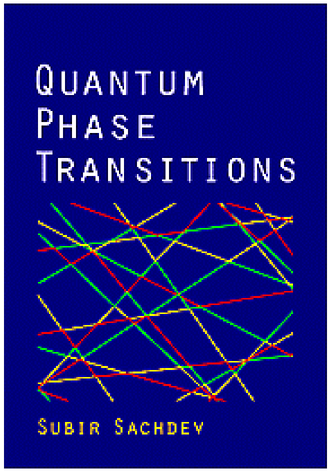
Competing orders and quantum criticality in the cuprate superconductors

Subir Sachdev

Science **286**, 2479 (1999).



Quantum Phase Transitions
Cambridge University Press

Transparencies online at
http://pantheon.yale.edu/~subir

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Outline

I. **Coupled Ladder Antiferromagnet**
   A. Ground states in limiting regimes
   B. Coherent state path integral
   C. Quantum field theory for critical point

II. Berry phases and duality in one dimension
   S=1/2 quantum XY model.

III. Berry phases and duality in two dimensions
   Bond-centered charge ("spin-Peierls") order.

Chapter 13 from Quantum Phase Transitions, and cond-mat/0109419 (sections I–III)

IV. Magnetic transitions in *d*-wave superconductors
   A. Theory of SC+SDW to SC quantum transition
   B. Phase diagrams of Mott insulators and superconductors in an applied magnetic field
   C. Comparison of predictions with experiments

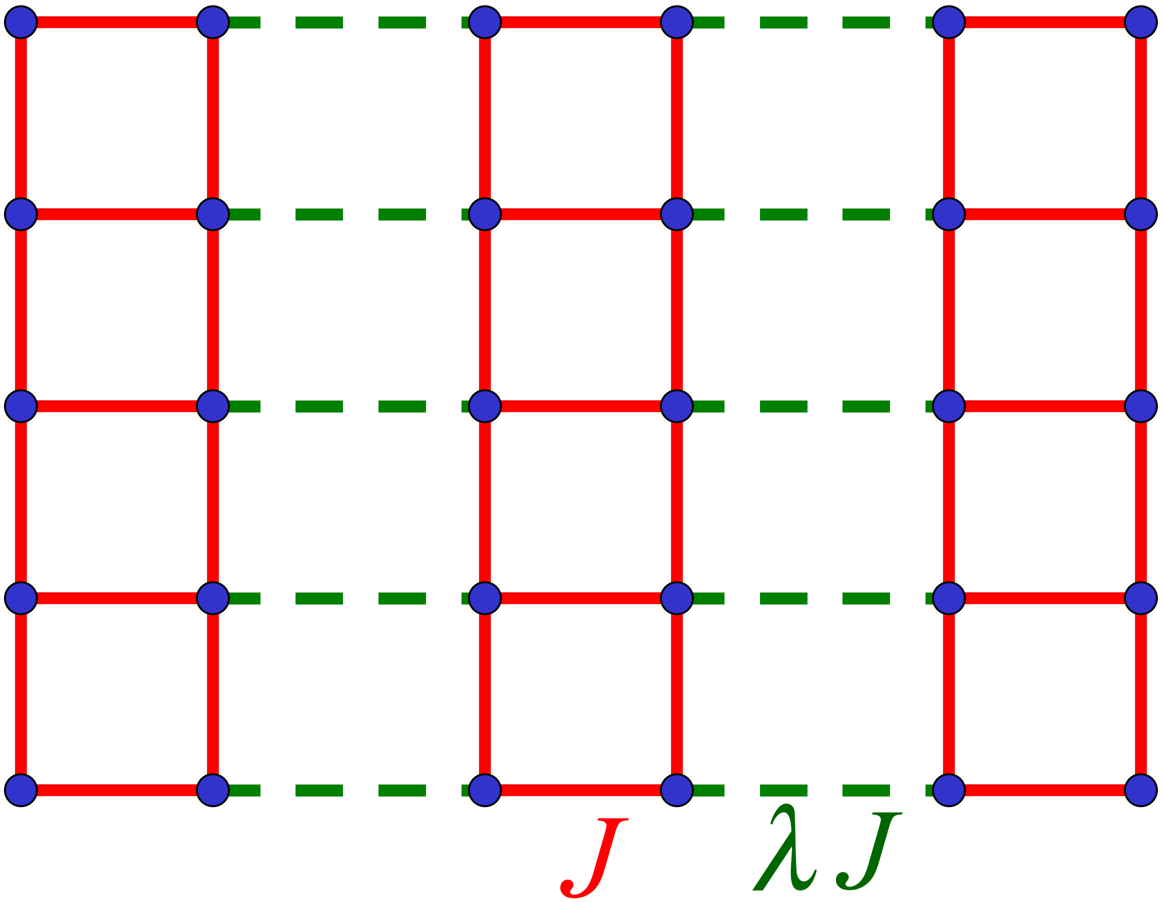
cond-mat/0112343 (section IV)

V. Conclusions

I.A Coupled Ladder Antiferromagnet

N. Katoh and M. Imada, J. Phys. Soc. Jpn. **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, Phys. Rev. B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, cond-mat/0107115.

S=1/2 spins on coupled 2-leg ladders



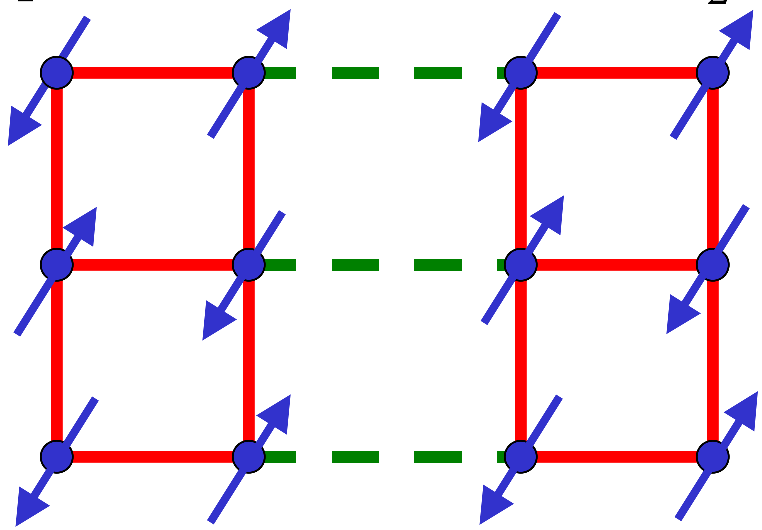
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



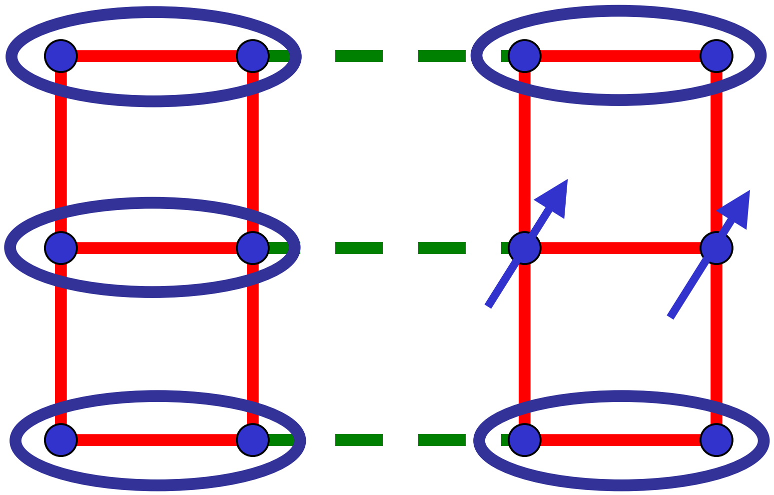
Ground state has long-range magnetic (Neel) order

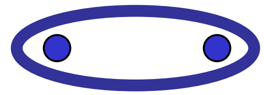
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves $\varepsilon_k = \sqrt{c_x{}^2 k_x{}^2 + c_y{}^2 k_y{}^2}$

λ close to 0

Weakly coupled ladders





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

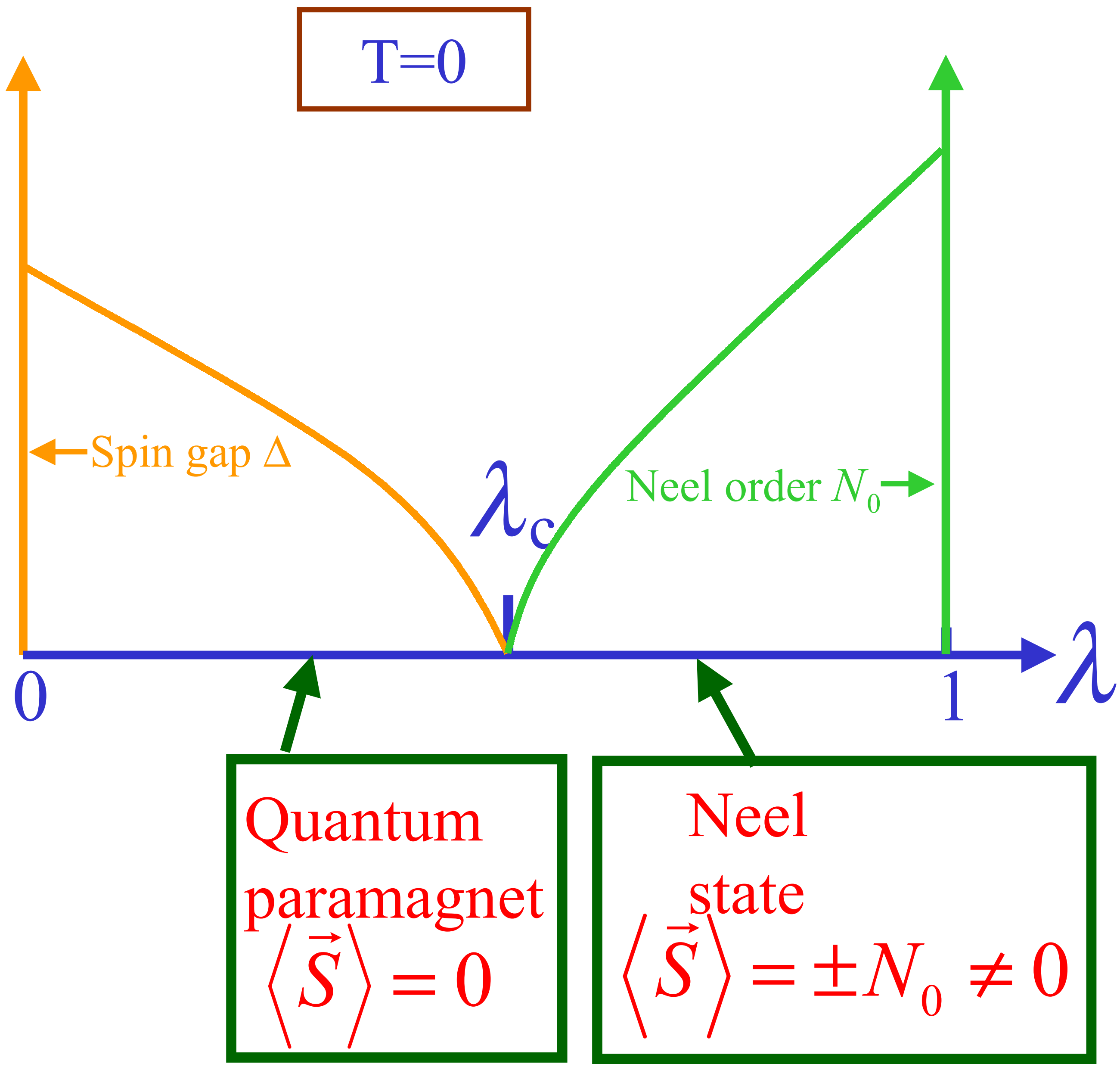
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0$$

Excitation: S=1 ***exciton*** (spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_k = \Delta + \frac{c_x^{\,2} k_x^{\,2} + c_y^{\,2} k_y^{\,2}}{2\Delta}$$



T=0
Spin gap Δ
λ_c
Neel order N_0
0
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle = \pm N_0 \neq 0$

I.B Coherent state path integral

See Chapter 13 of *Quantum Phase Transitions*, S. Sachdev, Cambridge University Press (1999).

Path integral for a single spin

$$Z = \mathrm{Tr}\left(e^{-H[\boldsymbol{S}]/T}\right)$$

$$= \int \mathcal{D}\boldsymbol{N}(\tau)\,\delta\left(\boldsymbol{N}^2 - 1\right)\exp\left(-iS\int A_\tau(\tau)\,d\tau - \int d\tau\, H\left[S\boldsymbol{N}(\tau)\right]\right)$$

$A_\tau(\tau)\,d\tau =$ Oriented area of triangle on surface of unit sphere bounded by $\boldsymbol{N}(\tau), \boldsymbol{N}(\tau + d\tau)$, and a fixed reference $\boldsymbol{N}_0$

Action for lattice antiferromagnet

$$\boldsymbol{N}_j(\tau) = \eta_j \boldsymbol{n}(x_j, \tau) + \boldsymbol{L}(x_j, \tau)$$

$\eta_j = \pm 1$ identifies sublattices

n and ***L*** vary slowly in space and time

Integrate out ***L*** and take the continuum limit

$$Z = \int \mathcal{D}\boldsymbol{n}(x,\tau)\,\delta\left(\boldsymbol{n}^2 - 1\right)\exp\left(-iS\sum_j \int \eta_j A_\tau(x_j,\tau)\,d\tau - \frac{1}{2g}\int d^2x\,d\tau\left(\left(\partial_\tau \boldsymbol{n}\right)^2 + c^2\left(\nabla_x \boldsymbol{n}\right)^2\right)\right)$$

$\eta_j = \pm 1$ identifies sublattices

Small g: long-range Neel order and spin-wave theory

Large g: duality mapping to the **Quantum Dimer Model**

I.C Quantum field theory for critical point

λ close to λ_c : neglect Berry phases and use "soft spin" field

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

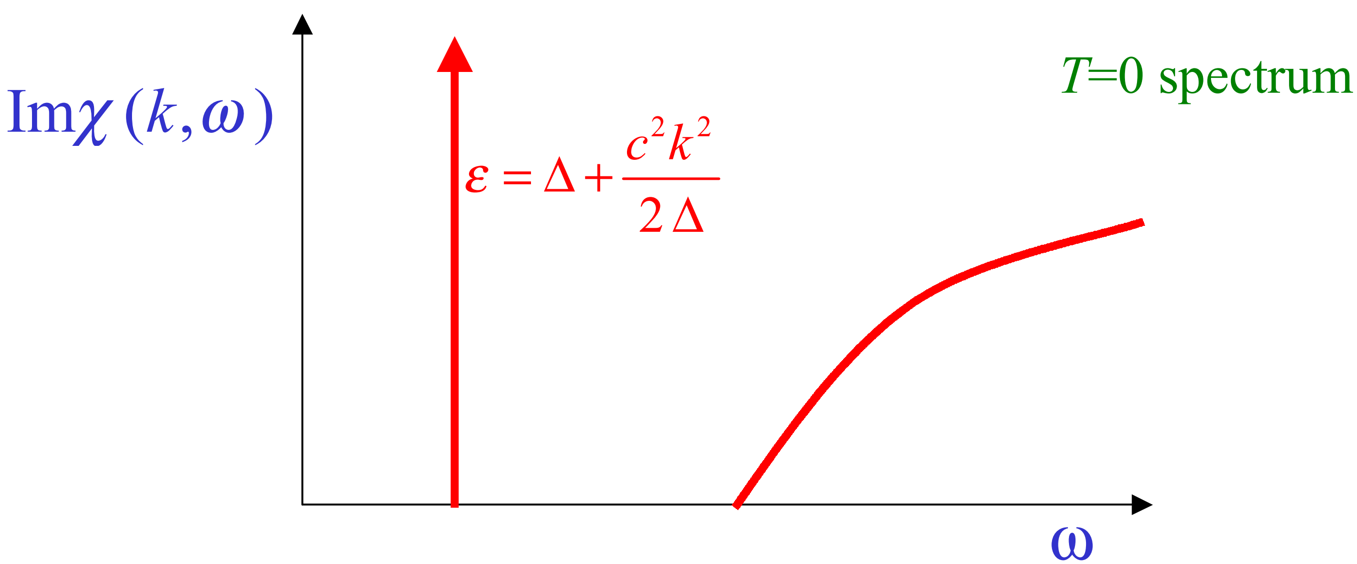
ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$ → $\lambda < \lambda_c$

$r < 0$ → $\lambda > \lambda_c$

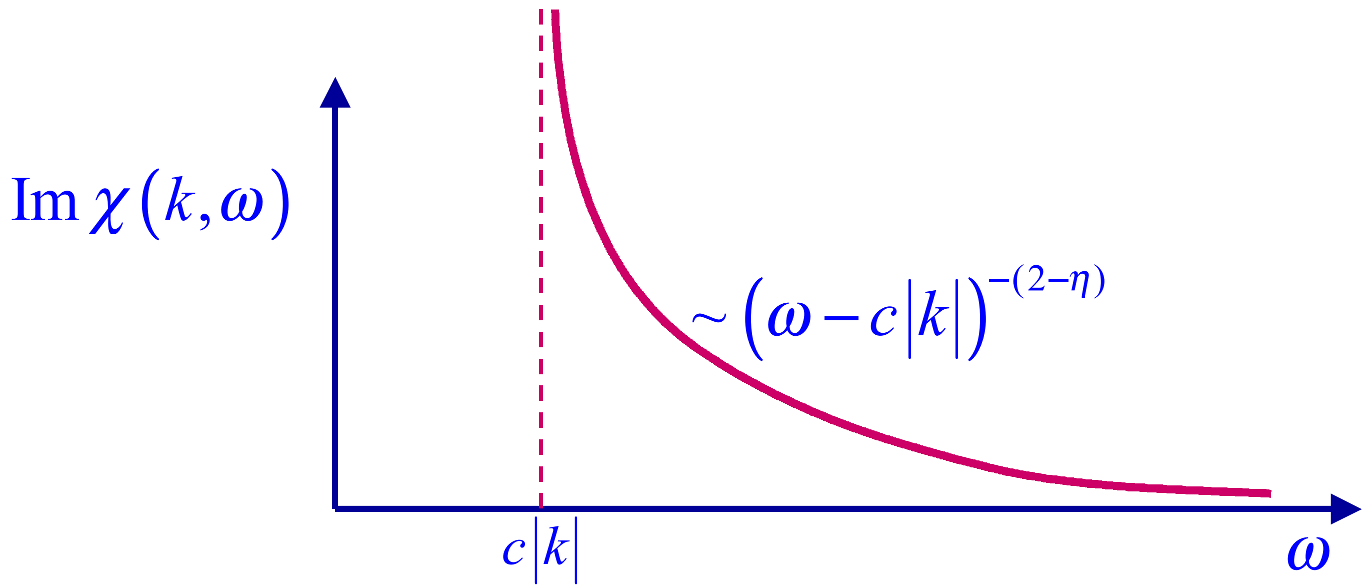
Oscillations of ϕ_α about zero (for $r > 0$)

→ spin-1 collective mode



Critical coupling $(\lambda = \lambda_c)$

Dynamic spectrum at the critical point



No quasiparticles --- dissipative critical continuum

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II. Quantum XY model in one dimension

S. Sachdev and K. Park, cond-mat/0108214

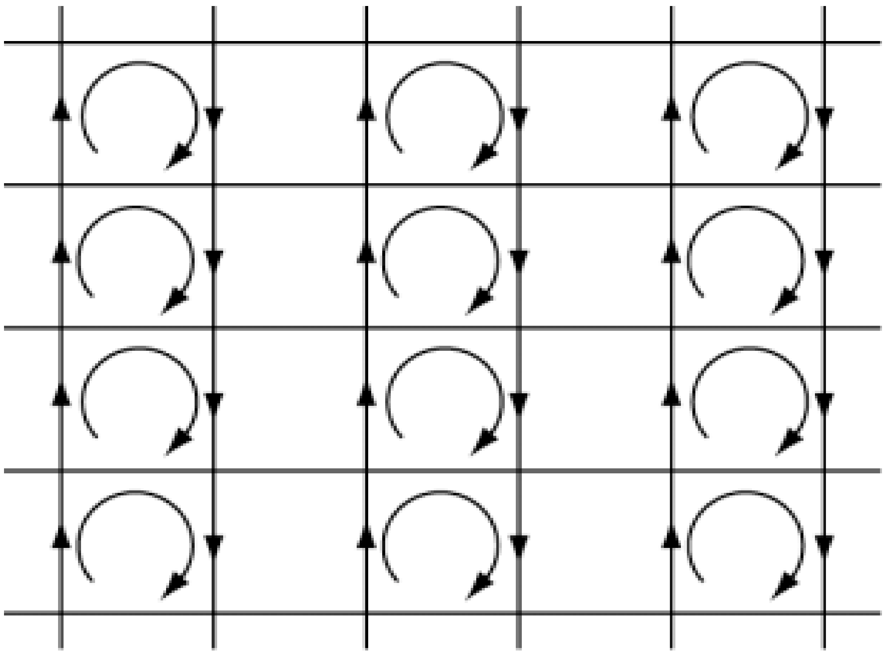
$$H_{XY} = J_1 \sum_j \left(S_{xj} S_{x,j+1} + S_{yj} S_{y,j+1} + \lambda S_{zj} S_{z,j+1} \right) + J_2 \sum_j \left(S_{xj} S_{x,j+2} + S_{yj} S_{y,j+2} + \lambda S_{zj} S_{z,j+2} \right)$$

Write

$$\mathbf{n}_j = (\cos(\theta_j), \sin(\theta_j), 0)$$

where j is now a discrete spacetime index upon a square lattice.

Structure of Berry phase terms.



$$S \sum_j \eta_j A_{j\tau} = S \sum_j \ell_{\bar{j}} \epsilon_{\mu\nu} \Delta_\mu A_{j\nu}$$

where $\ell_{\bar{j}} = 0$ ($\ell_{\bar{j}} = 1$) on even (odd) columns.

For XY model, $S \epsilon_{\mu\nu} \Delta_\mu A_{j\nu} = (2\pi S) \times$ vortex number

Vortices in odd columns carry a factor $(-1)^{2S}$.

$\underline{S \text{ integer}}$.

$$Z'_{XY} = \prod_j \int d\theta_j \exp\left(\frac{1}{g}\sum_j \cos(\Delta_\mu \theta_j)\right)$$

where $\mu = x, \tau$.
This is the action for a "classical" XY model in $D = 2$.
Displays Kosterlitz-Thouless transition.
Dual height model for KT transition:

$$Z'_{XY} = \sum_{\{m_{j\mu}\}} \prod_j \int d\theta_j \exp\left(-\frac{1}{2g}\sum_j (\Delta_\mu \theta_j - 2\pi m_{j\mu})^2\right)$$

This is the Villain periodic Gaussian form. Poisson summation leads to

$$Z'_{XY} = \sum_{\{p_{\bar{\jmath}}\}} \exp\left(-\frac{g}{2}\sum_{\bar{\jmath}} (\Delta_\mu p_{\bar{\jmath}})^2\right)$$

Height (or roughening) model on the square lattice
All heights are integers.

For XY model, the A 'flux' measures vortex number.
In the periodic Gaussian formulation

$$S\epsilon_{\mu\nu}\Delta_\mu A_{j\nu} = \pi\epsilon_{\mu\nu}\Delta_\mu m_{j\nu}$$

So with Berry phases partition function of $S = 1/2$ quantum XY model is

$$Z_{XY} = \sum_{\{m_{j\mu}\}} \prod_j \int d\theta_j \exp\left(-\frac{1}{2g}\sum_j(\Delta_\mu\theta_j - 2\pi m_{j\mu})^2 + i\pi\sum_j \ell_{\bar{j}}\epsilon_{\mu\nu}\Delta_\mu m_{j\nu}\right)$$

Vortices in odd columns contribute a factor (-1) to the partition function
Weights are not positive.
Dual height model for Z_{XY}

$$Z_{XY} = \sum_{\{p_{\bar{j}}\}} \exp\left(-\frac{g}{2}\sum_{\bar{j}}\left(\Delta_\mu p_{\bar{j}} - \frac{1}{2}\Delta_\mu \ell_{\bar{j}}\right)^2\right)$$

Height model in which the heights are integers (half-integers) on even (odd) columns.

Phases of height model

Rough interface

Tomonaga-Luttinger liquid with power-law spin correlations:

$$\langle n_{ix} n_{jx} \rangle = \langle n_{iy} n_{jy} \rangle \sim \frac{1}{|i-j|^{g/(2\pi)}}$$

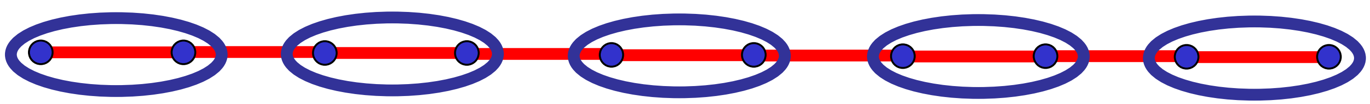
$$\langle n_{iz} n_{jz} \rangle \sim \frac{1}{|i-j|^{2\pi/g}}$$

Smooth interface

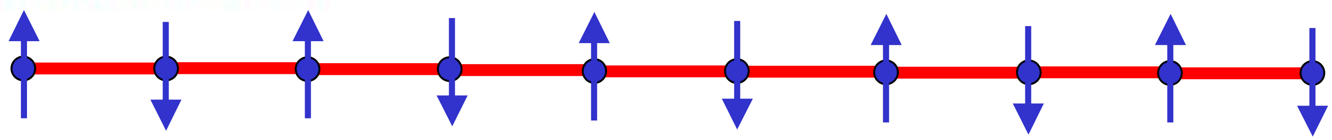
All correlations decay exponentially → there is a gap to all excitations

$\langle p_{\bar{\jmath}} - \ell_{\bar{\jmath}}/2 \rangle$ has a definite value: any such definite value breaks a discrete symmetry of the Hamiltonian.

$\langle p_{\bar{\jmath}} - \ell_{\bar{\jmath}}/2 \rangle = 0, 1/2$ (plus integer) are states with **bond-centered charge order** *i.e.* neighboring links have valence bonds with distinct probabilities

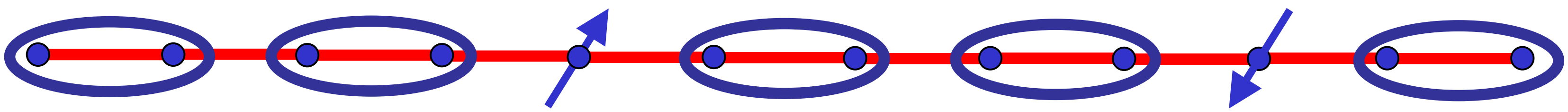


$\langle p_{\bar{\jmath}} - \ell_{\bar{\jmath}}/2 \rangle = 1/4, 3/4$ (plus integer) are the two states with Ising antiferromagnetic order.

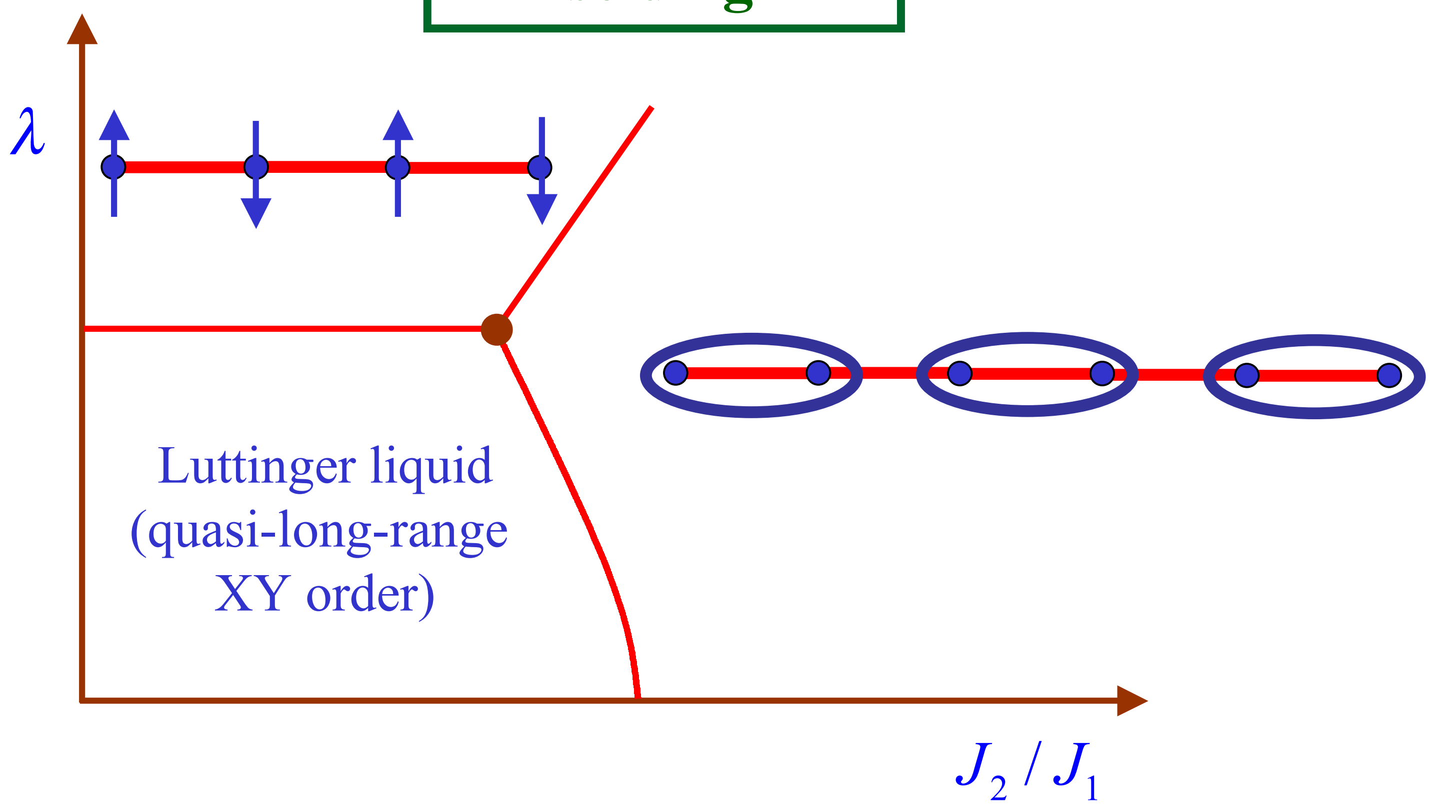


Excitations of paramagnet with bond-charge-order

Deconfined S=1/2 spinons



Phase diagram



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III. Berry phases and the square lattice antiferromagnet

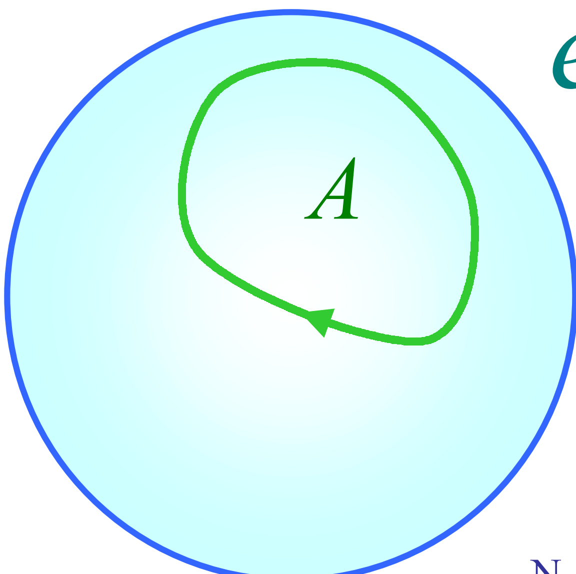
$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Action: $$S_b = \int d^2x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \phi_\alpha \right)^2 + c^2 \left(\partial_\tau \phi_\alpha \right)^2 \right) + V\left(\phi_\alpha^2 \right) \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. B **39**, 2344 (1989).

Missing: Spin Berry Phases

e^{iSA}



Berry phases induce
bond charge order
in quantum "disordered"
phase with $\langle \phi_\alpha \rangle = 0$;
"Dual order parameter"

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

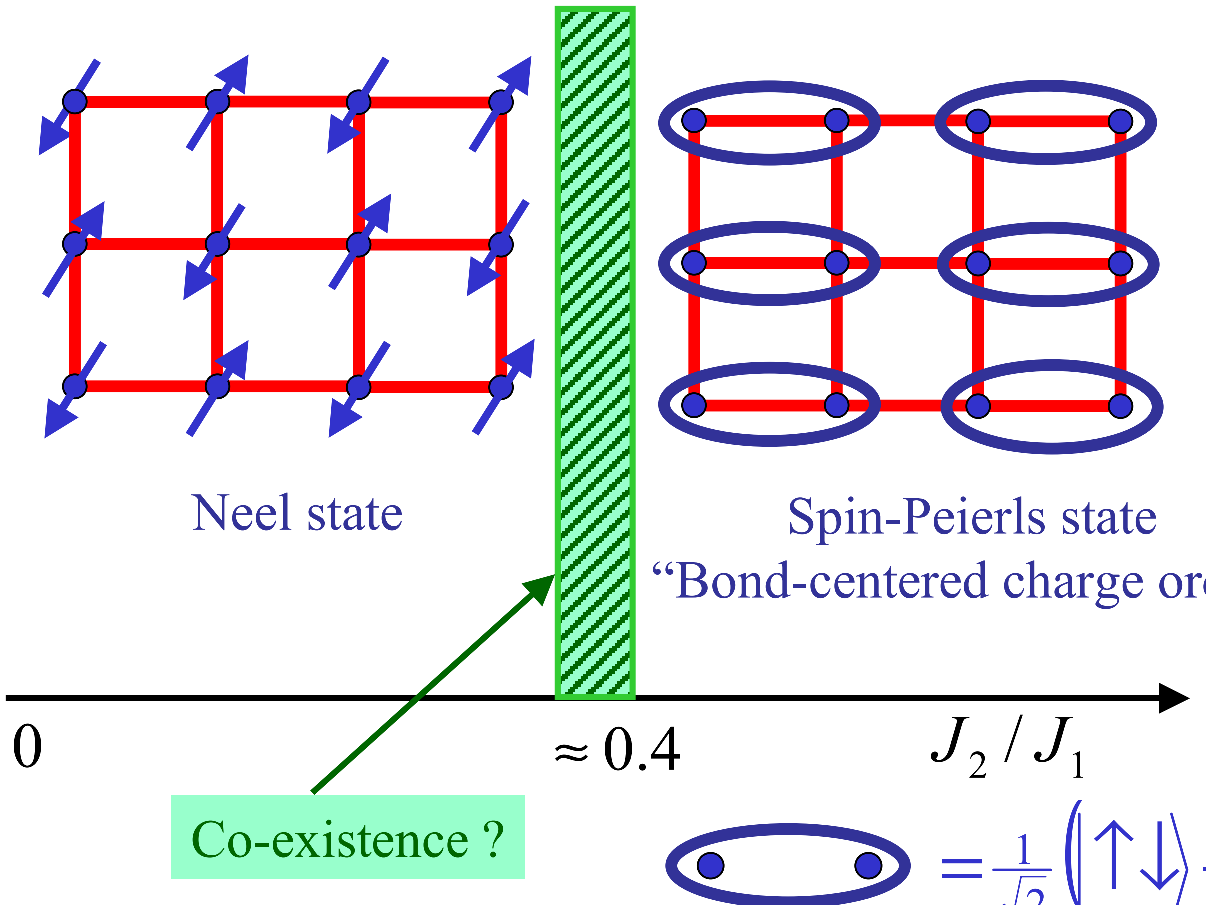
Square lattice with first (J_1) and second (J_2) neighbor exchange interactions

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, Phys. Rev. B **63**, 104420 (2001).

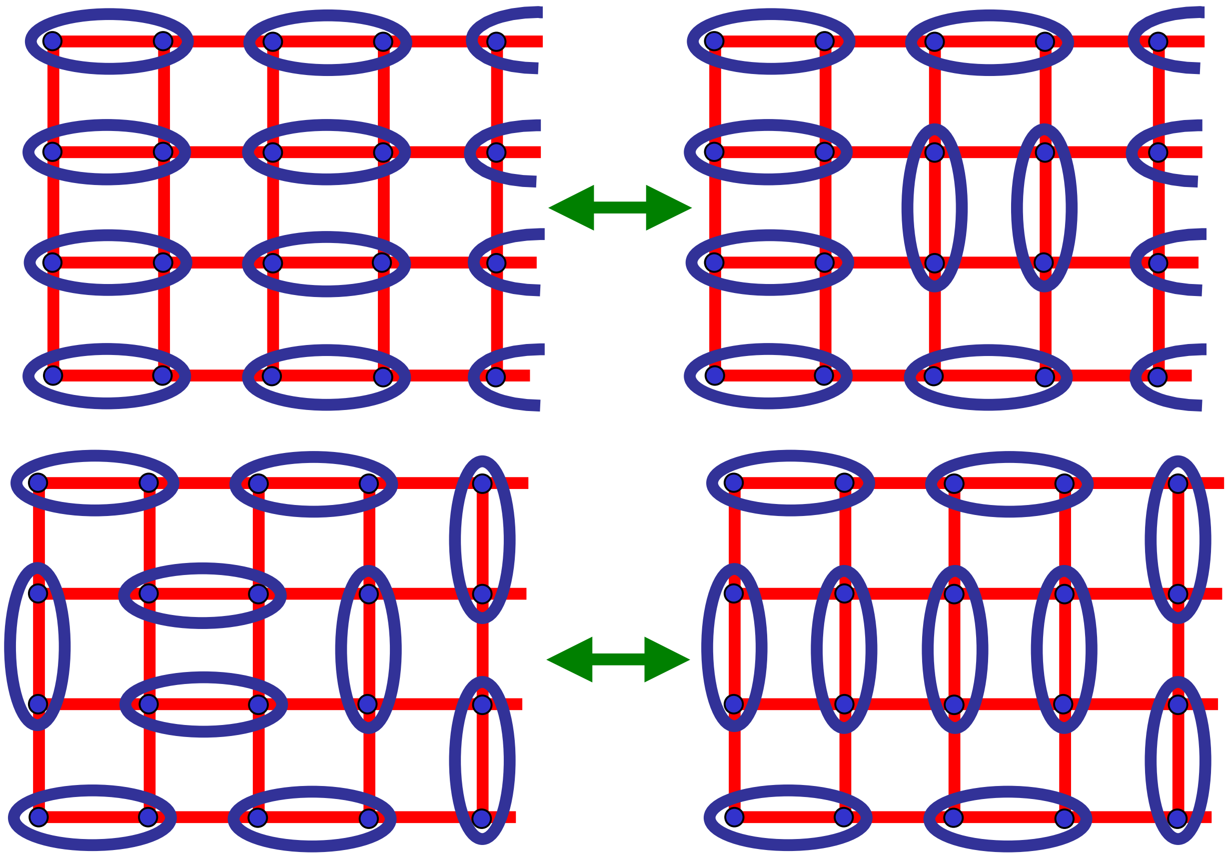
M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, Phys. Rev. B **62**, 14844 (2000).

S. Sachdev and K. Park, cond-mat/0108214.



$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Quantum dimer model –
D. Rokhsar and S. Kivelson Phys. Rev. Lett. **61**, 2376 (1988)



Quantum “entropic” effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.
These effects lead to a broken square lattice symmetry near the transition to the Neel state.

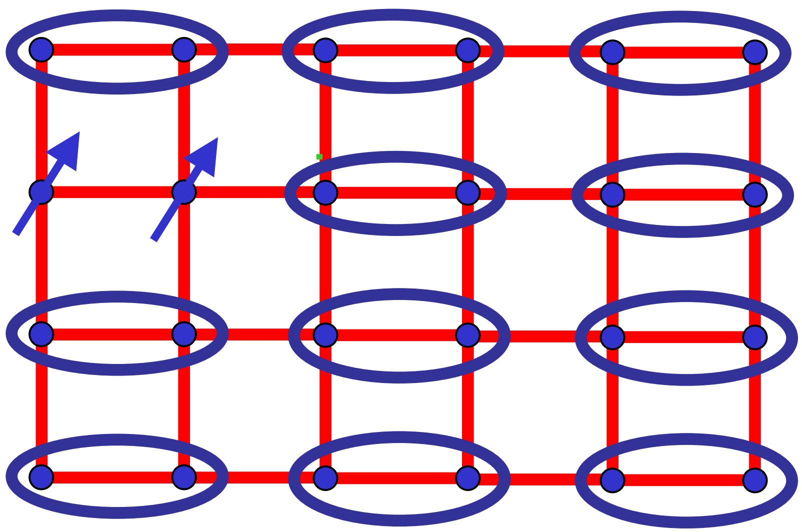
N. Read and S. Sachdev Phys. Rev. B **42**, 4568 (1990).

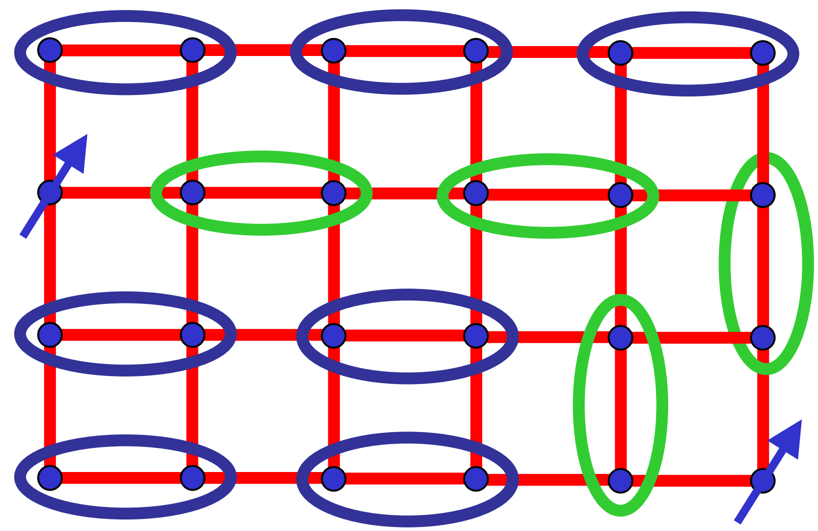
Properties of paramagnet with bond-charge-order

Stable S=1 spin exciton – quanta of 3-component ϕ_α

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

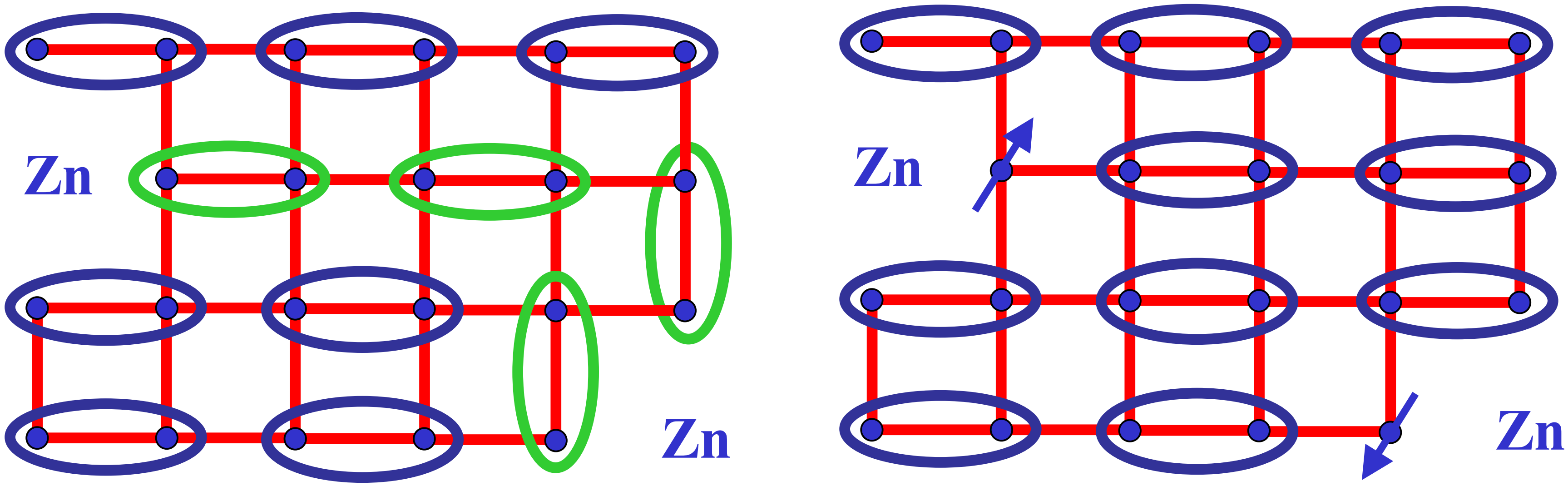
$\Delta \rightarrow$ Spin gap





S=1/2 spinons are ***confined*** by a linear potential.

Effect of static non-magnetic impurities (Zn or Li)



Spinon confinement implies that free S=1/2
moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, Physica C **168**, 370 (1990).
J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **86**, 4116 (2001).

Field theory of bond order

Discretize coherent state path integral on a cubic lattice in spacetime:

$$Z = \prod_j \int d\mathbf{n}_j \delta(\mathbf{n}_j^2 - 1) \exp\left(-\frac{1}{2g} \sum_{j,\mu} \mathbf{n}_j \cdot \mathbf{n}_{j+\hat{\mu}} - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right)$$

where $\mu = x, y, \tau$, and we assume henceforth that $S = 1/2$.
For large g, perform a "high temperature" expansion to obtain an effective action for the $A_{j\mu}$. This action must be invariant under the 'gauge' transformation

$$A_{j\mu} \to A_{j\mu} - \Delta_\mu \gamma_j$$

associated with the change in choice of $\mathbf{n}_0$ (γ_j is the oriented area of the spherical triangle formed by $\mathbf{n}_j$ and the two choices for $\mathbf{n}_0$). Also, it should be invariant under

$$A_{j\mu} \to A_{j\mu} + 4\pi$$

because area of triangle is uncertain modulo 4π.
Simplest large g effective model

$$Z = \prod_j \int dA_{j\mu} \exp\left(\frac{1}{e^2} \sum_{\square} \cos\left(\frac{1}{2} \epsilon_{\mu\nu\lambda} \Delta_\nu A_{j\lambda} \right) - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right)$$

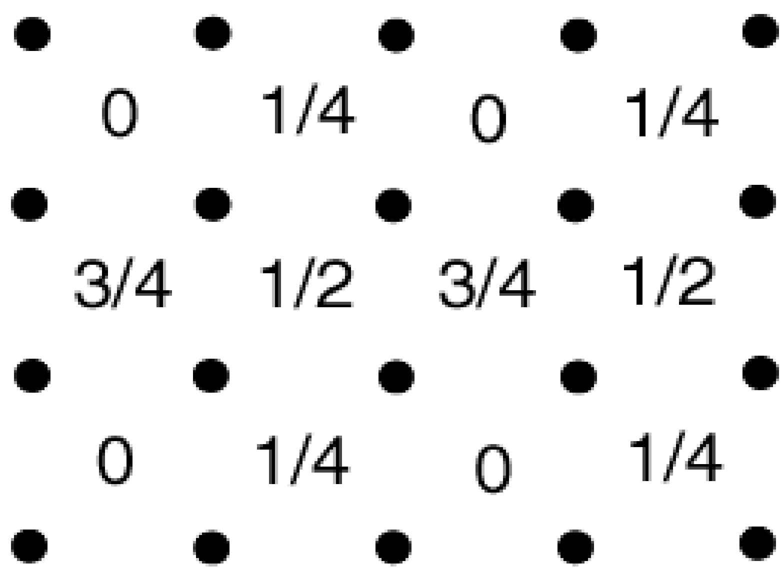
with $e^2 \sim g^2$.
This is compact QED in 2+1 dimensions with Berry phases.

Exact duality transform on periodic Gaussian ("Villain") action for compact QED yields

$$Z = \sum_{\{h_{\bar{\jmath}}\}} \exp\left(-\frac{e^2}{2}\sum_{\bar{\jmath}}\left(\Delta_\mu h_{\bar{\jmath}} - \Delta_\mu \mathcal{X}_{\bar{\jmath}}\right)^2\right)$$

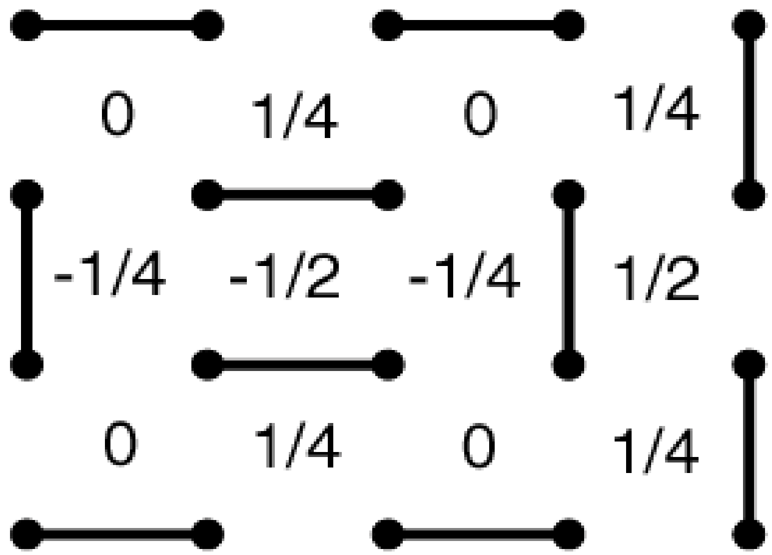
with $h_{\bar{\jmath}}$ integer.

Height model in 2+1 dimensions with 'offsets' $\mathcal{X}_{\bar{\jmath}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



For large e^2 , low energy height configurations are in exact one-to-one correspondence with dimer coverings of the square lattice

⟹ 2+1 dimensional height model is the path integral of the quantum dimer model



There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

⟹ There is a definite average height of the interface

⟹ **Ground state has bond-charge order.**

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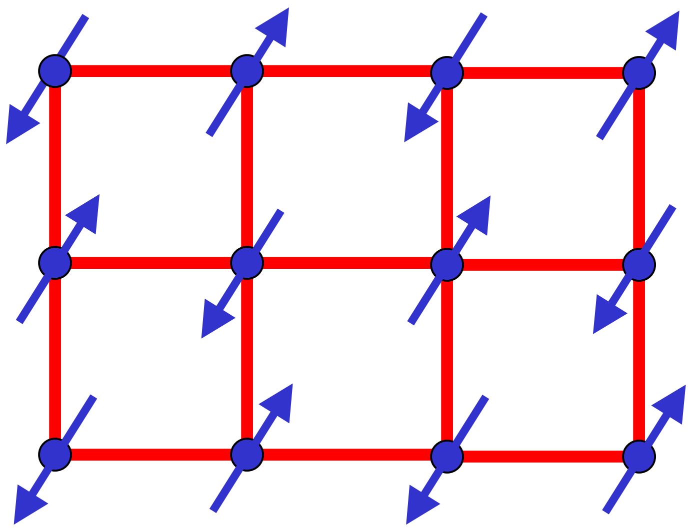
Parent compound of the high temperature superconductors: La_2CuO_4

Mott insulator: square lattice antiferromagnet



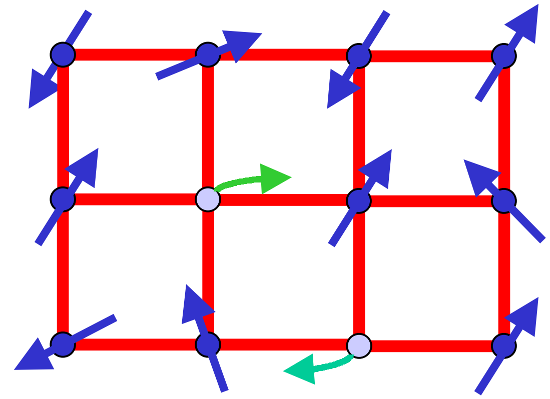
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Ground state has long-range magnetic (Néel) order

Néel order parameter: $\phi_\alpha = (-1)^{i_x + i_y} S_{i\alpha} \; ; \; \alpha = x, y, z$

$$\langle \phi_\alpha \rangle \neq 0$$

Introduce mobile carriers of density δ by substitutional doping of out-of-plane ions *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$



Exhibits superconductivity below a high critical temperature T_c

Superconductivity in a doped Mott insulator

⇕ ?

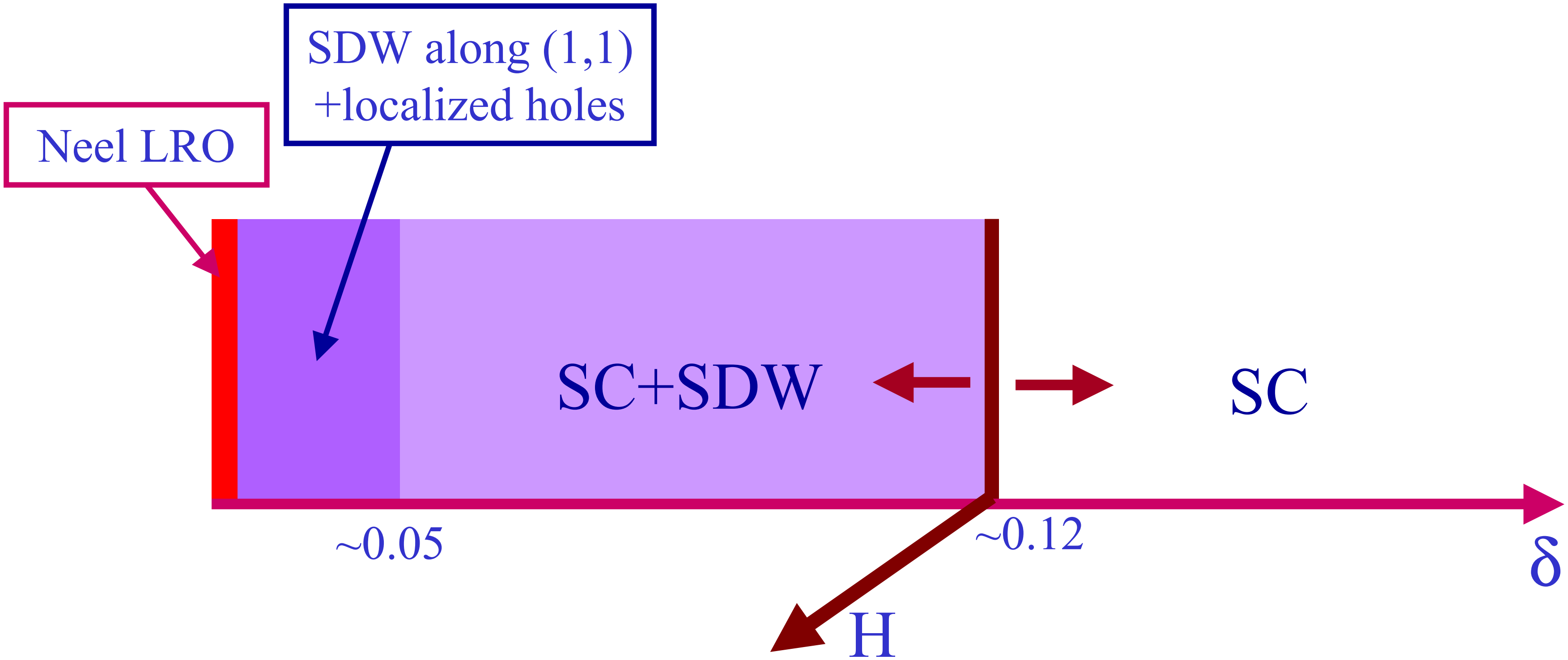
BCS superconductor obtained by the Cooper instability of a metallic Fermi liquid

Quantum numbers of ground state and low energy quasiparticles are the same, but characteristics of the Mott insulator are revealed in the vortices.

S. Sachdev, Phys. Rev. B **45**, 389 (1992); K. Park and S. Sachdev Phys. Rev. B **64**, 184510 (2001).

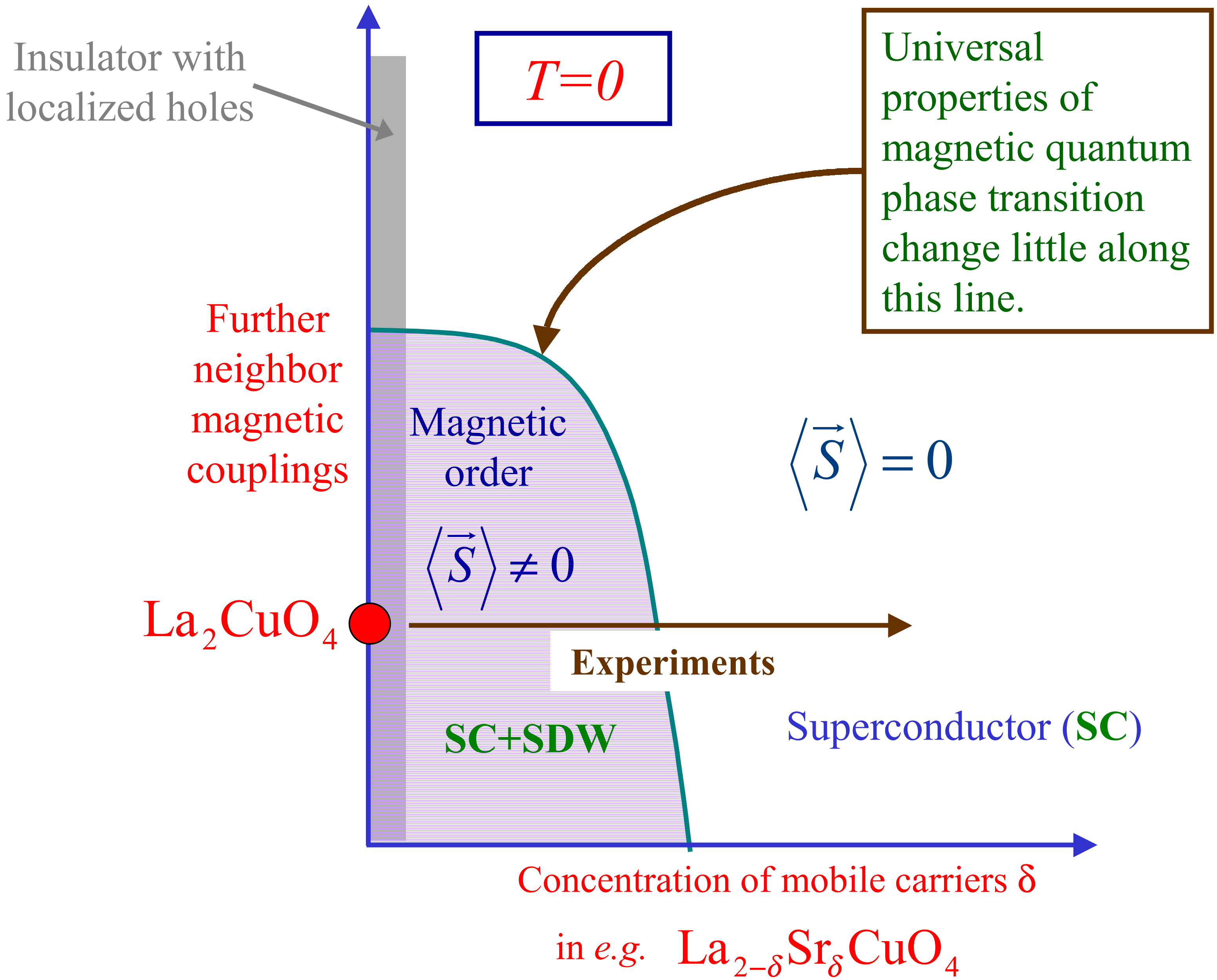
STM measurement of J.E. Hoffman *et al*., Science, Jan 2002.

Zero temperature phases of the cuprate superconductors as a function of hole density



Theory for a system with strong interactions: describe SC and SC+SDW phases by expanding in the deviation from the quantum critical point between them.

B. Keimer *et al*. Phys. Rev. B **46**, 14034 (1992).
S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al.,* Phys. Rev. B **60**, 3643 (1999).
J. E. Sonier *et al*., cond-mat/0108479.
C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.

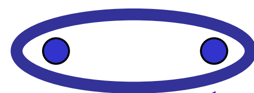


S. Sachdev and J. Ye, *Phys. Rev. Lett*. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev*. B **49**, 11919 (1994)

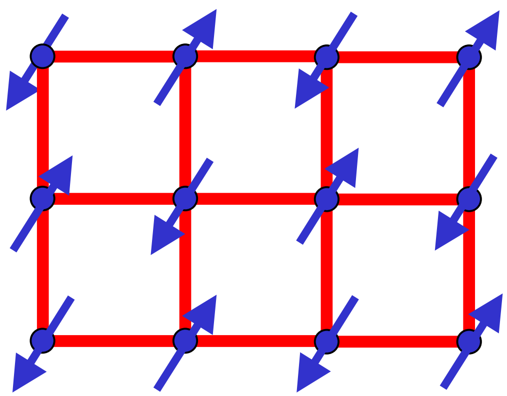
Magnetic ordering transitions in the insulator

Square lattice with first(J_1) and second (J_2) neighbor exchange interactions (say)

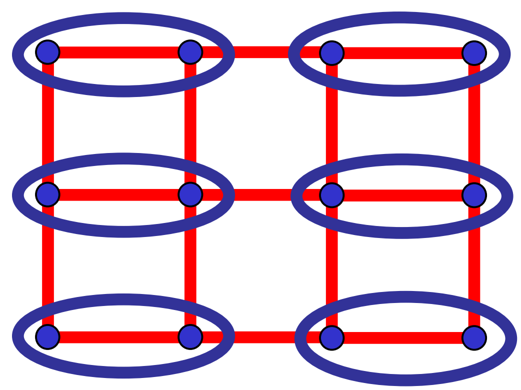
$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$



$$= \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big)$$



Neel state



Spin-Peierls (or plaquette) state
"Bond-centered charge order"

J_2 / J_1

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, *Phys. Rev.* B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, *Phys. Rev.* B **62**, 14844 (2000).

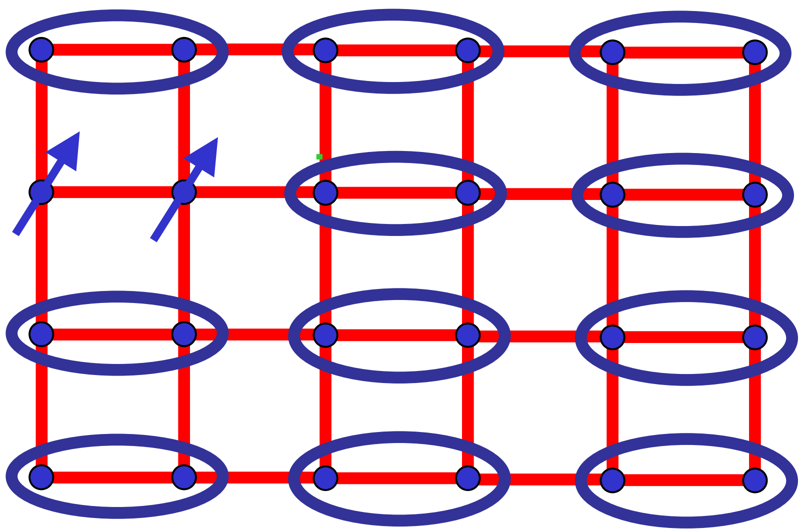
See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204 .

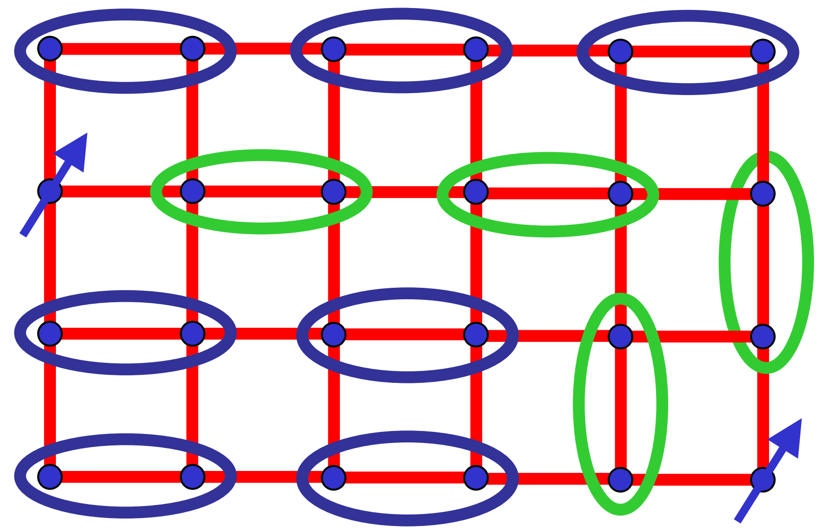
Properties of paramagnet with bond-charge-order

Stable S=1 spin exciton – quanta of 3-component ϕ_α

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

$\Delta \rightarrow$ Spin gap



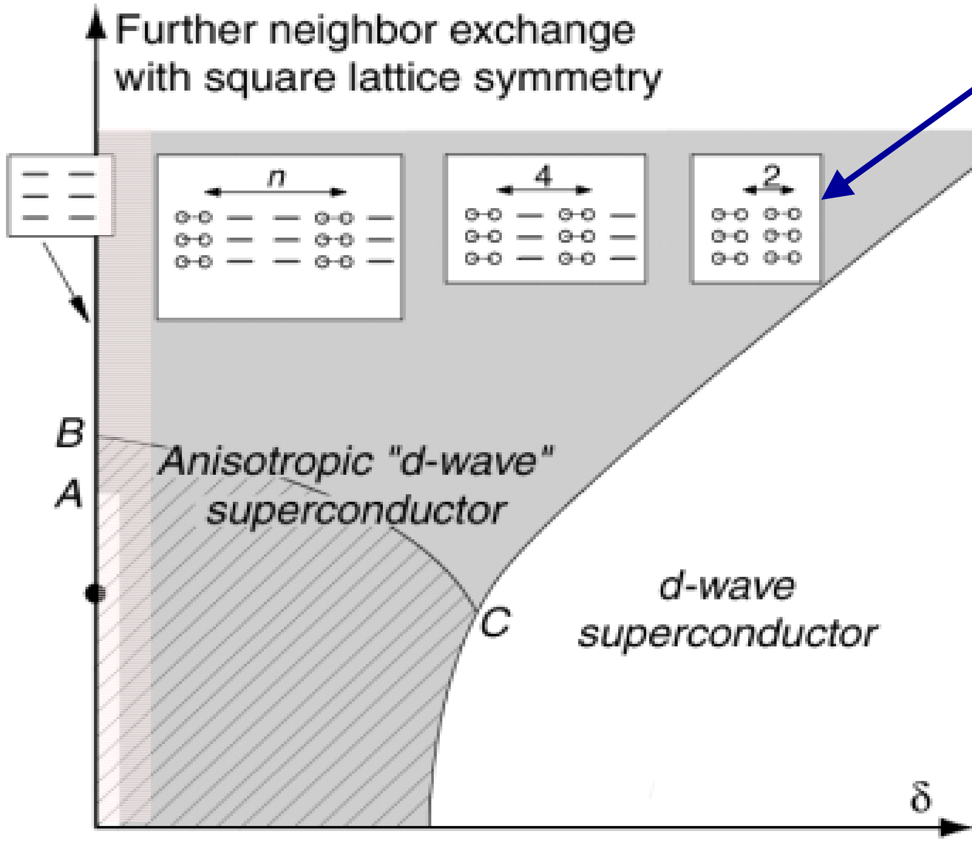


S=1/2 spinons are ***confined*** by a linear potential.

Transition to Neel state $\Rightarrow$ Bose condensation of ϕ_α

Develop quantum theory of SC+SDW to SC transition driven by condensation of a S=*1* boson (spin exciton)

Doping the paramagnetic Mott insulator

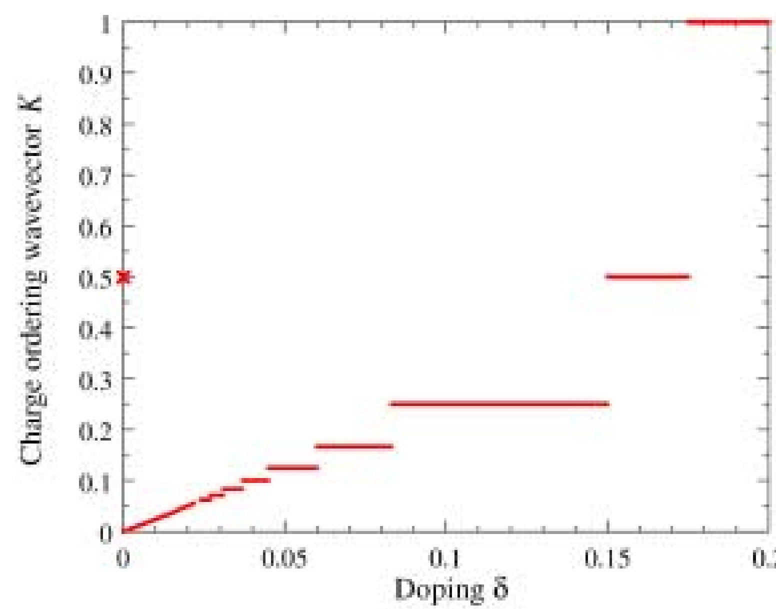


"Large *N*" theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).

M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999).

M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000).



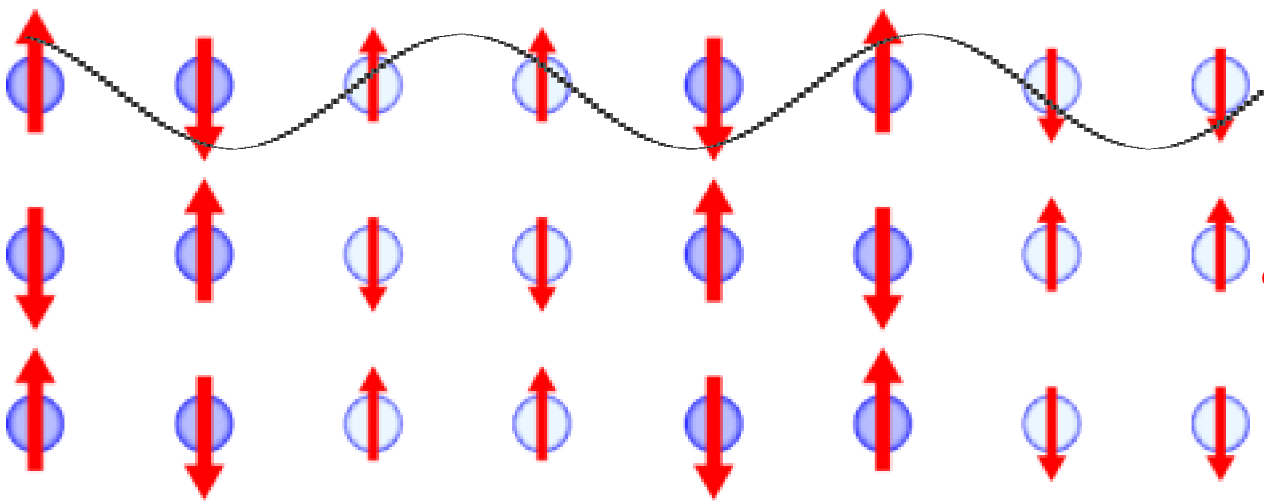
See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett.* **80**, 1272 (1998).

IV.A Theory of SC+SDW to SC quantum transition

Spin density wave order parameter for general ordering wavevector

$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a complex field $\left(\text{except for } \boldsymbol{K}=(\pi,\pi) \text{ when } e^{i\boldsymbol{K}\cdot\boldsymbol{r}} = (-1)^{r_x+r_y}\right)$



Wavevector $\boldsymbol{K}$=(3π/4,π)

Exciton wavefunction $\Phi_\alpha(\boldsymbol{r})$ describes *envelope* of this order. Phase of $\Phi_\alpha(\boldsymbol{r})$ represents sliding degree of freedom

Associated "charge" density wave order

$$\delta\rho(\boldsymbol{r}) \propto S_\alpha^2(\boldsymbol{r}) = \sum_\alpha \Phi_\alpha^2(\boldsymbol{r})e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

J. Zaanen and O. Gunnarsson, Phys. Rev. B **40**, 7391 (1989).
H. Schulz, J. de Physique **50**, 2833 (1989).
O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).

Action for SDW ordering transition in the superconductor

$$\mathcal{S} = \int d^2r d\tau \left[\left| \nabla_r \Phi_\alpha \right|^2 + c^2 \left| \partial_\tau \Phi_\alpha \right|^2 + V\left(\Phi_\alpha \right) \right]$$

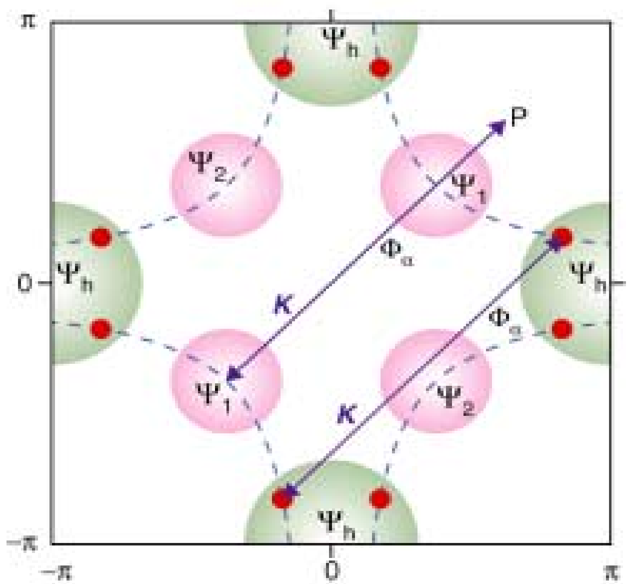
Similar terms present in action for SDW ordering in the insulator

Coupling to the S=1/2 Bogoliubov quasiparticles of the d-wave superconductor

Trilinear "Yukawa" coupling

$$\int d^2r d\tau \, \Phi_\alpha \Psi \Psi$$

is prohibited unless ordering wavevector is fine-tuned.

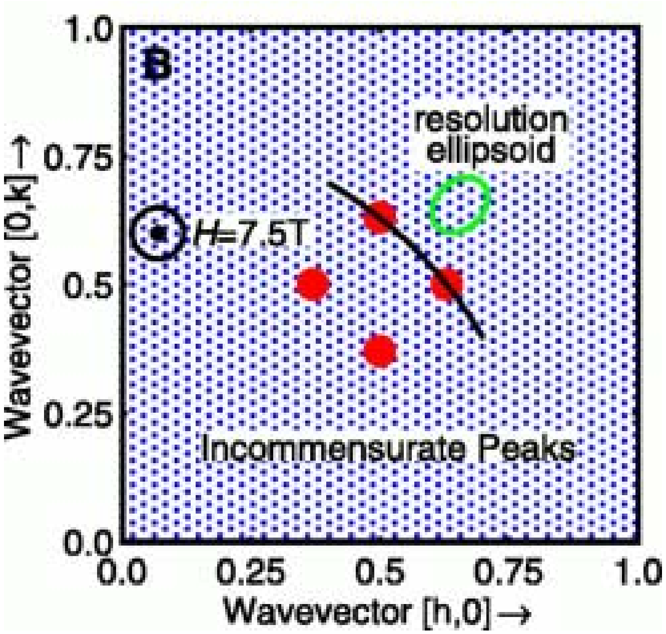


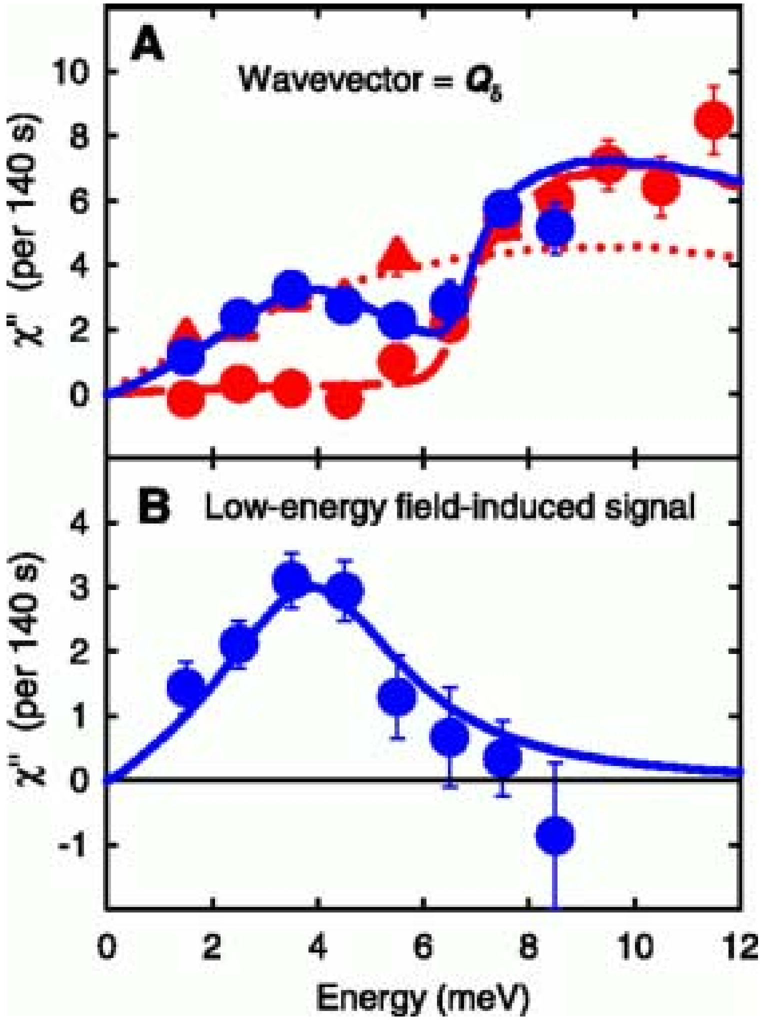
$$\kappa \sum_\alpha \int d^2r d\tau \left| \Phi_\alpha \right|^2 \Psi^\dagger \Psi \text{ is allowed}$$

Scaling dimension of $\kappa = (1/\nu - 2) < 0 \;\Rightarrow\;$ irrelevant.

Neutron scattering measurements of dynamic spin correlations of the superconductor (SC) in a magnetic field

B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder, *Science* **291**, 1759 (2001).





Peaks at $(0.5, 0.5) \pm (0.125, 0)$

and $(0.5,0.5) \pm (0,0.125)$

$\Rightarrow$ dynamic SDW of period 8

Neutron scattering off $La_{2\text{-}\delta}Sr_{\delta}CuO_4$ ($\delta = 0.163$, ***SC phase***)

at low temperatures in H=0 (red dots) and H=7.5T (blue dots)

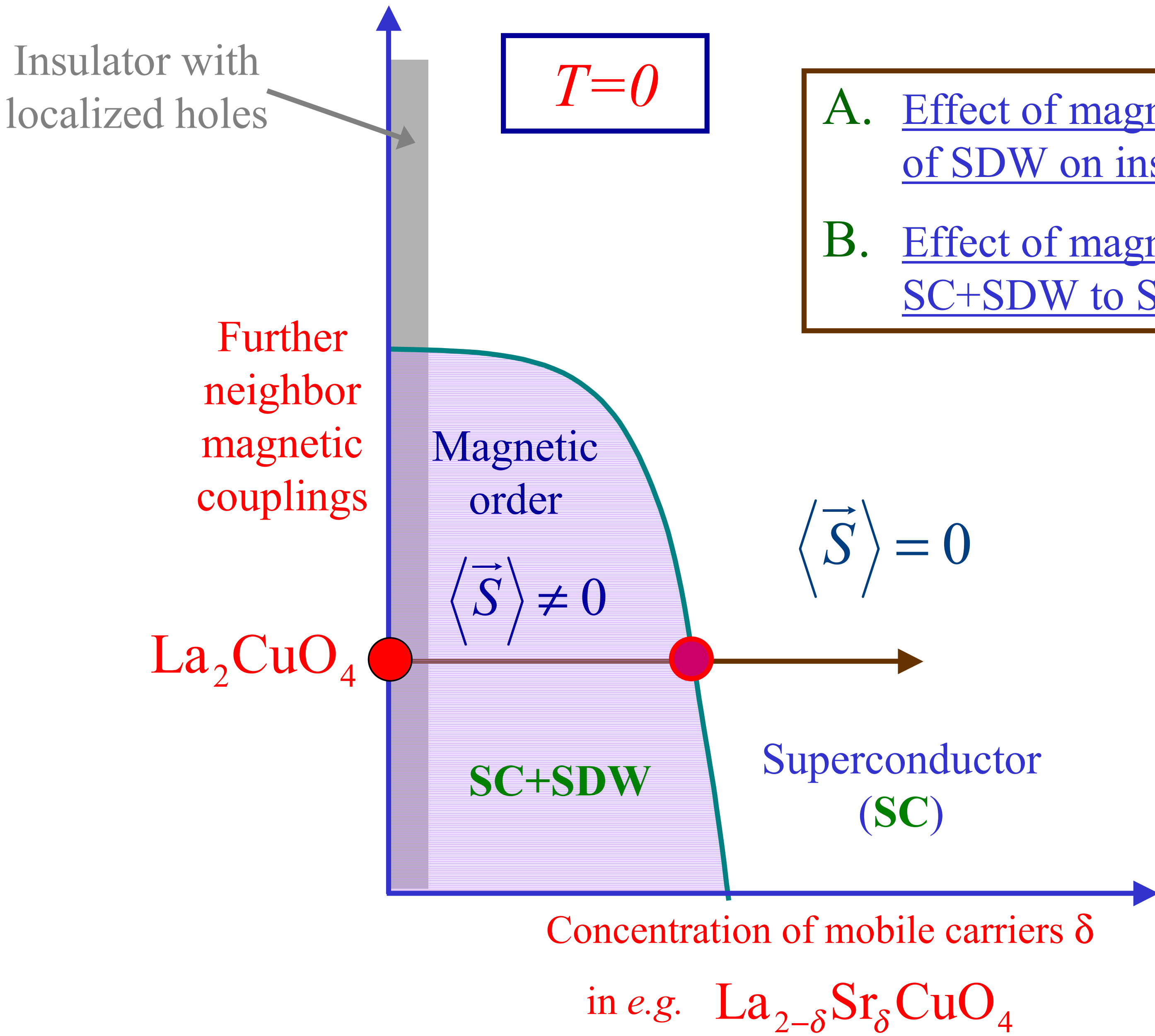
S. Sachdev, *Phys. Rev.* B **45**, 389 (1992), and N. Nagaosa and P.A. Lee, *Phys. Rev.* B **45**, 966 (1992), suggested an enhancement of dynamic spin-gap correlations (as in a spin-gap Mott insulator) in the cores of vortices in the underdoped cuprates. In the simplest mean-field theory, this enhancement appears most easily for vortices with flux *hc/e*.

D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) suggested static Néel order in the cores of vortices (SC order “rotates” into Néel order in SO(5) picture) .

Using a picture of “dynamically fluctuating spins in the vortices”, the amplitude of the field-induced signal, and the volume-fraction of vortex cores (~10%), Lake *et al.* estimated that in such a model each spin in the vortex core would have a low-frequency moment equal to that in the insulating state at $\delta=0$ ($0.6\ \mu_B$).

Observed field-induced signal is much larger than anticipated.

IV.B Phase diagrams in a magnetic field.



A. Effect of magnetic field on onset of SDW on insulator

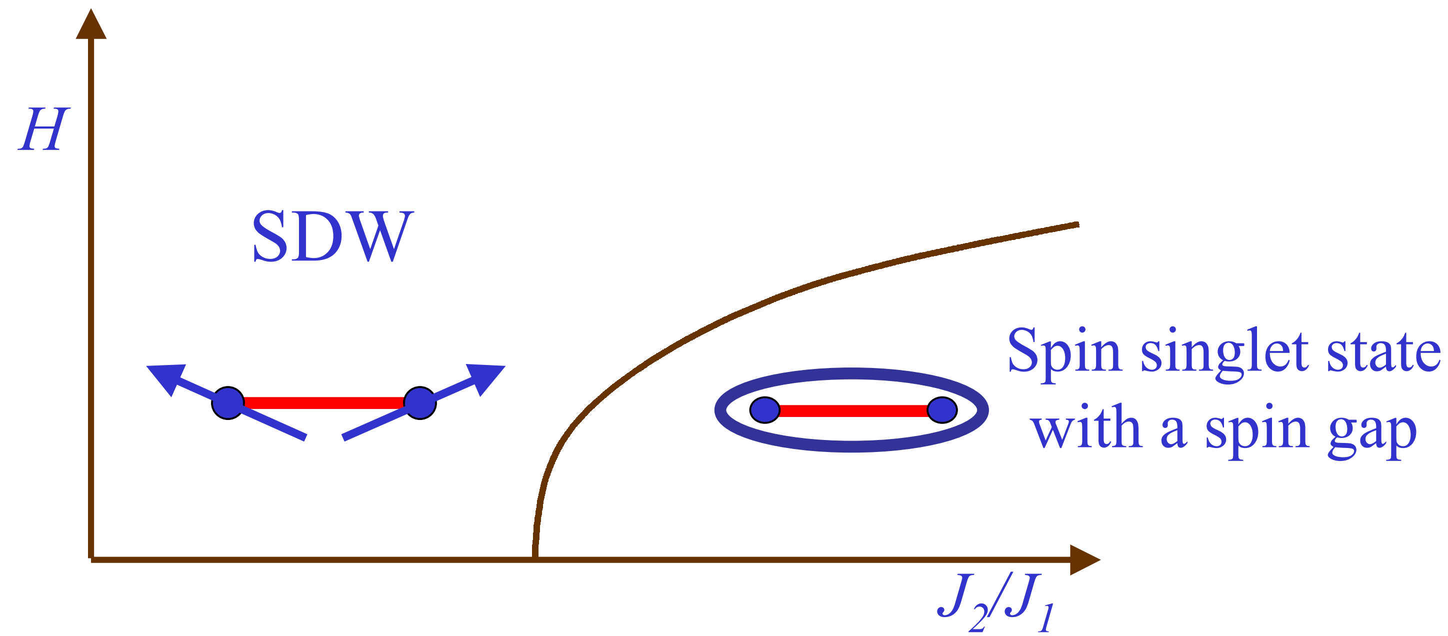
B. Effect of magnetic field on SC+SDW to SC transition

IV.B Phase diagrams in a magnetic field.

A. Effect of magnetic field on onset of SDW in the insulator

H couples via the Zeeman term

$$\left|\partial_\tau \Phi_\alpha\right|^2 \Rightarrow \left(\partial_\tau \Phi_\alpha^* - i\varepsilon_{\alpha\sigma\rho} H_\sigma \Phi_\rho\right)\left(\partial_\tau \Phi_\alpha - i\varepsilon_{\alpha\beta\gamma} H_\beta \Phi_\gamma\right)$$



Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

Related theory applies to spin gap systems in a field and to double layer quantum Hall systems at ν=2

IV.B Phase diagrams in a magnetic field.

(extreme Type II superconductivity)

B. Effect of magnetic field on SDW+SC to SC transition

Infinite diamagnetic susceptibility of *non-critical* superconductivity leads to a strong effect.

- Theory should account for *dynamic* quantum spin fluctuations
- All effects are ~ H^2 except those associated with H induced superflow.
- Can treat SC order in a *static* Ginzburg-Landau theory

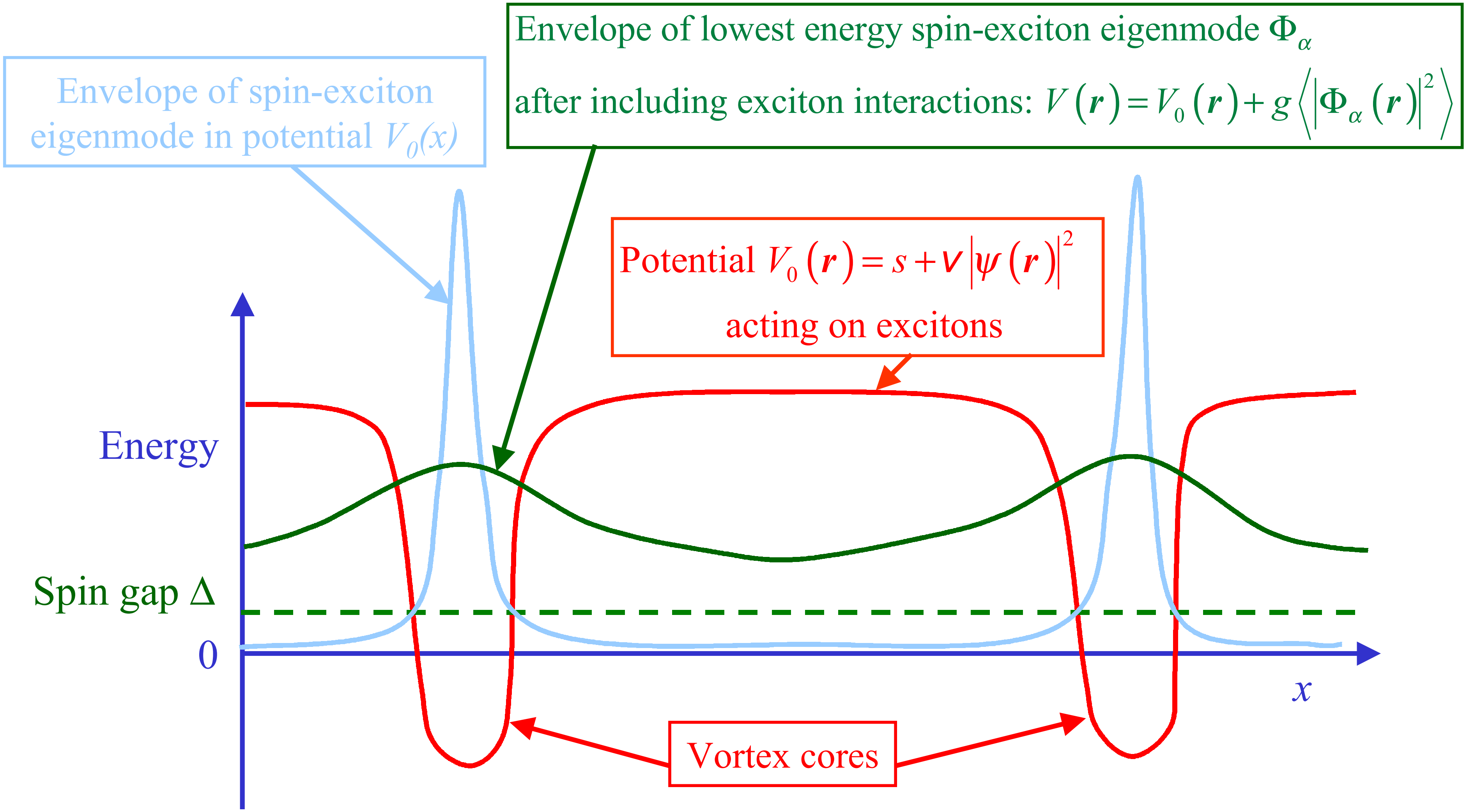
$$S_b = \int d^2 r \int_0^{1/T} d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + s\left|\Phi_\alpha\right|^2 + \frac{g_1}{2}\left(\left|\Phi_\alpha\right|^2\right)^2 + \frac{g_2}{2}\left|\Phi_\alpha^2\right|^2 \right]$$

$$S_c = \int d^2 r d\tau \left[\frac{v}{2} \left|\Phi_\alpha\right|^2 \left|\psi\right|^2 \right]$$

$$F_{GL} = \int d^2 r \left[-\left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2} + \left|\left(\nabla_r - iA\right)\psi\right|^2 \right]$$

$$Z\left[\psi(r)\right] = \int D\Phi(r,\tau) e^{-F_{GL} - S_b - S_c}$$

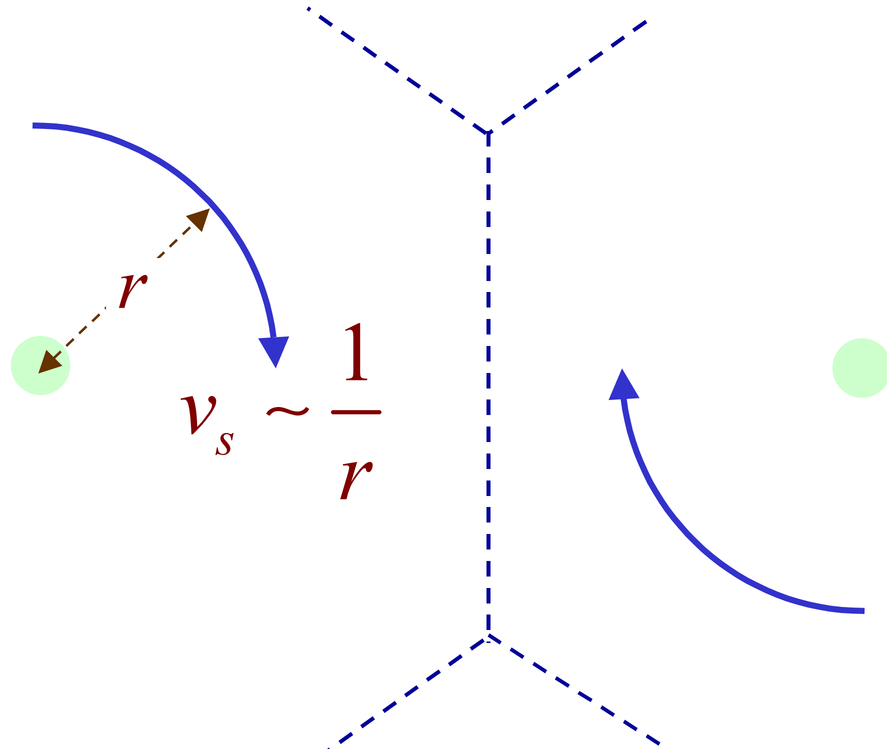
$$\frac{\delta \ln Z\left[\psi(r)\right]}{\delta \psi(r)} = 0$$



Strongly relevant repulsive interactions between excitons imply that low energy excitons must be extended.

A.J. Bray and M.A. Moore, J. Phys. C **15**, L7 65 (1982).
J.A. Hertz, A. Fleishman, and P.W. Anderson, Phys. Rev. Lett. **43**, 942 (1979).

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



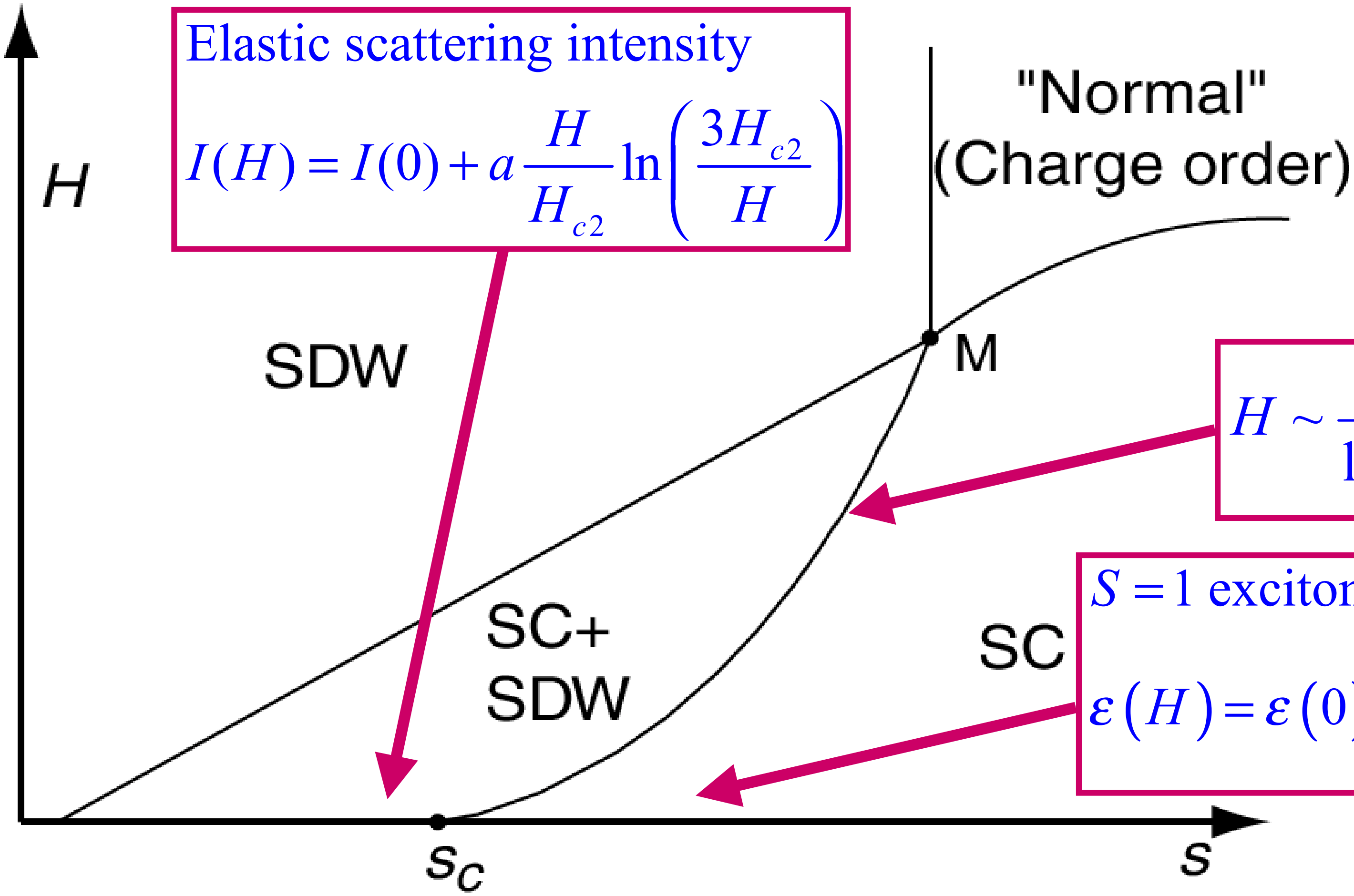
Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Tuning parameter s replaced by $s_{eff}\left(H\right) = s - C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$

Main results

T=0



Elastic scattering intensity

$$I(H) = I(0) + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$H \sim \frac{(s - s_c)}{\ln\left(1/(s - s_c)\right)}$$

$S = 1$ exciton energy

$$\varepsilon(H) = \varepsilon(0) - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

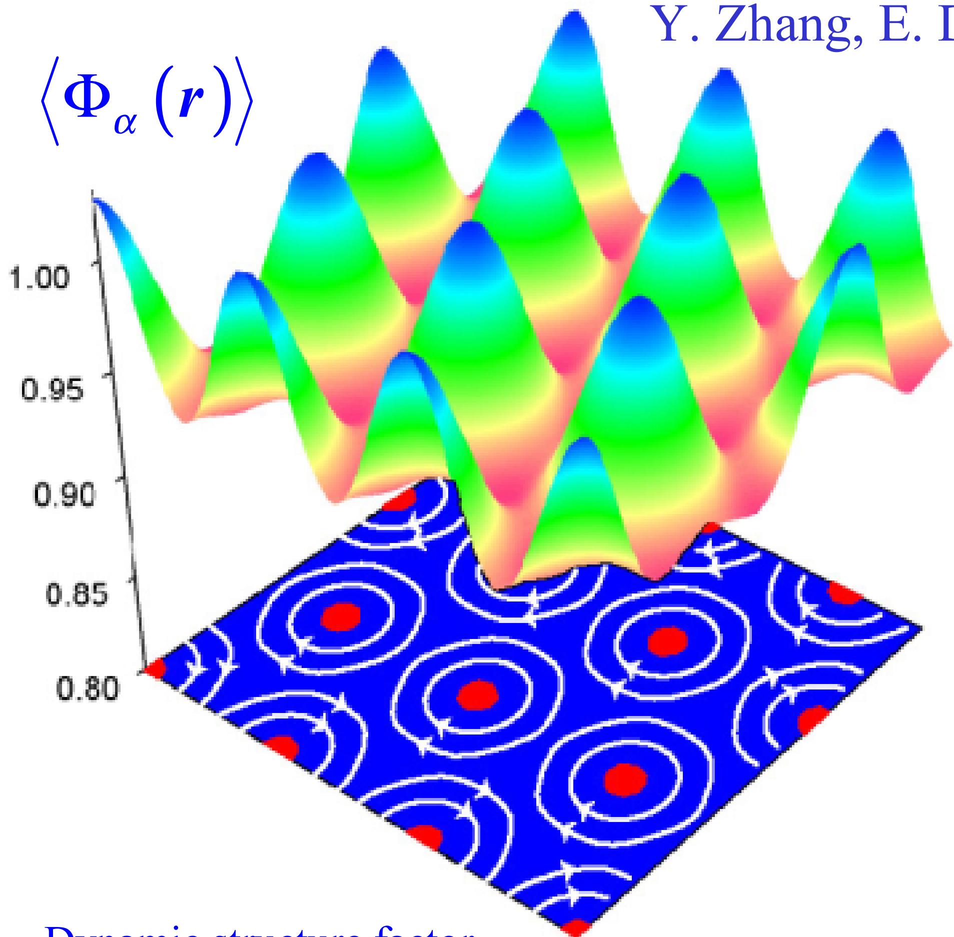
- All functional forms are **exact**.

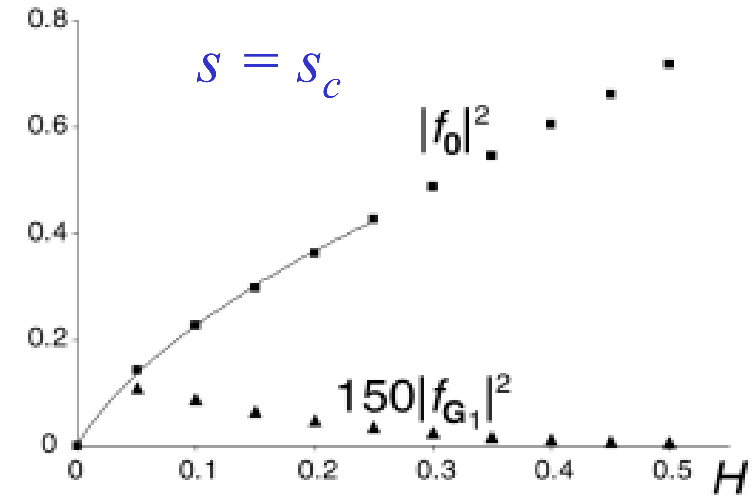
E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

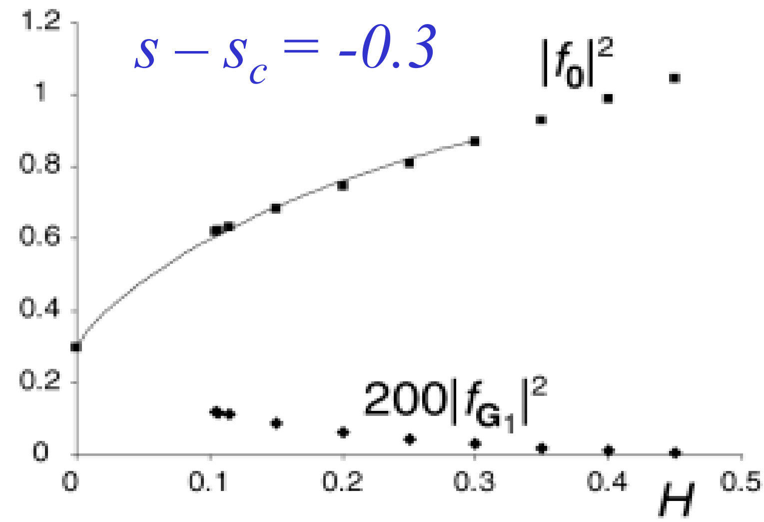
Structure of *long-range* SDW order in SC+SDW phase

Computation in a self-consistent "large *N*" theory

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343







Dynamic structure factor

$$S(\boldsymbol{k},\omega) = (2\pi)^3 \delta(\omega) \sum_{\boldsymbol{G}} |f_{\boldsymbol{G}}|^2 \delta(\boldsymbol{k}-\boldsymbol{G}) + \cdots$$

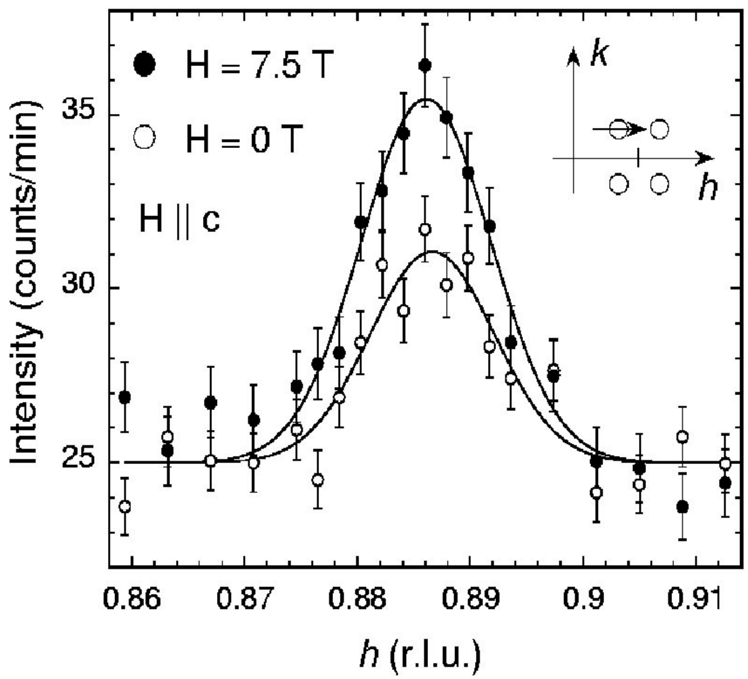
$\boldsymbol{G} \rightarrow$ reciprocal lattice vectors of vortex lattice.

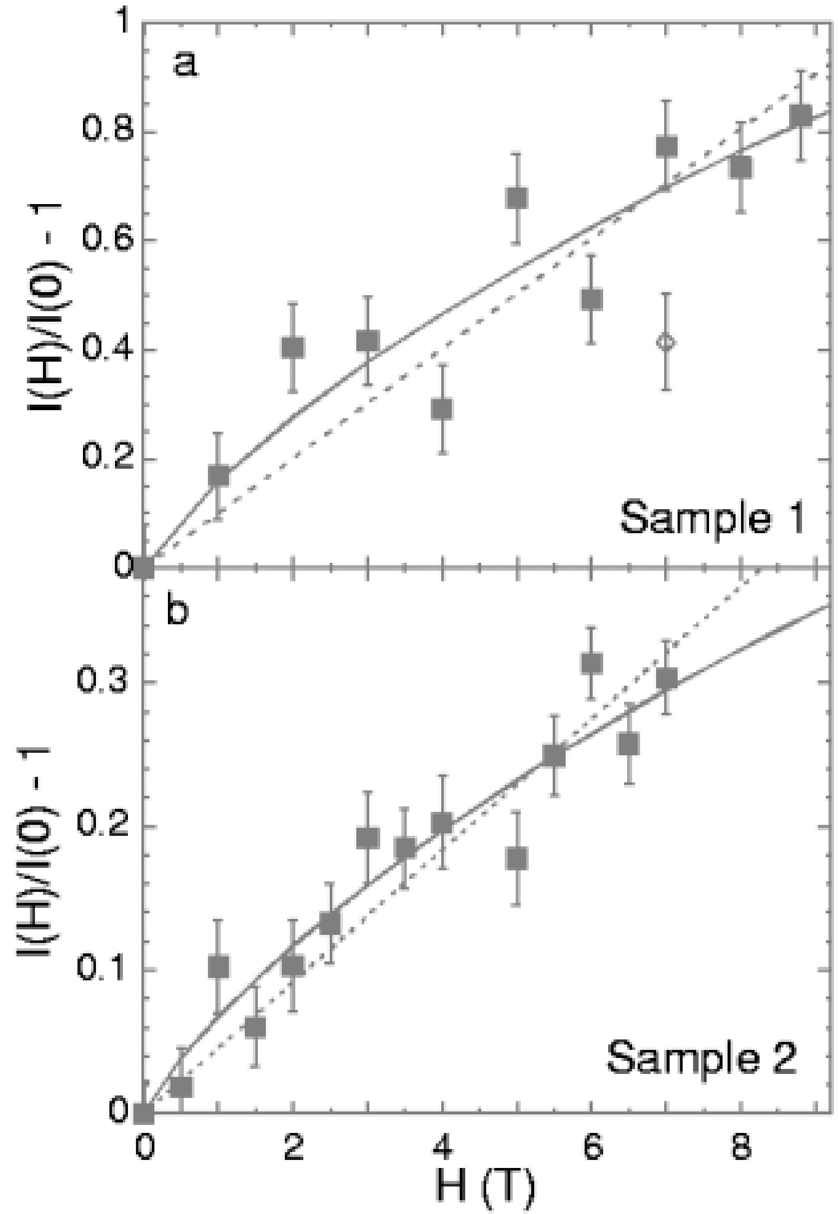
$\boldsymbol{k}$ measures deviation from SDW ordering wavevector $\boldsymbol{K}$

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, cond-mat/0112505.





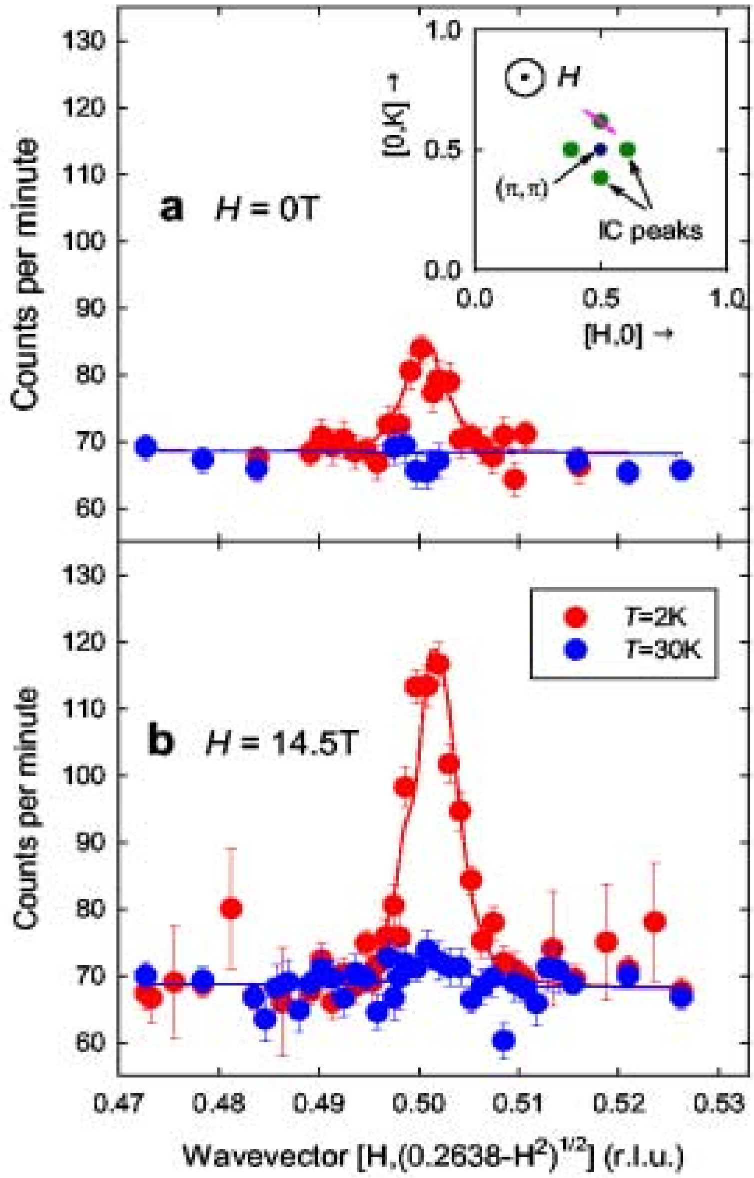
Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

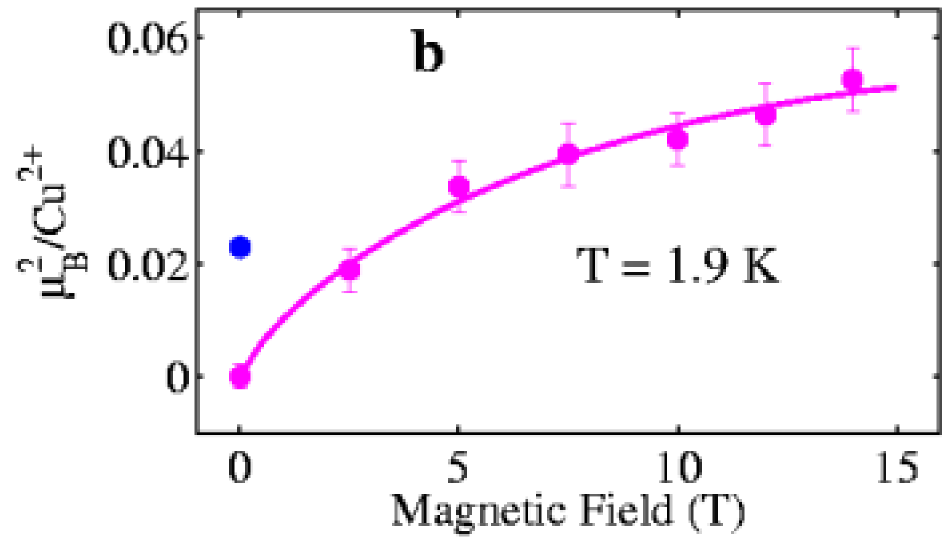
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1

B. Lake, G. Aeppli, *et al.*, Nature, Jan 2002.

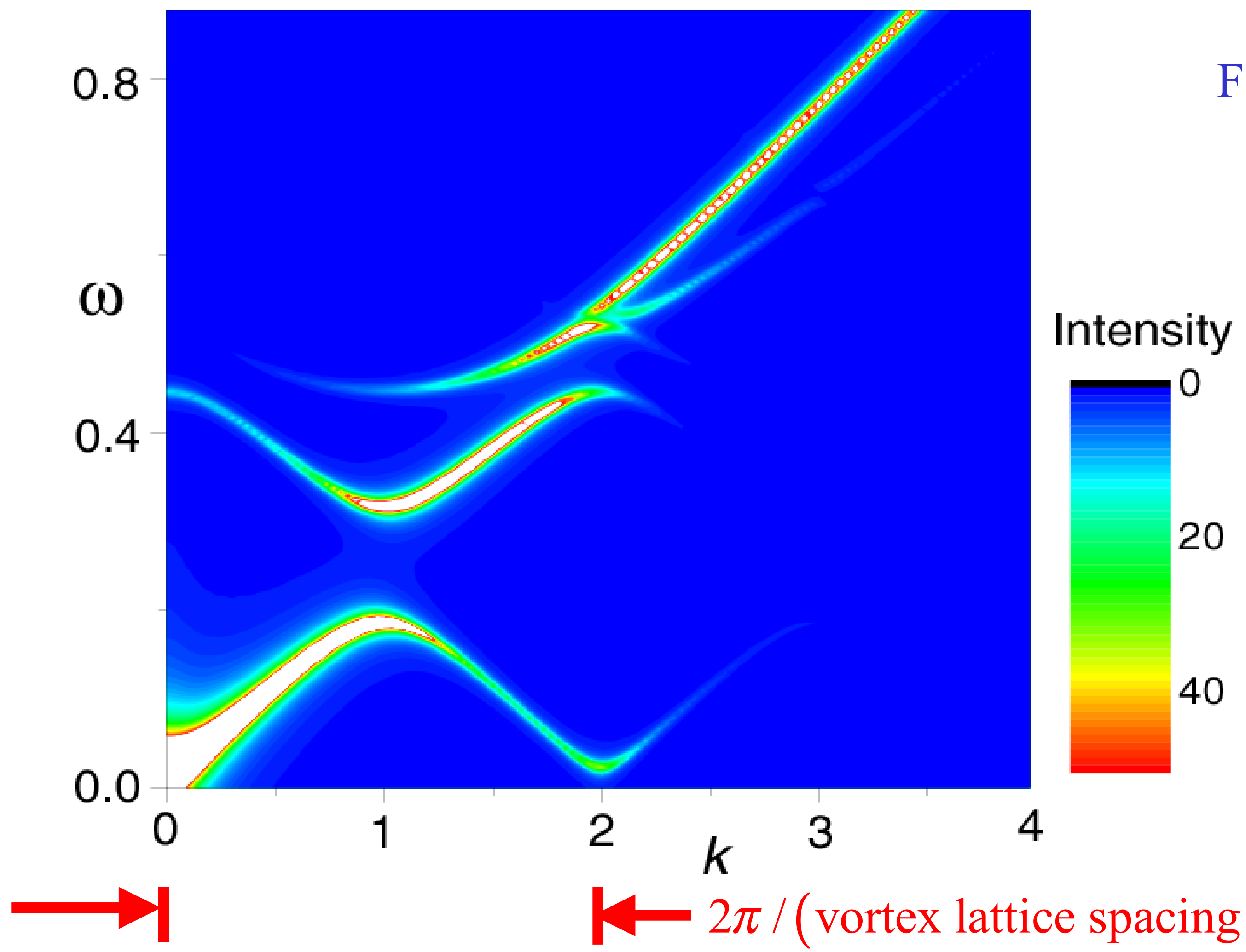




Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

Dynamic SDW fluctuations in the SC phase

Computation of spin susceptibility $\chi''(k,\omega)$ in self-consistent large N theory of Φ_α fluctuations



$2\pi/(\text{vortex lattice spacing})$

Field H chosen to place the system close to boundary to SC+SDW phase

Prediction of static CDW order by vortex cores in SC phase, with dynamic SDW correlations

"Spin gap" state in vortex core appears by a "local quantum disordering transition" of magnetic order: by our generalized phase diagram, charge order should appear in this region.

K. Park and S. Sachdev

Physical Review B **64**, 184510 (2001).

In addition to fluctuating charge order modes detected in phonon scattering, it would also be useful to study systems in which the charge order is required to be static; in such situations, atomic-resolution scanning tunneling microscopy (STM) studies should yield much useful information on the microstructure of the charge order. Our physical picture implies that static charge order should be present in situations in which both magnetic and superconducting order have been suppressed (systems with one of these orders may only have fluctuating charge order). A convenient way to achieve this is by application of a strong magnetic field on underdoped samples.[59] A phenomenological theory of the phase diagram in a magnetic field has been provided recently in Ref. 60: the "normal" state in this phase diagram is a very attractive candidate to bond-centered charge order. It would be especially interesting to conduct STM measurements on the strongly underdoped YBCO crystals that have become available recently,[61] after superconductivity has been suppressed by a static magnetic field. An alternative is to look for charge order in STM studies in which the superconductivity has only been locally suppressed, as is the case in the cores of vortices in the superconducting order.[62,63] However, the short-range nature of the suppression means that charge order is not required to appear, and may remain dynamic—this makes this approach less attractive. Recent indications[64] of mesoscale self-segregation of charge carriers in bulk samples also naturally raise the possibility of bond charge order in the lower density regions which (presumably) have suppressed superconductivity.

Pinning of static CDW order by vortex cores in SC phase, with dynamic SDW correlations

A.Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, cond-mat/0110329
Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343

Superflow reduces energy of dynamic spin exciton, but action so far does not lead to static CDW order because all terms are invariant under the "sliding" symmetry:

$$\Phi_\alpha(\boldsymbol{r}) \to \Phi_\alpha(\boldsymbol{r}) e^{i\theta}$$

Small vortex cores break this sliding symmetry on the lattice scale, and lead to a pinning term, which picks particular phase of the local CDW order

$$\mathcal{S}_{\text{pin}} = \zeta \sum_{\text{All } \boldsymbol{r}_v \text{ where } \psi(\boldsymbol{r}_v)=0} \int_0^{1/T} d\tau \left[\Phi_\alpha^2(\boldsymbol{r}_v) e^{i\theta} + \text{c.c.} \right]$$

With this term, SC phase has static CDW but dynamic SDW

$$\left\langle \Phi_\alpha^2(\boldsymbol{r}) \right\rangle \neq 0 \quad ; \quad \left\langle \Phi_\alpha(\boldsymbol{r}) \right\rangle = 0$$

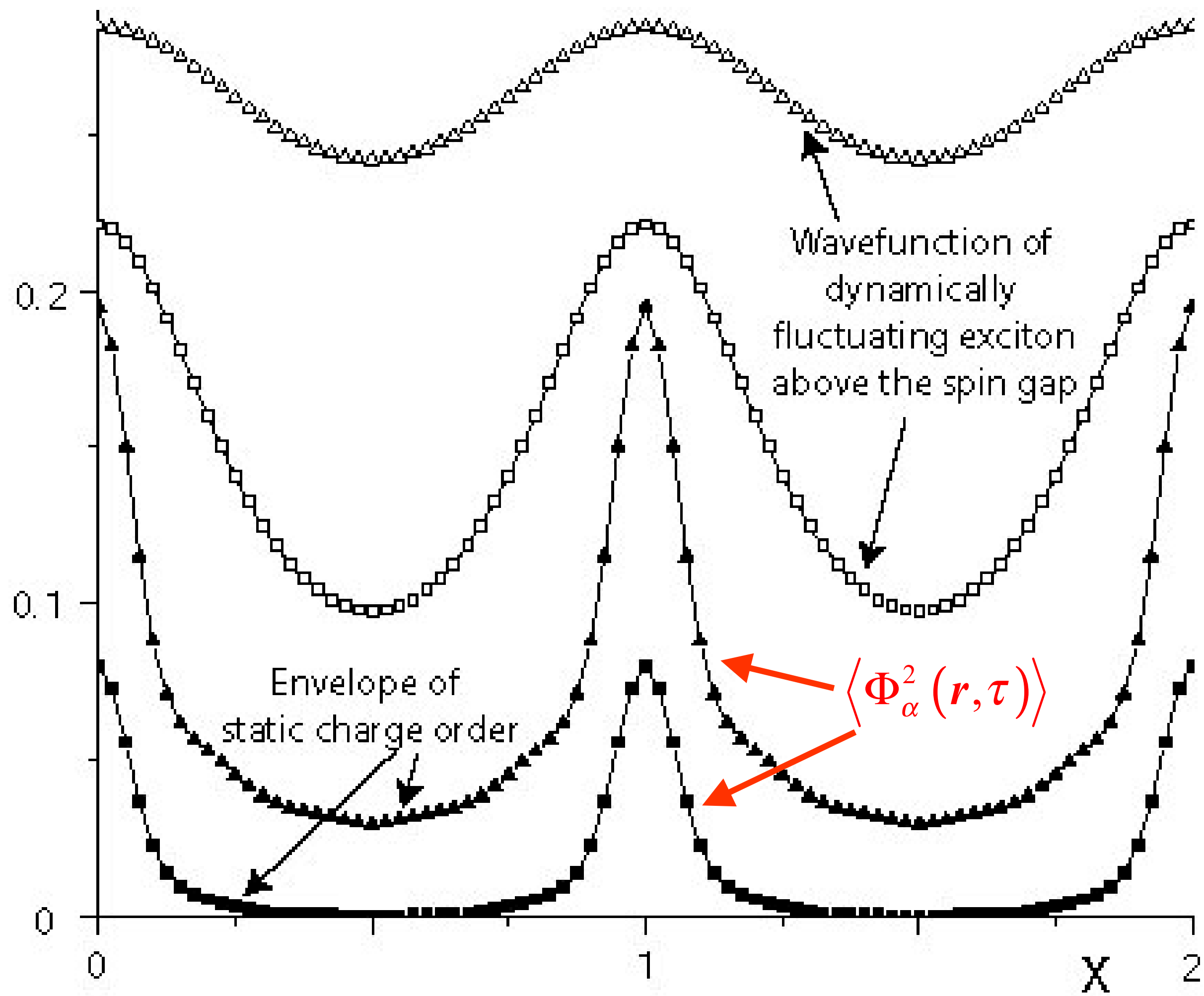
$$\delta\rho(\boldsymbol{r}) \propto \sum_\alpha \Phi_\alpha^2(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.} \quad ; \quad S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

"Friedel oscillations of a doped spin-gap antiferromagnet"

Pinning of CDW order by vortex cores in SC phase

Computation in self-consistent large N theory

$$\left\langle \Phi_\alpha^2(\mathbf{r},\tau)\right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\mathbf{r},\tau)\Phi_\alpha^*(\mathbf{r}_v,\tau_1)\right\rangle^2$$

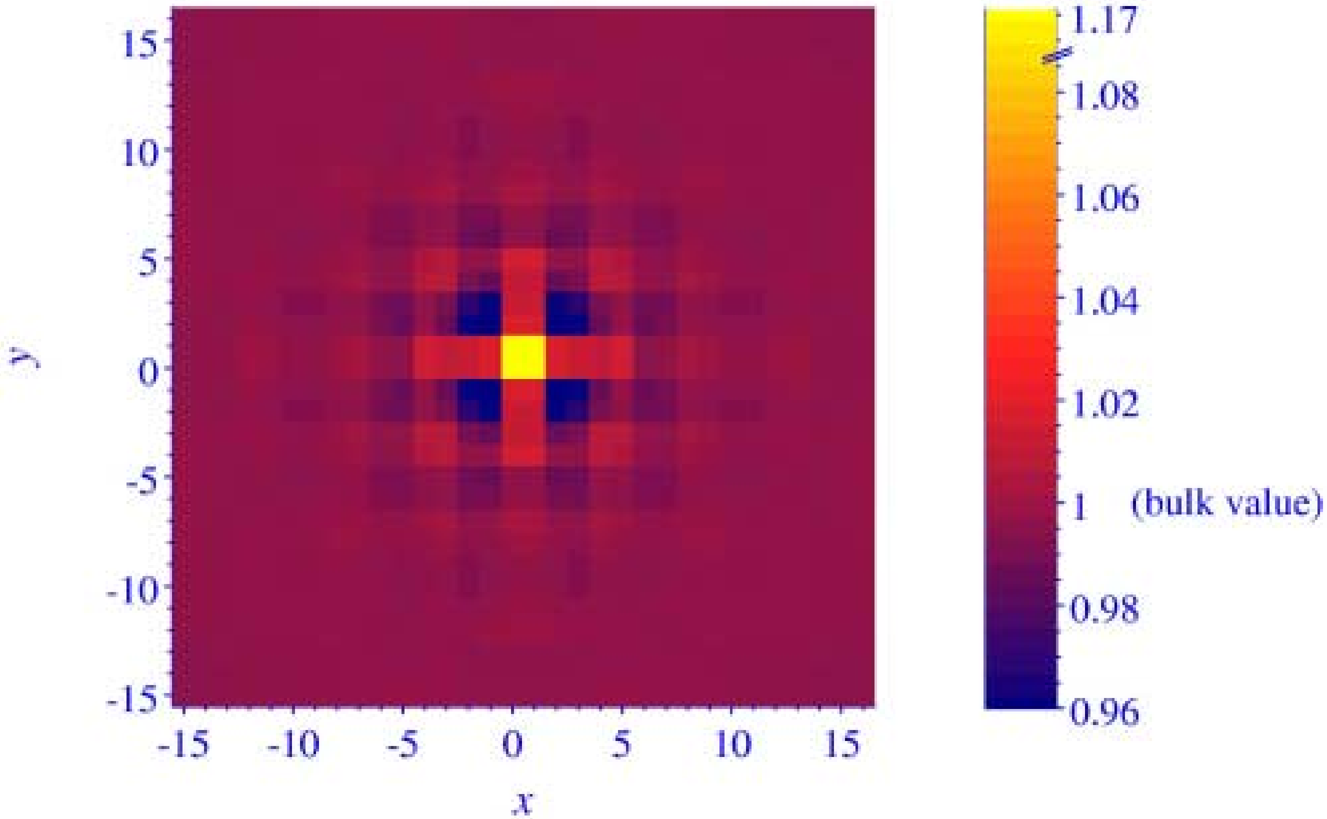


□⟶ low magnetic field

△⟶ high magnetic field near the boundary to the SC+SDW phase

Simplified theoretical computation of modulation in local density of states at low energy due to CDW order induced by superflow and pinned by vortex core

A. Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, cond-mat/0110329



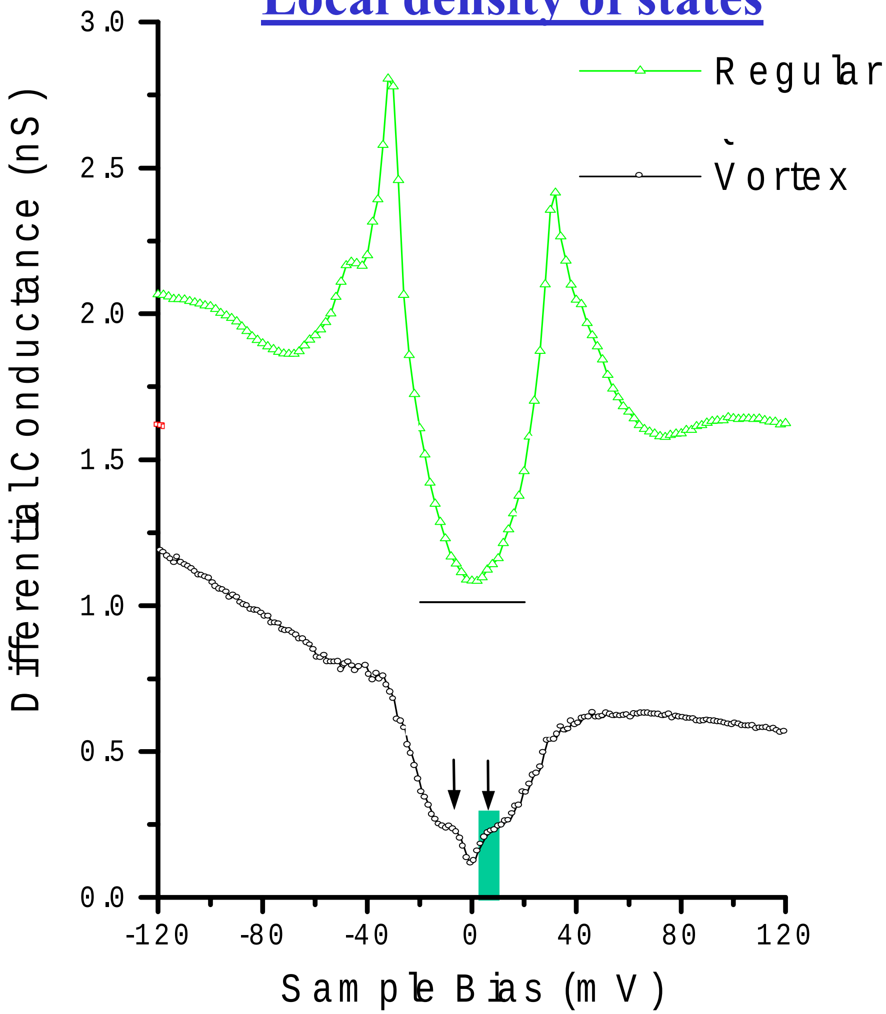
$$H = \sum_{ij} \left(-t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \Delta_{ij} c_{i\sigma}^{\dagger} c_{j,-\sigma}^{\dagger} + h.c. \right) + \sum_{i} \left[v(\boldsymbol{r}_i) - \mu \right] c_{i\sigma}^{\dagger} c_{i\sigma},$$

$$v(\boldsymbol{r}) = v_1 \left\{ \cos\left[\boldsymbol{K}_{cx} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) \right] + \cos\left[\boldsymbol{K}_{cy} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) \right] \right\} e^{-|\boldsymbol{r}-\boldsymbol{r}_0|/\xi_c} \left(|\boldsymbol{r} - \boldsymbol{r}_0|^2 + 1 \right)^{-3/4}$$

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan,
H. Eisaki, S. Uchida, and J. C. Davis, *Science,* Jan 2002

Local density of states



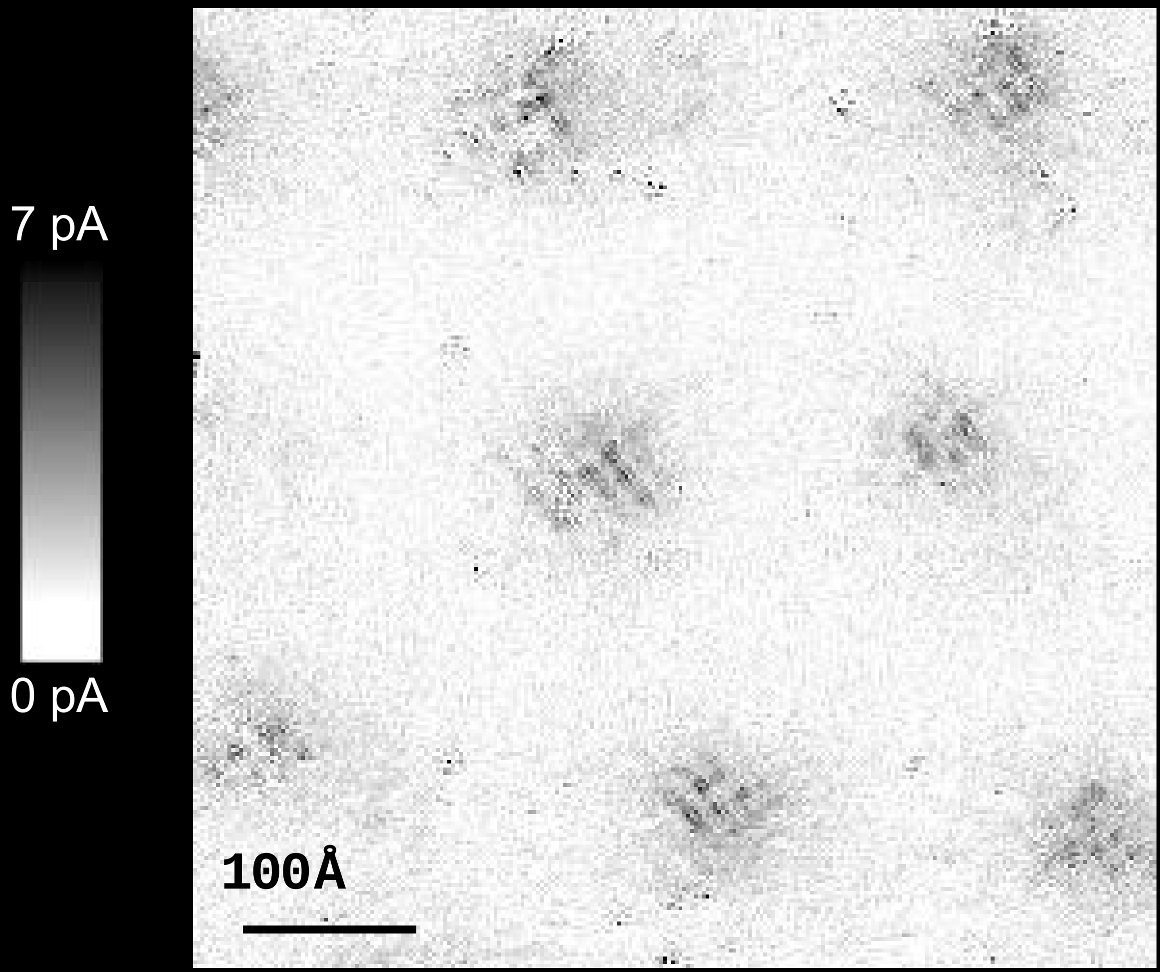
S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

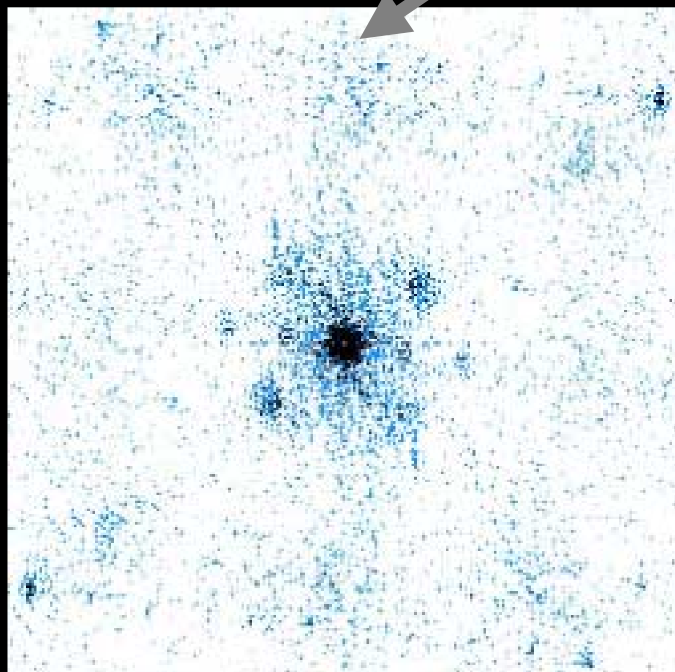
Berkeley University of California

Vortex-induced LDOS integrated from 1meV to 12meV

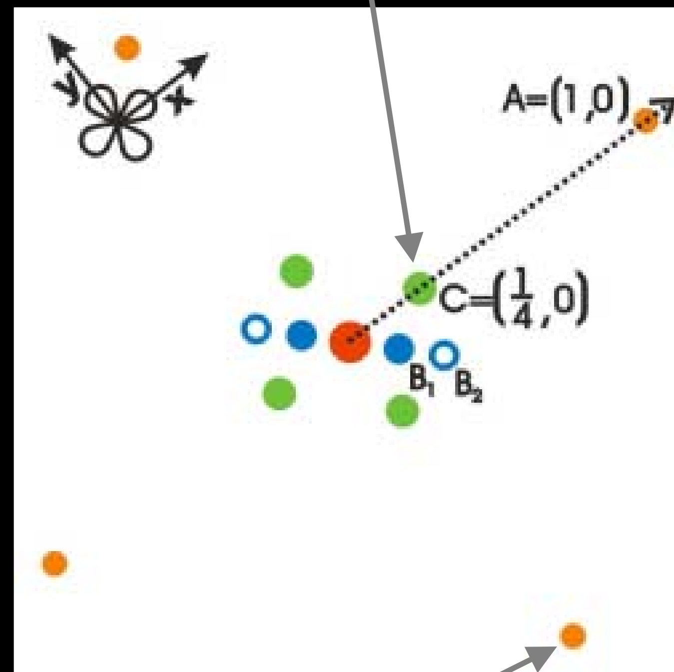


J. **Hoffman** *et al,* Science, Jan 2002.

Fourier Transform of Vortex-Induced LDOS map



K-space locations of vortex induced LDOS

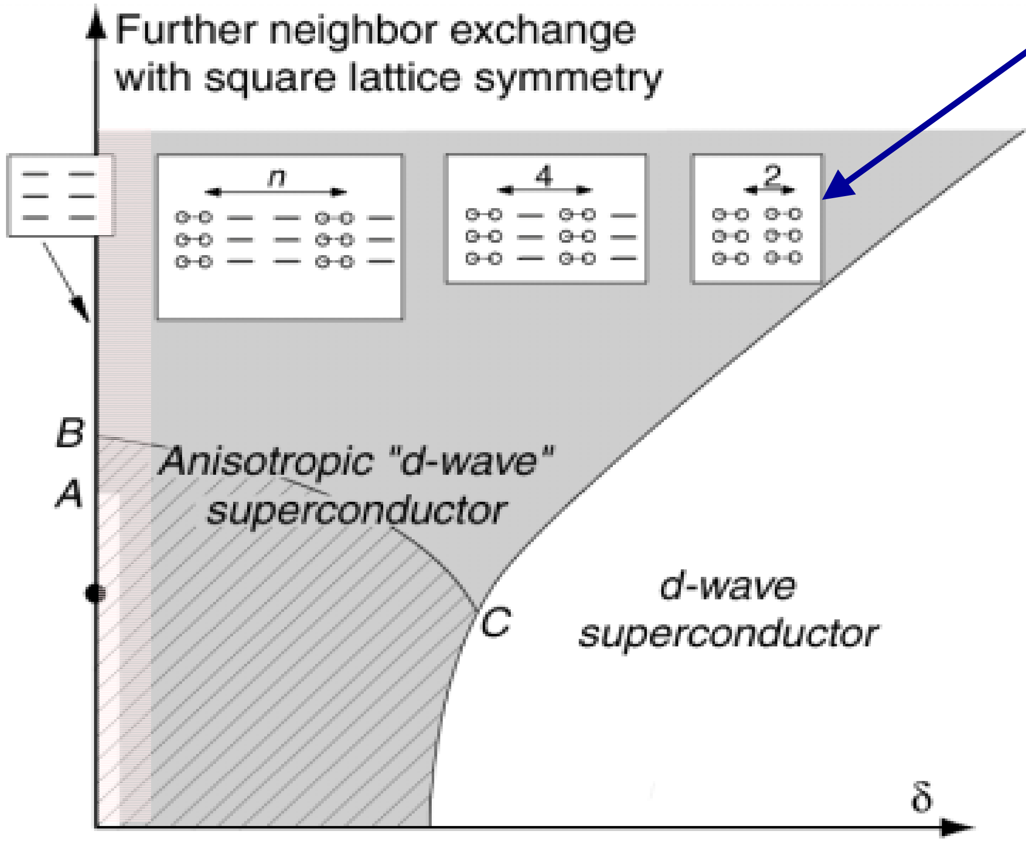


K-space locations of Bi and Cu atoms

Distances in k –space have units of $2\pi/a_0$
a_0=3.83 Å is Cu-Cu distance

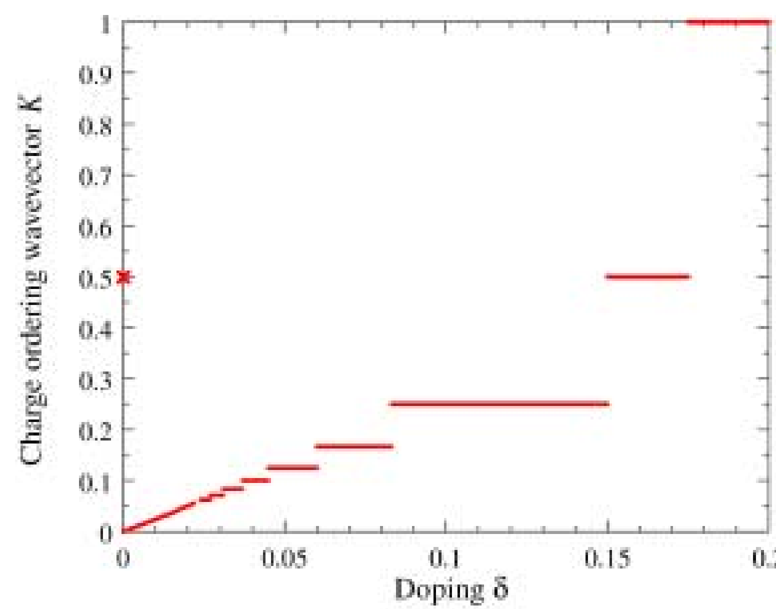
J. Hoffman *et al* Science, Jan 2002.

Doping the paramagnetic Mott insulator



"Large N" theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).



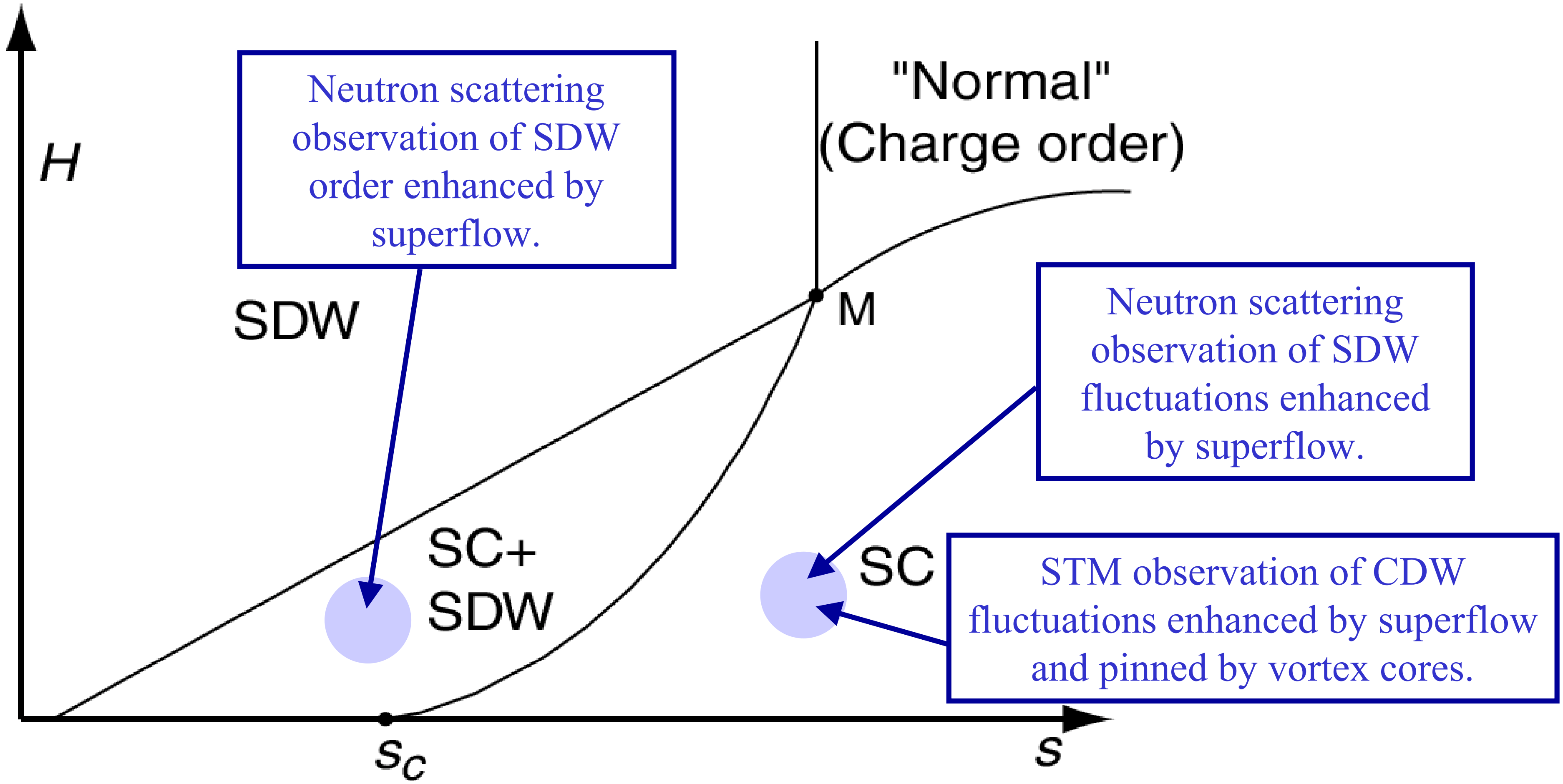
See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

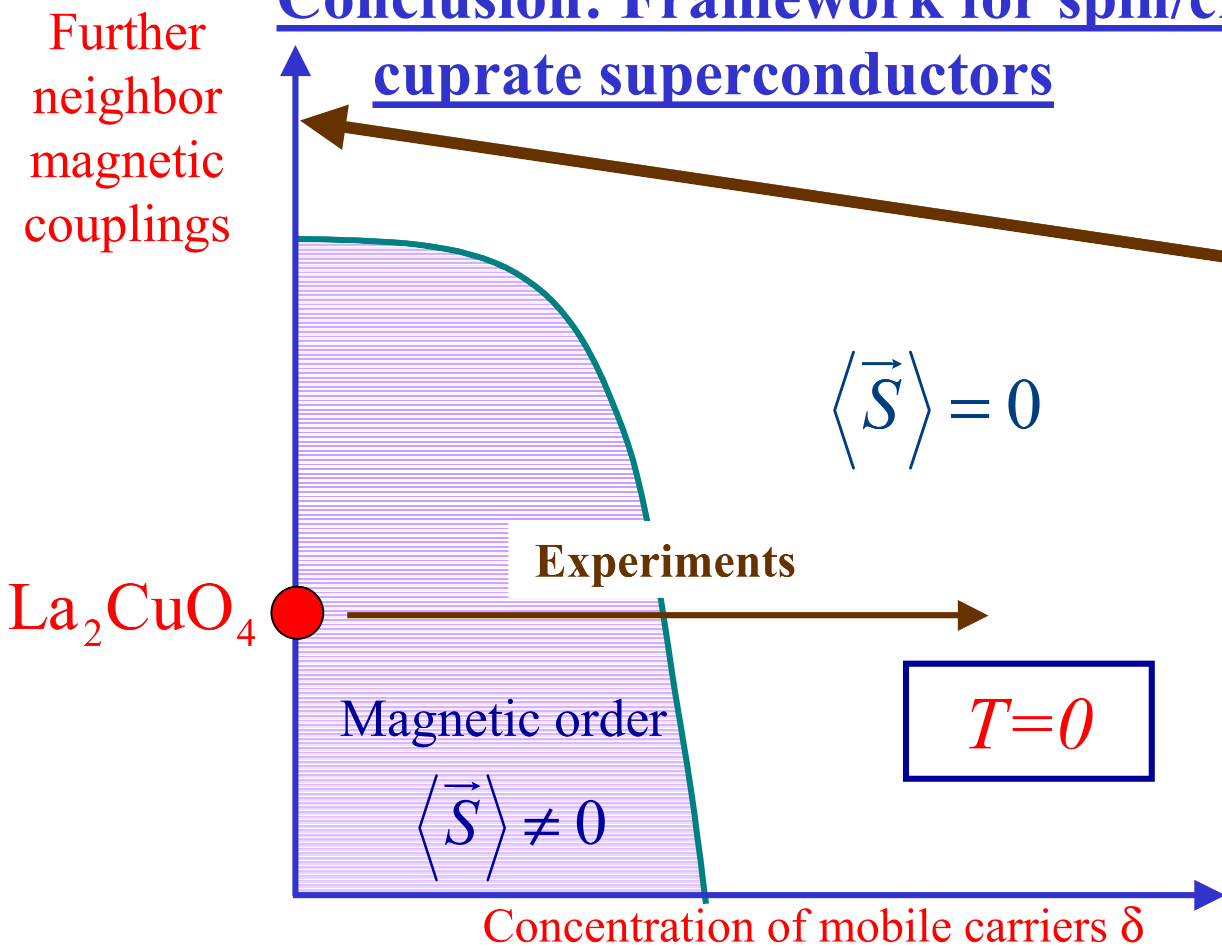
T=0

Main results



Prospects for studying quantum critical point between SC and SC+SDW phases by tuning H ?

Conclusion: Framework for spin/charge order in cuprate superconductors



Confined, paramagnetic Mott insulator has

1. Stable S=1 spin exciton ϕ_α .
2. Broken translational symmetry:- bond-centered charge order.
3. S=1/2 moments near non-magnetic impurities

Theory of magnetic ordering quantum transitions in antiferromagnets and superconductors leads to quantitative theories for

- Spin correlations in a magnetic field
- Effect of Zn/Li impurities on collective spin excitations