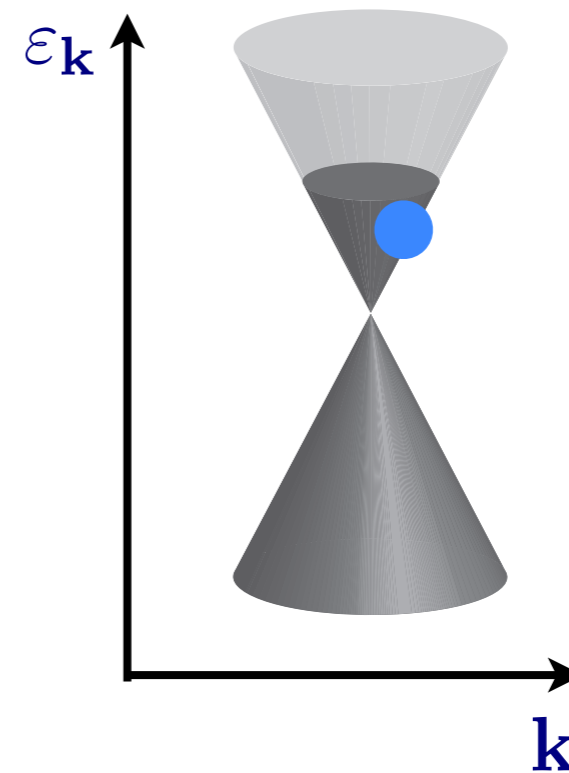
Holes



Momentum



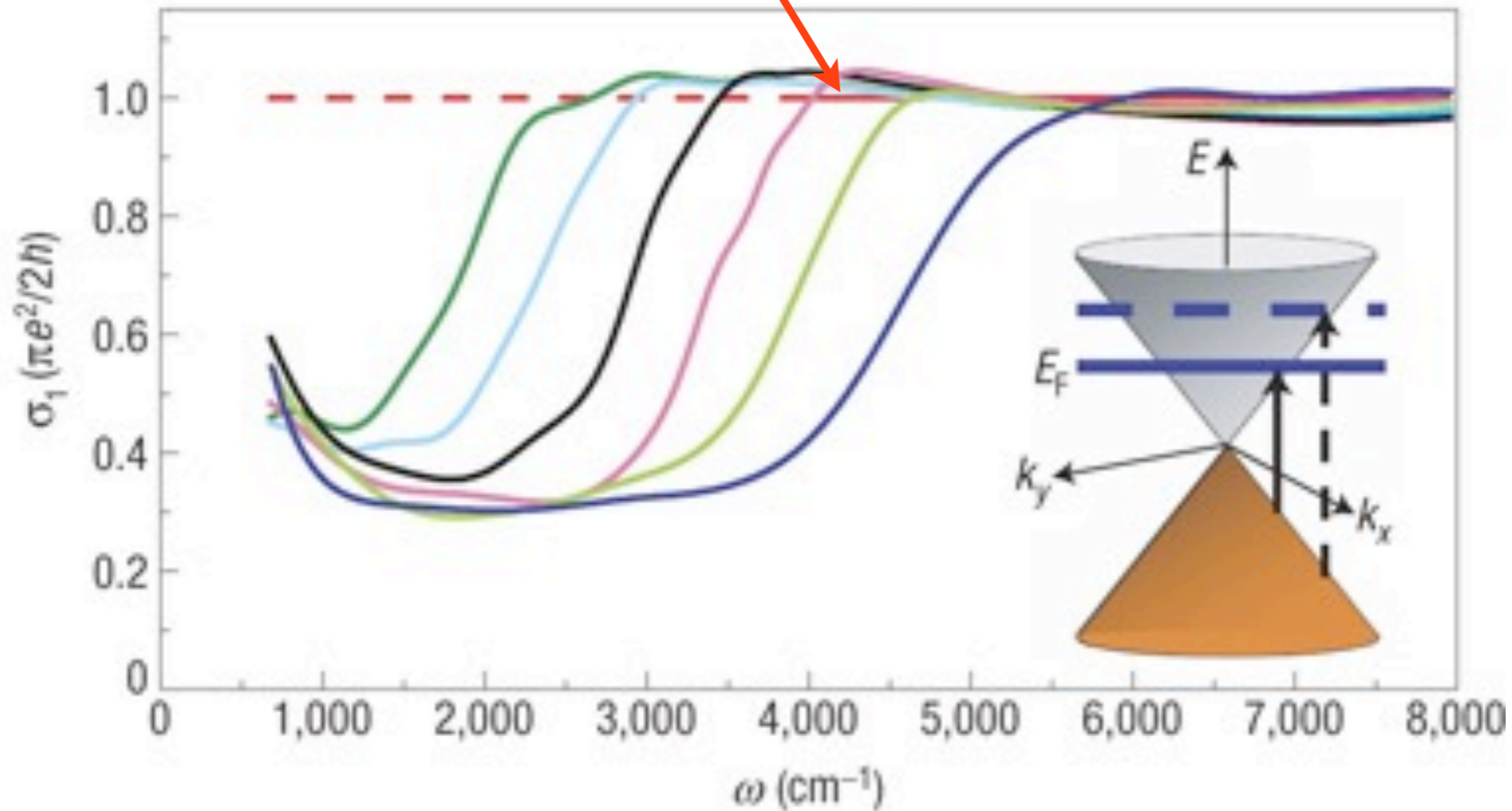
Current



Current carrying state has non-zero momentum, and collisions cannot relax current to zero

Optical conductivity of graphene

Undoped graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

Compressible quantum matter

A. Fermi liquids: graphene

*B. Holography: Reissner - Nördstrom
solution*

*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*

Compressible quantum matter

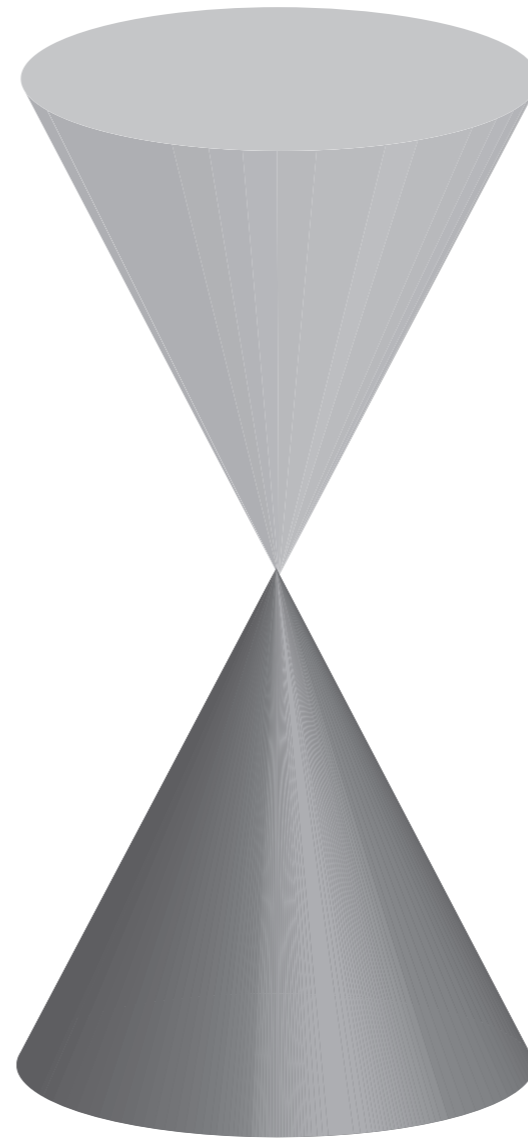
A. Fermi liquids: graphene

B. Holography: Reissner - Nördstrom solution

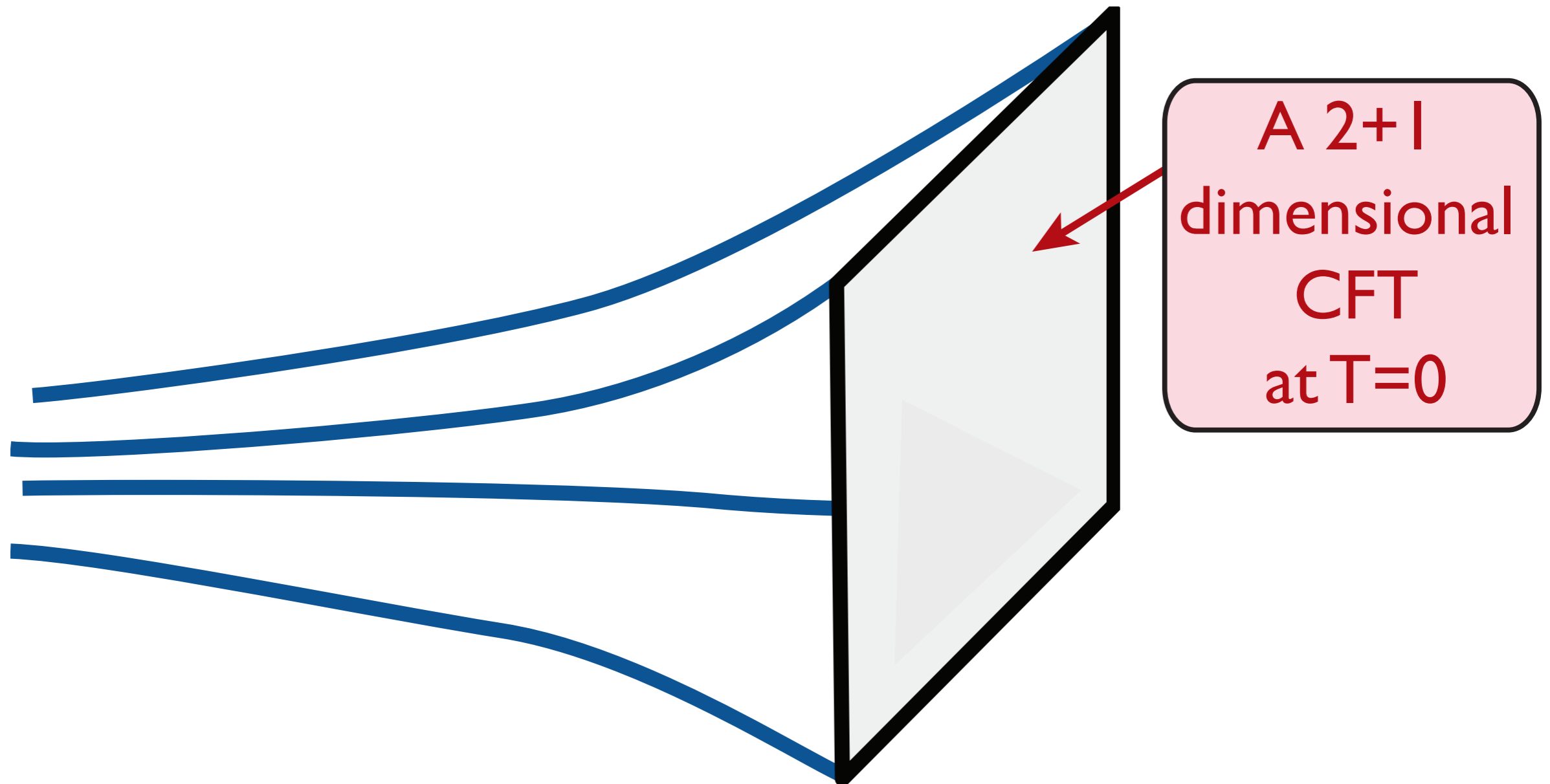
*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*

Begin with a CFT

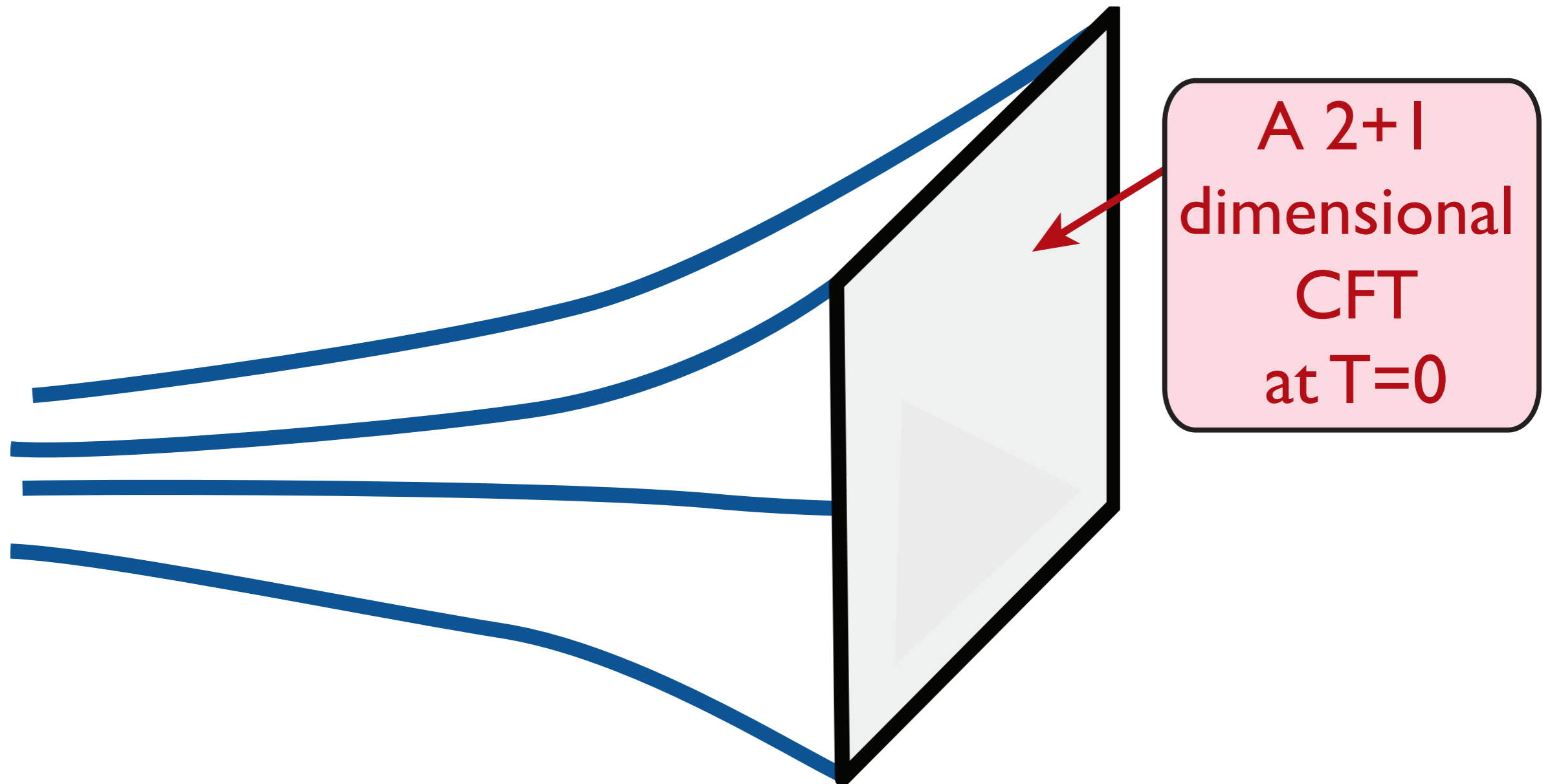


Holographic representation: AdS₄



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

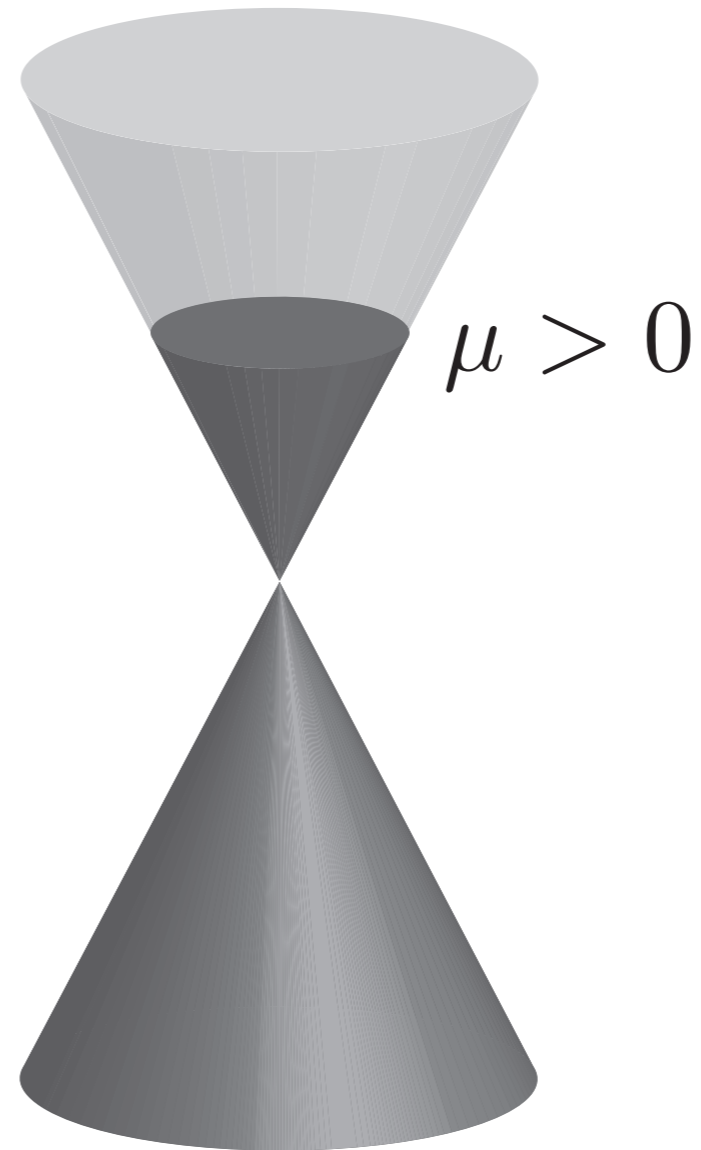
Holographic representation: AdS₄



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1$

Apply a chemical potential



AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right].$$

This is to be solved subject to the constraint

$$A_\mu(r \rightarrow 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where \mathcal{A}_μ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right].$$

This is to be solved subject to the constraint

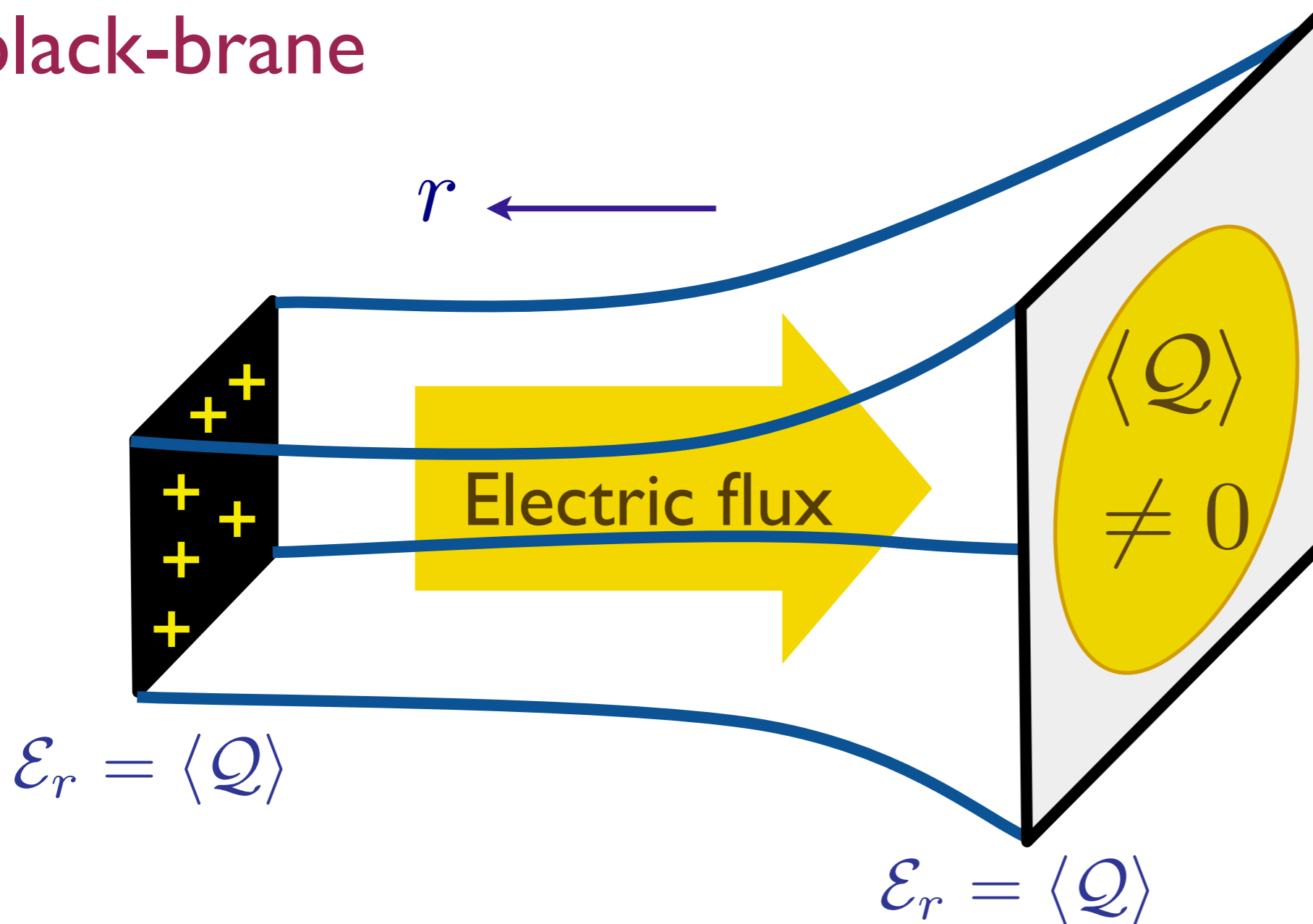
$$A_\mu(r \rightarrow 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where \mathcal{A}_μ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

At non-zero chemical potential we simply require $\mathcal{A}_\tau = \mu$.

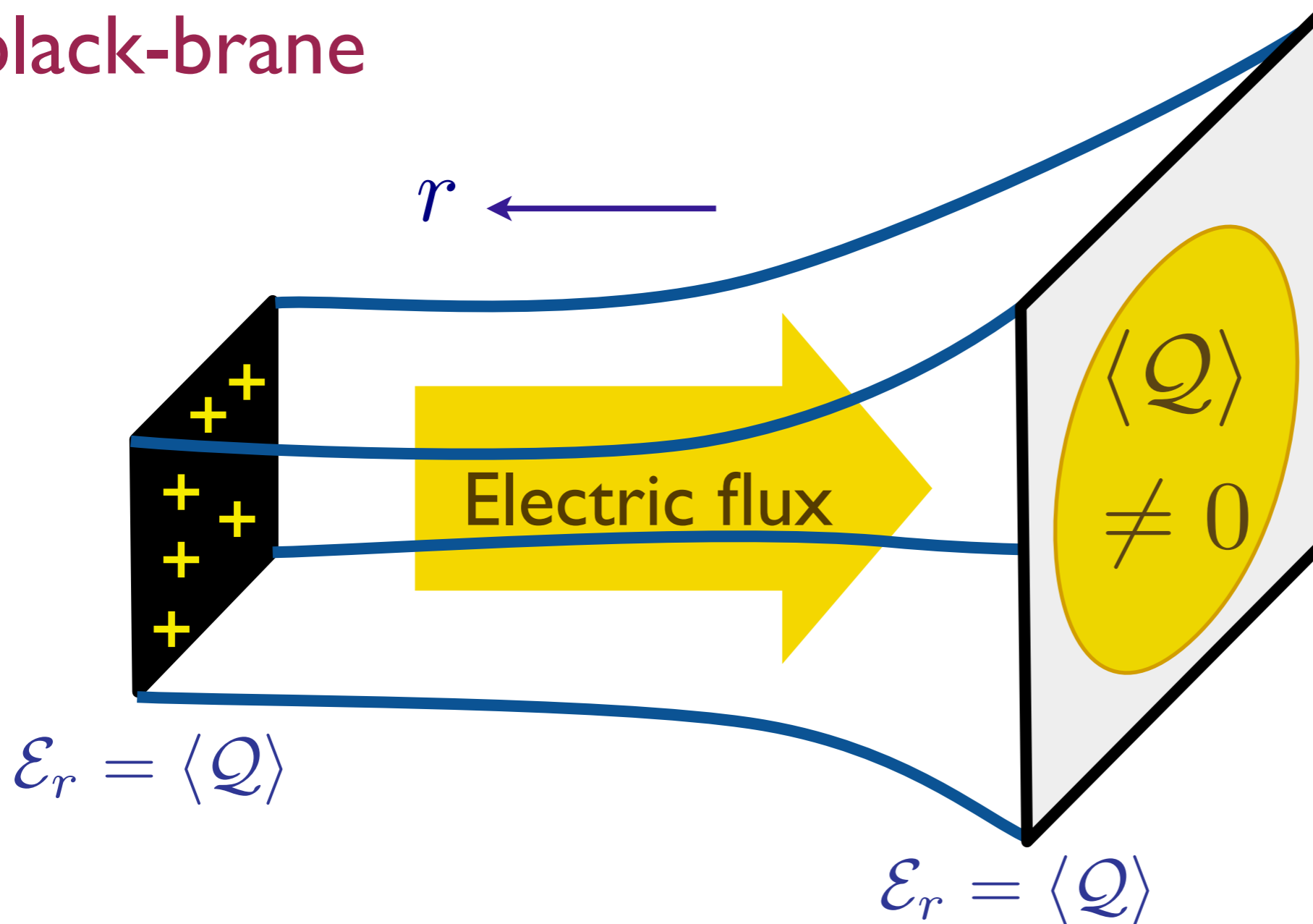
The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_4^2} F_{ab} F^{ab} \right]$$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

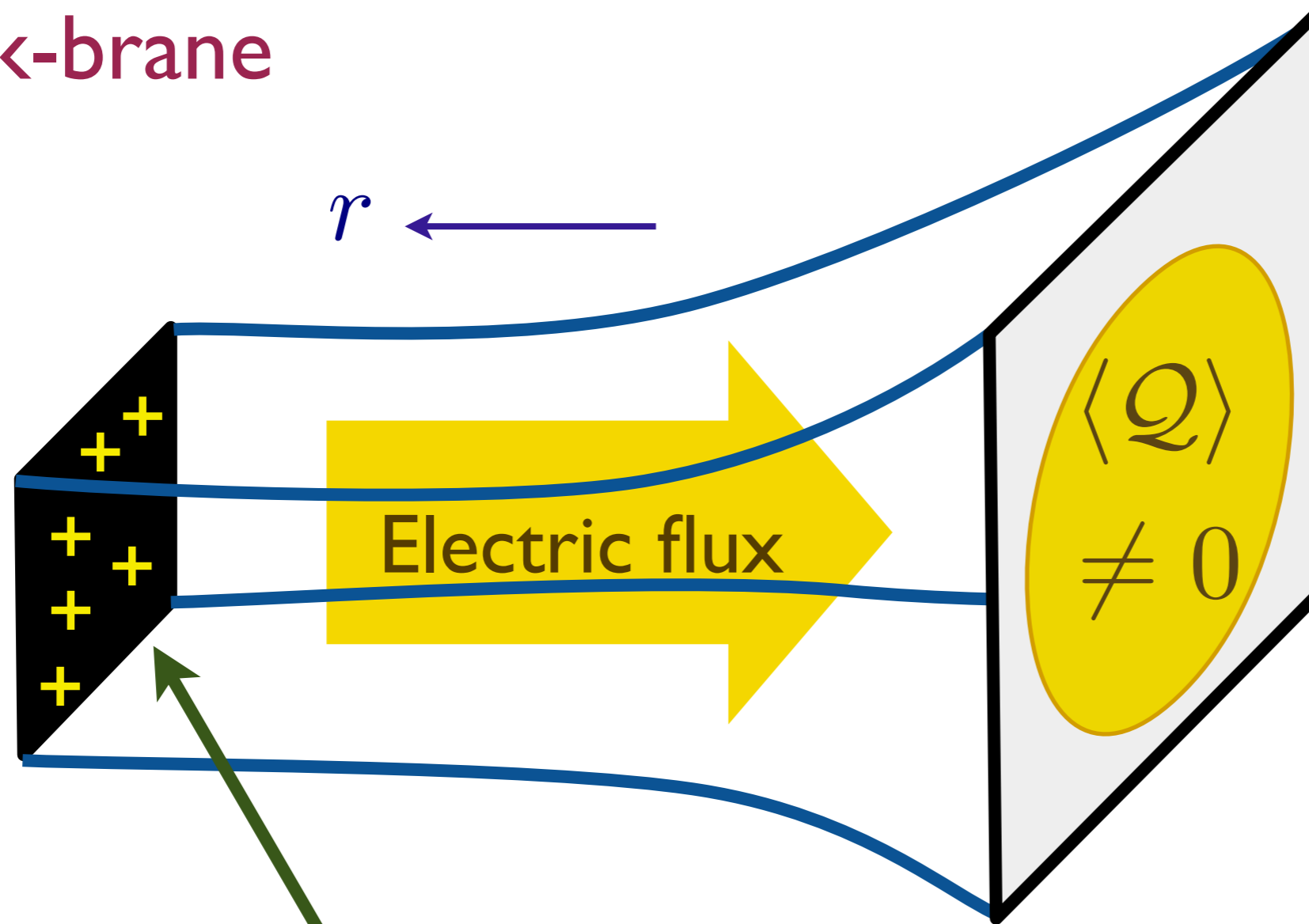
The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

$$\text{with } f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right) \text{ and } R = \frac{\sqrt{6}Lg_4}{\kappa\mu}, \text{ and } A_\tau = \mu \left(1 - \frac{r}{R}\right)$$

The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane

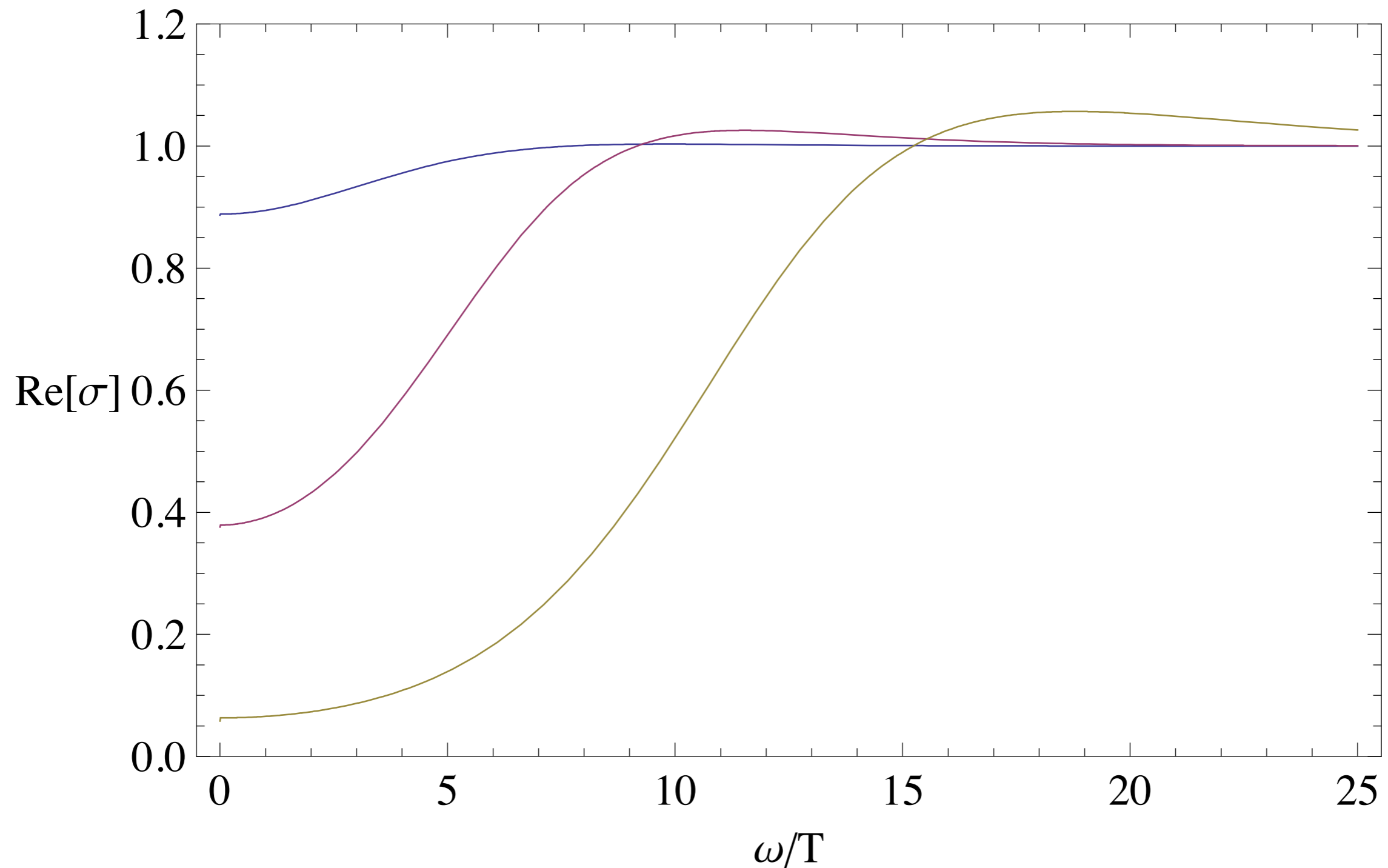


At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $AdS_2 \times R^2$

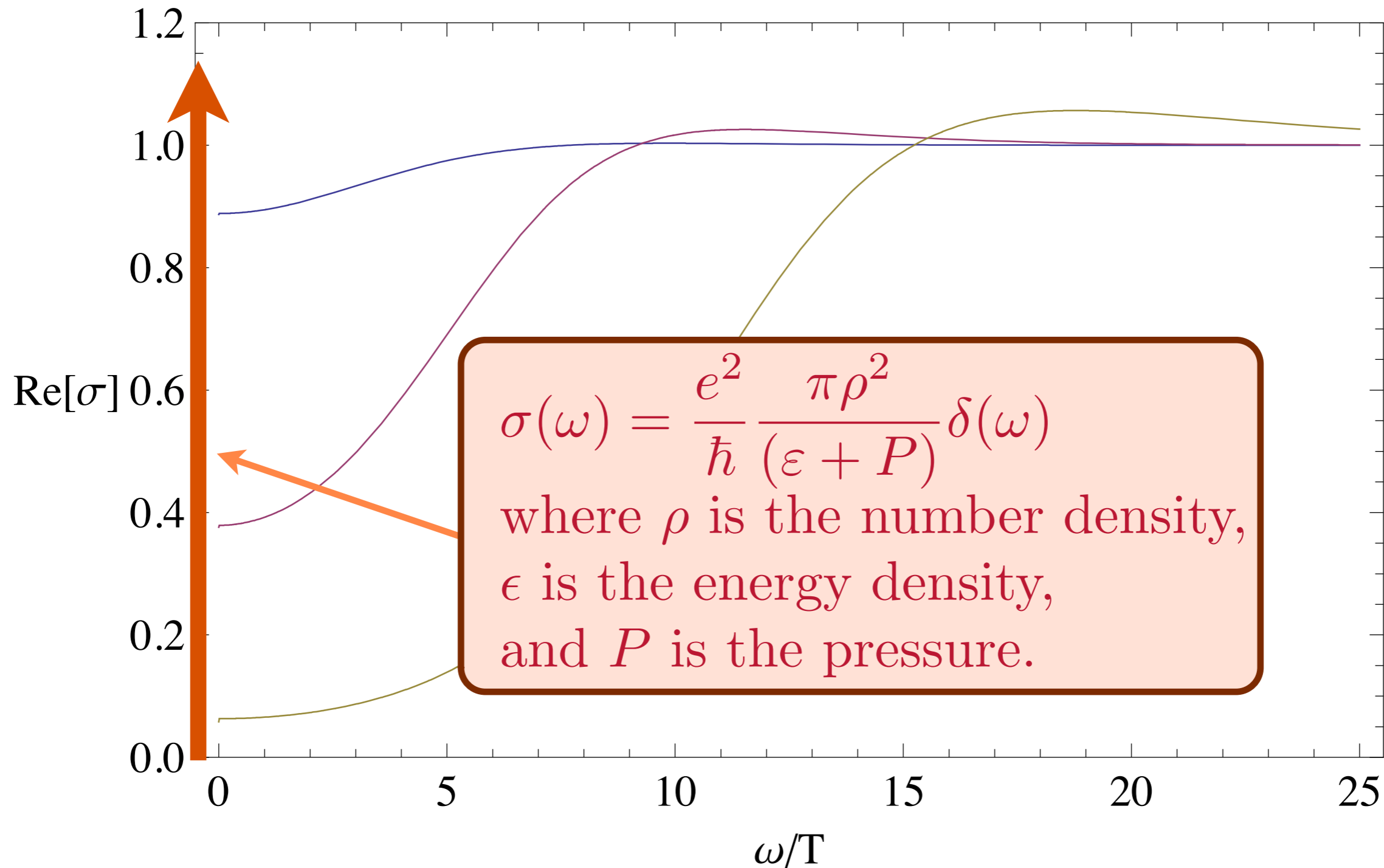
$$ds^2 = \frac{L^2}{6} \left(\frac{-dt^2 + dr^2}{r^2} \right) + dx^2 + dy^2$$

T. Faulkner, H. Liu,
J. McGreevy,
and D. Vegh,
arXiv:0907.2694

Compute conductivity using response to a time-dependent vector potential as a function of ω/T and μ/T



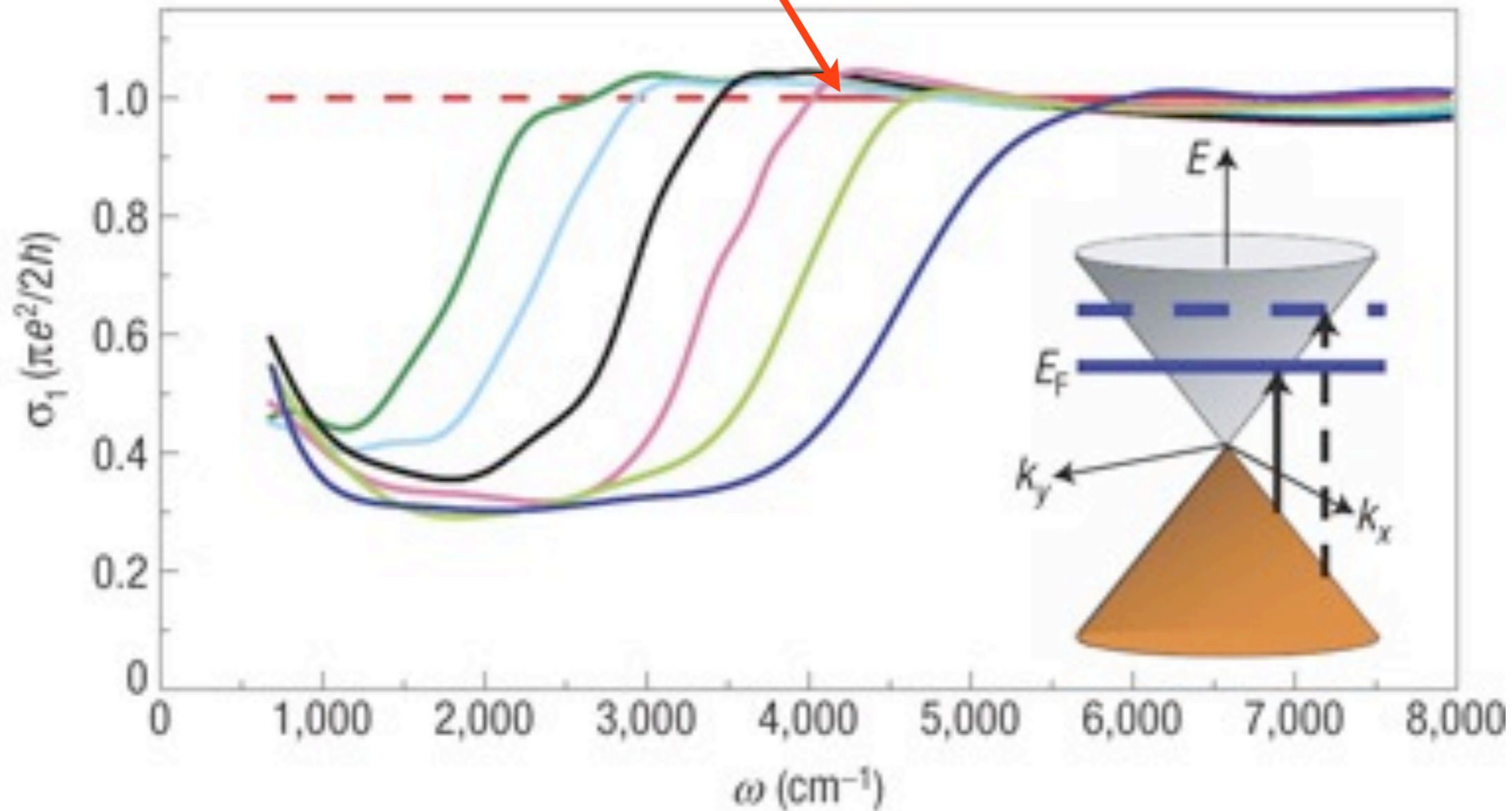
Compute conductivity using response to a time-dependent vector potential as a function of ω/T and μ/T



S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

Optical conductivity of graphene

Undoped graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

Features of $\text{AdS}_2 \times R^2$

- Has non-zero entropy density at $T = 0$, and “volume” law for entanglement entropy.
- Green’s function of a probe fermion (a *mesino*) can have a Fermi surface, but self energies are momentum independent, and the singular behavior is the same on and off the Fermi surface
- Deficit of order $\sim N^2$ in the volume enclosed by the mesino Fermi surfaces: presumably associated with “hidden Fermi surfaces” of gauge-charged particles (the *quarks*).

S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);

M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

Compressible quantum matter

A. Fermi liquids: graphene

*B. Holography: Reissner - Nördstrom
solution*

*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*

Compressible quantum matter

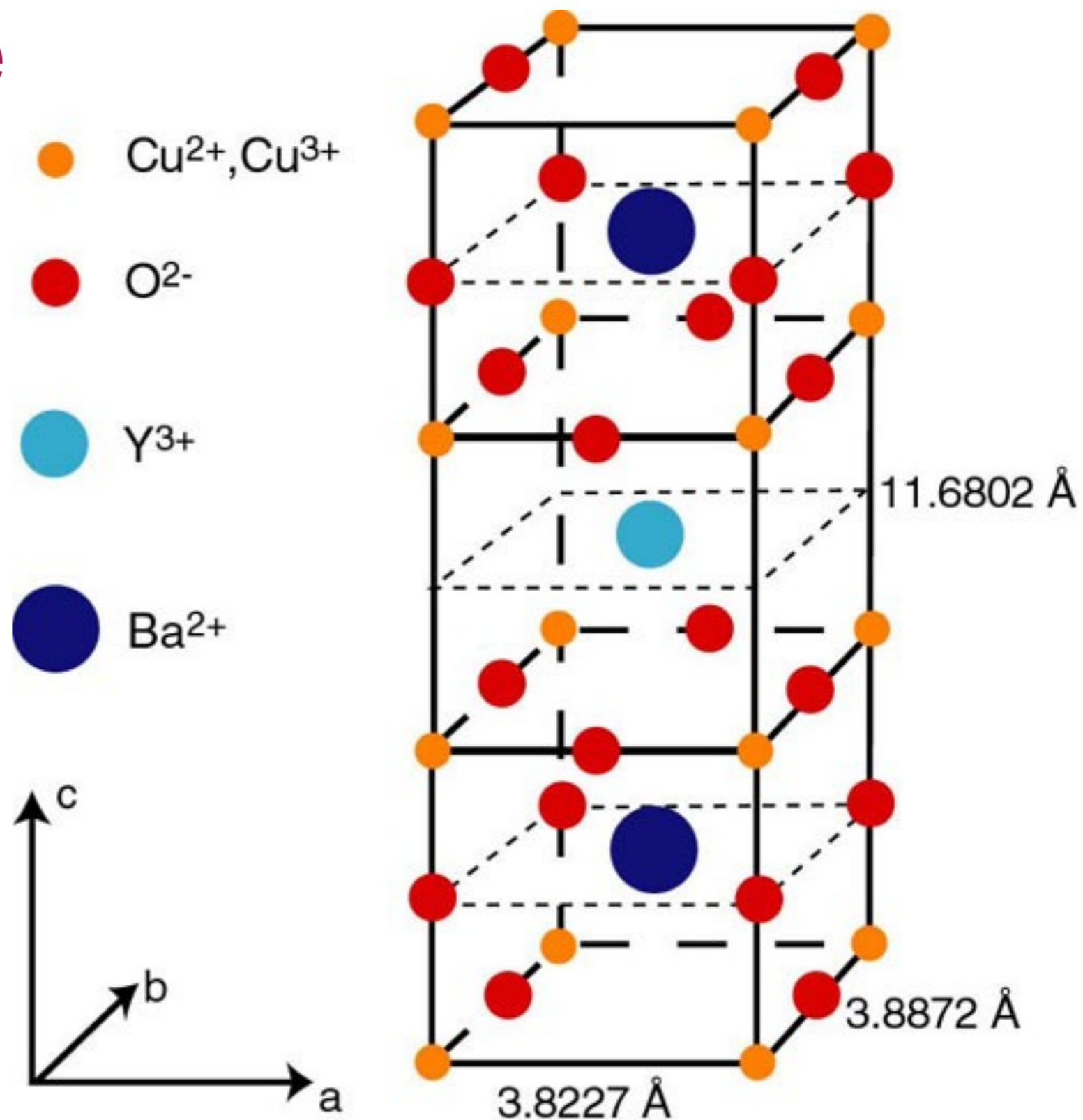
A. Fermi liquids: graphene

B. Holography: Reissner - Nördstrom solution

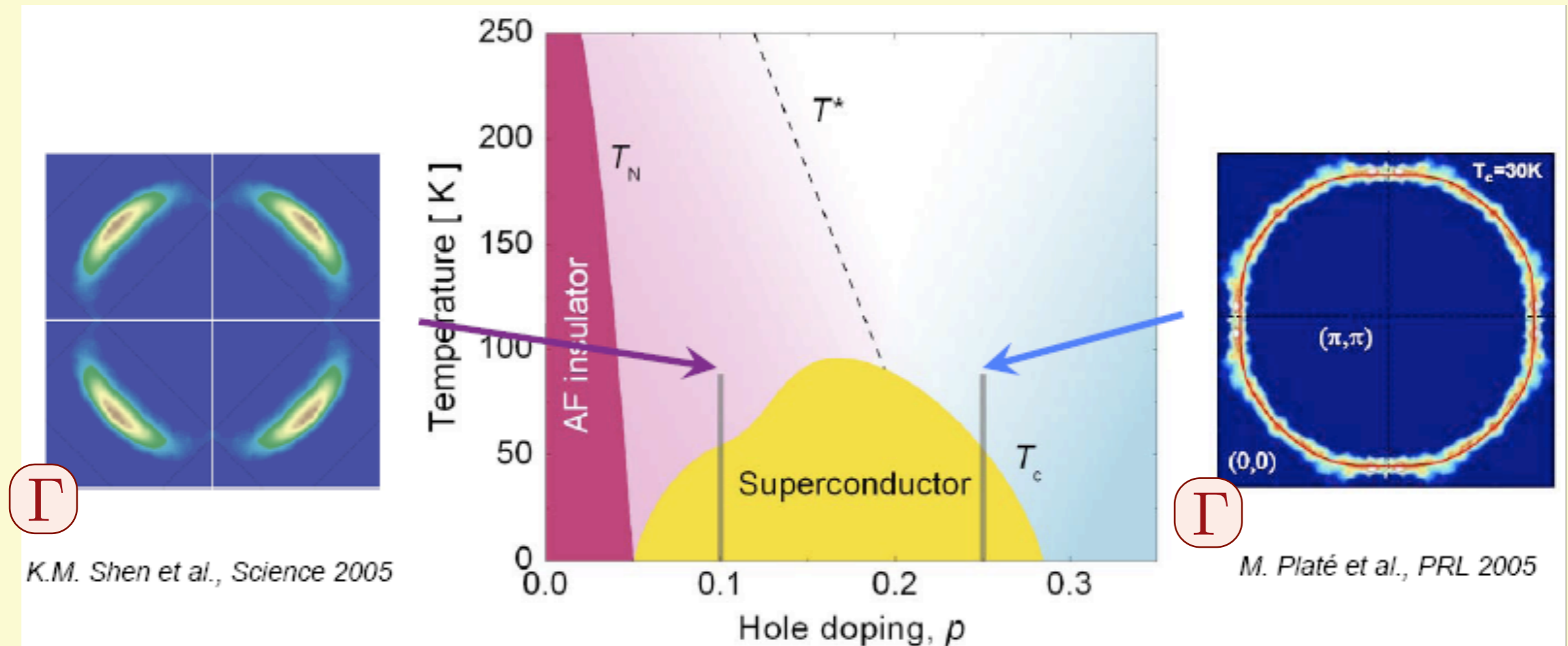
*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

D. Holography: scaling arguments for entropy and entanglement entropy

High temperature superconductors



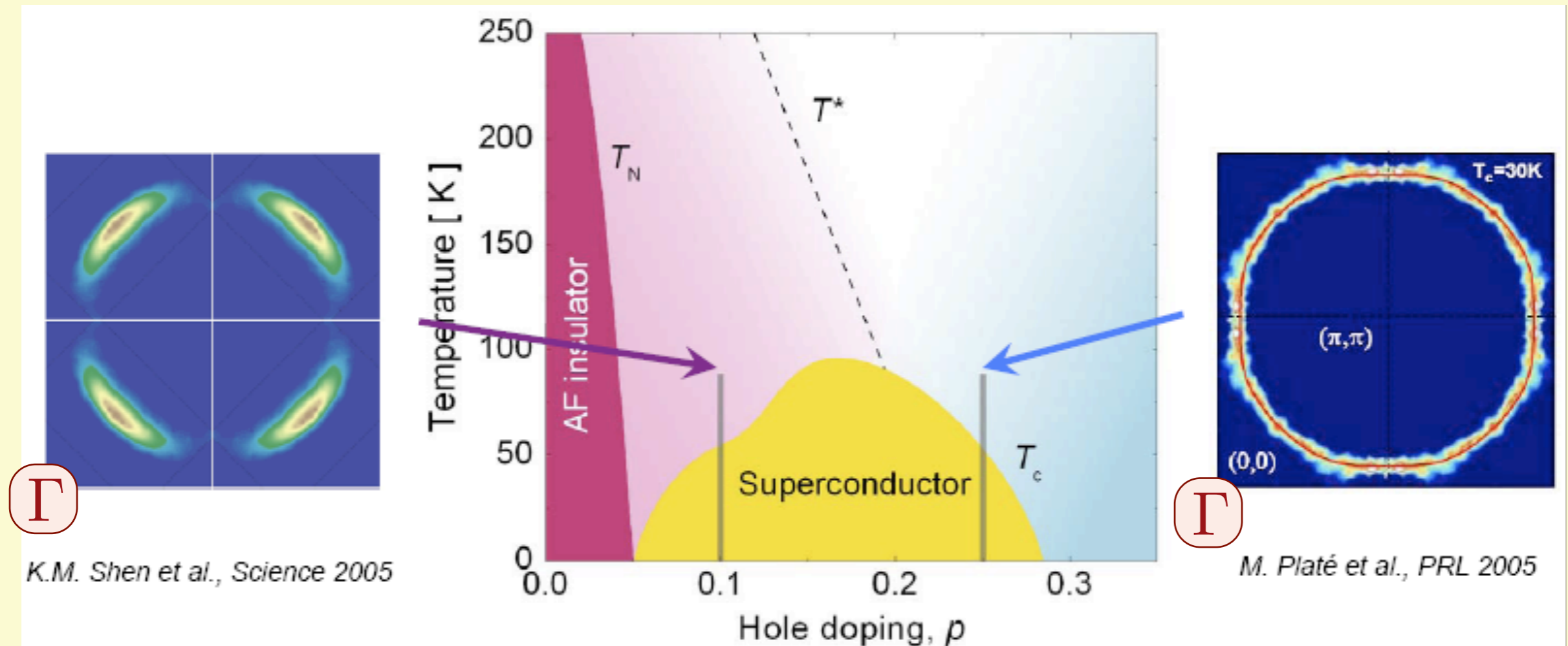
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole
Fermi-pockets

Large hole
Fermi surface

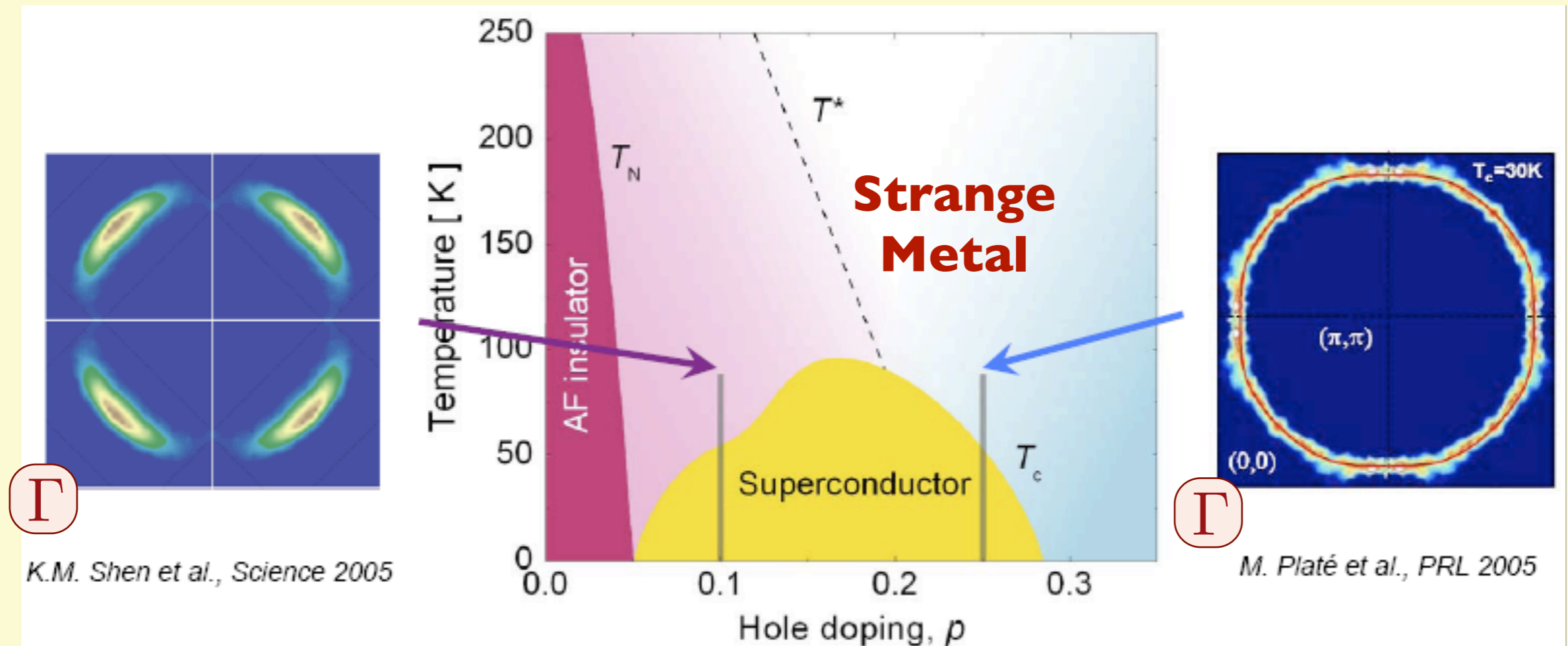
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole
Fermi-pockets

Large hole
Fermi surface

Evolution of the (ARPES) Fermi surface on the cuprate phase diagram

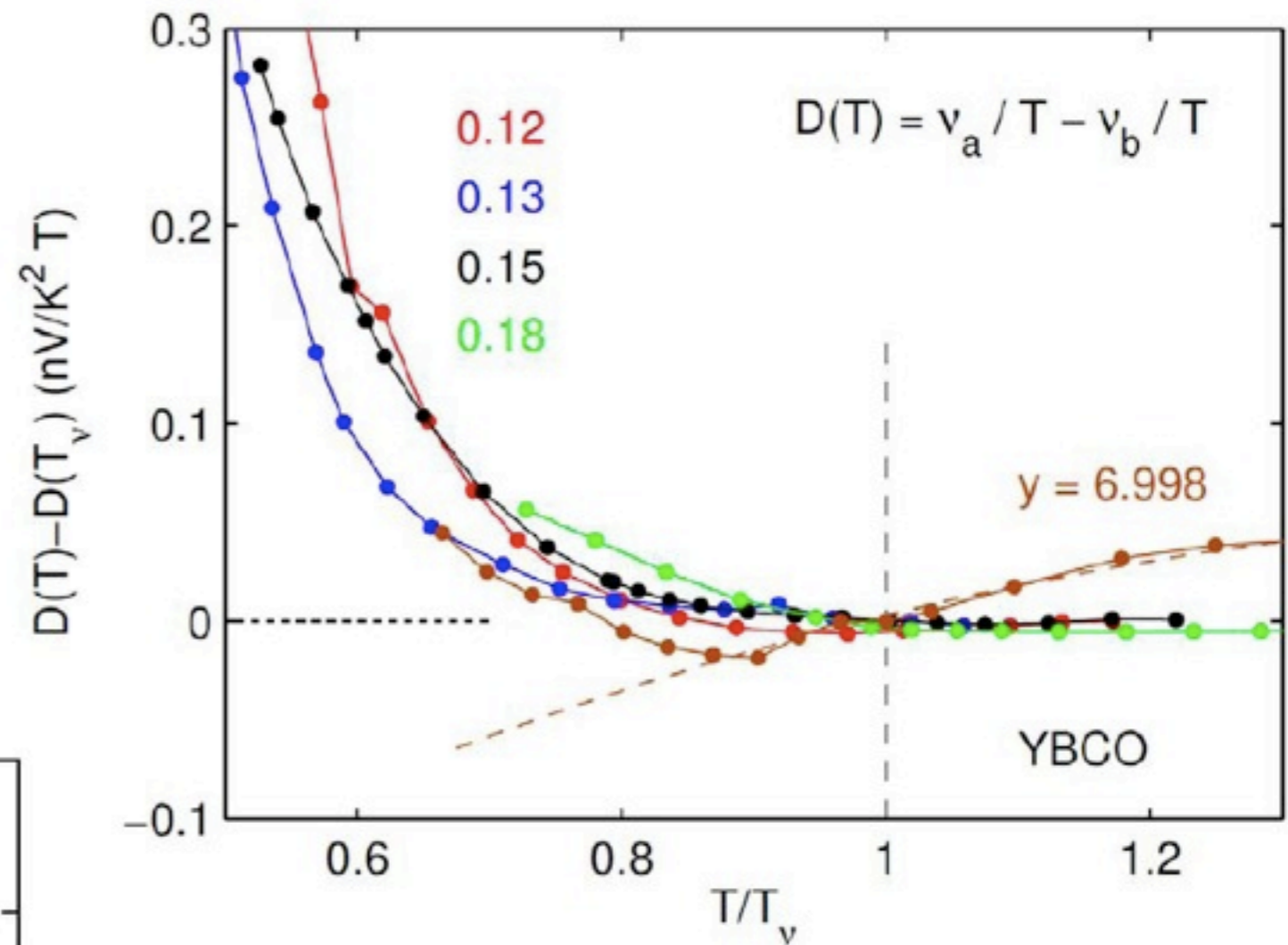
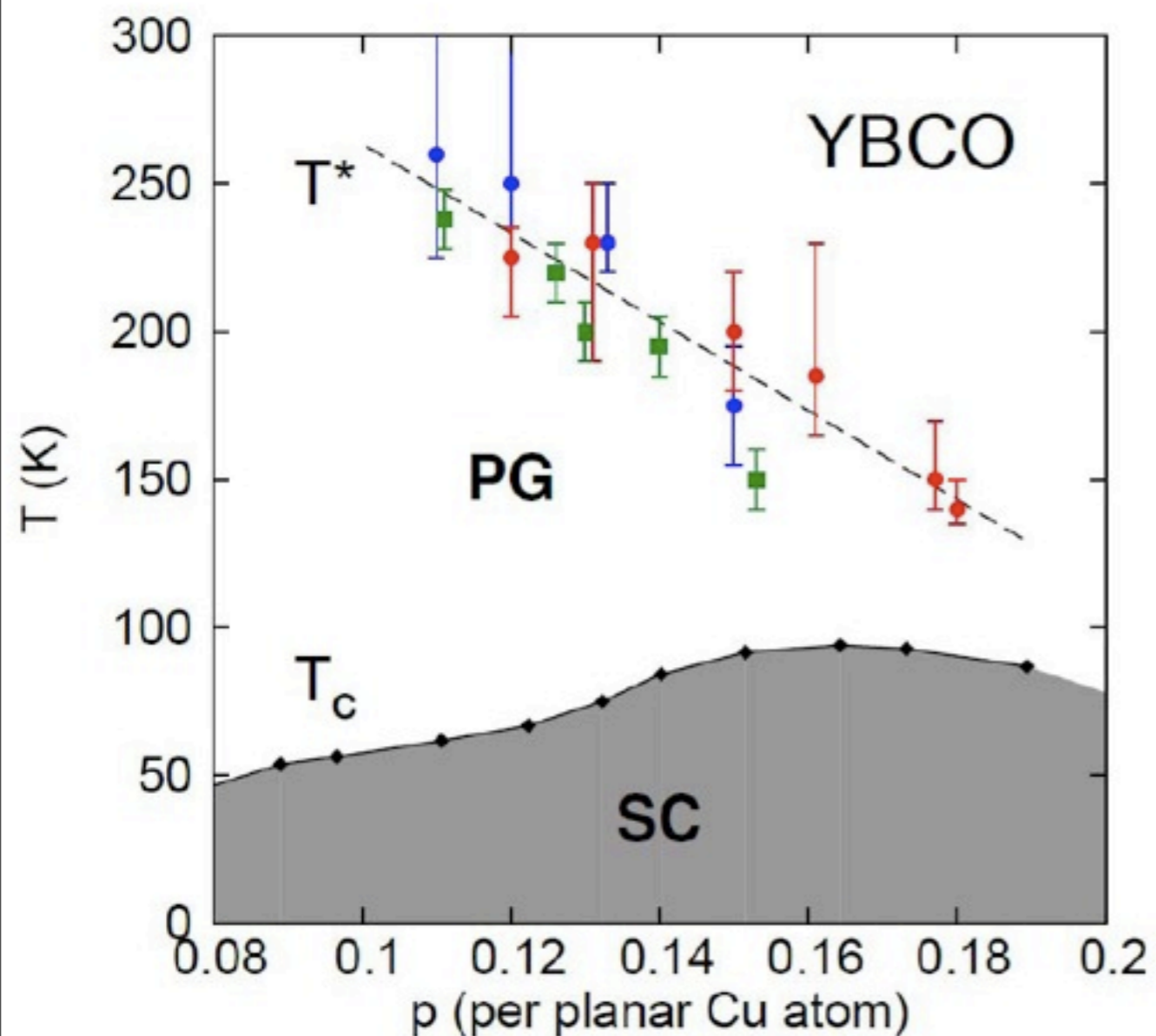


Smaller hole
Fermi-pockets

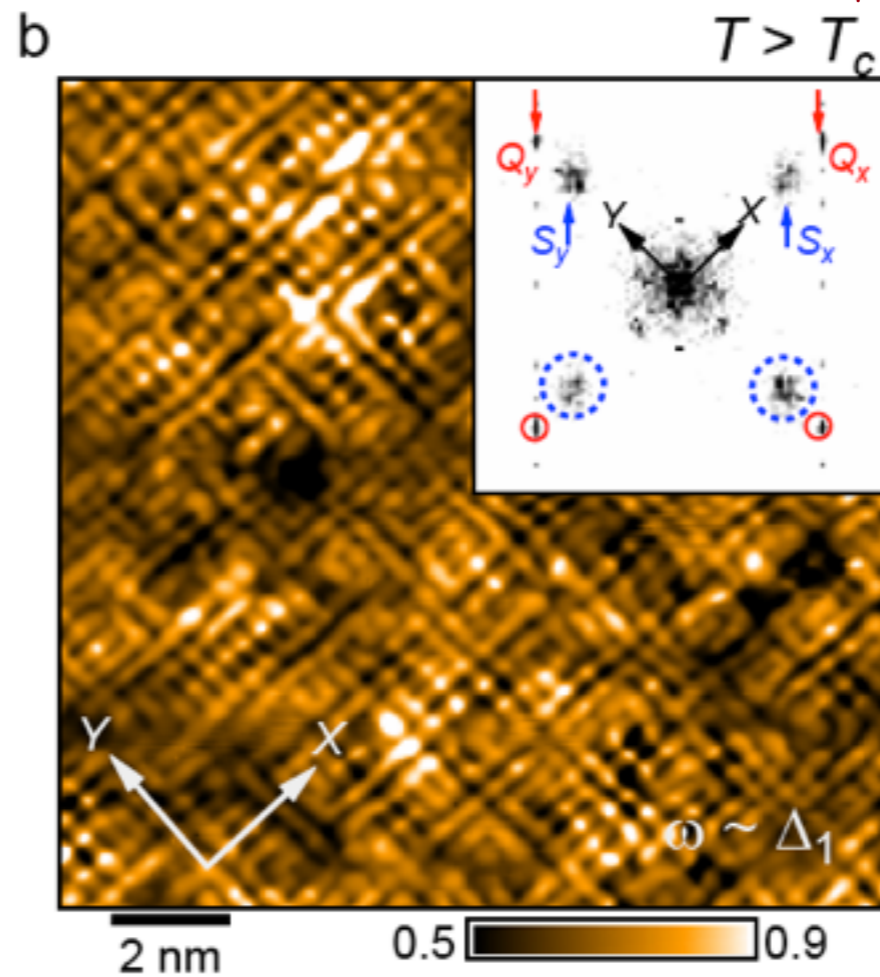
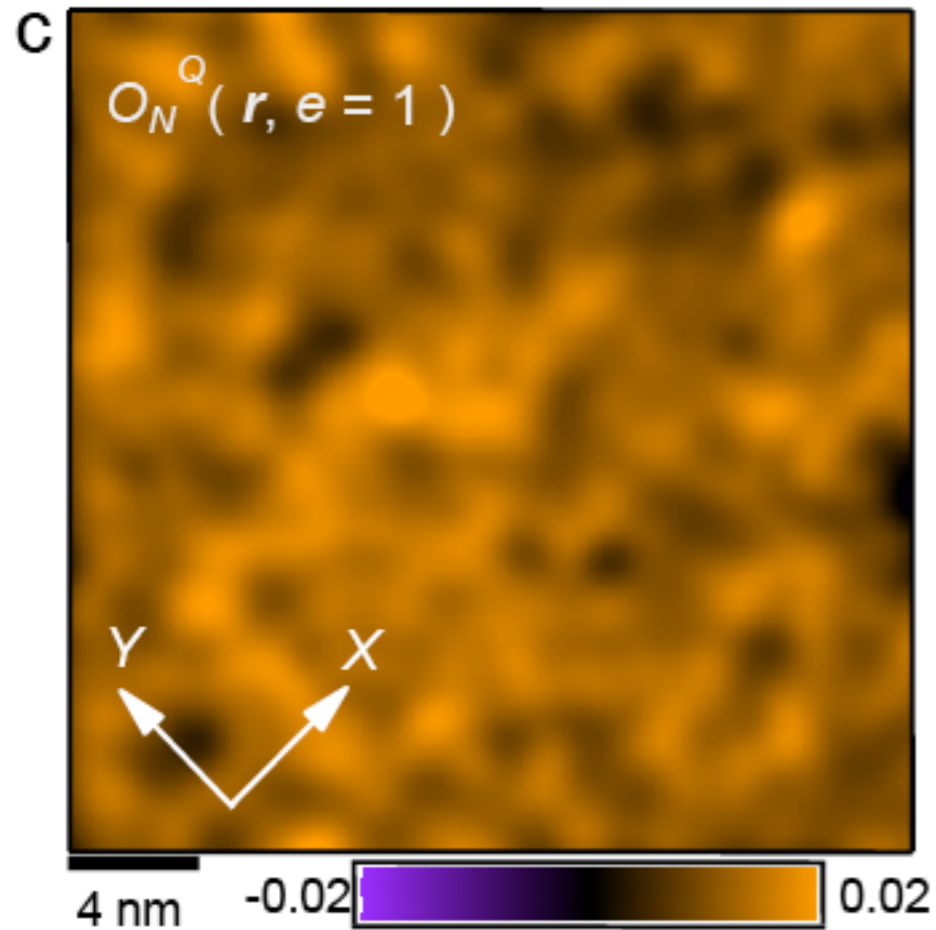
Large hole
Fermi surface

Broken rotational symmetry in the pseudogap phase of a high- T_c superconductor

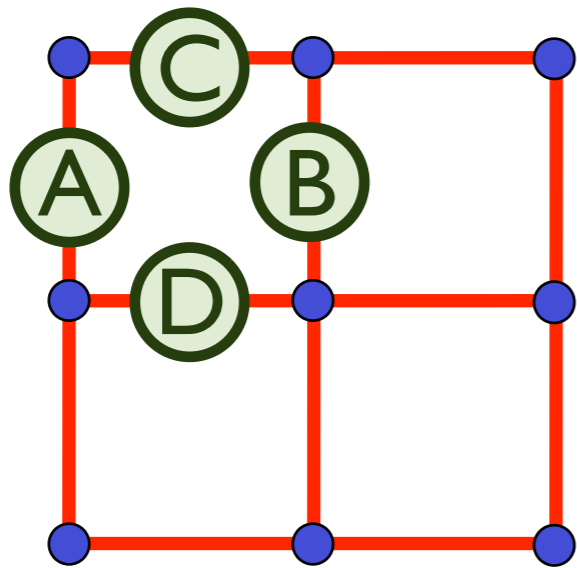
R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
Nature, **463**, 519 (2010).



STM measurements of $Z(r)$, the energy asymmetry in density of states in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$.



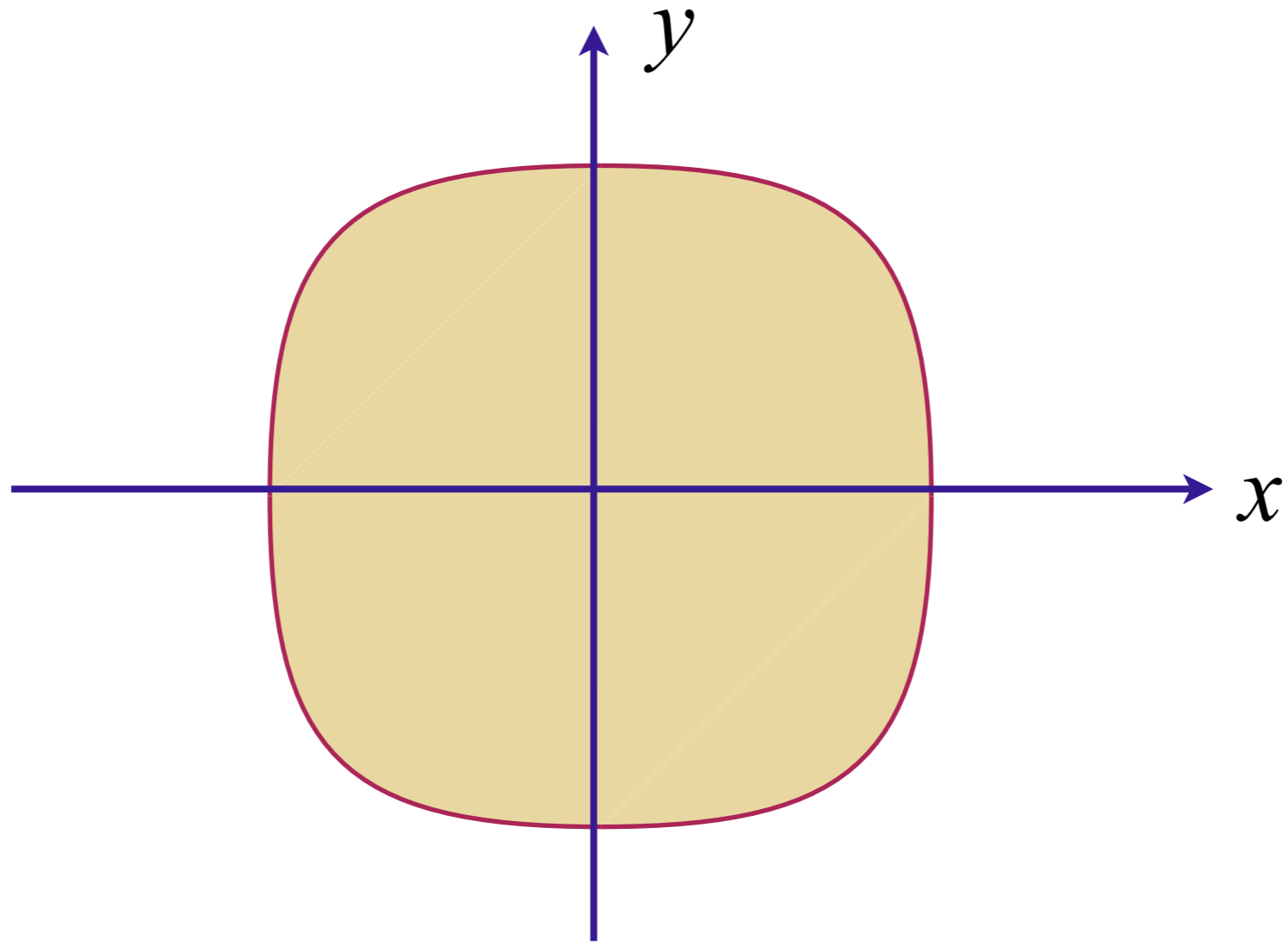
M. J. Lawler, K. Fujita, Jinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, *Nature* **466**, 347 (2010)



$$O_N = Z_A + Z_B - Z_C - Z_D$$

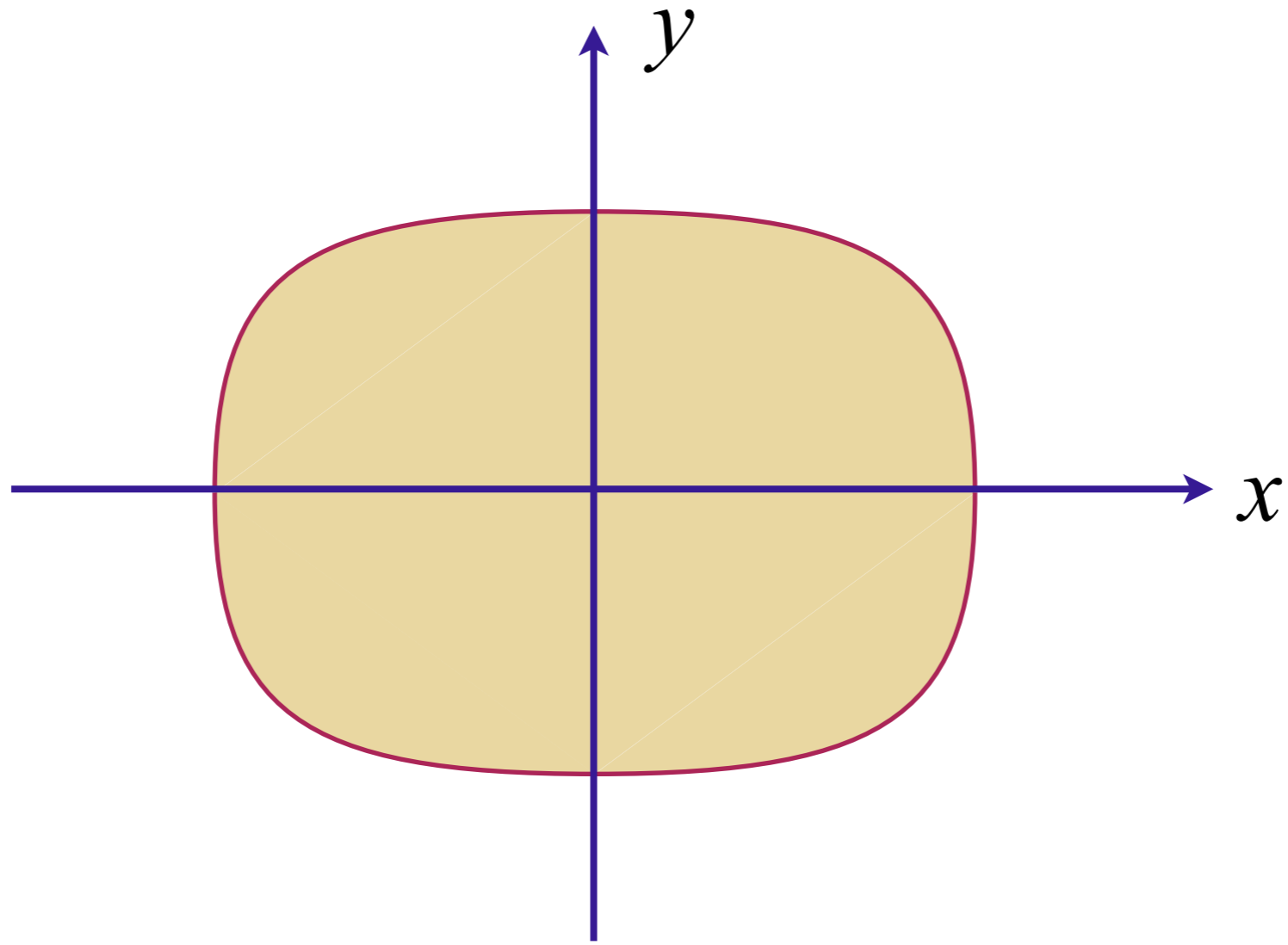
Strong anisotropy of electronic states between x and y directions:
Electronic “Ising-nematic” order

Quantum criticality of Ising-nematic ordering



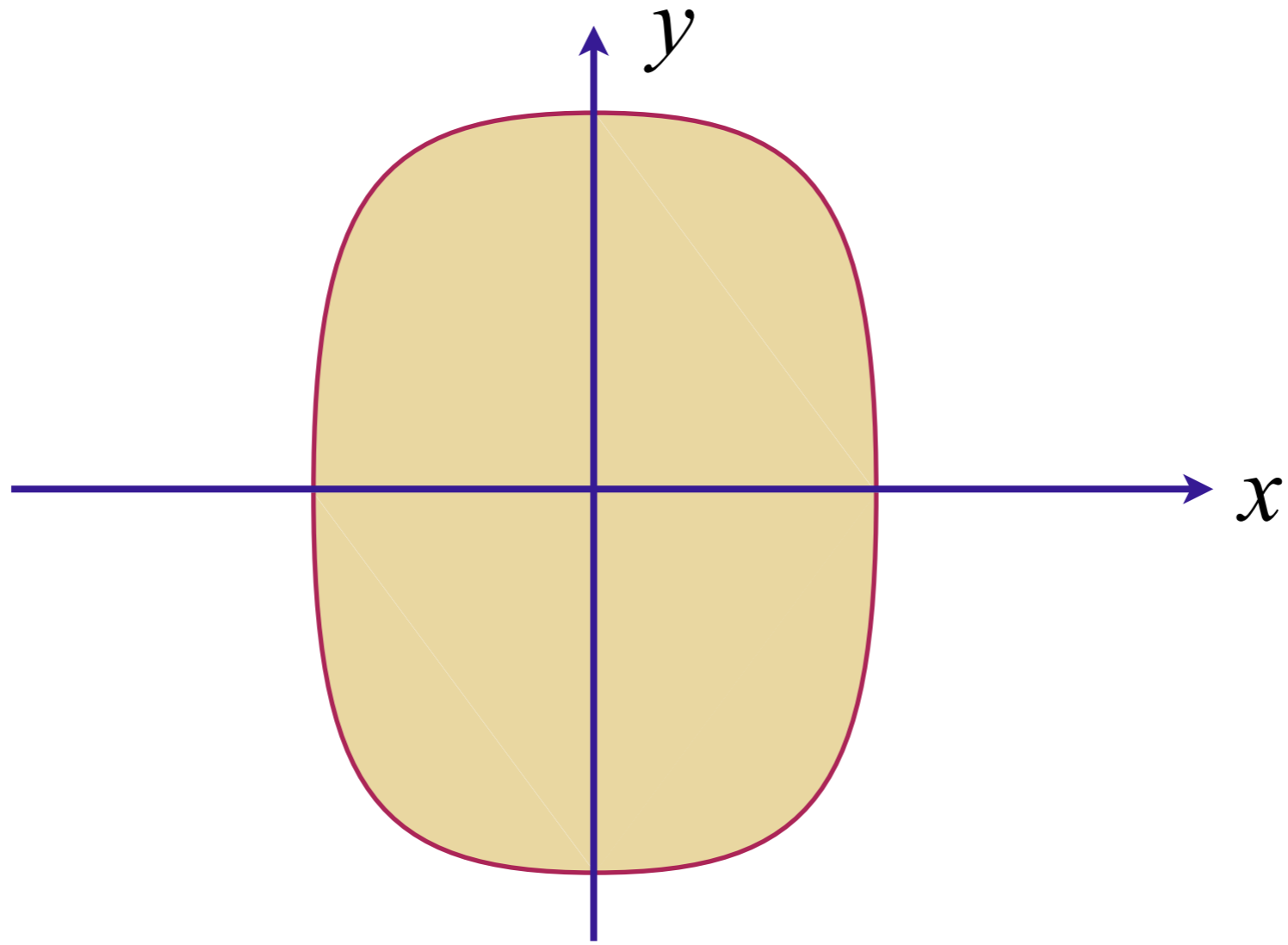
Fermi surface with full square lattice symmetry

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:

Quantum criticality of Ising-nematic ordering



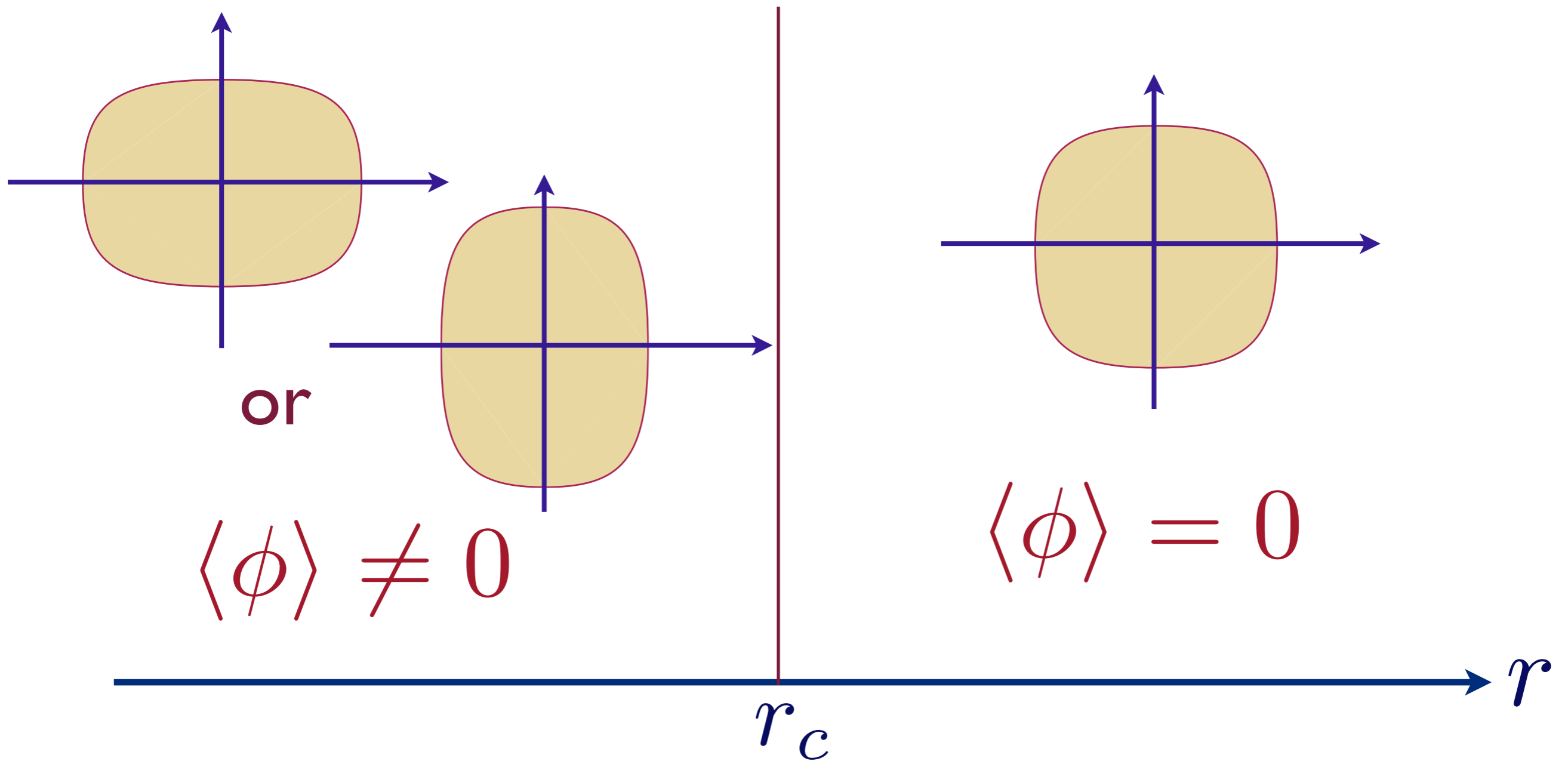
Spontaneous elongation along y direction:

Ising-nematic order parameter

$$\phi \sim \int d^2k (\cos k_x - \cos k_y) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

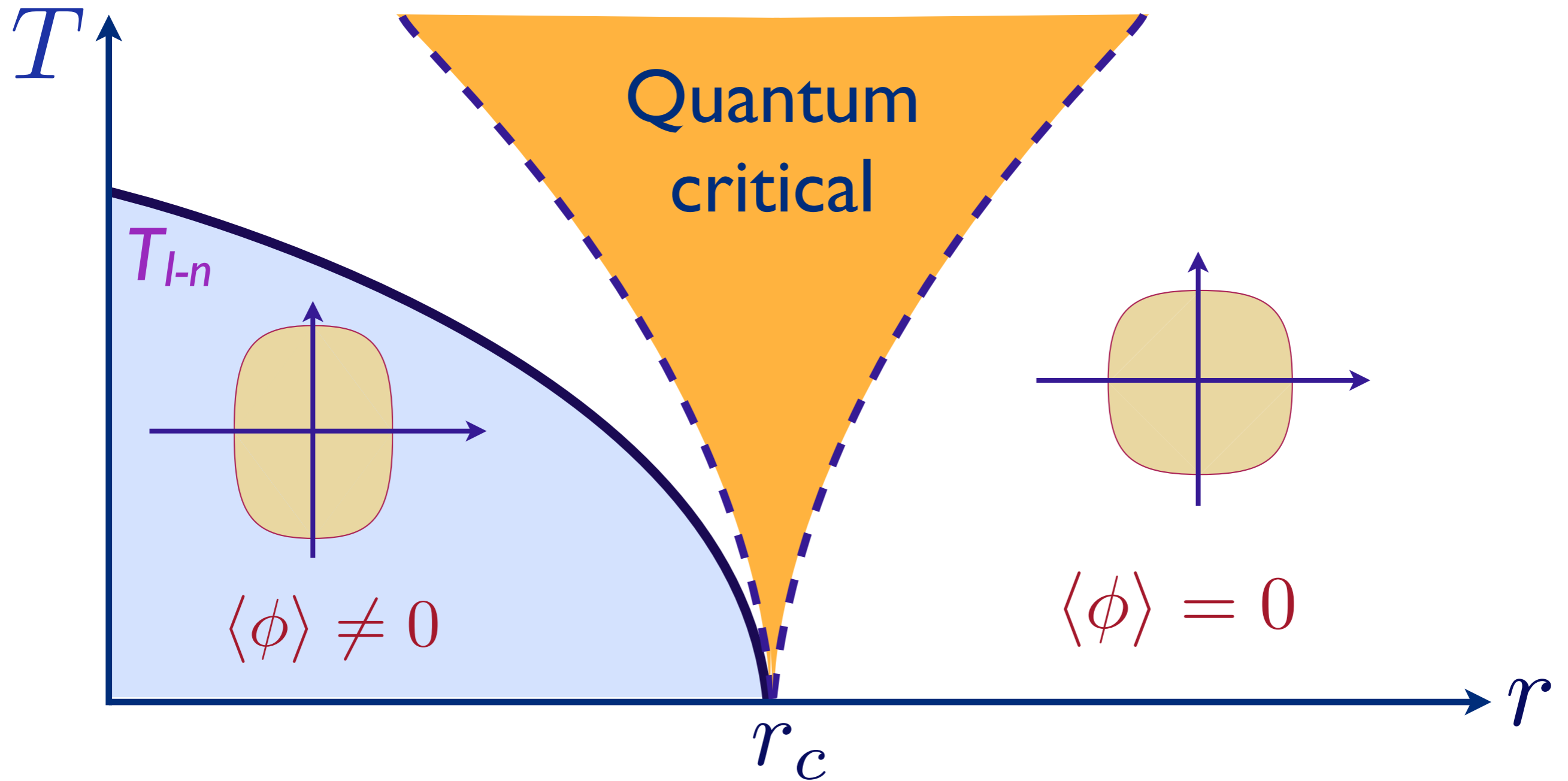
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering



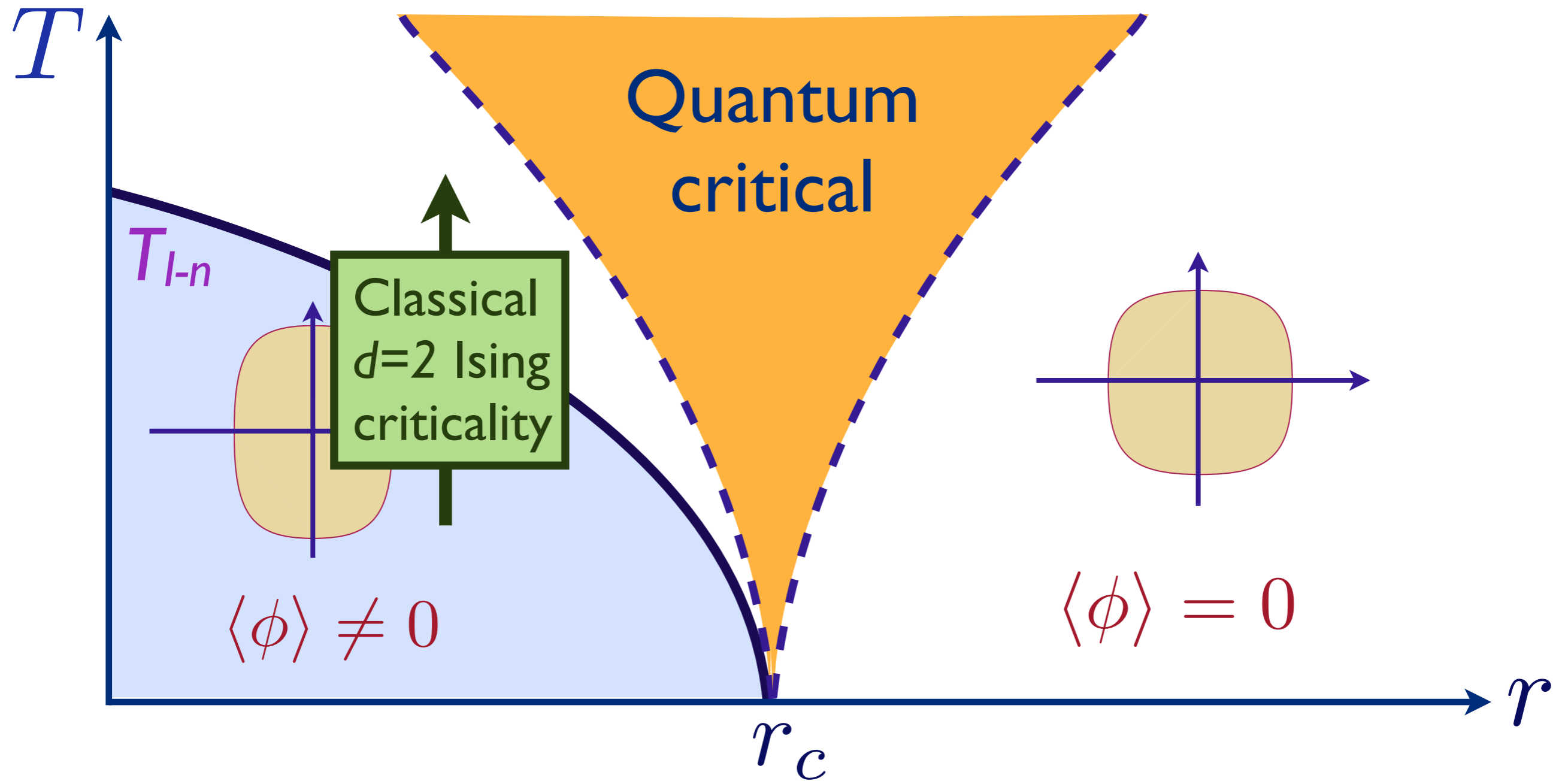
Pomeranchuk instability as a function of coupling r

Quantum criticality of Ising-nematic ordering



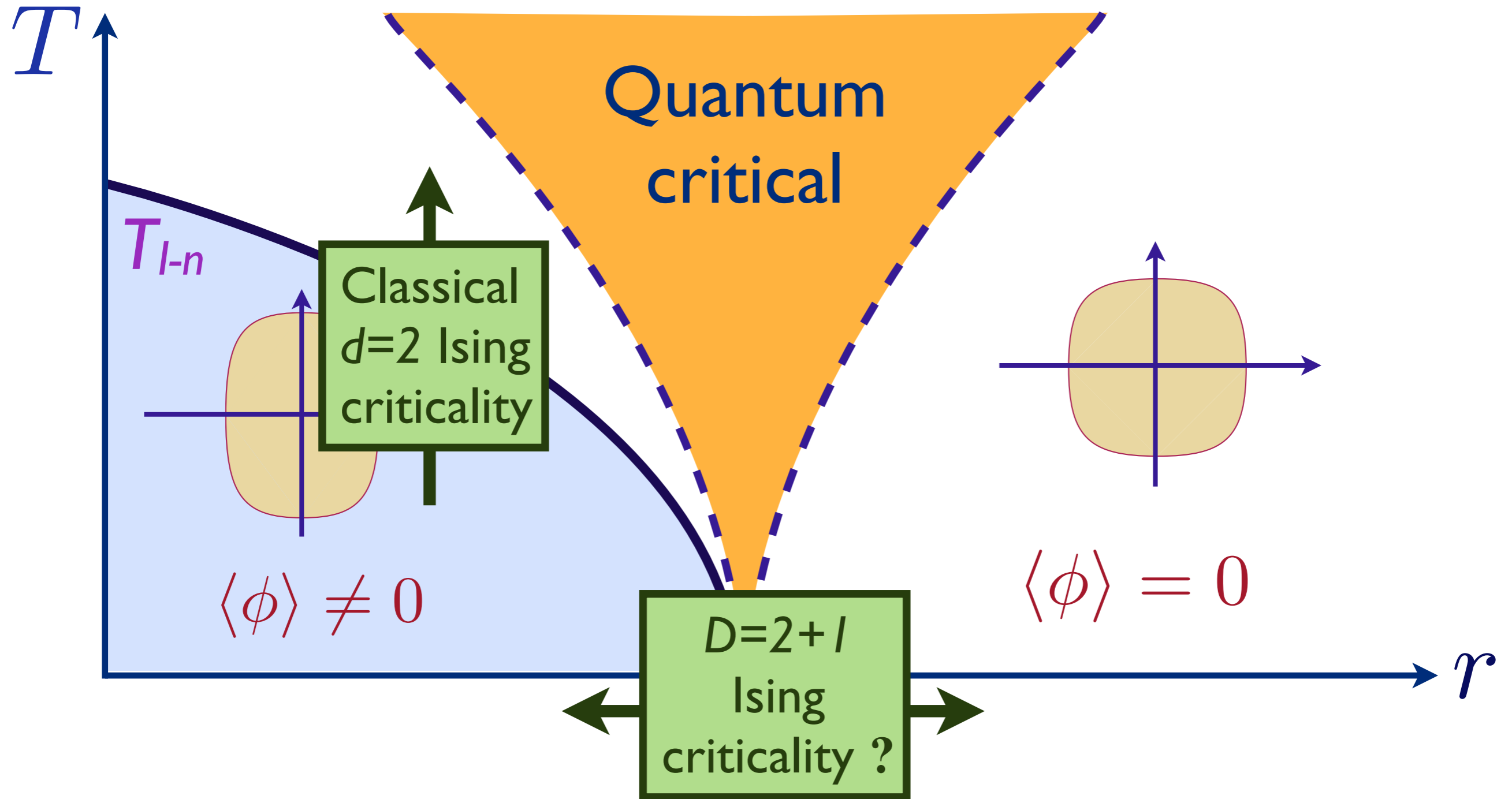
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



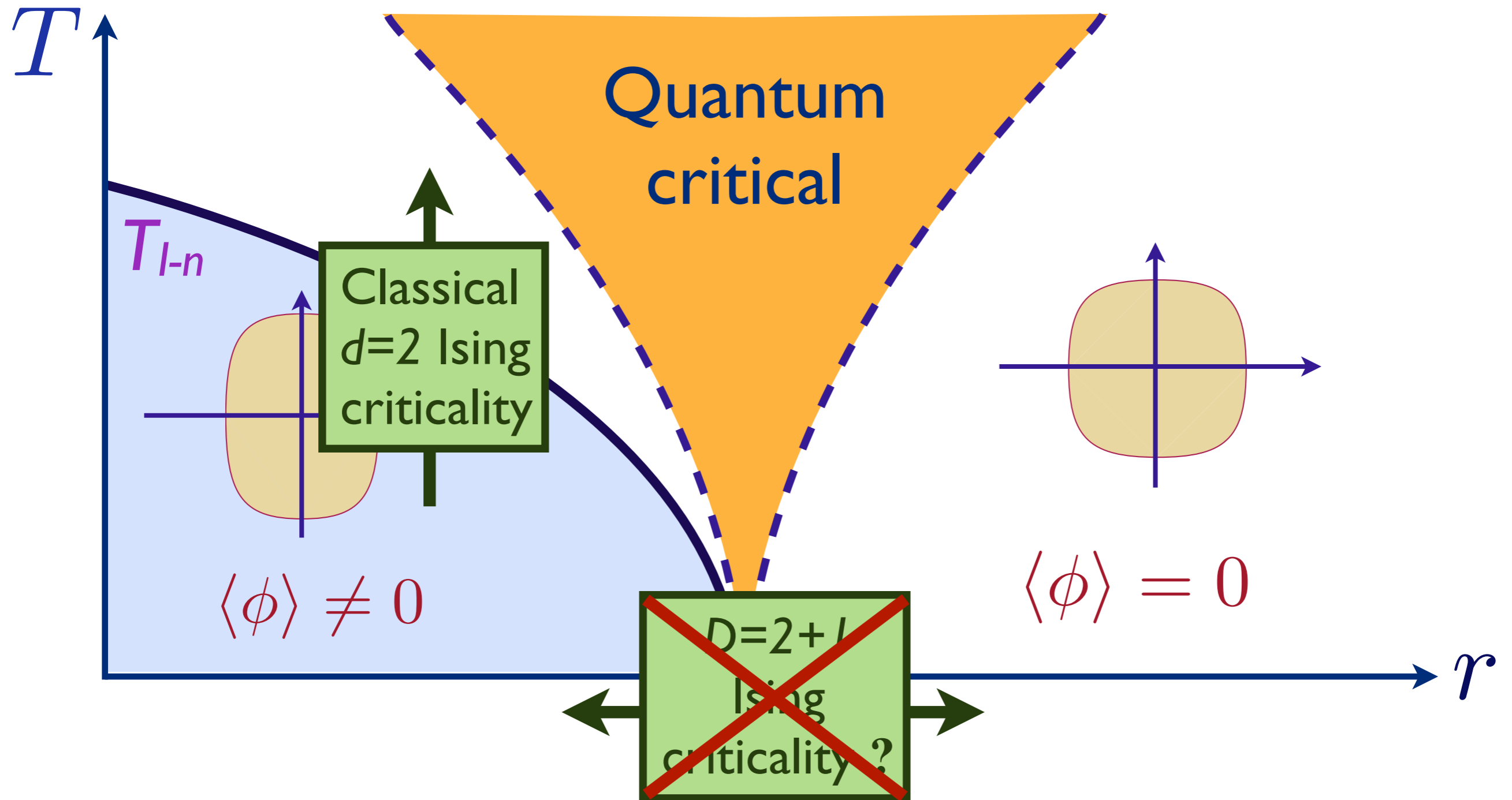
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



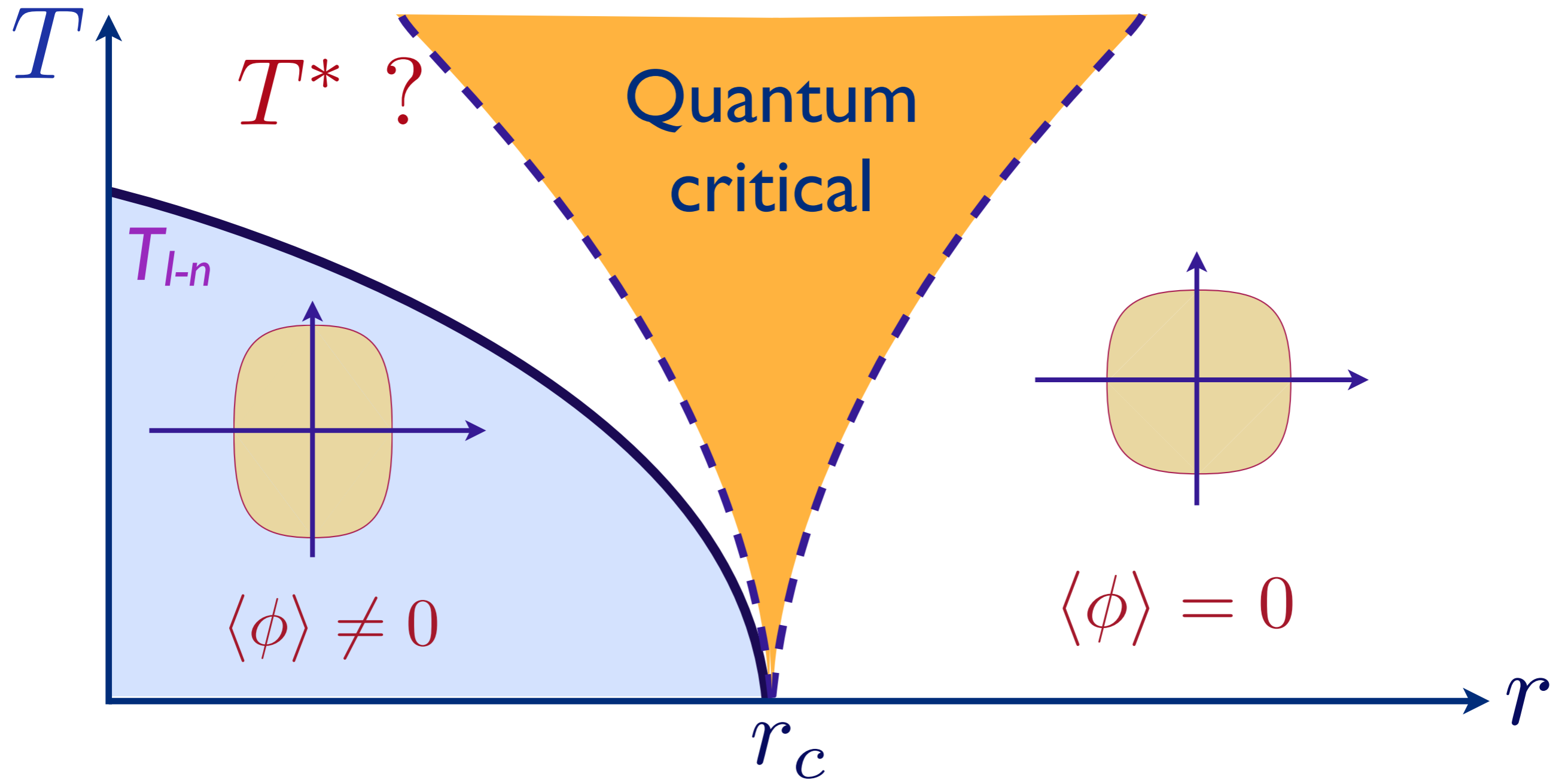
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



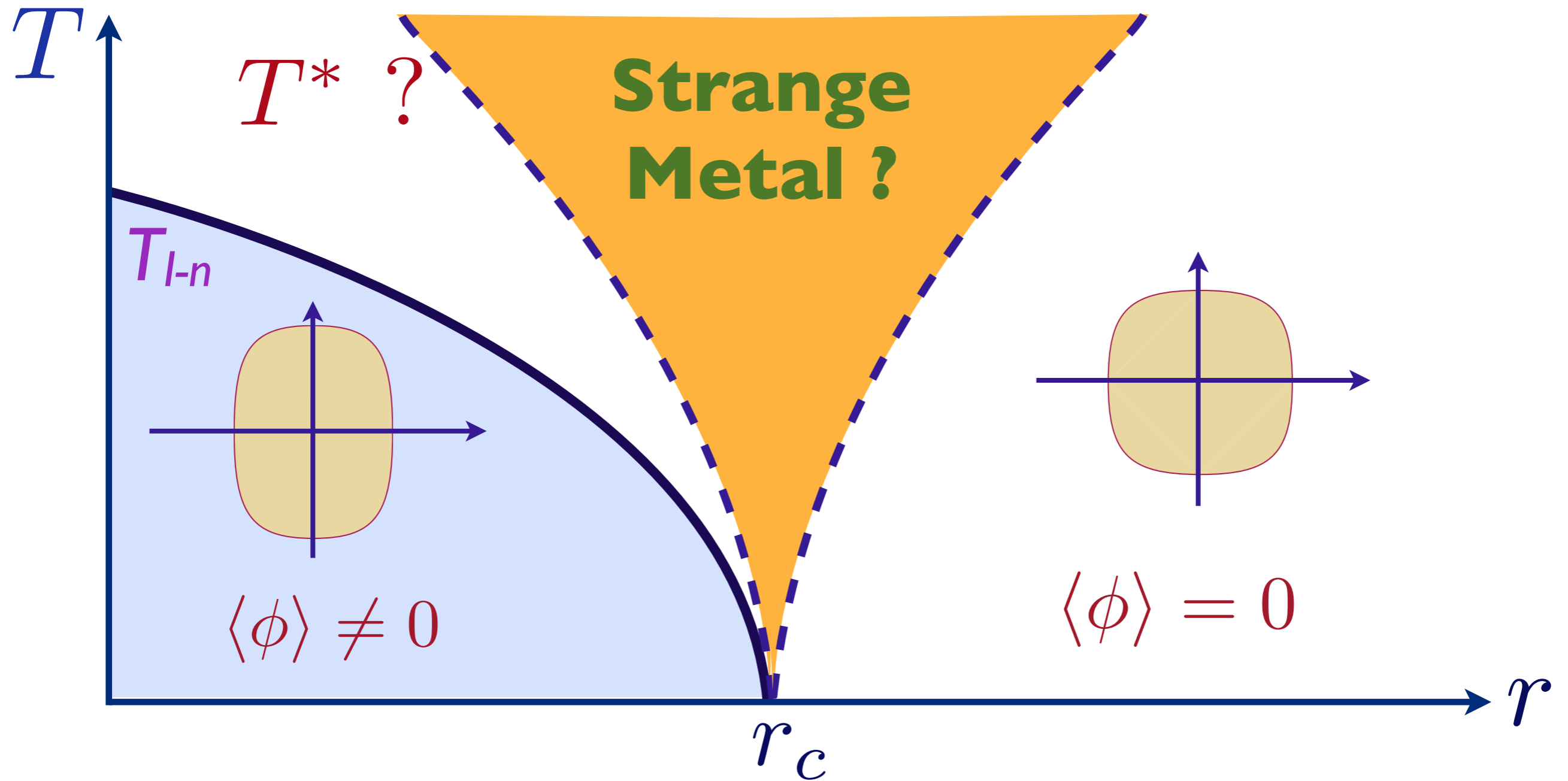
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

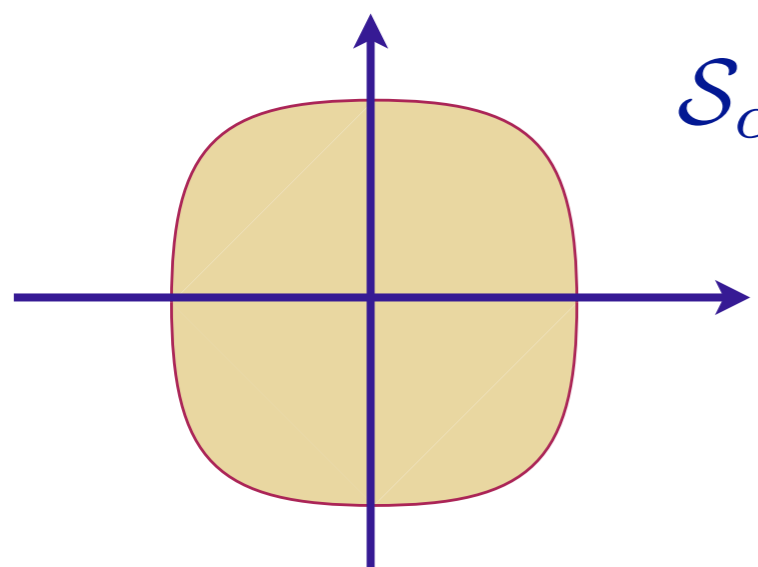
$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

Effective action for electrons:

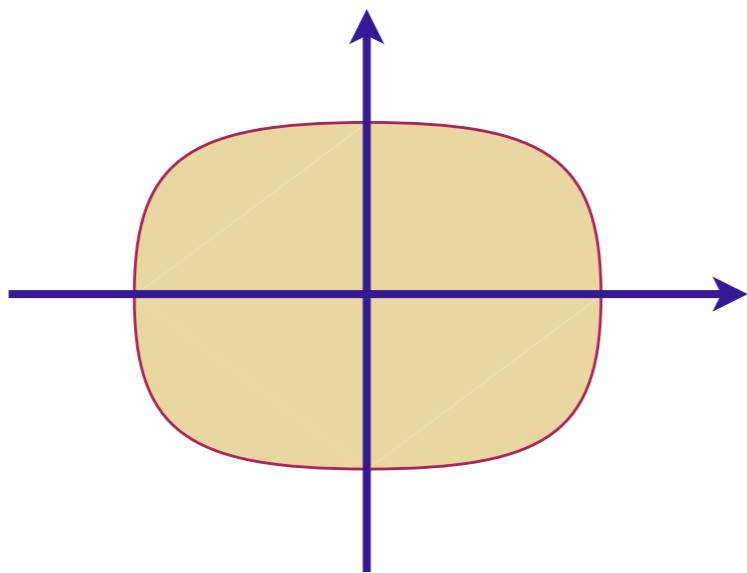

$$\begin{aligned} \mathcal{S}_c &= \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \right] \\ &\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha} \end{aligned}$$

Quantum criticality of Ising-nematic ordering

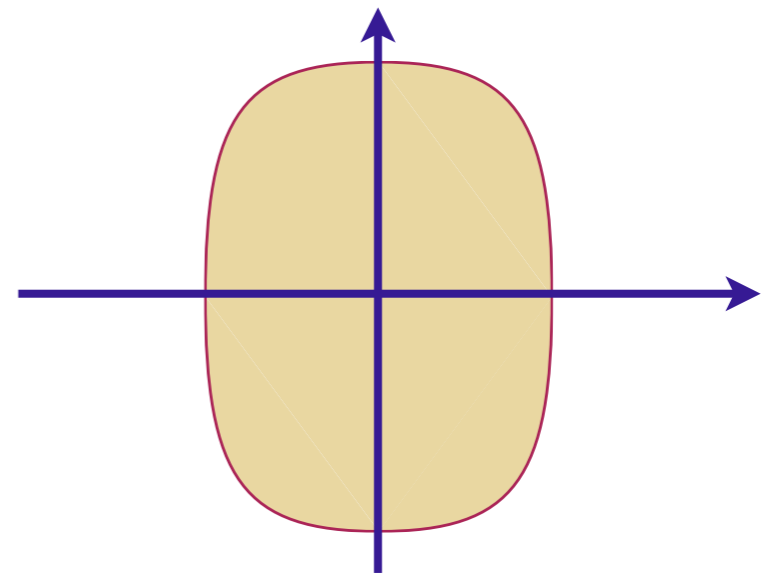
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

for spatially dependent ϕ



$$\langle \phi \rangle > 0$$



$$\langle \phi \rangle < 0$$

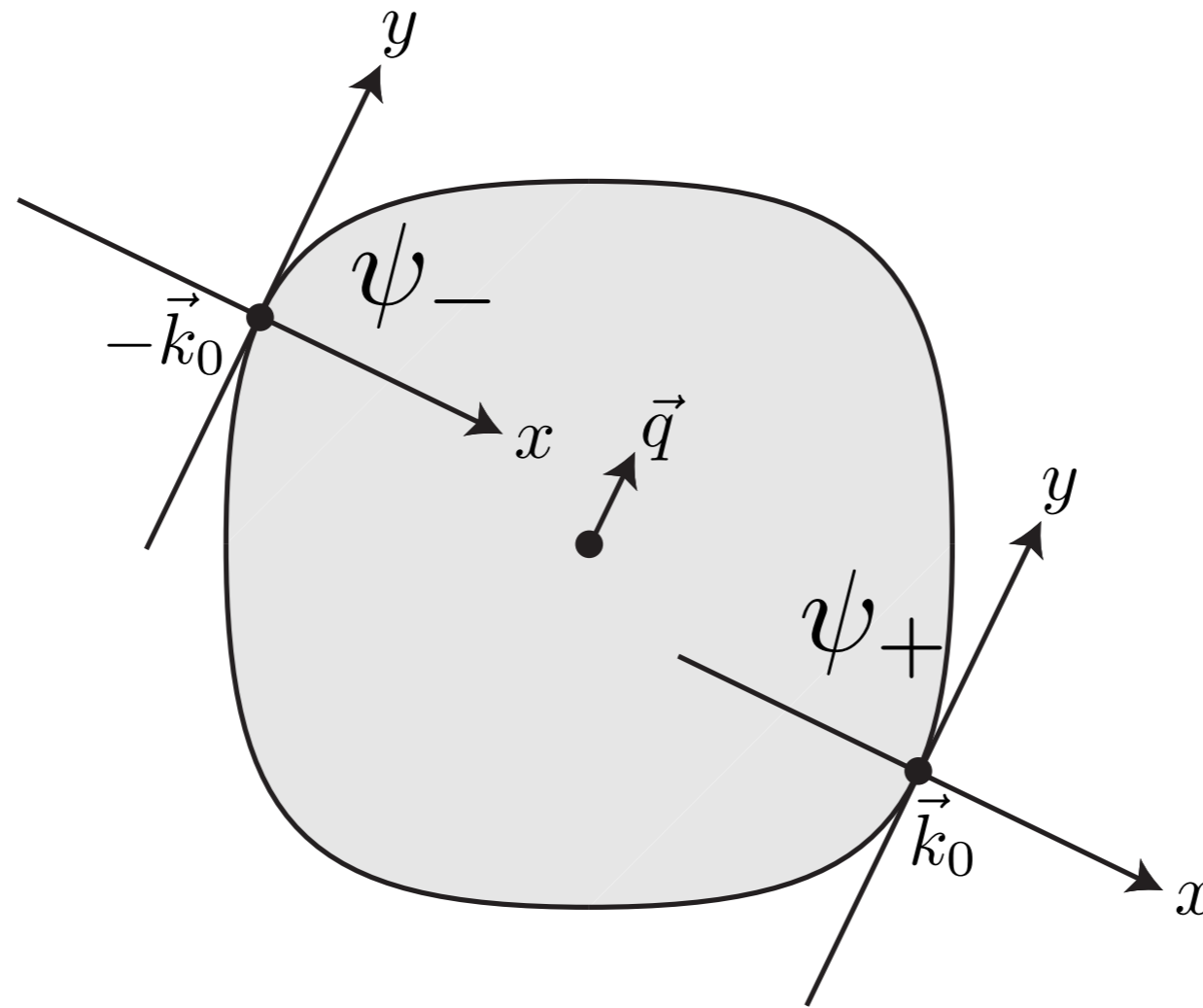
Quantum criticality of Ising-nematic ordering

$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha}$$

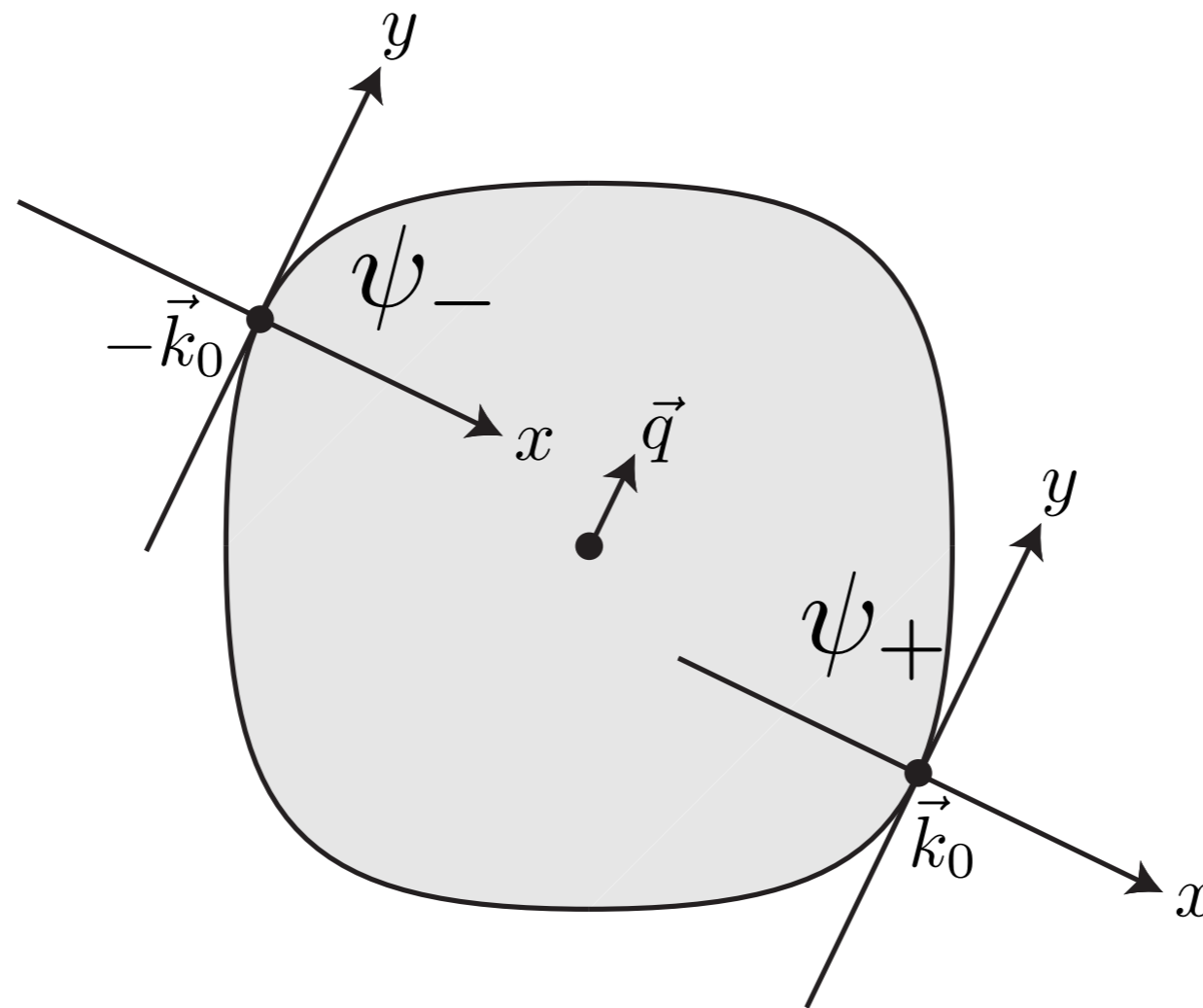
$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

Quantum criticality of Ising-nematic ordering



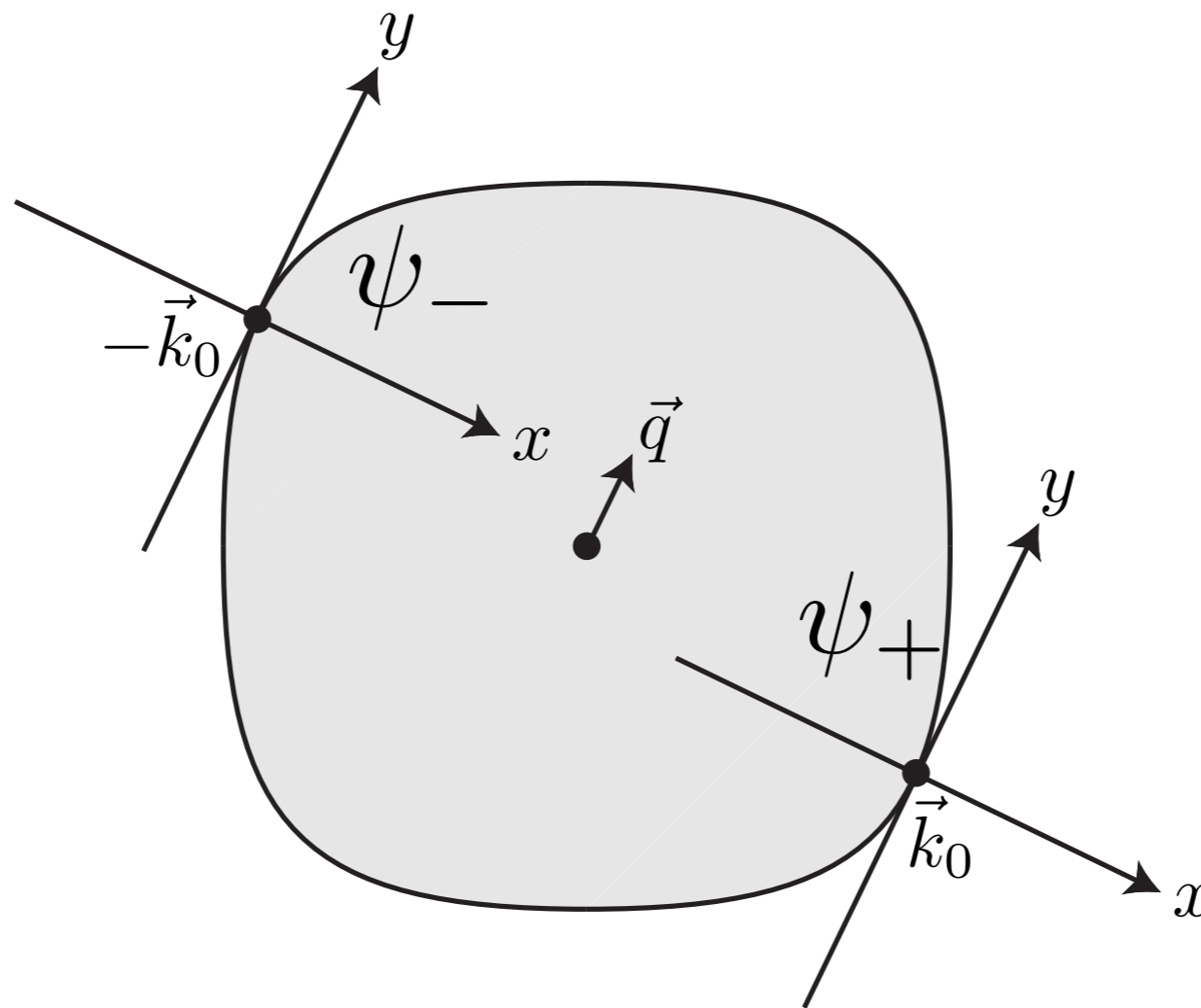
- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.

Quantum criticality of Ising-nematic ordering



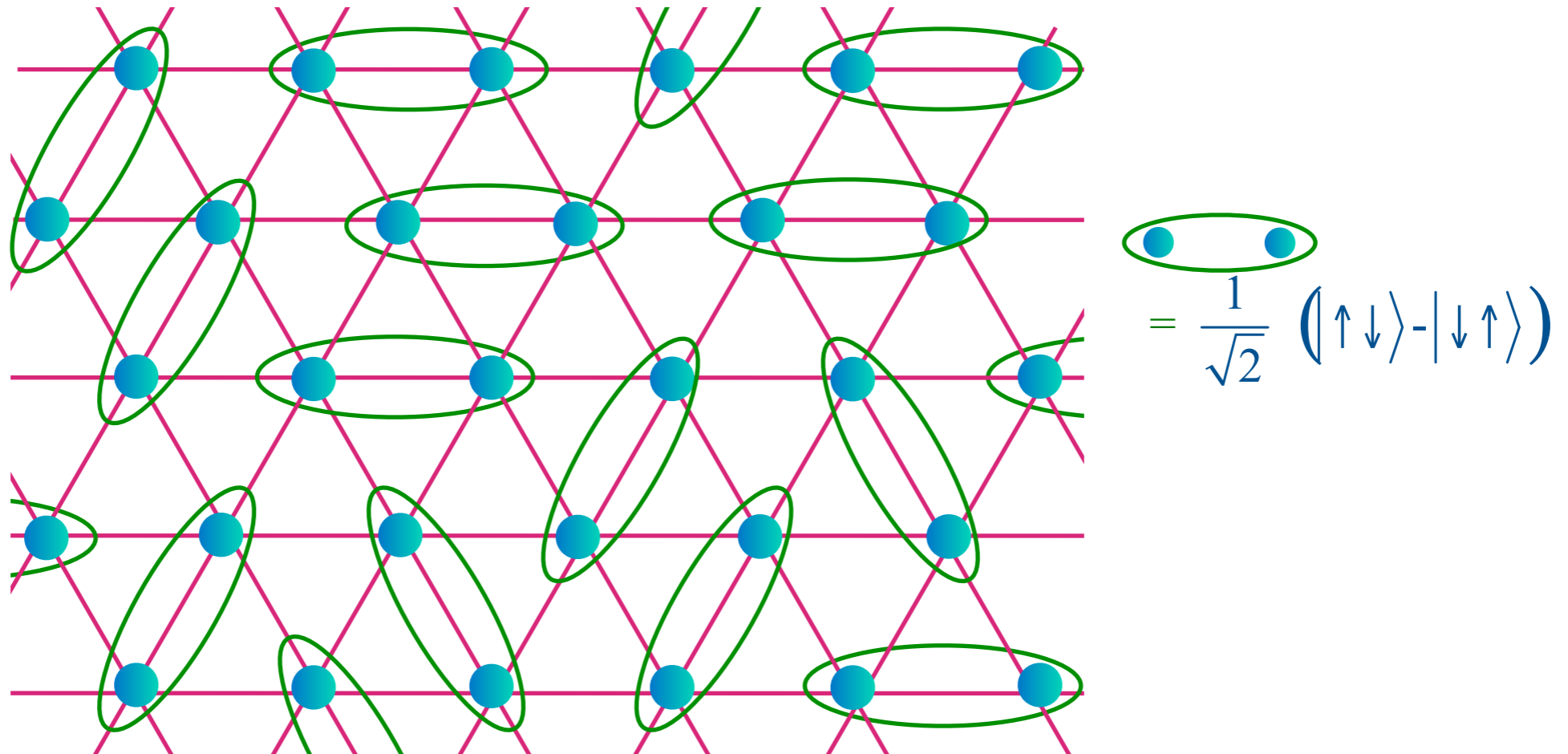
- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering



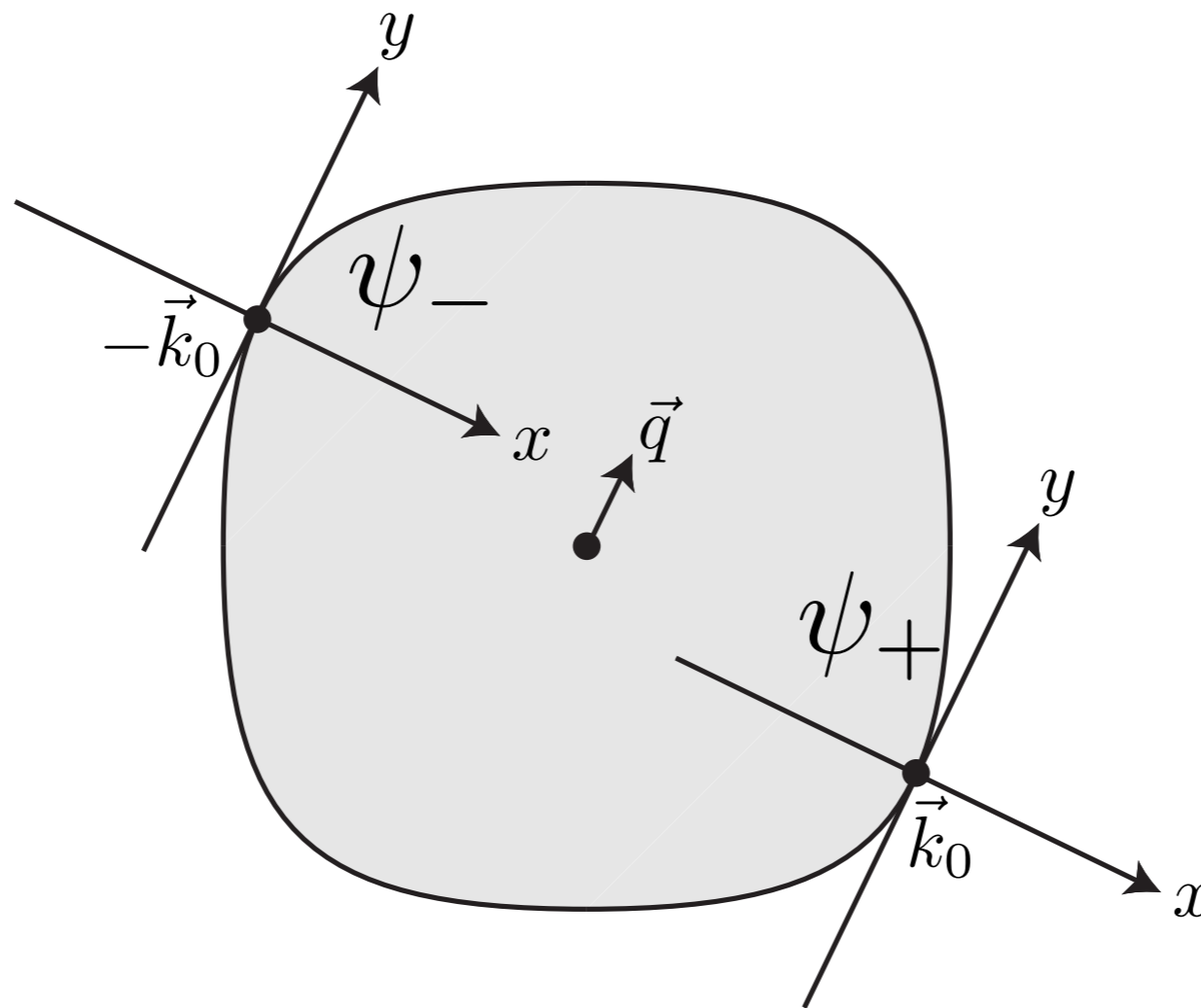
$$\begin{aligned} \mathcal{L}[\psi_{\pm}, \phi] = & \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ & - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \end{aligned}$$

- Model of a spin liquid (“Bose metal”): couple fermions to a dynamical gauge field A_μ .



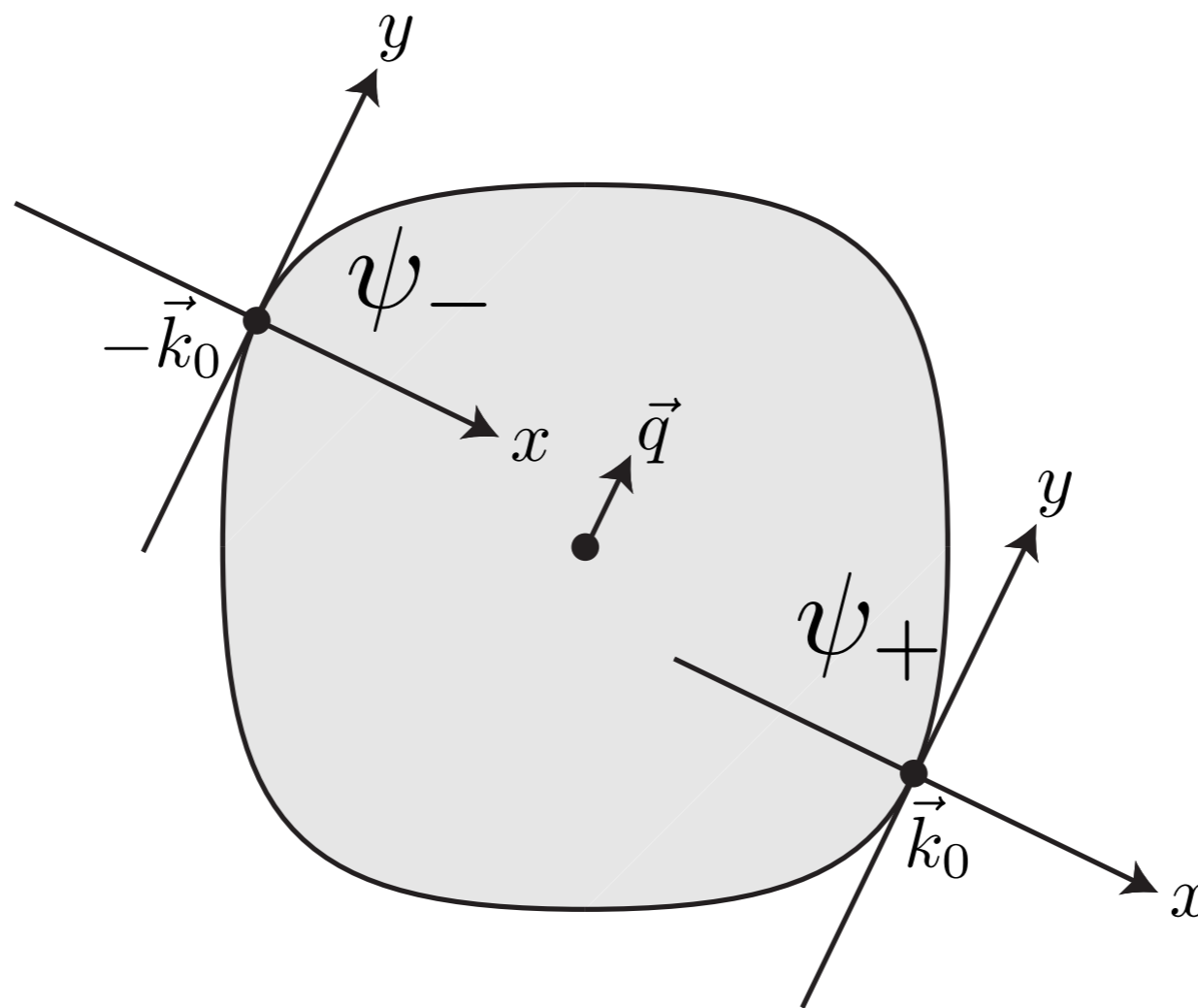
$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

Quantum criticality of Ising-nematic ordering



$$\mathcal{L}[\psi_{\pm}, \phi] = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$

Field theory of U(1) spin liquid

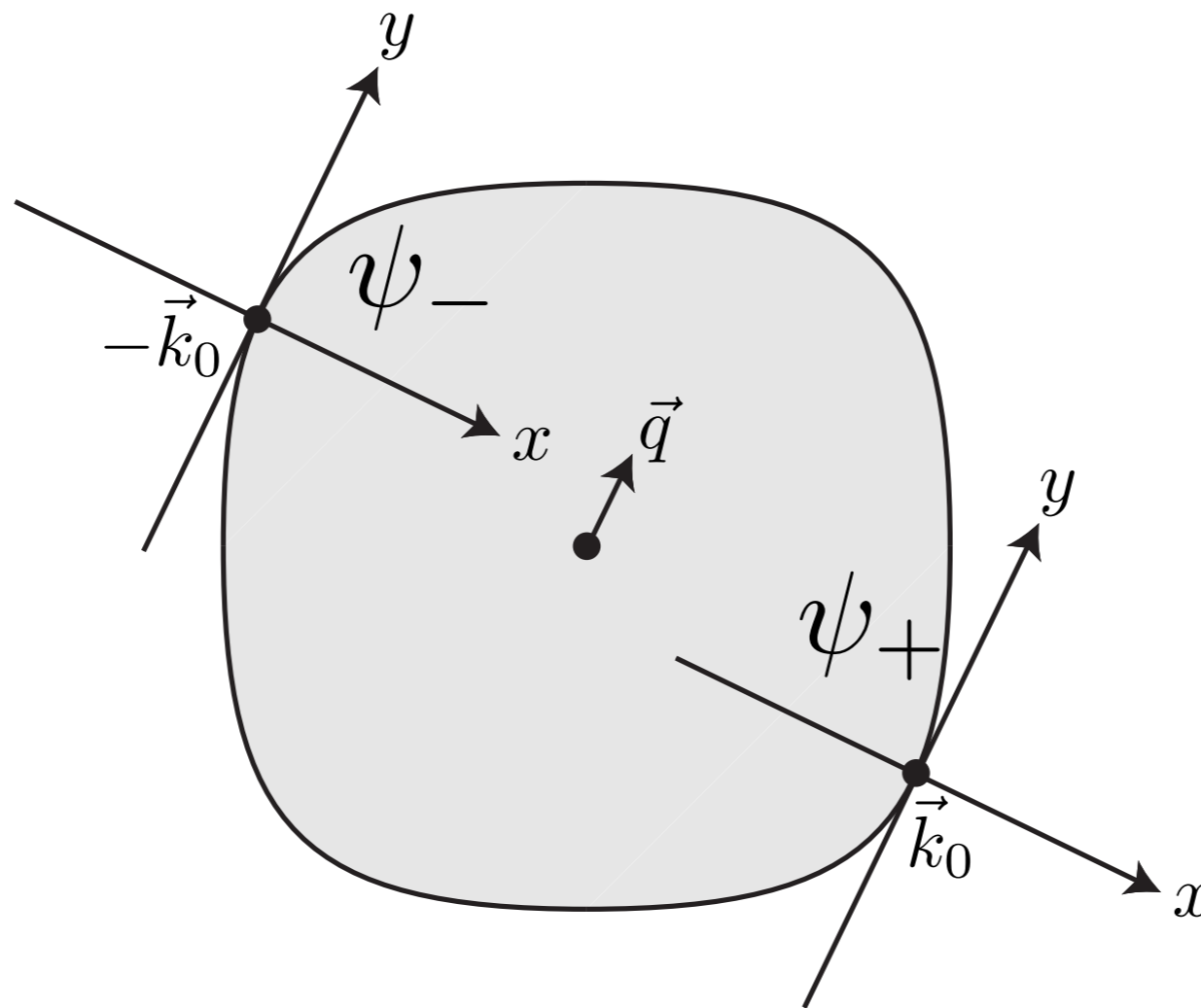


$$\mathcal{L}[\psi_{\pm}, a] =$$

$$\psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_-$$

$$-a (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) + \frac{1}{2g^2} (\partial_y a)^2$$

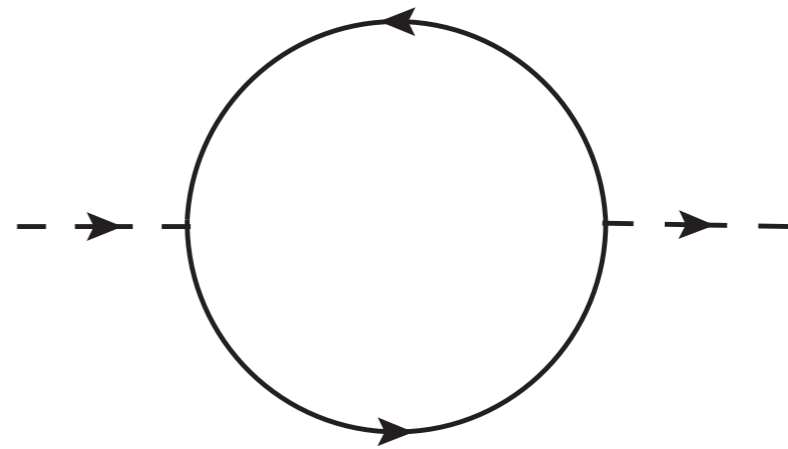
Quantum criticality of Ising-nematic ordering



$$\mathcal{L}[\psi_{\pm}, \phi] = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$



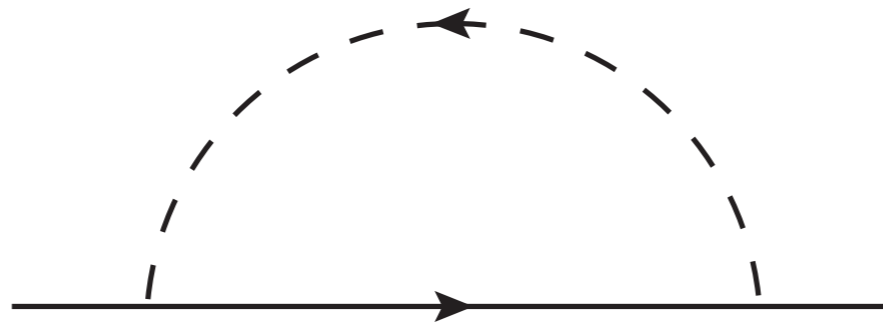
One loop ϕ self-energy with N_f fermion flavors:

$$D(\vec{q}, \omega) = N_f \int \frac{d^2 k}{4\pi^2} \frac{d\Omega}{2\pi} \frac{1}{[-i(\Omega + \omega) + k_x + q_x + (k_y + q_y)^2] [-i\Omega - k_x + k_y^2]}$$
$$= \frac{N_f}{4\pi} \frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$



Electron self-energy at order $1/N_f$:

$$\begin{aligned} \Sigma(\vec{k}, \Omega) &= -\frac{1}{N_f} \int \frac{d^2q}{4\pi^2} \frac{d\omega}{2\pi} \frac{1}{[-i(\omega + \Omega) + k_x + q_x + (k_y + q_y)^2] \left[\frac{q_y^2}{g^2} + \frac{|\omega|}{|q_y|} \right]} \\ &= -i \frac{2}{\sqrt{3}N_f} \left(\frac{g^2}{4\pi} \right)^{2/3} \text{sgn}(\Omega) |\Omega|^{2/3} \end{aligned}$$

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$

Schematic form of ϕ and fermion Green's functions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \frac{|\omega|}{|q_y|}}, \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i \text{sgn}(\omega) |\omega|^{2/3} / N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering

$$\begin{aligned} \mathcal{L} = & \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ & - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \end{aligned}$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\begin{aligned} \mathcal{L}_{\text{scaling}} = & \psi_+^\dagger (-i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (+i\partial_x - \partial_y^2) \psi_- \\ & - g\phi \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + (\partial_y \phi)^2 \end{aligned}$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\begin{aligned} \mathcal{L}_{\text{scaling}} = & \psi_+^\dagger (-i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (+i\partial_x - \partial_y^2) \psi_- \\ & - g \phi \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + (\partial_y \phi)^2 \end{aligned}$$

Simple scaling argument for $z = 3/2$.

Under the rescaling $x \rightarrow x/s$, $y \rightarrow y/s^{1/2}$, and $\tau \rightarrow \tau/s^z$, we find invariance provided

$$\begin{aligned} \phi & \rightarrow \phi s^{(2z+1)/4} \\ \psi & \rightarrow \psi s^{(2z+1)/4} \\ g & \rightarrow g s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$

Schematic form of ϕ and fermion Green's functions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \frac{|\omega|}{|q_y|}}, \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i \text{sgn}(\omega) |\omega|^{2/3} / N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

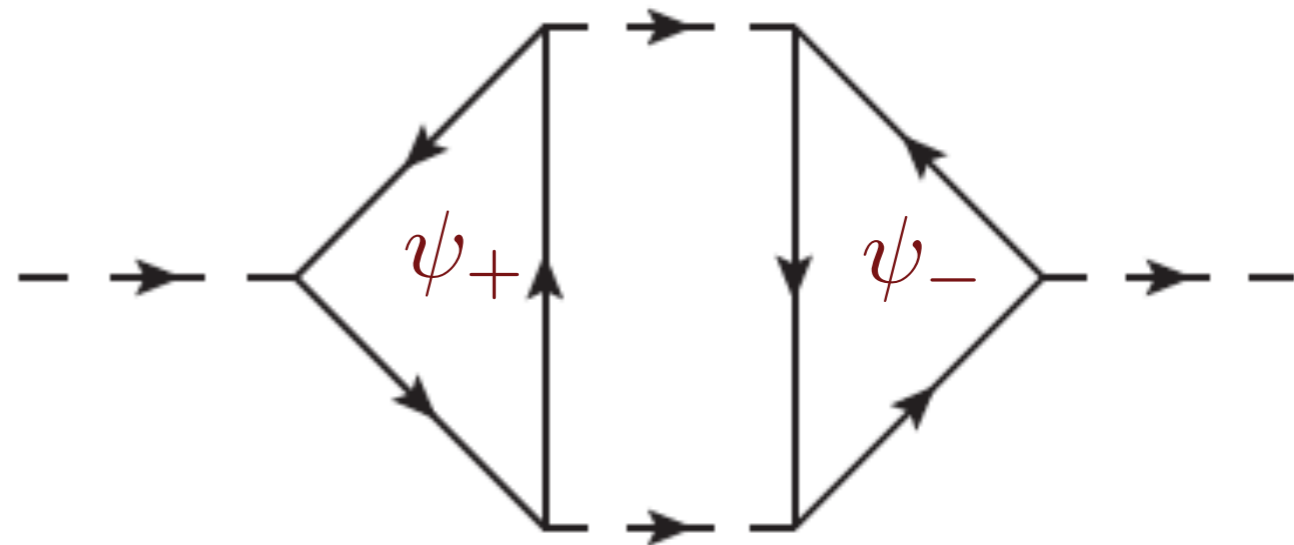
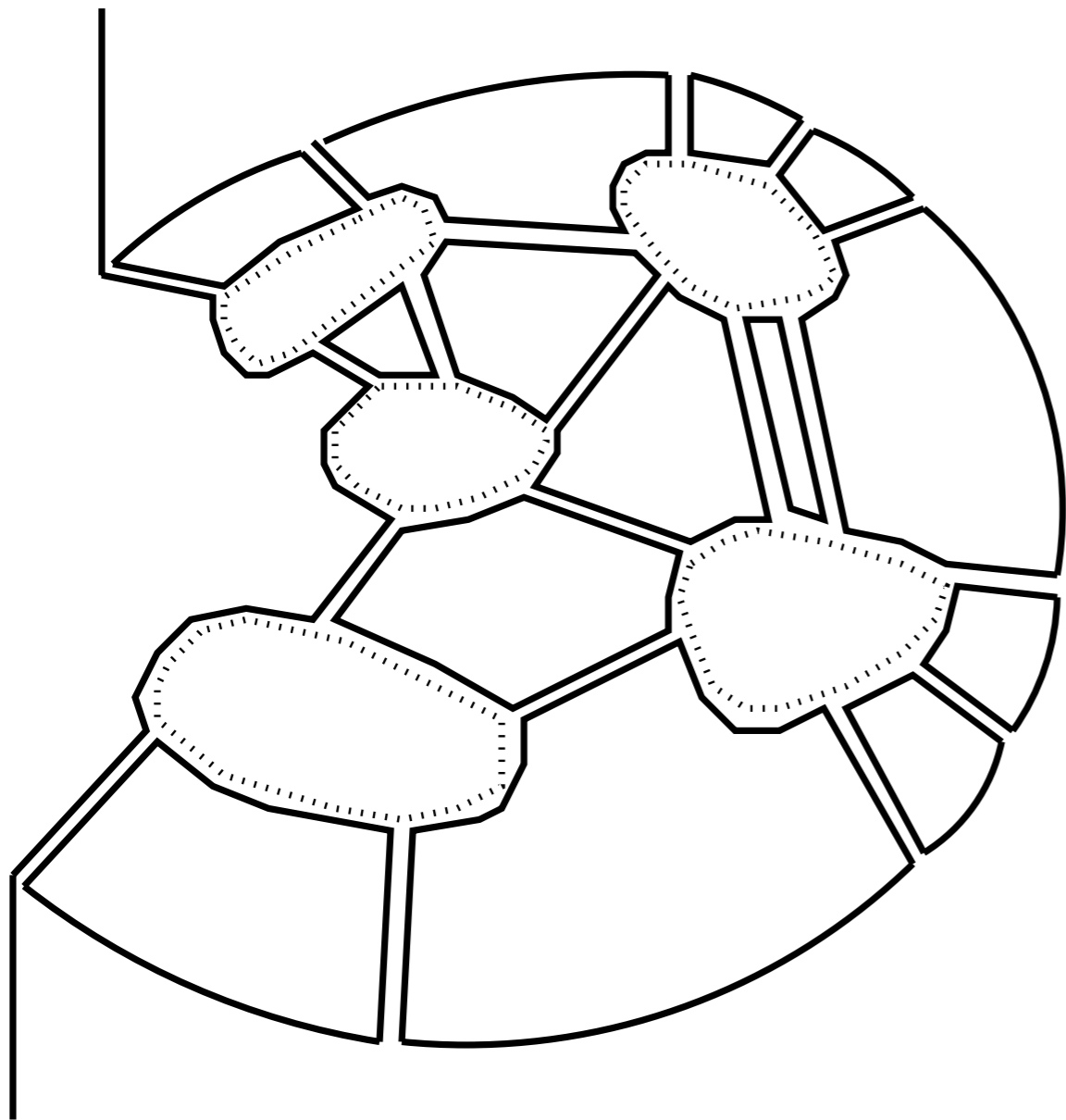
Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering

$$\begin{aligned} \mathcal{L} = & \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ & - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \end{aligned}$$

The $1/N_f$ expansion is *not* determined by counting fermion loops, because of infrared singularities created by the Fermi surface. The $|\omega|^{2/3}/N_f$ fermion self-energy leads to additional powers of N_f , and a breakdown in the loop expansion.

Computations in the $1/N$ expansion



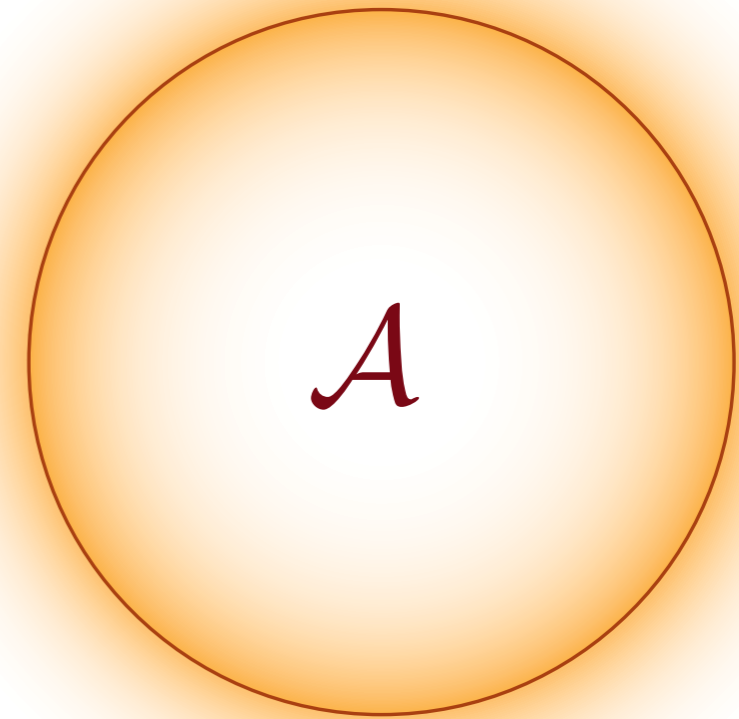
Graph mixing ψ_+ and ψ_- is $\mathcal{O}(N^{3/2})$ (instead of $\mathcal{O}(N)$), violating genus expansion

All planar graphs of ψ_+ alone are as important as the leading term

M. A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review B* **80**, 165102 (2009)

Properties of the strange metal at the Ising-nematic critical point



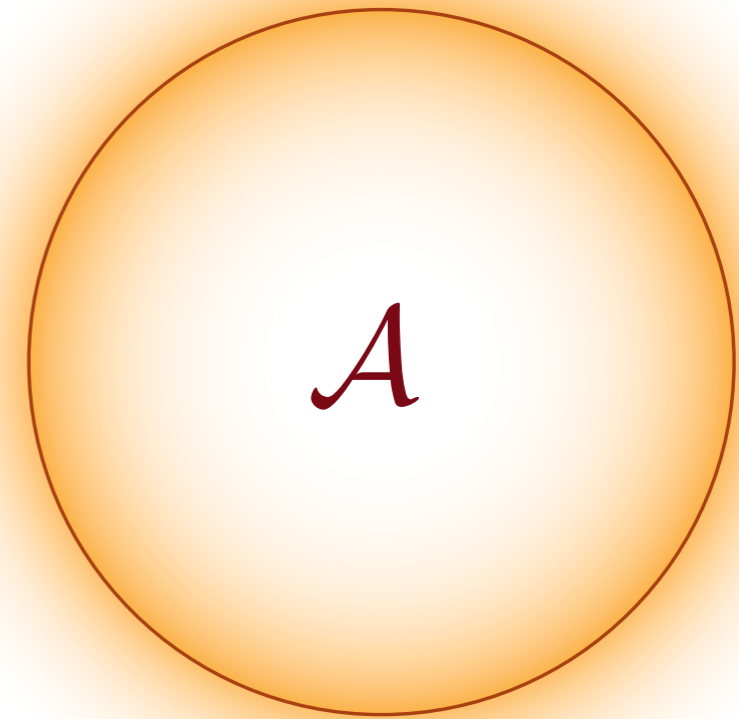
- There is a sharp Fermi surface defined by the fermion Green's function: $G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

Properties of the strange metal at the Ising-nematic critical point



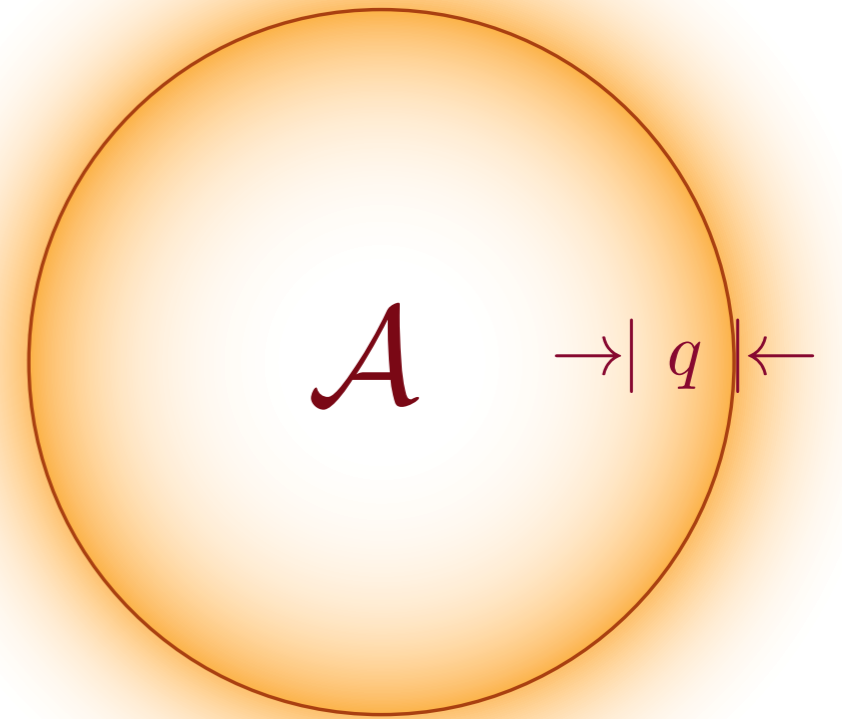
- There is a sharp Fermi surface defined by the fermion Green's function: $G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0$.
- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

Properties of the strange metal at the Ising-nematic critical point



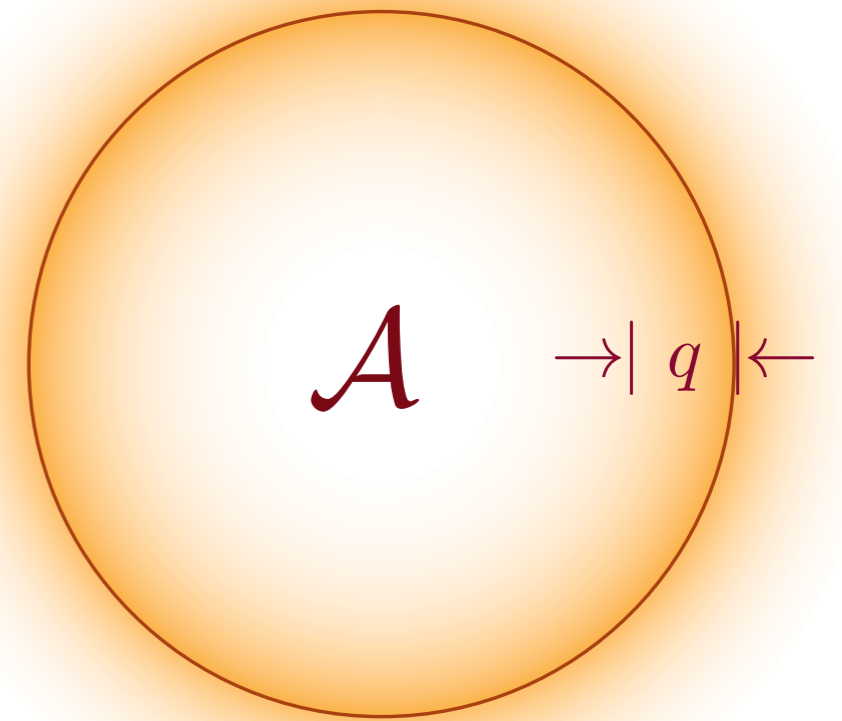
- There is a sharp Fermi surface defined by the fermion Green's function: $G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0$.
- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density
- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

Properties of the strange metal at the Ising-nematic critical point



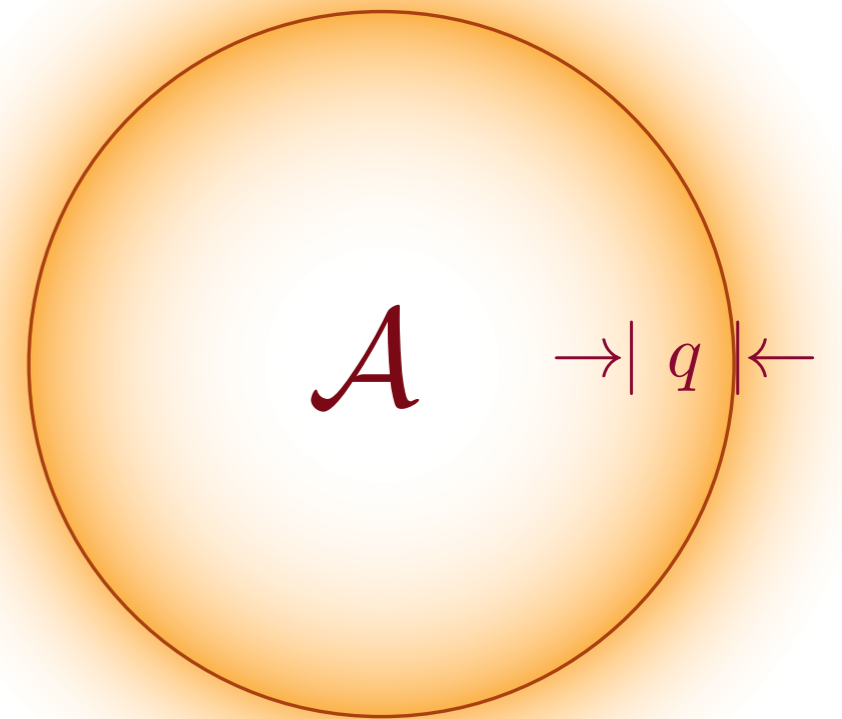
- Fermion Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

Properties of the strange metal at the Ising-nematic critical point



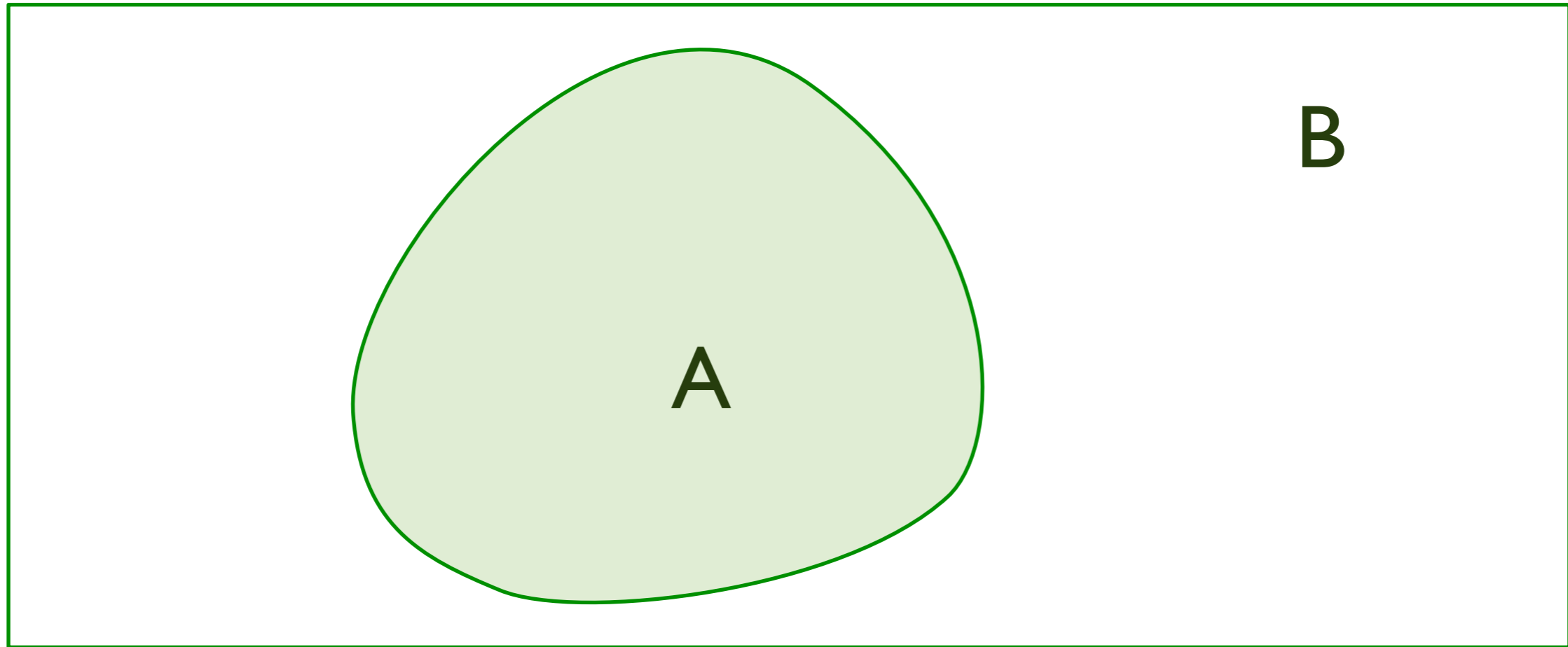
- Fermion Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\text{eff}}/z}$ with $d_{\text{eff}} = 1$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

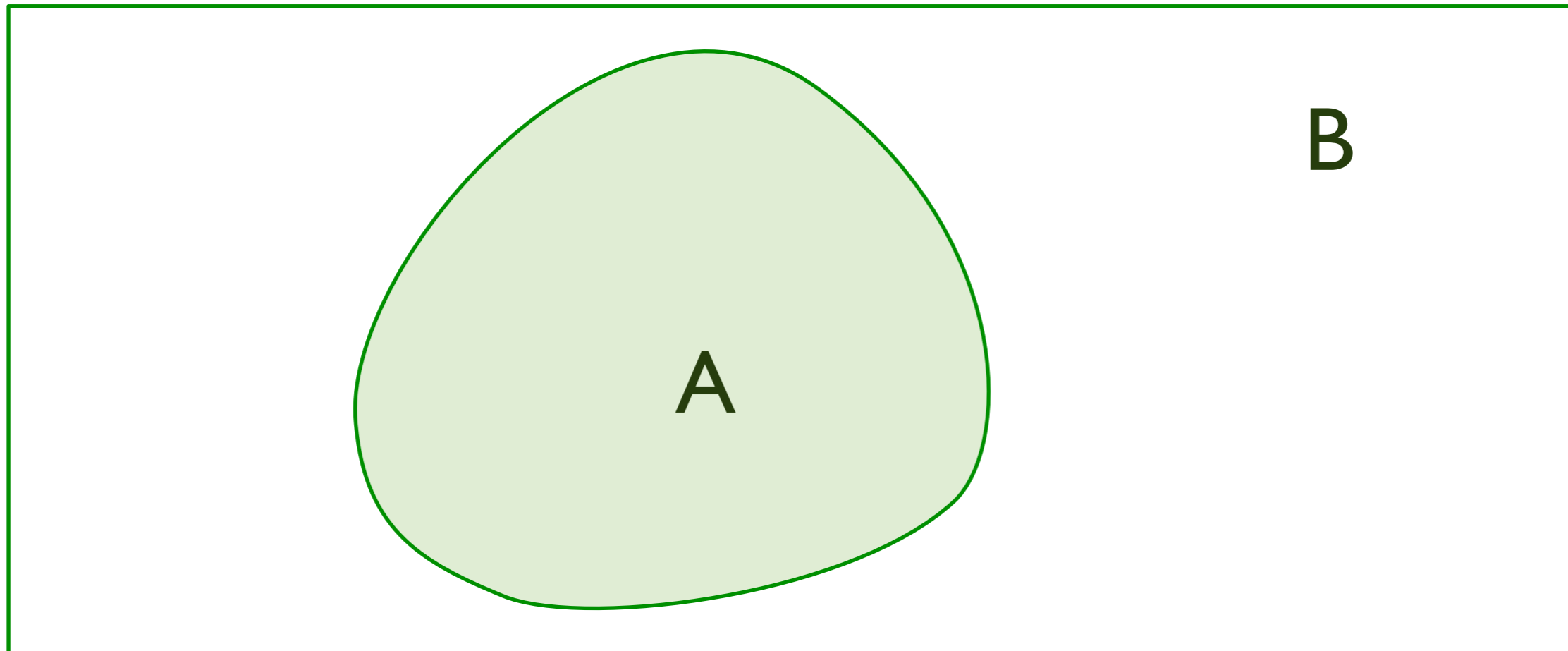
Entanglement entropy



Measure strength of quantum entanglement of region A with region B .

$\rho_A = \text{Tr}_B \rho =$ density matrix of region A
Entanglement entropy $S_{EE} = -\text{Tr}(\rho_A \ln \rho_A)$

Entanglement entropy of Fermi surfaces



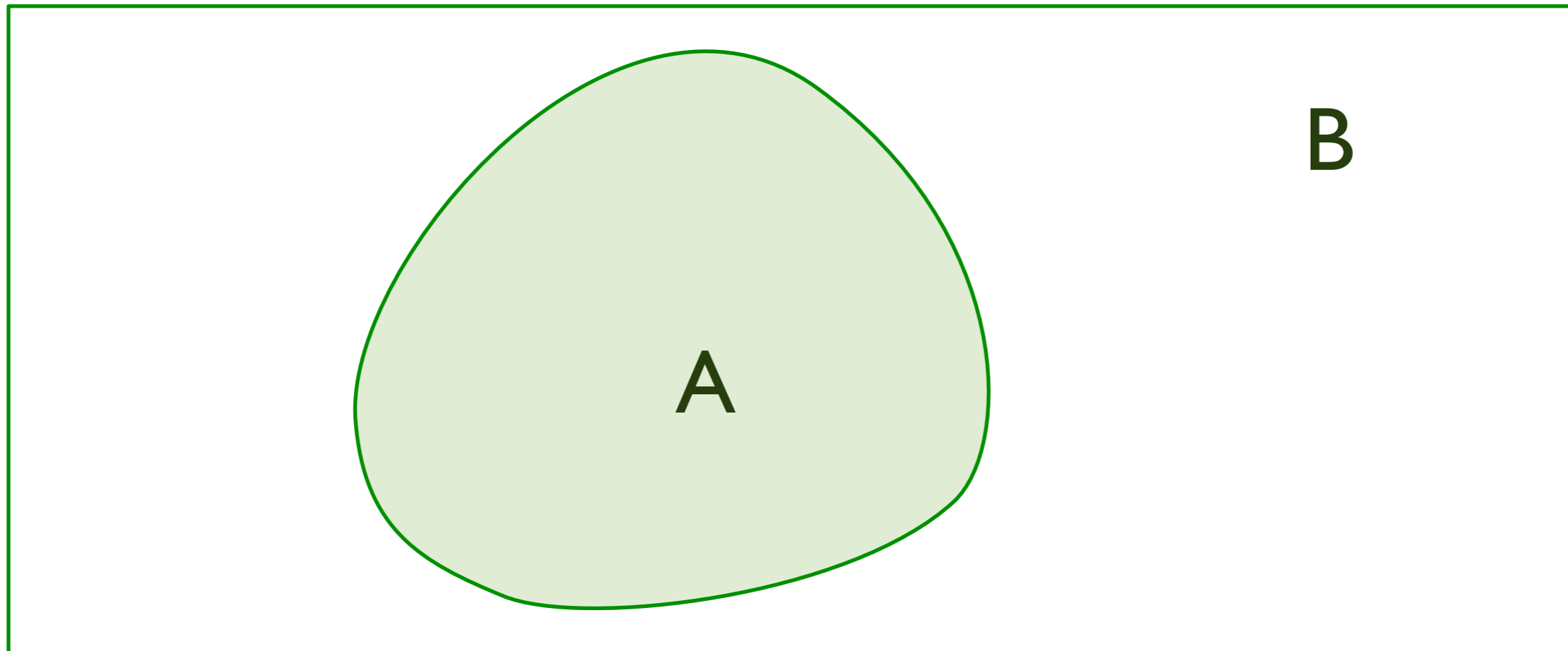
Logarithmic violation of “area law”: $S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F , where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of Fermi surfaces



Logarithmic violation of “area law”: $S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F , where P is the perimeter of region A with an arbitrary smooth shape.

Non-Fermi liquids have, at most, the “1/12” prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Compressible quantum matter

A. Fermi liquids: graphene

*B. Holography: Reissner - Nördstrom
solution*

*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*

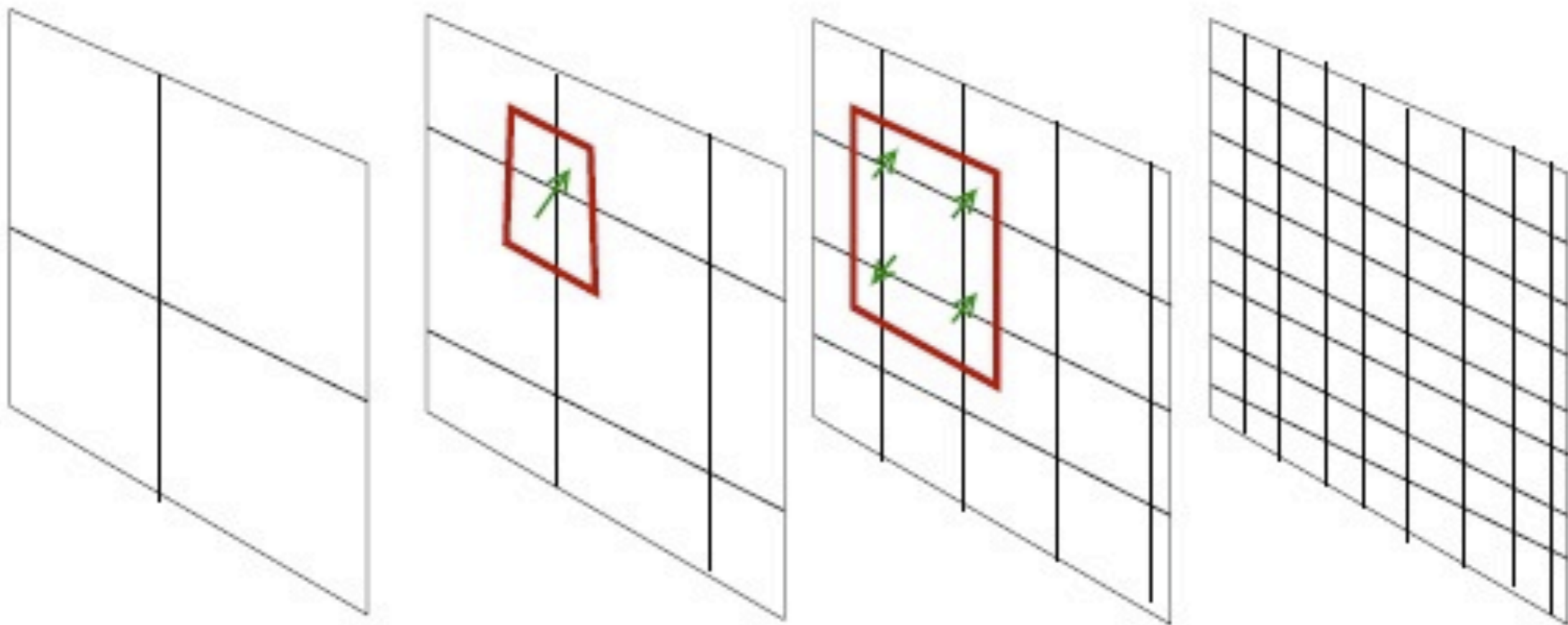
Compressible quantum matter

A. Fermi liquids: graphene

*B. Holography: Reissner - Nördstrom
solution*

*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*



r ←

Consider the metric which transforms under rescaling as

$$\begin{aligned}x_i &\rightarrow \zeta x_i \\t &\rightarrow \zeta^z t \\ds &\rightarrow \zeta^{\theta/d} ds.\end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points).

θ is the violation of hyperscaling exponent.

Consider the metric which transforms under rescaling as

$$\begin{aligned}x_i &\rightarrow \zeta x_i \\t &\rightarrow \zeta^z t \\ds &\rightarrow \zeta^{\theta/d} ds.\end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points).

θ is the violation of hyperscaling exponent.

The most general choice of such a metric is

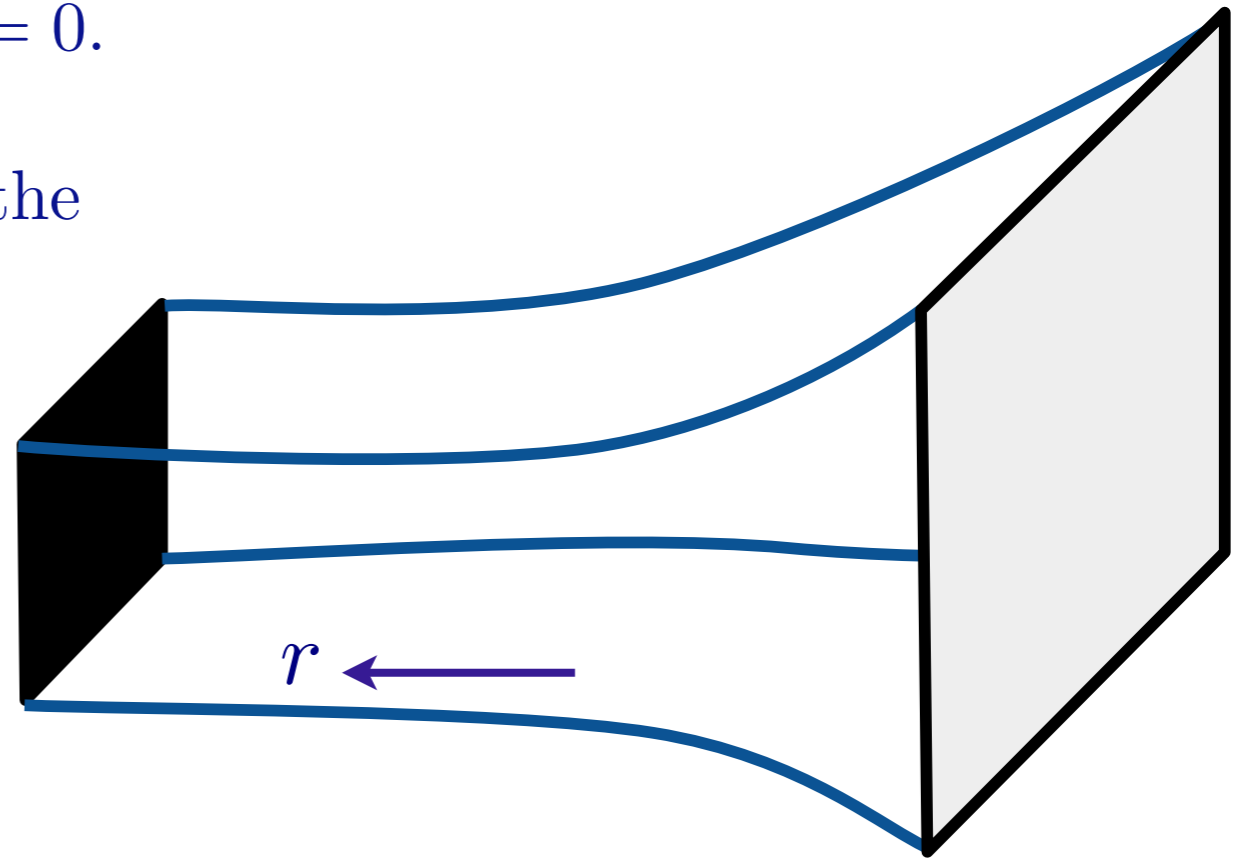
$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

At $T > 0$, there is a “black-brane” at $r = r_h$.

The Bekenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

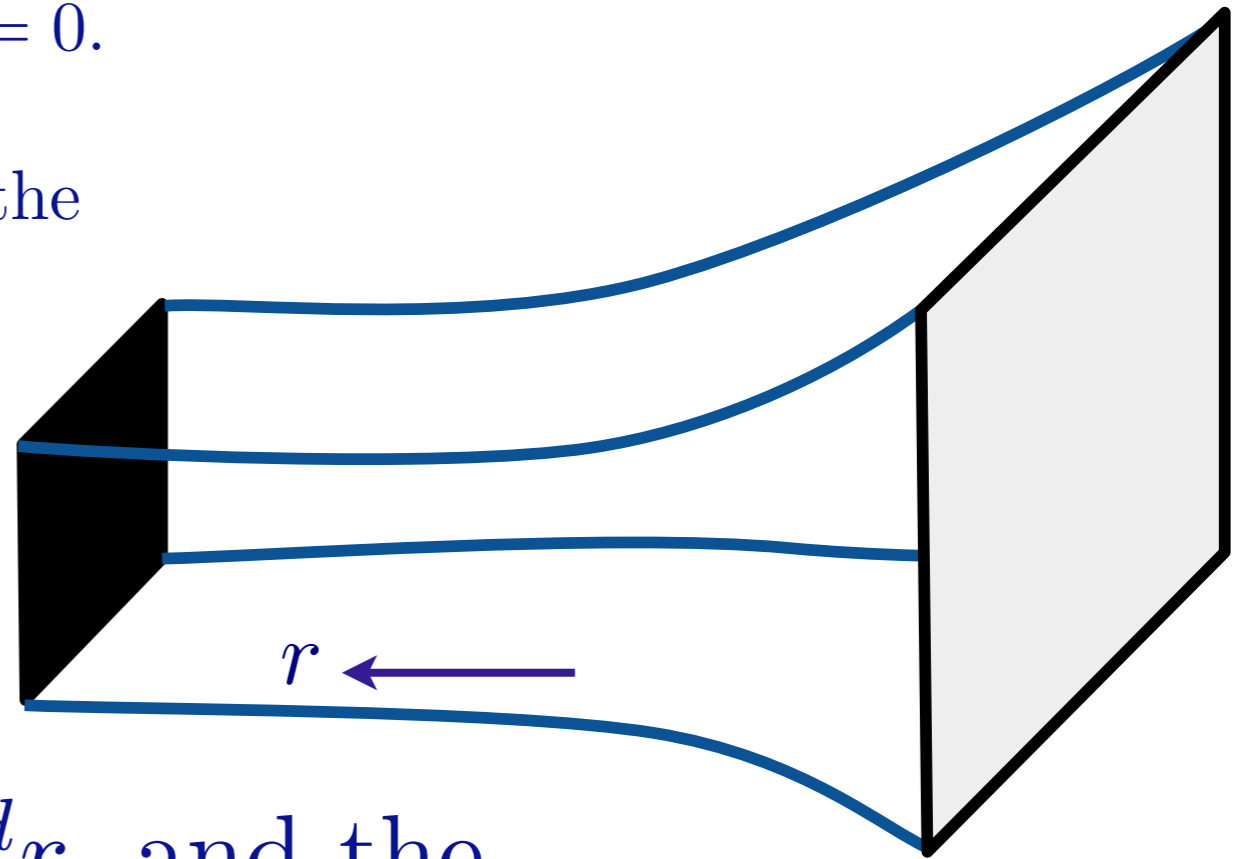
The entropy density, S , is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$



At $T > 0$, there is a “black-brane” at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S , is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$



Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$ measures “dimension deficit” in the phase space of low energy degrees of a freedom.

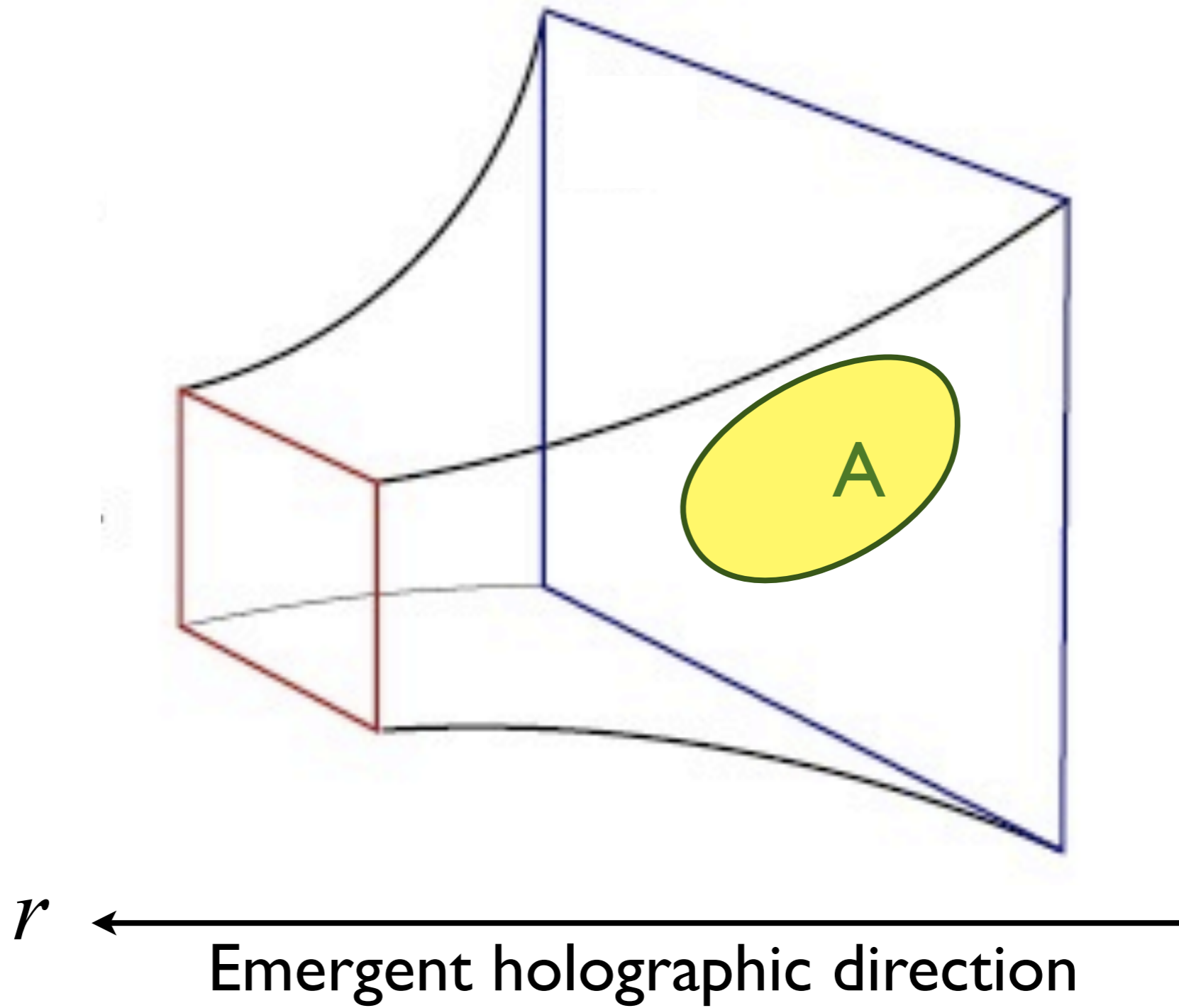
$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

- The thermal entropy density scales as

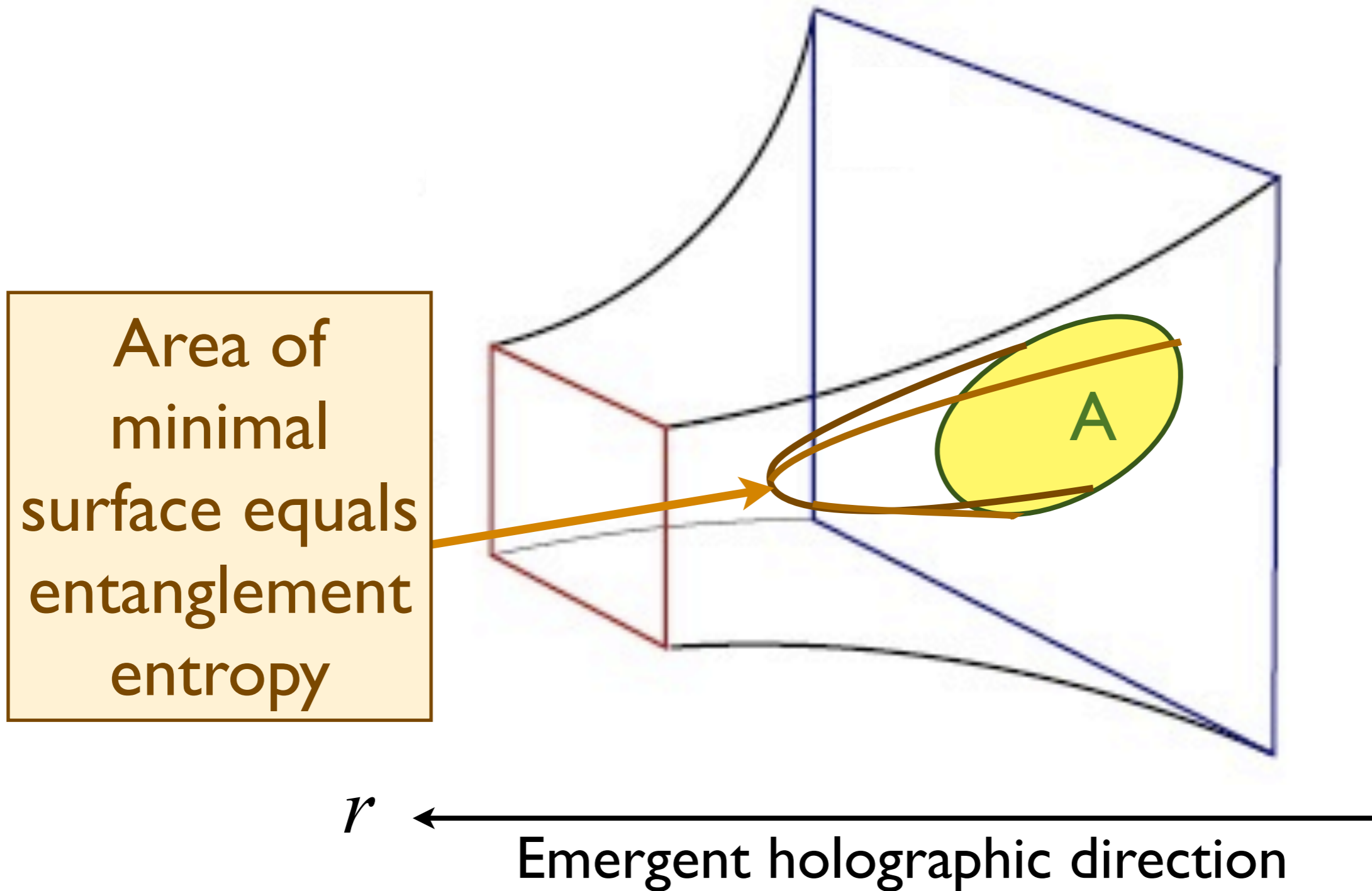
$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

Holographic entanglement entropy



Holographic entanglement entropy



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

- The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

- The entanglement entropy, S_E , of an entangling region with boundary surface ‘area’ P scales as

$$S_E \sim \begin{cases} P & , \text{ for } \theta < d - 1 \\ P \ln P & , \text{ for } \theta = d - 1 \\ P^{\theta/(d-1)} & , \text{ for } \theta > d - 1 \end{cases}$$

All local quantum field theories obey the “area law” (upto log violations) and so $\theta \leq d - 1$.

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

- The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

- The entanglement entropy, S_E , of an entangling region with boundary surface ‘area’ P scales as

$$S_E \sim \begin{cases} P & , \text{ for } \theta < d - 1 \\ P \ln P & , \text{ for } \theta = d - 1 \\ P^{\theta/(d-1)} & , \text{ for } \theta > d - 1 \end{cases}$$

All local quantum field theories obey the “area law” (upto log violations) and so $\theta \leq d - 1$.

- The null energy condition implies $z \geq 1 + \frac{\theta}{d}$.

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

- The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

- The entanglement entropy, S_E , of an entangling region with boundary surface ‘area’ P scales as

$$S_E \sim \begin{cases} P & , \text{ for } \theta < d - 1 \\ P \ln P & , \text{ for } \theta = d - 1 \\ P^{\theta/(d-1)} & , \text{ for } \theta > d - 1 \end{cases}$$

All local quantum field theories obey the “area law” (upto log violations) and so $\theta \leq d - 1$.

- The null energy condition implies $z \geq 1 + \frac{\theta}{d}$.

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d .

Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d .

Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of non-Fermi liquids

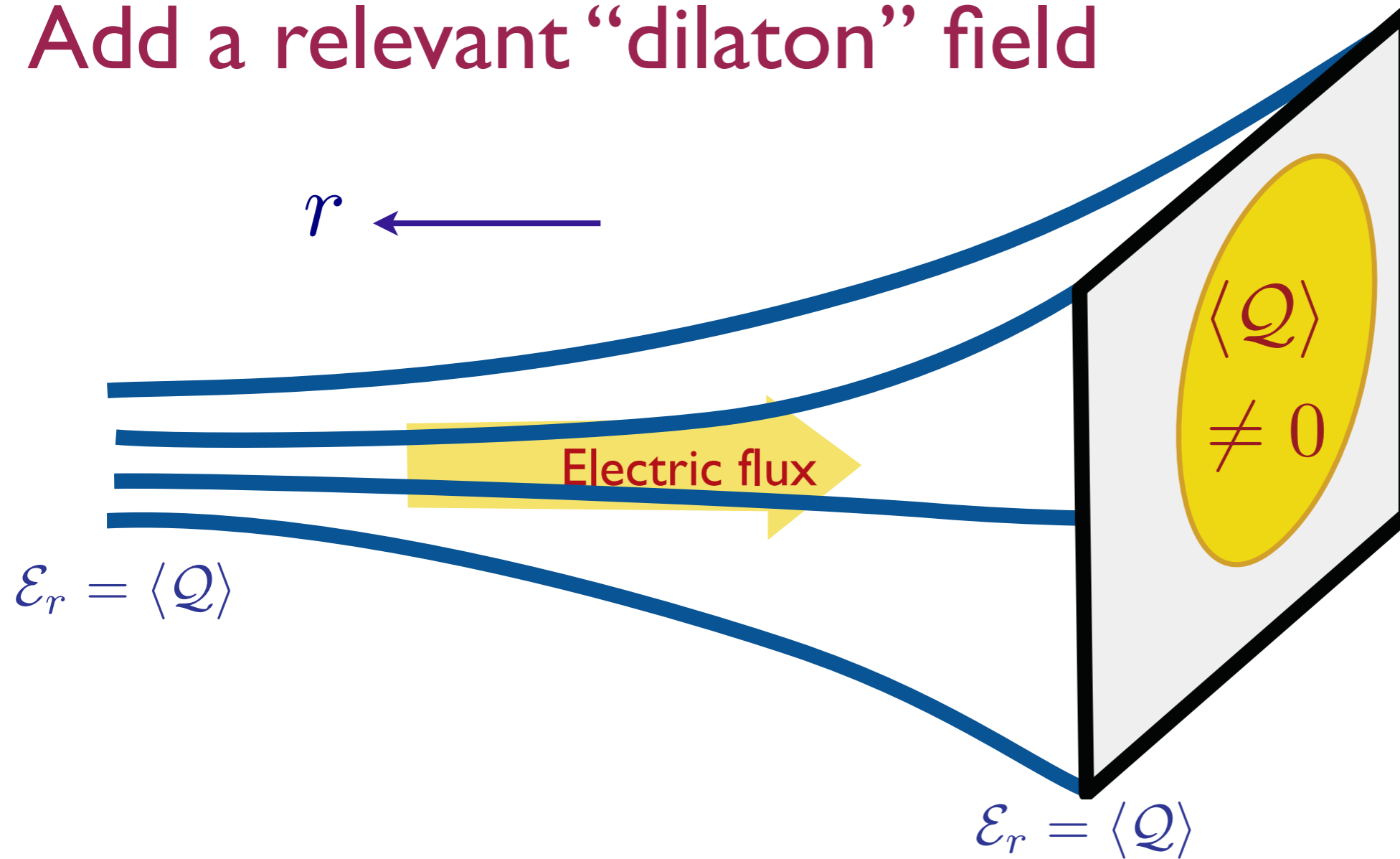
$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!
- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



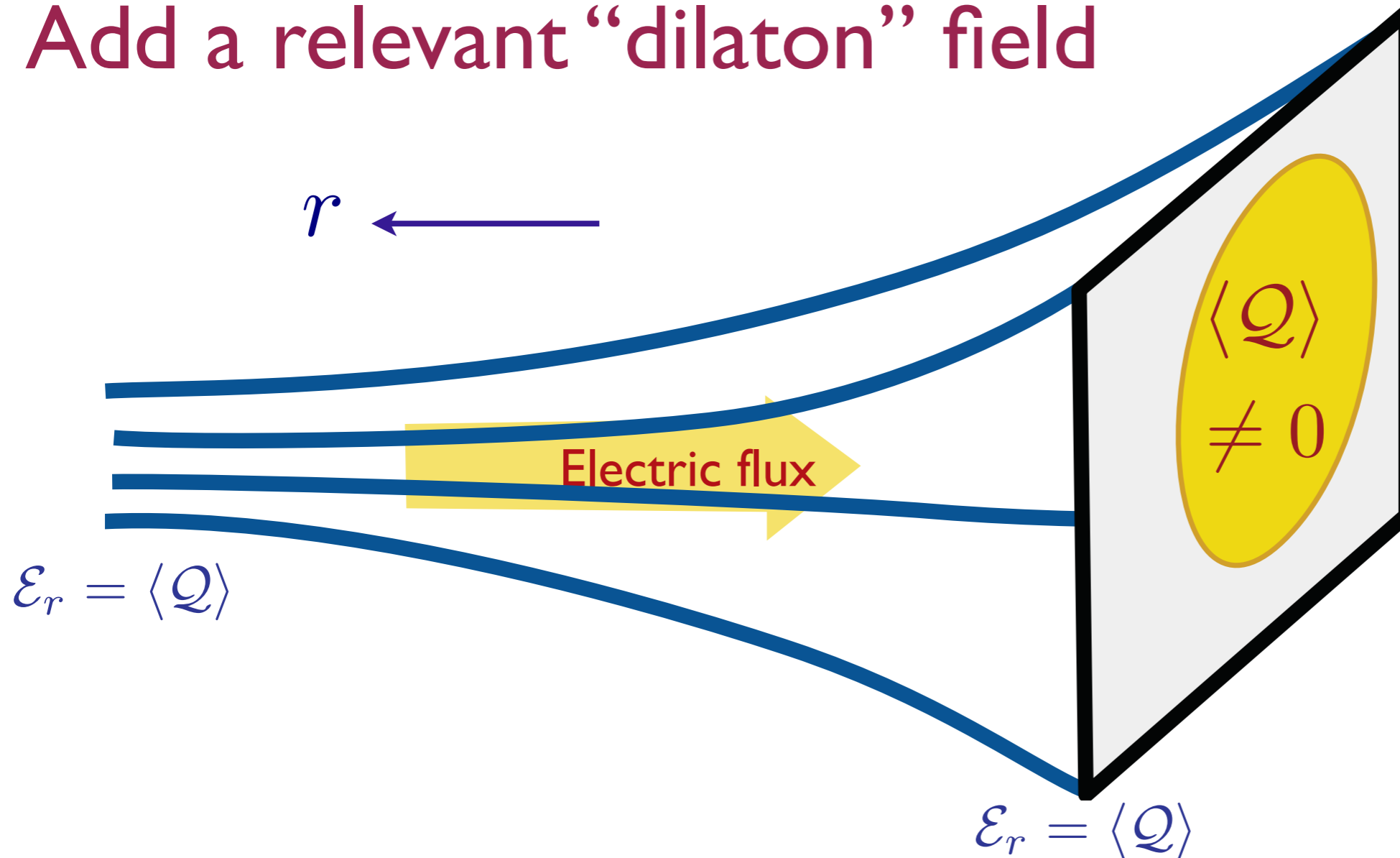
$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \rightarrow \infty$.

- C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



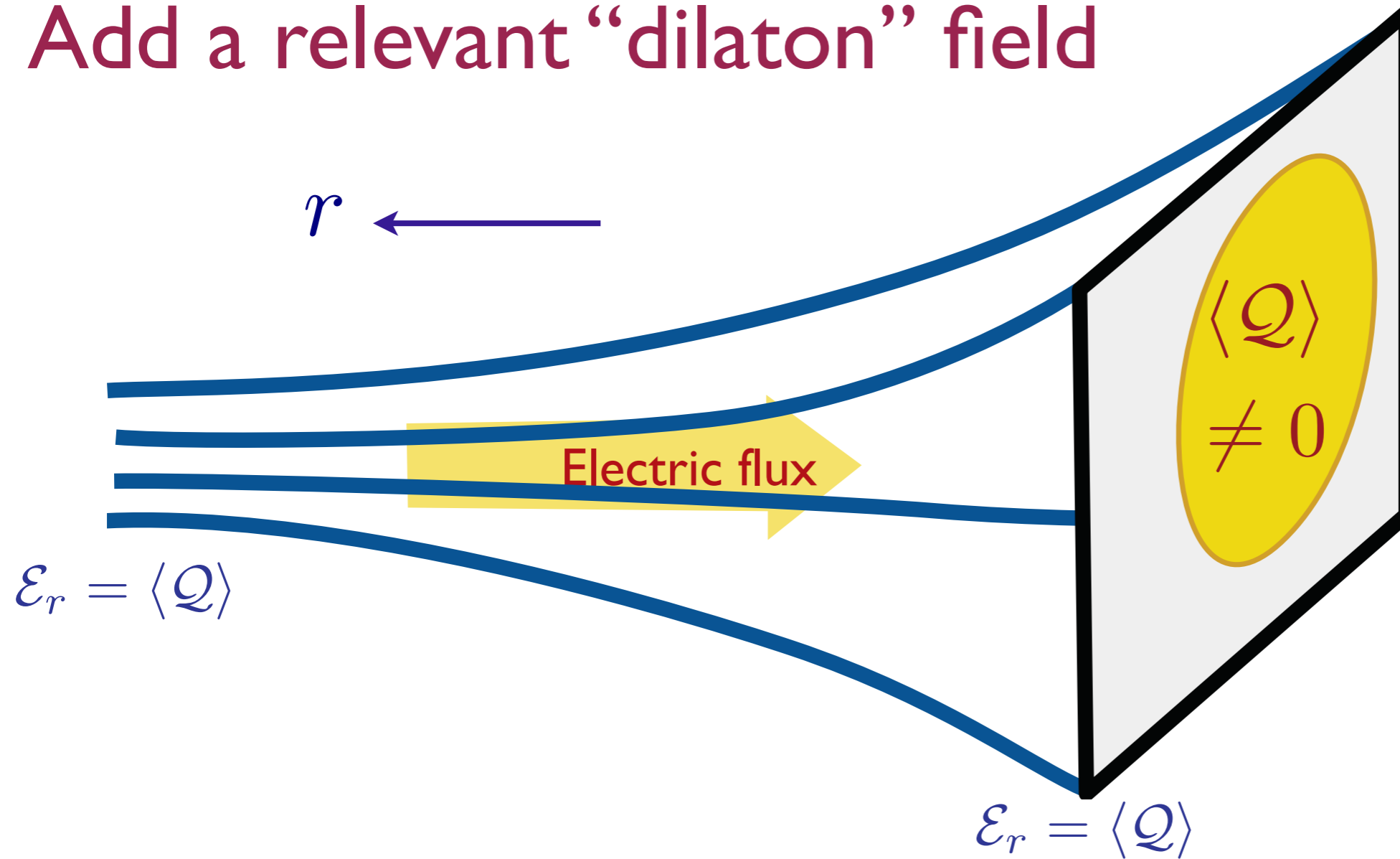
$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \rightarrow \infty$.

This is a “bosonization” of the Fermi surface

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$
with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^\delta$, $\Phi(r) \sim \ln(r)$ as $r \rightarrow \infty$.

- C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

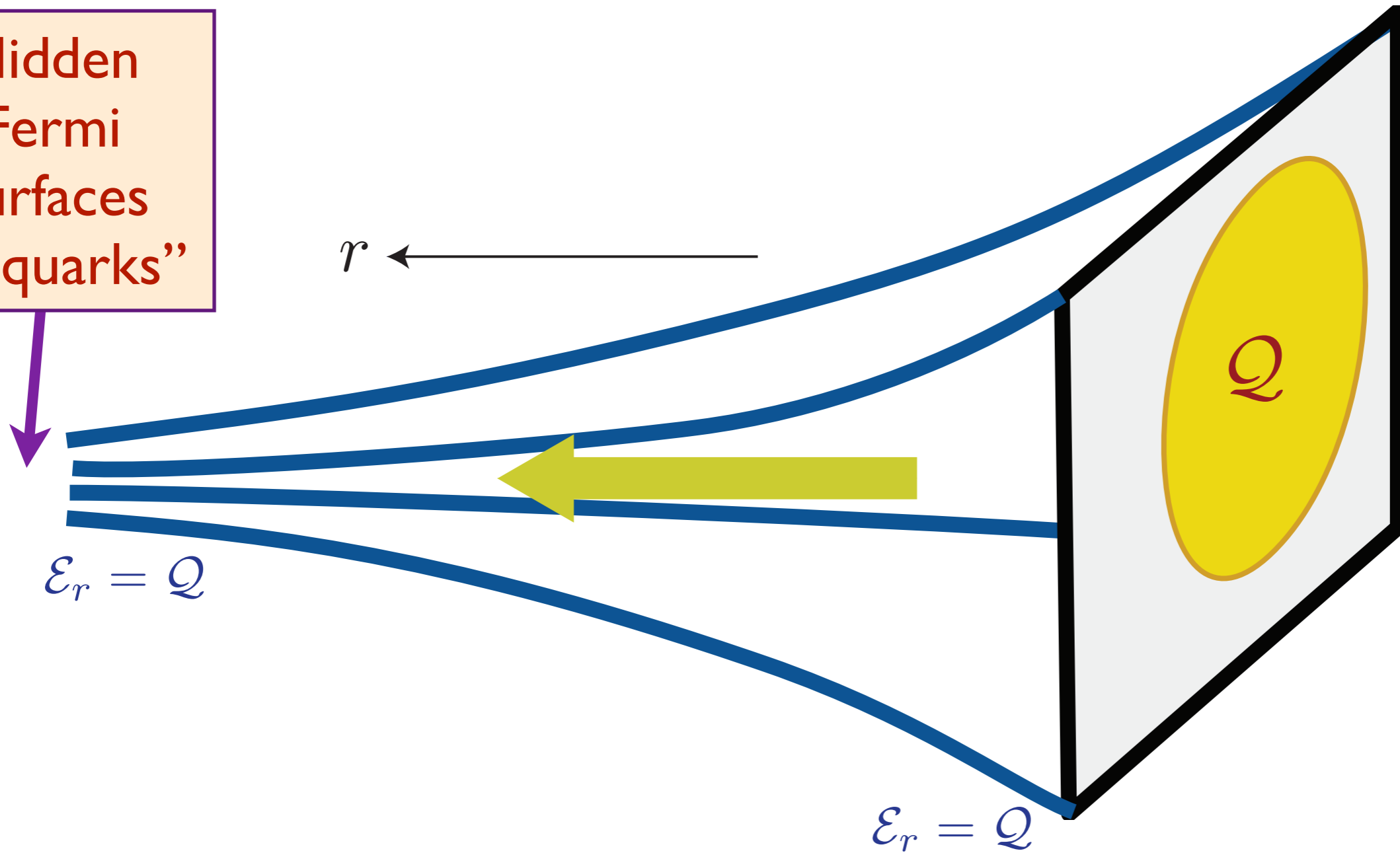
- The entanglement entropy has log-violation of the area law

$$S_E = \Xi Q^{(d-1)/d} P \ln P.$$

where P is surface area of the entangling region, and Ξ is a dimensionless constant which is **independent of all UV details**, of Q , and of any property of the entangling region. Note $Q^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

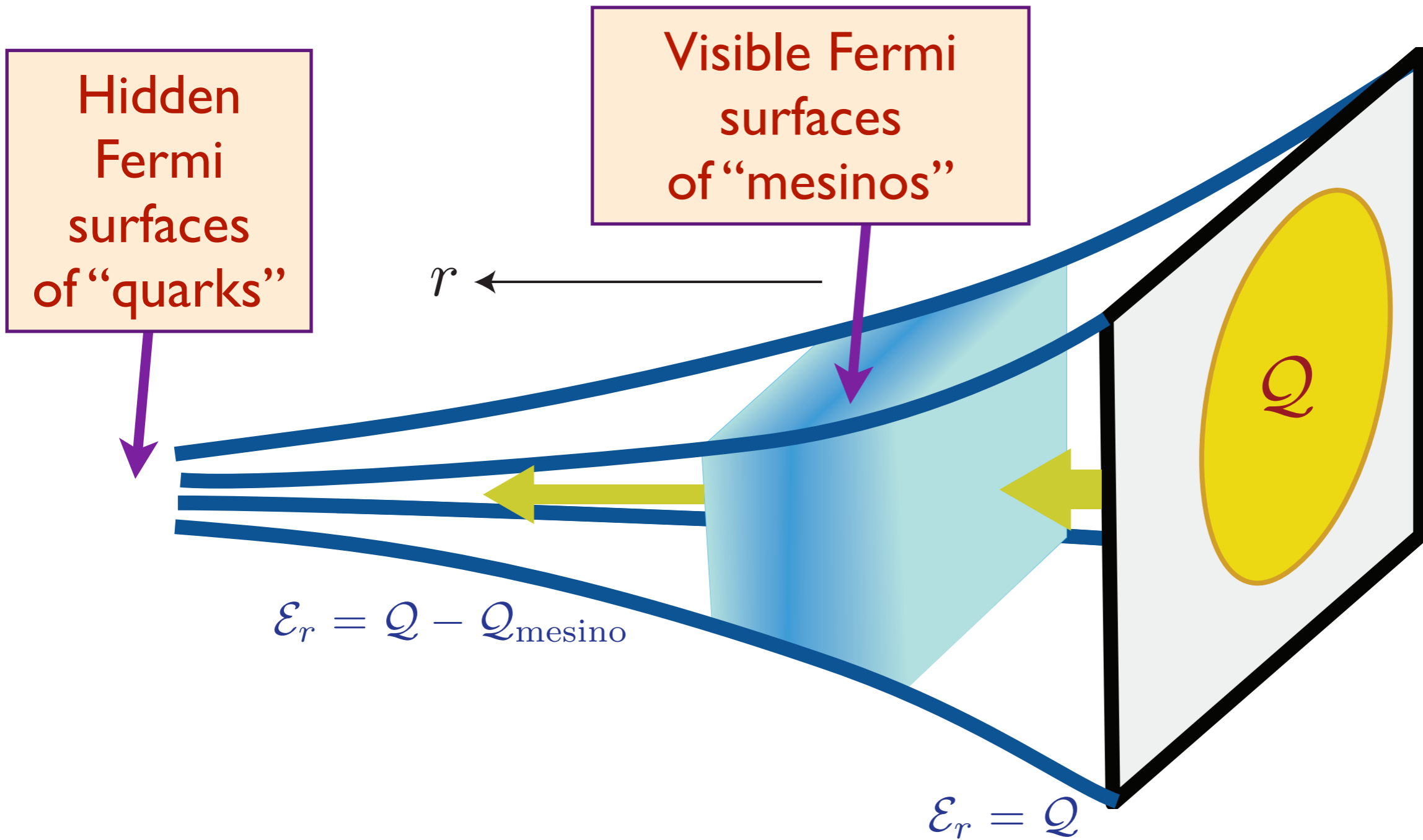
Holographic theory of a non-Fermi liquid (NFL)

Hidden Fermi surfaces of “quarks”



Gauss Law and the “attractor” mechanism
 \Leftrightarrow Luttinger theorem on the boundary

Holographic theory of a fractionalized-Fermi liquid (FL*)

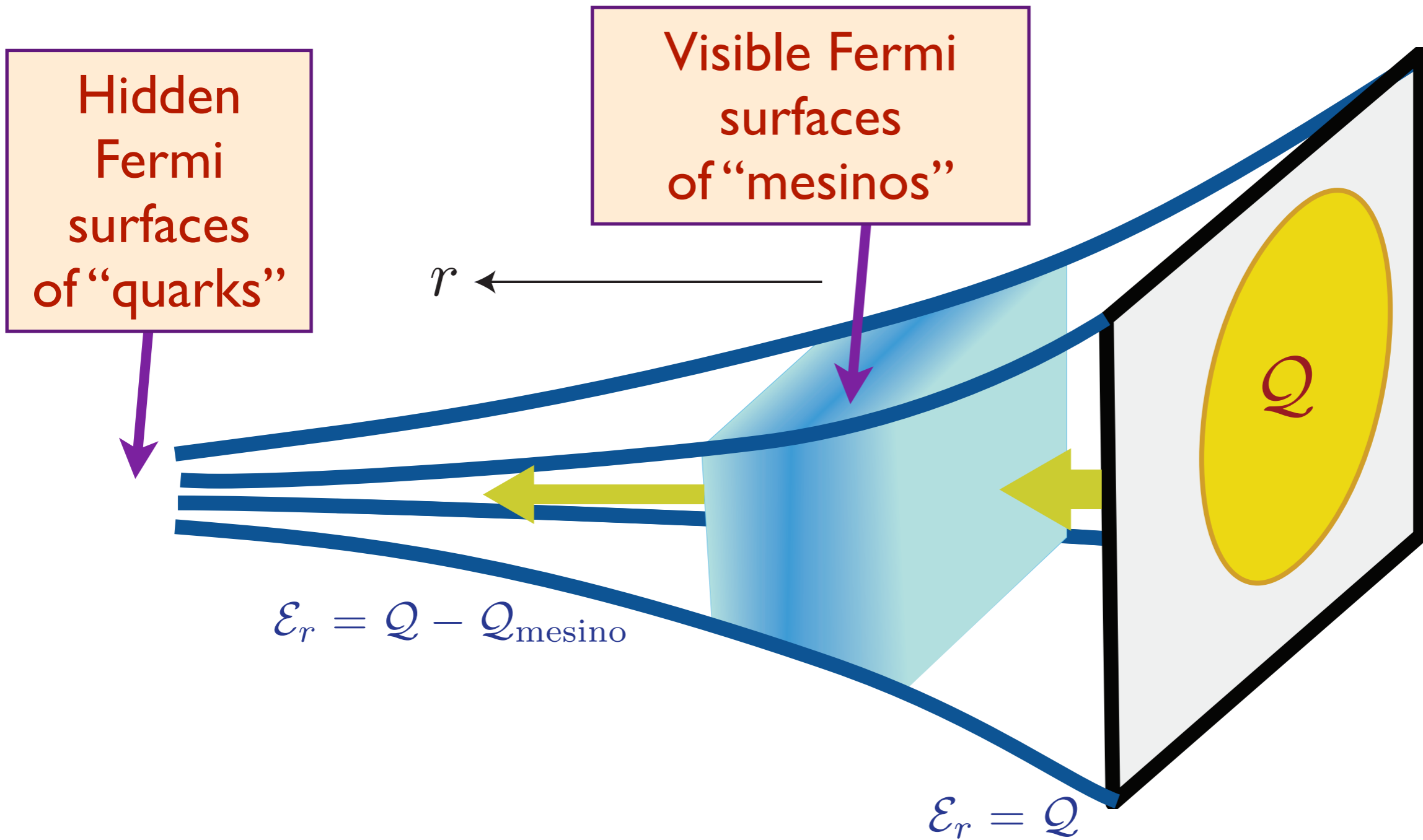


A state with *partial* confinement

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)

S. Sachdev, *Physical Review D* **84**, 066009 (2011)

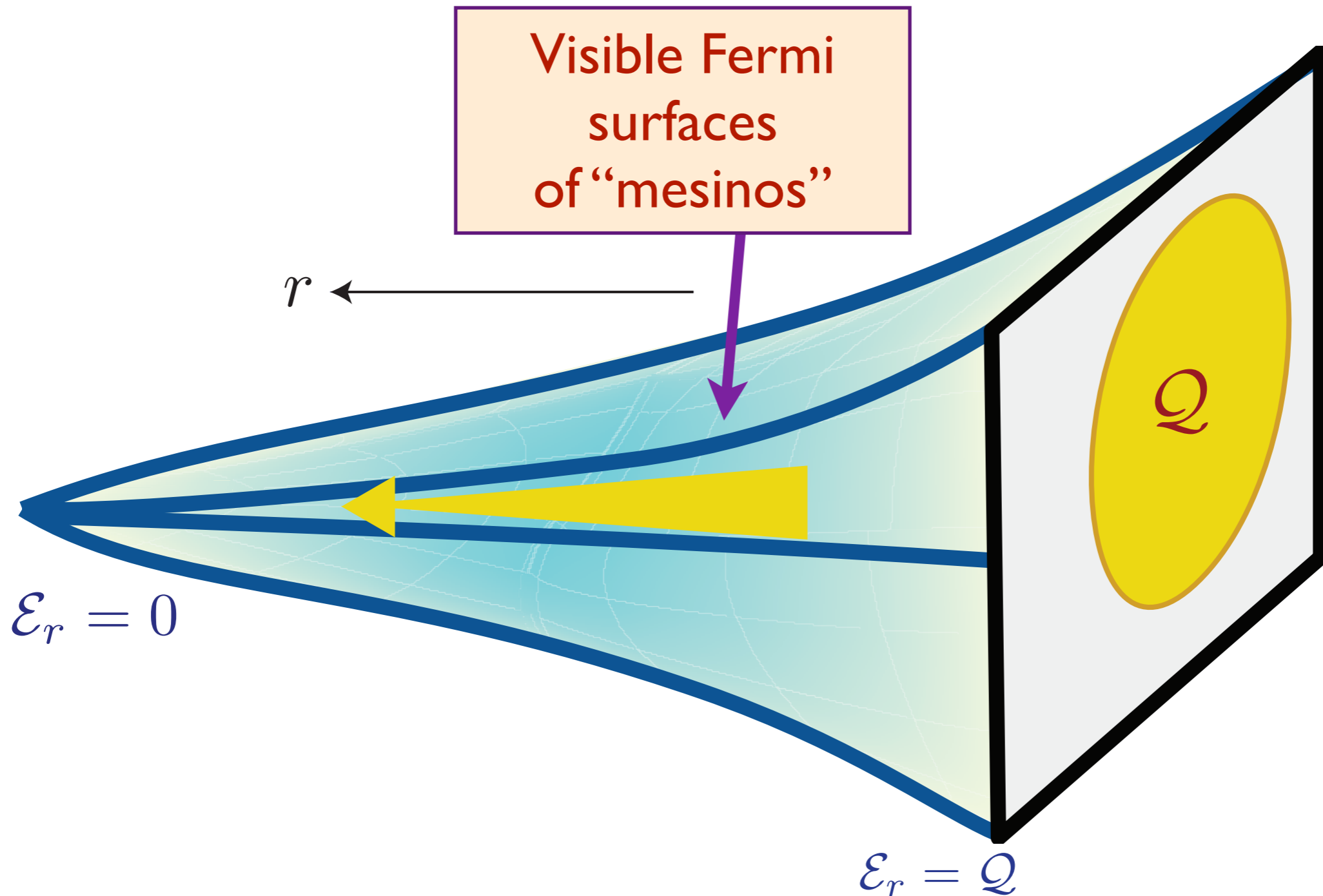
Holographic theory of a fractionalized-Fermi liquid (FL*)



The “mesinos” corresponds to the Fermi surfaces obtained in the early probe fermion computation (S.-S. Lee, Phys. Rev. D **79**, 086006 (2009); H. Liu, J. McGreevy, and D. Vegh, arXiv:0903.2477; M. Čubrović, J. Zaanen, and K. Schalm, Science **325**, 439 (2009)).

These are spectators, and are expected to have well-defined quasiparticle excitations.

Holographic theory of a Fermi liquid (FL)




- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)

Conclusions

Compressible quantum matter

 Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.

Conclusions

Compressible quantum matter

● Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.

After fixing $\theta = d - 1$ to obtain thermal entropy density $S \sim T^{1/z}$, we found

- Log violation of the area law in entanglement entropy, S_E .

Conclusions

Compressible quantum matter

● Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.

After fixing $\theta = d - 1$ to obtain thermal entropy density $S \sim T^{1/z}$, we found

- Log violation of the area law in entanglement entropy, S_E .
- Leading-log S_E independent of shape of entangling region.

Conclusions

Compressible quantum matter

● Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.

After fixing $\theta = d - 1$ to obtain thermal entropy density $S \sim T^{1/z}$, we found

- Log violation of the area law in entanglement entropy, S_E .
- Leading-log S_E independent of shape of entangling region.
- The $d = 2$ bound $z \geq 3/2$, compared to $z = 3/2$ in three-loop field theory.

Conclusions

Compressible quantum matter

● Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.

After fixing $\theta = d - 1$ to obtain thermal entropy density $S \sim T^{1/z}$, we found

- Log violation of the area law in entanglement entropy, S_E .
- Leading-log S_E independent of shape of entangling region.
- The $d = 2$ bound $z \geq 3/2$, compared to $z = 3/2$ in three-loop field theory.
- Evidence for Luttinger theorem in prefactor of S_E .

Conclusions

Compressible quantum matter

- Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.
- Fermi liquid (FL) state described by a confining holographic geometry

Conclusions

Compressible quantum matter

- Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.
- Fermi liquid (FL) state described by a confining holographic geometry
- Hidden Fermi surfaces can co-exist with Fermi surfaces of mesinos, leading to a state with partial confinement: the fractionalized Fermi liquid (FL*)