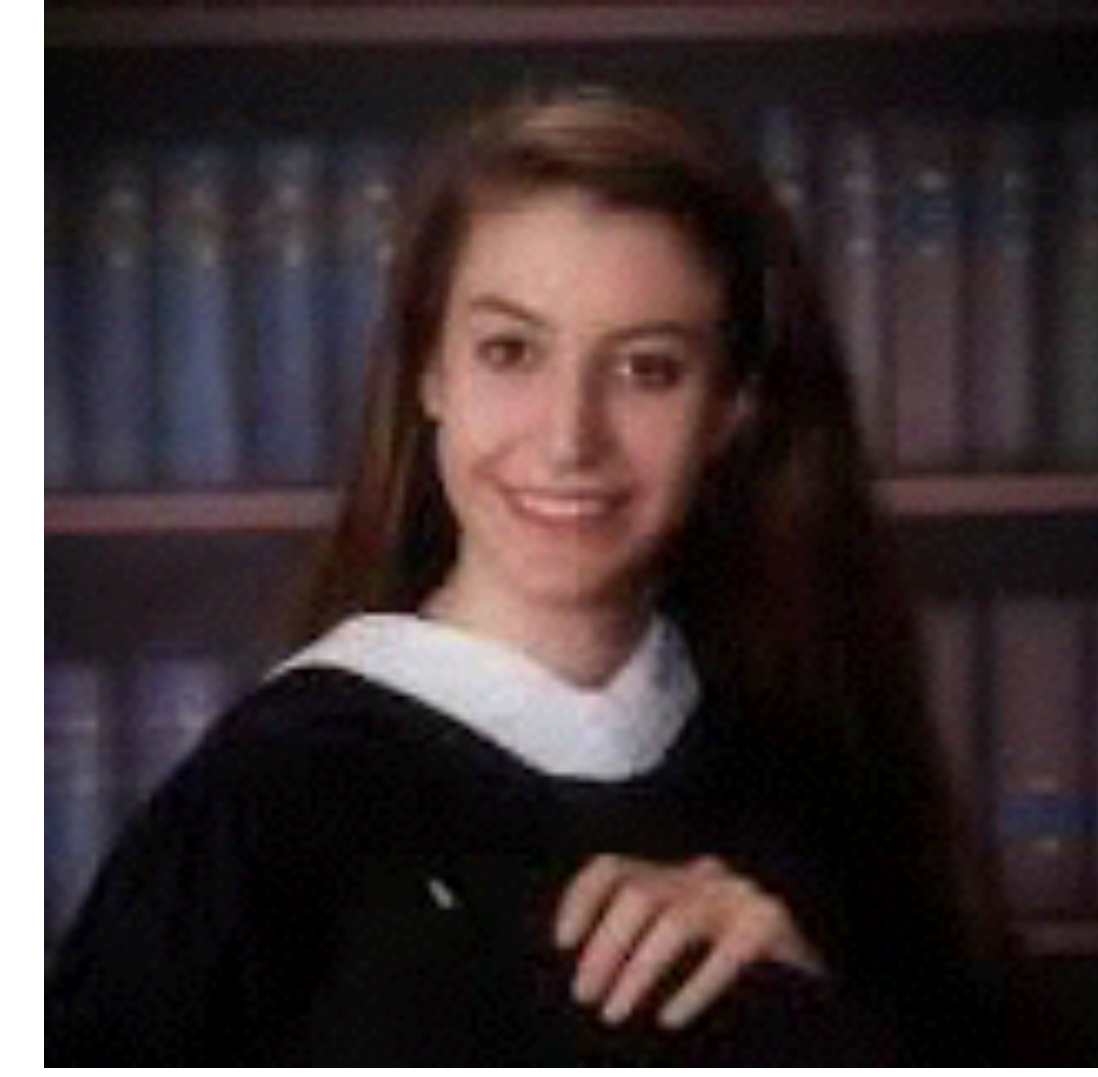
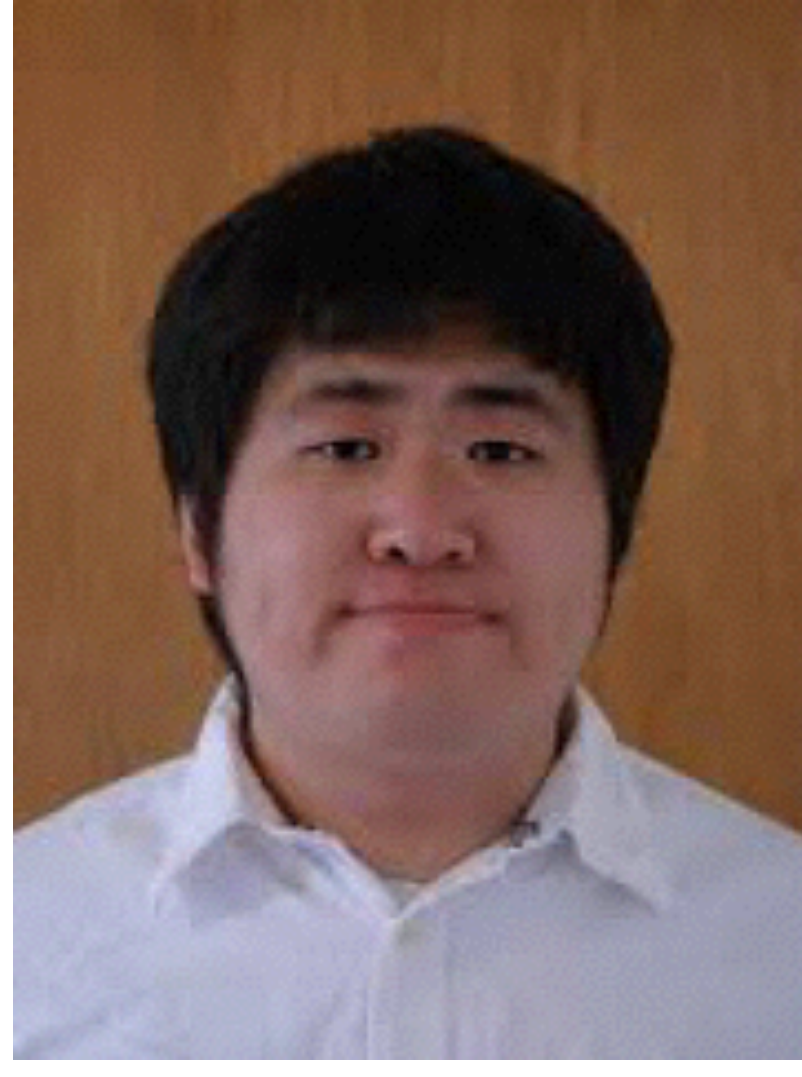
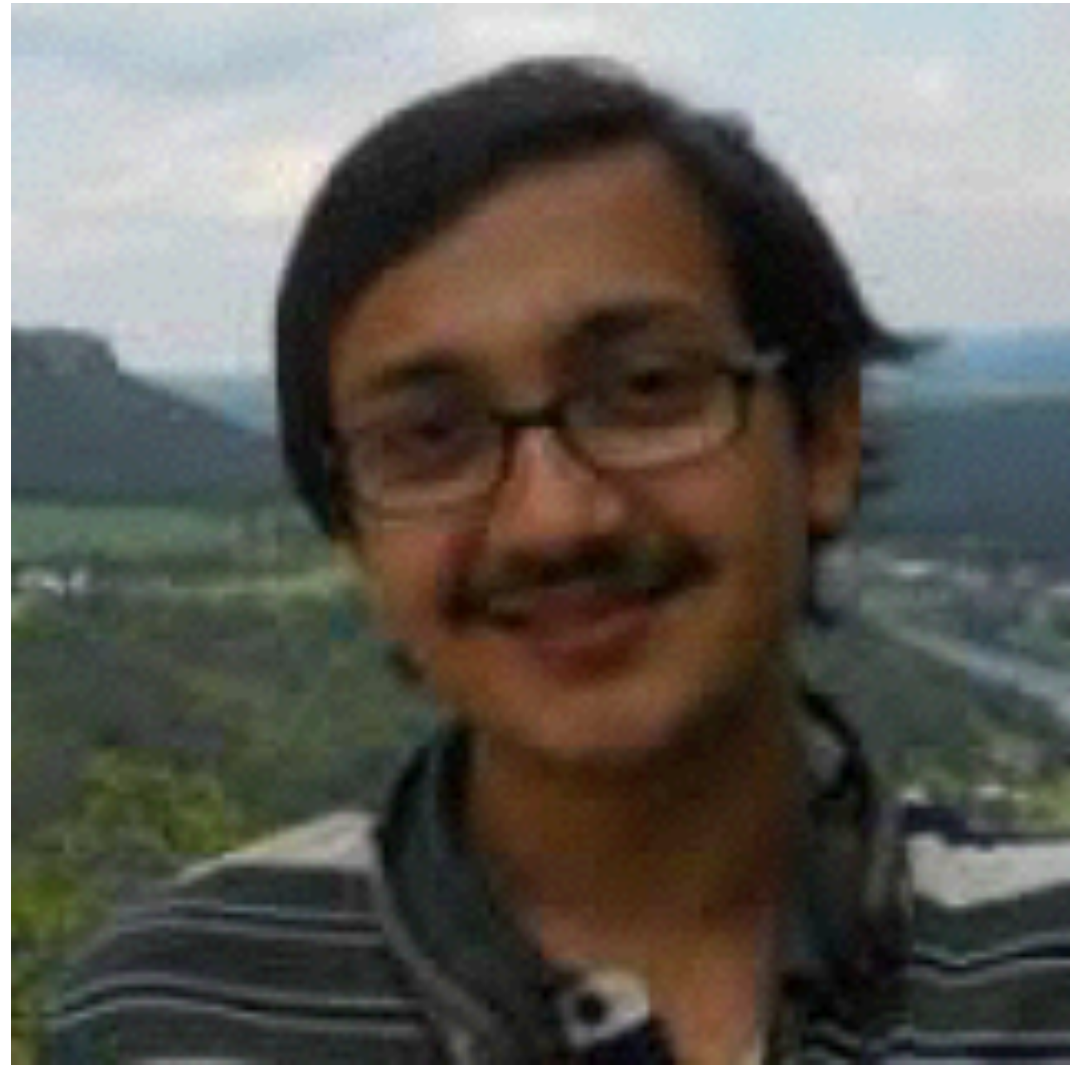


# Confinement transition out of a FL\* metal with $Z_2$ topological and Ising-nematic order



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## 1. $Z_2$ spin liquid on the square lattice

N. Read and S. Sachdev, PRL 66, 1773 (1991)

- Start with the semiclassical ground state of the  $J_1$ - $J_2$ - $J_3$  antiferromagnet on the square lattice which has co-planar spiral antiferromagnetic order at the wavevector  $(Q, 0)$ .
- Quantum fluctuations across a continuous phase transition lead to a spin liquid state with  $Z_2$  topological order and long-range Ising-nematic order
- This state can be efficiently described by Schwinger boson mean field theory.

$$H_{MF}^b = - \sum_{ij} (Q_{ij} \epsilon_{\alpha\beta} b_{i\alpha}^\dagger b_{j\beta}^\dagger + h.c.) + \sum_i \lambda_i b_{i\alpha}^\dagger b_{i\alpha}$$

$$|\Psi^b\rangle = P_G \exp \left[ \sum_{ij} \xi_{ij} \epsilon_{\alpha\beta} b_{i\alpha}^\dagger b_{j\beta}^\dagger \right] |0\rangle$$

- Excitations of the  $Z_2$  spin liquid are
  - bosonic spinons  $z_\alpha \sim b_\alpha + b_\alpha^\dagger$  and
  - bosonic visons, which are  $Z_2$  vortices in the  $Q_{ij}$ .
- The bosonic spinons and visons are mutual semions.

## 2. From bosonic to fermionic spinons

### Purely topological properties of $Z_2$ spin liquids:

- 4 kinds of excitations,  $e$ ,  $m$ ,  $\epsilon$  and the trivial local excitation 1
- Have the following fusion rules:

$$e \times e = m \times m = \epsilon \times \epsilon = 1$$

$$1 \times 1 = 1, e \times 1 = e, m \times 1 = m, \epsilon \times 1 = \epsilon$$

$$e \times m = \epsilon, e \times \epsilon = m, m \times \epsilon = e$$

- In the context of spin liquids,  $e$  and  $\epsilon$  are bosonic and fermionic spinons, and  $m$  is the vison, 1 is a local excitation with integer spin

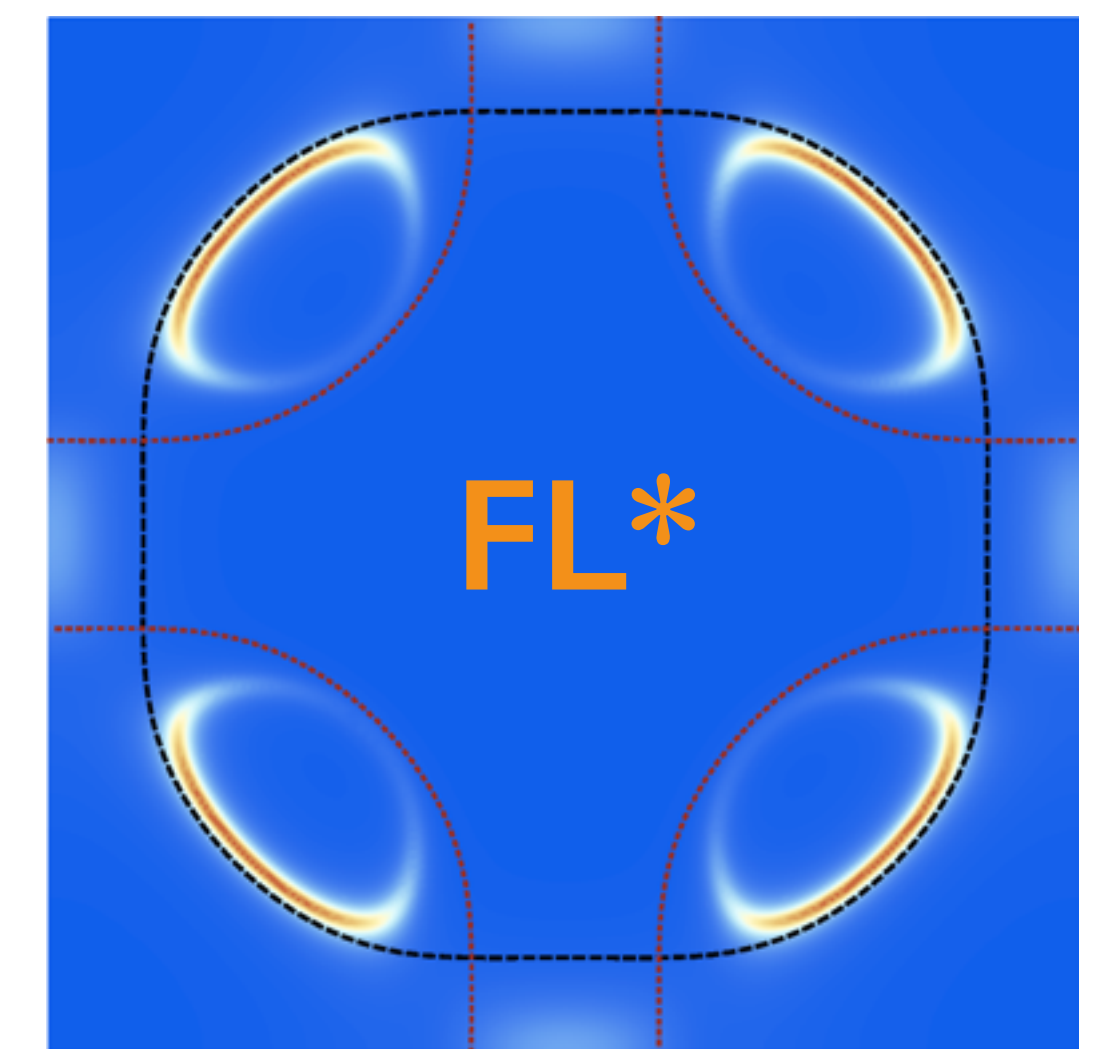
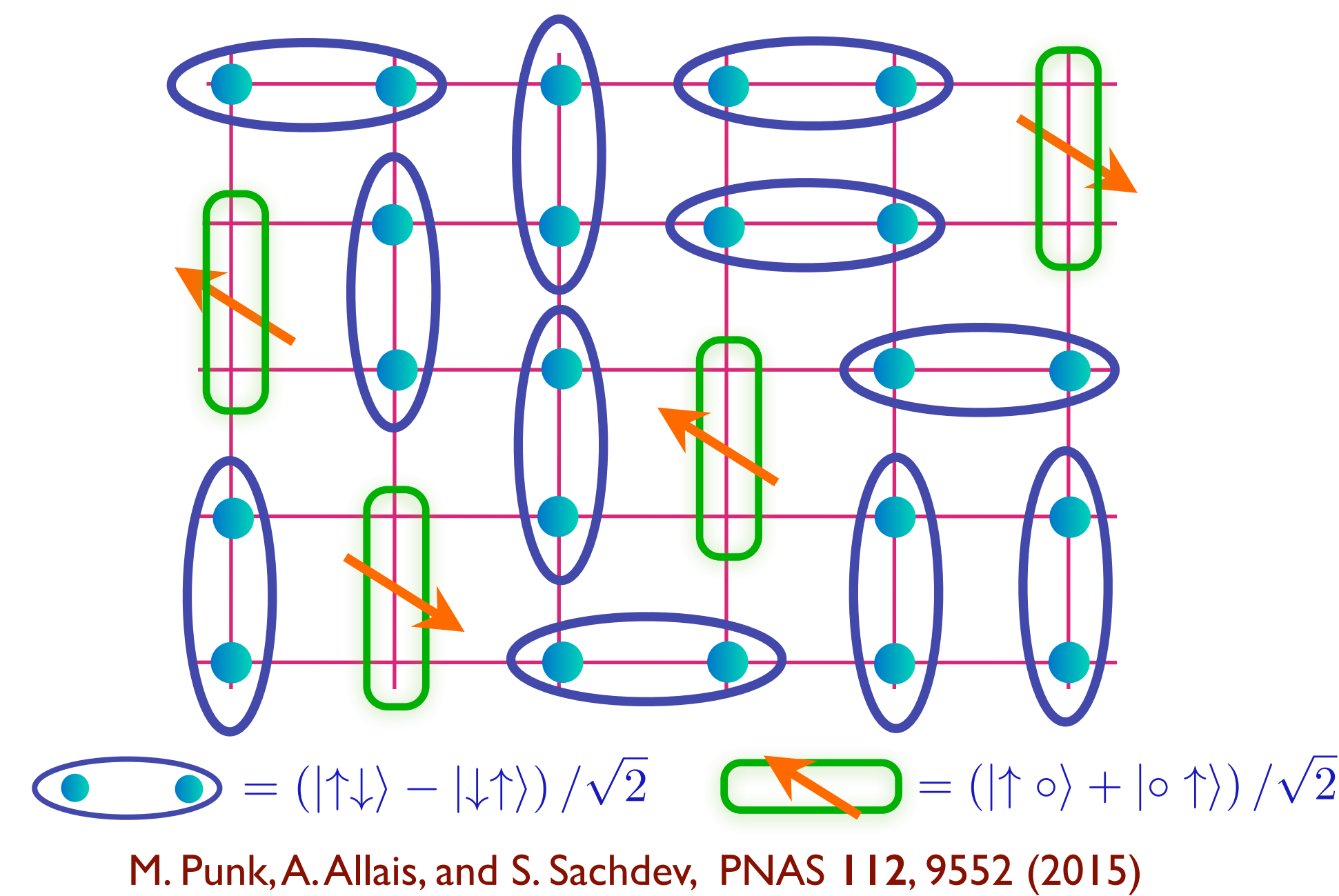
From the projective transformations of bosonic spinons ( $e$  particle) and the vison (the  $m$  particle) under space-group symmetries of the antiferromagnet, we can determine the projective symmetry transformations of the fermionic spinons ( $\epsilon$  particle). Finally, we can determine the effective Hamiltonian of the fermionic spinons  $f_\alpha$ .

A.M. Essin and M. Hermele, PRB 87, 104406 (2013)  
Y.-M. Lu, G.Y. Cho, A. Vishwanath, 1403.0575

### Symmetry relations for spin liquids on the rectangular lattice

Commutation relation	Bosonic PSG	Fermionic PSG	Vison PSG	Twist factor	Relation
$T_x^{-1} T_y^{-1} T_x T_y$	$(-1)^{p_1}$	$\eta_{T_x T_y}$	-1	1	$(-1)^{p_1+1} = \eta_{T_x T_y}$
$P_x^{-1} T_x P_x T_x$	$(-1)^{p_2}$	$\eta_{P_x T_x}$	1	1	$(-1)^{p_2} = \eta_{P_x T_x}$
$P_y^{-1} T_y P_y T_y$	$(-1)^{p_3}$	$\eta_{P_y T_y}$	-1	1	$(-1)^{p_3+1} = \eta_{P_y T_y}$
$P_x^{-1} T_x^{-1} P_x T_x$	$(-1)^{p_4}$	$\eta_{P_x T_x}$	-1	1	$(-1)^{p_4+1} = \eta_{P_x T_x}$
$P_y^{-1} T_y^{-1} P_y T_y$	$(-1)^{p_5}$	$\eta_{P_y T_y}$	1	1	$(-1)^{p_5} = \eta_{P_y T_y}$
$P_x^2$	$(-1)^{p_6}$	$\eta_{P_x}$	1	-1	$(-1)^{p_6+1} = \eta_{P_x}$
$P_y^2$	$(-1)^{p_7}$	$\eta_{P_y}$	1	-1	$(-1)^{p_7+1} = \eta_{P_y}$
$P_x^{-1} P_y^{-1} P_x P_y$	1	$\eta_{P_x P_y}$	-1	-1	$1 = \eta_{P_x P_y}$
$\mathcal{T}^2$	-1	-1	1	1	$1 = 1$
$T_x^{-1} \mathcal{T}^{-1} T_x \mathcal{T}$	$(-1)^{p_8}$	$\eta_{\mathcal{T} T_x}$	1	1	$(-1)^{p_8} = \eta_{\mathcal{T} T_x}$
$T_y^{-1} \mathcal{T}^{-1} T_y \mathcal{T}$	$(-1)^{p_9}$	$\eta_{\mathcal{T} T_y}$	1	1	$(-1)^{p_9} = \eta_{\mathcal{T} T_y}$
$P_x^{-1} \mathcal{T}^{-1} P_x \mathcal{T}$	$(-1)^{p_6}$	$\eta_{\mathcal{T} P_x}$	1	-1	$(-1)^{p_6+1} = \eta_{\mathcal{T} P_x}$
$P_y^{-1} \mathcal{T}^{-1} P_y \mathcal{T}$	$(-1)^{p_7}$	$\eta_{\mathcal{T} P_y}$	1	-1	$(-1)^{p_7+1} = \eta_{\mathcal{T} P_y}$

## 3. FL\* metal from a $Z_2$ spin liquid



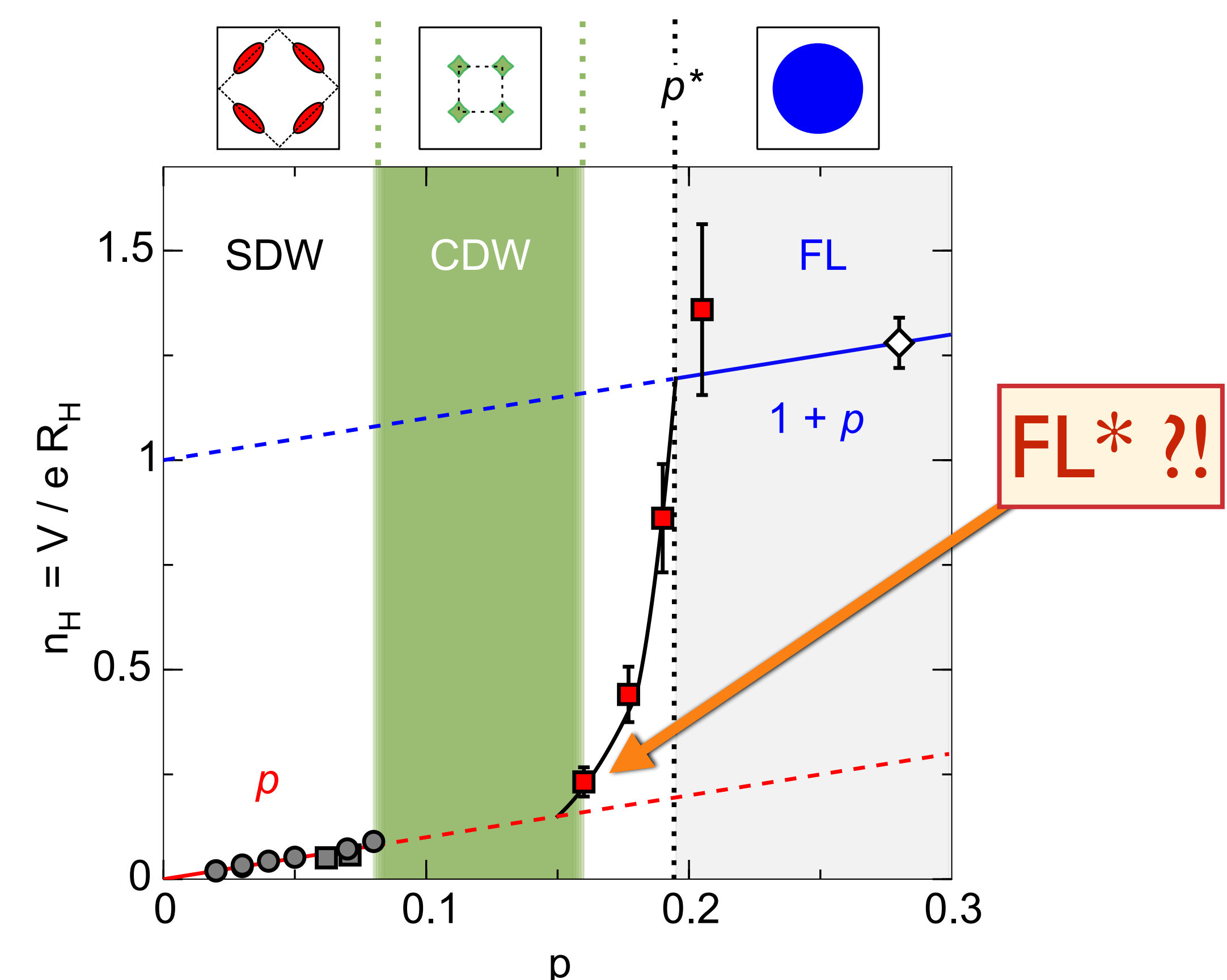
Y. Qi and S. Sachdev, PRB 81, 115129 (2010)

M. Punk, A. Allais, and S. Sachdev, PNAS 112, 9552 (2015)

- Dopants in a FL\* metal are fermions,  $c_\alpha$ , with charge  $+e$  and spin  $S = 1/2$  (the green dimers). So there need not be any low energy fractionalized excitations.
- The dopants form a Fermi surface of size equal to the dopant density  $p$ .
- The emergent gauge excitations of the  $Z_2$  spin liquid, *i.e.* visons, survive in the FL\* metal. Note that the green and blue dimers together have the same topological properties as the undoped dimer model.
- The violation of the Luttinger theorem in the FL\* metal is justified by the presence of emergent gauge excitations (*i.e.* topological order).

Recent evidence for pseudogap metal as FL\* in YBCO

Proust-Taillefer-UBC collaboration, Badoux et al. arXiv:1511.08162



## 4. Confinement transition of a FL\* metal

- Confinement can be induced by the condensation of the bosonic bilinear  $B \sim f_\alpha^\dagger c_\alpha$  or  $\epsilon_{\alpha\beta} f_\alpha c_\beta$ .
- This is a “Higgs” transition leading to confinement because  $B$  carries electric  $Z_2$  charge.
- The  $B$ -condensed (Higgs) state is a superconductor because the pairing of the  $f_\alpha$  fermions in the  $Z_2$  spin liquid now induces a pairing of the  $c_\alpha$  fermions. This pairing can have  $d_{x^2-y^2} + s$  symmetry.
- The  $c_\alpha$  fermions have trivial space group transformations, and so the projective space group transformations of  $B$  can be deduced from those of the fermionic spinons  $f_\alpha$ .
- In many cases, the projective transformations of  $B$  also imply density-wave order in the superconductor. This implies there can be a direct second-order transition from the FL\* metal to a confining Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) state.