Gauge-gravity duality and its applications

Liza Huijse

Leiden, June 14 2011

Talk online: sachdev.physics.harvard.edu

PHYSICS
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Outline

1. Quantum criticality and conformal field theories in condensed matter

2. Quantum transport and Einstein-Maxwell theory on AdS_4

3. Compressible quantum matter

   A. Strange metals: experiments and theoretical framework

   B. ABJM (-like) theory at non-zero density

   C. Gauge-gravity duality

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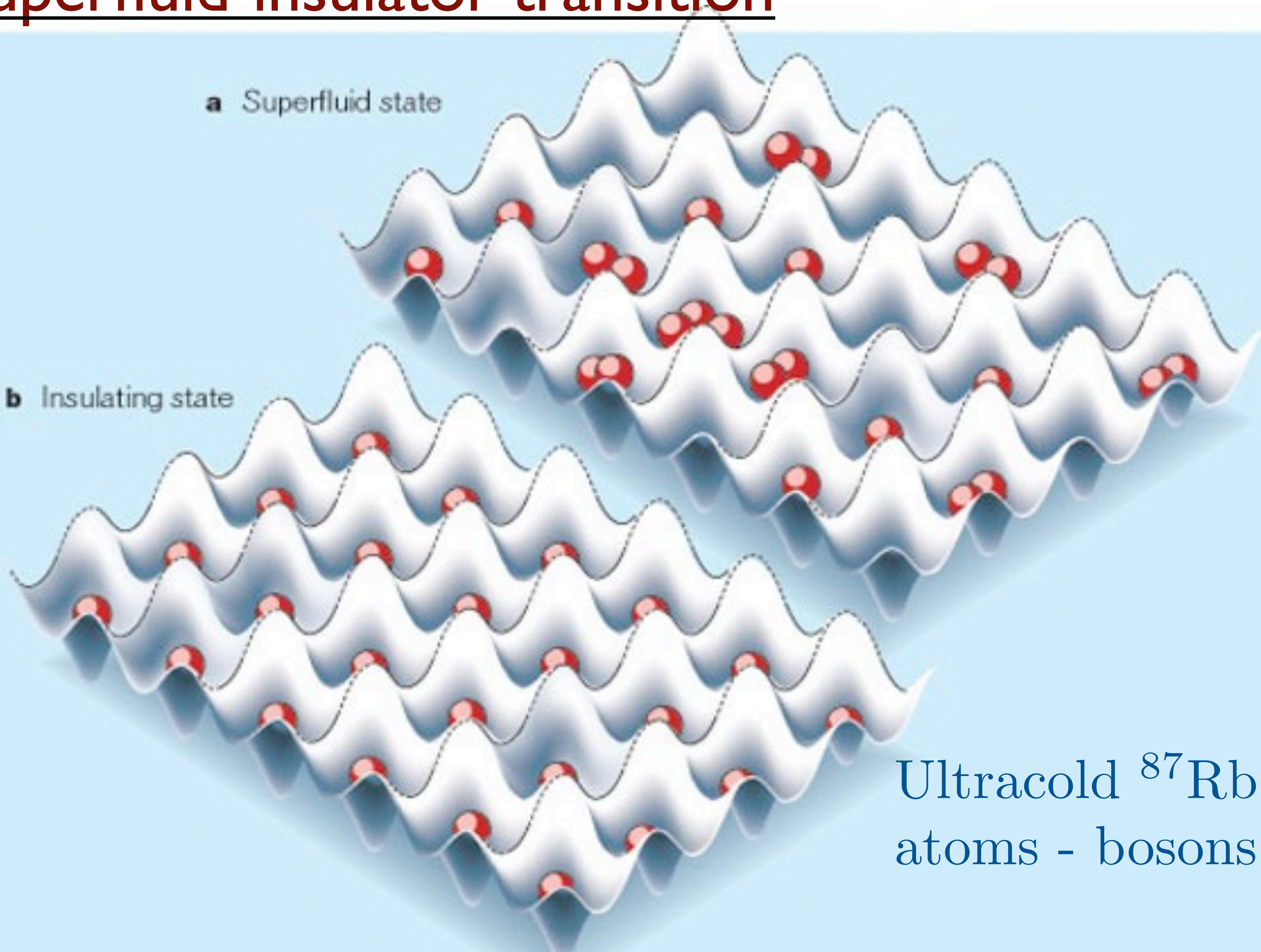
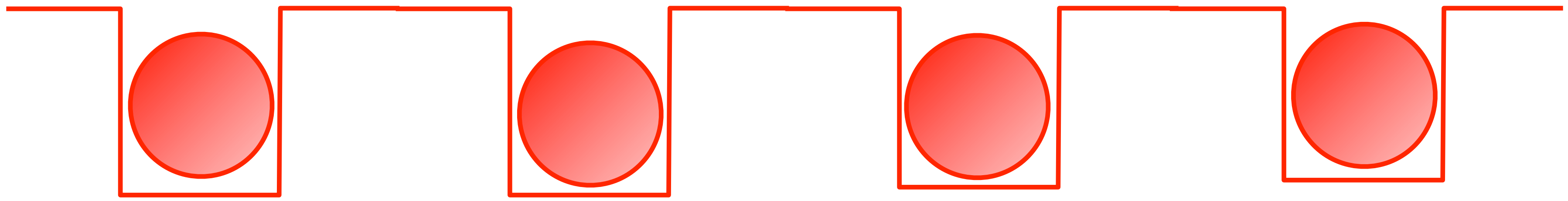
Superfluid-insulator transition



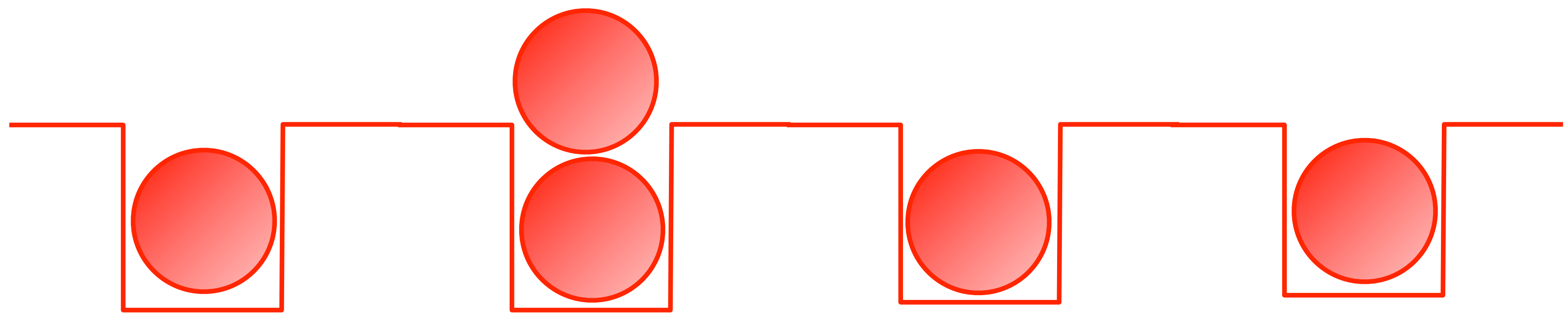
Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



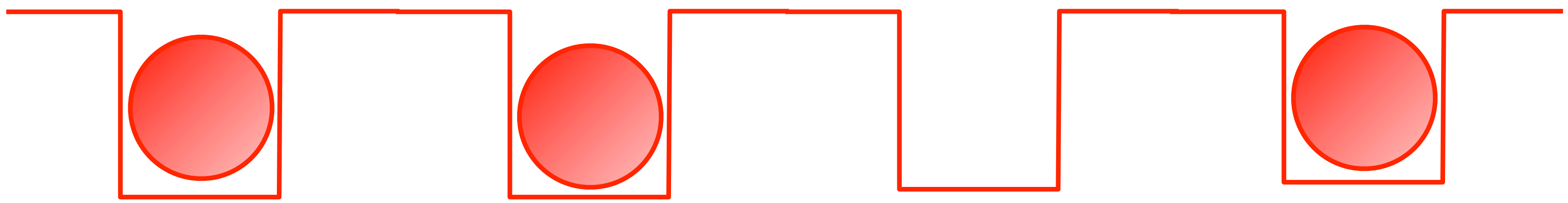
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



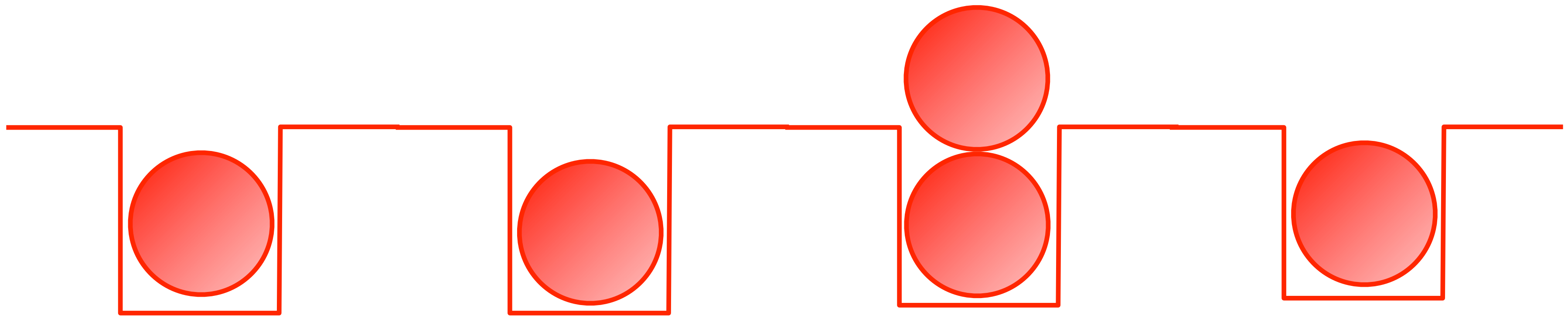
Particles ~ $\psi^\dagger$

Excitations of the insulator:



Holes $\sim \psi$

Excitations of the insulator:



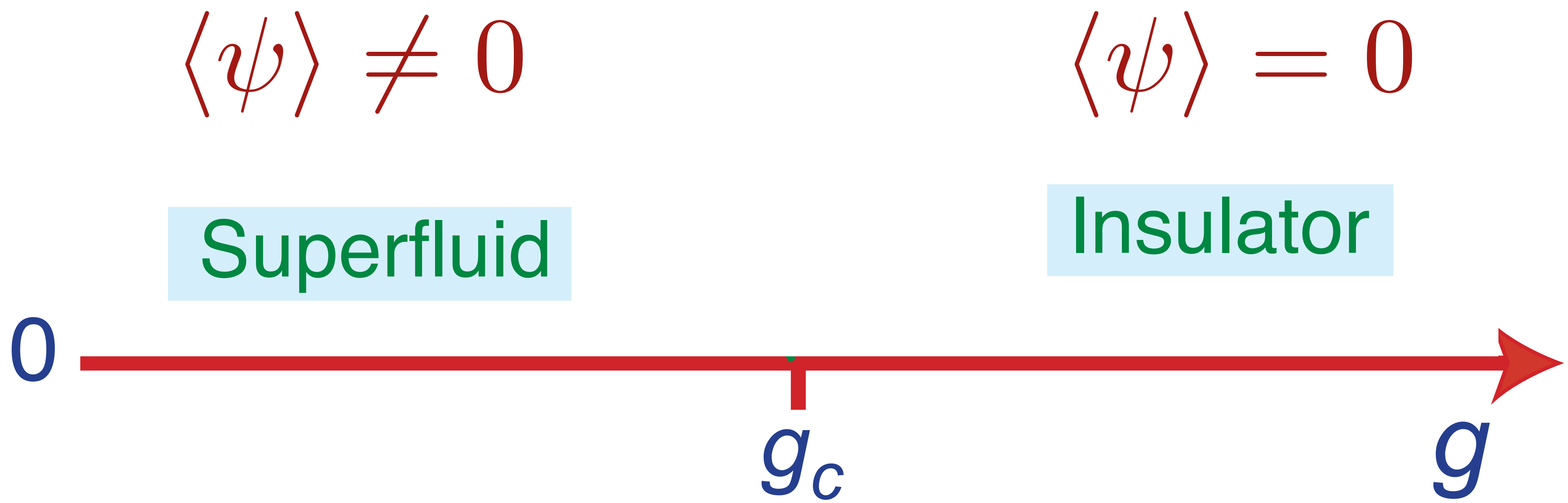
Particles $\sim \psi^\dagger$

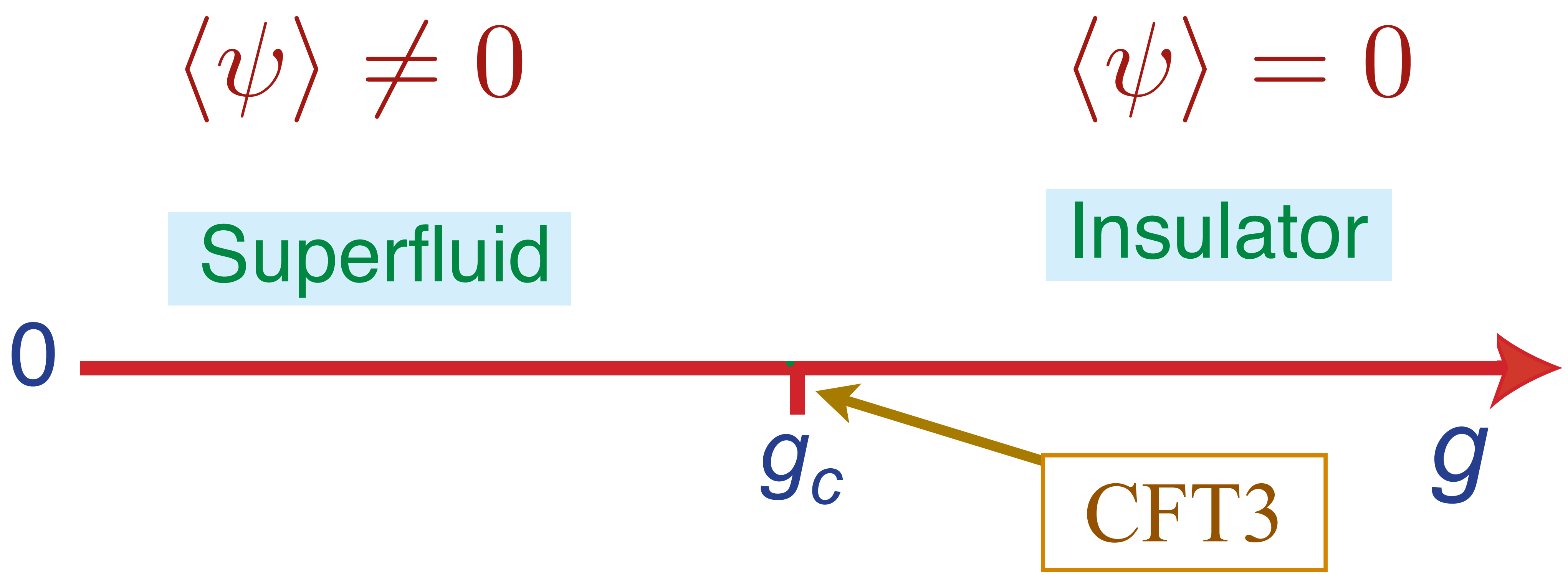
Holes $\sim \psi$

Density of particles = density of holes ⇒
"relativistic" field theory for ψ:

.

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$


$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g


$\langle \psi \rangle \neq 0$
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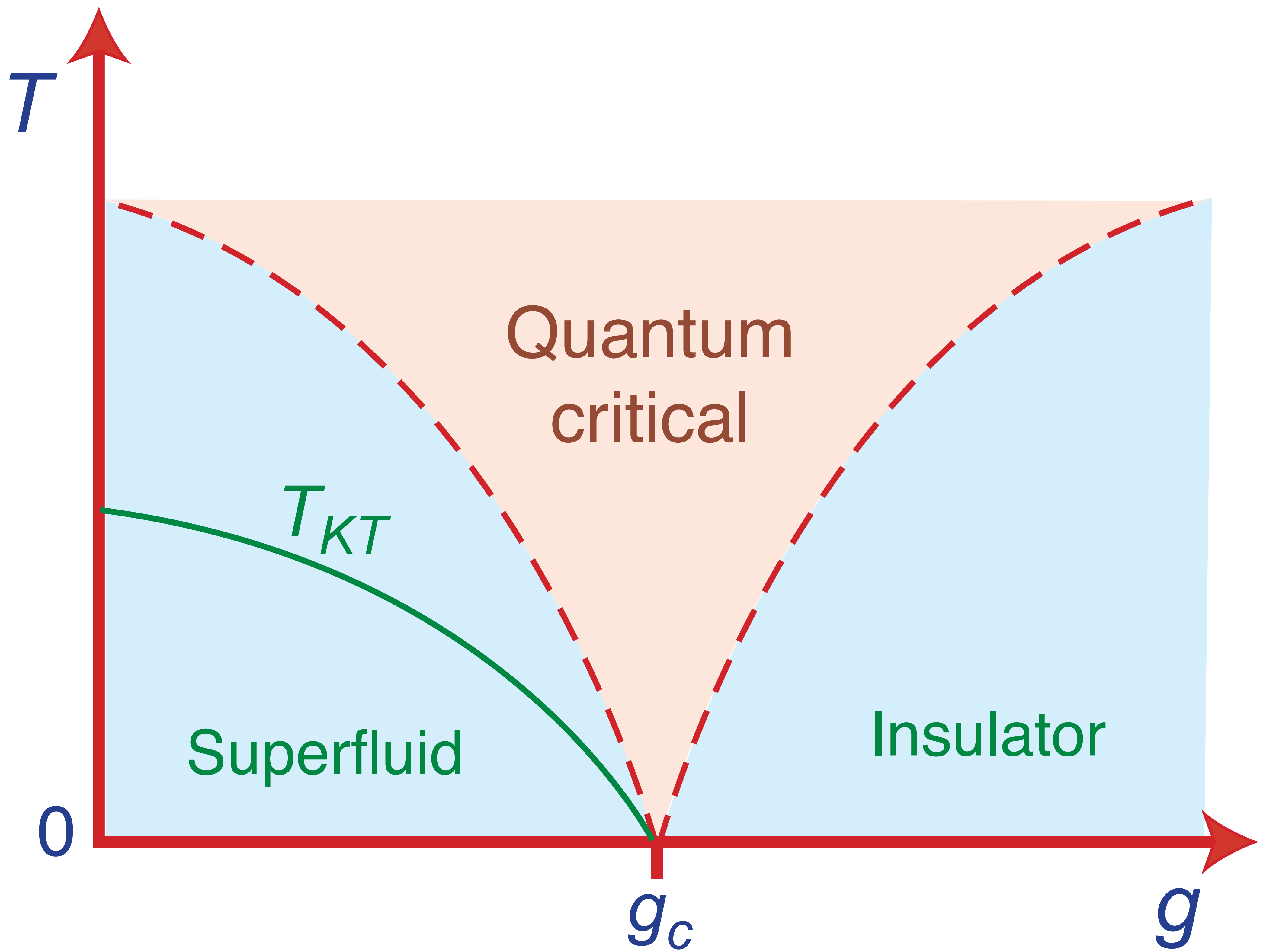
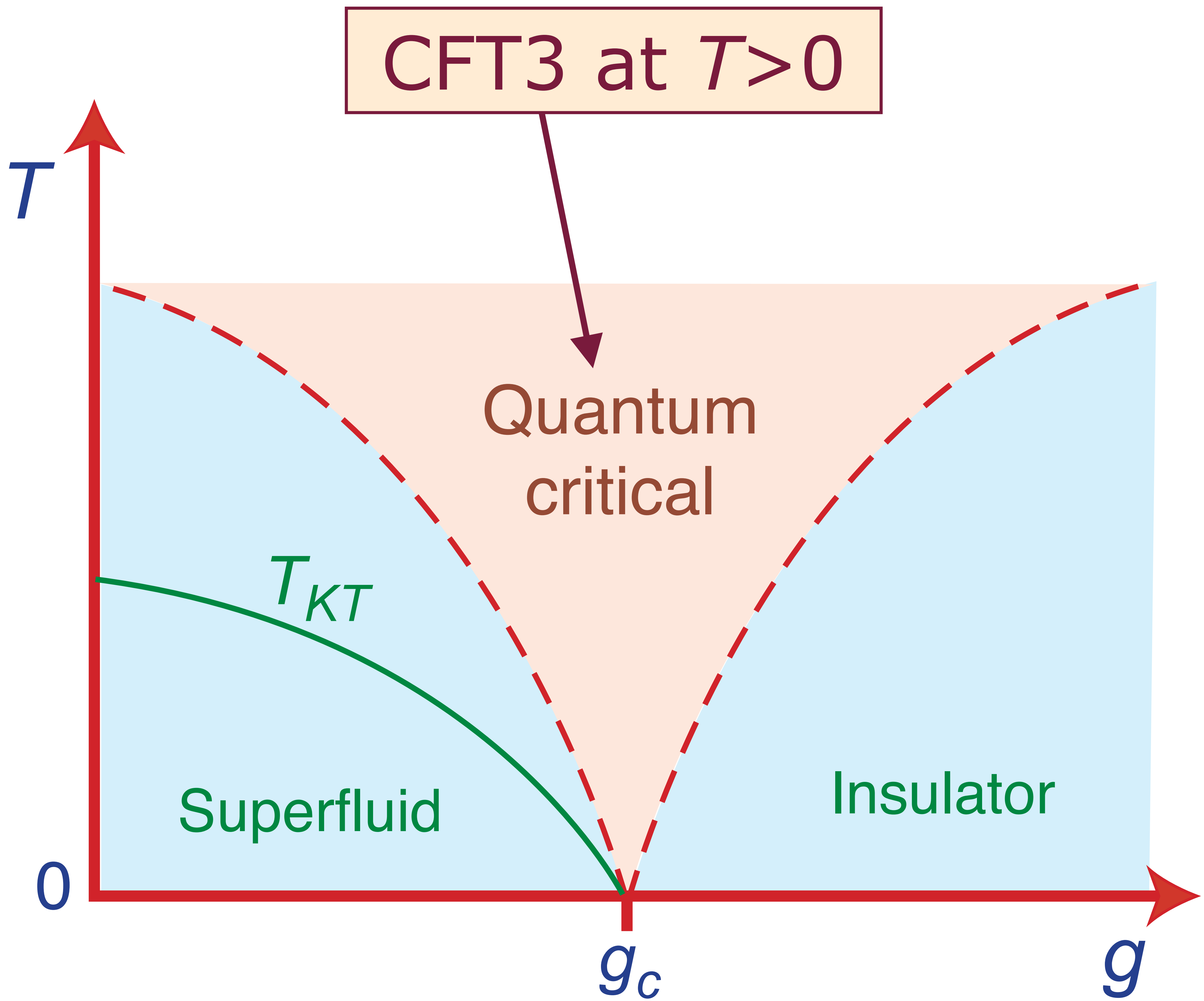
Superfluid
Insulator
0
g_c
CFT3
g



T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g



CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"* with shortest possible equilibration time, $\tau_{\rm eq}$

$$\tau_{\rm eq} = \mathcal{C}\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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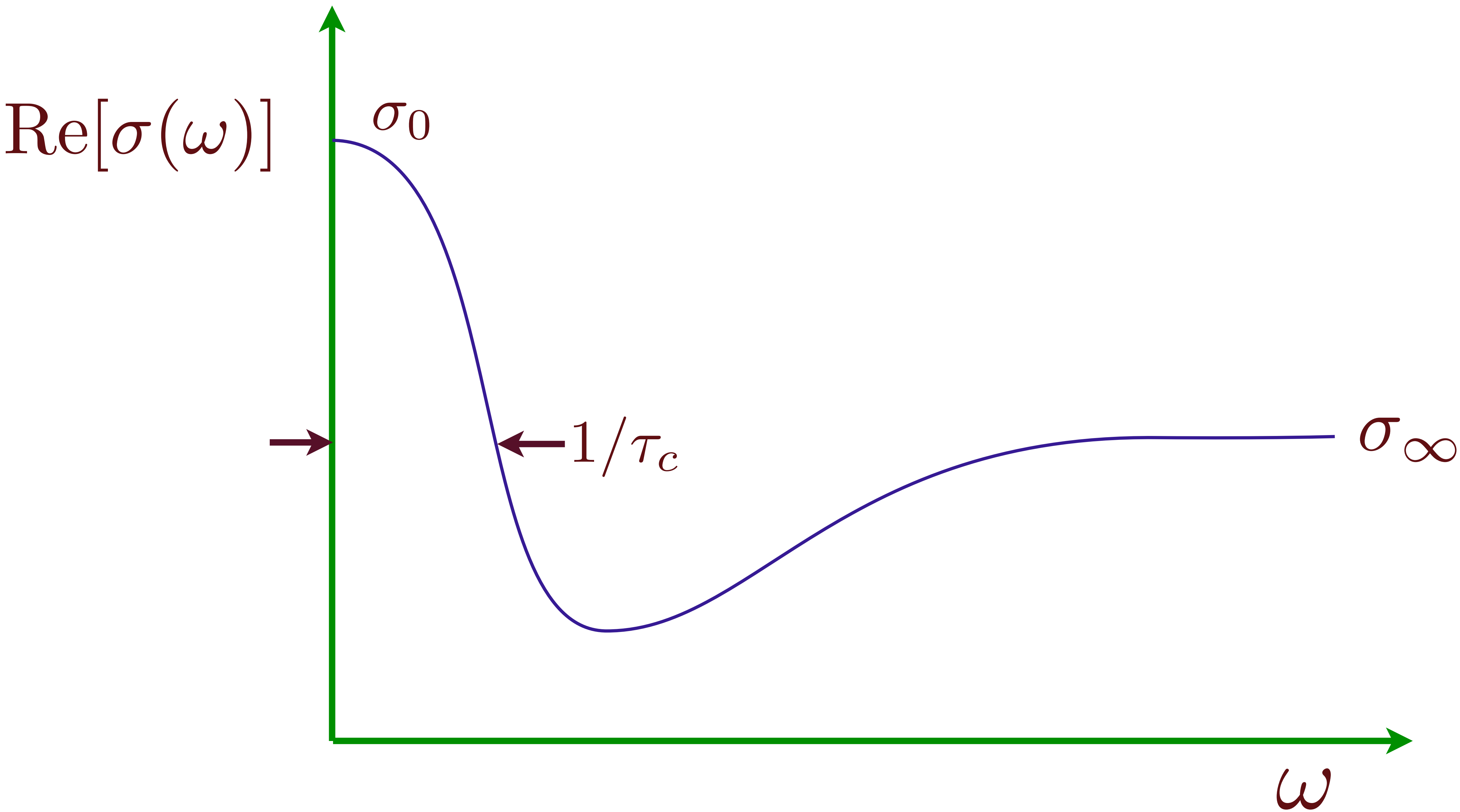
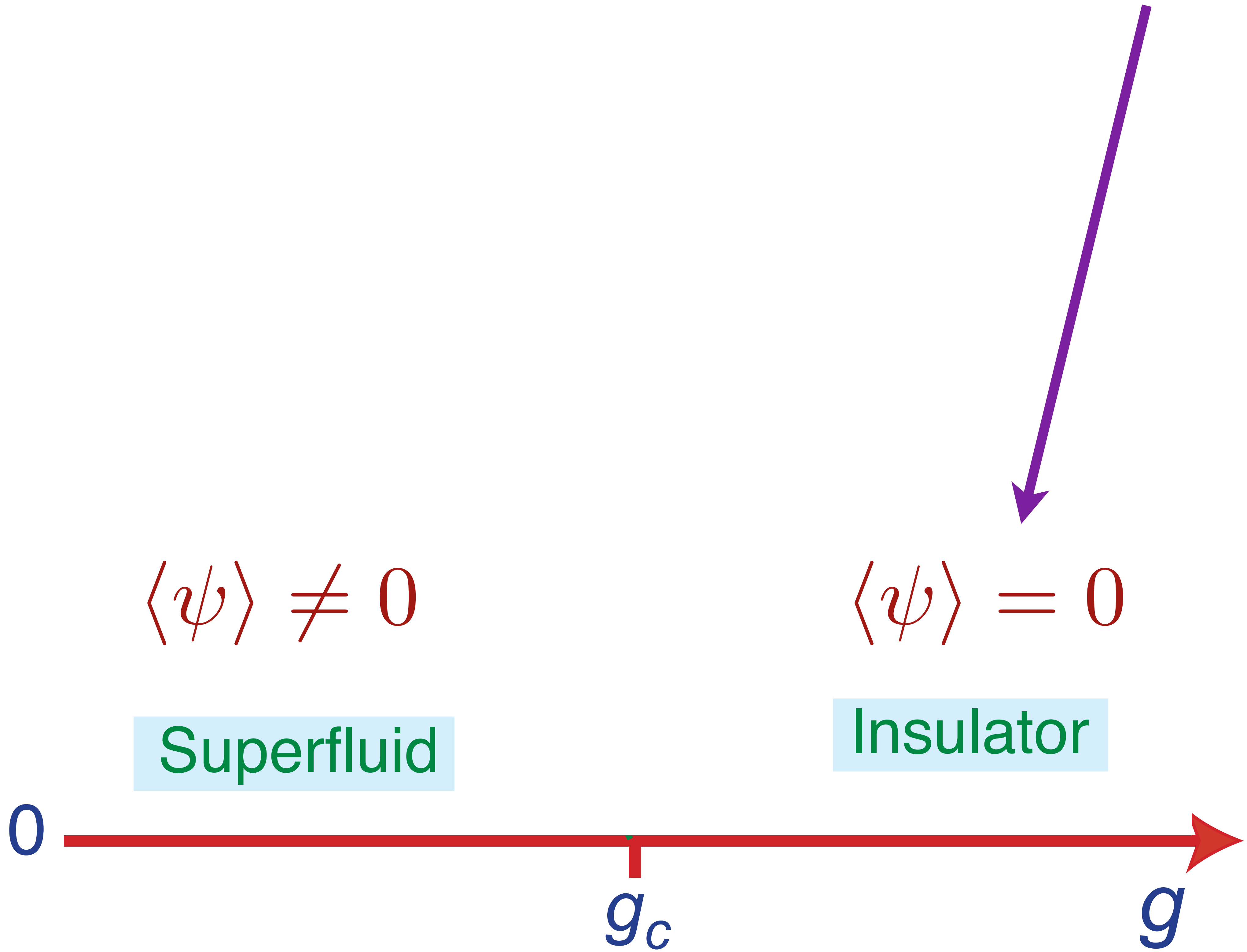
Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

Boltzmann theory of bosons

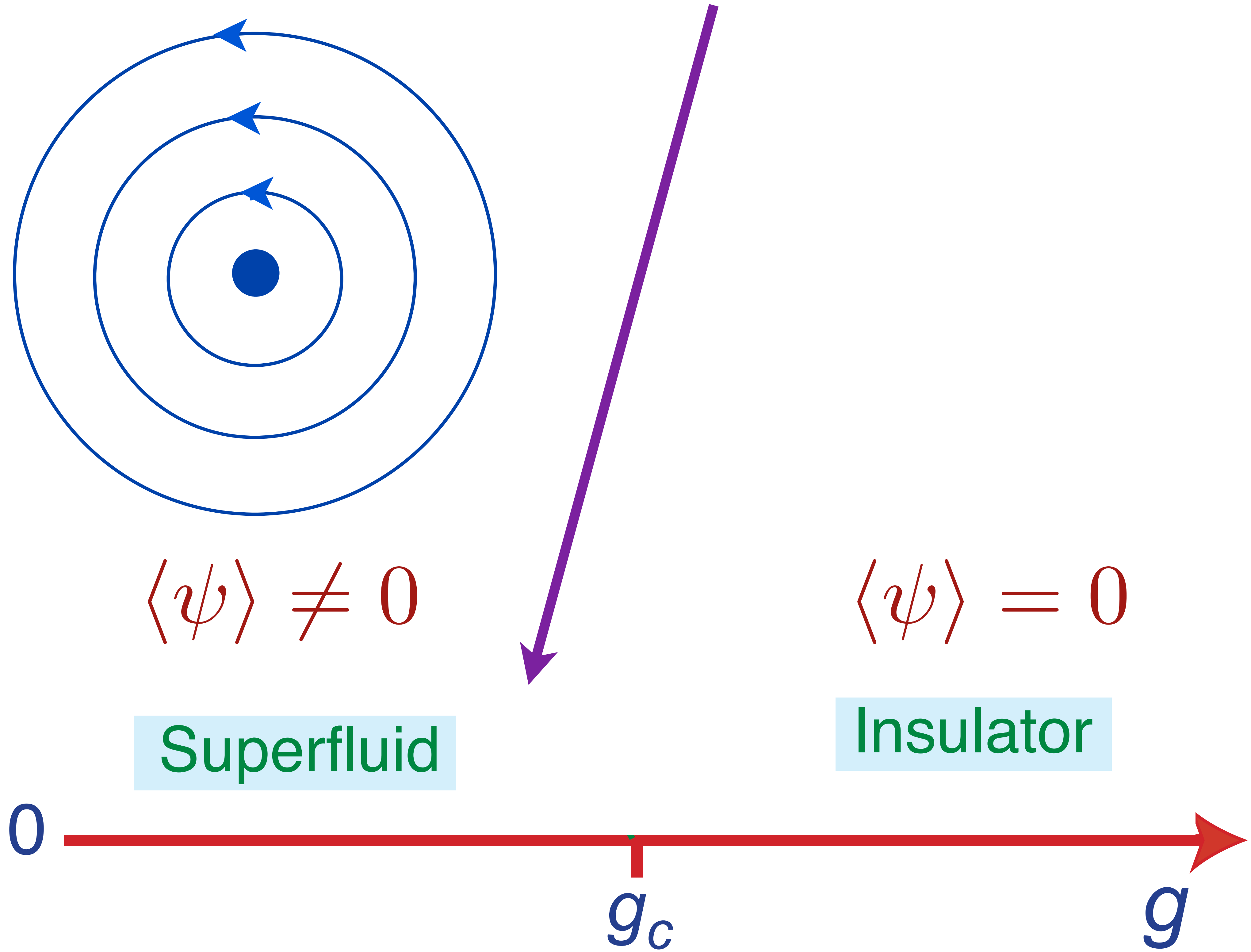


K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



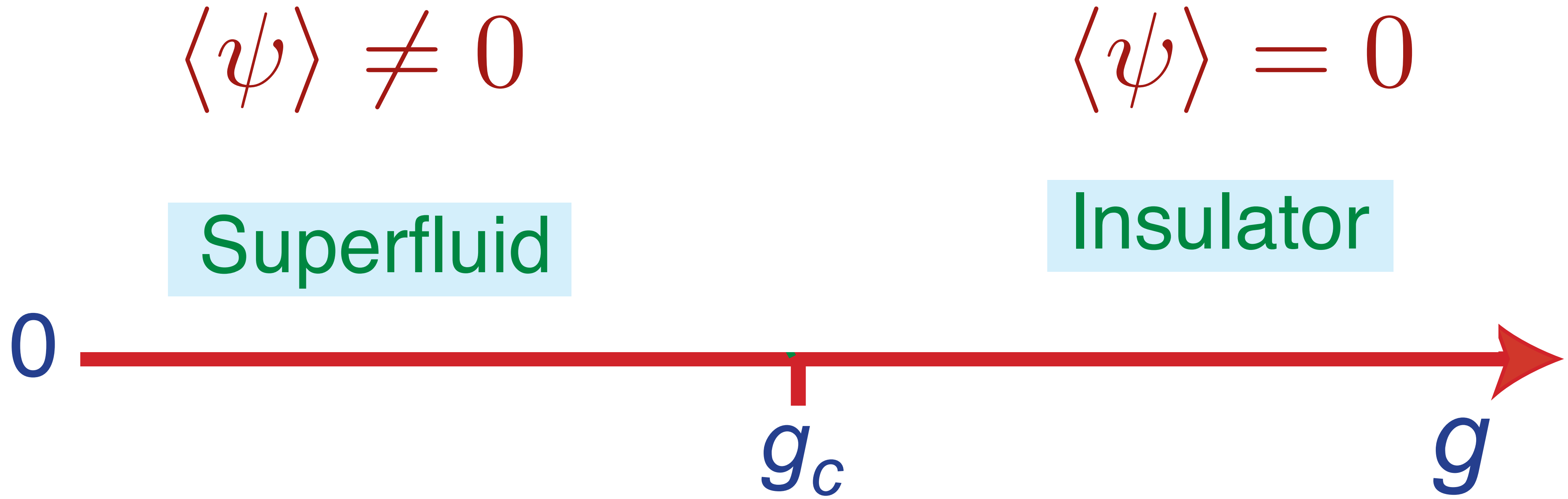
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



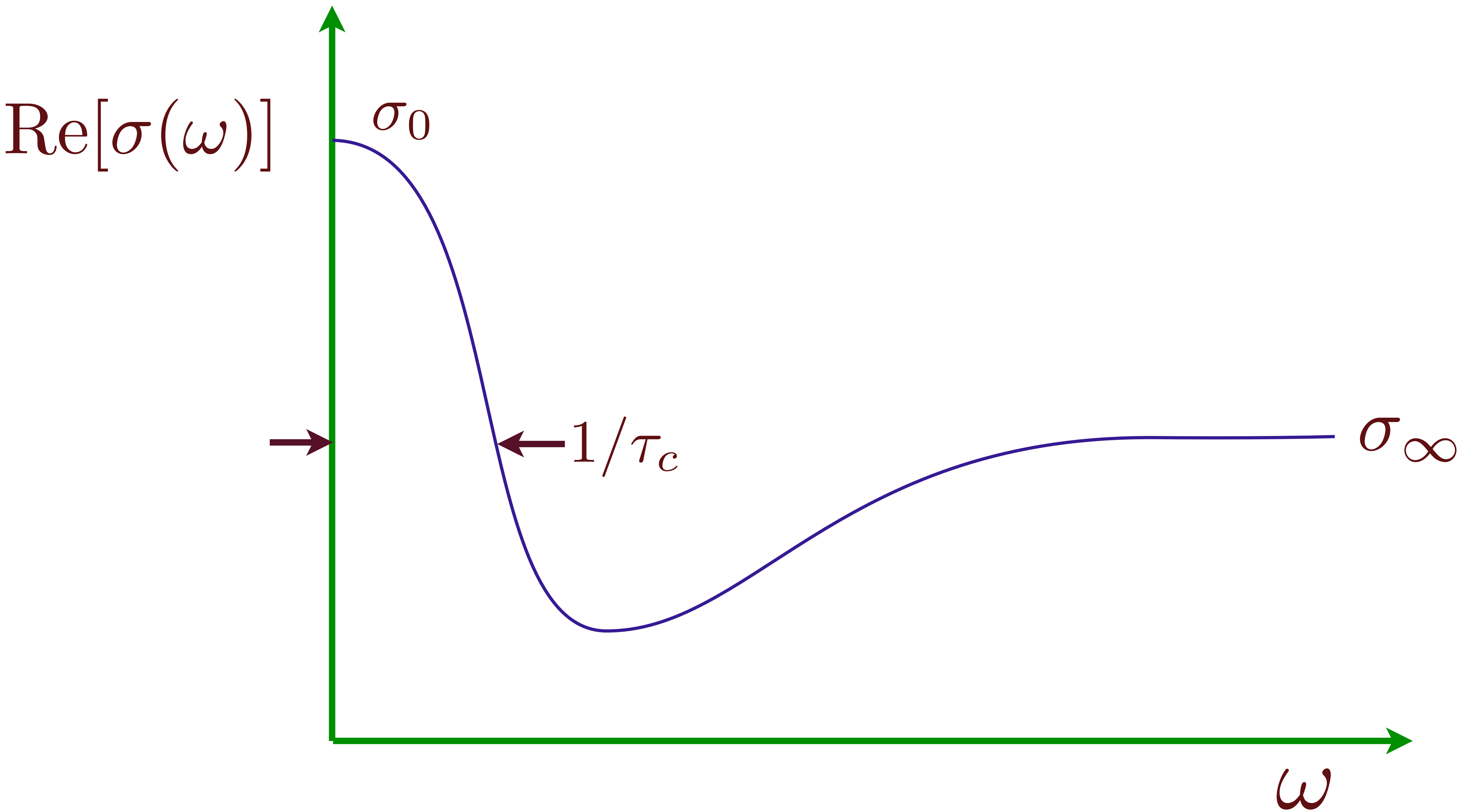
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

$$\text{Conductivity} = \text{Resistivity of vortices}$$



Boltzmann theory of bosons



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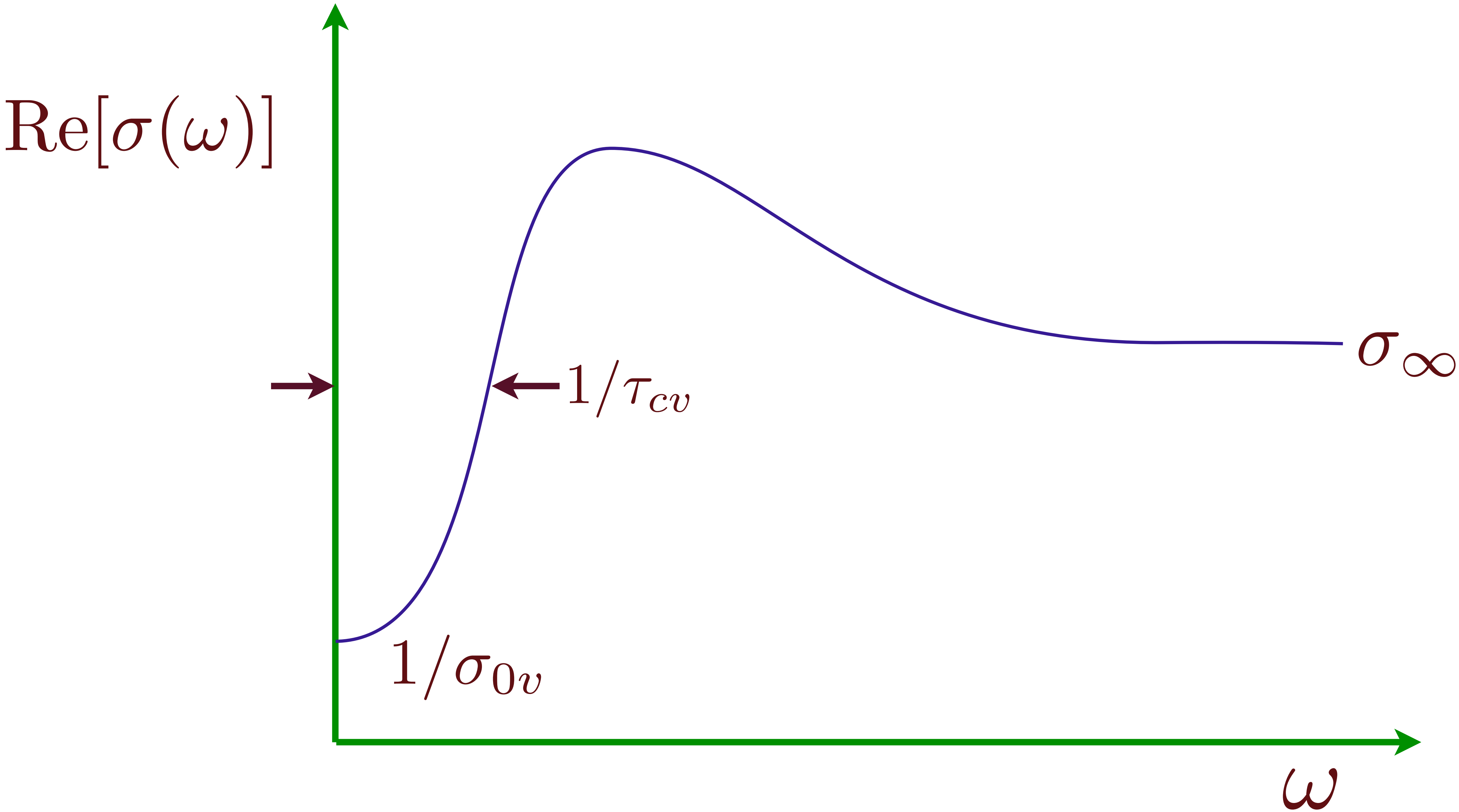
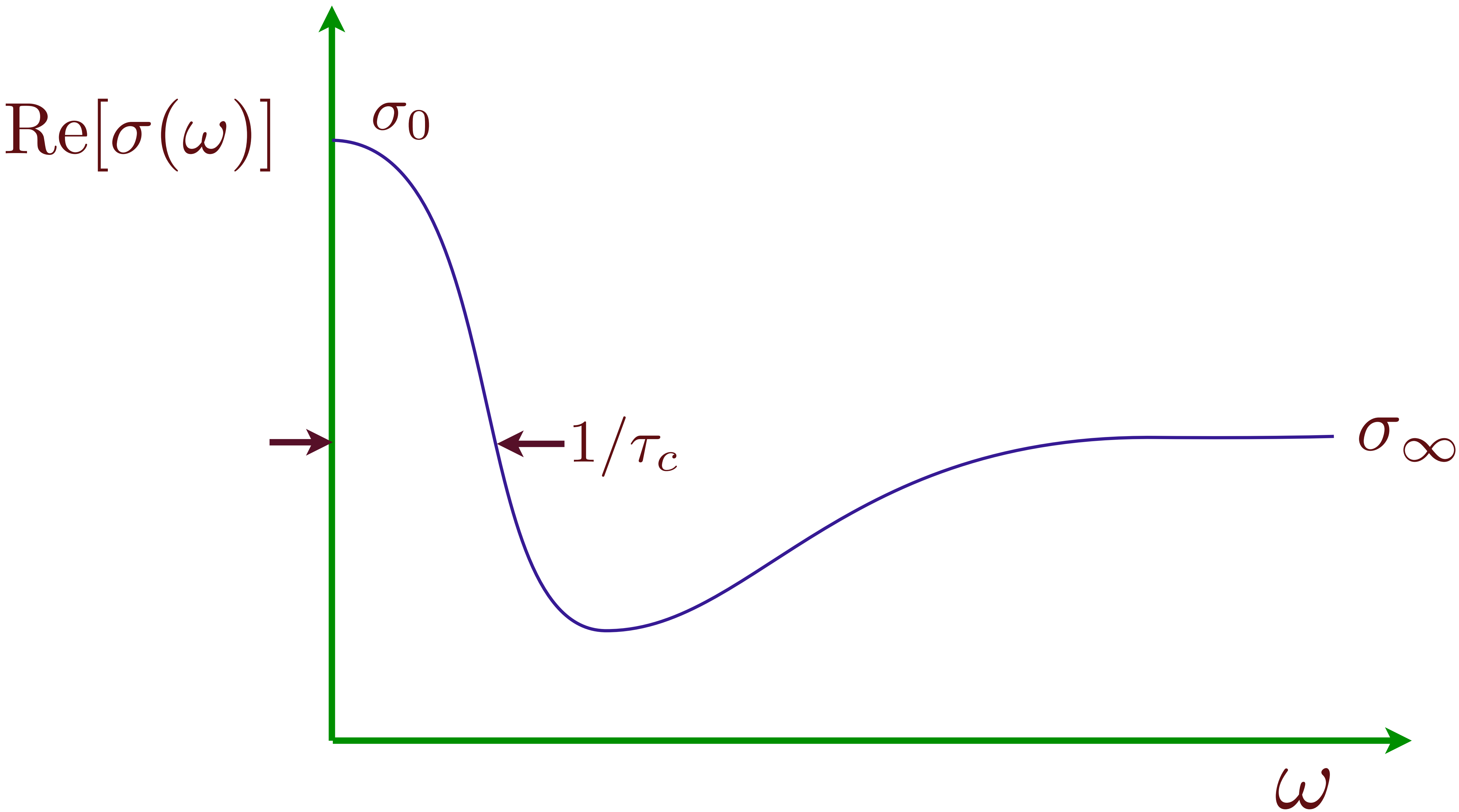
Boltzmann theory of vortices



Boltzmann theory of bosons



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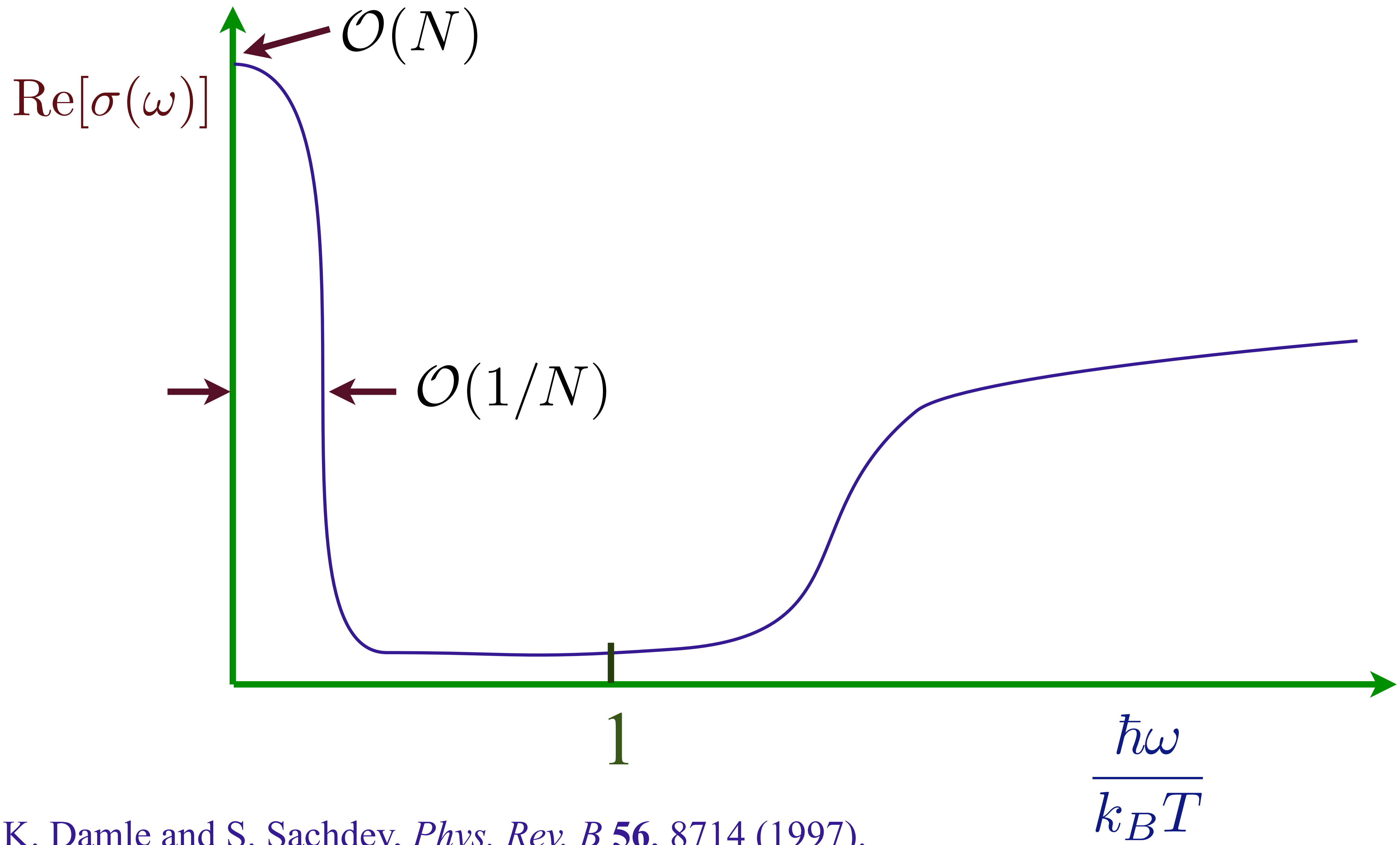
Vector large N expansion for CFT3

$$\sigma = \frac{Q^2}{h} \Sigma \left(\frac{\hbar \omega}{k_B T} \right); \quad \Sigma \rightarrow \text{a universal function}$$



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Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value

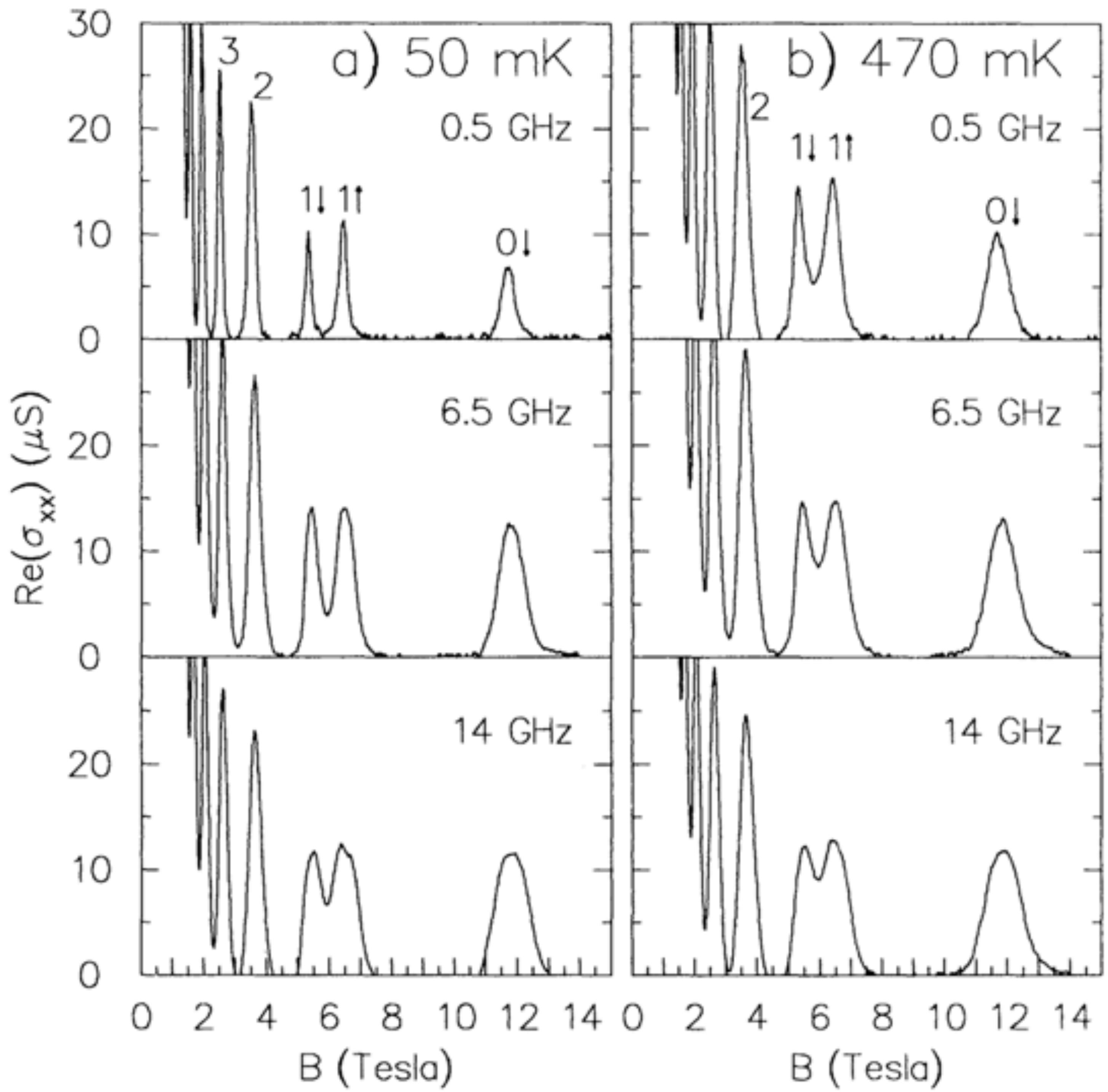

FIG. 3. Re(σ_{xx}) vs B at three frequencies and two temperatures. Peaks are marked with Landau level index N and spin.

L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui, *Physical Review Letters* **71**, 2638 (1993).

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   - *A. Strange metals: experiments and theoretical framework*
   - *B. ABJM (-like) theory at non-zero density*
   - *C. Gauge-gravity duality*

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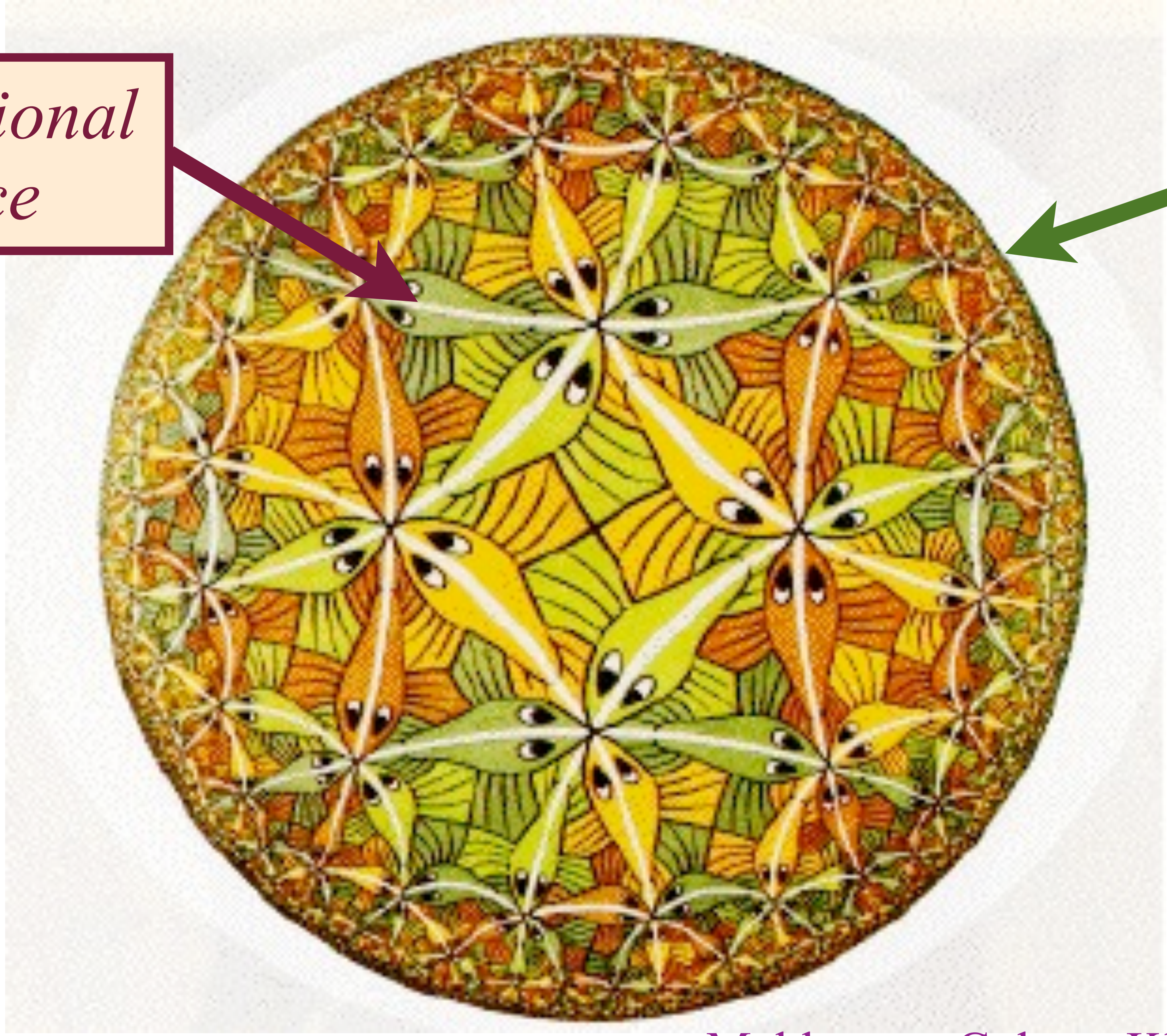
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

3+1 dimensional AdS space



A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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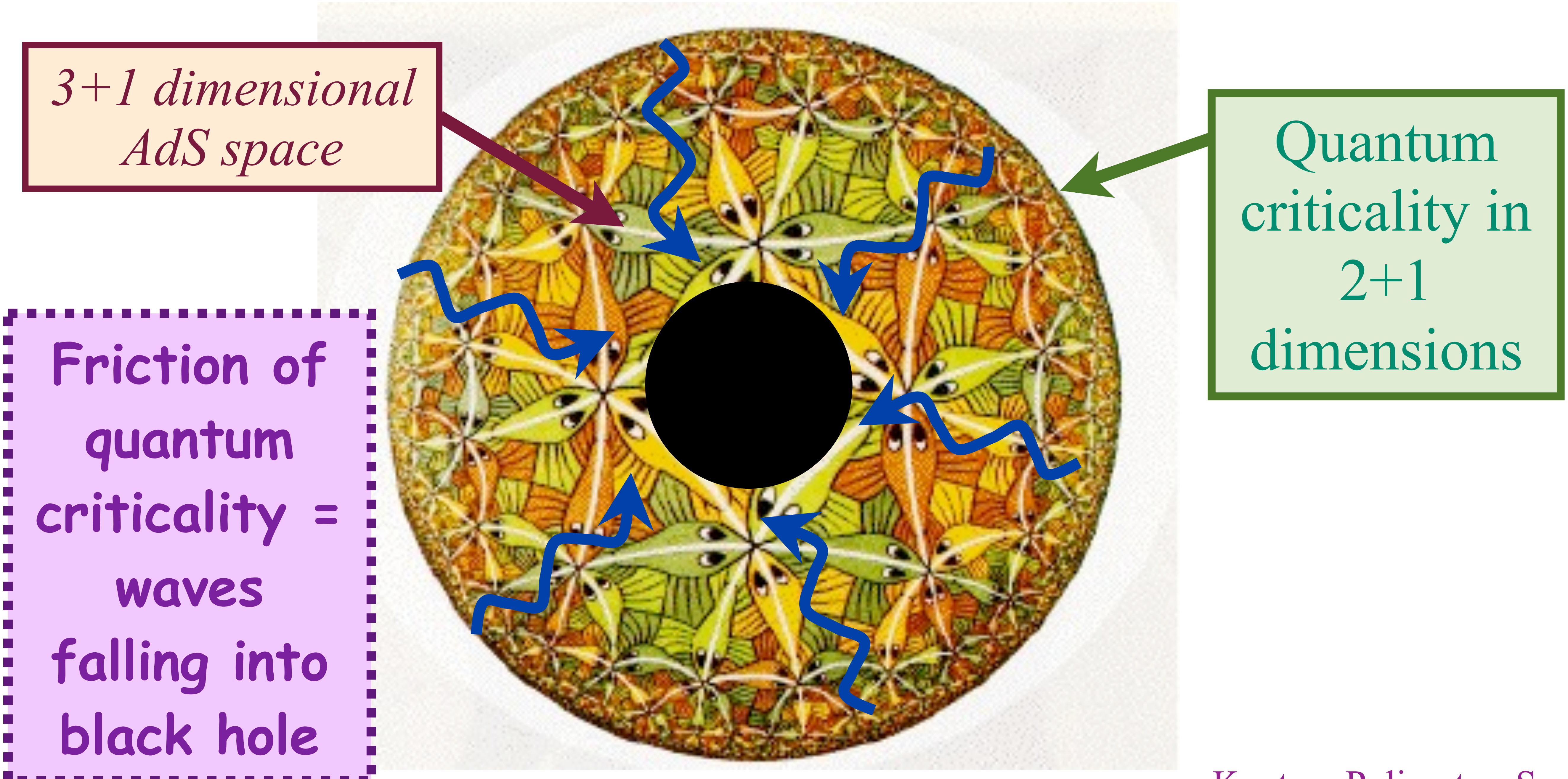

Kovtun, Policastro, Son

AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$

$$\mathcal{S}_{EM} = \frac{1}{g_4^2} \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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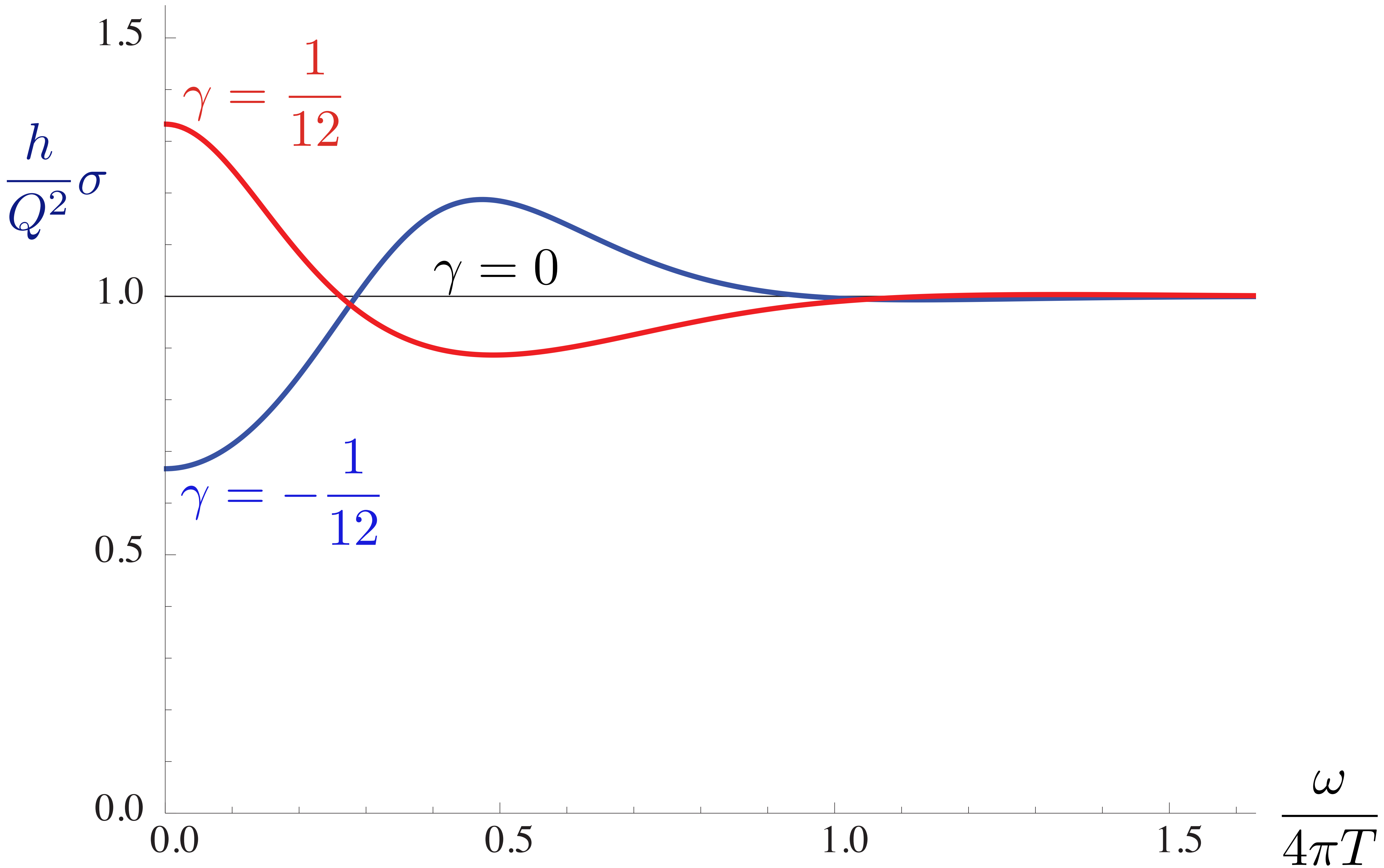
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

$$\mathcal{S} = \frac{1}{g_4^2} \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{ab} F^{ab} + \gamma \, L^2 C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS_4 theory of strongly interacting "perfect fluids"



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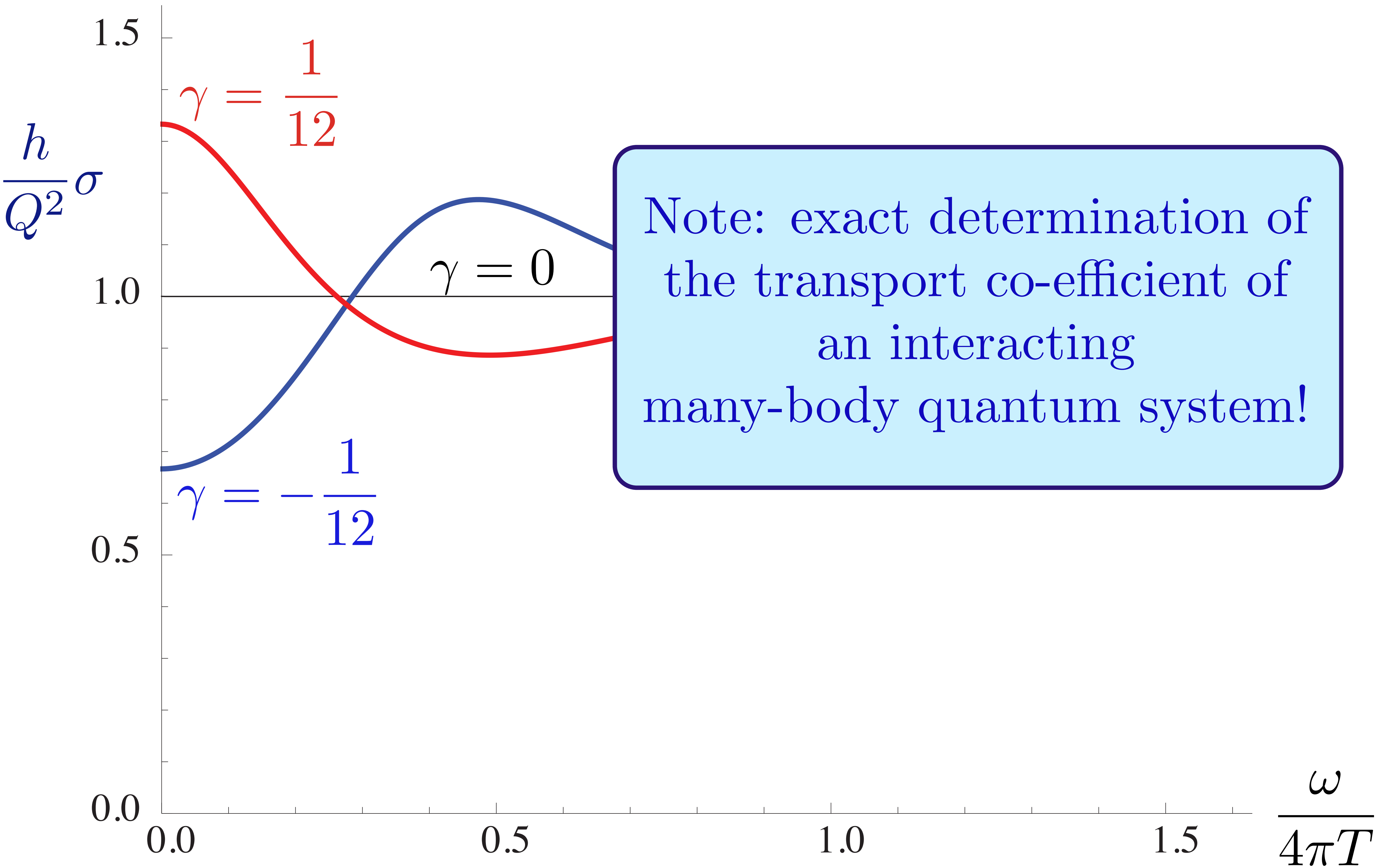

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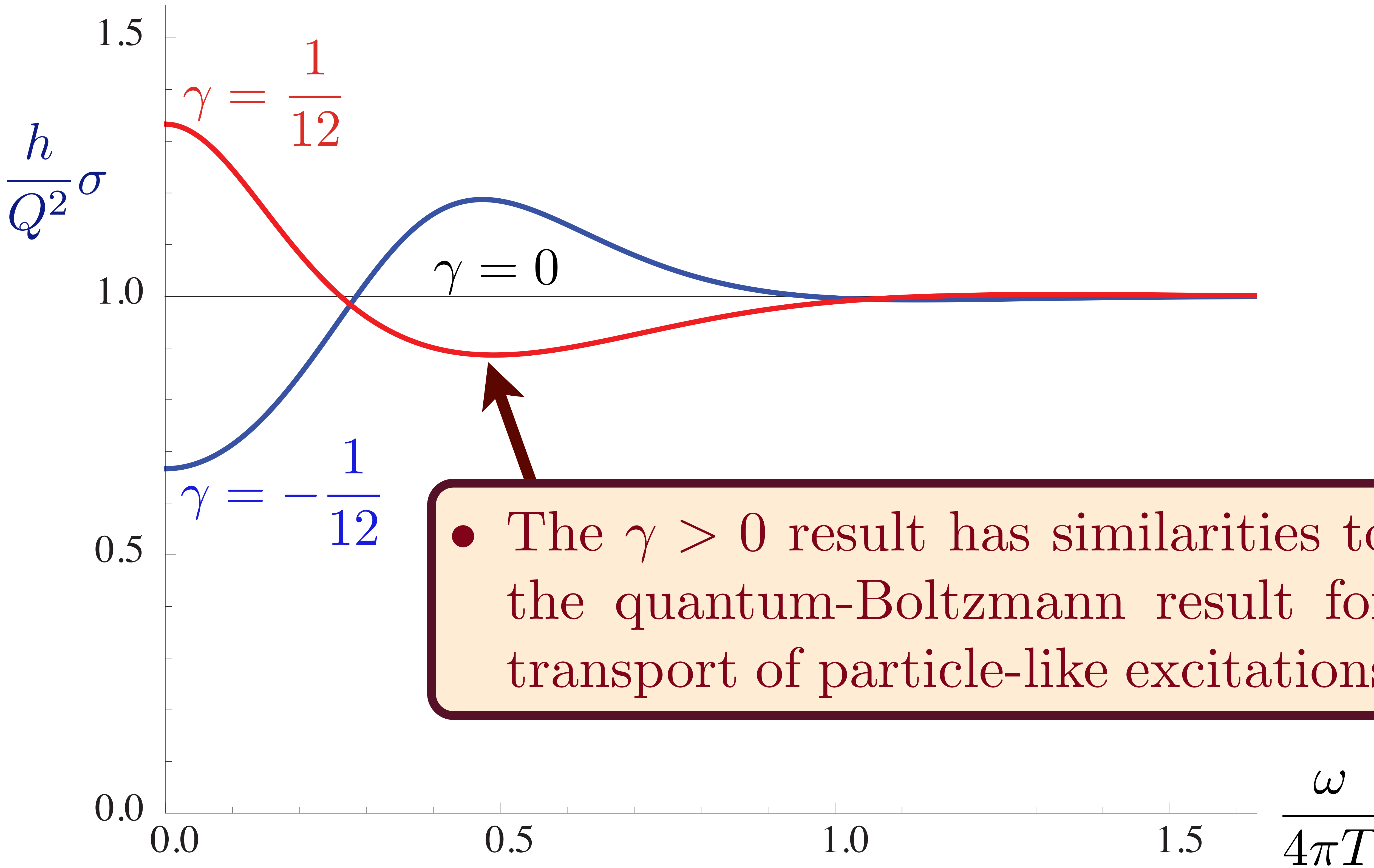

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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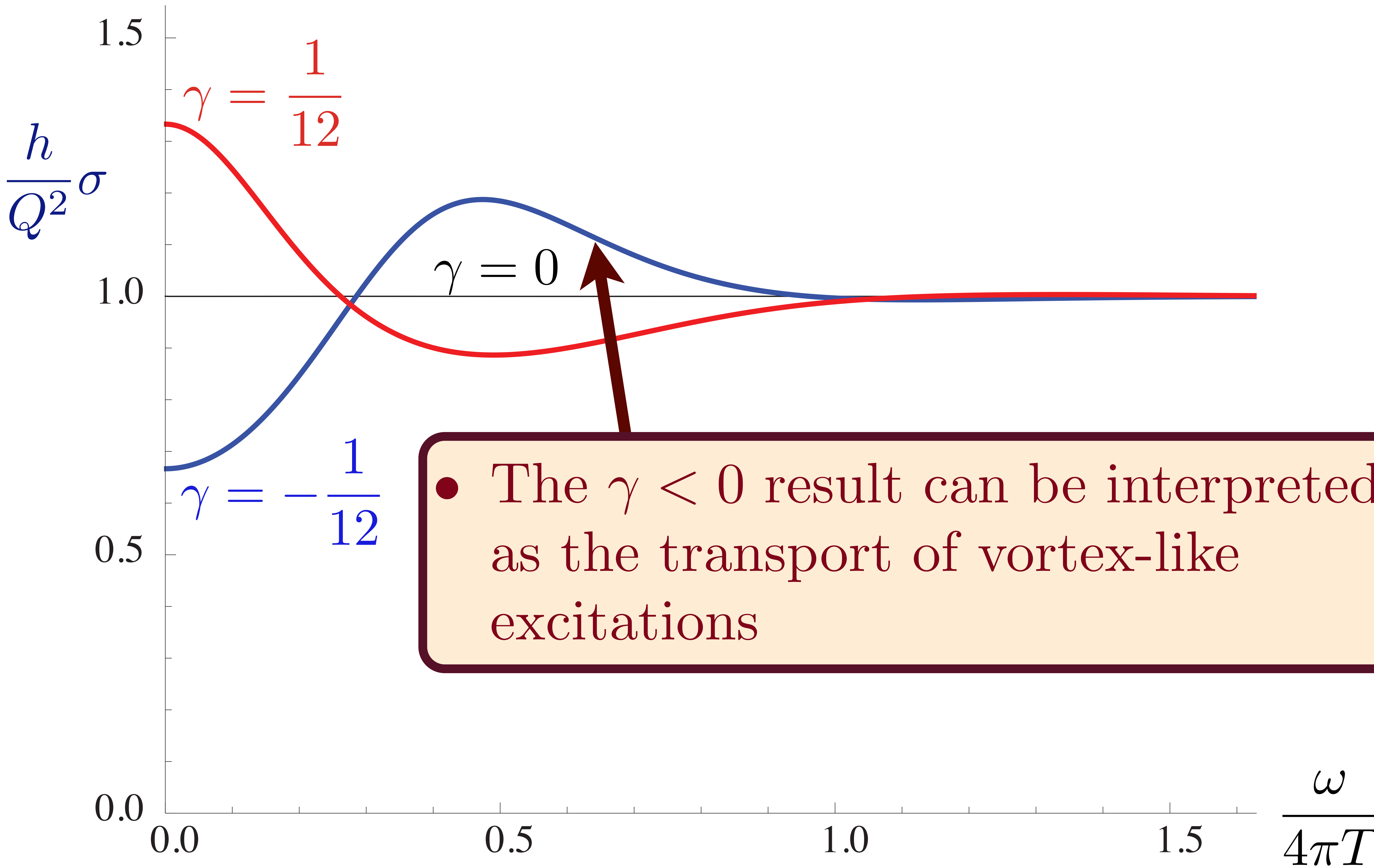

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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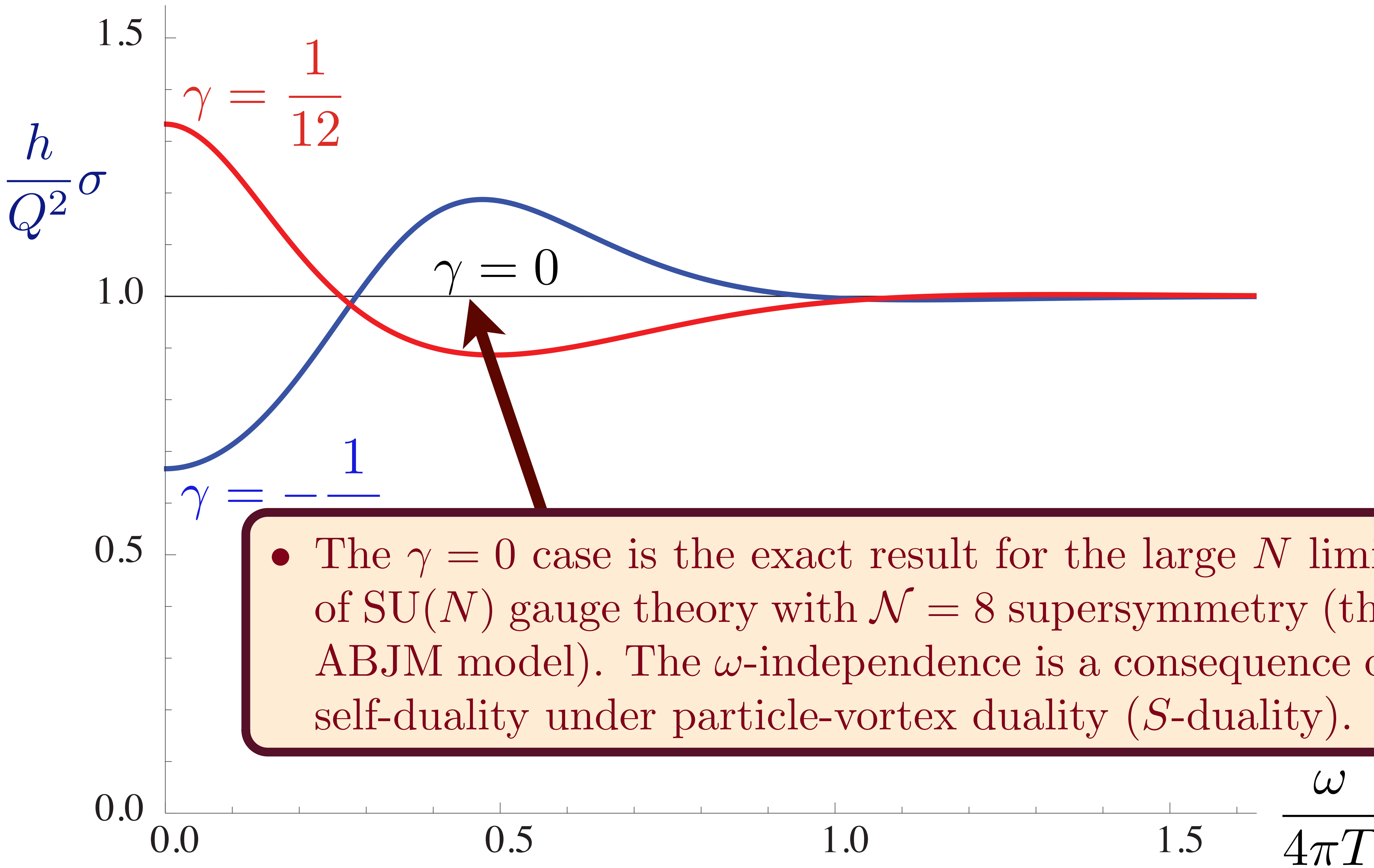

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

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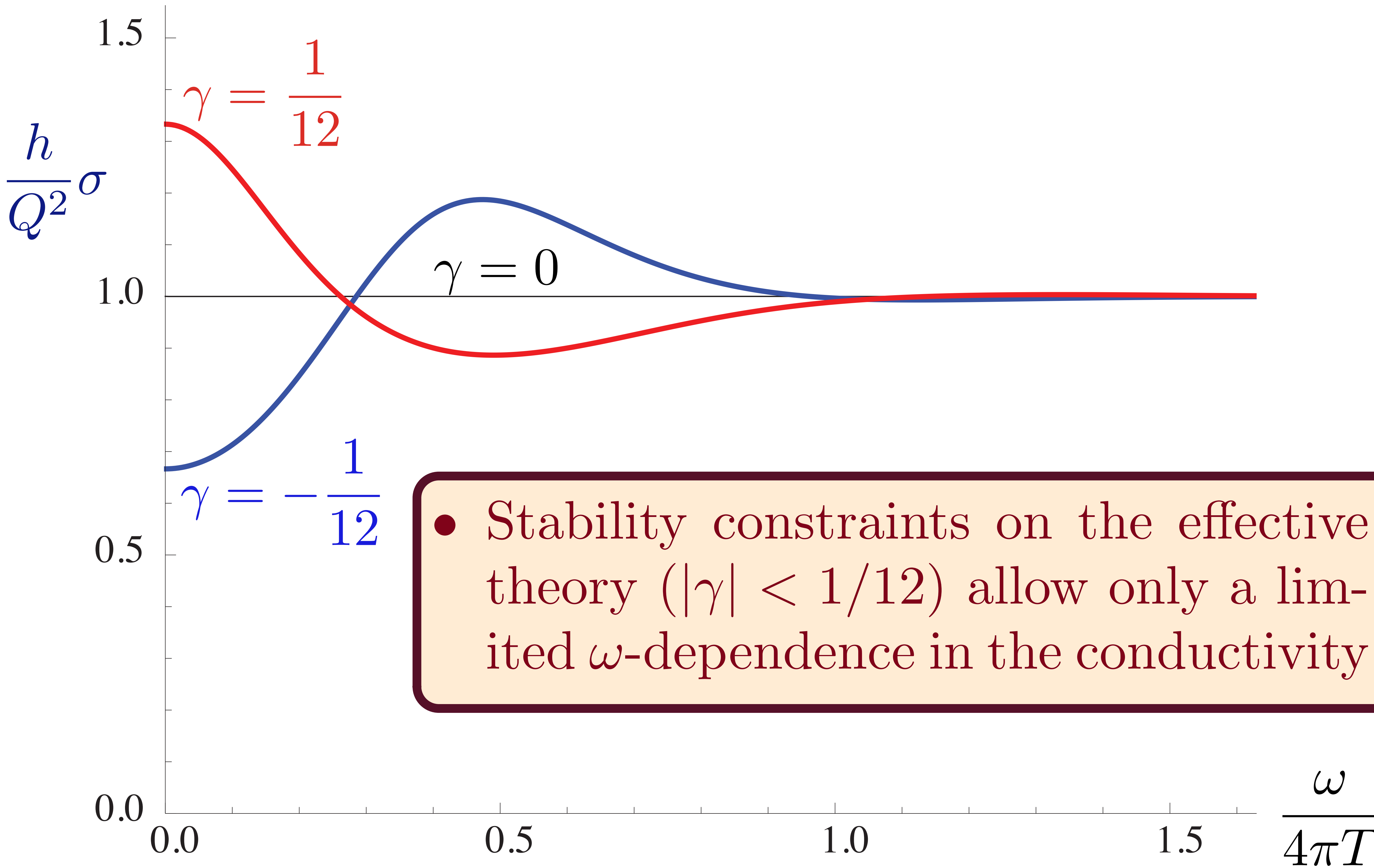
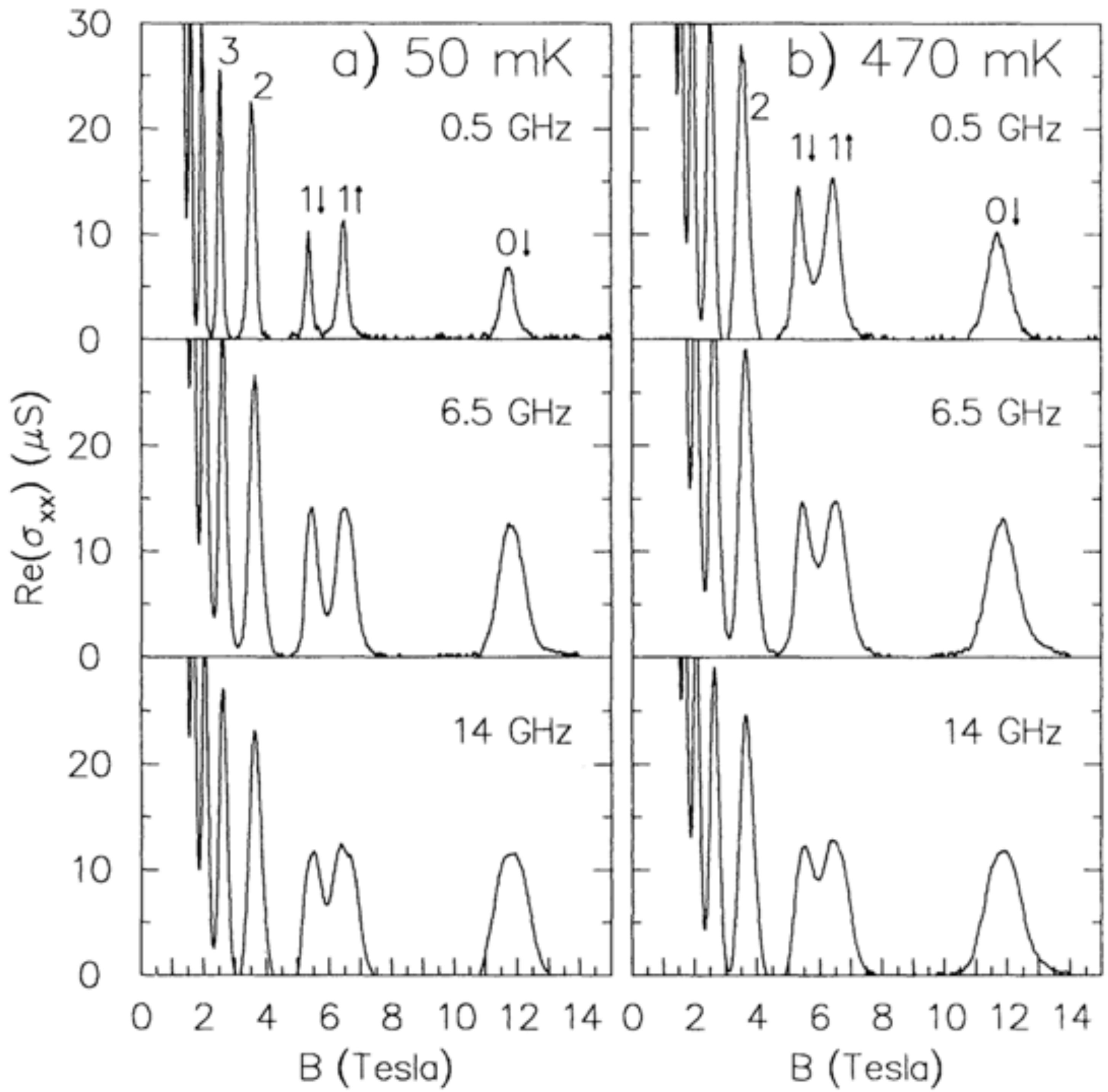


- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value



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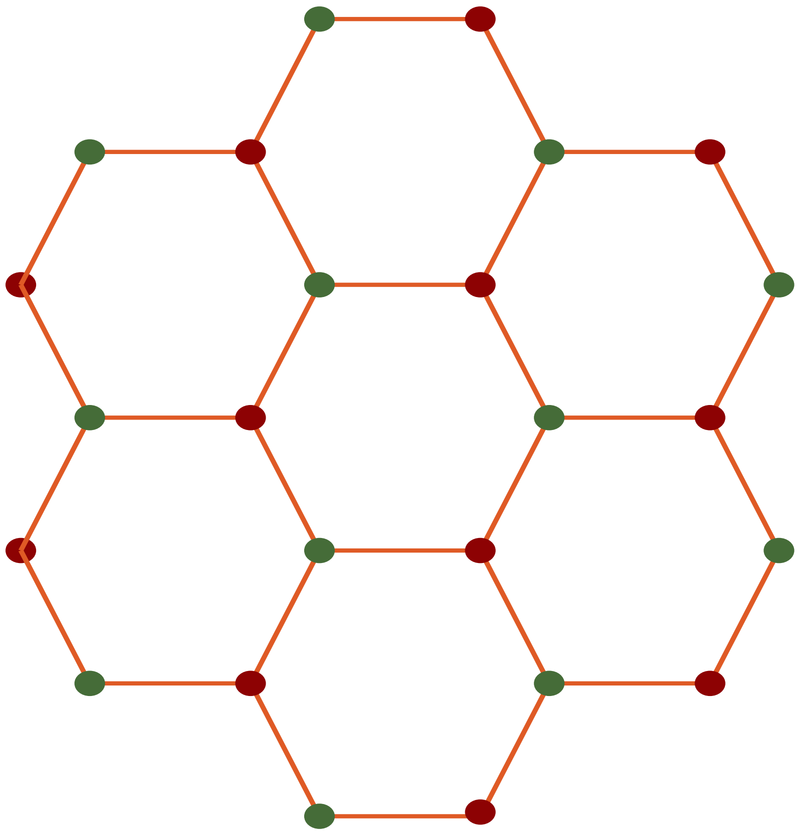
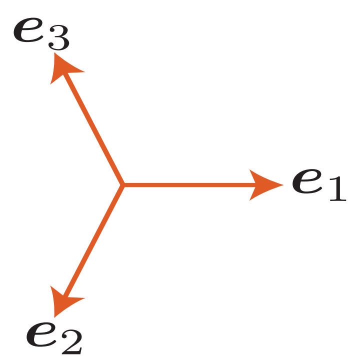
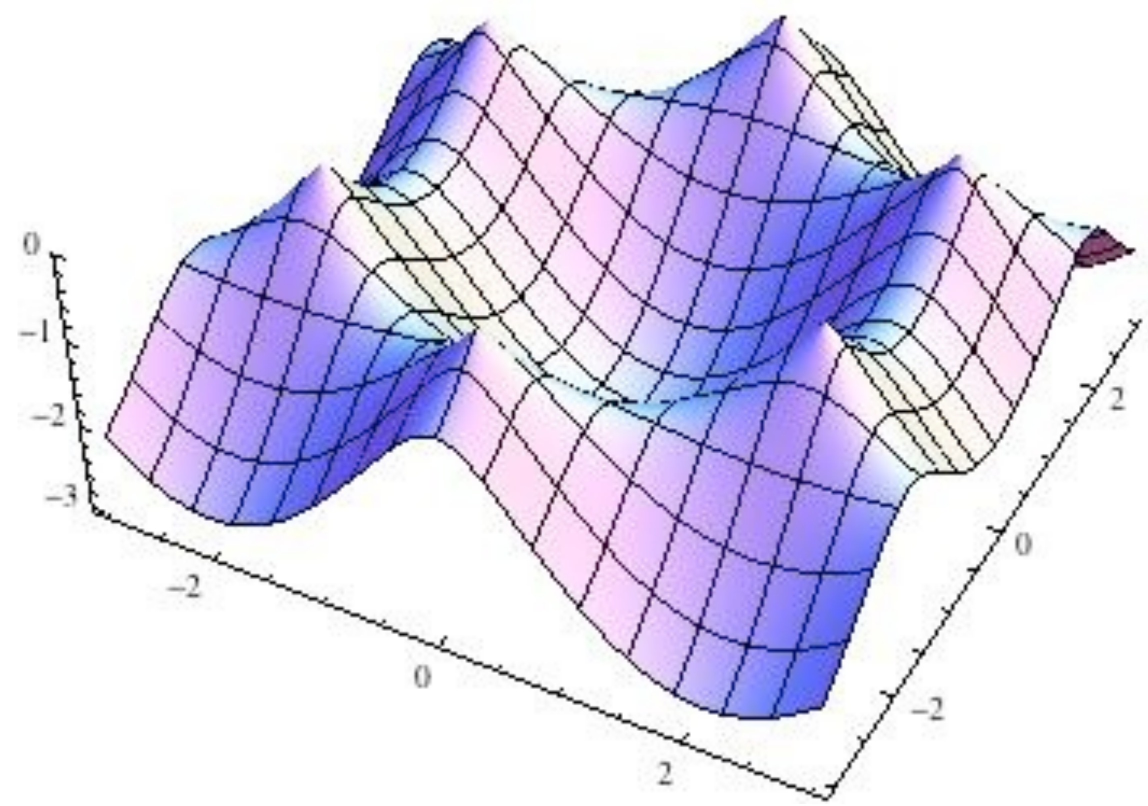
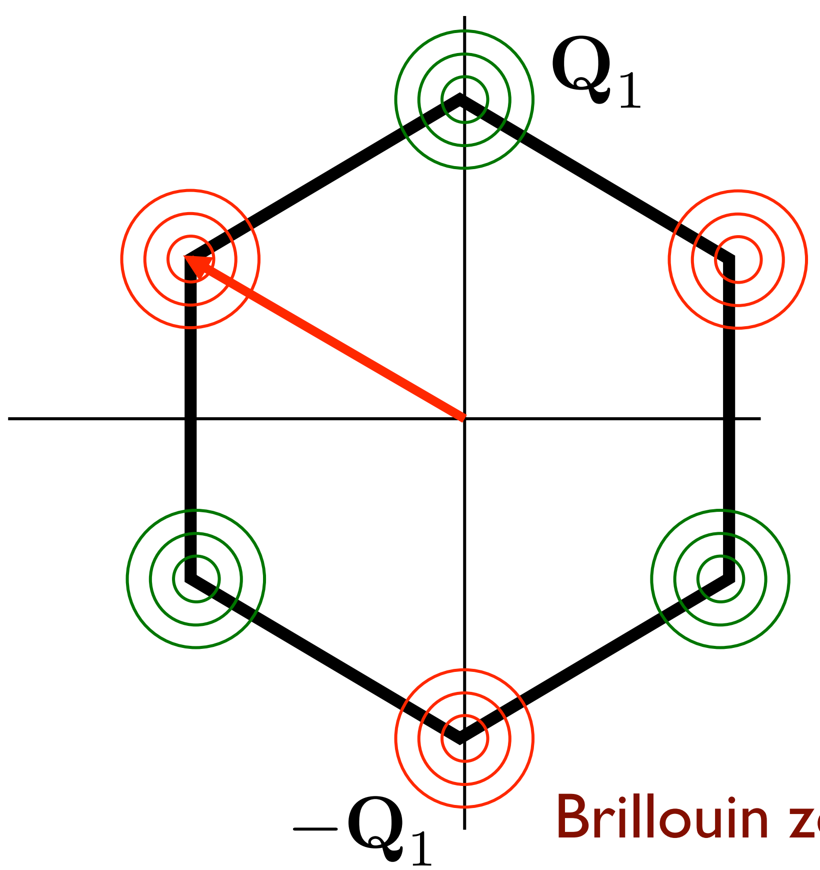
Graphene





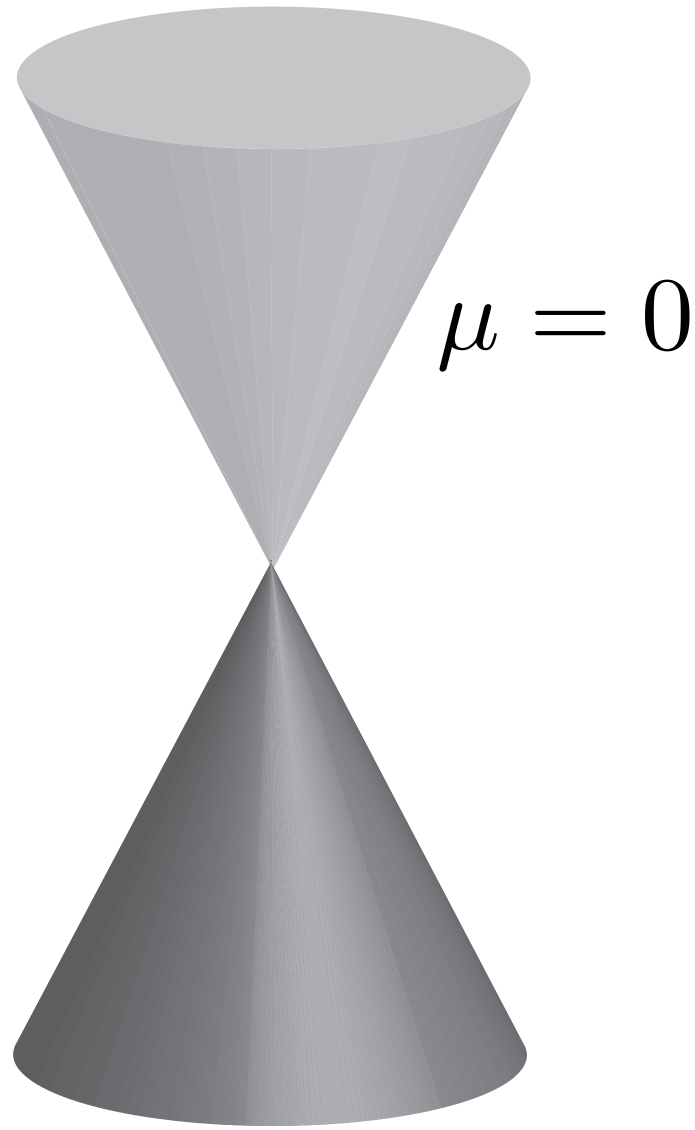




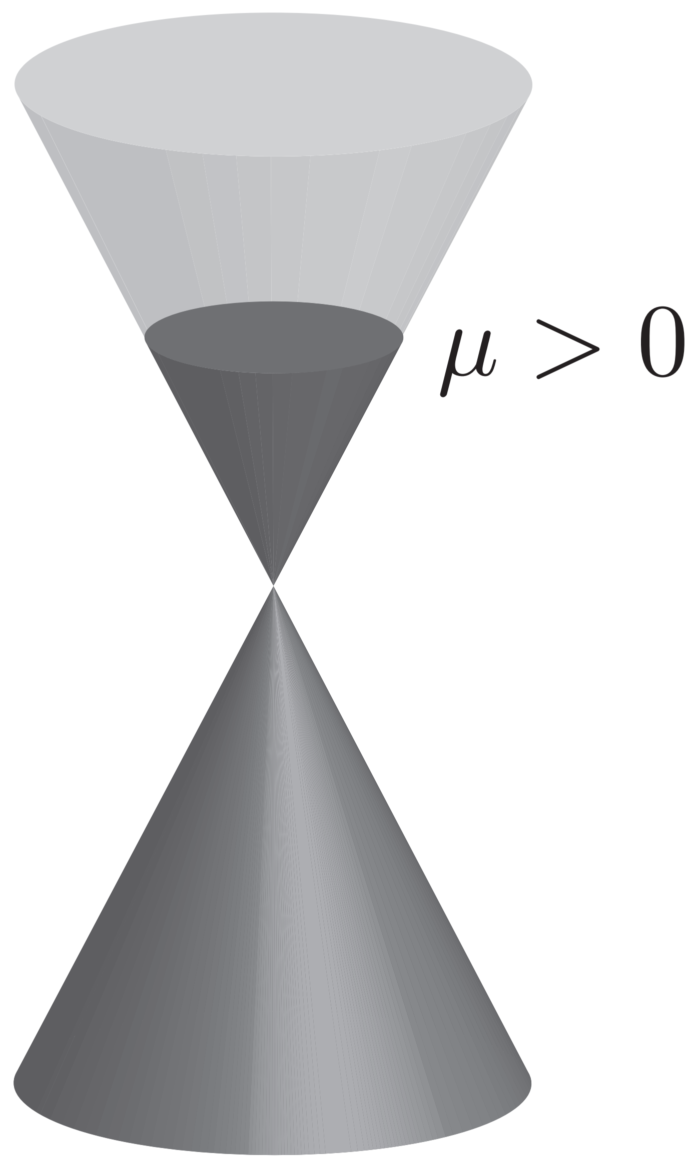
Brillouin zone

Semi-metal with
massless Dirac fermions

Turn on a chemical potential on a CFT

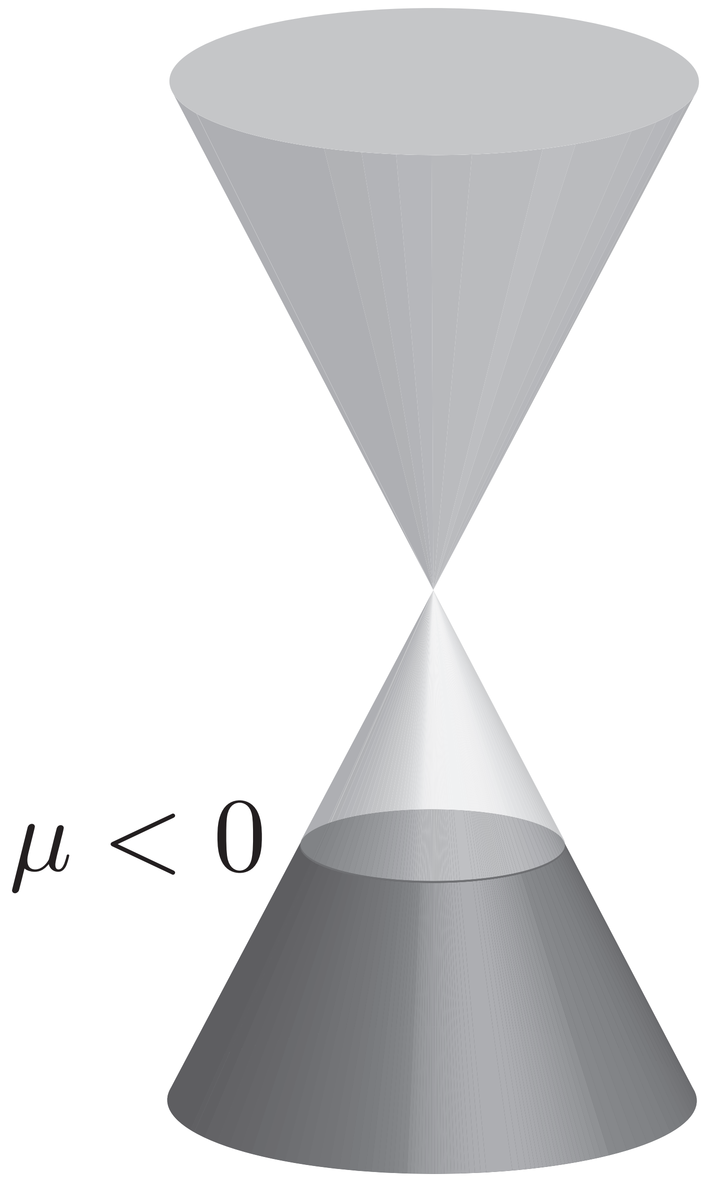


Turn on a chemical potential on a CFT



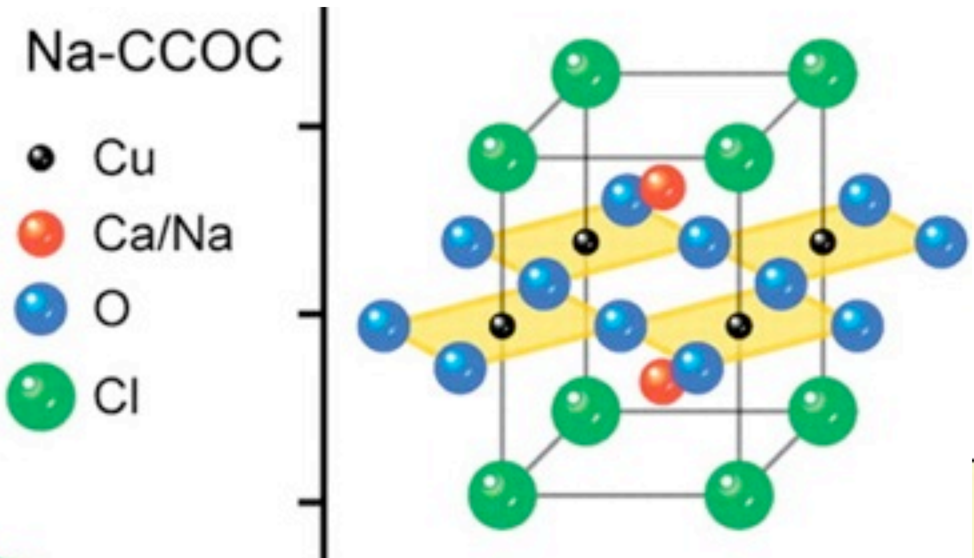
Electron
Fermi surface

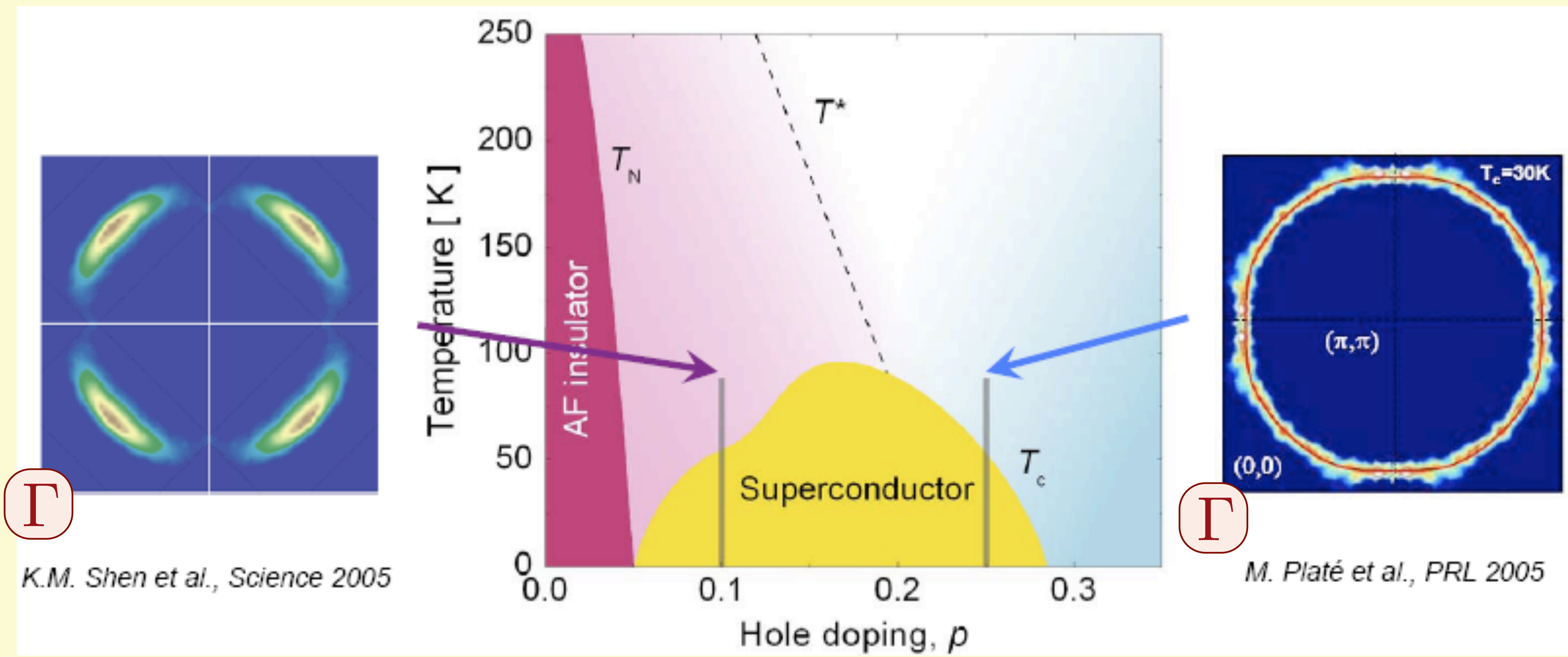
Turn on a chemical potential on a CFT



Hole
Fermi surface

The cuprate superconductors

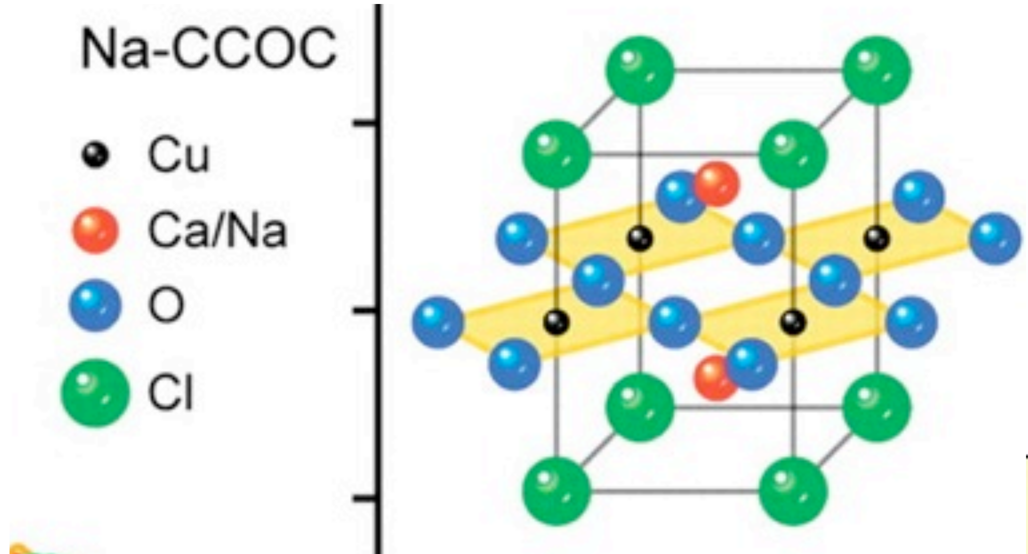


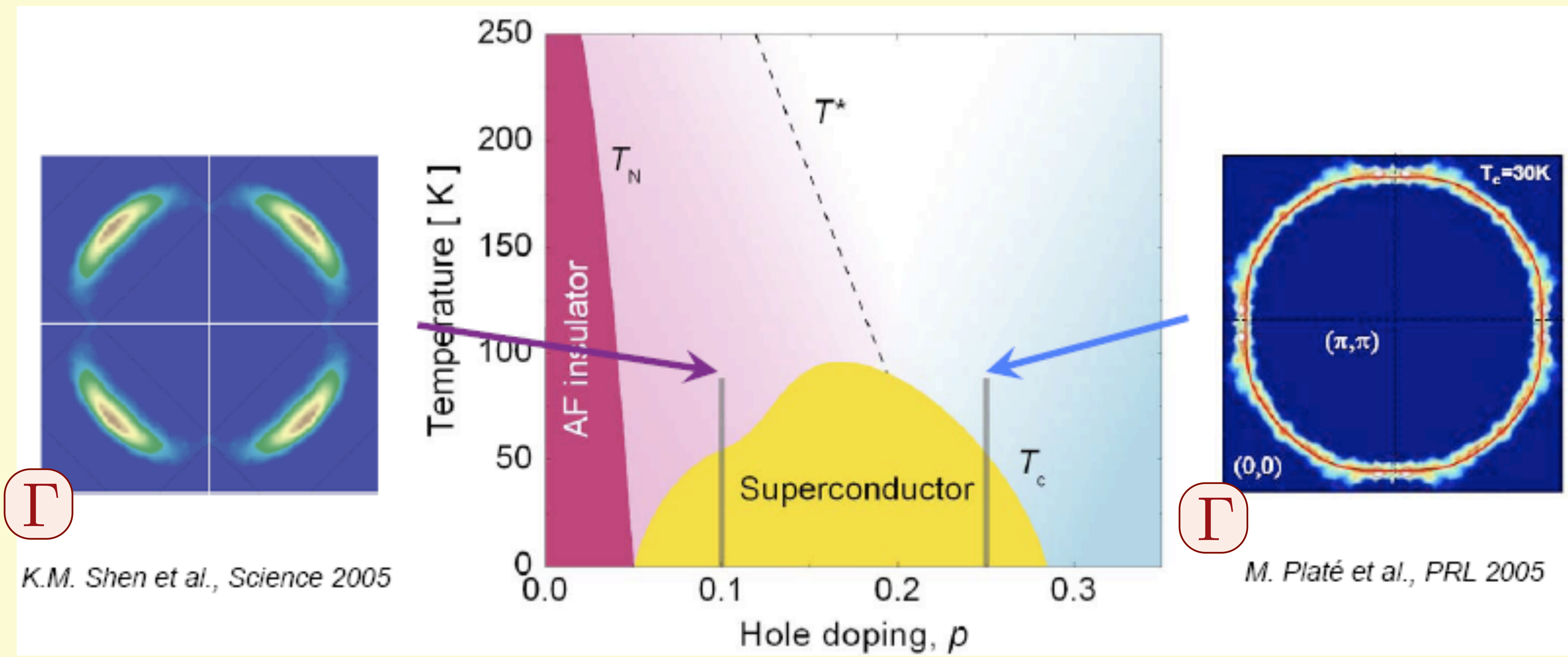


Smaller hole Fermi-pockets

Large hole Fermi surface

The cuprate superconductors





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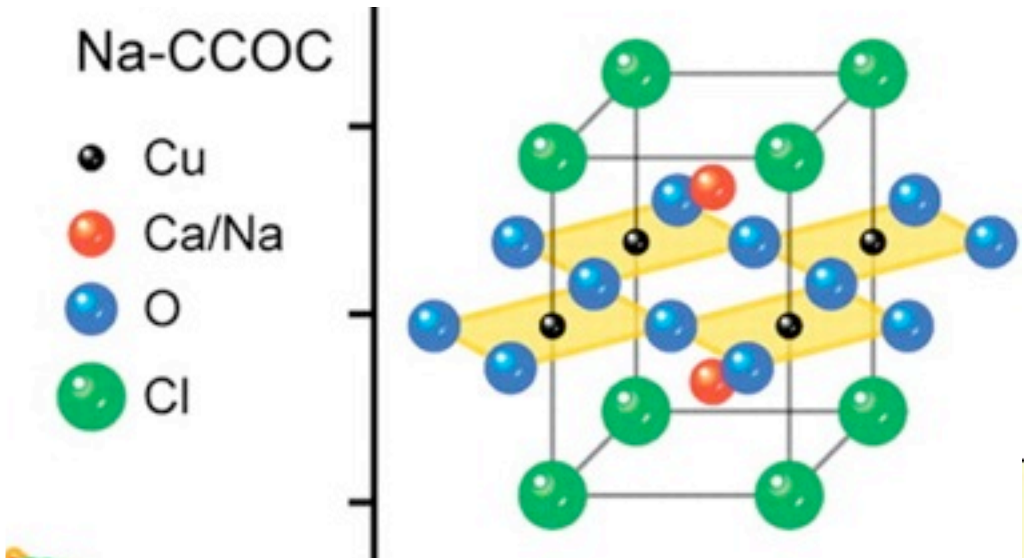
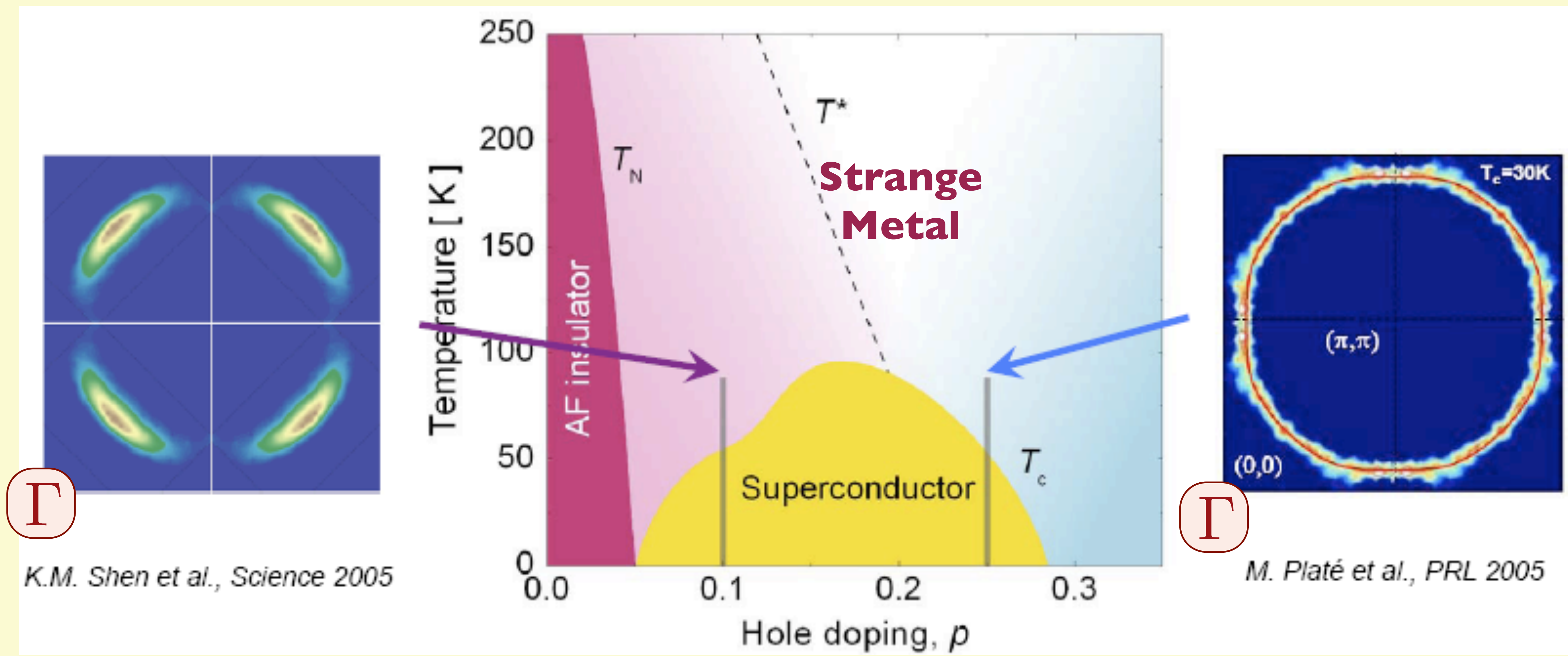




Smaller hole Fermi-pockets

Large hole Fermi surface

Electron-doped cuprate superconductors

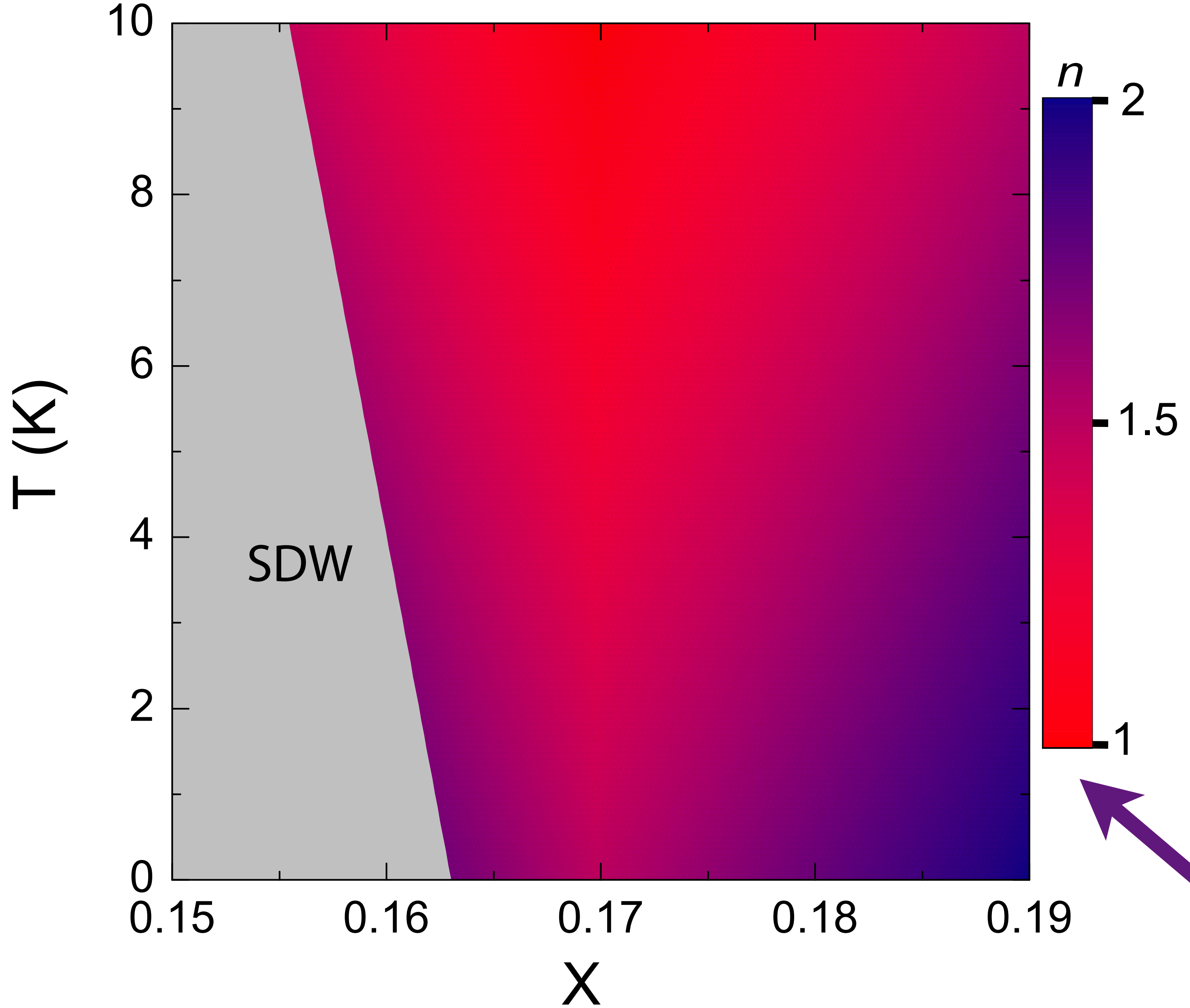
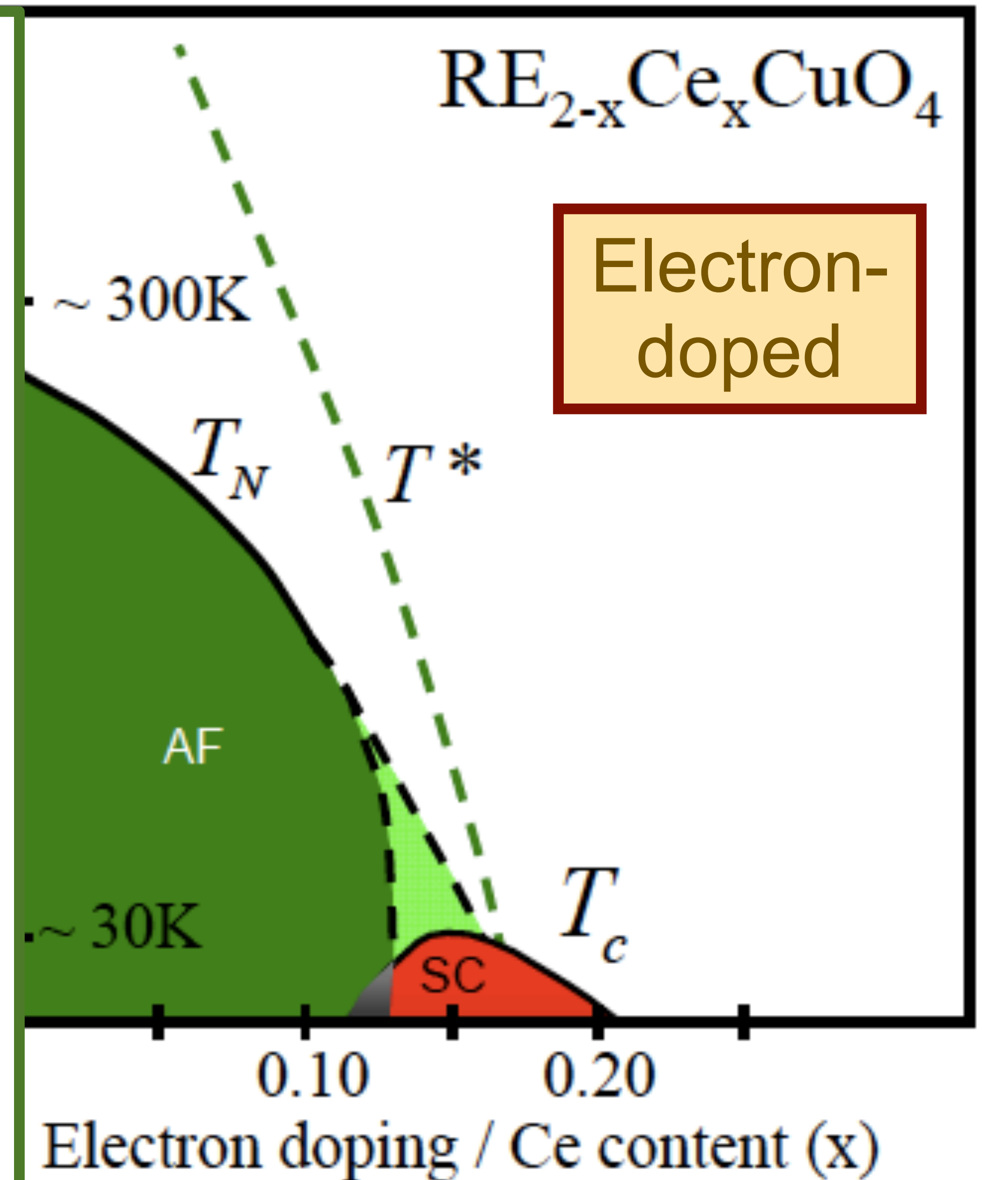


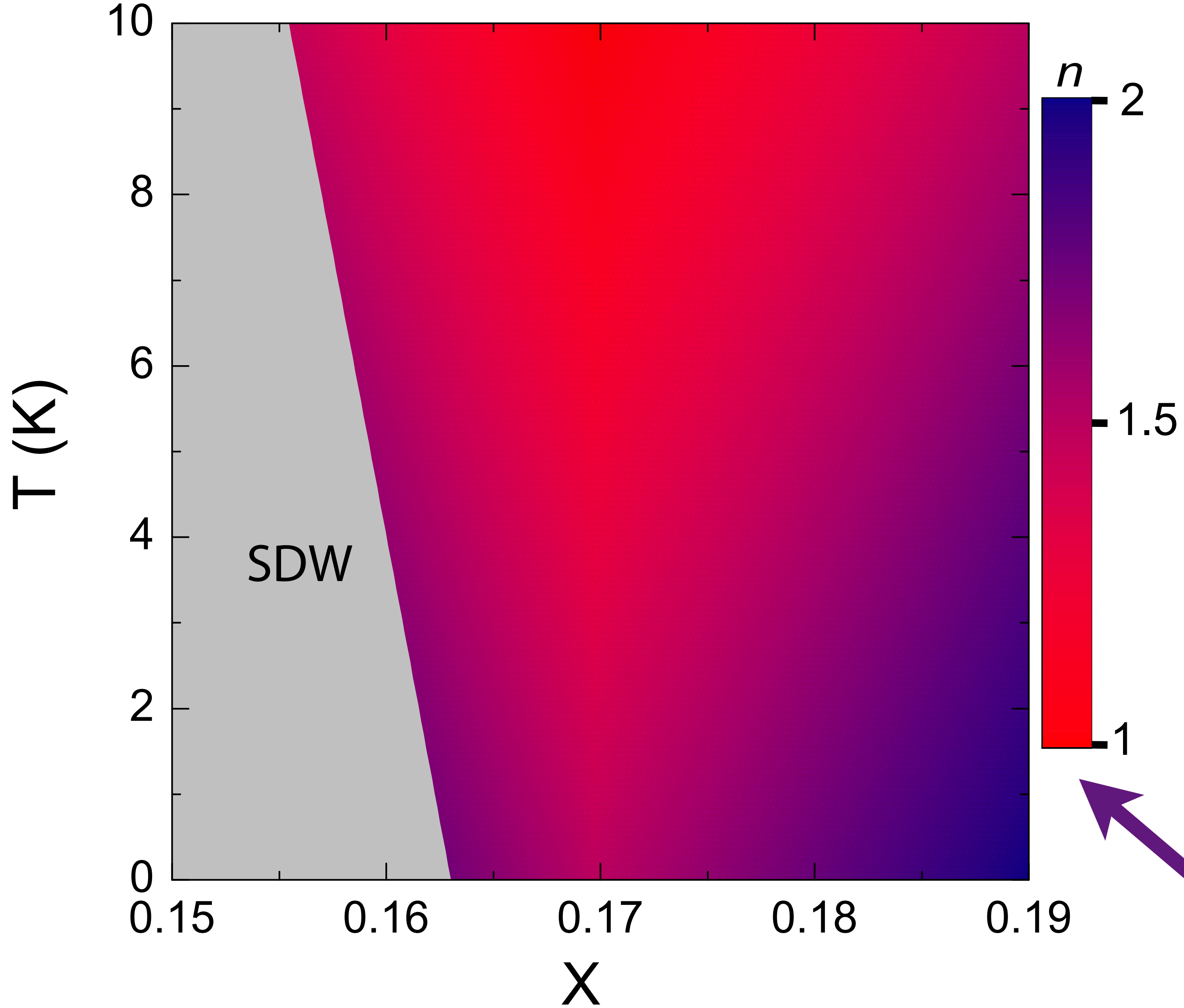
Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity $\sim \rho_0 + AT^n$

Wednesday, June 22, 2011

Electron-doped cuprate superconductors



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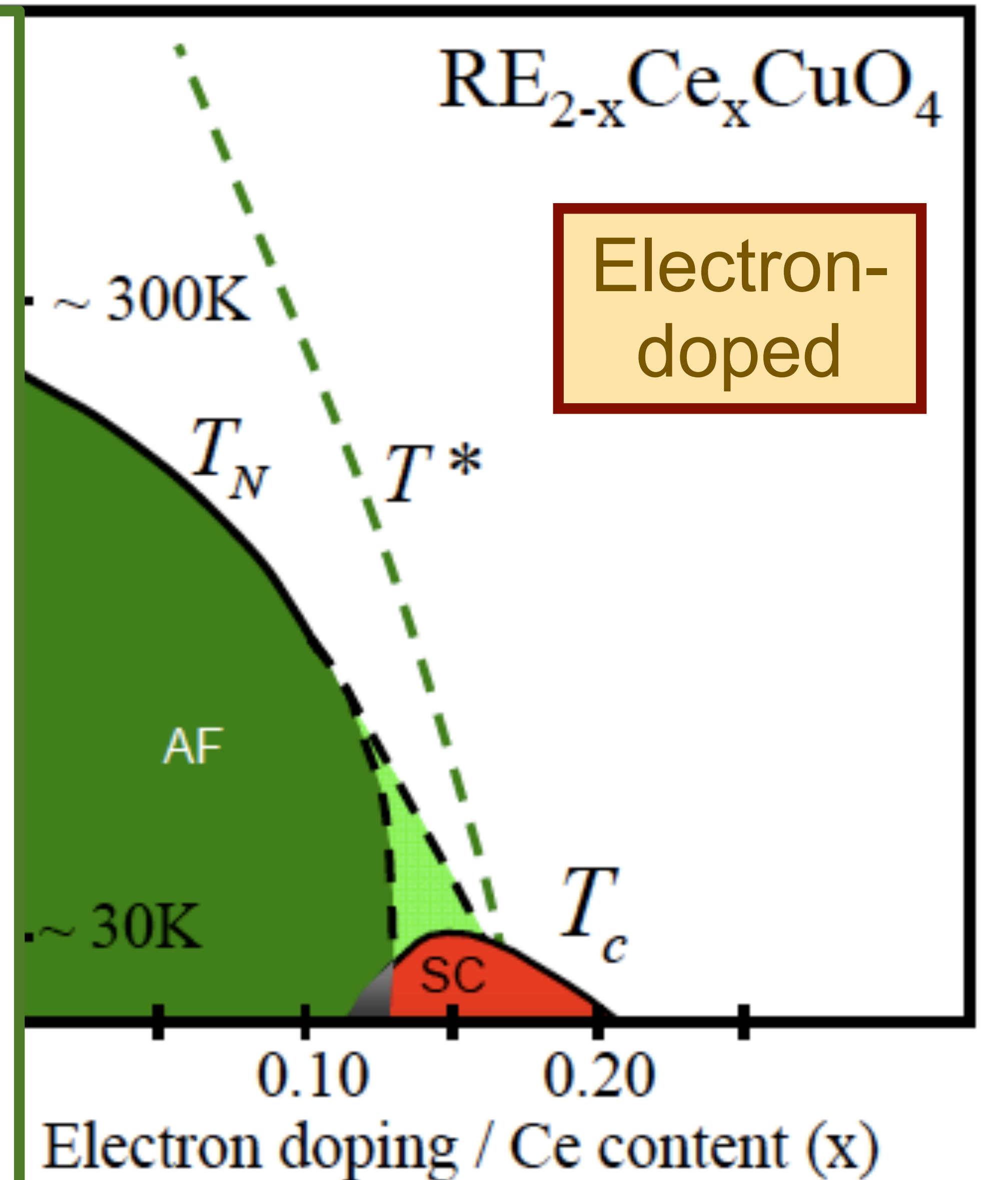

Resistivity

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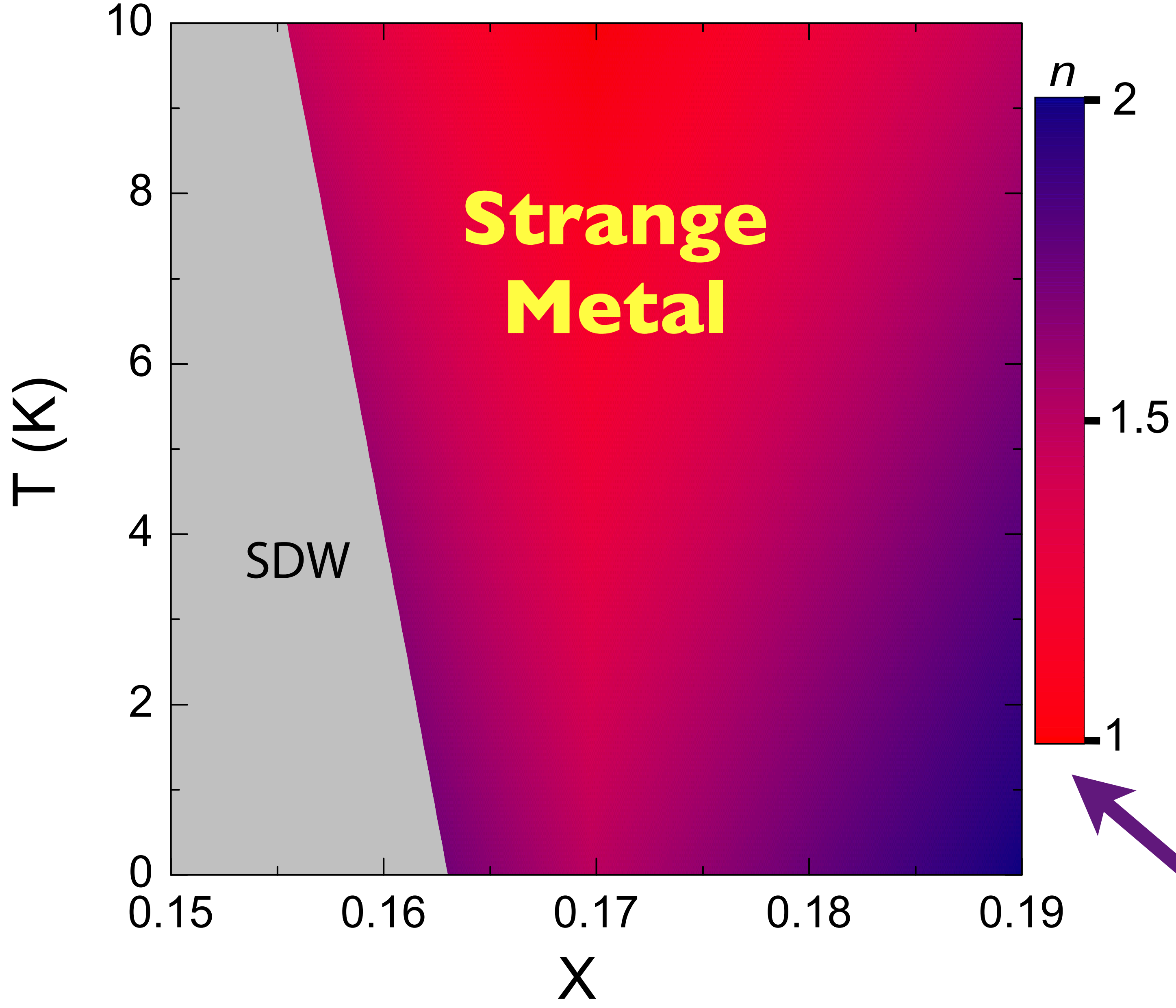
Electron-doped cuprate superconductors



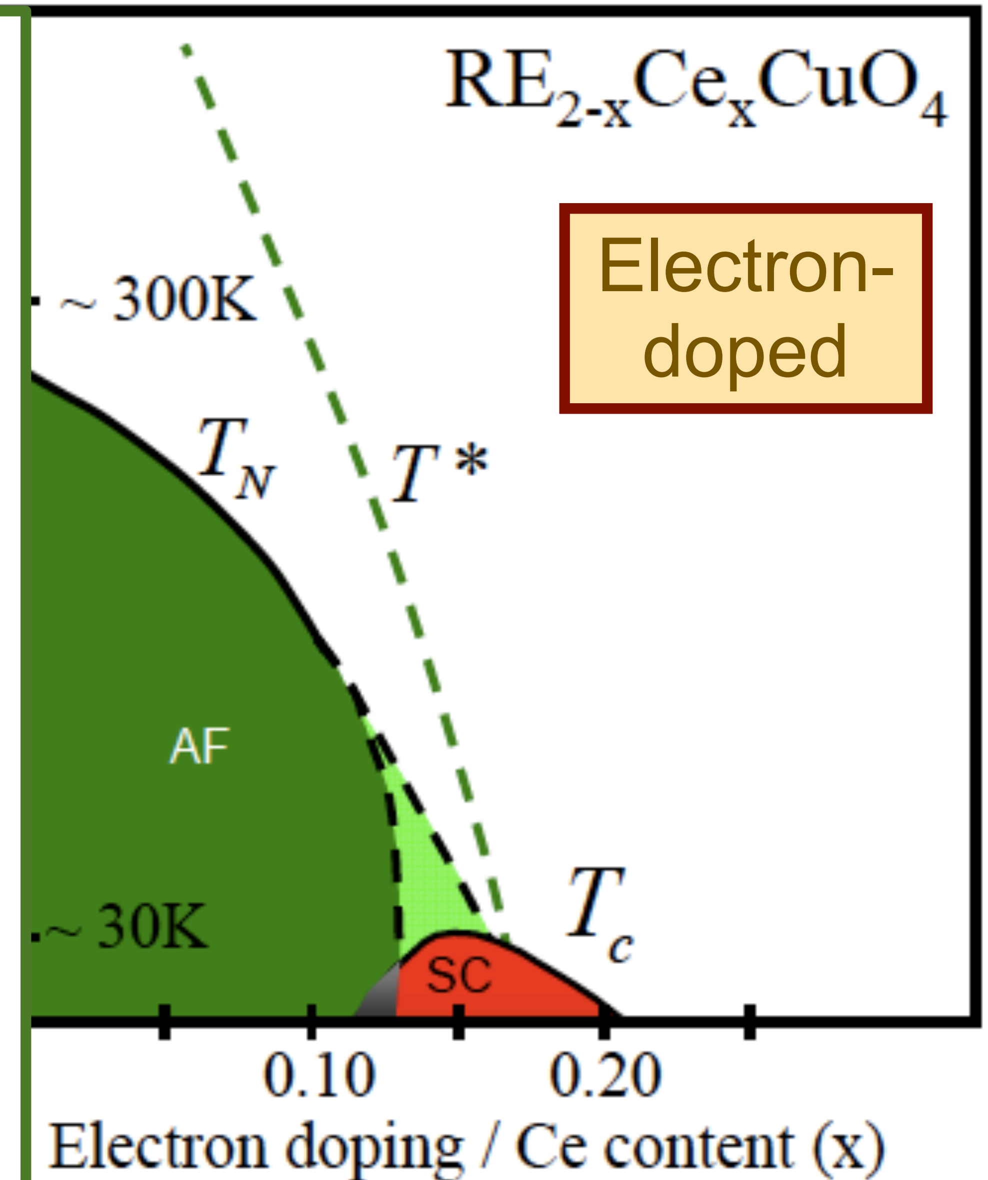
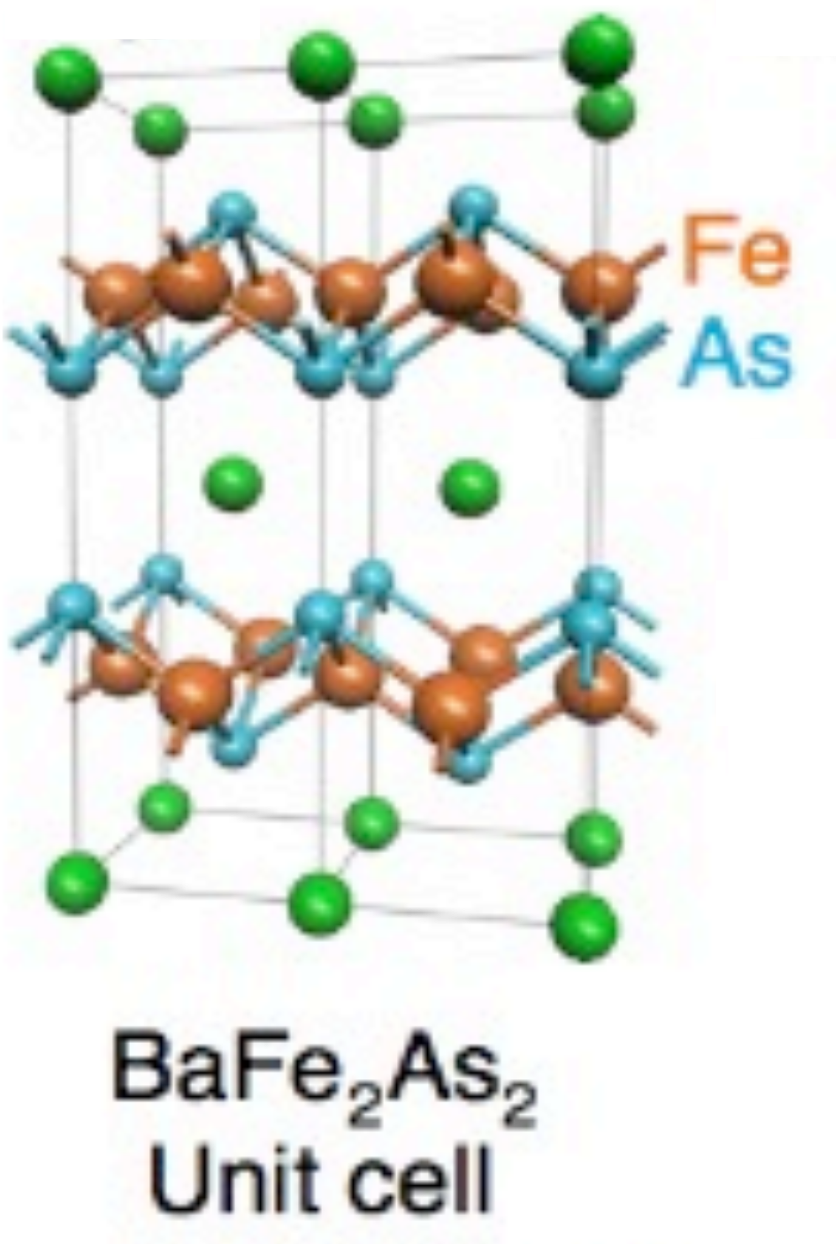


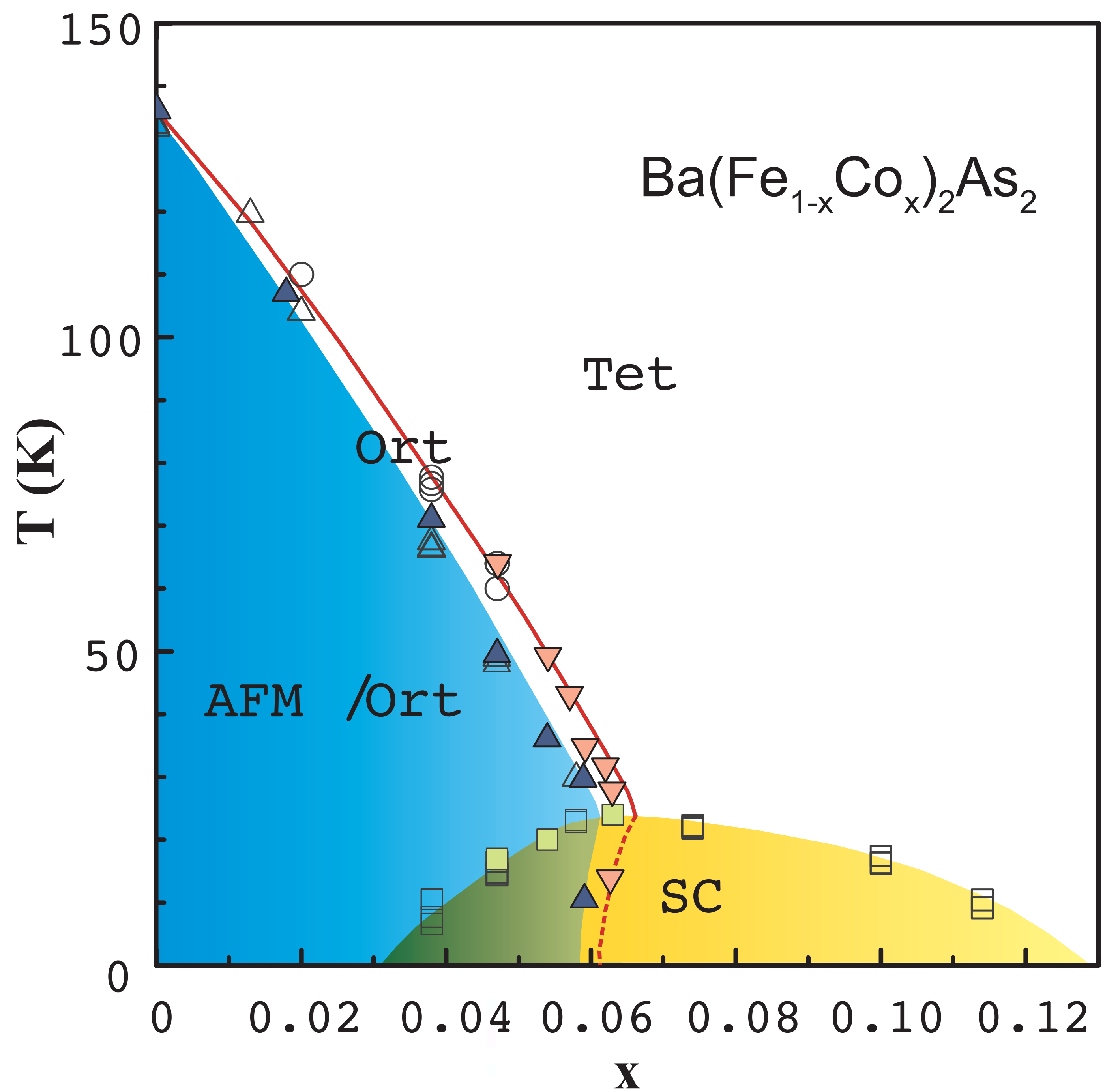
Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

Resistivity $\sim \rho_0 + AT^n$

Iron pnictides:

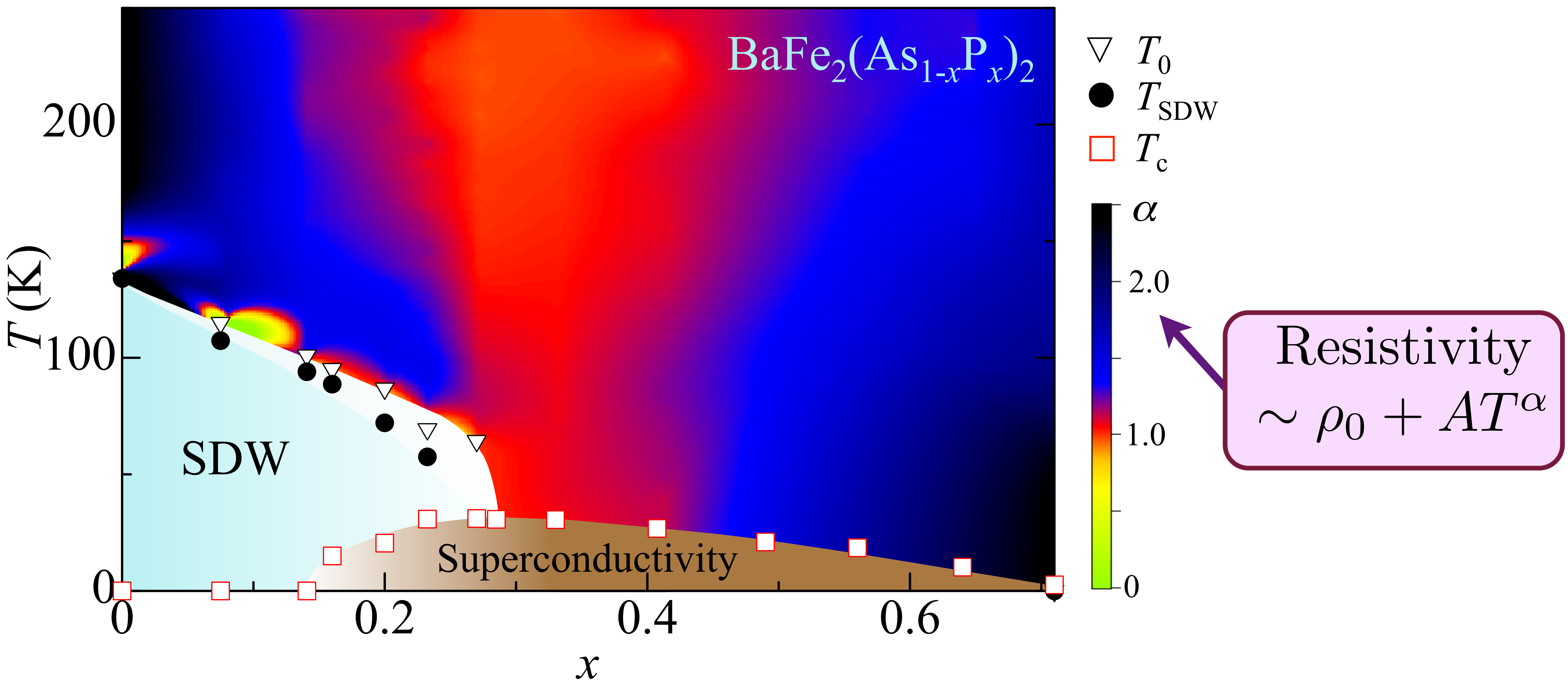
a new class of high temperature superconductors





S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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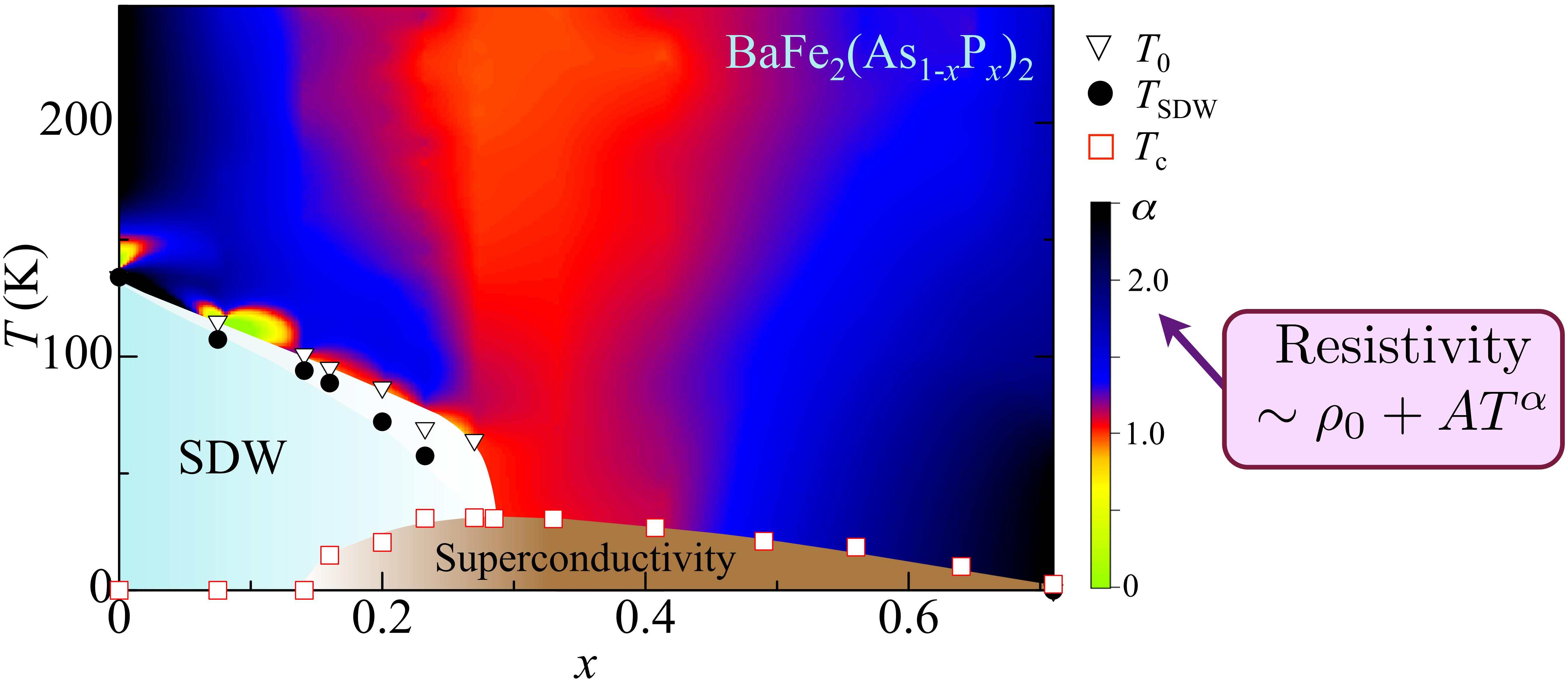

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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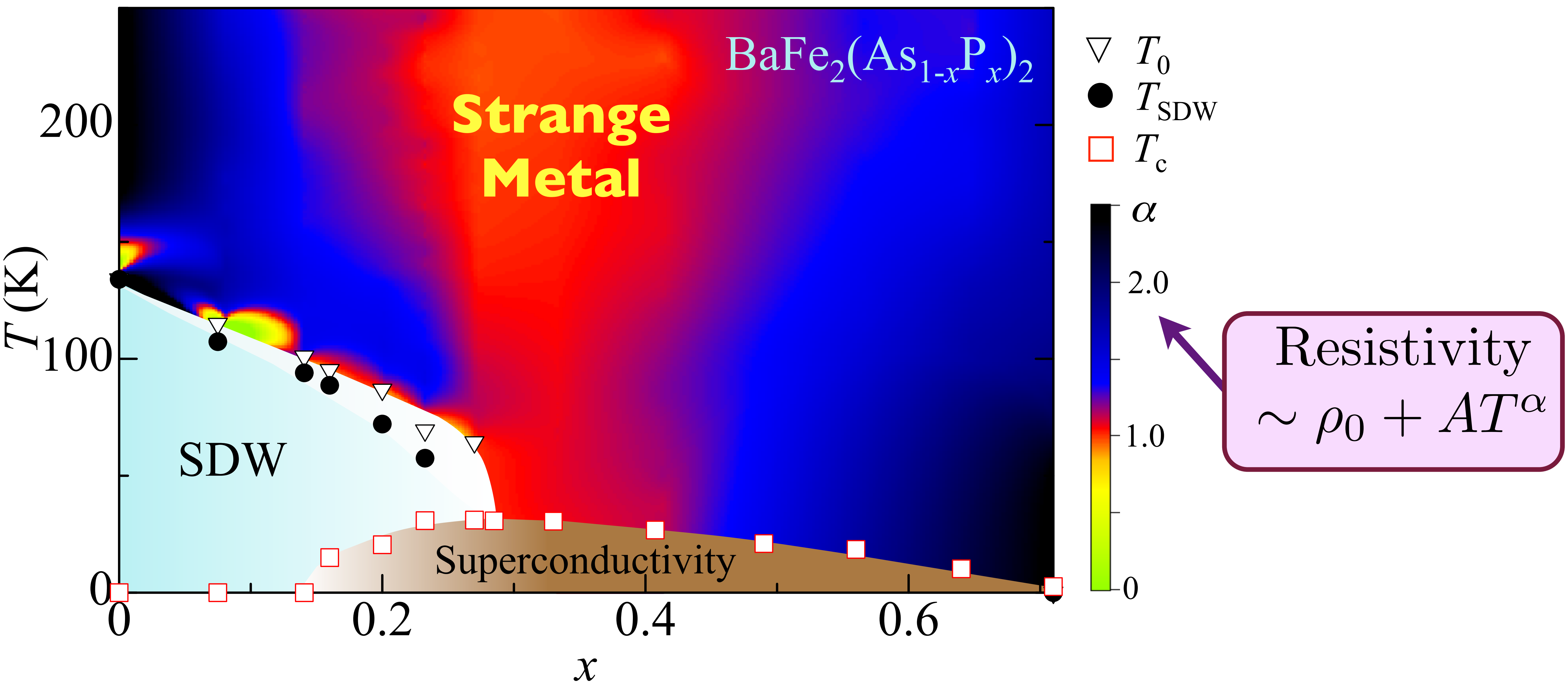
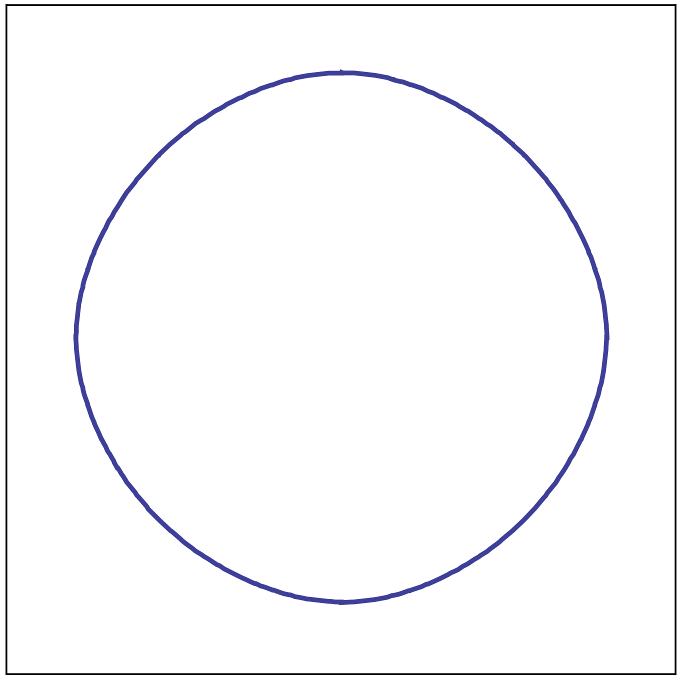
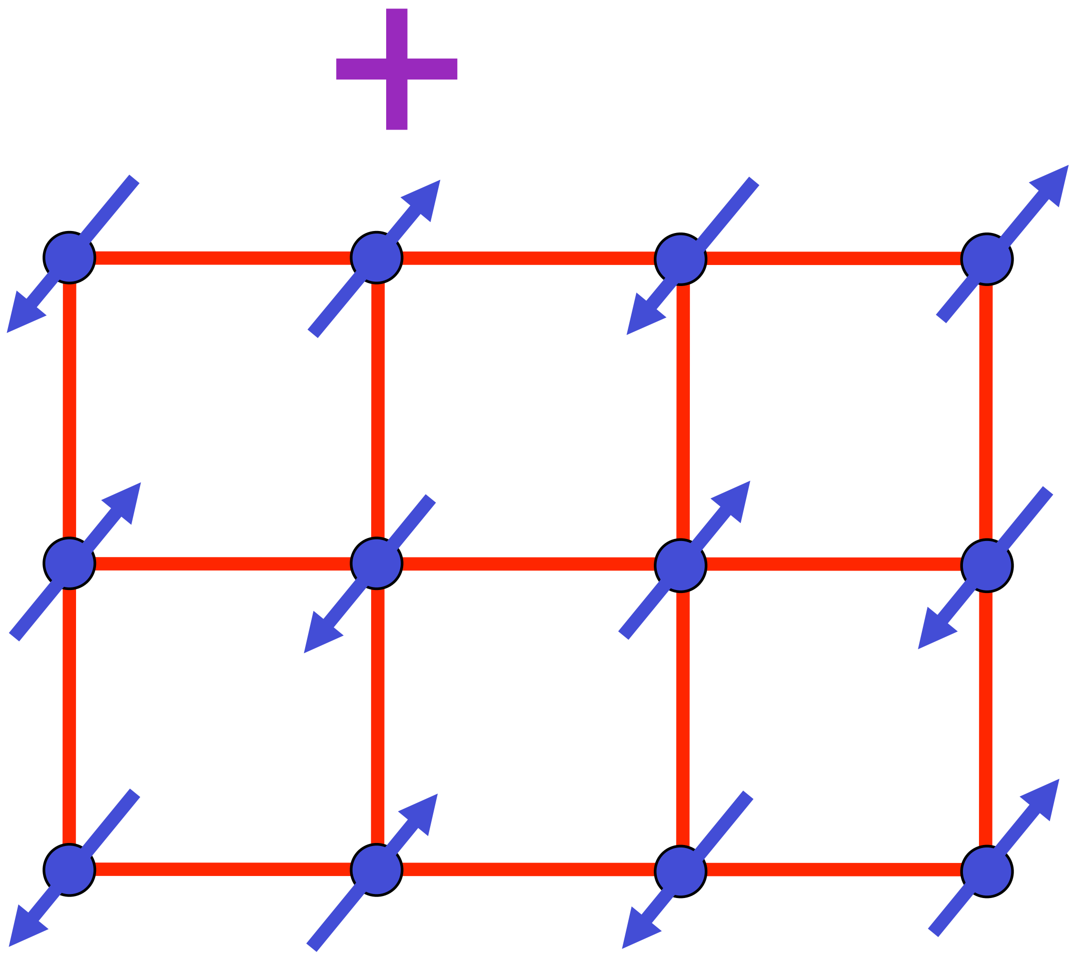


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



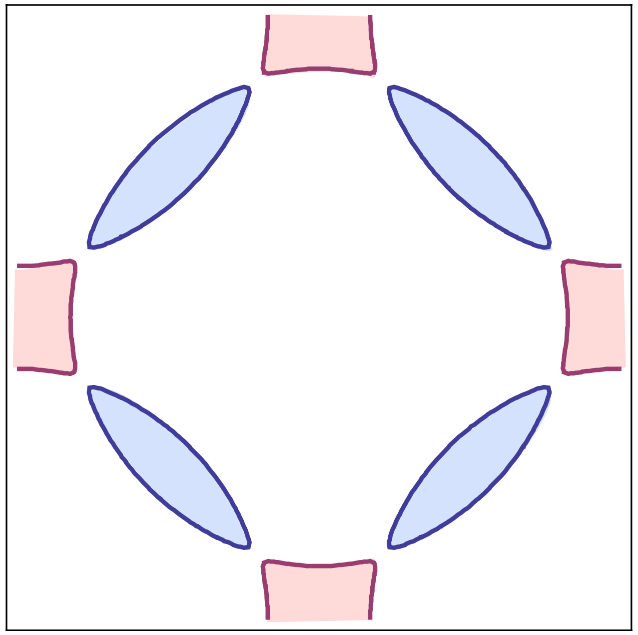


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

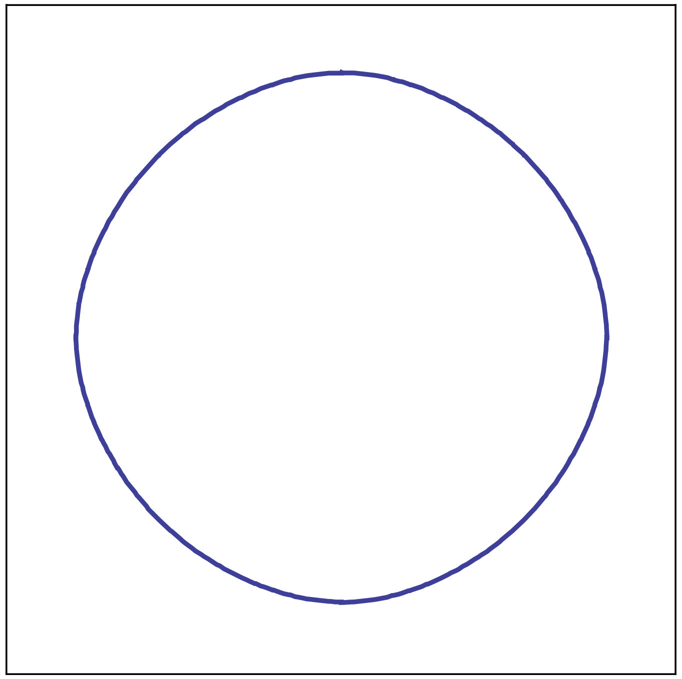
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large" Fermi surface

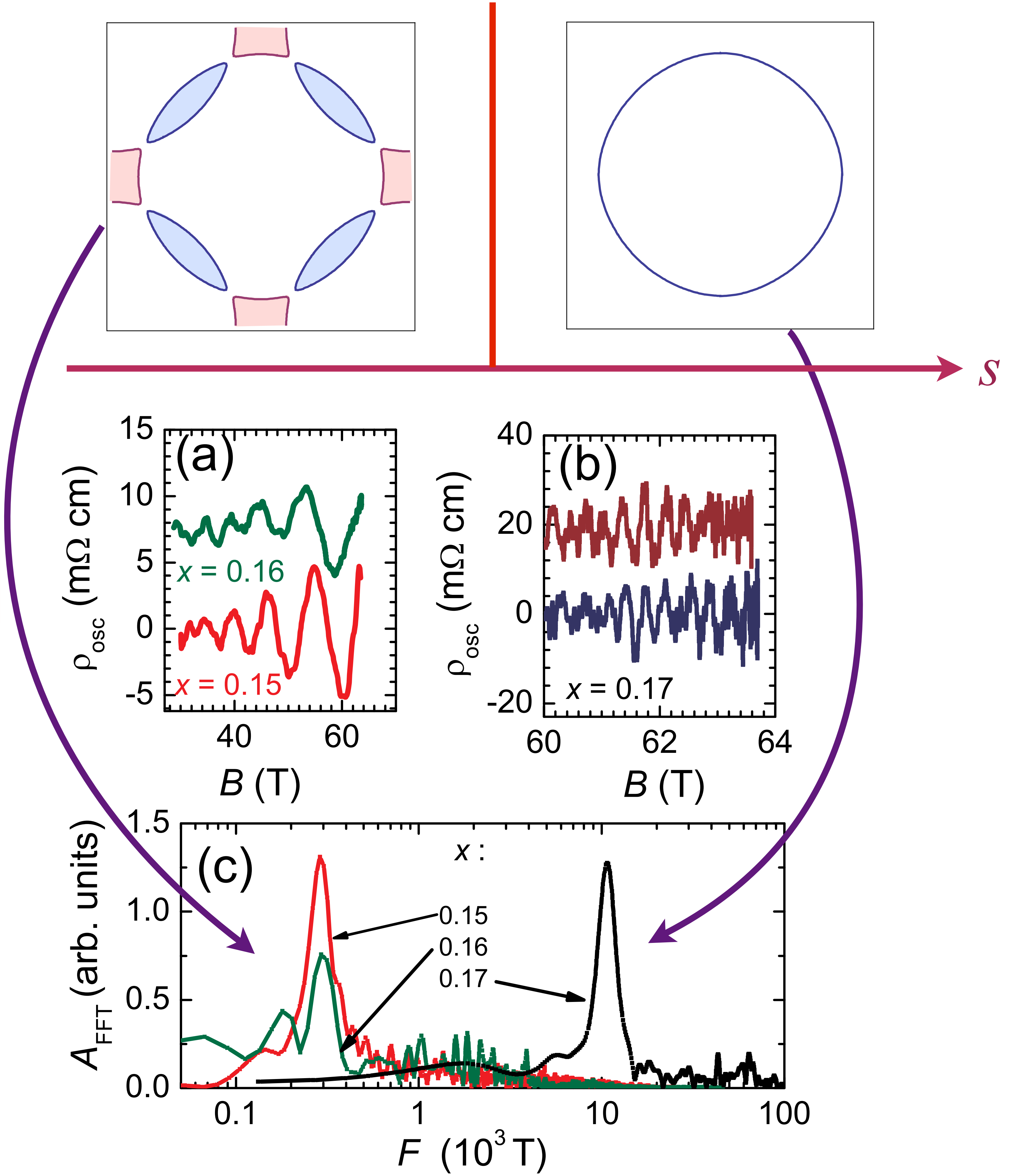
Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

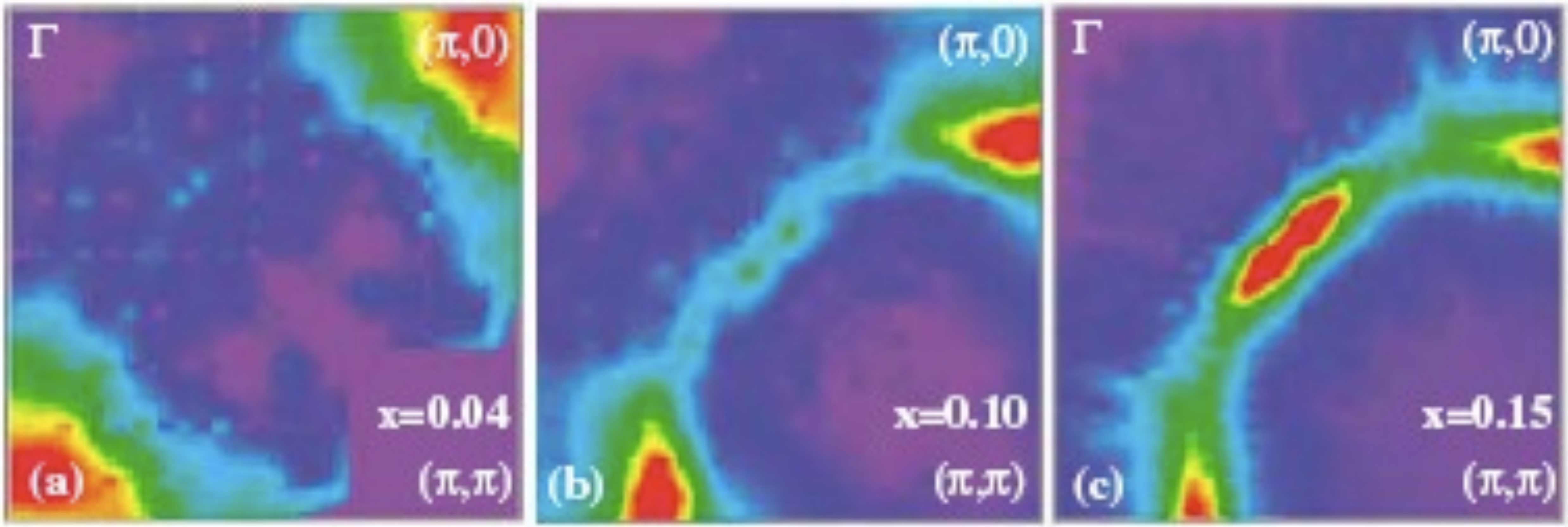
Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).



Photoemission in $Nd_{2-x}Ce_xCuO_4$



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Fermi surface + antiferromagnetism:
SU(2) gauge theory

Transform electrons to a
"rotating reference frame",
quantizing spins in the direction of the
local antiferromagnetic order

Fermi surface + antiferromagnetism:
SU(2) gauge theory

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

Fermi surface + antiferromagnetism:

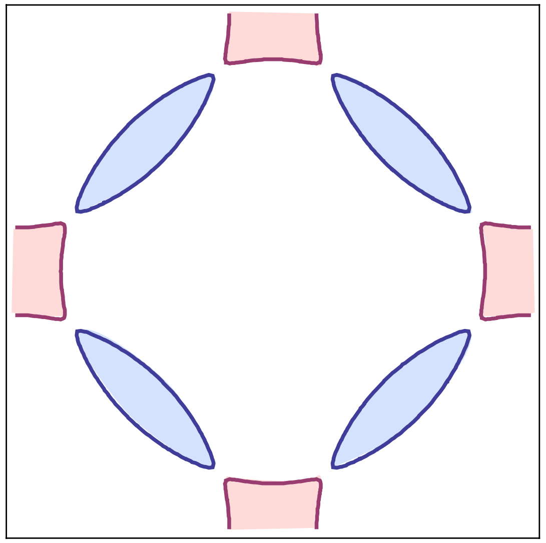
SU(2) gauge theory

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \bullet \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$U \times U^{-1}$

$\mathrm{SU(2)}_{\mathrm{gauge}}$

Phases of SU(2) gauge theory

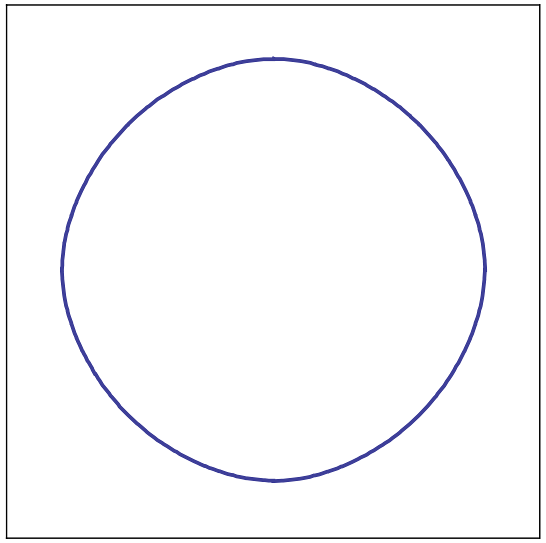


$\langle \vec{\varphi} \rangle \neq 0$

SDW order
small Fermi pockets

Fermi liquid

$\langle \vec{\varphi} \rangle = 0$

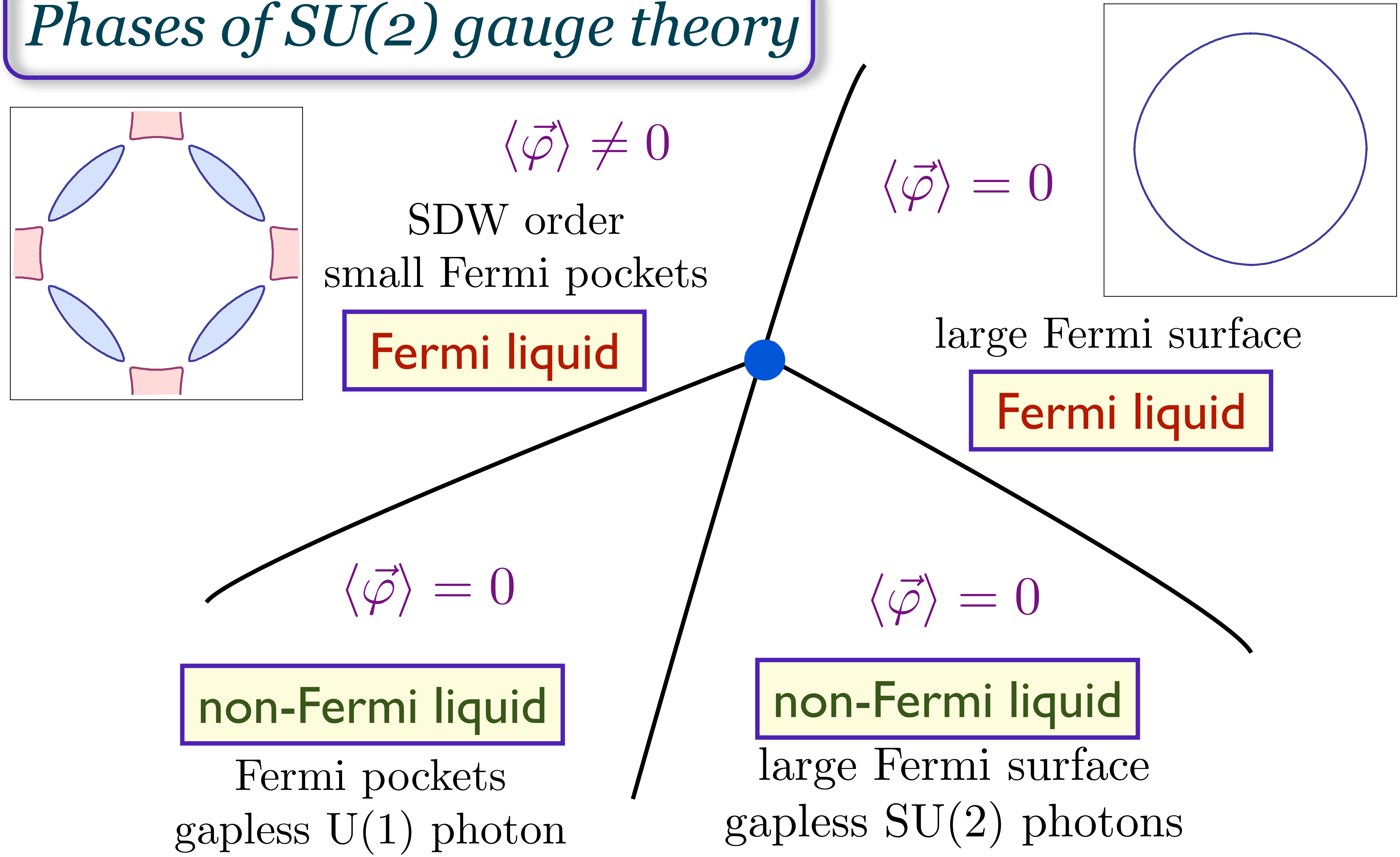


large Fermi surface

Fermi liquid

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Phases of SU(2) gauge theory



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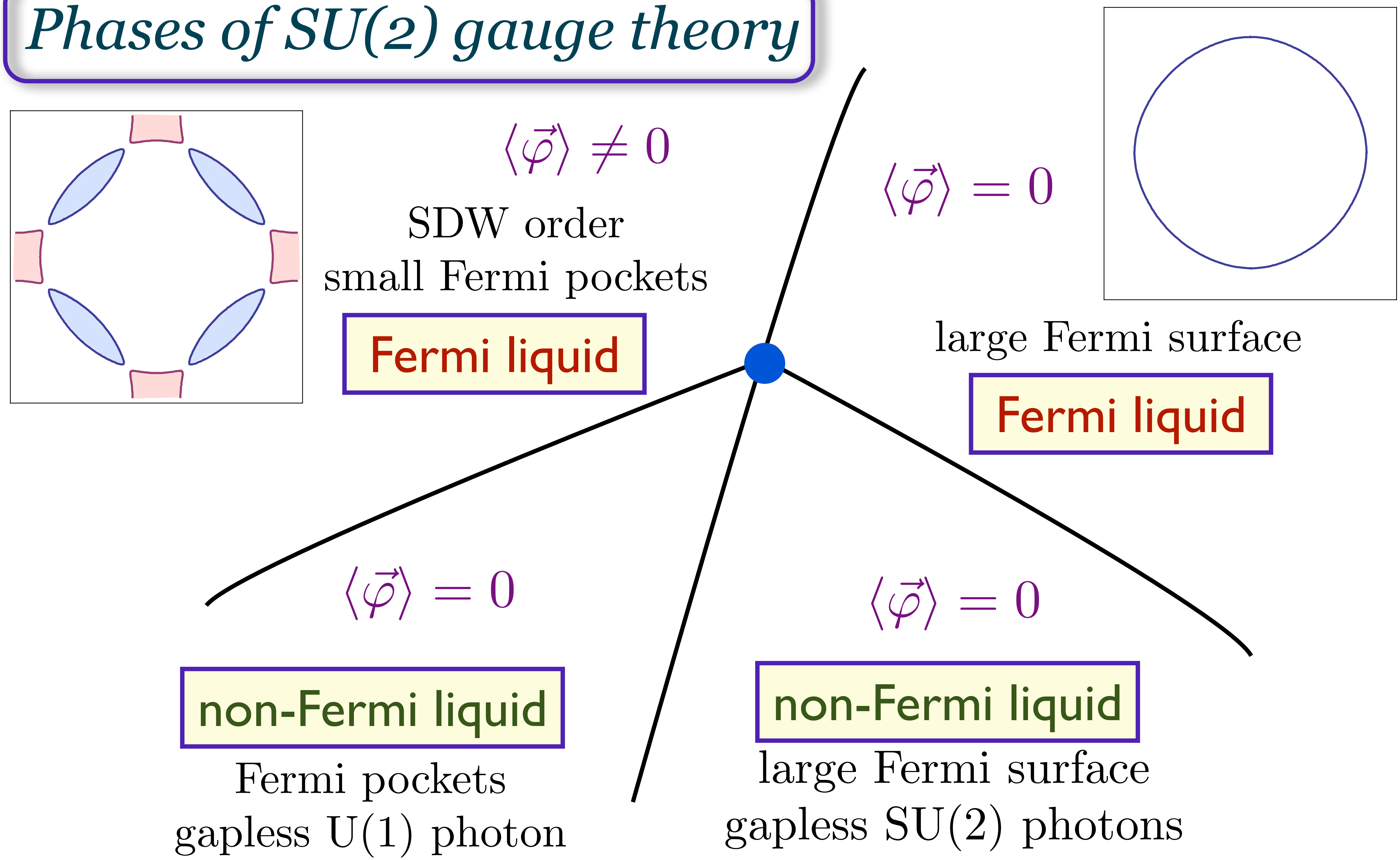

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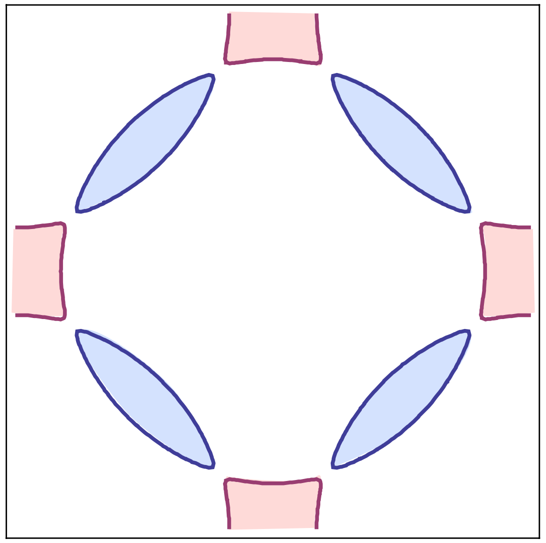


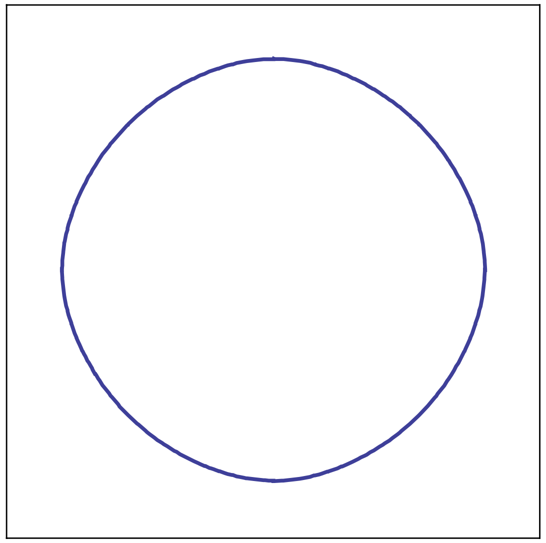
$\langle \vec{\varphi} \rangle \neq 0$

SDW order
small Fermi pockets

Fermi liquid

Electron doped?

$\langle \vec{\varphi} \rangle = 0$



large Fermi surface

Fermi liquid

$\langle \vec{\varphi} \rangle = 0$

non-Fermi liquid

Fermi pockets
gapless U(1) photon

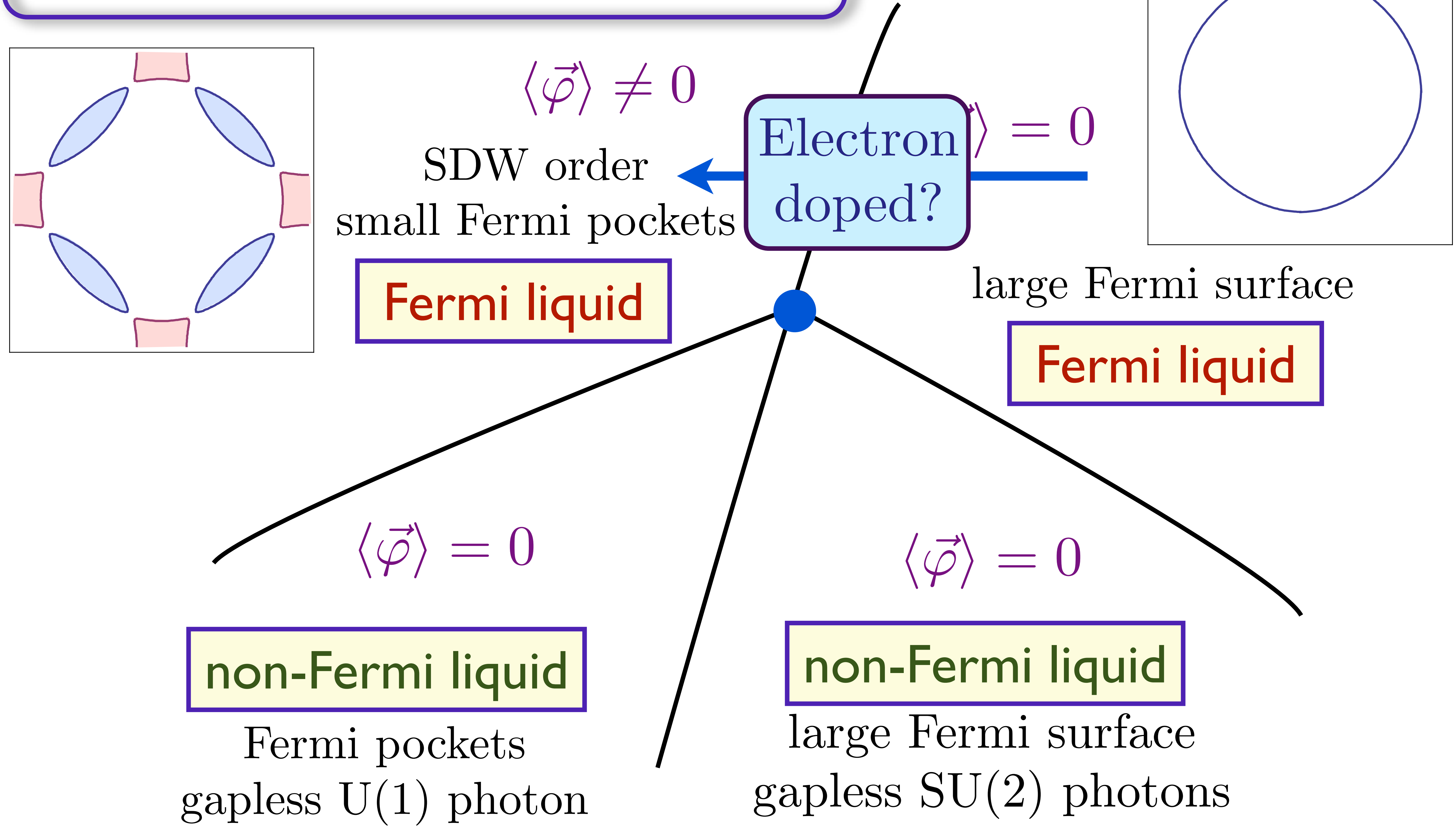
$\langle \vec{\varphi} \rangle = 0$

non-Fermi liquid

large Fermi surface
gapless SU(2) photons

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Phases of SU(2) gauge theory



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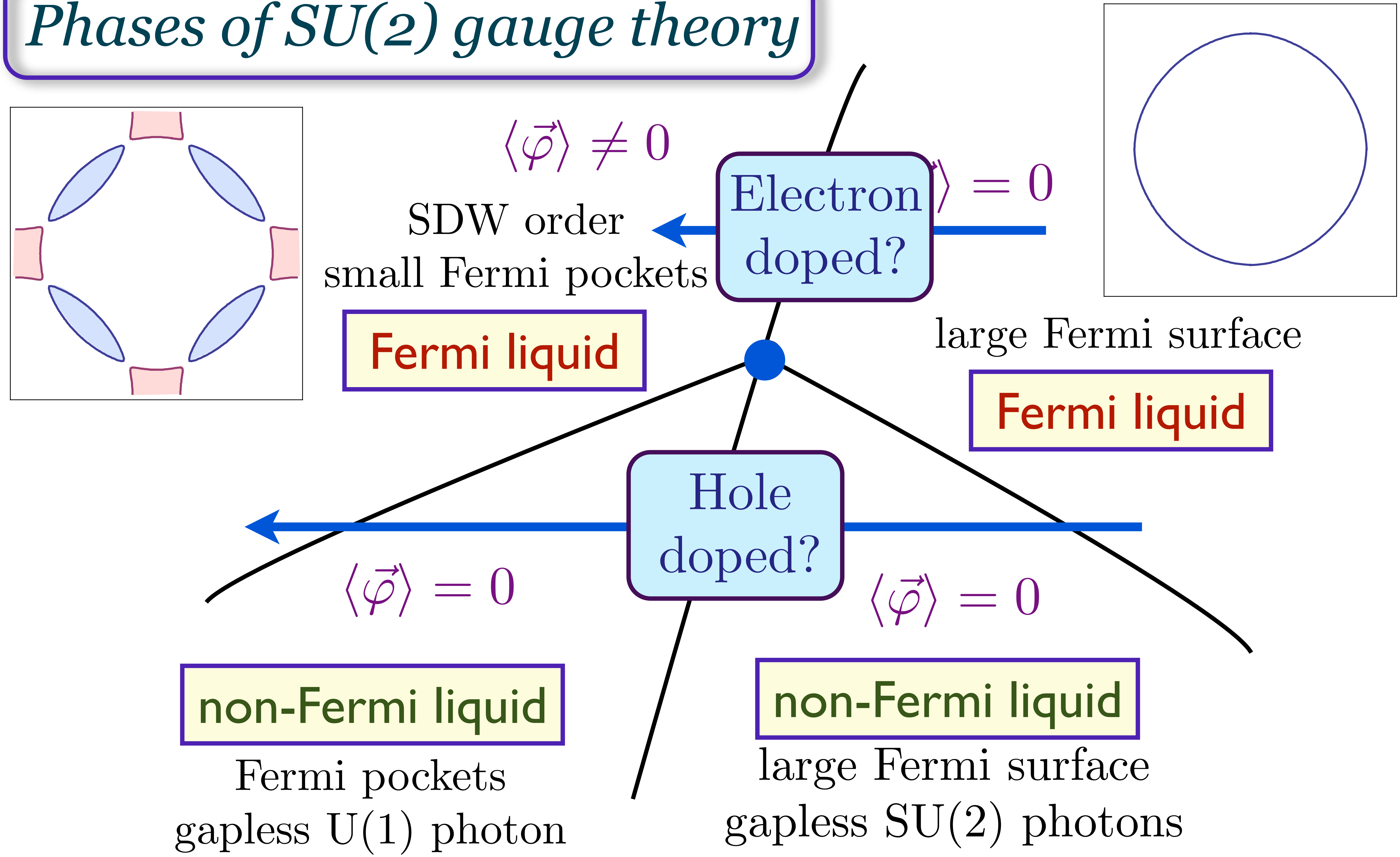

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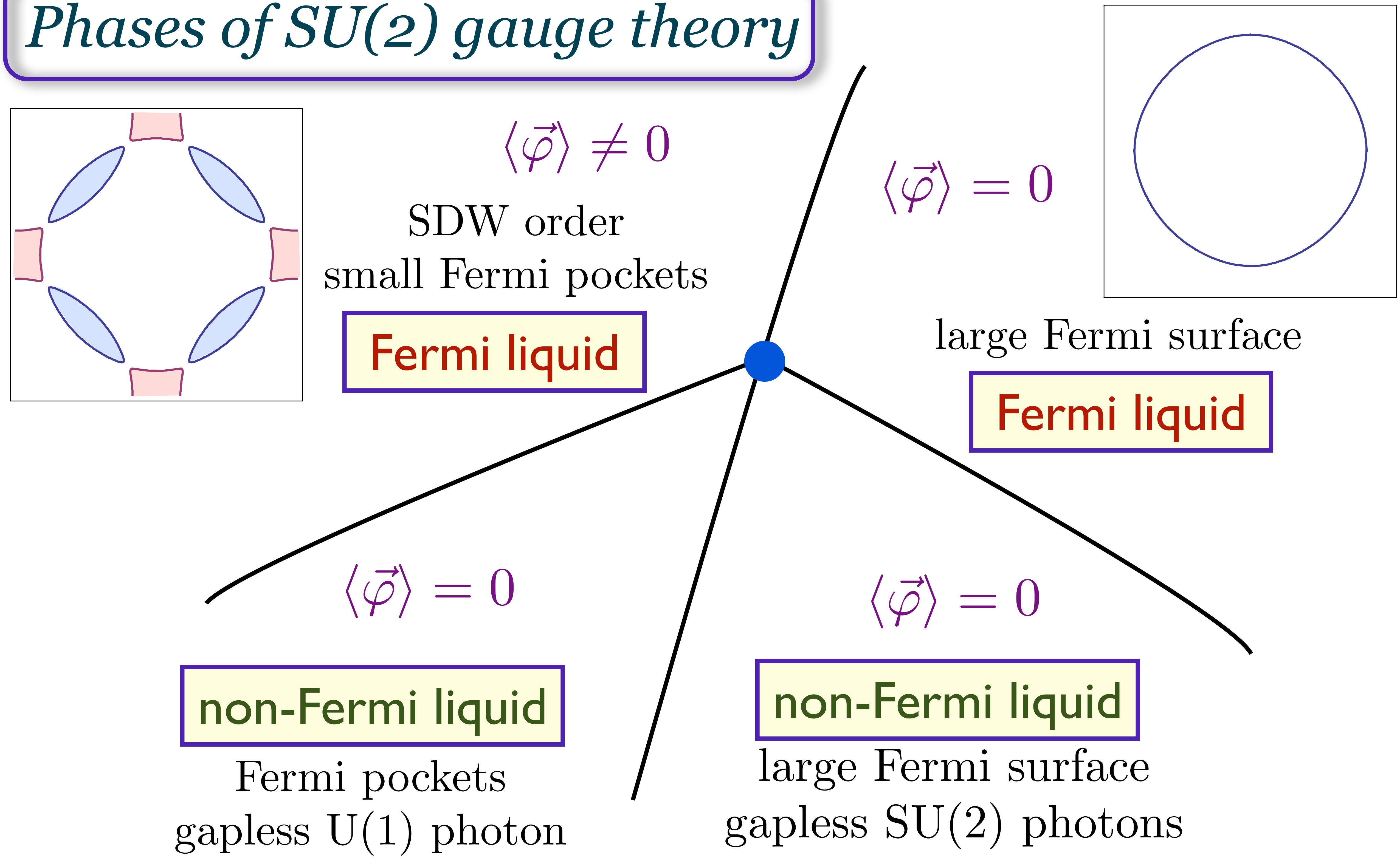

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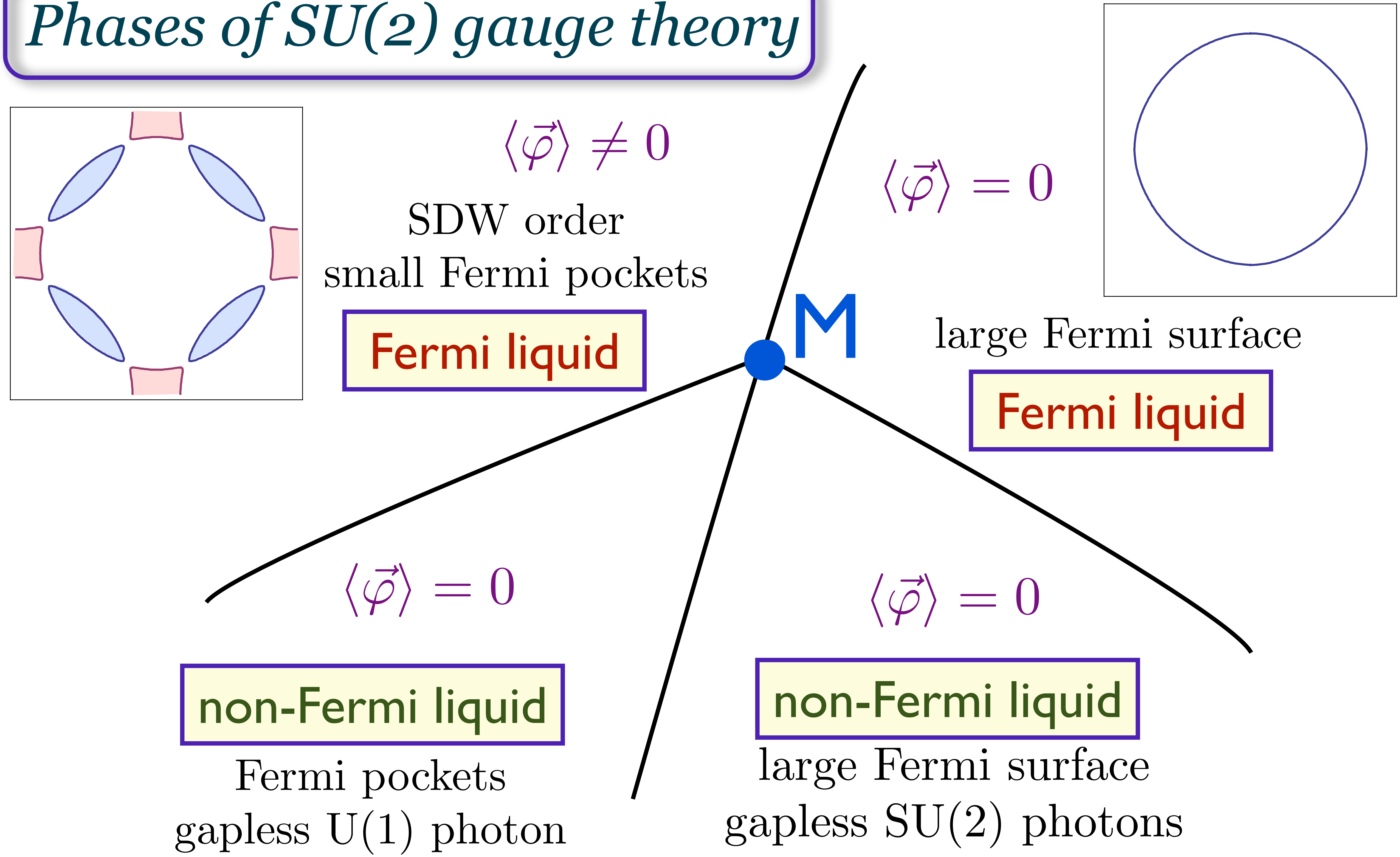

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Theory near M

SU(2) gauge field coupled to matter transforming under $\mathrm{SU}(2)_{\mathrm{gauge}} \times \mathrm{SU}(2)_{\mathrm{spin}} \times \mathrm{U}(1)_{\mathrm{charge}}$

- Fermi surfaces of $\psi : (\mathbf{2}, \mathbf{1}, 1)$,
- Scalar $z : (\bar{\mathbf{2}}, \mathbf{2}, 0)$
- Adjoint scalar $\vec{N} : (\mathbf{3}, \mathbf{1}, 0)$

Note: chemical potential couples only to ψ.

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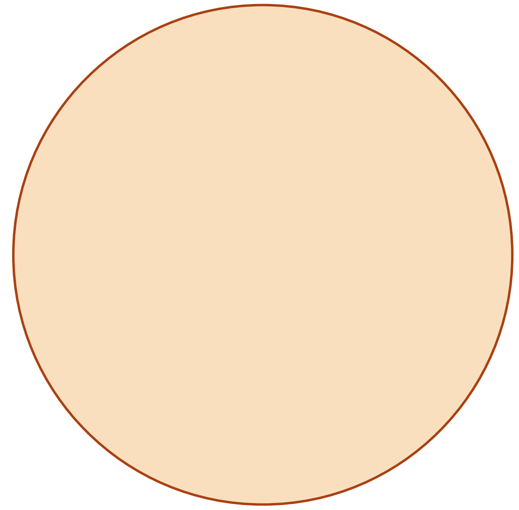
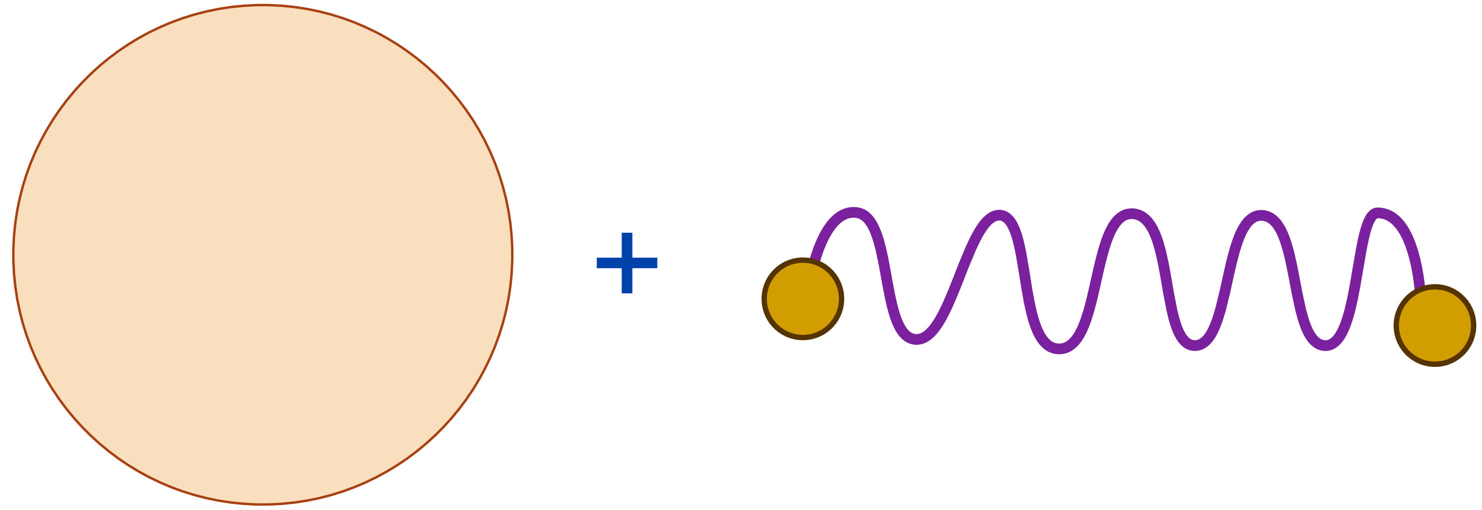
Yukawa coupling: $\int d^2r \, e^{i\mathbf{K}\cdot\mathbf{r}} \, \vec{N} \cdot \psi^\dagger \vec{\sigma} \psi$:
spatial oscillation hard to realize holographically, but perhaps be ignored for some properties of the strange metal.

Basic ingredient in many models of strange metals:



- Begin with fermions with short-range interactions. This leads to a Fermi liquid, with sharp fermionic quasiparticles near the Fermi surface.

Basic ingredient in many models of strange metals:



- Couple fermions to a gauge field (physics turns out to be similar for abelian or non-Abelian gauge fields). This is an "emergent" gauge field, found in many analyses of Hubbard or Kondo models of correlated electrons.

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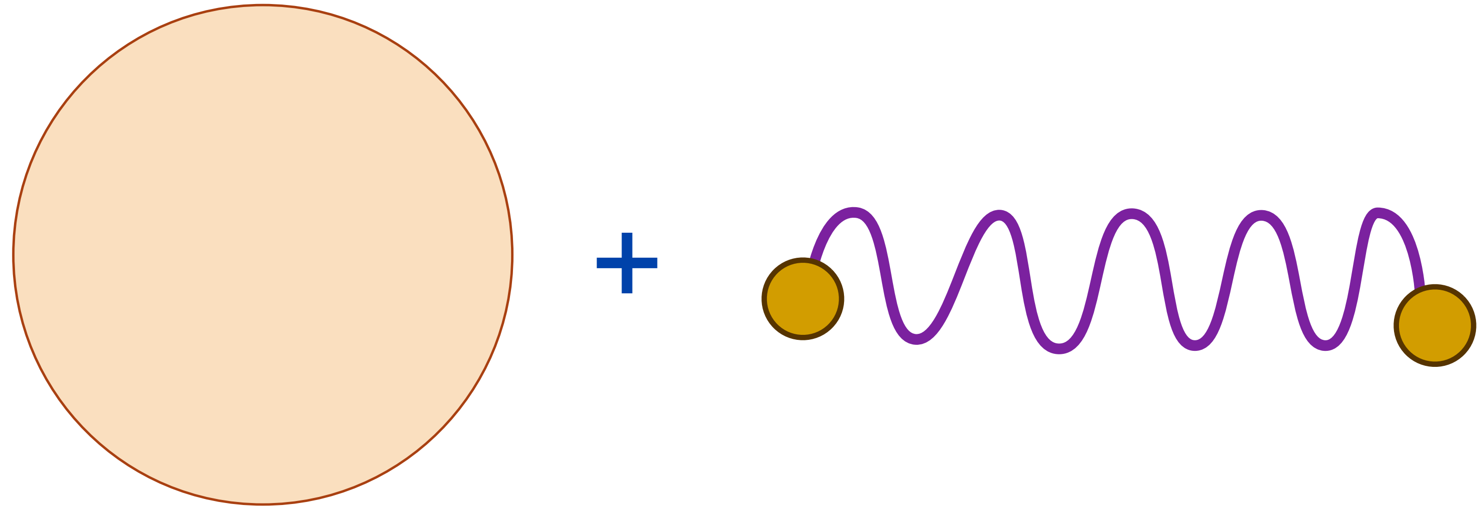

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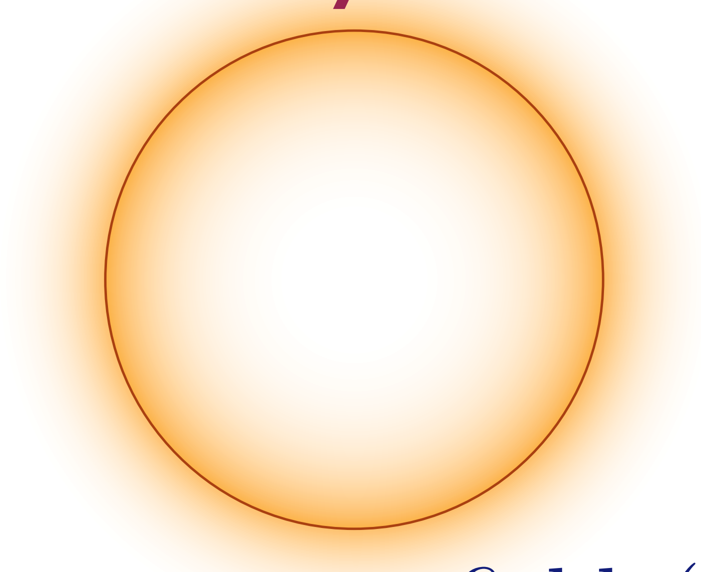
- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions.*

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Basic ingredient in many models of strange metals:



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- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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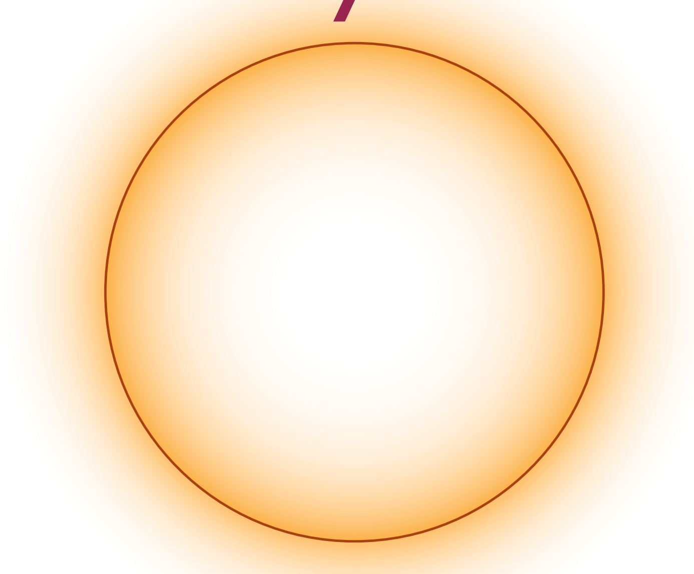

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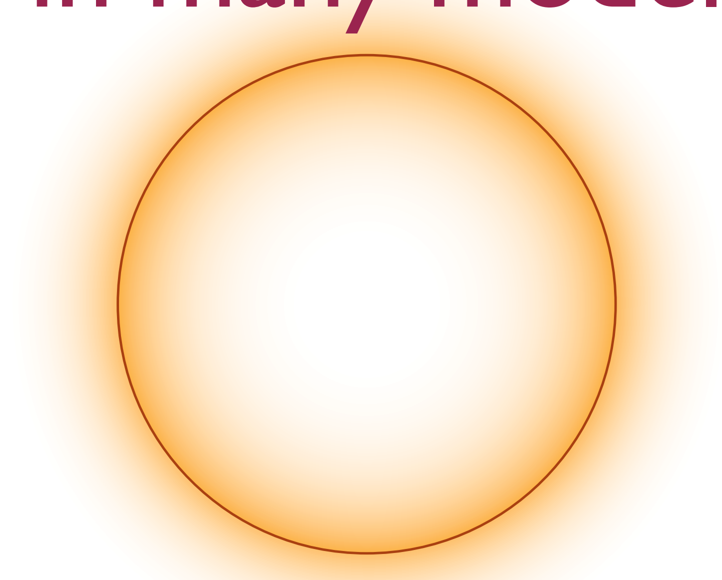

- Couple fermions to a gauge field (physics turns out to be similar for abelian or non-Abelian gauge fields). This is an "eme ... ubbard or K ...

Needed: a complete theory of this, and related non-Fermi liquid states, and of their instabilities

- Long ... mions. But ... nd are Lanc ... e. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions.*

- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

Outline

1. Quantum criticality and conformal field theories in condensed matter

2. Quantum transport and Einstein-Maxwell theory on AdS_4

3. Compressible quantum matter
   A. Strange metals: experiments and theoretical framework
   B. ABJM (-like) theory at non-zero density
   C. Gauge-gravity duality

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ABJM theory in D=2+1 dimensions

- $4N^2$ Weyl fermions carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $4N^2$ complex bosons carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $\mathcal{N} = 6$ supersymmetry

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Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance

Theory similar to ABJM

- U(1) gauge invariance and U(1) global symmetry
- Fermions, f_+ and f_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- Bosons, b_+ and b_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- No supersymmetry

L. Huijse and S. Sachdev, arXiv:1104.5022

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- No supersymmetry
- Fermions, c, gauge-invariant bound states of fermions and bosons carrying global U(1) charge 2.

L. Huijse and S. Sachdev, arXiv:1104.5022

Theory similar to ABJM

$$\mathcal{L} = f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma$$
$$+ \quad b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma$$
$$+ \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right)$$

The index $\sigma = \pm 1$

L. Huijse and S. Sachdev, arXiv:1104.5022

Theory similar to ABJM

$$\begin{aligned}
\mathcal{L} &= f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma \\
&+ b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma \\
&\quad + \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right) \\
&+ c^\dagger \left[\partial_\tau - \frac{\nabla^2}{2m_c} + \epsilon_2 - 2\mu \right] c \\
&\quad - g_2 \left[c^\dagger \left(f_+ b_- + f_- b_+ \right) + \text{H.c.} \right]
\end{aligned}$$

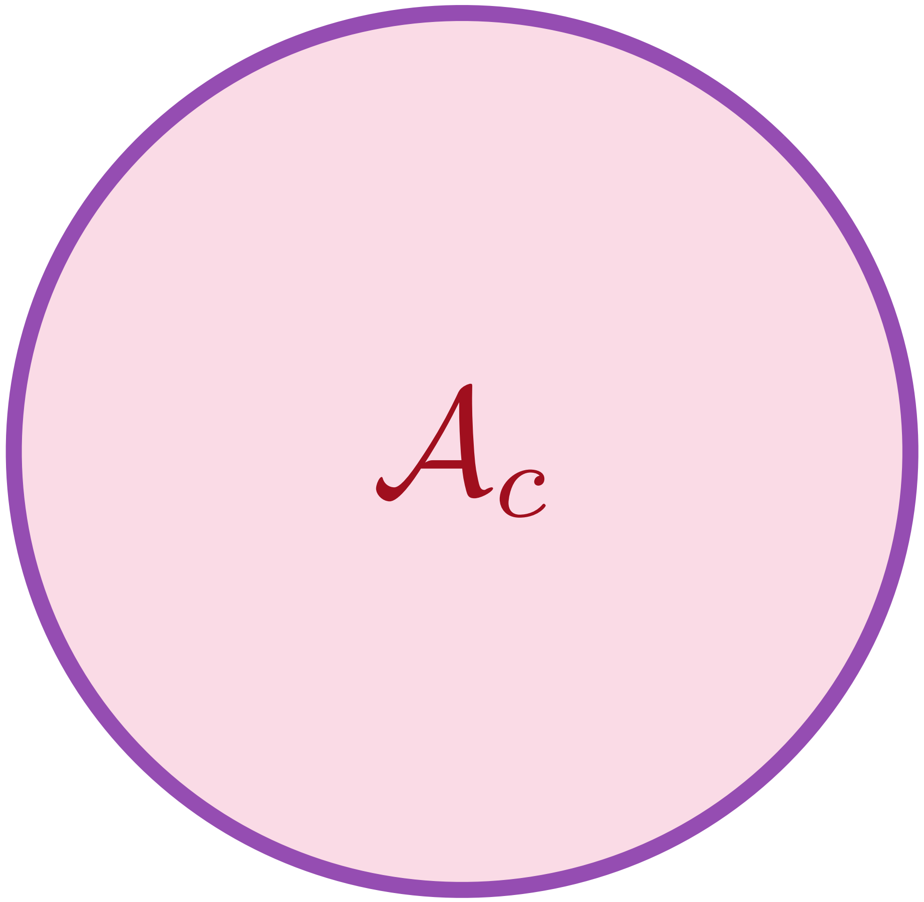
The index $\sigma = \pm 1$, and $\epsilon_{1,2}$ are tuning parameters of phase diagram

Conserved U(1) charge: $\mathcal{Q} = f_\sigma^\dagger f_\sigma + b_\sigma^\dagger b_\sigma + 2c^\dagger c$

L. Huijse and S. Sachdev, arXiv:1104.5022

Phases of ABJM-like theories

$$\langle b_\pm \rangle = 0$$



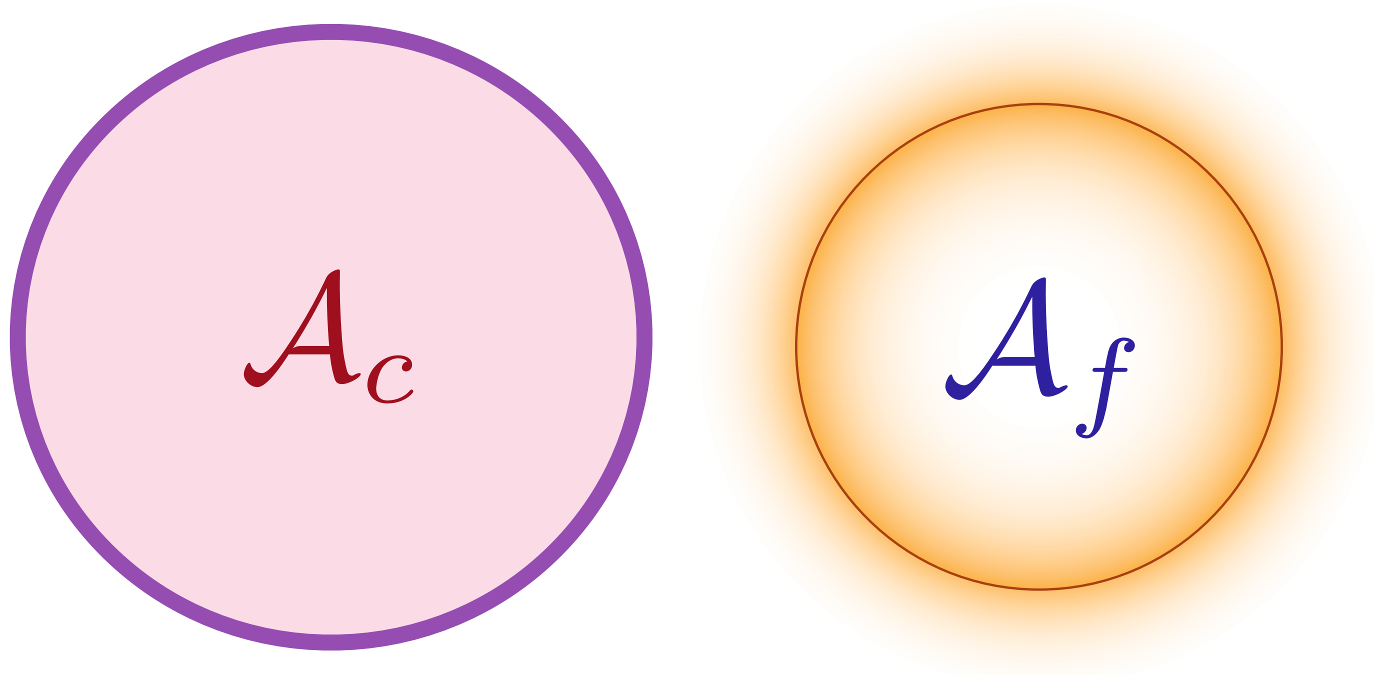
$$2\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

Fermi liquid (FL) of gauge-neutral particles

U(1) gauge theory is in confining phase

Phases of ABJM-like theories

$\langle b_{\pm} \rangle = 0$



$$2\mathcal{A}_c + 2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

Fractionalized Fermi liquid (FL*)

U(*1*) gauge theory is in deconfined phase

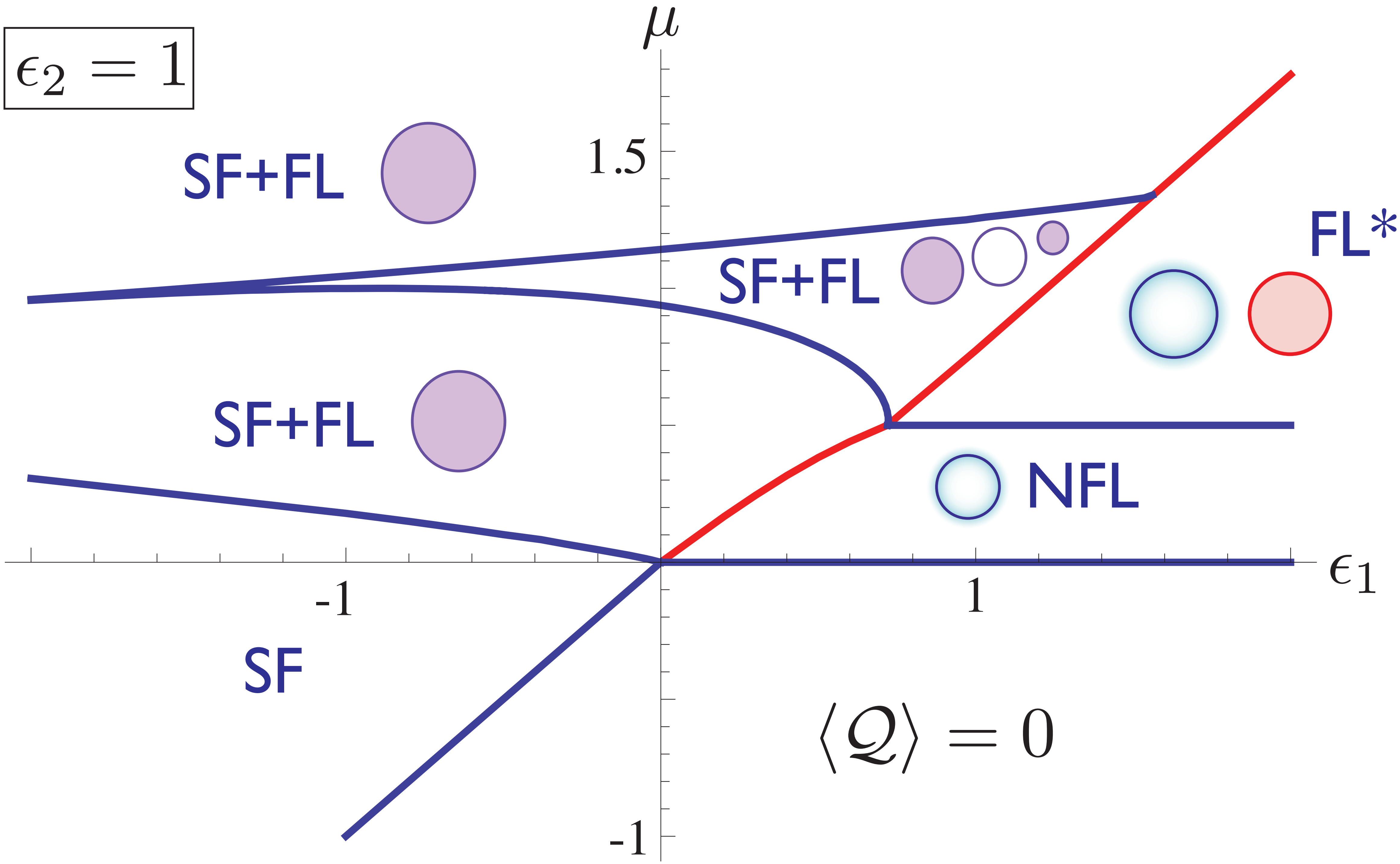
Phases of ABJM-like theories

$$\langle b_\pm \rangle \neq 0$$
$$\langle b_+ b_- \rangle \neq 0$$

No constraint on Fermi surface area,
which can be zero

Superfluid (SF)

U(*1*) gauge theory is in Higgs phase,
due to condensation of fermion pairs,
and global U(1) is broken



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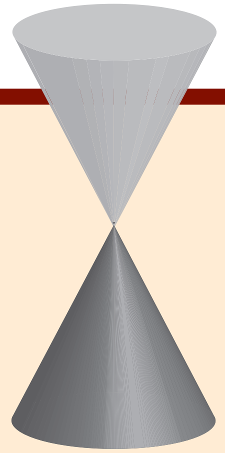
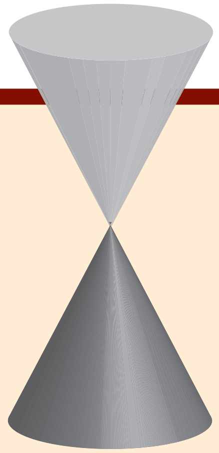
Gauge-gravity duality



- Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry

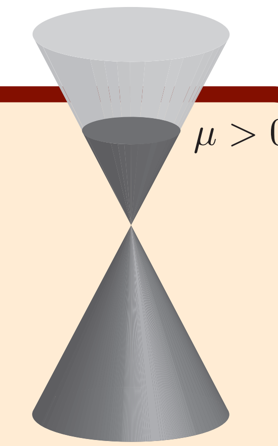
Gauge-gravity duality



- Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry

- The CFT is dual to a gravity theory on $AdS_4 \times S^7$

Gauge-gravity duality



- Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry
- Add some SU(4) charge by turning on a chemical potential (this breaks the SU(4) symmetry): we obtain Fermi surfaces coupled to non-Abelian gauge fields, similar to many correlated electron models

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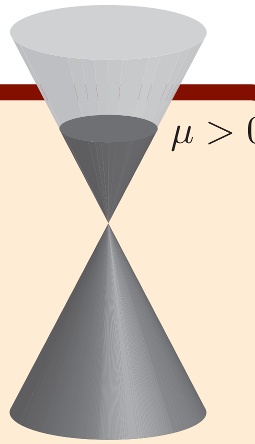
L. Huijse and S. Sachdev, arXiv:1104.5022

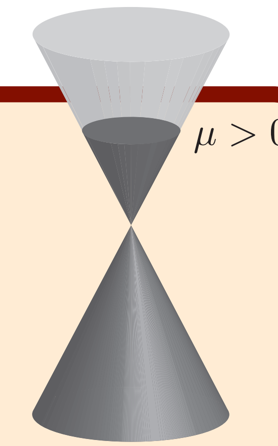
Gauge-gravity duality



- Begin with a CFT *e.g.* the ABJM theory with a SU(4) global symmetry
- Add some SU(4) charge by turning on a chemical potential (this breaks the SU(4) symmetry): we obtain Fermi surfaces coupled to non-Abelian gauge fields, similar to many correlated electron models

- The CFT is dual to a gravity theory on $AdS_4 \times S^7$
- In the Einstein-Maxwell theory, the chemical potential leads at T=0 to an extremal Reissner-Nordtrom black hole in the AdS_4 spacetime.

Gauge-gravity duality



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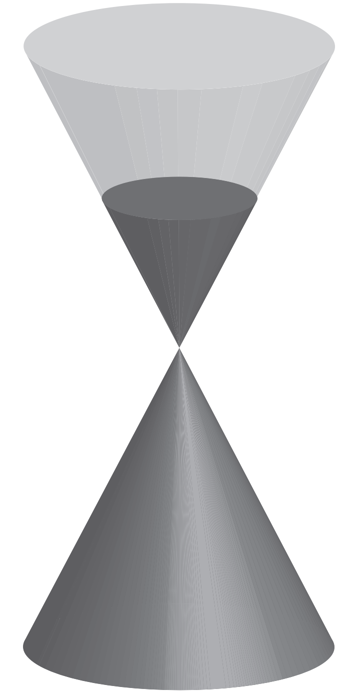
- The near-horizon geometry of the RN black hole is $AdS_2 \times R^2$. This factorization leads to finite ground state entropy density

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Examine the free energy and Green's function of a probe particle



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
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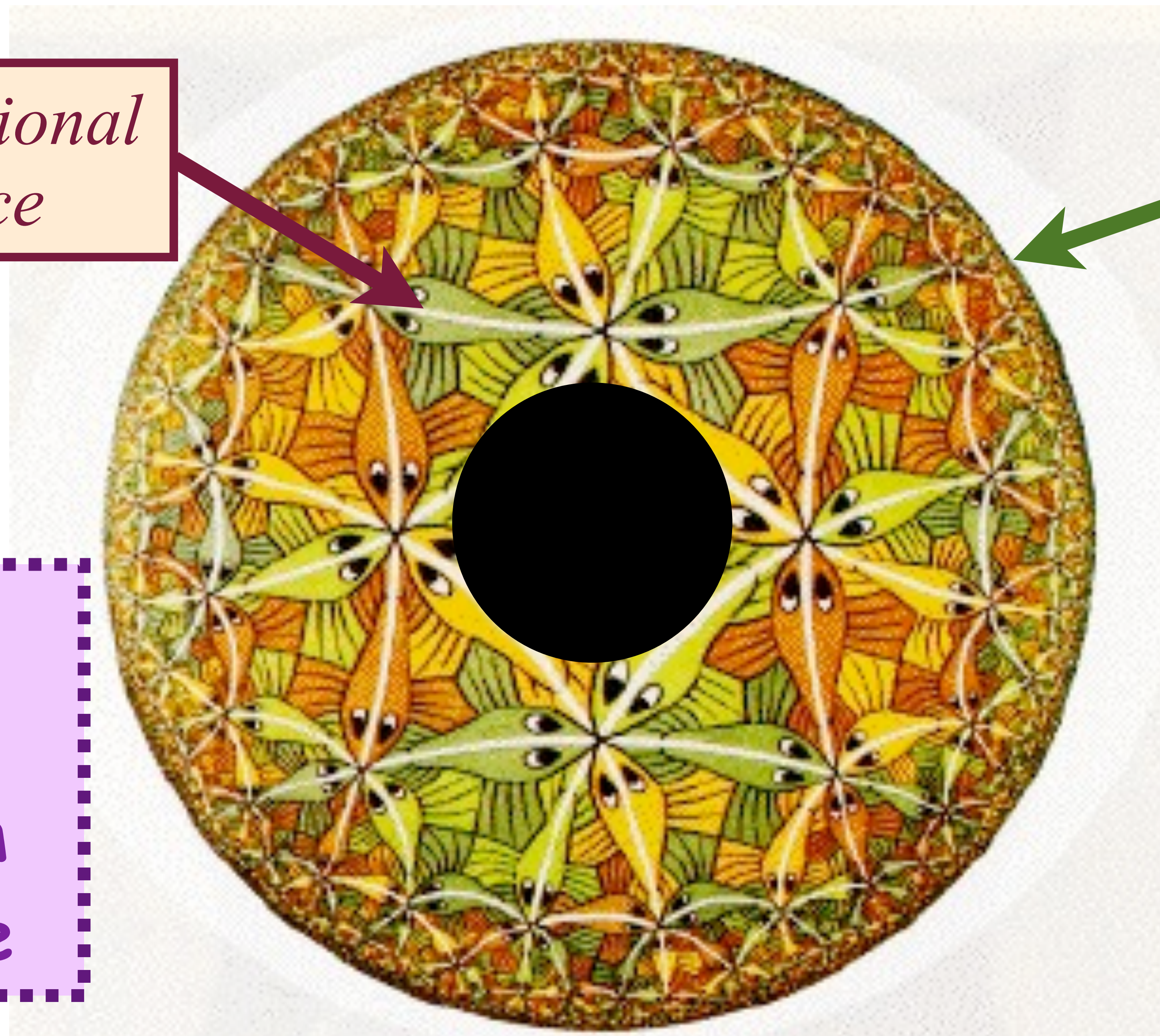
3+1 dimensional AdS space

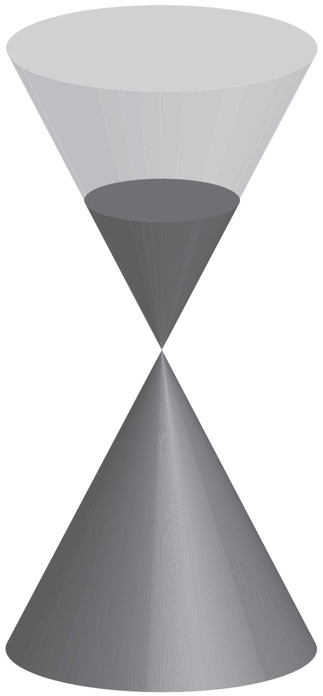
Finite density matter in 2+1 dimensions

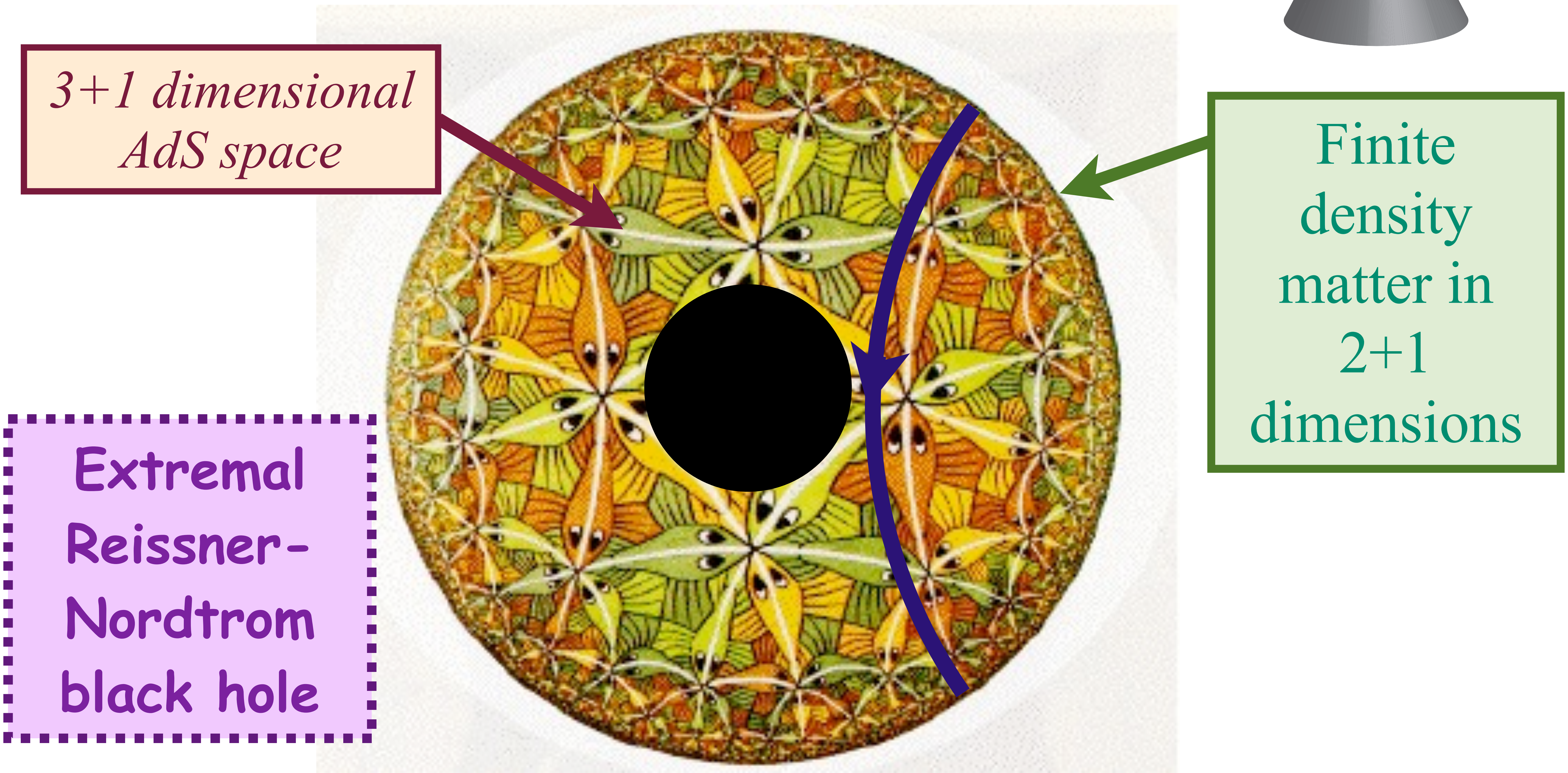


Extremal Reissner-Nordtrom black hole

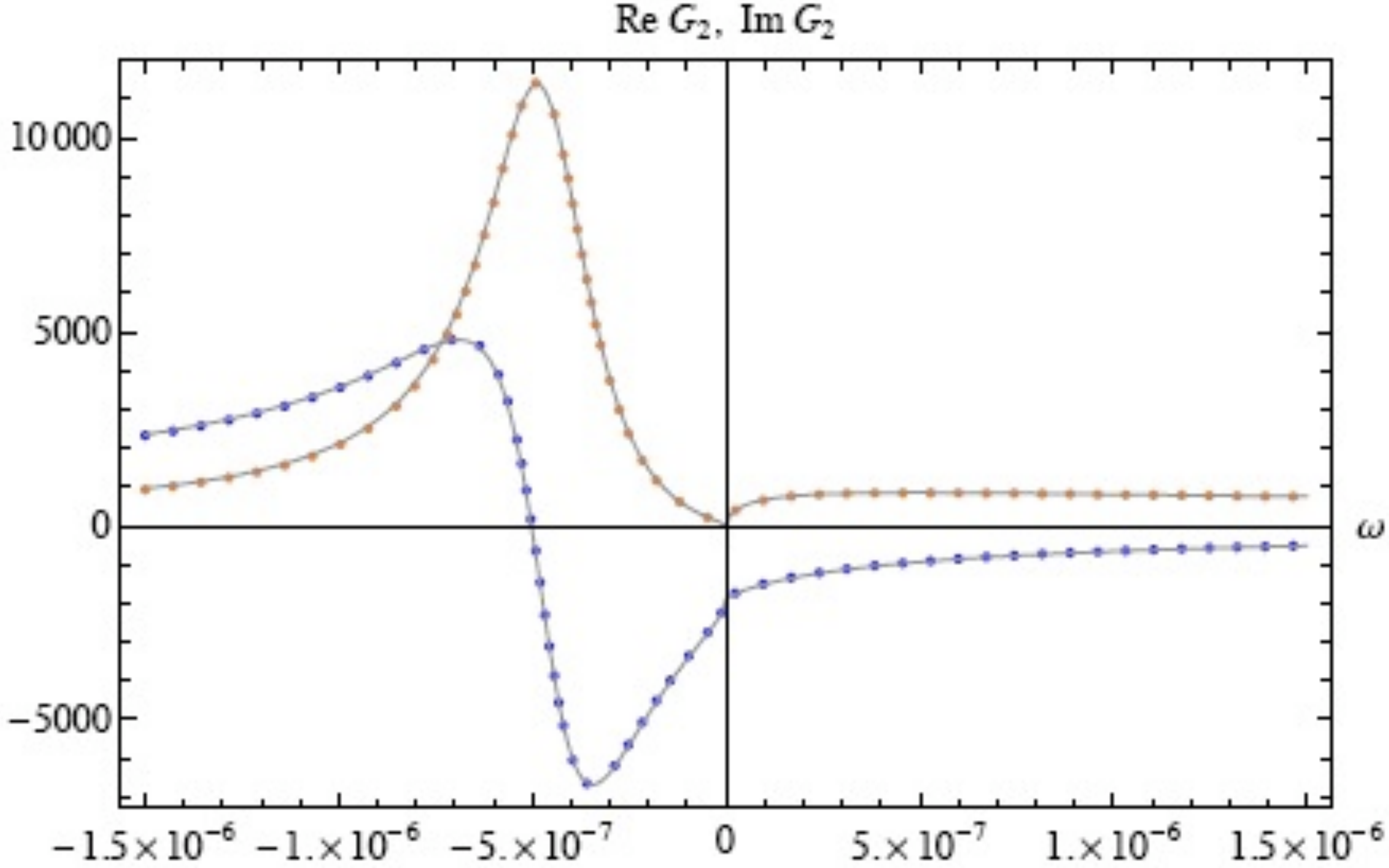
Examine the free energy and Green's function of a probe particle

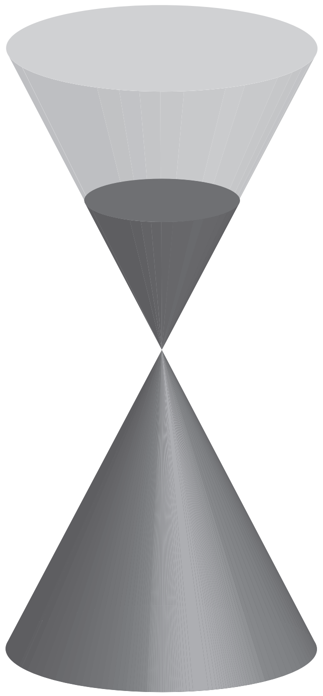
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788





Green's function of a fermion





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See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
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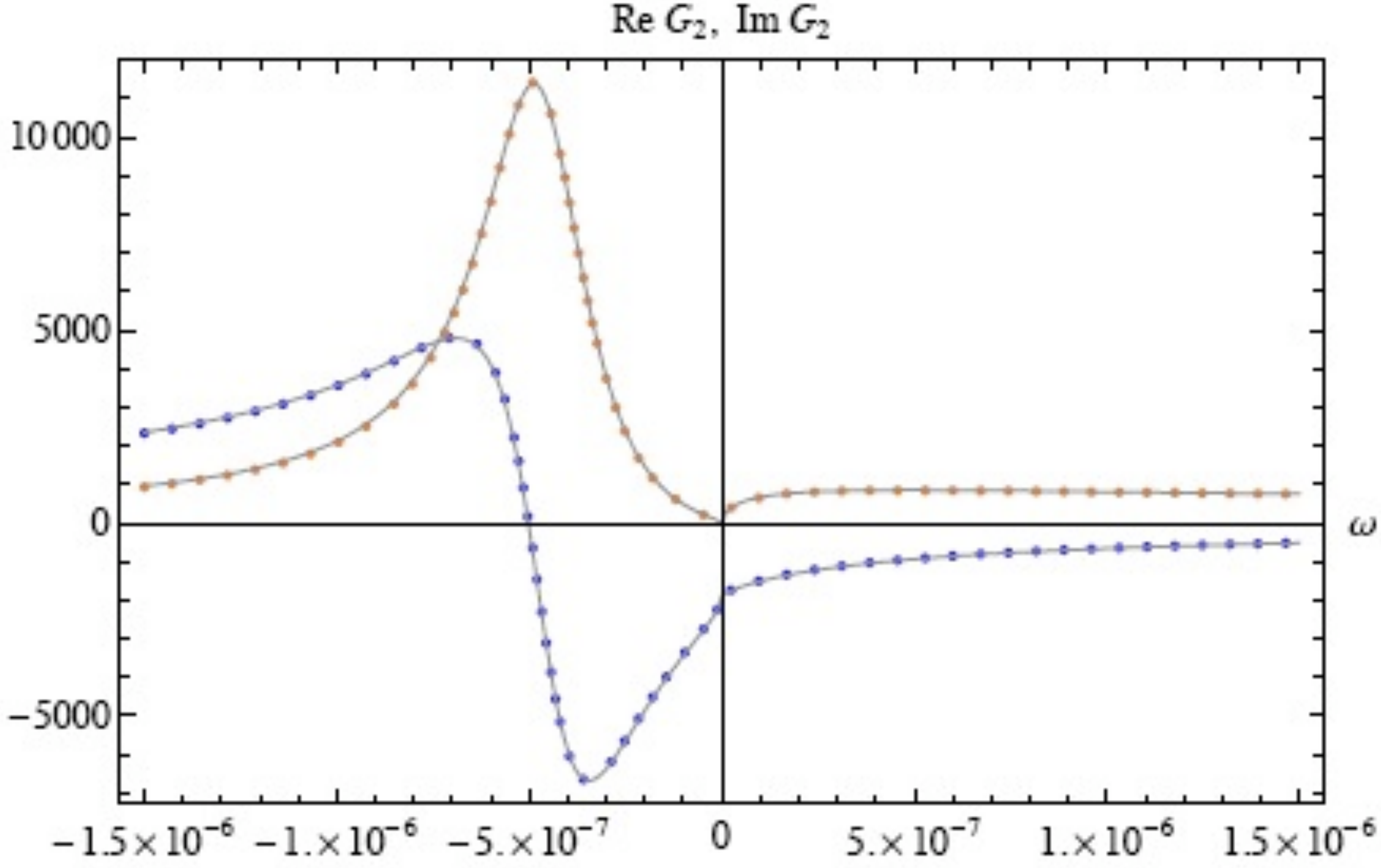

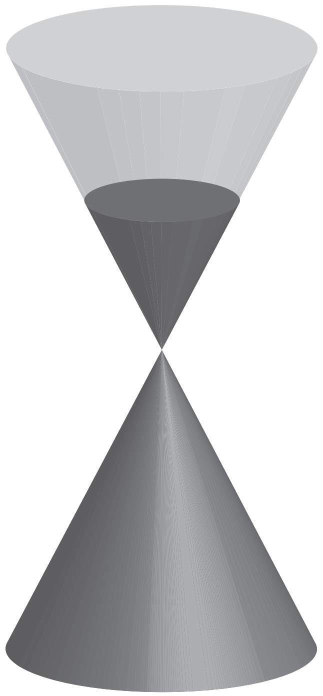

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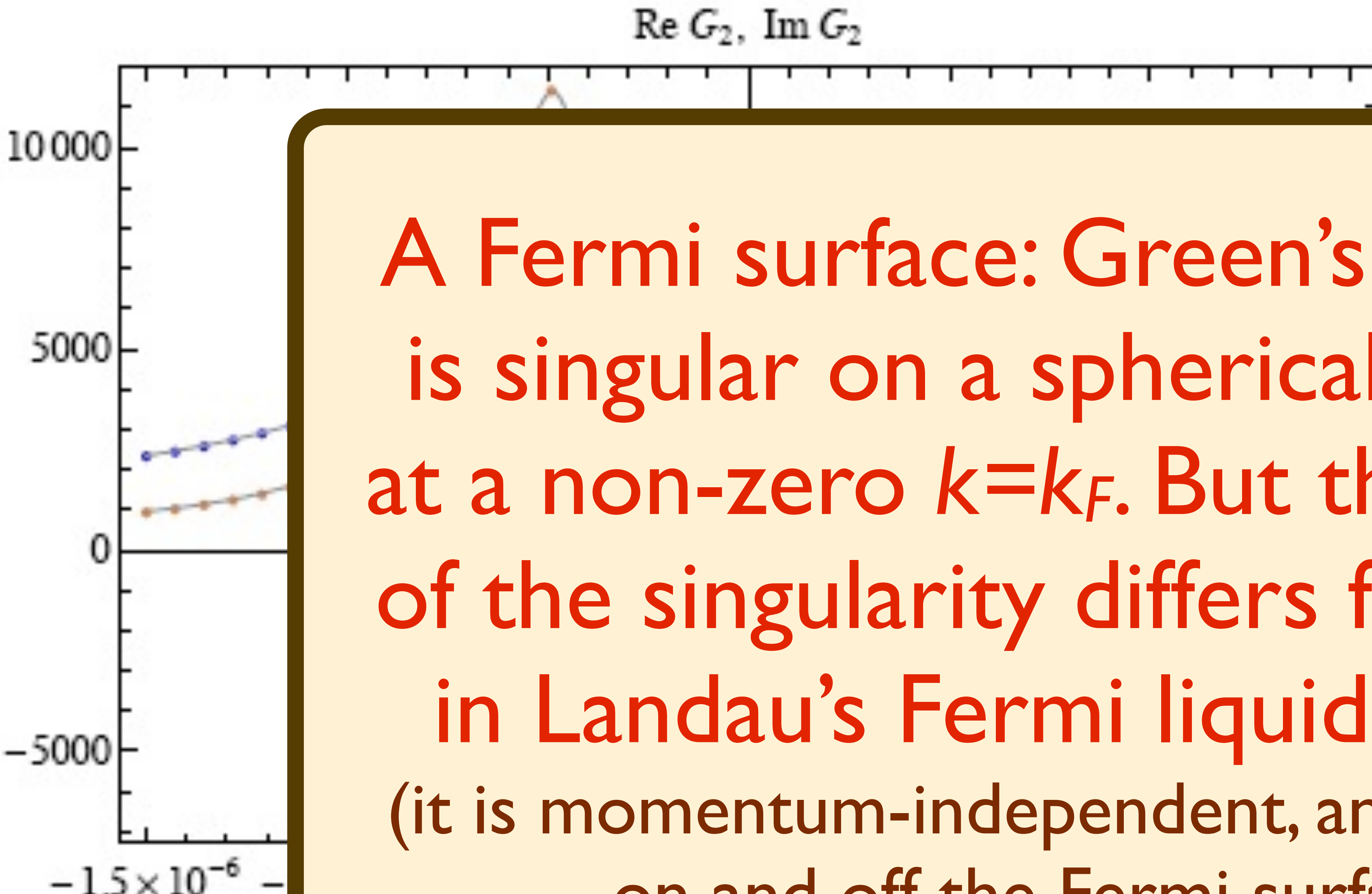

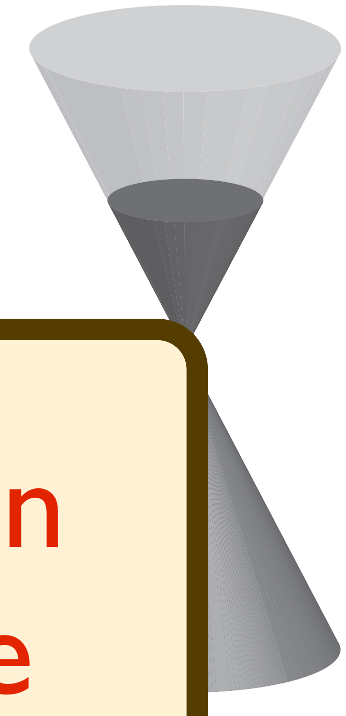

A Fermi surface: Green's function is singular on a spherical surface at a non-zero $k=k_F$. But the nature of the singularity differs from that in Landau's Fermi liquid theory

(it is momentum-independent, and the same on and off the Fermi surface)

H. Liu,
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7.2694

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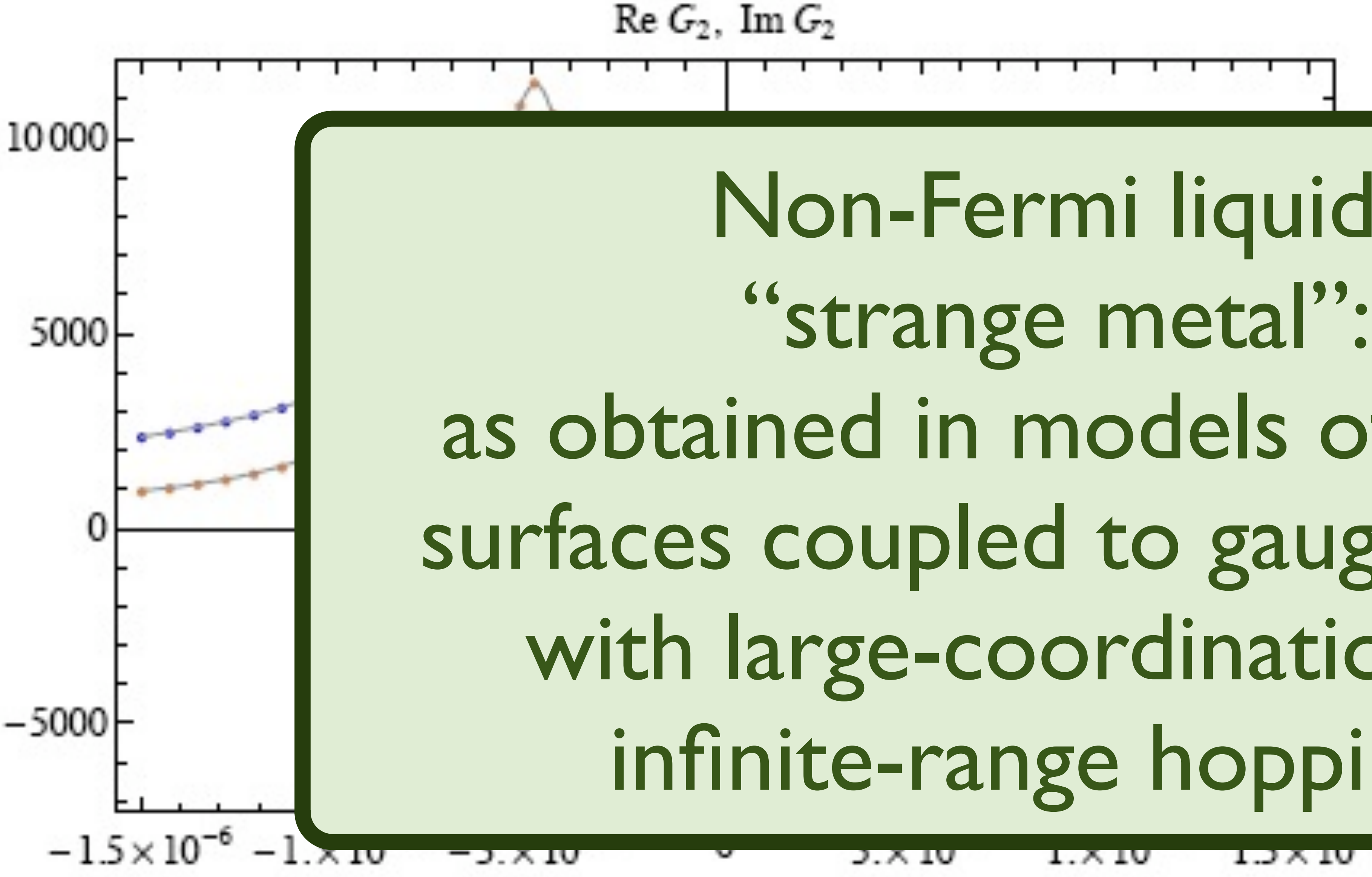

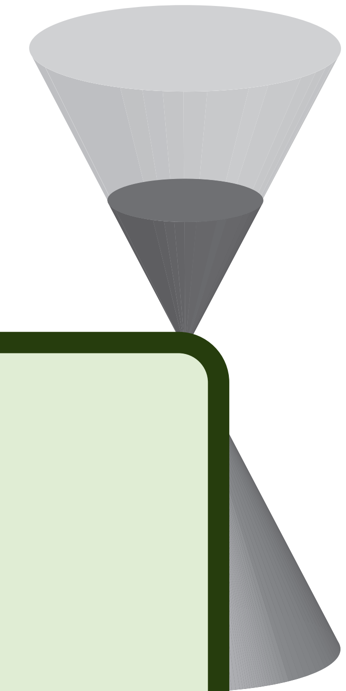

Non-Fermi liquid
"strange metal":
as obtained in models of Fermi
surfaces coupled to gauge fields,
with large-coordination or
infinite-range hopping.

H. Liu,
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.2694

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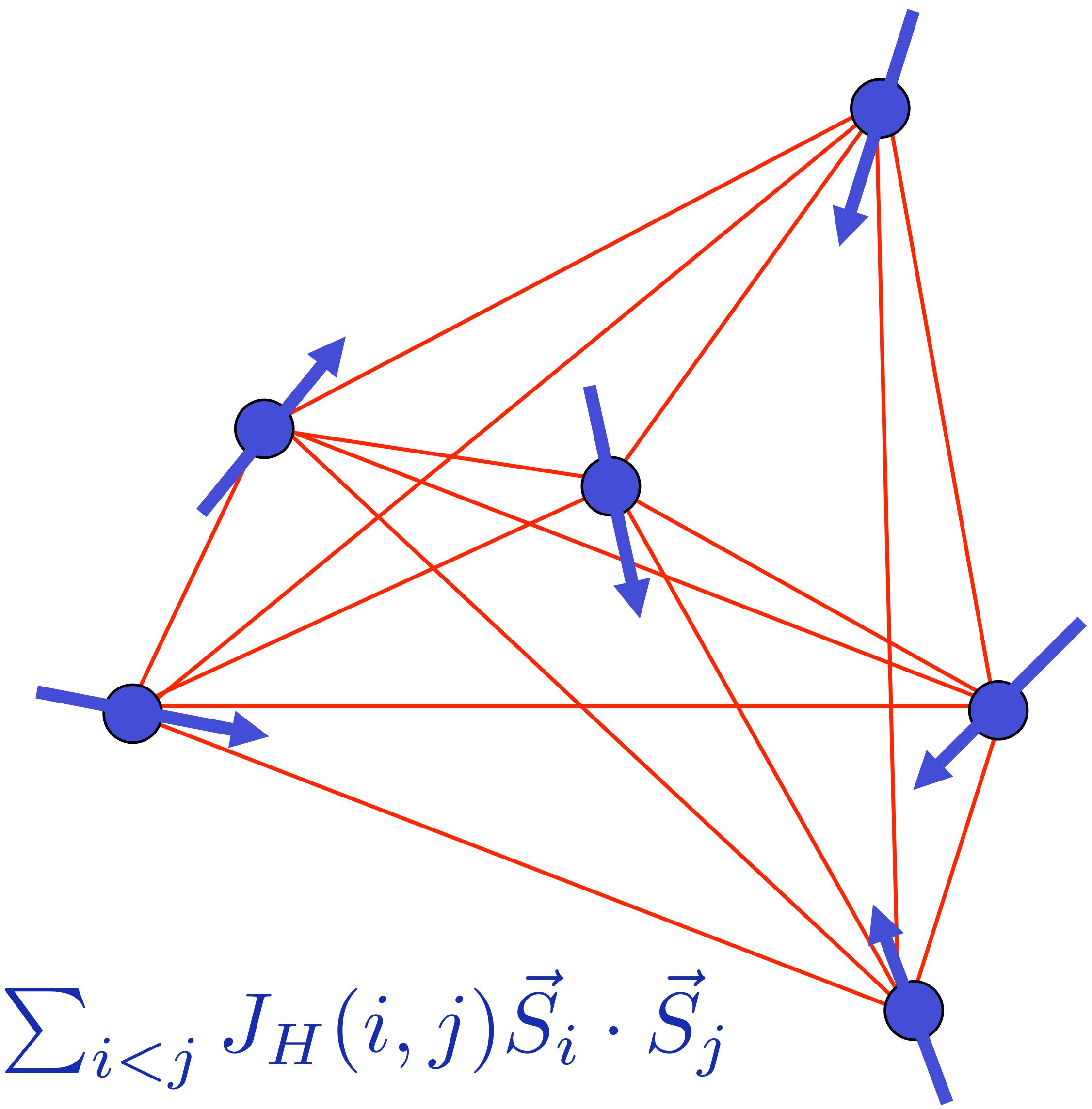
A Kondo lattice model for the $AdS_2 \times R^d$ region of an extremal Reissner-Nordstrom black hole



$$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$$

$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick model of SU(N) spins.

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

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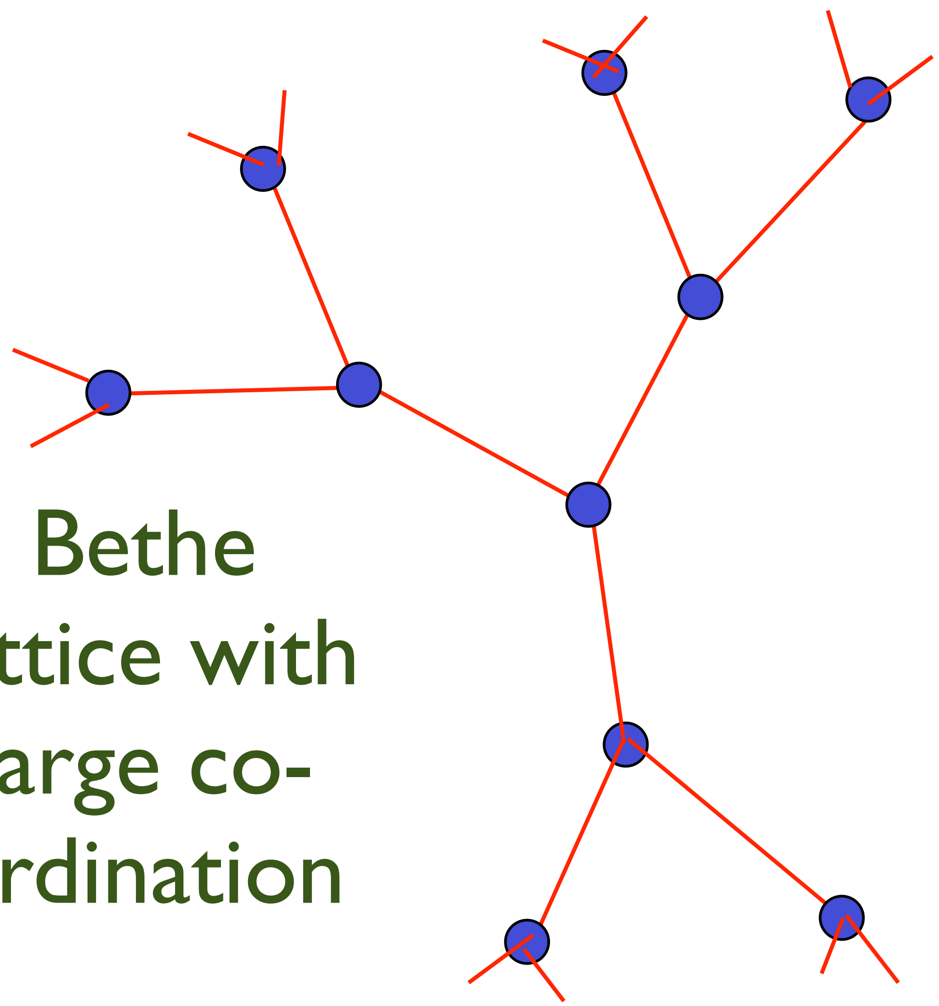

Bethe lattice with large co-ordination

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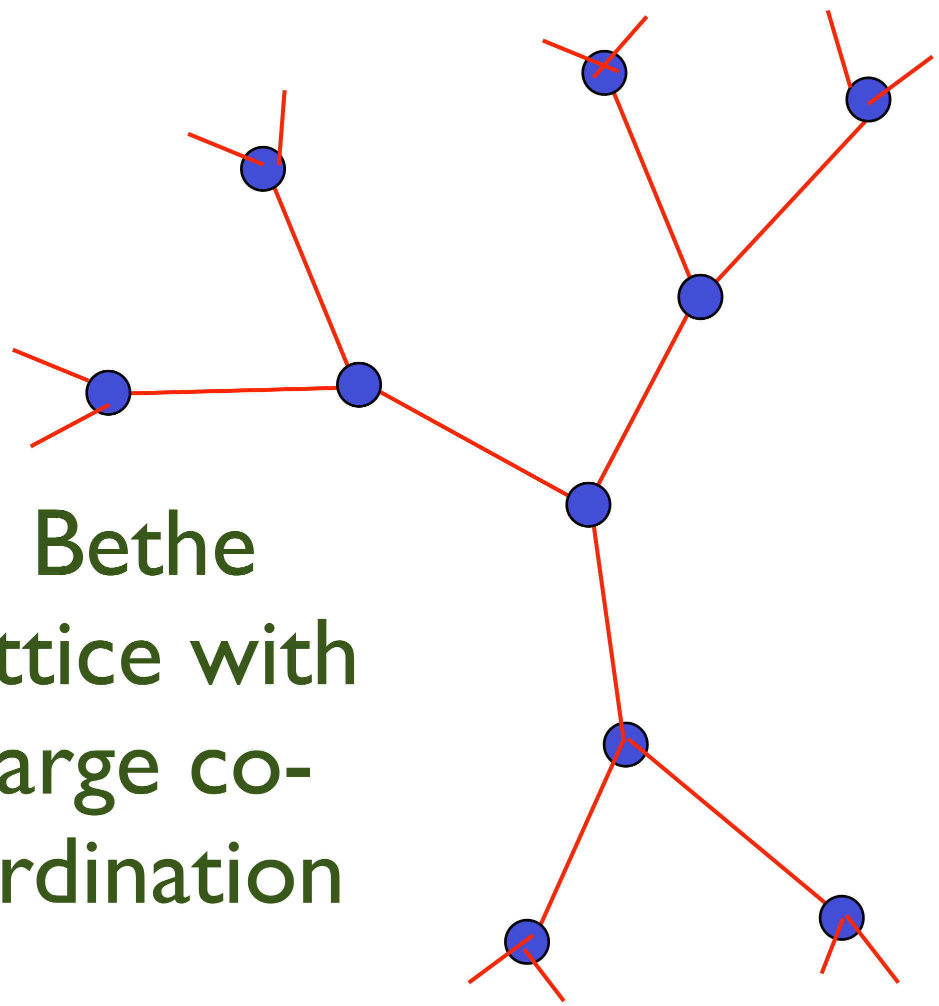

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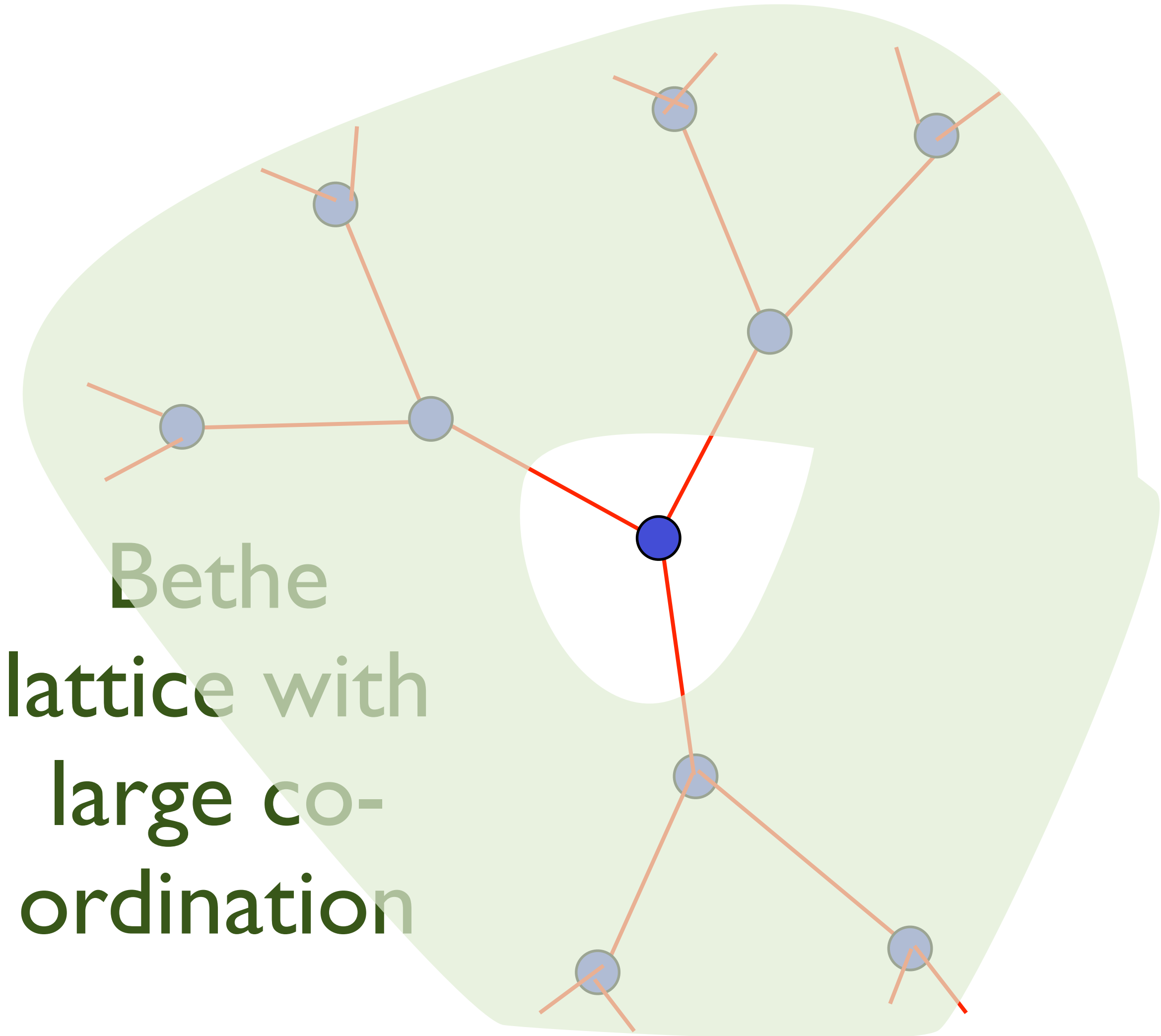

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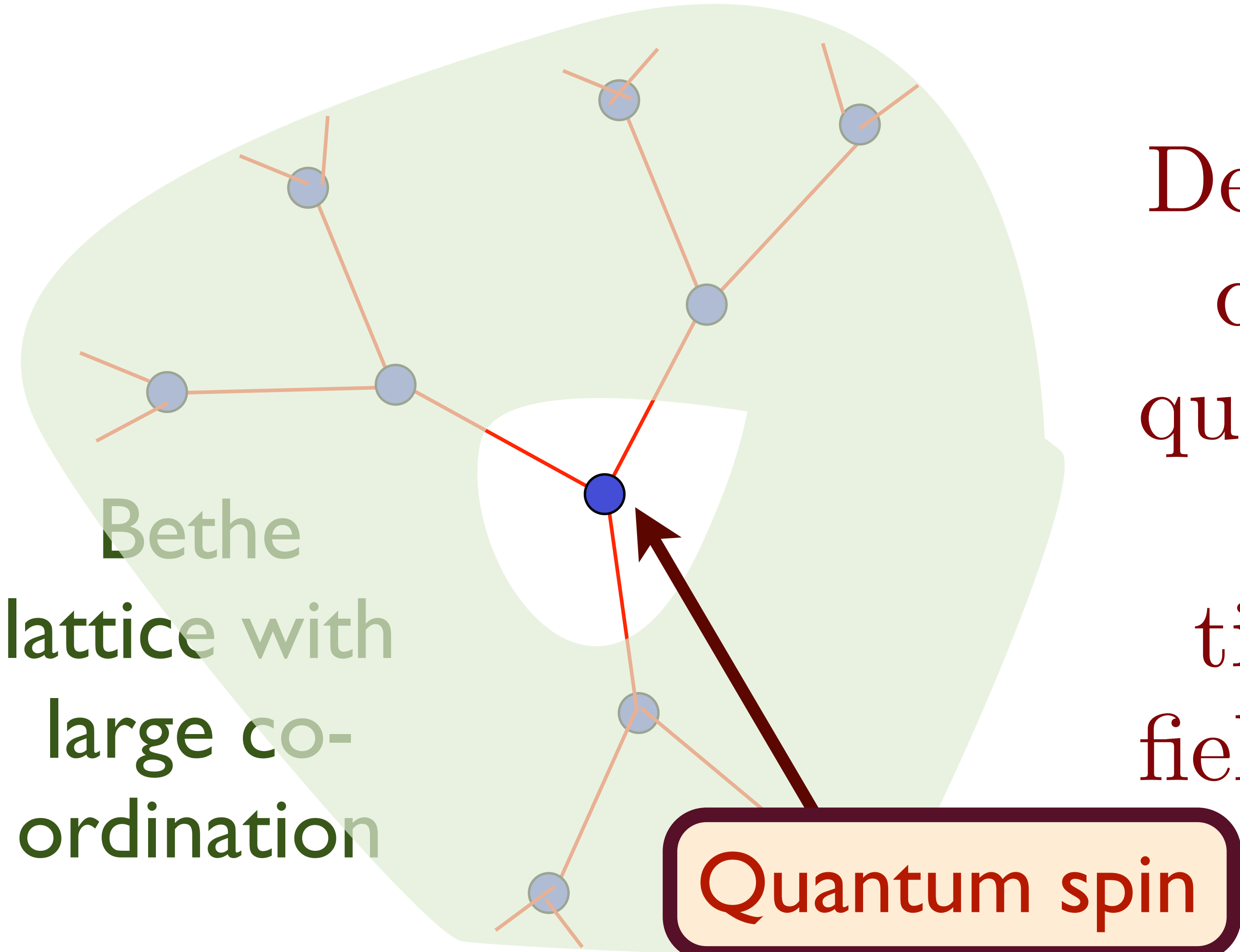

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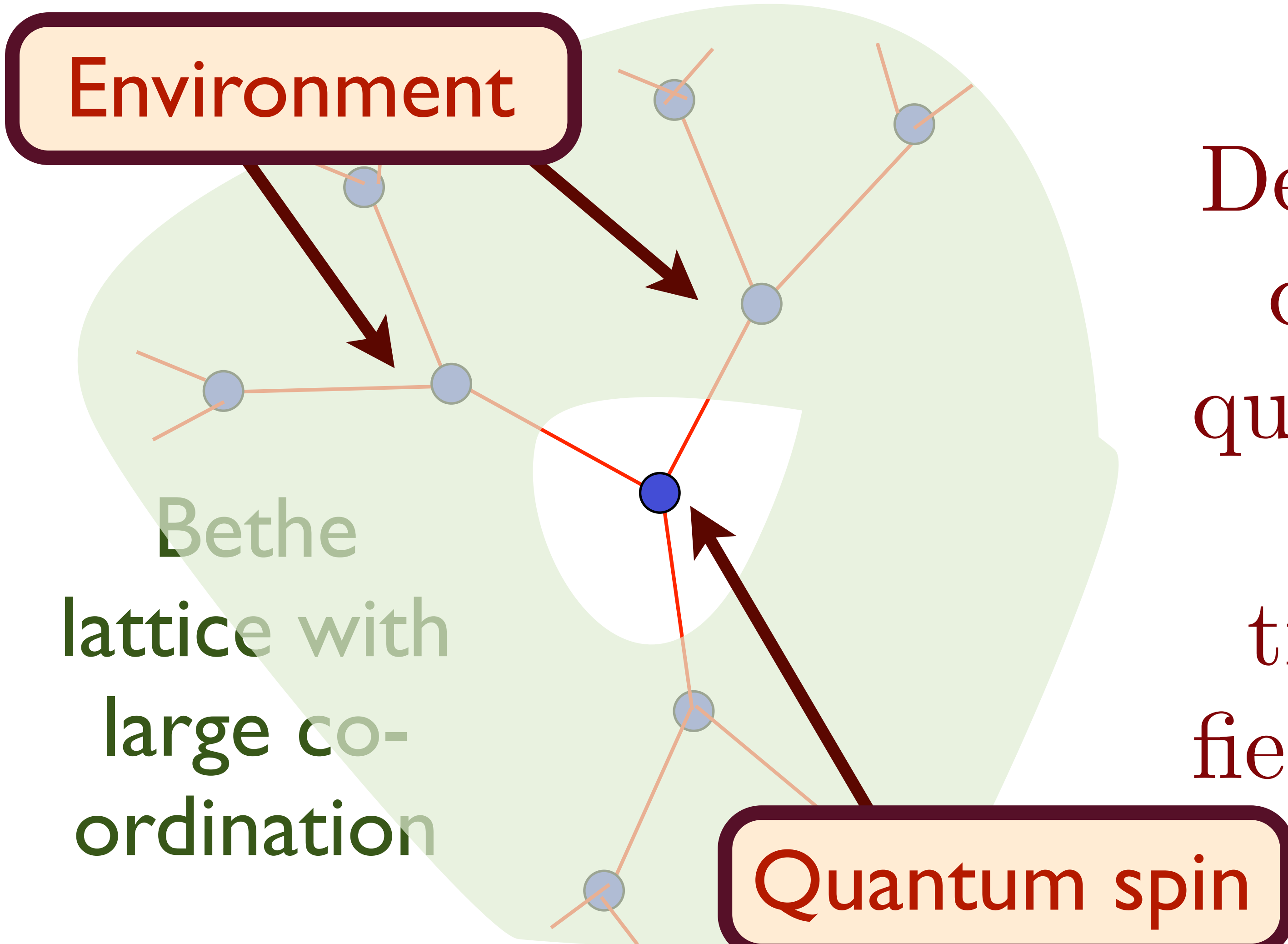

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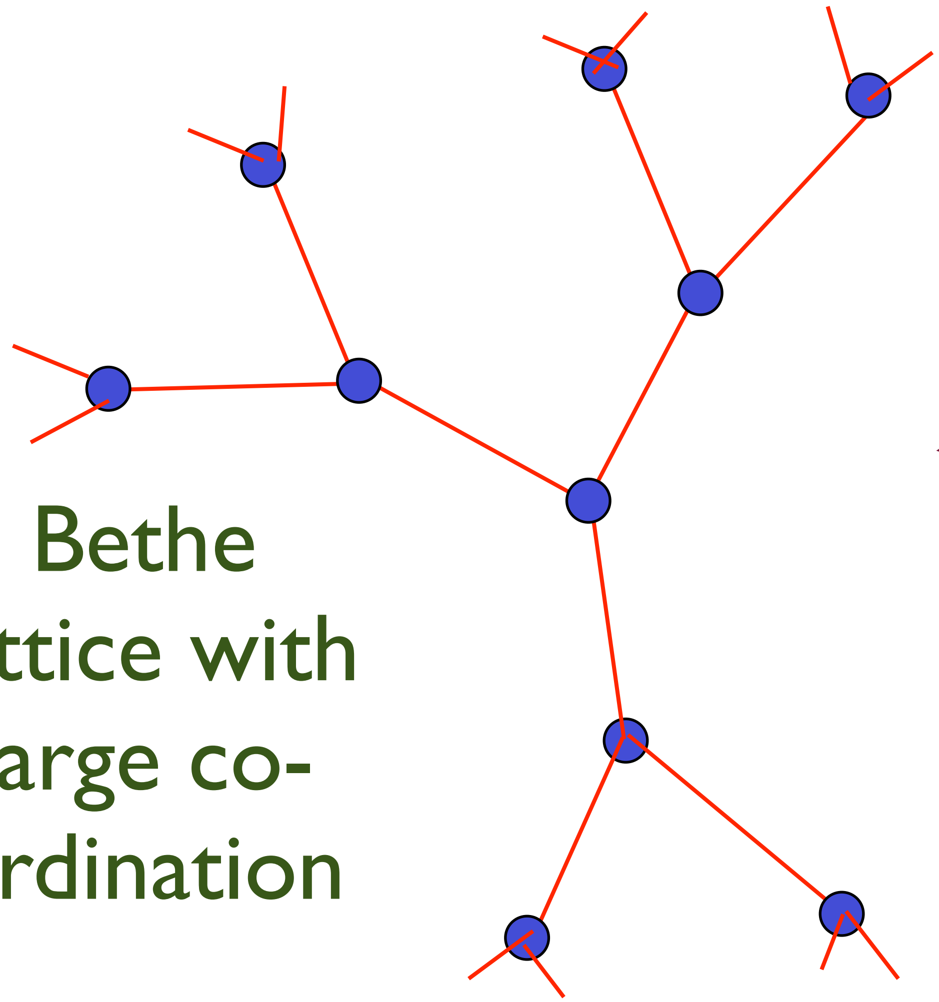
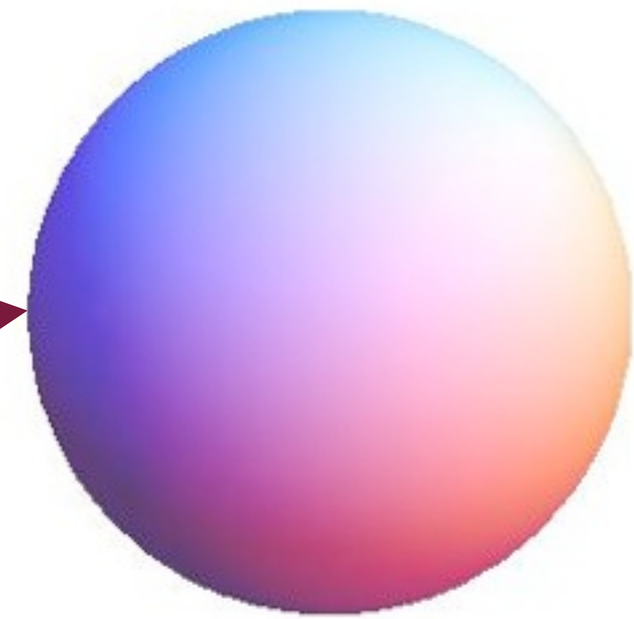


Bethe lattice with large co-ordination

Kondo exchange J_K



A small Fermi surface of probe fermions

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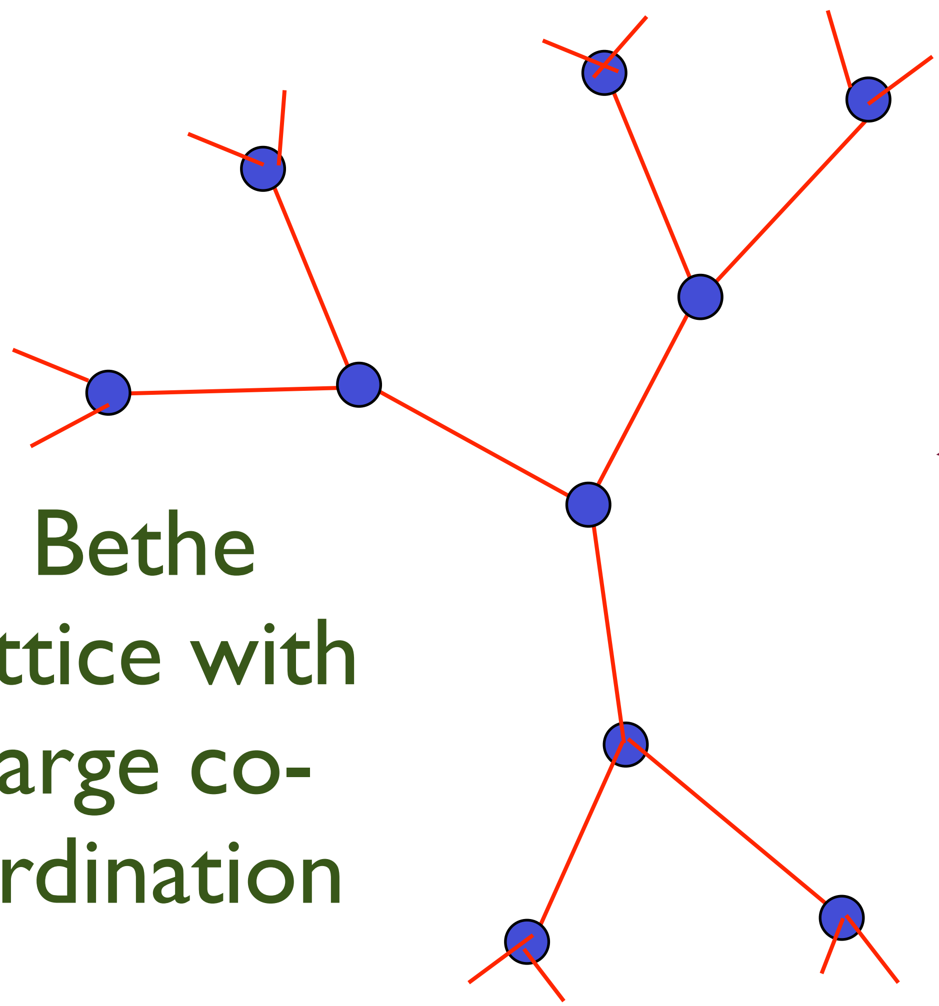

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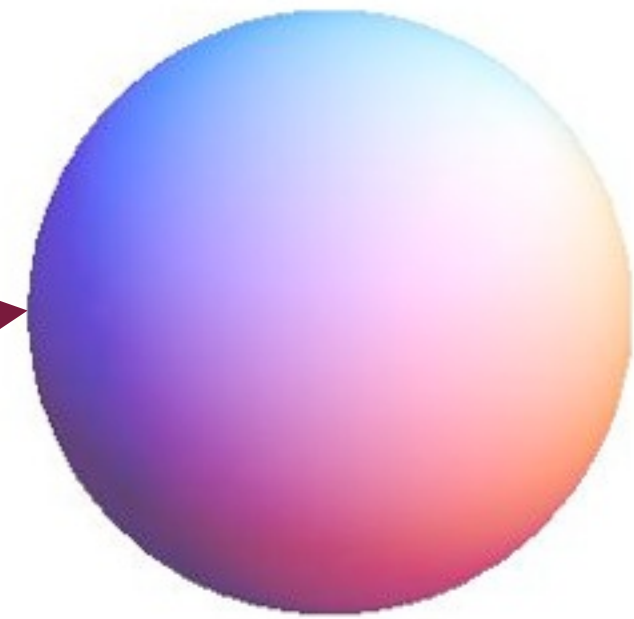

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Low energy properties of the Sherrington-Kirkpatrick-Kondo model map onto the near-horizon physics of an extremal Reissner-Nordstrom black hole

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S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

- The infinite-range Sherrington-Kirkpatrick-Kondo model has properties which match with those of the $AdS_2 \times R^2$ holographic solutions:
  - A non-zero ground state entropy.
  - Single fermion self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
  - A marginal Fermi liquid spectrum for the conduction electrons (for the holographic solution, this requires tuning a free parameter).
  - The low energy sector has conformally invariant correlations which are consistent with the AdS_2 geometry.

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

Much work remains in extending these solvable models to a realistic theory of the cuprate superconductors. Considerable recent progress in gravitational theories which include back-reaction of Fermi surfaces on the AdS metric

Conclusions

New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

Conclusions

The AdS/CFT correspondence offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density