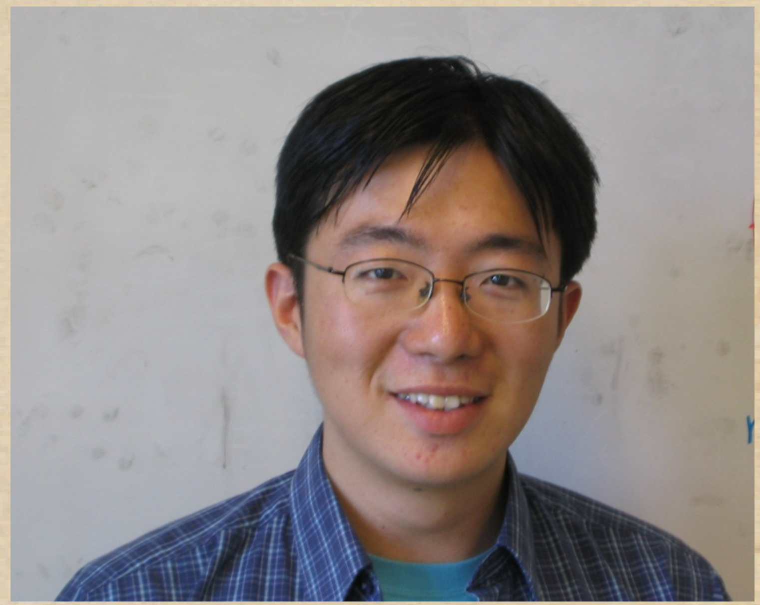
Loss of Neel order in insulators and superconductors

Ribhu Kaul

Max Metlitski



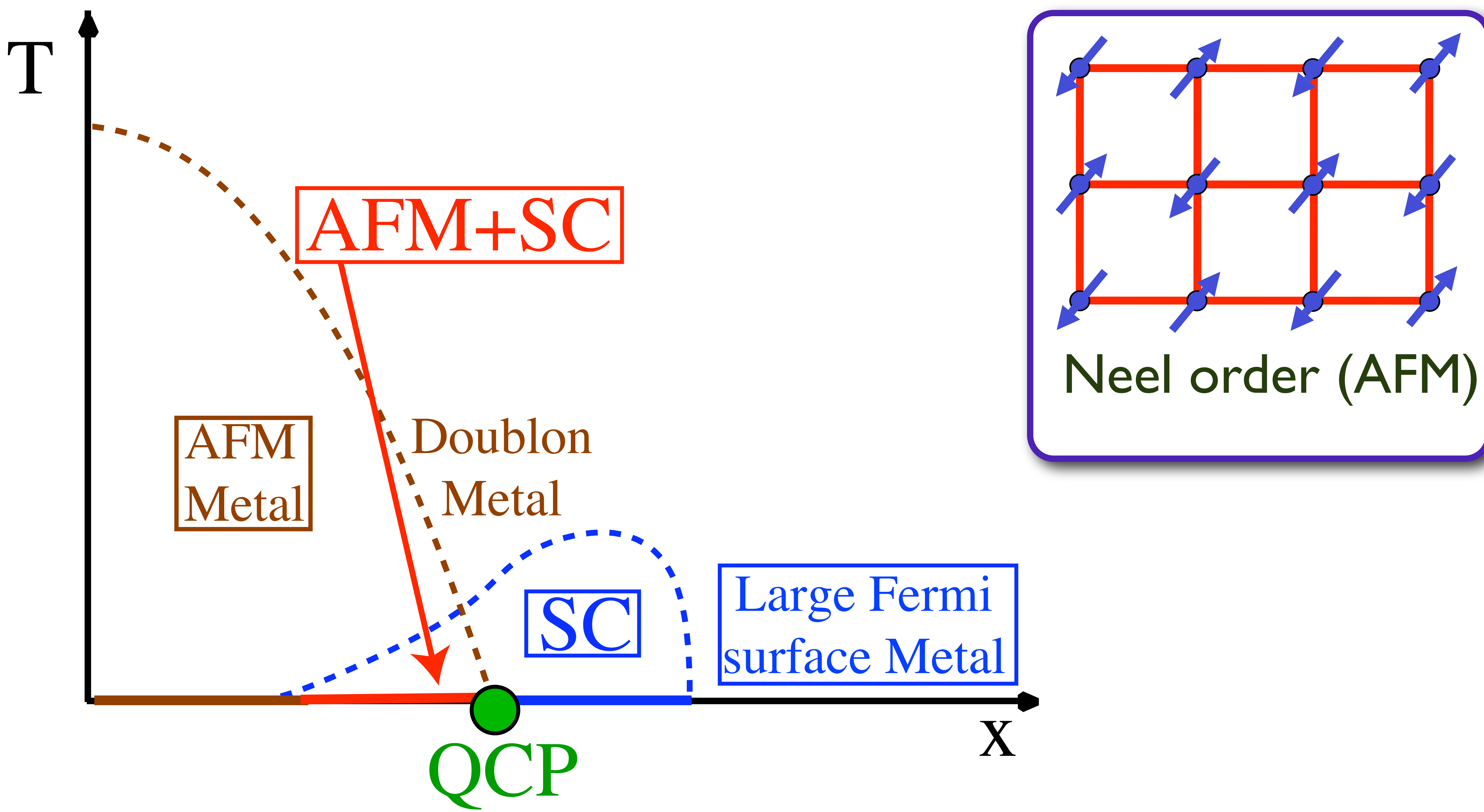
Cenke Xu

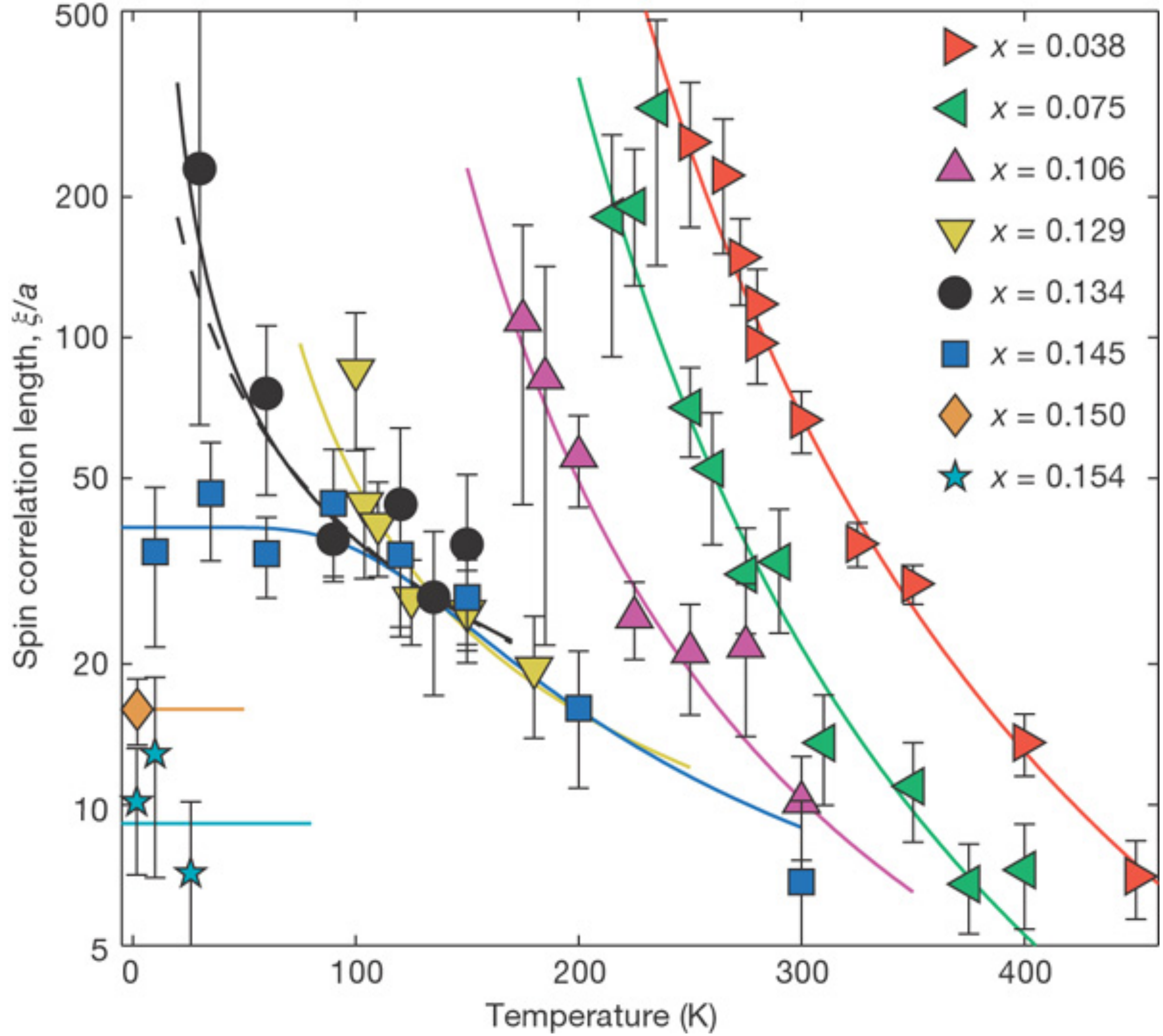
Subir Sachdev

arXiv:0804.1794

Phase diagram of electron-doped superconductors

$Nd_{2-x}Ce_xCuO_{4-y}$ and $Pr_{2-x}Ce_xCuO_{4-y}$



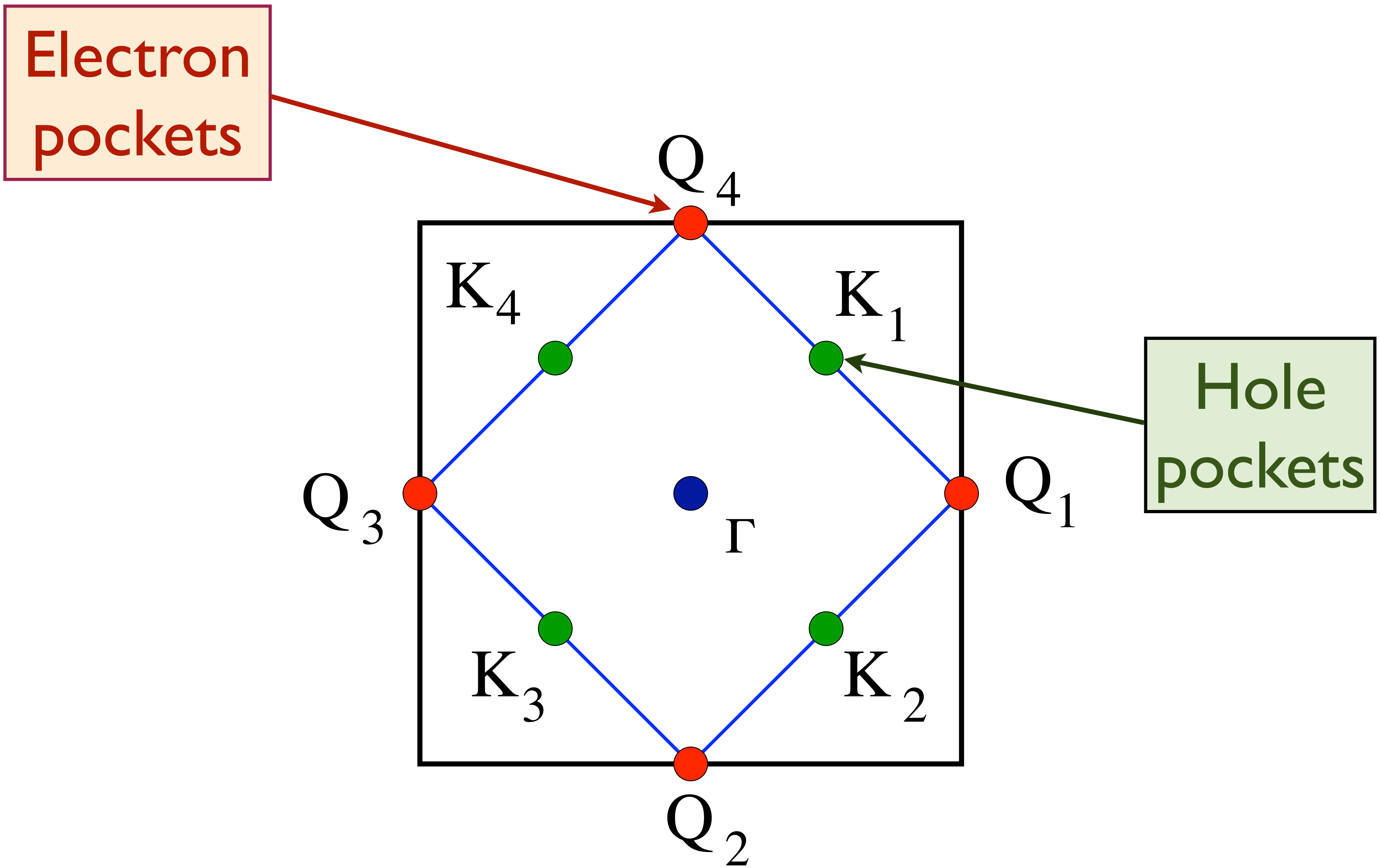


Spin correlations in the electron-doped high-transition-temperature superconductor $Nd_{2-x}Ce_xCuO_4$

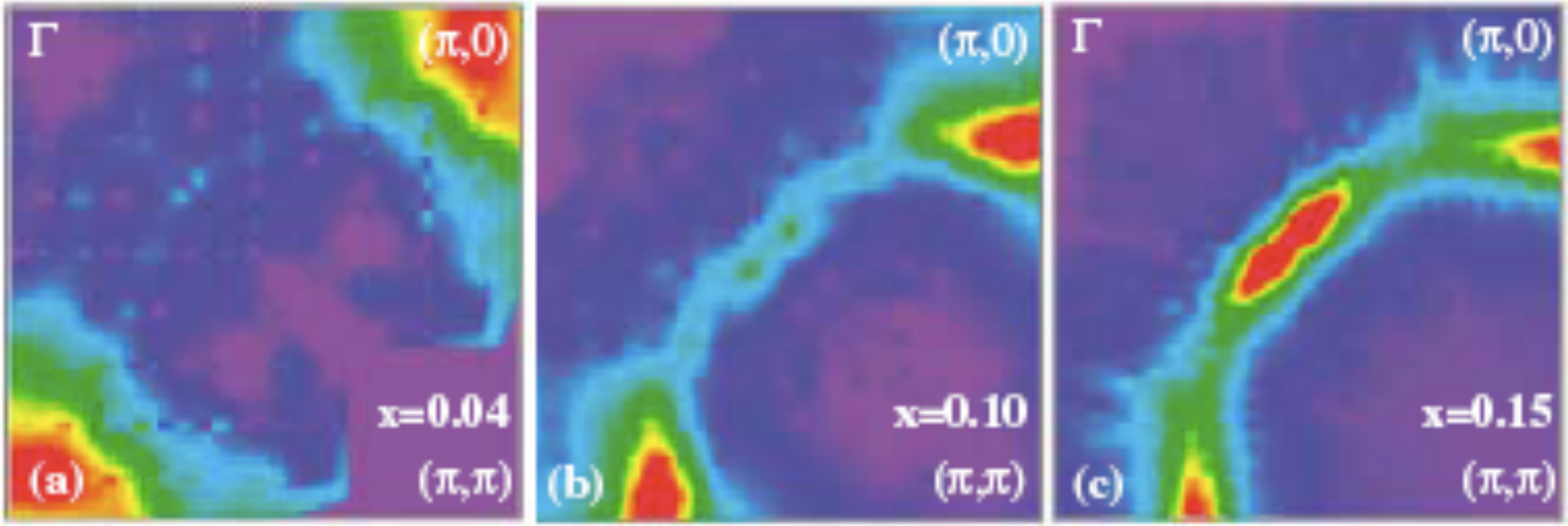
E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang and M. Greven

Nature **445**, 186-189(11 January 2007)

Charge carriers in the lightly-doped cuprates with Neel order

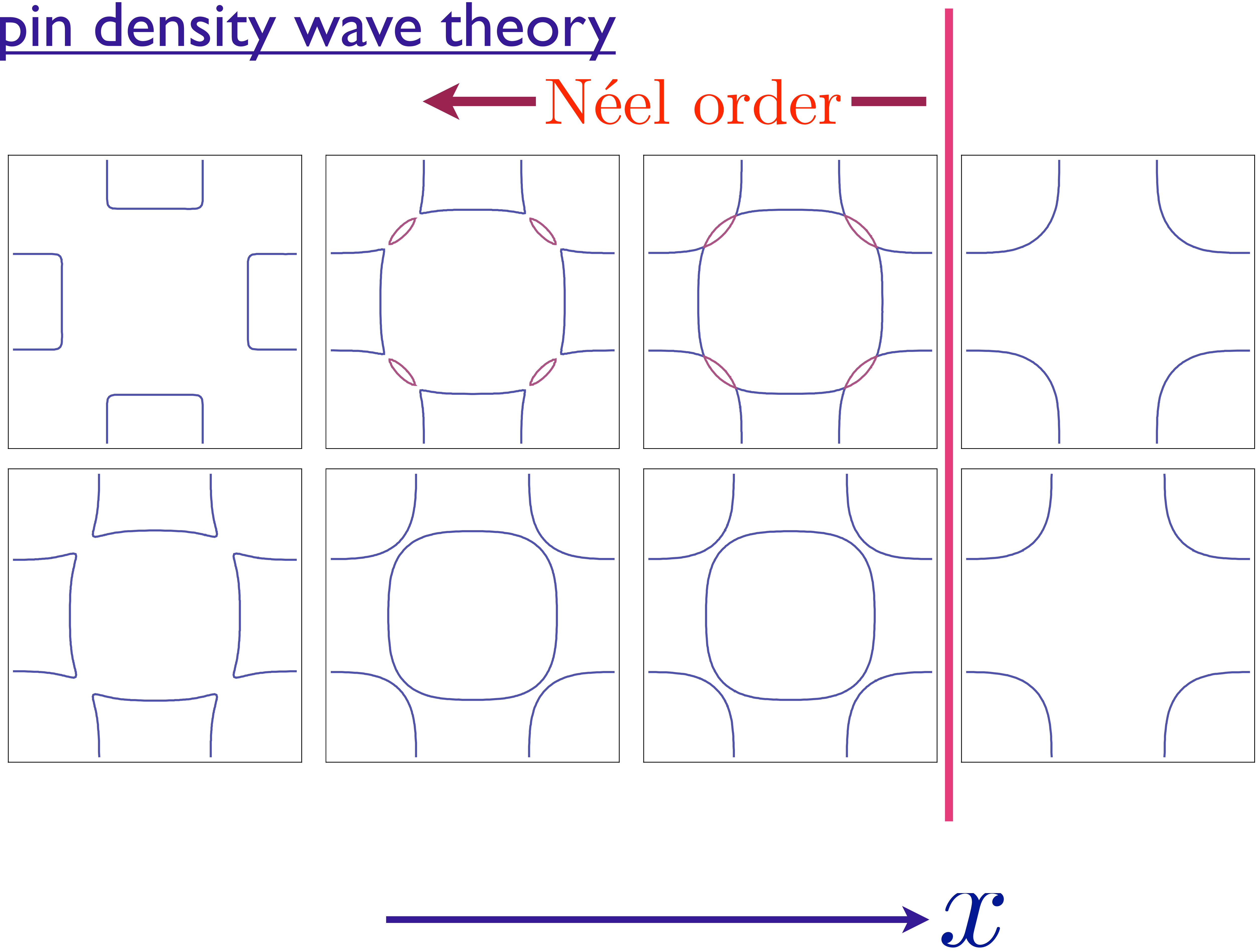


Photoemission in NCCO

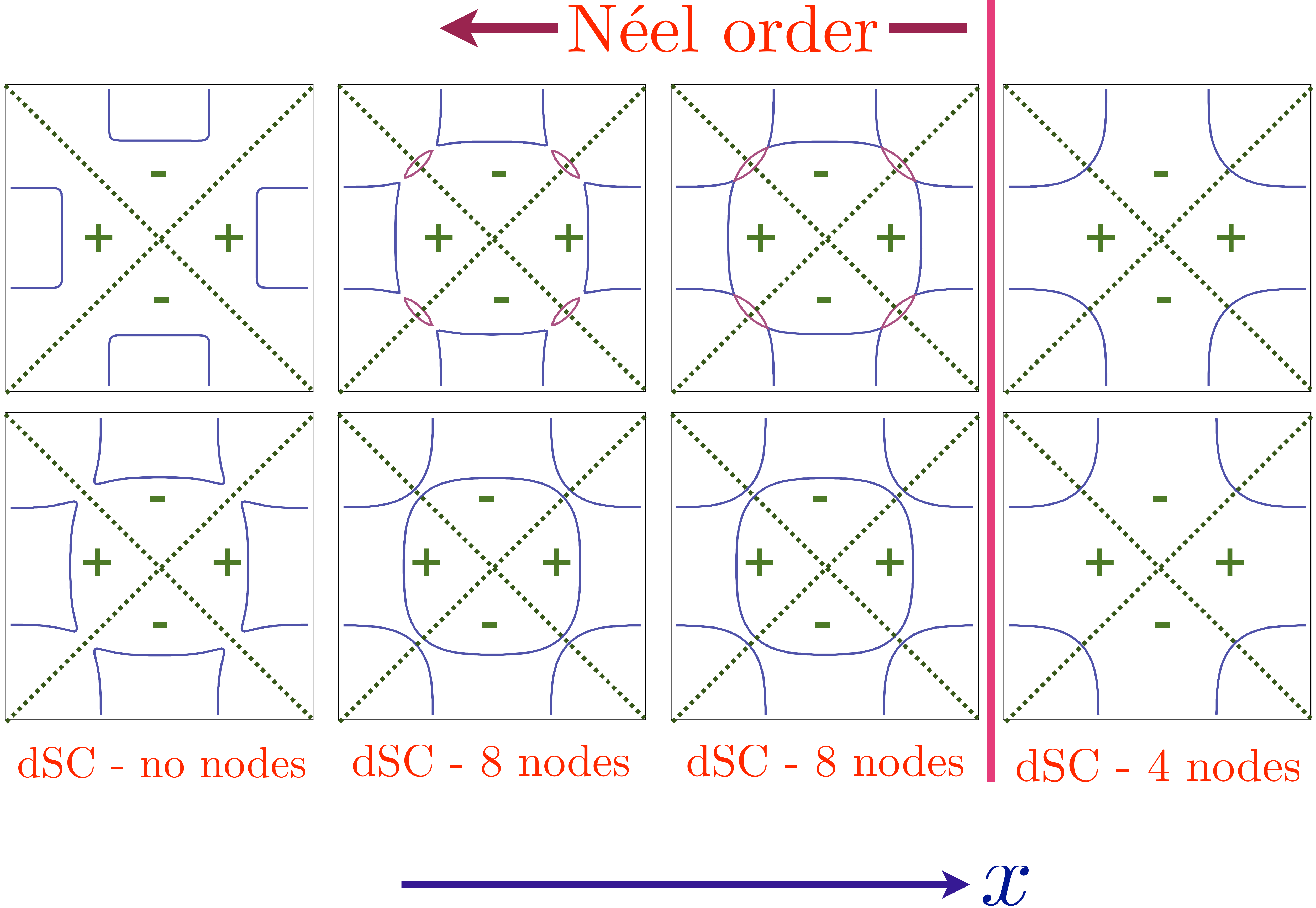


N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Spin density wave theory



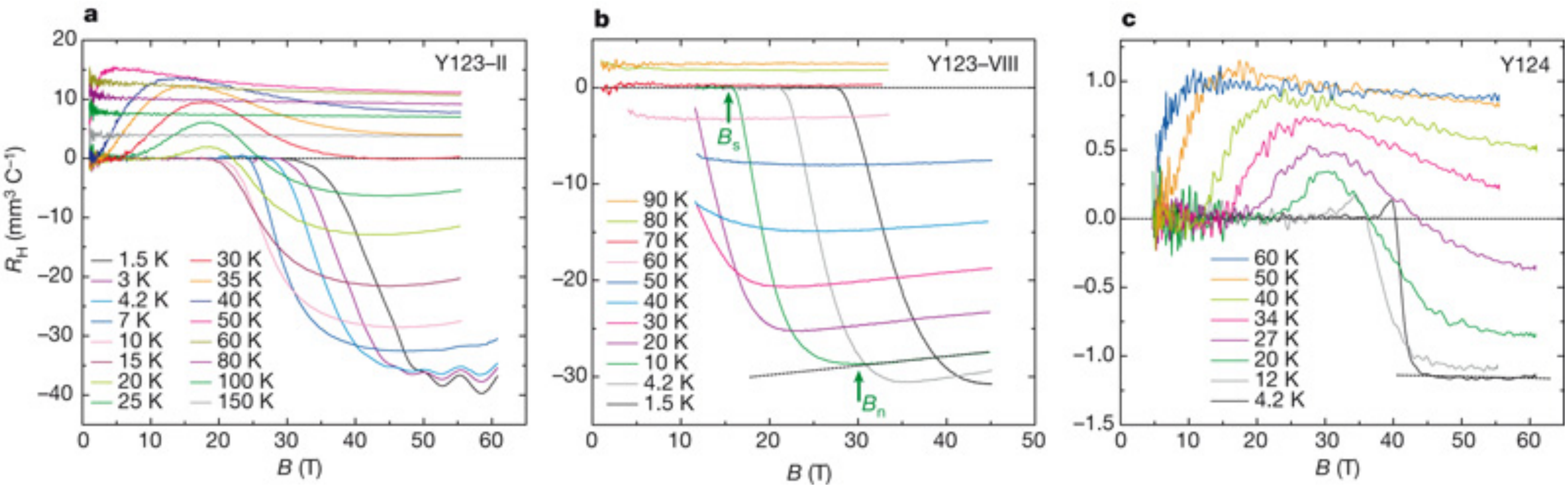
Spin density wave theory



Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



Outline

1. Loss of Neel order in insulating square lattice antiferromagnets

   Distinct universality classes for half-integer, odd integer, and even integer spin S

2. Low energy theory for doped S=1/2 antiferromagnets

   d-wave superconductivity

3. Loss of Neel order in the d-wave superconductor

   Same universality classes as in general S insulators, leading to transitions to a valence bond supersolid, a nematic superconductor, or a vanilla superconductor

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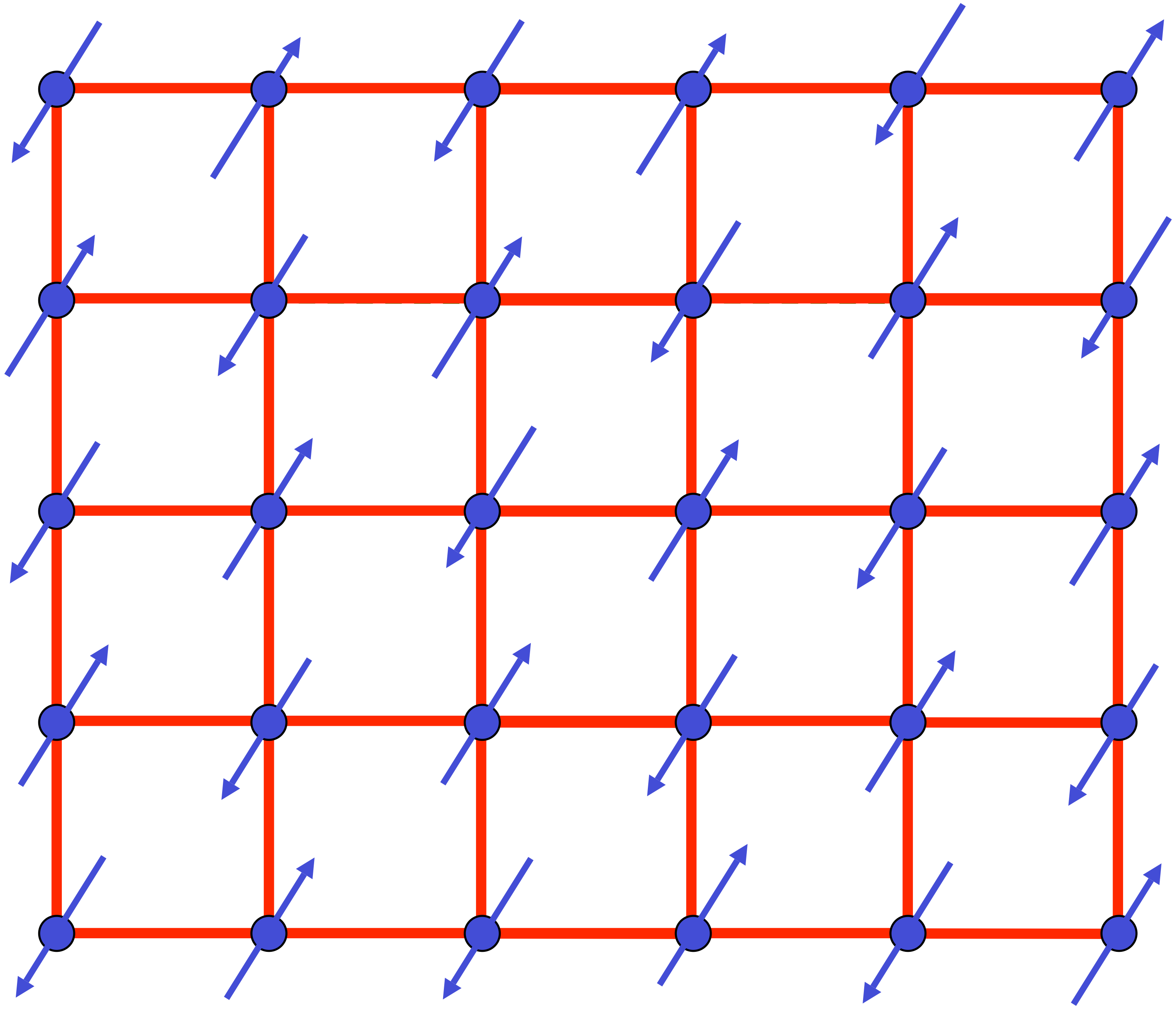


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

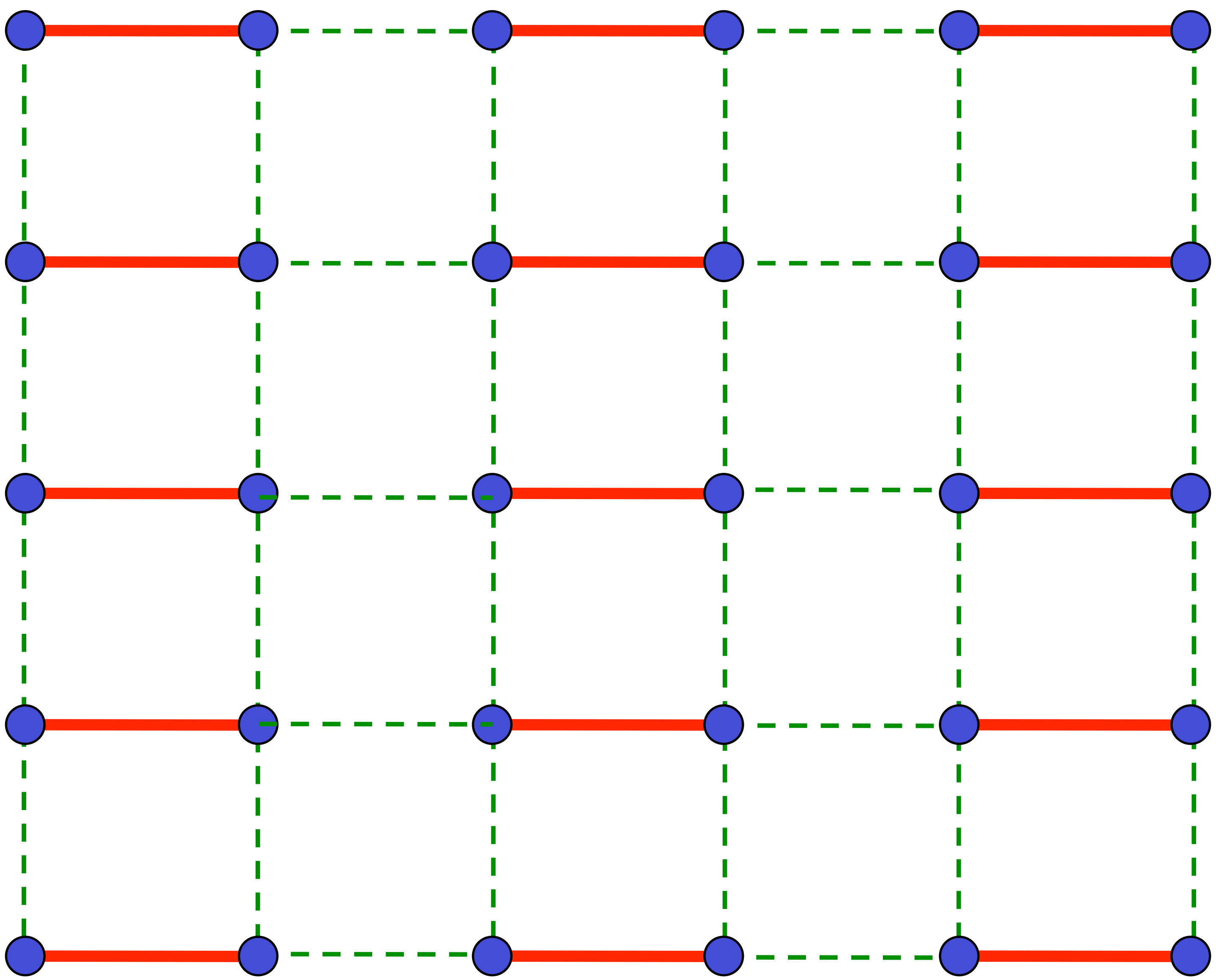
$\vec{S}_i \Rightarrow$ spin operator

with $\vec{S}_i^2 = S(S+1)$.

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.



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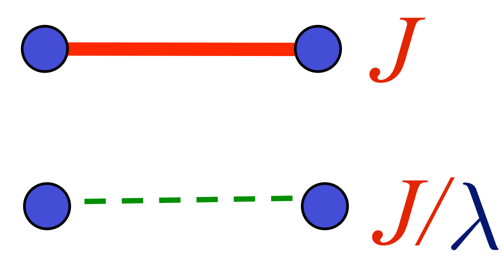
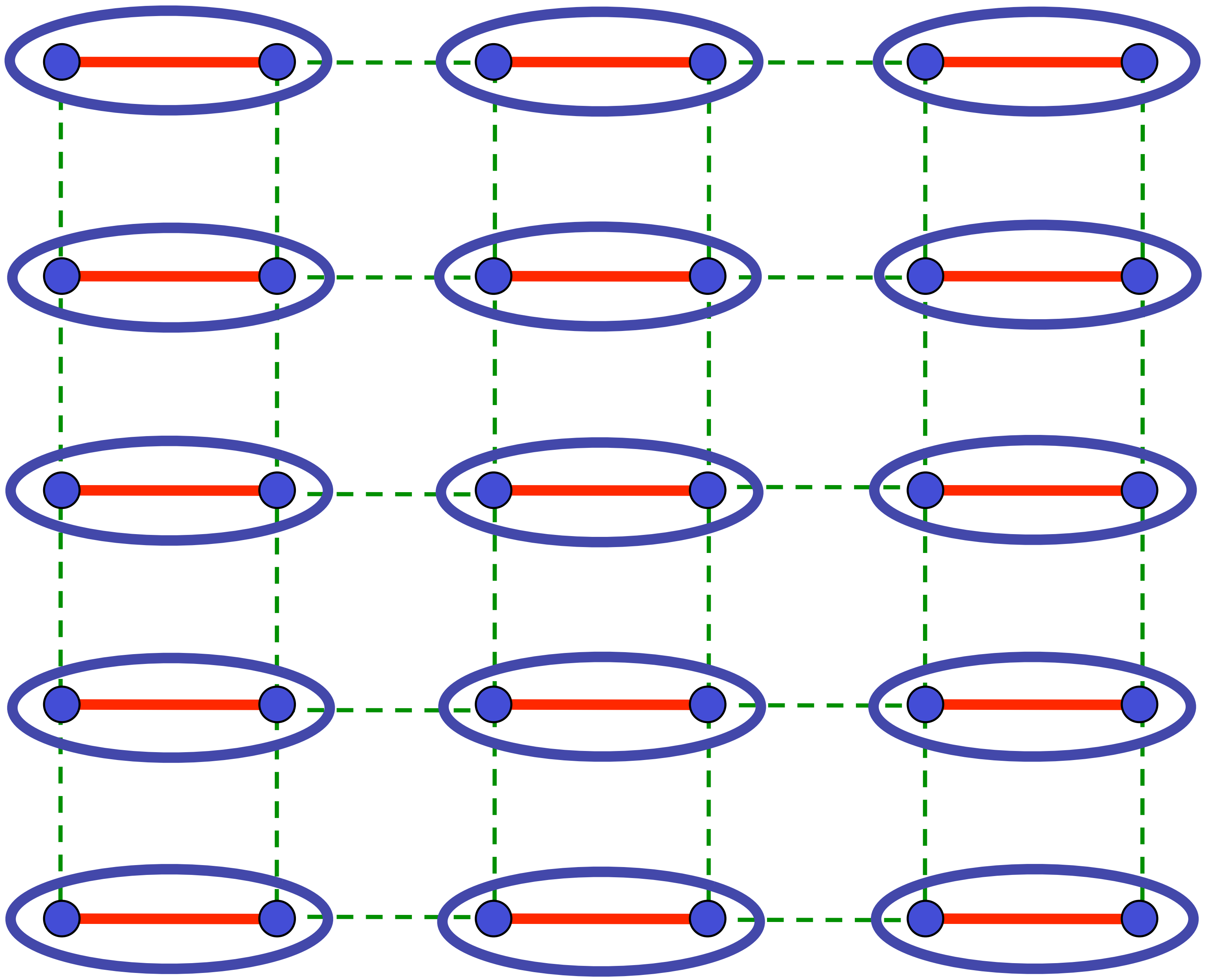


Bond modulation weakens Neel order



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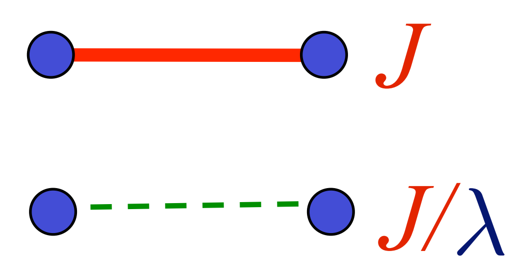
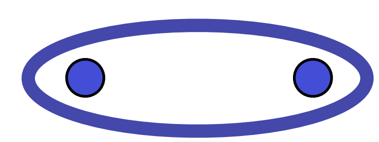
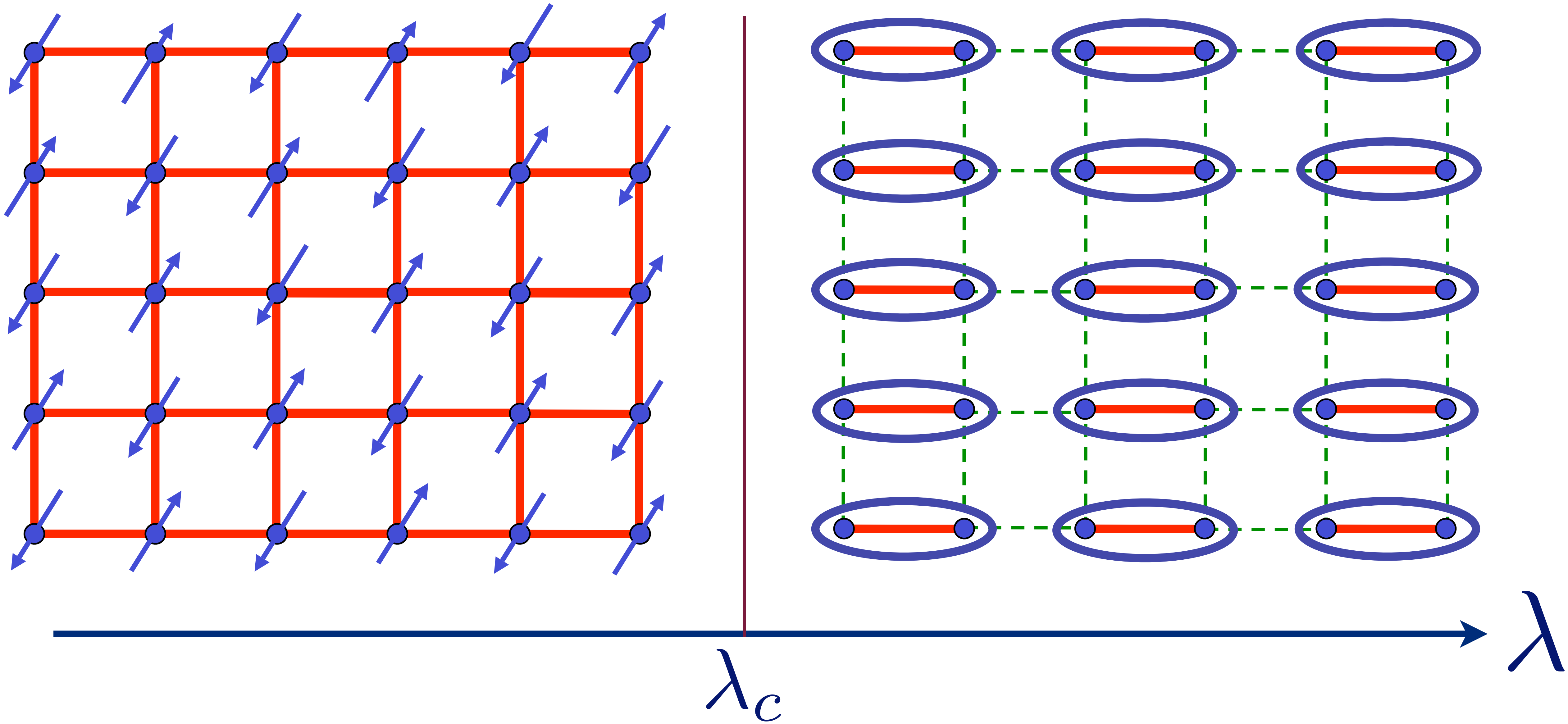




$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Ground state is a product of pairs of entangled spins.

Coupled dimer antiferromagnet



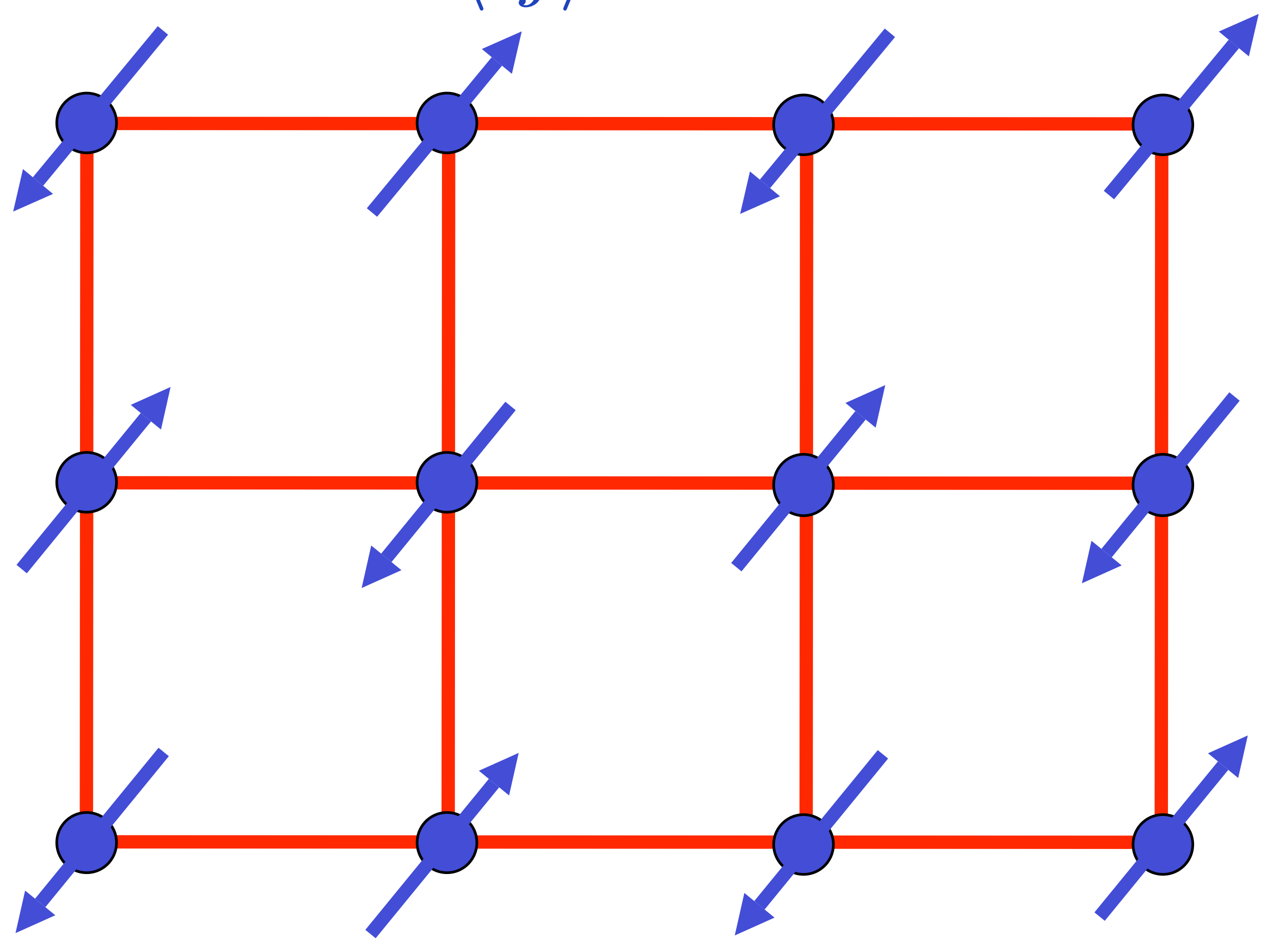
Landau-Ginzburg-Wilson theory for
O(3) order parameter $\vec{\varphi} = (-1)^i \vec{S}_i$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_x \vec{\varphi})^2 + s \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.*B **65**, 014407 (2002).

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

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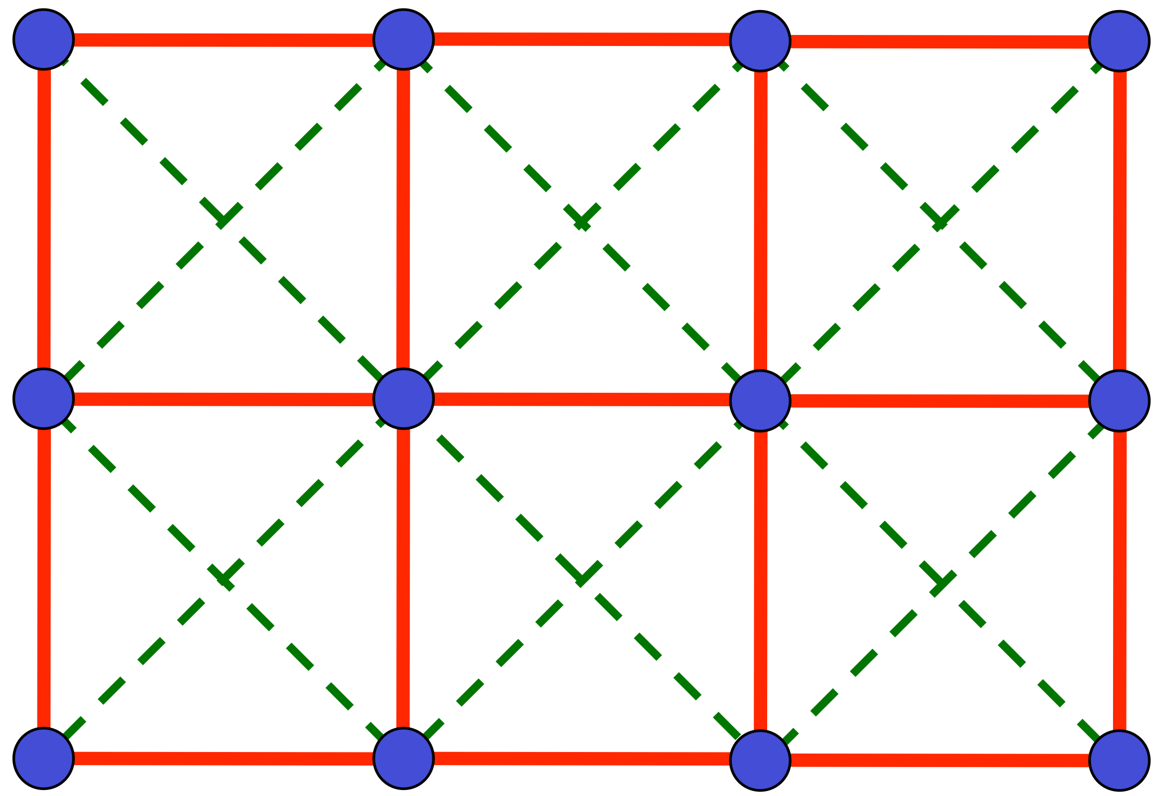
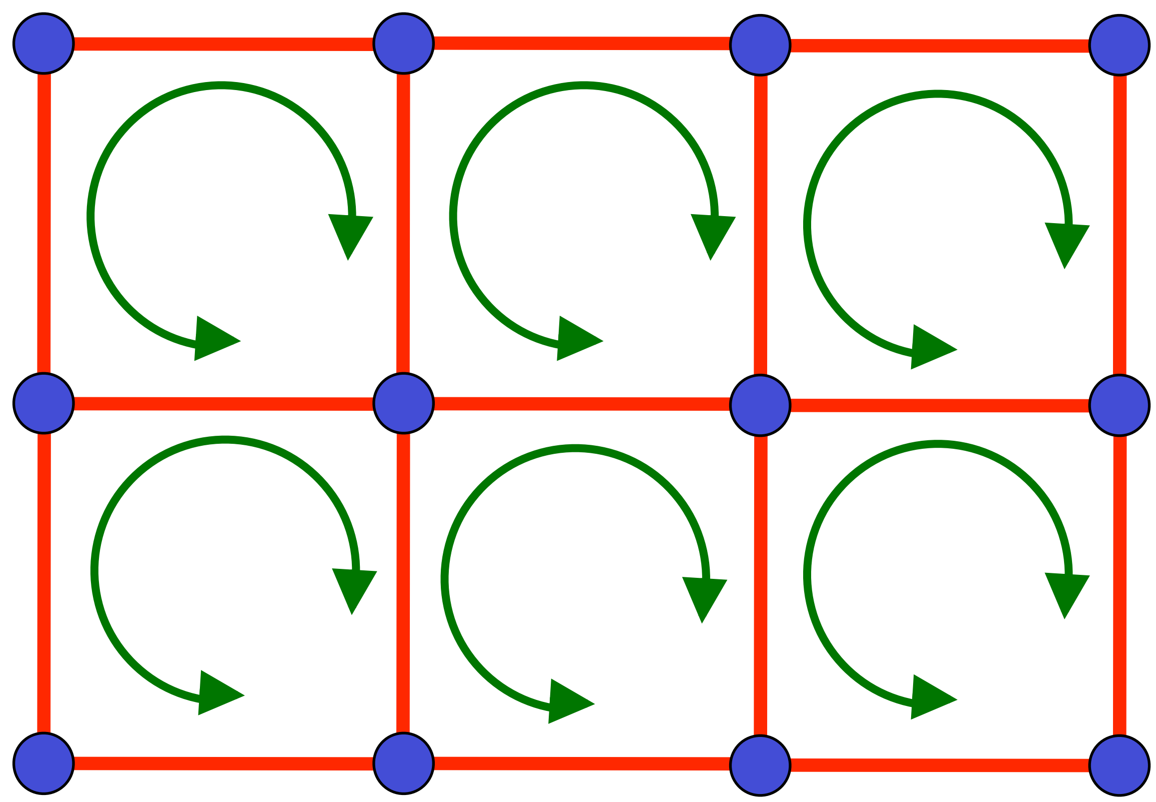


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What are possible states with $\langle \vec{\varphi} \rangle = 0$?

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What are possible states with $\langle \vec{\varphi} \rangle = 0$?

Theory for loss of Neel order

Write the spin operator in terms of
Schwinger bosons (spinons) $z_{i\alpha}$, $\alpha = \uparrow, \downarrow$:

$$\vec{S}_i = z_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} z_{i\beta}$$

where $\vec{\sigma}$ are Pauli matrices, and the bosons obey the local constraint

$$\sum_\alpha z_{i\alpha}^\dagger z_{i\alpha} = 2S$$

Effective theory for spinons must be invariant under the U(1) gauge transformation

$$z_{i\alpha} \rightarrow e^{i\theta} z_{i\alpha}$$

Perturbation theory

Low energy spinon theory for "quantum disordering" the Néel state is the CP^1 model

$$\mathcal{S}_z = \int d^2x d\tau \left[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s\left|z_\alpha\right|^2 \right.$$
$$\left. + u\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda\right)^2 \right]$$

where A_μ is an emergent U(1) gauge field which describes low-lying spin-singlet excitations.

Phases:

$\langle z_\alpha \rangle \neq 0$	$\Rightarrow$	Néel (Higgs) state
$\langle z_\alpha \rangle = 0$	$\Rightarrow$	Spin liquid (Coulomb) state

Perturbation theory

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Distinct universality from O(3) model

Non-perturbative effects

Have to account for Dirac monopoles in A_μ.
Evaulation of the boson number constraint

$$i \int d\tau \sum_i \eta_i A_{i\tau} \left(z_{i\alpha}^\dagger z_{i\alpha} - 2S \right)$$

for a Dirac monopole in A_μ yields the *monopole Berry phase*:

$$\mathcal{S}_B = \int d\tau \sum_i \zeta_i^{2S} V_i(\tau) \ + \ \text{c.c.}$$

where V_i creates a monopole on the dual lattice site i.

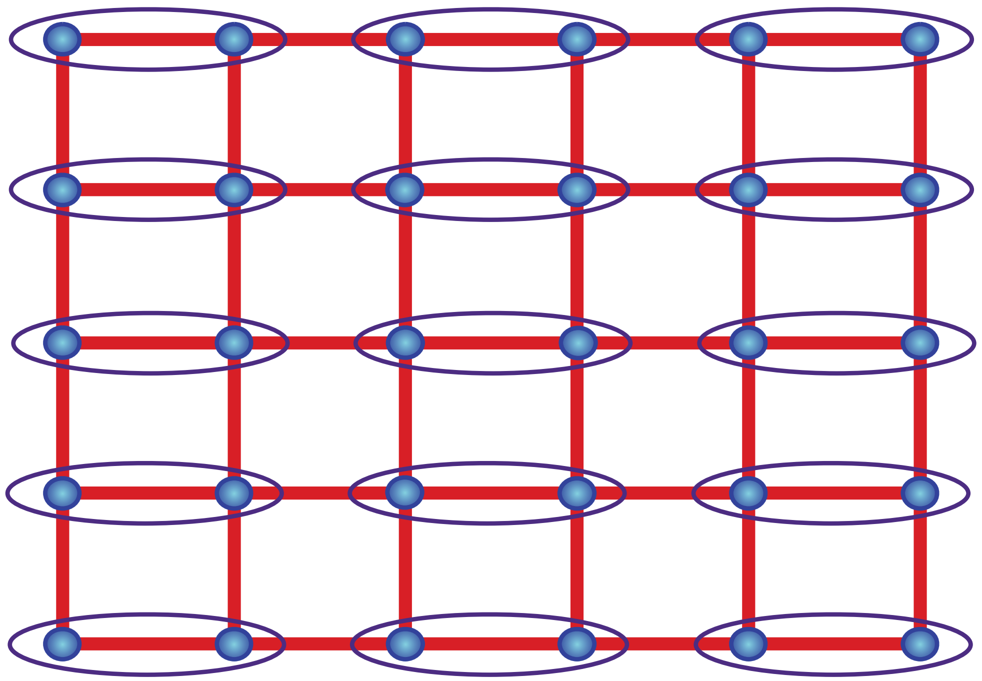
$\zeta_i =$

1	i	1	i
-i	-1	-i	-1
1	i	1	i
-i	-1	-i	-1

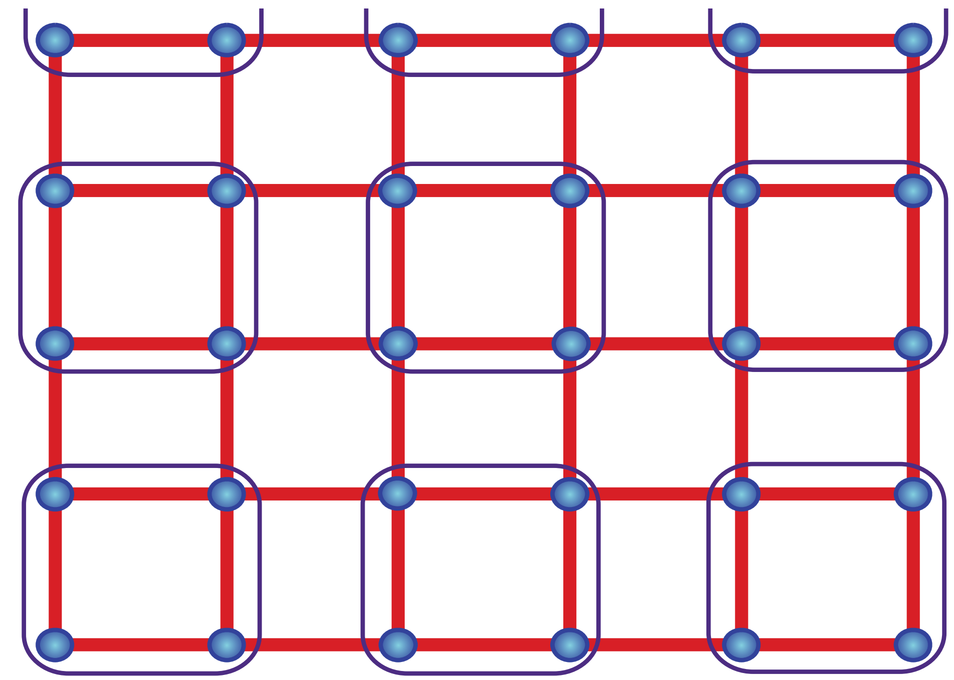
F. D. M. Haldane, *Phys. Rev. Lett*. **61**, 1029 (1988).

Half-odd-integer S

Loss of Neel order leads to a Valence Bond Solid (VBS)



or



Continuum theory only has a quadrupled monopole term

$$\mathcal{S} = \ldots + \lambda_4 \int d^2 r d\tau V^4(r,\tau).$$

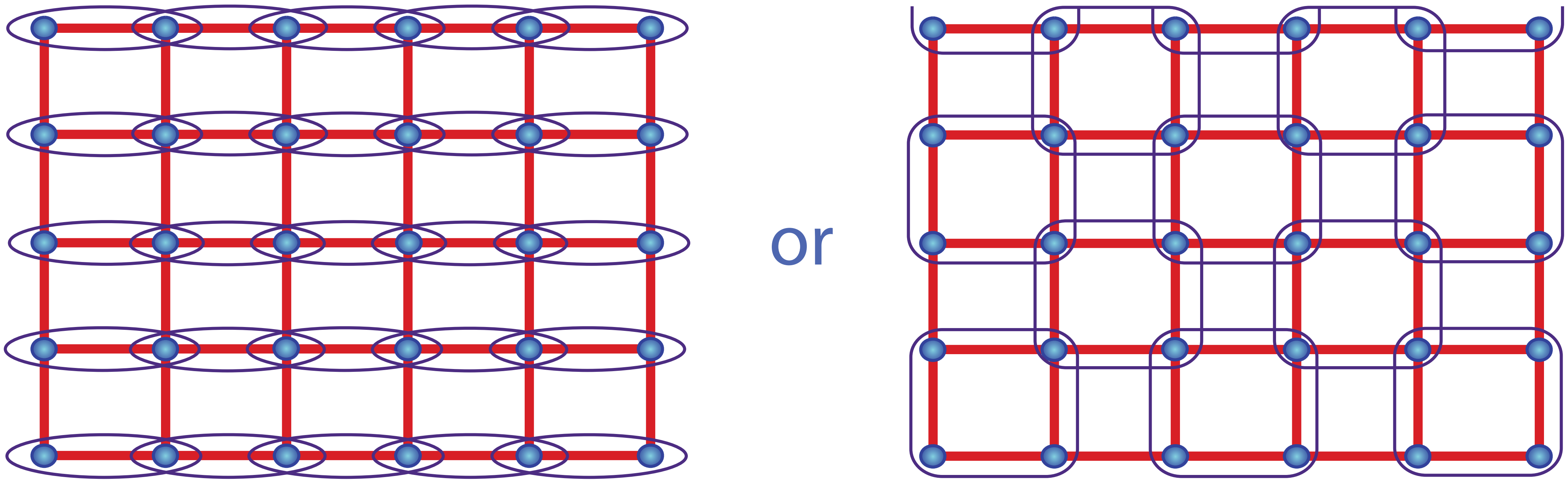
λ_4 is likely irrelevant at the critical point.

$\Rightarrow$ Critical theory is non-compact CP^1 model

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

Odd integer S

Loss of Neel order leads to a (charge) nematic



Nematic state can be viewed as a set of Haldane-gap spin chains.
Continuum theory only has a doubled monopole term

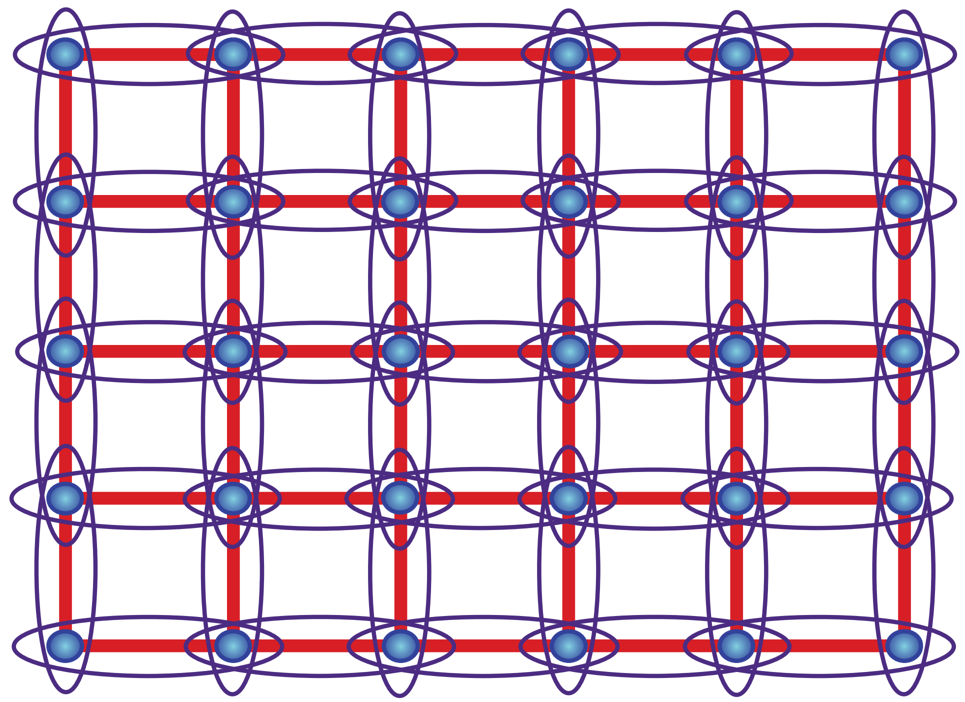
$$\mathcal{S} = \ldots + \lambda_2 \int d^2 r d\tau V^2(r,\tau).$$

Fate of λ_2 and resulting critical theory is unknown.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

Even integer S

Loss of Neel order leads to a state with no broken symmetry



This is the Affleck-Kennedy-Leib-Tasaki (AKLT) spin gap state. Continuum theory only has a single monopole term

$$\mathcal{S} = \ldots + \lambda_1 \int d^2 r d\tau V(r,\tau).$$

Monopoles proliferate at the transition, and the critical theory is the Landau-Ginzburg-Wilson O(3) model.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

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Hole dynamics in an antiferromagnet across a deconfined quantum critical point,
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,
Physical Review B **75** , 235122 (2007).

Algebraic charge liquids and the underdoped cuprates,
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil,
Nature Physics **4**, 28 (2008).

• Begin with the representation of the quantum antiferromagnet as the lattice CP^1 model:

$$\mathcal{S}_z = -\frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau}$$

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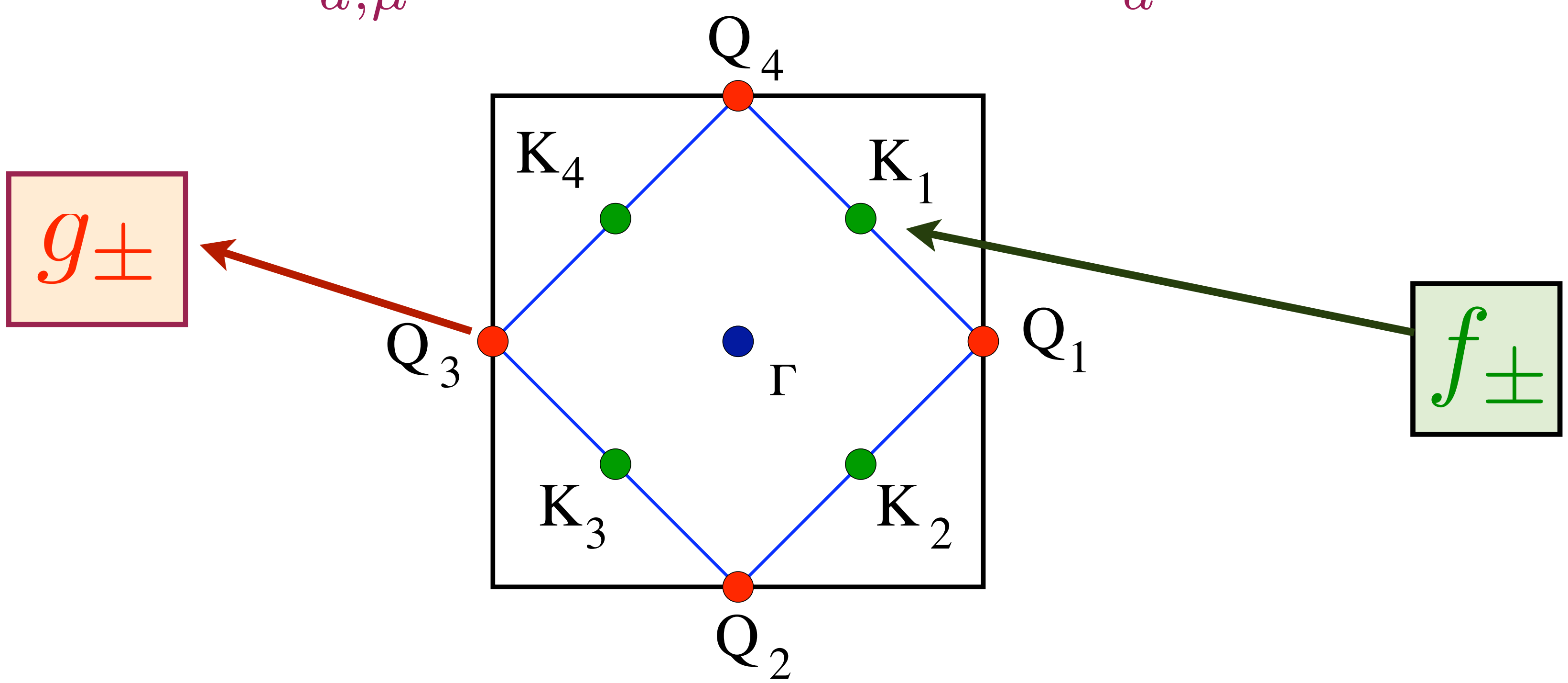


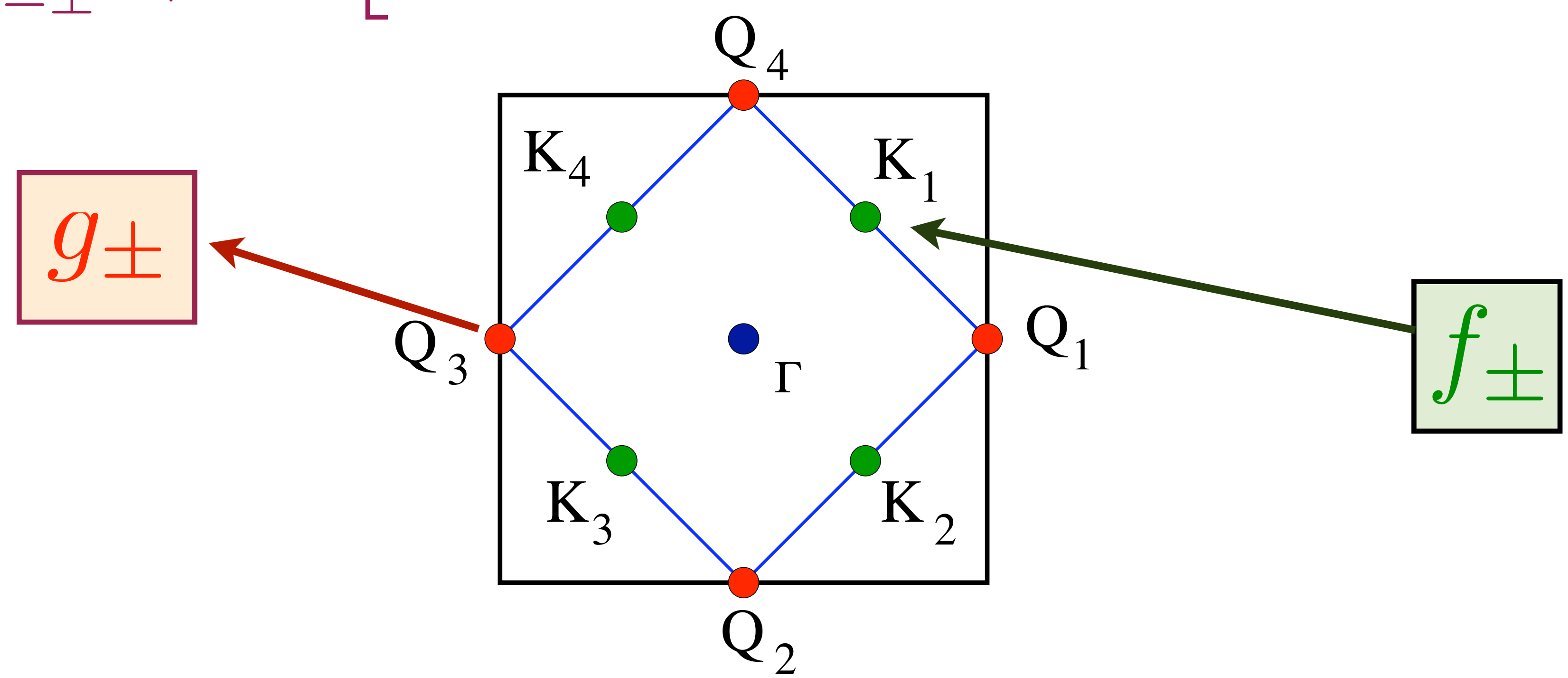
• Write the electron operator at site r, $c_\alpha(r)$ in terms of **doublon** operators $g_\pm$ and **holon** operators $f_\pm$

$$c_\alpha(r) = \begin{cases} (g_+(r) + f_+^\dagger(r))z_{r\alpha} & \text{for } r \text{ on sublattice A} \\ \varepsilon_{\alpha\beta}(g_- + f_-^\dagger(r))(r) z^*_{r\beta} & \text{for } r \text{ on sublattice B} \end{cases}$$

Note that the fermions g_s, f_s have charge s under the U(1) gauge field A_μ.

• Choose the fermion dispersions to match the positions on electron/hole pockets

$$\mathcal{S}_f = \int d\tau \sum_{s=\pm} \int_\diamond \frac{d^2k}{4\pi^2} \left[g_s^\dagger(\vec{k}) \left(\partial_\tau - isA_\tau + \epsilon(\vec{k} - s\vec{A}) \right) g_s(\vec{k}) + (g \to f) \right]$$



• Include the hopping between opposite sublattices (Shraiman-Siggia term):

$$\mathcal{S}_t = -t \int d\tau \sum_{\langle rr' \rangle} c_\alpha^\dagger(r) c_\alpha(r') + \text{h.c.}$$

• Complete theory for doped antiferromagnet:

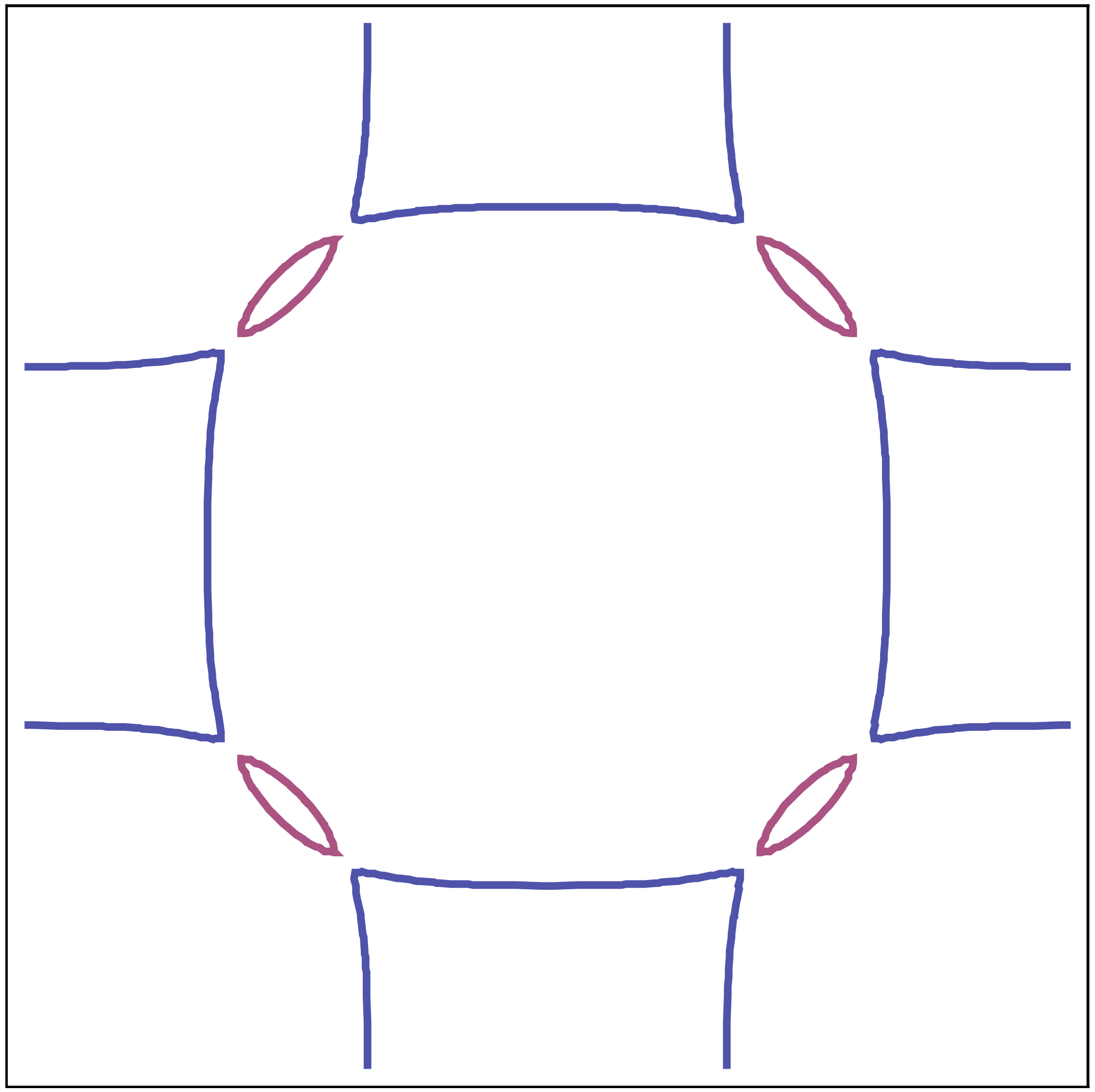
$$\mathcal{S} = \mathcal{S}_z + \mathcal{S}_f + \mathcal{S}_t$$

Conventional phases

AF Metal

$\langle z_\alpha \rangle \neq 0 \quad , \quad$ Fermi surfaces of $f_\pm$ and/or $g_\pm$

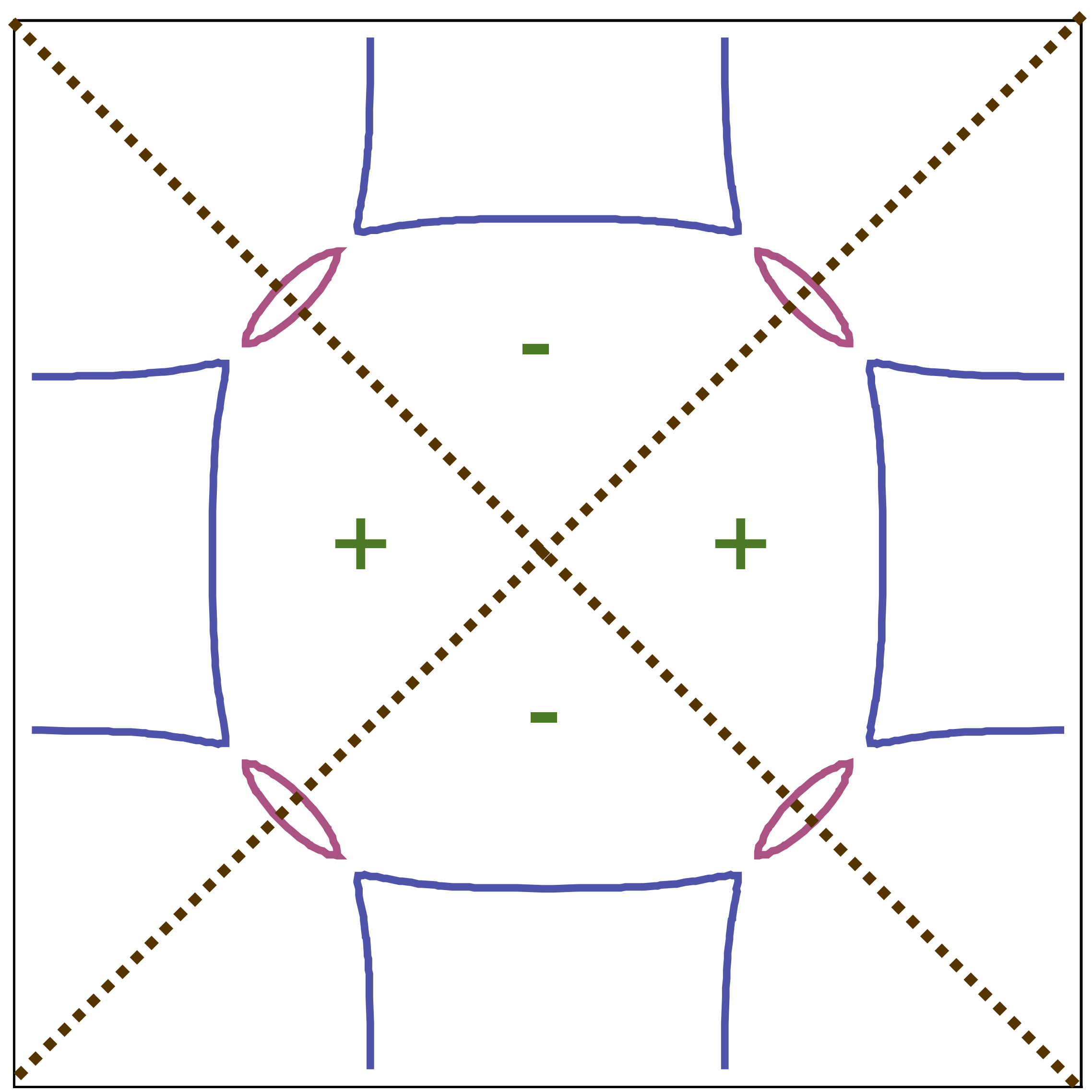
"Meissner" effect ties U(1) gauge charge to conserved spin along the direction of Néel order



Conventional phases

AF d-wave superconductor

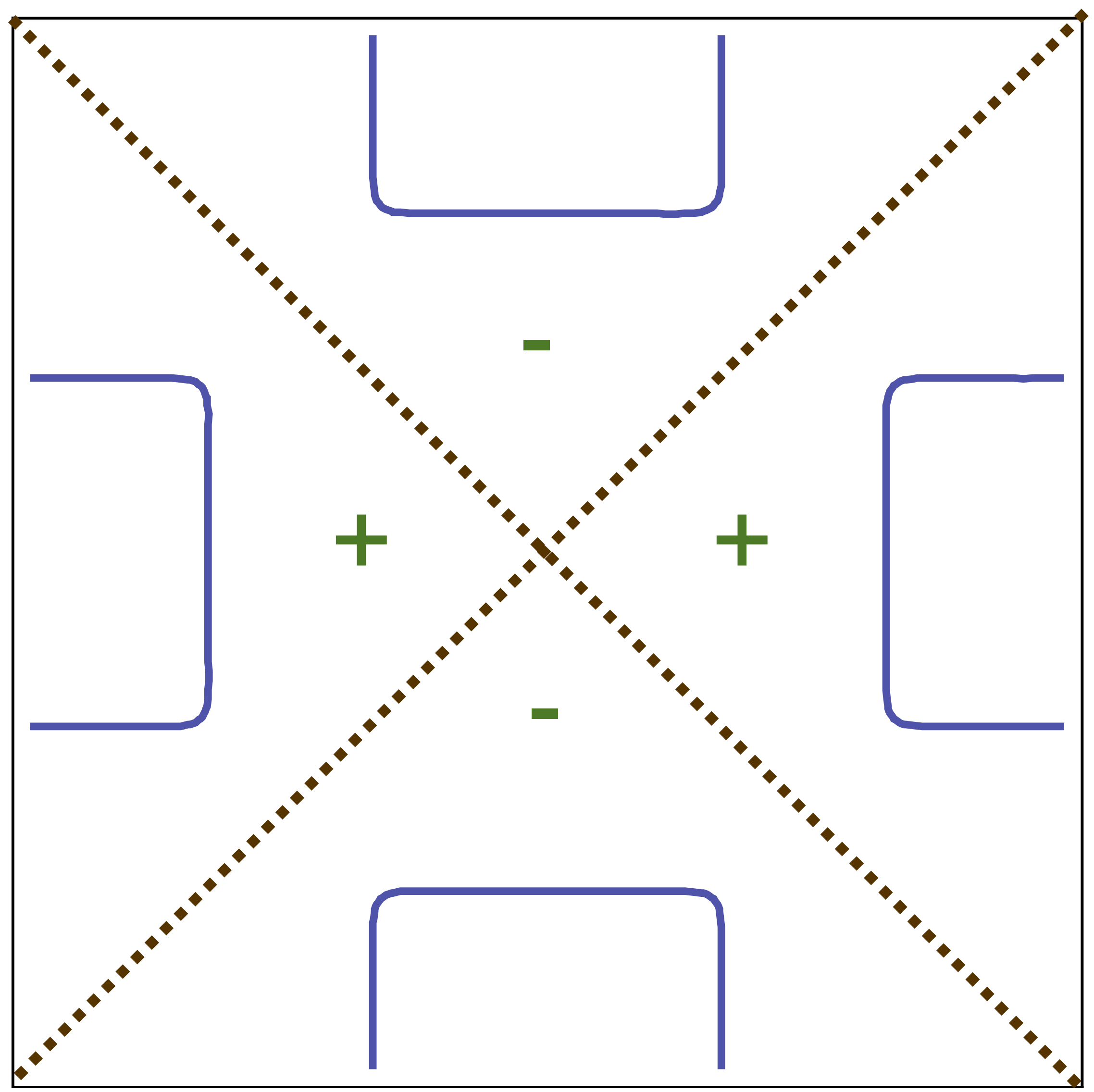
$$\langle z_\alpha \rangle \neq 0$$
$$\langle g_+ g_- \rangle \neq 0 \ , \ s - \text{wave pairing}$$
$$\langle f_+ f_- \rangle \neq 0 \ , \ p - \text{wave pairing}$$



4 Dirac points
(+ 4 shadows)

AF d-wave superconductor

$$
\begin{aligned}
\langle z_\alpha \rangle &\neq 0 \\
\langle g_+ g_- \rangle &\neq 0 \ , \ s-\text{wave pairing} \\
\langle f_+ f_- \rangle &\neq 0 \ , \ p-\text{wave pairing}
\end{aligned}
$$



Fermions fully gapped

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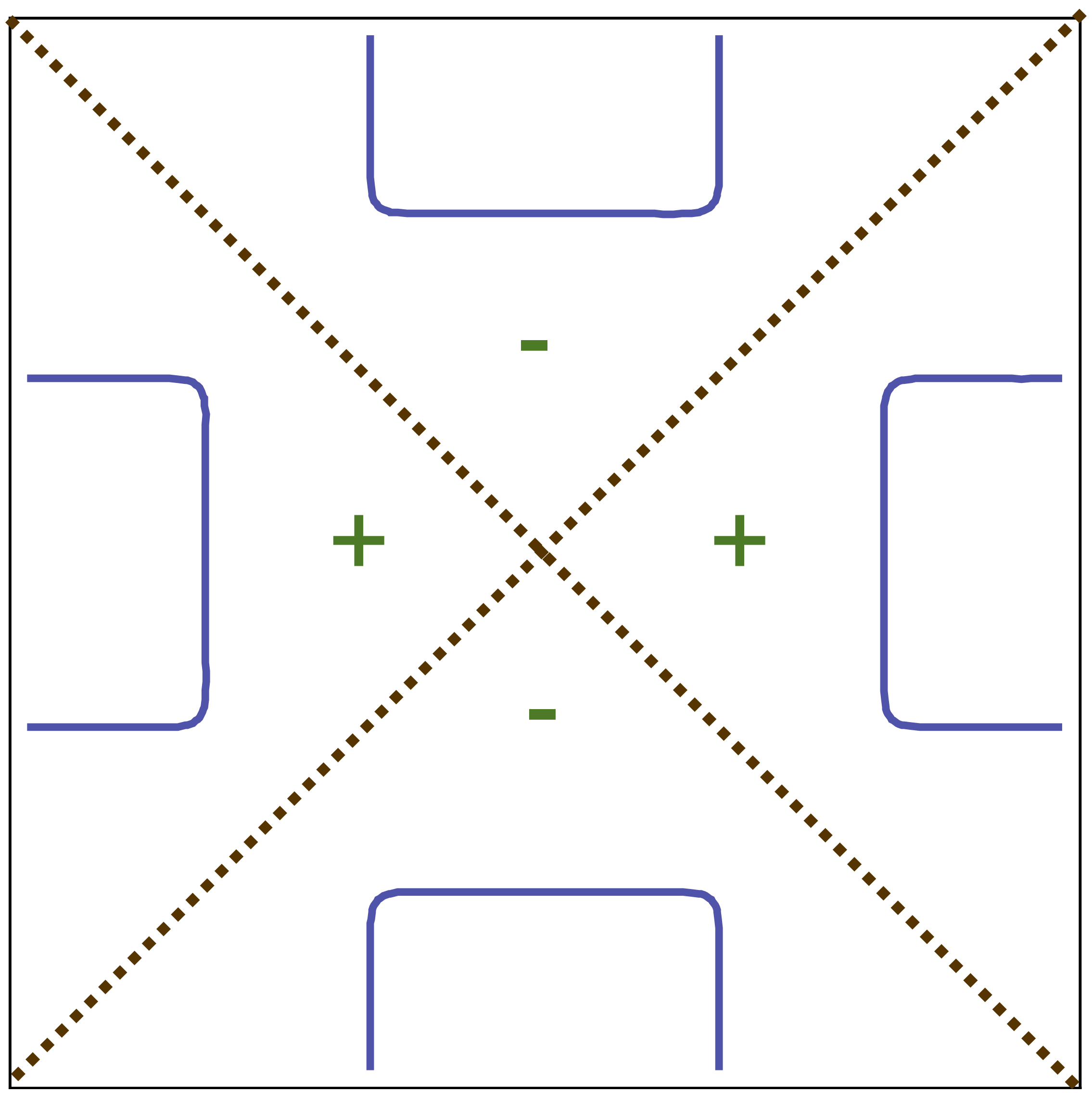
Loss of Neel order in AFM d-wave superconductor

$$\langle z_\alpha \rangle \neq 0 \quad \Rightarrow \quad \langle z_\alpha \rangle = 0 \;, \text{ Higgs to Coulomb transition}$$
$$\langle g_+ g_- \rangle \quad \neq \quad 0 \;, \; s - \text{wave pairing}$$
$$\langle f_+ f_- \rangle \quad \neq \quad 0 \;, \; p - \text{wave pairing}$$



Fermions fully gapped

Perturbation theory

Because fermions are gapped, low energy theory for spinons is the same as that in the insulator: the CP^1 model:

$$\mathcal{S}_z = \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s \left|z_\alpha\right|^2 + u \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

Phases:

$\langle z_\alpha \rangle \neq 0$	$\Rightarrow$	AFM+dSC (Higgs) state
$\langle z_\alpha \rangle = 0$	$\Rightarrow$	dSC (Coulomb) state

Recall non-perturbative effects in insulator

Have to account for Dirac monopoles in A_μ.
Evaulation of the boson number constraint

$$i \int d\tau \sum_i \eta_i A_{i\tau} \left(z^\dagger_{i\alpha} z_{i\alpha} - 2S \right)$$

for a Dirac monopole in A_μ yields the *monopole Berry phase*:

$$\mathcal{S}_B = \int d\tau \sum_i \zeta_i^{2S} V_i(\tau) \; + \; \text{c.c.}$$

where $V_i(\tau)$ creates a monopole on the dual lattice site i.

$\zeta_i =$

1	i	1	i
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F. D. M. Haldane, *Phys. Rev. Lett.* **61**, 1029 (1988).

Non-perturbative effects in superconductor

Have to account for Dirac monopoles in A_μ.
Now the boson number constraint is

$$i \int d\tau \sum_i \eta_i A_{i\tau} \left(z_{i\alpha}^\dagger z_{i\alpha} + g_i^\dagger g_i + f_i^\dagger f_i - 1 \right).$$

Non-perturbative effects in superconductor

Have to account for Dirac monopoles in A_μ.
Now the boson number constraint is

$$i \int d\tau \sum_i \eta_i A_{i\tau} \left(z_{i\alpha}^\dagger z_{i\alpha} + g_i^\dagger g_i + f_i^\dagger f_i - 1 \right).$$

In the dual formulation, this modifies the Berry phase to:

$$\mathcal{S}_B = \int d\tau \sum_i \zeta_i V_i \sum_{\ell,m} \mathcal{C}_{\ell m}^i \Phi_{\ell i}^\dagger \Phi_{mi} \; + \; \text{c.c.}$$

where Φ_ℓ annihilates a vortex in the superconducting order with flux $h/(2e)$. These vortices come in multiple flavors, ℓ, determined by the density of fermions. The $\mathcal{C}_{\ell m}^i$ are oscillatory numerical coefficients which can be determined from space-group symmetry considerations.

These results can be justified by an explicit duality transformation on a toy model in which the $g_\pm$ are treated as bosons.

Non-perturbative effects in superconductor

Key question: What are the allowed values of $\langle \Phi_\ell^\dagger \Phi_m \rangle$, the vortex-anti-vortex condensate, which preserve square lattice symmetry (upto spin rotations) in the AFM+dSC phase ?

Answer: In addition to the obvious $\langle \Phi_\ell^\dagger \Phi_m \rangle \sim \delta_{\ell m}$, there are a finite number of other choices, which we can enumerate. These choices lead to distinct AFM+dSC states, which are indistinguishable in their symmetry properties, but have distinct low-energy vortex fluctuations ("topological order").

Non-perturbative effects in superconductor

Key question: What are the allowed values of $\langle \Phi_\ell^\dagger \Phi_m \rangle$, the vortex-anti-vortex condensate, which preserve square lattice symmetry (upto spin rotations) in the AFM+dSC phase ?

Answer: In addition to the obvious $\langle \Phi_\ell^\dagger \Phi_m \rangle \sim \delta_{\ell m}$, there are a finite number of other choices, which we can enumerate. These choices lead to distinct AFM+dSC states, which are indistinguishable in their symmetry properties, but have distinct low-energy vortex fluctuations ("topological order").

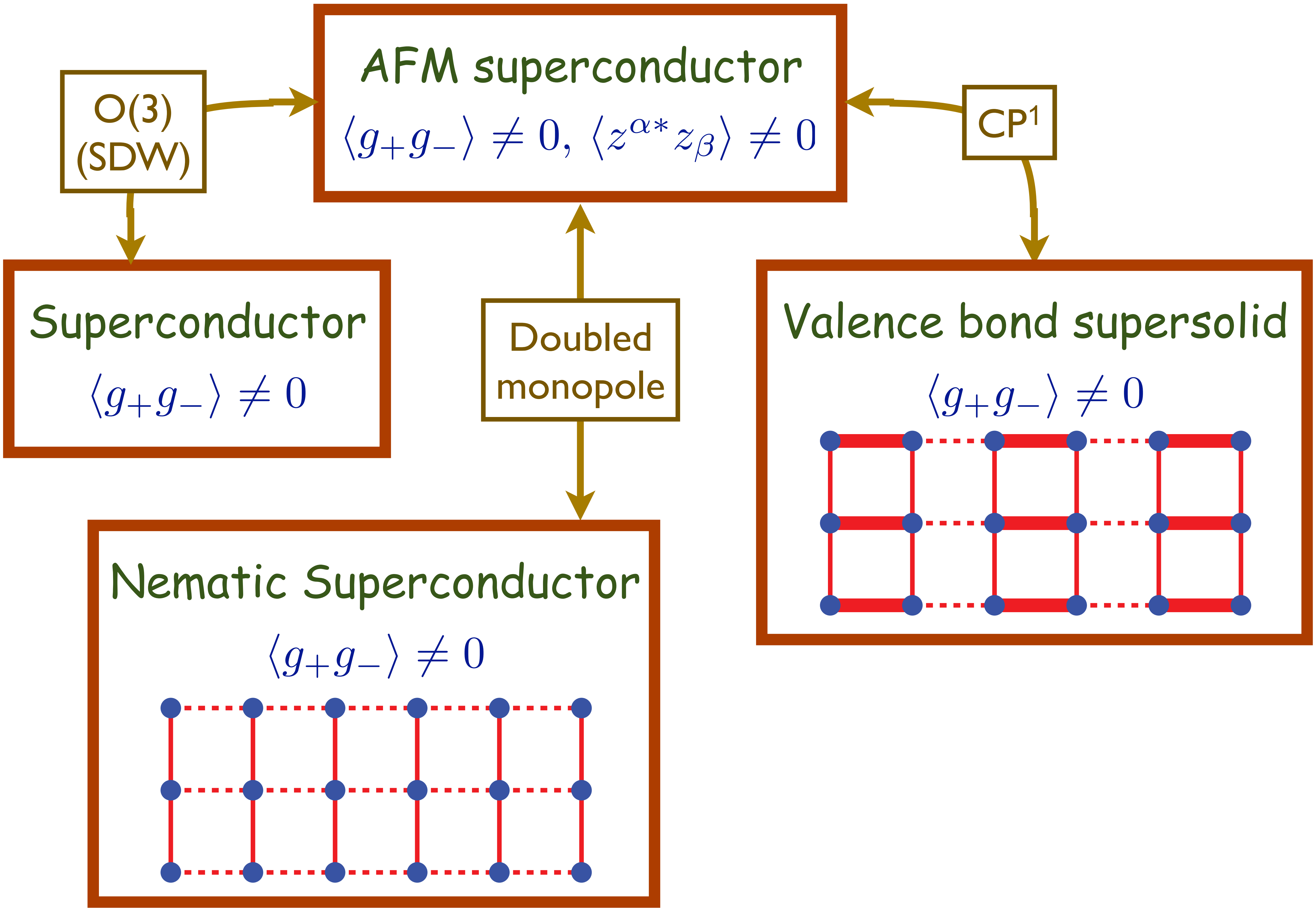
In the continuum limit, after integrating over Φ_ℓ, these choices lead to 3 possibilities for effective monopole term:

$$\int d^2 r d\tau V^4(r,\tau),$$

$$\text{or} \quad \int d^2 r d\tau V^2(r,\tau),$$

$$\text{or} \quad \int d^2 r d\tau V(r,\tau)$$

Non-perturbative effects in superconductor



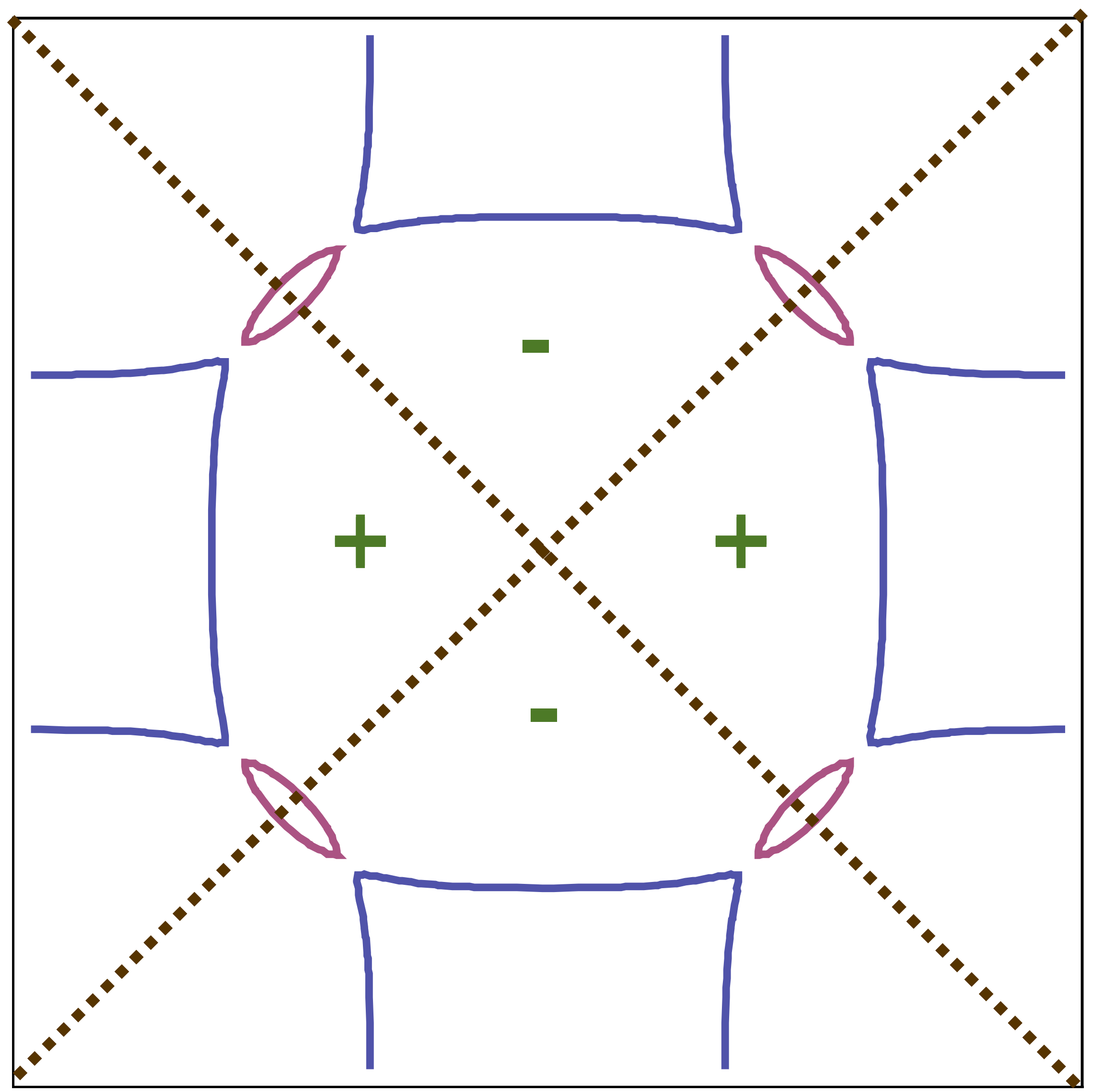
Transitions match those found in insulators for different S

Loss of Neel order in AFM d-wave superconductor *with gapless fermions*

$$\langle z_\alpha \rangle \neq 0 \quad \Rightarrow \quad \langle z_\alpha \rangle = 0 \; , \; \text{Higgs to Coulomb transition}$$

$$\langle g_+ g_- \rangle \quad \neq \quad 0 \; , \; s - \text{wave pairing}$$

$$\langle f_+ f_- \rangle \quad \neq \quad 0 \; , \; p - \text{wave pairing}$$



4 Dirac points (+ 4 shadows)

Perturbation theory

Have to include Dirac fermions, and so the low energy theory for spinons is *not* the CP^1 model on the insulator:

$$\mathcal{S}_z = \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s \left|z_\alpha\right|^2 + u \left(\left|z_\alpha\right|^2\right)^2 + i\overline{\psi}_a \gamma_\mu (\partial_\mu - iA_\mu)\psi_a + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

Phases:

$\langle z_\alpha \rangle \neq 0$	$\Rightarrow$	AFM+dSC (Higgs)
$\langle z_\alpha \rangle = 0$	$\Rightarrow$	Holon superconductor (Algebraic charge liquid)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, *Nature Physics* **4**, 28 (2008)

Non-perturbative effects in superconductor

Have to account for Dirac monopoles in A_μ.

Monopole Berry phases are modified by fermion zero modes . . . . . . ?

Conclusions

- Theory for the loss of Neel order in d-wave superconductors
- For superconductors with gapped Bogoliubov quasiparticles, we found 3 distinct transitions, to a valence bond supersolid, a nematic superconductor, and a vanilla superconductor
- These transitions in the compressible superconductor of $S=1/2$ electrons match the classification of transitions in the incompressible Mott insulator of general S spins.