Universal theory of complex SYK models and extremal charged black holes

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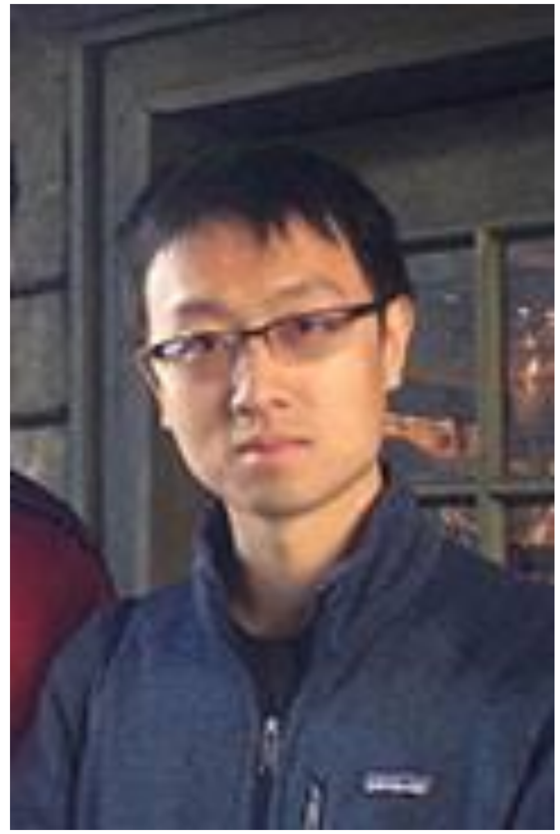
Chaos and Order: from Strongly Correlated Systems to Black Holes,
Kavli Institute for Theoretical Physics
University of California, Santa Barbara,
December 7, 2018

PHYSICS
HARVARD

Wenbo Fu



Yingfei Gu

Grigory Tarnopolsky

DEPARTMENT OF ENERGY · UNITED STATES OF AMERICA

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PHYSICS
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1. Quantum matter without quasiparticles:
the complex SYK model

2. Einstein-Maxwell theory of charged
black holes in AdS space

3. Fluctuations, and the Schwarzian

4. Supersymmetric models

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The complex SYK model

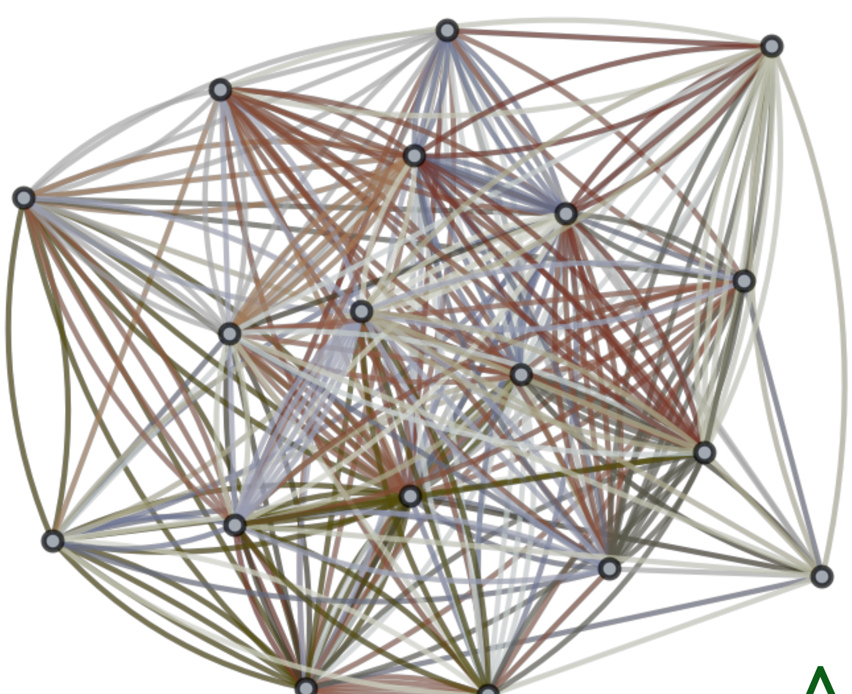
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



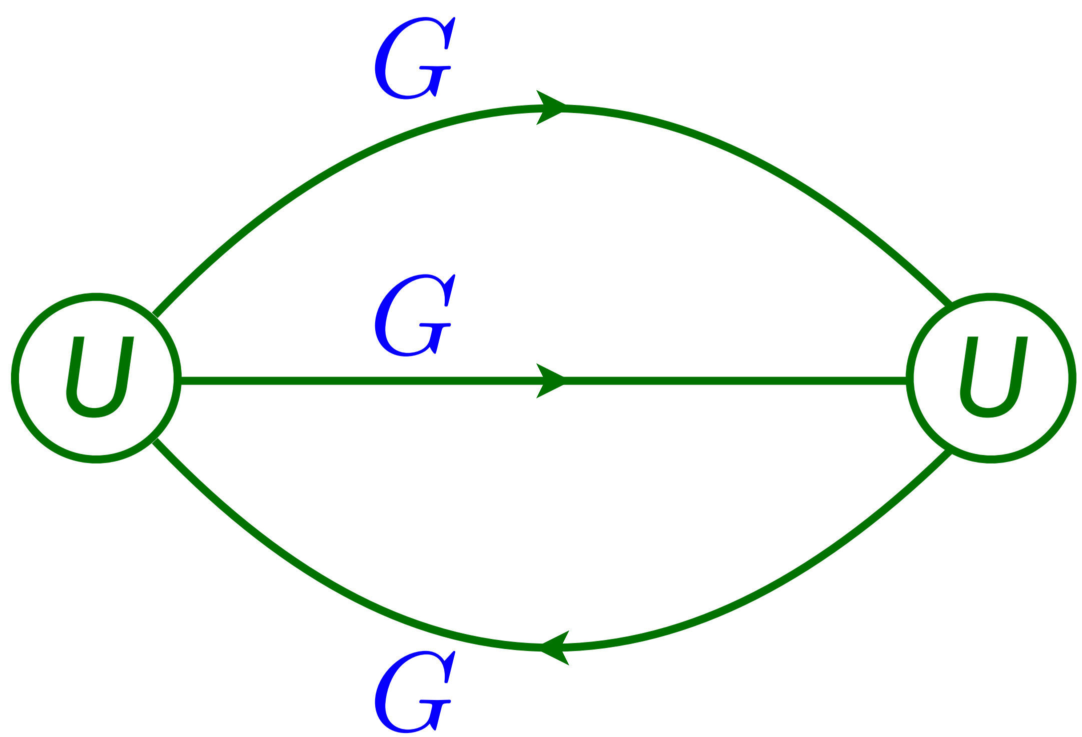
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

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Solution of these equations and corresponding evaluation of the free energy yields the following universal results (*i.e.* all results are *quantitatively* unchanged by adding additional higher q fermion terms):

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- At general charge $\mathcal{Q}$, there is a spectral symmetry determined by a parameter $\mathcal{E}$:

$$G(\tau) \sim \begin{cases} -\tau^{-2\Delta} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-2\Delta} & \tau < 0 \end{cases} \quad , \quad T = 0$$

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- There is a universal 'Luttinger relation' between $-\infty < \mathcal{E} < \infty$ and the total charge $0 < \mathcal{Q} < 1$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$$

$$\mathcal{Q} = \frac{1}{2} - \frac{\theta}{\pi} + \left(\Delta - \frac{1}{2}\right)\frac{\sin(2\theta)}{\sin(2\pi\Delta)}$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

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- There is a non-vanishing entropy in the zero temperature limit

$$S(T \to 0) = N s_0 + \ldots$$

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- The saddle point equations imply the relation

$$\boxed{\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}}$$

Integrating this relation from $s_0 = 0$, $\mathcal{Q} = 0$, allows us to compute s_0 as a function of $\mathcal{Q}$.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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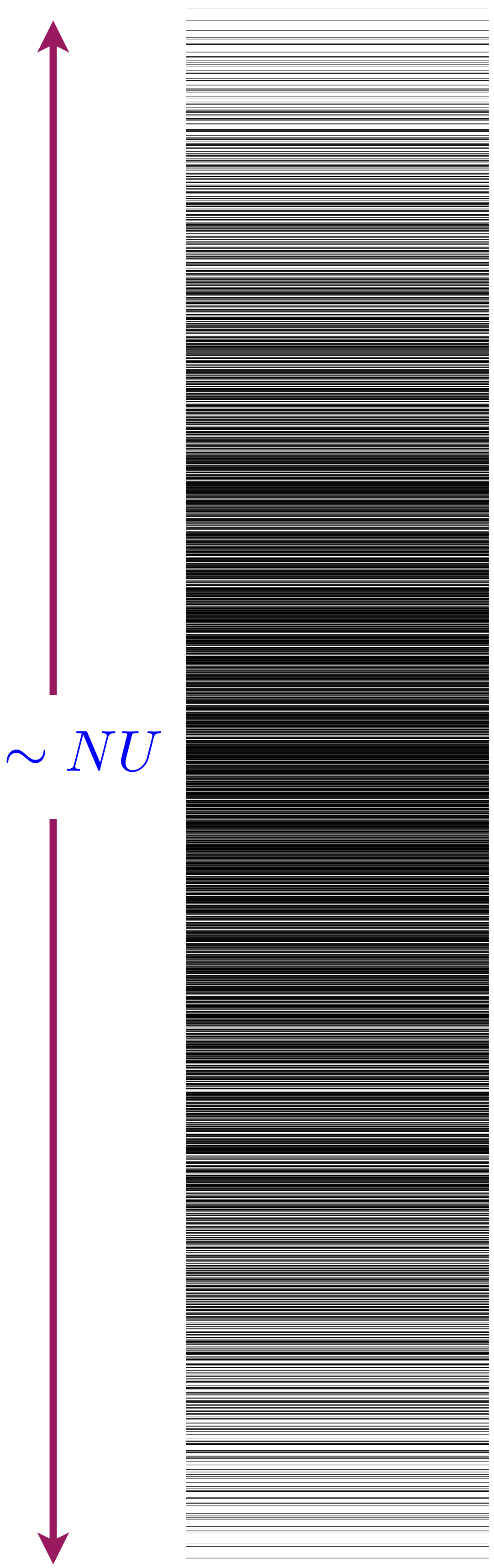

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

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At $\mathcal{Q} = 1/2$,

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where G is Catalan's constant.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

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Solution of these equations and corresponding evaluation of the free energy yields the following universal results (*i.e.* all results are *quantitatively* unchanged by adding additional higher q fermion terms):

- At $T > 0$, we obtain a solution with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi \mathcal{E} T \tau}}{\sqrt{1 + e^{-4\pi \mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \, ,$$

where the 'particle-hole asymmetry' is determined by $\mathcal{E}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

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S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
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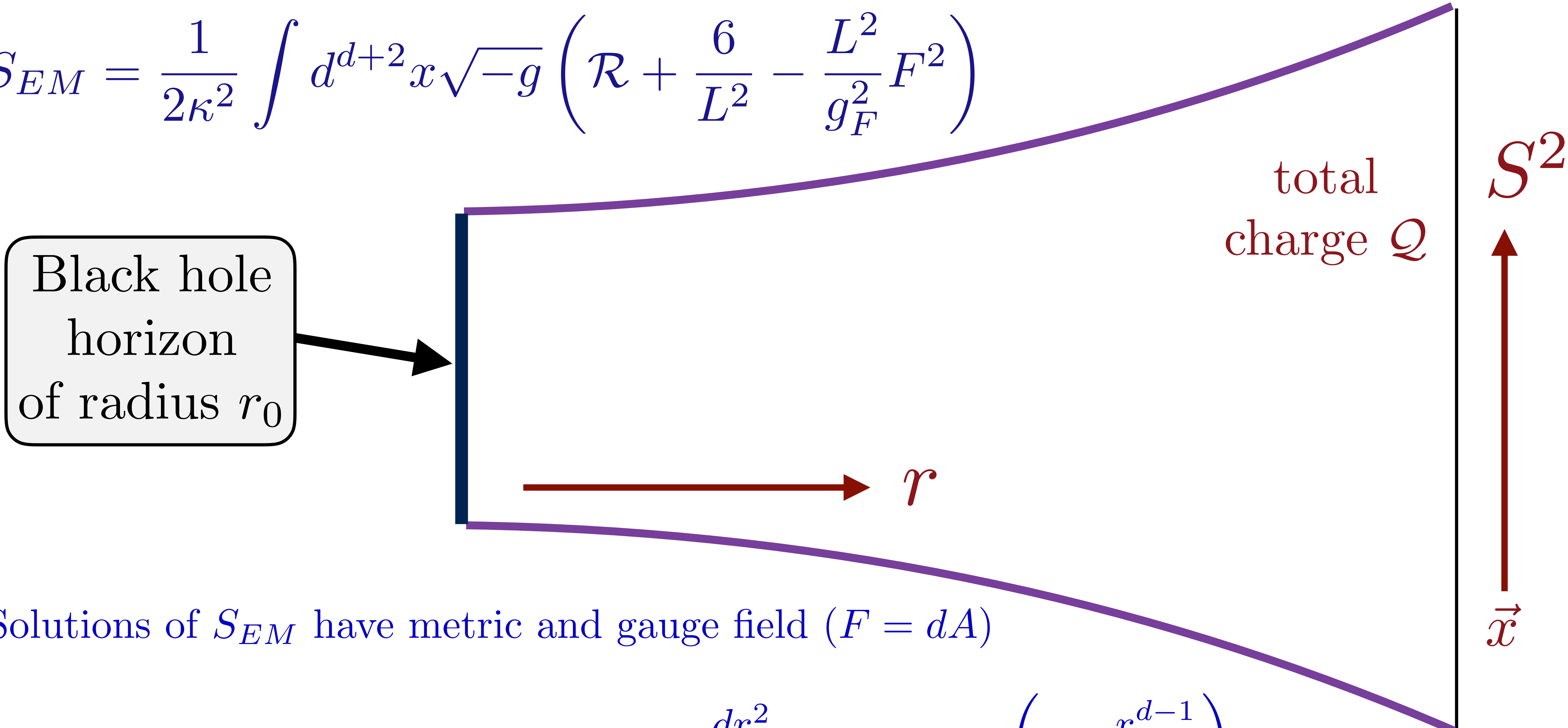
Charged black holes

$$S_{EM} = \frac{1}{2\kappa^2}\int d^{d+2}x\sqrt{-g}\left(\mathcal{R} + \frac{6}{L^2} - \frac{L^2}{g_F^2}F^2\right)$$



Solutions of S_{EM} have metric and gauge field ($F = dA$)

$$ds^2 = -V(r)dt^2 + r^2 d\Omega_d^2 + \frac{dr^2}{V(r)} \quad , \quad A = \mu\left(1 - \frac{r_0^{d-1}}{r^{d-1}}\right)dt$$

$$V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^{2d-2}} - \frac{M}{r^{d-1}}.$$

where $d\Omega_d^2$ is the metric of the d-sphere. All parameters of the solution are determined in terms of the chemical potential μ, and the Hawking temperature of horizon, T.

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

Charged black holes

In the $T \to 0$ limit, at fixed μ, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of R_h

- The total charge in the black hole is

$$\mathcal{Q} = \frac{R_h^{d-1}\sqrt{2d\left[(d+1)R_h^2 + (d-1)L^2\right]}}{\kappa^2 g_F}$$

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- The Bekenstein-Hawking entropy remains finite as $T \to 0$ (s_d is the area of the d-dimensional surface of a unit sphere)

$$S(T \to 0) = s_0 + \ldots \quad , \quad s_0 = \frac{2\pi s_d}{\kappa^2} R_h^d$$

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- In the near-horizon region, we change co-ordinates from r to ζ so that

$$r - R_h = \frac{R_2^2}{\zeta} \quad , \quad R_2 = \frac{L R_h}{\sqrt{d(d+1)R_h^2 + (d-1)^2 L^2}}.$$

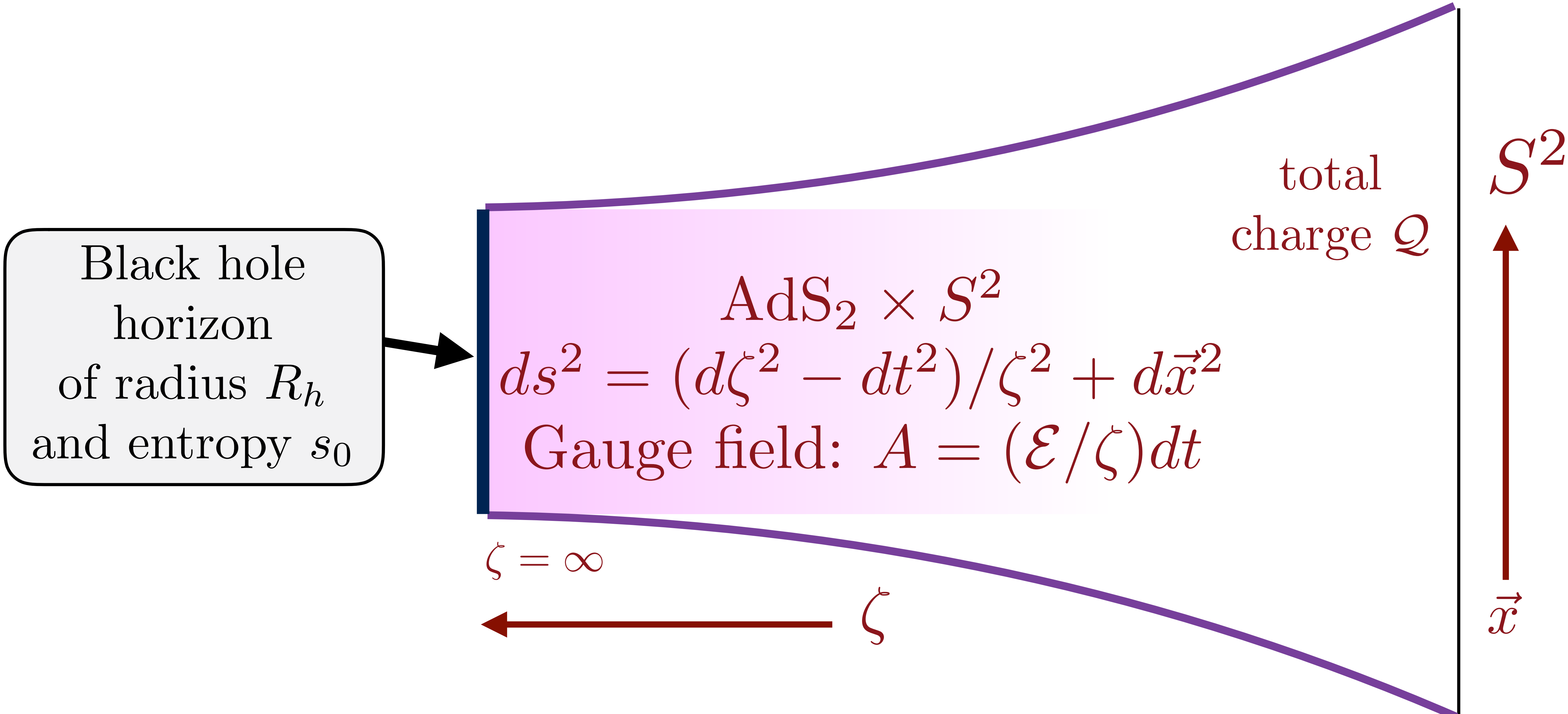
Then the near-horizon metric becomes AdS$_2\times$ S$_d$, with

$$ds^2 = R_2^2 \left[\frac{-dt^2 + d\zeta^2}{\zeta^2} \right] + R_h^2 d\Omega_d^2 \quad , \quad A = \frac{\mathcal{E}}{\zeta} dt \, .$$

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = \frac{g_F R_h \sqrt{2d \left[(d+1) R_h^2 + (d-1) L^2 \right]}}{2 \left[d(d+1) R_h^2 + (d-1)^2 L^2 \right]}.$$

Charged black holes



- The entropy s_0, the charge $\mathcal{Q}$, and the dimensionless electric field $\mathcal{E}$ obey

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen, JHEP **0509**, 038 (2005)

Charged black holes

In the $T \to 0$ limit, at fixed μ, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of R_h

- A probe fermion has a near-horizon Green's function with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi \mathcal{E} T \tau}}{\sqrt{1 + e^{-4\pi \mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \, ,$$

where the 'particle-hole asymmetry' is determined by $\mathcal{E}$. This is identical to the complex SYK model.

T. Faulkner, Hong Liu, J. McGreevy, D. Vegh, PRD **83**, 125002 (2011)

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Fluctuations

- The fluctuations can be expressed as a path integral over a Green's function $G(\tau_1, \tau_2)$ which is bilocal in time. At frequencies $\ll U$, the action is invariant under reparameterizations $f(\tau)$ and gauge transformations $\phi(\tau)$

$$\tau \to f(\sigma)$$

$$G(\tau_1, \tau_2) \to [f'(\sigma_1) f'(\sigma_2)]^{-\Delta} \, e^{-i(\phi(\sigma_1) - \phi(\sigma_2))} \, G(\sigma_1, \sigma_2)$$

A. Kitaev, 2015

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

Fluctuations

- The saddle-point

$$G(\tau_1 - \tau_2) = -A \frac{e^{-2\pi \mathcal{E} T(\tau_1 - \tau_2)}}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T(\tau_1 - \tau_2))} \right)^{2\Delta}$$

is invariant only under PSL(2, R) transformations which map the thermal circle onto itself, and an associated gauge transformation

$$\frac{\tan(\pi T f(\tau))}{\pi T} = \frac{a \dfrac{\tan(\pi T \tau)}{\pi T} + b}{c \dfrac{\tan(\pi T \tau)}{\pi T} + d} \quad , \quad ad - bc = 1,$$

$$-i\phi(\tau) = -i\phi_0 + 2\pi \mathcal{E} T(\tau - f(\tau))$$

A. Kitaev, 2015

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

Fluctuations

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f,\phi] = \frac{NK}{2}\int_0^{1/T} d\tau(\partial_\tau\phi + i(2\pi\mathcal{E}T)\partial_\tau f)^2 - \frac{N\gamma}{4\pi^2}\int_0^{1/T} d\tau\,\{\tan(\pi T f(\tau)),\tau\},$$

where $f(\tau)$ is a monotonic map from $[0,1/T]$ to $[0,1/T]$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g,\tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 .$$

Specifically, an argument constraining the effective at $T=0$ is

$$S_{\text{eff}}\left[f(\tau) = \frac{a\tau+b}{c\tau+d}, \phi(\tau)=0\right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017);
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746

Fluctuations

An *exact* path integral over the effective action leads to the following physical consequences

- The ground state energy with fermion number $N\mathcal{Q} + p$ (p integer) varies as

$$E_p = E_0 + \frac{p^2}{2NK}$$

This identifies K with the compressibility $K = d\mathcal{Q}/d\mu$ at $T = 0$.

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- The low temperature corrections to the entropy at *fixed* $\mathcal{Q}$ are

$$S(T \to 0, \mathcal{Q}) = N\Big[s_0 + \gamma T + \ldots\Big] + 2\ln(U/T)\ldots$$

  This defines γ as the co-efficient of the linear-in-T specific heat (at fixed $\mathcal{Q}$)

D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S. Josephine Suh, arXiv:1711.08467

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An *exact* path integral over the effective action leads to the following physical consequences

- The *many*-body density of states, $D(E)$, is related to the grand potential, $\Omega(T)$ by

$$Z = e^{-\Omega(T)/T} = \int_{-\infty}^{\infty} dE D(E) e^{-E/T}$$

We obtain

$$D(E) = \sum_{p=-\infty}^{\infty} e^{2\pi p \mathcal{E}} \, d\left(E - E_p\right)$$

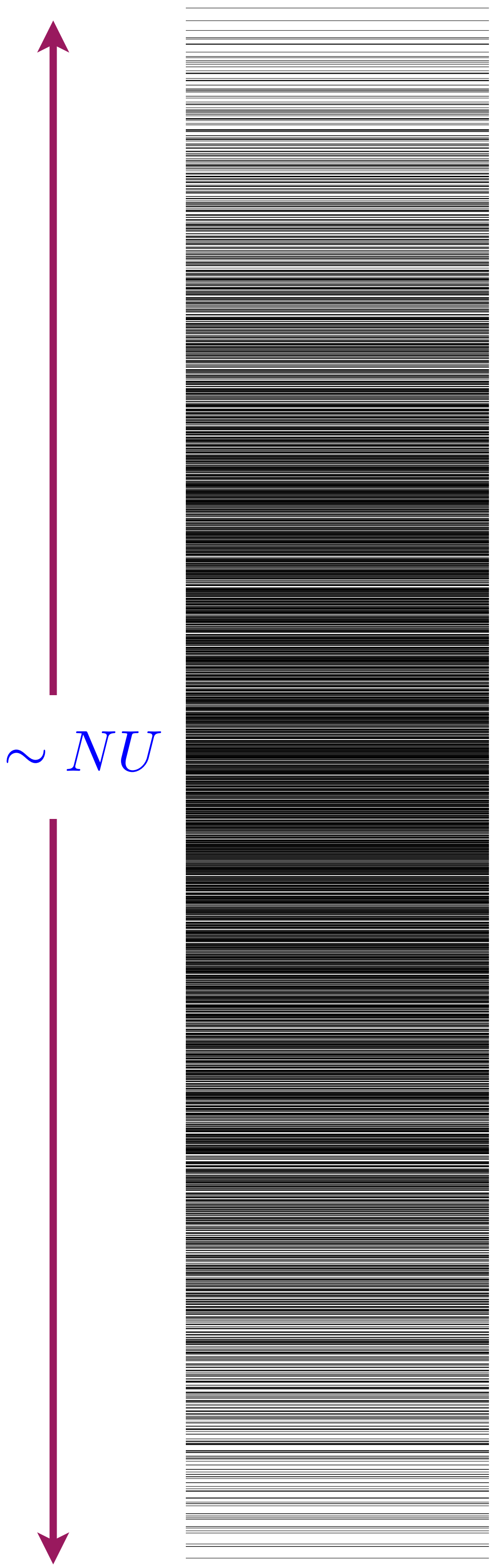
where $N\mathcal{Q} + p$ is the integer fermion number,

$$d(E) \sim \exp\left(N s_0\right) \sinh\left(\sqrt{2N\gamma E}\right) \quad , \quad E > 0 \; , \; e^{-cN} \ll \gamma E \ll N$$

There are exponentially more low energy states than for the quasiparticle case, and $D(E)$ self-averages down to energies exponentially small in N.

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, arXiv:1611.04650; A.M. Garcia-Garcia and J.J.M. Verbaarschot, arXiv:1701.06593; D. Bagrets, A. Altland, and A. Kamenev, arXiv:1702.08902; D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S.J. Suh, arXiv:1711.08467

The complex SYK model



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D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S.J. Suh, arXiv:1711.08467

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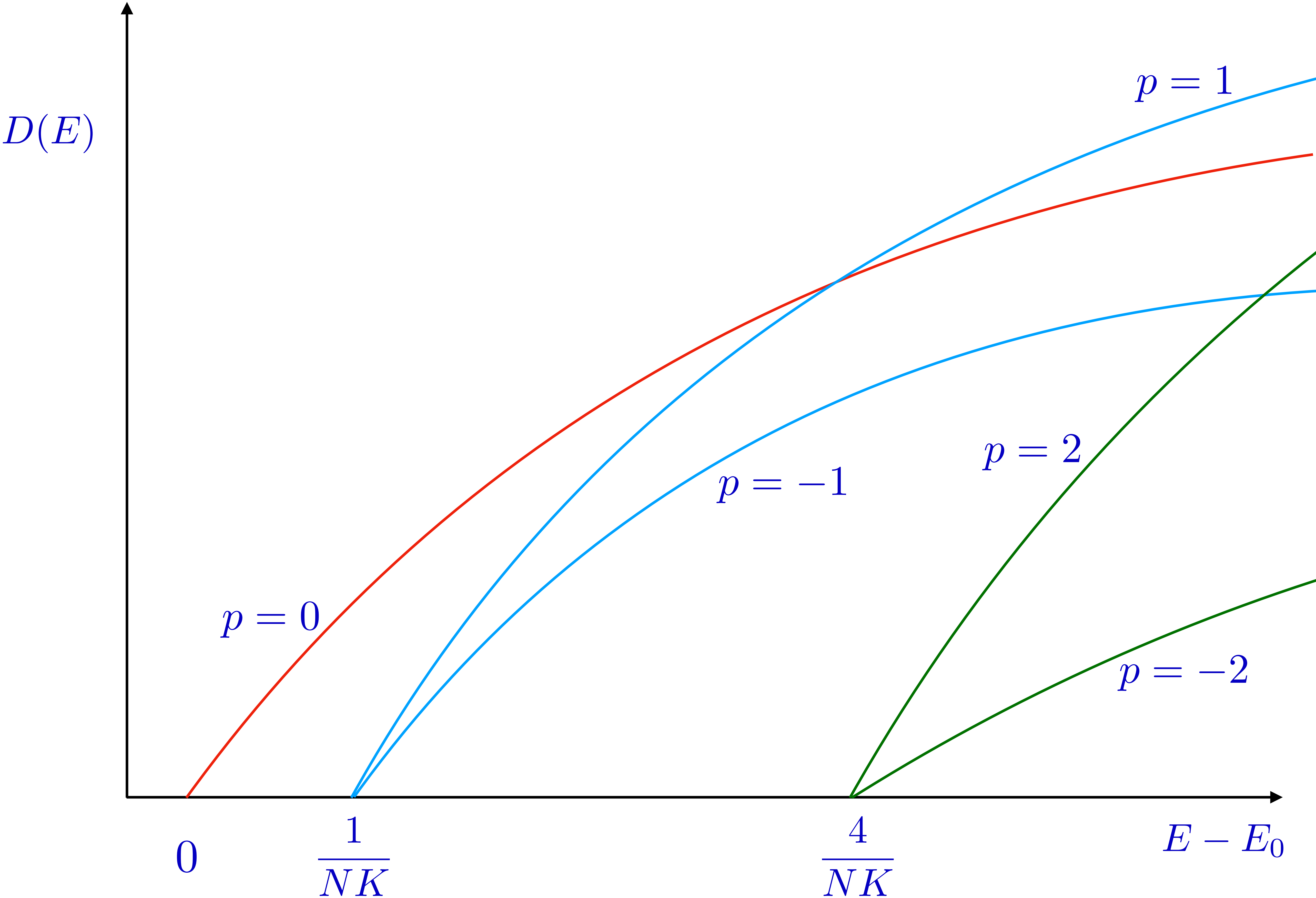
- At charge $N\mathcal{Q} + p$, the prefactor of the $\sinh(\sqrt{2N\gamma(E - E_p)})$ term is

$$\exp\left[N s_0(\mathcal{Q}) + 2\pi p \mathcal{E}\right] \approx \exp\left[N s_0(\mathcal{Q} + p/N)\right]$$

using

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}$$



$D(E)$
$p=1$
$p=-1$
$p=2$
$p=0$
$p=-2$
0
$\frac{1}{NK}$
$\frac{4}{NK}$
$E-E_0$

The Schwarzian theory and black holes

- Reparameterization invariance is a defining property of Einstein's theory of gravity

- In imaginary time, AdS_2 is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1.$$

A. Kitaev, 2015

J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

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Semiclassical fluctuations about the saddle-point of Einstein-Maxwell theory of a charged black holes in $d \geq 2$ spatial dimensions lead to the same Schwarzian+phase theory of fluctuations.

P. Nayak, A. Shukla, R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547
U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239
P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746

The Schwarzian theory and black holes

- The Einstein-Maxwell theory leads to the following parameters for the Schwarzian+phase theory

$$K = \left.\frac{d\mathcal{Q}}{d\mu}\right|_{T=0} = \frac{2(d-1)L^2 s_d R_h^{d-3}\left[d(d+1)R_h^2 + (d-1)^2 L^2\right]}{(d+1)g_F^2\kappa^2}$$

$$S(T \to 0, \mathcal{Q}) = s_0 + \gamma T + \dots$$

$$\gamma = \frac{4\pi^2 d s_d L^2 R_h^{d+1}}{\kappa^2(d(d+1)R_h^2 + (d-1)^2 L^2)}\,.$$

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SYK with $\mathcal{N} = 2$ SUSY

$$Q = i \sum_{1 \leq i < j < k \leq N} C_{ijk} c_i c_j c_k$$

$$H = QQ^\dagger + Q^\dagger Q = \sum_{i,j,k,\ell}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

C_{ijk} are independent random variables with $\overline{C_{ijk}} = 0$ and $\overline{|C_{ijk}|^2} = 2U/N^2$.

Wenbo Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

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There is a $\text{U}(1)_R$ charge

$$\mathcal{Q}_R = \frac{1}{3} \sum_i \left(c_i^\dagger c_i - \frac{1}{2} \right)$$

The smallest absolute values of are $\mathcal{Q}_R = 0$ (for N even) and $\mathcal{Q}_R = \pm 1/6$ (for N odd) and we focus on these cases: $\mathcal{N} = 2$ SUSY is preserved by the ground states. Cases with extensive $\mathcal{Q}_R$ charge, $\lim_{N \to \infty} \mathcal{Q}_R / N \neq 0$, break SUSY and require a more complex analysis.

Wenbo Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

SYK with $\mathcal{N} = 2$ SUSY

- There is an *exact*, exponentially large degeneracy of zero energy states for any C_{ijk}. Consequently, $e^{Ns_0} \equiv g(N, \mathcal{Q}_R)$ is an integer for all N (there was no such requirement for the non-SUSY case). The exact non-zero values for small $\mathcal{Q}_R$ are

$$g(N,0) = 2 \times 3^{N/2-1} \quad , \quad g(N, \pm 1/3) = 3^{N/2-1}\,, \quad \text{for } N \text{ even}$$
$$g(N, \pm 1/6) = 3^{(N-1)/2}\,, \quad \text{for } N = 3 \bmod 4$$
$$g(N, \pm 1/6) = 3^{N/2-1} \quad , \quad g(N, \pm 1/2) = 1 \text{ or } 3\,, \quad \text{for } N = 1 \bmod 4$$

There are no zero energy states for other values of $\mathcal{Q}_R$. So in the large N limit, the entropy $s_0 = (1/2)\ln 3$ for $\lim_{N \to \infty} \mathcal{Q}_R/N = 0$.

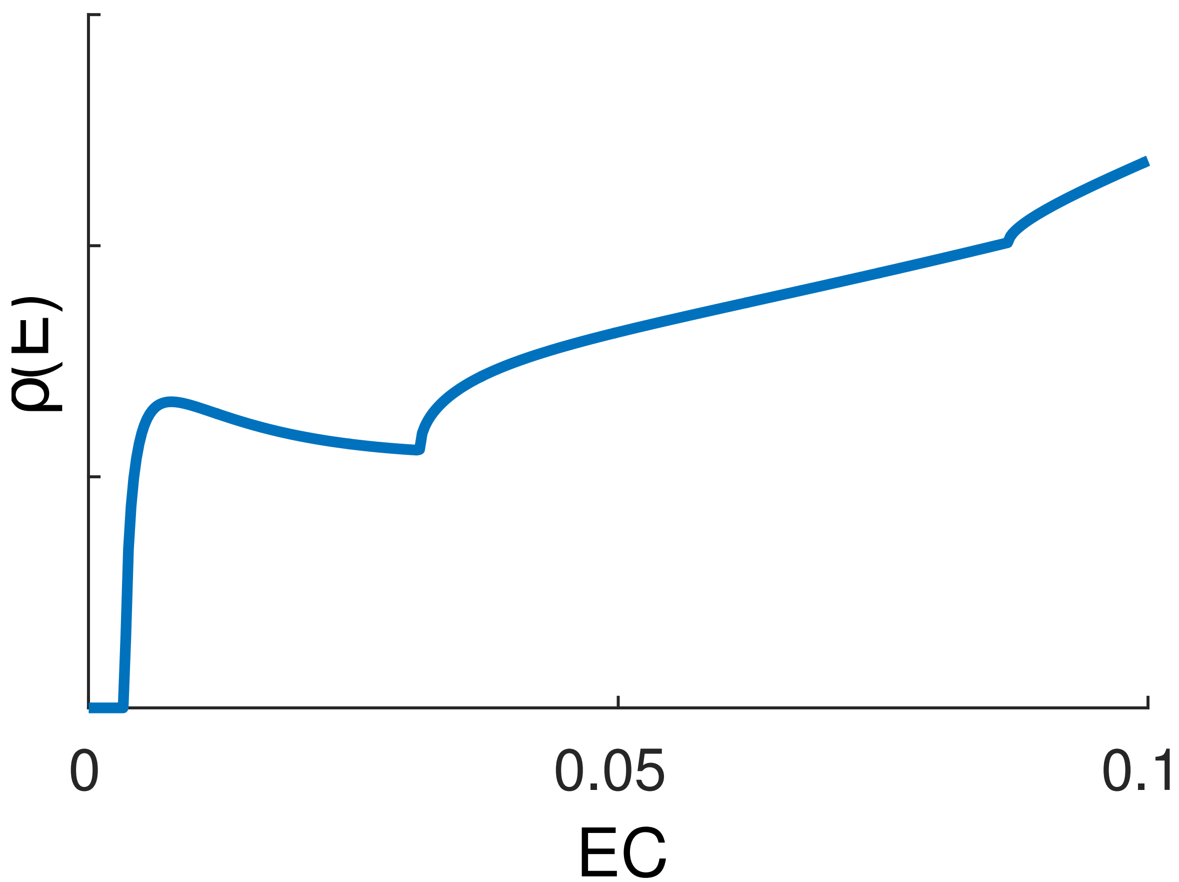
Wenbo Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

SYK with $\mathcal{N} = 2$ SUSY

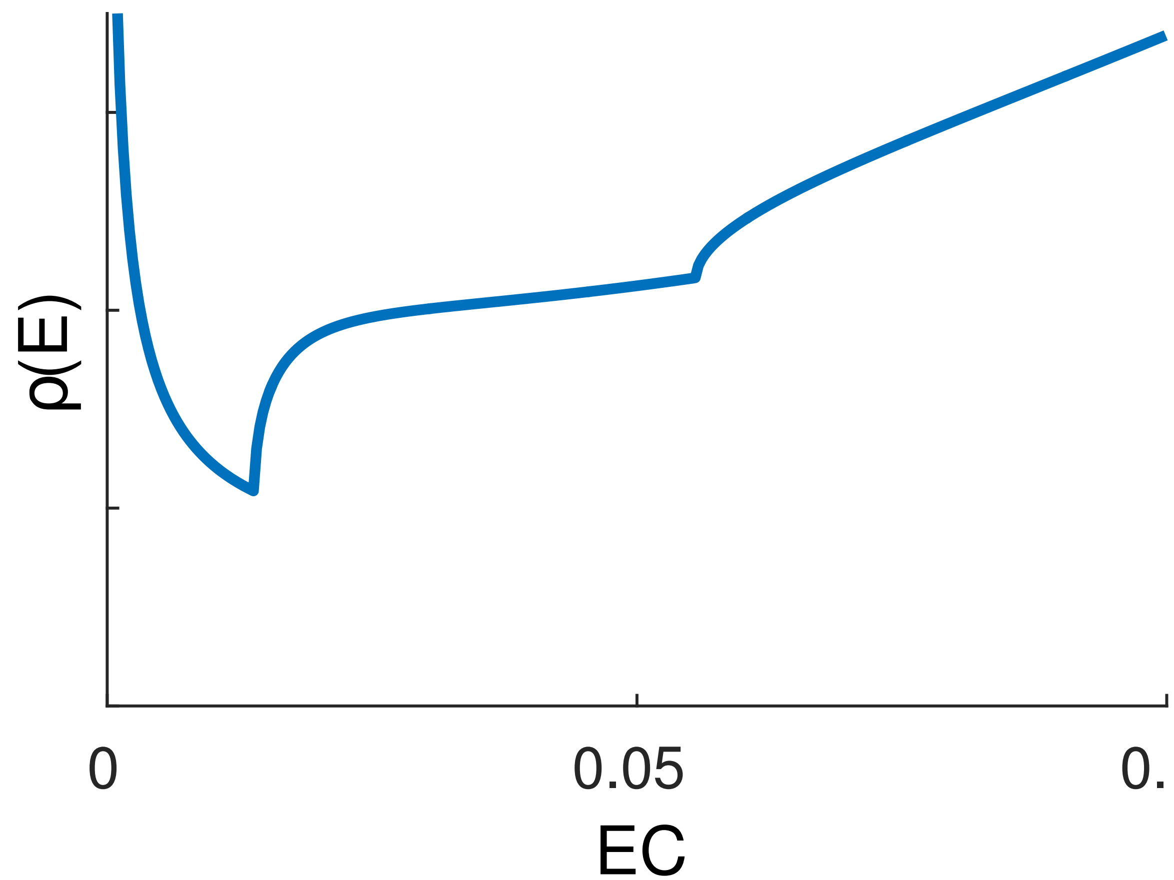
- Fluctuations are described by a super-Schwarzian theory for $\lim_{N\to\infty} \mathcal{Q}_R/N = 0$. The density of states has delta function at zero energy, and a continuum contribution at non-zero energies, both of order $e^{s_0 N}$. There is a gap between the ground states and the excited states $\sim 1/N$ for even N.

D. Stanford and E. Witten, arXiv:1703.04612

$\mathcal{N} = 2$ (N even)



$\mathcal{N} = 2$ (N odd)



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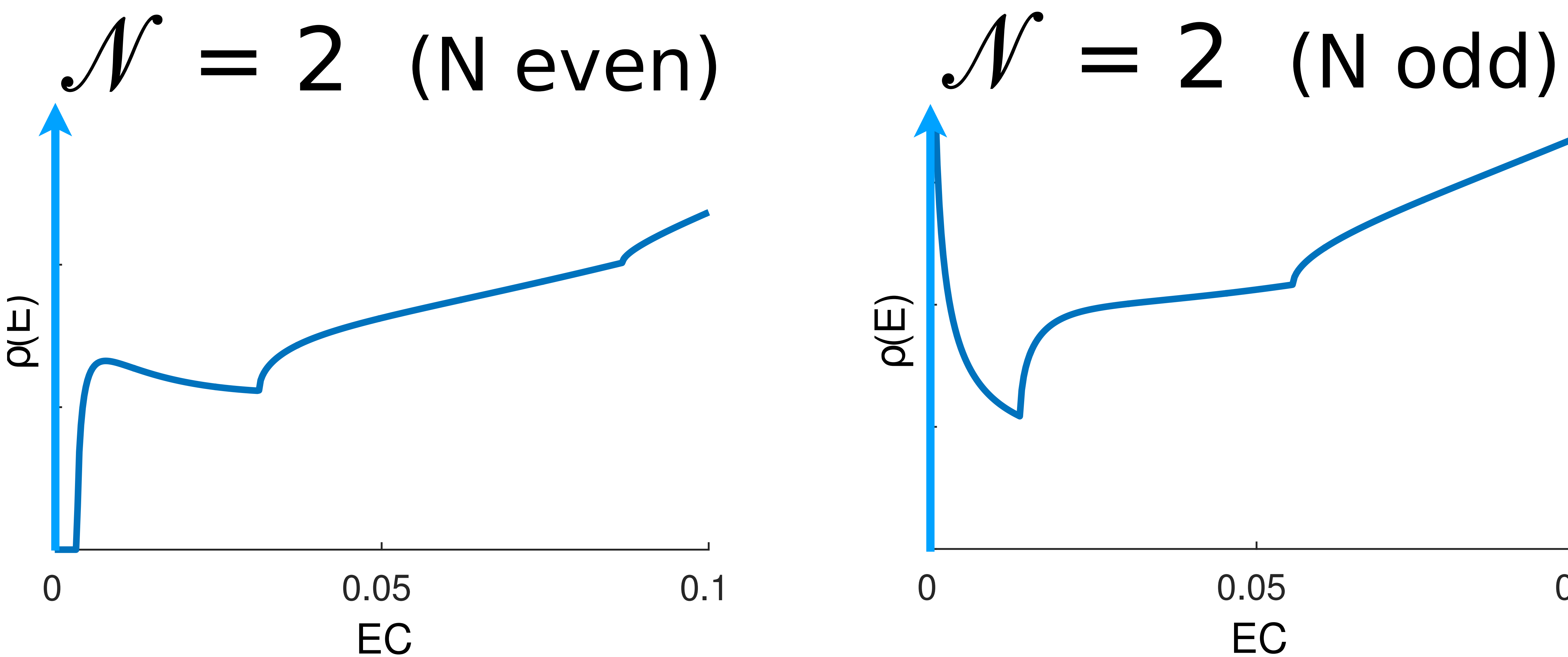

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- For $\lim_{N\to\infty} \mathcal{Q}_R/N \neq 0$, we expect that the density of states of the generic complex SYK model will apply.

Wenbo Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

Quantum matter without quasiparticles

- Planckian dynamics is realized in the 'solvable' SYK models
- Black holes thermalize in a time $\sim \hbar/(k_B T_H)$, where T_H is the Hawking temperature.
- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, describes the quantum dynamics of
  - the SYK models
  - black holes with near-extremal AdS_2 horizons