From the pseudogap to the strange metal

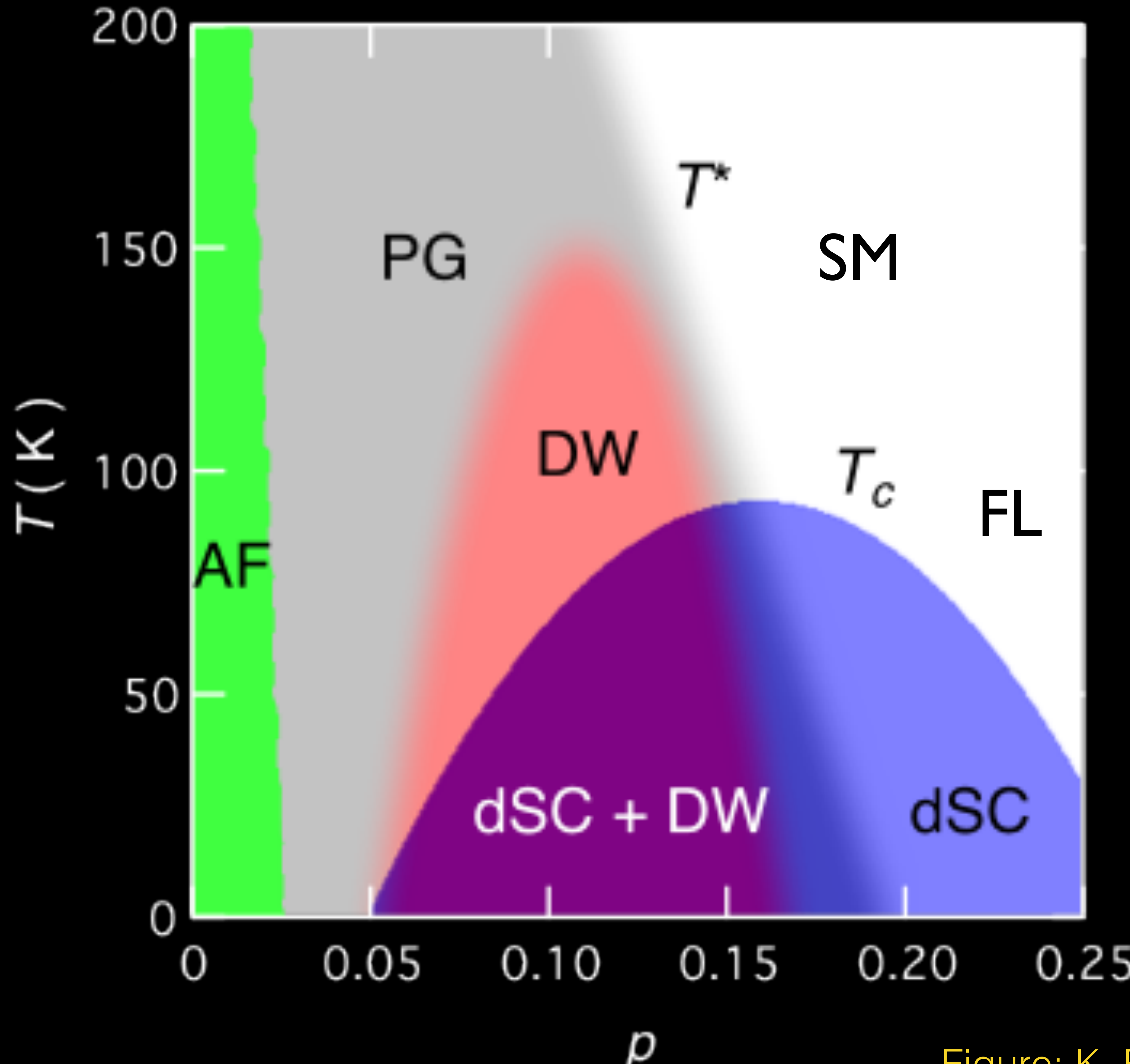
Common Threads in the Electronic Phase Diagram
of Unconventional Superconductors,
Lorentz Institute, Leiden

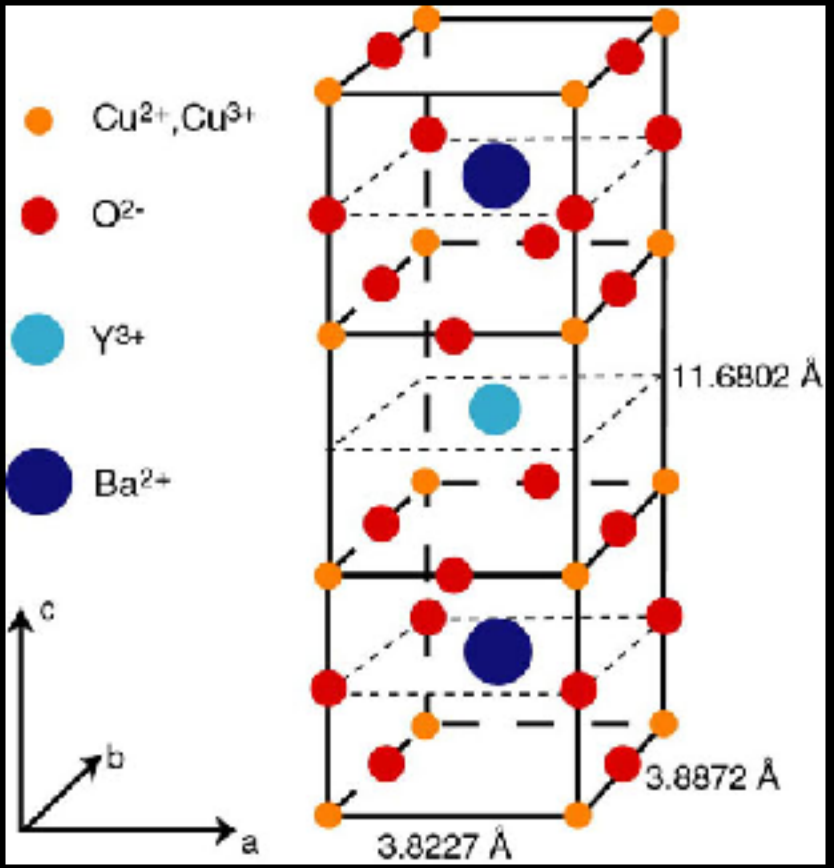
Subir Sachdev
March 2, 2017

Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

PHYSICS
VE RI TAS
HARVARD

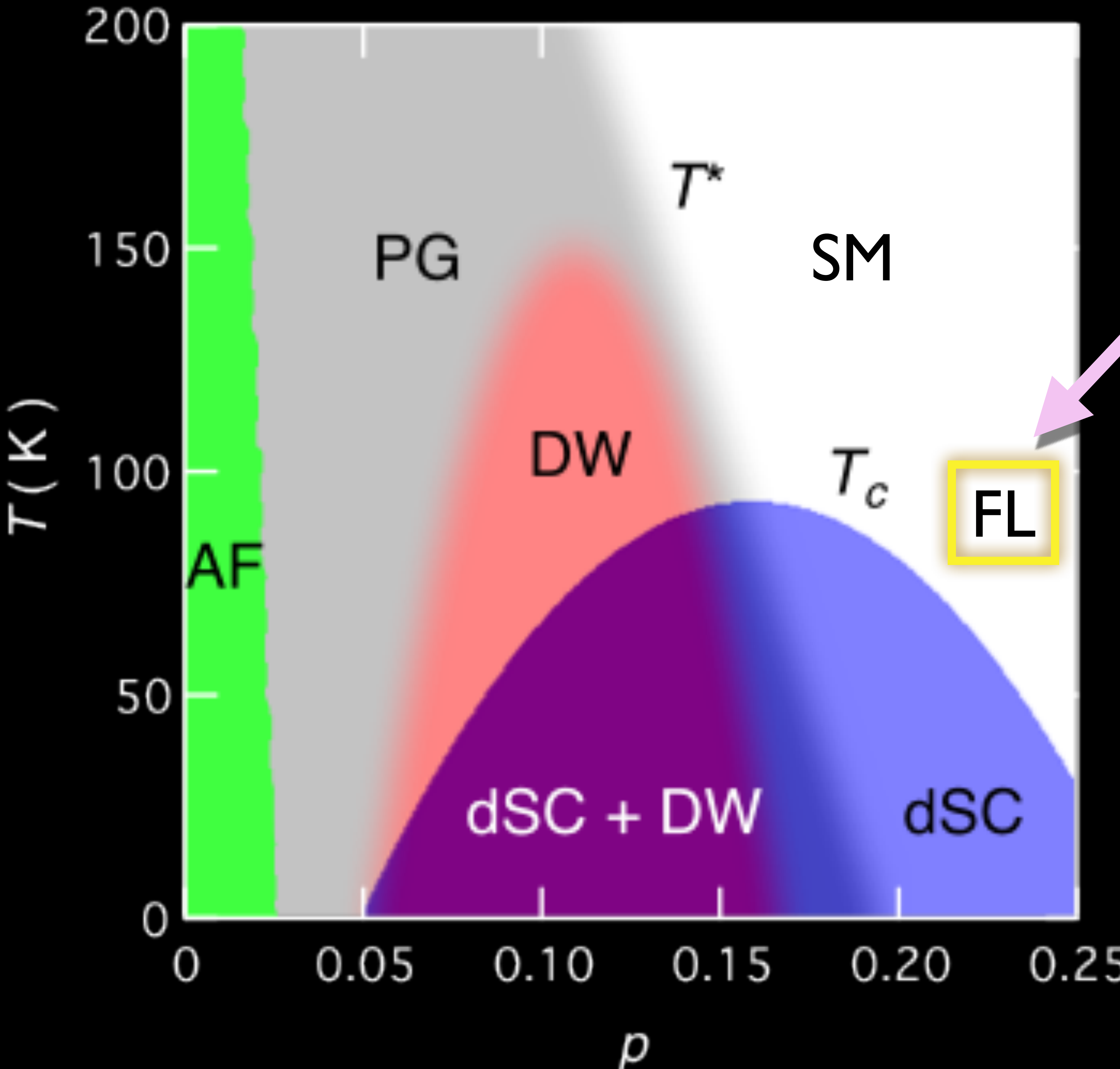


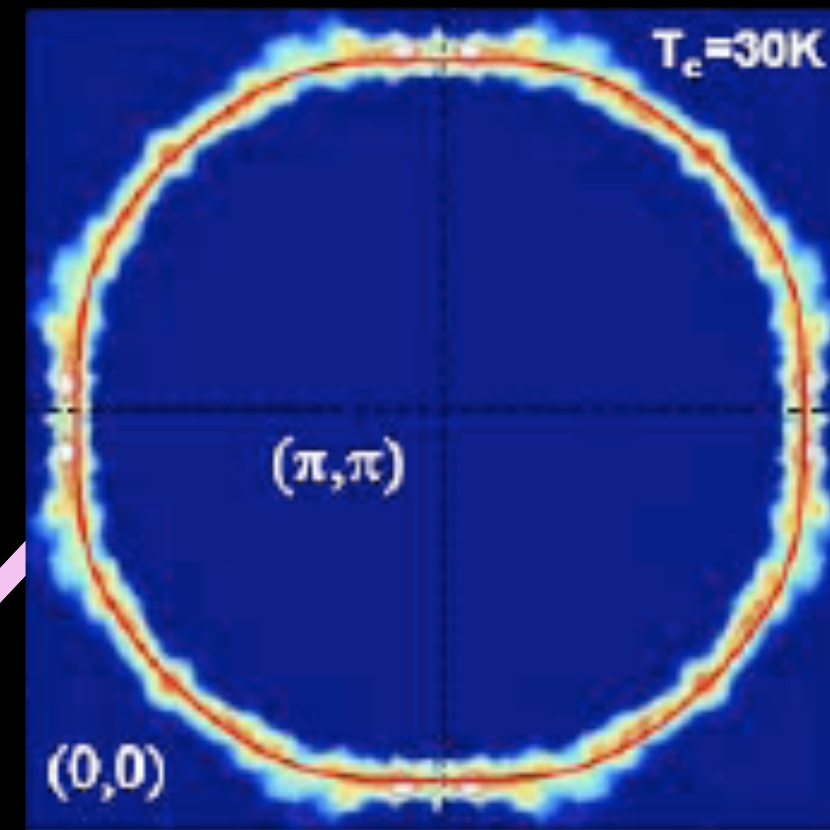


$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

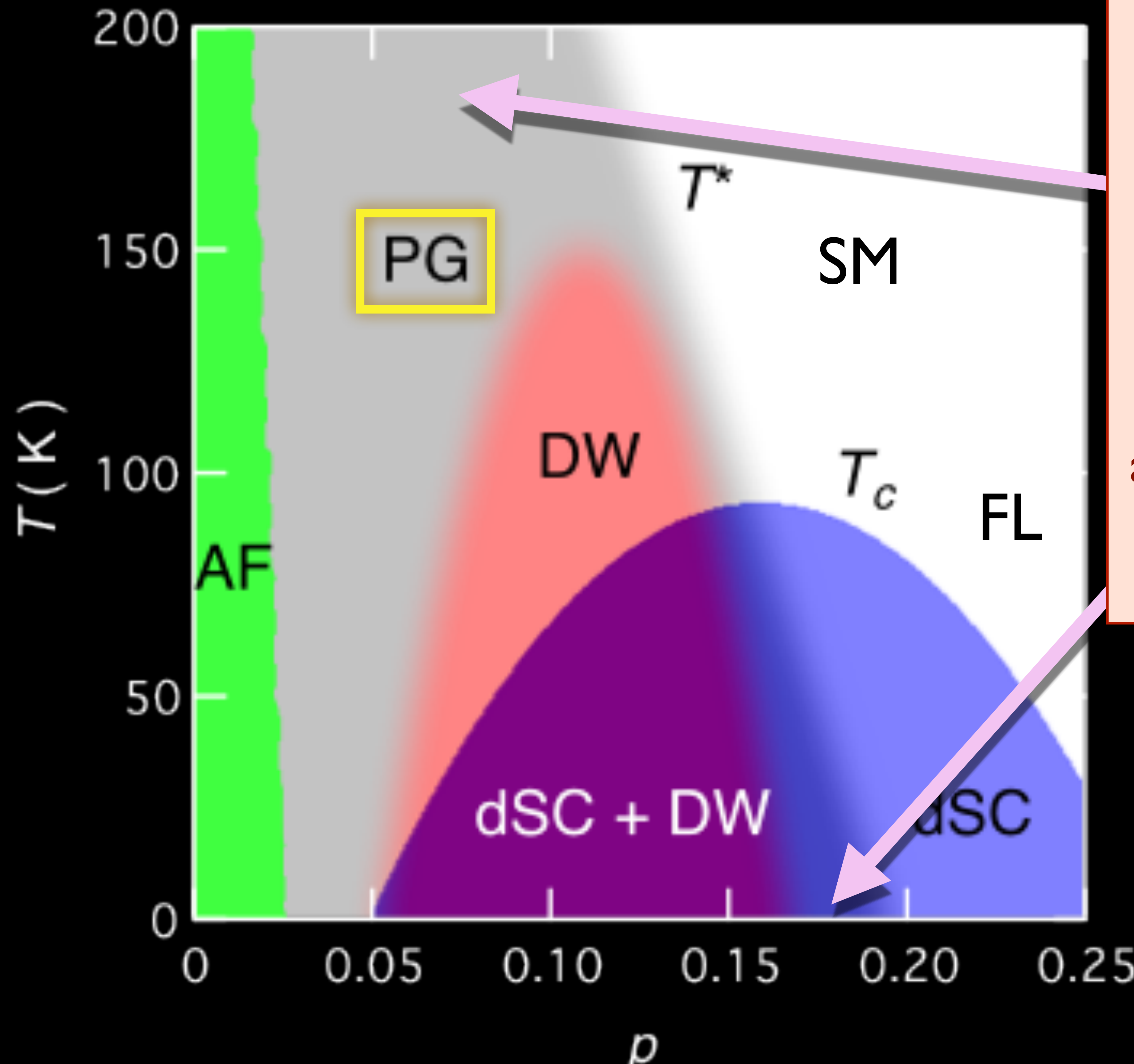
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





A conventional metal: the Fermi liquid with Fermi surface of size *1+p*

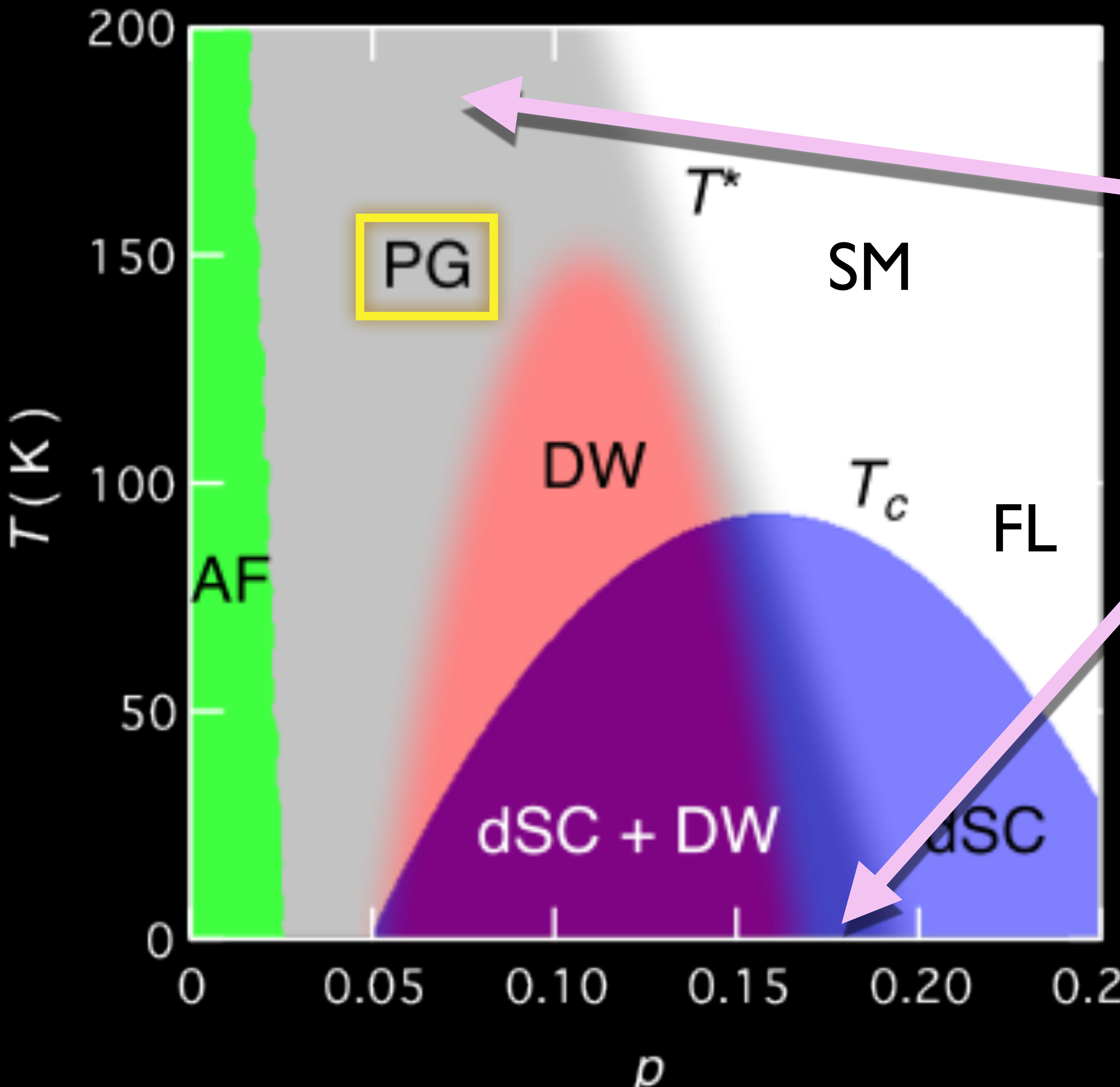
S. Badoux, W. Tabis, F. Laliberté, G. Grissonnanche, B. Vignolle, D. Vignolles, J. Béard, D.A. Bonn, W.N. Hardy, R. Liang, N. Doiron-Leyraud, L. Taillefer, and C. Proust, Nature **531**, 210 (2016).



Pseudogap metal at low p

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

T. Senthil, M. Vojta and S. Sachdev, PRB **69**, 035111 (2004)

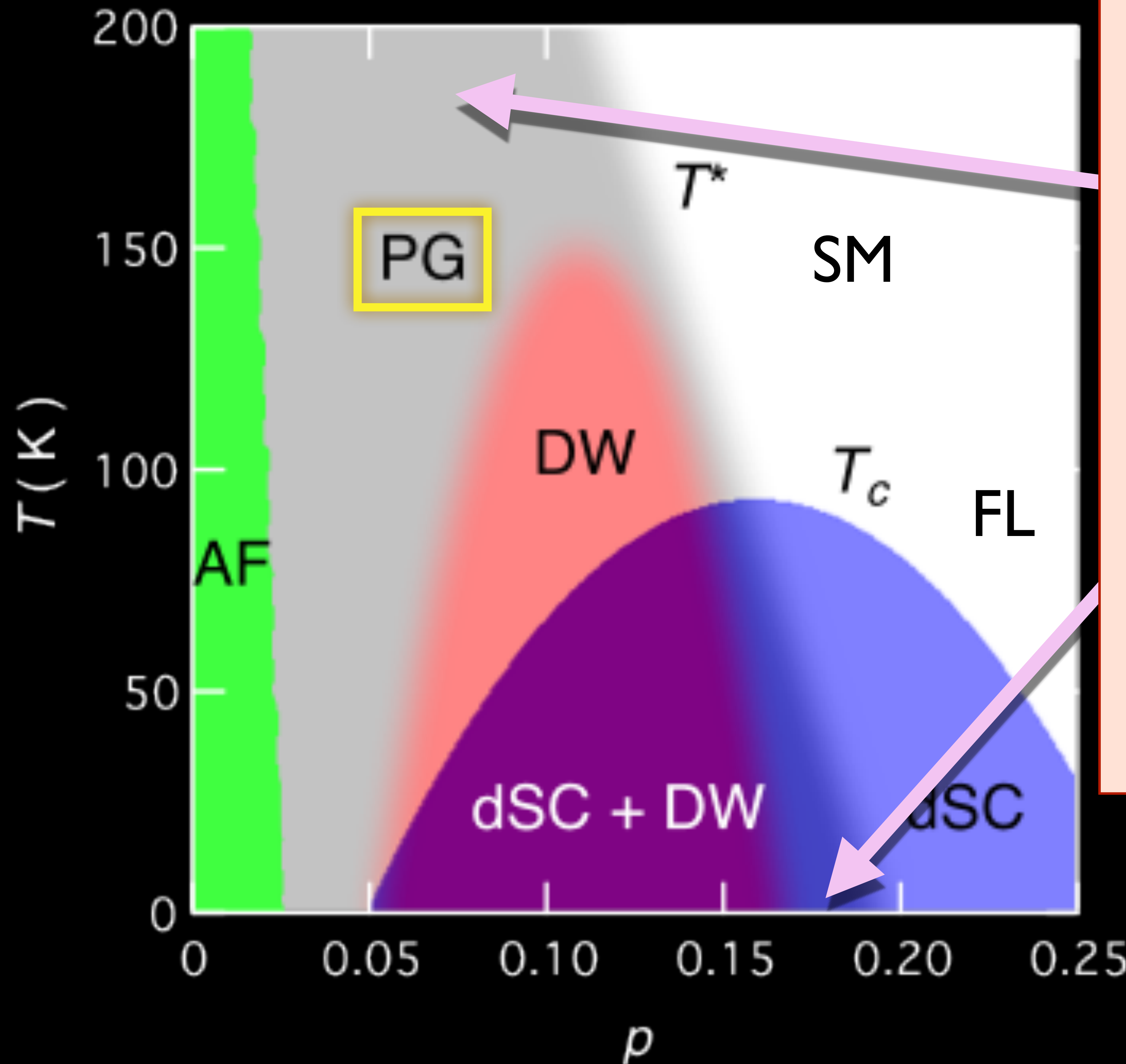


Pseudogap metal at low p

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

If present at $T=0$, a metal with a size p Fermi surface (and translational symmetry preserved) *must* have *topological order*

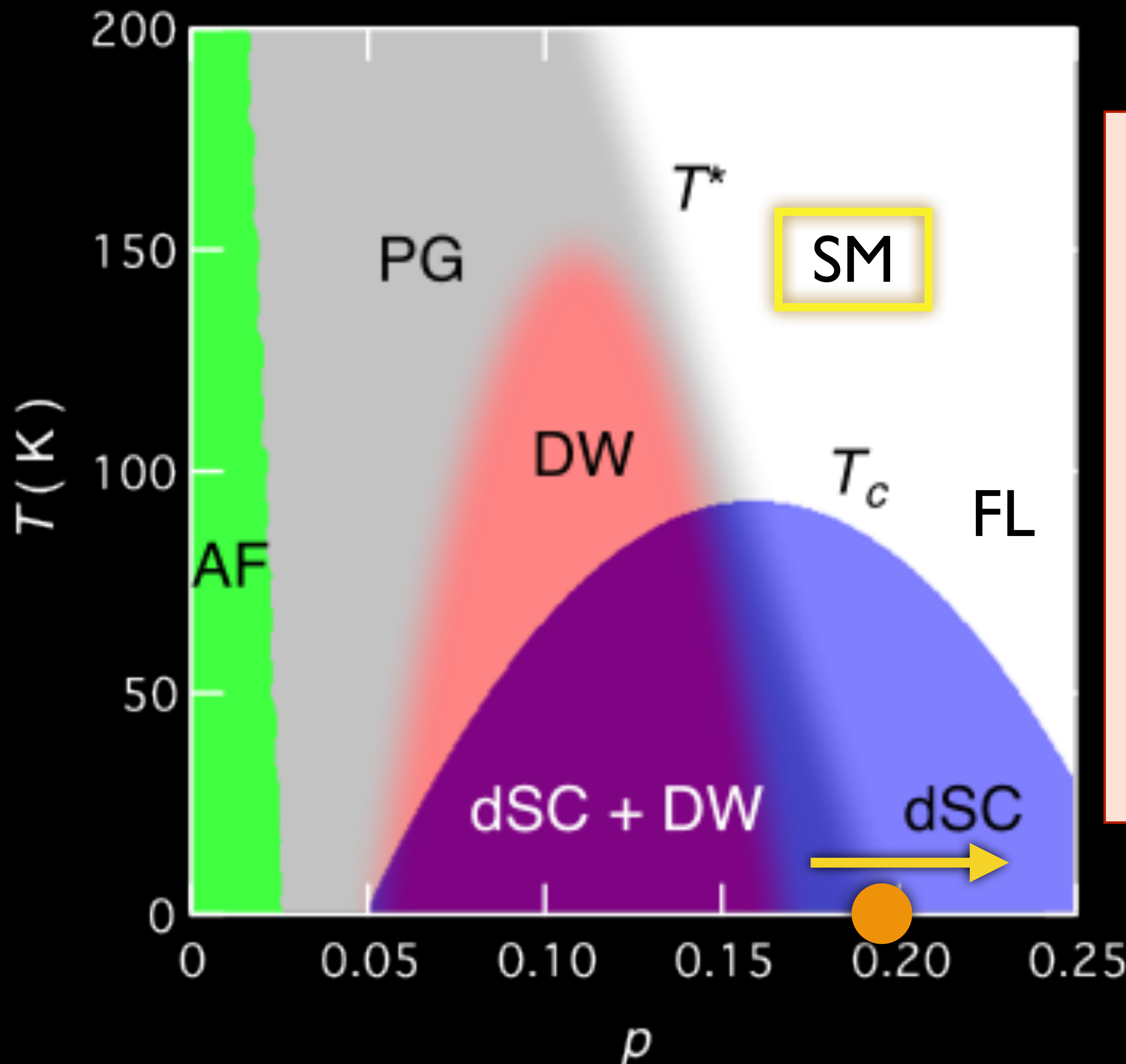
S. Sachdev and S. Chatterjee, arXiv:1703.00014



Pseudogap metal at low p

Lattice gauge theory for a metal with topological order co-existing with broken time-reversal and inversion symmetries, and Ising-nematic order

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009); D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579.



Gauge theory for a topological phase transition, and for the strange metal (SM)

Insulators and metals with
topological order
and breaking of
time-reversal/inversion/lattice-rotation
symmetry

Begin with the "spin-fermion" model. **Electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\rm int}$$

are coupled to an **antiferromagnetic order parameter** $\Phi^\ell(i)$**,** $\ell = x, y, z$

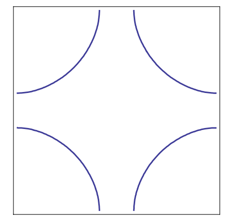
$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i \Phi^\ell(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^\ell c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices.

When $\Phi^\ell(i)$ =constant independent of i, we have long-range AFM, and a gap in the fermion spectrum at the anti-nodes.

Begin with the "spin-fermion" model. **Electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c^\dagger_{i,\alpha} c_{i+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{i+\boldsymbol{v}_\rho,\alpha} c_{i,\alpha} \right) - \mu \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} + \mathcal{H}_{\rm int}$$



are coupled to an **antiferromagnetic order parameter** $\Phi^\ell(i)$**,** $\ell = x, y, z$

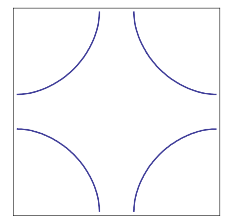
$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i \Phi^\ell(i) c^\dagger_{i,\alpha} \sigma^\ell_{\alpha\beta} c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices.

When $\Phi^\ell(i)$ =constant independent of i, we have long-range AFM, and a gap in the fermion spectrum at the anti-nodes.

Begin with the "spin-fermion" model. **Electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c^\dagger_{i,\alpha} c_{i+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{i+\boldsymbol{v}_\rho,\alpha} c_{i,\alpha} \right) - \mu \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} + \mathcal{H}_{\text{int}}$$

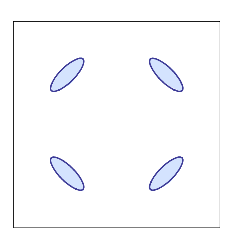


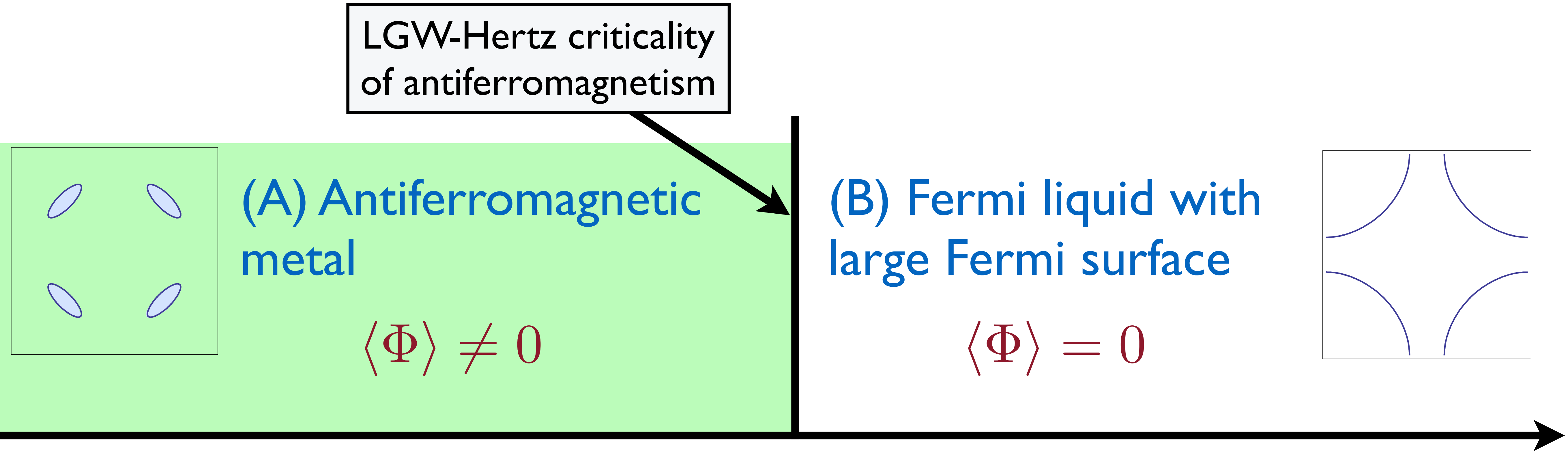
are coupled to an **antiferromagnetic order parameter** $\Phi^\ell(i)$**,** $\ell = x, y, z$

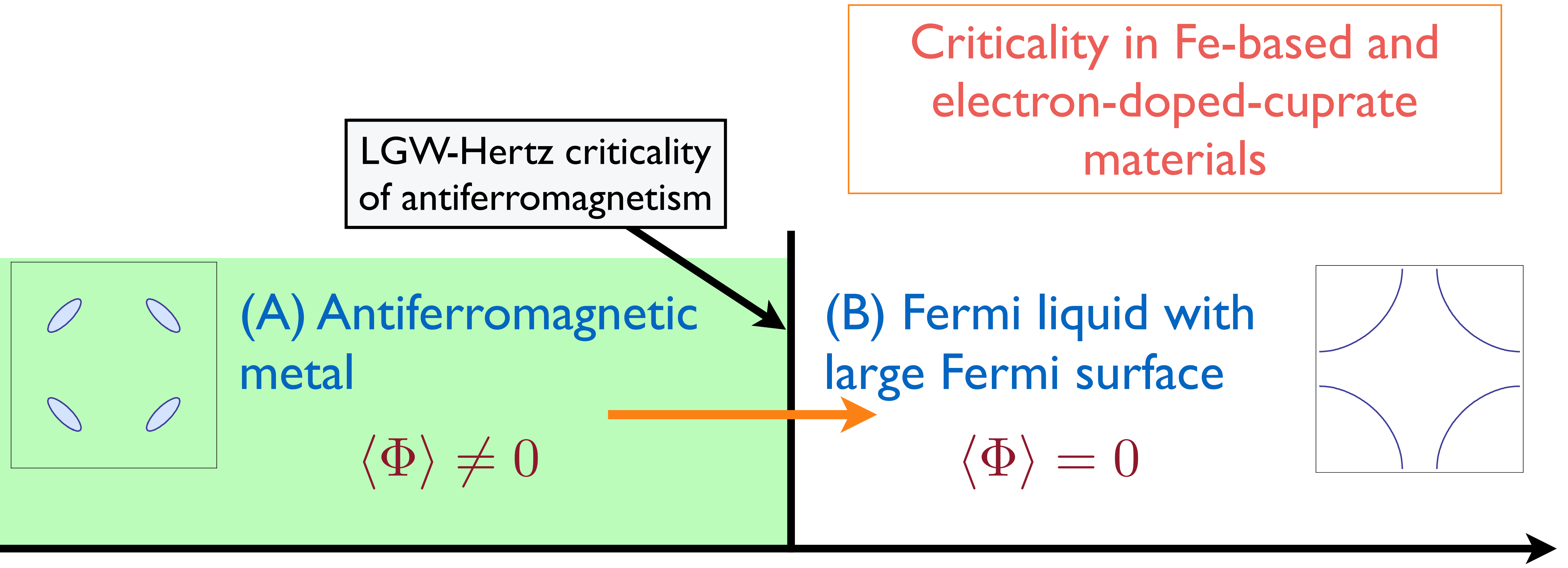
$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i \Phi^\ell(i) c^\dagger_{i,\alpha} \sigma^\ell_{\alpha\beta} c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices.

When $\Phi^\ell(i)$ =constant independent of i, we have long-range AFM, and a gap in the fermion spectrum at the anti-nodes.




LGW-Hertz criticality
of antiferromagnetism
(A) Antiferromagnetic
metal
$\langle \Phi \rangle \neq 0$
(B) Fermi liquid with
large Fermi surface
$\langle \Phi \rangle = 0$


Criticality in Fe-based and electron-doped-cuprate materials
LGW-Hertz criticality of antiferromagnetism
(A) Antiferromagnetic metal
$\langle \Phi \rangle \neq 0$
(B) Fermi liquid with large Fermi surface
$\langle \Phi \rangle = 0$

Can we get a stable zero temperature state with “fluctuating antiferromagnetism” and a small Fermi surface (and so a gap near the anti-nodes) ?

Can we get a stable zero temperature state with “fluctuating antiferromagnetism” and a small Fermi surface (and so a gap near the anti-nodes) ?

Yes

For fluctuating antiferromagnetism, we transform to a rotating reference frame using the SU(2) rotation R_i

$$\begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} = R_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix},$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the AFM order in the rotating reference frame.

For fluctuating antiferromagnetism, we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the AFM order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right) \rightarrow V_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right)$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger.$$

Fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the AFM order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma^a_{ss'} \psi_{i,s'} + V_H$$

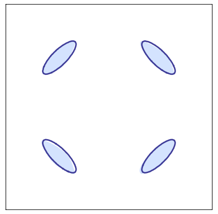
Fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the AFM order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\rm int}$$

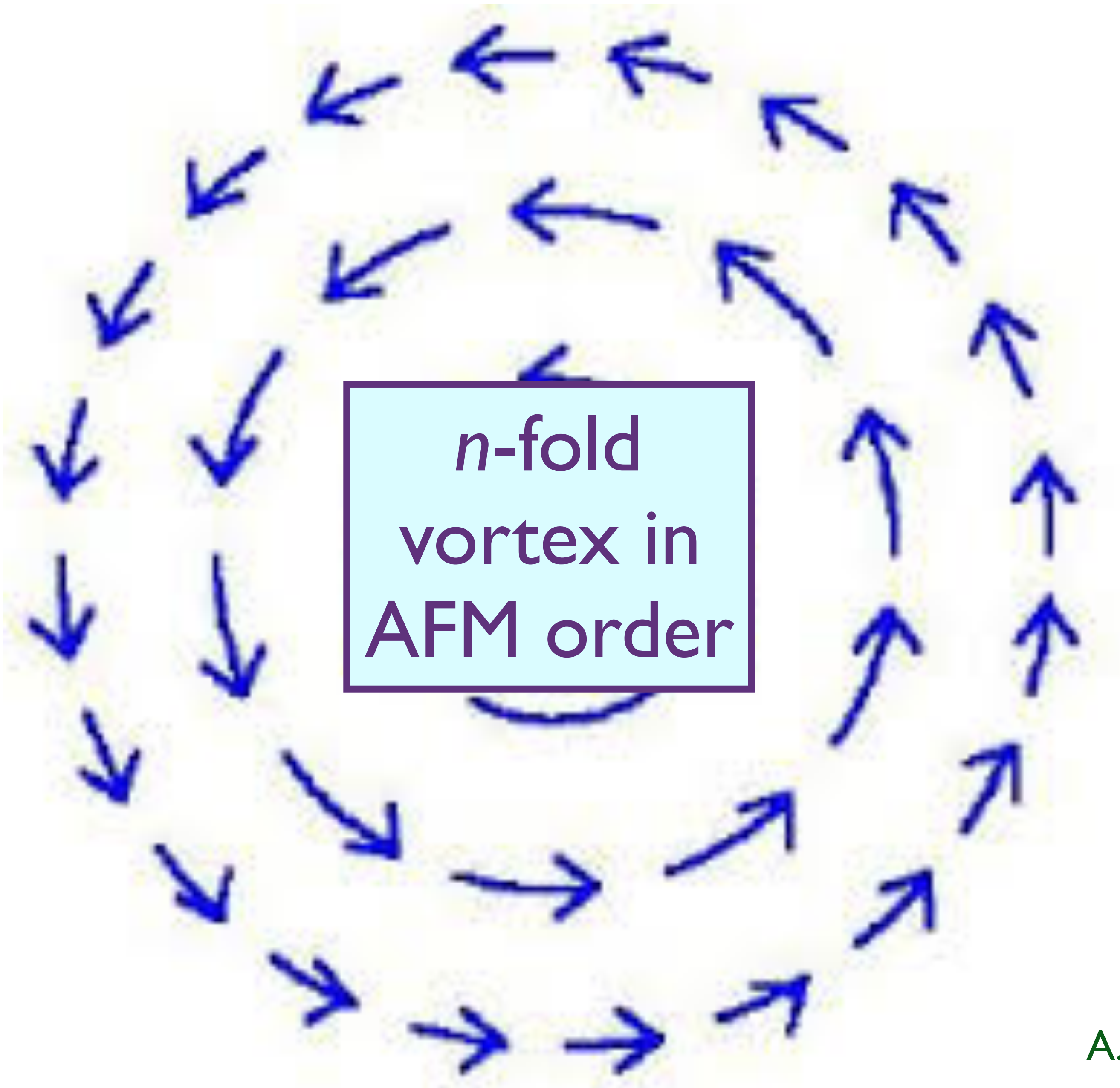
$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V_H$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of fluctuating AFM will inherit the anti-nodal gap of the electrons in the presence of static AFM.



Fluctuating antiferromagnetism

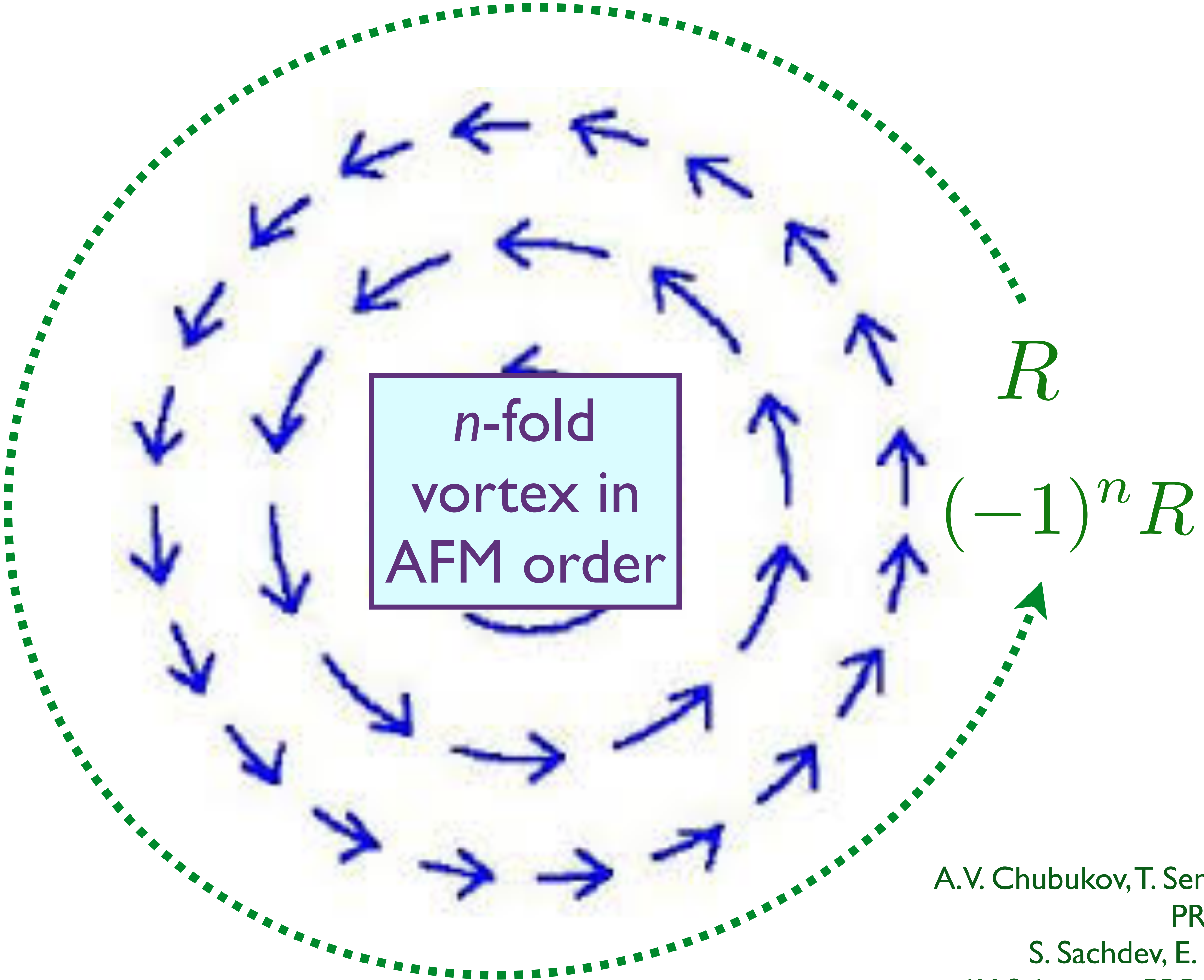
We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !



A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994);
S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

Fluctuating antiferromagnetism

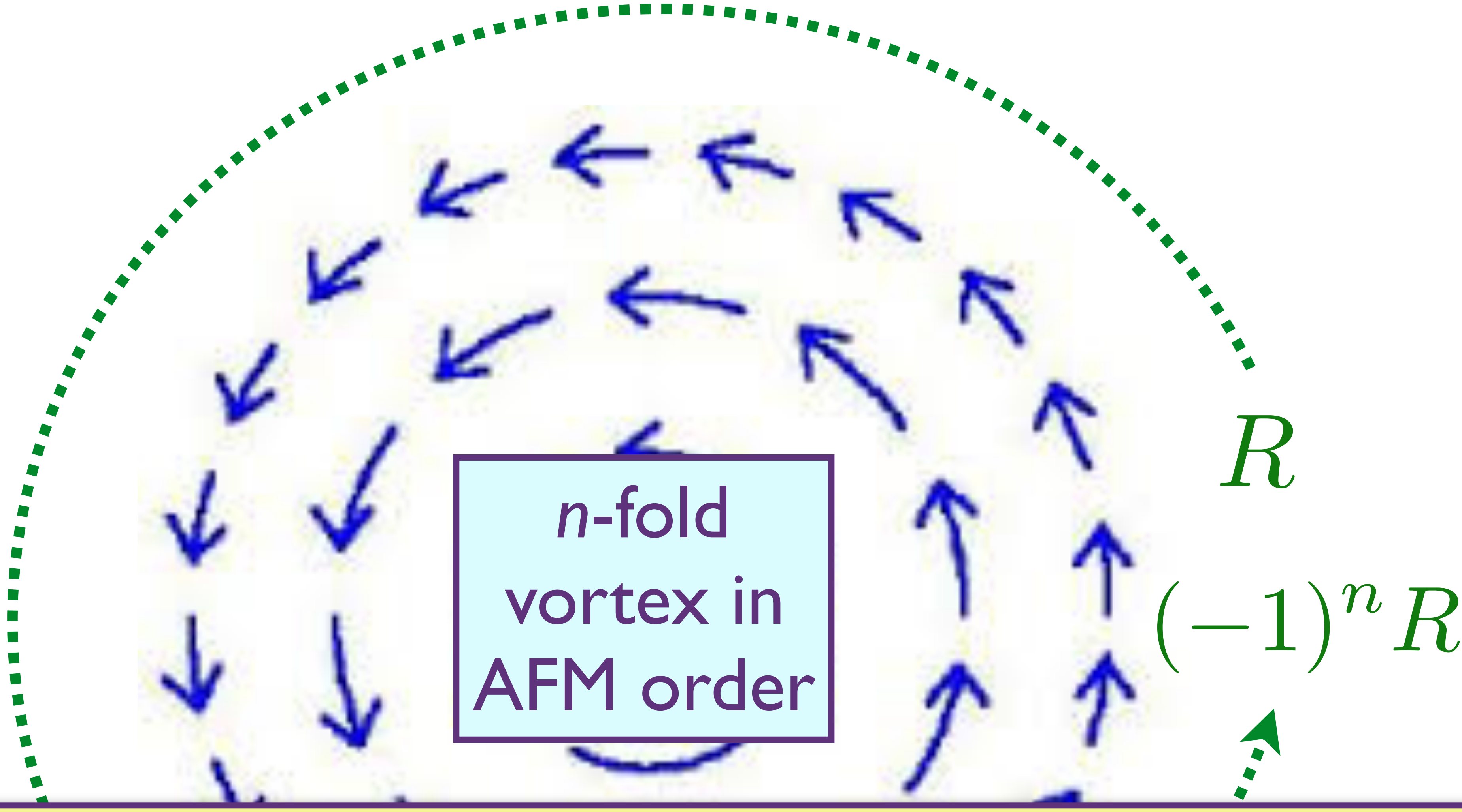
We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !



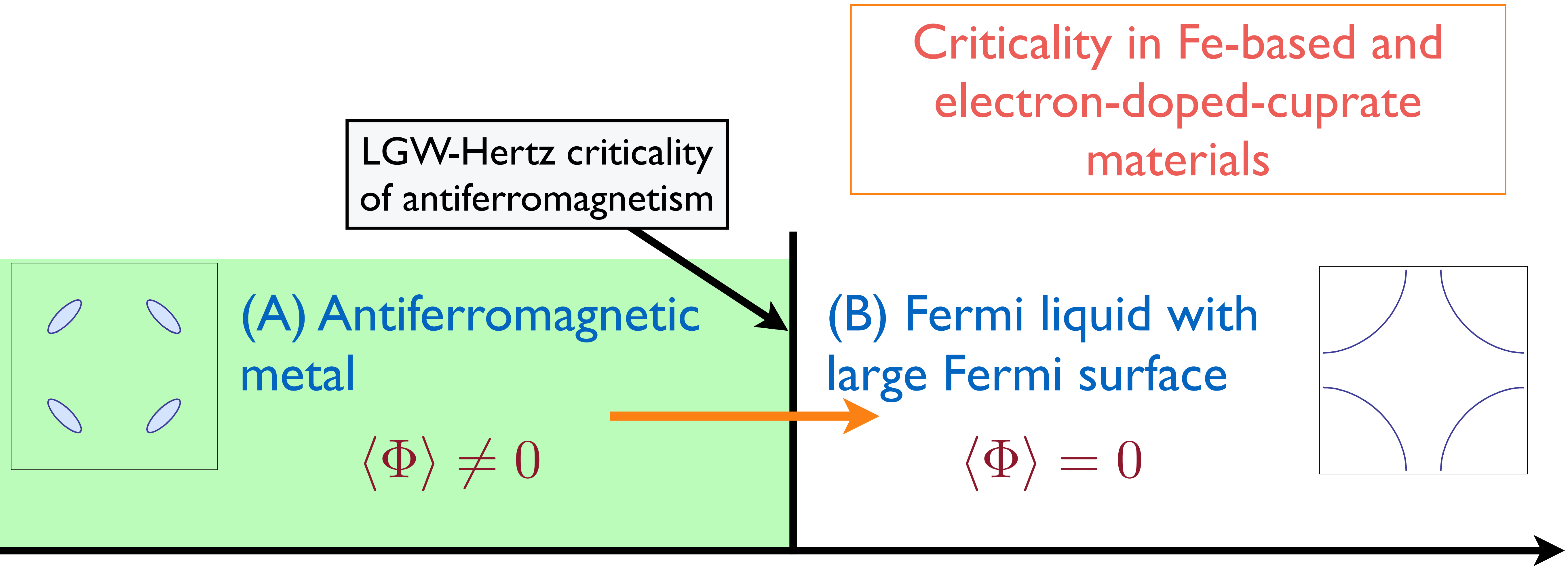
A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994);
S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

Topological order

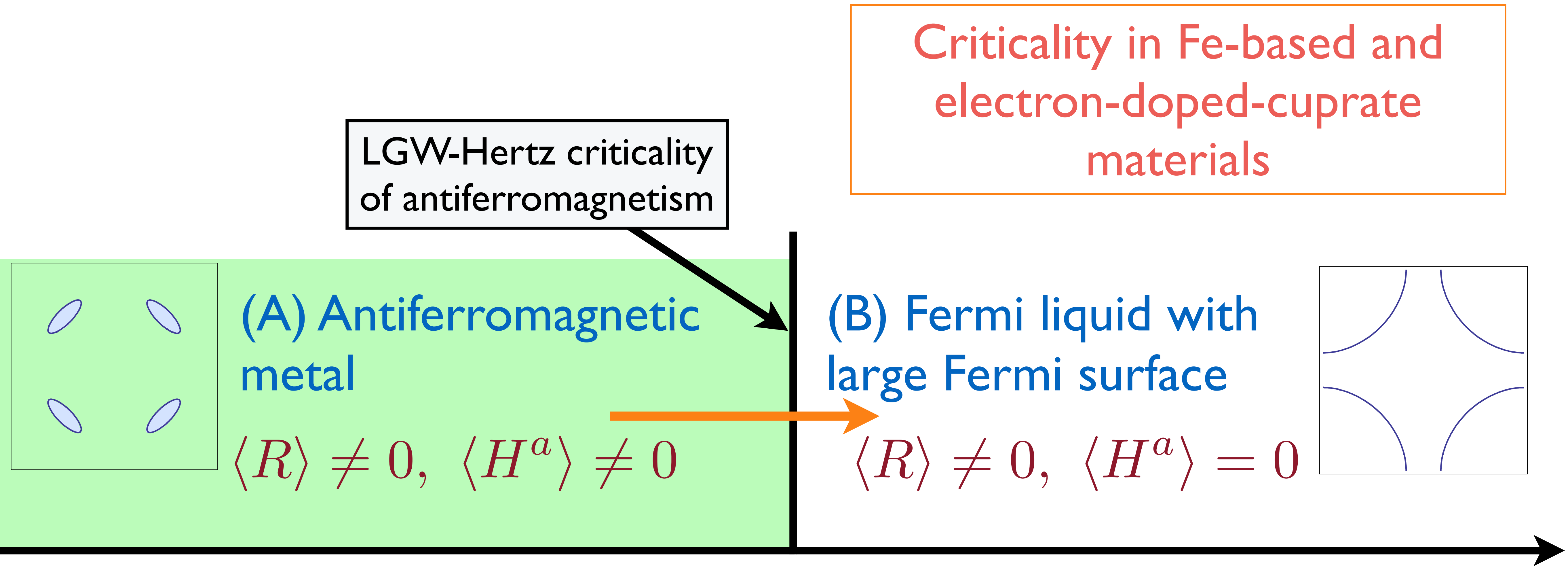
We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !



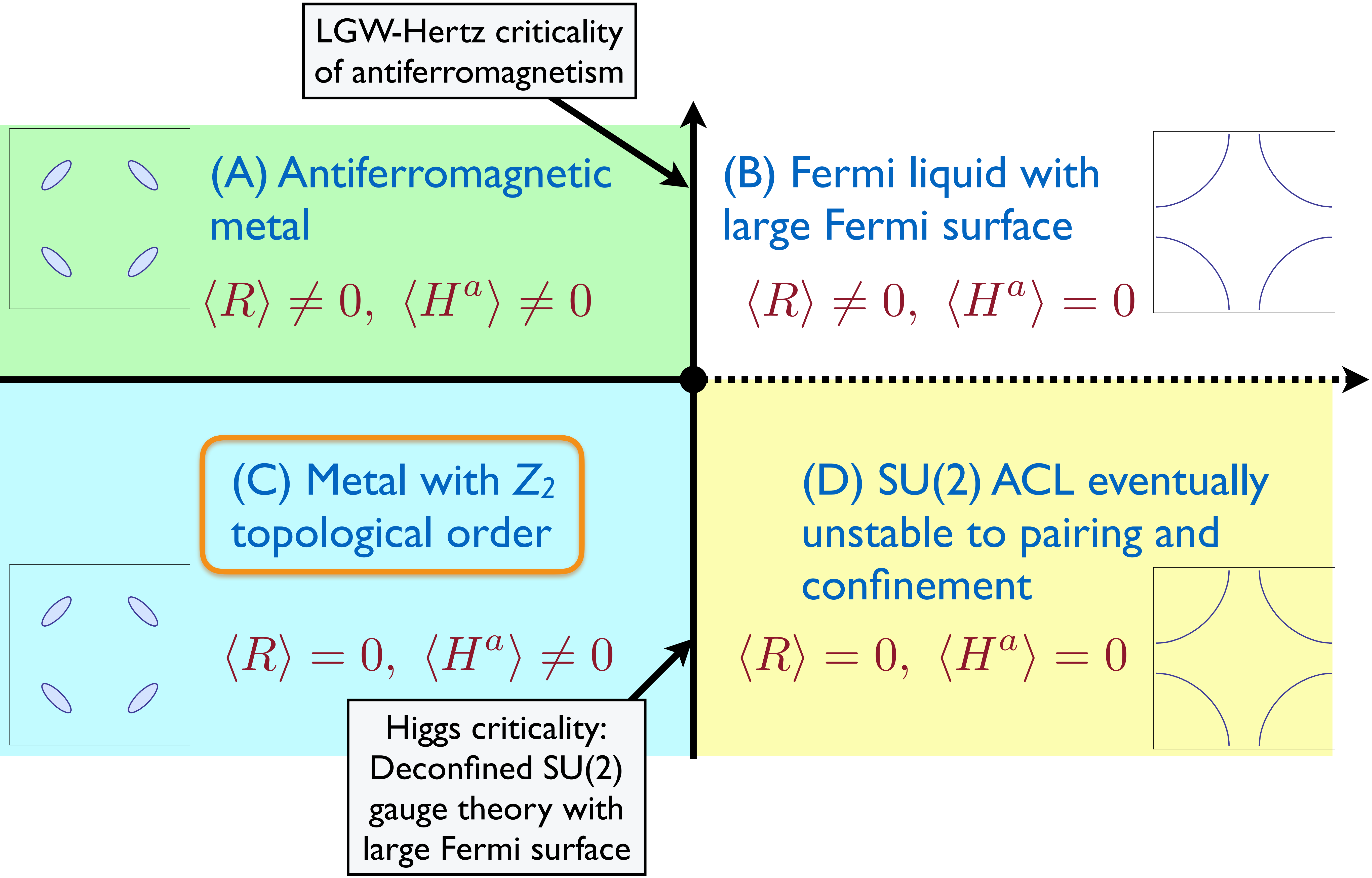
Vortices with n odd must be suppressed: such a metal with "fluctuating antiferromagnetism" has $\mathbb{Z}_2$ TOPOLOGICAL ORDER and fermions which inherit the "pocket" Fermi surfaces of the antiferromagnetic metal *i.e.* a pseudogap.



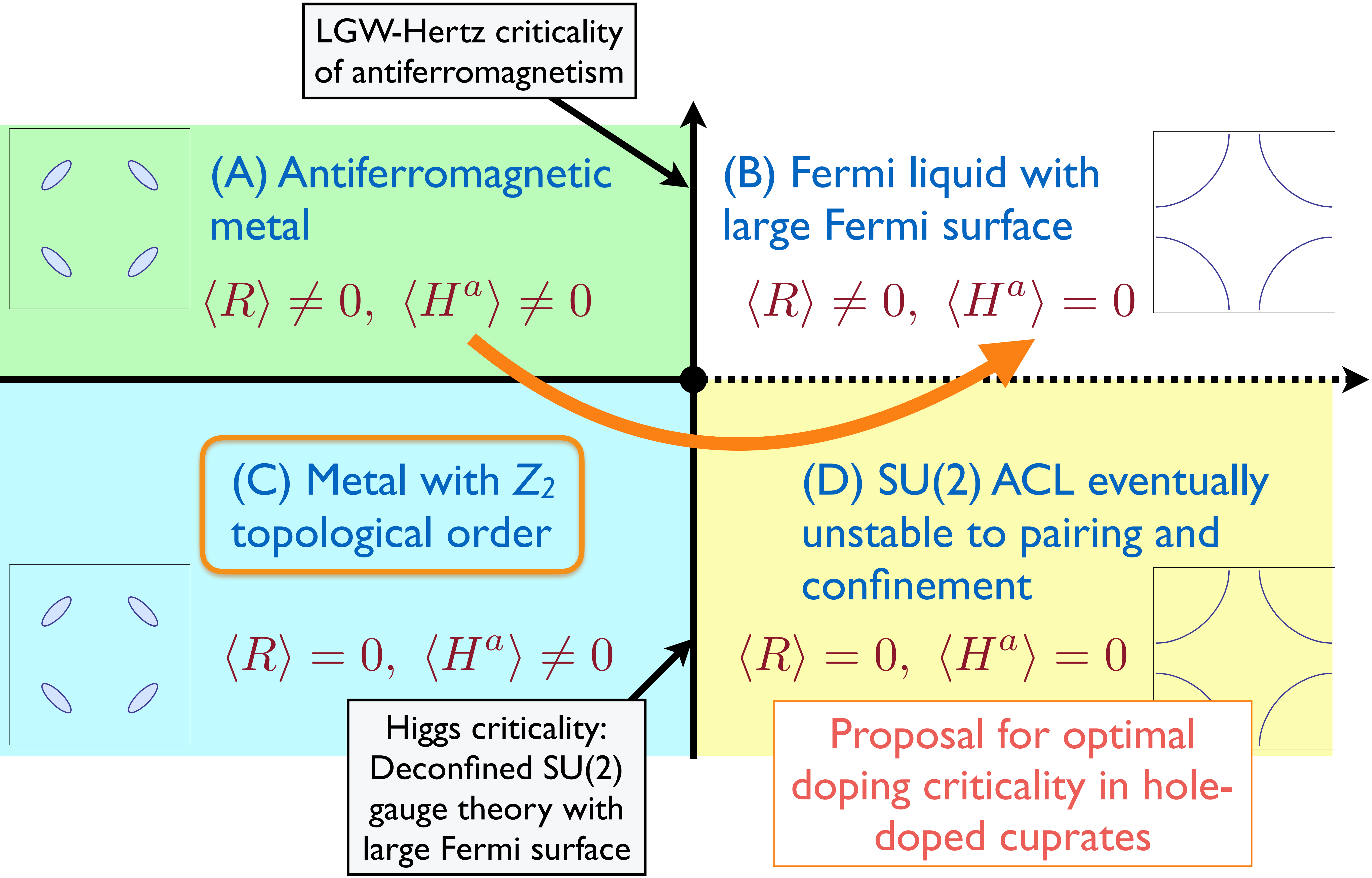
Criticality in Fe-based and electron-doped-cuprate materials
LGW-Hertz criticality of antiferromagnetism
(A) Antiferromagnetic metal
$\langle \Phi \rangle \neq 0$
(B) Fermi liquid with large Fermi surface
$\langle \Phi \rangle = 0$


Criticality in Fe-based and electron-doped-cuprate materials
LGW-Hertz criticality of antiferromagnetism
(A) Antiferromagnetic metal
$\langle R \rangle \neq 0, \ \langle H^a \rangle \neq 0$
(B) Fermi liquid with large Fermi surface
$\langle R \rangle \neq 0, \ \langle H^a \rangle = 0$

Global phase diagram



Global phase diagram



Topological order

More generally, the effective Hamiltonian for the fermionic chargons can also have non-trivial SU(2) gauge connections $U^\rho(i)$ along with the Higgs field $H^a(i)$.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho, s'} + \text{H.c.} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

Topological order

S. Sachdev and S. Chatterjee, arXiv:1703.00014

More generally, the effective Hamiltonian for the fermionic chargons can also have non-trivial SU(2) gauge connections $U^\rho(i)$ along with the Higgs field $H^a(i)$.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \, U^\rho_{ss'}(i) \, \psi_{i+\boldsymbol{v}_\rho, s'} + \text{H.c.} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

Topological order

S. Sachdev and S. Chatterjee, arXiv:1703.00014

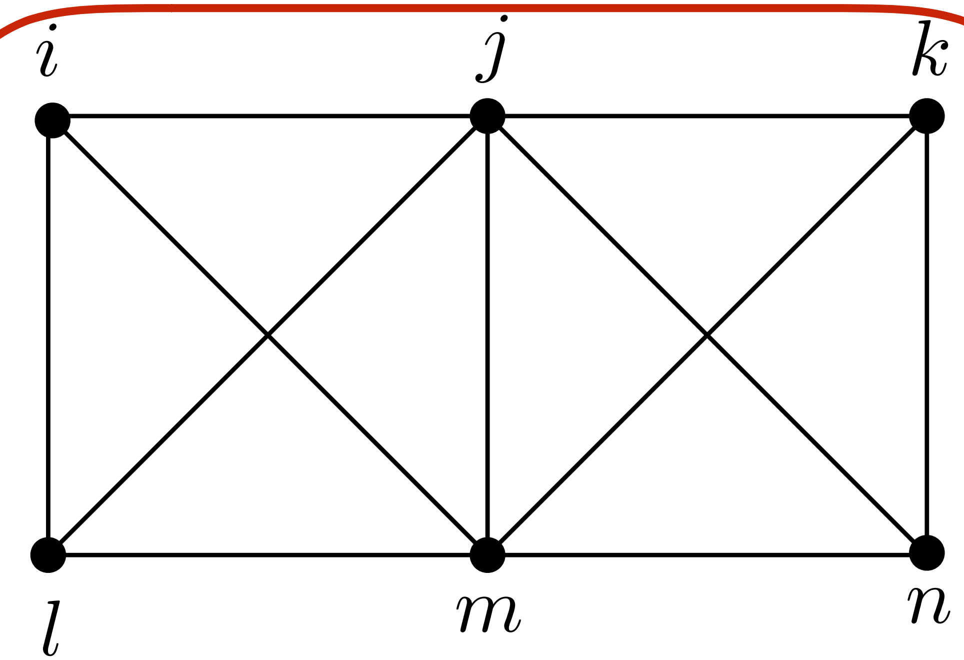
More generally, the effective Hamiltonian for the fermionic chargons can also have non-trivial SU(2) gauge connections $U^\rho(i)$ along with the Higgs field $H^a(i)$.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \, U^\rho_{ss'}(i) \, \psi_{i+\boldsymbol{v}_\rho, s'} + \text{H.c.} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

Such a gauge-connection can induce various gauge-invariant fluxes which can break one or more of time-reversal, inversion, and lattice rotation symmetries.

Topological order



Gauge-invariant combinations of Higgs fields and gauge connections which are proportional to the electrical current on links

$$
\begin{aligned}
O_{mj} = & \; i\,\mathrm{Tr}\left(\sigma^a U_{mj} U_{jk} U_{km}\right) H^a(m) \\
& - i\,\mathrm{Tr}\left(\sigma^a U_{jm} U_{mn} U_{nj}\right) H^a(j) \\
& + i\,\mathrm{Tr}\left(\sigma^a U_{mj} U_{ji} U_{im}\right) H^a(m) \\
& - i\,\mathrm{Tr}\left(\sigma^a U_{jm} U_{ml} U_{lj}\right) H^a(j)
\end{aligned}
$$

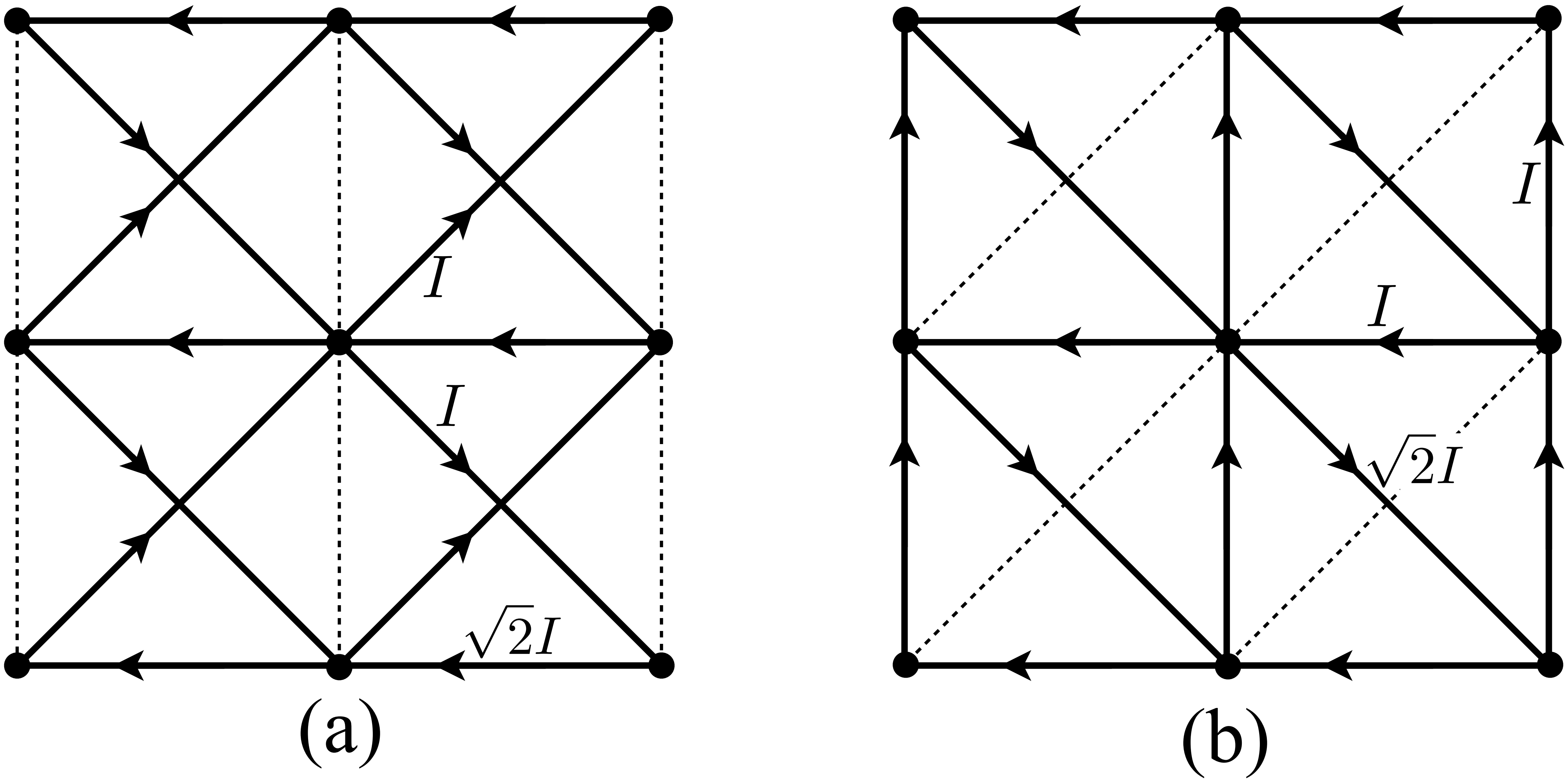
$$
\begin{aligned}
O_{mk} = & \; i\,\mathrm{Tr}\left(\sigma^a U_{mj} U_{jk} U_{km}\right) H^a(m) \\
& - i\,\mathrm{Tr}\left(\sigma^a U_{kj} U_{jm} U_{mk}\right) H^a(k) \\
& + i\,\mathrm{Tr}\left(\sigma^a U_{mn} U_{nk} U_{km}\right) H^a(m) \\
& - i\,\mathrm{Tr}\left(\sigma^a U_{kn} U_{nm} U_{mk}\right) H^a(k)
\end{aligned}
$$

S. Sachdev and S. Chatterjee, arXiv:1703.00014

Such a gauge-connection can induce various gauge-invariant fluxes which can break one or more of time-reversal, inversion, and lattice rotation symmetries.

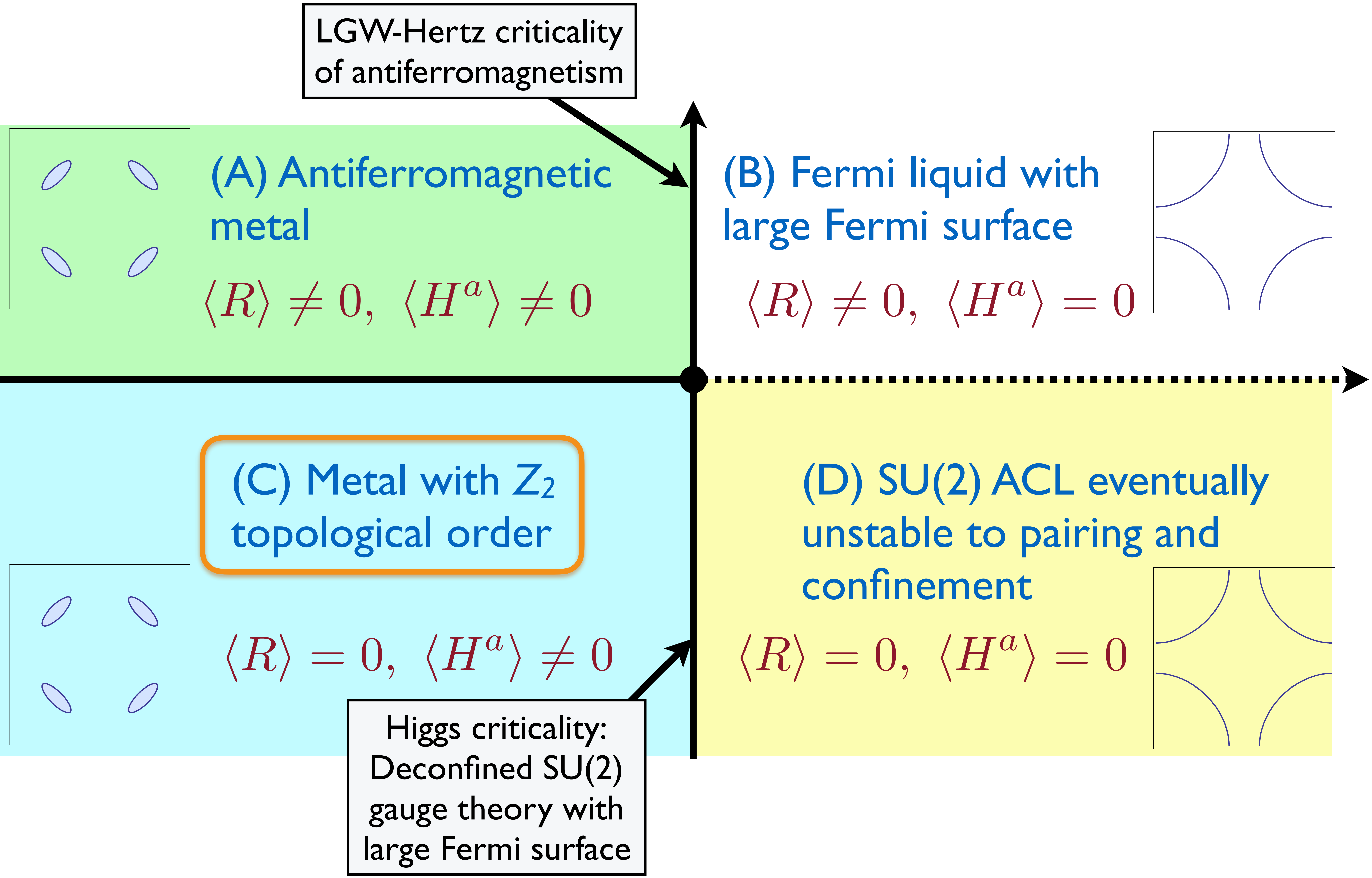
Topological order

S. Sachdev and S. Chatterjee, arXiv:1703.00014

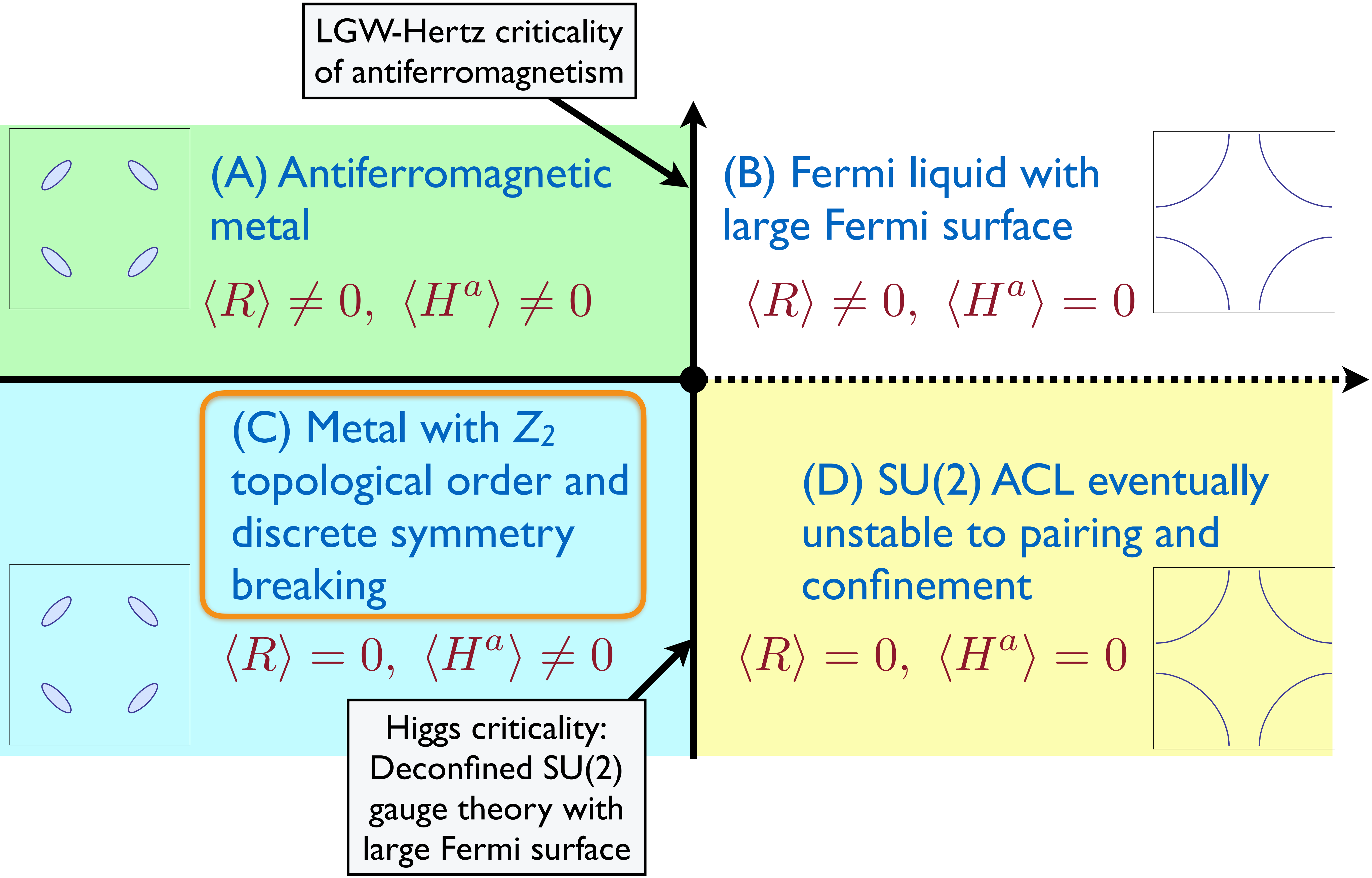


States with topological order can have these patterns of spontaneous currents, while preserving translational symmetry. Both patterns are consistent with present neutron and light scattering experiments. Both patterns have Ising-nematic order: the Ising-nematic order of (a) is similar to that observed in the cuprates.

Global phase diagram



Global phase diagram



Solvable models of disordered metals without quasiparticle excitations

Quantum matter <u>without</u> quasiparticles:

No quasiparticle structure to excitations.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some exotic quasiparticles inaccessible to current experiments……..

Consider how rapidly the system loses "phase coherence", reaches local thermal equilibrium, or becomes "chaotic"

Local thermal equilibration or phase coherence time, τ_φ:

- There is an *lower bound* on τ_φ in all many-body quantum systems as $T \to 0$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

where C is a T-independent constant. Systems *without* quasiparticles have $\tau_\varphi \sim \hbar/(k_B T)$.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T;
e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)
J. Zaanen, Nature **430**, 512 (2004)

A bound on quantum chaos:

- In classical chaos, we measure the sensitivity of the position at time t, $q(t)$, to variations in the initial position, $q(0)$, *i.e.* we measure

$$\left(\frac{\partial q(t)}{\partial q(0)}\right)^2 = \left(\{q(t), p(0)\}_{\text{P.B.}}\right)^2$$

- By analogy, we define τ_L as the LYAPUNOV TIME over which the wave-function of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$\left\langle \left| [\hat{A}(x,t), \hat{B}(0,0)] \right|^2 \right\rangle \sim \exp\left(\frac{1}{\tau_L}\left[t - \frac{|x|}{v_B}\right]\right),$$

where v_B is the 'butterfly velocity'. This time τ_L was argued to obey lower bound

$$\tau_L \geq \frac{1}{2\pi}\frac{\hbar}{k_B T}.$$

There is no analogous bound in classical mechanics.

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

A bound on quantum chaos:

- In classical chaos, we measure the sensitivity of the position at time t, $q(t)$, to variations in the initial position, $q(0)$, *i.e.* we measure

$$\left(\frac{\partial q(t)}{\partial q(0)}\right)^2 = \left(\{q(t), p(0)\}_{\text{P.B.}}\right)^2$$

- By analogy, we define τ_L as the Lyapunov time over which the wave-function of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$\left\langle \left| [\hat{A}(x,t), \hat{B}(0,0)] \right|^2 \right\rangle \sim \exp\left(\frac{1}{\tau_L}\left[t - \frac{|x|}{v_B}\right]\right),$$

where v_B is the 'butterfly velocity'. This time τ_L was argued to obey lower bound

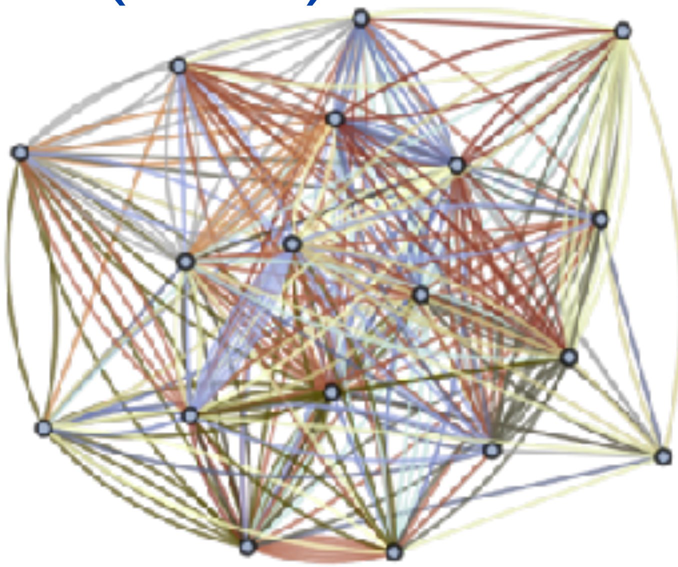
$$\tau_L \geq \frac{1}{2\pi}\frac{\hbar}{k_B T}.$$

Quantum matter without quasiparticles
$\approx$ fastest possible many-body quantum chaos

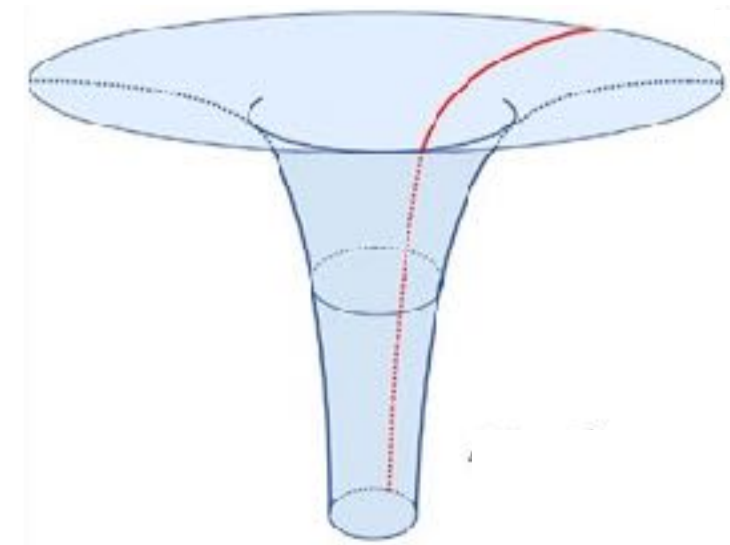
P **28**, 6 (1969)
iv:1503.01409

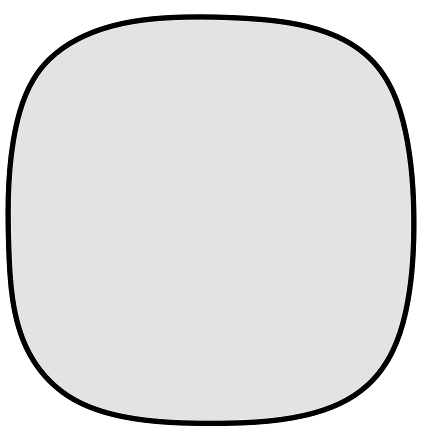
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons



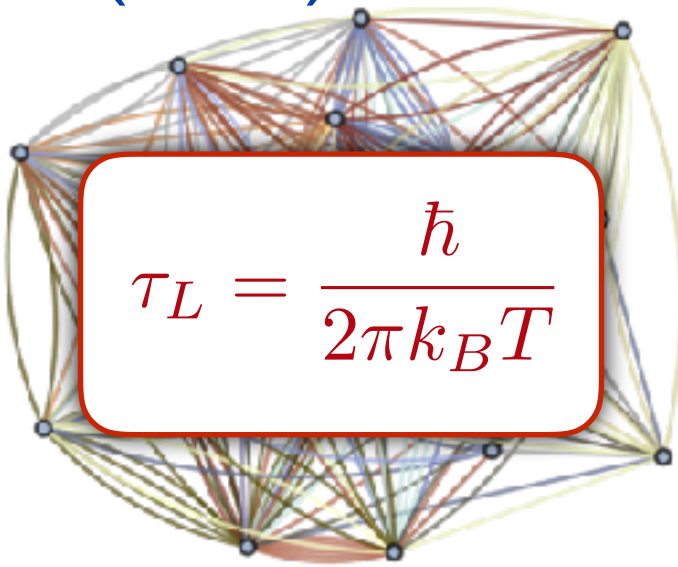


Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

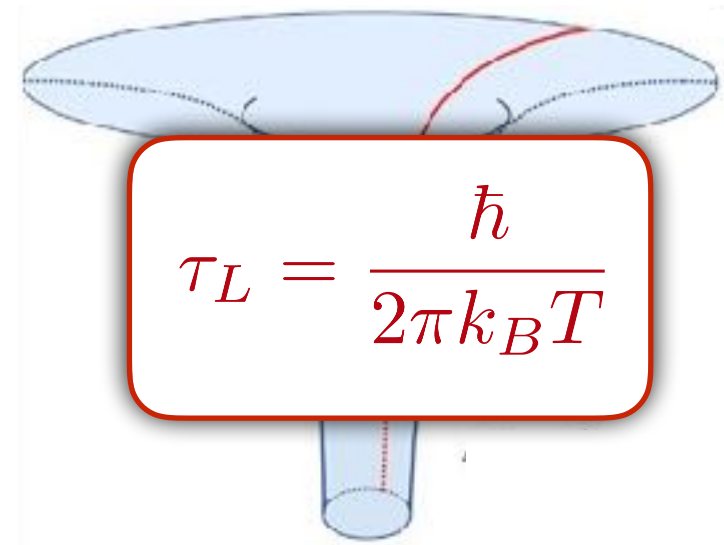
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



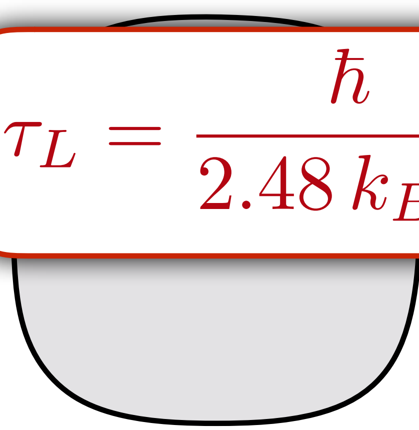
$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Black holes with AdS_2 horizons



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Fermi surface coupled to a gauge field



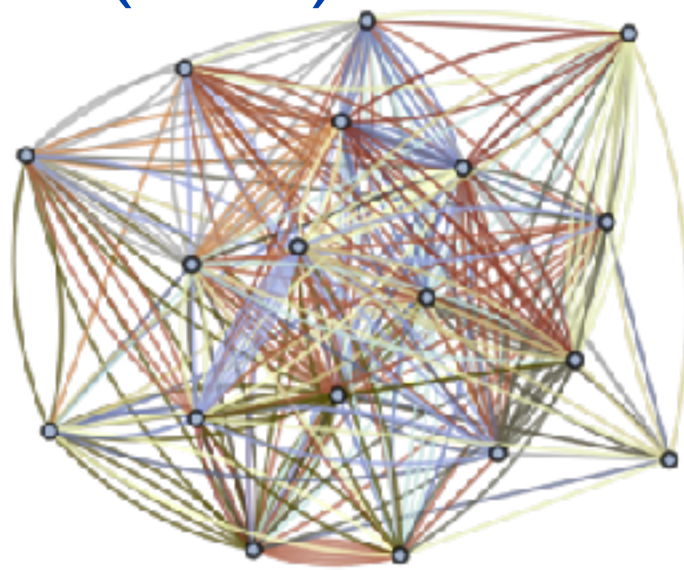
$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

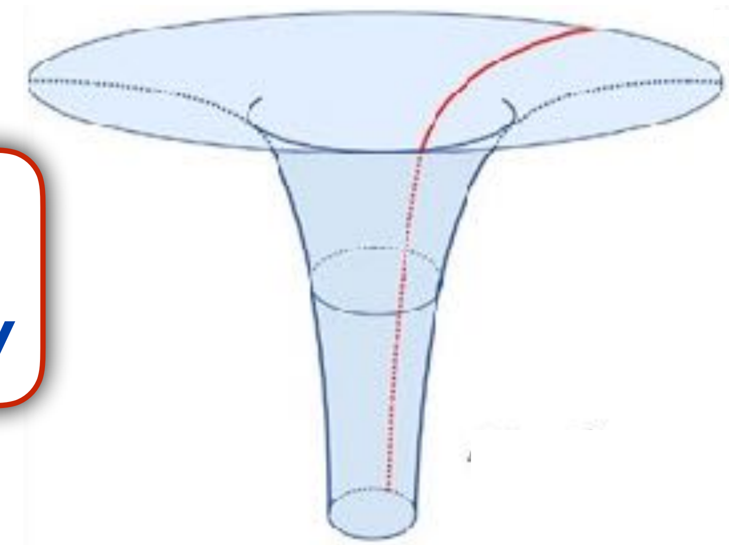
Quantum matter without quasiparticles:

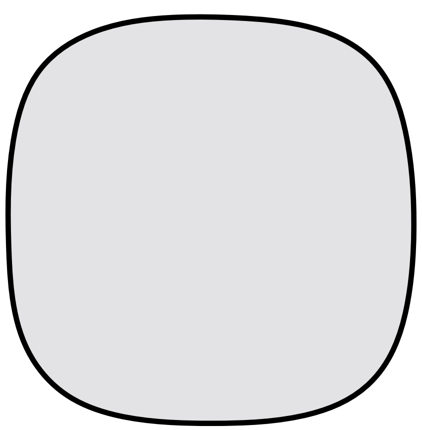
The Sachdev-Ye-Kitaev (SYK) models



Same low energy theory

Black holes with AdS_2 horizons





Fermi surface coupled to a gauge field

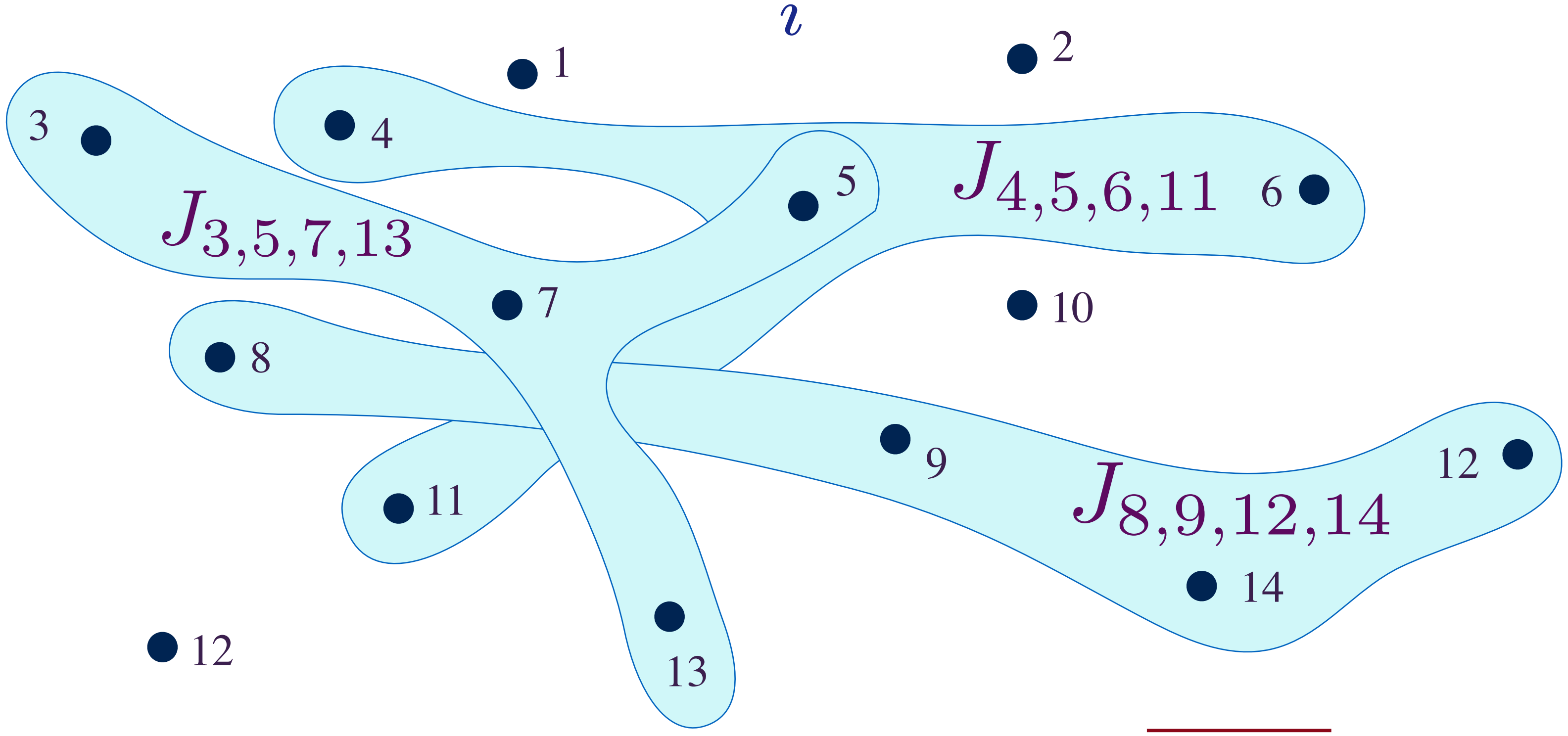
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK and AdS_2

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$ **Not a ground state degeneracy**: due to an exponentially small (in N) many-body level spacing at all energies down to the ground state energy.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

SYK and AdS_2

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$ **Not a ground state degeneracy**: due to an exponentially small (in N) many-body level spacing at all energies down to the ground state energy.

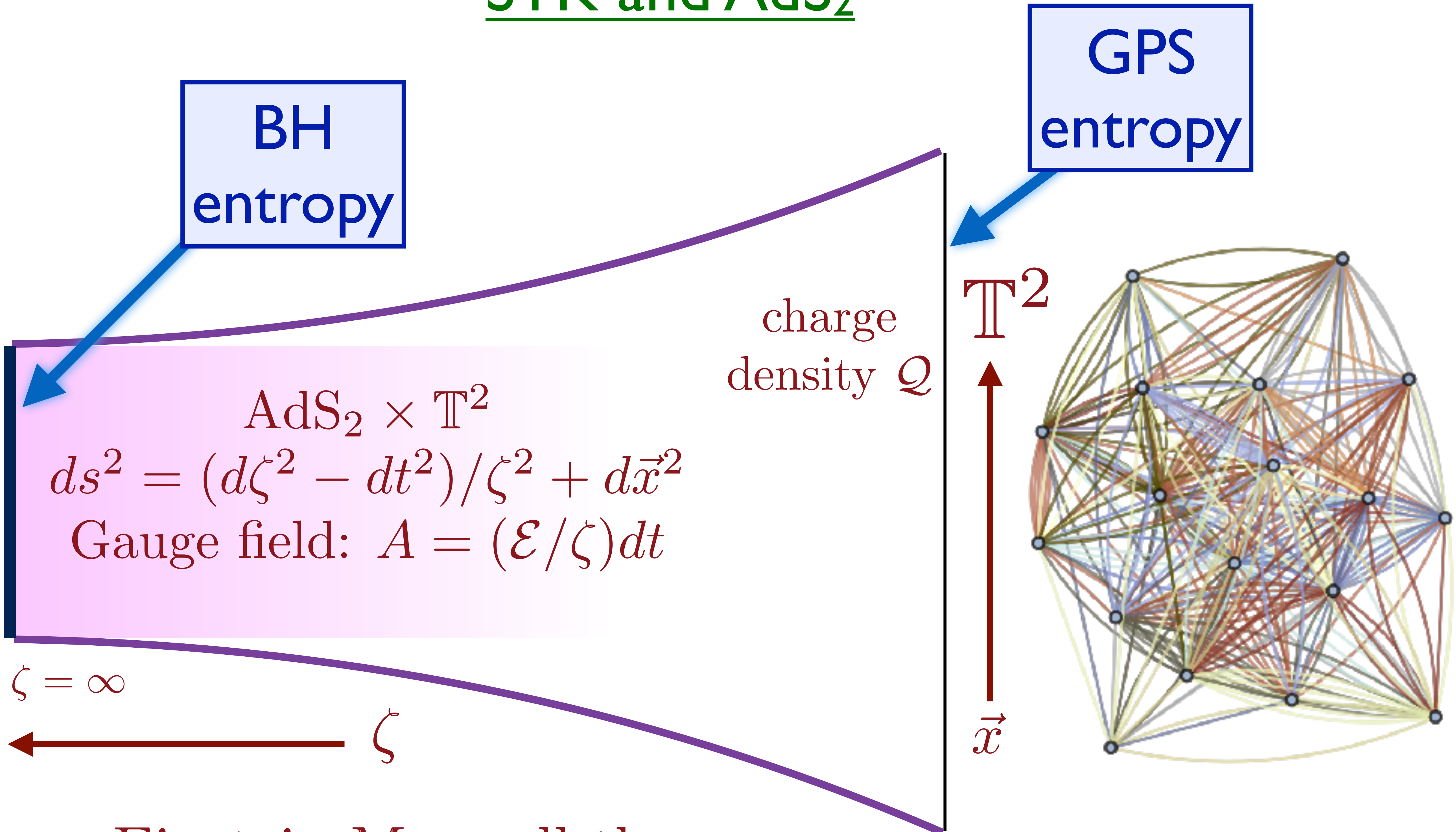
A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

- This entropy, and other dynamic correlators of the SYK models, imply that the SYK model is holographically dual to black holes with an AdS_2 horizon. The Bekenstein-Hawking entropy of the black hole equals $N S_0$:

$$\mathbf{GPS} = \mathbf{BH}.$$

S. Sachdev, PRL **105**, 151602 (2010)

SYK and AdS_2



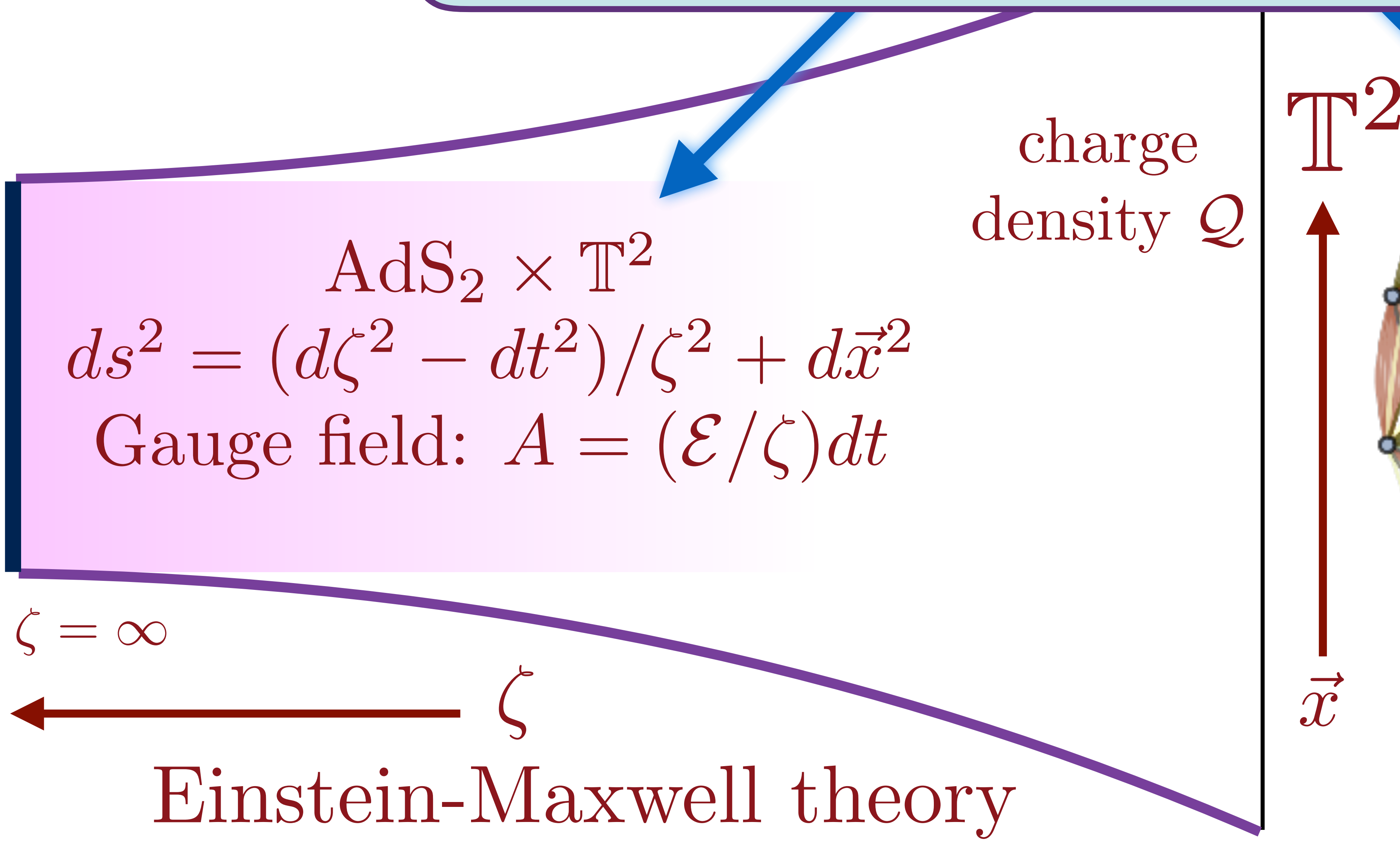
Einstein-Maxwell theory
+ cosmological constant

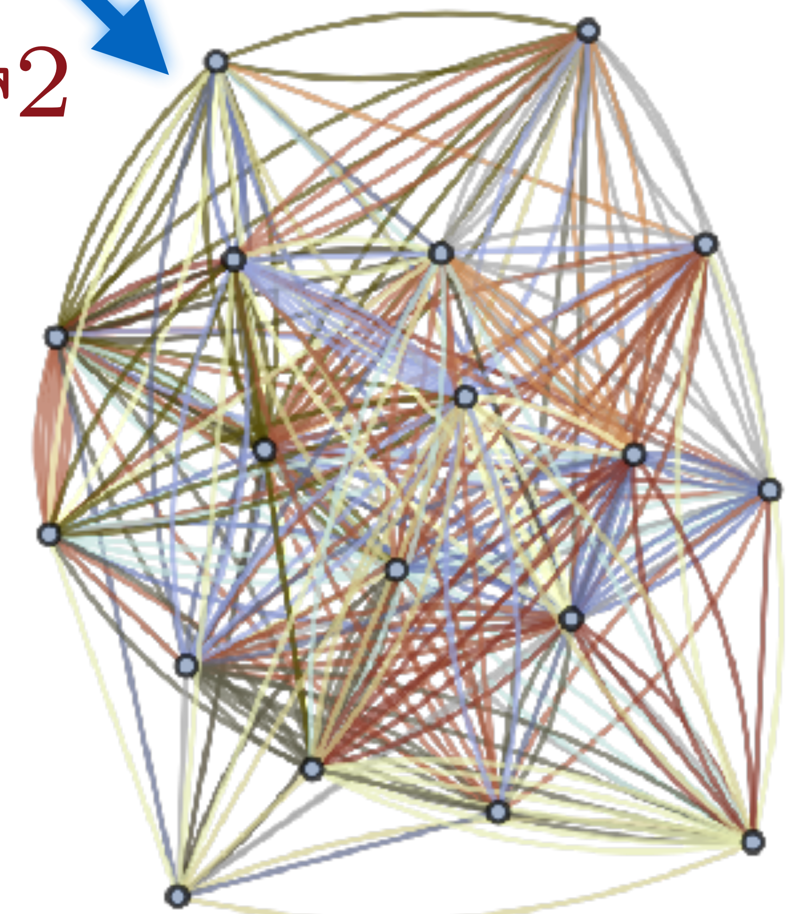
S. Sachdev, PRL **105**, 151602 (2010)

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

SYK and AdS_2

Same long-time effective action for energy and number fluctuations, involving Schwarzian derivatives of $f(\tau)$.



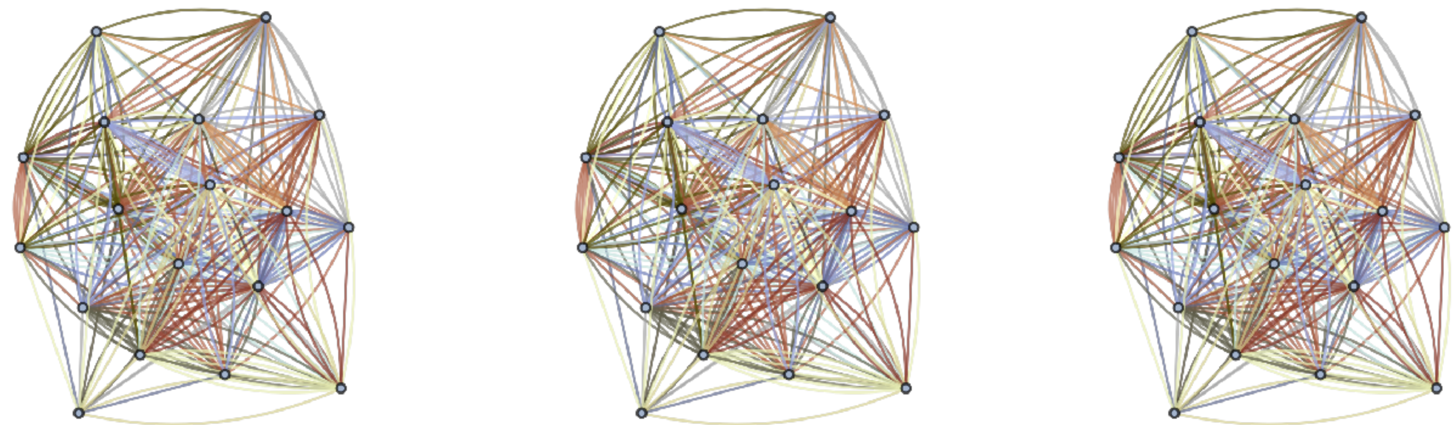


Einstein-Maxwell theory + cosmological constant

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

A. Kitaev, KITP talk (2015; J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Coupled SYK models



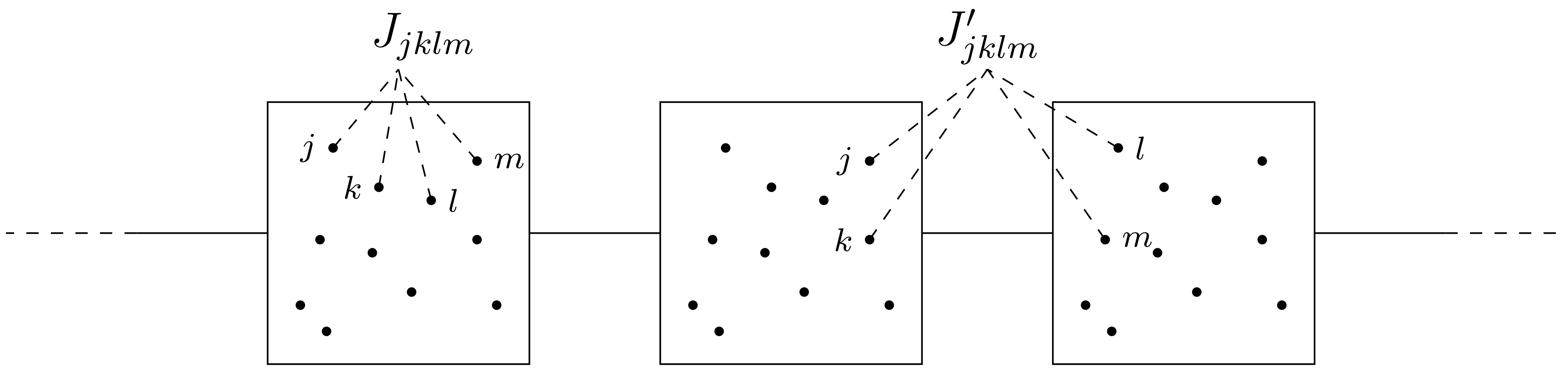
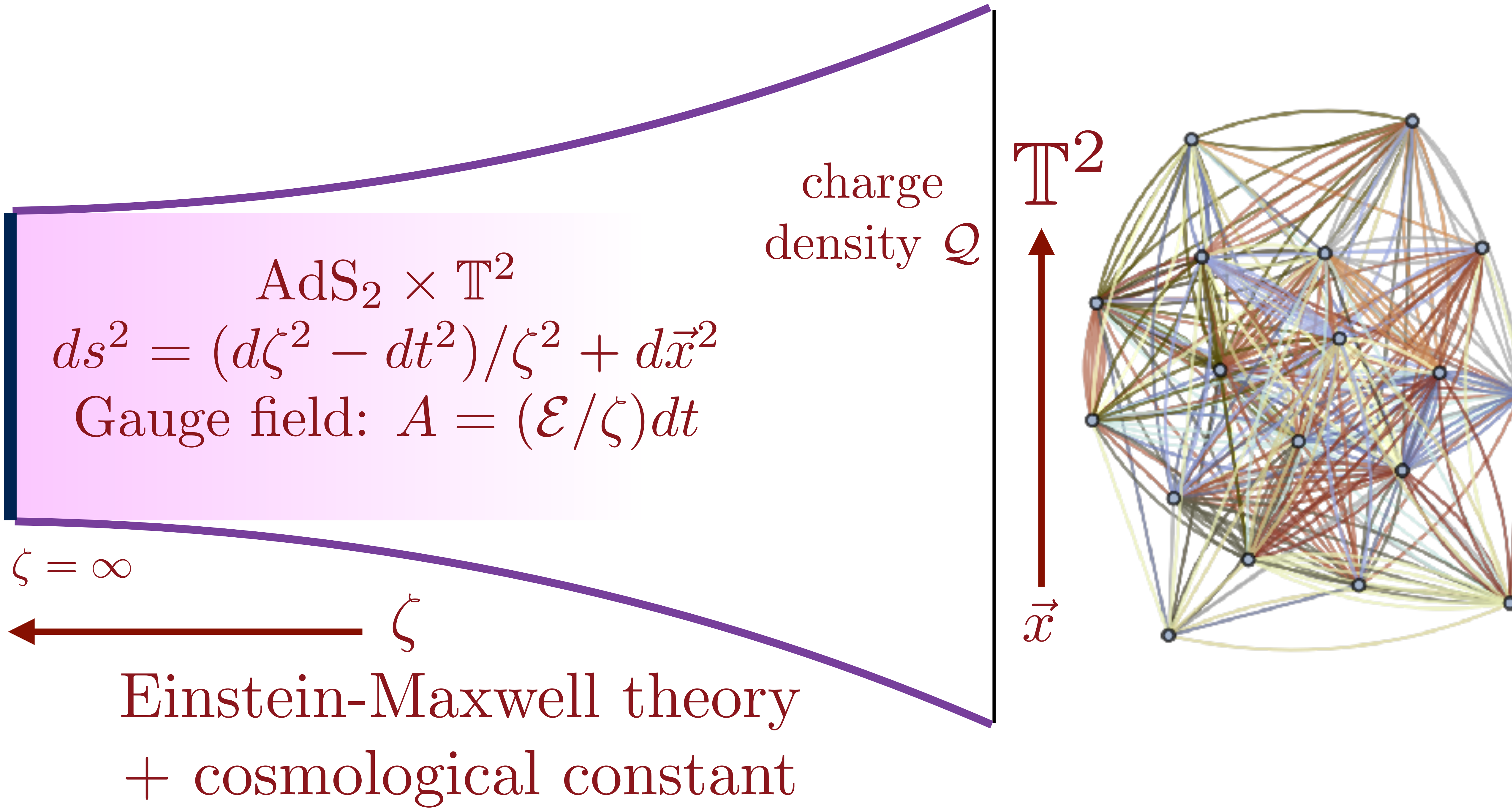


Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

SYK and AdS_2

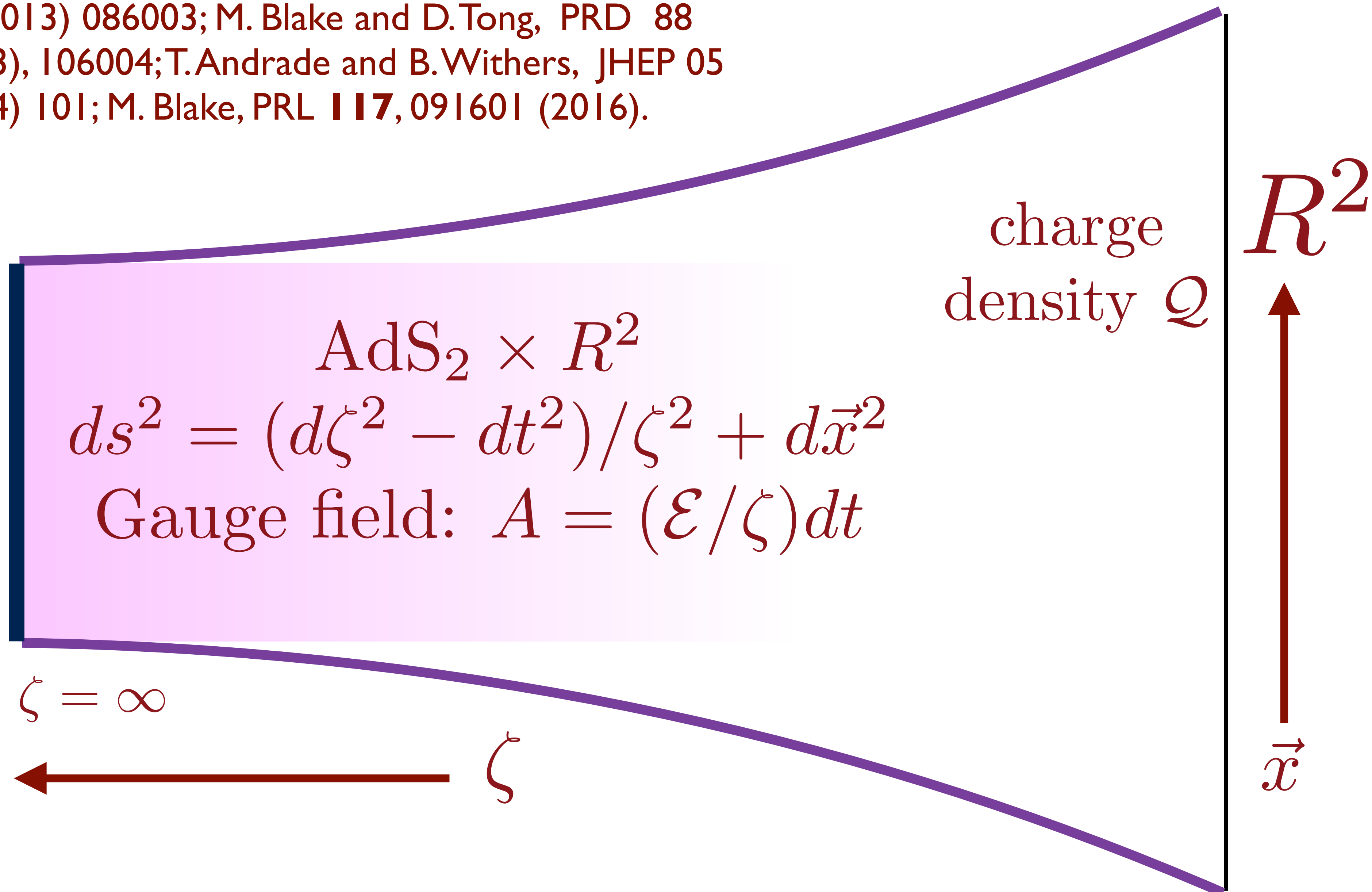


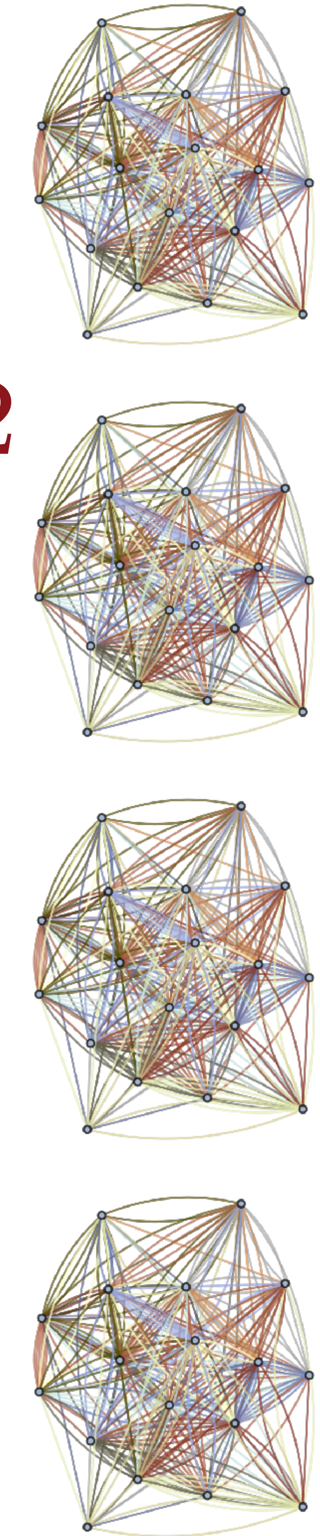
Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

S. Sachdev, PRL **105**, 151602 (2010)

Coupled SYK and AdS_4

Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054; D. Vegh, arXiv:1301.0537; R. A. Davison, PRD 88 (2013) 086003; M. Blake and D. Tong, PRD 88 (2013), 106004; T. Andrade and B. Withers, JHEP 05 (2014) 101; M. Blake, PRL **117**, 091601 (2016).





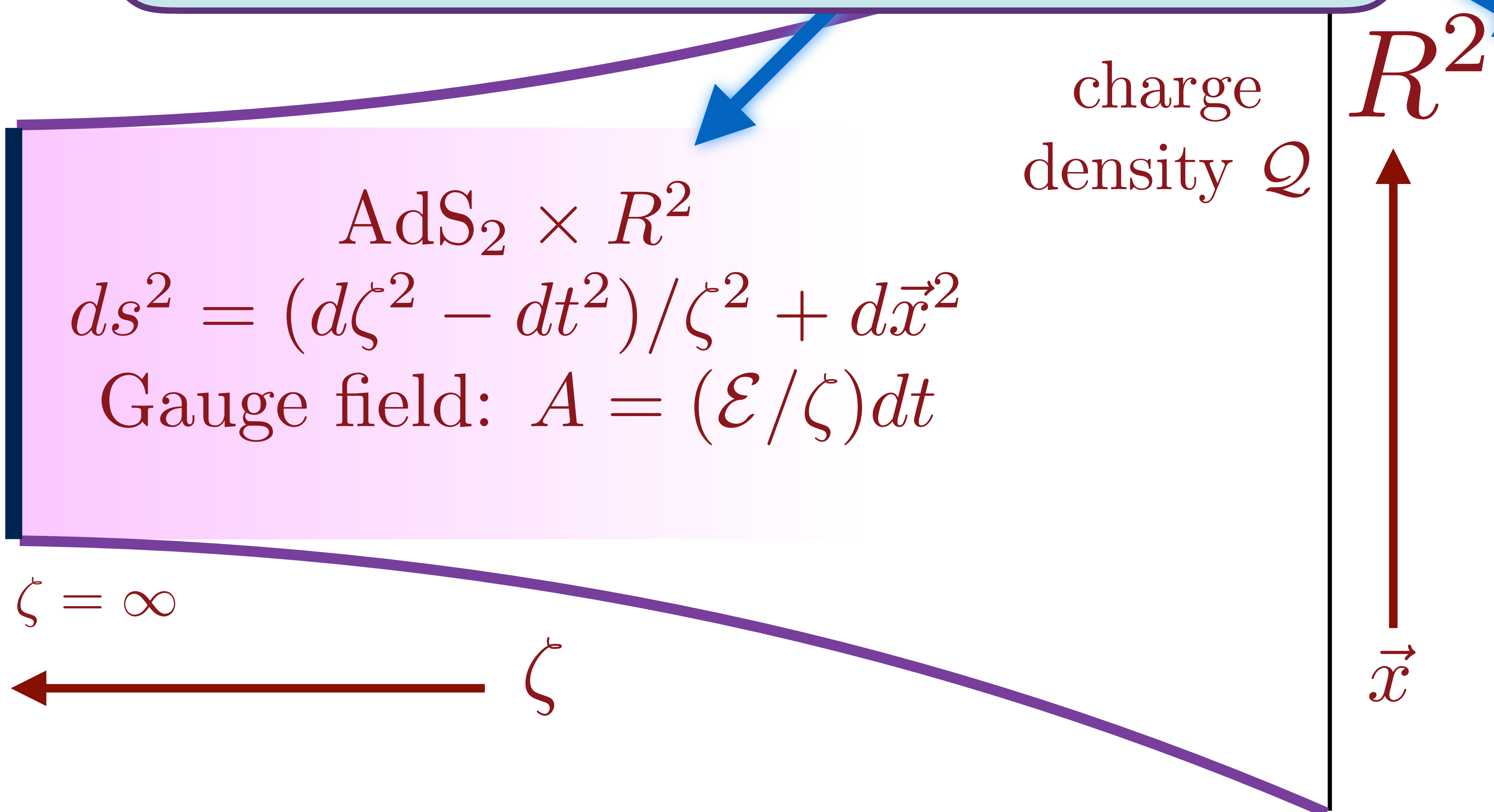
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv. 1612.00849

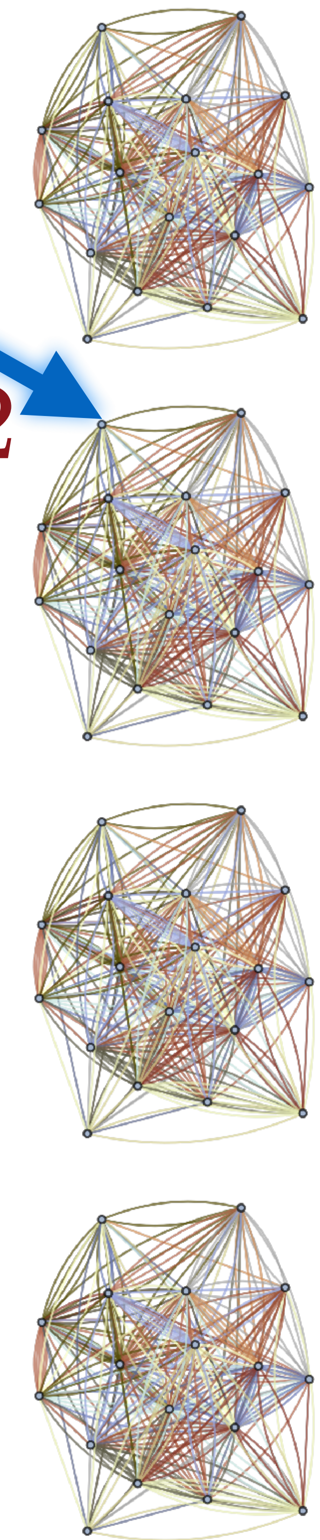
$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = k x_i$ leading to momentum disspation

Coupled SYK and AdS_4

Matching correlators for thermoelectric diffusion,
and quantum chaos
$\tau_L = \hbar/(2\pi k_B T)$, $v_B \sim T^{1/2}$,
and thermal diffusivity $D_E = v_B^2 \tau_L$





R. Davison,
Wenbo Fu,
A. Georges,
Yingfei Gu,
K. Jensen,
S. Sachdev,

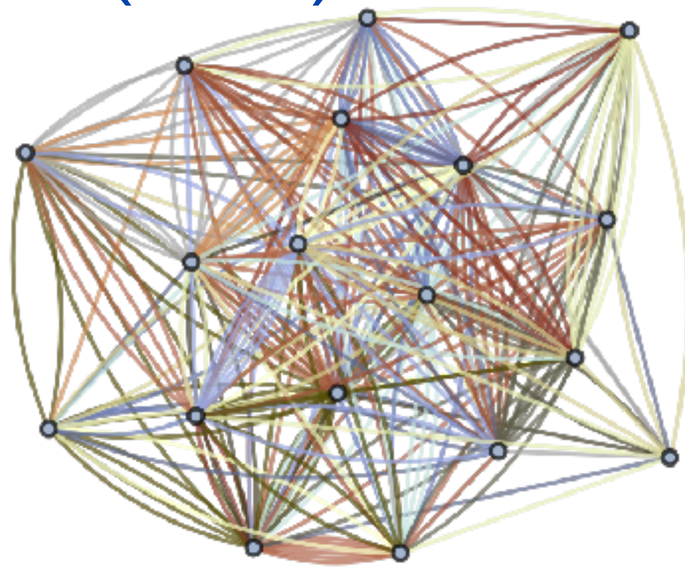
arXiv.
1612.00849

$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2}\sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} \right),$$

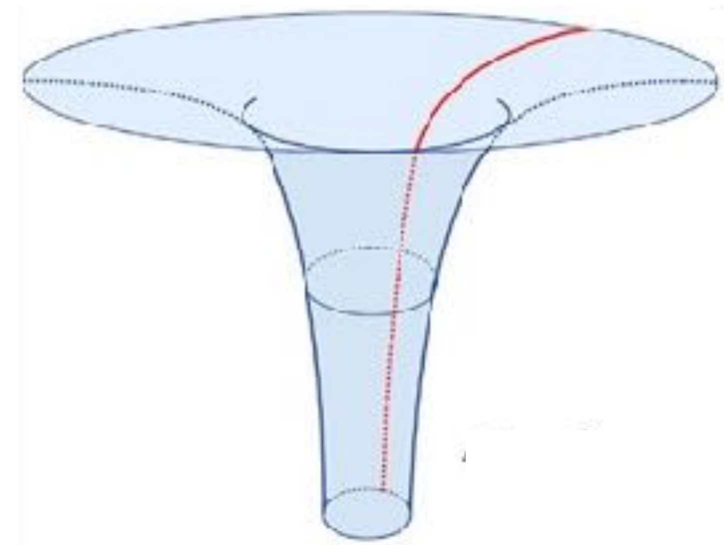
Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = k x_i$
leading to momentum disspation

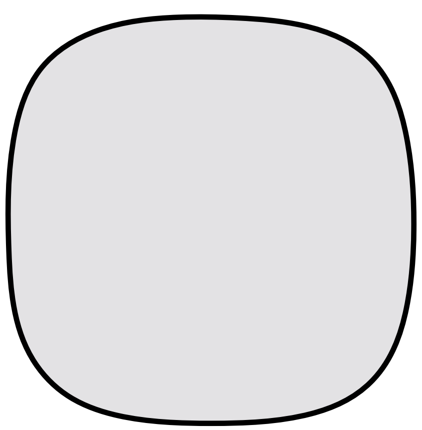
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons



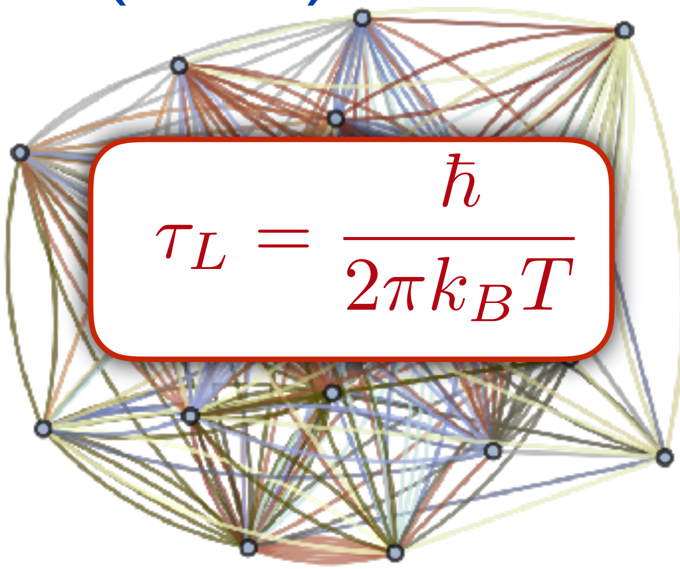


Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

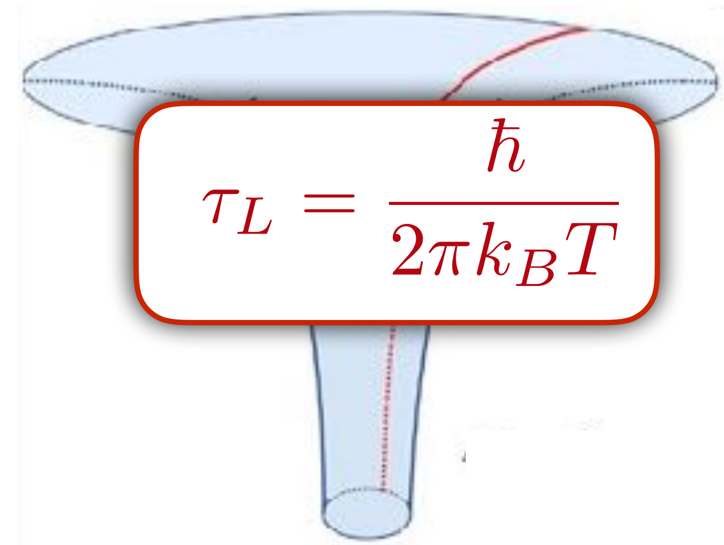
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

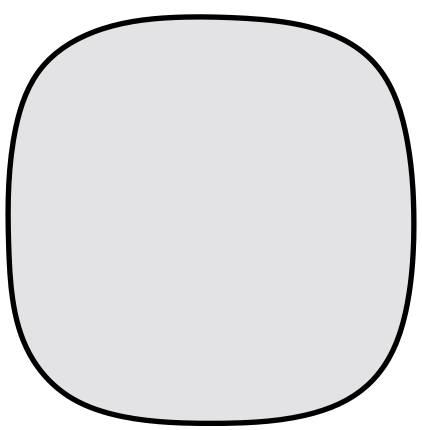


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Black holes with AdS_2 horizons



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$



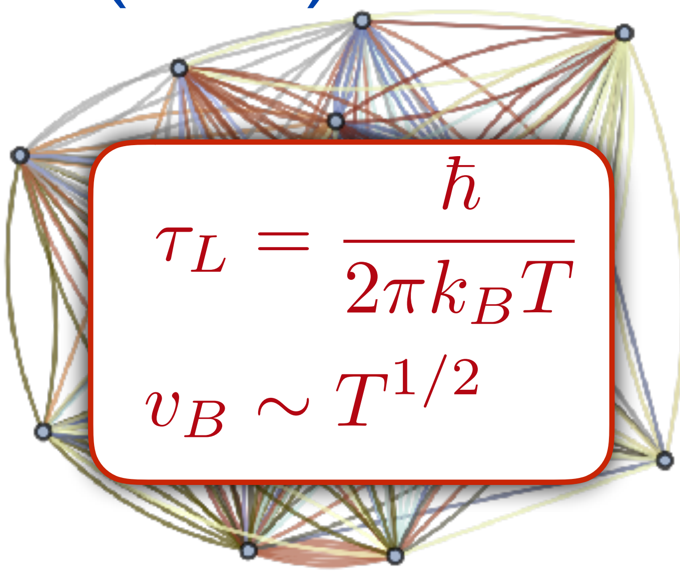
Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

Quantum matter without quasiparticles:

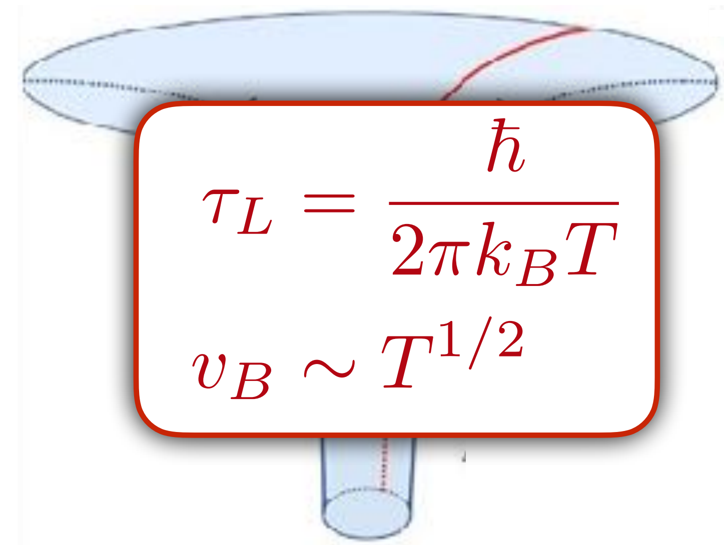
The Sachdev-Ye-Kitaev (SYK) models



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

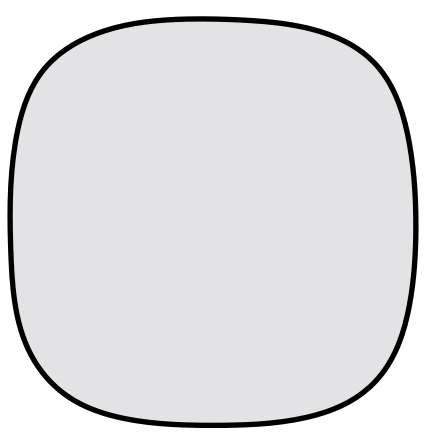
$$v_B \sim T^{1/2}$$

Black holes with AdS_2 horizons



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$



Fermi surface coupled to a gauge field

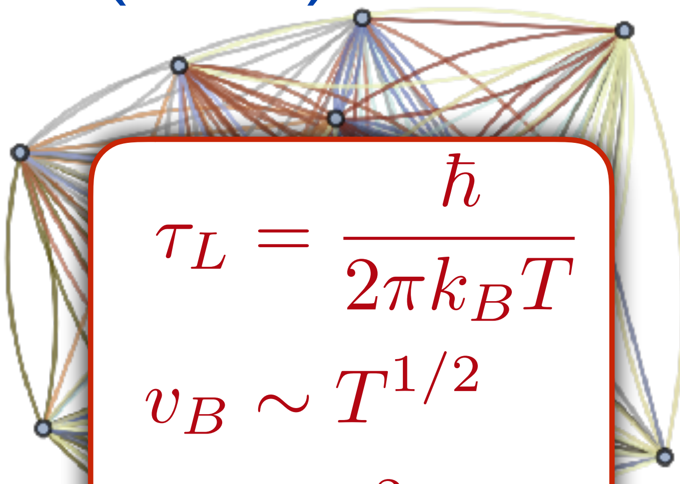
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

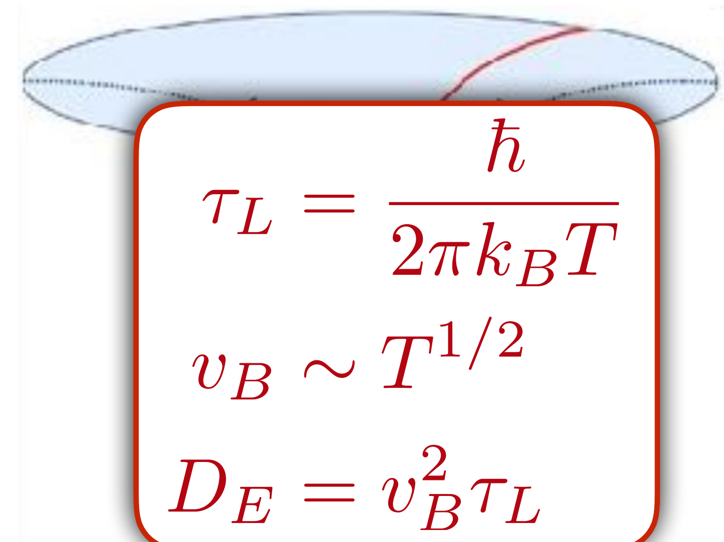


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

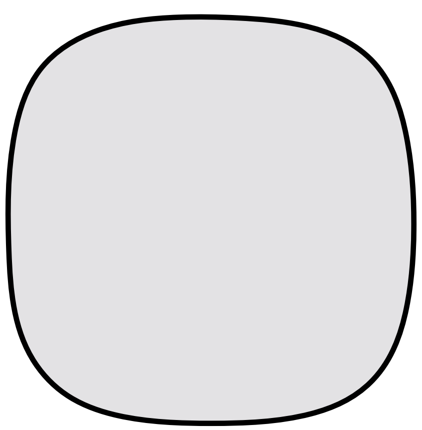
Black holes with AdS_2 horizons



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$



Fermi surface coupled to a gauge field

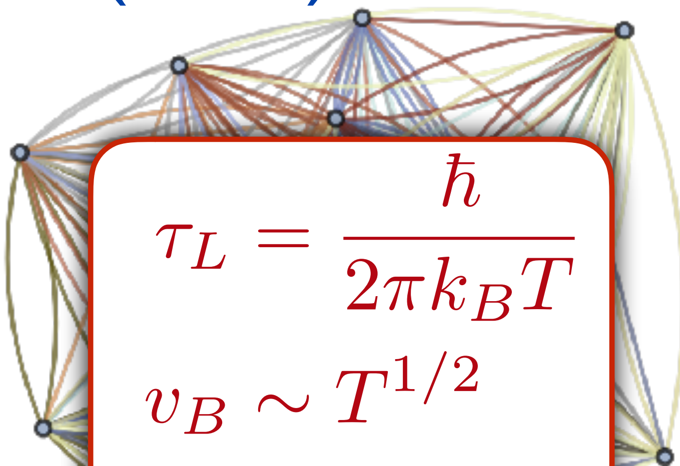
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

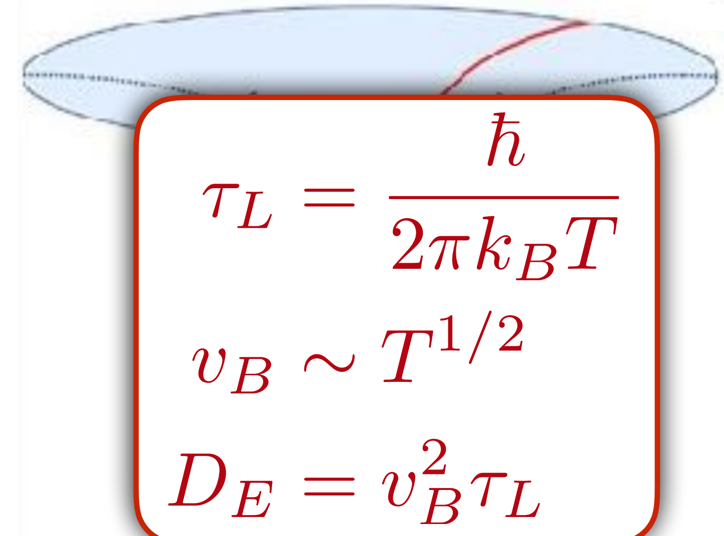


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

Black holes with AdS_2 horizons

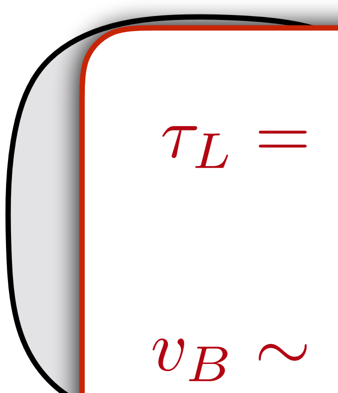


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

A. A. Patel and S. Sachdev, arXiv: 1611.00003



ermi surface coupled to a gauge field

$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$v_B \sim \frac{N v_F^{5/3}}{e^{4/3}\gamma^{1/3}} T^{1/3}$$

$$D_E = 0.42 v_B^2 \tau_L$$

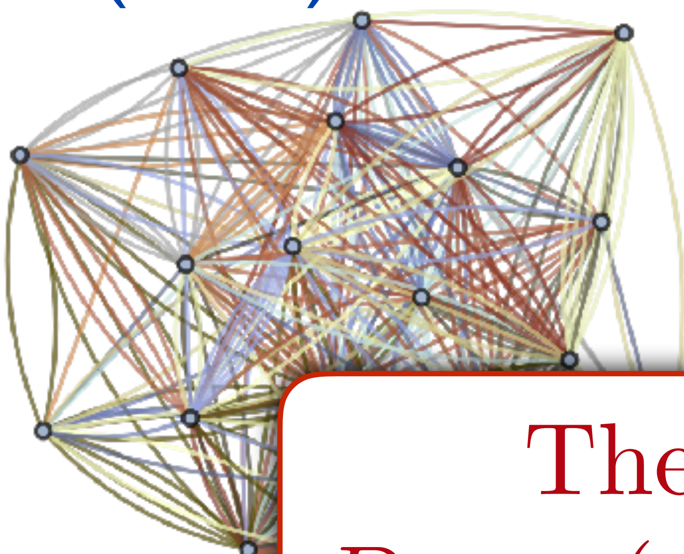
$$\mathcal{L}[\Psi \quad \frac{(\nabla - i\vec{a})^2}{2m} - \mu \Big) \Psi + \frac{1}{2g^2}(\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

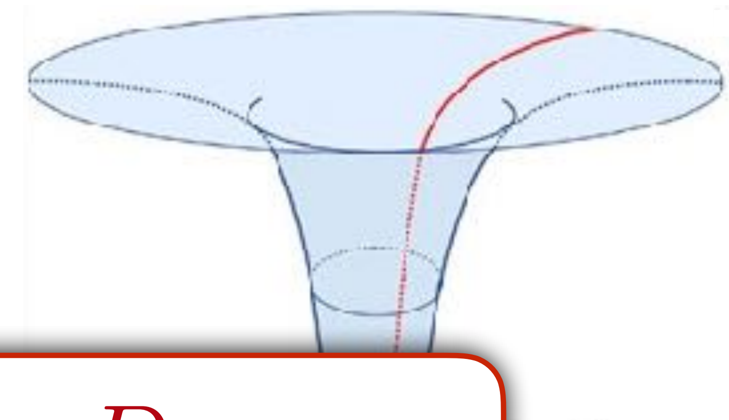
v_B: the "butterfly velocity" for the spatial propagation of chaos

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



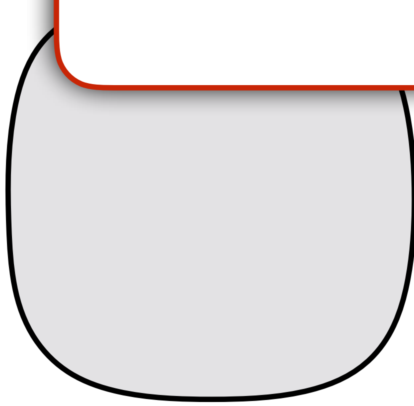
Black holes with AdS_2 horizons



Thermal diffusivity, D_E:

$D_E =$ (universal number) $\times v_B^2 \tau_L$

in all three models



Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

- Quantum chaos is intimately linked to the loss of phase coherence from electron-electron interactions. As the time derivative of the local phase is determined by the local energy, phase fluctuations and chaos are linked to interaction-induced energy fluctuations, and hence thermal diffusivity.