Gauge-gravity duality and its applications

Marc Kac Memorial Lectures, May 5 2011

Talk online: sachdev.physics.harvard.edu



<u>Outline</u>

I. Quantum criticality and conformal field theories in condensed matter

2. The AdS/CFT correspondence Quantum criticality and black holes

3. Quantum transport and Einstein-Maxwell theory on AdS₄

4. Compressible quantum matter Fermi surfaces

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Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$ $\eta_i = \pm 1$ on two sublattices $\langle \vec{\varphi} \rangle \neq 0$ in Néel state.







Weaken some bonds to induce spin entanglement in a new quantum phase

<u>Square lattice antiferromagnet</u>



Ground state is a "quantum paramagnet" with spins locked in valence bond singlets

$$=\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle-\left|\downarrow\uparrow\right\rangle\right)$$









 $\boxed{} = \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$











Superfluid-insulator transition



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415, 39 (2002).

Insulator (the vacuum) at large U

Excitations:



Excitations:





$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$







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4. Compressible quantum matter Fermi surfaces Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

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where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

Key idea: \Rightarrow Implement *u* as an extra dimension, and map to a local theory in *D*+1 dimensions.

At the RG fixed point, $\beta(g) = 0$, the *D* dimensional field theory is invariant under the scale transformation

$$x^{\mu} \to x^{\mu}/b$$
 , $u \to b u$

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This is an invariance of the *metric* of the theory in D + 1 dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^{\mu} dx_{\mu} + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^{2} = L^{2} \frac{dx^{\mu} dx_{\mu} + dz^{2}}{z^{2}}.$$

This is the space AdS_{D+1} , and L is the AdS radius.



Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

J. McGreevy, arXiv0909.0518



Maldacena, Gubser, Klebanov, Polyakov, Witten



3+1 dimensional AdS space

Black hole temperature

temperature of quantum criticality



Quantum criticality in 2+1 dimensions

Maldacena, Gubser, Klebanov, Polyakov, Witten

3+1 dimensional AdS space

Black hole entropy = entropy of quantum criticality



Quantum criticality in 2+1 dimensions

3+1 dimensional AdS space

Quantum critical dynamics = waves in curved space



Quantum criticality in 2+1 dimensions

Maldacena, Gubser, Klebanov, Polyakov, Witten

3+1 dimensional AdS space

Friction of quantum criticality = waves falling into black hole



Quantum criticality in 2+1 dimensions

Kovtun, Policastro, Son

3+1 dimensional AdS space

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Quantum criticality in 2+1 dimensions

Kovtun, Policastro, Son

<u>AdS/CFT correspondence</u> The quantum theory of a black hole in a 3+1-		
o holo	Strong coupling problem: General solution of spin and	of a
3+1 di AdS	magneto-thermo-electric transport in quantum critical region.	tum
Fricti	 C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, <i>Phys. Rev.</i> D 75, 085020 (2007). S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, 	1 sions
quan critica	Phys. Rev. B 76, 144502 (2007).	
wav falling black	es into hole Kovtun. Pol	licastro, Son

Thursday, May 5, 2011
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Quantum critical transport

Quantum "nearly perfect fluid" with shortest possible relaxation time, τ_R



where \mathcal{C} is a *universal* constant

S. Sachdev, Quantum Phase Transitions, Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Spin/charge conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)]$$

(Q is the quantum of spin/charge)

K. Damle and S. Sachdev, Phys. Rev. B 56, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature



P. Kovtun, D. T. Son, and A. Starinets, Phys. Rev. Lett. 94, 11601 (2005)

 ω

 $\omega = i2\pi nk_B T/\hbar,$ *n* integer: computable in perturbative analysis of conformal field theory about free field theory

 ω

$\omega \ll k_B T/\hbar,$

hydrodynamic regime: requires computation in dual gravity theory

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To leading order in a gradient expansion, an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4

$$\mathcal{S}_{EM} = \frac{1}{g_4^2} \int d^4 x \sqrt{-g} \left[-\frac{1}{4} F_{ab} F^{ab} \right]$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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$$\mathcal{S}_{EM} = \frac{1}{g_4^2} \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{ab} F^{ab} \right]$$

This theory is self-dual under $F_{ab} \rightarrow \epsilon_{abcd} F^{cd}$, and this leads to some artifacts in the properties of the CFT3

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (*L* is the radius of AdS₄):

$$S = \frac{1}{g_4^2} \int d^4 x \sqrt{-g} \left[-\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] \,,$$

where C_{abcd} is the Weyl curvature tensor. Stability and causality constraints restrict $|\gamma| < 1/12$.







R. C. Myers, S. Sachdev, and A. Singh, arXiv:1010.0443

ω

2.0 $4\pi T$









R. C. Myers, S. Sachdev, and A. Singh, arXiv:1010.0443

Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value



FIG. 3. $\operatorname{Re}(\sigma_{xx})$ vs *B* at three frequencies and two temperatures. Peaks are marked with Landau level index *N* and spin.

L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui, *Physical Review Letters* **71**, 2638 (1993).

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Graphene

 \boldsymbol{e}_3

 e_2





Semi-metal with massless Dirac fermions

Turn on a chemical potential on a CFT



Turn on a chemical potential on a CFT



Electron Fermi surface

• Consider a continuum quantum system with a globally conserved U(1) charge Q (the "electron density") in spatial dimension d > 1.

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There are only a few established examples of such phases in condensed matter physics. However, they appear naturally as duals of gravitational theories, and we want to interpret them in the gauge theory.

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All known examples of such phases have a <u>Fermi Surface</u>

(even in systems with only bosons in the Hamiltonian)
The Fermi surface

This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge Q.

$$G_{\text{fermion}}^{-1}(k=k_F,\omega=0)=0.$$

Luttinger relation: The total "volume (area)" \mathcal{A} enclosed by Fermi surfaces of fermions carrying charge \mathcal{Q} is equal to $\langle \mathcal{Q} \rangle$. This is a *key* constraint which allows extrapolation from weak to strong coupling.



Aharony-Bergman-Jafferis-Maldacena (ABJM) CFT3

- $U(N) \times U(N)$ gauge field.
- $4N^2$ Weyl fermions carrying fundamental charges of $U(N) \times U(N) \times SU(4)_R$.
- $4N^2$ complex bosons carrying fundamental charges of $U(N) \times U(N) \times SU(4)_R$.
- $\mathcal{N} = 6$ supersymmetry

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- $4N^2$ complex bosons carrying fundamental charges of $U(N) \times U(N) \times SU(4)_R$.
- $\mathcal{N} = 6$ supersymmetry
- Add a chemical potential μ coupling to a global $\mathrm{SU}(4)_R$ charge \mathcal{Q} .

Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance



 $2\mathcal{A}_c = \langle \mathcal{Q} \rangle$

Fermi liquid (FL) of gauge-neutral particles Gauge theory is in confining phase



Fermi surface of gauge charged particles, f, which quench gauge forces and lead to deconfinement

 $2\mathcal{A}_c + 2\mathcal{A}_f = \langle \mathcal{Q} \rangle$

Fractionalized Fermi liquid (FL*) Gauge theory is in deconfined phase



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$2\mathcal{A}_c + 2\mathcal{A}_f = \langle \mathcal{Q} \rangle$ Fractionalized Fermi liquid (FL*) Gauge theory is in deconfined phase

Claim: this is the phase underlying recent holographic theories of compressible metallic states. However, a number of artifacts appear in the classical gravity approximation. Gauge-gravity duality

SU(4) global symmetry
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 \bigcirc The CFT is dual to a gravity theory on AdS₄ x S⁷

Gauge-gravity duality

Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry
 Add some SU(4) charge by turning on a chemical potential (this breaks the SU(4) symmetry)

The CFT is dual to a gravity theory on AdS₄ x S⁷
 In the Einstein-Maxwell theory, the chemical potential leads at T=0 to an extremal Reissner-Nordtrom black hole in the AdS₄ spacetime.

T. Faulkner, H. Liu, J. McGreevy, and D.Vegh, arXiv:0907.2694

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 In the Einstein-Maxwell theory, the chemical potential leads at T=0 to an extremal Reissner-Nordtrom black hole in the AdS₄ spacetime.
 The RN black hole describes compressible quantum matter with Fermi surfaces, but with infinite range hopping: this leads to numerous artifacts.



Solution Compressible quantum matter is characterized by Fermi surfaces.

Summary

Sompressible quantum matter is characterized by Fermi surfaces.

Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

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Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

Sermi liquids are everywhere.

There is evidence that FL* phases have been recently been observed in some intermetallic compounds. The FL* and related phases are attractive candidates for "strange metals" in the higher temperature superconductors

<u>Summary</u>

Sompressible quantum matter is characterized by Fermi surfaces.

Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

Gauge-gravity duality is a very promising approach to solving strong-coupling problems associated with FL*-like phases.

FL* and related phases are attractive candidates for "strange metals" in the higher temperature superconductors

Conclusions

New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle picture.

Conclusions

The AdS/CFT correspondence offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density