Quantum phase transitions and the Luttinger theorem.

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Outline

A. Bose-Fermi mixtures

Depleting the Bose-Einstein condensate in trapped ultracold atoms

B. The Kondo Lattice

The heavy Fermi liquid (FL) and the fractionalized Fermi liquid (FL)*

C. *Detour*: Deconfined criticality in insulators

Landau forbidden quantum transitions

D. Deconfined criticality in the Kondo lattice ?

A. Bose-Fermi mixtures

Depleting the Bose-Einstein condensate in trapped ultracold atoms

Mixture of bosons *b* and fermions *f*

(*e.g.* ^{7}Li+^{6}Li, ^{23}Na+^{6}Li, ^{87}Rb+^{40}K)

Tune to the vicinity of a Feshbach resonance associated with a molecular state ψ

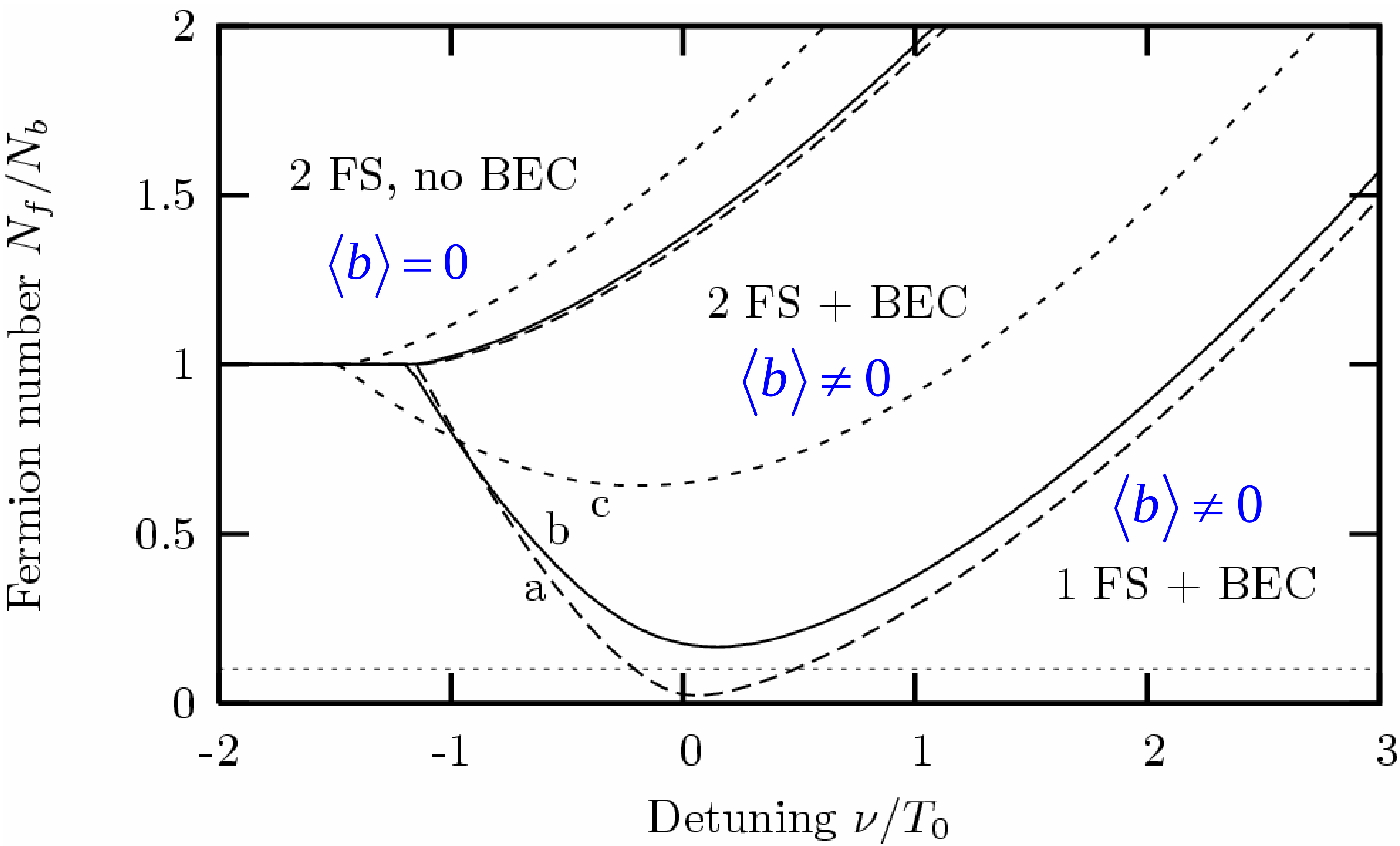
Conservation laws:

$$b^\dagger b + \psi^\dagger \psi = N_b$$

$$f^\dagger f + \psi^\dagger \psi = N_f$$

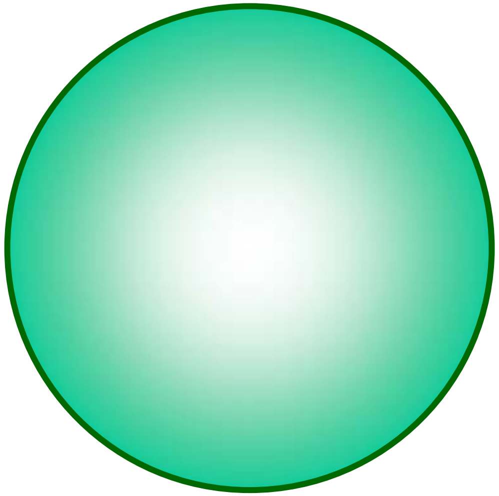
$$f^\dagger f - b^\dagger b = N_f - N_b$$

Phase diagram

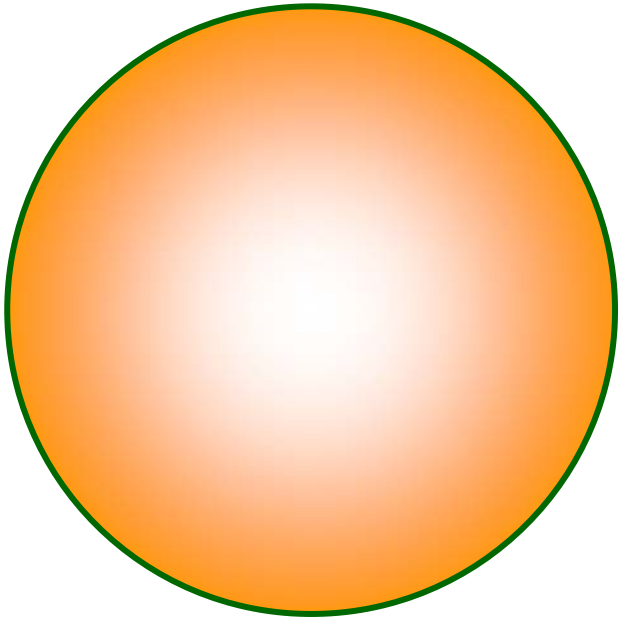


2 FS, no BEC phase

"molecular" Fermi surface



"atomic" Fermi surface



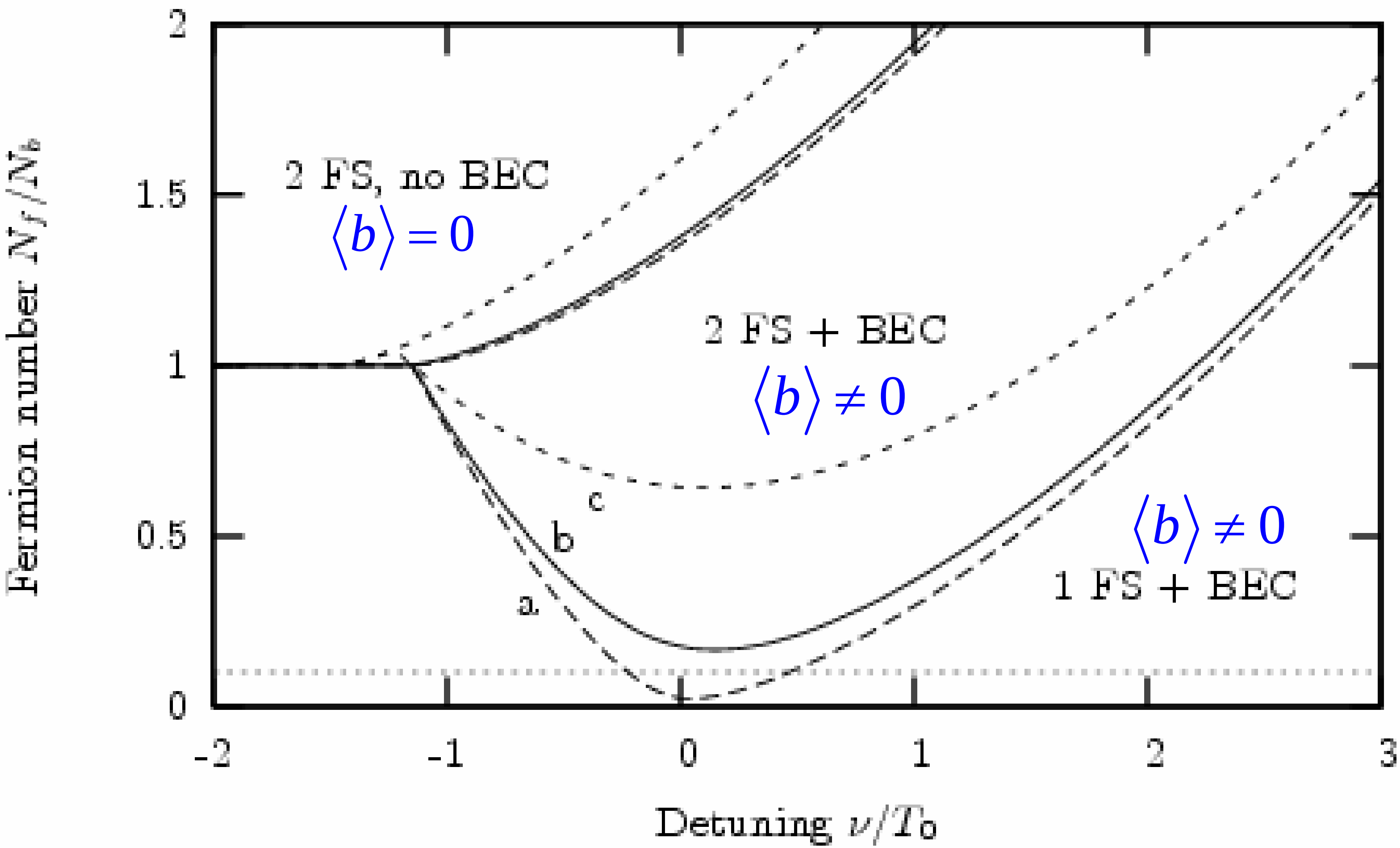
$$\langle b \rangle = 0$$

$$\text{Volume} = N_b$$

$$\text{Volume} = N_f - N_b$$

2 Luttinger theorems; volume within both Fermi surfaces is conserved

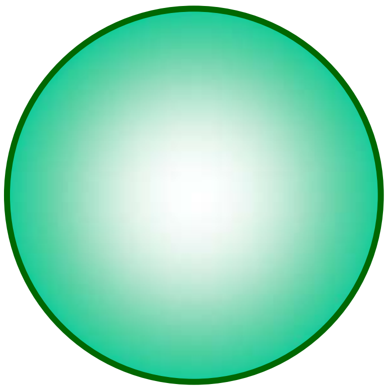
Phase diagram



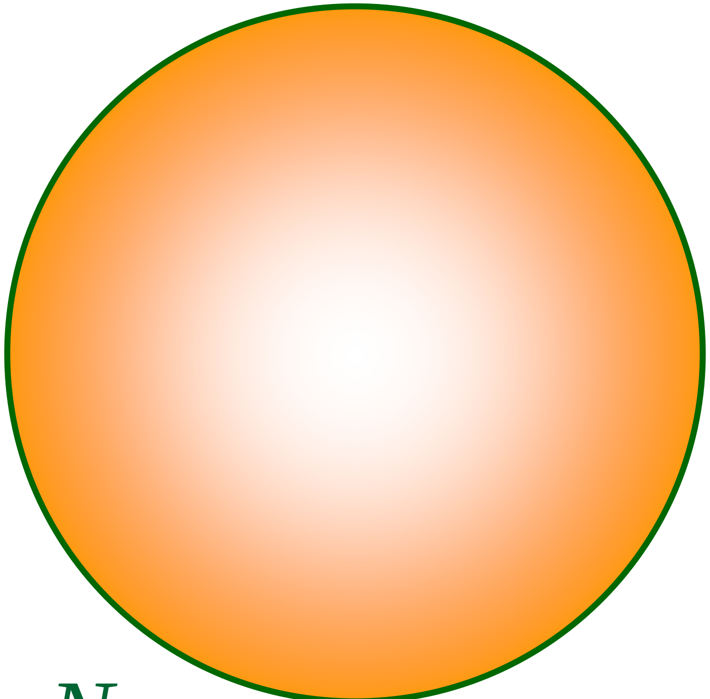
2 FS + BEC phase

“atomic” Fermi surface

“molecular” Fermi surface



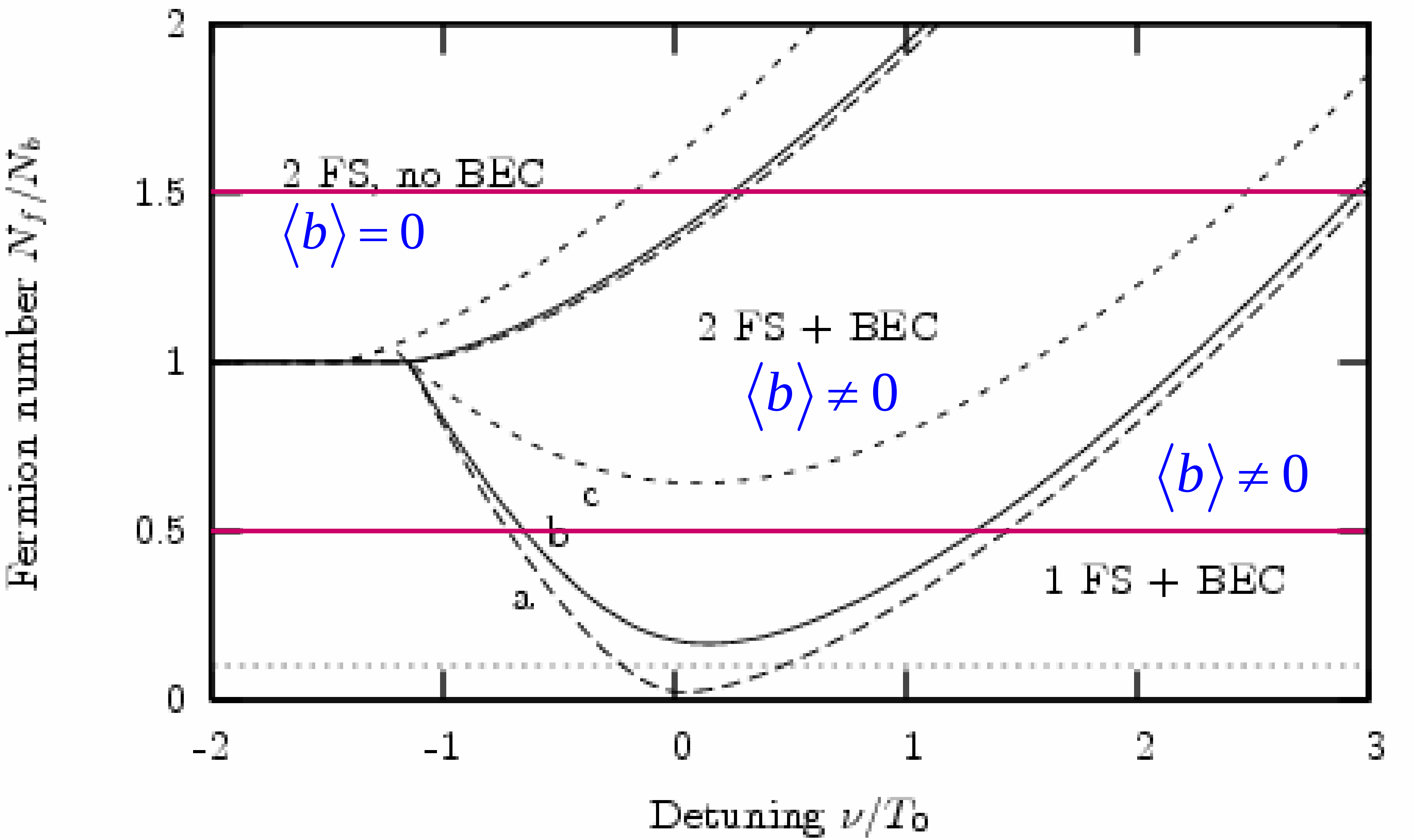
$$\langle b \rangle \neq 0$$



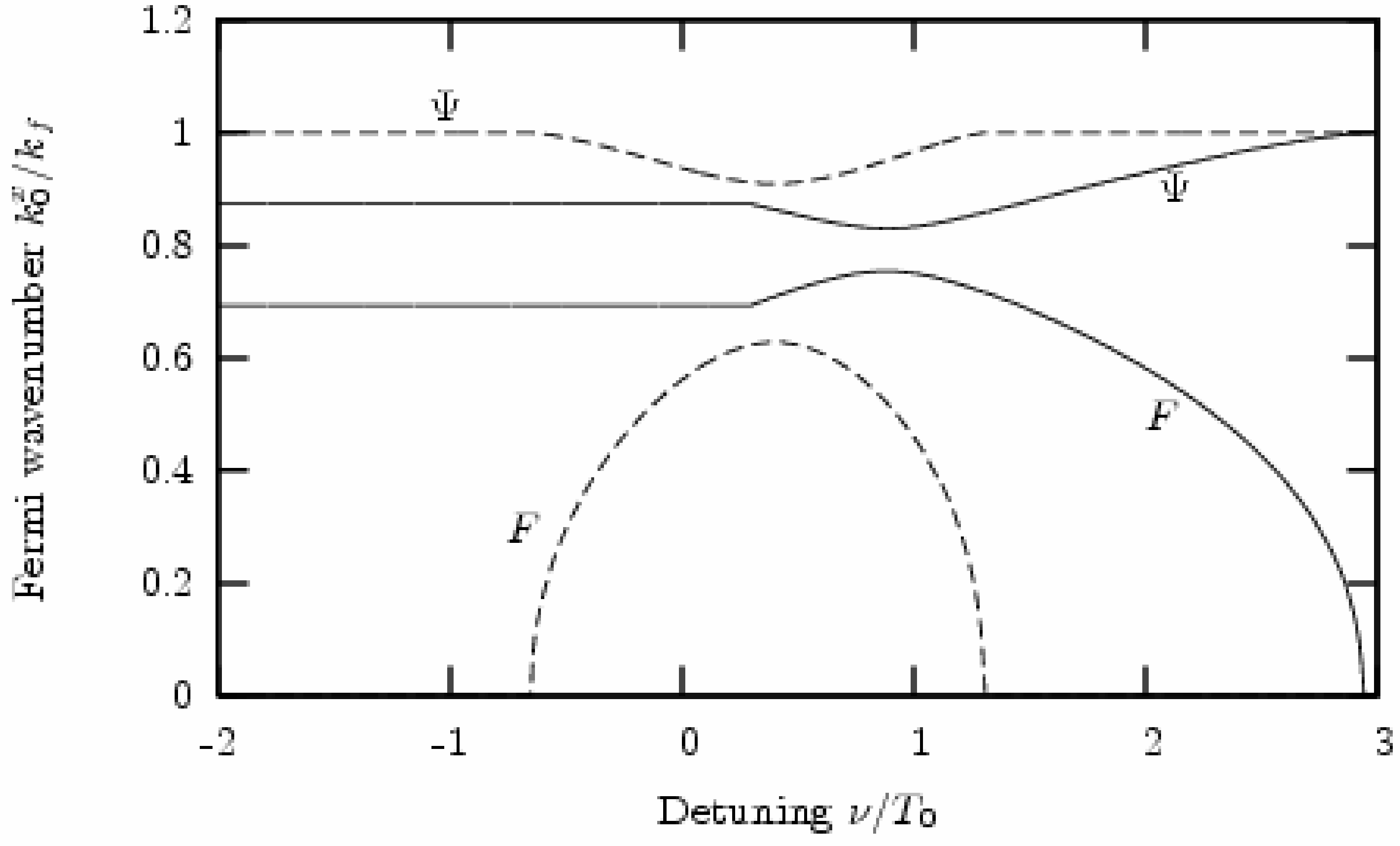
$$\text{Total volume} = N_f$$

1 Luttinger theorem; only total volume within Fermi surfaces is conserved

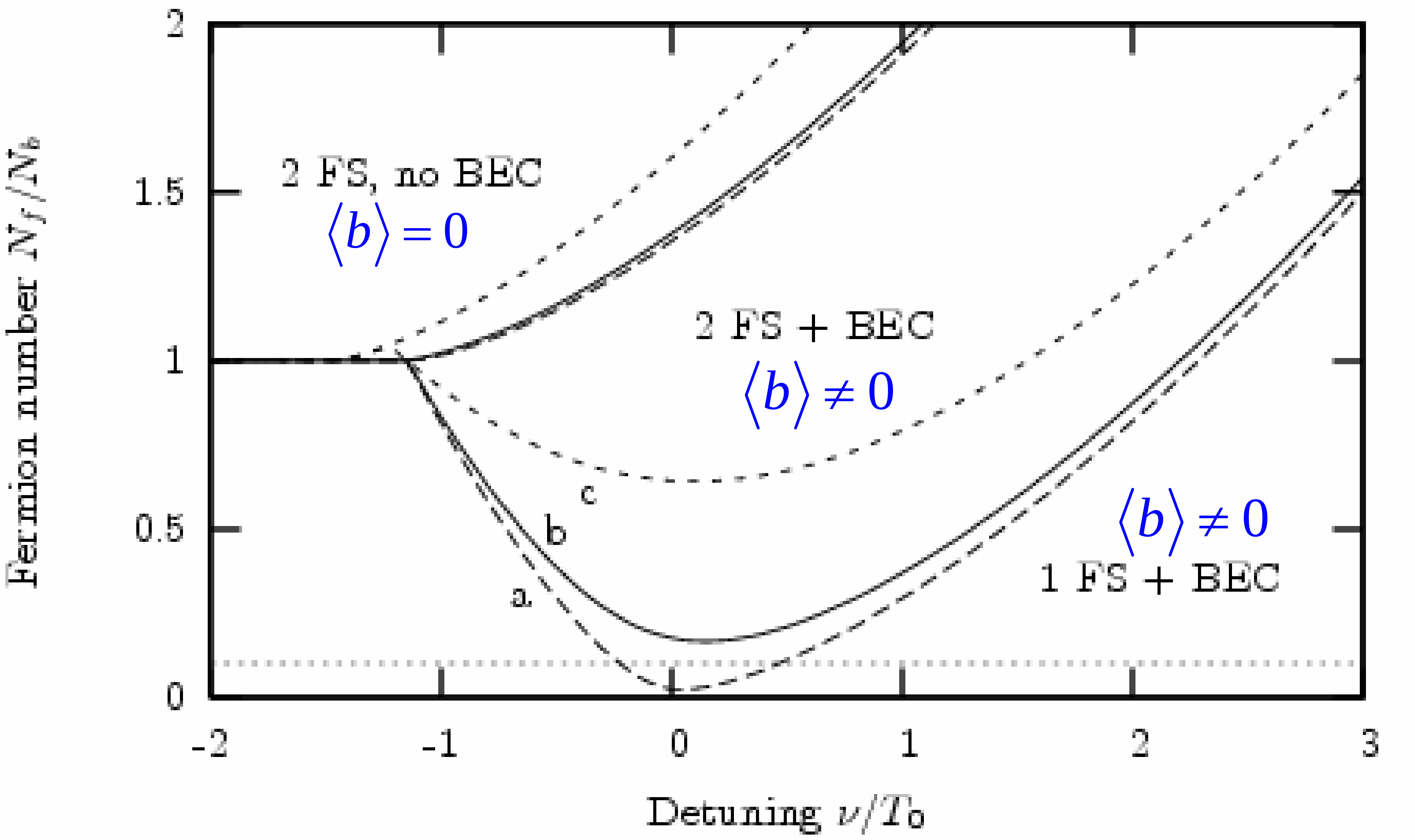
Phase diagram



Fermi wavevectors

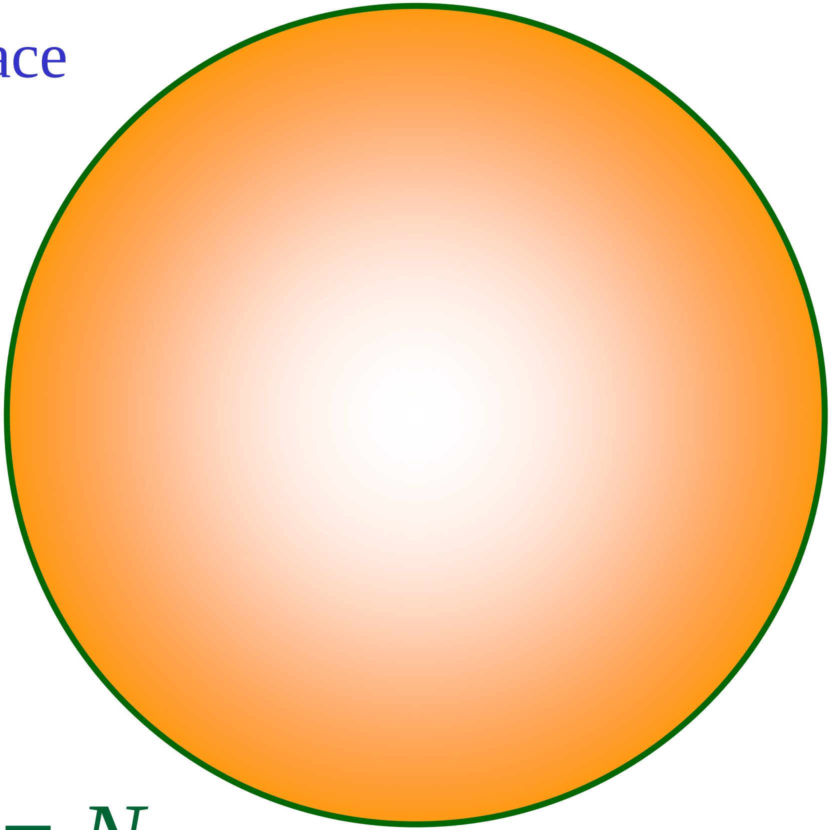


Phase diagram



1 FS + BEC phase

"atomic" Fermi surface



$$\langle b \rangle \neq 0$$

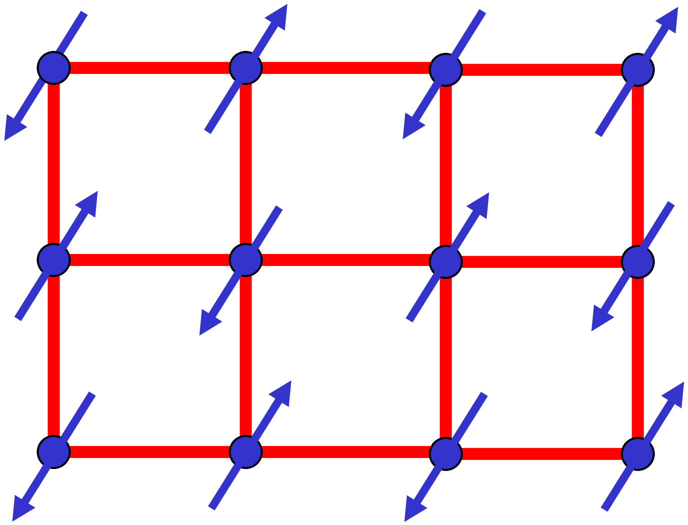
Total volume = N_f

1 Luttinger theorem; only total volume within Fermi surfaces is conserved

B. The Kondo Lattice

The heavy Fermi liquid (FL) and the fractionalized Fermi liquid (FL)*

The Kondo lattice



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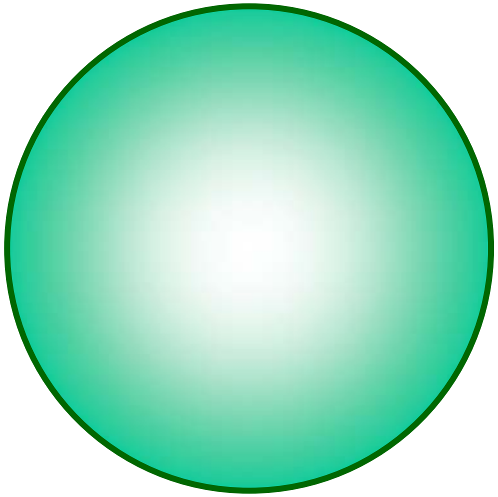


Local moments f_σ

Conduction electrons c_σ

$$H_K = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + J_K \sum_i c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Number of f electrons per unit cell = $n_f = 1$

Number of c electrons per unit cell = n_c

Define a bosonic field which measures the hybridization between the two bands:

$$b_i \sim \sum_\sigma c_{i\sigma}^\dagger f_{i\sigma}$$

Analogy with Bose-Fermi mixture problem:

$c_{i\sigma}$ is the analog of the "molecule" ψ

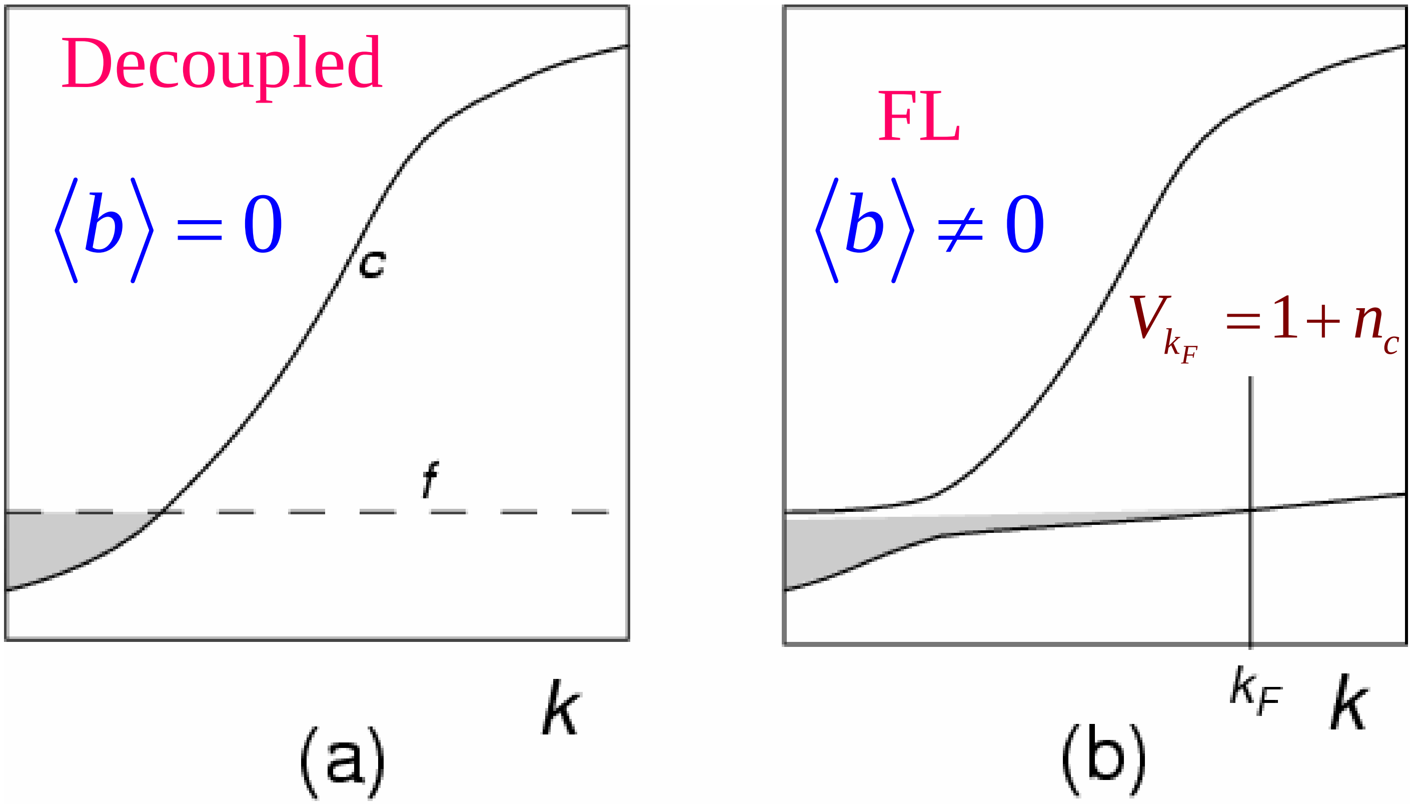
Conservation laws:

$$f_\sigma^\dagger f_\sigma + c_\sigma^\dagger c_\sigma = 1 + n_c \quad \text{(Global)}$$

$$f_\sigma^\dagger f_\sigma + b^\dagger b = 1 \quad \text{(Local)}$$

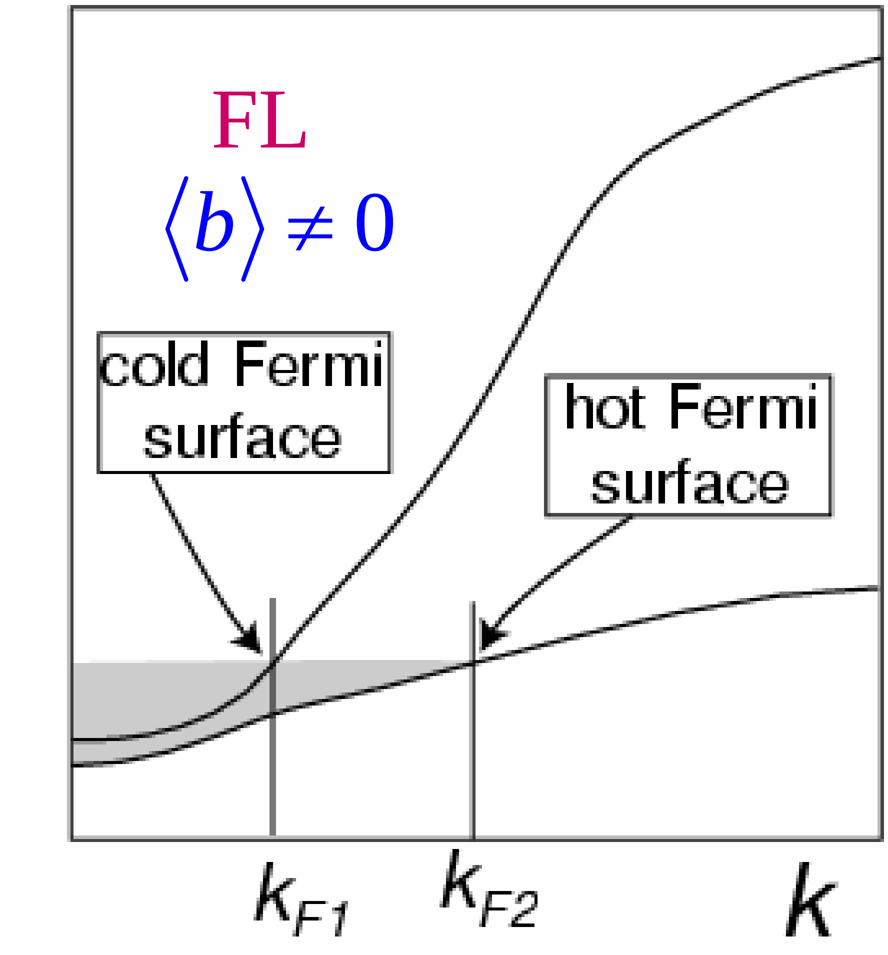
Main difference: second conservation law is *local* so there is a U(1) gauge field.

1 FS + BEC ⇔ Heavy Fermi liquid (FL) ⇔ Higgs phase



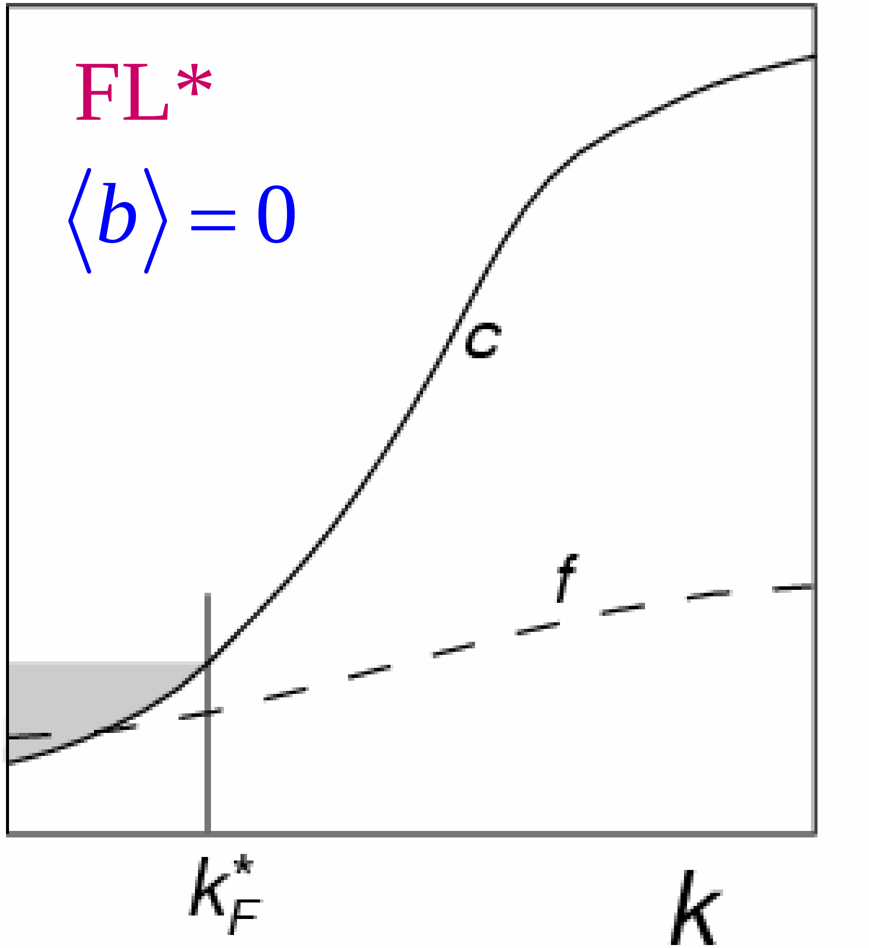
If the f band is dispersionless in the decoupled case, the ground state is always in the 1 FS FL phase.

2 FS + BEC ⇔ Heavy Fermi liquid (FL) ⇔ Higgs phase



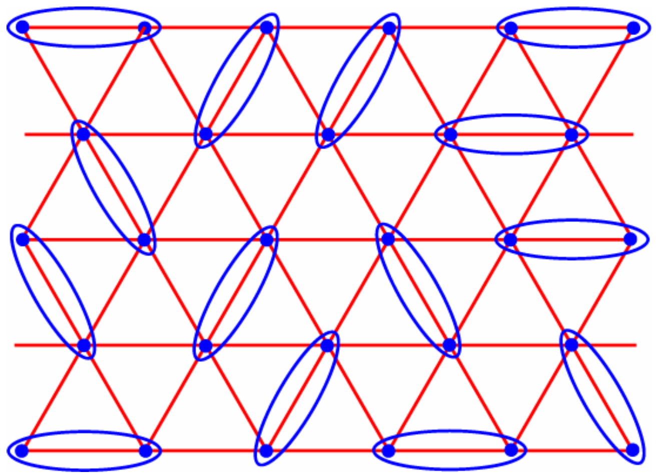
A bare f dispersion (from the RKKY couplings) allows a 2 FS FL phase.

2 FS, no BEC ⇔ Fractionalized Fermi liquid (FL*) ⇔ Deconfined phase



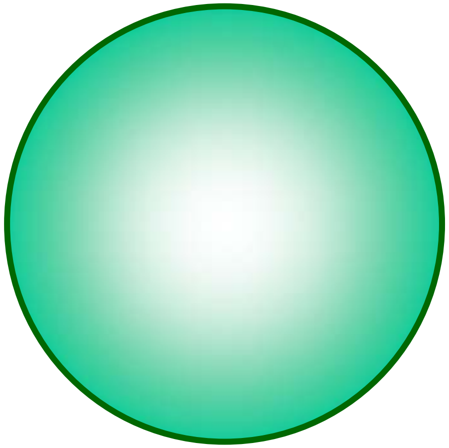
The f band "Fermi surface" realizes a spin liquid (because of the local constraint)

Another perspective on the FL* phase



Local moments f_σ

+



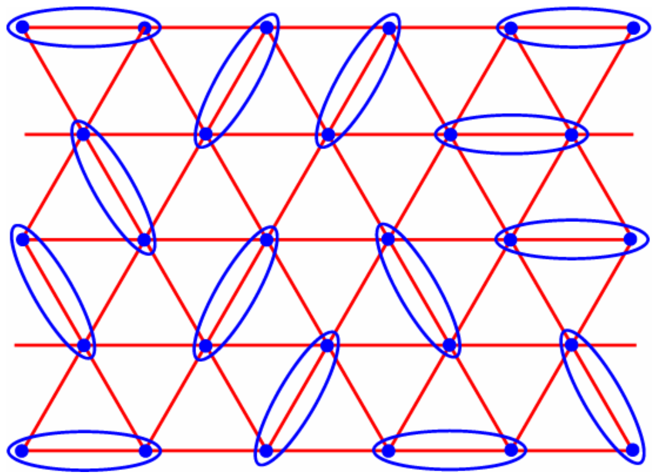
Conduction electrons c_σ

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left(J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

Choose J_H so that ground state of antiferromagnet is a Z_2 or U(1) spin liquid

Influence of conduction electrons



Local moments f_σ

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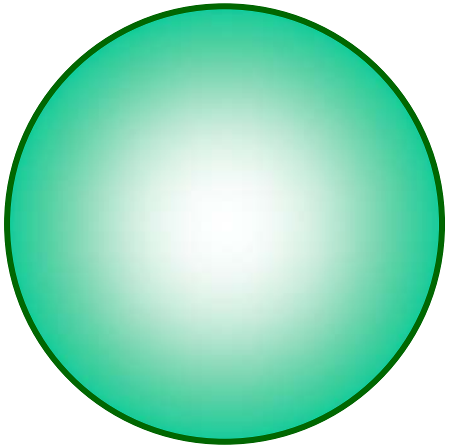


Conduction electrons c_σ

At J_K= 0 the conduction electrons form a Fermi surface on their own with volume determined by n_c.

Perturbation theory in J_K is regular, and so this state will be stable for finite J_K.

So volume of Fermi surface is determined by **$(n_c+n_f\ -1)= n_c$(mod 2),** and does not equal the Luttinger value.

The (U(1) or Z_2) FL* state

A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

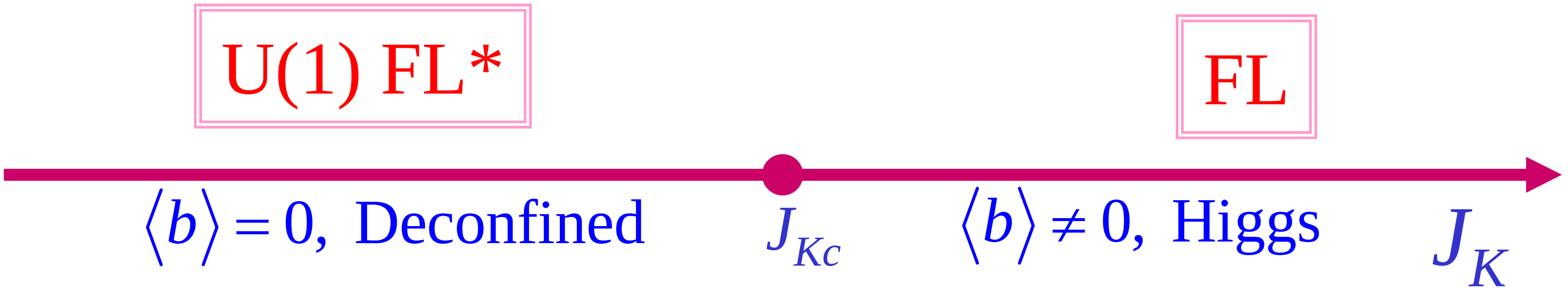
The state has "*topological order*" and associated neutral excitations. The topological order can be detected by the violation of Luttinger's Fermi surface volume. It can only appear in dimensions $d > 1$

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

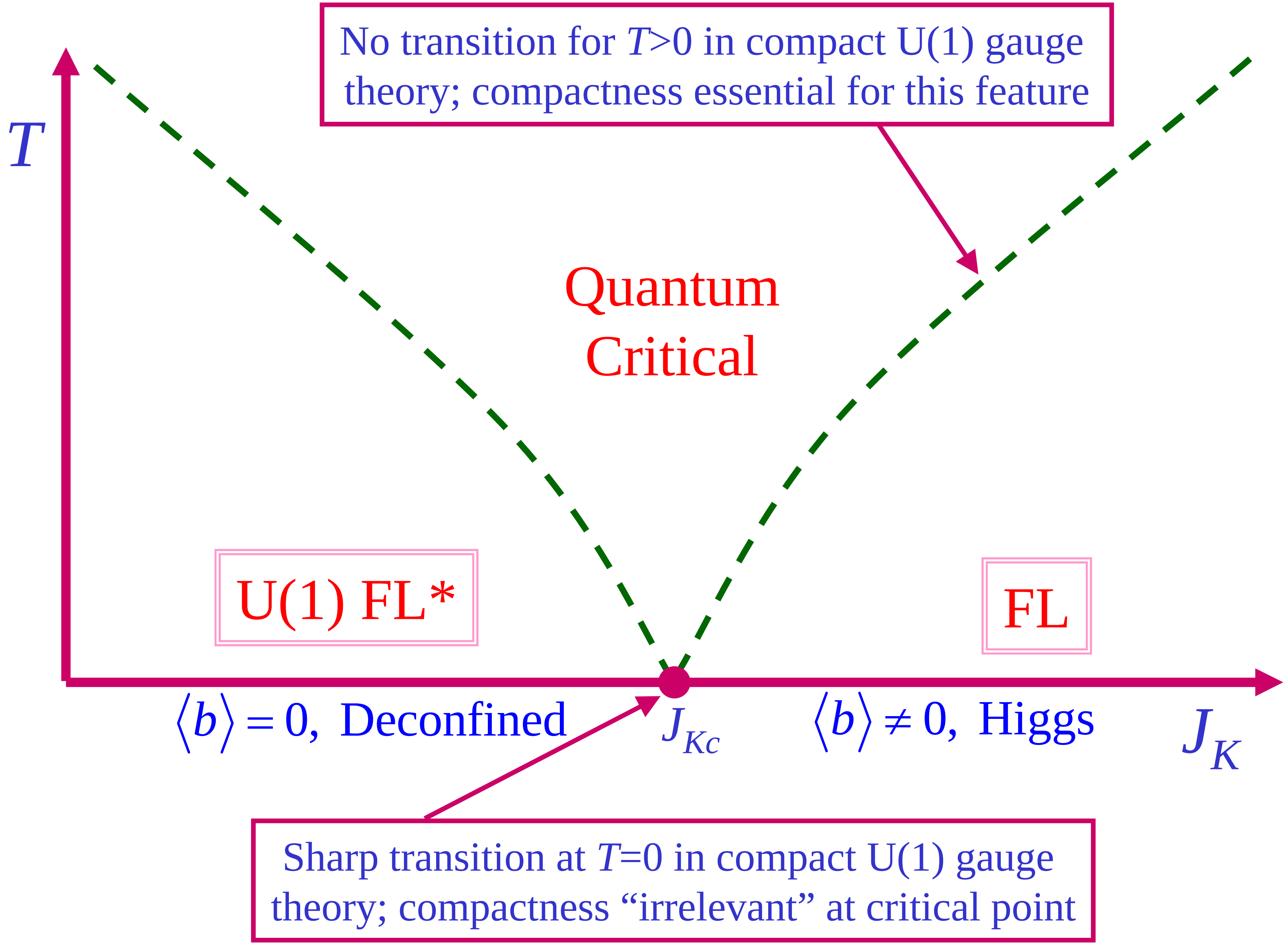
$$= n_c \, (\text{mod } 2)$$

Precursors: N. Andrei and P. Coleman, *Phys. Rev. Lett*. **62**, 595 (1989).
Yu. Kagan, K. A. Kikoin, and N. V. Prokof'ev, *Physica* B **182**, 201 (1992).
Q. Si, S. Rabello, K. Ingersent, and L. Smith, *Nature* **413**, 804 (2001).
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev*. B 66, 045111 (2002).
L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev*. B **60**, 1654, (1999);
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).
F. H. L. Essler and A. M. Tsvelik, *Phys. Rev*. B **65**, 115117 (2002).

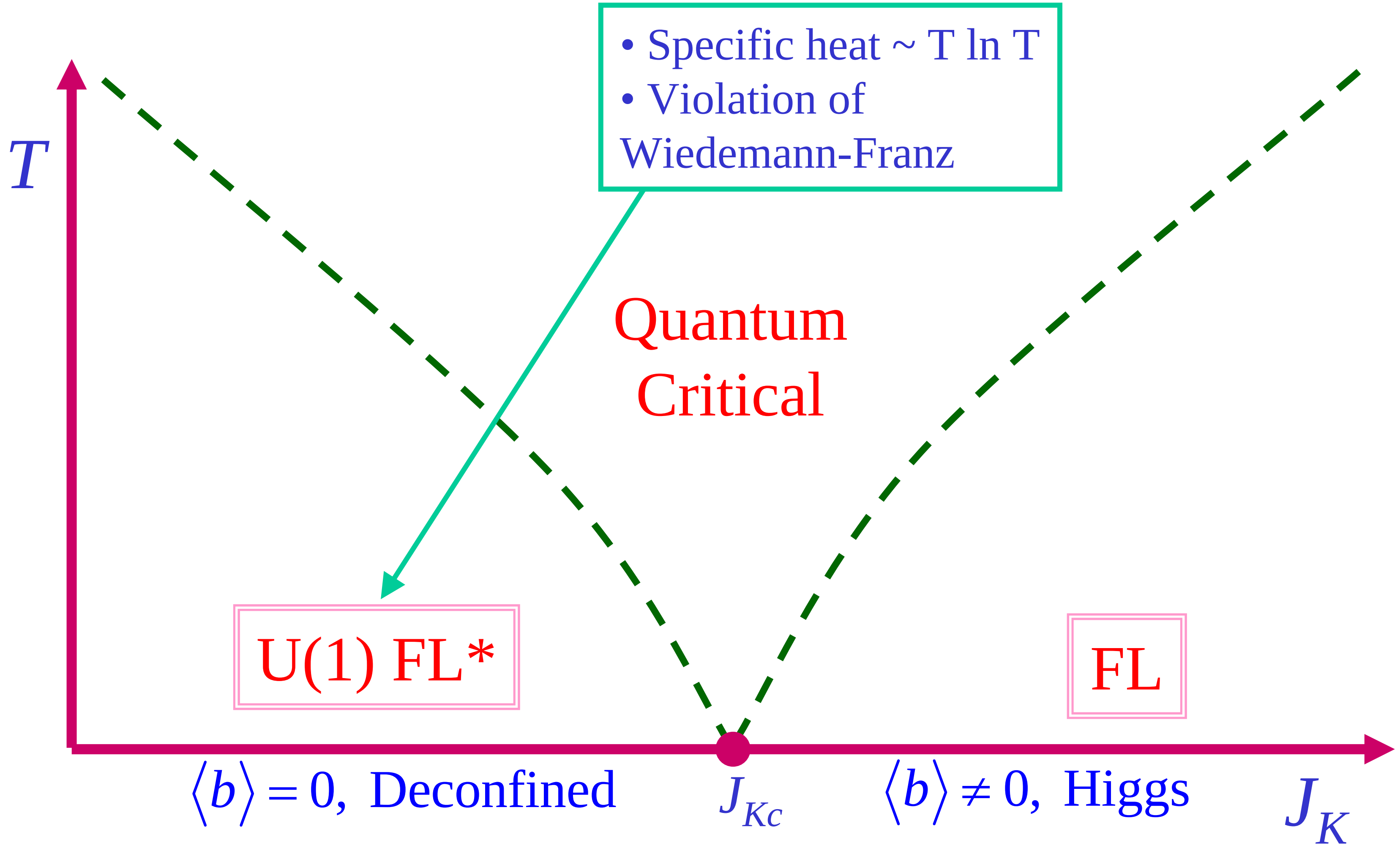
Phase diagram



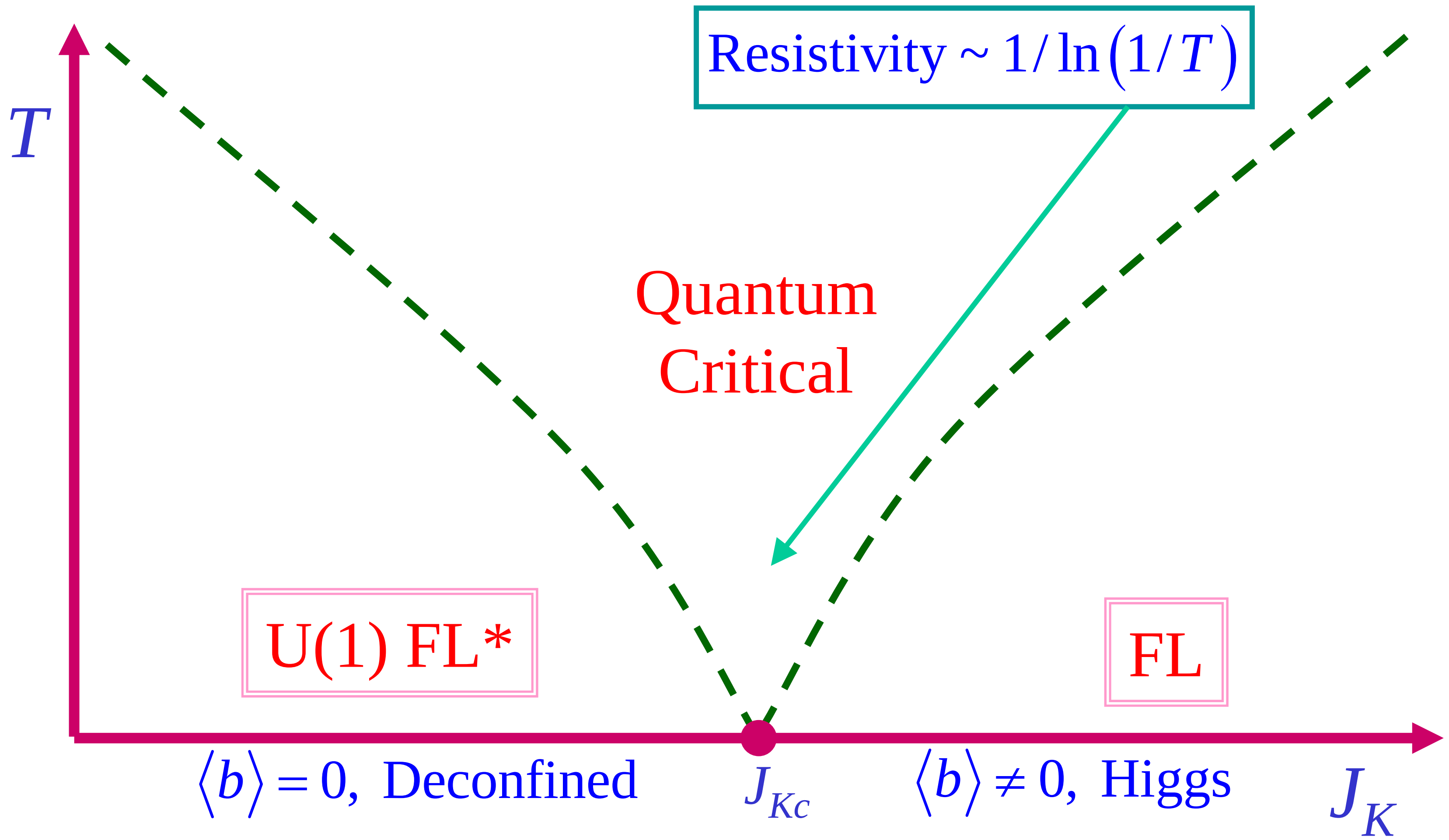
Phase diagram



Phase diagram



Phase diagram

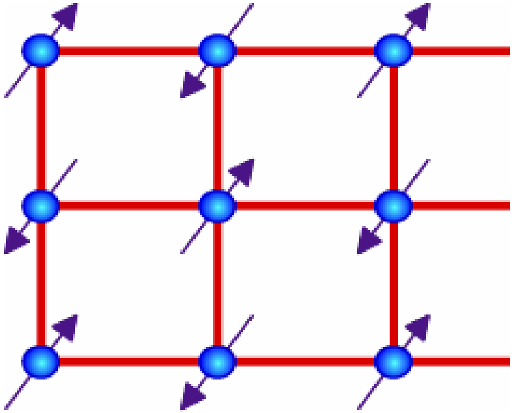


Is the U(1) FL* phase unstable to the LMM metal at the lowest energy scales ?

C. Detour: Deconfined criticality in insulating antiferromagnets

Landau forbidden quantum transitions

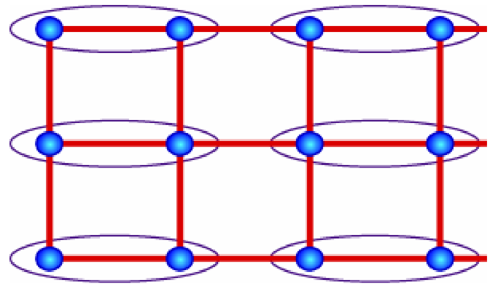
Phase diagram of S=1/2 square lattice antiferromagnet



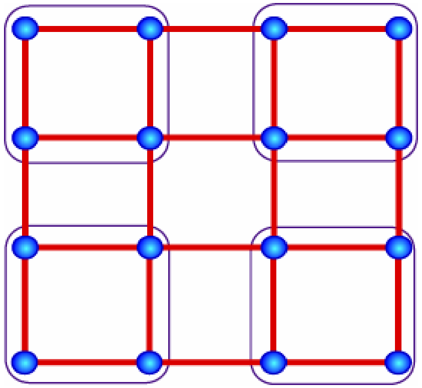
Neel order

$\vec{\varphi} \sim z_{\alpha}^{*} \vec{\sigma}_{\alpha\beta} z_{\beta} \neq 0$

(Higgs)



or



VBS order $\Psi_{\mathrm{VBS}} \neq 0$,

$S = 1/2$ spinons z_{α} confined,

$S = 1$ triplon excitations

s

Deconfined critical point described by a theory of spinons

$$\mathcal{S}_{\mathrm{critical}} = \int d^2x d\tau \left[|(\partial_\mu - i A_\mu) z_\alpha|^2 + s\,|z_\alpha|^2 + \frac{u}{2} \left(|z_\alpha|^2 \right)^2 + \frac{1}{4e^2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 \right]$$

Landau-forbidden transition between phases which break "unrelated" symmetries

Attempted theory for the destruction of Néel order

Express Néel order $\vec{\varphi}$ in terms of $S = 1/2$ bosonic spinons z_α by

$$\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta.$$

This introduces a U(1) gauge invariance under $z_\alpha \to z_\alpha e^{i\phi(x,\tau)}$.
Field theory for the z_α spinons:

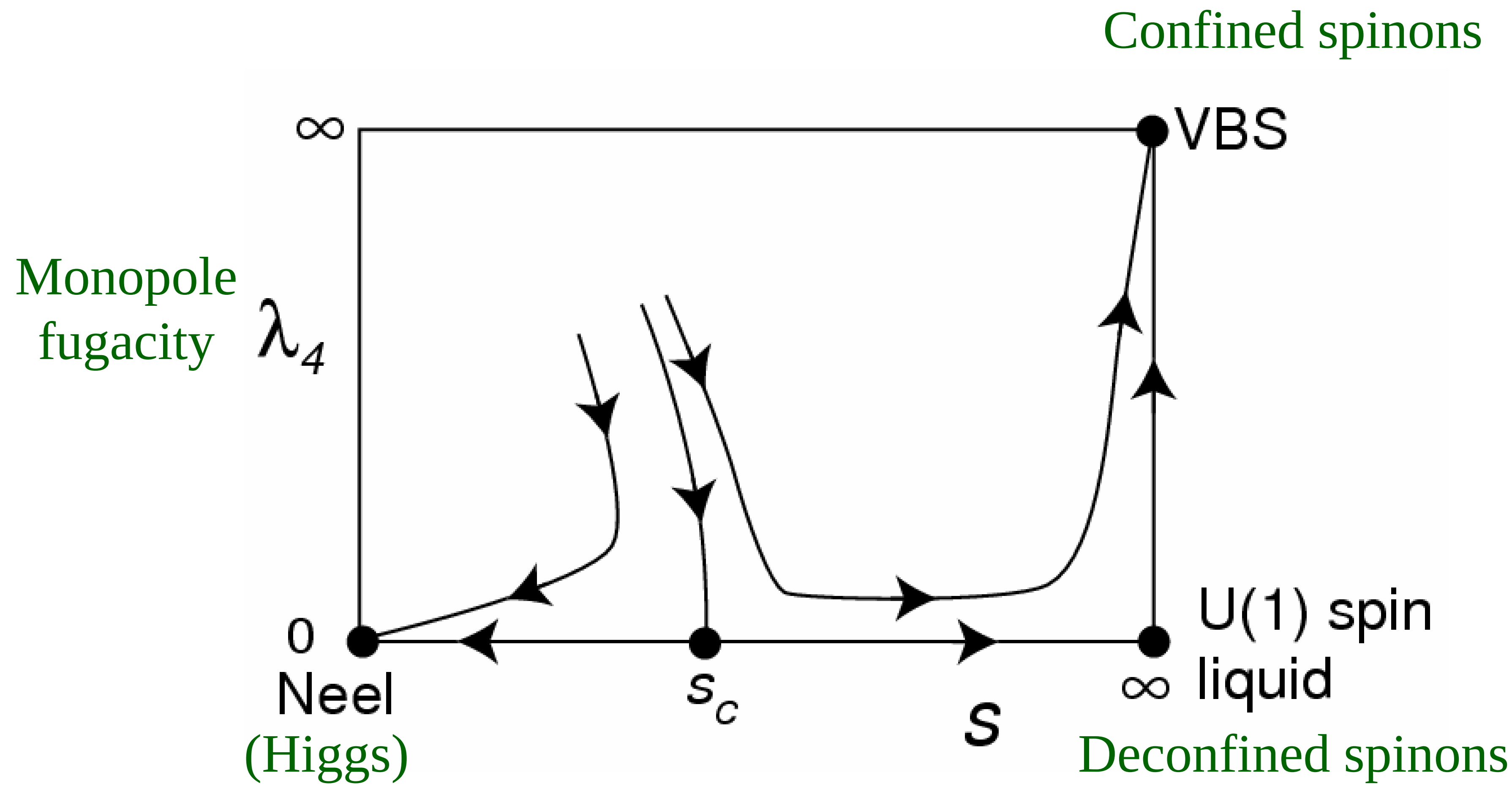
$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + s\, |z_\alpha|^2 + \frac{u}{2} \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

where A_μ is a U(1) gauge field.

Phases of theory

$s < s_c \;\Rightarrow\;$ Néel (Higgs) phase with $\langle z_\alpha \rangle \neq 0$

$s > s_c \;\Rightarrow\;$ Deconfined U(1) spin liquid with $\langle z_\alpha \rangle = 0$



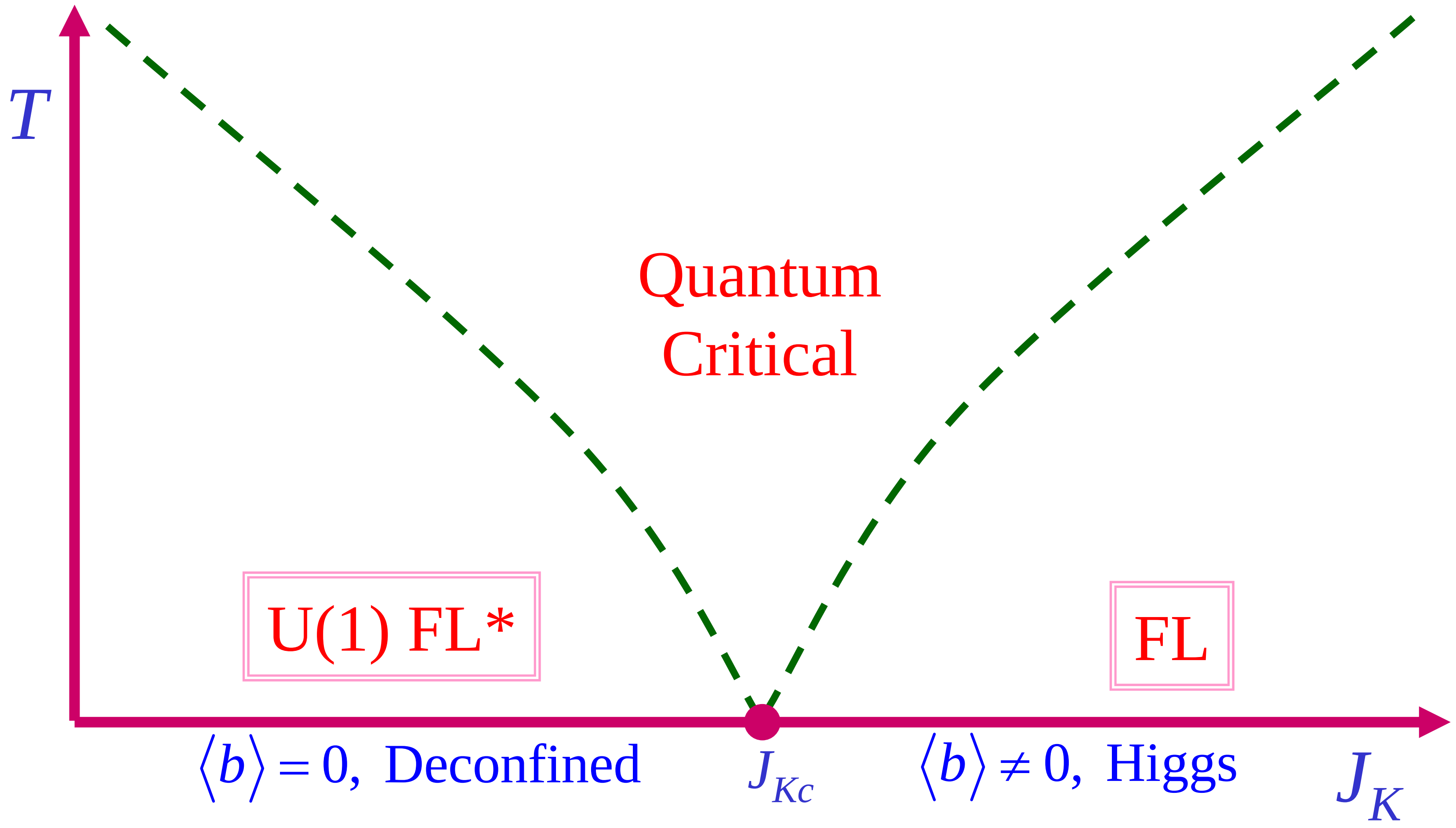
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

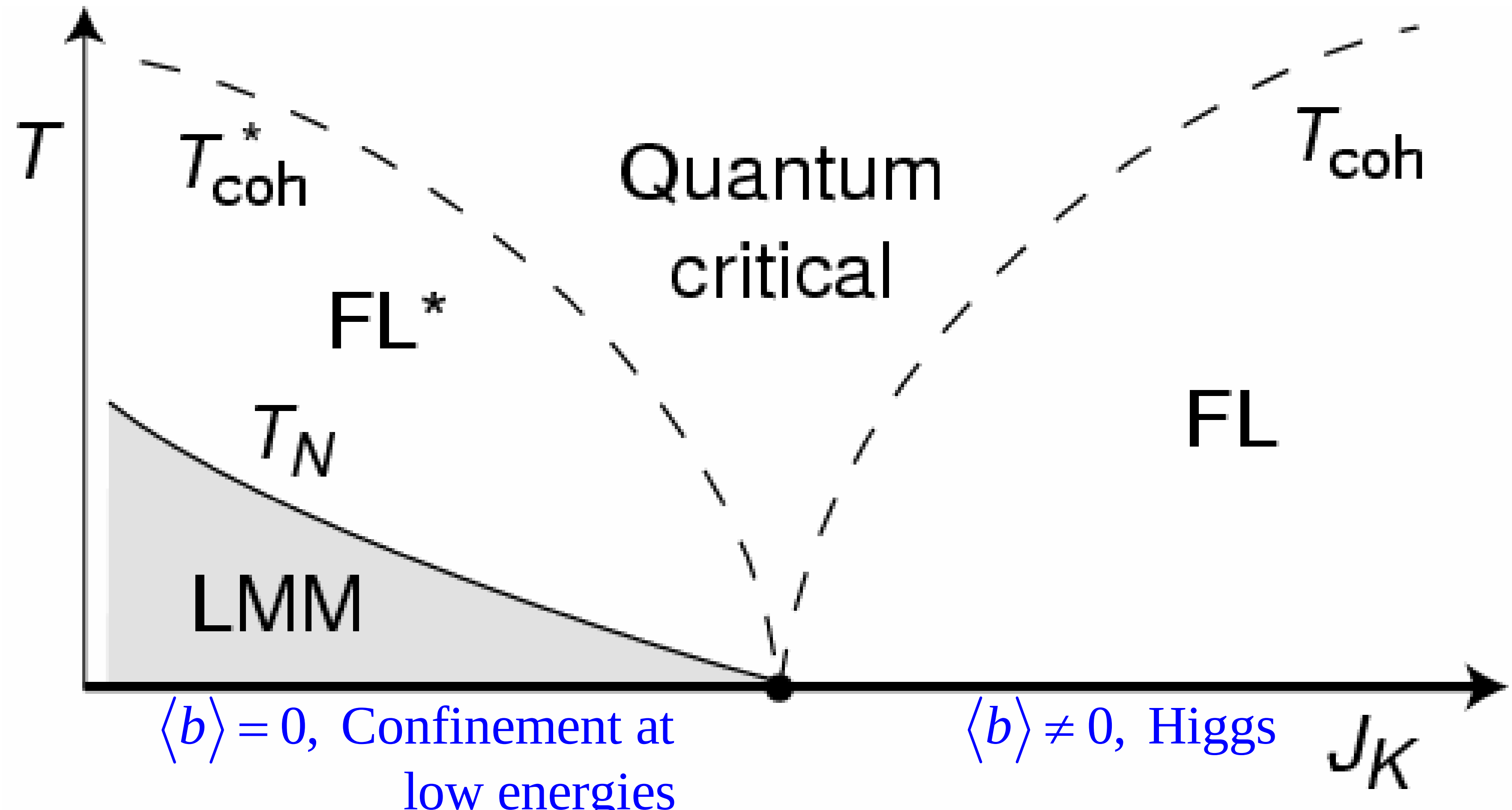
F. Deconfined criticality in the Kondo lattice ?

Phase diagram



Is the U(1) FL* phase unstable to the LMM metal at the lowest energy scales ?

Phase diagram ?



U(1) FL* phase generates magnetism at energies much lower than the critical energy of the FL to FL* transition