

A simple model of many-particle entanglement: how it describes black holes and superconductors



KAIST, Korea
February 17, 2021

Subir Sachdev



PHYSICS



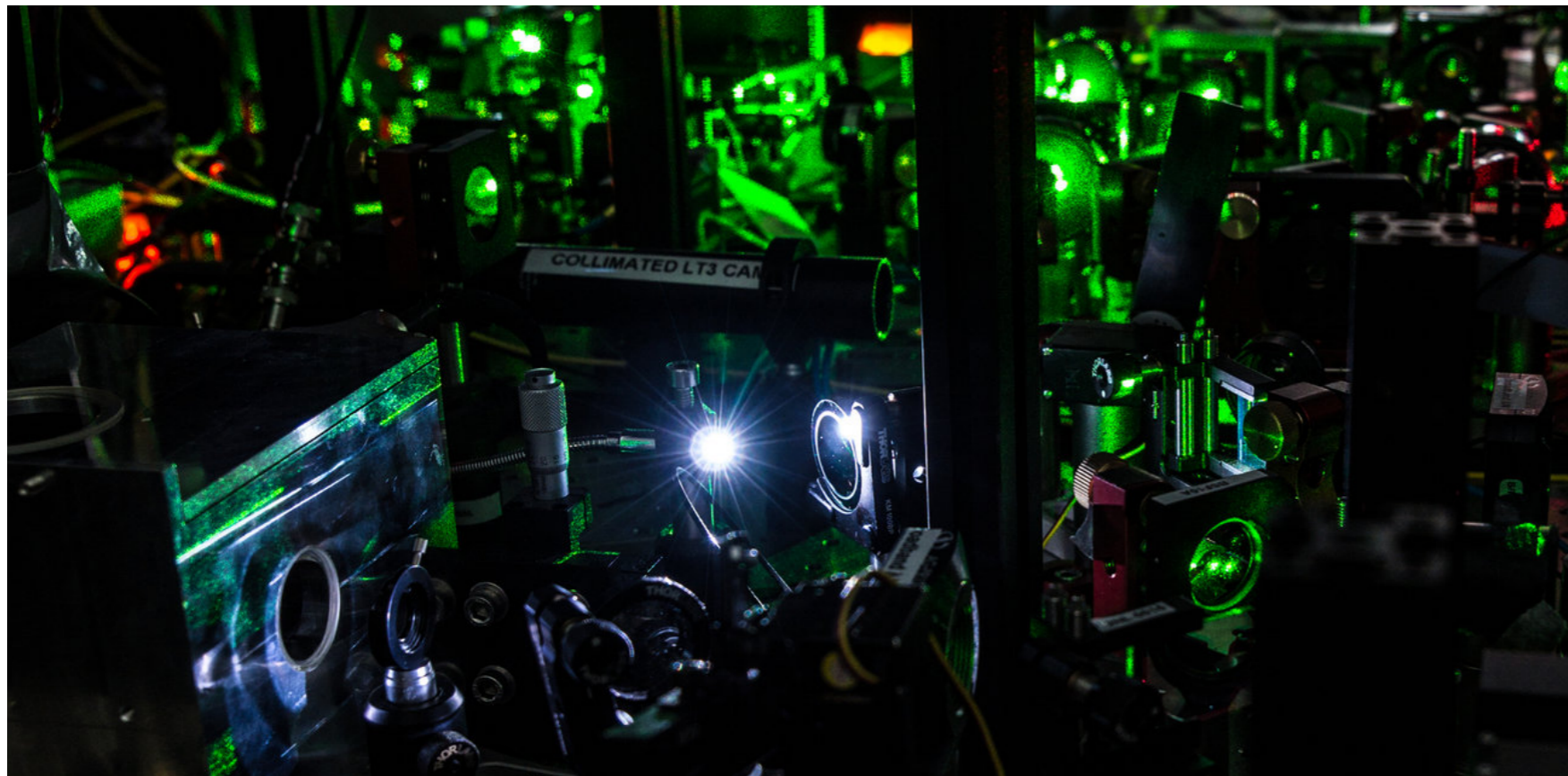
HARVARD

Talk online: sachdev.physics.harvard.edu

Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

By **JOHN MARKOFF** OCT. 21, 2015

In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.



Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

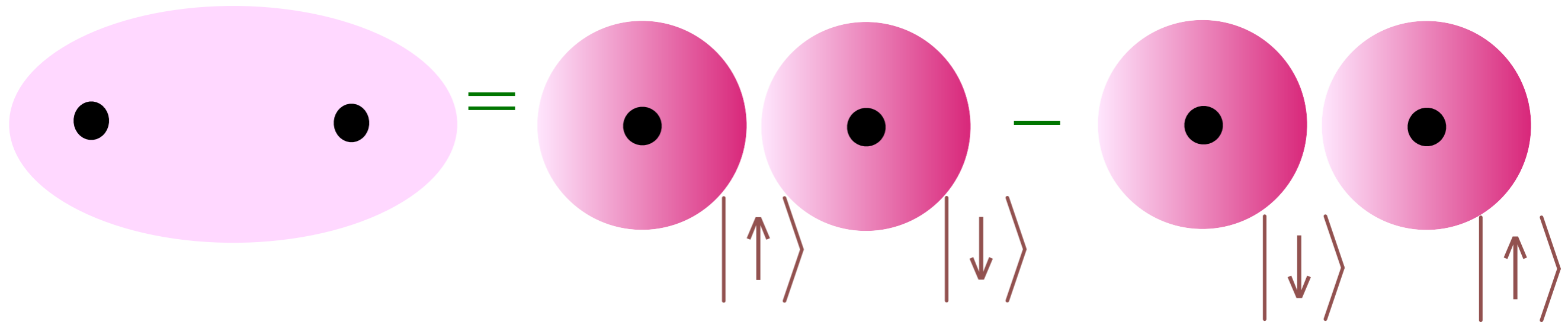
Quantum entanglement

Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle



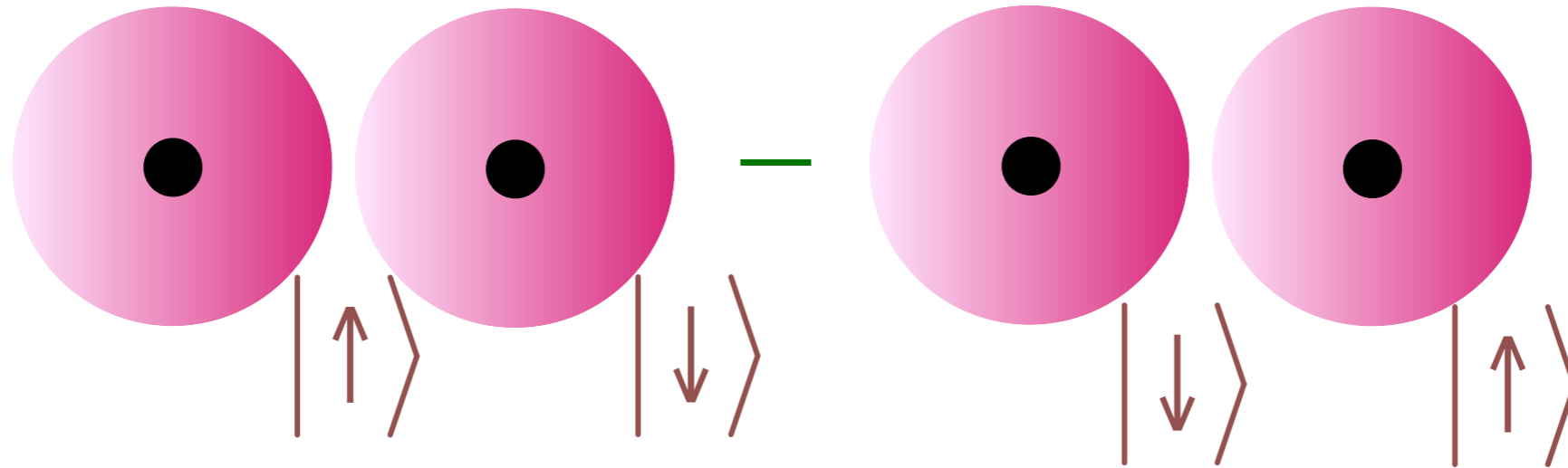
Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

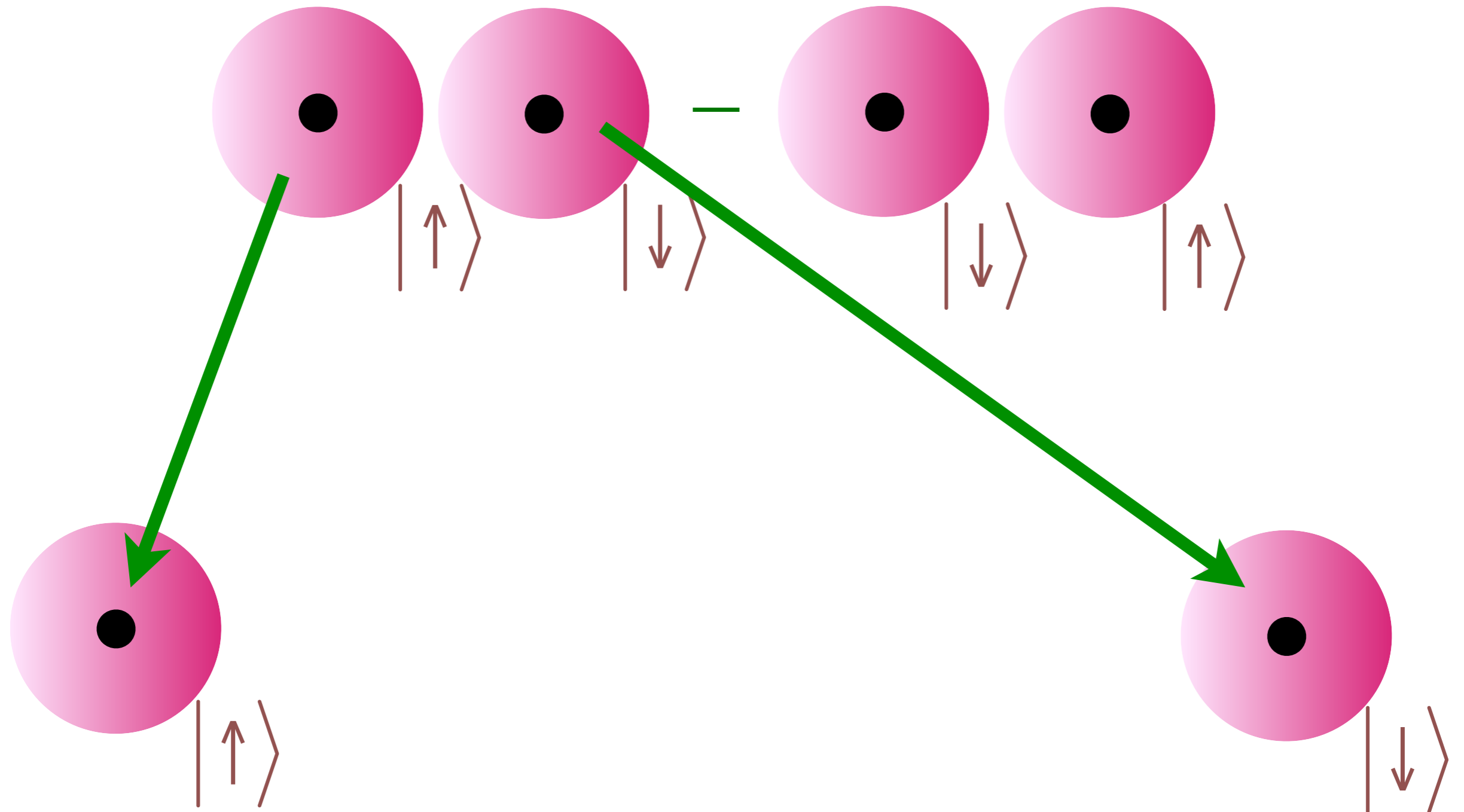
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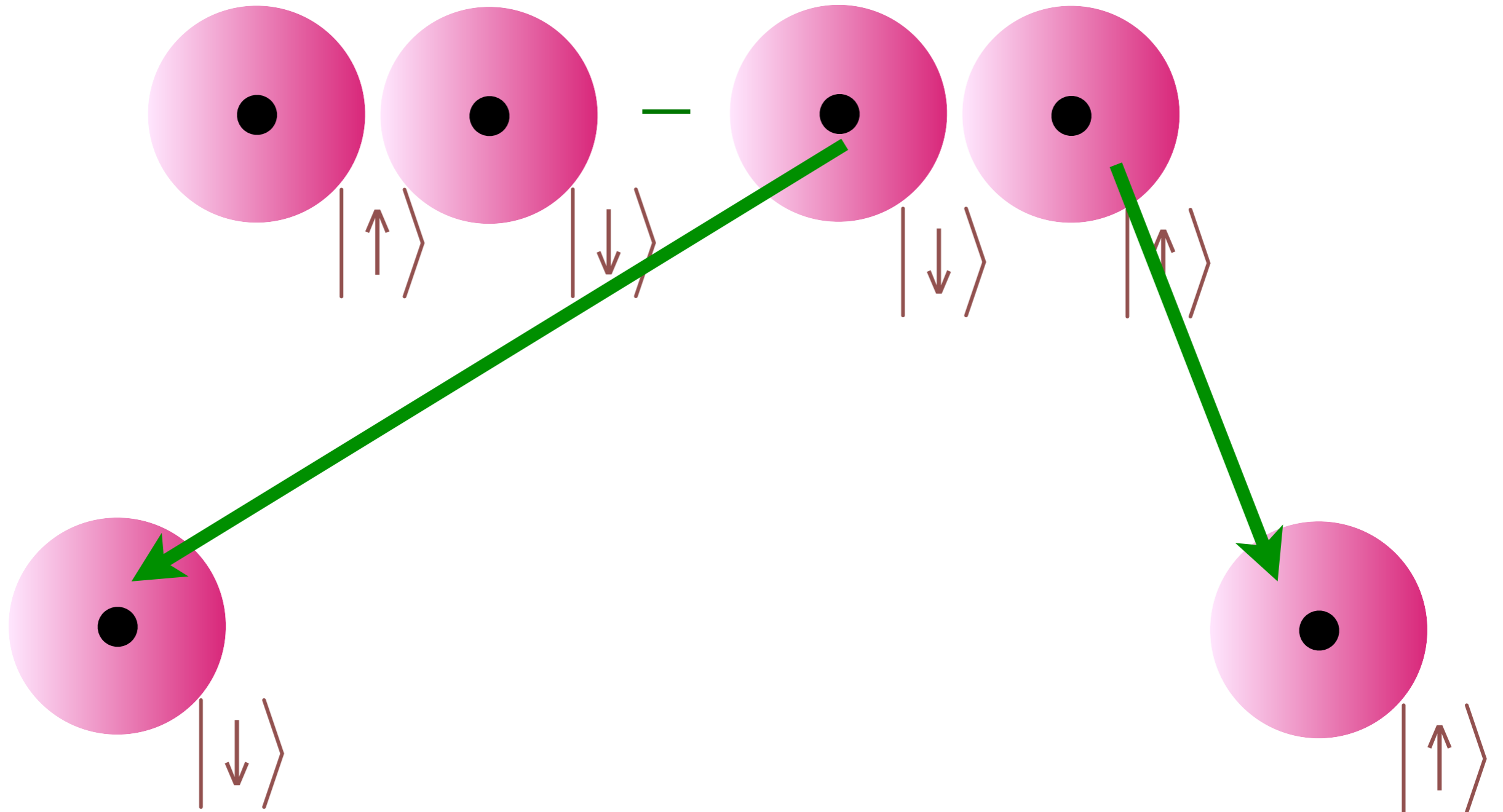
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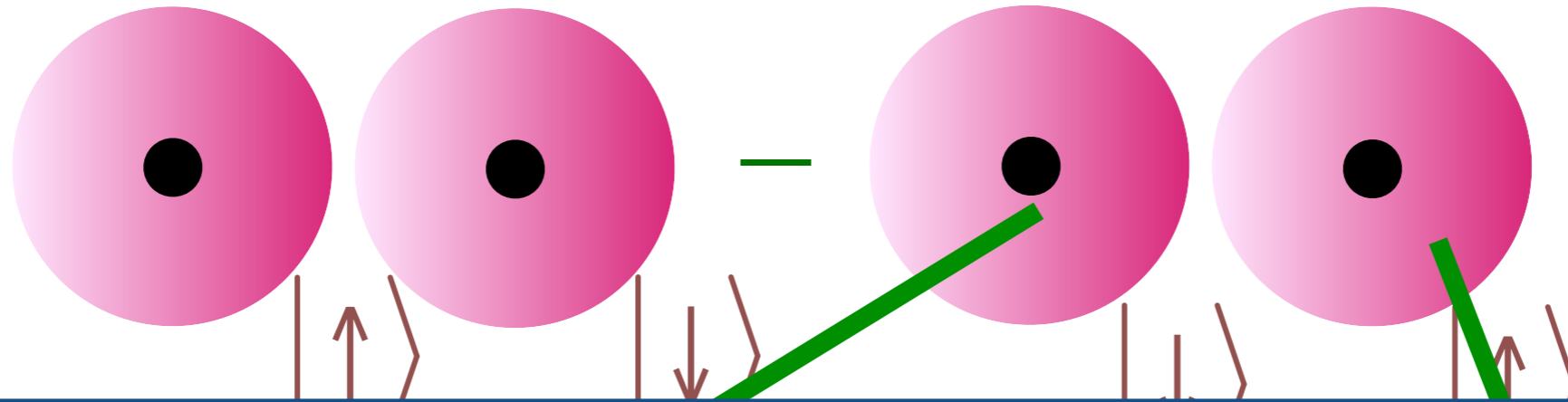
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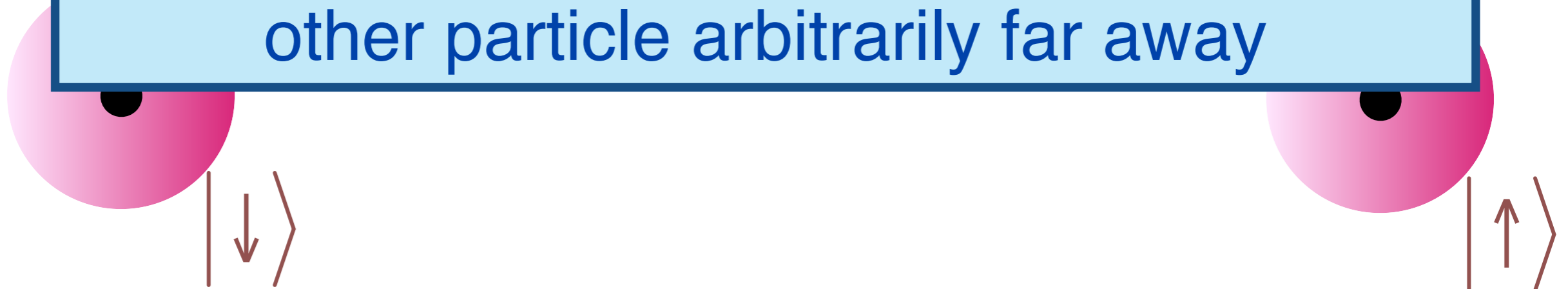


Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle



Einstein-Podolsky-Rosen “paradox” (1935):
Measurement of one particle
instantaneously determines the state of the
other particle arbitrarily far away



Quantum entanglement

Quantum entanglement

A simple
many-particle
(SYK) model

Ordinary metals



Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

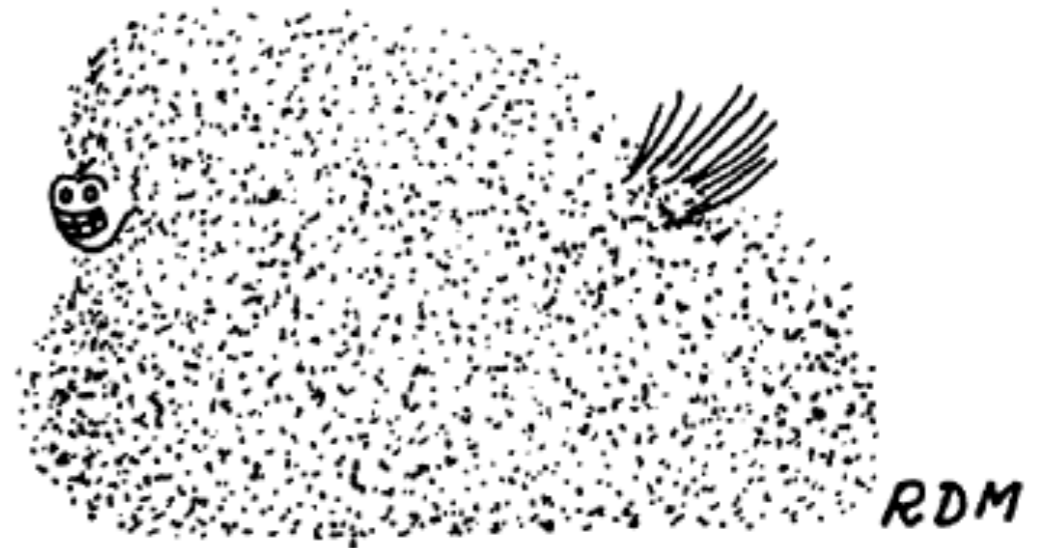
Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle. The existence of quasiparticles implies limited many-particle entanglement

← ●
real particle

← ○
quasi particle



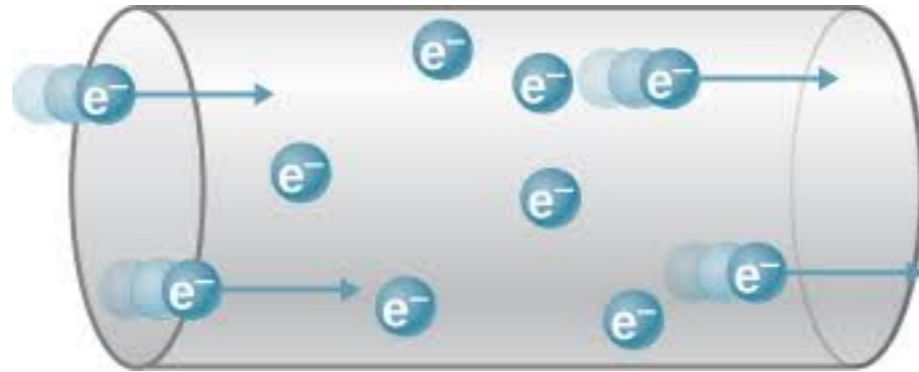
real horse



quasi horse

R.D. Mattuck

Current flow with quasiparticles

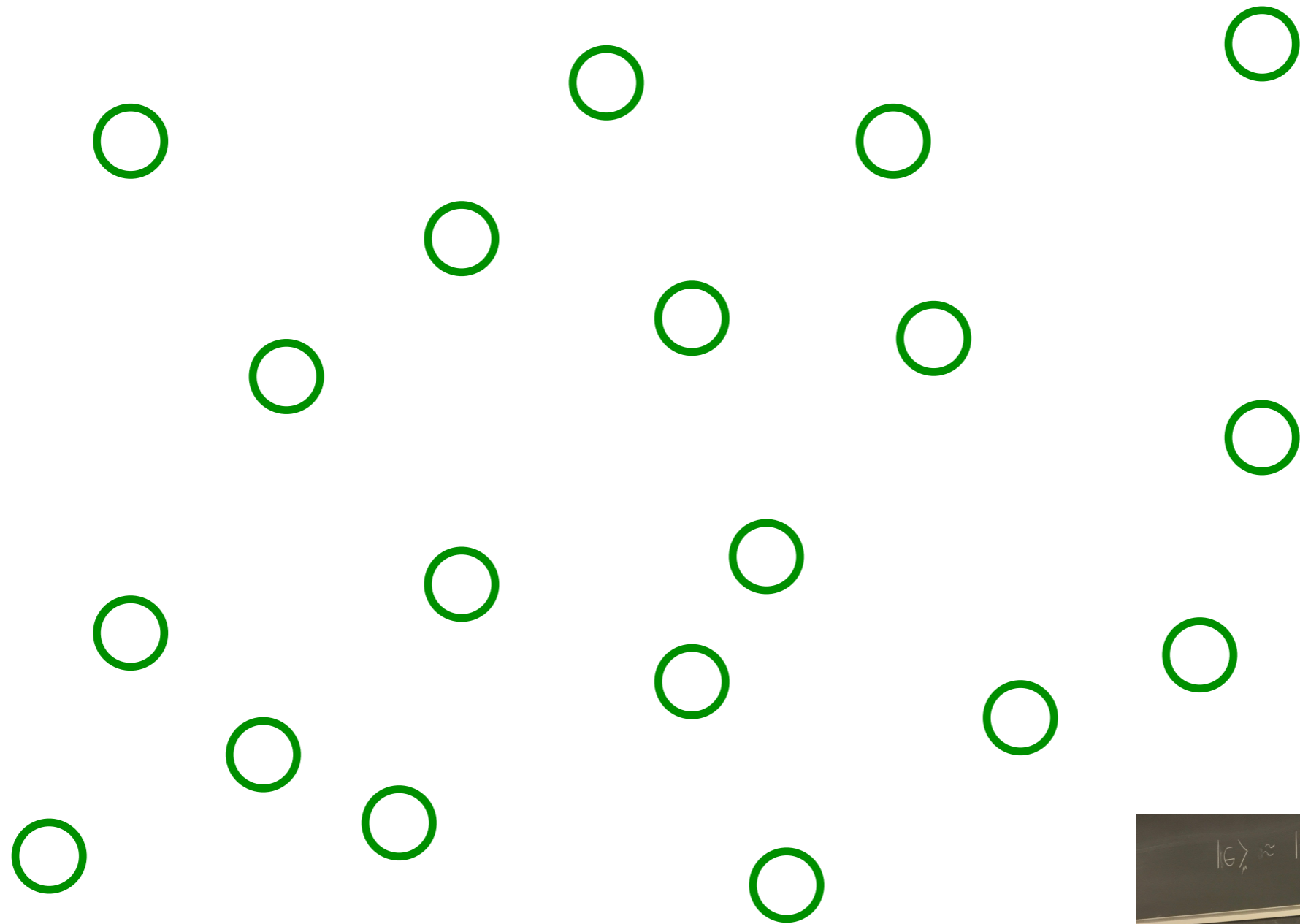


Flowing quasiparticles scatter off each other in a typical scattering time τ

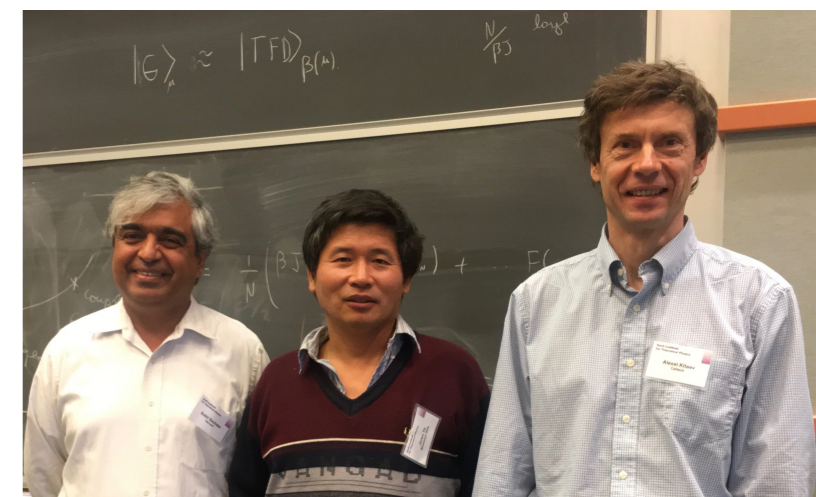
This time is much longer than a limiting
'Planckian time' $\frac{\hbar}{k_B T}$.

The long scattering time implies that quasiparticles are well-defined.

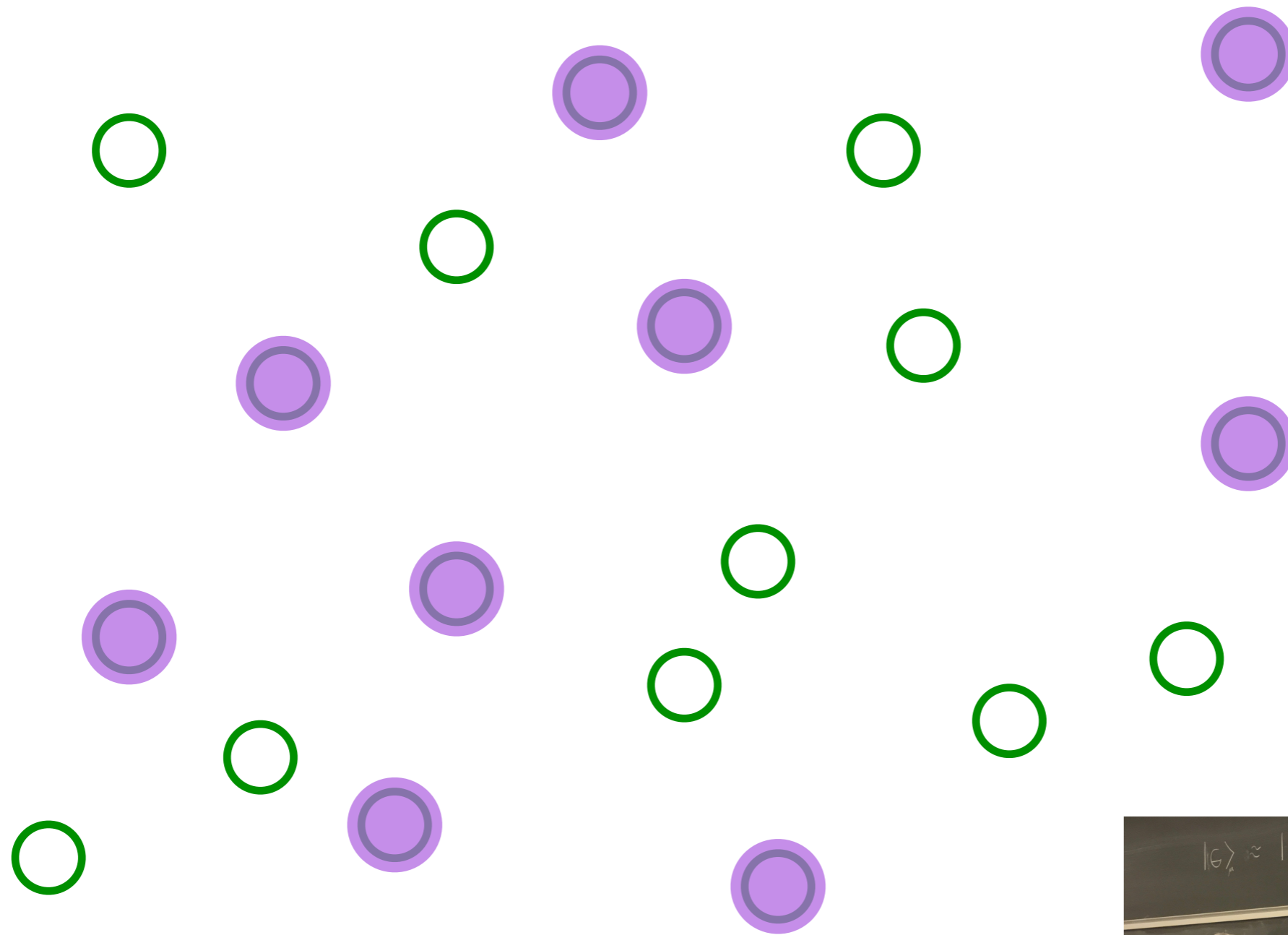
The Sachdev-Ye-Kitaev (SYK) model



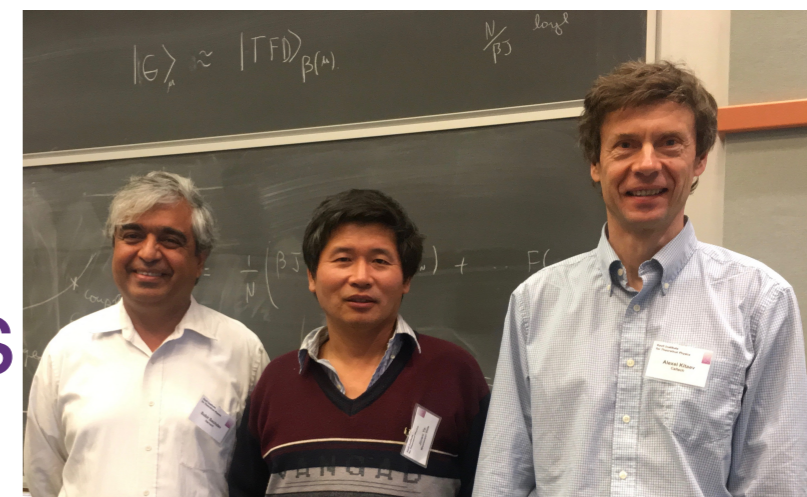
Pick a set of random positions



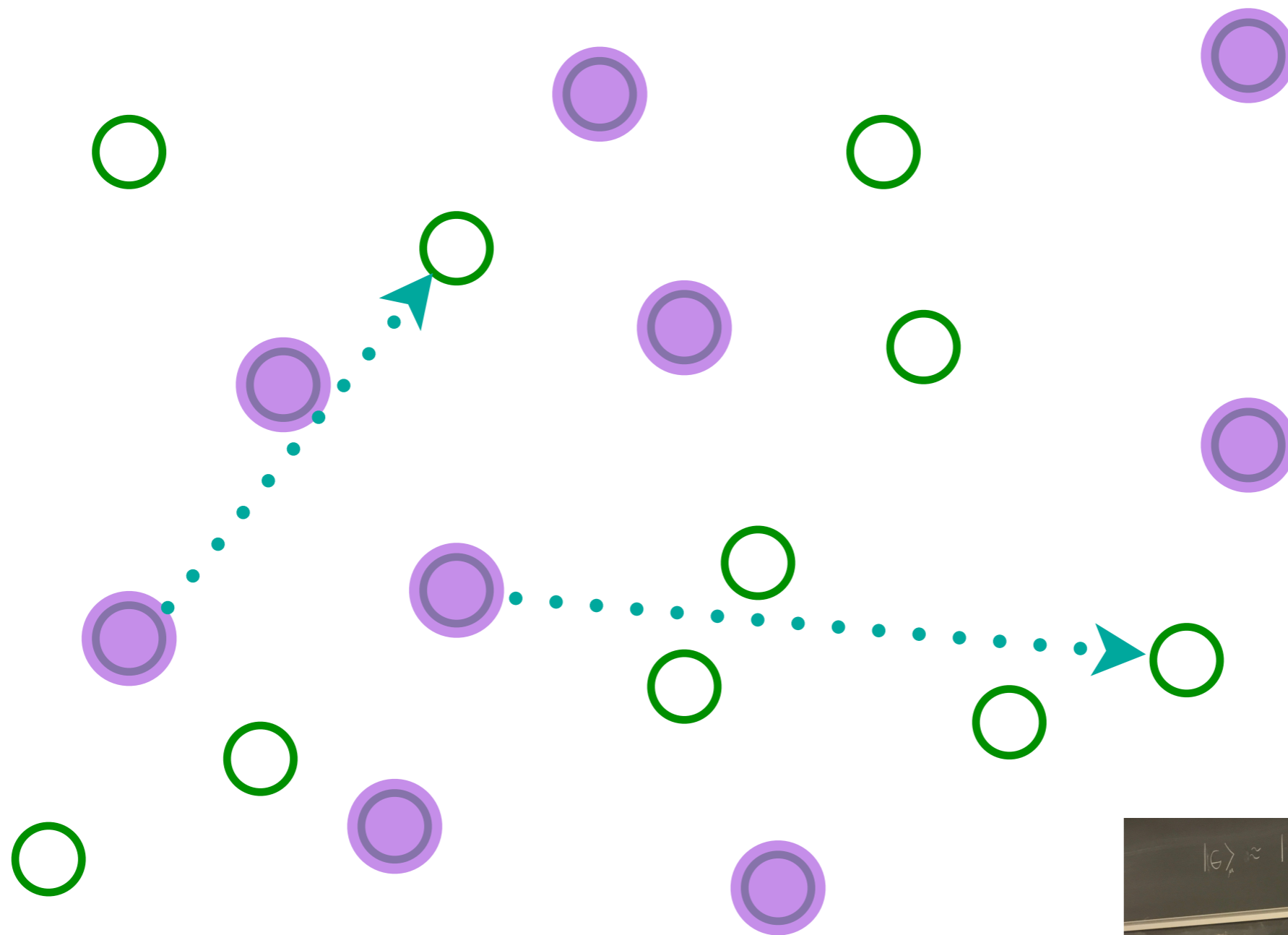
The SYK model



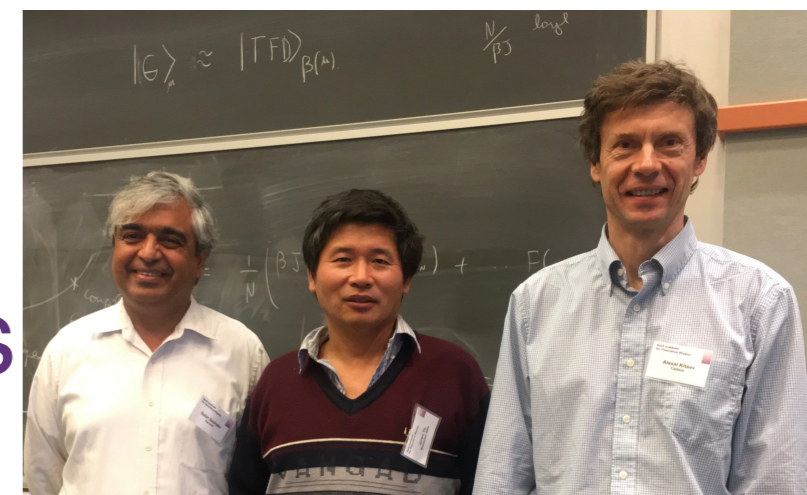
Place electrons randomly on some sites



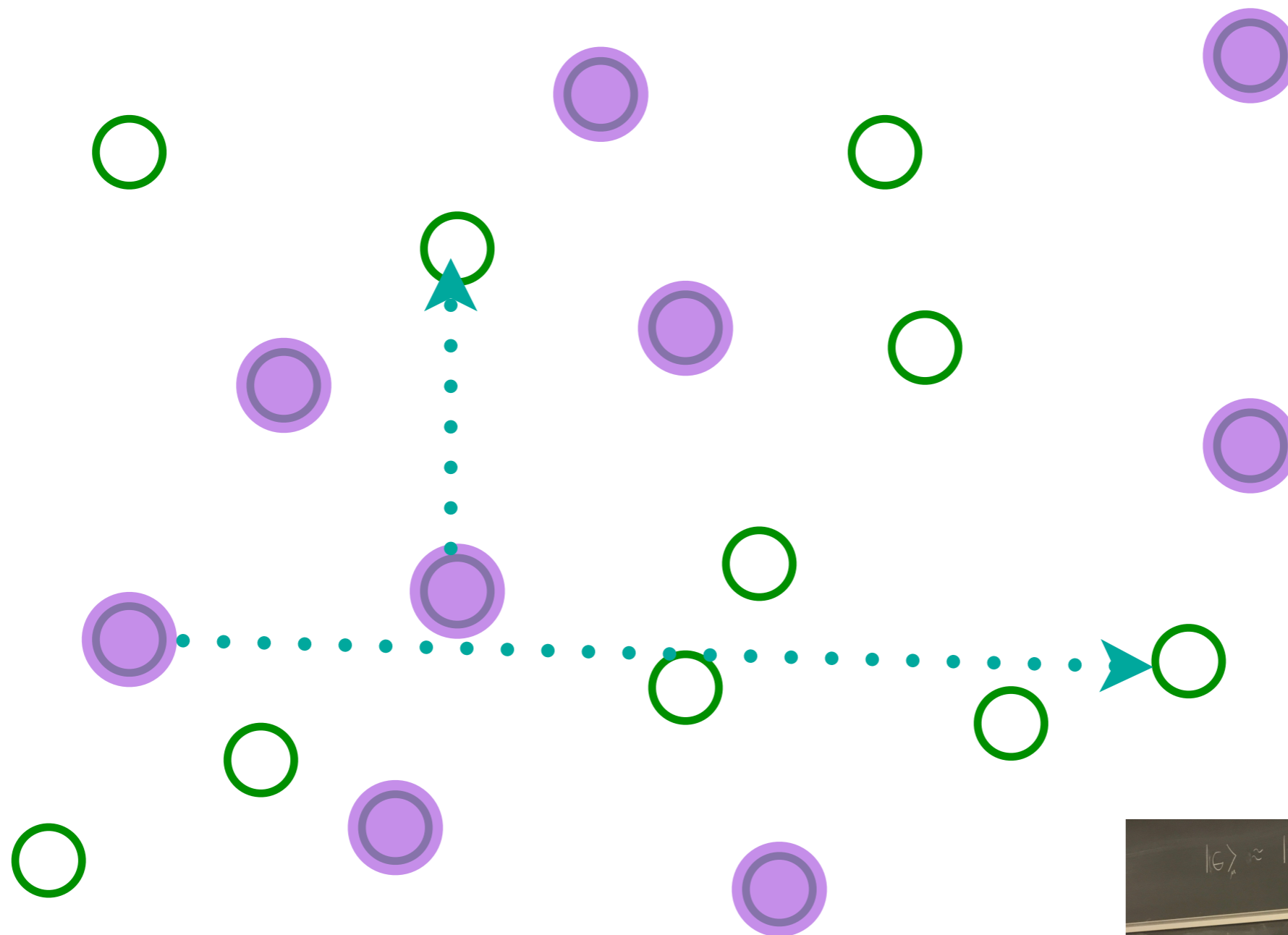
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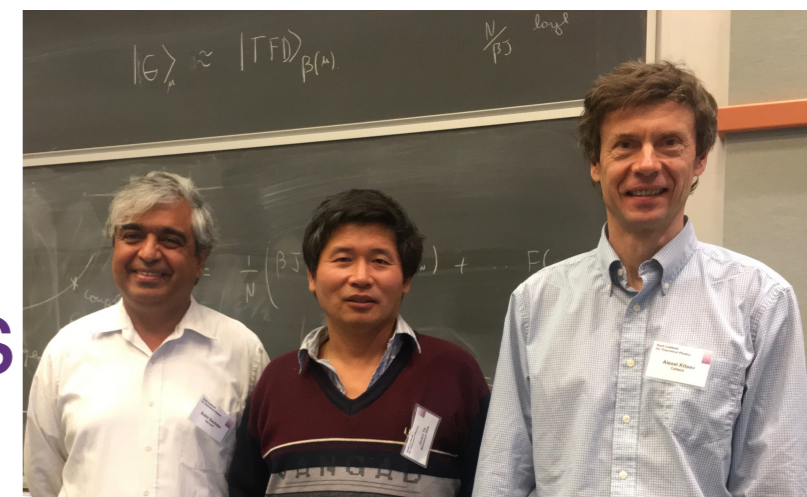
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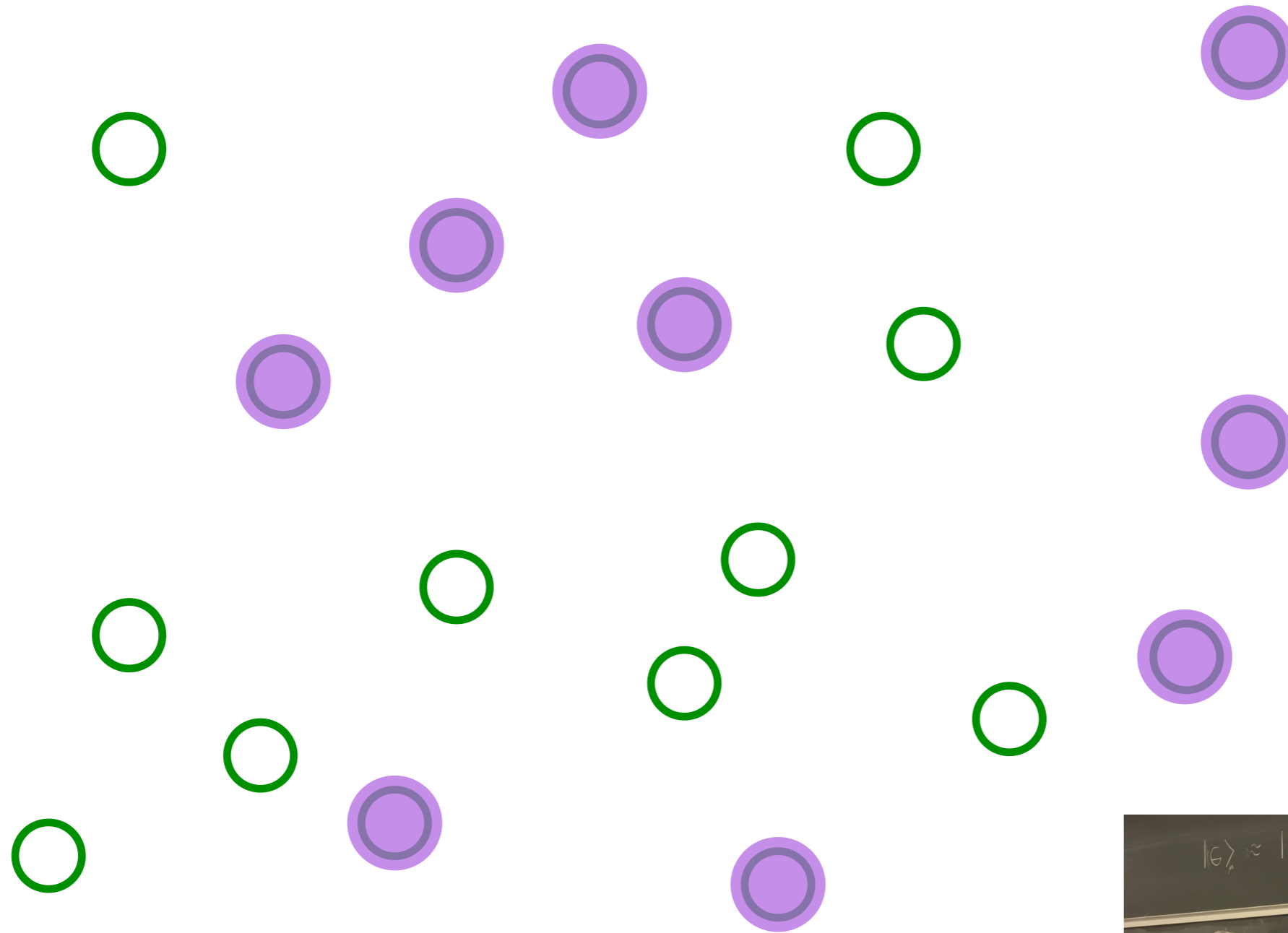
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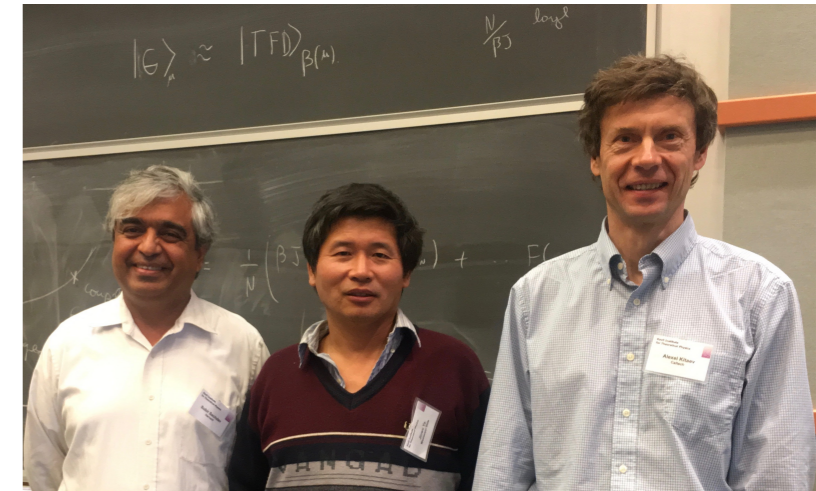
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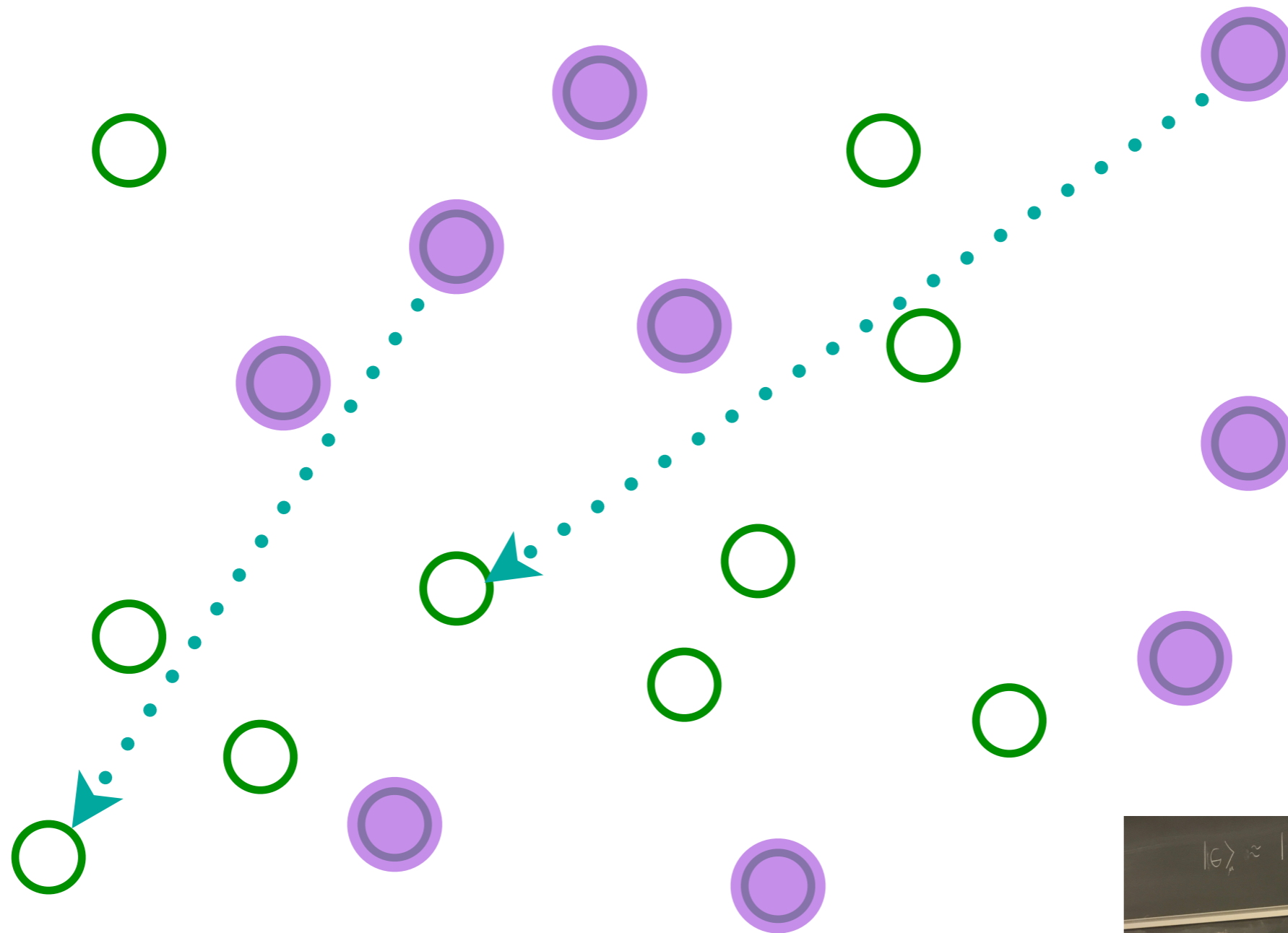
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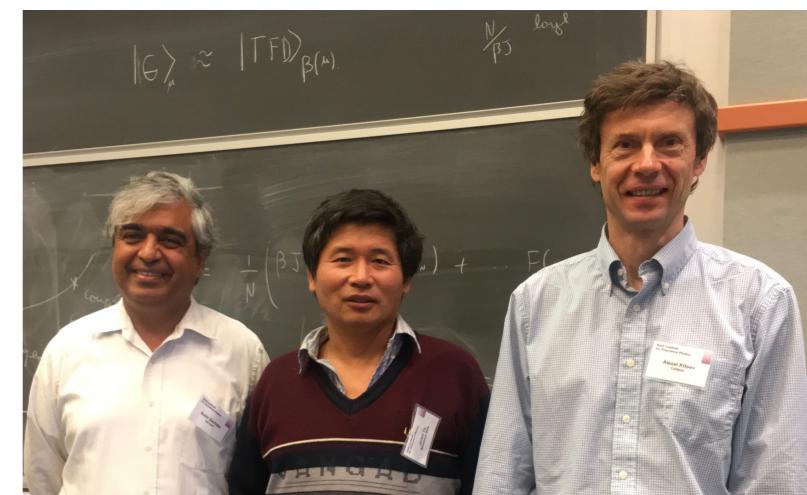
Entangle electrons pairwise randomly



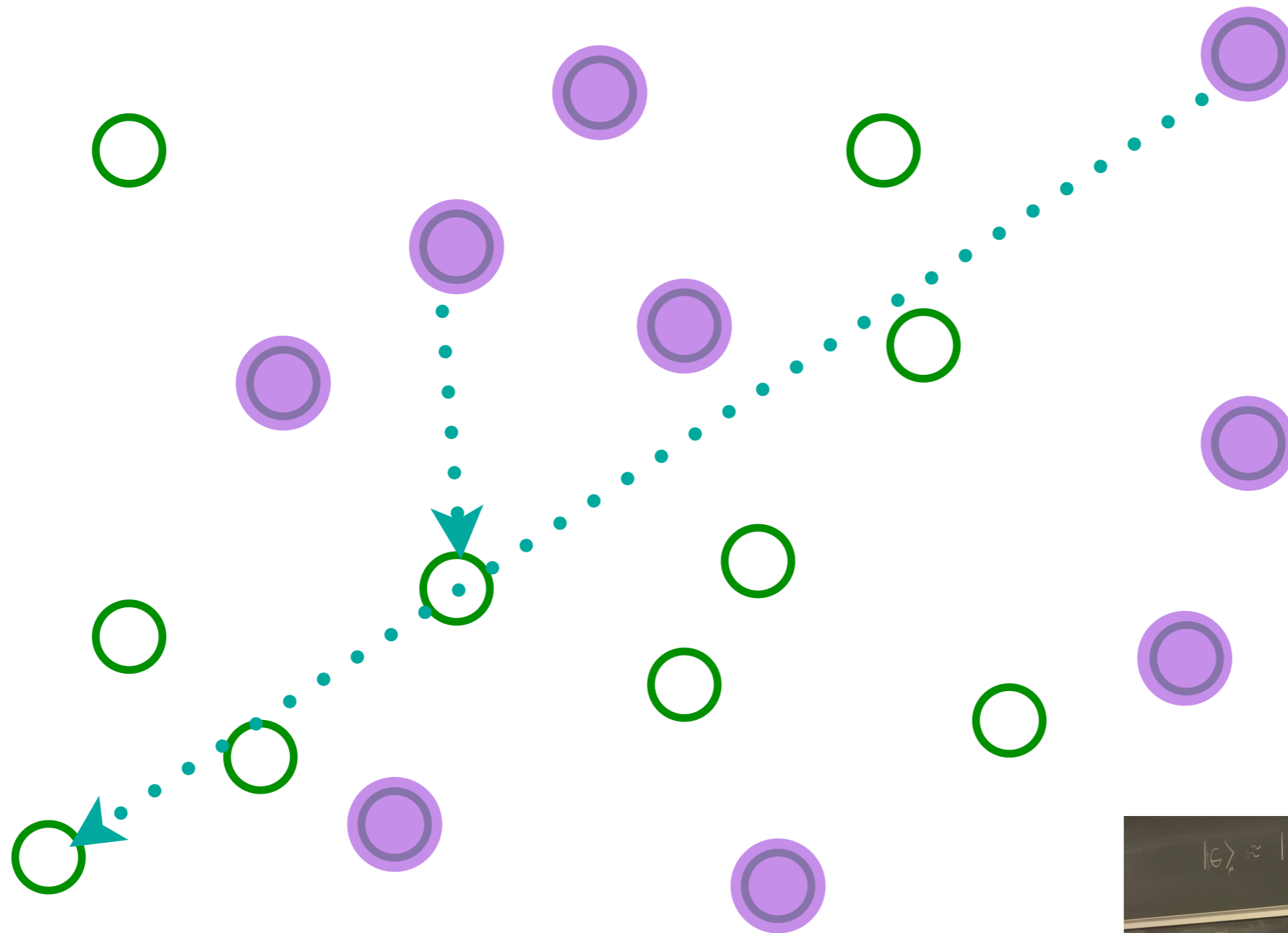
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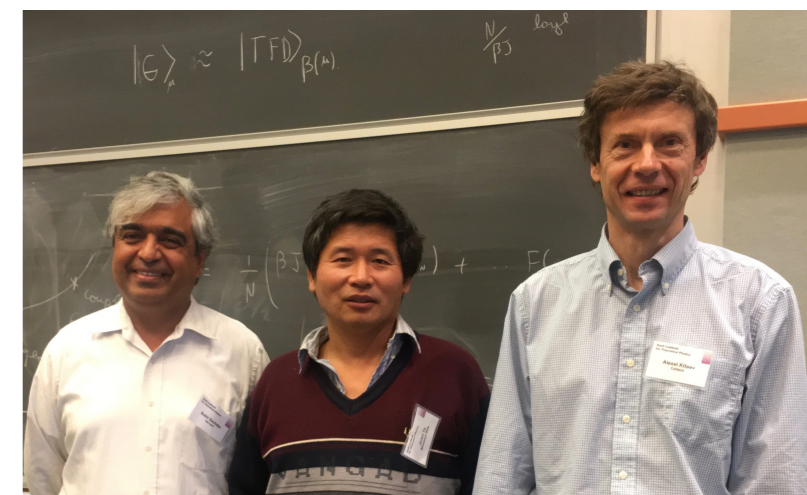
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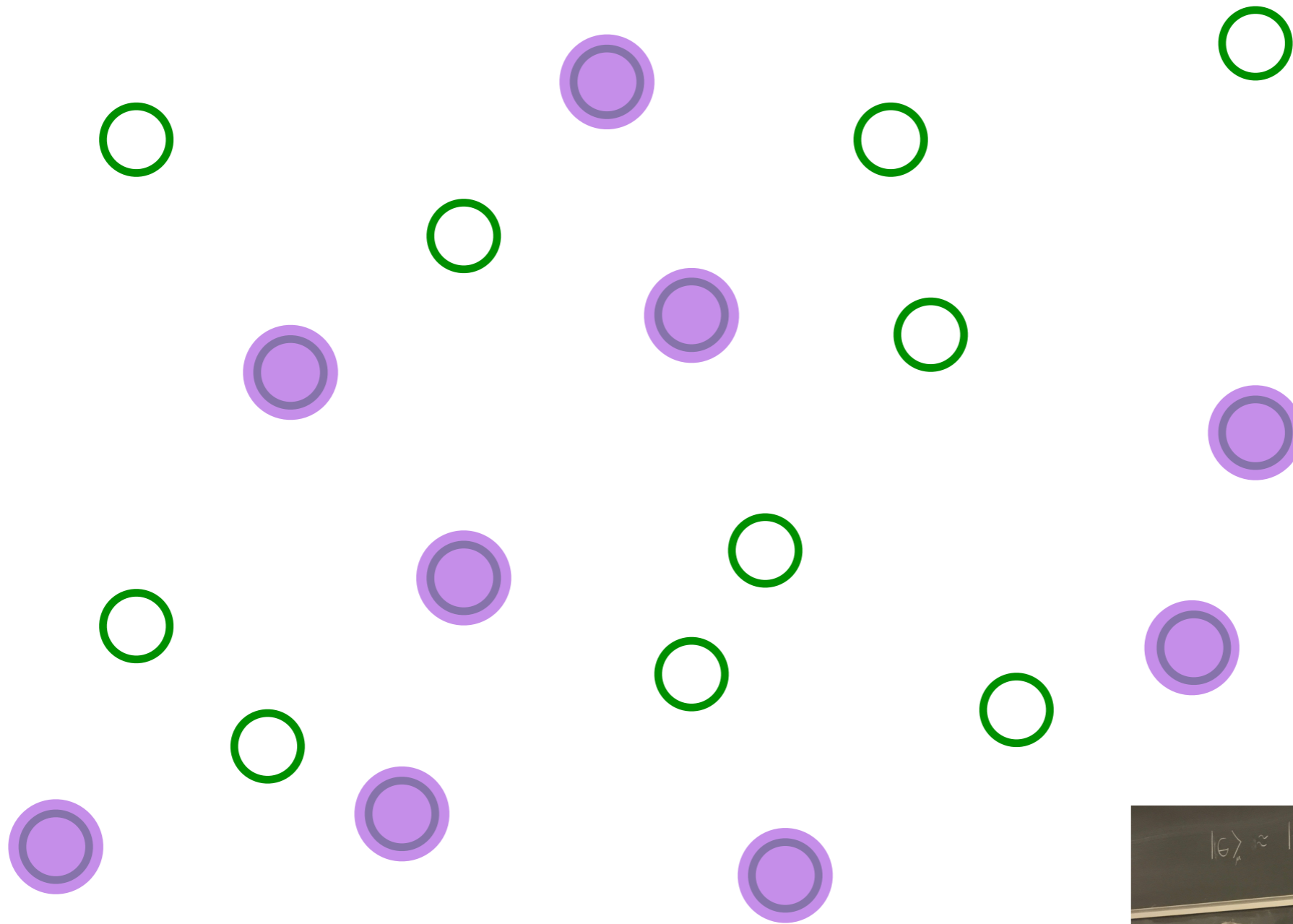
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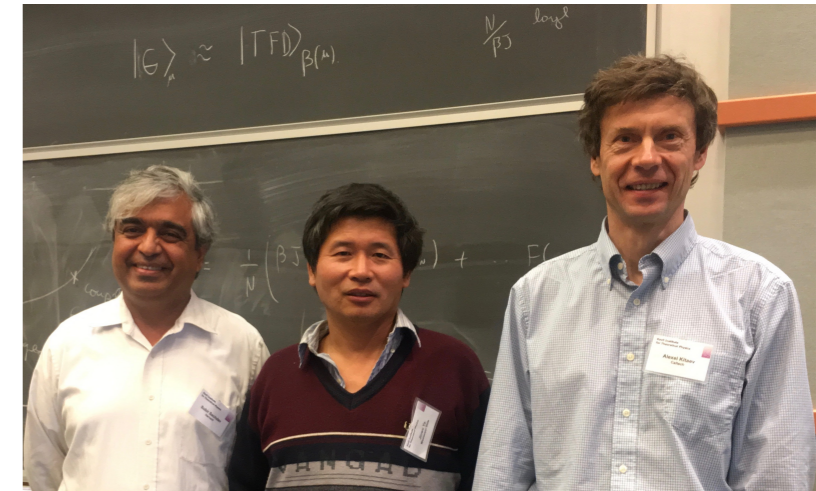
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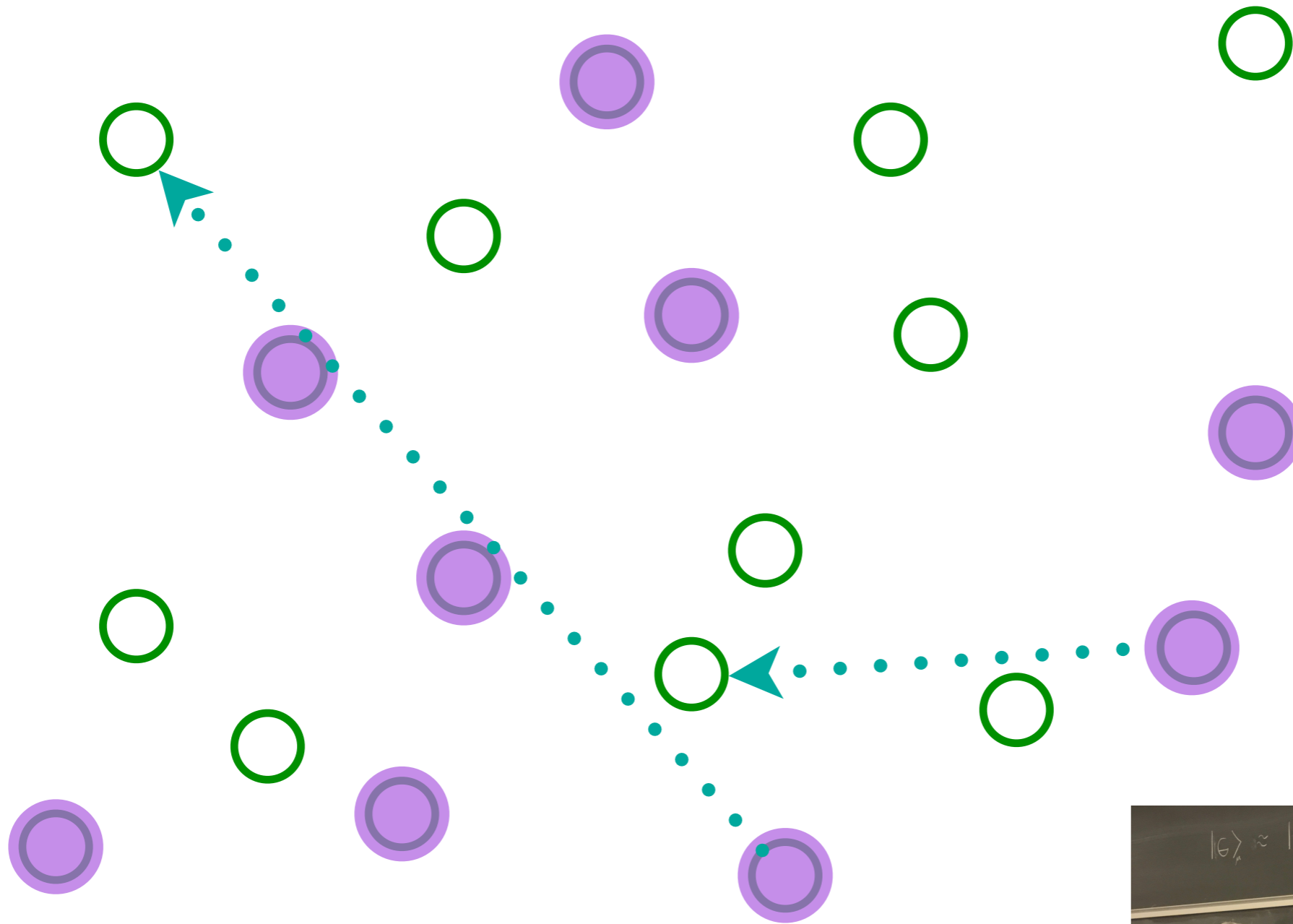
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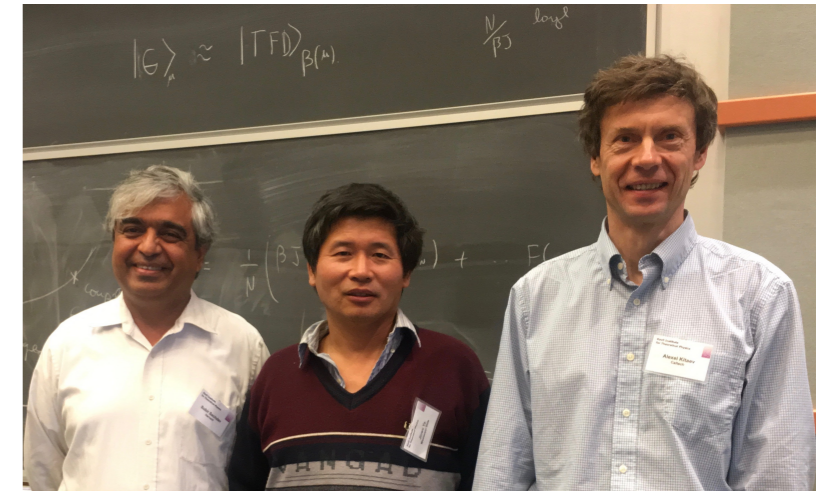
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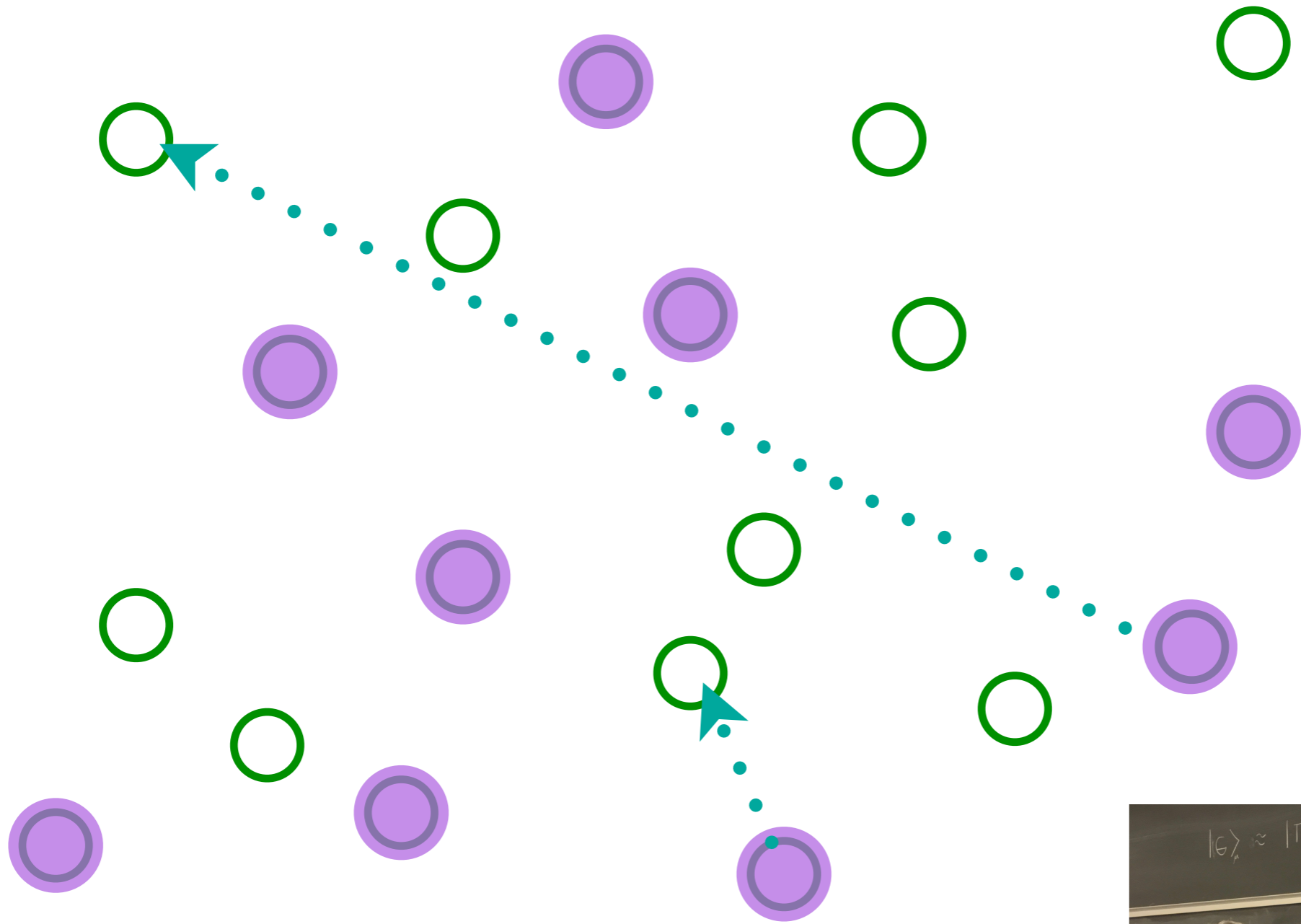
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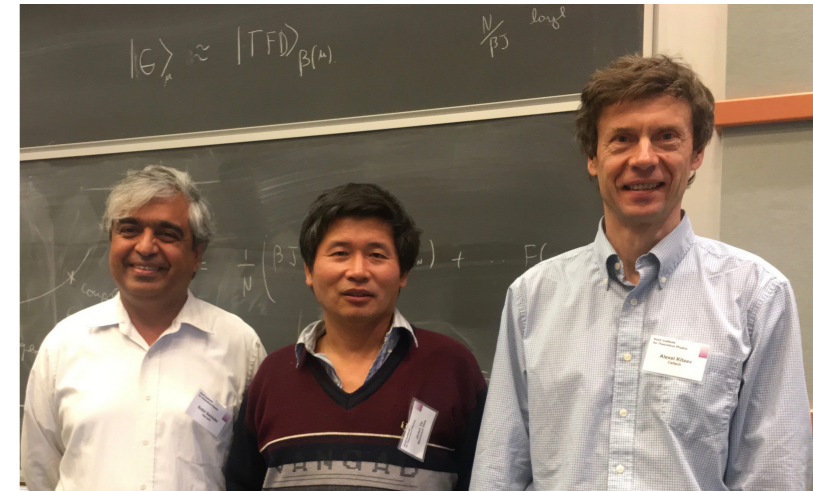
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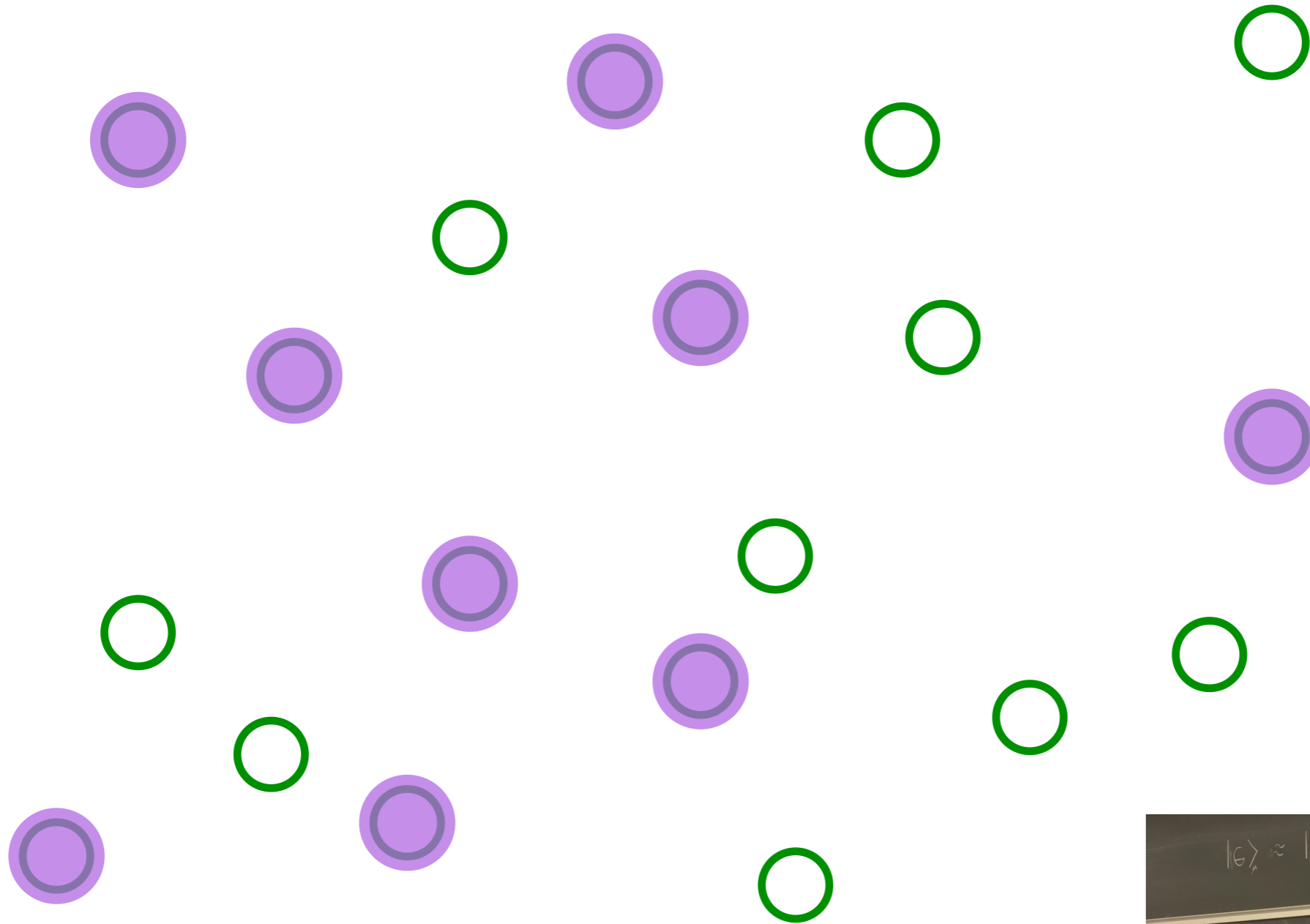
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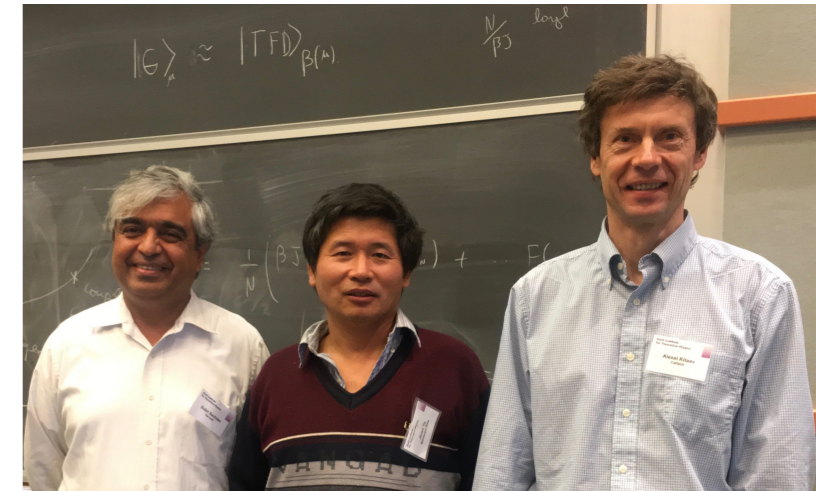
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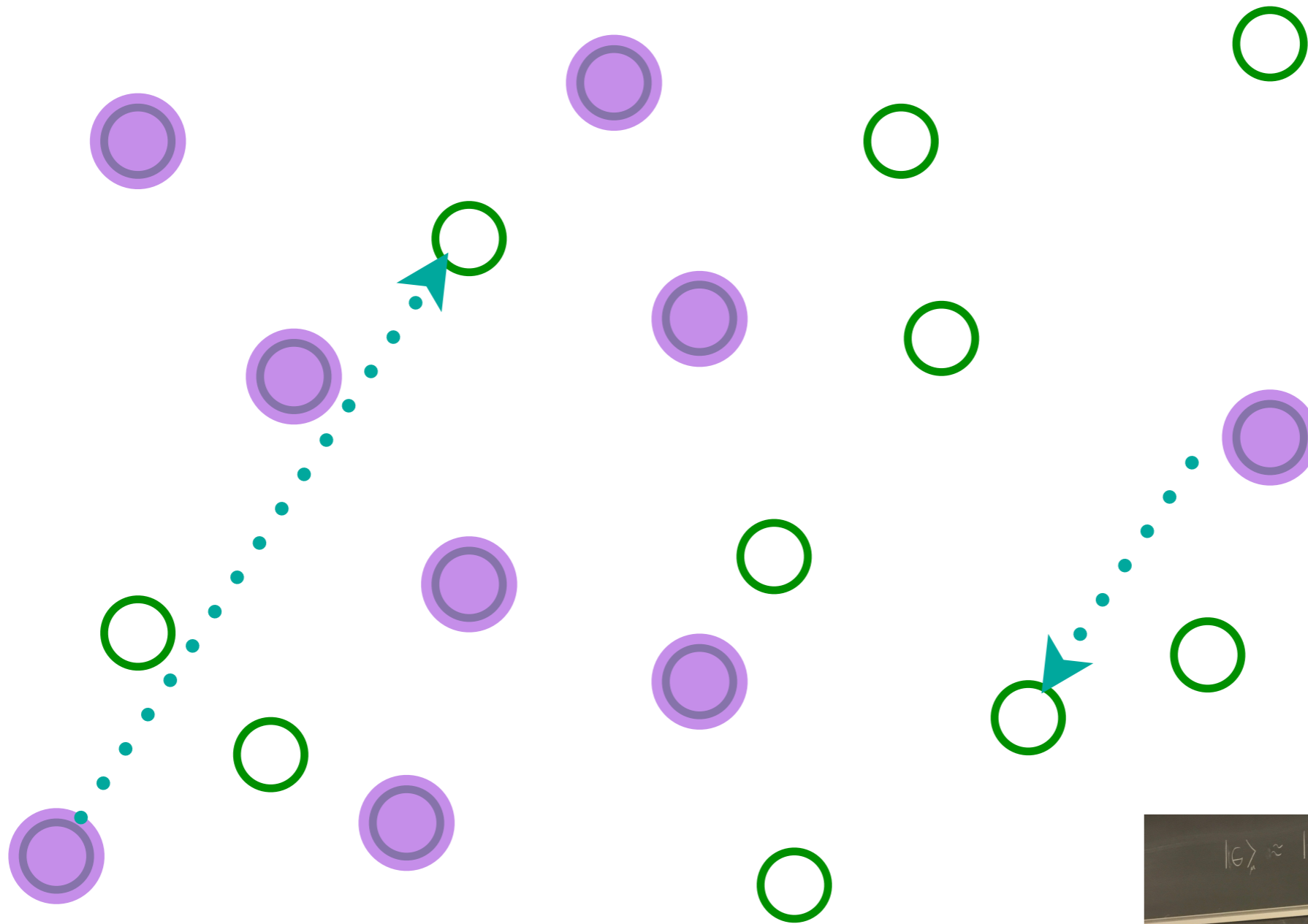
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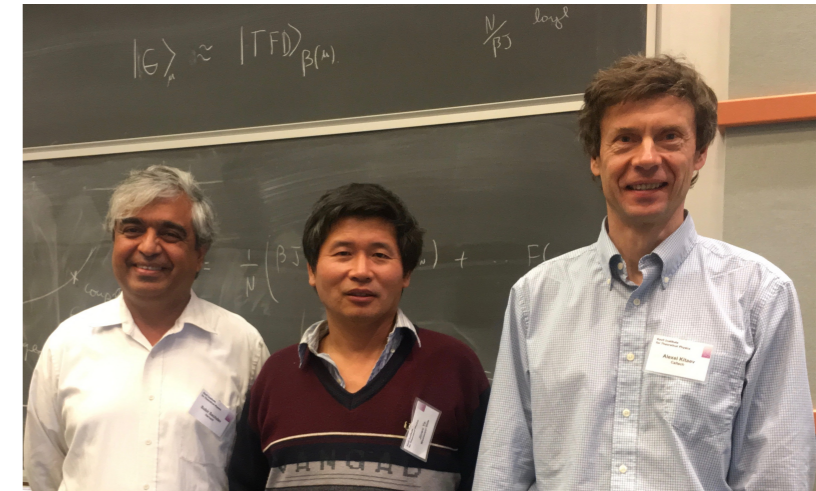
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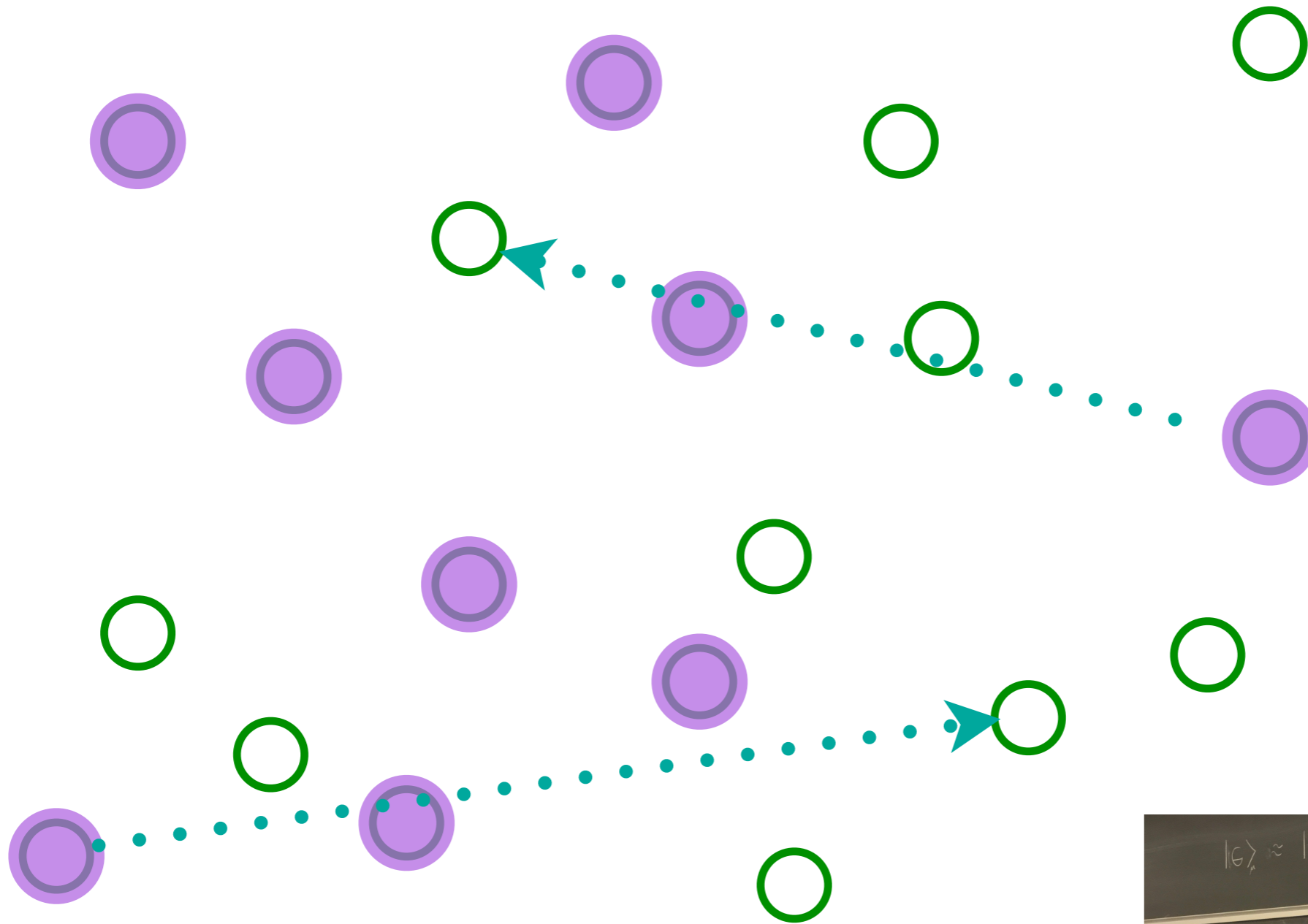
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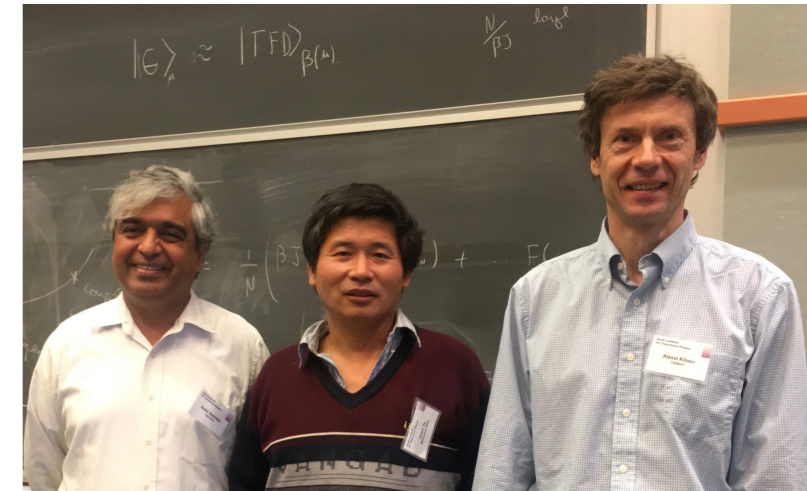
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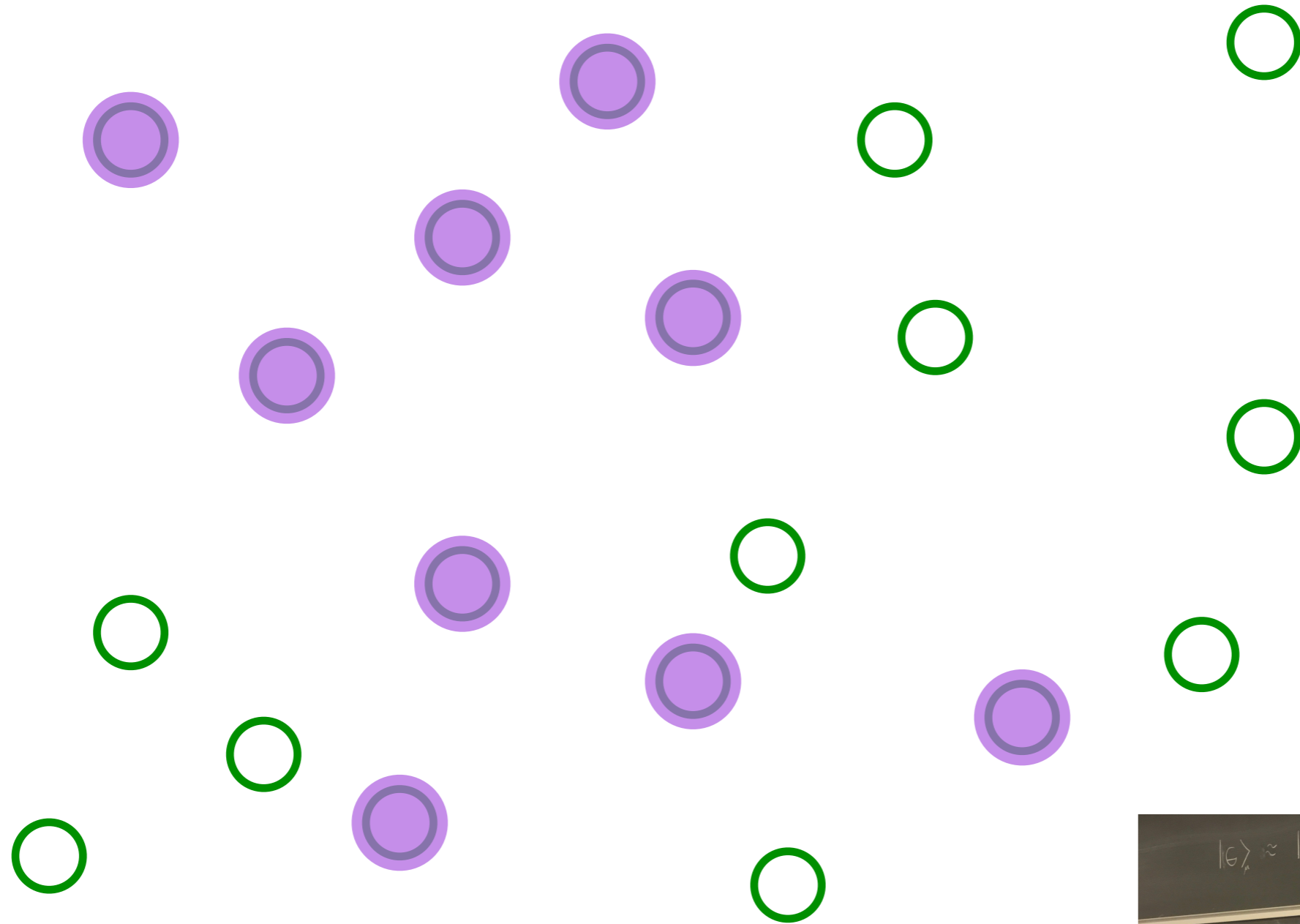
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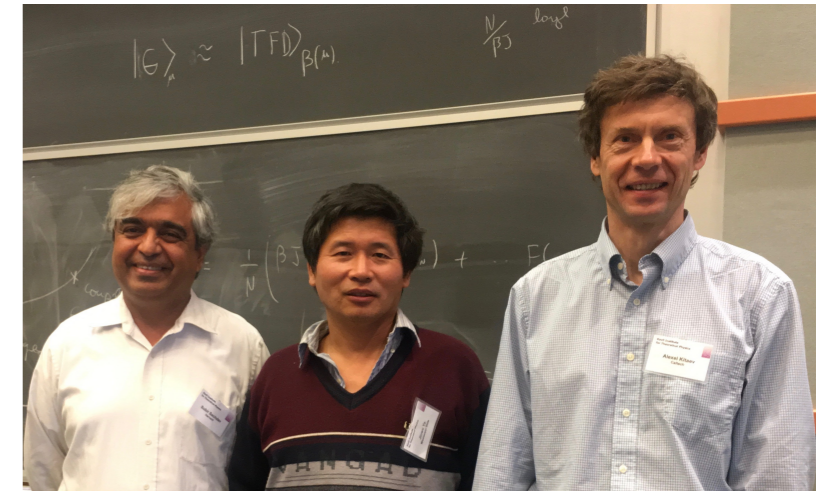
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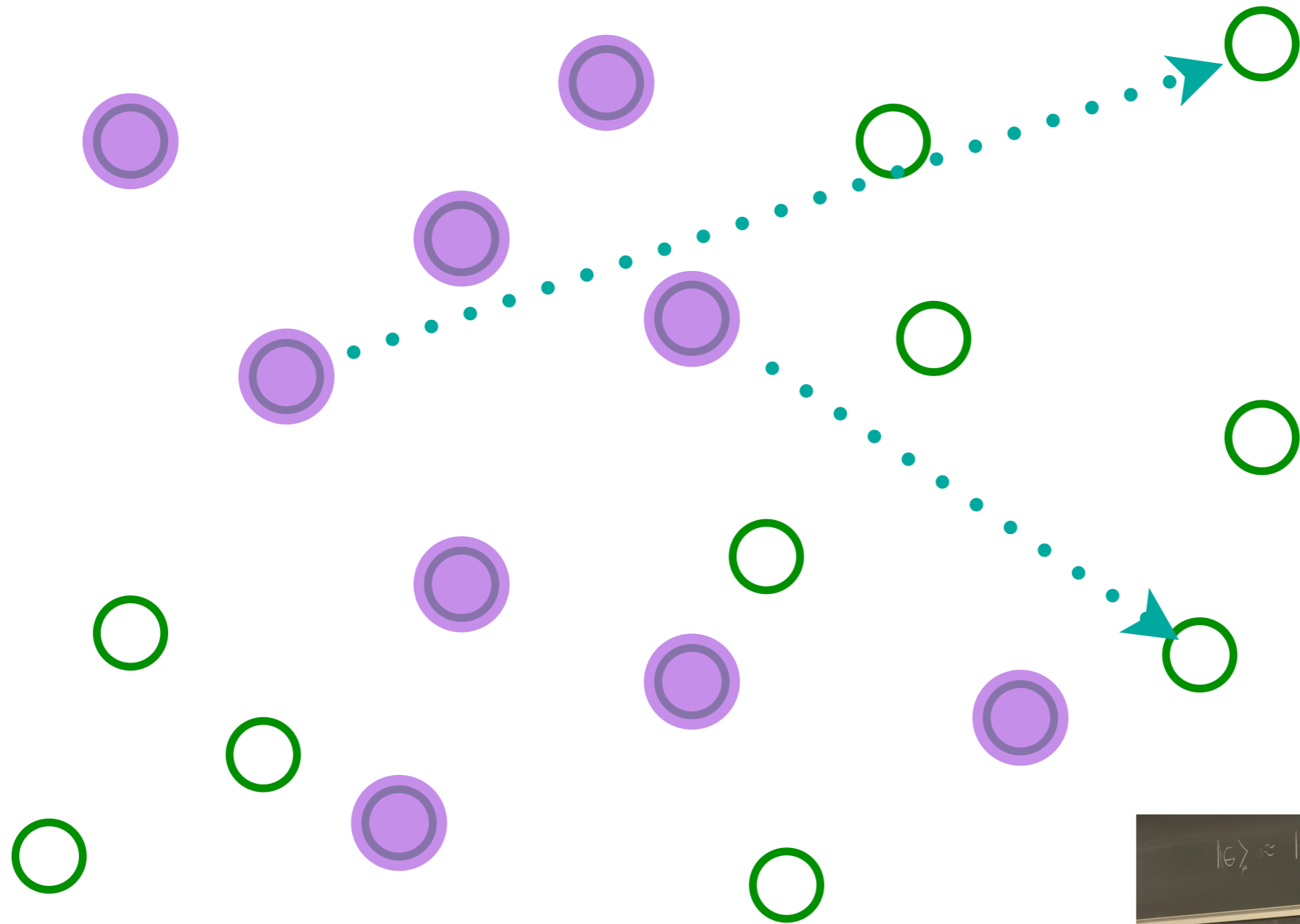
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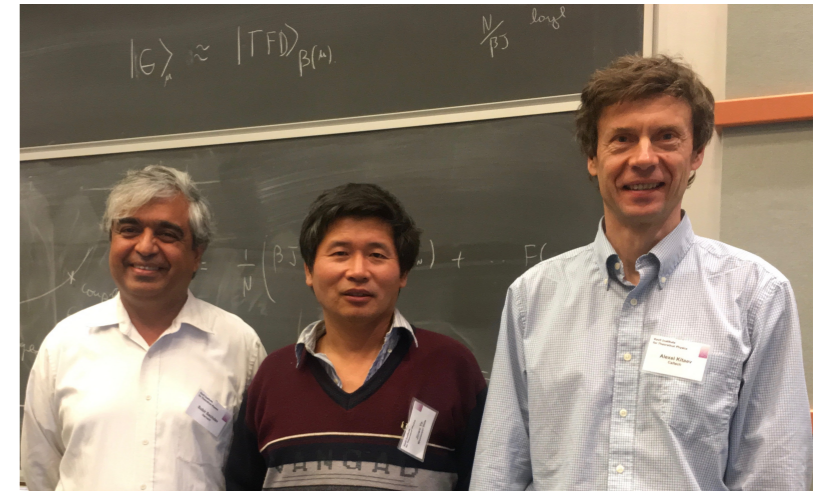
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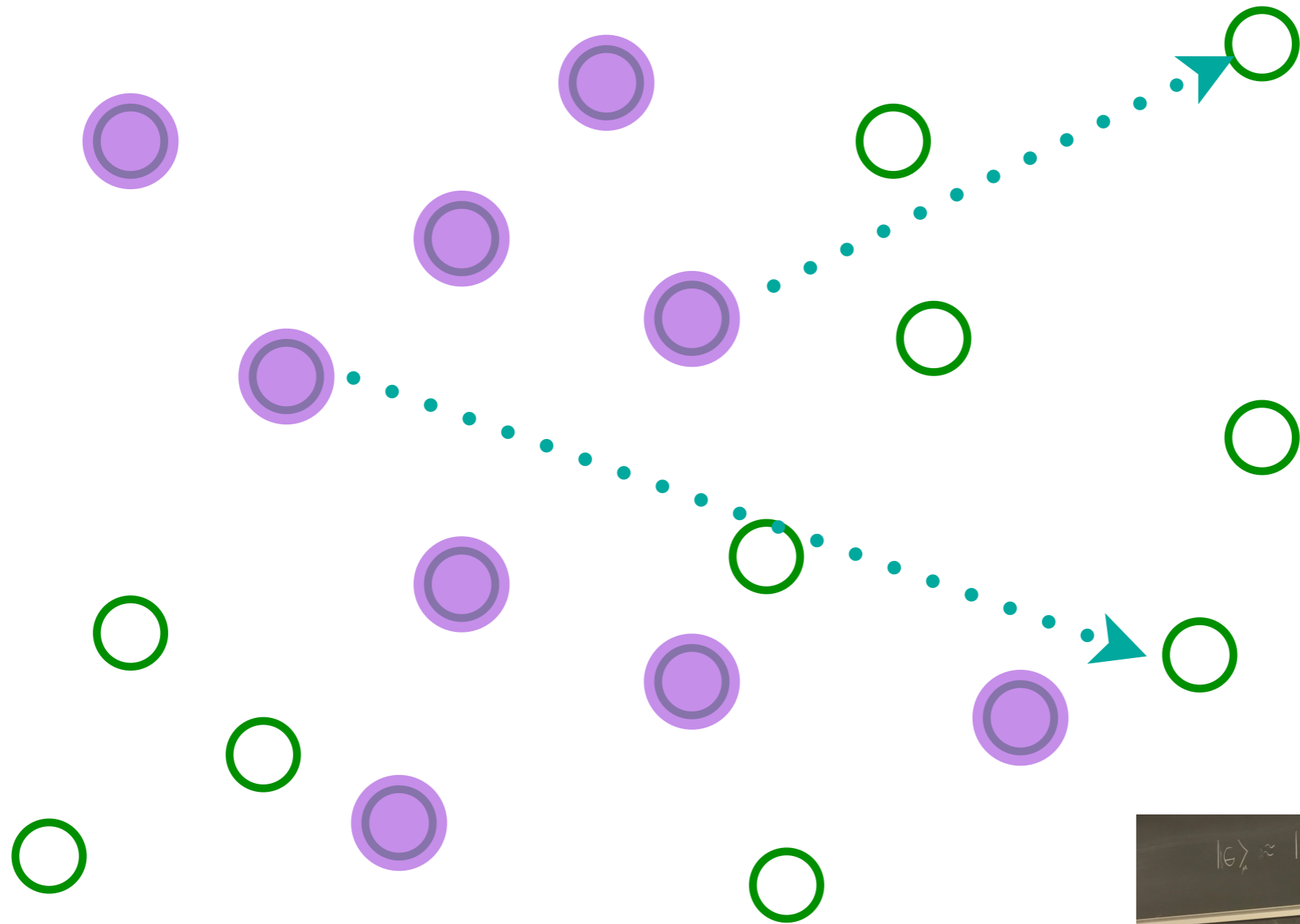
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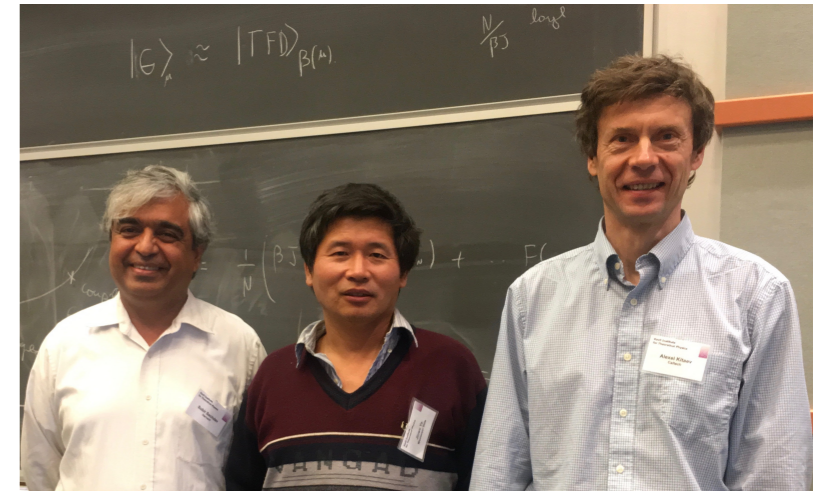
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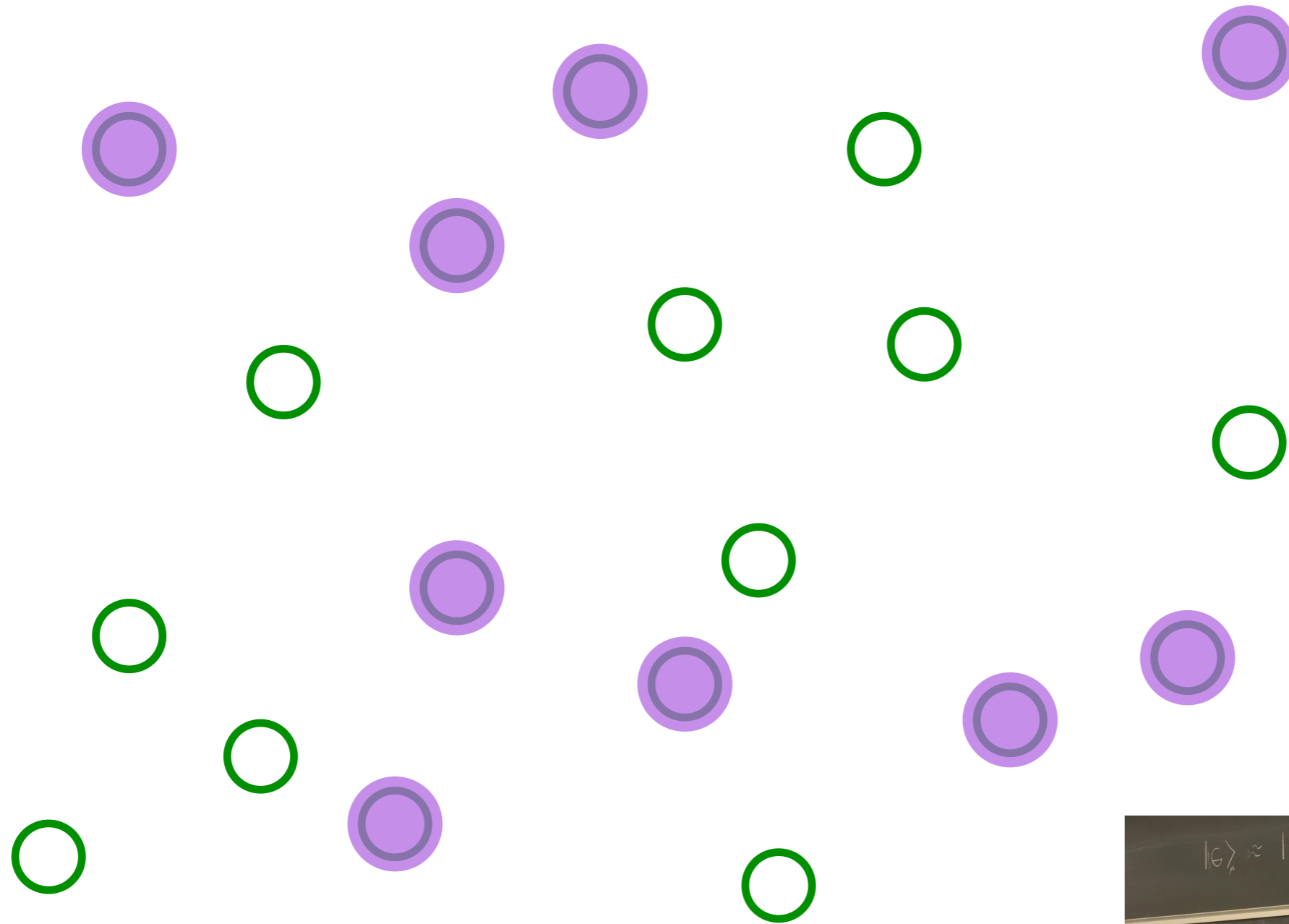
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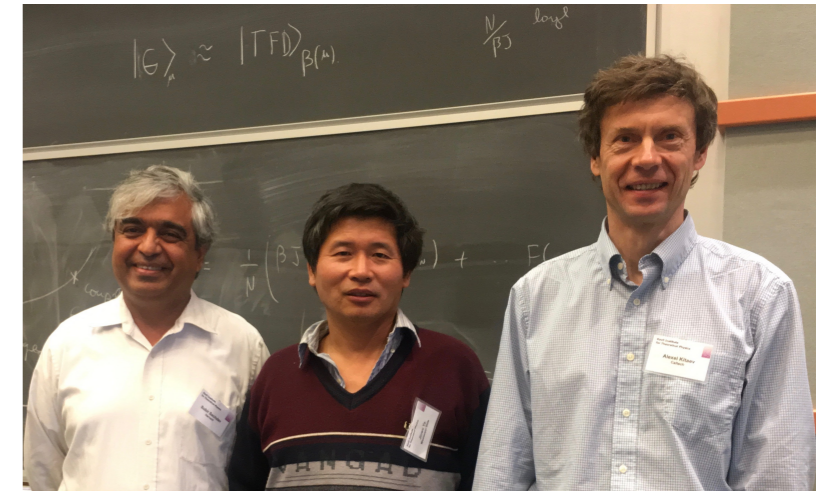
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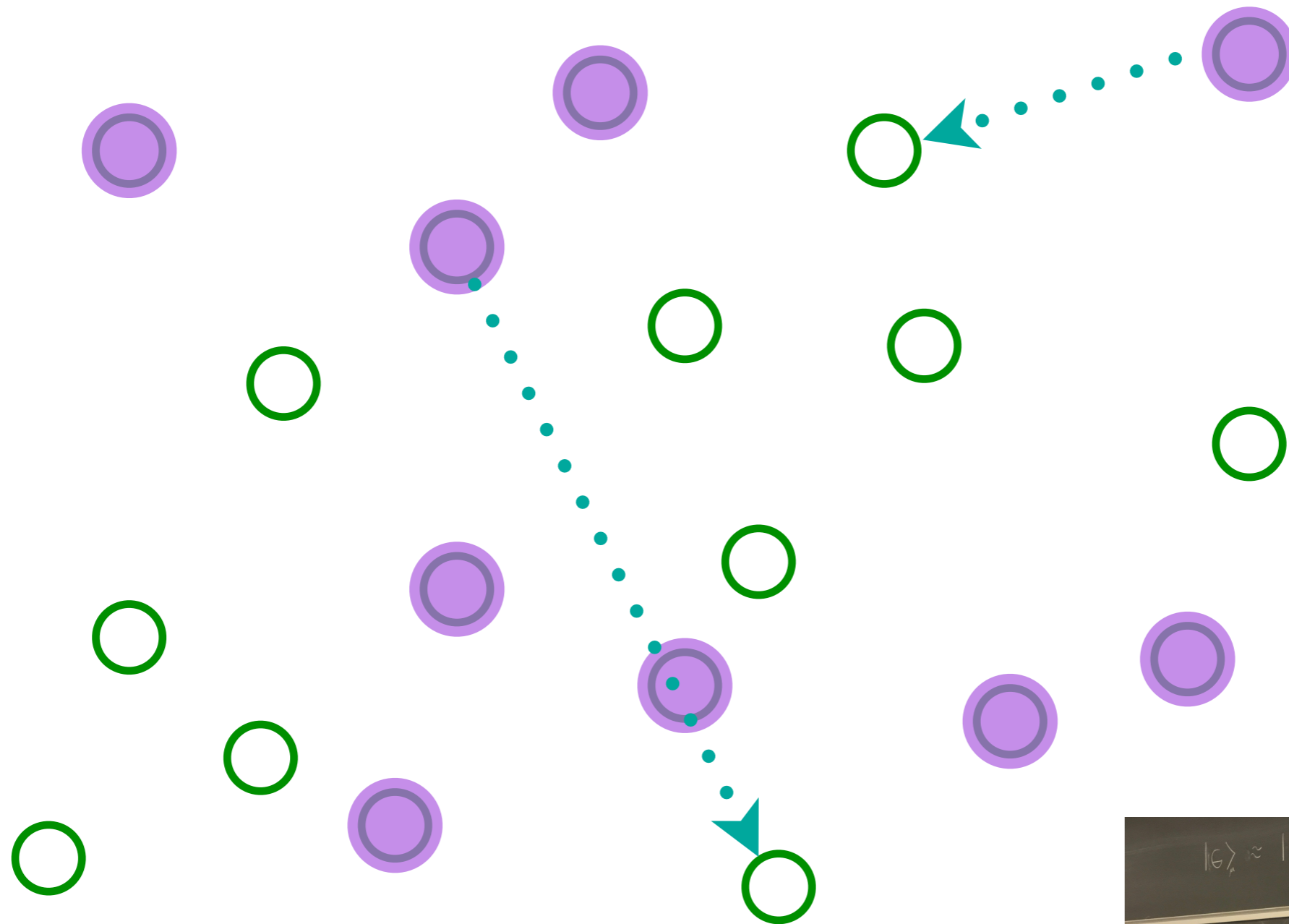
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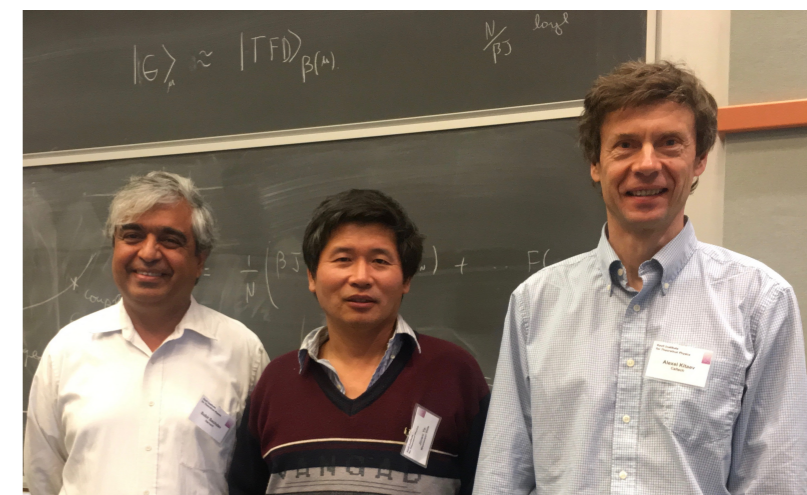
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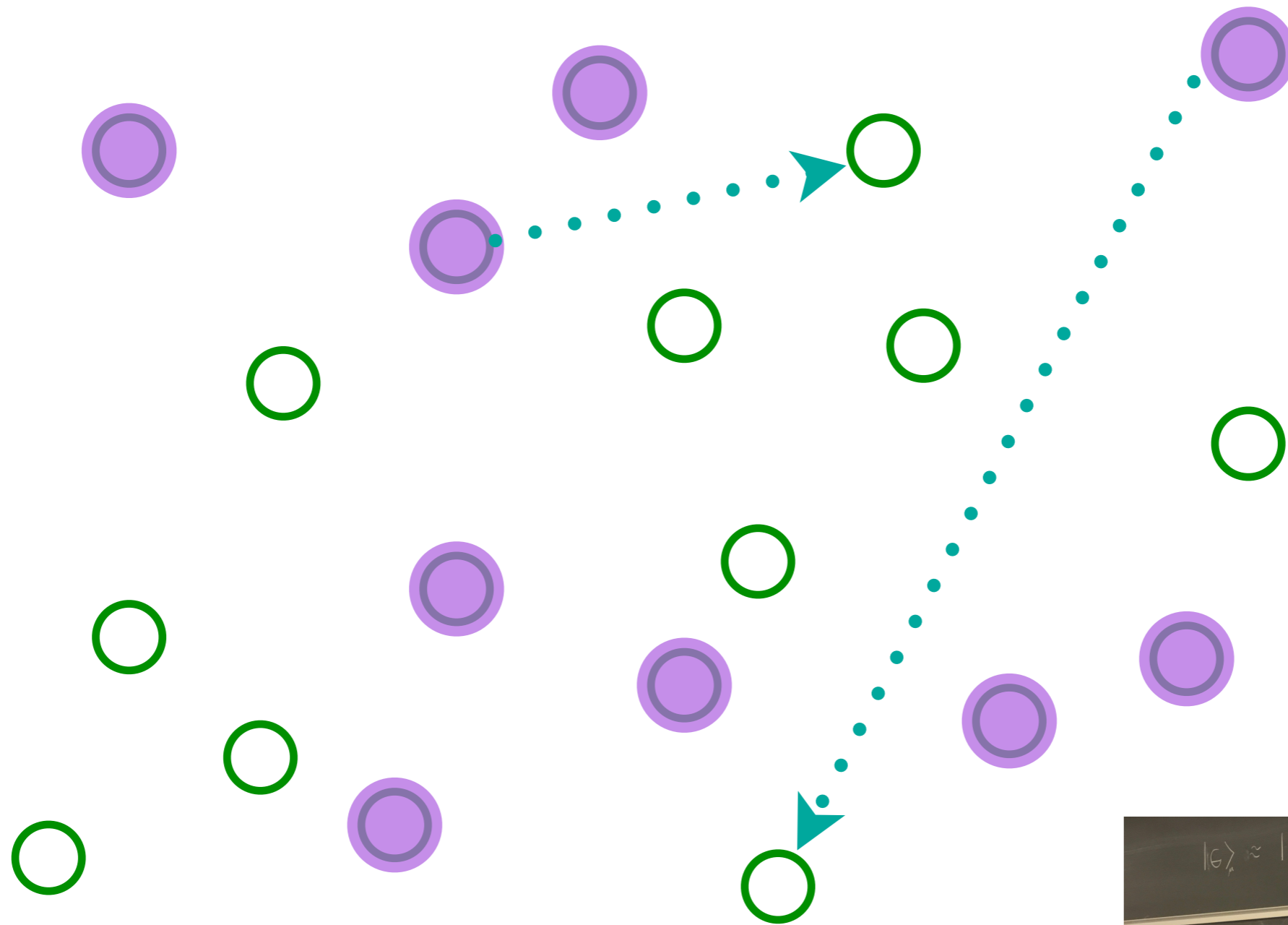
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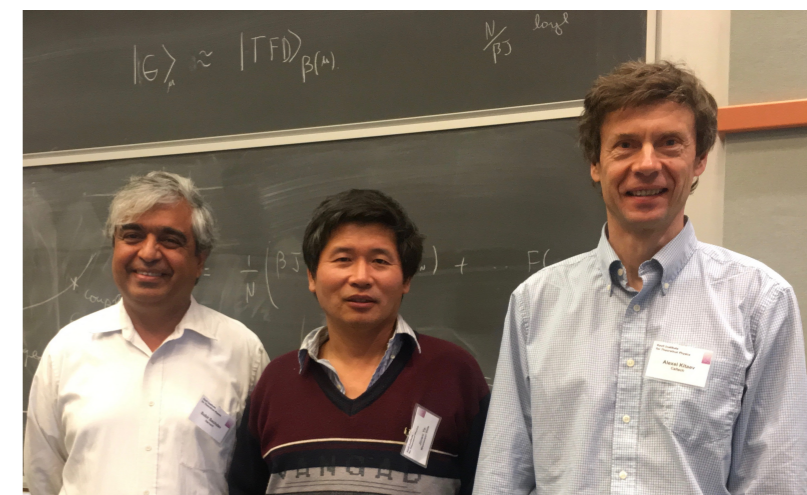
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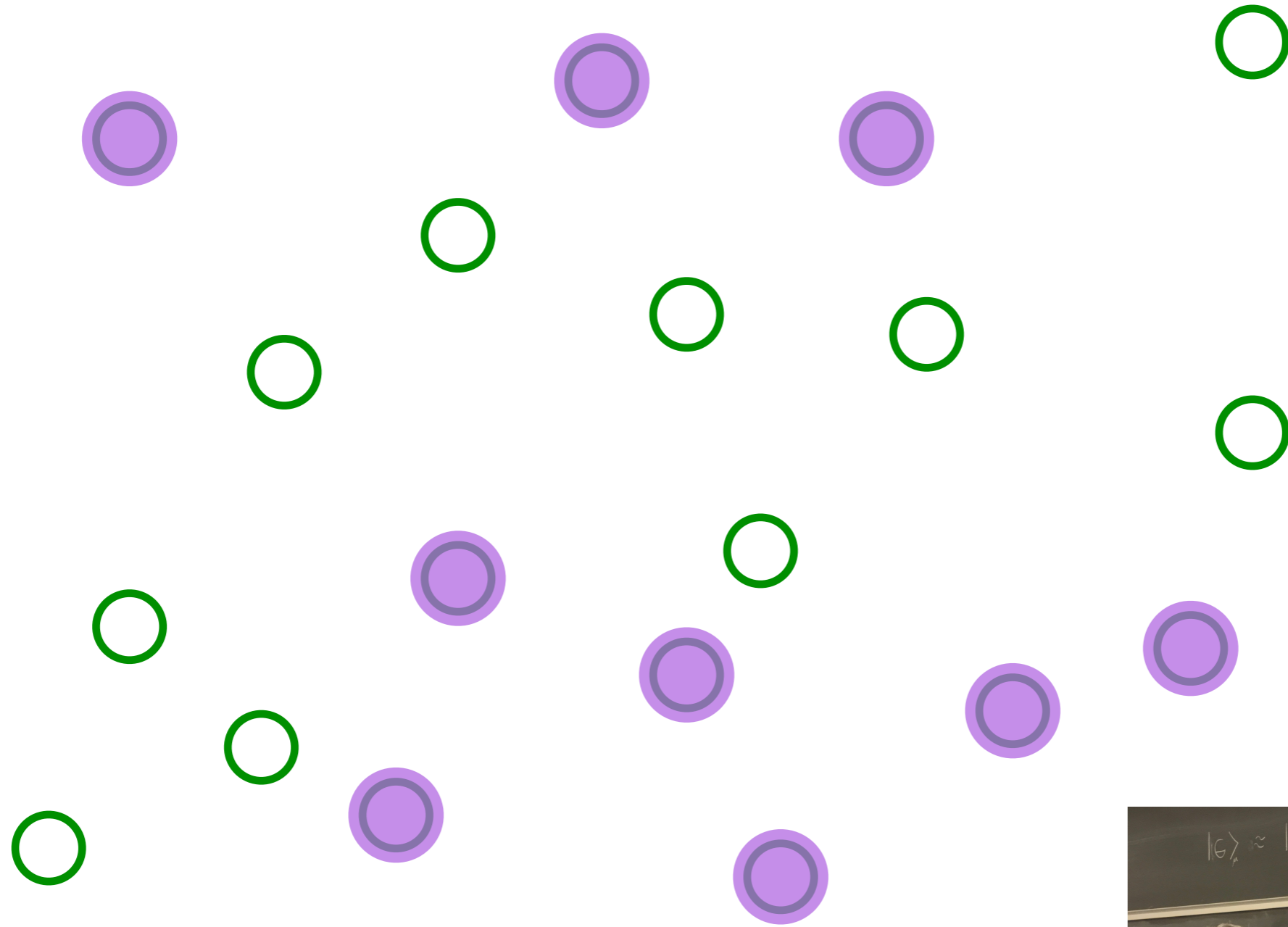
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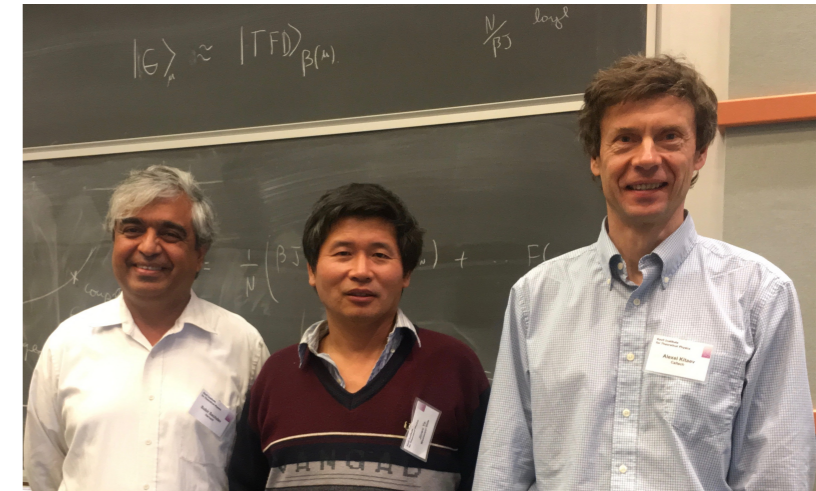
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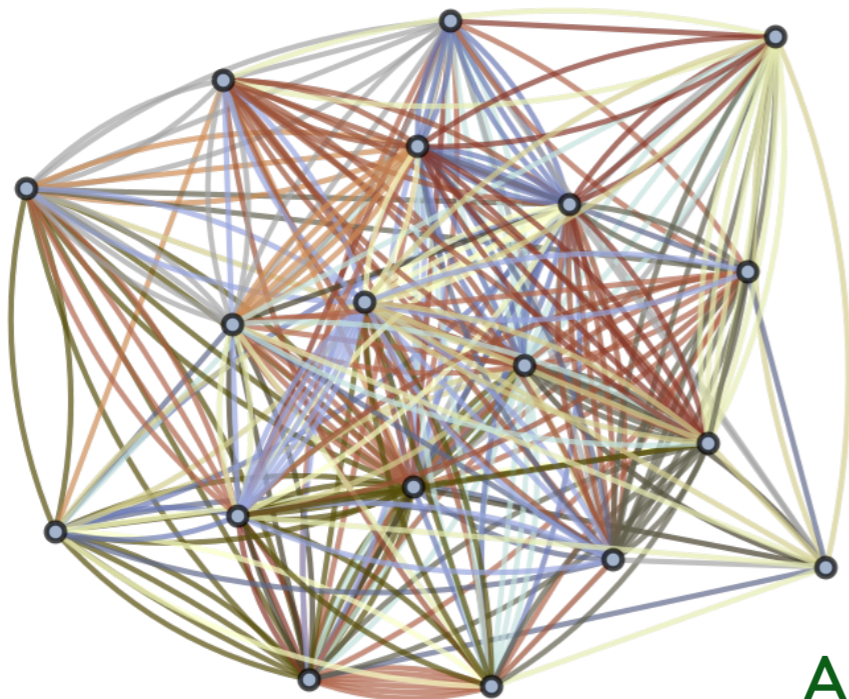
(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N U_{ij;k\ell} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

Complex multi-particle entanglement in the SYK model leads to a state without ‘quasiparticle’ excitations; *i.e.*

multiple excitations cannot be built by composing an elementary set of ‘quasiparticle’ excitations.

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Many-body chaos and thermal equilibration in the shortest possible Planckian time $\sim \frac{\hbar}{k_B T}$.

Main result I

Low temperature thermodynamics: for $k_B T \ll U$

$$\begin{aligned} \mathcal{Z} &= \text{Tr} \exp \left(-\frac{\mathcal{H}}{k_B T} \right) \\ &= \exp \left(N \frac{S_0}{k_B} \right) \int \mathcal{D}f(\tau) \exp \left(-\frac{1}{\hbar} \mathcal{S}_{2\text{D-gravity}} [f(\tau)] \right) \end{aligned}$$

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

A. Kitaev (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016)

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S_0 is the $T \rightarrow 0$ entropy of the SYK model.
This entropy is a consequence of the exponentially large number of ways of entangling many particles.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

$S(T) = S_0 + \dots$ will map on to the Bekenstein-Hawking entropy of black holes

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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- Feynman path integral over $f(\tau)$, the reparameterization of the time of the SYK model.
- $f(\tau)$ is also the fluctuation of the boundary of a theory of 2D-gravity in 1+1 spacetime dimensions: a ‘boundary graviton’:
the SYK model is a ‘hologram’ of quantum gravity.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

A. Kitaev (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016)

Quantum
entanglement

A simple
many-particle
(SYK) model

Low temperatures

Quantum gravity in
1+1 dimensions

**Quantum
entanglement**

**Black
holes**

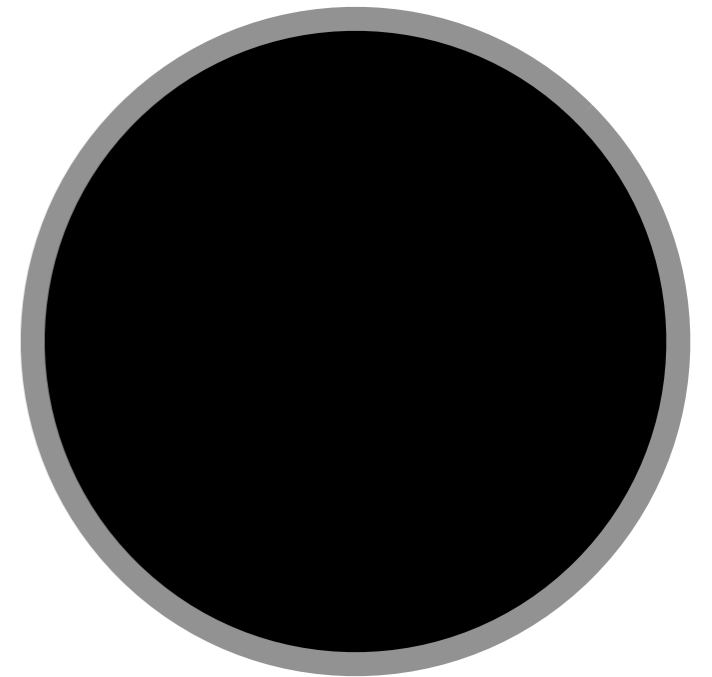
**A simple
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Black Holes

Objects so dense that light is gravitationally bound to them.

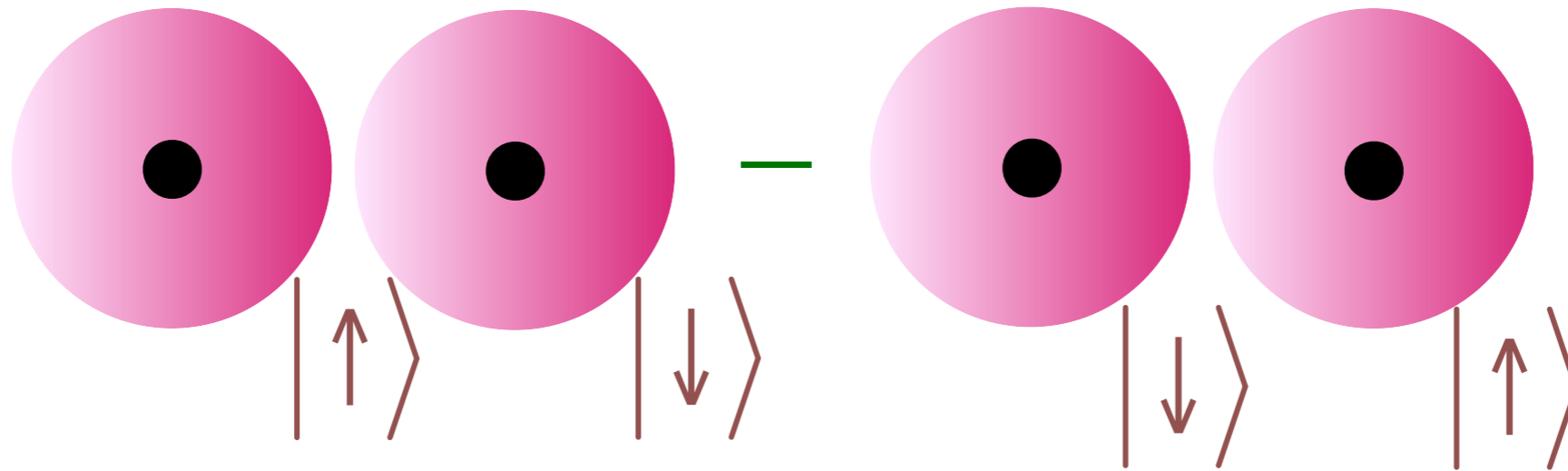
In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

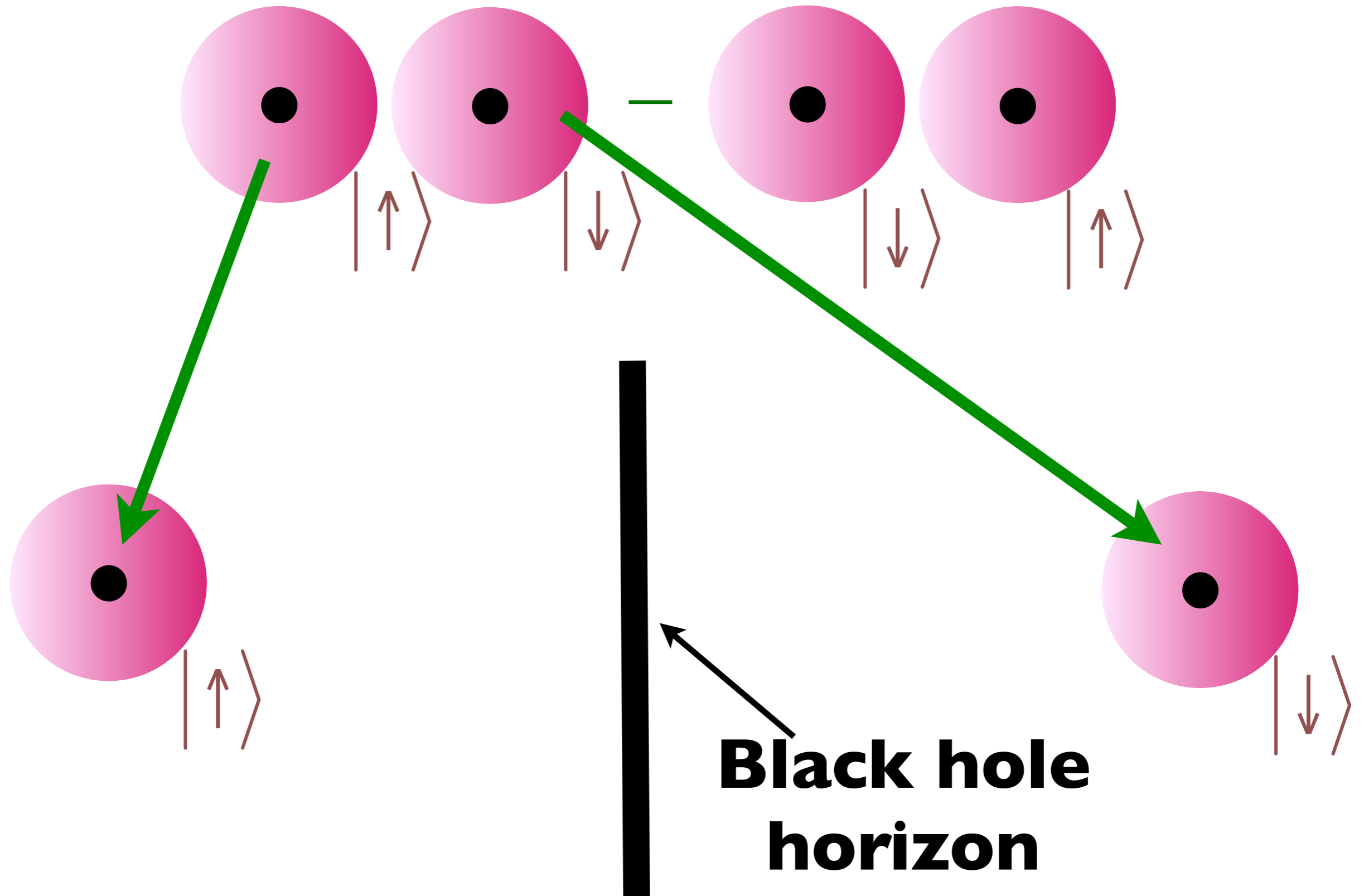


G Newton's constant, c velocity of light, M mass of black hole

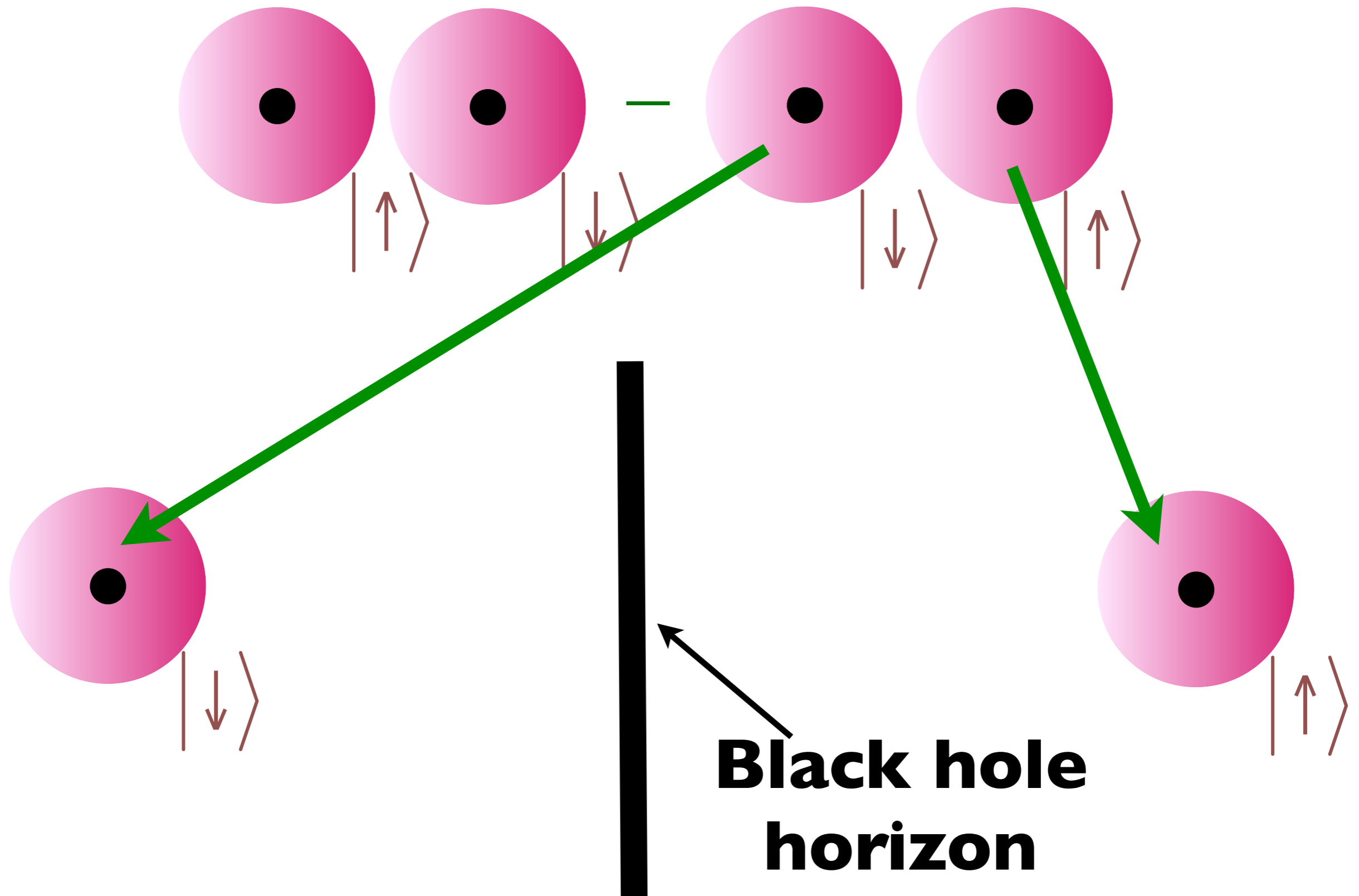
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

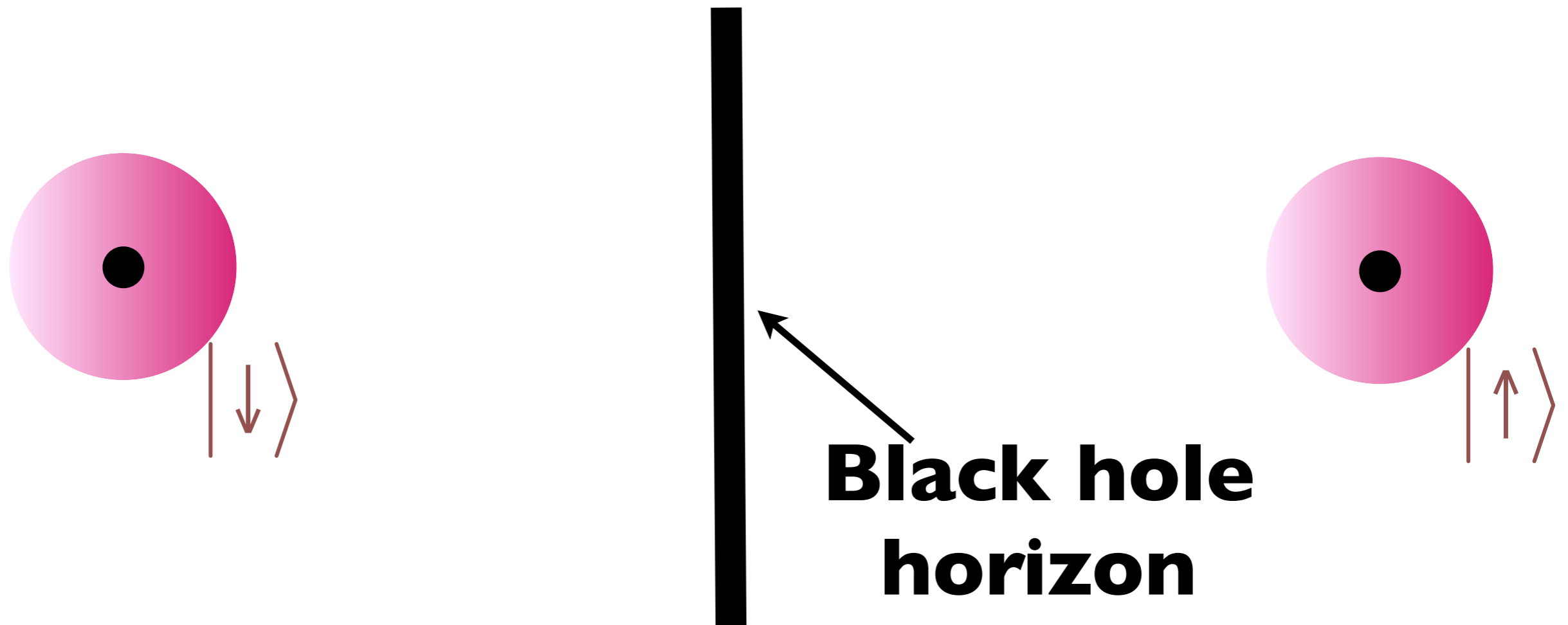


Quantum Entanglement across a black hole horizon



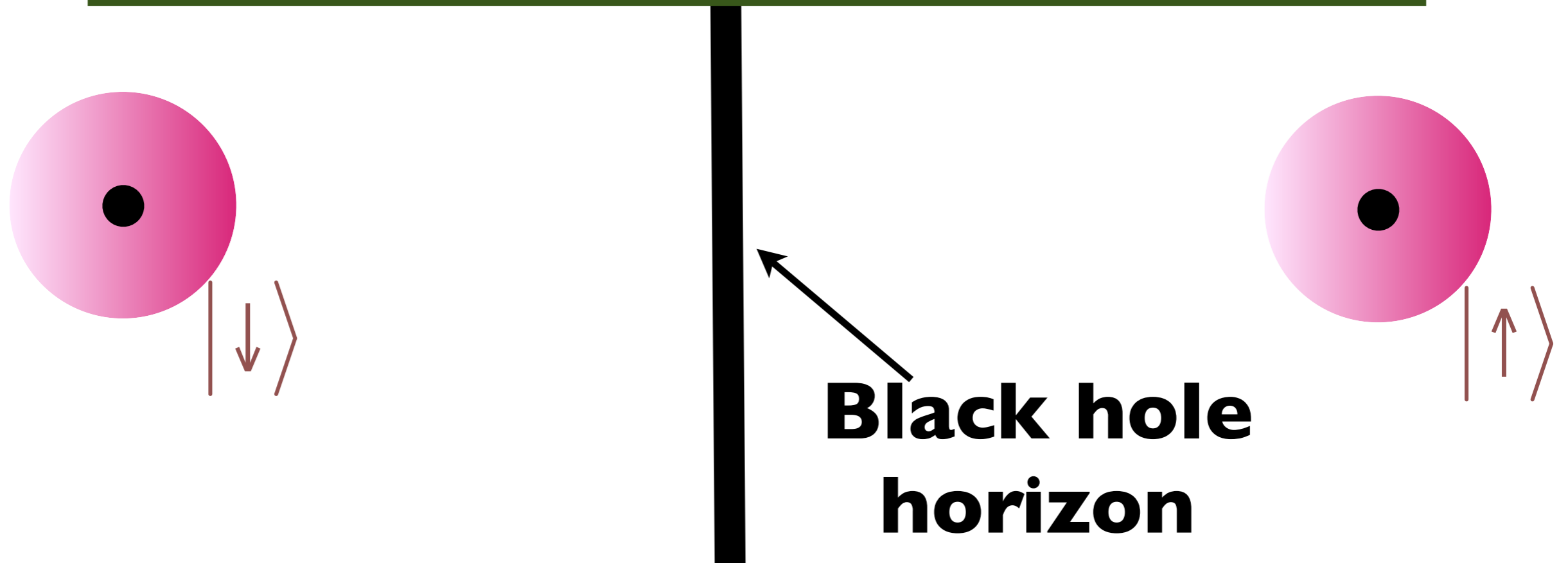
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature
(because to an outside observer, the state of the electron inside the black hole is an unknown)



Quantum Black holes

- Black holes have an entropy and a temperature, T_H .
- The entropy, S_{BH} is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

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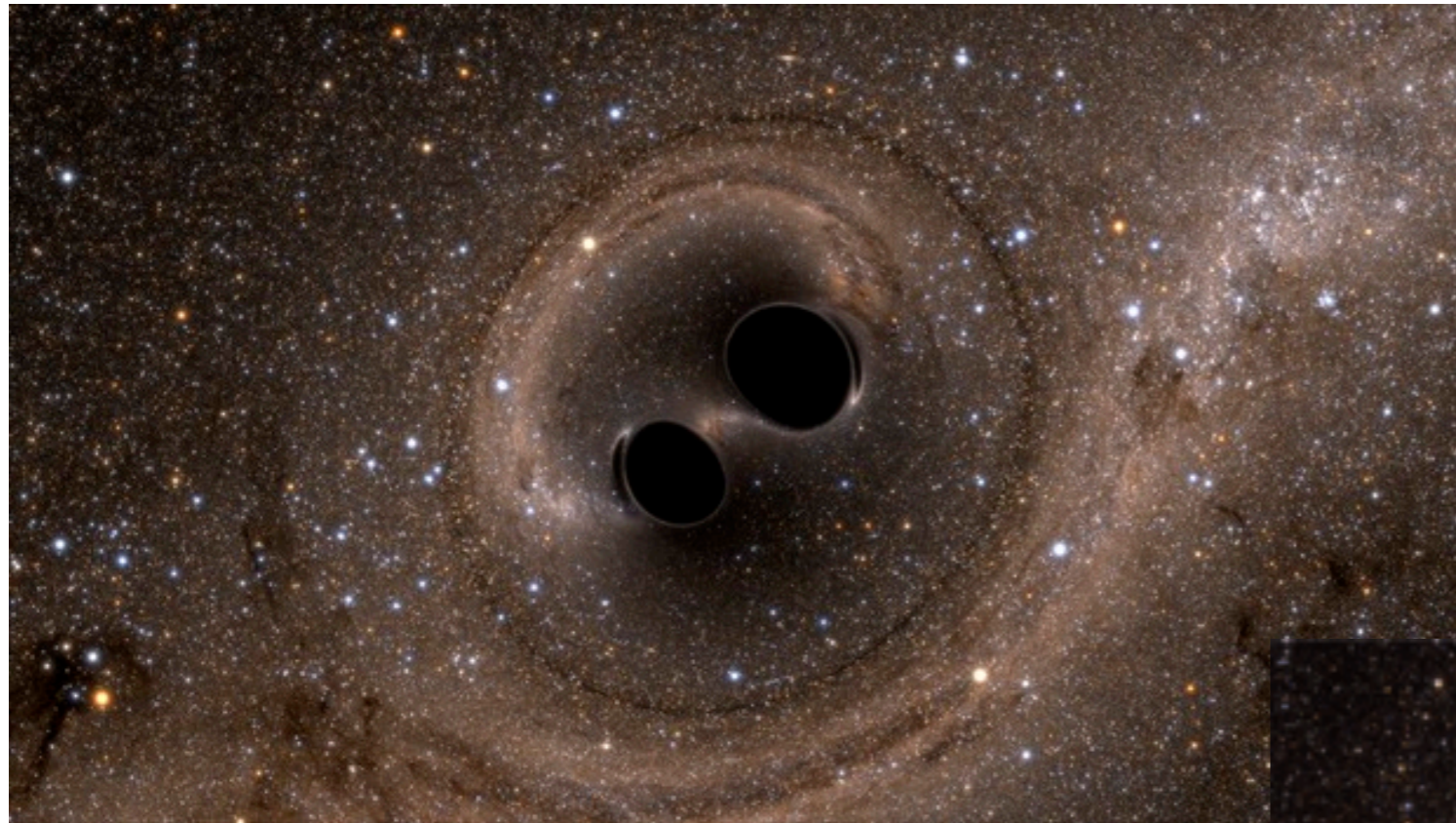
J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

All many-body quantum systems (without quantum gravity) have an entropy proportional to their volume.

Holography: Black holes have an entropy proportional to their surface area, and so they can be represented as a 'hologram' by a quantum many-body system in one lower dimension.

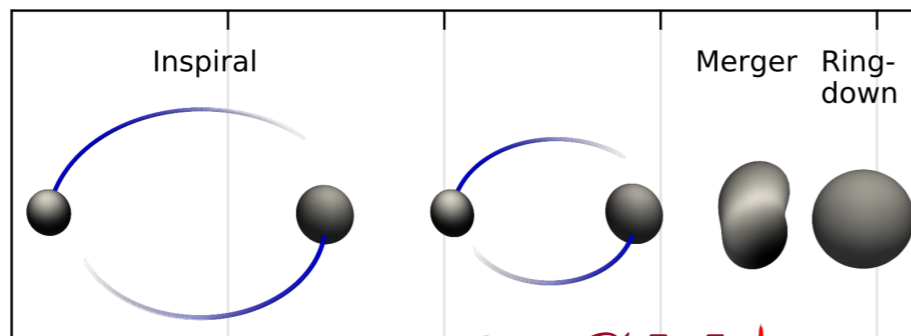
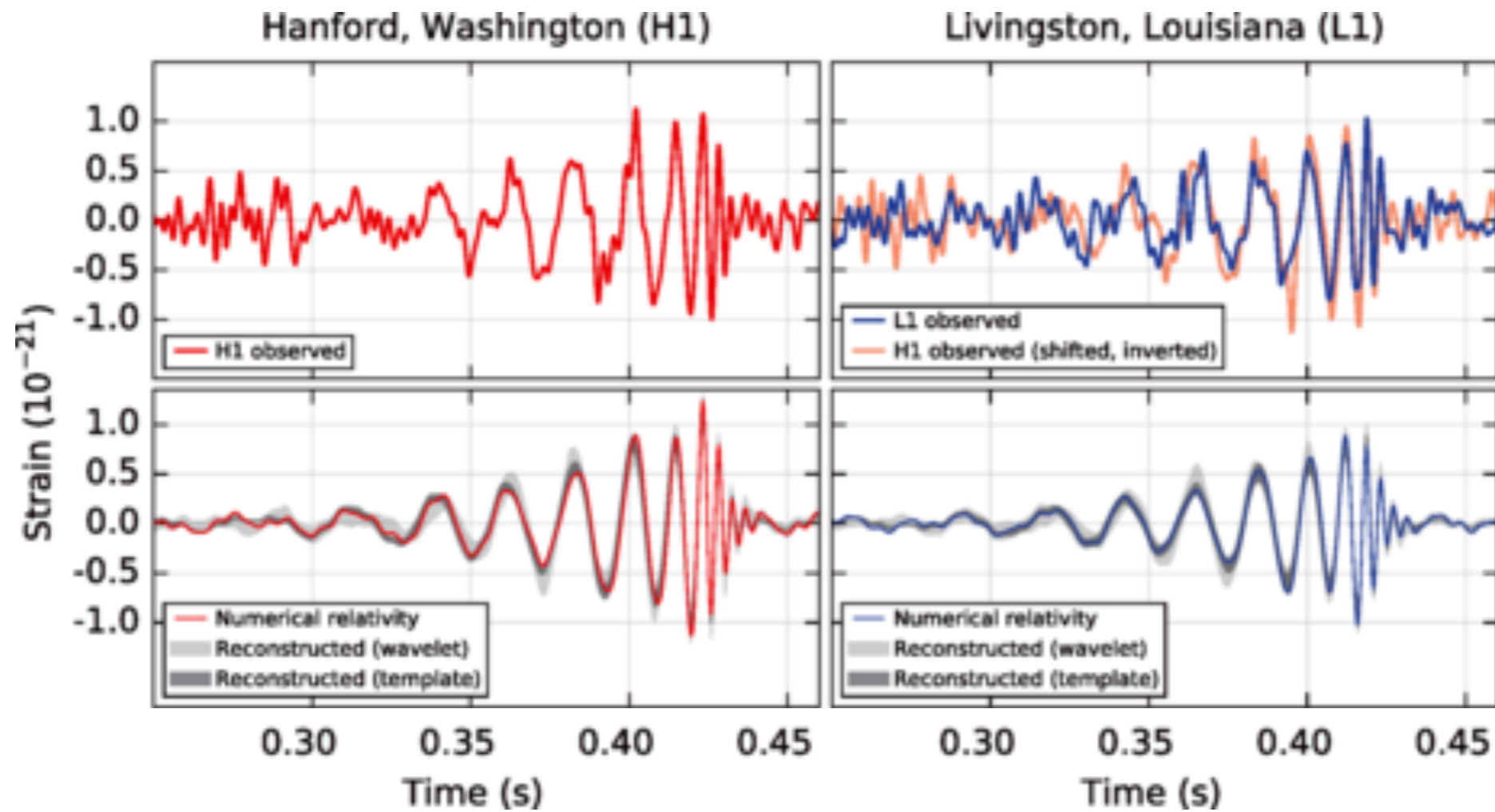
Susskind, Maldacena.....

On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



0.1 seconds later !

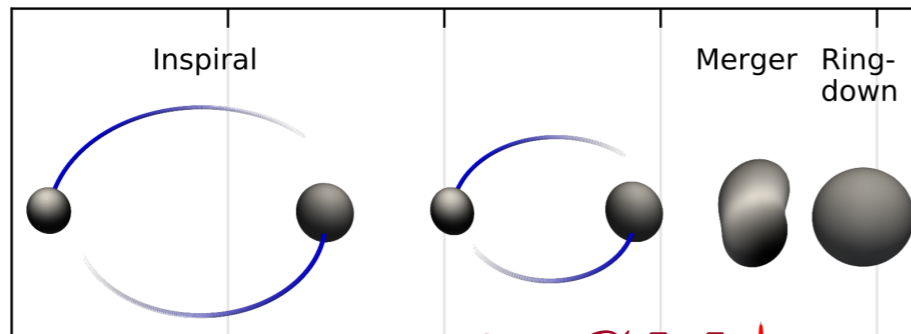
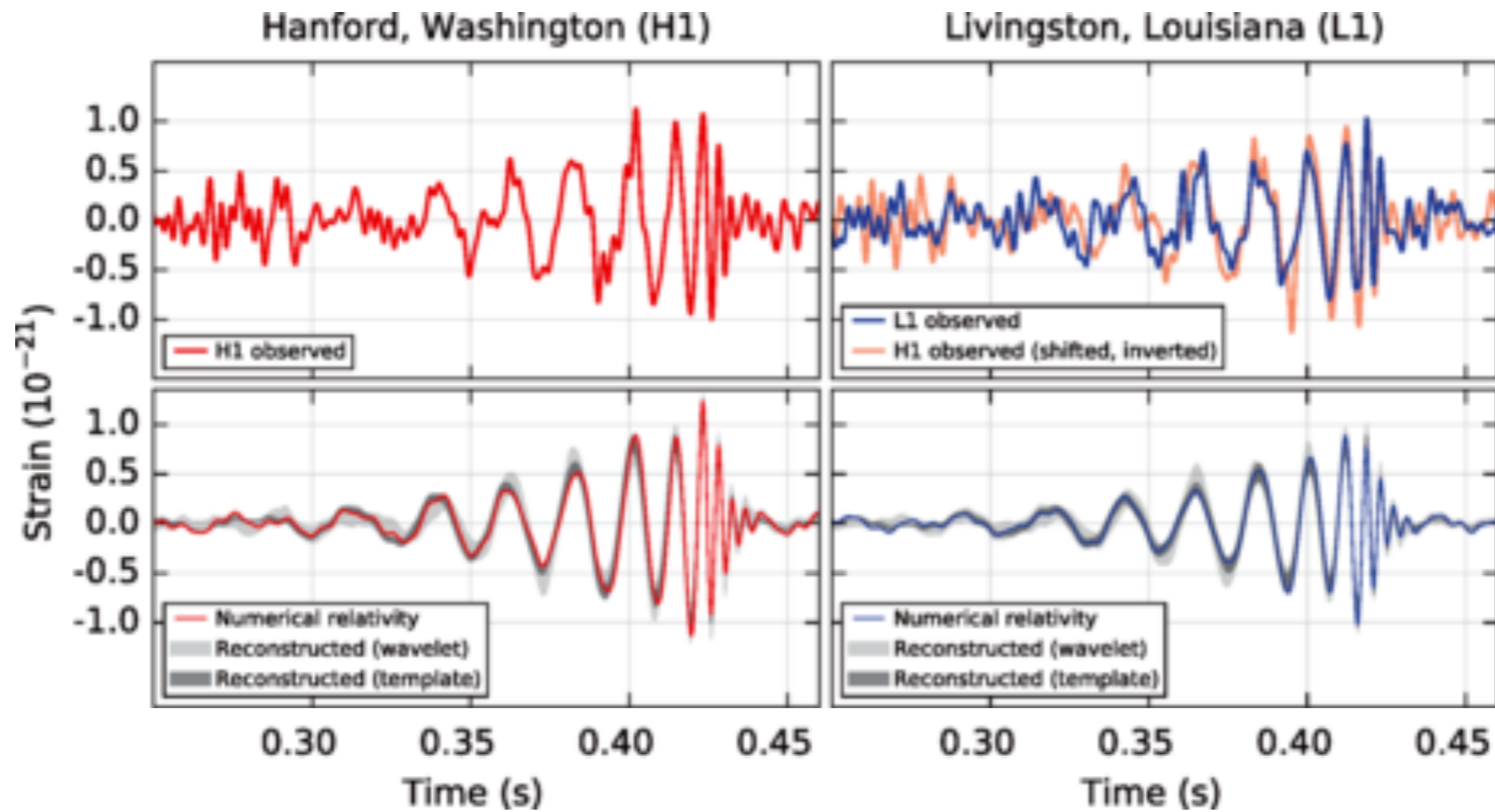




LIGO
September 14, 2015

- The ring-down time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals

$$\frac{\hbar}{k_B T_H}, \quad \hbar \text{ Planck's constant, } k_B \text{ Boltzmann's constant}$$



LIGO
September 14, 2015

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$$\frac{\hbar}{k_B T_H}$$

\hbar Planck's constant, k_B Boltzmann's constant

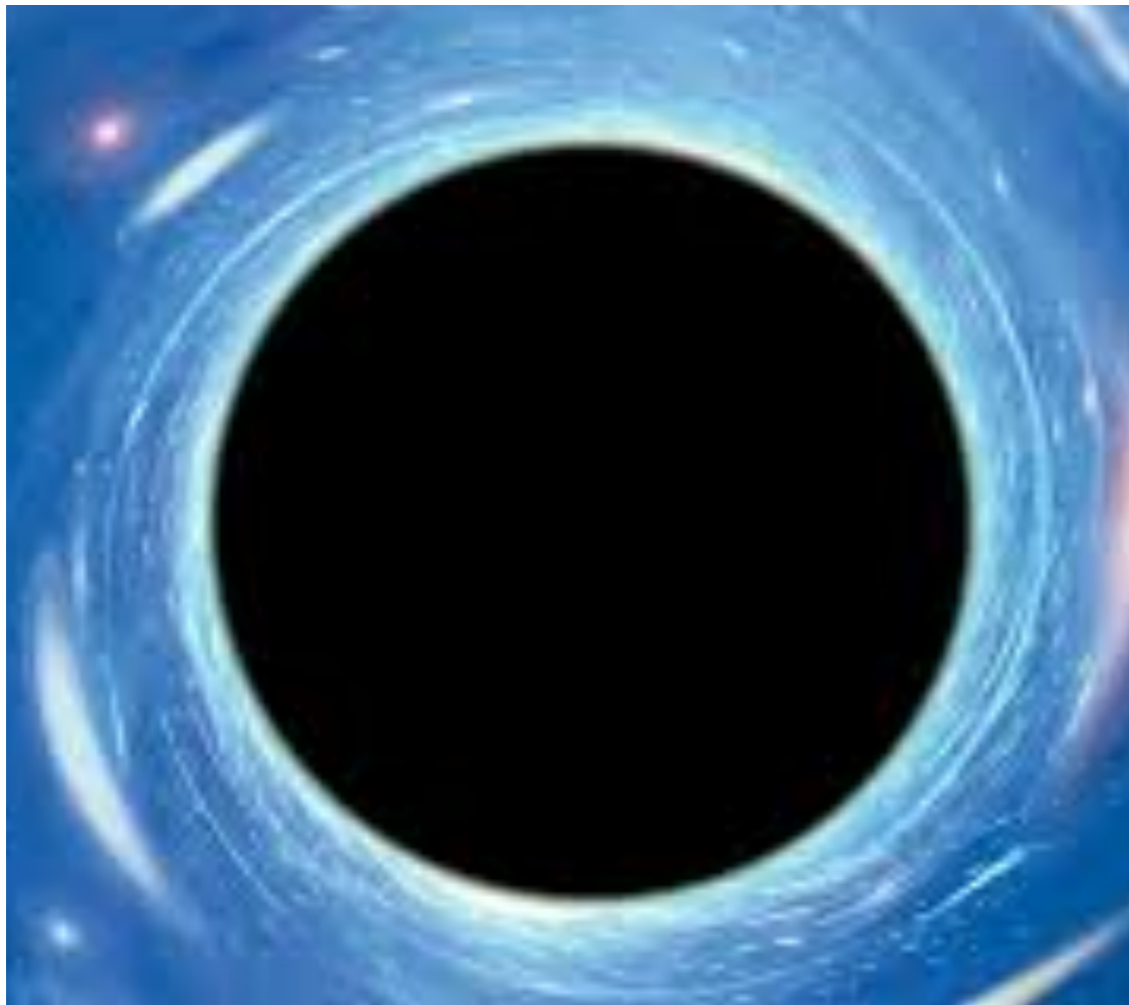
Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.



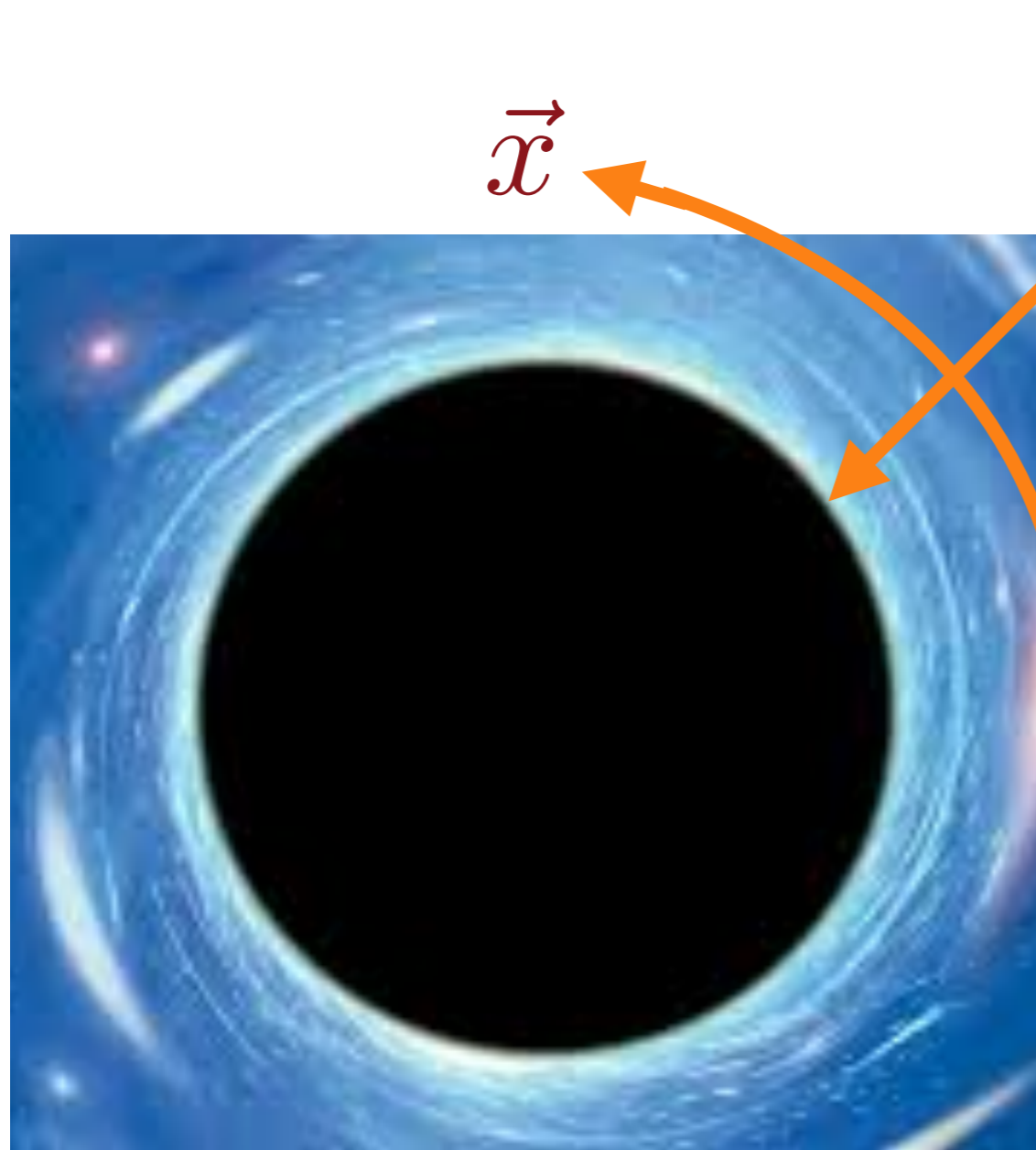


Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge





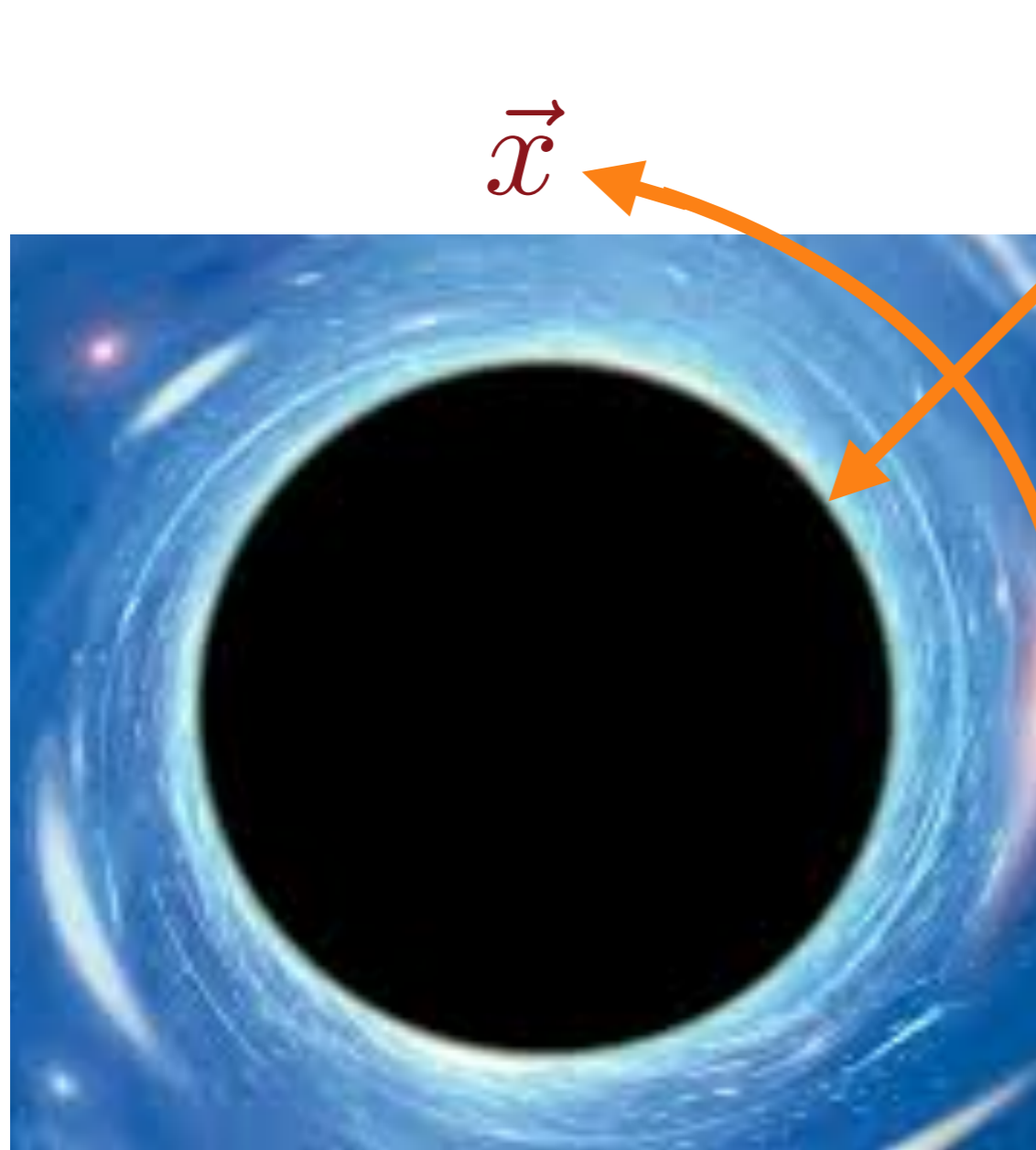
Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space (ζ) and one time dimension

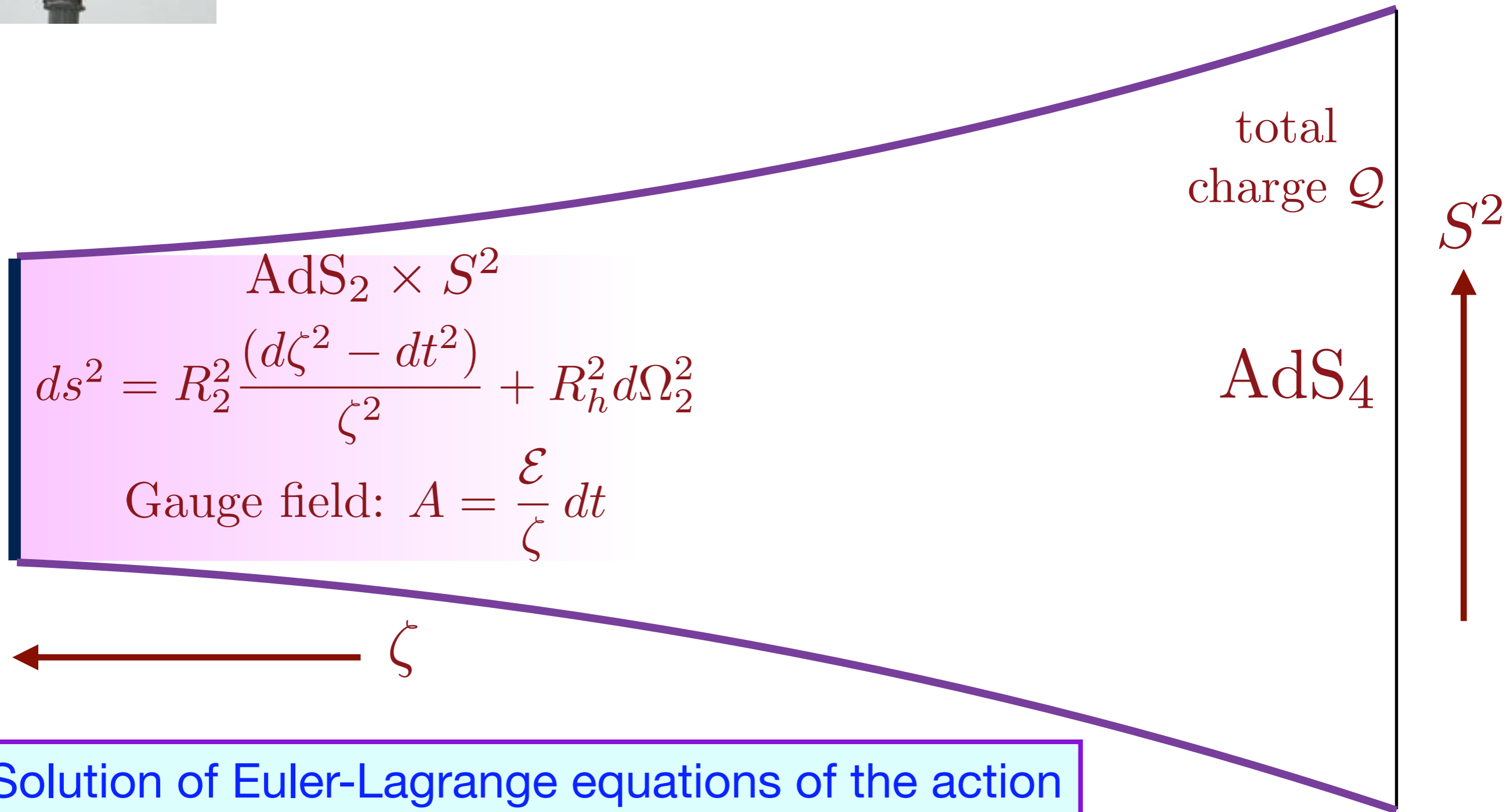


Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



This 2D-gravity theory
is precisely that
appearing in the low T
limit of the
Sachdev-Ye-Kitaev
(SYK) models

SYK model and charged black holes



Solution of Euler-Lagrange equations of the action of Einstein gravity and Maxwell electromagnetism

SYK model and charged black holes



Horizon

Bekenstein-Hawking entropy
connected to the
 S_0 entropy of SYK model

$\text{AdS}_2 \times S^2$

$$ds^2 = R_2^2 \frac{(d\zeta^2 - dt^2)}{\zeta^2} + R_h^2 d\Omega_2^2$$

$$\text{Gauge field: } A = \frac{\mathcal{E}}{\zeta} dt$$

total
charge Q

AdS_4

S^2

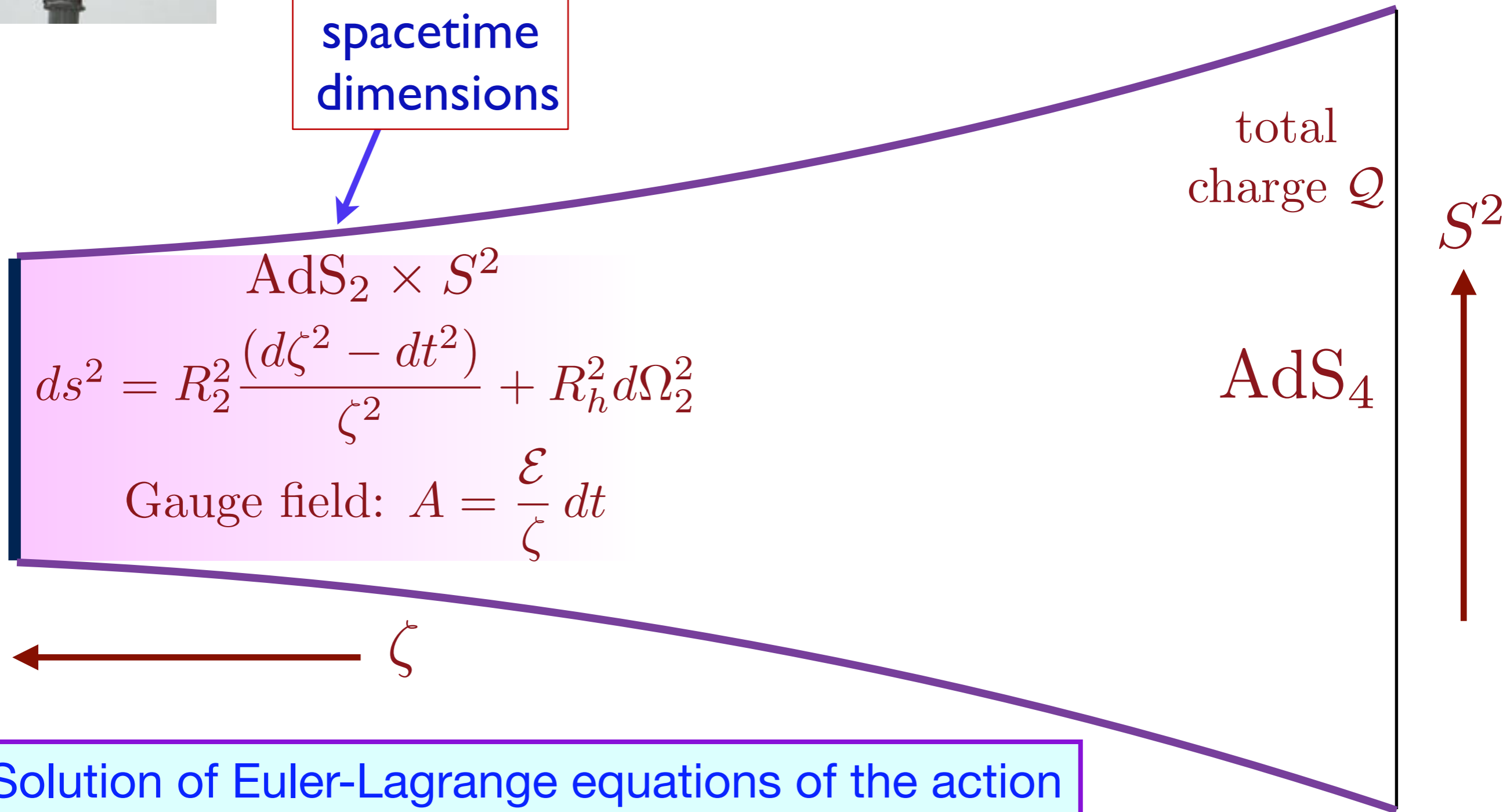
ζ

Solution of Euler-Lagrange equations of the action
of Einstein gravity and Maxwell electromagnetism

SYK model and charged black holes



|+|
spacetime
dimensions



Solution of Euler-Lagrange equations of the action
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SYK model and charged black holes



1+1
spacetime
dimensions

3+1
spacetime
dimensions

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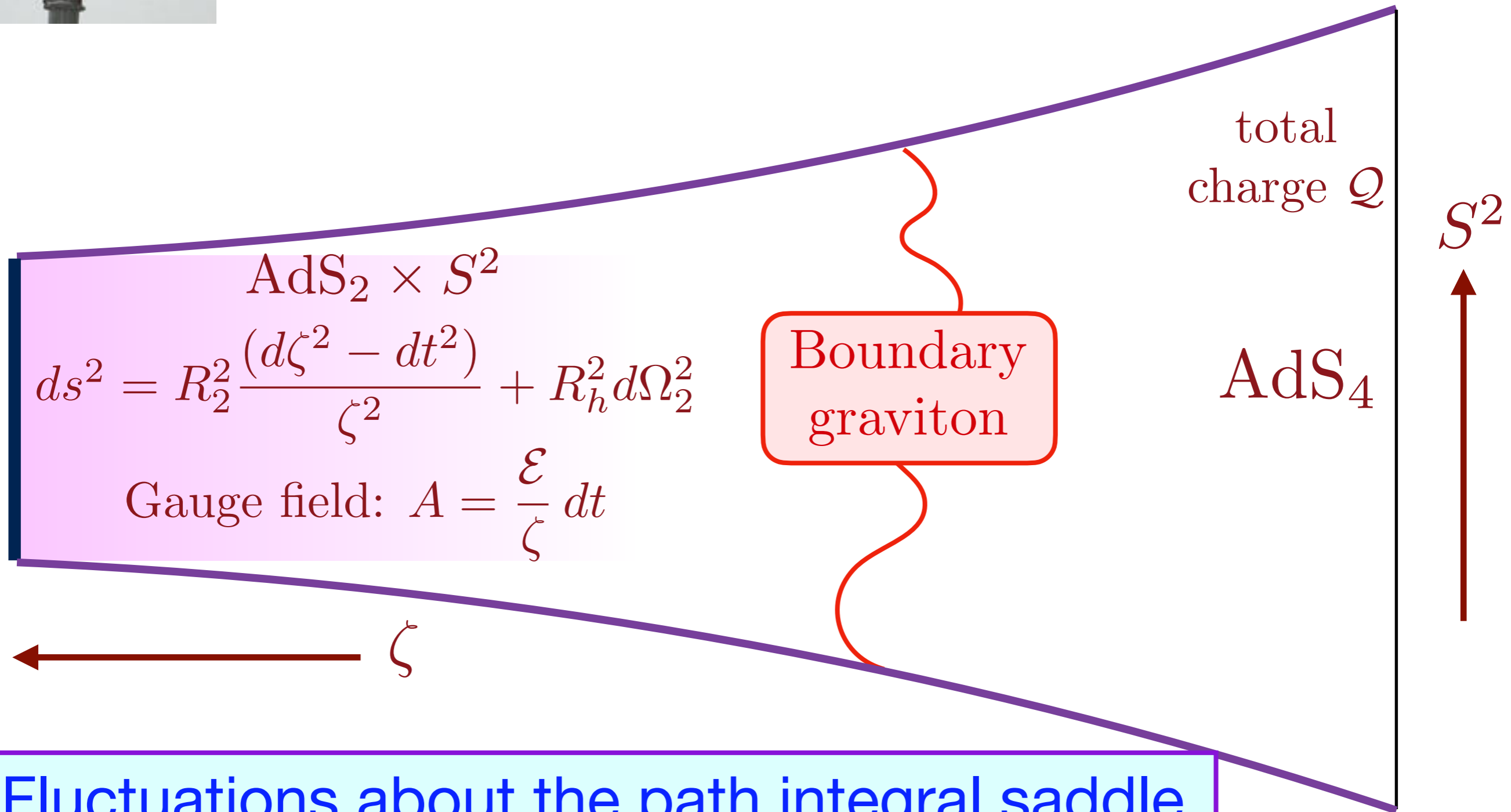
AdS_4

S^2

ζ

Solution of Euler-Lagrange equations of the action
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SYK model and charged black holes



Fluctuations about the path integral saddle of quantum gravity AND SYK model

Quantum
entanglement

A simple
many-particle
(SYK) model

Charged
black holes

Low temperatures

Quantum gravity in
1+1 dimensions

Quantum entanglement

A simple many-particle (SYK) model

Charged black holes

Low temperatures

Quantum gravity in 1+1 dimensions

Complex multi-particle entanglement
leads to quantum systems
without quasiparticle excitations.

Many-body chaos and
thermal equilibration
in the shortest possible
Planckian time $\sim \frac{\hbar}{k_B T}$.

Quantum entanglement

A simple many-particle (SYK) model

Charged black holes

Low temperatures

Quantum gravity in 1+1 dimensions

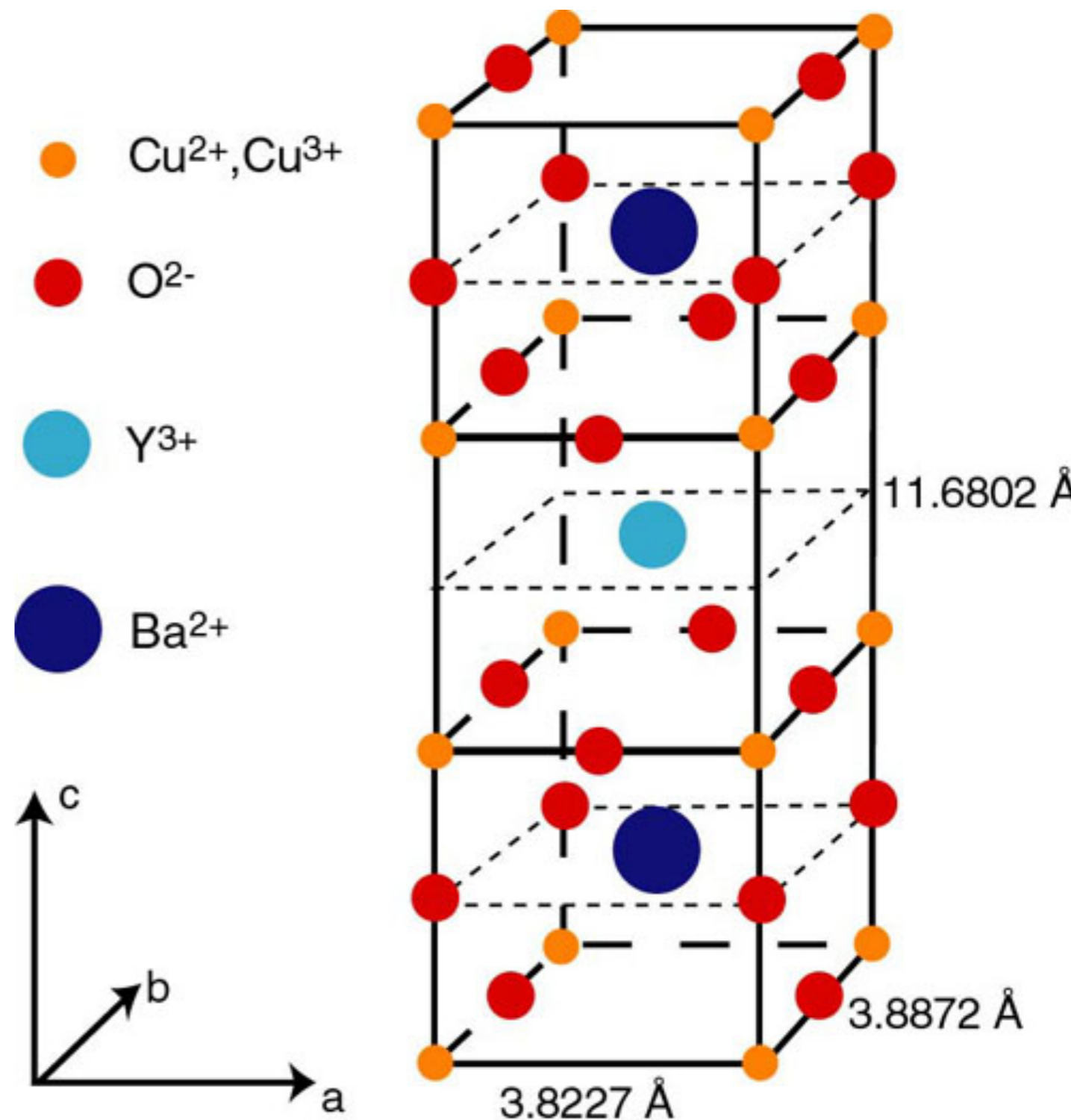
**Quantum
entanglement**

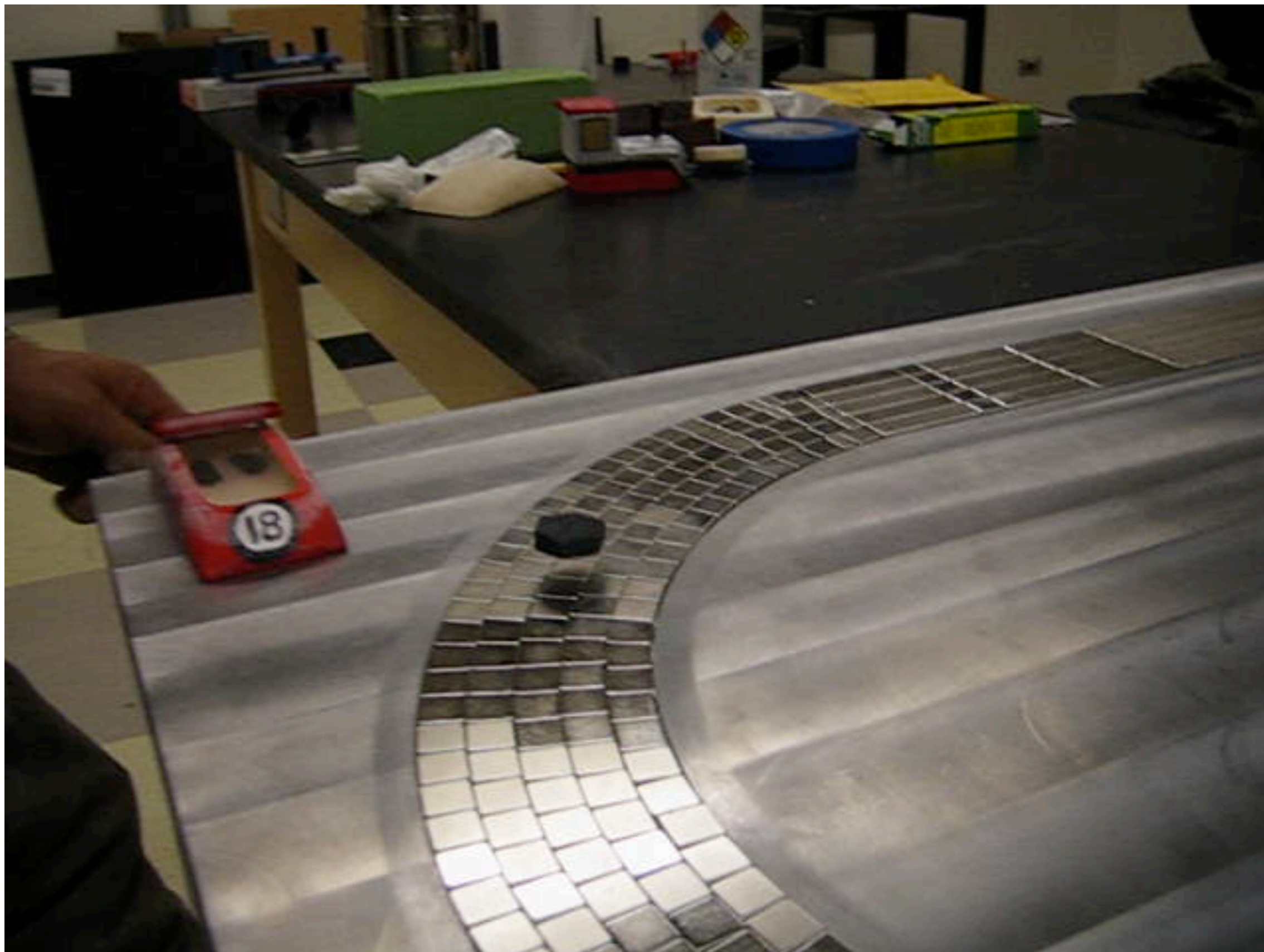
**Charged
black holes**

**A simple
many-particle
(SYK) model**

**Copper-based
superconductors**

High temperature superconductors

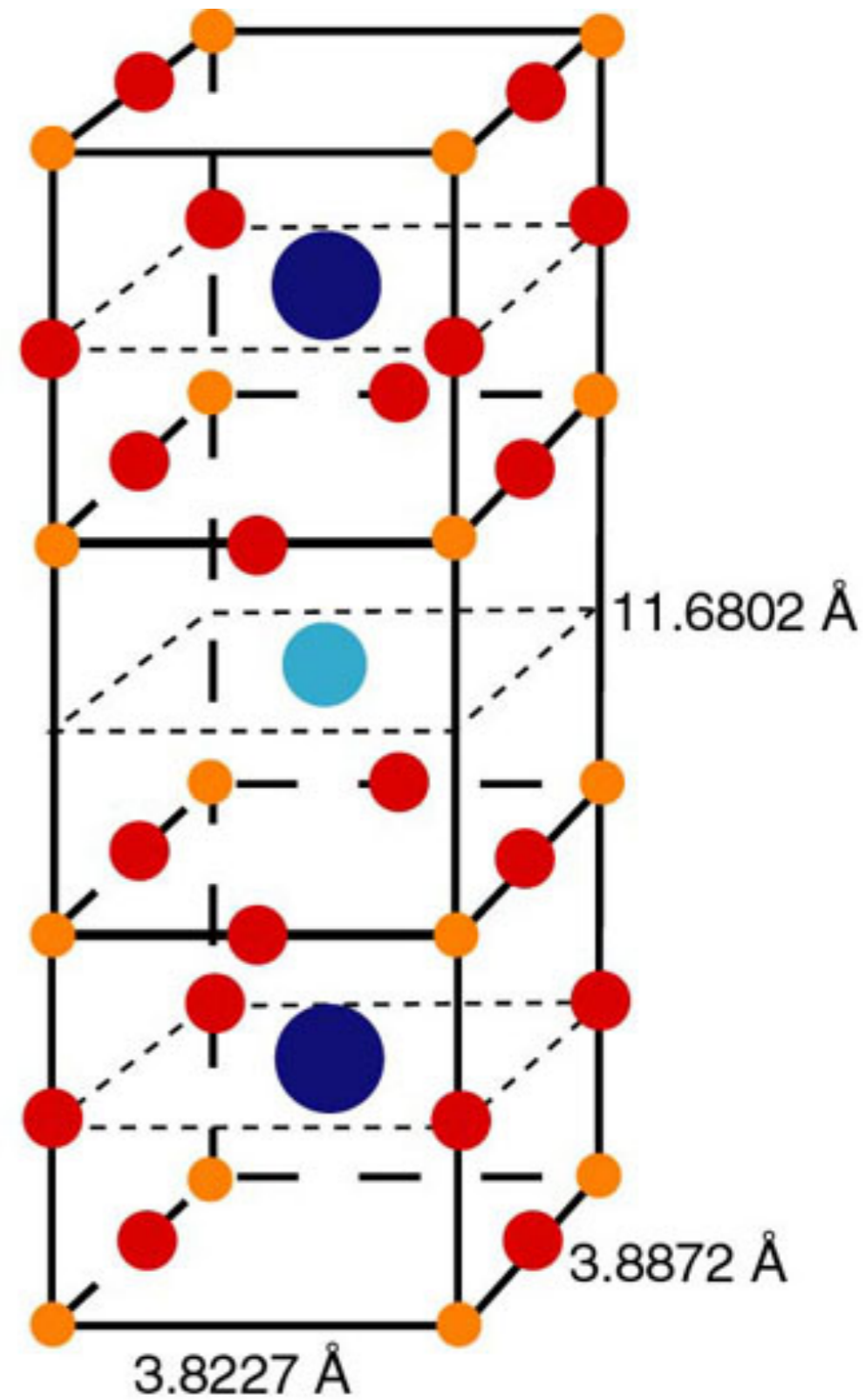
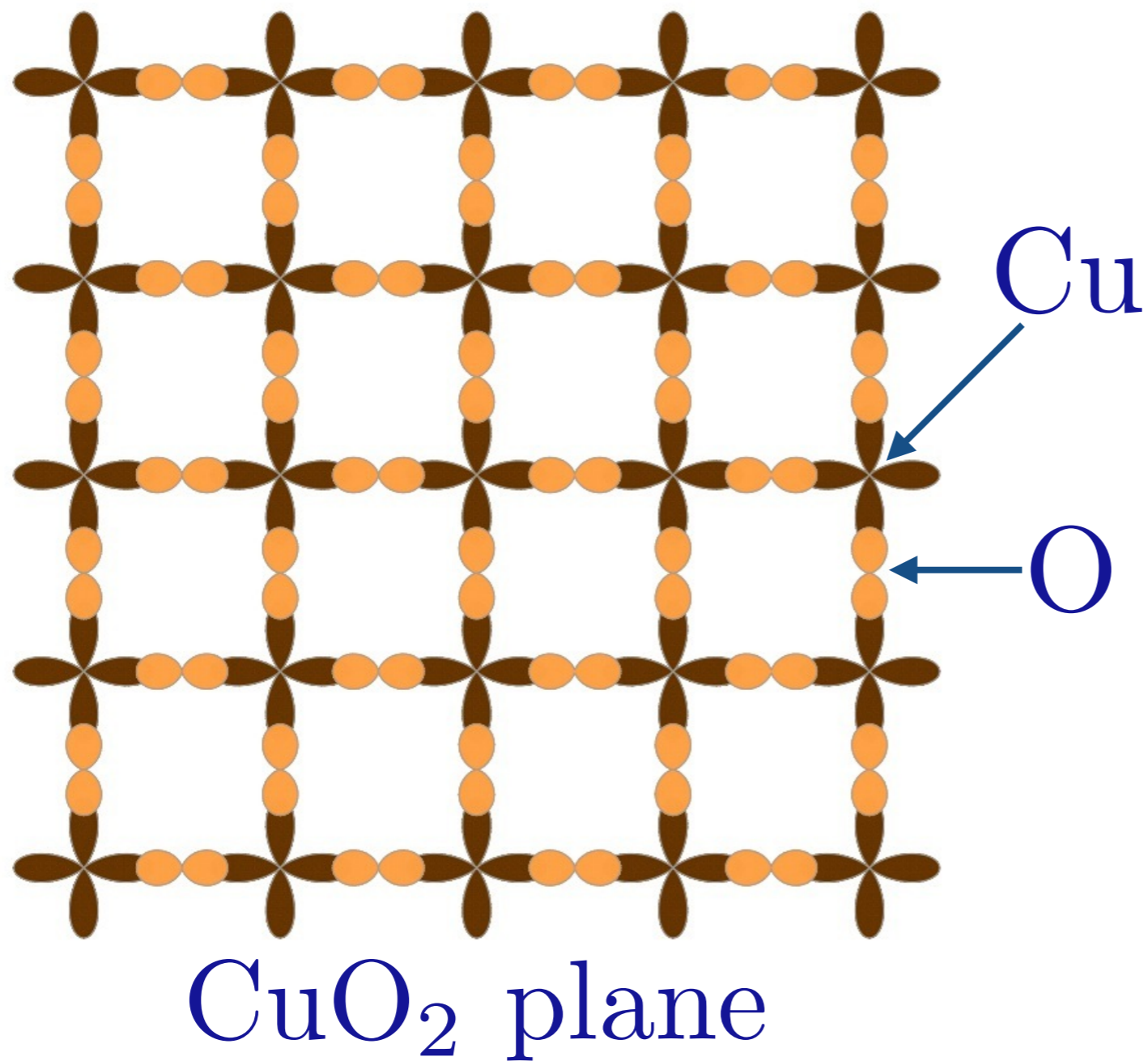




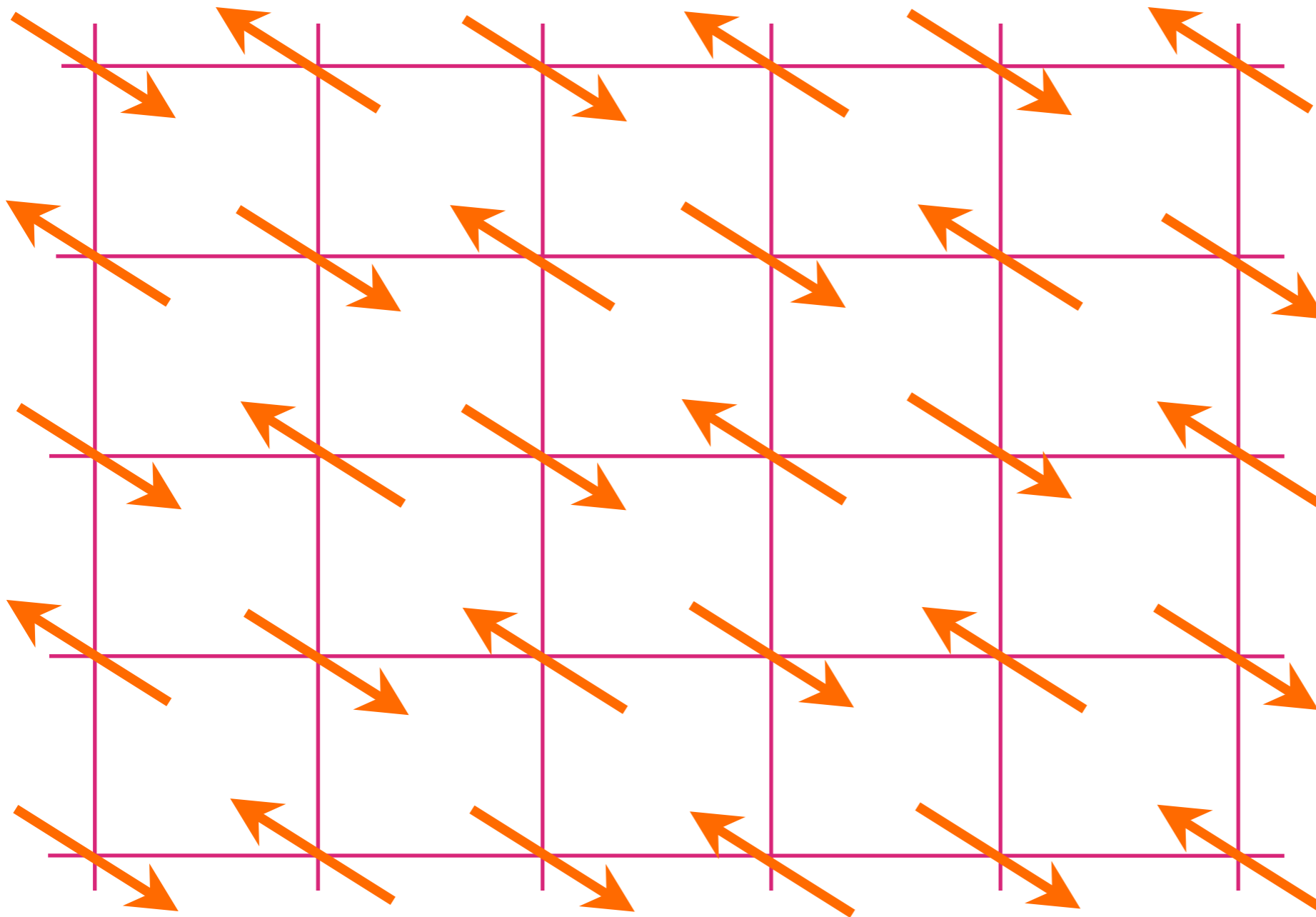
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

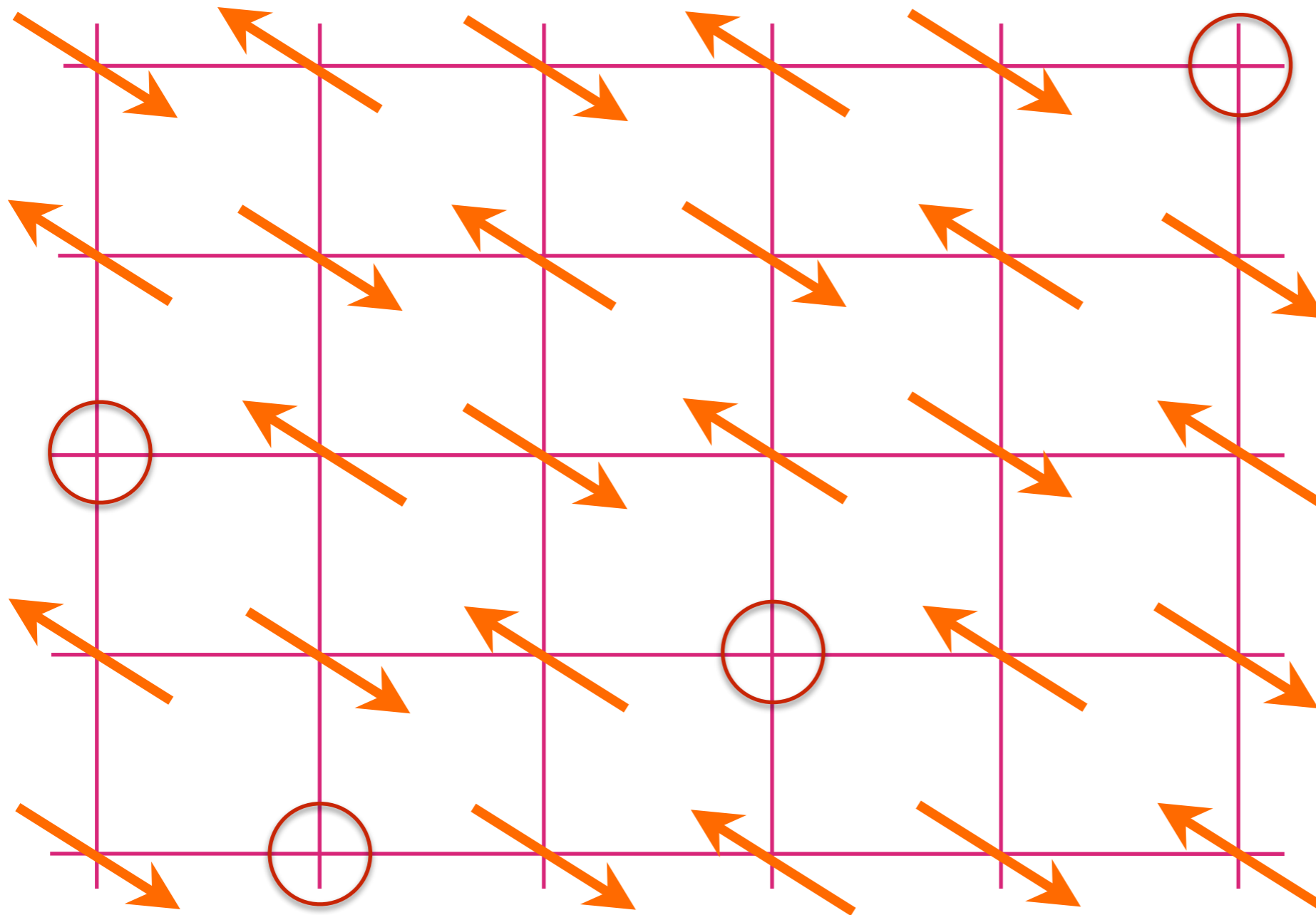
High temperature superconductors



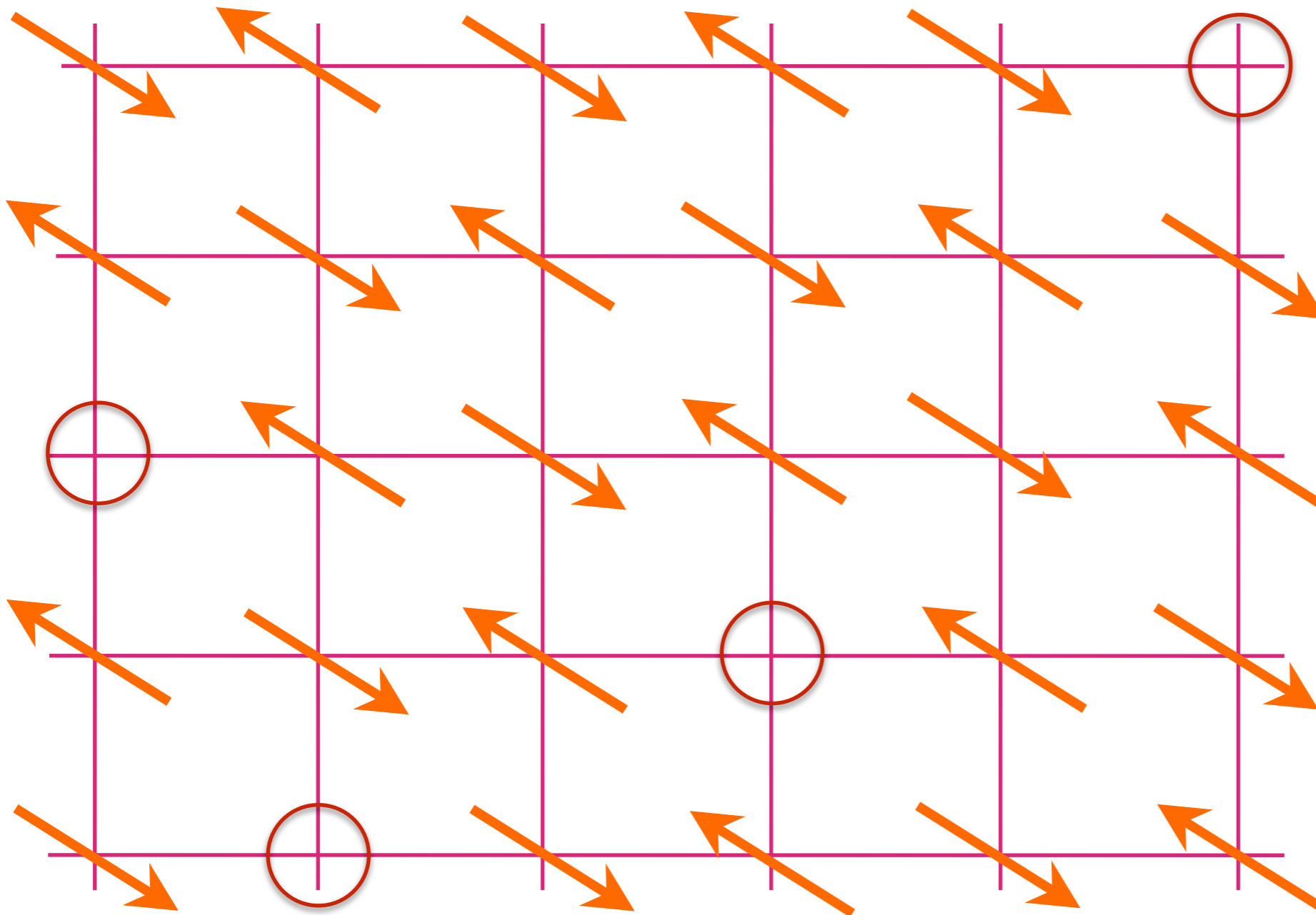
Insulating antiferromagnet



Antiferromagnet doped with hole density p

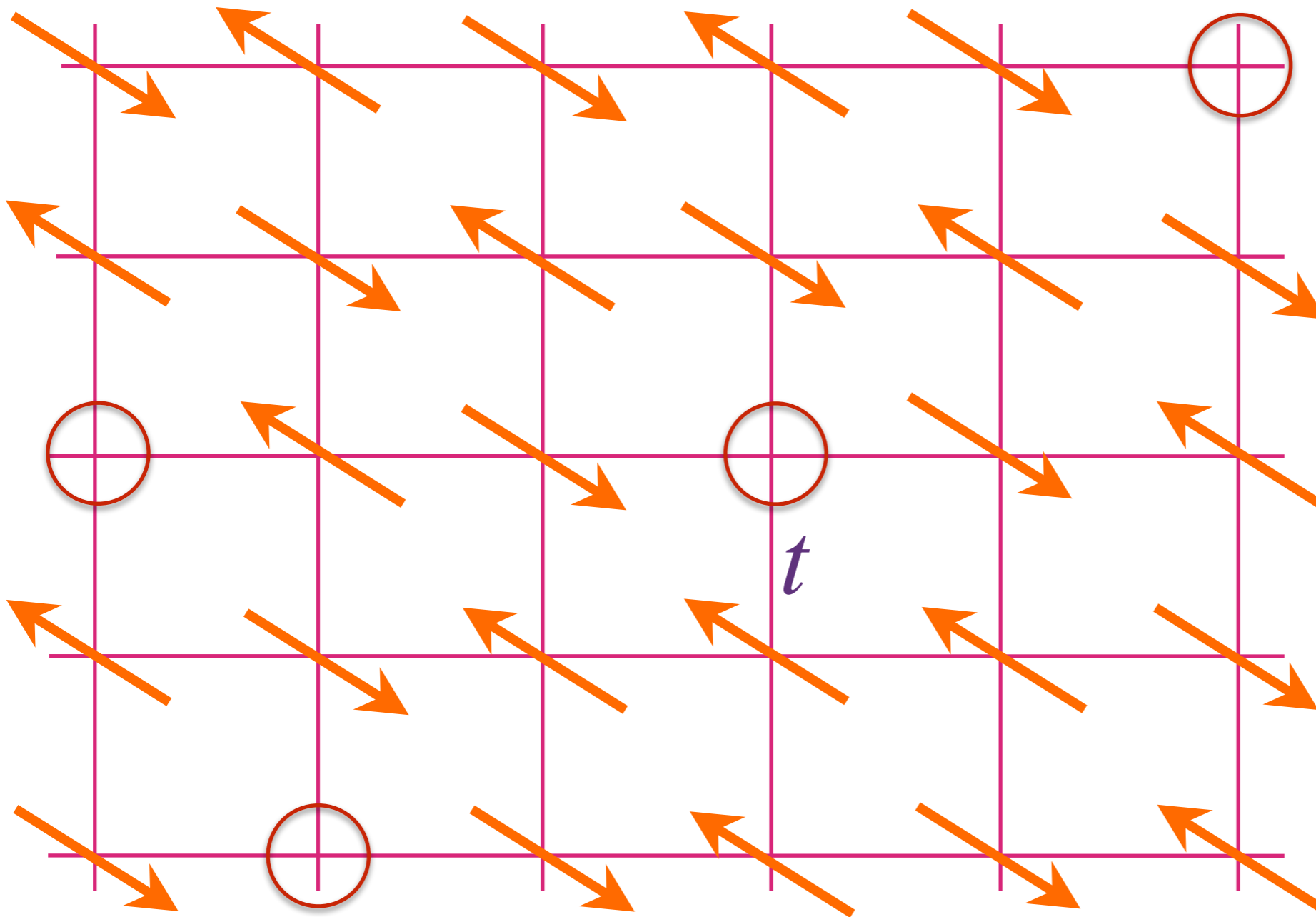


Real-space view



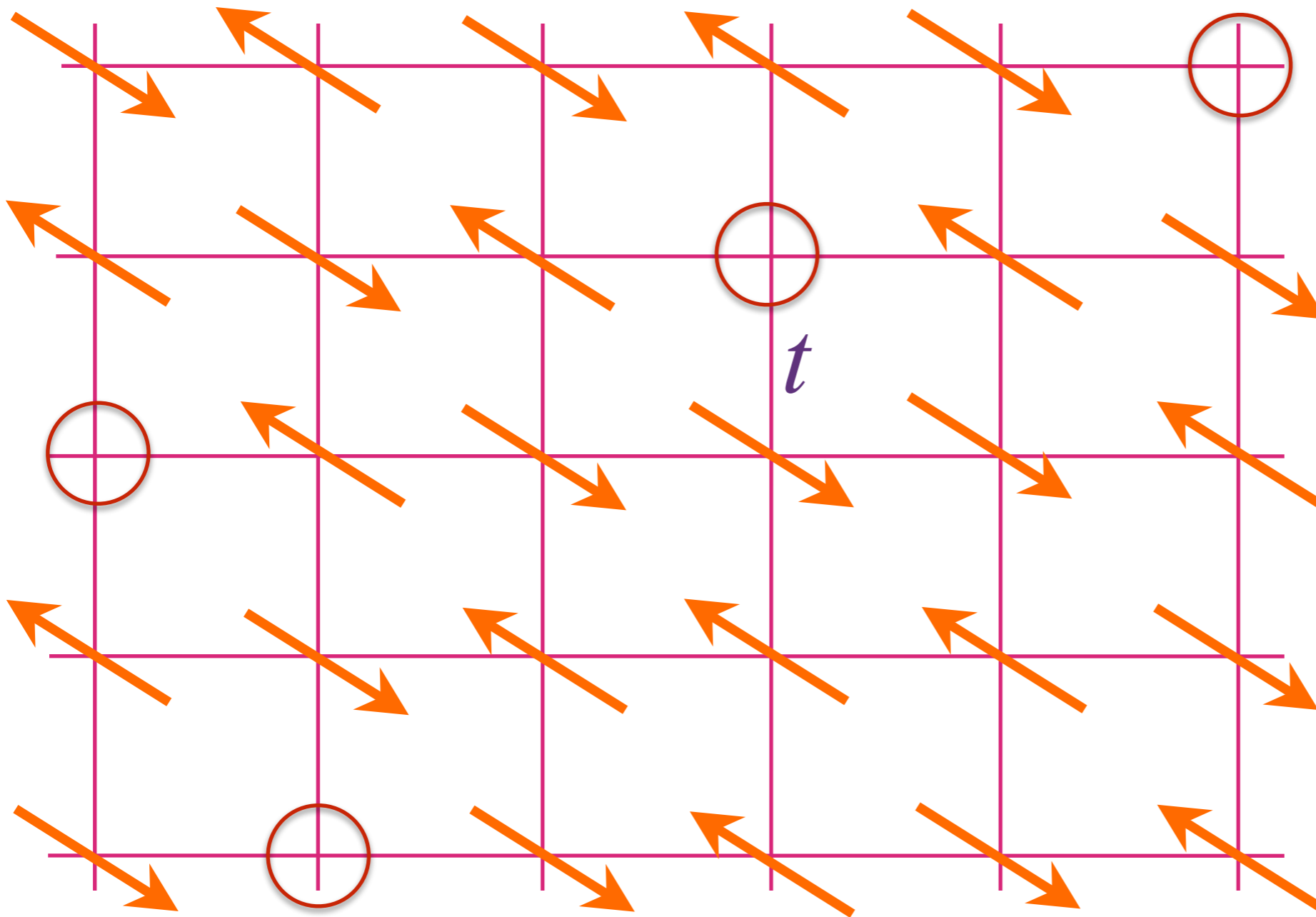
p mobile holes in a background of
fluctuating spins

Real-space view



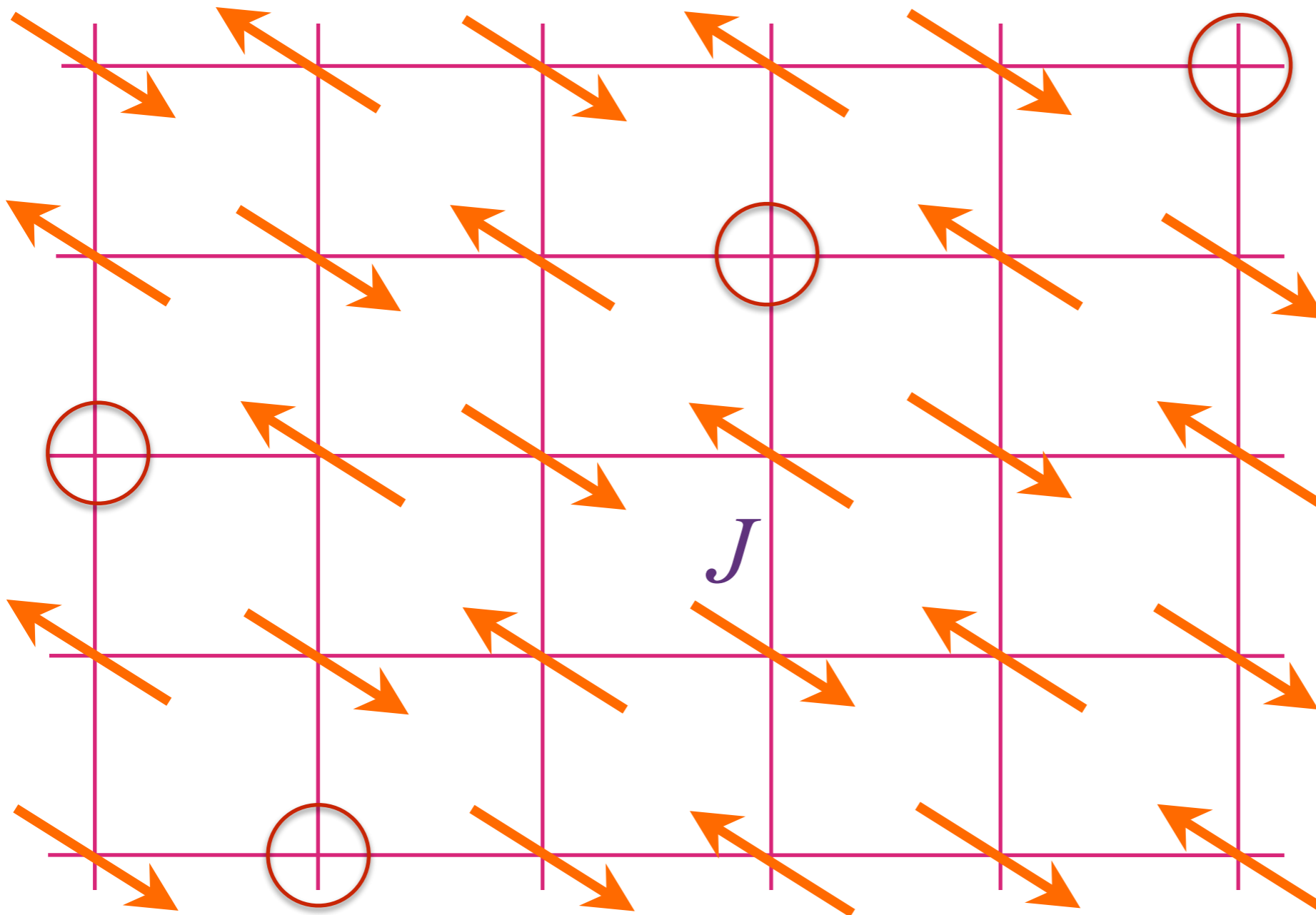
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Real-space view



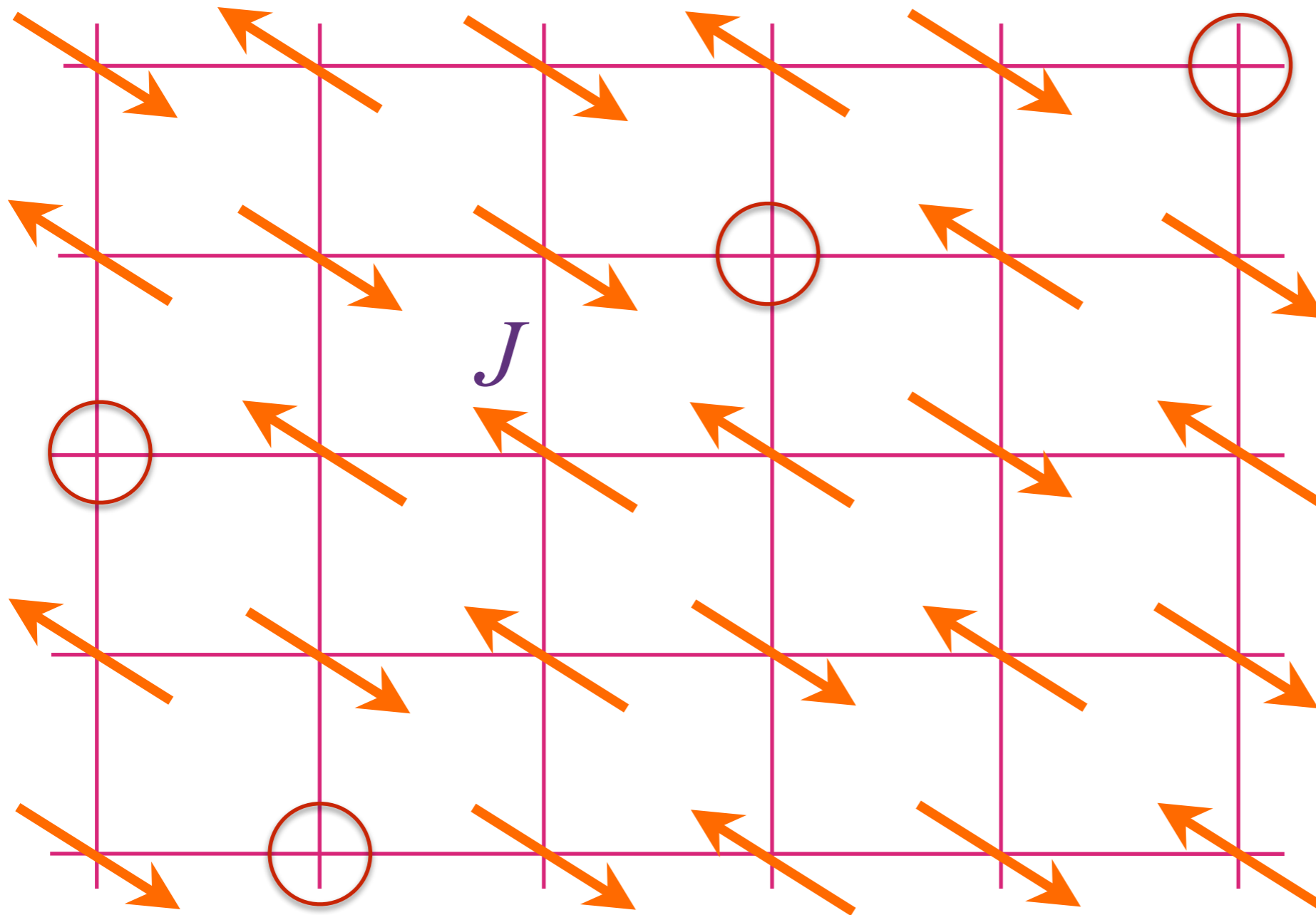
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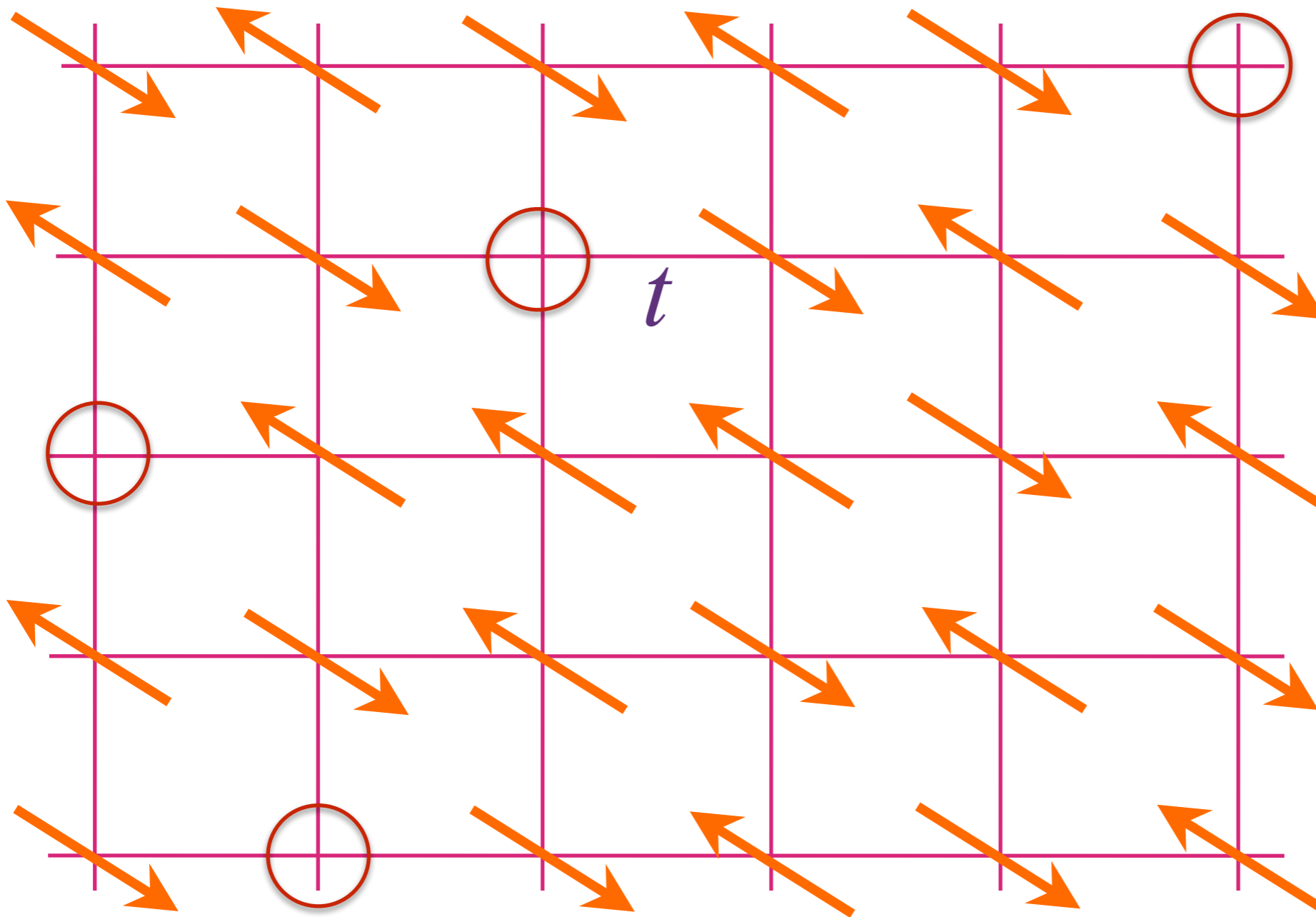
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Real-space view



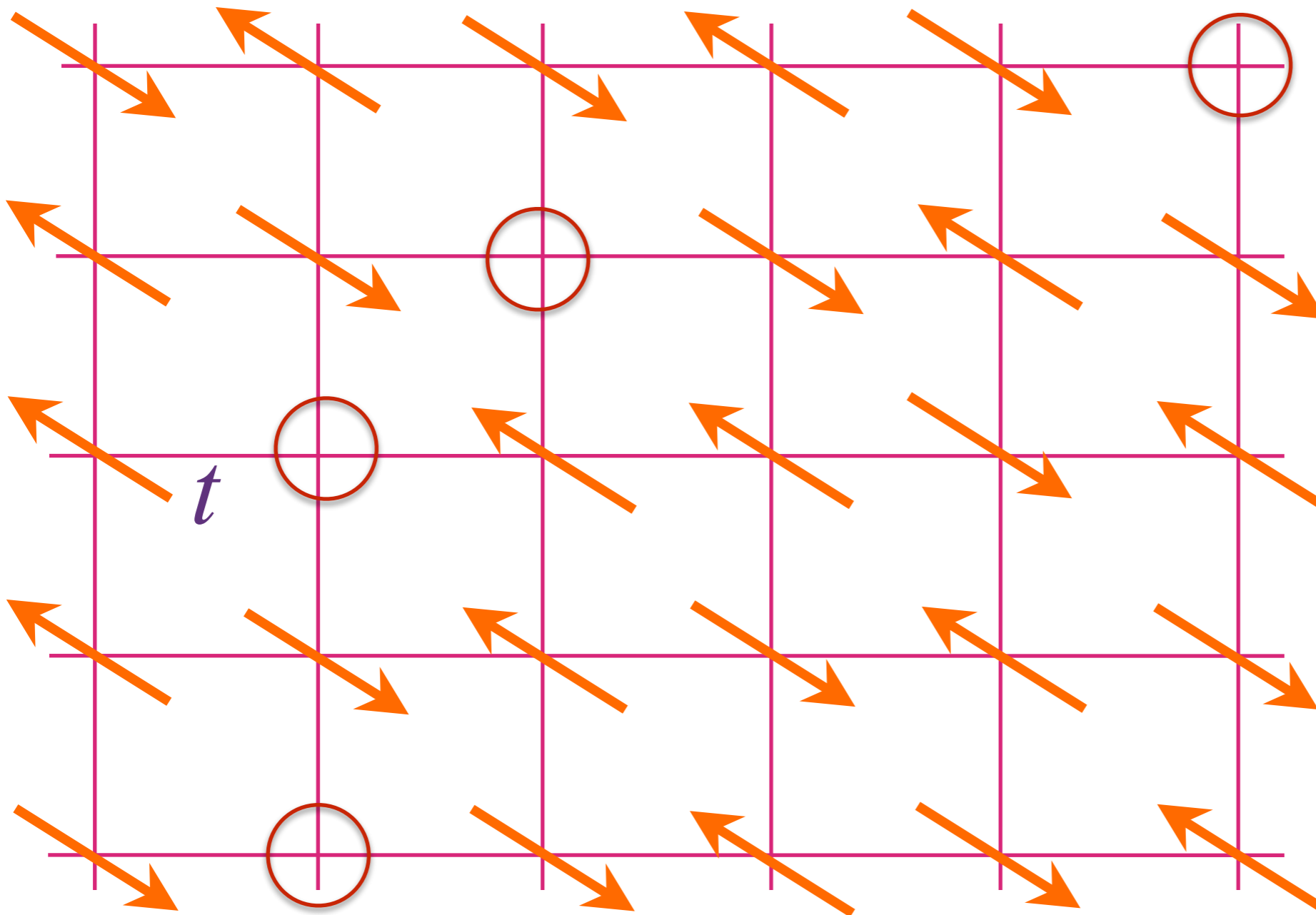
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Real-space view



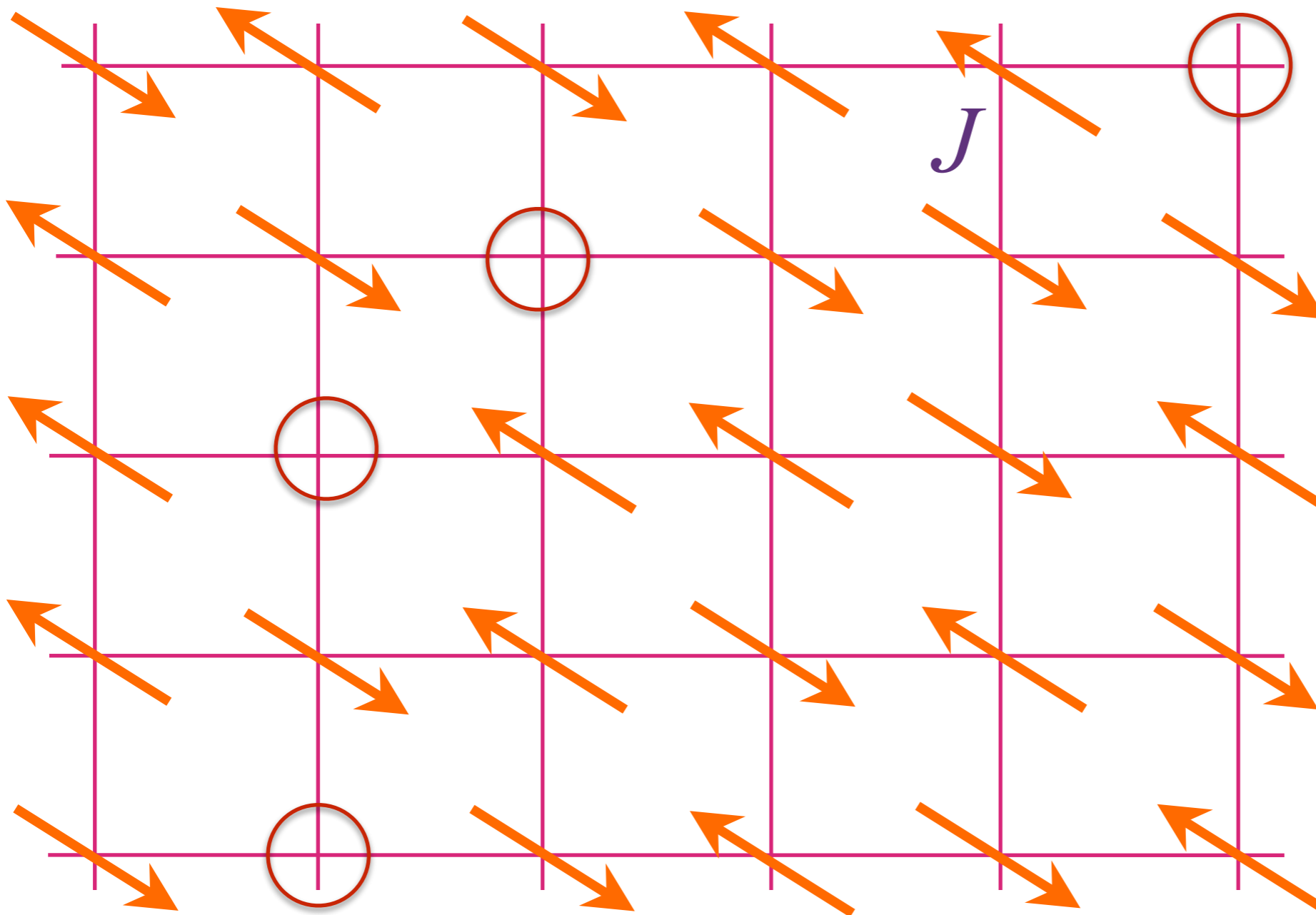
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Real-space view



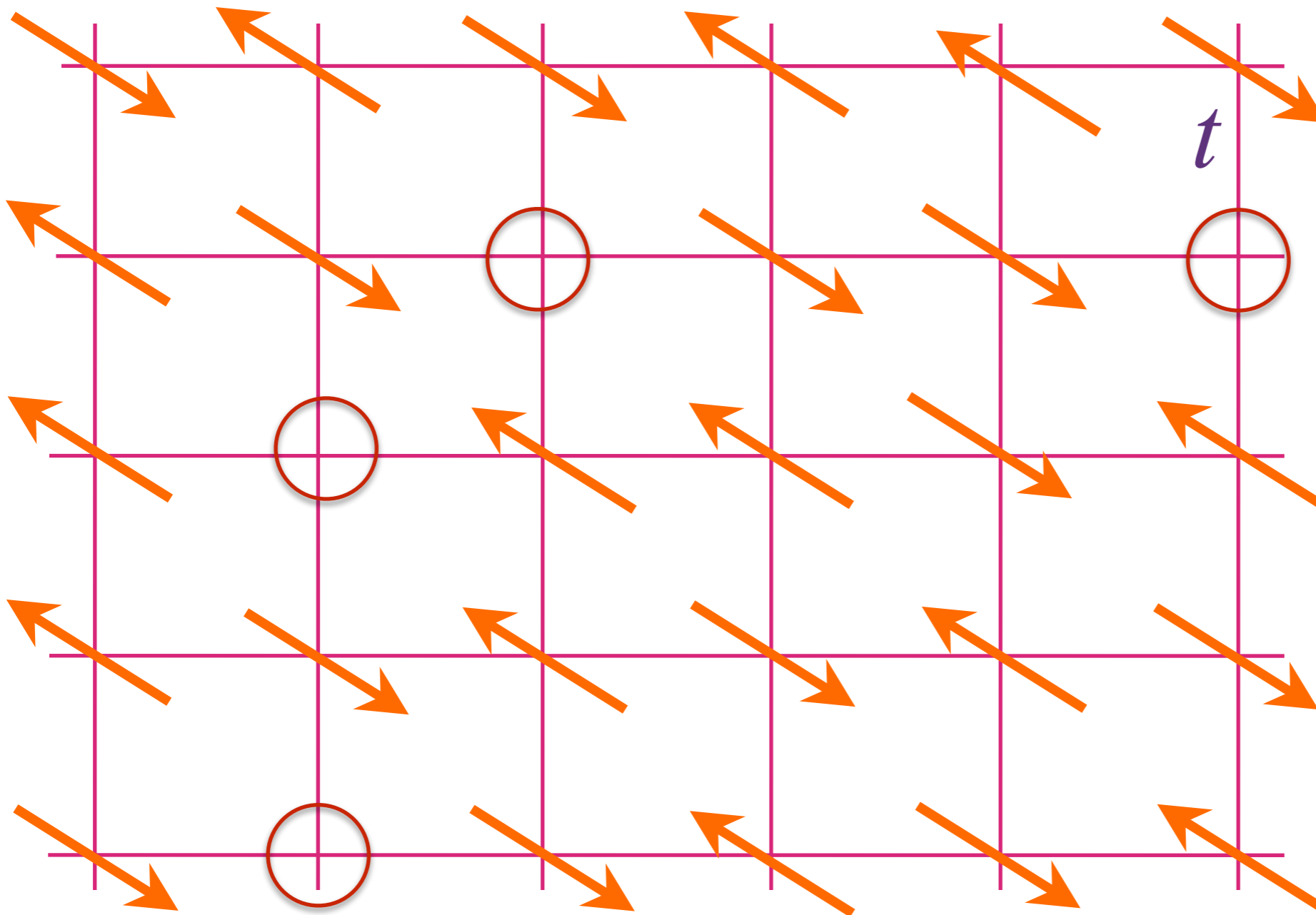
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Real-space view

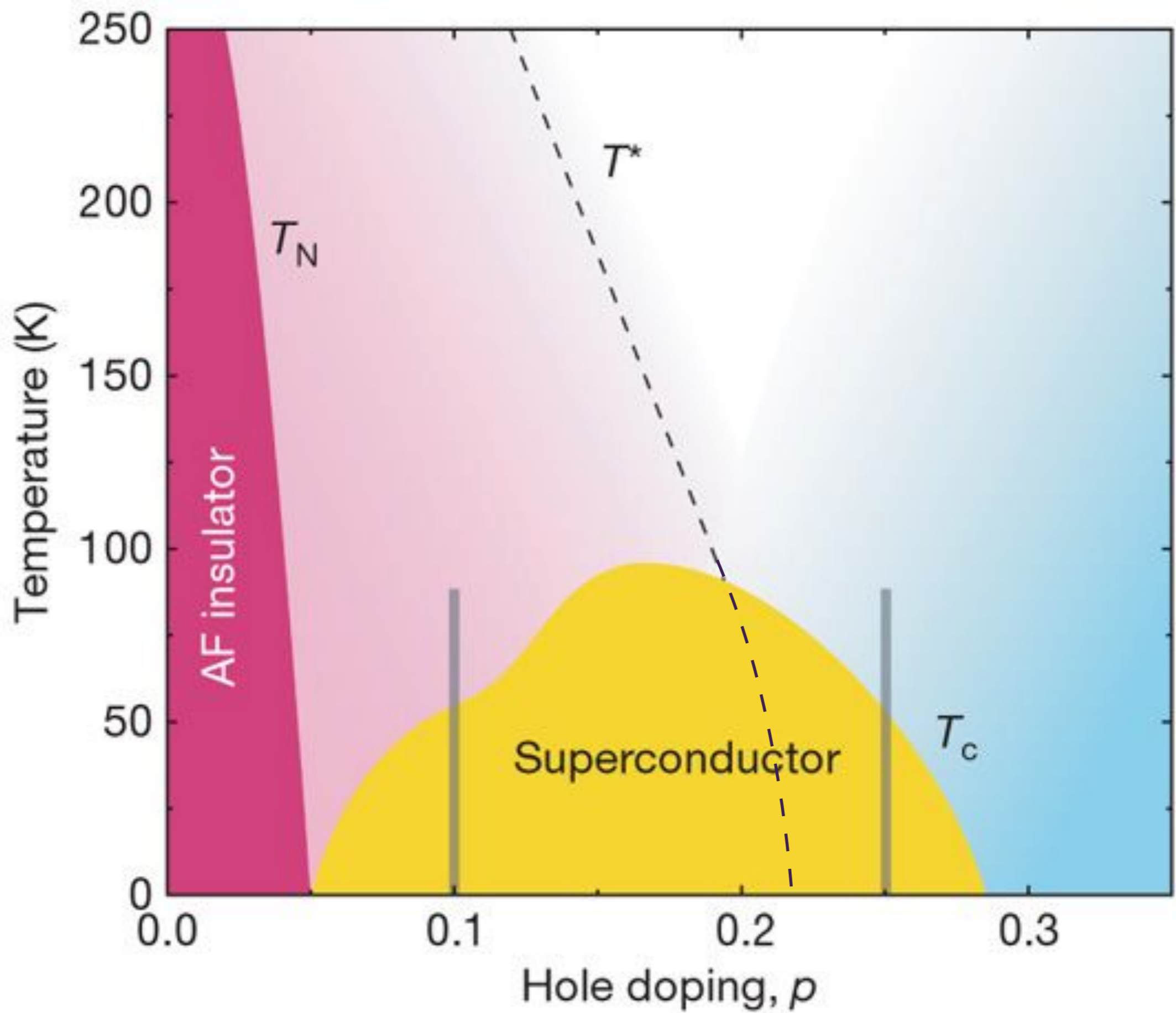


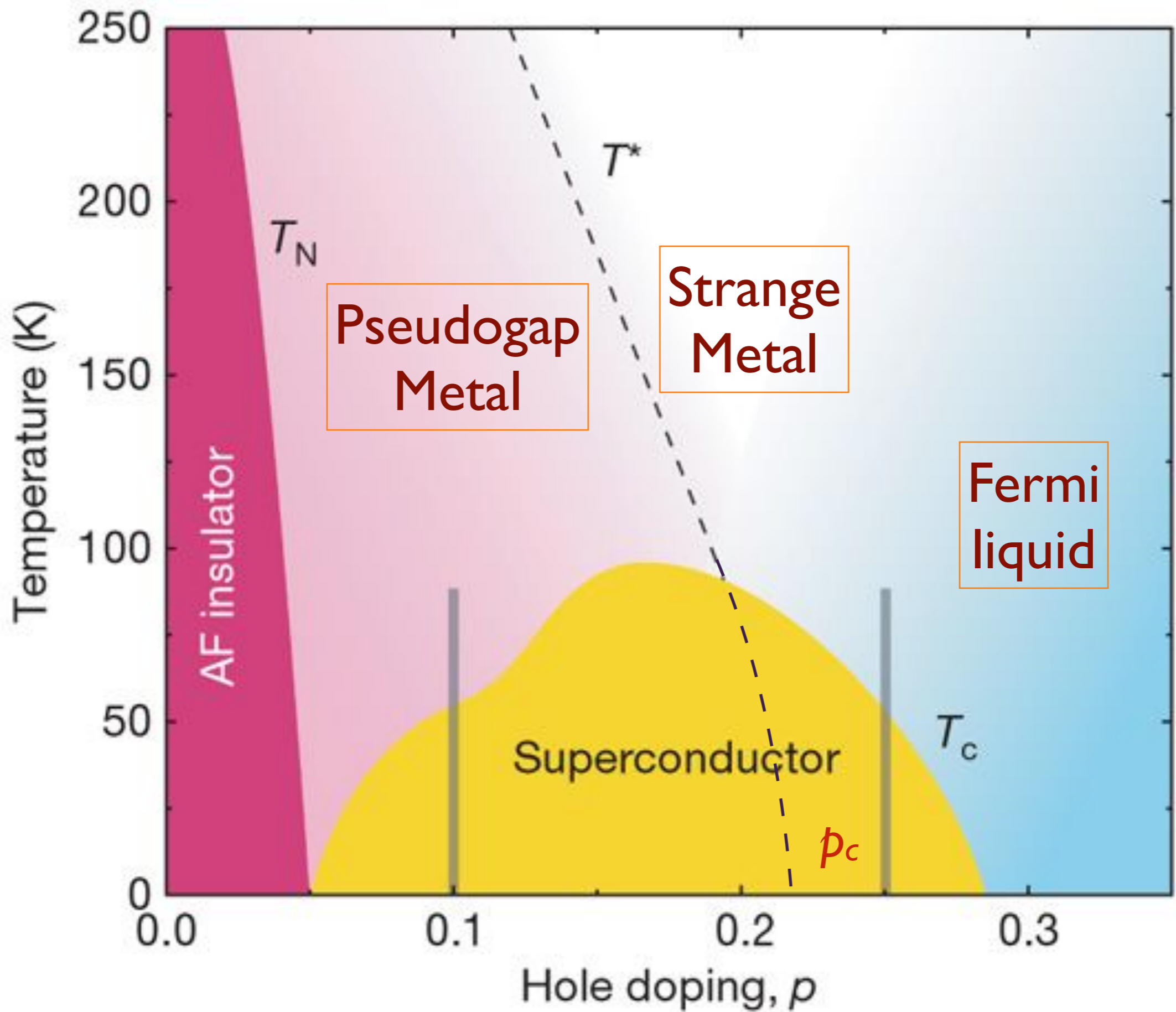
p mobile holes in a background of
fluctuating spins

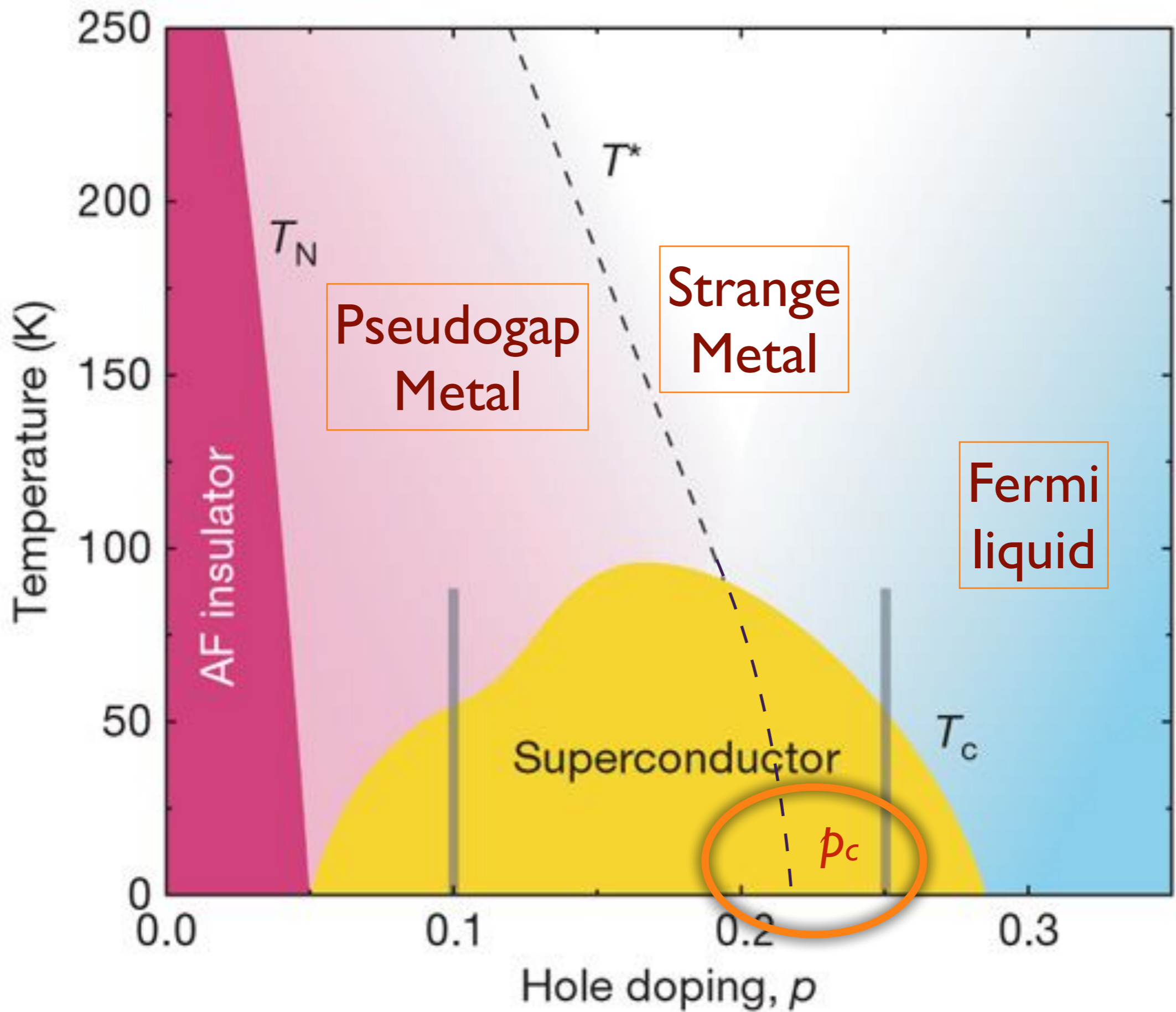
Real-space view



p mobile holes in a background of
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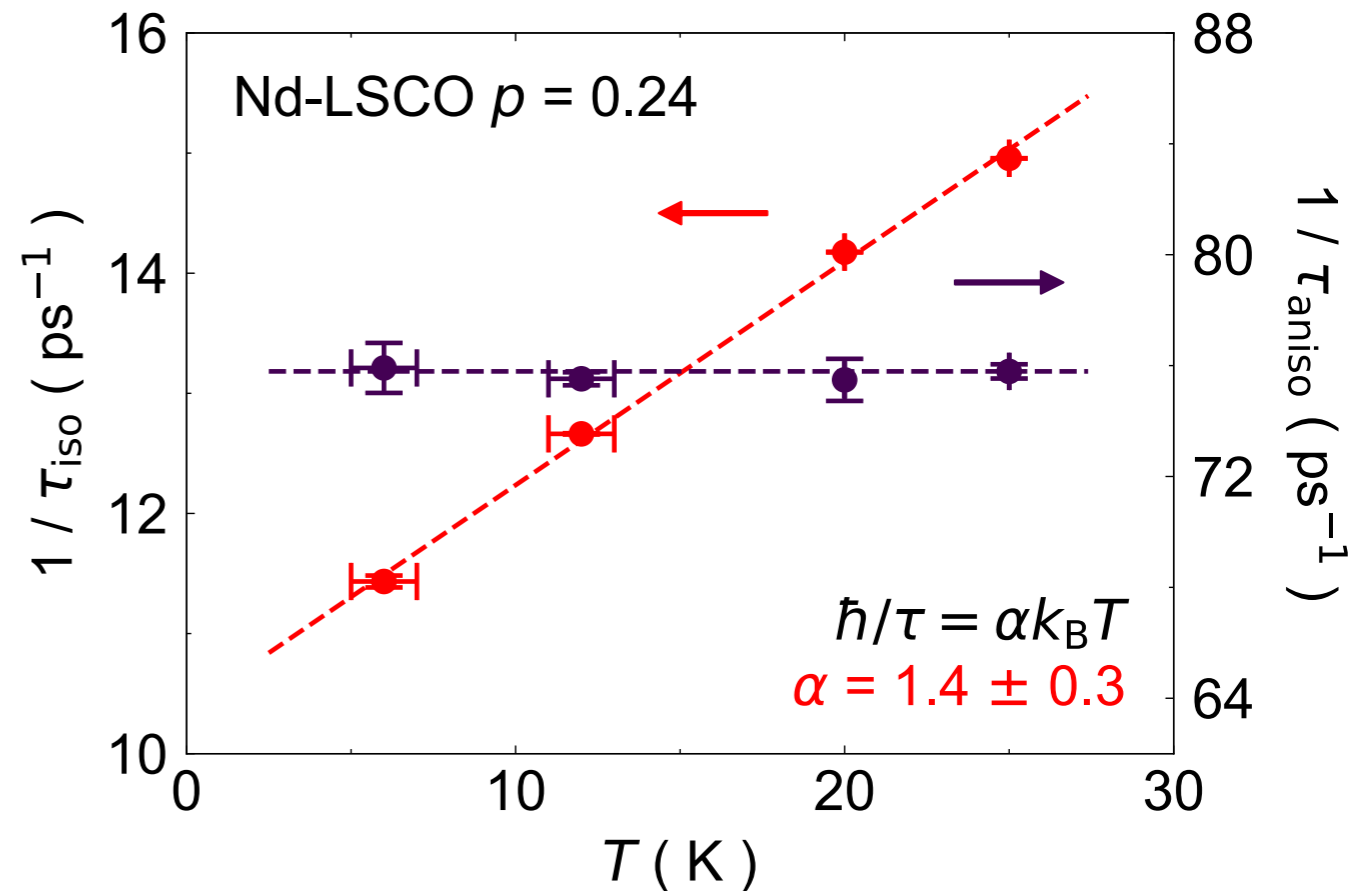
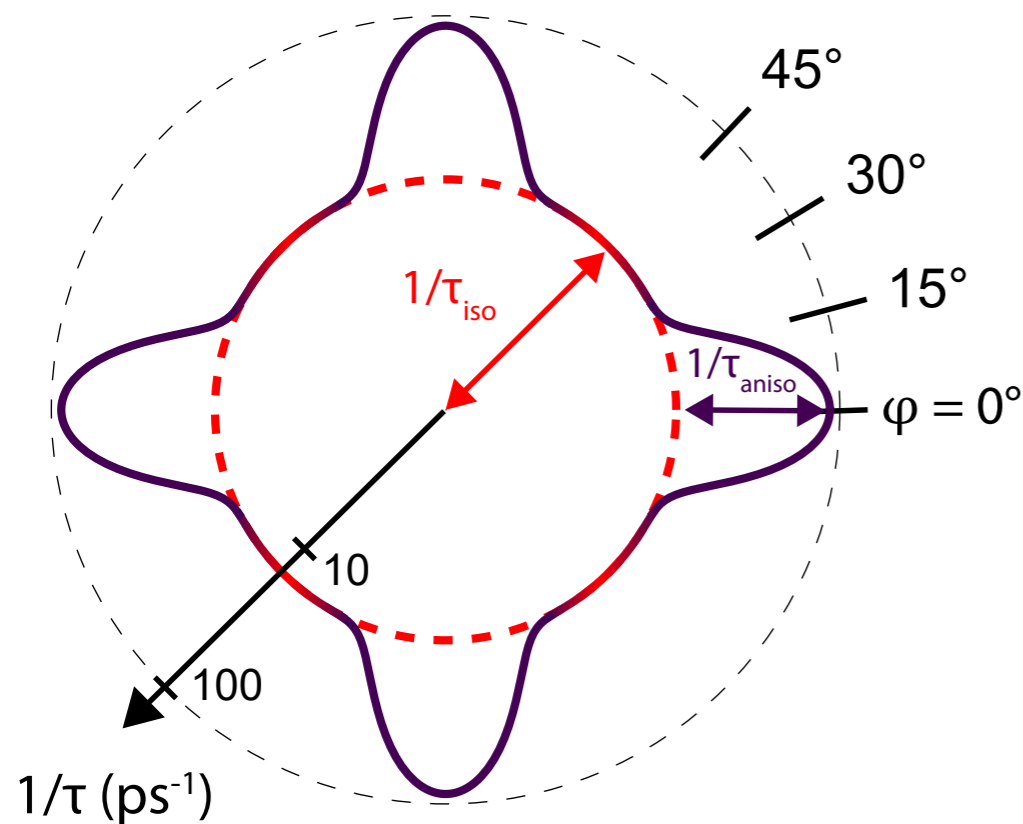




Measurement of the Planckian Scattering Rate

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw, arXiv:2011.13054

Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.

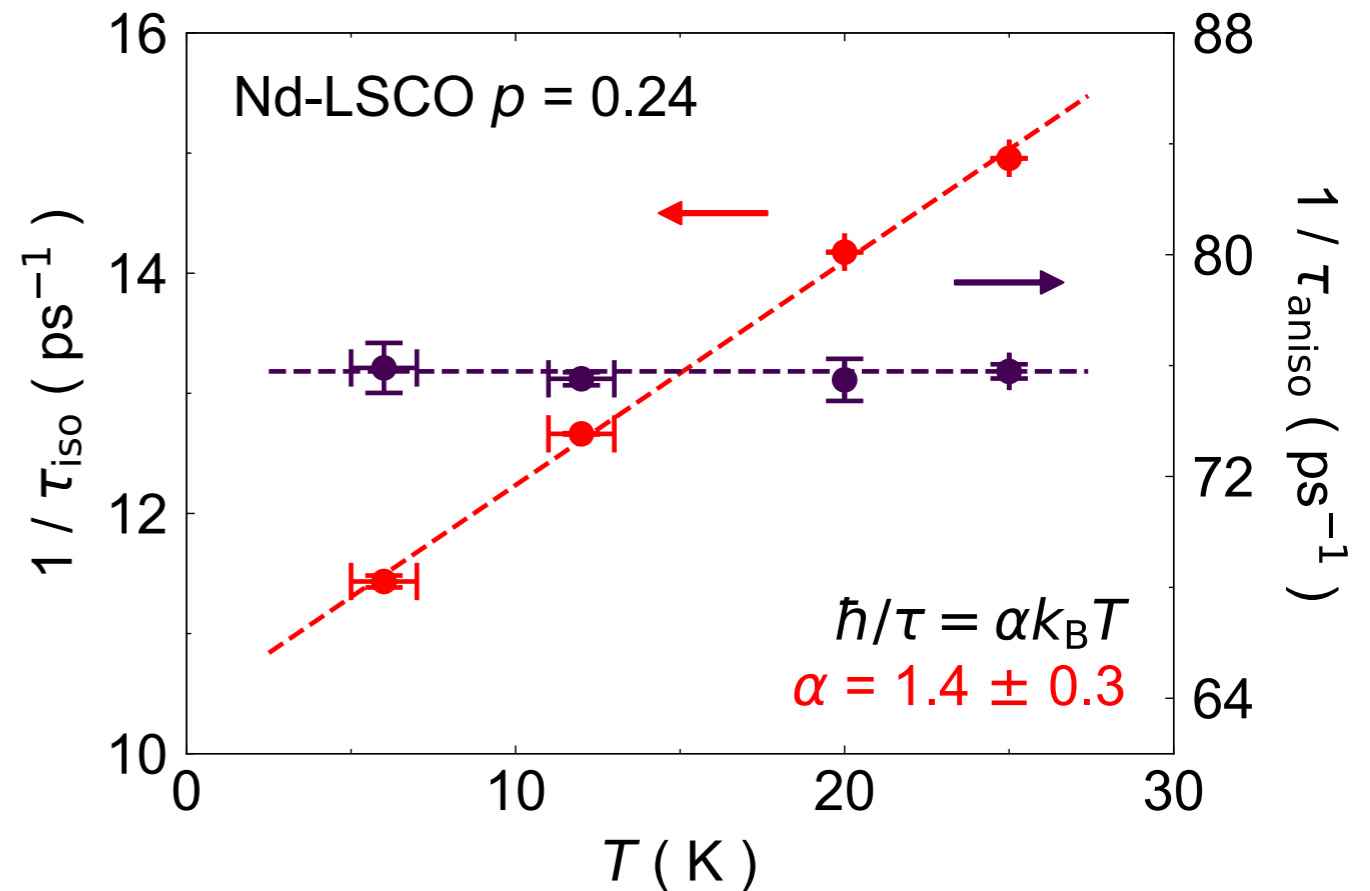
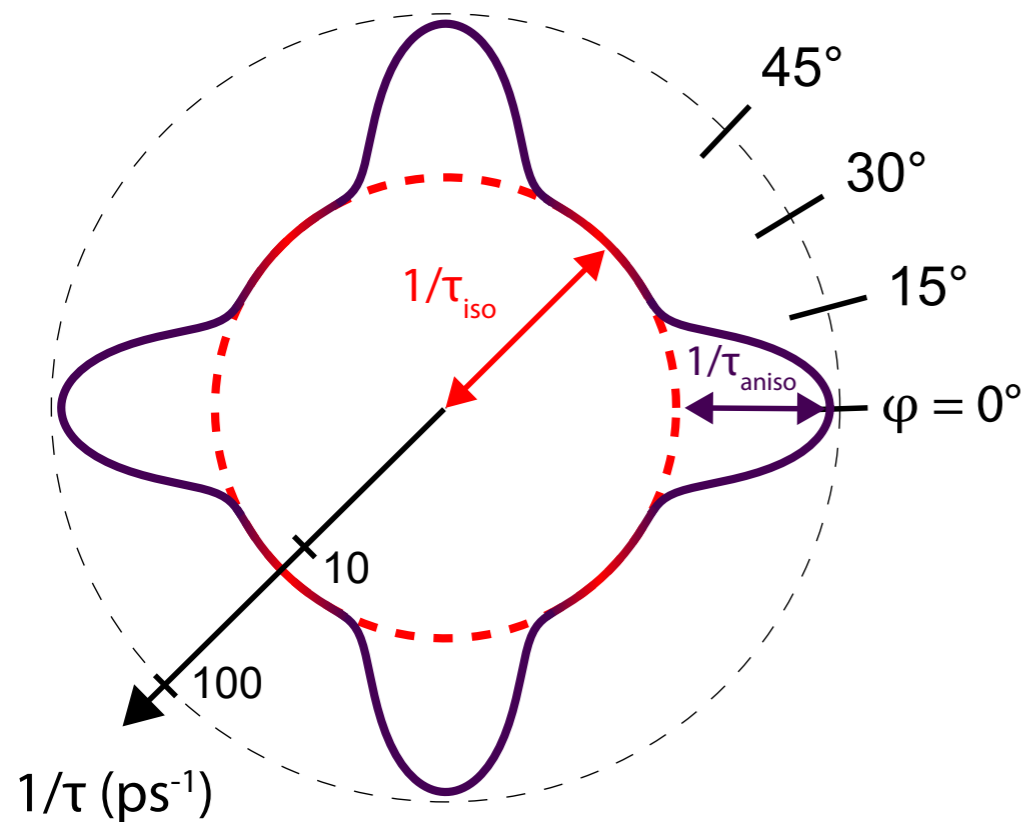


$$\frac{1}{\tau} = \frac{1}{\tau_{\text{aniso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T$$

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Henry Shackleton



Alexander Wietek



Antoine Georges

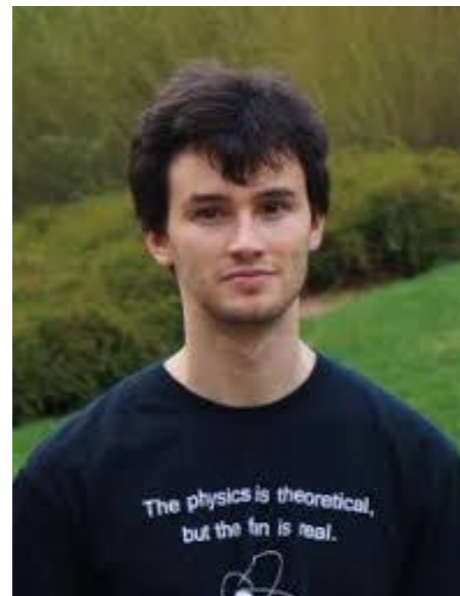
[arXiv:2012.06589](https://arxiv.org/abs/2012.06589)



Maria Tikhanovskaya



Haoyu Guo



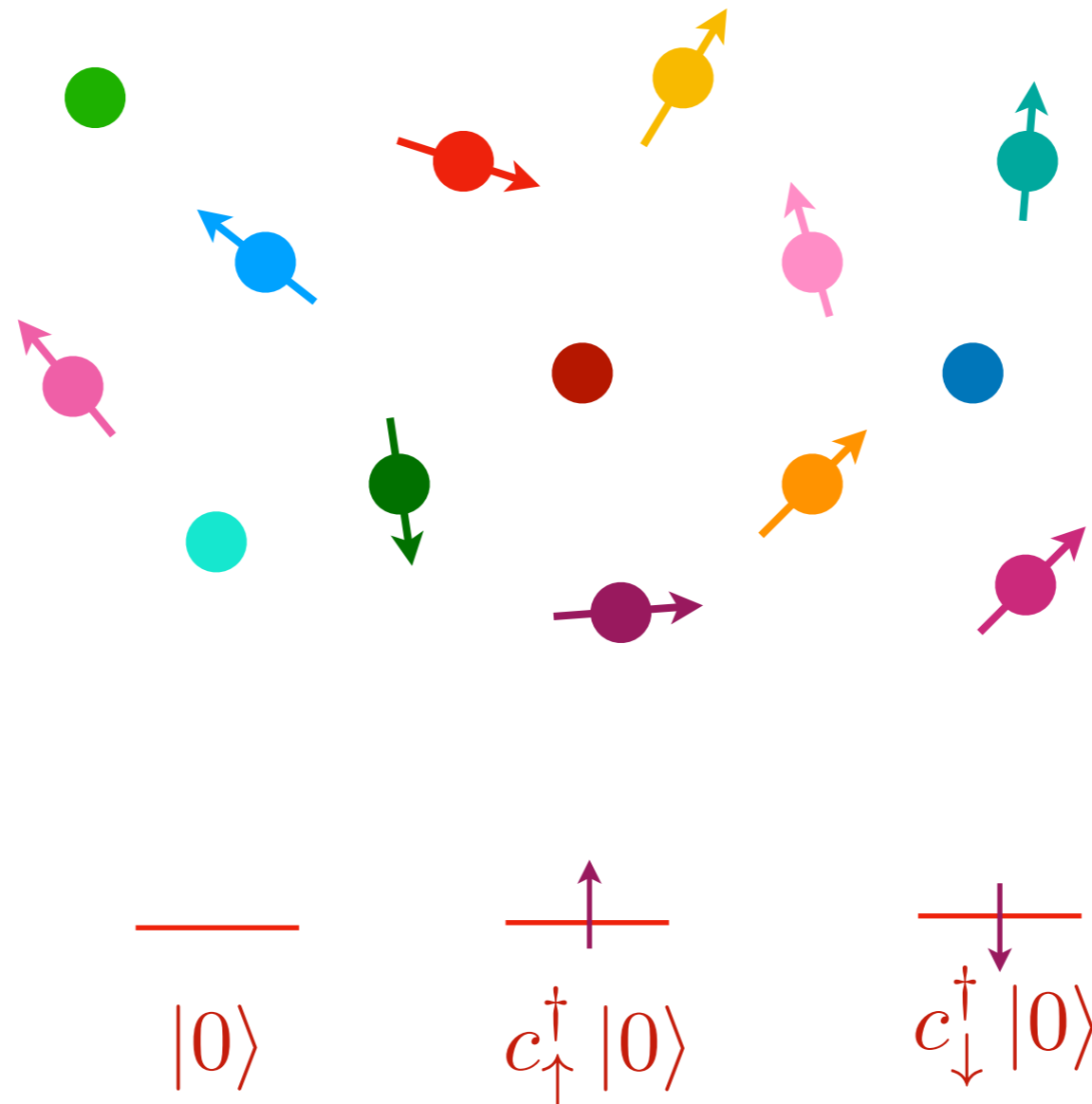
Grigory Tarnopolsky

arXiv:2010.09742
arXiv:2012.14449

Random t - J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

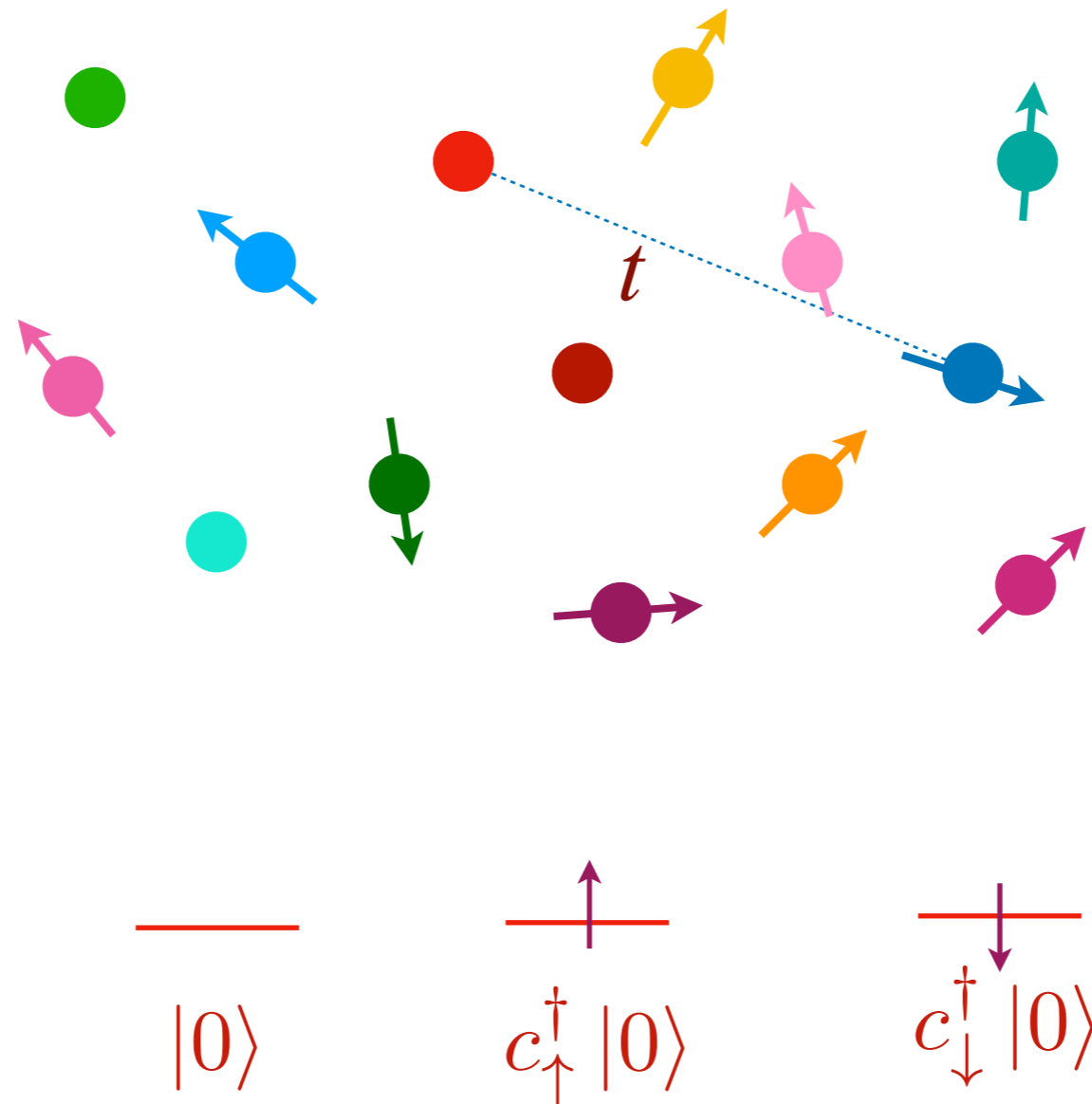
We consider the hole-doped case, with no double occupancy.



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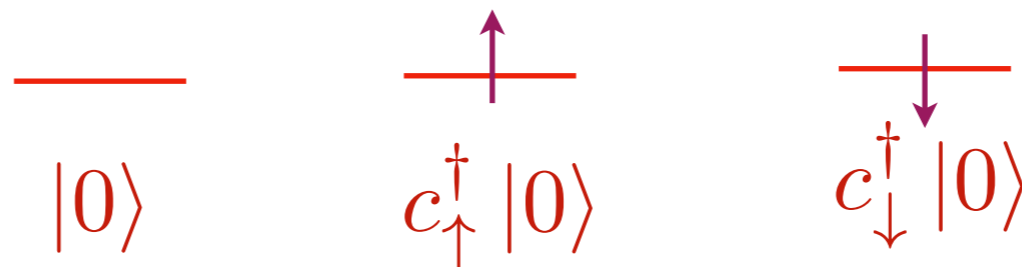
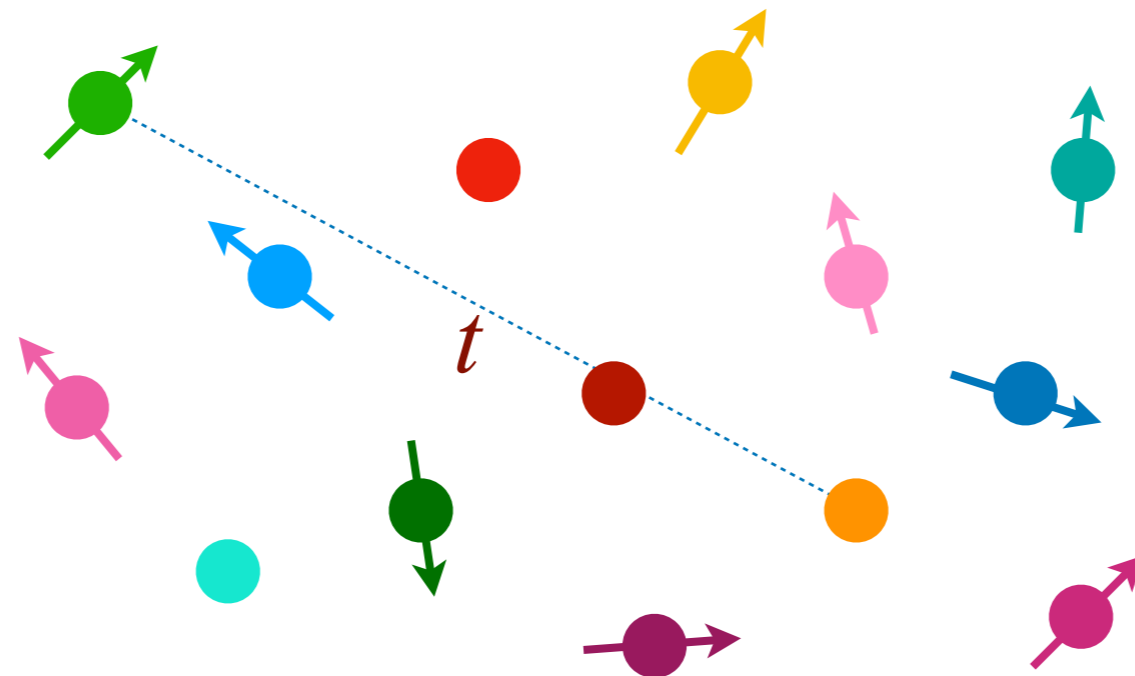
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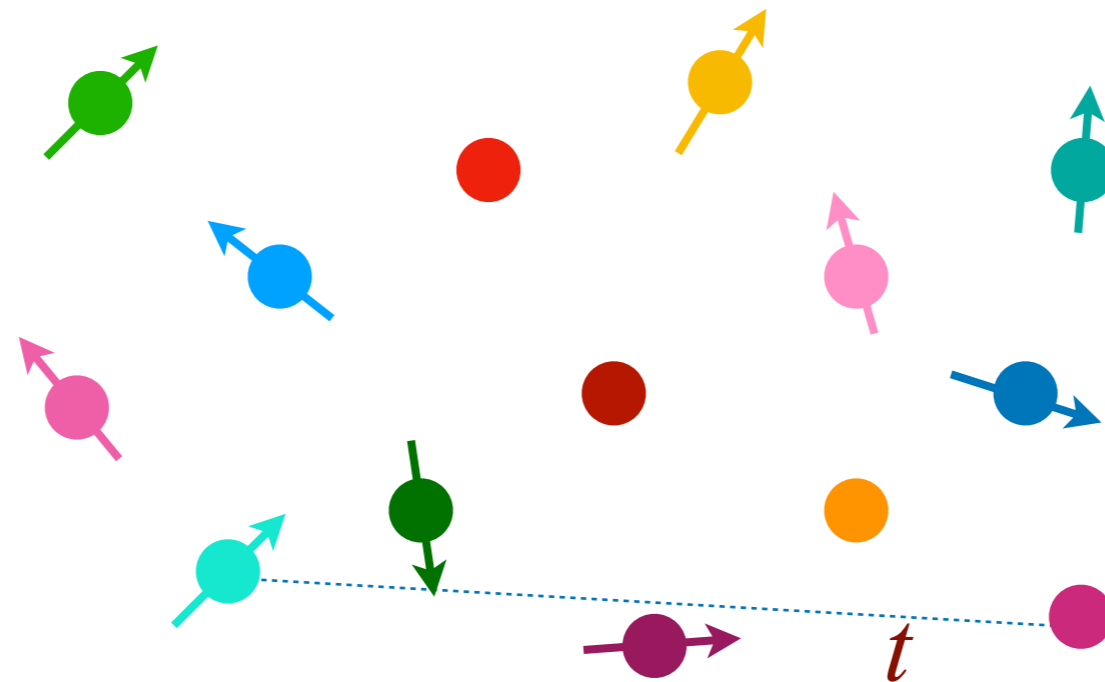
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$$\text{---} \\ |0\rangle$$

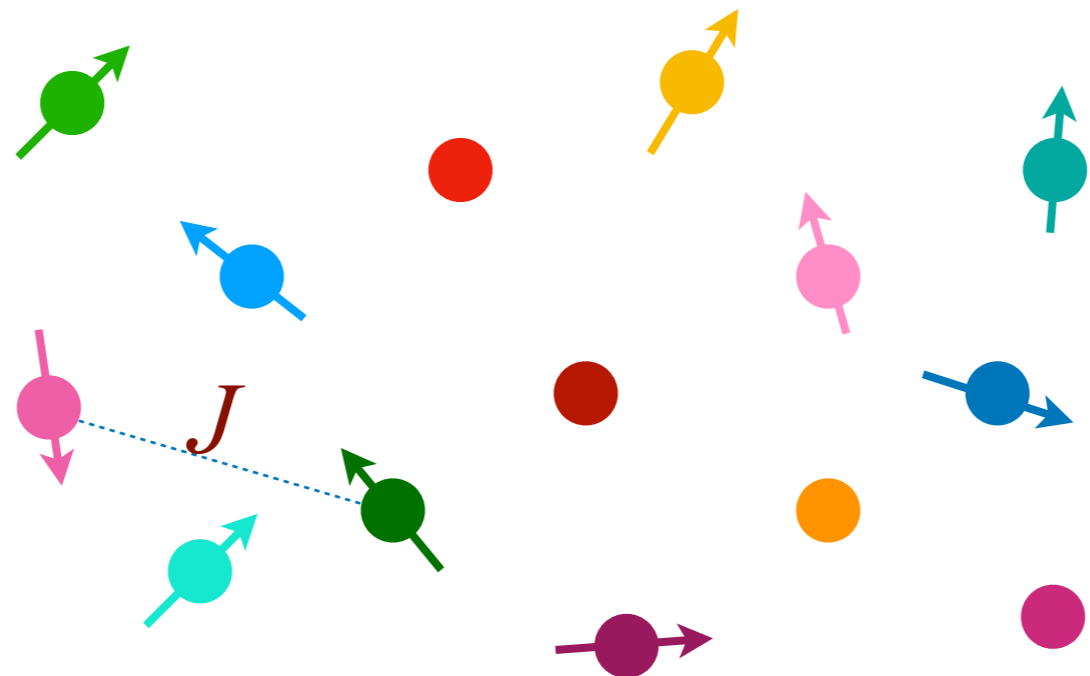
$$\text{---} \uparrow \\ c_{\uparrow}^\dagger |0\rangle$$

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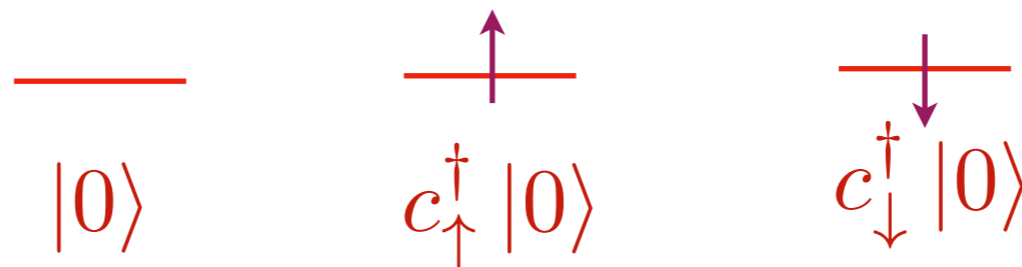
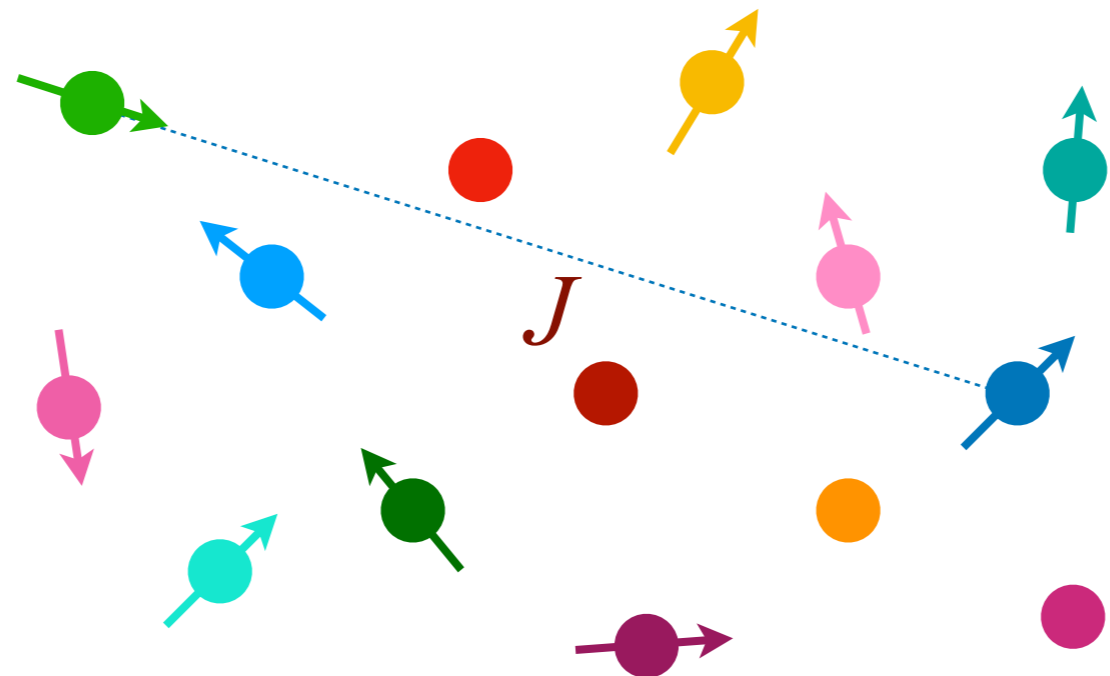
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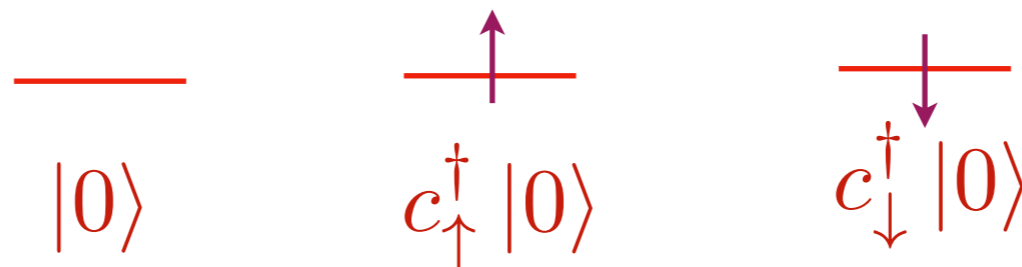
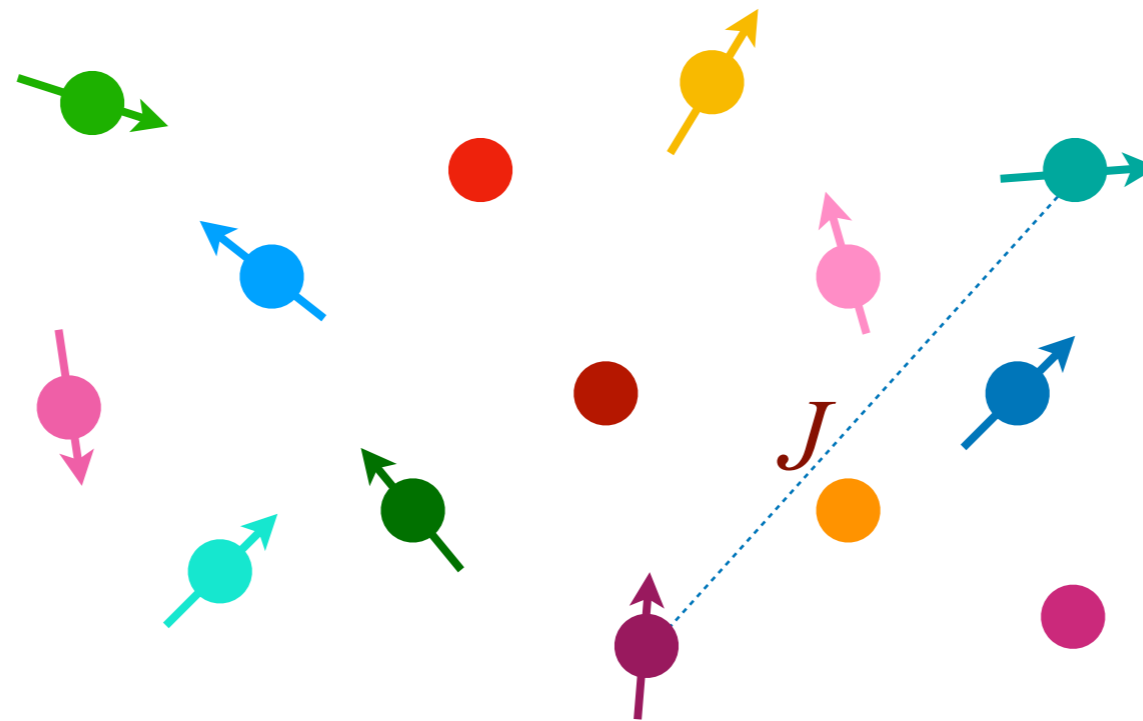
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Random t - J model

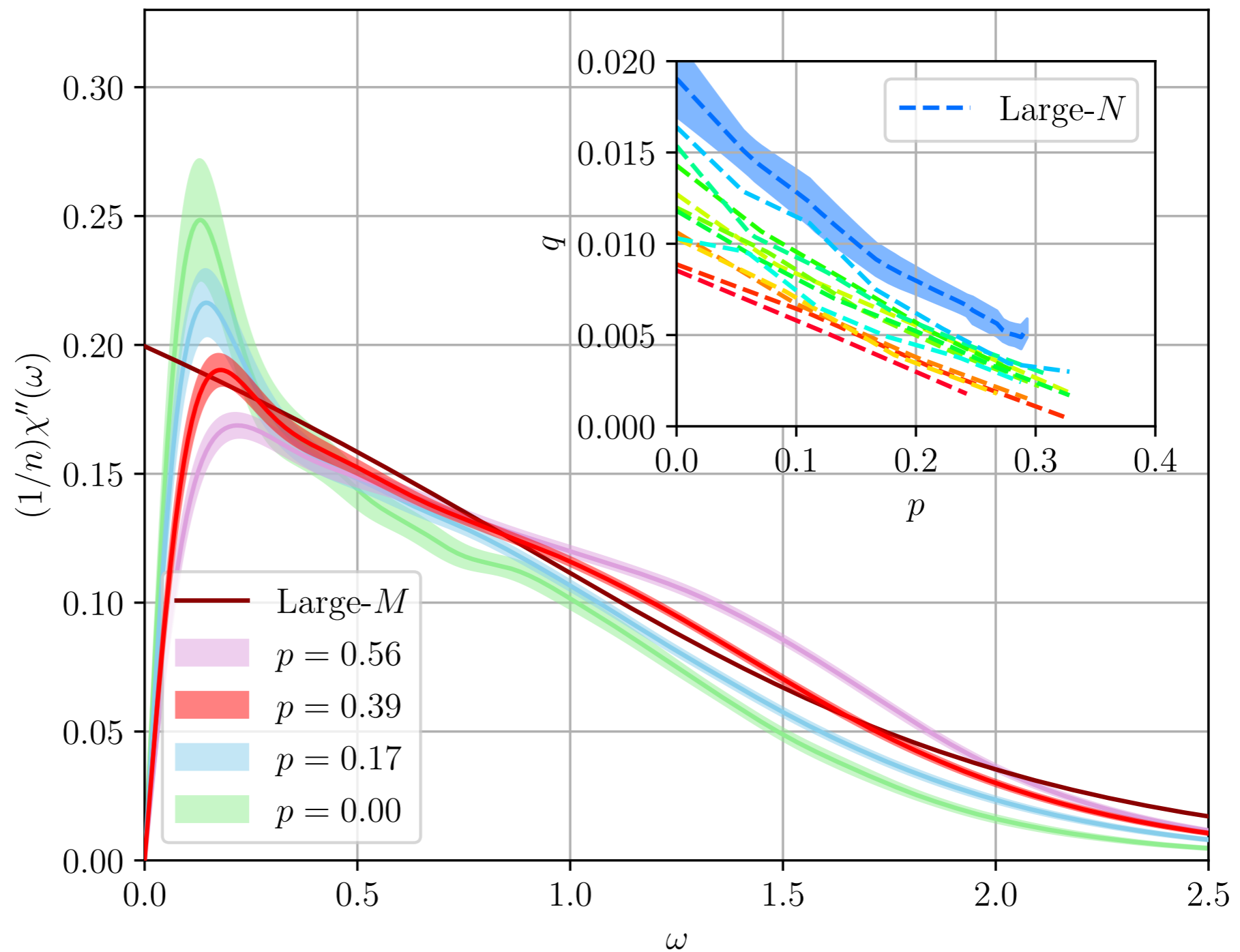
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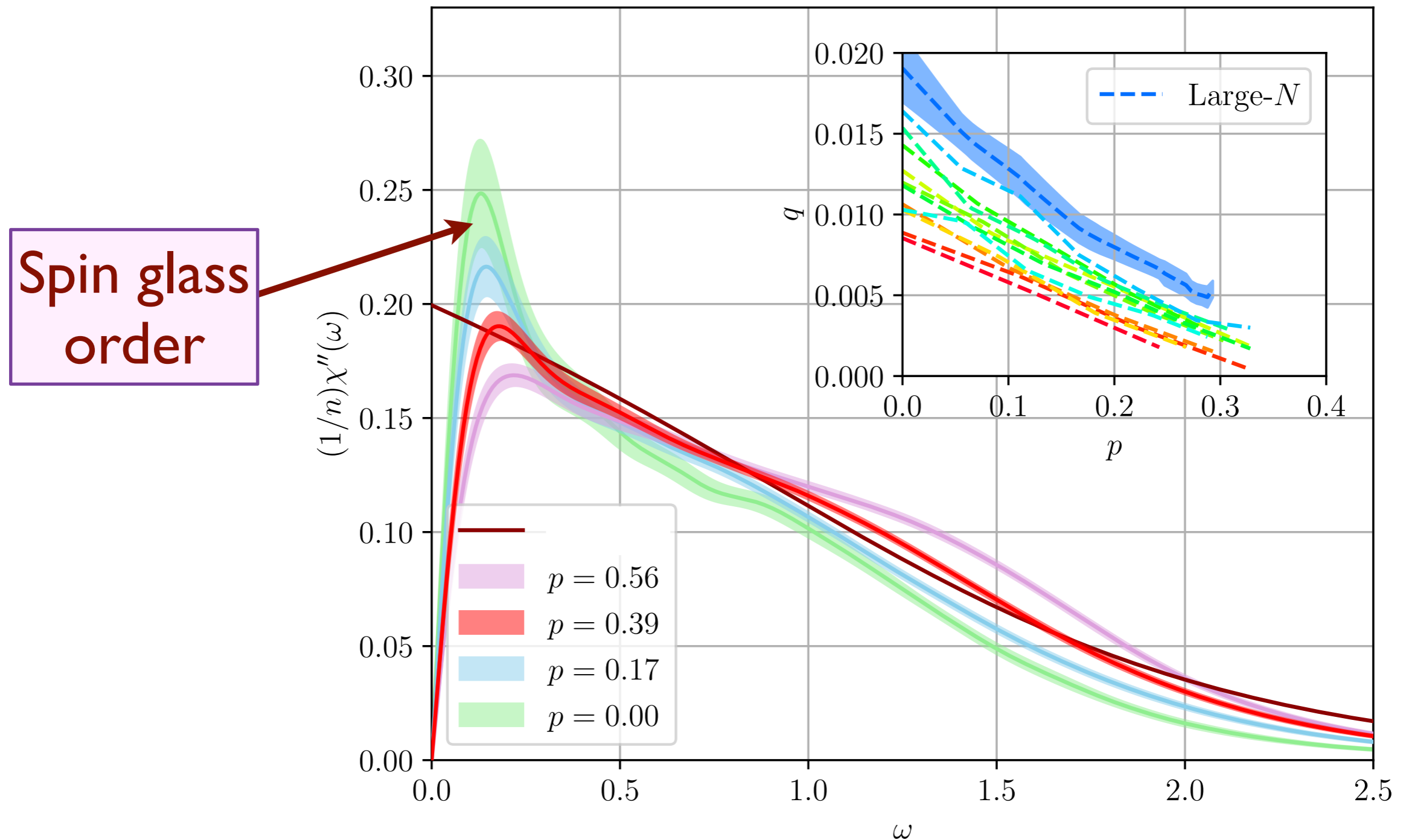
Dynamic spin susceptibility

$$\chi''(\omega) = \sum_n |\langle 0 | S_{+i} | n \rangle|^2 \delta(\hbar\omega - E_n + E_0), \quad (\text{at } T = 0)$$



Dynamic spin susceptibility

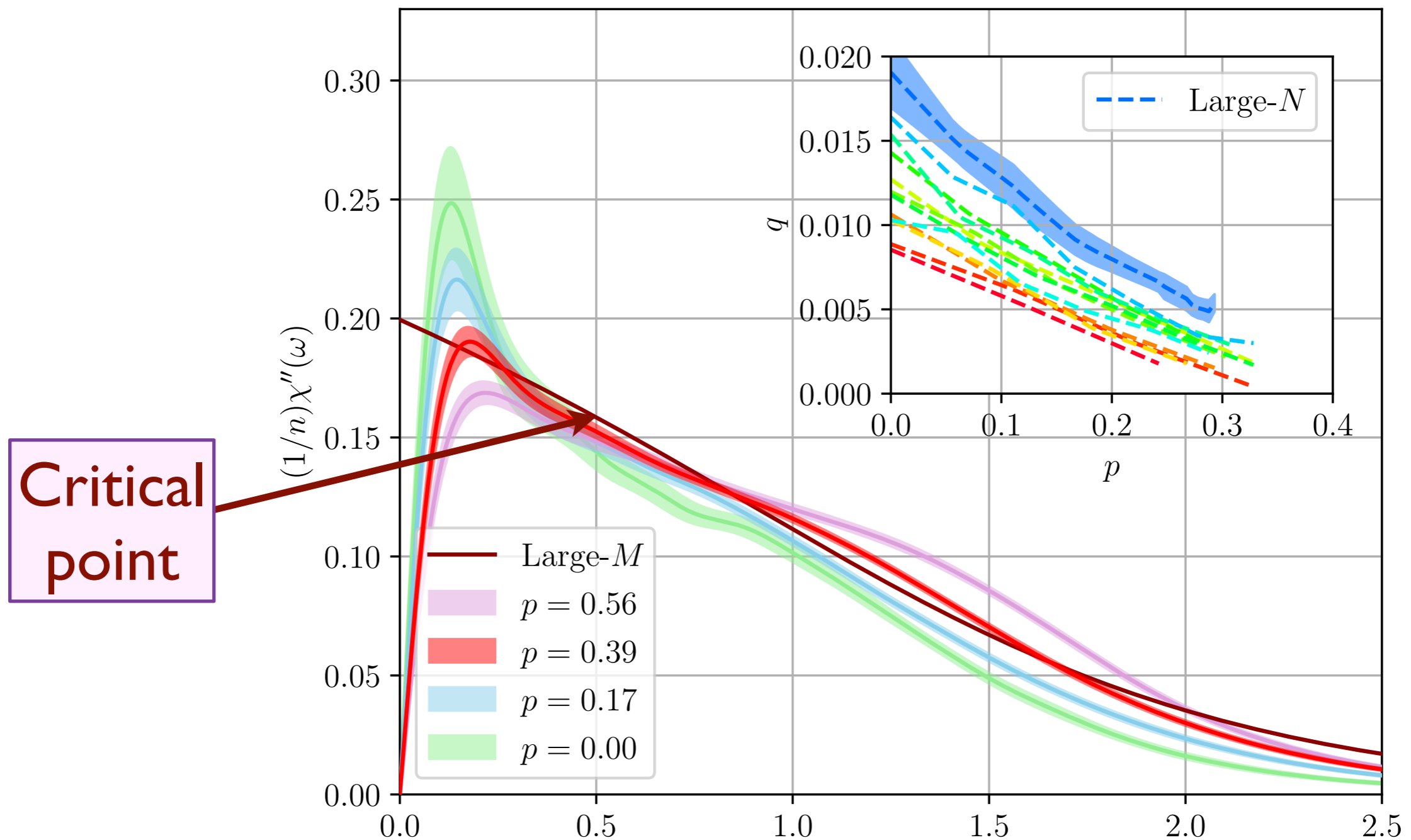
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Spin glass order q non-zero for $p < p_c \approx 0.4$

Dynamic spin susceptibility

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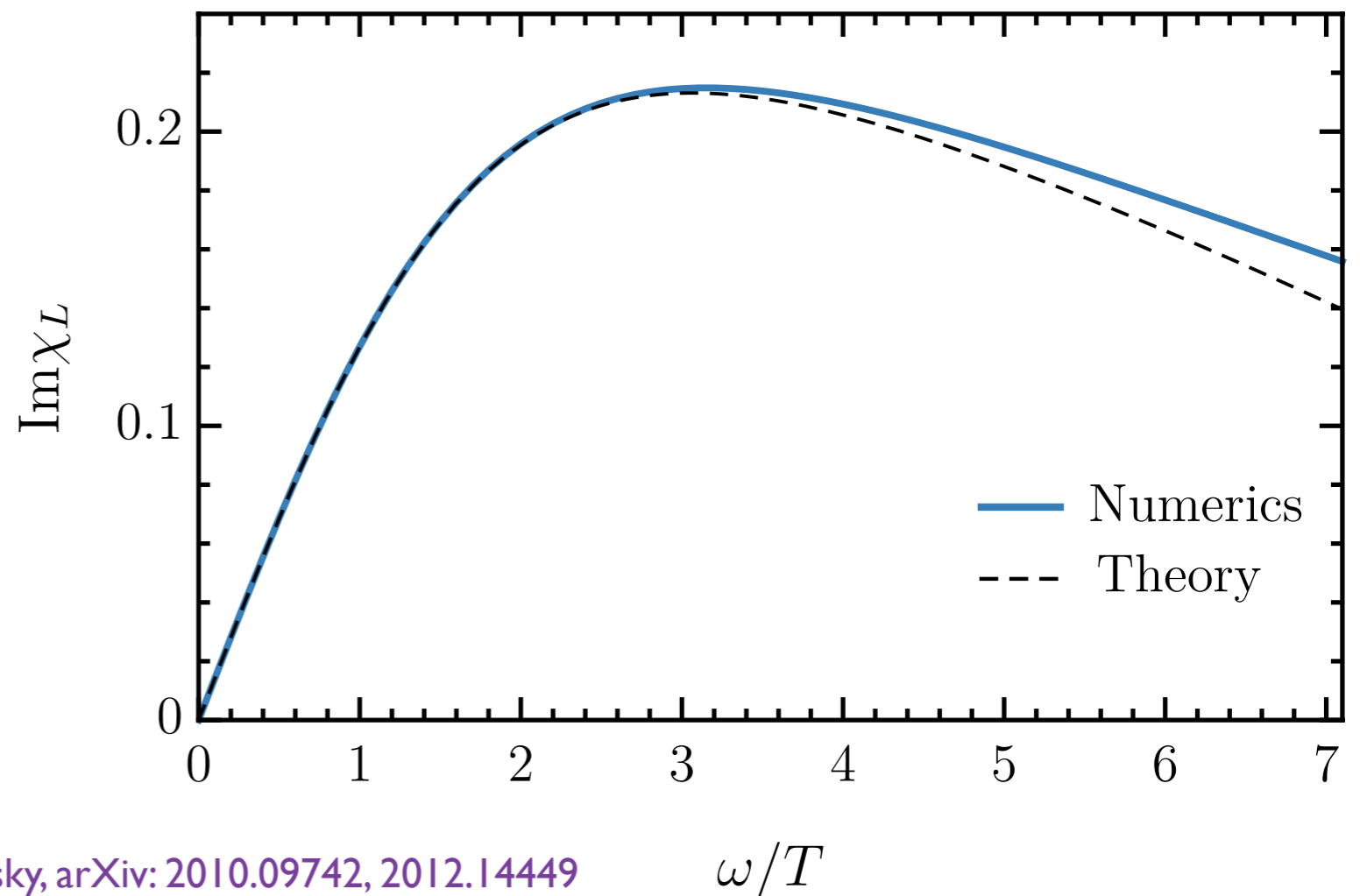
Critical spin susceptibility matches the SYK model!

$$\chi''(\omega) \sim \text{sgn}(\omega) [1 - C\gamma|\omega| + \dots]$$

Consequences of 2D-gravity for the SYK model

$$\chi''(\omega) = \sum_n |\langle 0 | S_{+i} | n \rangle|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\chi''(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right) \left[1 - C\gamma\omega \tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \dots \right]$$



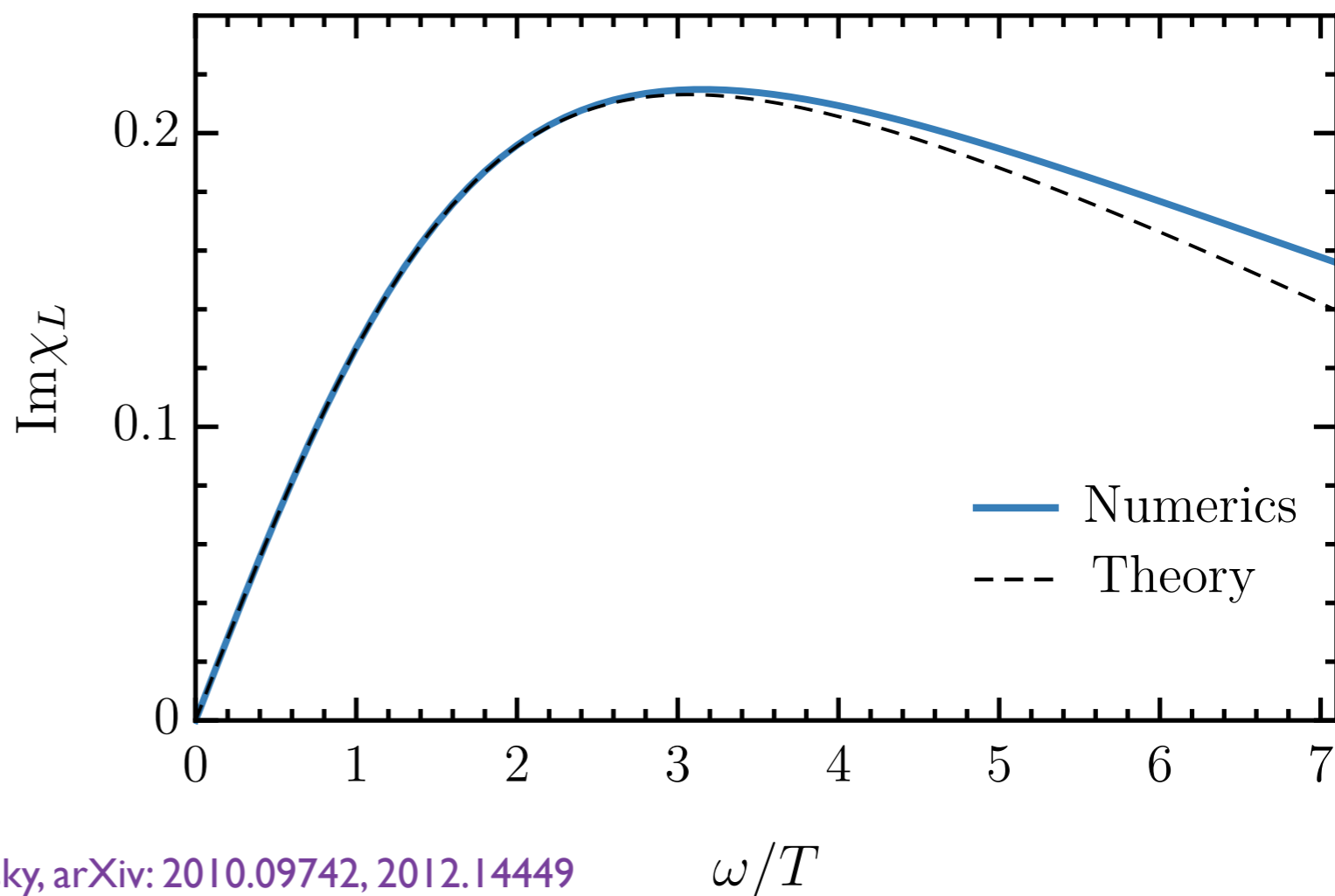
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Conformally (SL(2,R))
invariant result with
characteristic dissipative
time $\sim \hbar/(k_B T)$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

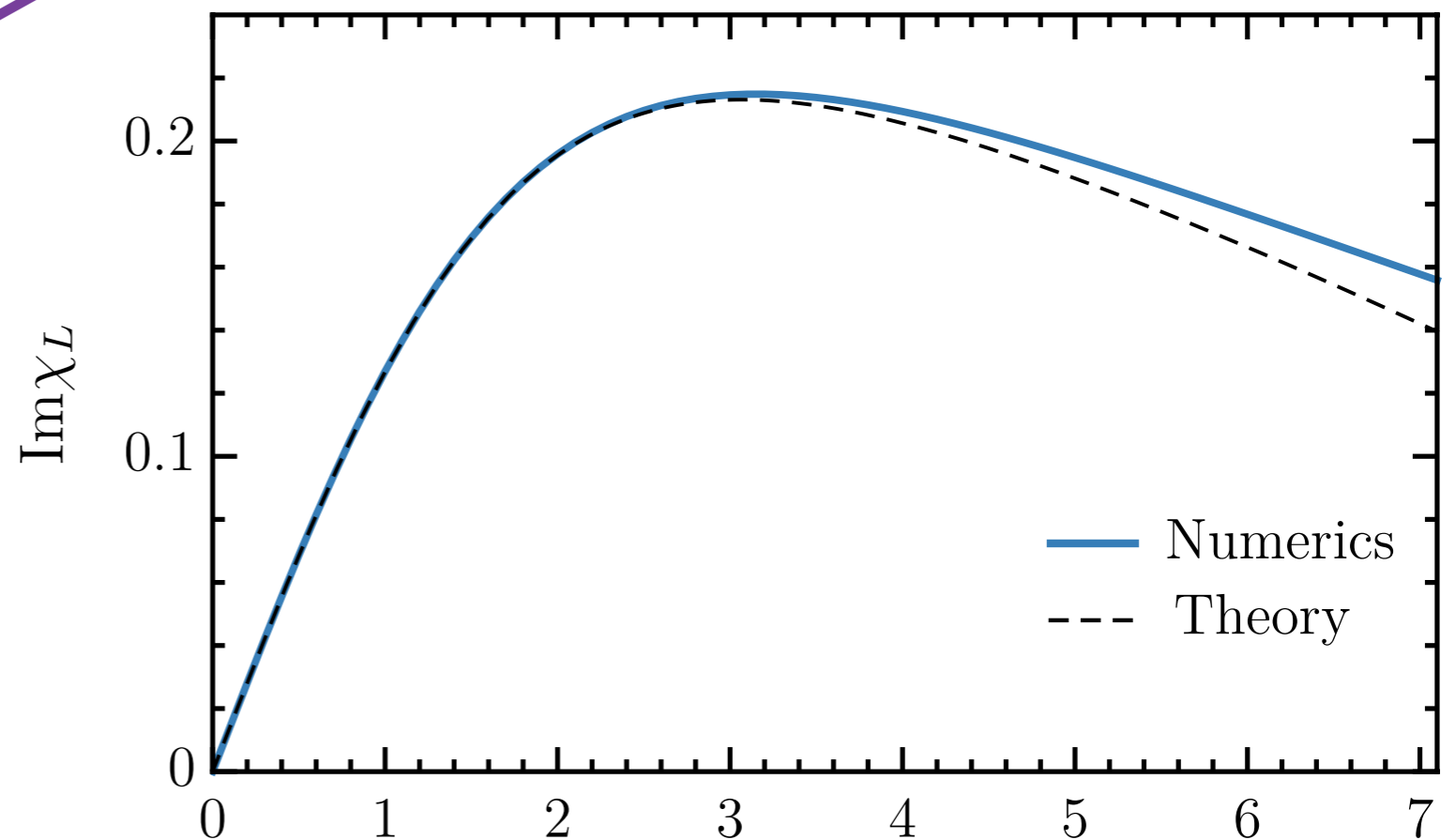


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Correction from
the boundary
graviton



The random t - J model has

- Spin glass order for $p < p_c$.
- Fermi liquid with “*large Fermi surface*” for $p > p_c$
- Maxima in entropy, specific heat, and entanglement entropy near $p = p_c$
- SYK-Planckian criticality near p_c .
- *Boundary graviton* correction in critical spin susceptibility!

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- SYK-Planckian criticality near p_c .
- *Boundary graviton* correction in critical spin susceptibility!
- SYK criticality can be understood in a model in which the electron fractionalizes into spinons and holons: then both the t and J terms map onto 4-particle SYK terms.

**Quantum
entanglement**

**Charged
black holes**

**A simple
many-particle
(SYK) model**

**Copper-based
superconductors**

**Quantum
entanglement**

**A simple
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(SYK) model**

**Charged
black holes**

2D
quantum
gravity

```
graph TD; A[2D quantum gravity] --> B[Quantum entanglement]; A --> C[Charged black holes]; A --> D[A simple many-particle (SYK) model]; E[Copper-based superconductors];
```

**Copper-based
superconductors**

Quantum
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Charged
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Copper-based
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