Spin correlations in Mott insulators and *d*-wave superconductors: implications of recent experiments

Subir Sachdev
Matthias Vojta (Augsburg)
Chiranjeeb Buragohain
Anatoli Polkovnikov
Kwon Park

Science **286**, 2479 (1999).

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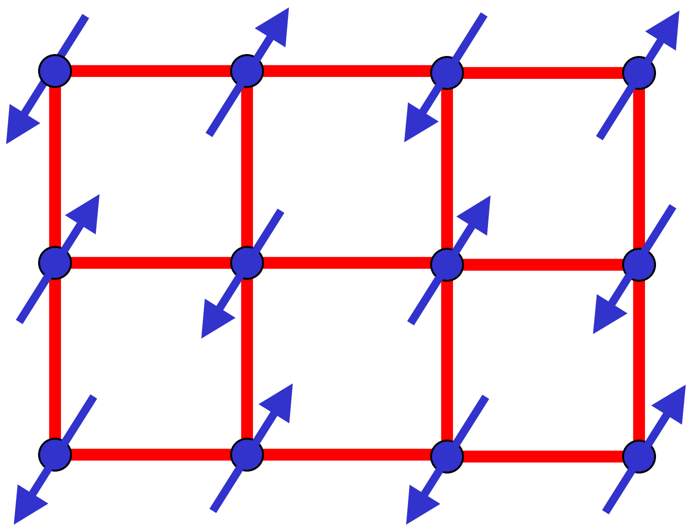
http://pantheon.yale.edu/~subir

Parent compound of the high temperature superconductors: La_2CuO_4

Square lattice antiferromagnet

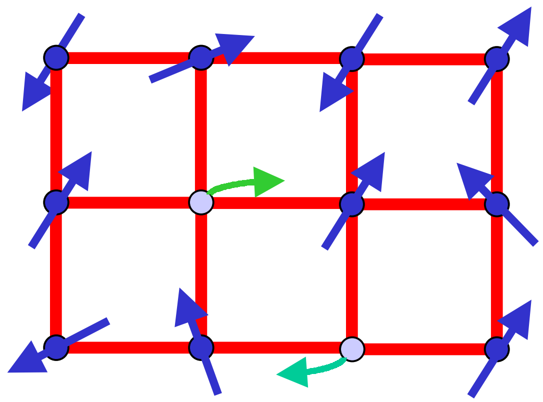


$$H = \sum_{<ij>} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

Ground state has long-range magnetic (Neel) order

$$\left\langle \vec{S}_i \right\rangle = (-1)^{i_x + i_y} N_0 \neq 0$$

Introduce mobile carriers of density δ by substitutional doping of out-of-plane ions *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$



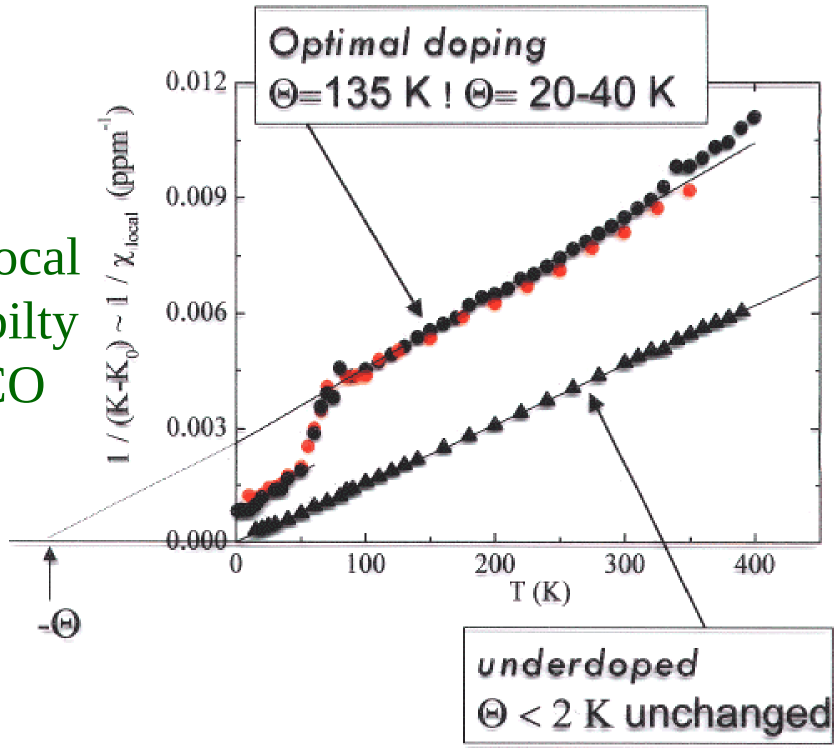
Exhibits superconductivity below a high critical temperature T_c

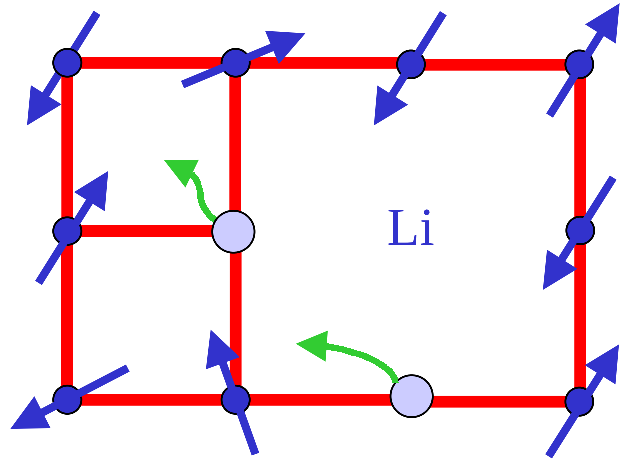
Almost all $T \to 0$ properties can be understood in the framework of a standard BCS theory in which the electrons form spin-singlet, *d*-wave Cooper pairs. However, many $T > T_c$ properties are anomalous.

$$\text{As } T \to 0, \quad \langle S_i \rangle = 0 \quad \text{and} \quad \chi_{\text{spin}} = 0$$

Measurement of spin susceptibility near non-magnetic (Zn/Li) impurities

Inverse local susceptibilty in YBCO





^{7}Li NMR below T$_c$

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, cond-mat/0010234.

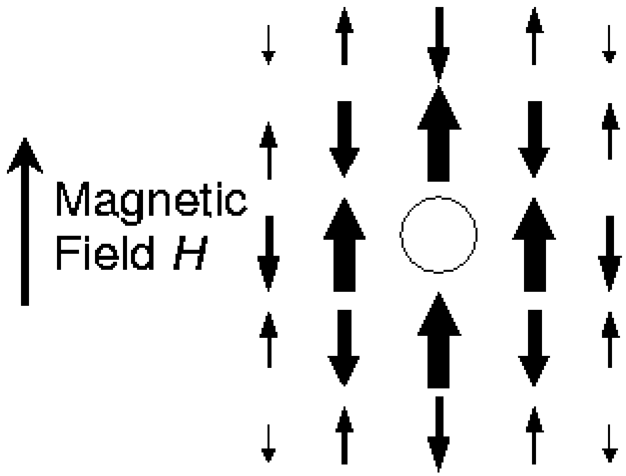
Measured $\chi_{\text{impurity}}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

Not expected from BCS theory, which predicts $\chi_{\text{impurity}}(T \to 0) \neq \infty$ for a non-magnetic impurity with strong potential scattering.

Zn impurity in $YBa_2Cu_3O_{6.7}$

Moments measured by analysis of Knight shifts

M.-H. Julien, T. Feher, M. Horvatic, C. Berthier, O. N. Bakharev, P. Segransan, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **84**, 3422 (2000); also earlier work of the group of H. Alloul and the original experiment of A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, and G.B. Teitel'baum, Physica C **168**, 370 (1990).



Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity: net uncancelled phase of S=1/2

S=1 resonance mode in YBCO

$YBa_2Cu_3O_7$

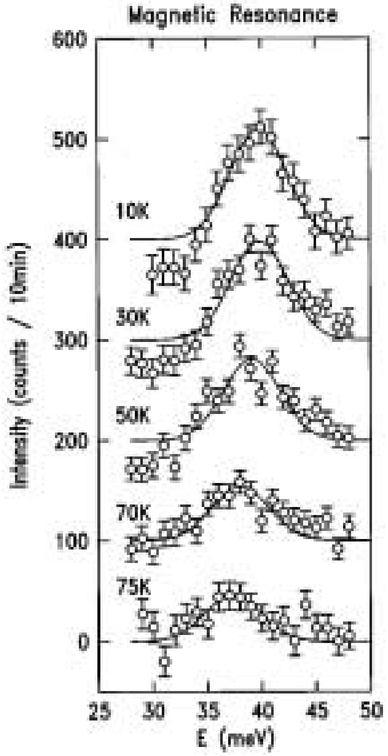
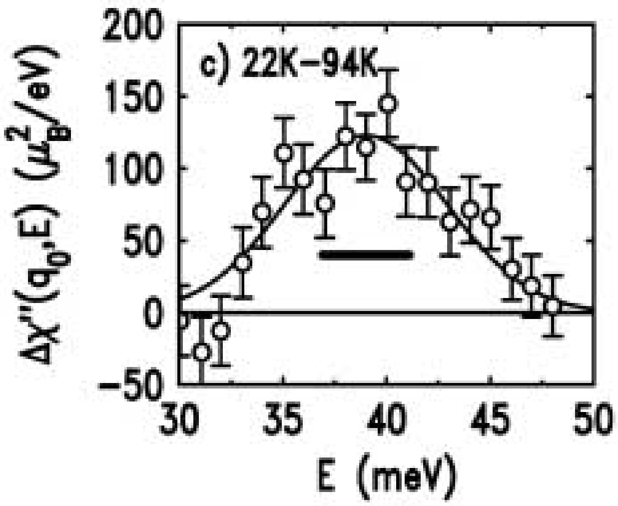


FIG. 8. Unpolarized beam, constant-**Q** data [**Q**=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the T=100 K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

Resolution limited width

H.F. Fong, B. Keimer, D. Reznik, D.L. Milius, and I.A. Aksay, Phys. Rev. B **54**, 6708 (1996)

$YBa_2Cu_3O_7 + 0.5\%$ Zn



Zn induced half-width = 4.25 meV

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)

Questions

I. Why do non-magnetic impurities acquire a *S=1/2* moment ?

II. What is the energy scale at which the collective spin resonance is broadened by Zn impurities ?

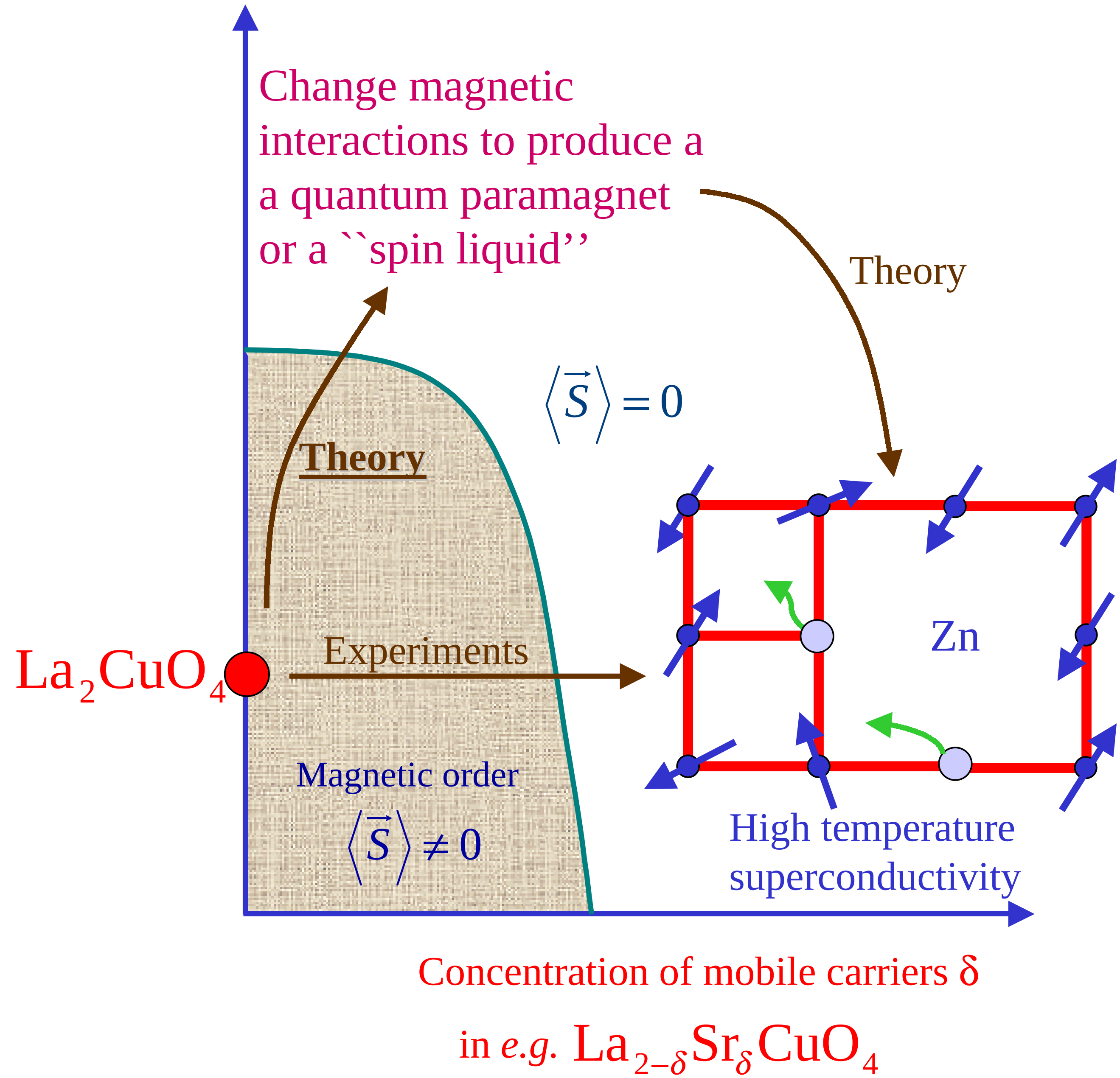
"Swiss cheese" model

Inverse Q of resonance = $\mathcal{C}\; n_{\text{imp}}\xi^2$

$\mathcal{C} \rightarrow$ universal number

III. Does the *S=1/2* moment near a non-magnetic impurity have any implications for STM measurements of a single Zn impurity ?

IV. Does our theoretical framework have any implications for order parameters which compete with the d-wave superconductivity ? Neutron scattering measurements of phonon spectra.

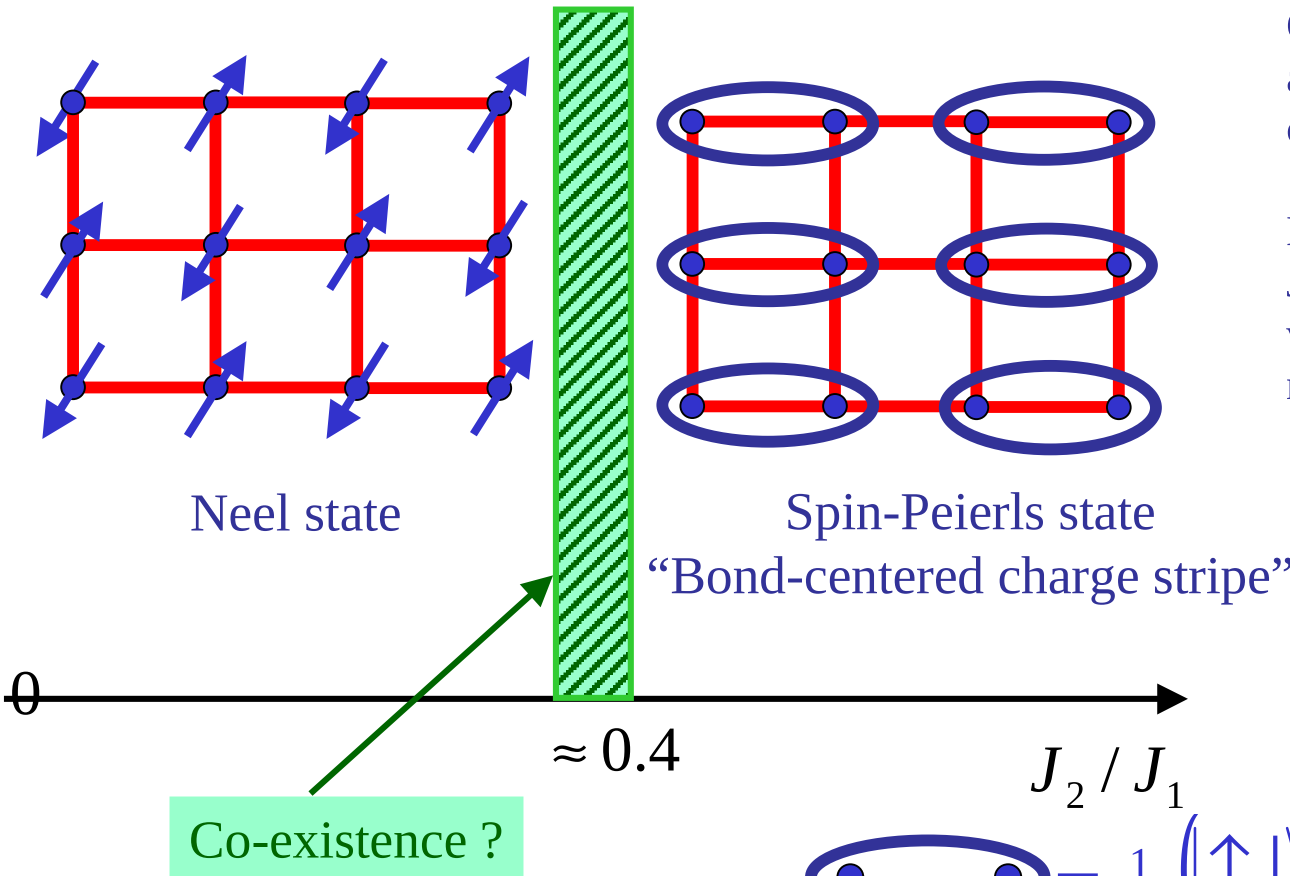


Change magnetic interactions to produce a a quantum paramagnet or a ``spin liquid''
Theory
$\langle \vec{S} \rangle = 0$
Theory
Zn
La_2CuO_4
Experiments
Magnetic order
$\langle \vec{S} \rangle \neq 0$
High temperature superconductivity
Concentration of mobile carriers δ
in *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$

I. Why do non-magnetic impurities acquire a *S=1/2* moment ?

I.A Insulating quantum paramagnets

Square lattice with first (J_1) and second (J_2) neighbor exchange interactions



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

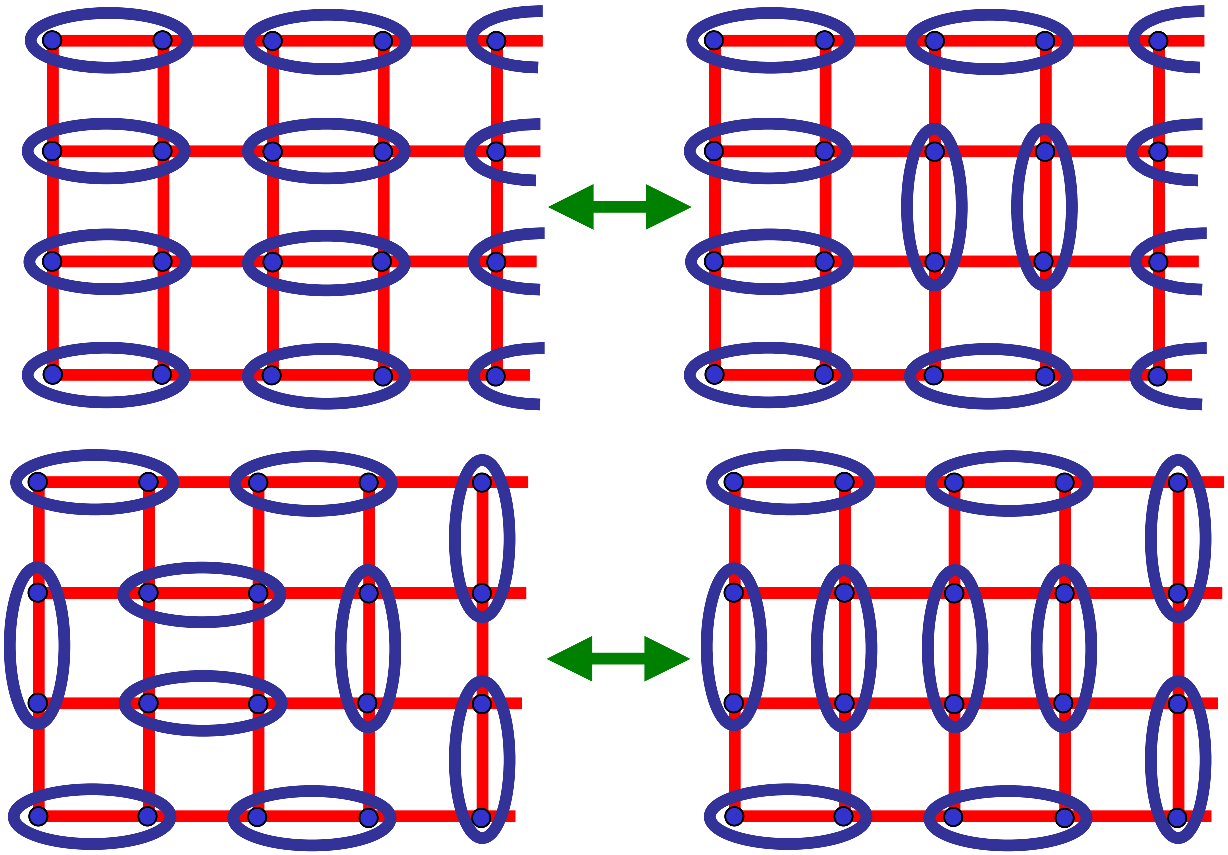
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, condmat/0007329.

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, cond-mat/0002116.

Quantum dimer model –

D. Rokhsar and S. Kivelson Phys. Rev. Lett. **61**, 2376 (1988)



Quantum "entropic" effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.

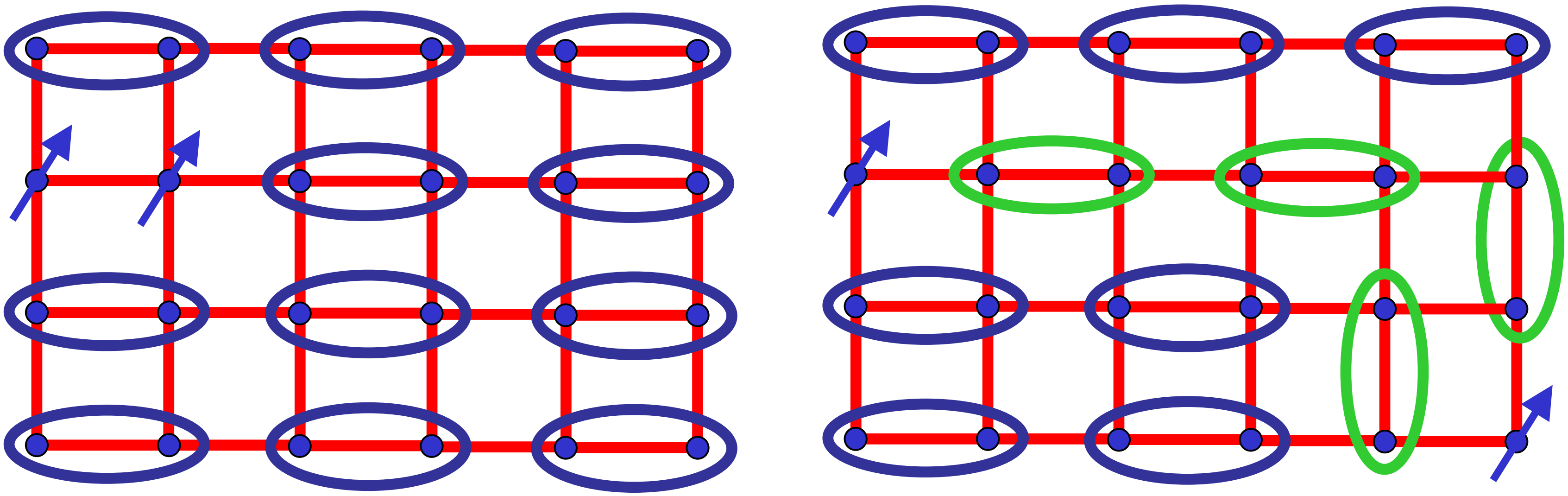
These effects always lead to a broken square lattice symmetry near the transition to the Neel state.

N. Read and S. Sachdev Phys. Rev. B **42**, 4568 (1990).

Excitations

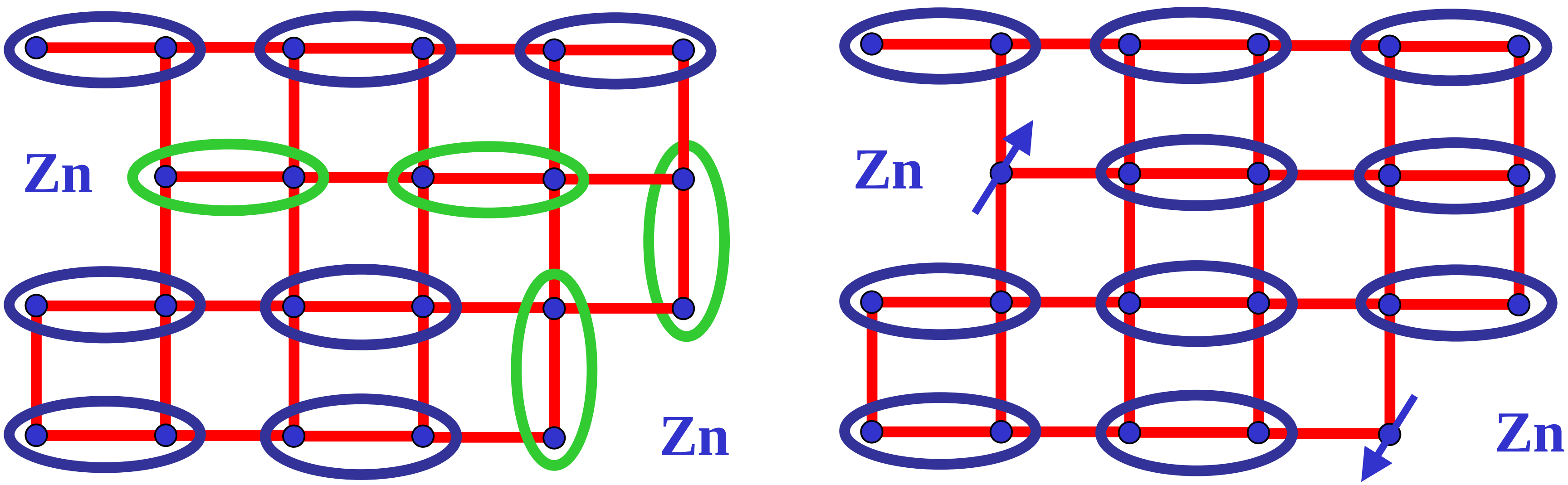
Stable S=1 particle

Energy dispersion $$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$ $\Delta \rightarrow$ Spin gap



S=1/2 spinons are linearly confined by the line of "defect" singlet pairs between them

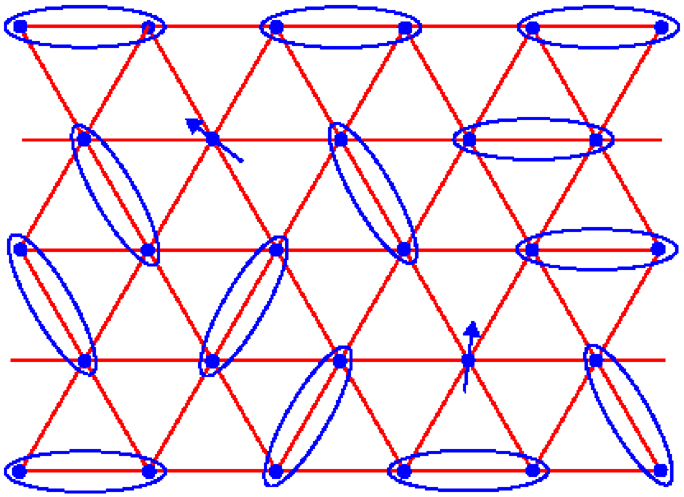
Effect of static non-magnetic impurities (Zn or Li)



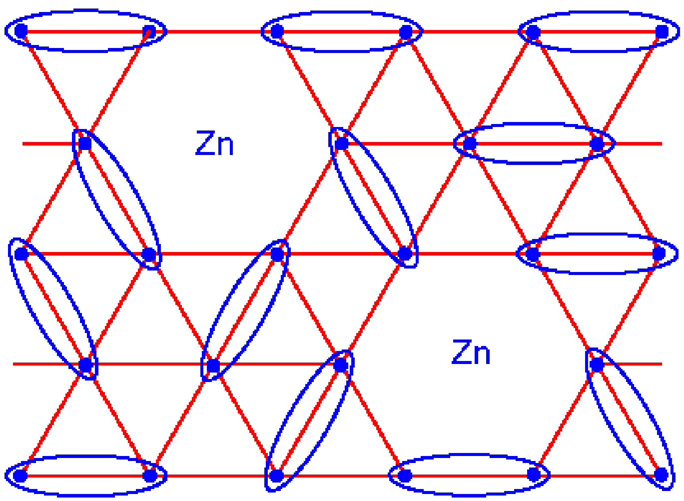
Spinon confinement implies that free S=1/2 moments **must** form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

Paramagnetic ground state with spinon deconfinement



Spinons are deconfined



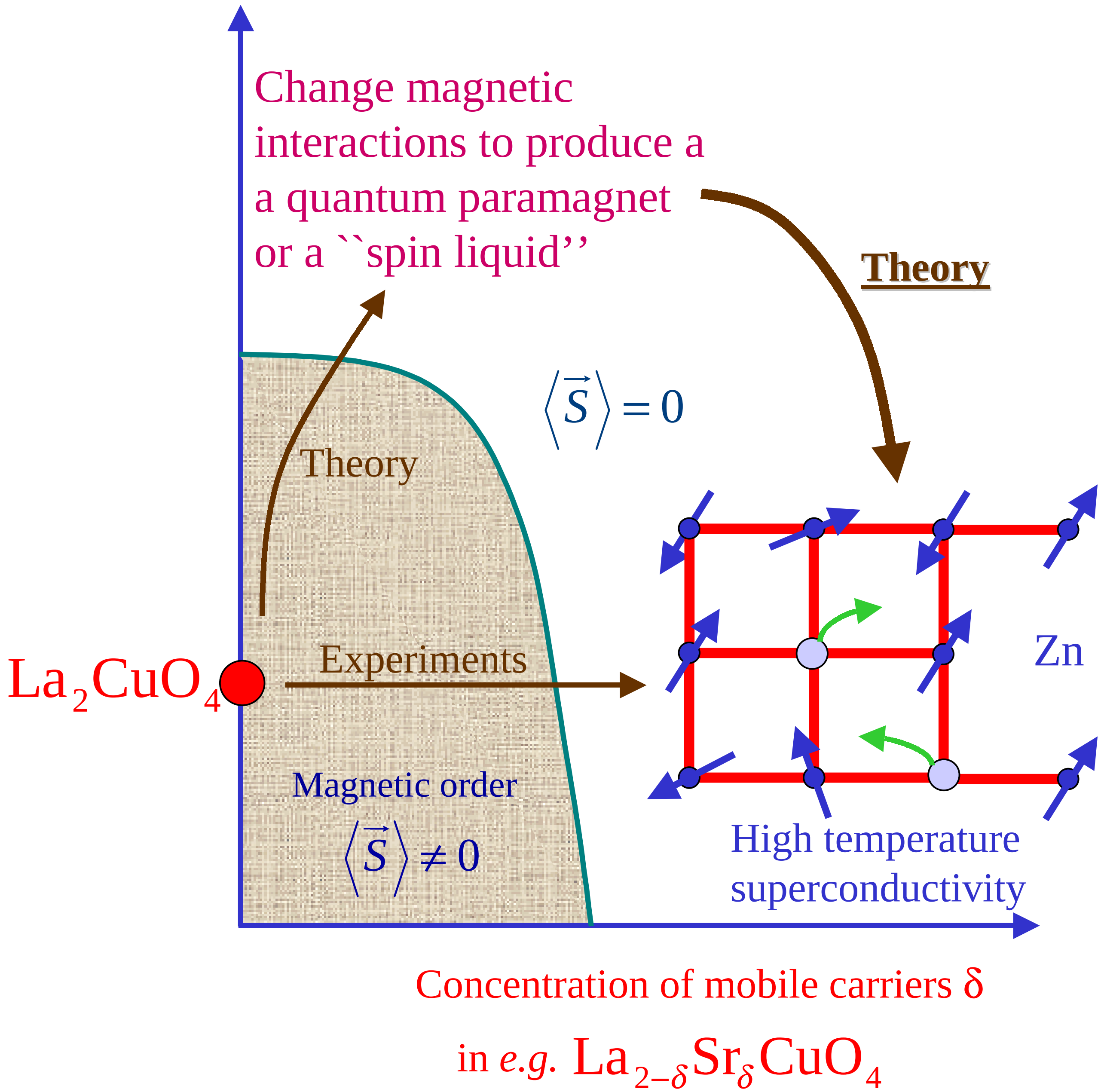
Free S=1/2 moments need not be present near the impurities

Translationally invariant "spin liquid" state obtained by a quantum transition from a magnetically ordered state with **co-planar** spin polarization. Can also appear in frustrated square lattice antiferromagnets – transition to confined states is described by a Z_2 gauge theory

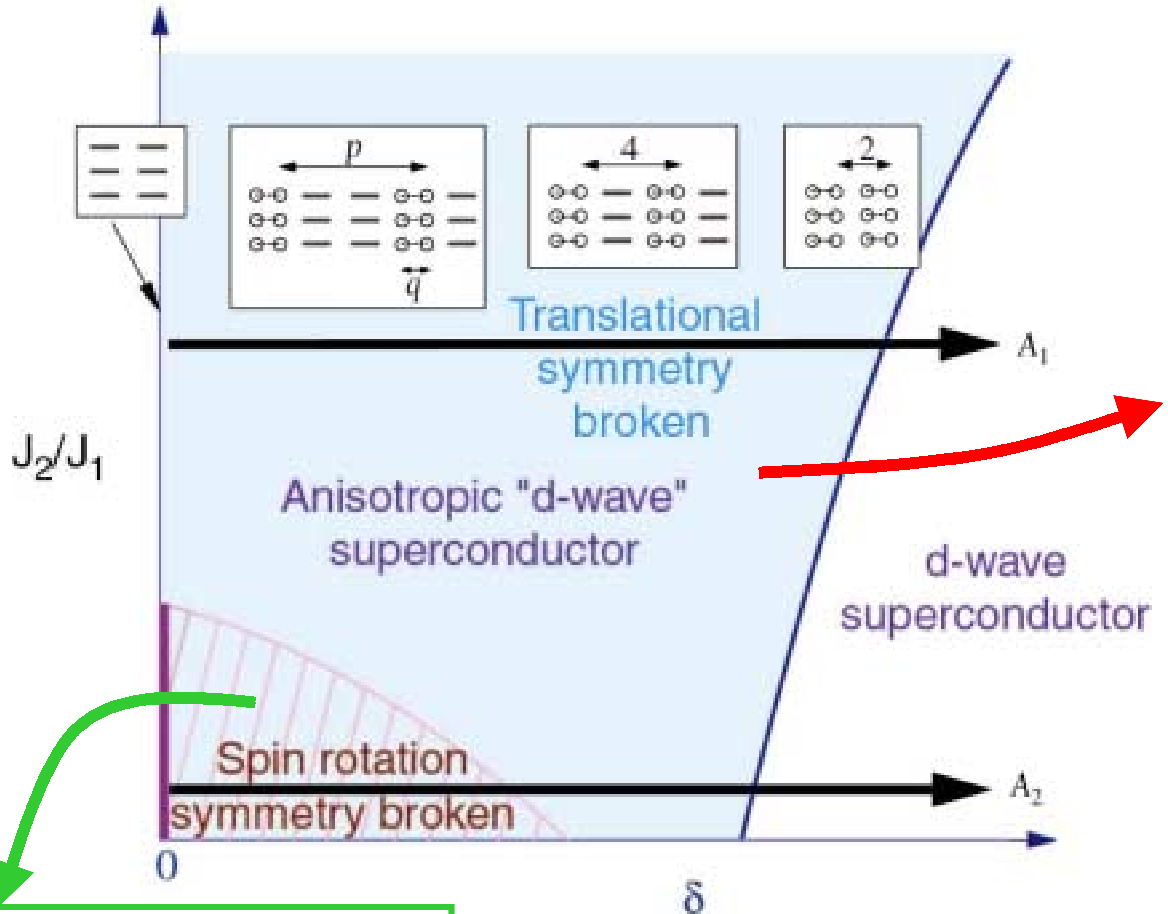
$$\chi_{\text{impurity}}(T \to 0) = 0$$

N. Read and S. Sachdev, Phys. Rev. Lett. **66**, 1773 (1991).
R. Jalabert and S. Sachdev, Phys. Rev. B **44**, 686 (1991).

P. Fazekas and P.W. Anderson, Phil Mag **30**, 23 (1974).
S. Sachdev, Phys. Rev. B **45**, 12377 (1992).
G. Misguich and C. Lhuillier, cond-mat/0002170.
R. Moessner and S.L. Sondhi, cond-mat/0007378.


Change magnetic
interactions to produce a
a quantum paramagnet
or a ``spin liquid''
Theory
$\langle \vec{S} \rangle = 0$
Theory
La_2CuO_4
Experiments
Zn
Magnetic order
$\langle \vec{S} \rangle \neq 0$
High temperature
superconductivity
Concentration of mobile carriers δ
in *e.g.* $La_{2-\delta}Sr_\delta CuO_4$

I.B Phase diagram for doping of confined Mott insulators



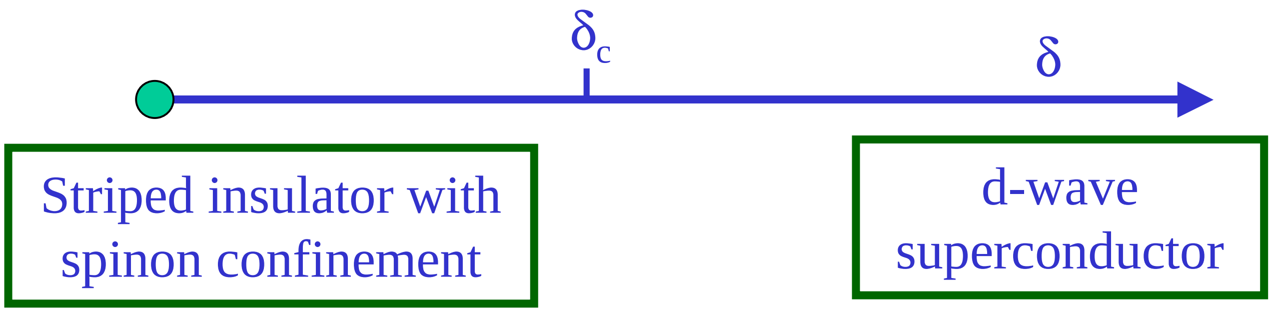
Superconductivity can coexist with *bond-centered* charge stripe order in region without magnetic order

Site-centered charge stripes likely in regions with magnetic order and suppressed superconductivity

S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).
M. Vojta and S. Sachdev, Phys. Rev. Lett. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. B **62**, 6721 (2000)
See also J. Zaanen, Physica C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, Nature **393**, 550 (1998),
S. White and D. Scalapino, Phys. Rev. Lett. **80**, 1272 (1998); **81**, 3227 (1998).
C. Lannert, M.P.A. Fisher, and T. Senthil cond-mat/0007002.

Zn or Li impurities in doped Mott insulators

Case A

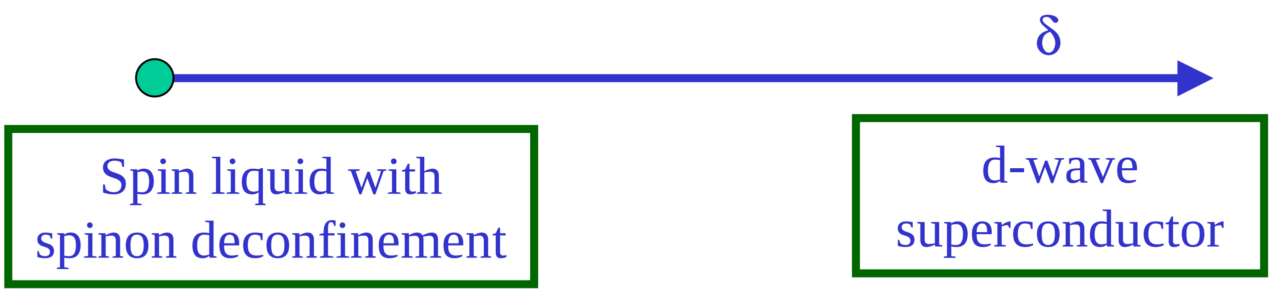


Moments form near each Zn or Li.

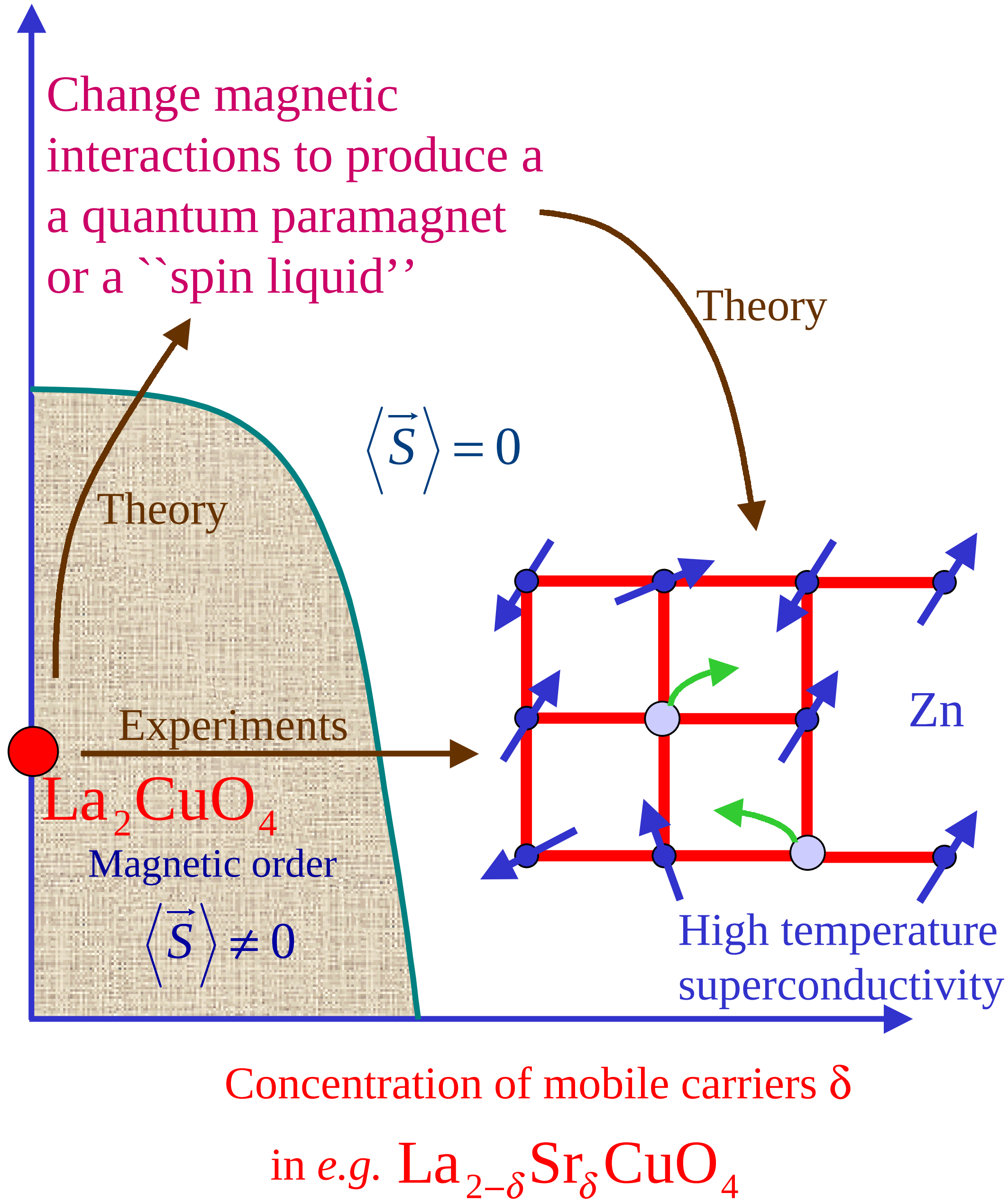
This moment is quenched at a quantum phase transition at $\delta=\delta_c$.

D. Withoff and E. Fradkin, Phys. Rev. Lett. **64**, 1835 (1990).
C. Gonzalez-Buxton and K. Ingersent, Phys. Rev. B **57**, 14254 (1998).

Case B



No moments form near Zn or Li ions substituted for Cu and impurity response evolves smoothly



Summary:

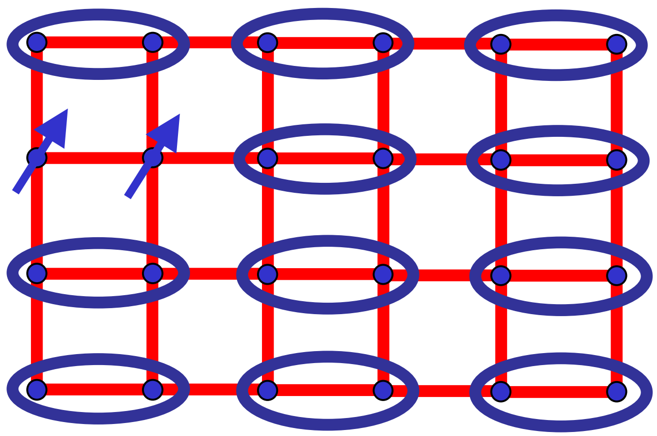
Confined, paramagnetic Mott insulator necessarily has

1. Stable S=1 spin collective mode.
2. Broken translational symmetry:- bond-centered charge stripe order.
3. S=1/2 moments near non-magnetic impurities.

These properties are expected to survive for a finite range of δ in the superconducting states

II. Effect of Zn impurities on spin resonance mode

S=1 resonance mode in YBCO



H.F. Fong, B. Keimer, D. Reznik, D.L. Milius, and I.A. Aksay, Phys. Rev. B **54**, 6708 (1996)

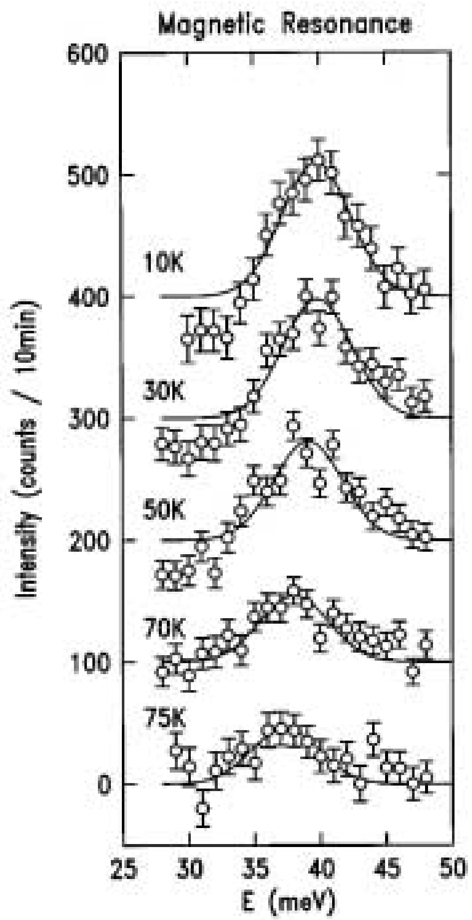


FIG. 8. Unpolarized beam, constant-**Q** data [**Q**=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the T=100 K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

Spin-1 collective mode in $YBa_2Cu_3O_7$- little observable damping at low T.

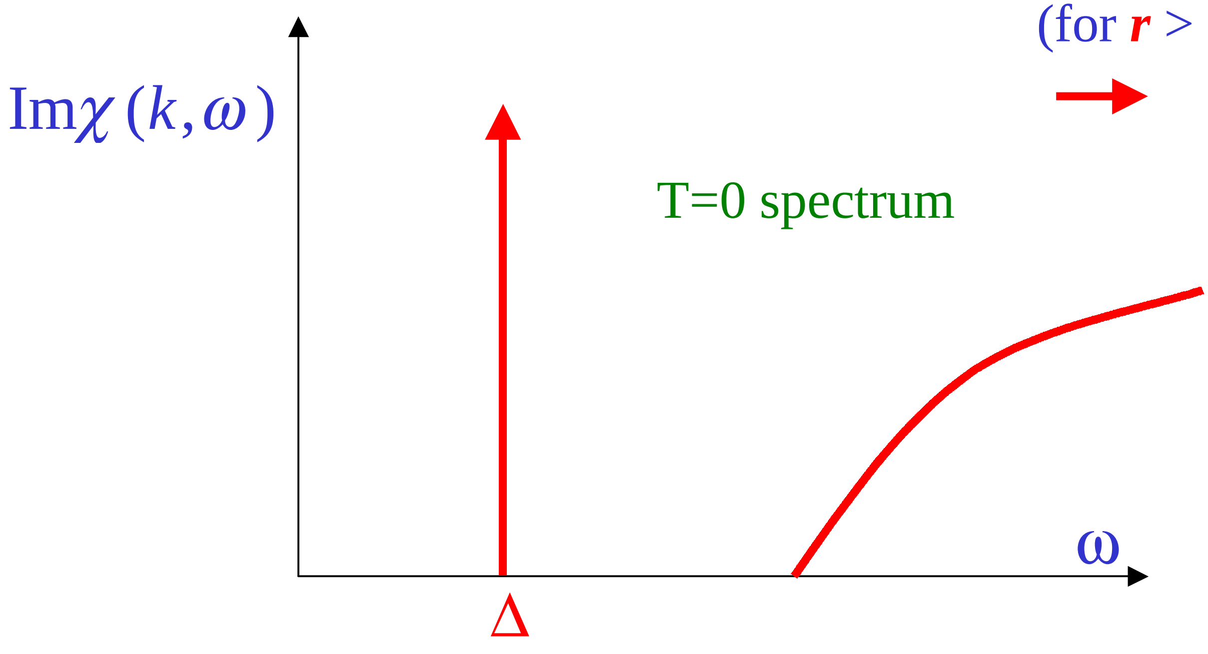
Coupling to superconducting quasiparticles unimportant

Quantum field theory for S=1 particle near magnetic ordering transition

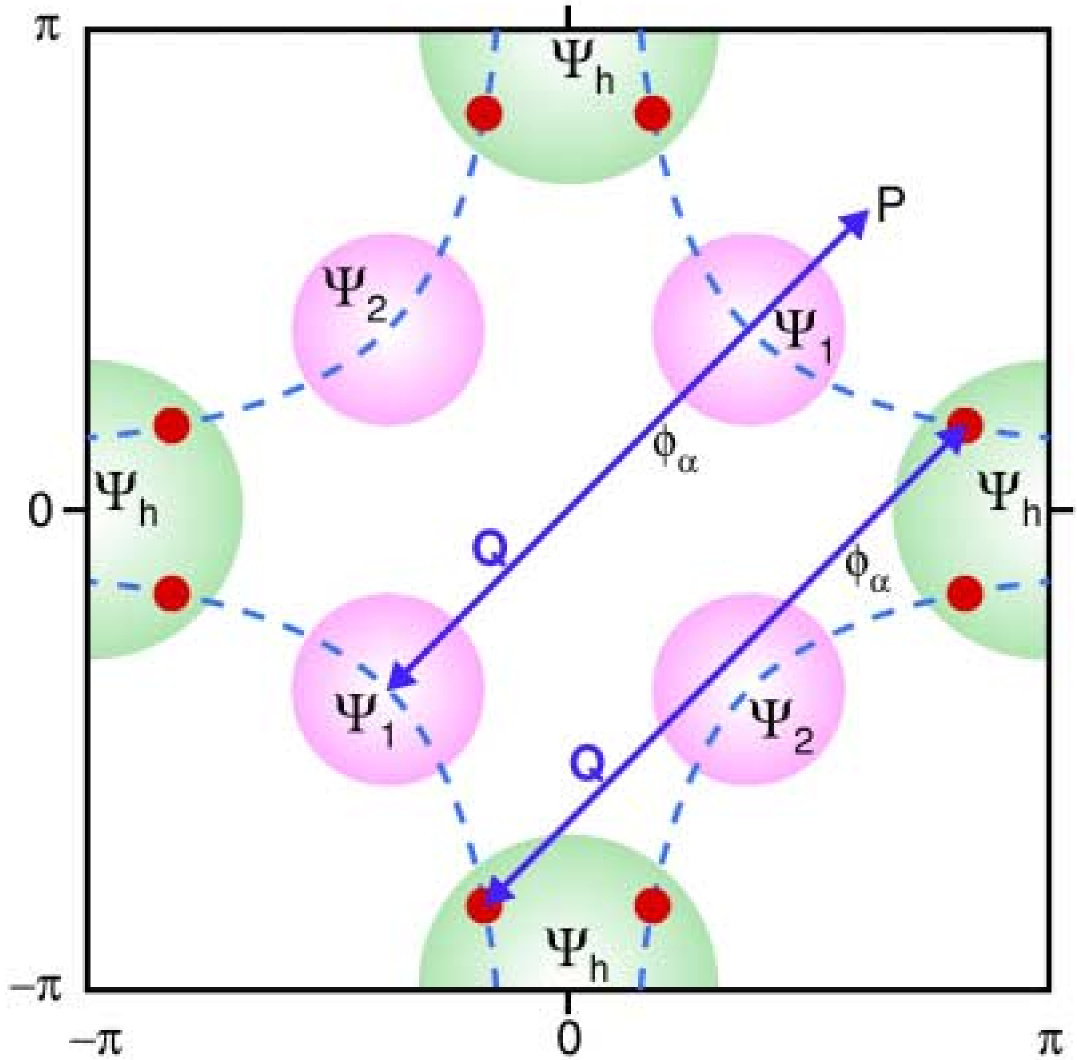
$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

Oscillations of ϕ_α about zero (for $\boldsymbol{r} > 0$)
→ spin-1 collective mode



Constraints from momentum conservation

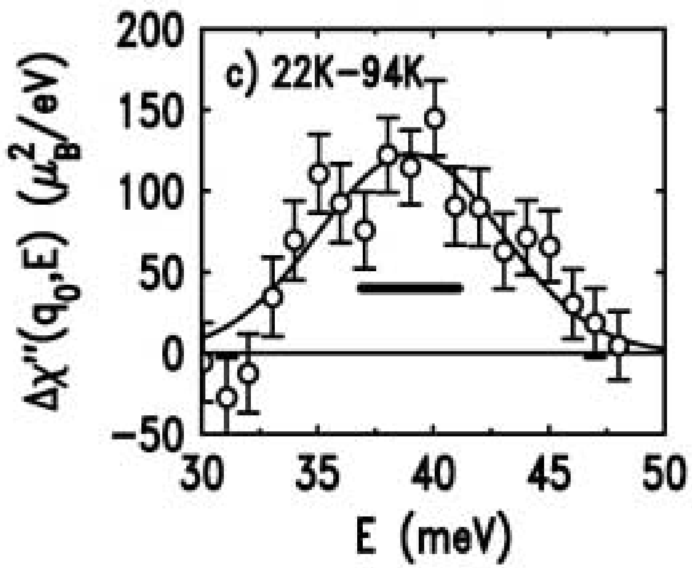


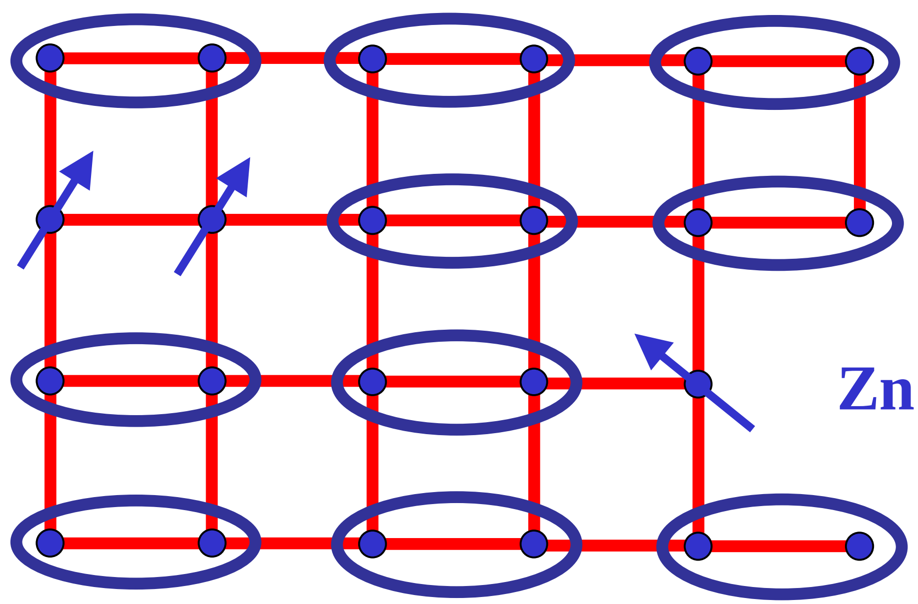
Ψ_h : *strongly coupled* to ϕ_α, but do not damp ϕ_α as long as $\Delta < 2\,\Delta_h$

$\Psi_{1,2}$: *decoupled* from ϕ_α

$YBa_2Cu_3O_7 + 0.5\% \ Zn$

H. F. Fong, P. Bourges,
Y. Sidis, L. P. Regnault,
J. Bossy, A. Ivanov,
D.L. Milius, I. A. Aksay,
and B. Keimer,
Phys. Rev. Lett. **82**, 1939
(1999)





Quantum field theory for *S=1* resonance in the presence of a non-magnetic impurity

Orientation of "impurity" spin -- $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$S_{\text{imp}} = \int d\tau \left[iSA_\alpha(n) \frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau) \phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n) \rightarrow$ Dirac monopole function

Boundary quantum field theory: $S_b + S_{\text{imp}}$

Recall -

$$S_b = \int d^2x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

Renormalization group analysis:
g and γ reach non-zero fixed point values

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993);
S. Sachdev, C. Buragohain, and M. Vojta, Science, **286**, 2479 (1999).
J.L. Smith and Q. Si, Europhys. Lett. **45**, 228 (1999).
A.M. Sengupta, Phys. Rev. B **61**, 4041 (2000);

β functions:

$$\beta(g) = -\varepsilon g + \frac{11g^2}{6} - \frac{23g^3}{12} + O(g^4)$$

$$\beta(\gamma) = -\frac{\varepsilon\gamma}{2} + \gamma^3 - \gamma^5 + \frac{5g^2\gamma}{144} + \pi^2\left(S(S+1) - \frac{1}{3}\right)g\gamma^3 + O\left(\left(\gamma, \sqrt{g}\right)^7\right)$$

No new relevant perturbations near the impurity;
All other boundary perturbations are irrelevant –

e.g. $\lambda \int d\tau \phi_\alpha^2(x=0,\tau)$

Δ and *c* completely determine spin dynamics near an impurity –
No new parameters are necessary !

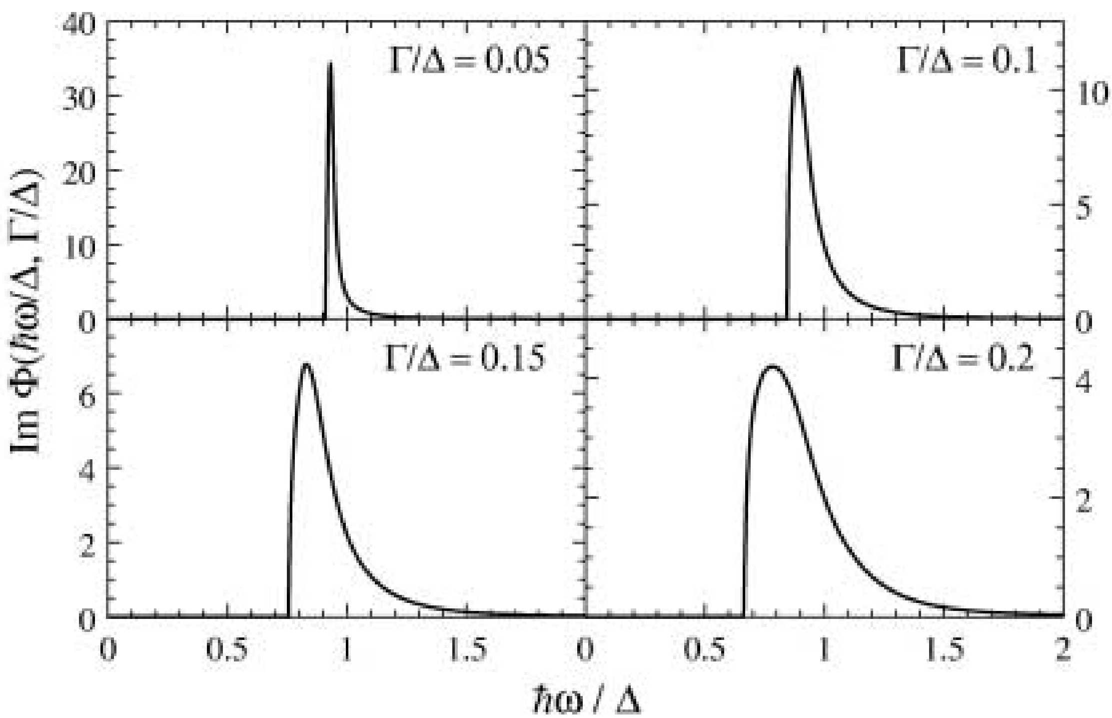
Predictions of quantum field theory

Without impurities $\chi(G,\omega) = \dfrac{A}{\Delta^2 - \omega^2}$

With impurities $\chi(G,\omega) = \dfrac{A}{\Delta^2}\Phi\left(\dfrac{\hbar\omega}{\Delta}, \dfrac{\Gamma}{\Delta}\right)$

$$\Gamma \equiv \frac{n_{\text{imp}} (\hbar c)^2}{\Delta}$$

$\Phi \longrightarrow$ *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation



Predictions:

Half-width of line $\approx \Gamma$

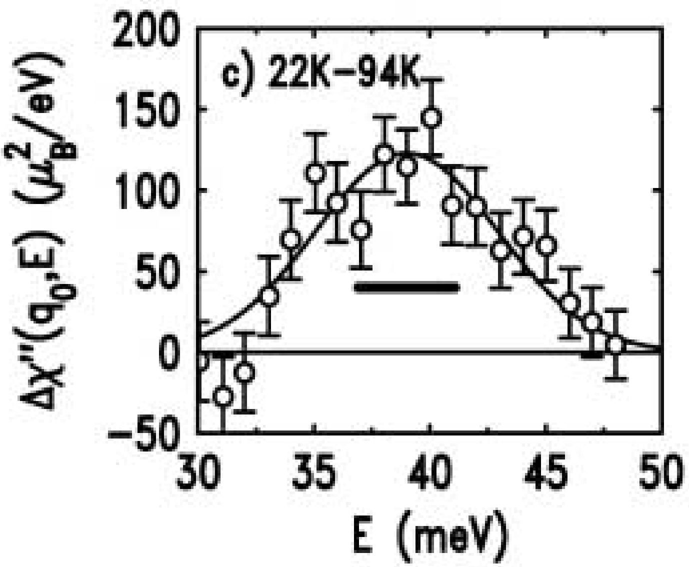
Universal asymmetric lineshape

S. Sachdev, C. Buragohain, M. Vojta, Science **286**, 2479 (1999).

M. Vojta, C. Buragohain, and S. Sachdev, Phys. Rev. B **61**, 15152 (2000).

$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



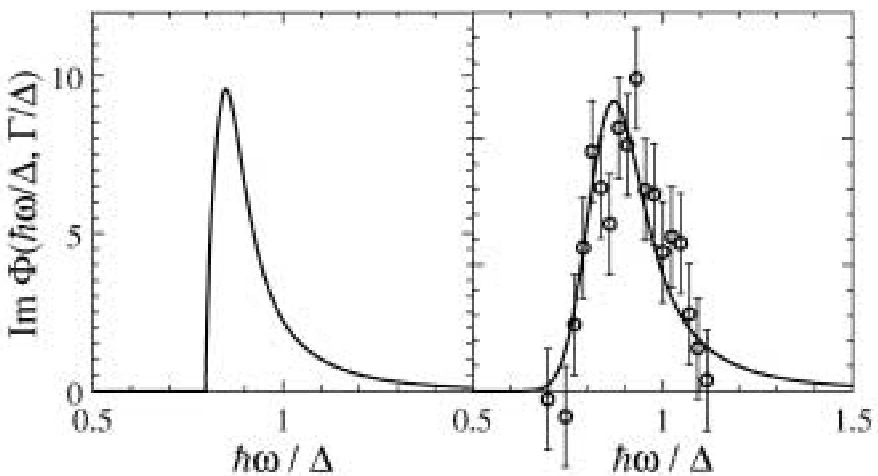
$$n_{\text{imp}} = 0.005$$

$$\Delta = 40\ \text{meV}$$

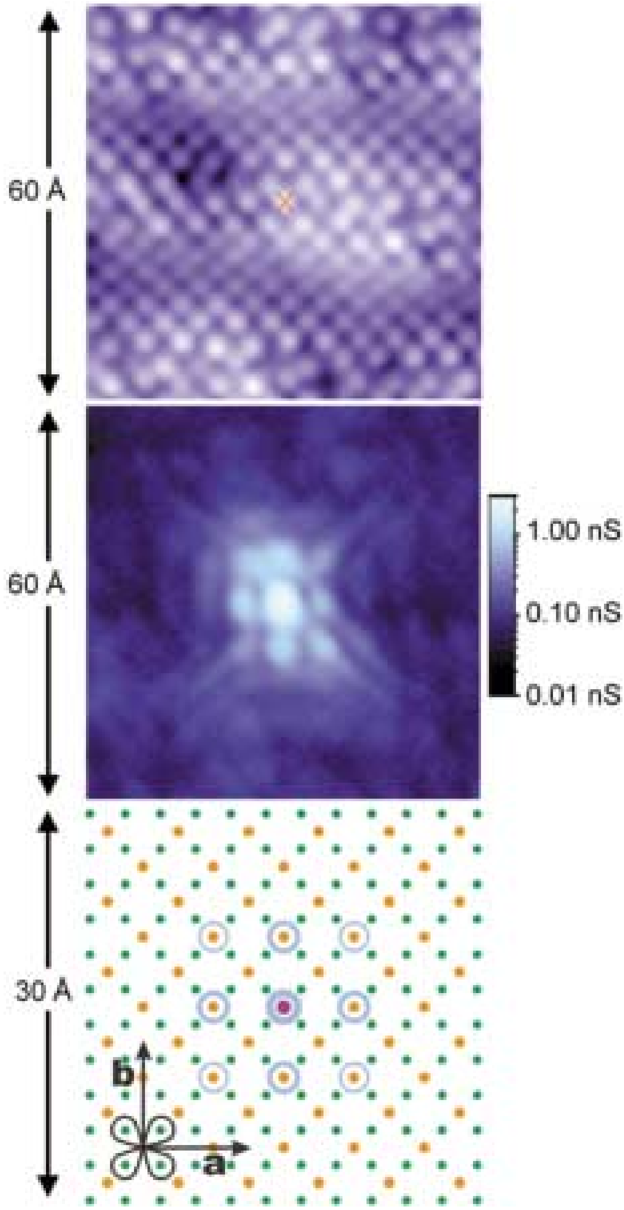
$$\hbar c = 0.2\ \text{eV}$$

$$\Rightarrow\quad \Gamma = 5\ \text{meV},\ \Gamma/\Delta = 0.125$$

Quoted half-width = 4.25 meV

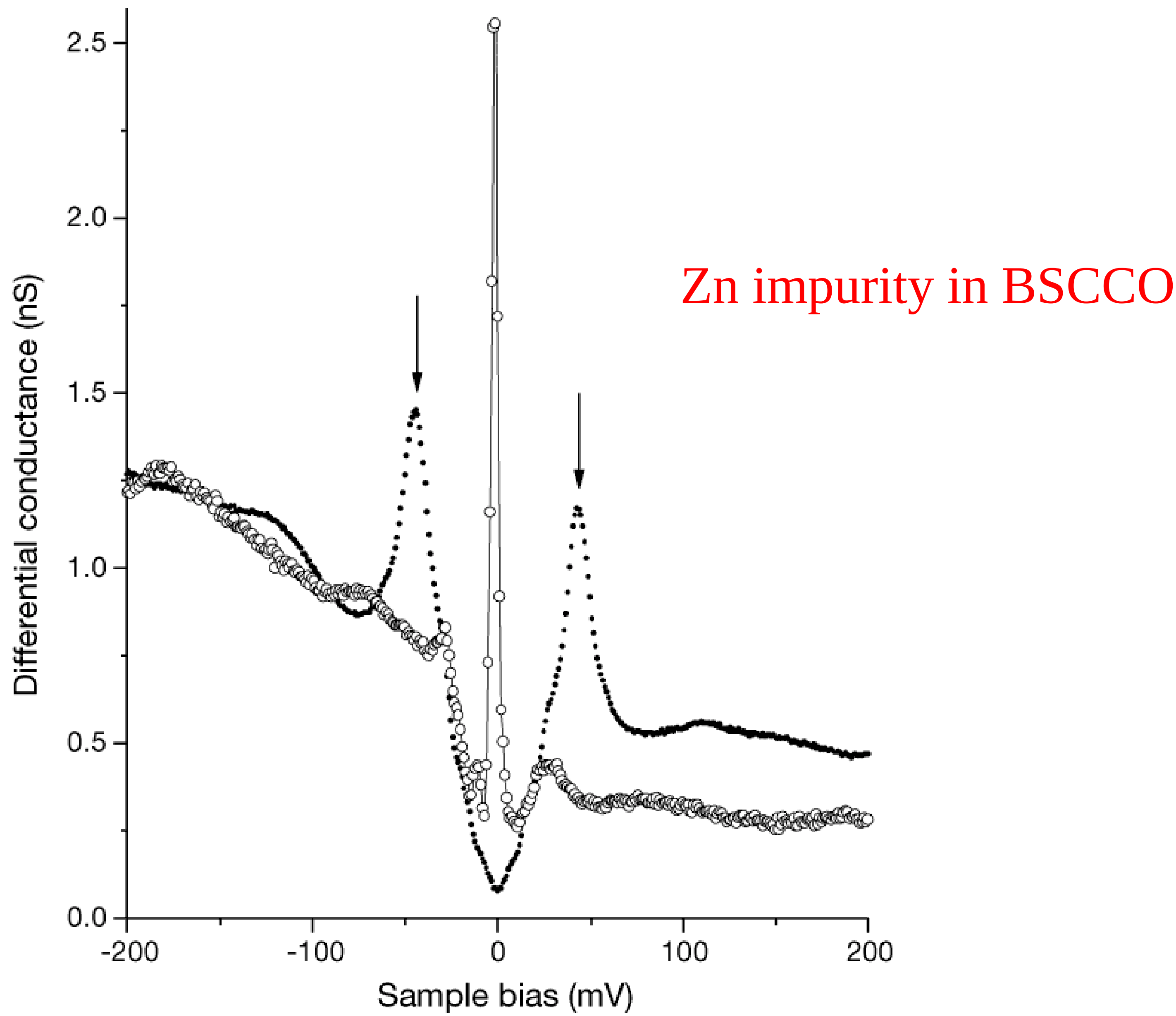


III. Implications for STM experiments



Zn impurities
in BSCCO

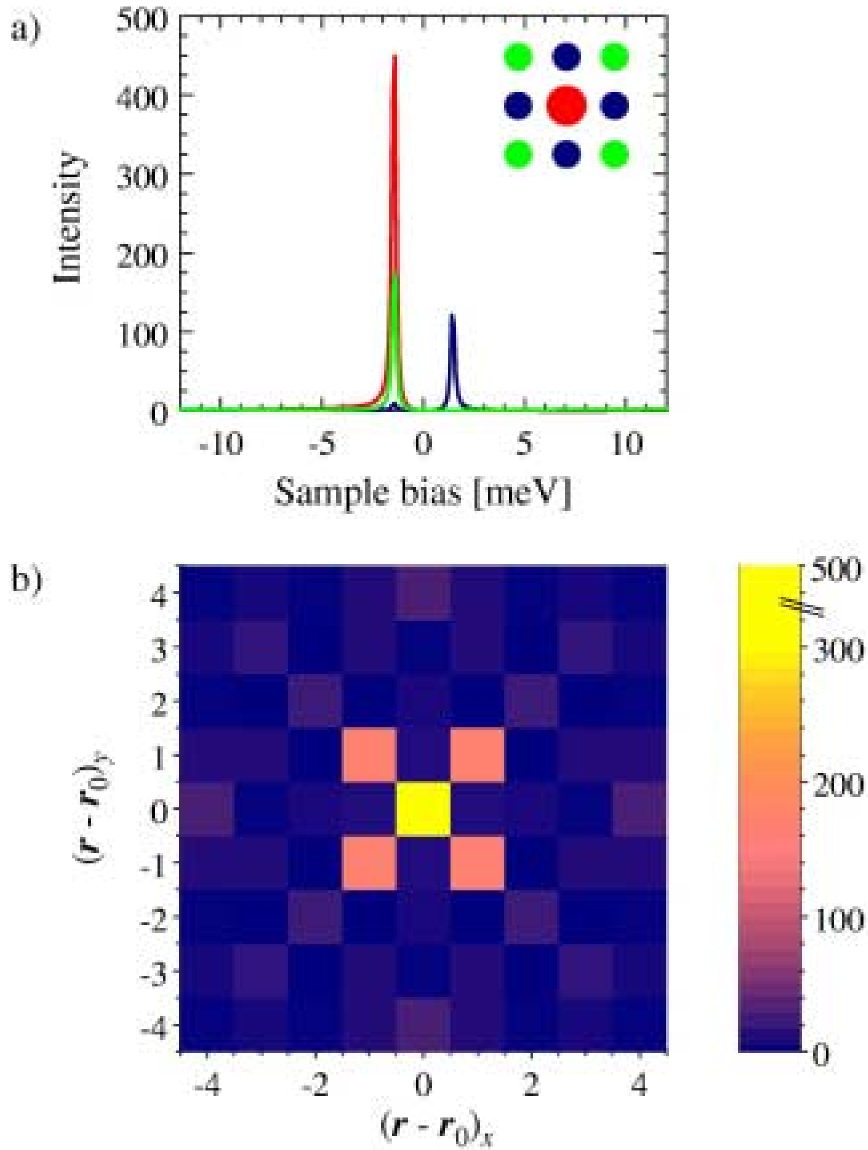
S. H. Pan *et al* Nature **403**, 746 (2000)



Zn impurity in BSCCO
Differential conductance (nS)
2.5
2.0
1.5
1.0
0.5
0.0
-200
-100
0
100
200
Sample bias (mV)

Theory: *S=1/2* local moment coupled to Bogoluibov quasiparticles of a *d*-wave superconductor

A. Polkovnikov, S. Sachdev, and M. Vojta, Phys. Rev. Lett. **86**, 296 (2001)

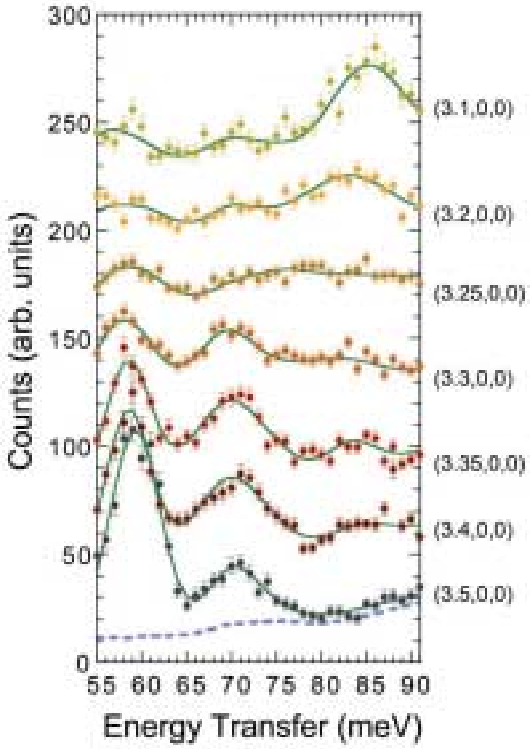


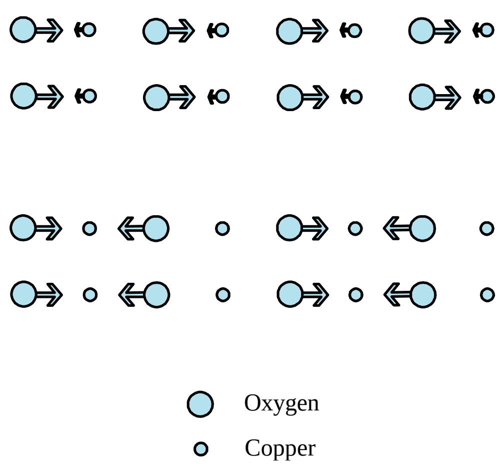
Peak bias ~ Kondo temperature for screening of moment by fermionic Bogoluibov quasiparticles.

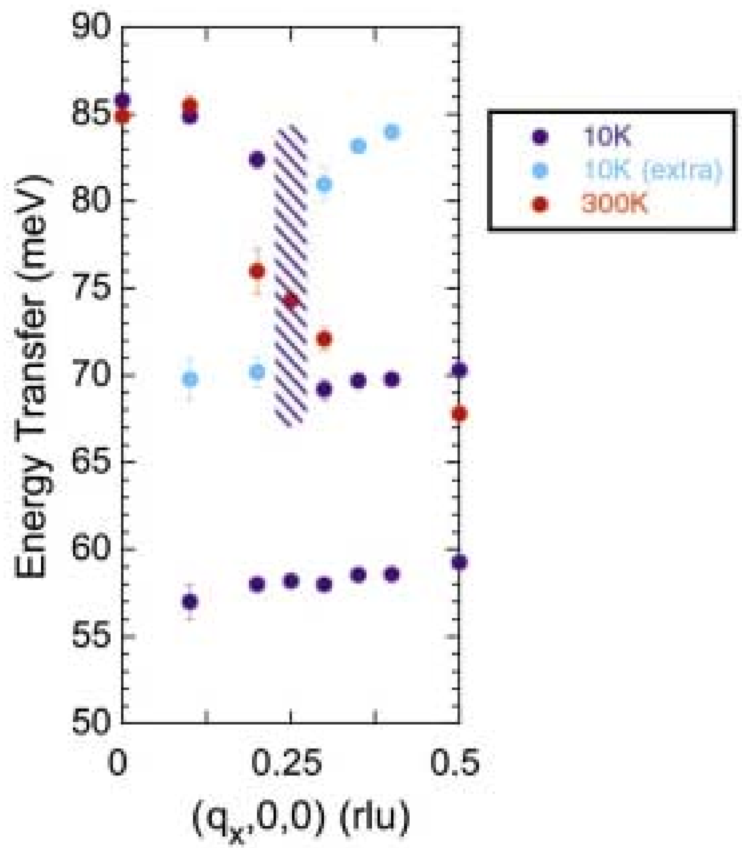
STM experiments imply that Kondo temperature ~ 15 K

Subsequent NMR experiments: Kondo temperature ~ 20-40 K

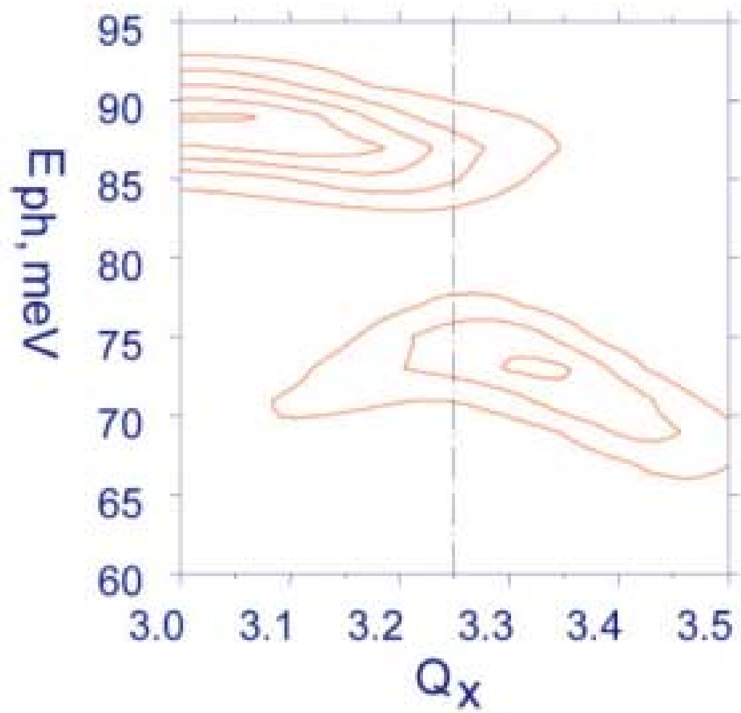
IV. Neutron scattering measurements of phonon spectra

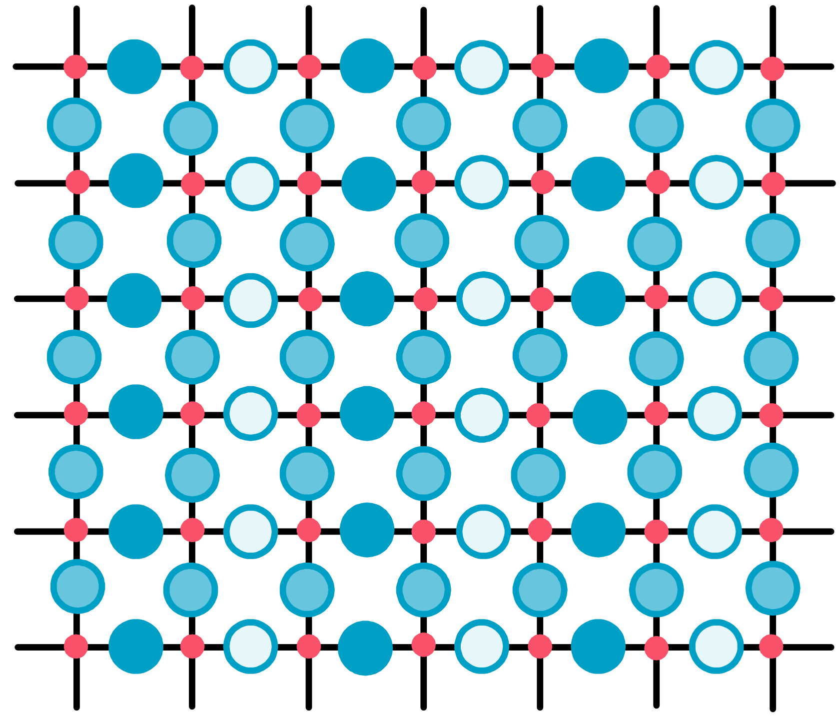






Neutron scattering measurements of phonon spectrum of superconducting $La_{1.85}Sr_{0.15}CuO_4$ by R. J. McQueeney, Y. Petrov, T. Egami, M. Yethiraj, G. Shirane, and Y. Endoh, Phys. Rev. Lett. **82**, 628 (1999)





Computation of phonon spectrum by McQueeney *et al* using a simple model based on lattice modulation on the right.

Evidence for coexistence of spin-Peierls order and "d-wave" superconductivity.

S. Sachdev & N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).

Conclusions

1. Strong experimental evidence for *S=1/2* moment near Zn and Li impurities in the underdoped high temperature superconductor.

2. This, and other properties of the high temperature superconductors (existing of *S=1* spin resonance mode, possible bond-centered charge stripe (spin-Peierls) order) are naturally understood by a theory of doping Mott insulators with confinement.

3. New boundary conformal quantum field theory in 2+1 dimensions describes scattering of spin resonance mode off “non-magnetic” impurities.