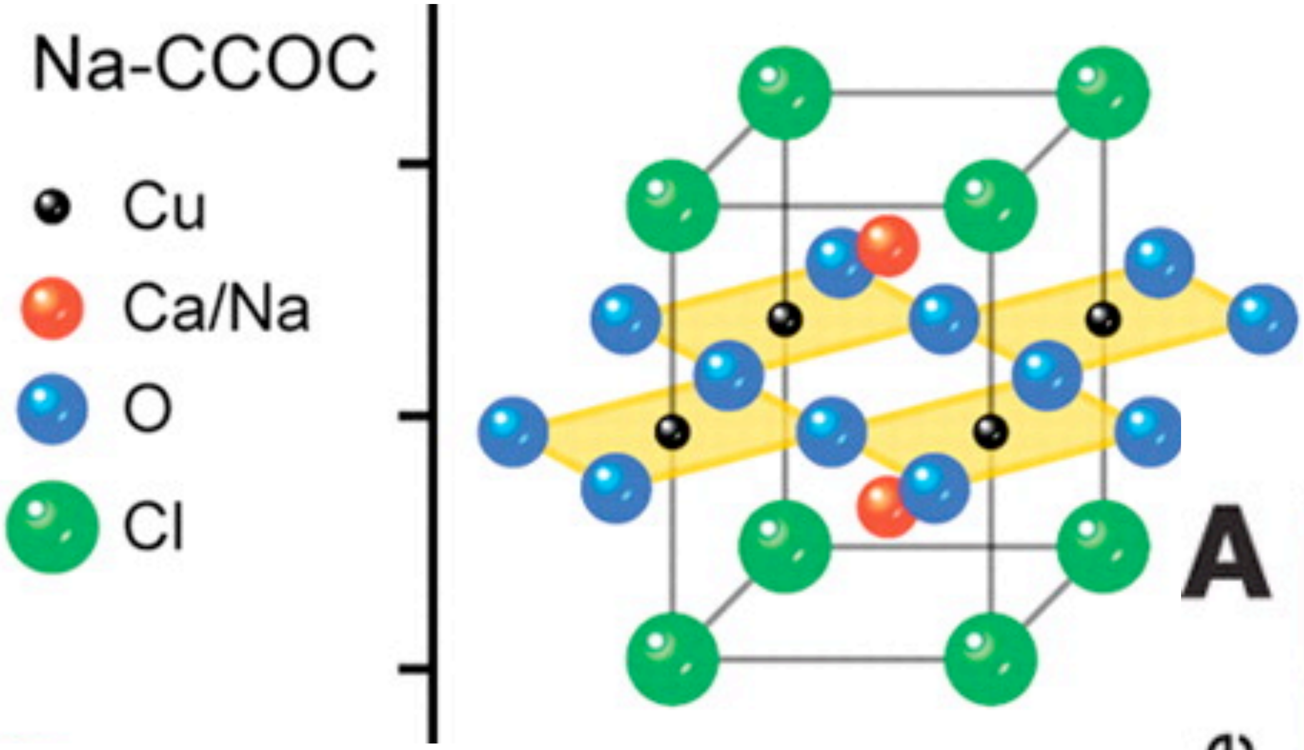
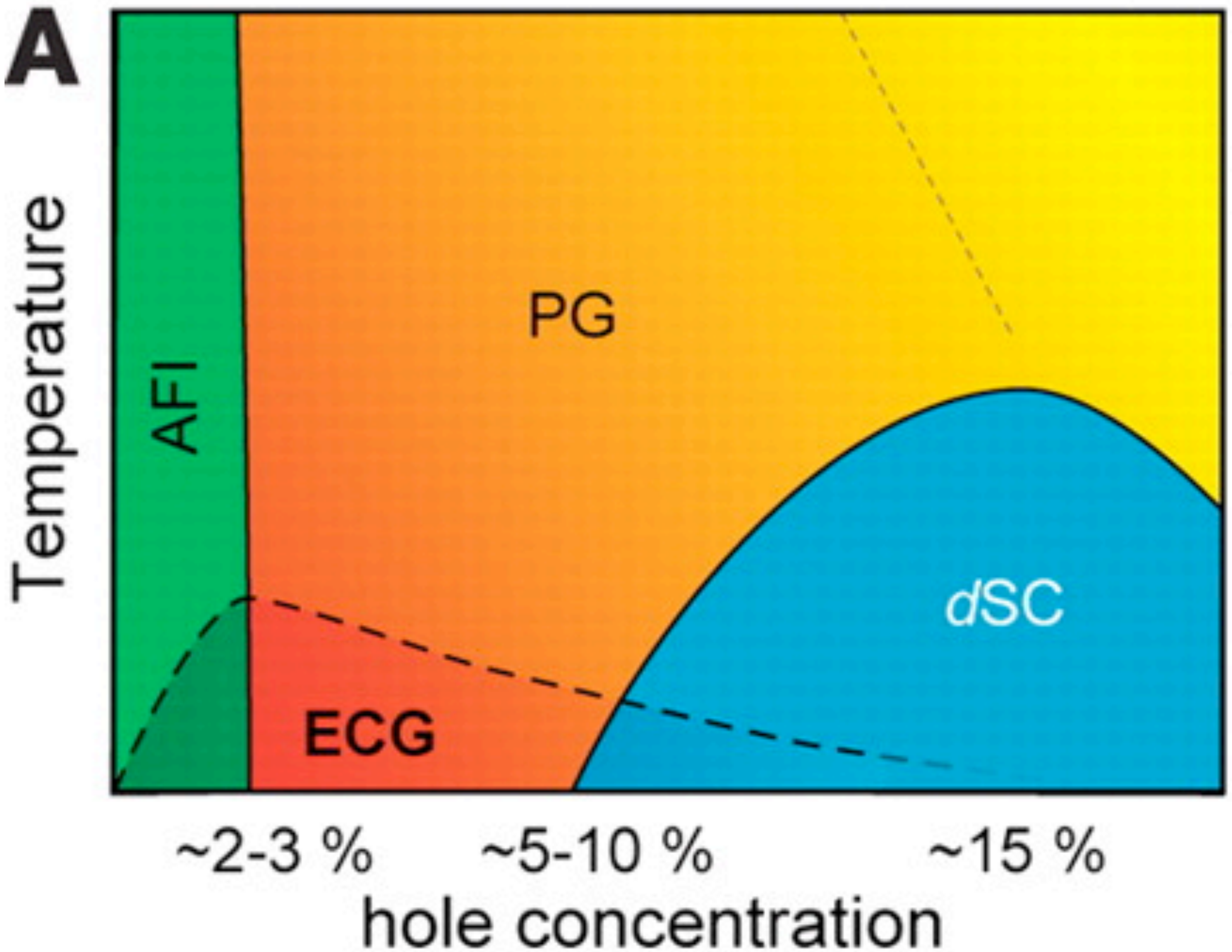
Quantum criticality in the cuprate superconductors

Talk online: sachdev.physics.harvard.edu

The cuprate superconductors





Destruction of Neel order in the cuprates by electron doping,
R. K. Kaul, M. Metlitksi, S. Sachdev, and C. Xu,
Physical Review B **78**, 045110 (2008).

Paired electron pockets in the underdoped cuprates,
V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Competition between spin density wave order and superconductivity in the underdoped cuprates,
Eun Gook Moon and S. Sachdev,
arXiv:0905.2608

PHYSICS

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Outline

1. Quantum criticality

   Coupled dimer antiferromagnets vs.
   the cuprate superconductors

2. Fermi surfaces in the hole-doped cuprates

   Observations of quantum oscillations

3. Superconductivity

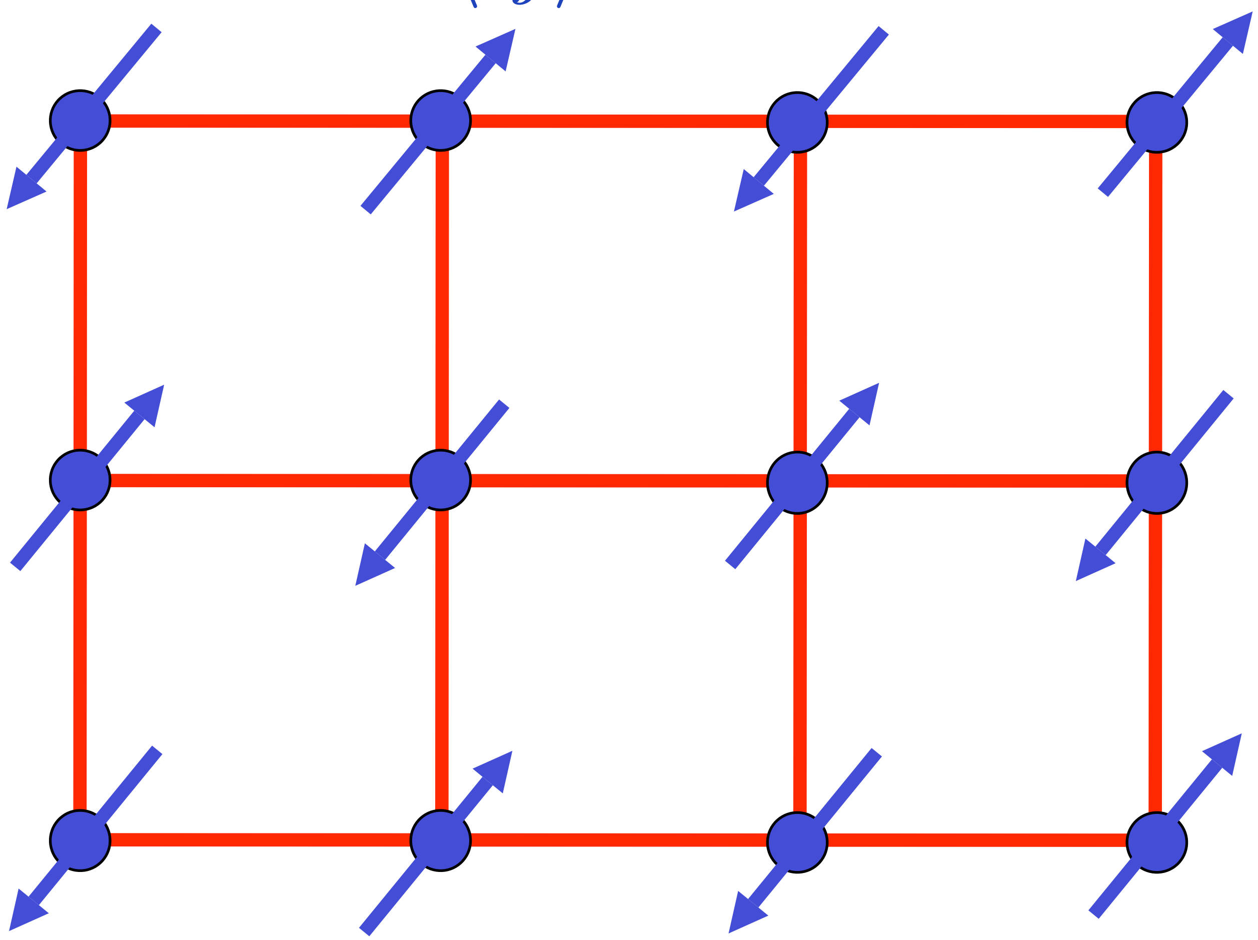
4. Competition between spin-density-wave order and superconductivity: phenomenological theory

5. Electronic theory of superconductivity and its competition with spin-density-wave order

Quantum criticality: coupled dimer antiferromagets vs. the cuprate supercondcutors

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

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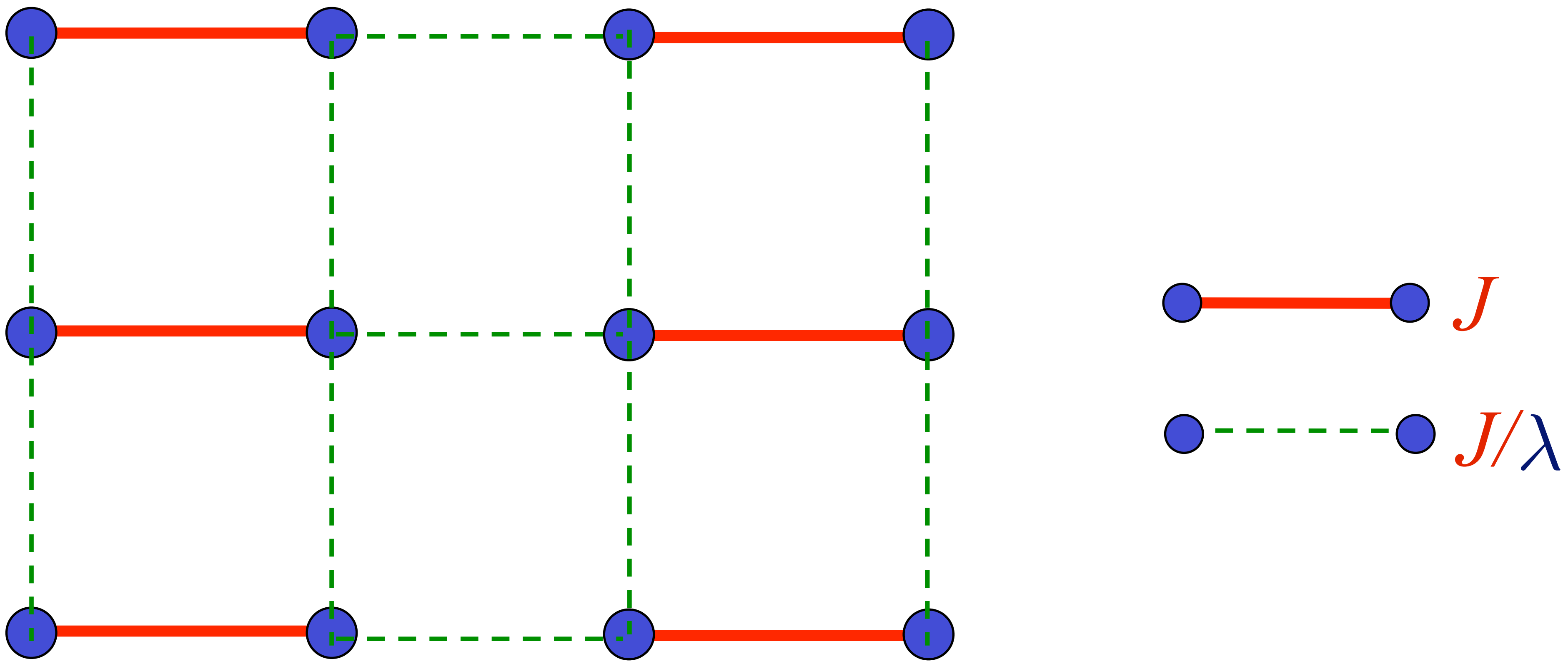
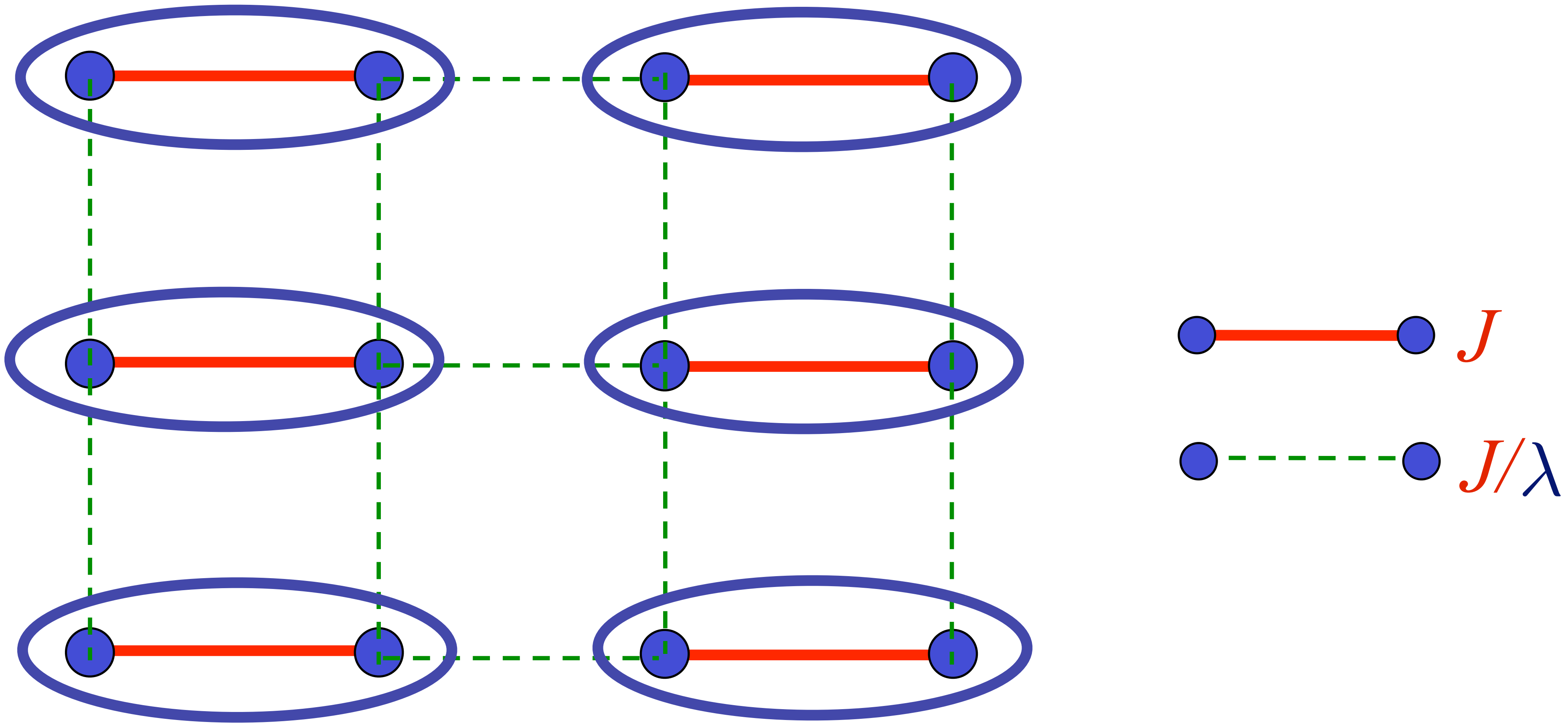


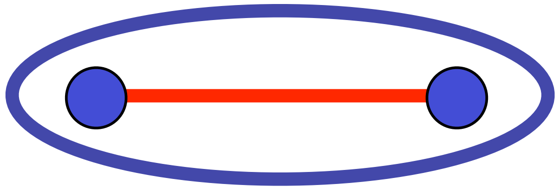
Weaken some bonds to induce spin entanglement in a new quantum phase

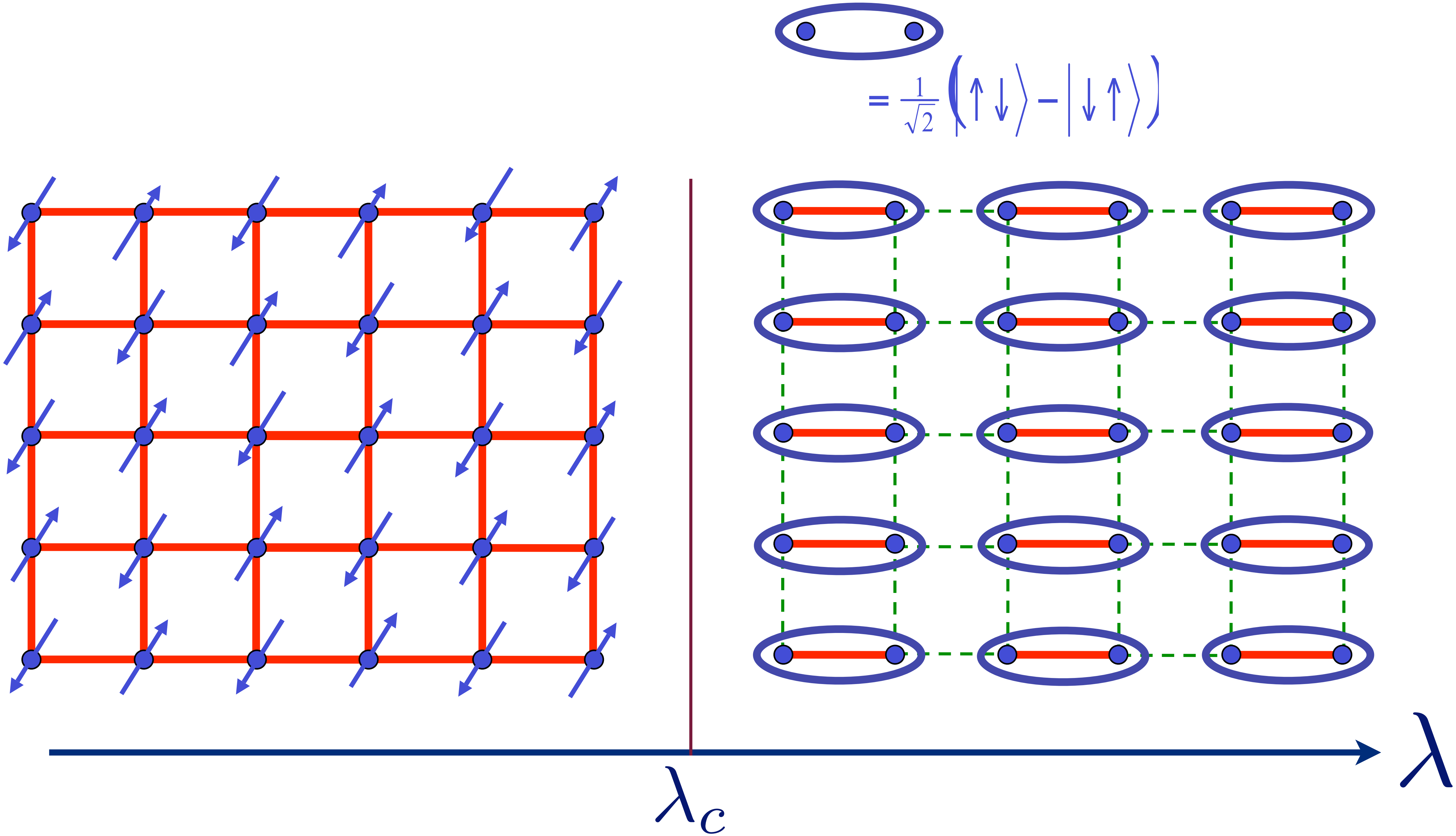
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

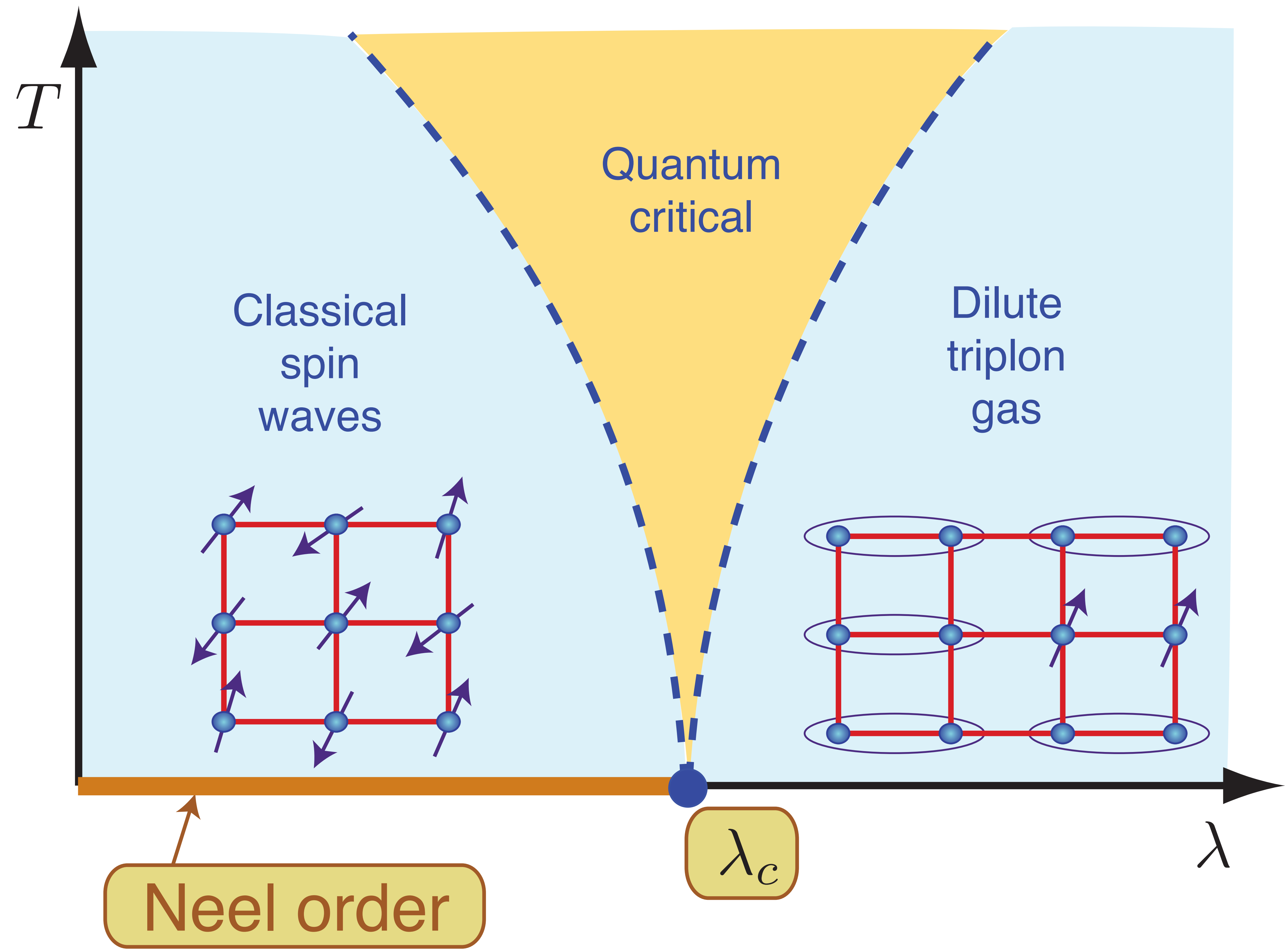


Ground state is a "quantum paramagnet" with spins locked in valence bond singlets

 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$

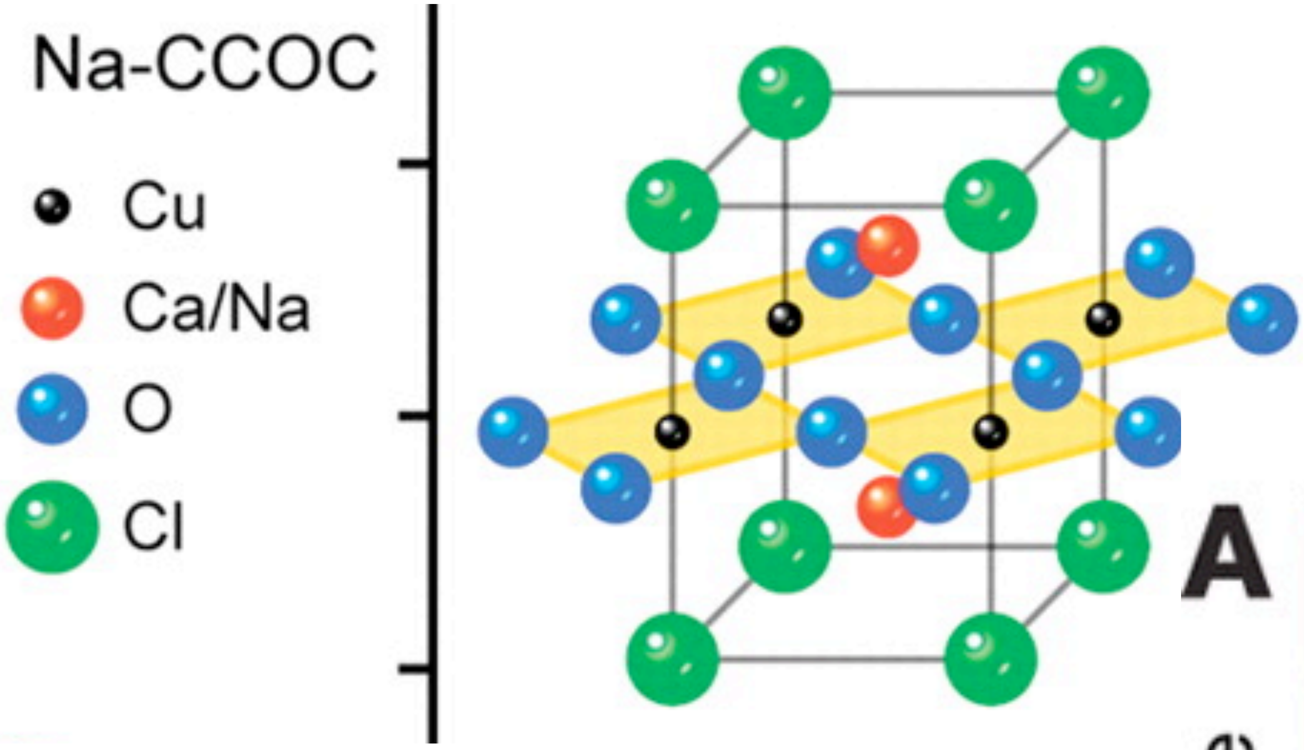


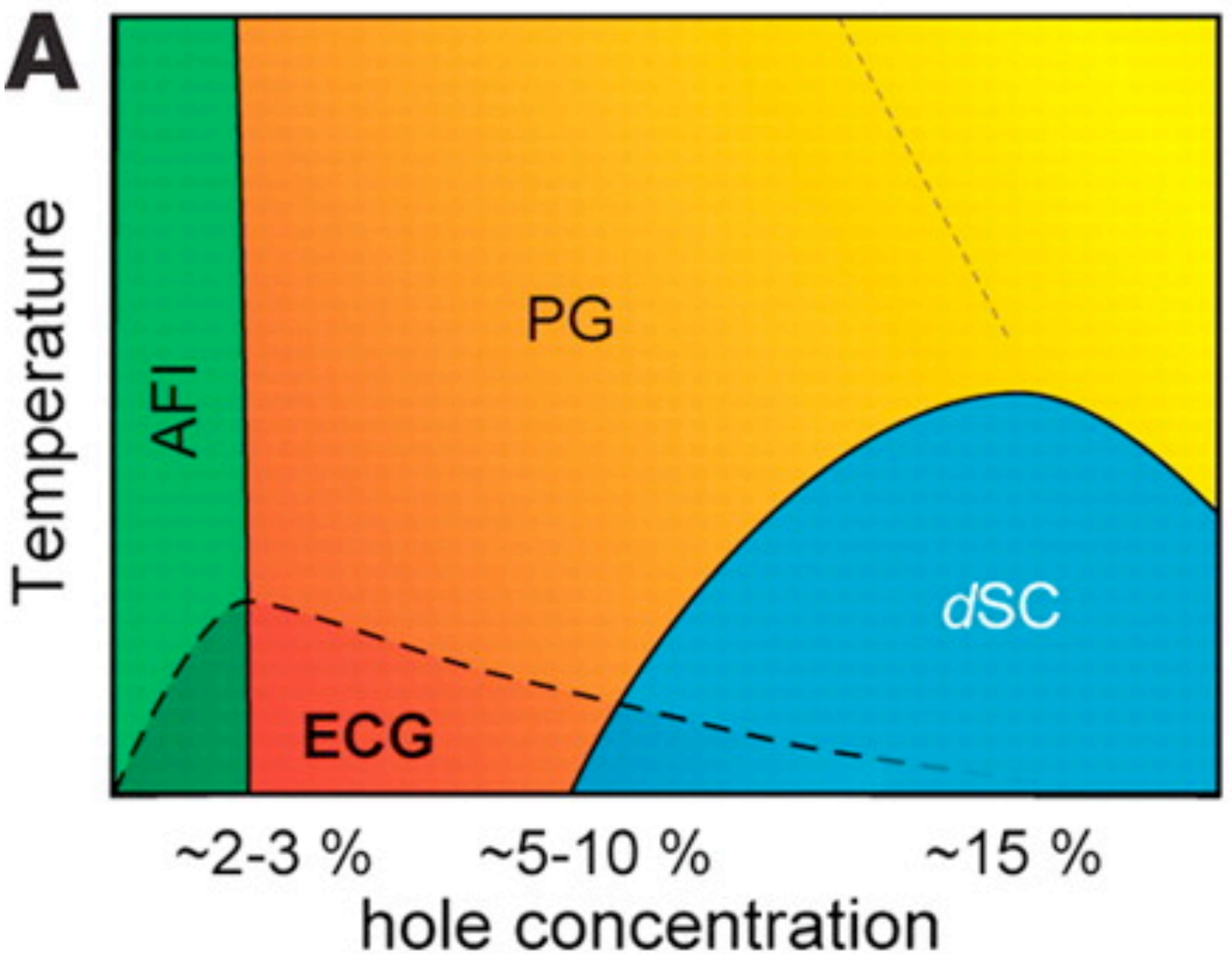
$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$
λ_c
λ

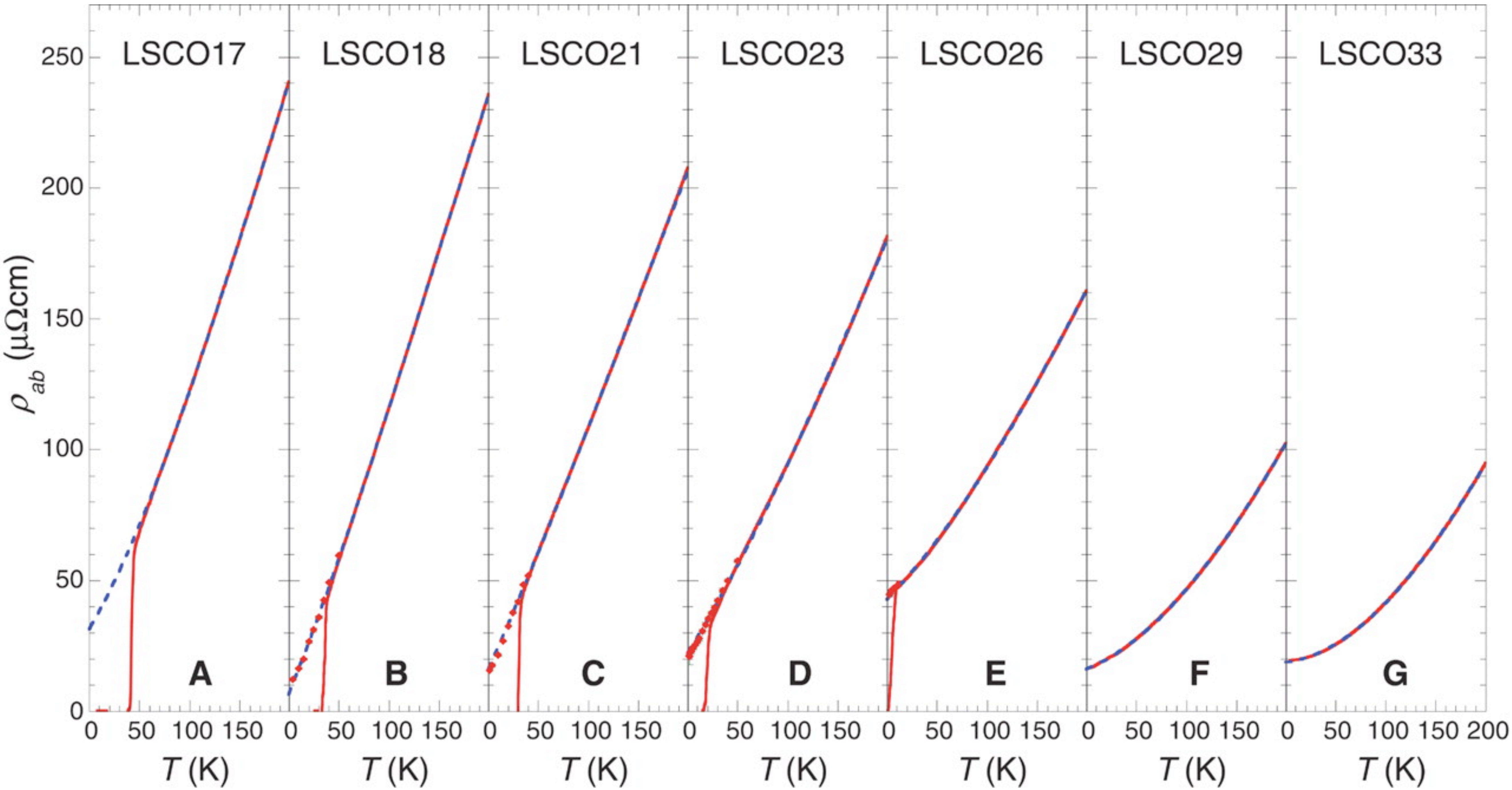


T
Quantum critical
Classical spin waves
Dilute triplon gas
λ
λ_c
Neel order

The cuprate superconductors

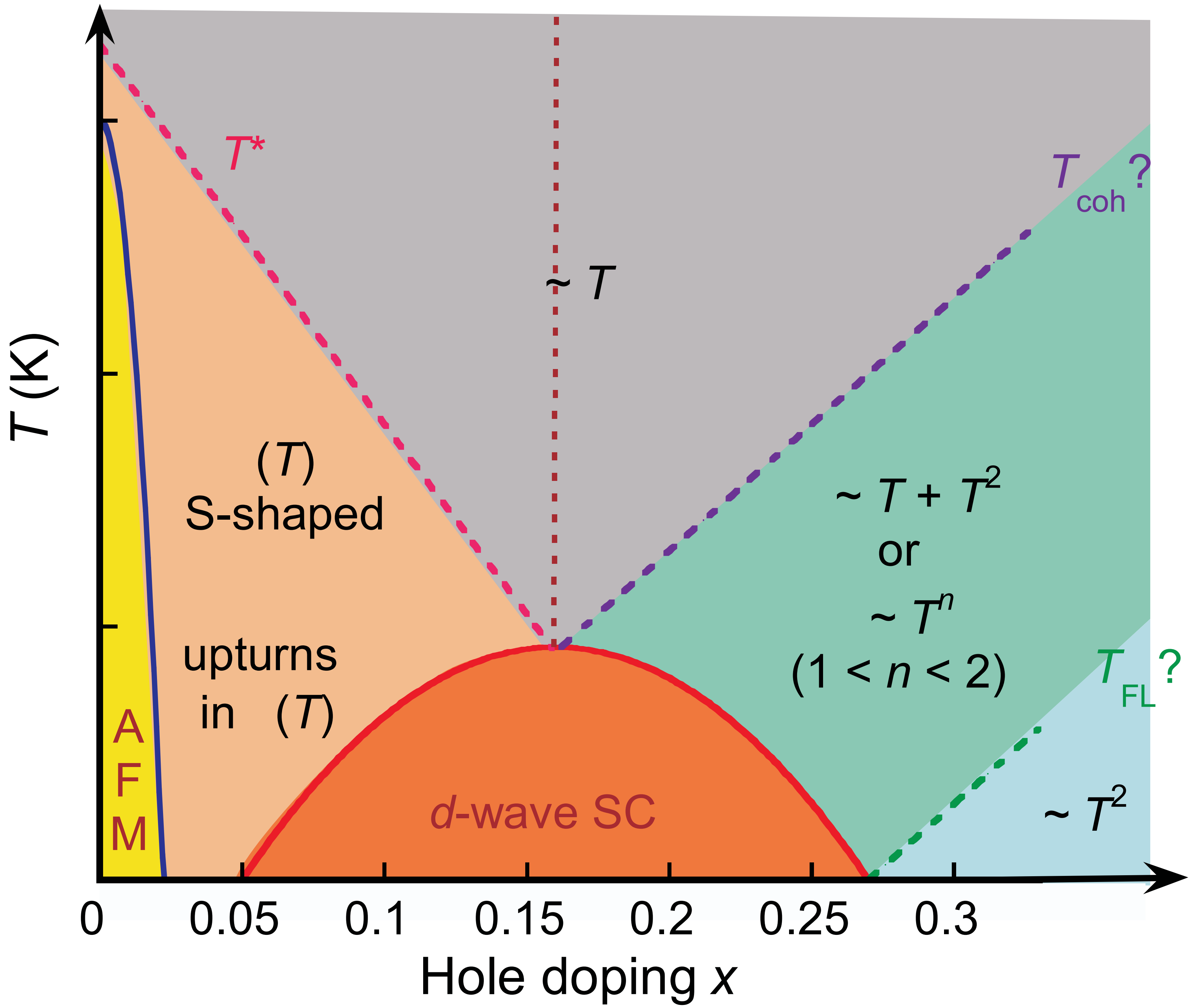






R. A. Cooper, Y. Wang, B. Vignolle, O. J. Lipscombe, S. M. Hayden, Y. Tanabe, T. Adachi, Y. Koike, M. Nohara, H. Takagi, Cyril Proust, N. E. Hussey,
Science, **323**, 603 (2009).

Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

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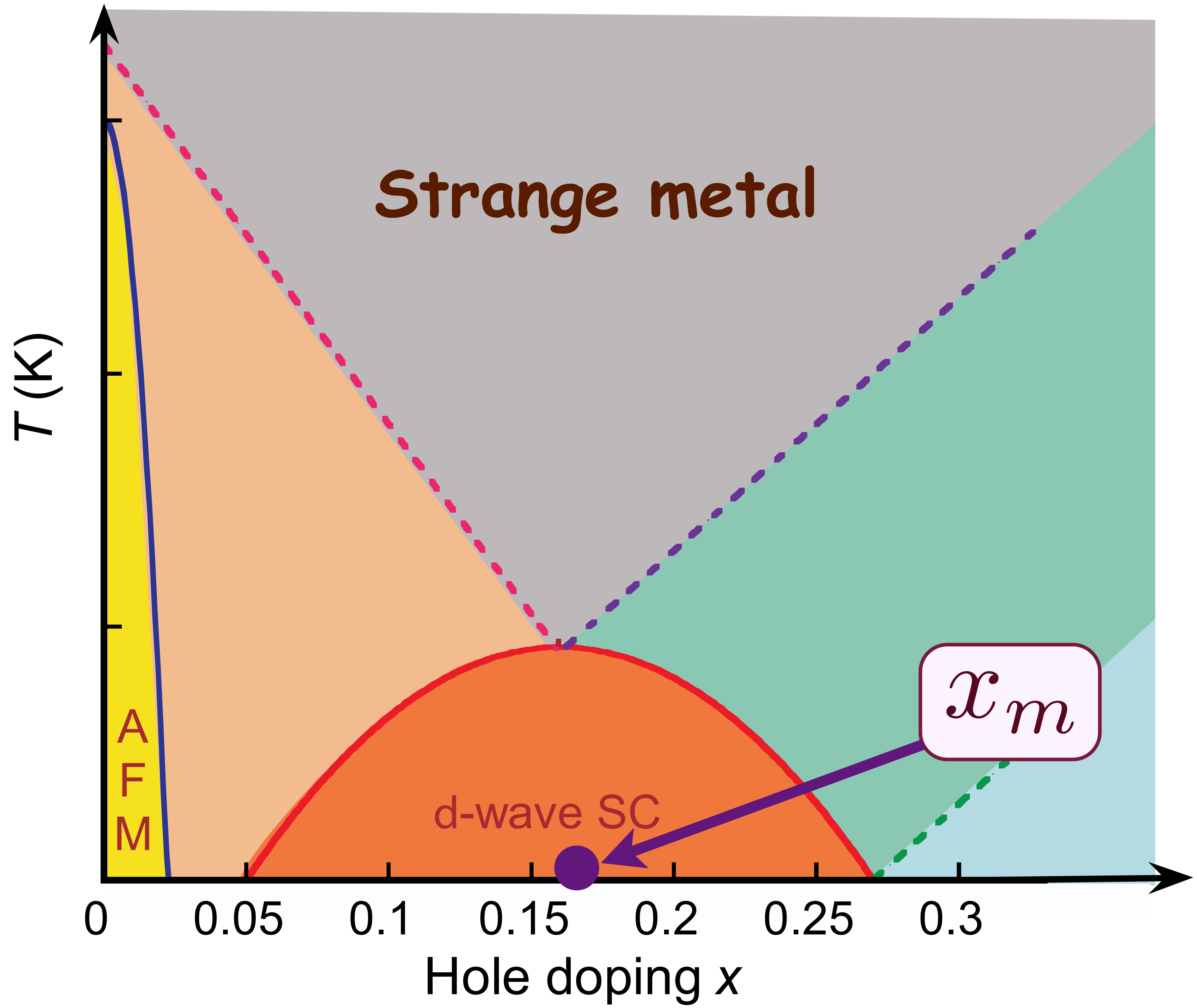
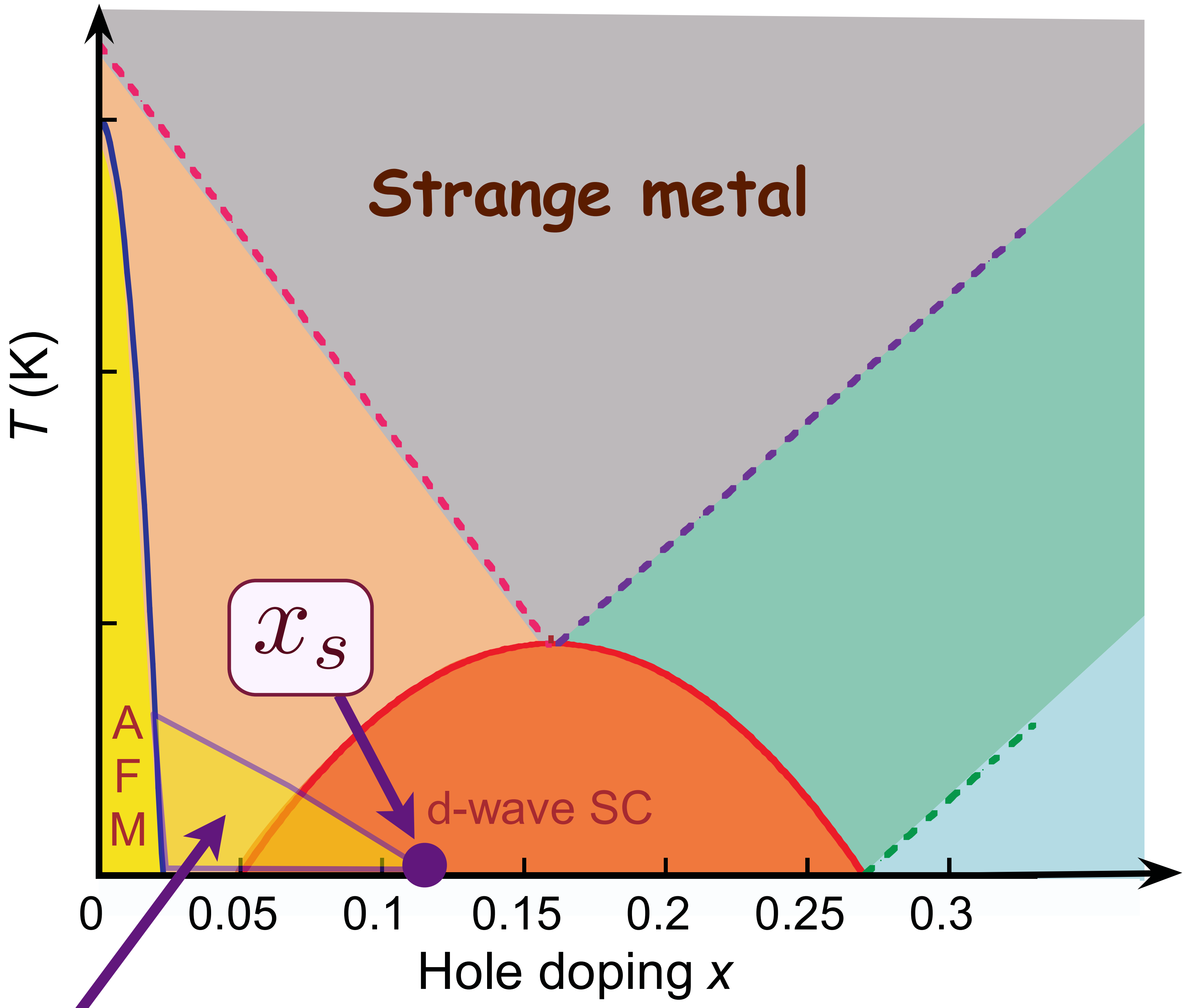


Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

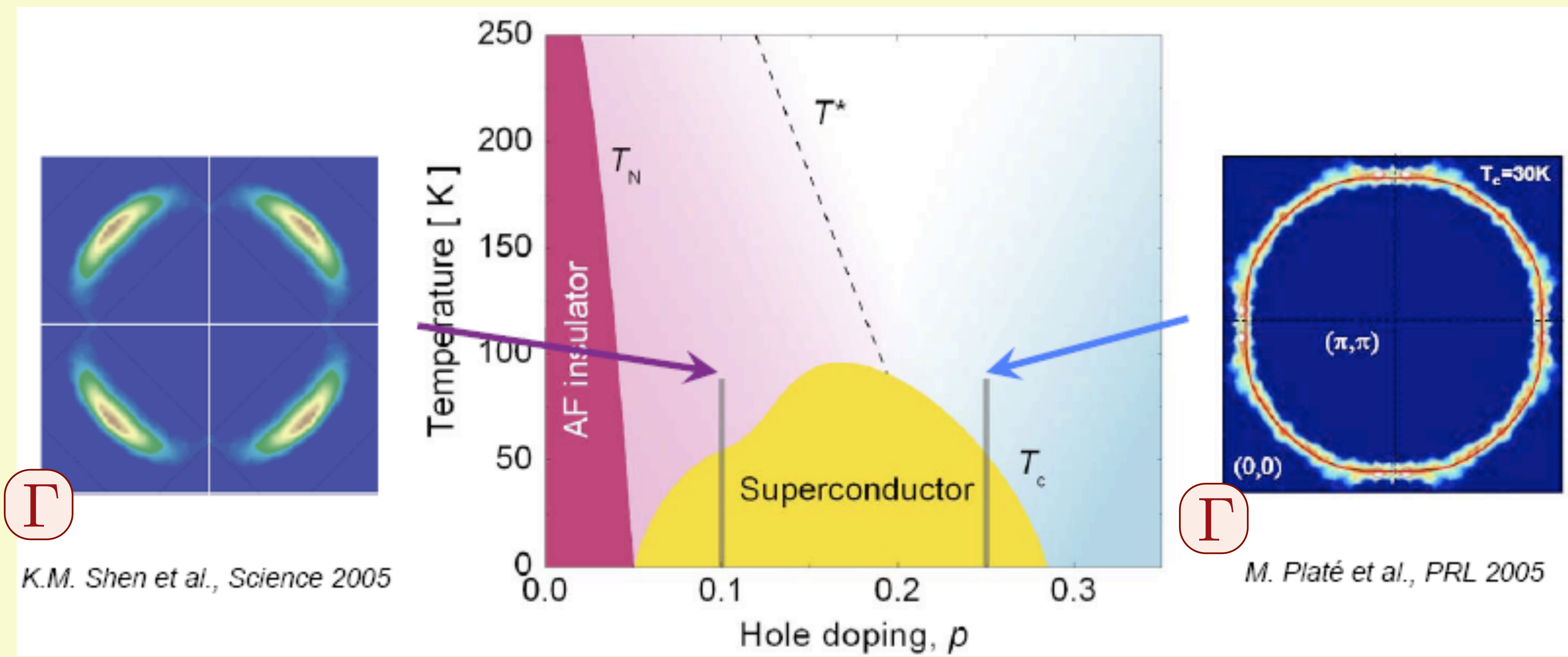
Only candidate quantum critical point observed at low T



Spin and charge density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Fermi surfaces in the hole-doped cuprates

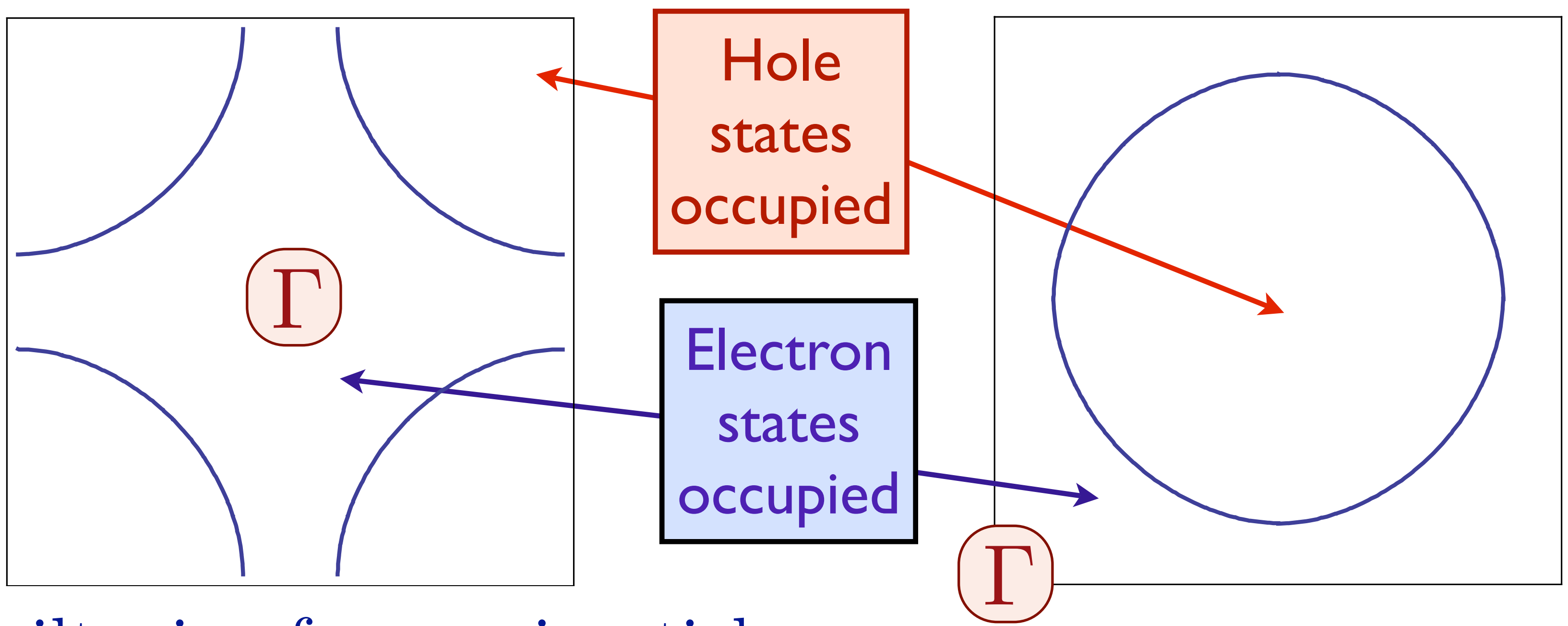
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole Fermi-pockets

Large hole Fermi surface

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-p) & \text{for hole-doping } p \\ 2\pi^2(1+x) & \text{for electron-doping } x \end{cases}$$

The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

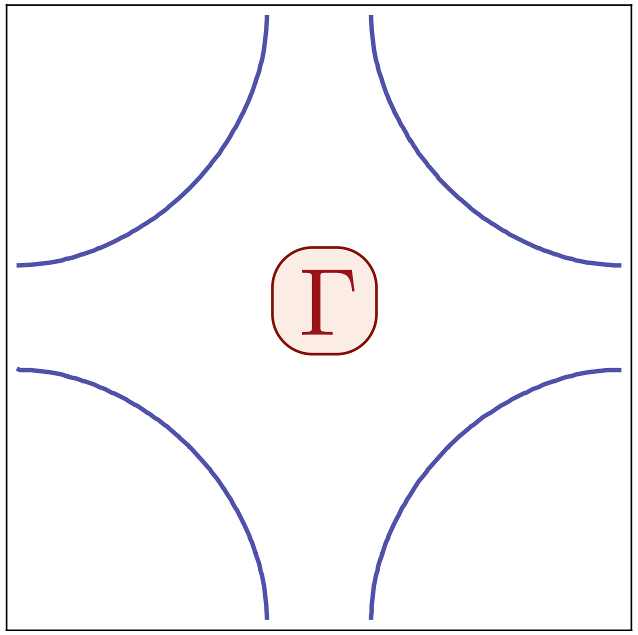
$$H_{\rm sdw} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\rm sdw}$ for $\vec{\varphi} \propto (0,0,1)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

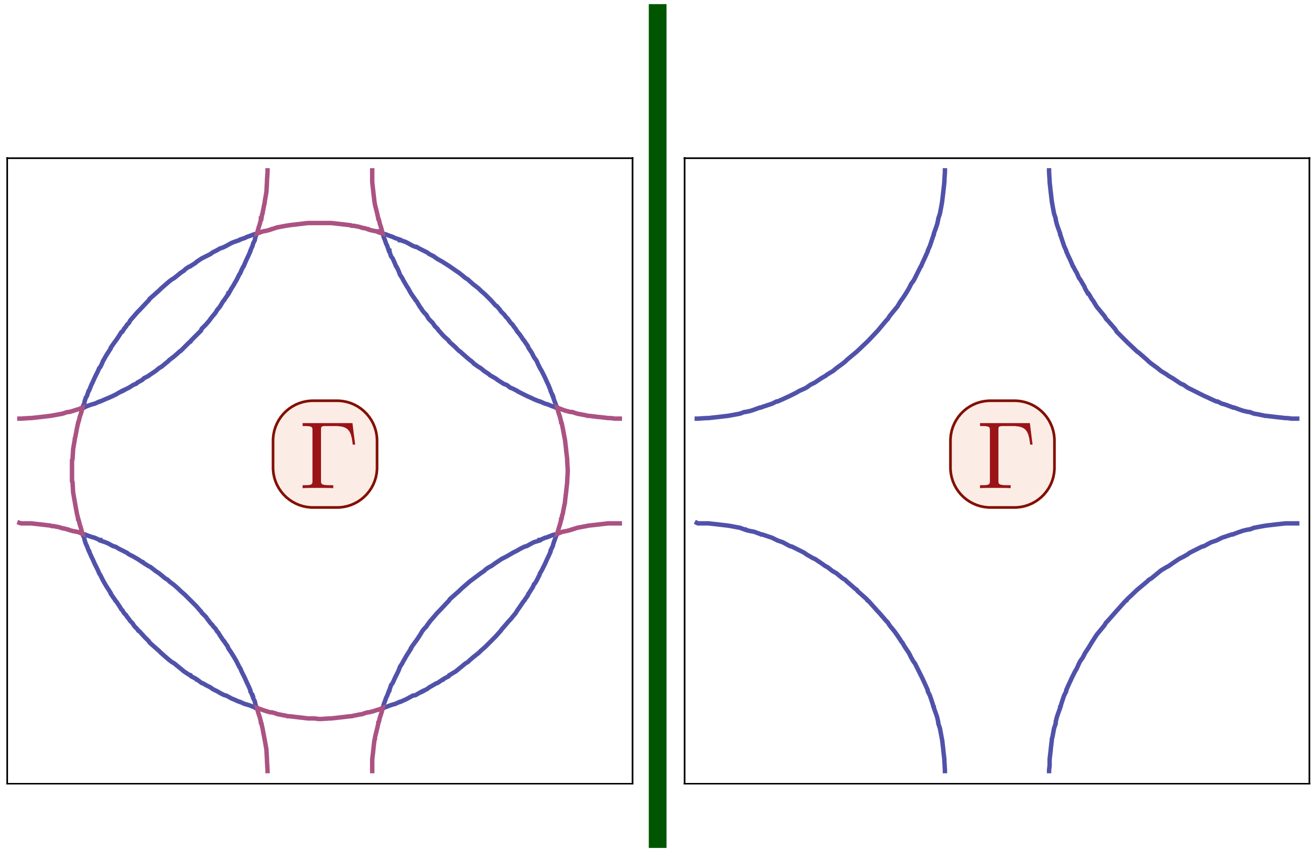
This leads to the Fermi surfaces shown in the following slides for hole doping.

Spin density wave theory in hole-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



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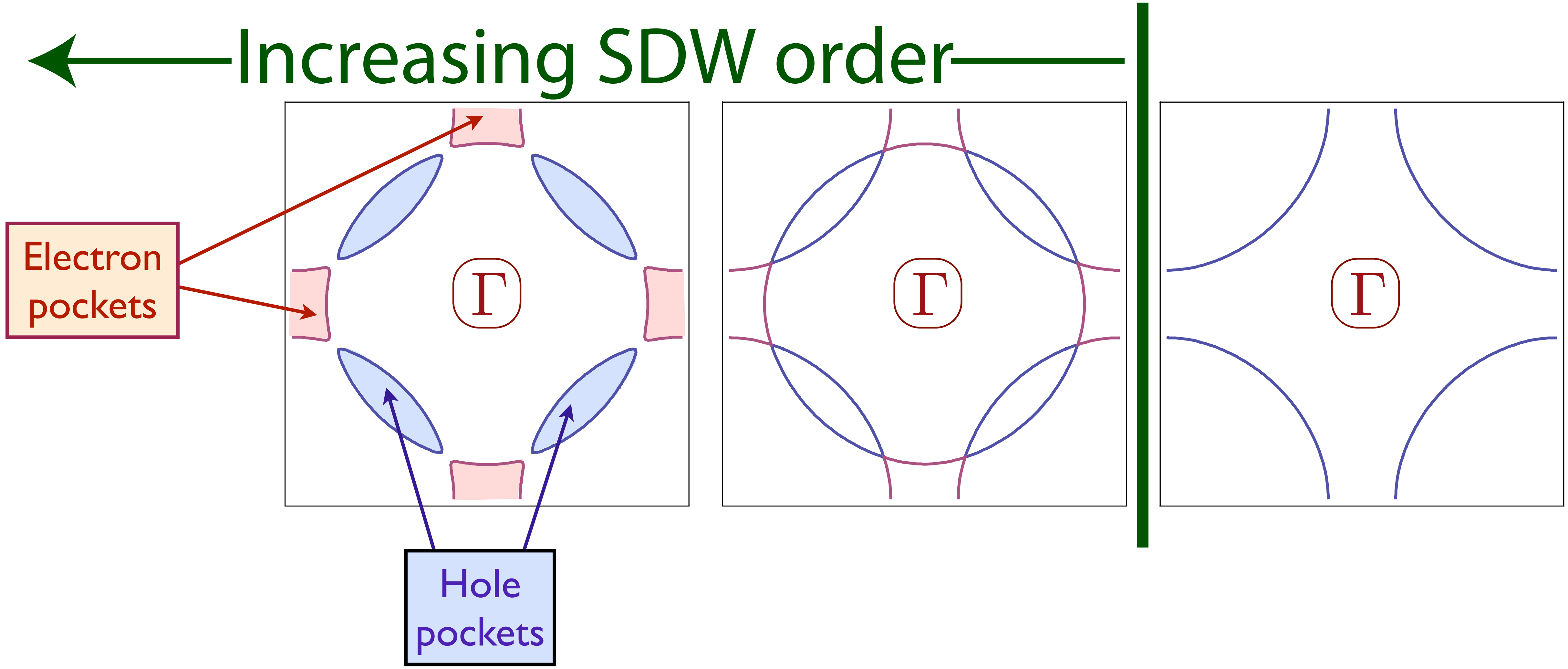

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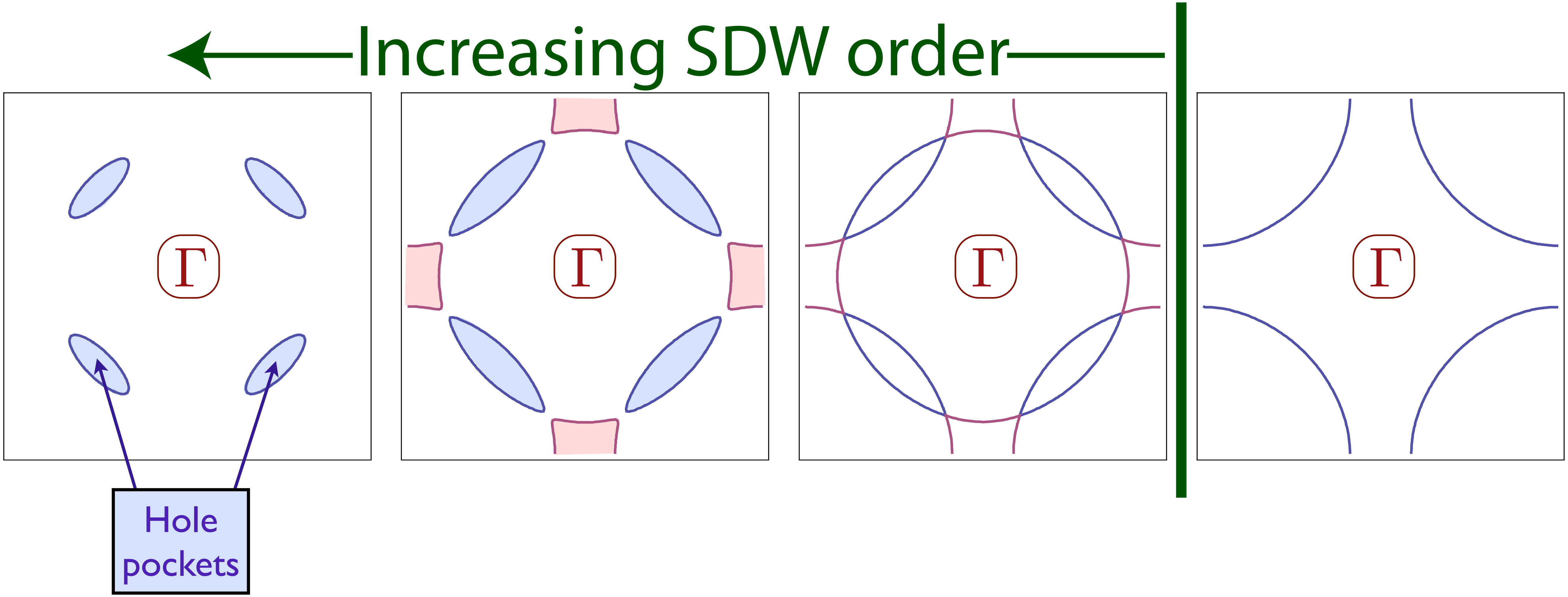

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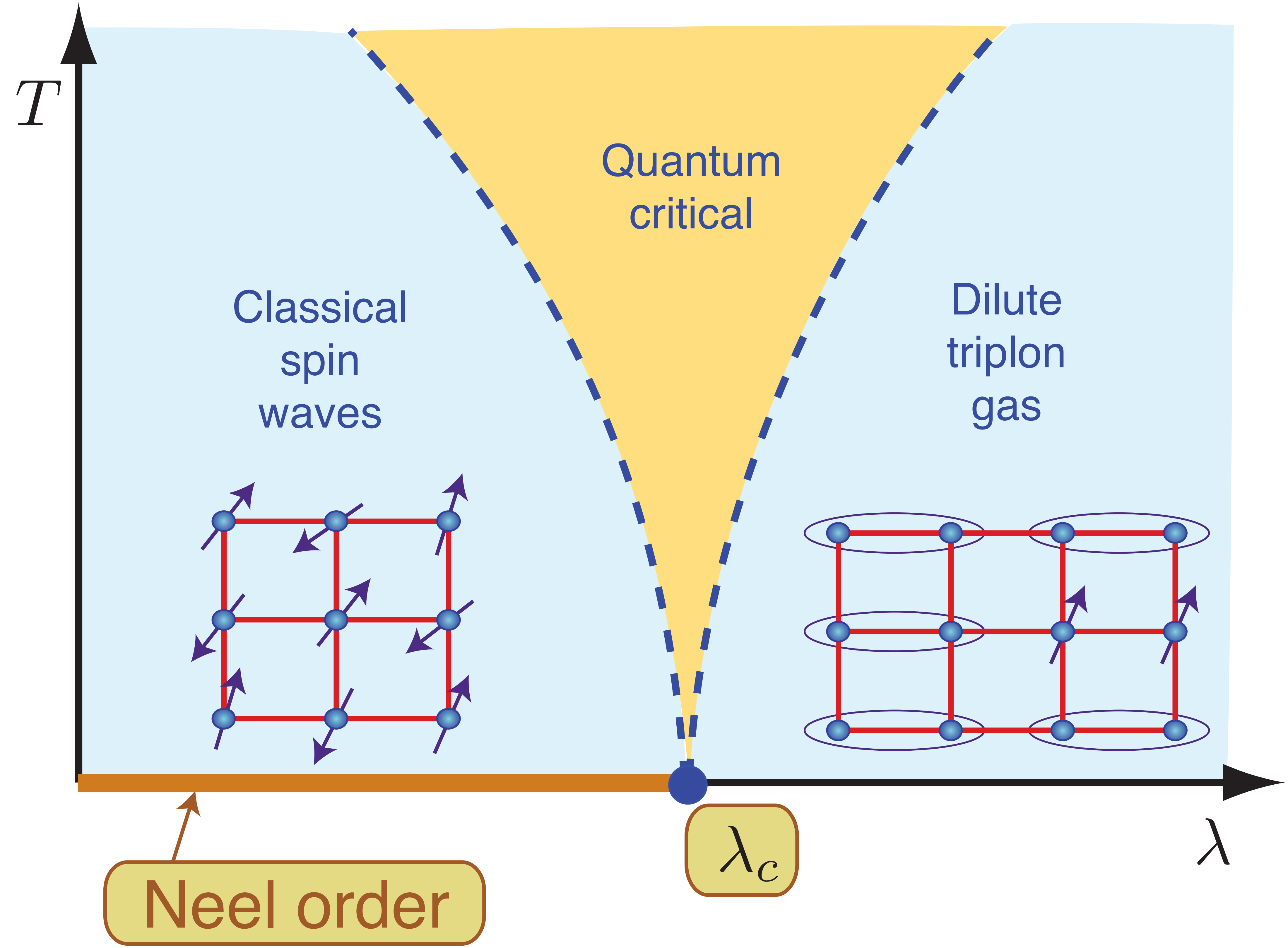

T
Quantum critical
Classical spin waves
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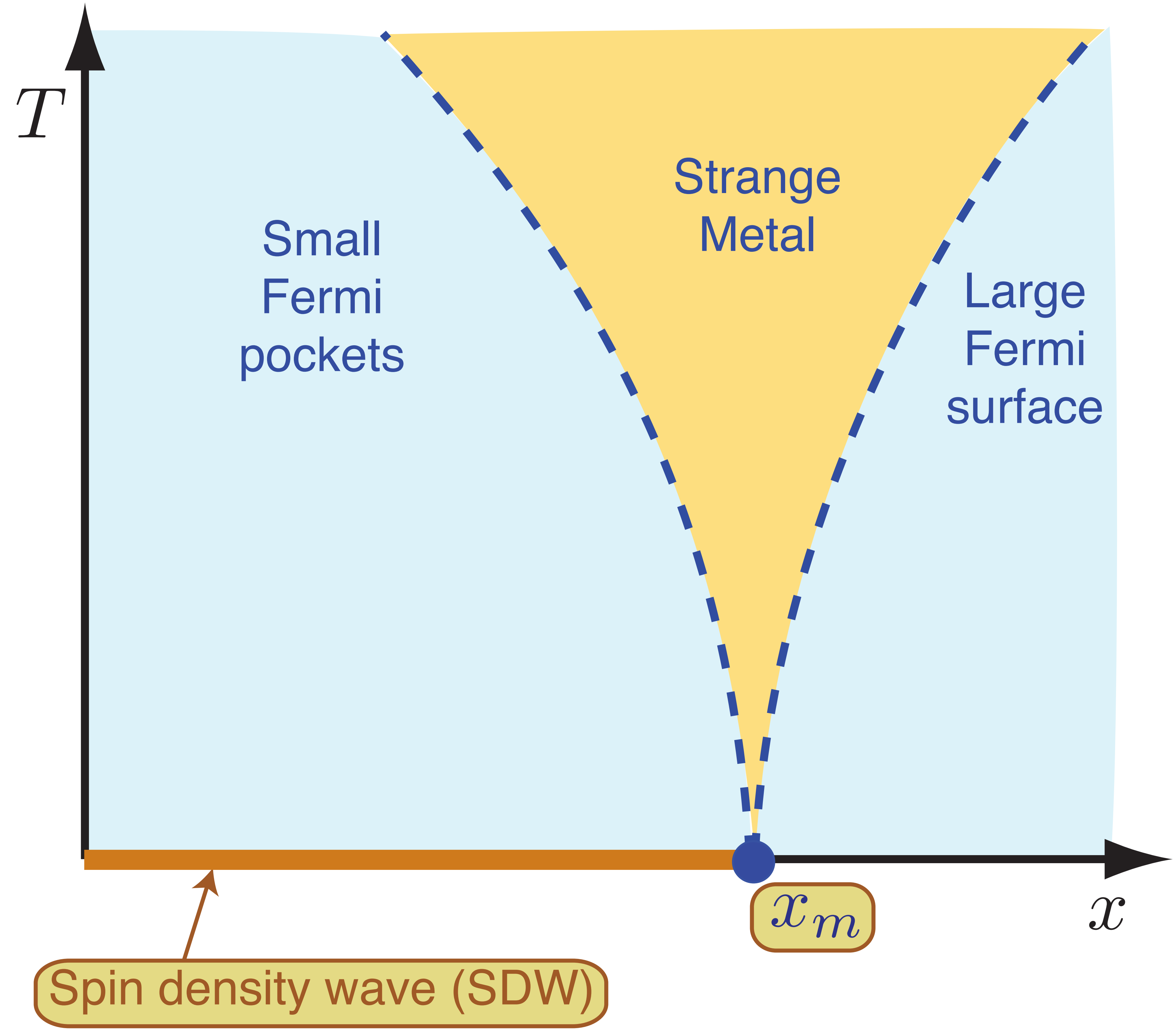
Neel order

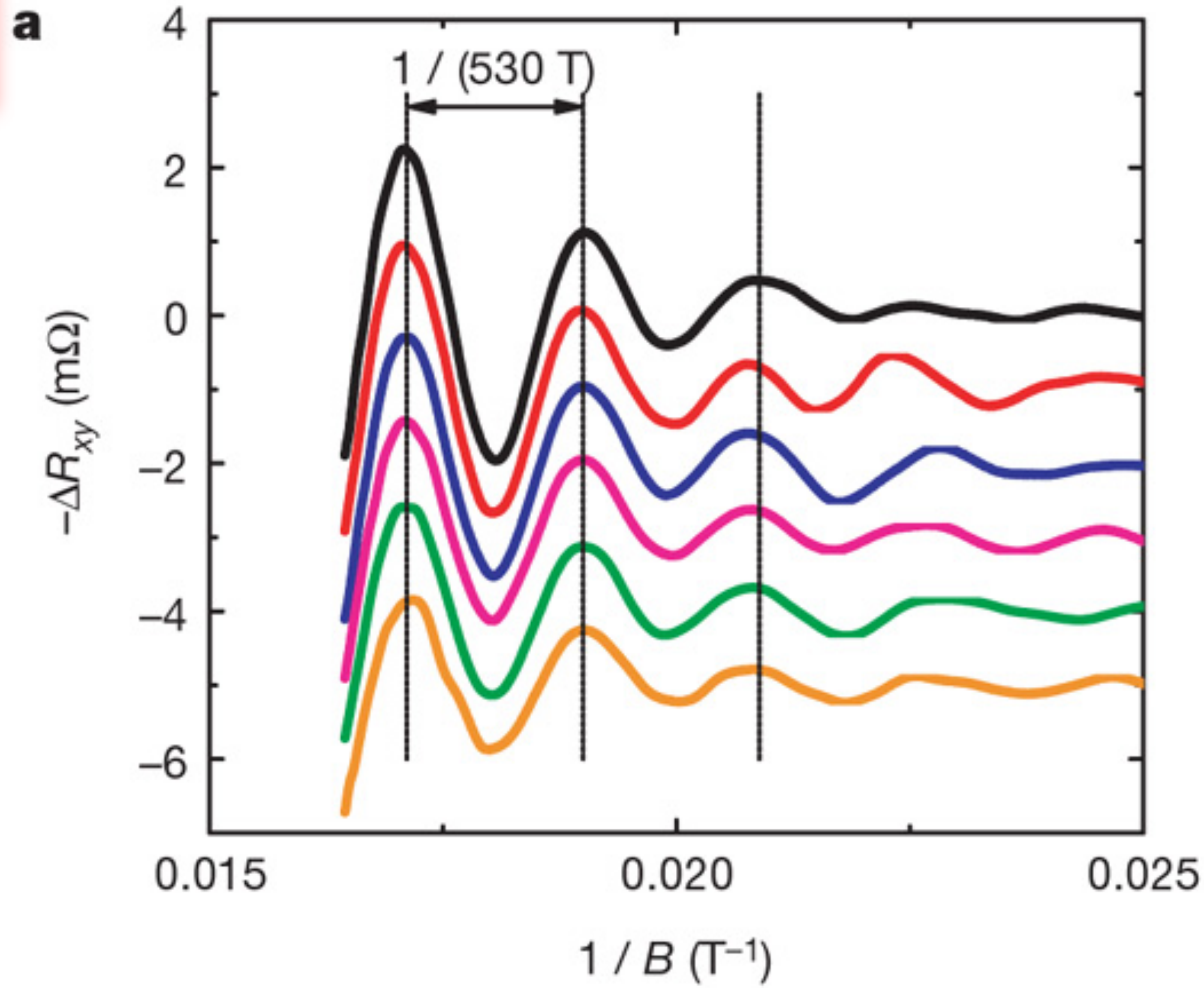
Theory of quantum criticality in the cuprates



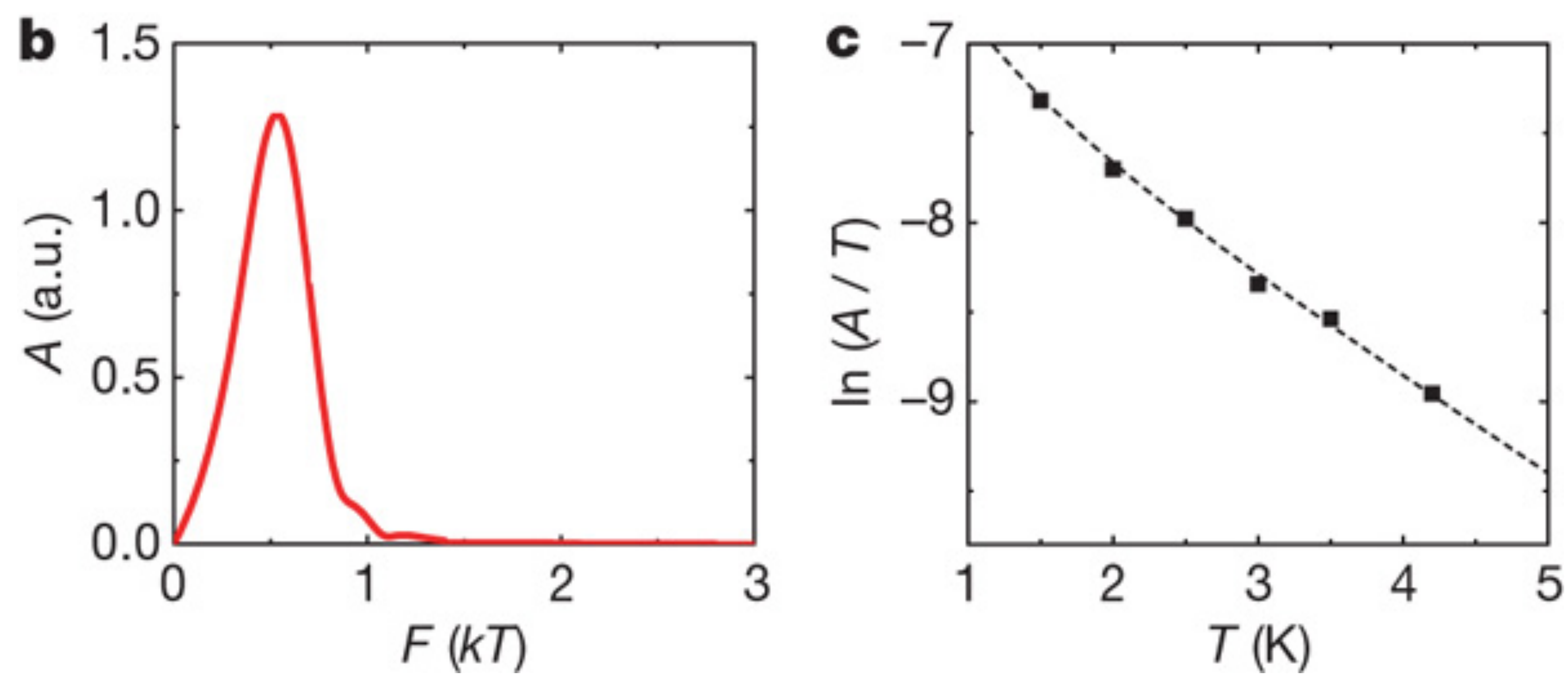
Underlying SDW ordering quantum critical point in metal at $x = x_m$

Quantum oscillations

Quantum oscillations and the Fermi surface in an underdoped high T_c superconductor (ortho-II ordered $YBa_2Cu_3O_{6.5}$). The period corresponds to a carrier density ≈ 0.076.



N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)



Quantum oscillations

Onsager-Lifshitz relation:

Frequency of oscillations in $1/H$

$$= \frac{hc}{2e} \frac{\text{(Area of Fermi surface)}}{2\pi^2}$$

Luttinger relation:

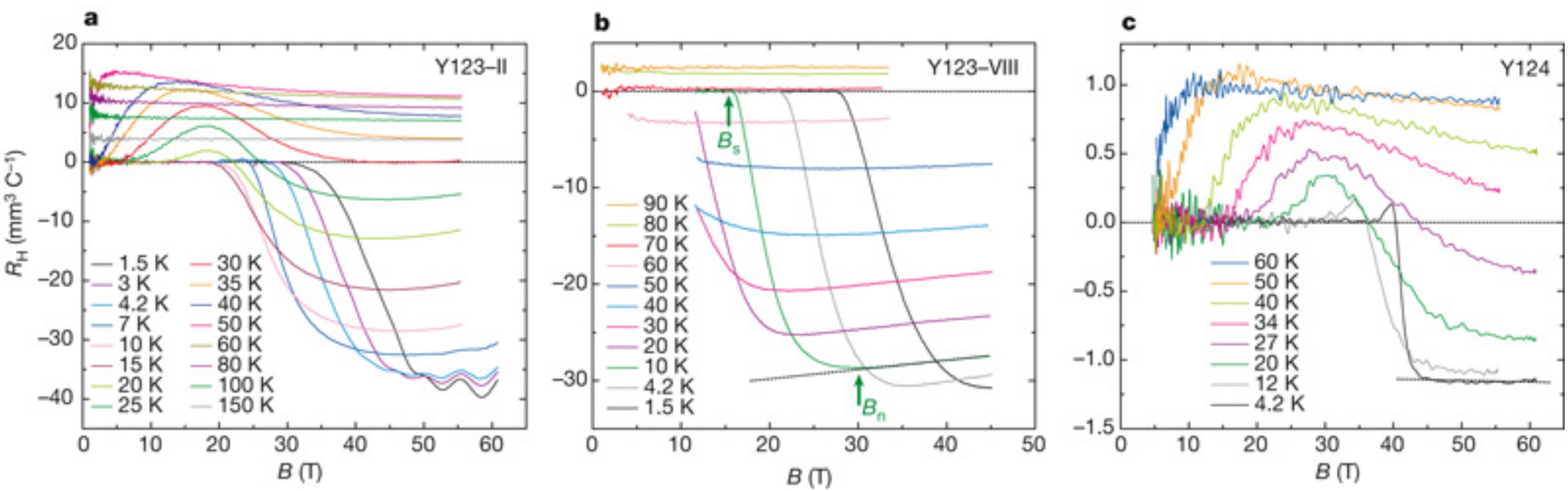
Area of Fermi surface $= 4\pi^3$(density of fermions)

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



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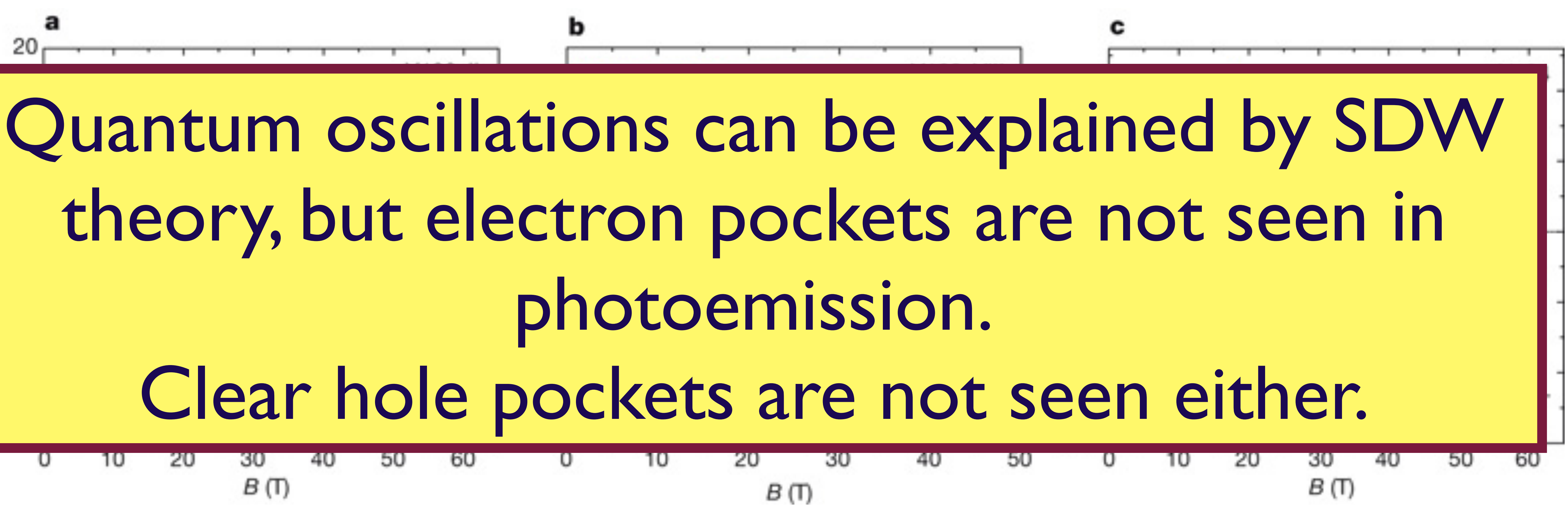
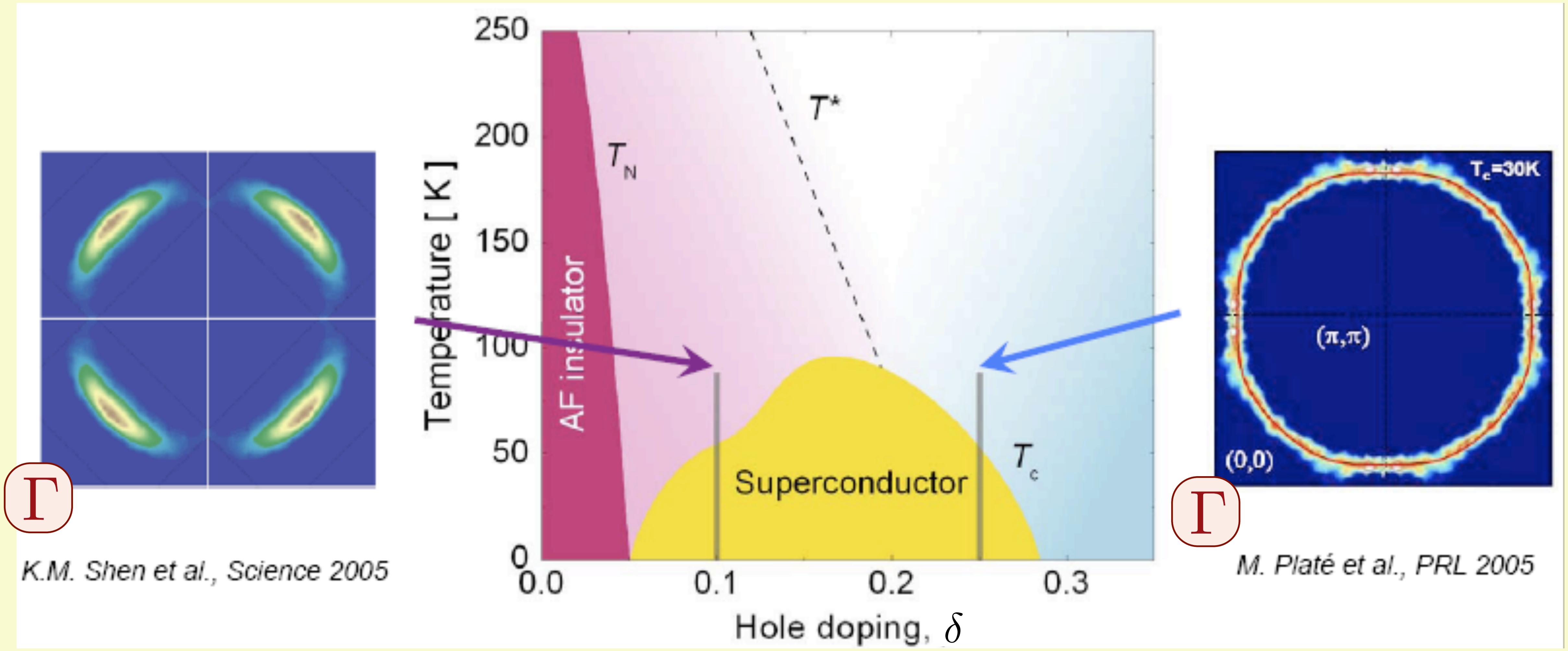


Quantum oscillations can be explained by SDW theory, but electron pockets are not seen in photoemission.
Clear hole pockets are not seen either.

Superconductivity in hole-doped cuprates

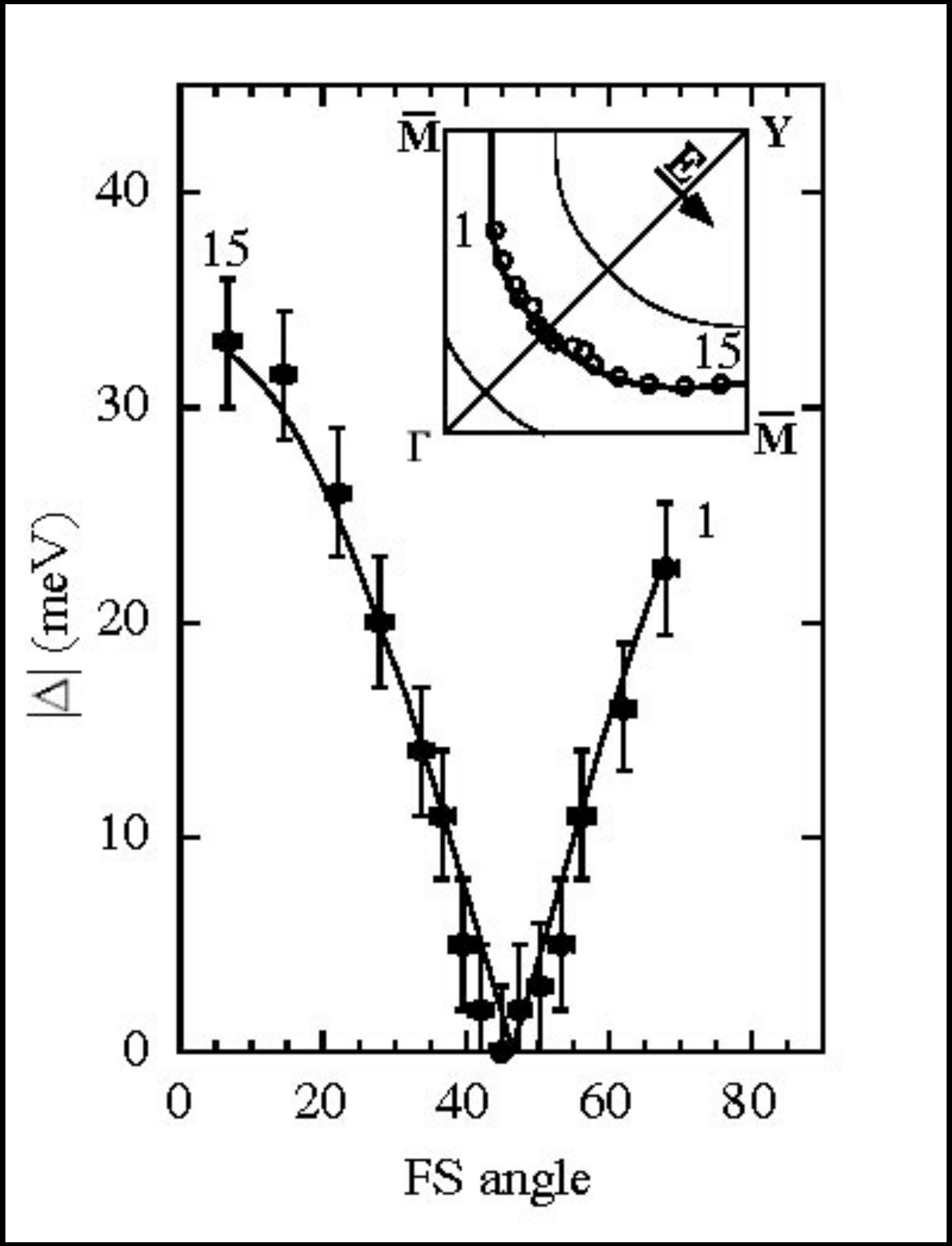
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



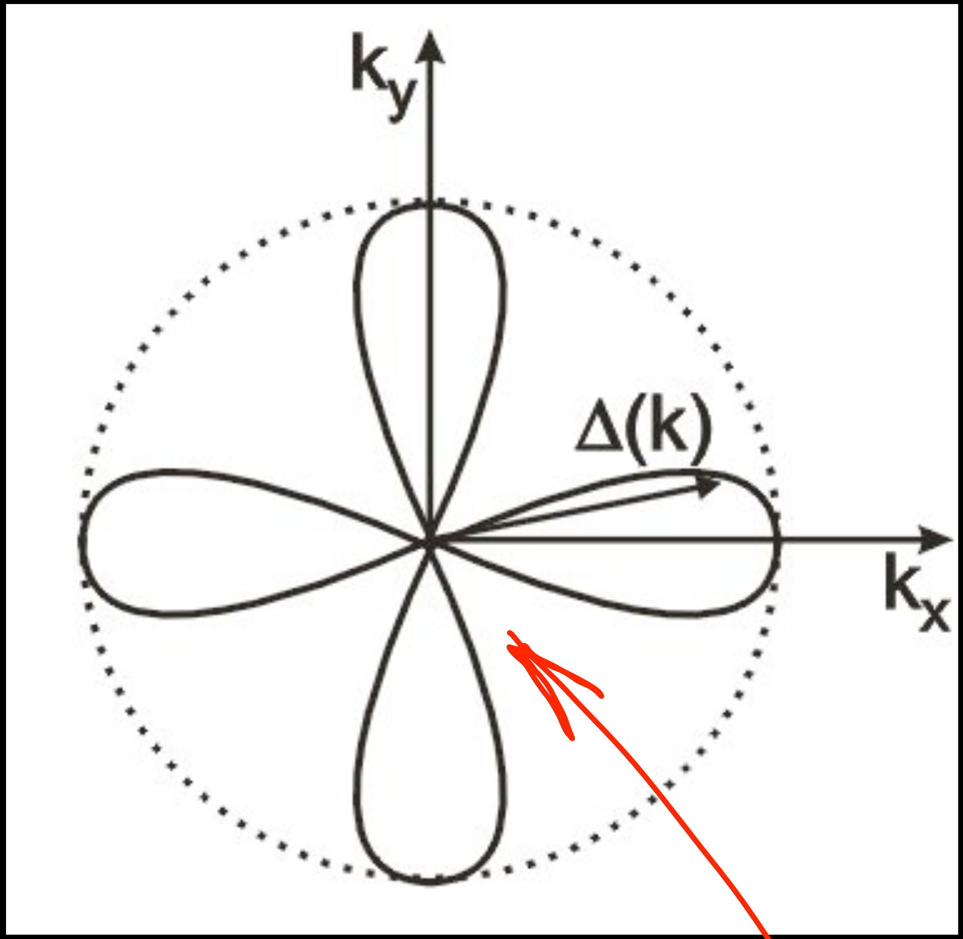
Smaller hole Fermi-pockets

Large hole Fermi surface

Overdoped SC State: Momentum-dependent Pair Energy Gap $\Delta(\vec{k})$

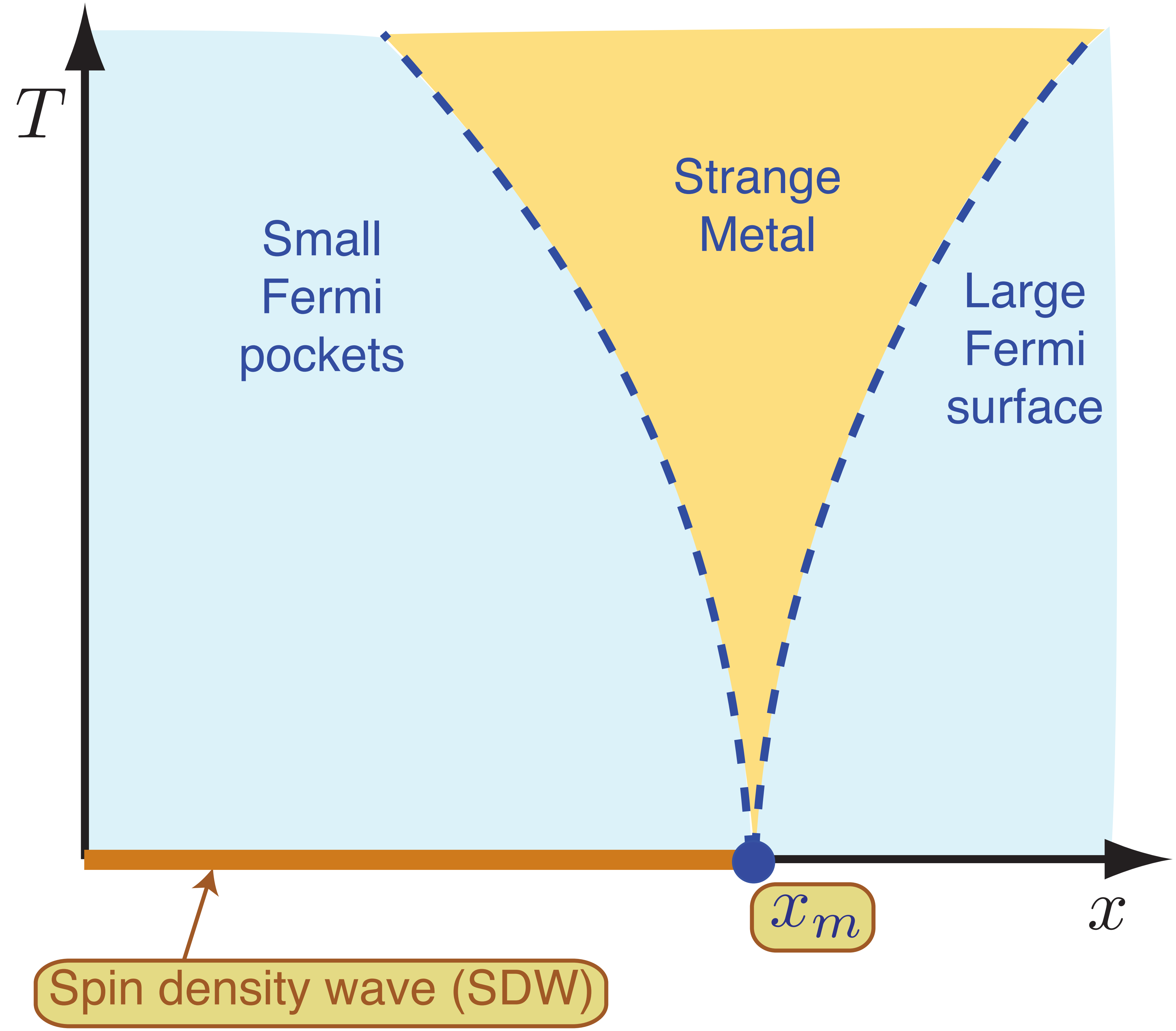


Shen et al PRL **70**, 3999 (1993)
Ding et al PRB **54** 9678 (1996)
Mesot et al PRL **83** 840 (1999)



The SC energy gap $\Delta(\vec{k})$ has four nodes.

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in metal at $x = x_m$

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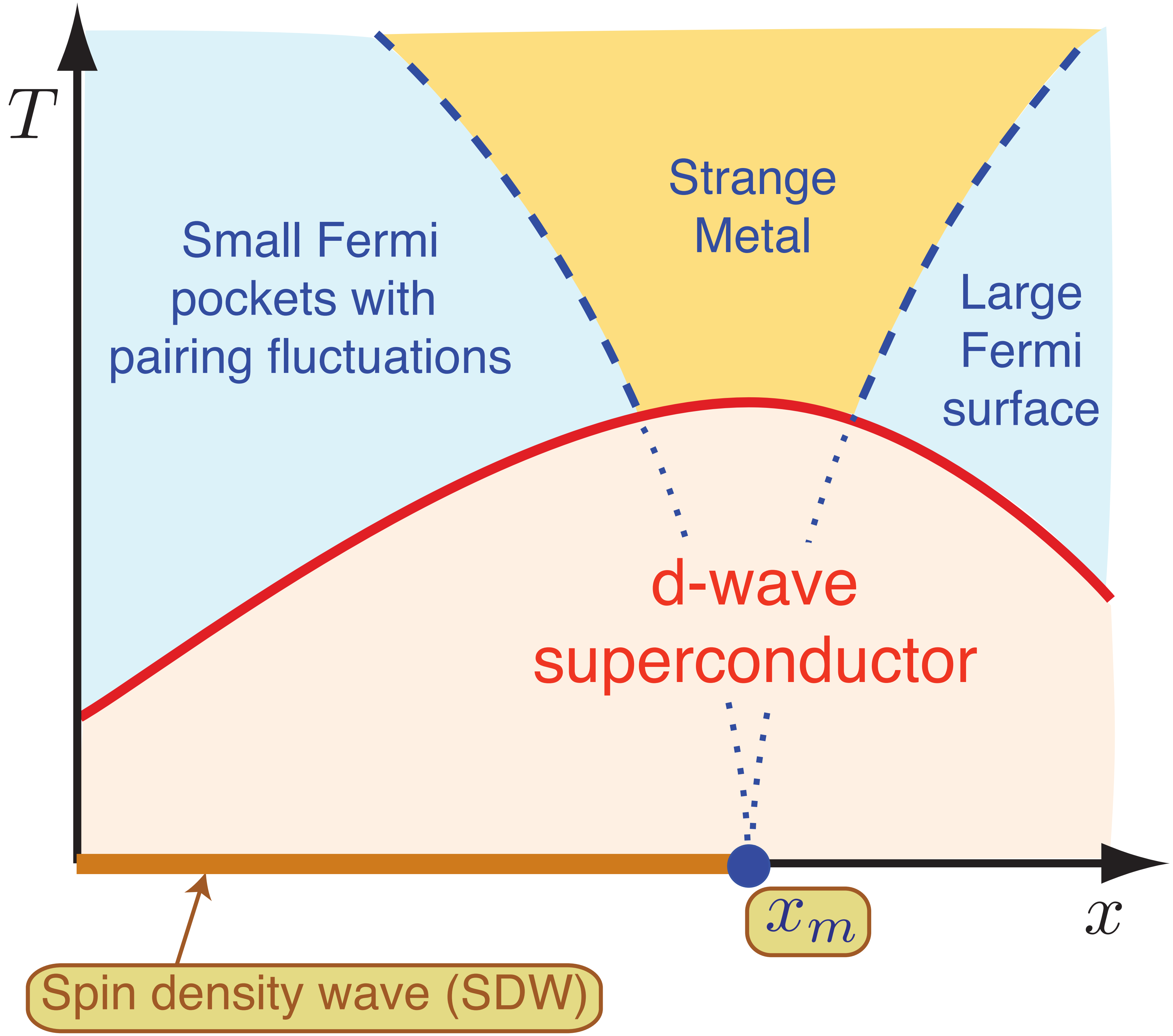
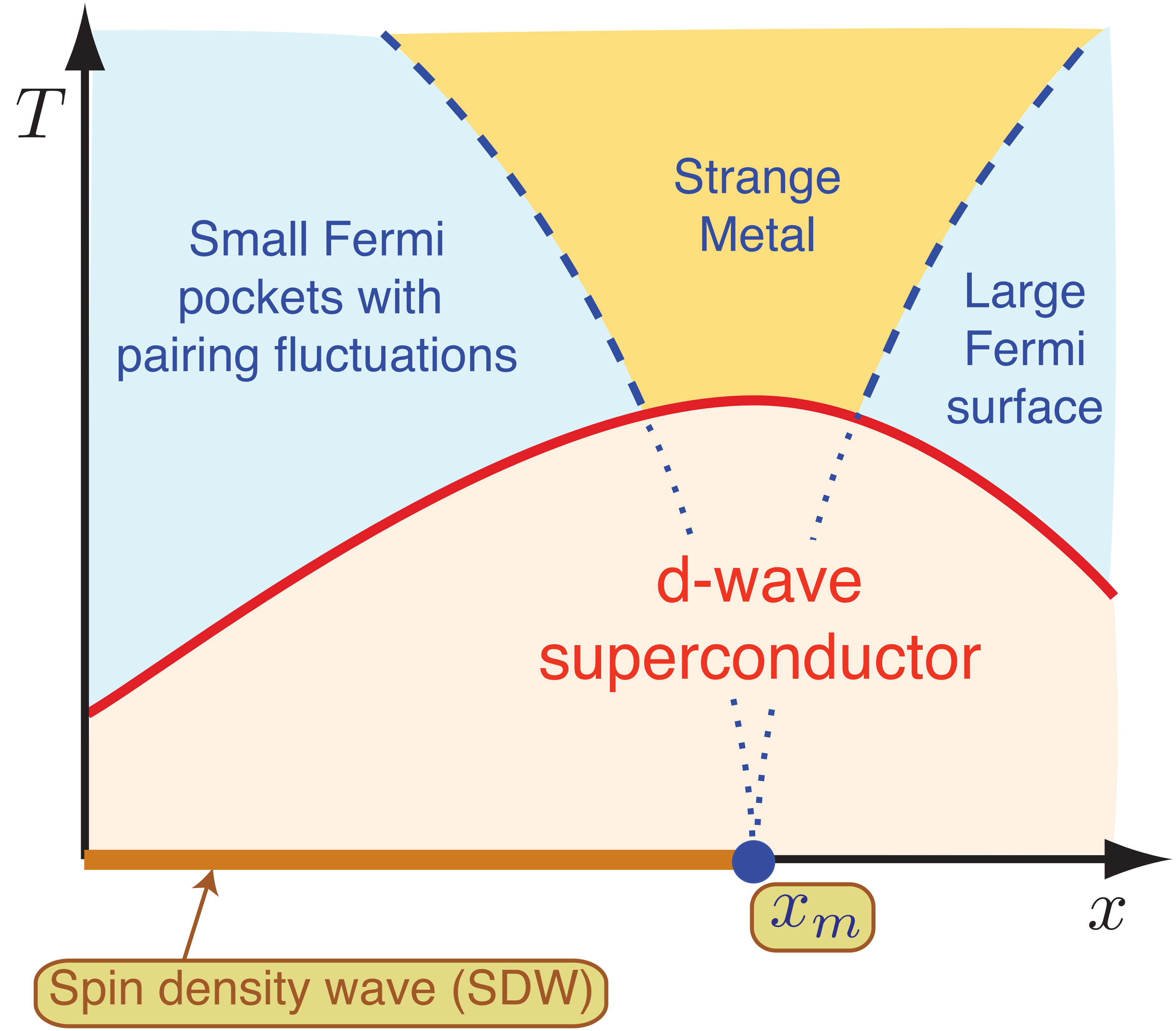


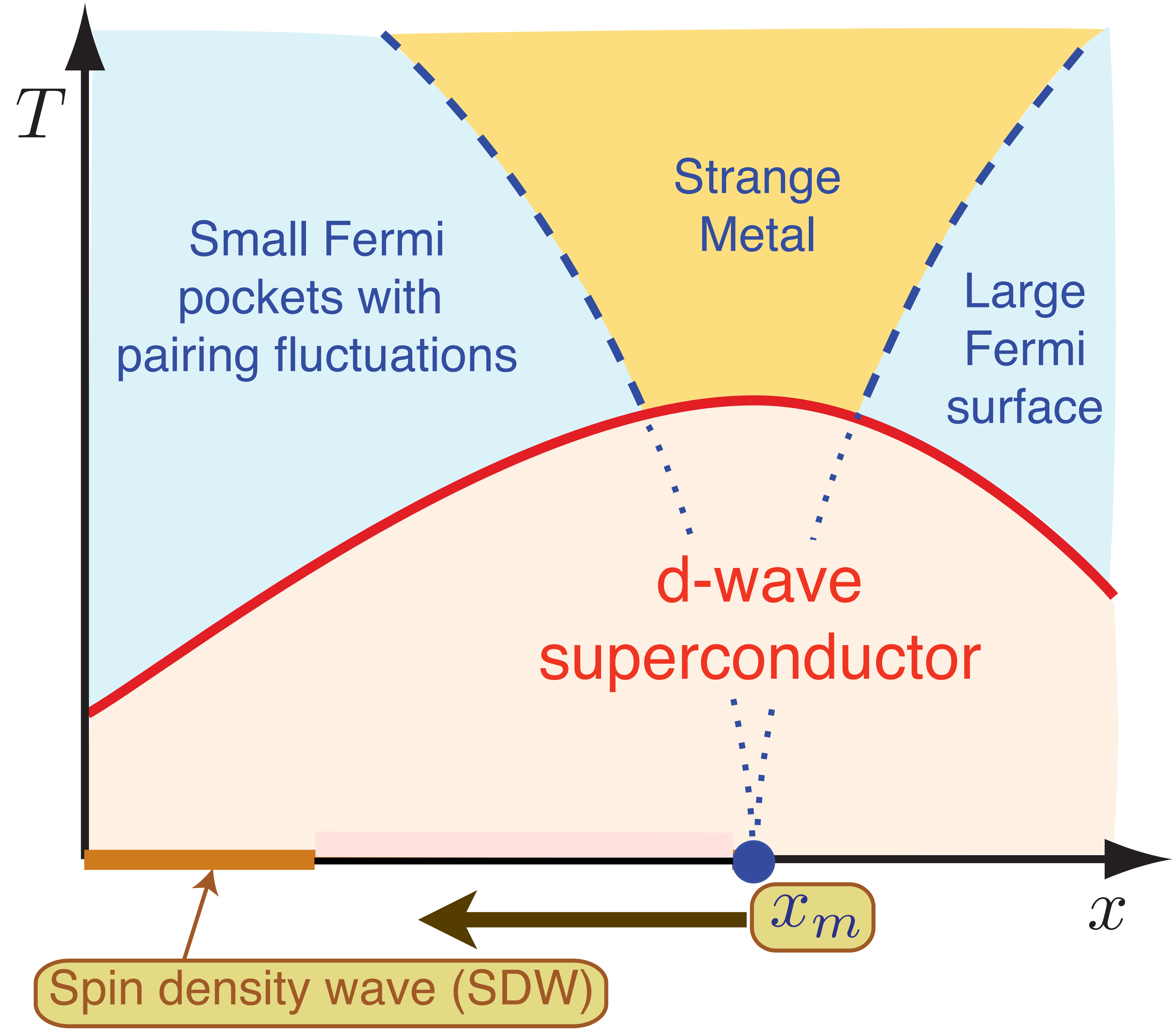
Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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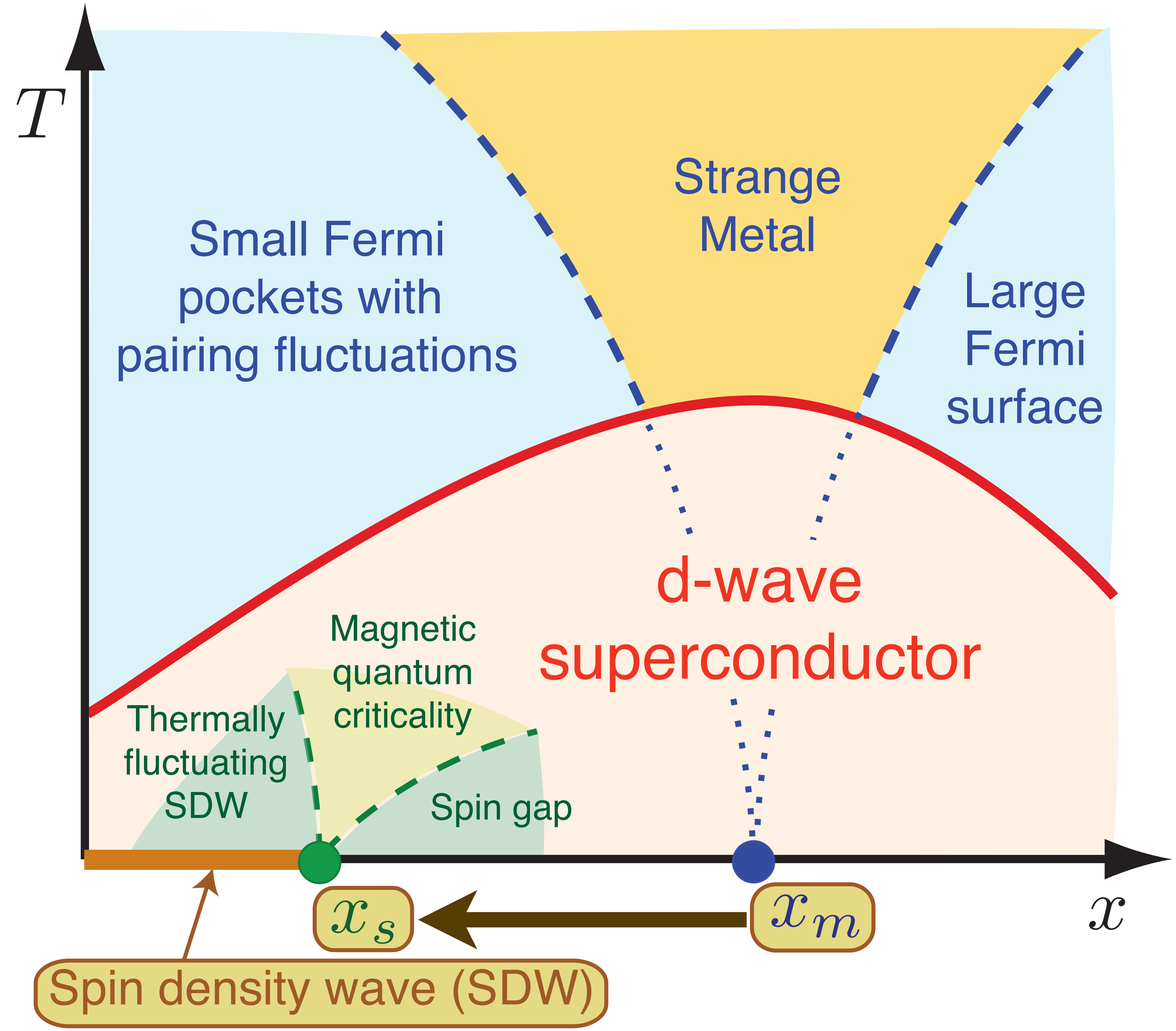

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Competition between SDW order and superconductivity: phenomenological theory

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 + \kappa\, \vec{\varphi}^2\, |\psi|^2 \right] + \int d^2r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
see also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004)

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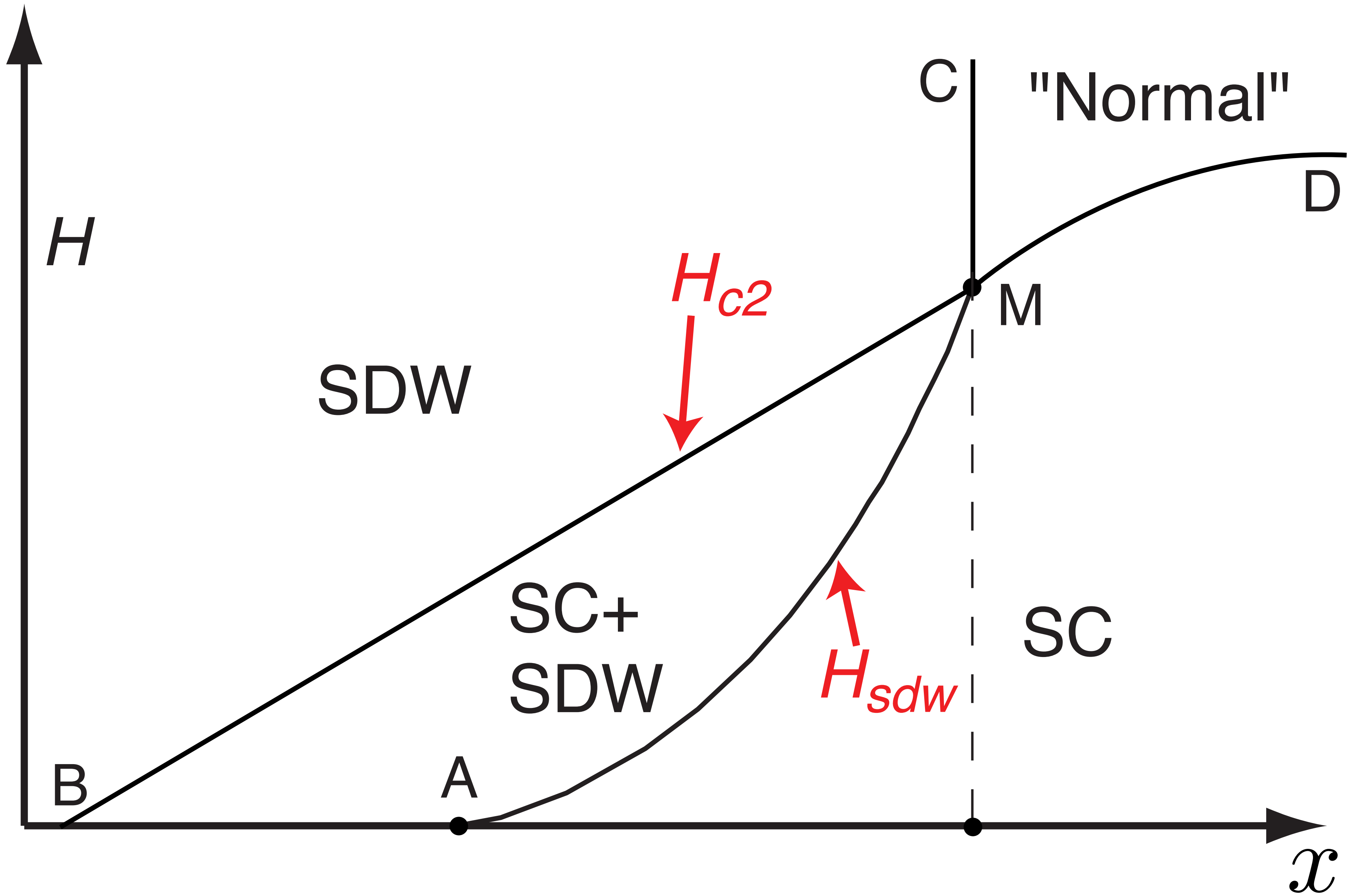

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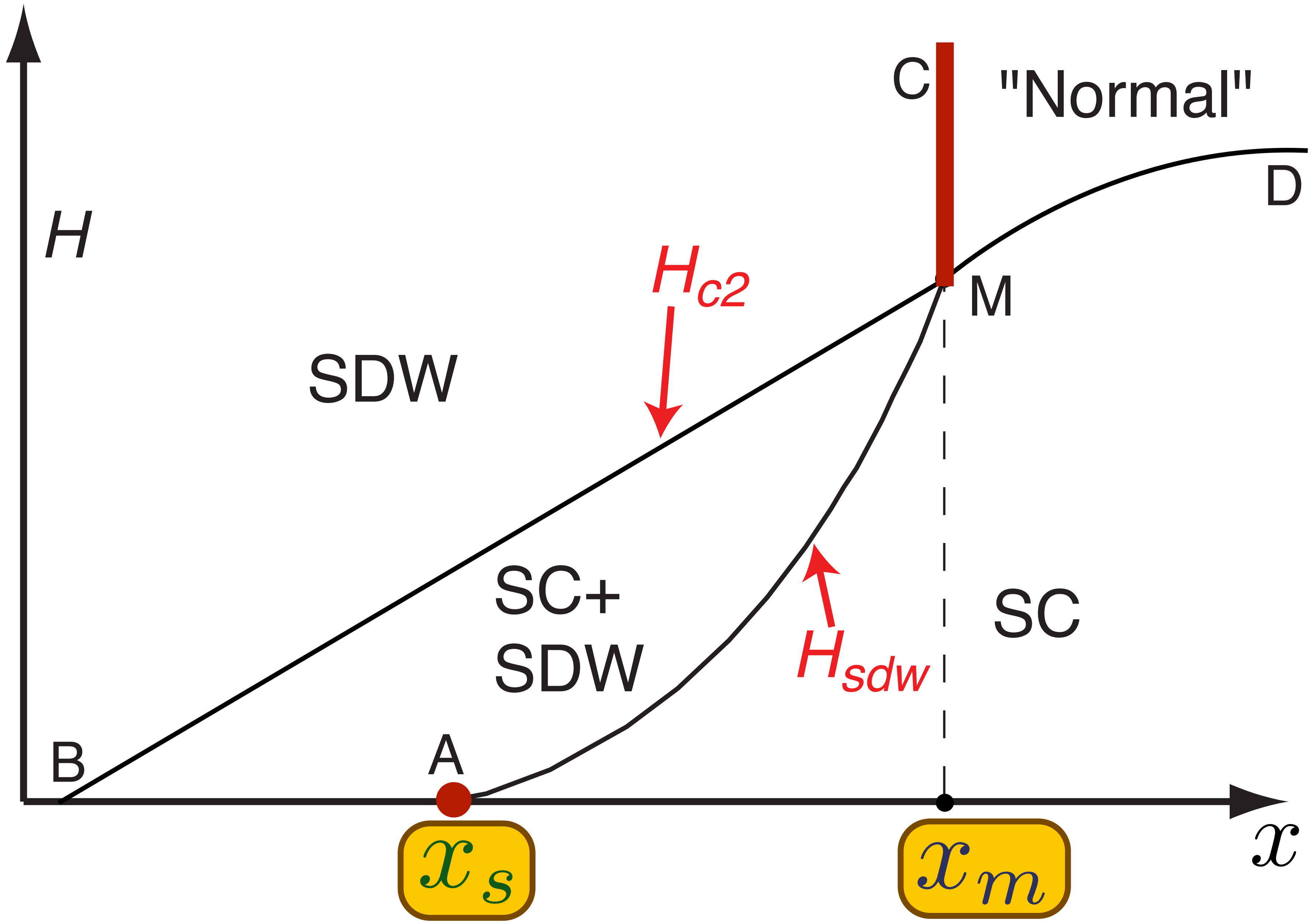

- SDW order is more stable in the metal than in the superconductor: $x_m > x_s$.

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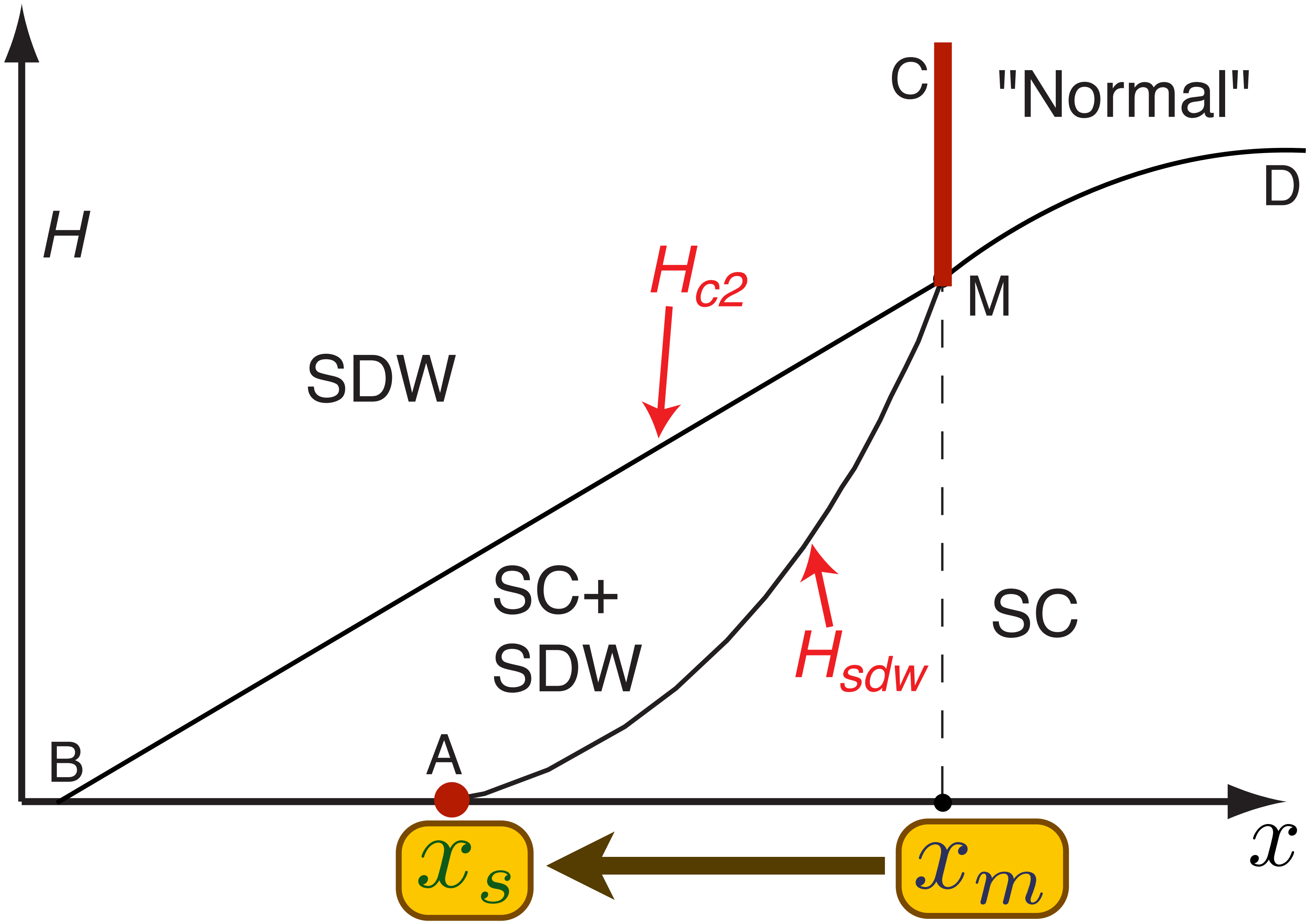

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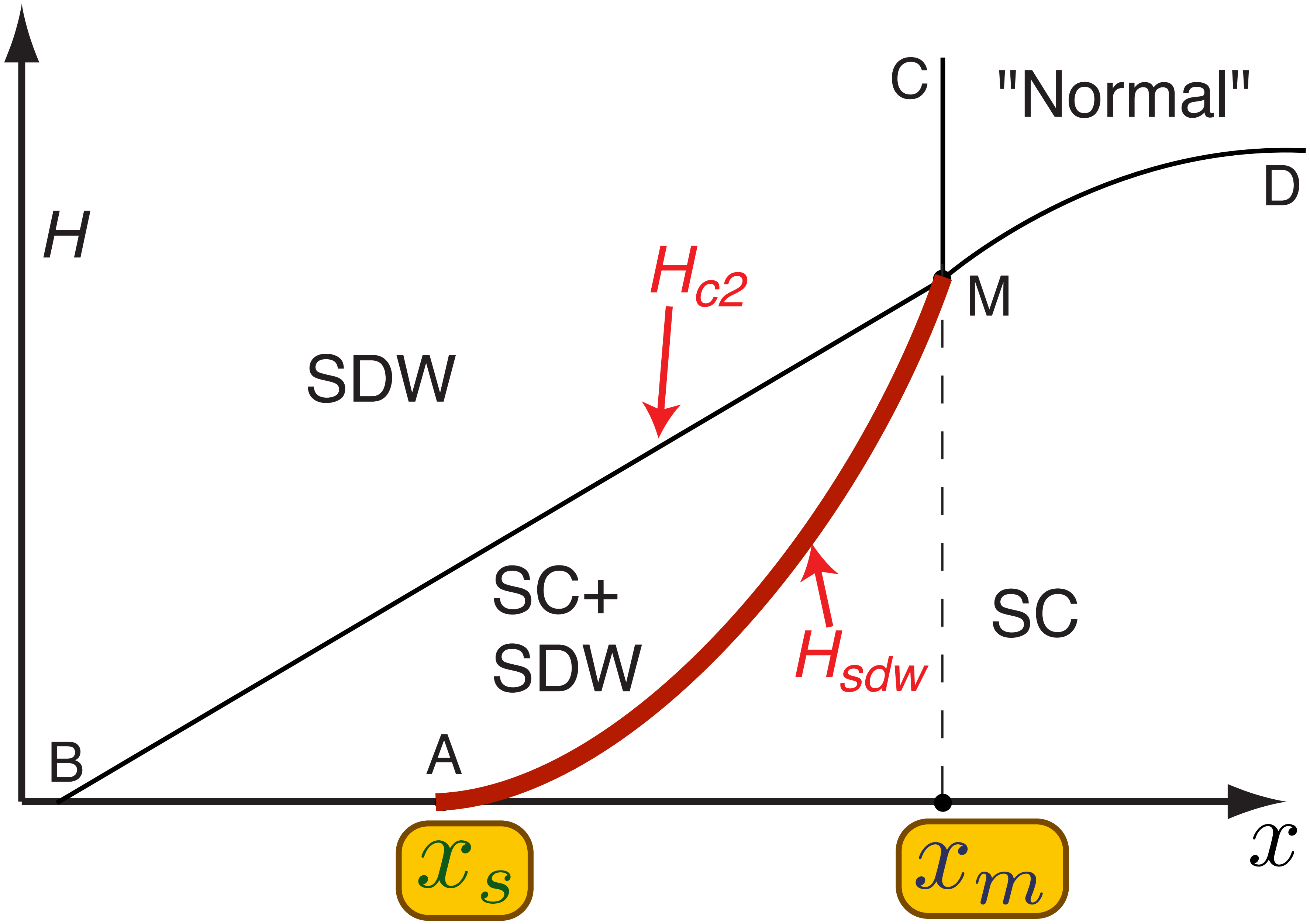

- For doping with $x_s < x < x_m$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

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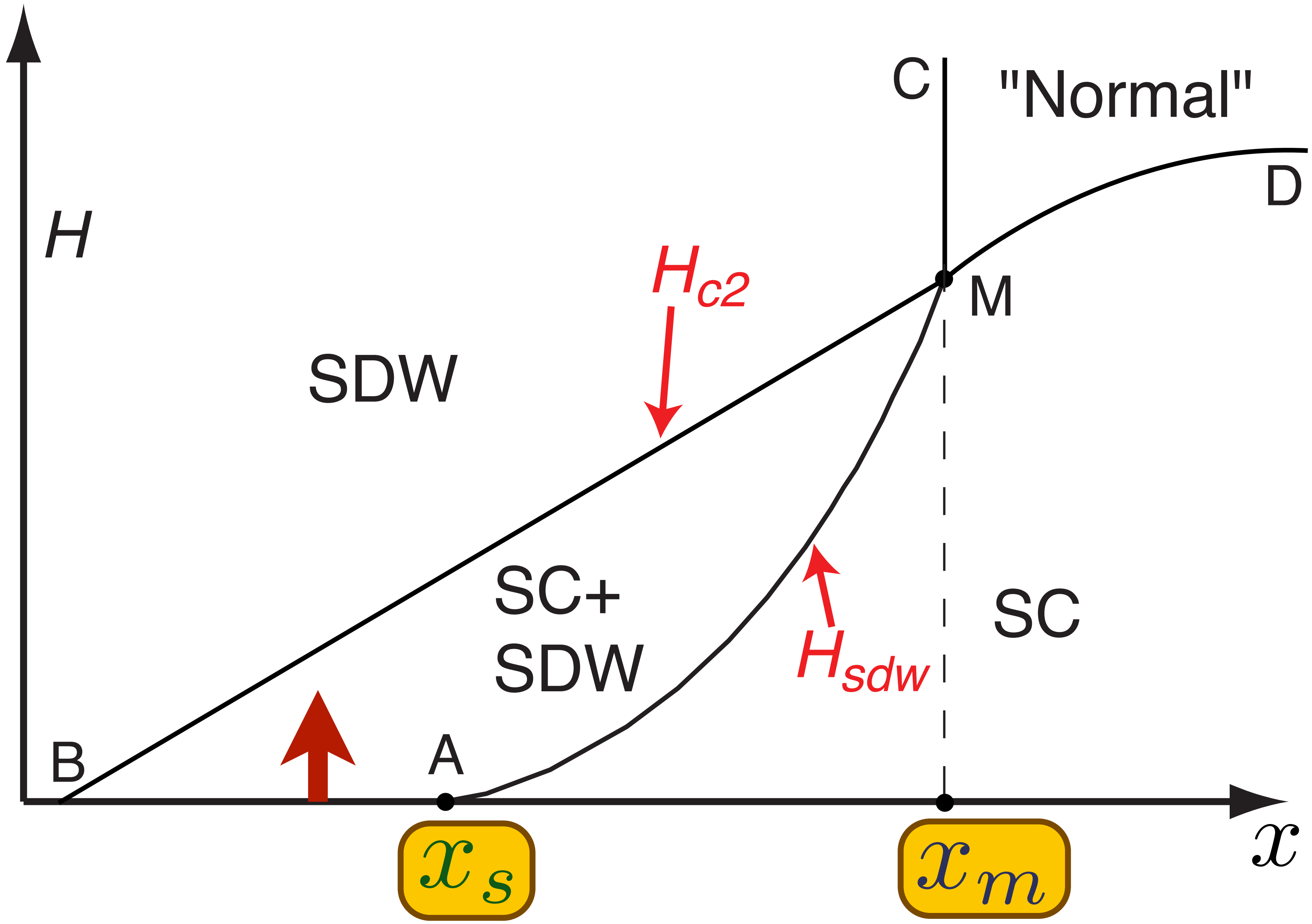

Neutron scattering on $La_{1.9}Sr_{0.1}CuO_4$
B. Lake *et al.*, *Nature* 415, 299 (2002)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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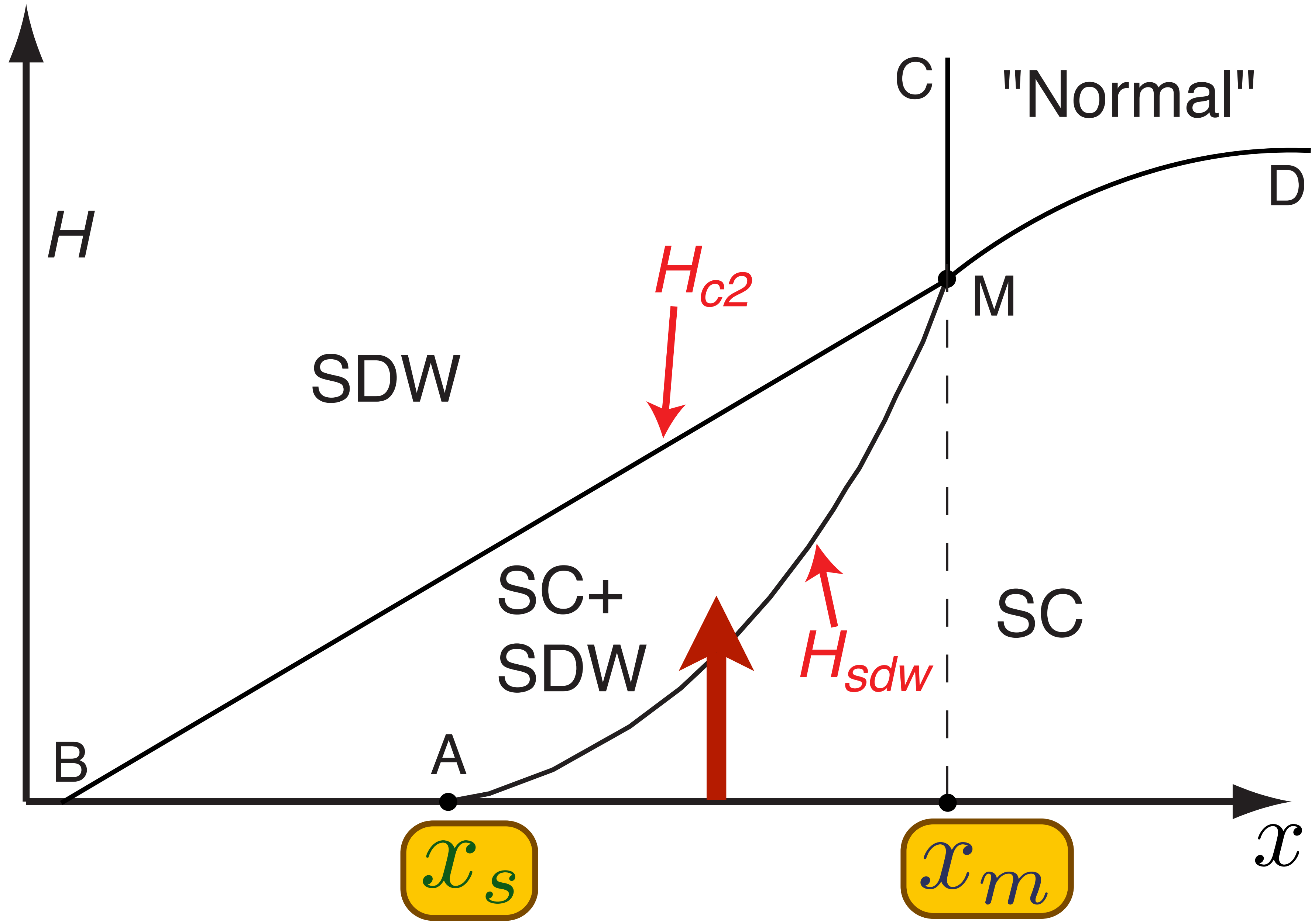

Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
J. Chang *et al.*, *Phys. Rev. Lett.* 102, 177006 (2009).

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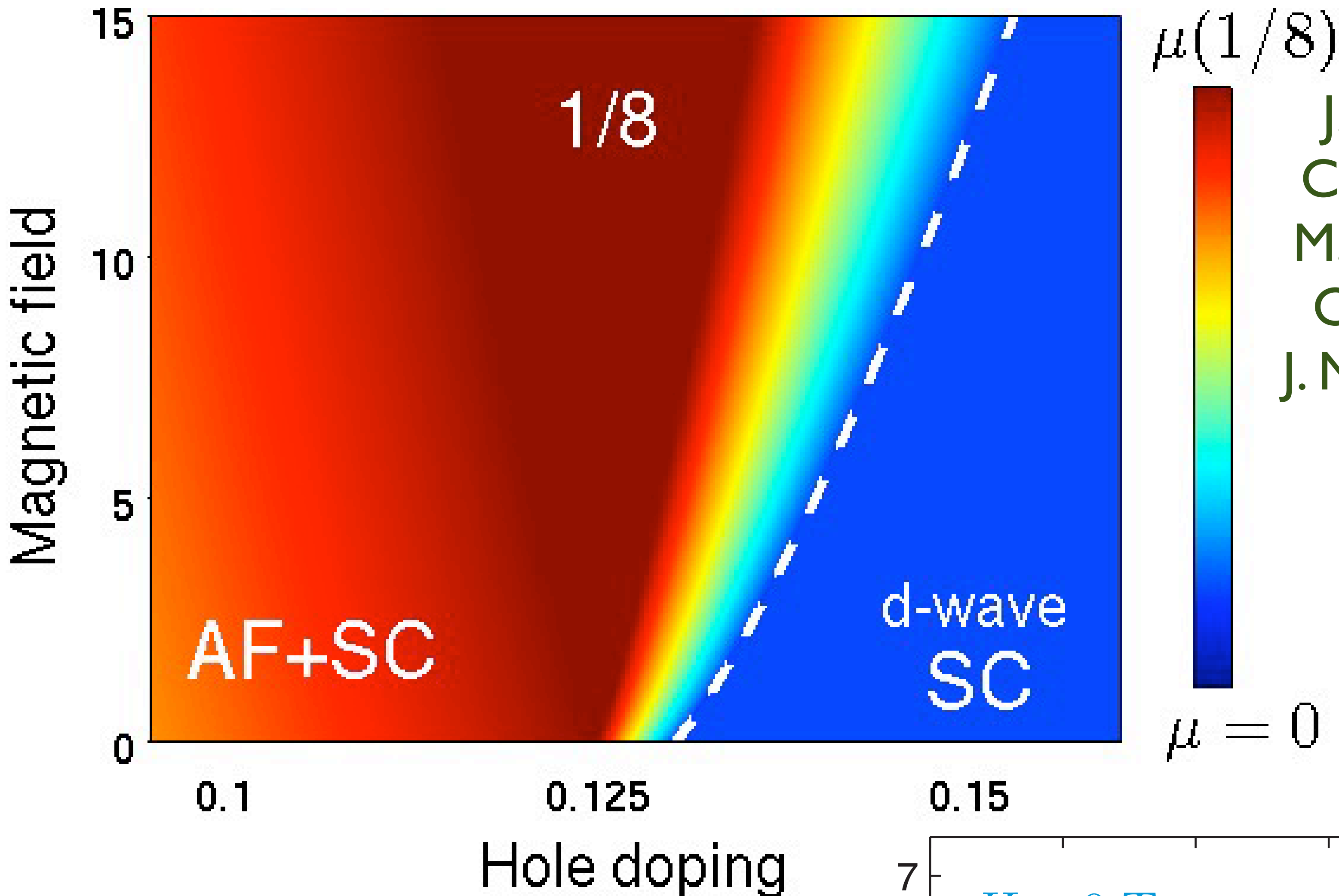
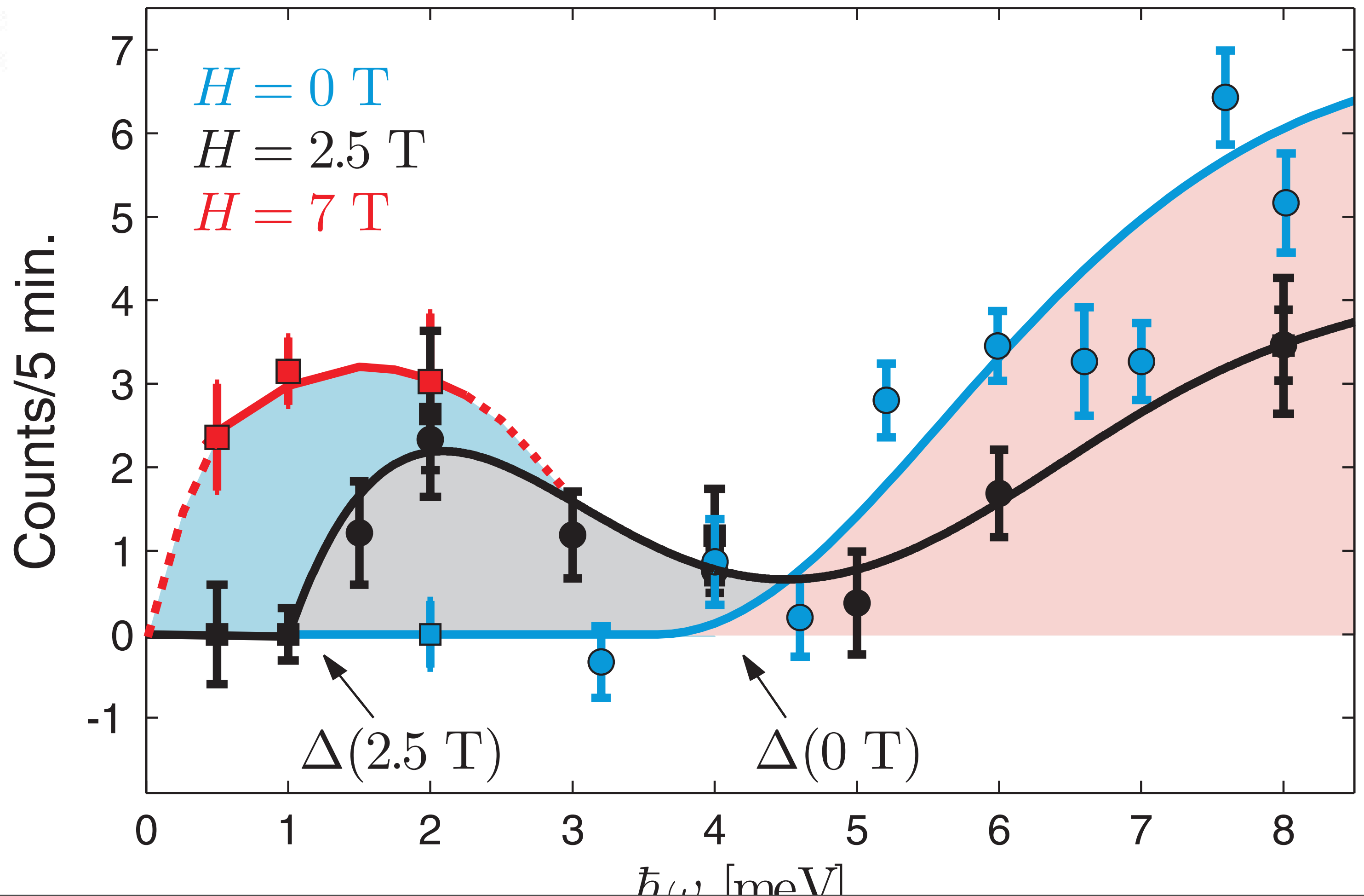


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).

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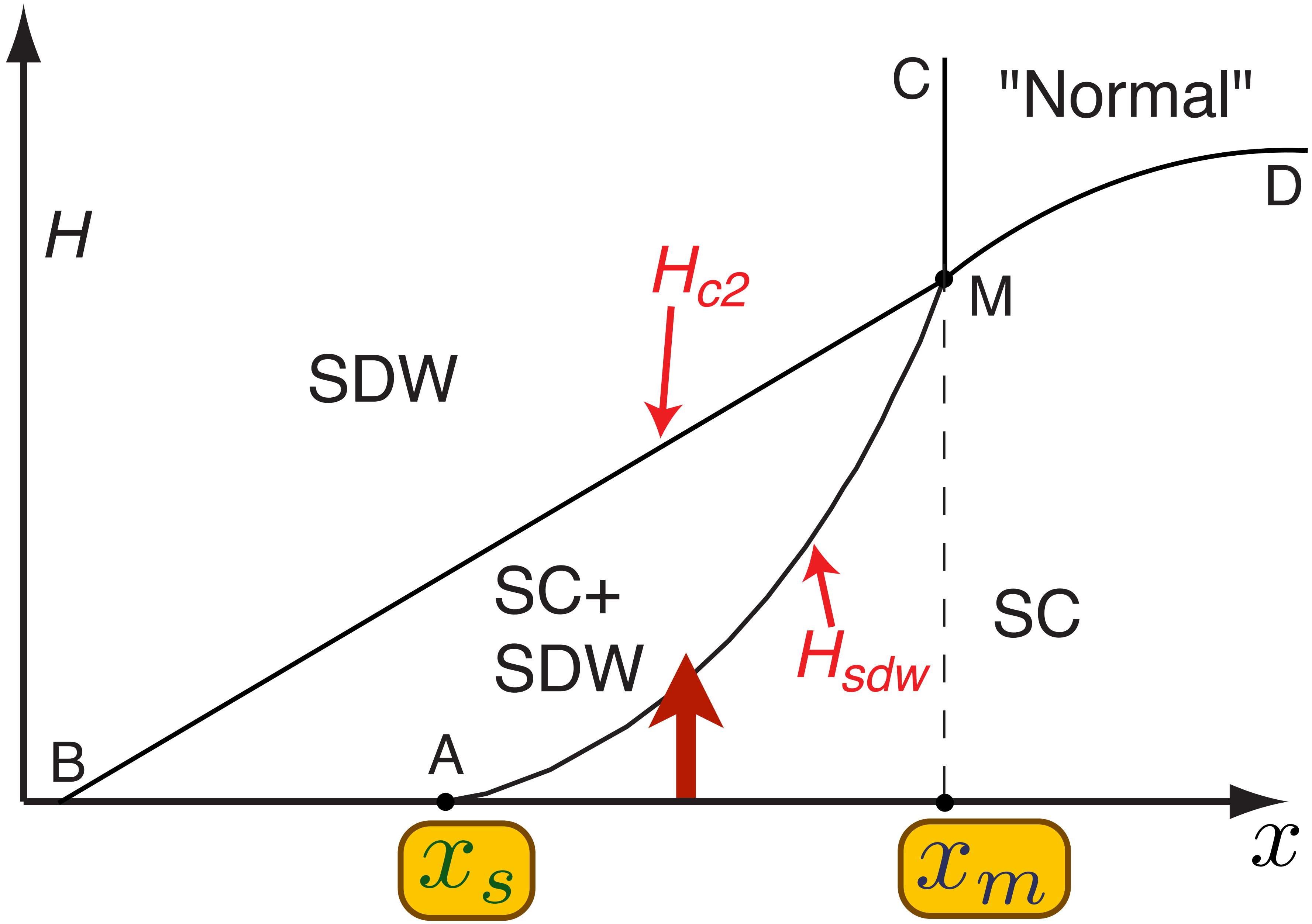
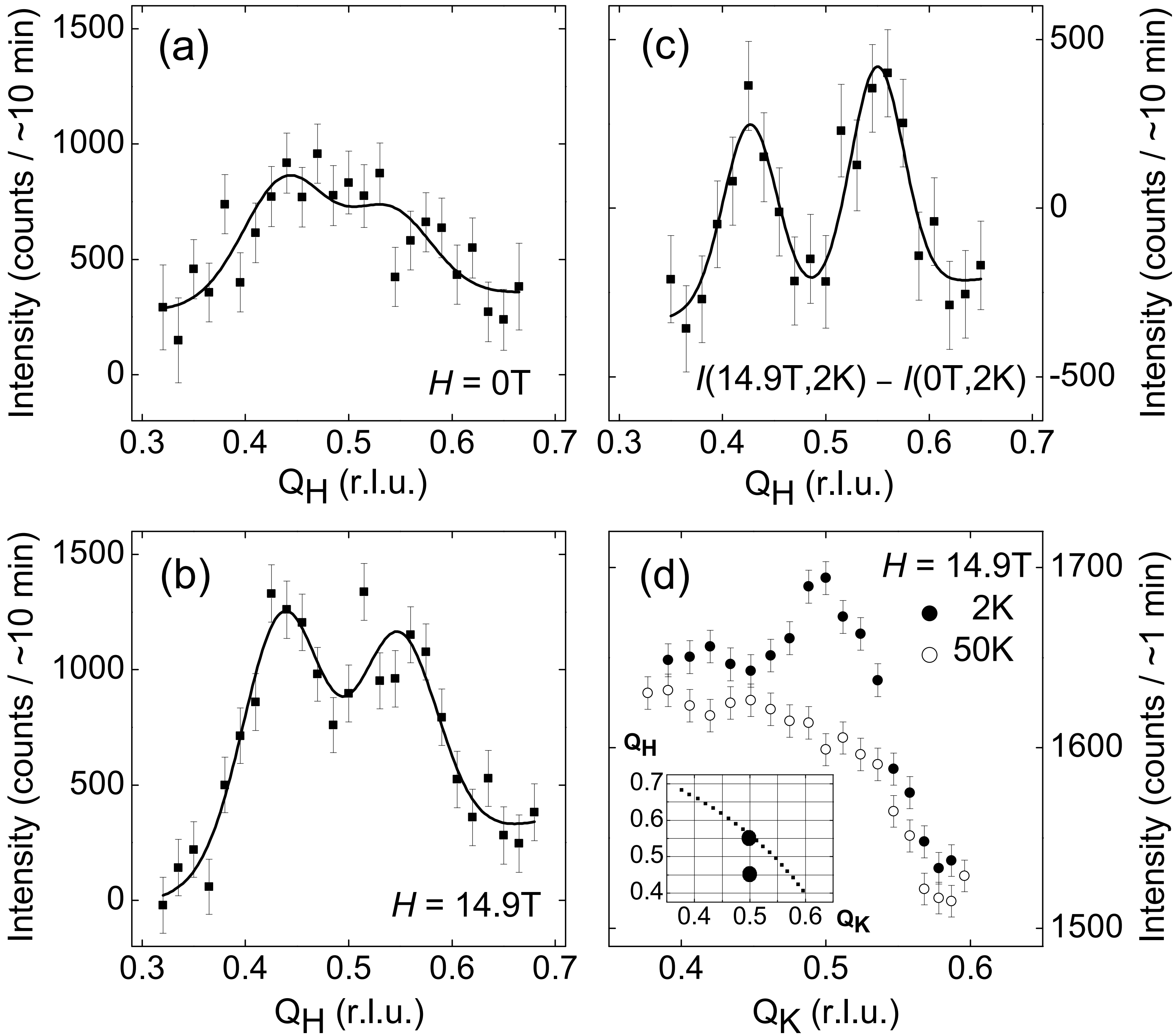


Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, arXiv:0902.3335

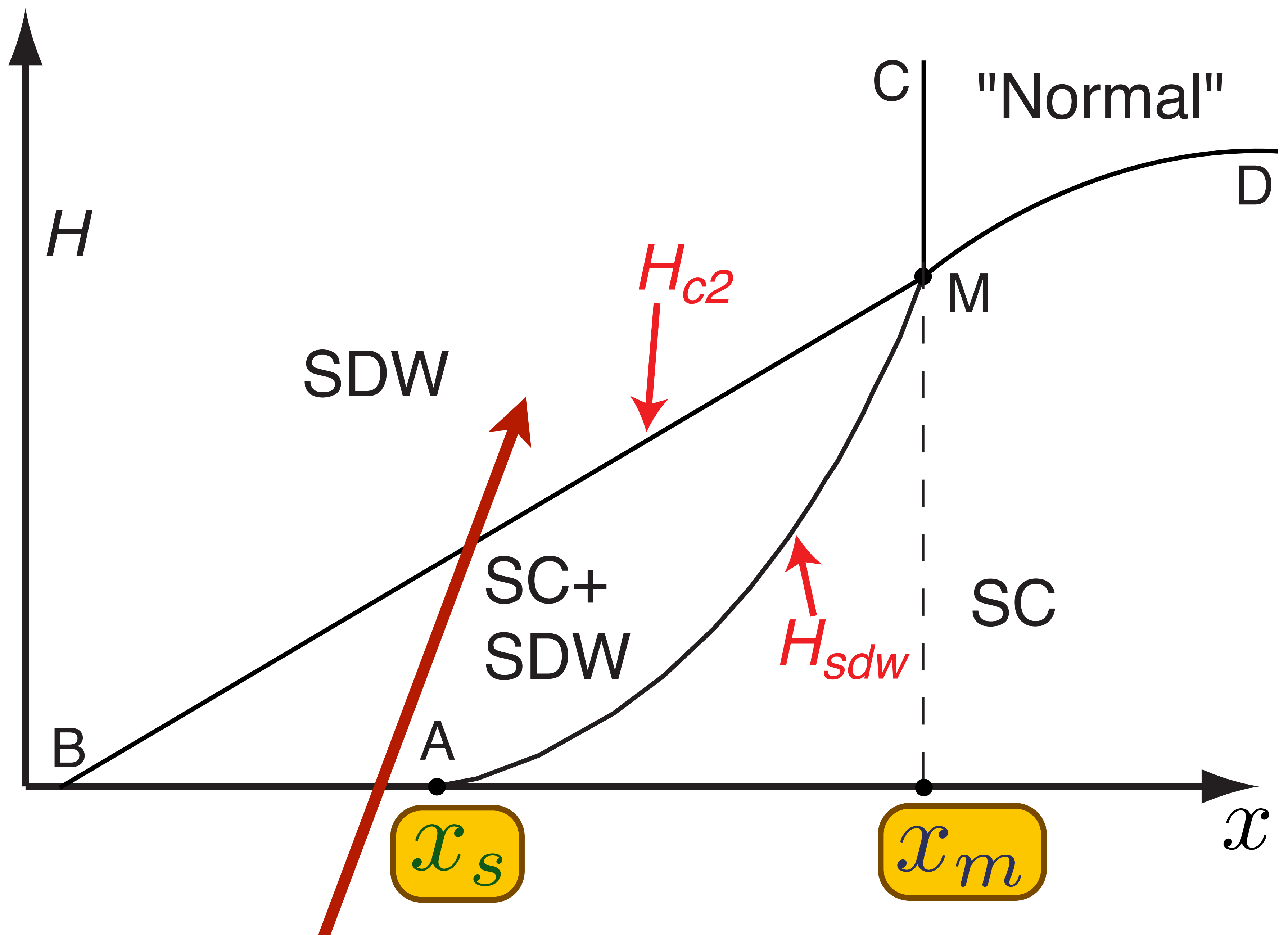
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *arXiv:0902.3335*.

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

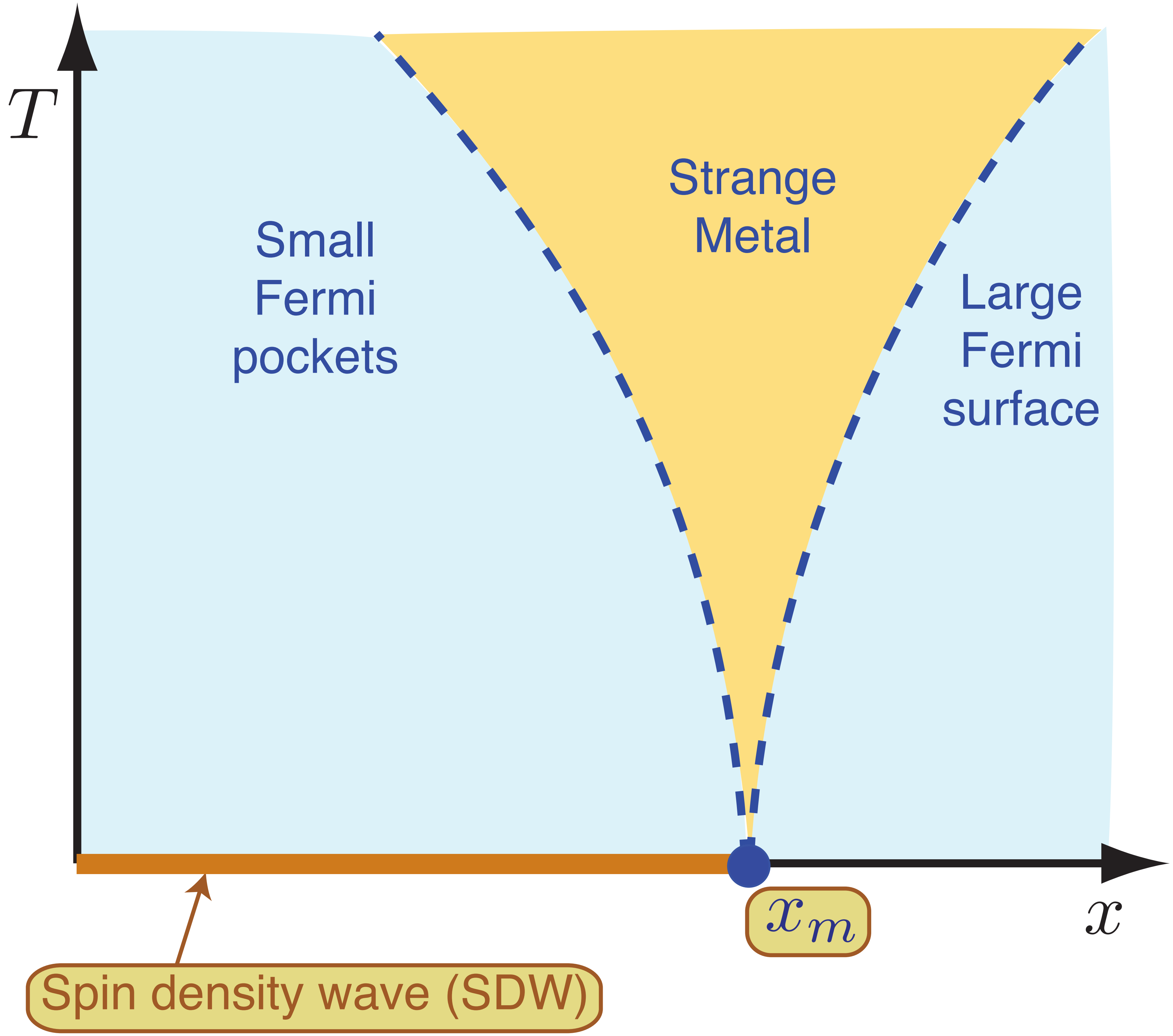


Quantum oscillations without Zeeman splitting

N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

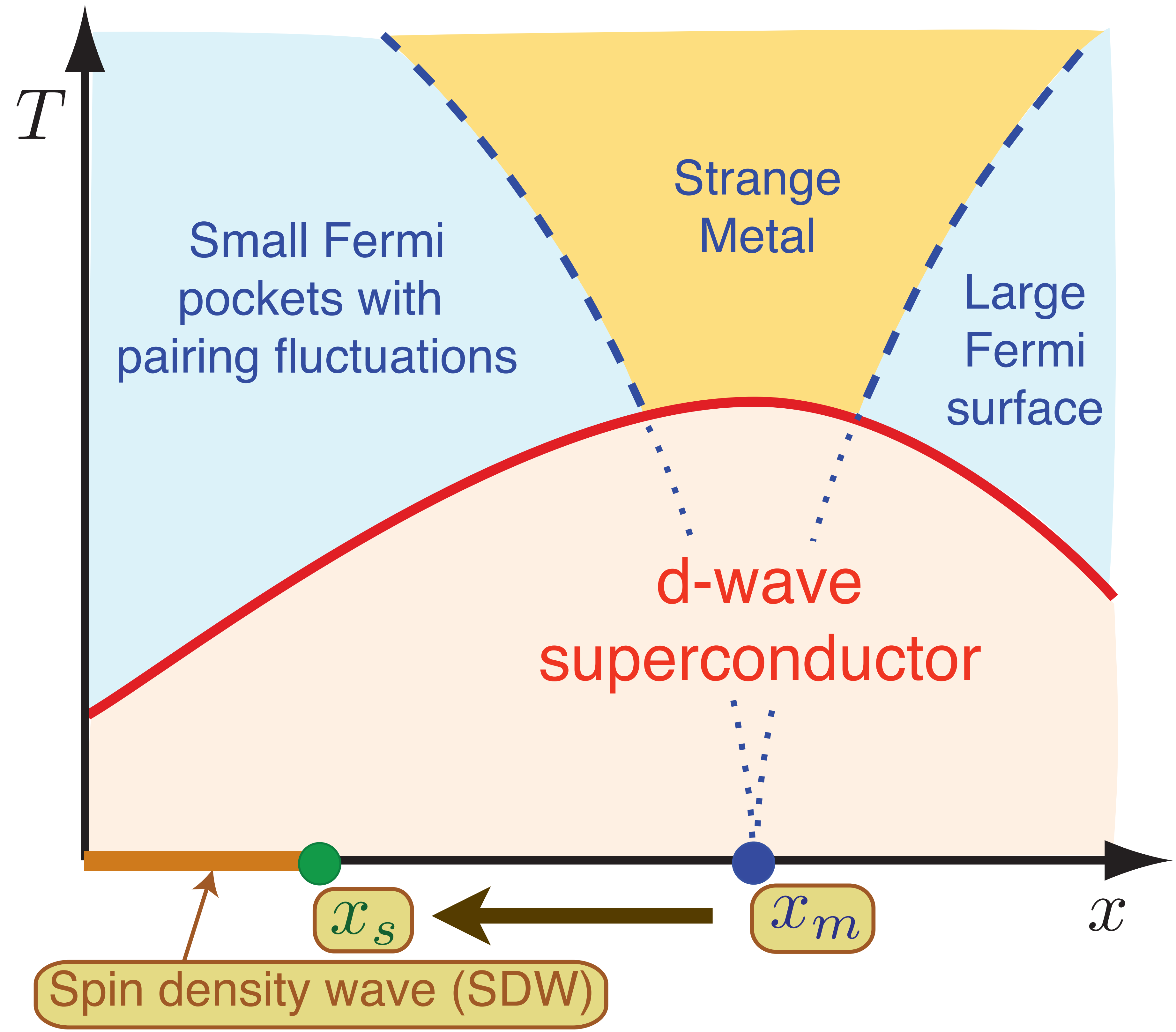
Electronic theory of superconductivity and its competition with spin-density wave order

Theory of quantum criticality in the cuprates



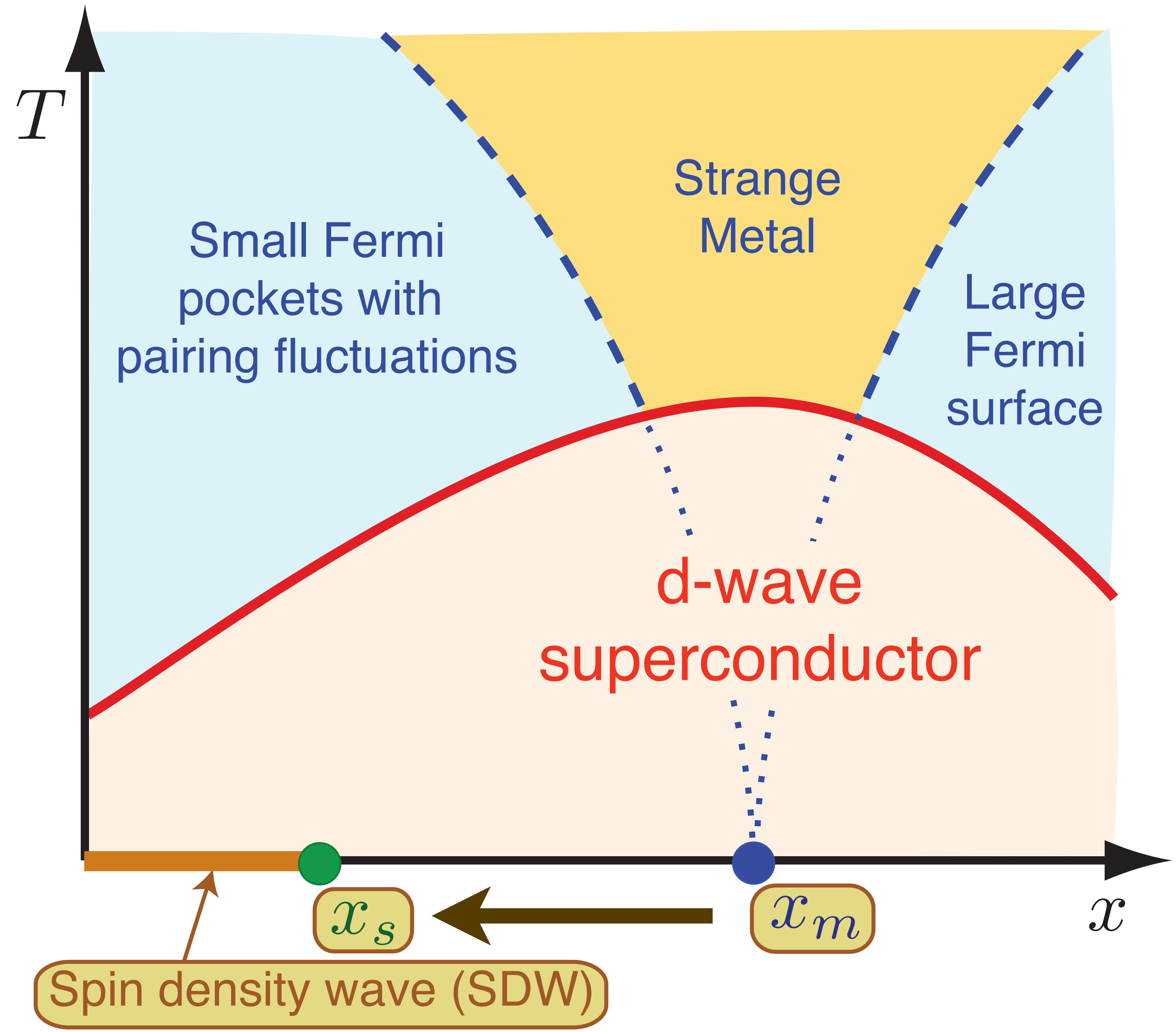
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Theory of quantum criticality in the cuprates



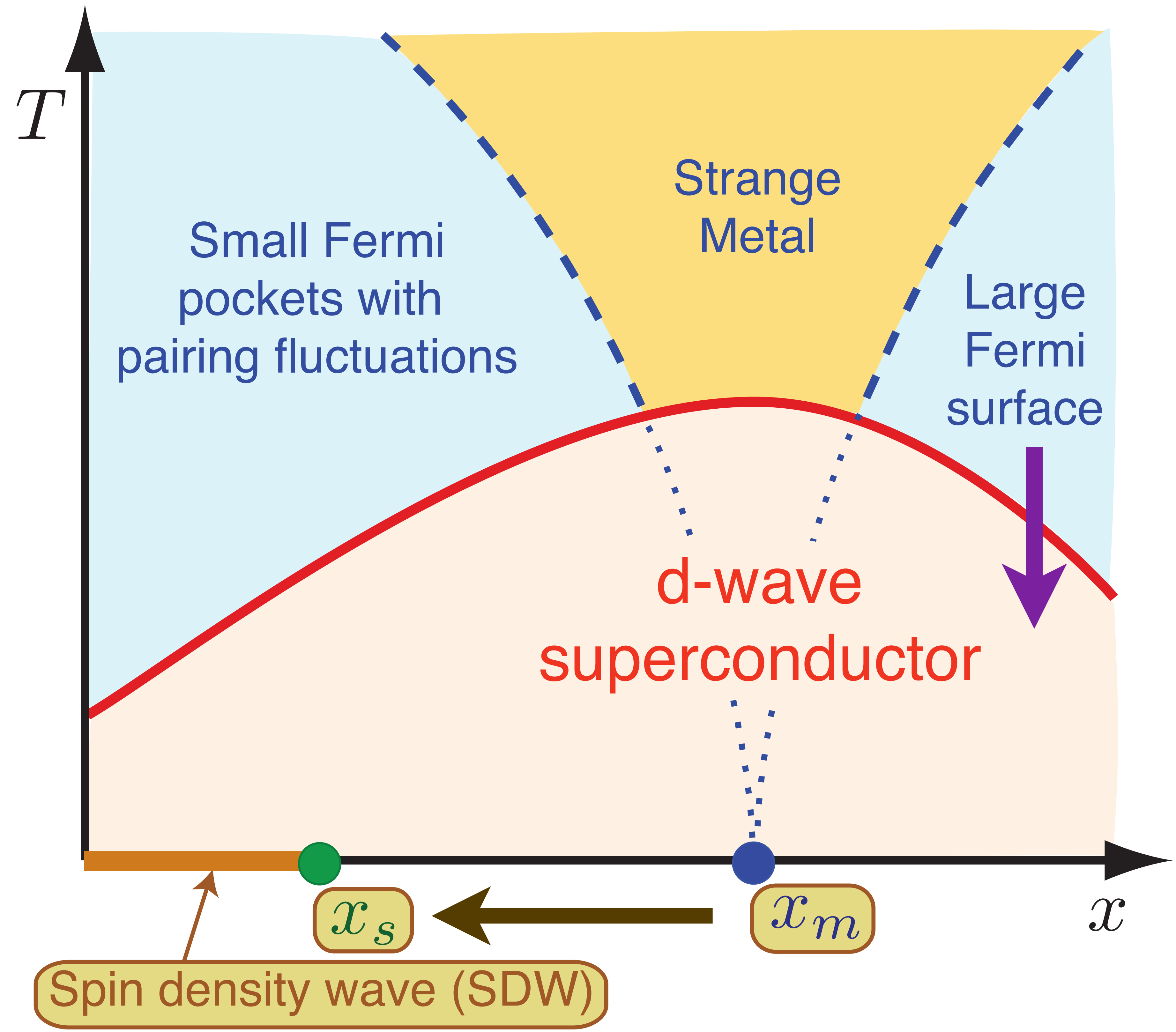
Onset of d-wave superconductivity hides the critical point at $x = x_m$ and moves it to $x = x_s < x_m$

Theory of quantum criticality in the cuprates



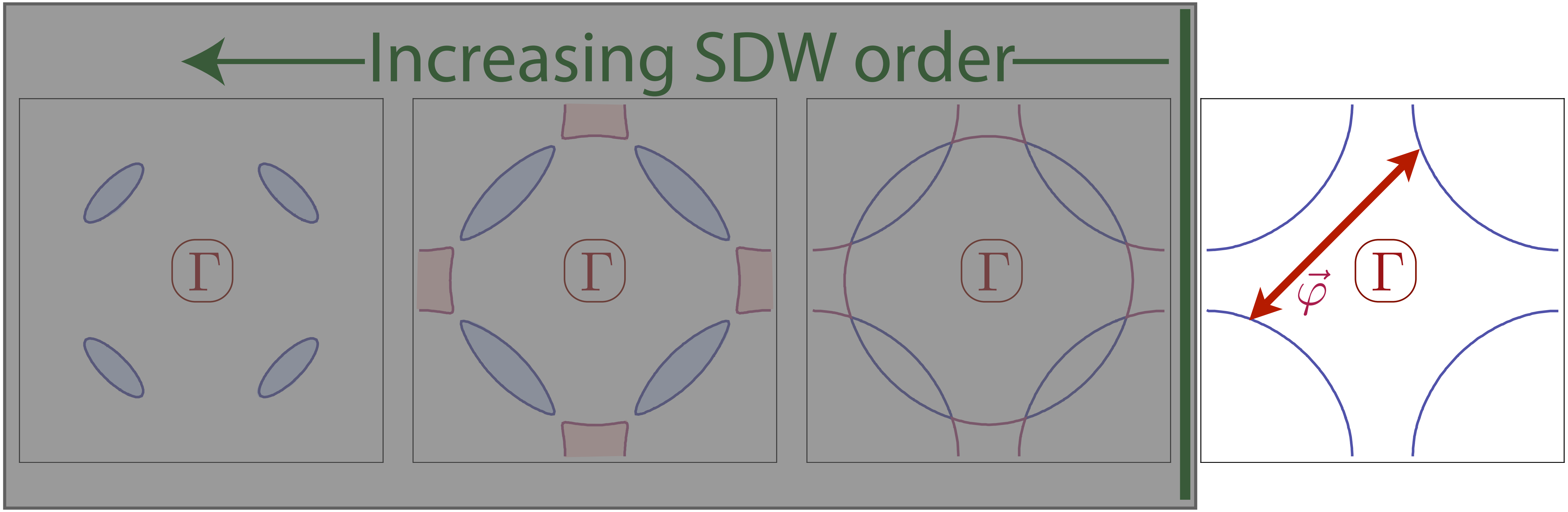
Theory of the onset of d-wave superconductivity from a large Fermi surface

Theory of quantum criticality in the cuprates



Theory of the onset of d-wave superconductivity from a large Fermi surface

Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

David Pines, Douglas Scalapino

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = - \sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^\dagger_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta} .$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

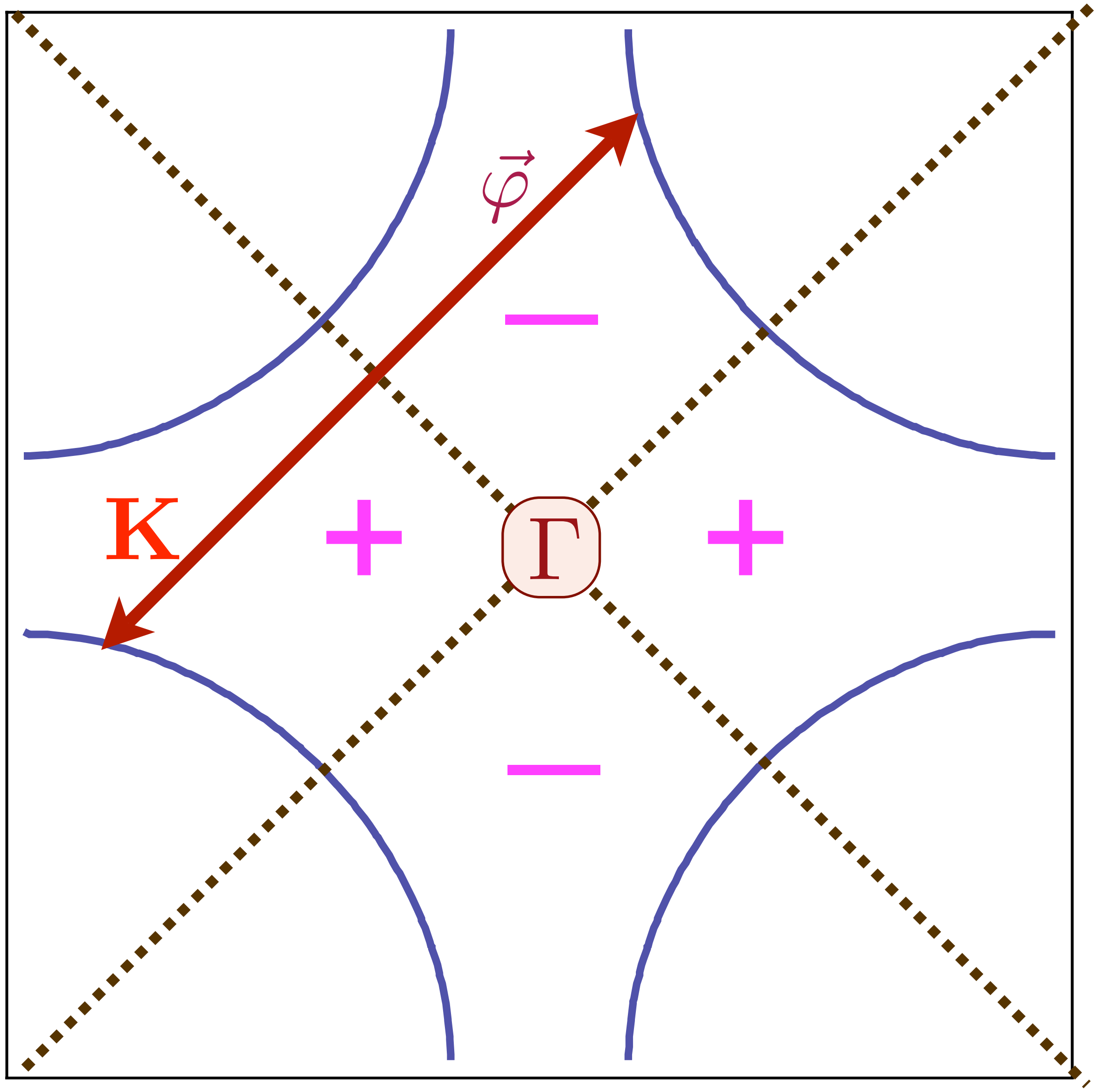
$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^\dagger_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^\dagger_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q}-\mathbf{K})^2},$$

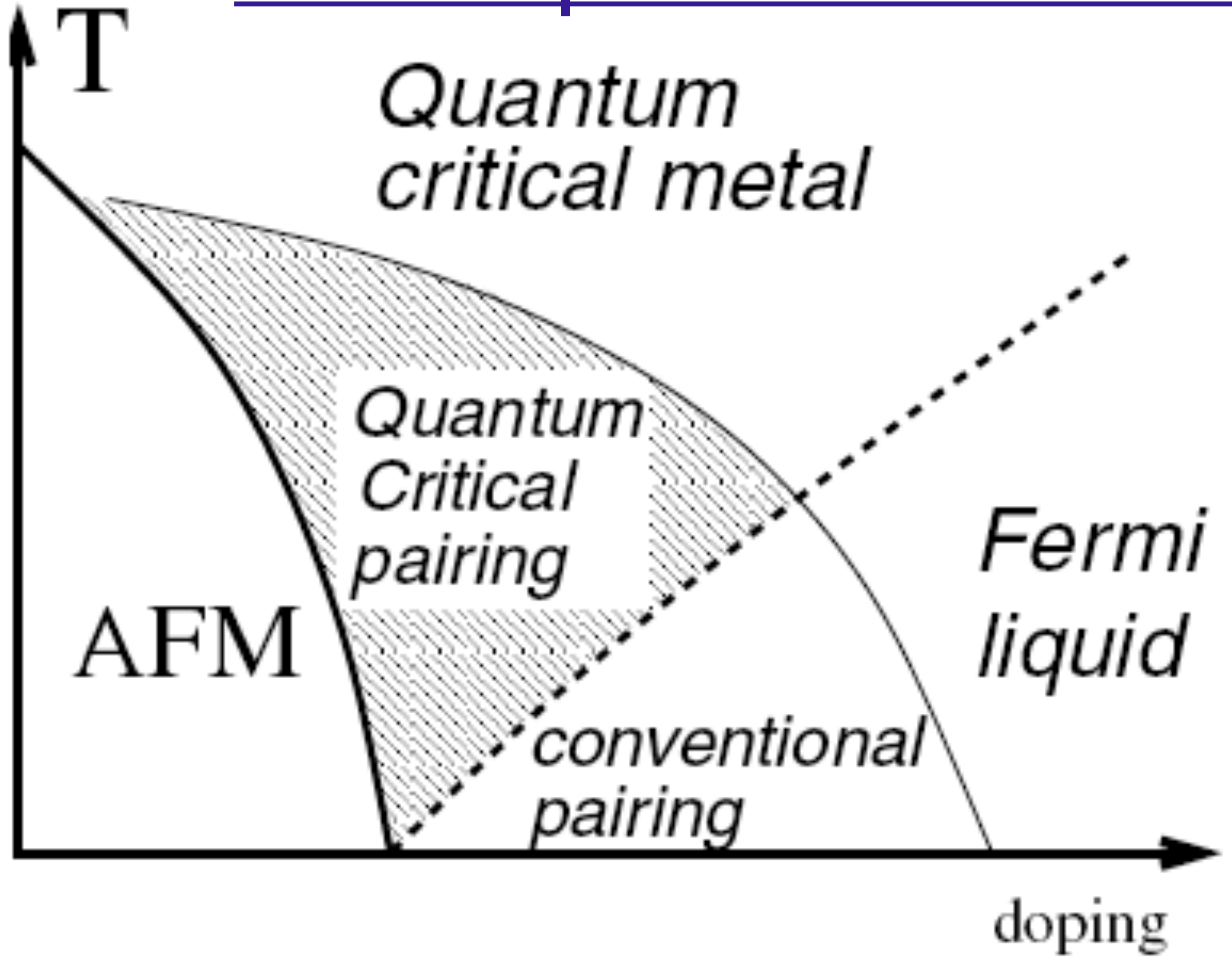
with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

d-wave pairing of the large Fermi surface



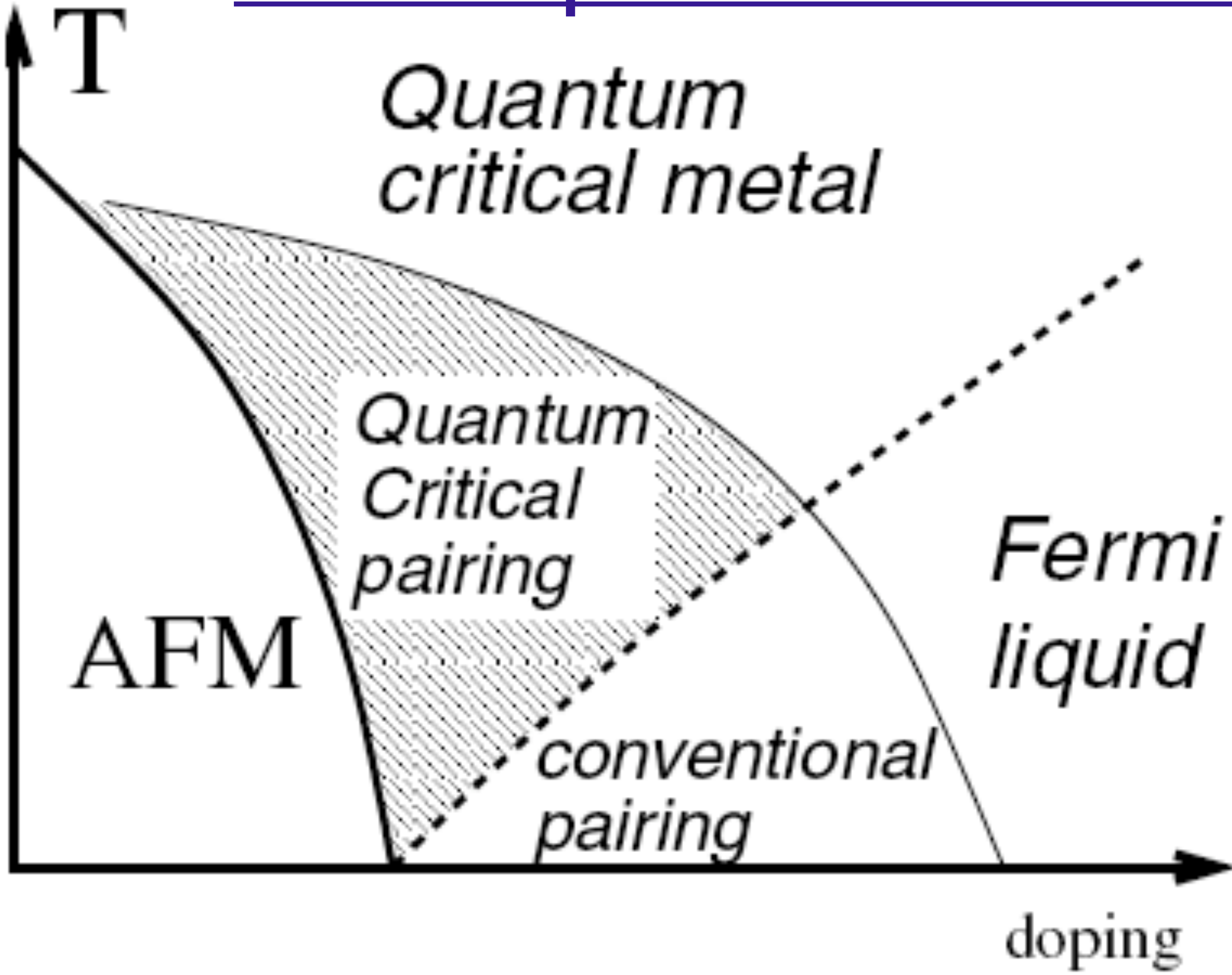
$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Ar. Abanov, A. V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Approaching the onset of antiferromagnetism in the spin-fluctuation theory

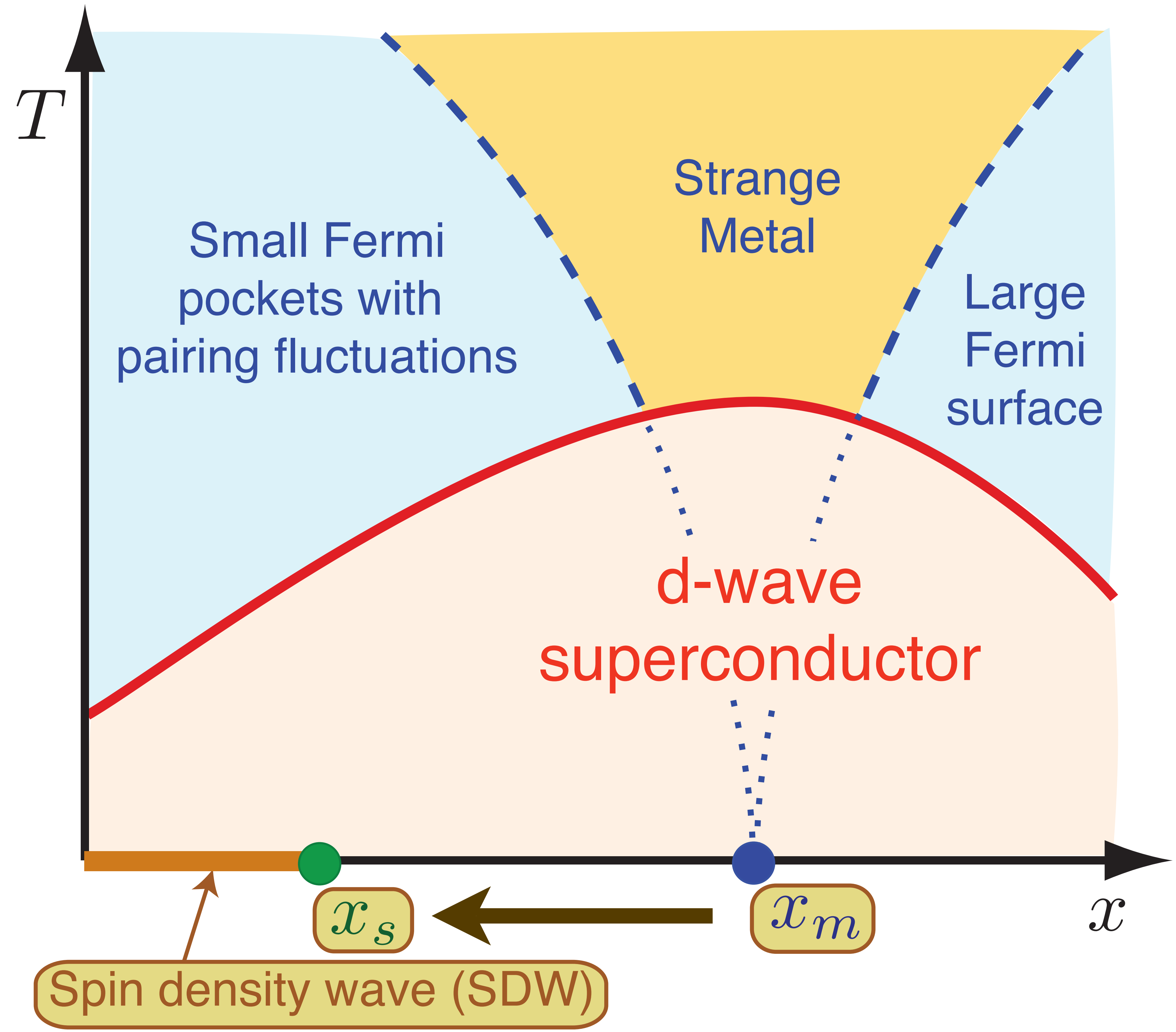


- T_c *increases* upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.

- No simple mechanism for nodal-anti-nodal dichotomy.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Theory of quantum criticality in the cuprates



Onset of d-wave superconductivity hides the critical point at $x = x_m$ and moves it to $x = x_s < x_m$

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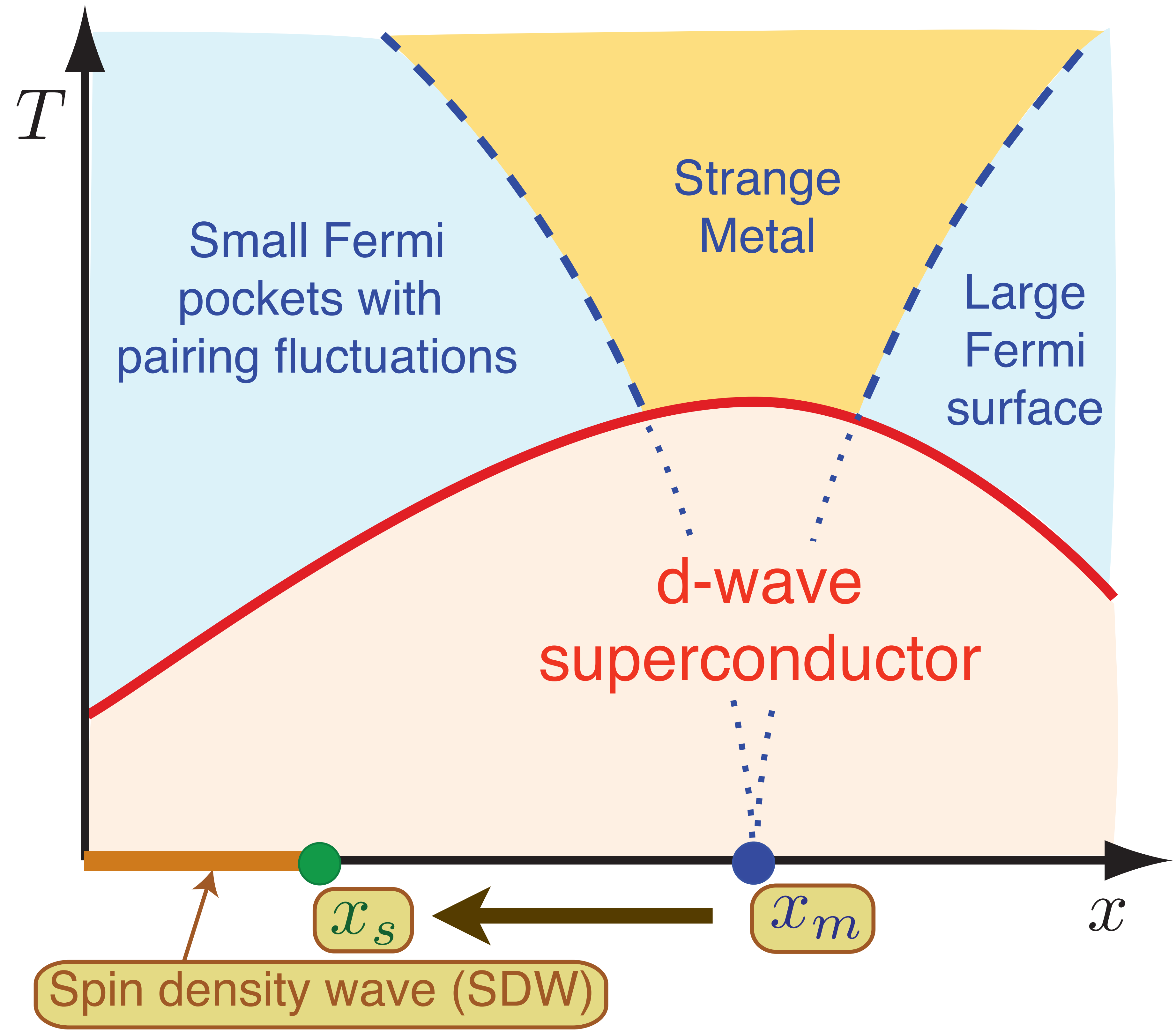
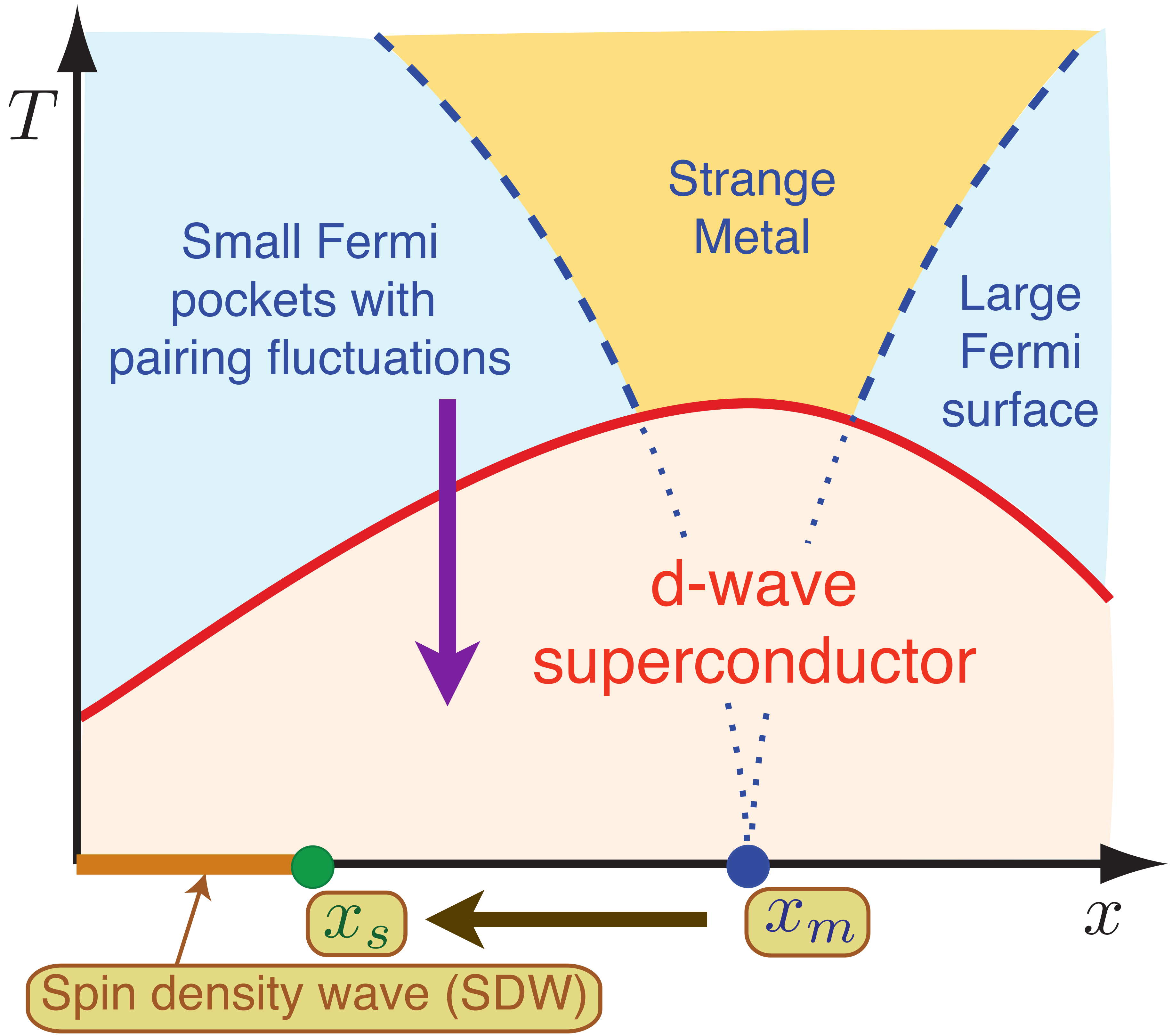


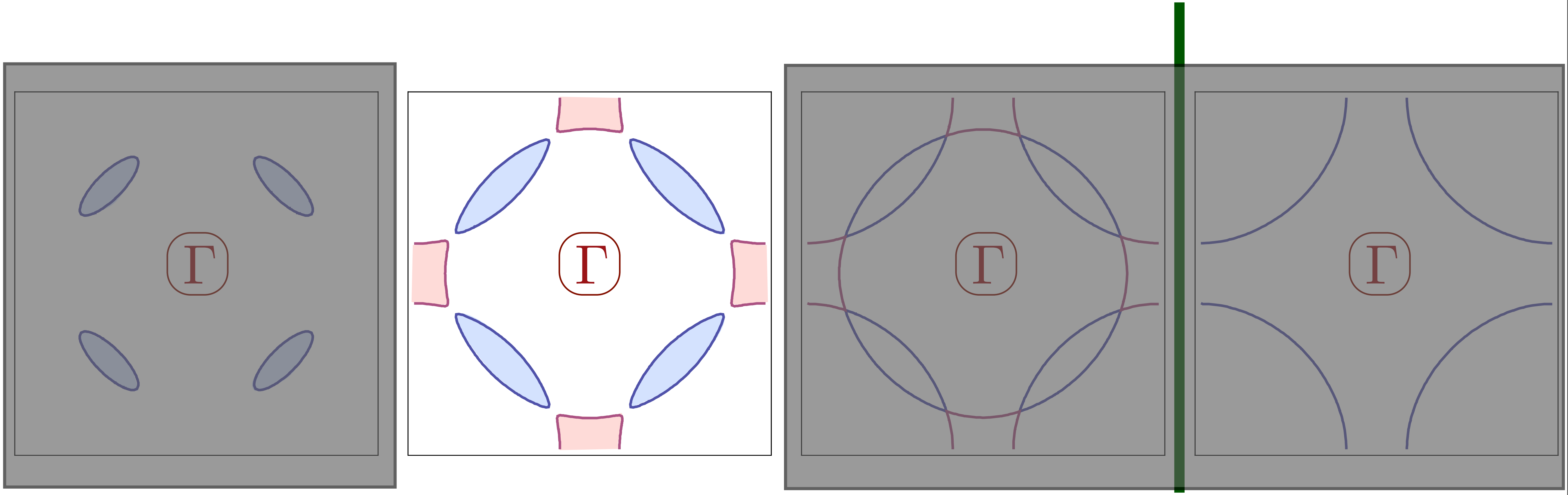
Theory of the onset of d-wave superconductivity from small Fermi pockets

Theory of quantum criticality in the cuprates



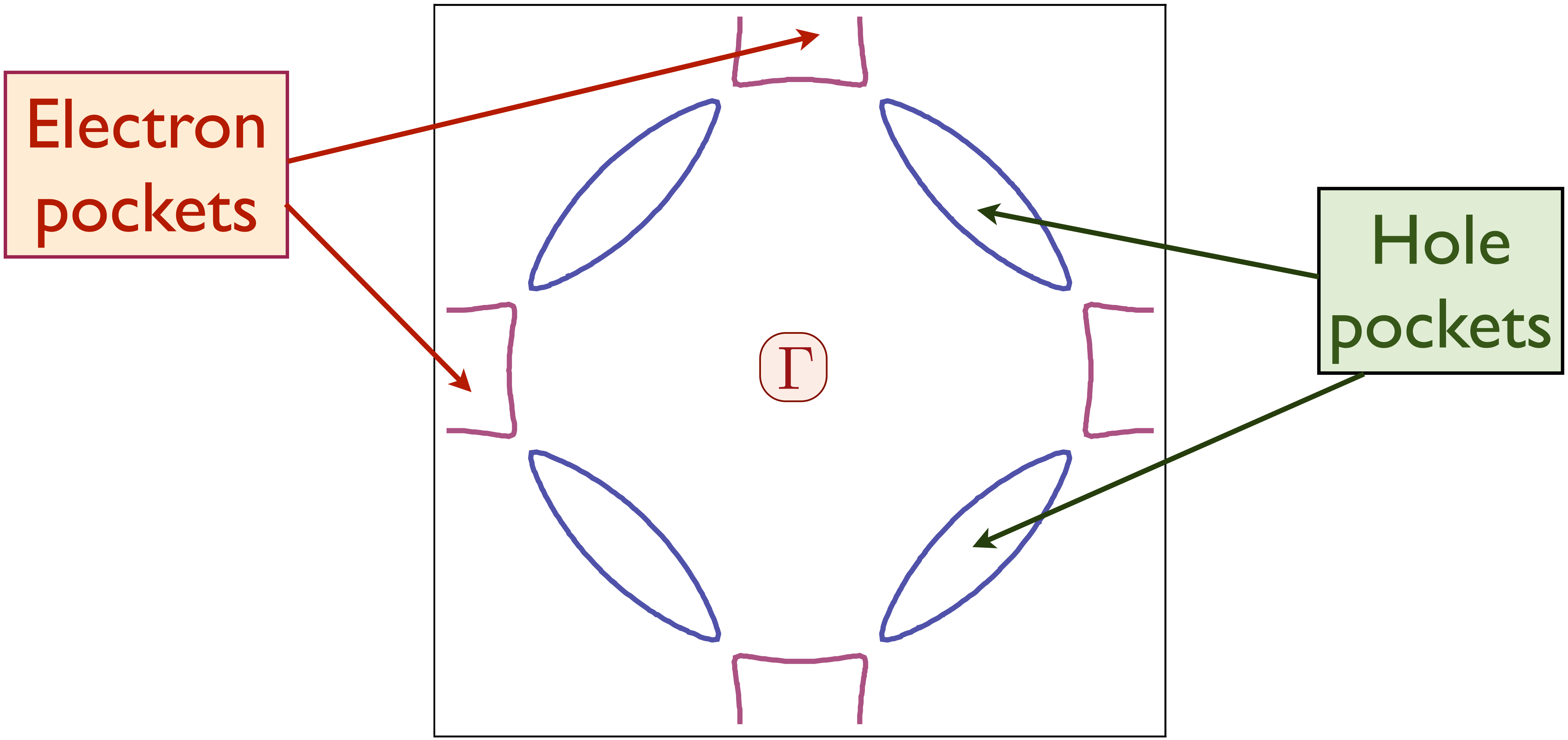
Theory of the onset of d-wave superconductivity from small Fermi pockets

Fermi pockets in hole-doped cuprates

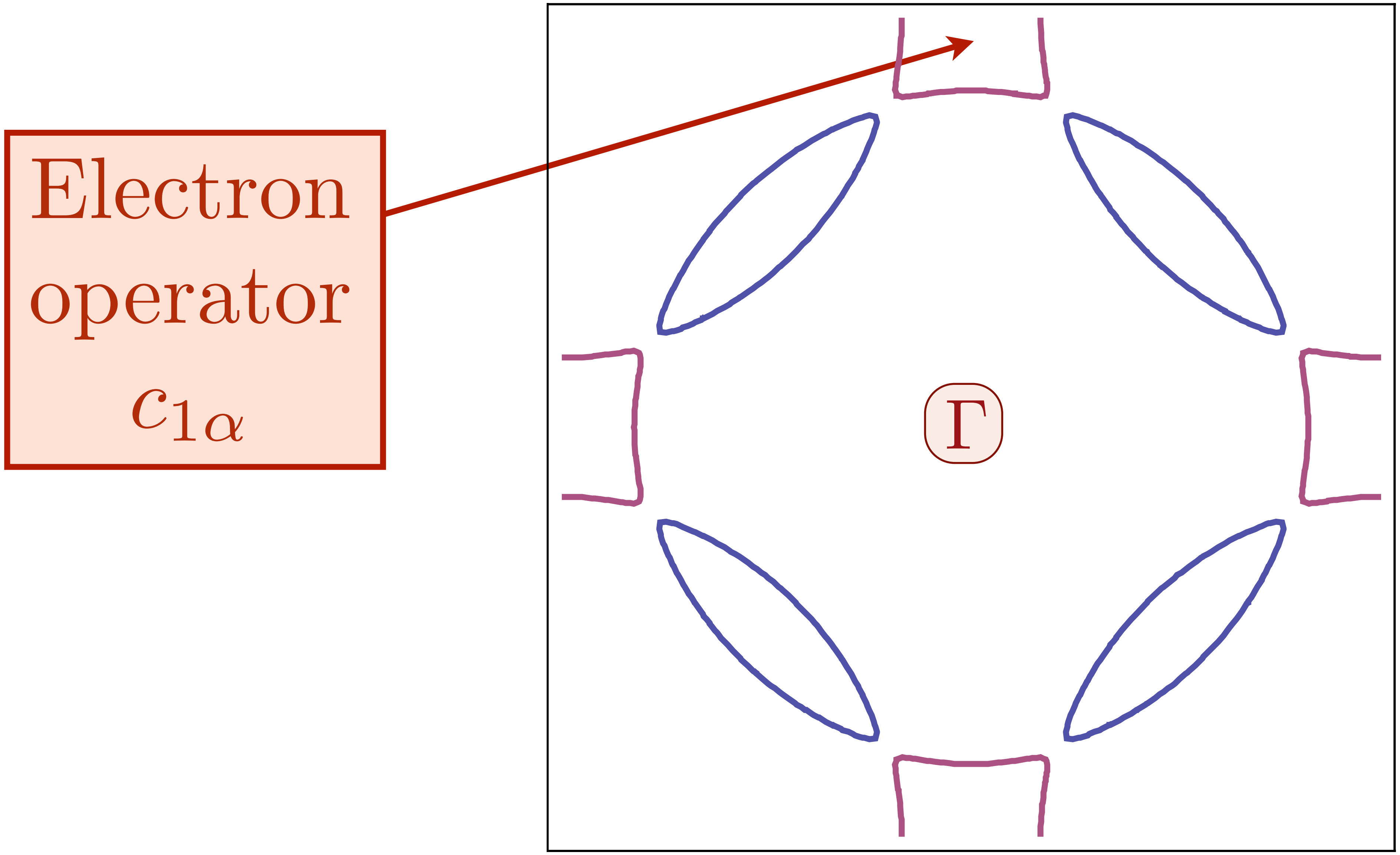


Begin with SDW ordered state, and focus on fluctuations in the *orientation* of $\vec{\varphi}$, by using a unit-length bosonic spinor z_α

$$\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$



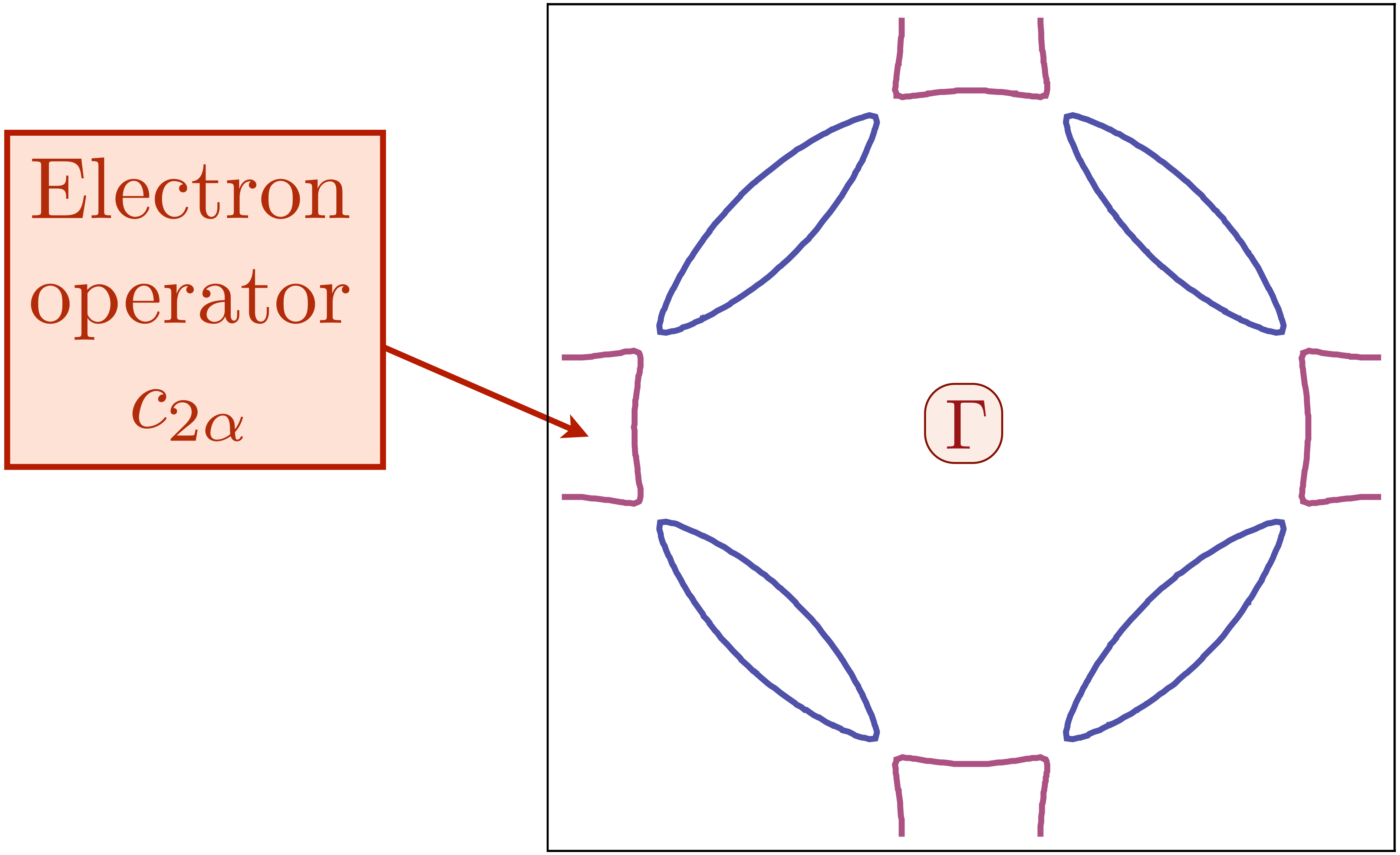
Charge carriers in the lightly-doped cuprates with Neel order



For a spacetime dependent SDW order, $\vec{\varphi} = z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z^*_\downarrow \\ z_\downarrow & z^*_\uparrow \end{pmatrix}.$$

So $g_\pm$ are the “up/down” electron operators
in a rotating reference frame defined by the local SDW order



For a spacetime dependent SDW order, $\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix} \quad ; \quad \mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

Same SU(2) matrix also rotates electrons in second pocket.

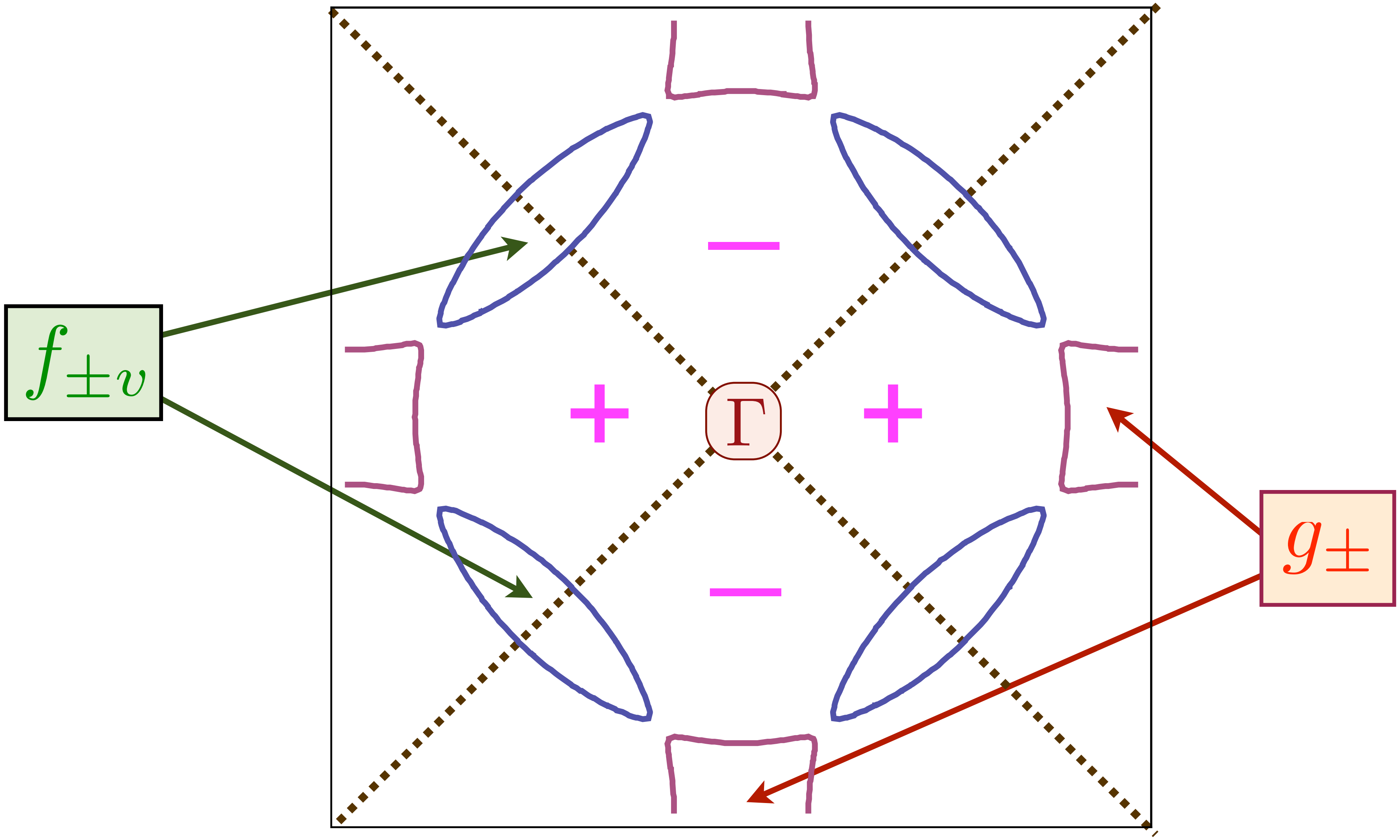
Low energy theory for spinless, charge $-e$ fermions $g_{\pm}$, and spinful, charge 0 bosons z_{α}:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_z + \mathcal{L}_g \\ \mathcal{L}_z &= \frac{1}{t}\Big[|(\partial_\tau - iA_\tau) z_\alpha|^2 + v^2 |\nabla - i\mathbf{A}) z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1)\Big] \\ &+ \text{ Berry phases of monopoles in } A_\mu.\end{aligned}$$

CP^1 field theory for z_α and an emergent U(1) gauge field A_μ. Coupling t tunes the strength of SDW orientation fluctuations.

$$\begin{aligned}\mathcal{L}_g &= g_+^\dagger \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu \right] g_+ \\ &\quad + g_-^\dagger \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu \right] g_-\end{aligned}$$

Two Fermi surfaces coupled to the emergent U(1) gauge field A_μ with opposite charges

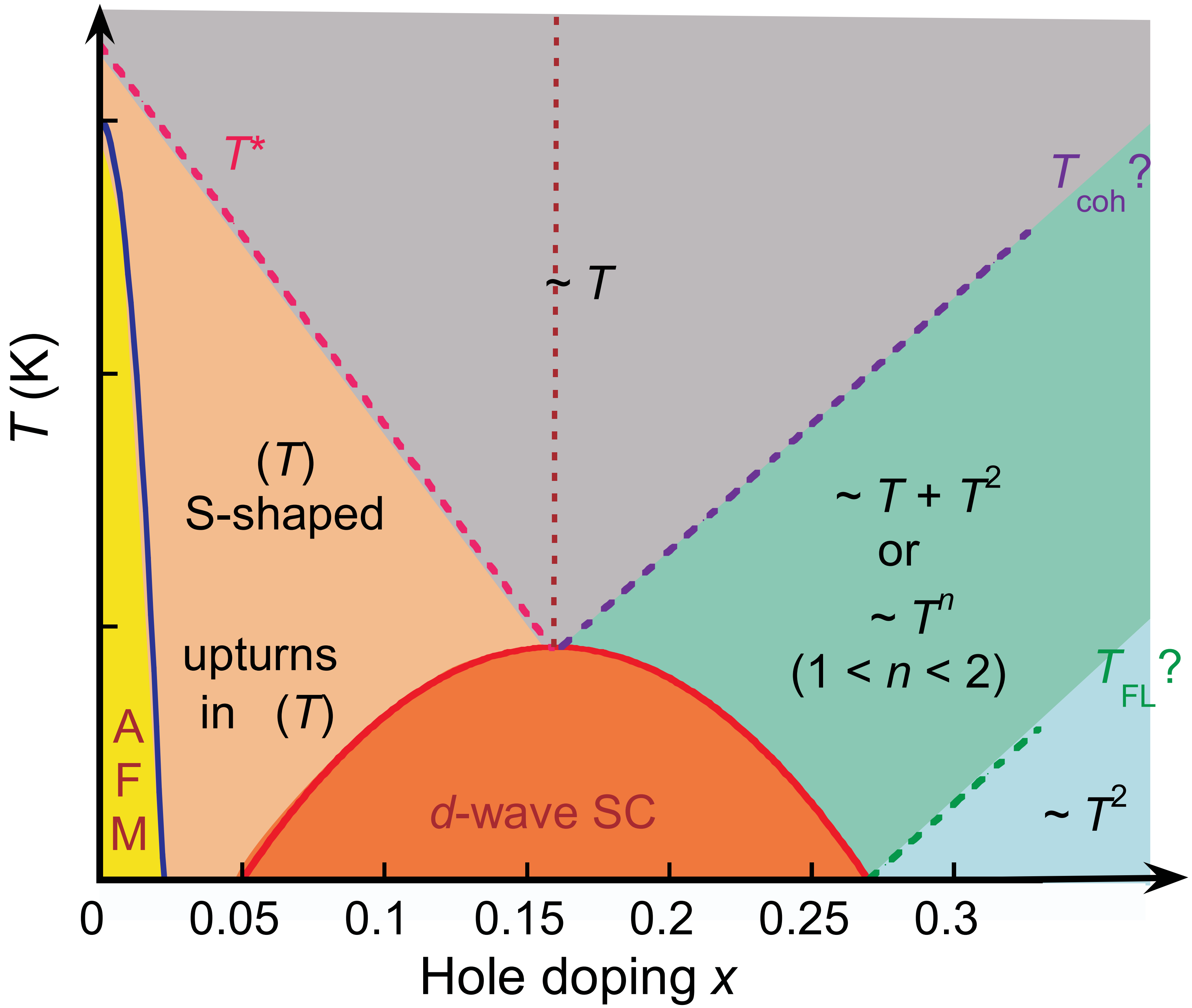


d-wave pairing of the electrons is associated with

- Strong *s*-wave pairing of $g_{\pm}$
- Weak *p*-wave pairing of $f_{\pm v}$.

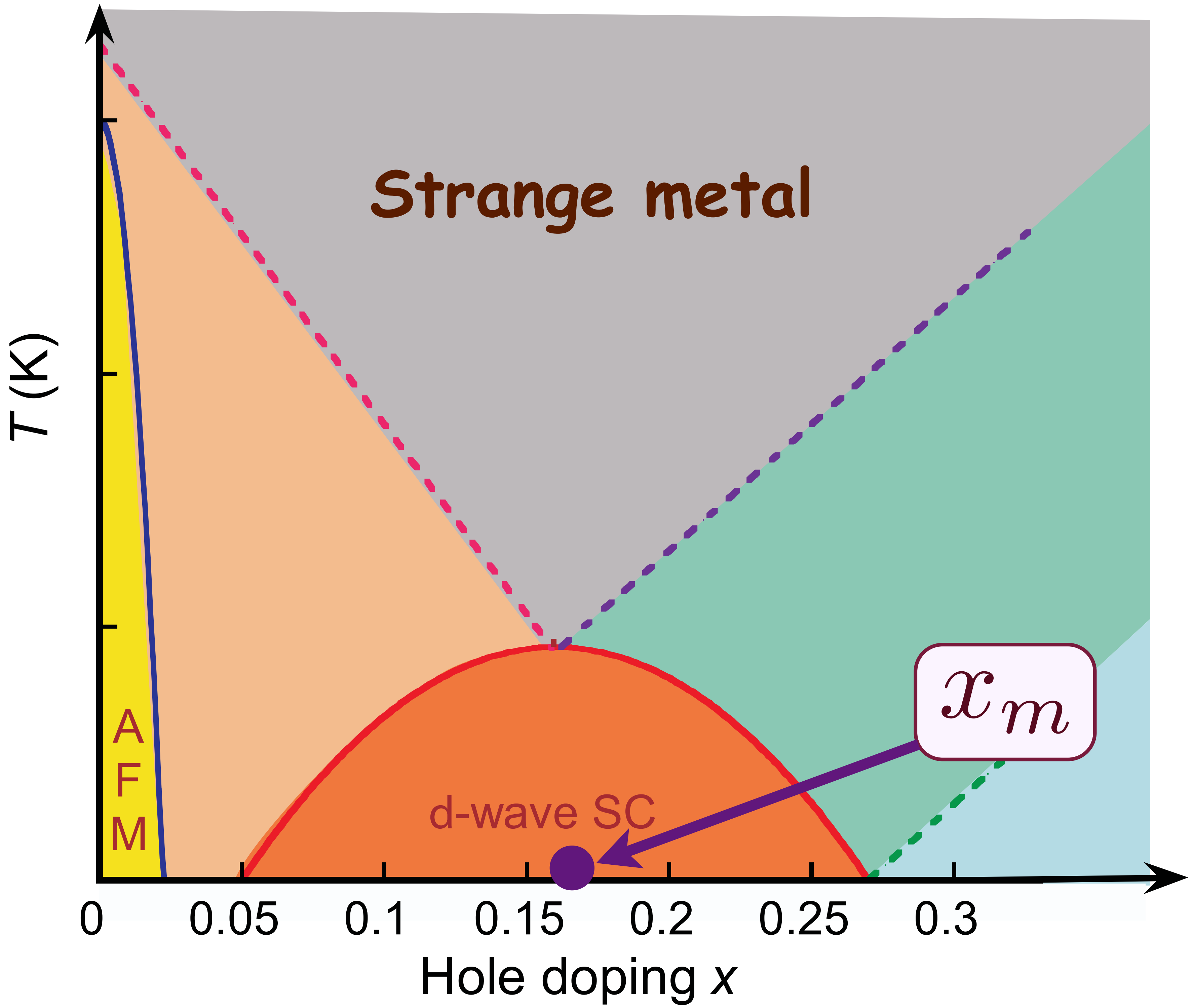
Conclusions

Crossovers in transport properties of hole-doped cuprates



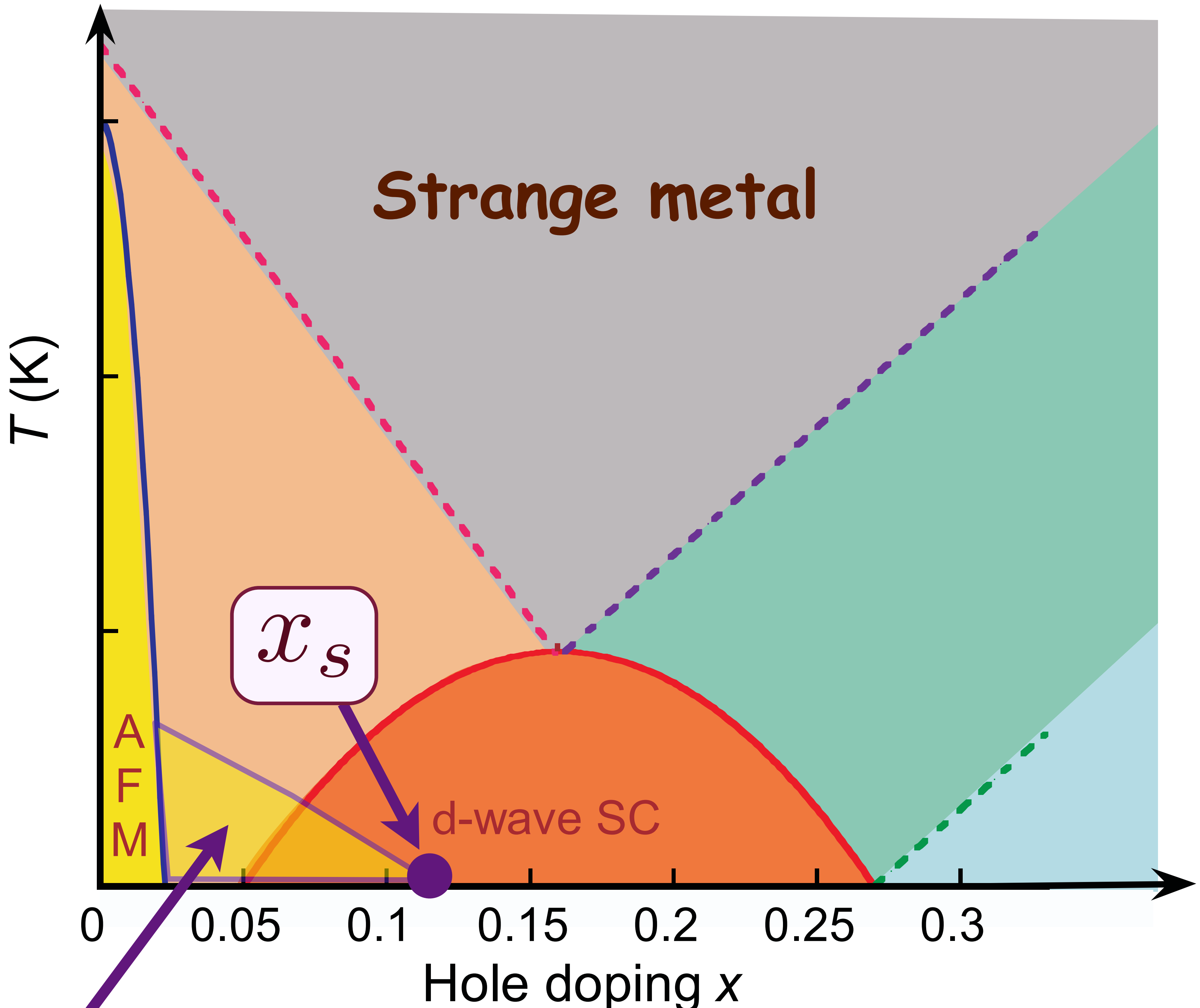
N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



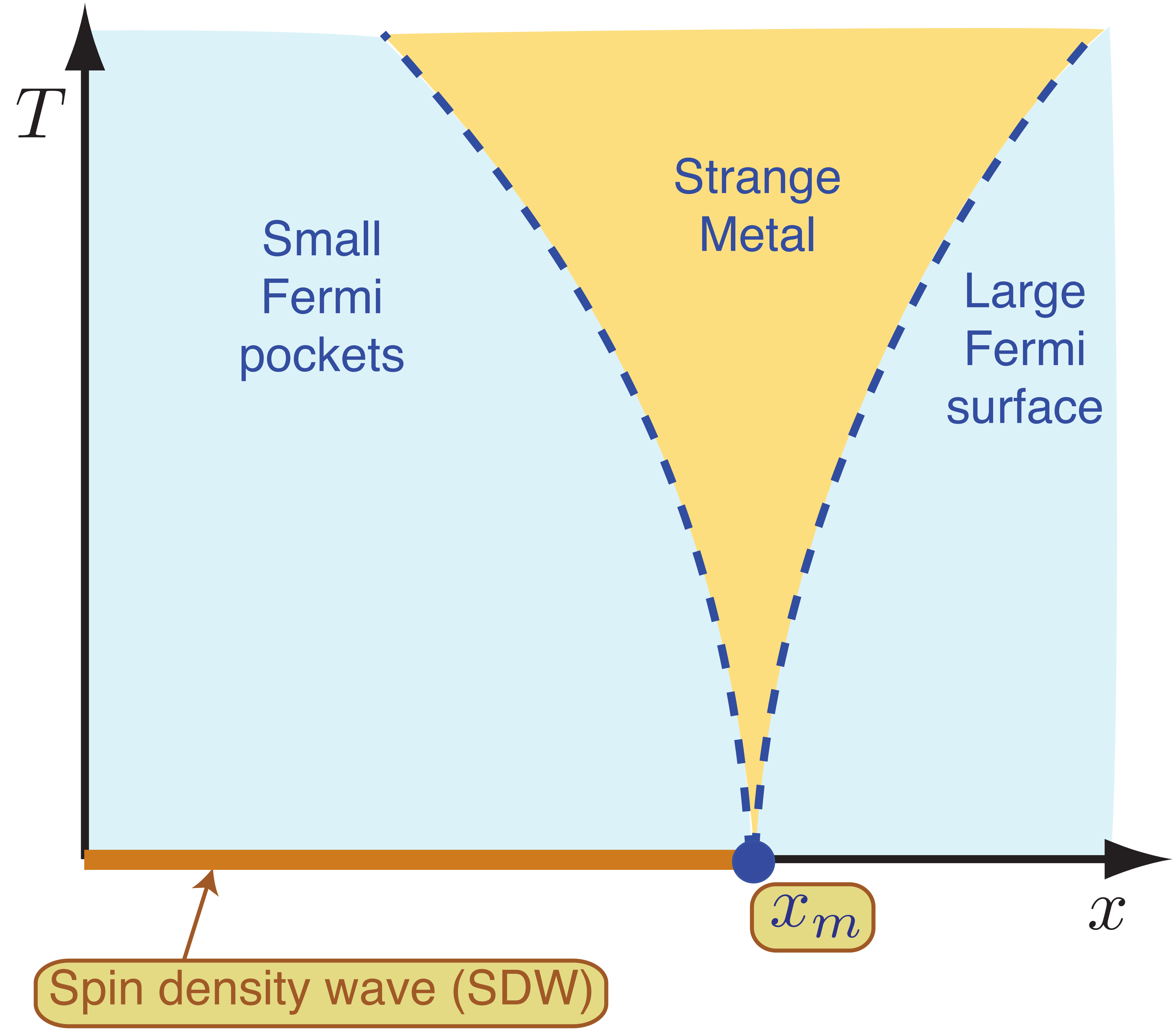
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



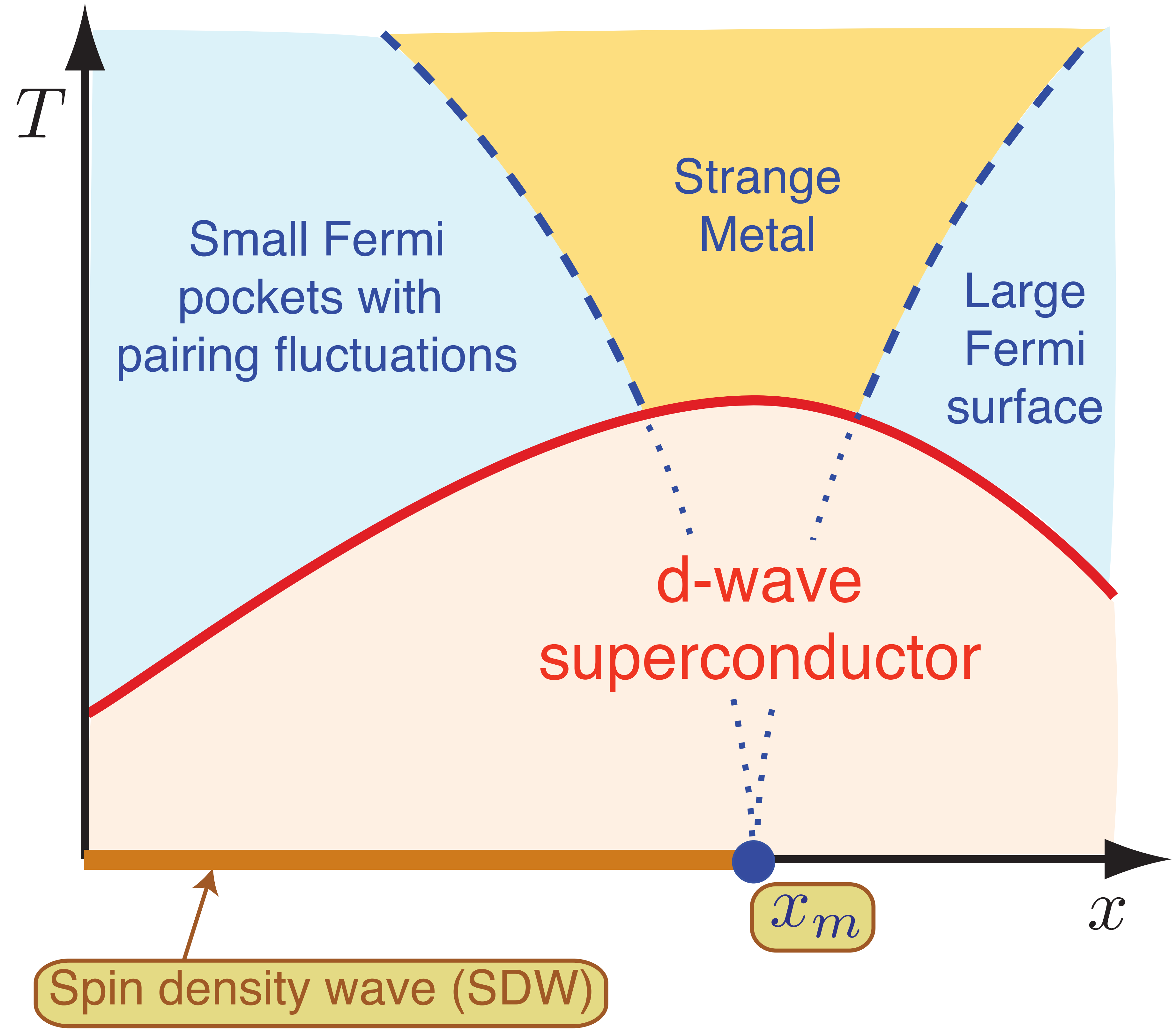
Spin and charge density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Theory of quantum criticality in the cuprates



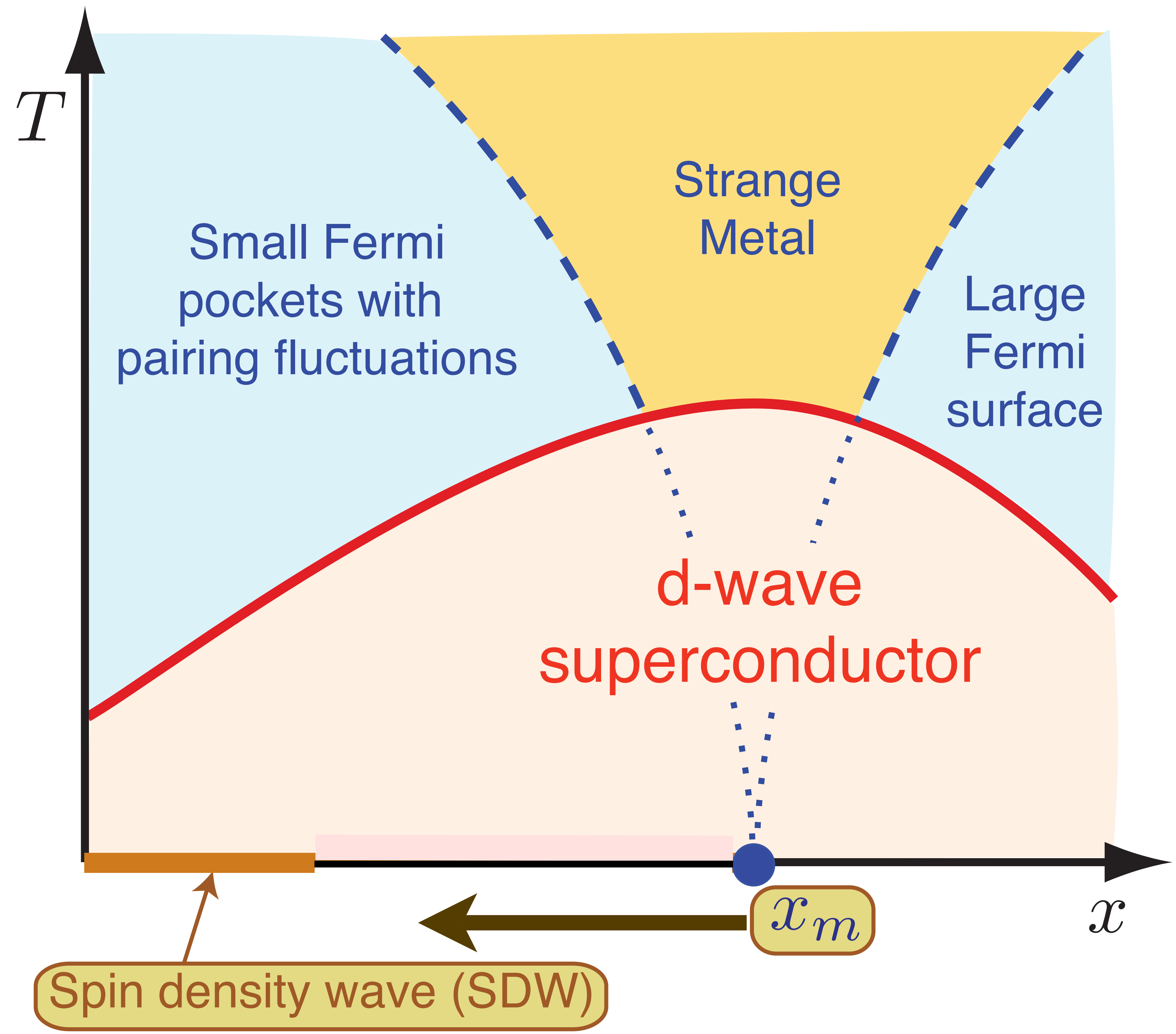
Underlying SDW ordering quantum critical point in metal at $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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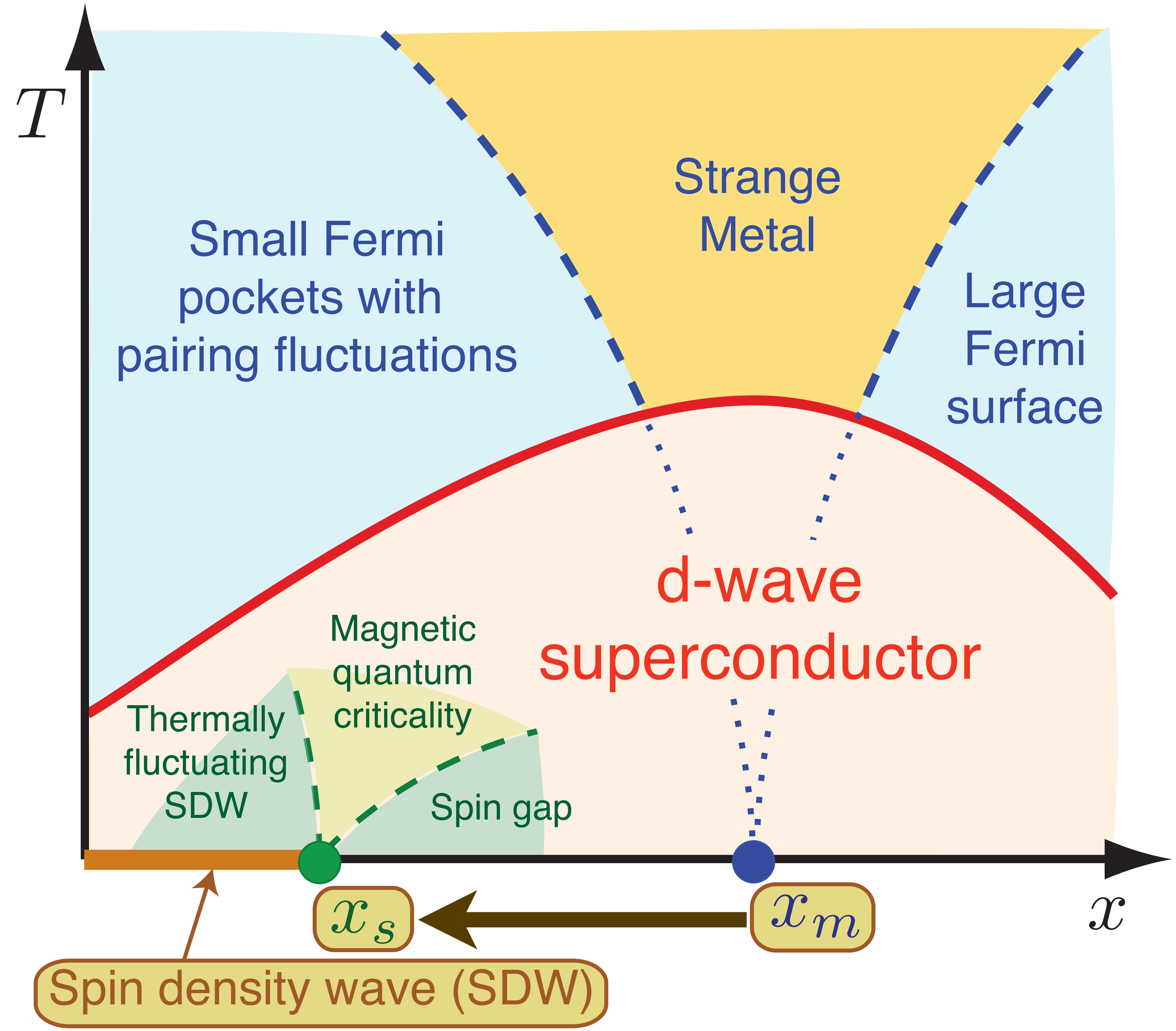

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Conclusions

- ★ Gauge theory for pairing in the underdoped cuprates, describing "angular" fluctuations of spin-density-wave order
- ★ Natural route to *d*-wave pairing with strong pairing at the antinodes and weak pairing at the nodes
- ★ Explains characteristic "competing order" features of field-doping phase diagram: SDW order is more stable in the metal than in the superconductor.