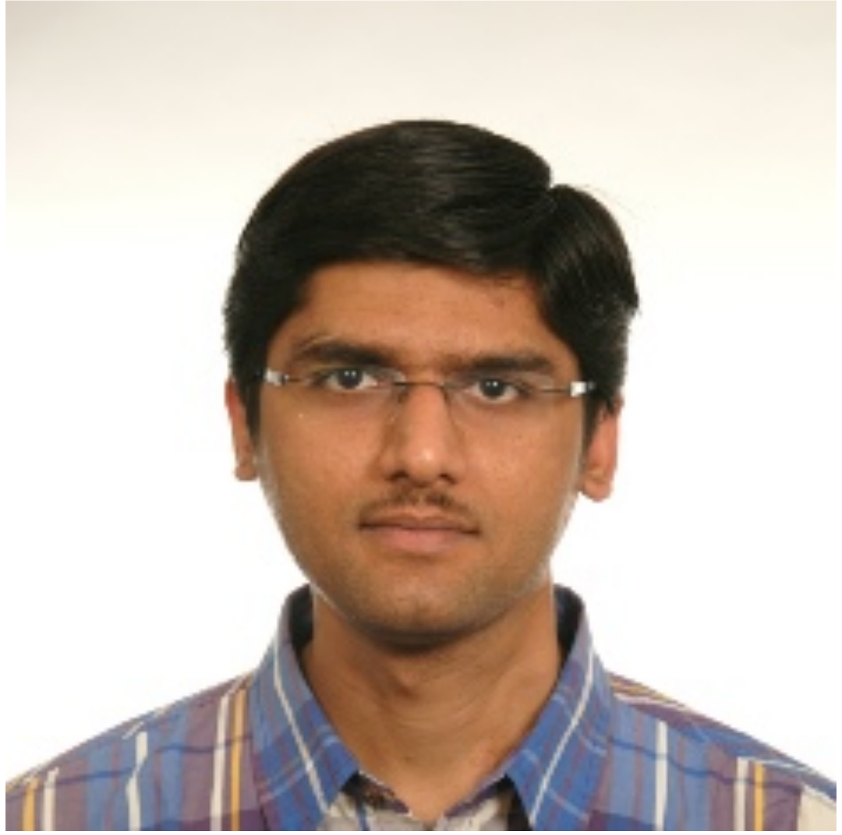
Deconfined quantum criticality in the cuprate high temperature superconductors

Indian Institute of Technology, Bombay
January 11, 2020

Subir Sachdev

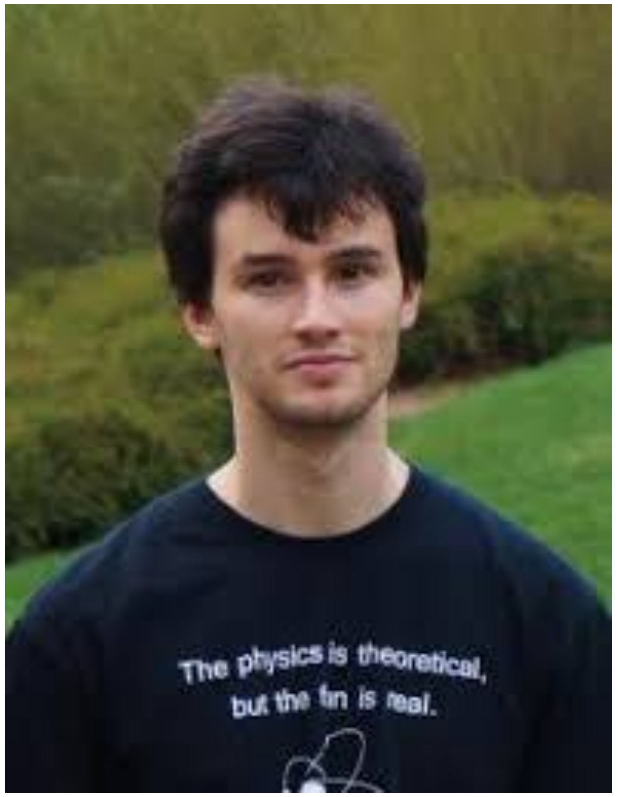
Talk online: sachdev.physics.harvard.edu



Darshan Joshi

Chenyuan Li

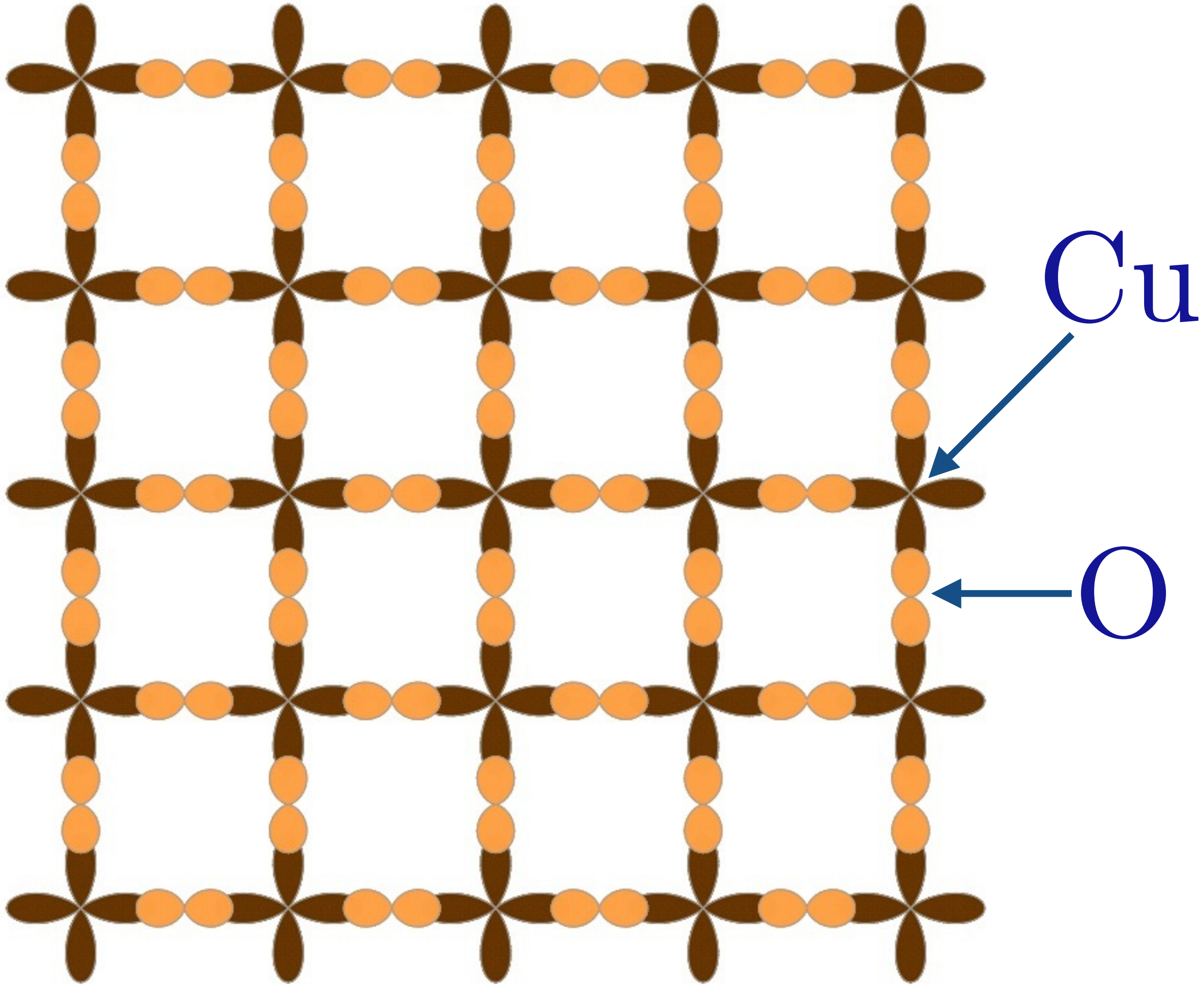
arXiv:1912.08822



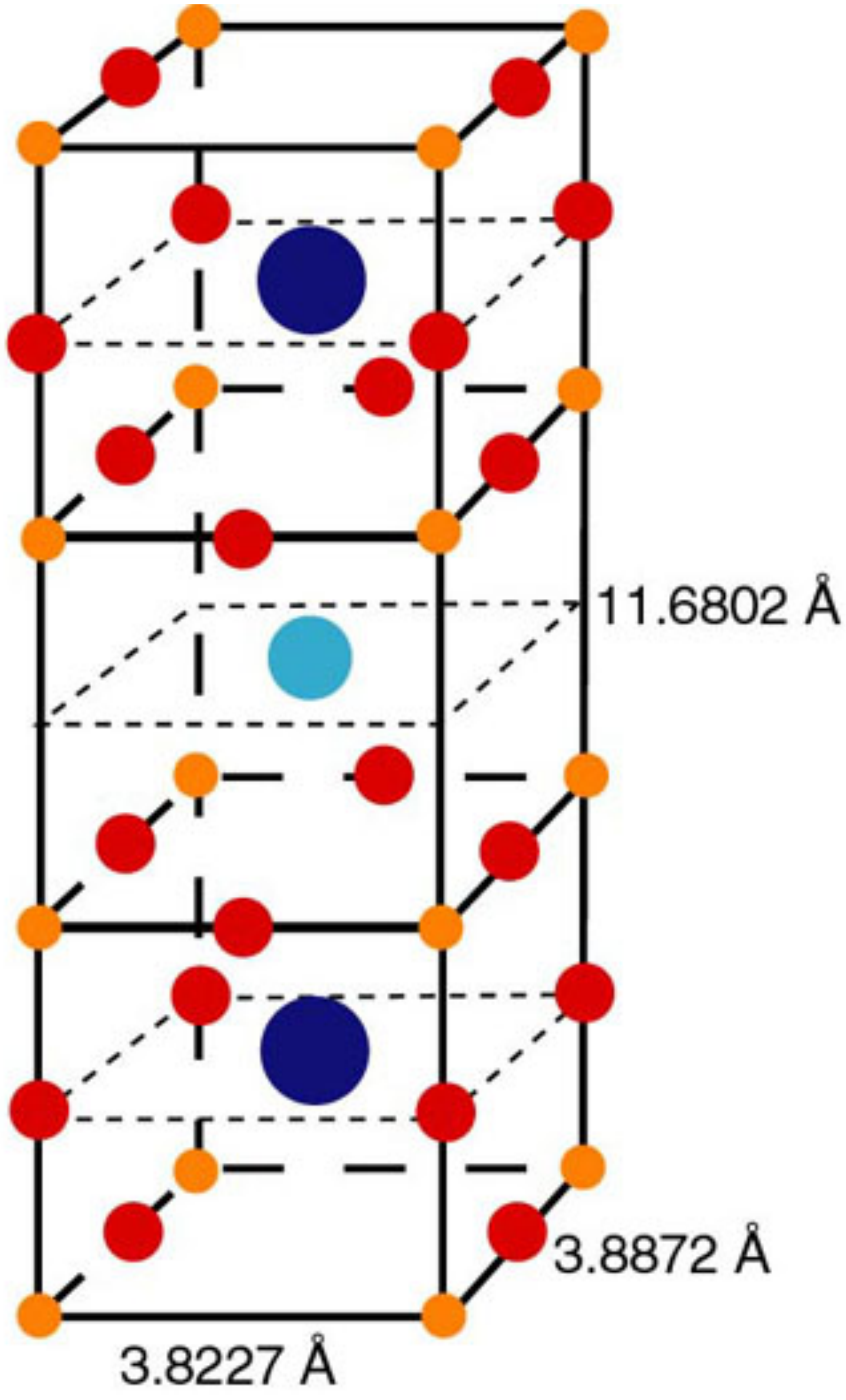
Grigory Tarnopolsky

Antoine Georges

High temperature superconductors

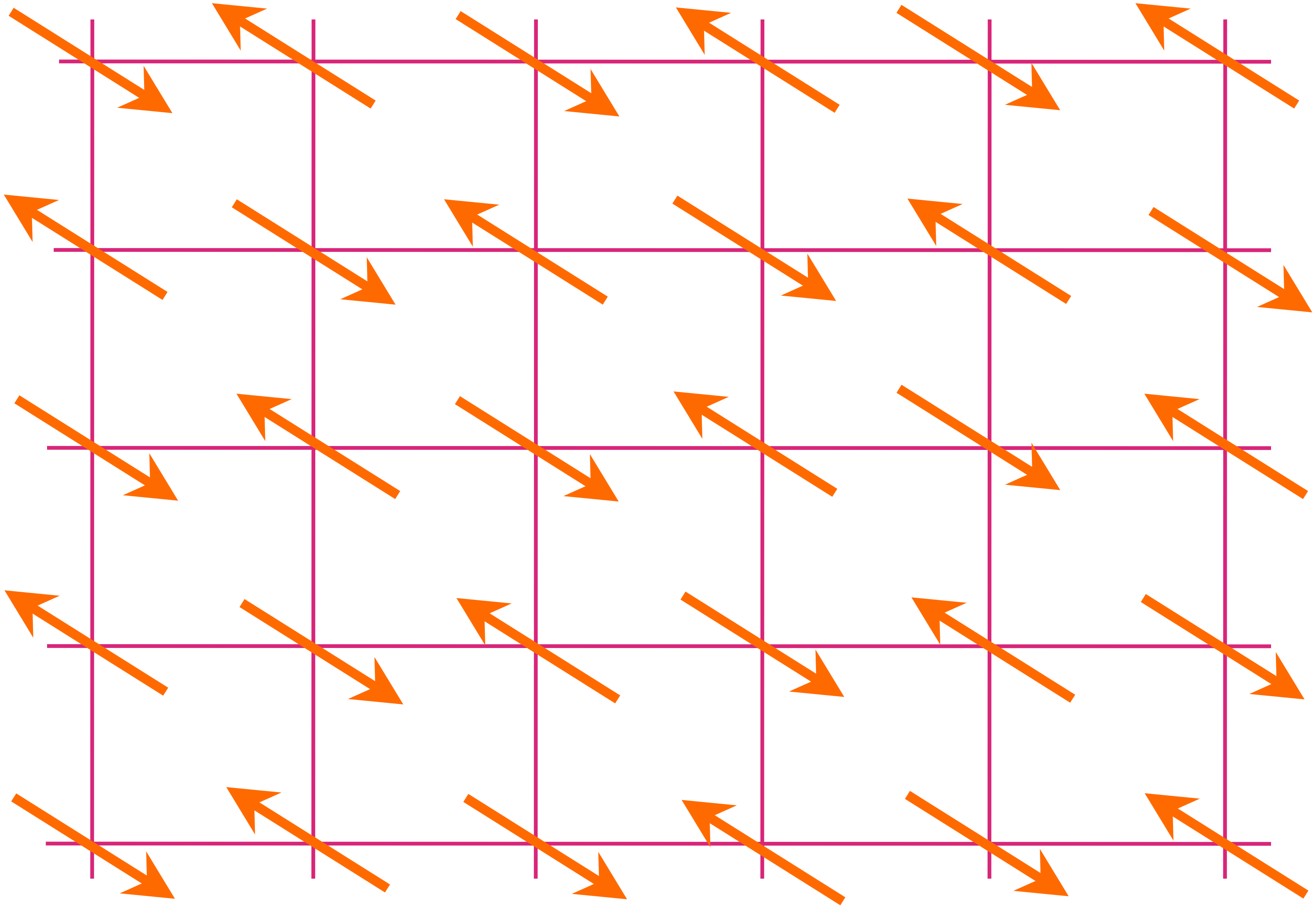


$\mathrm{CuO_2}$ plane

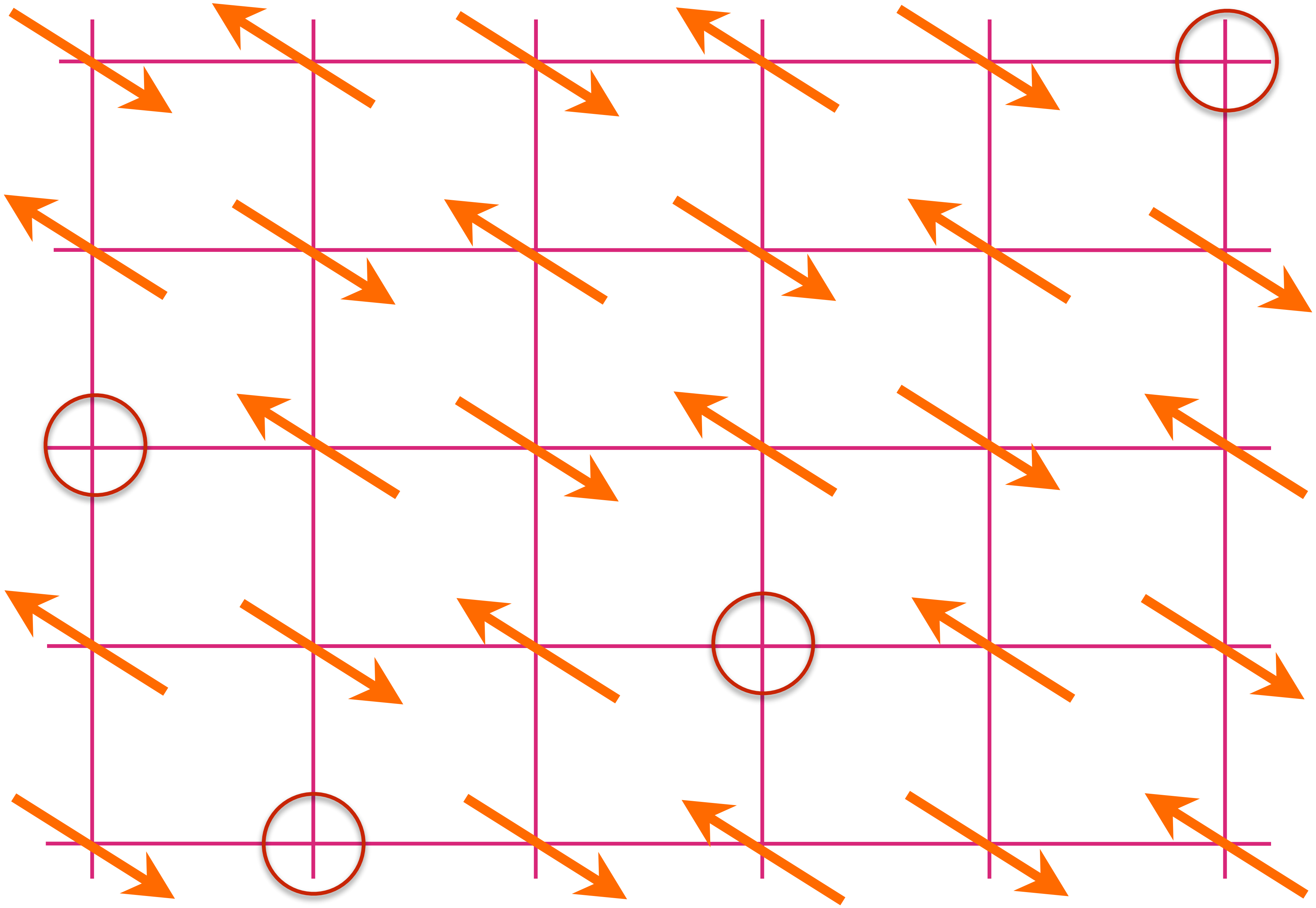


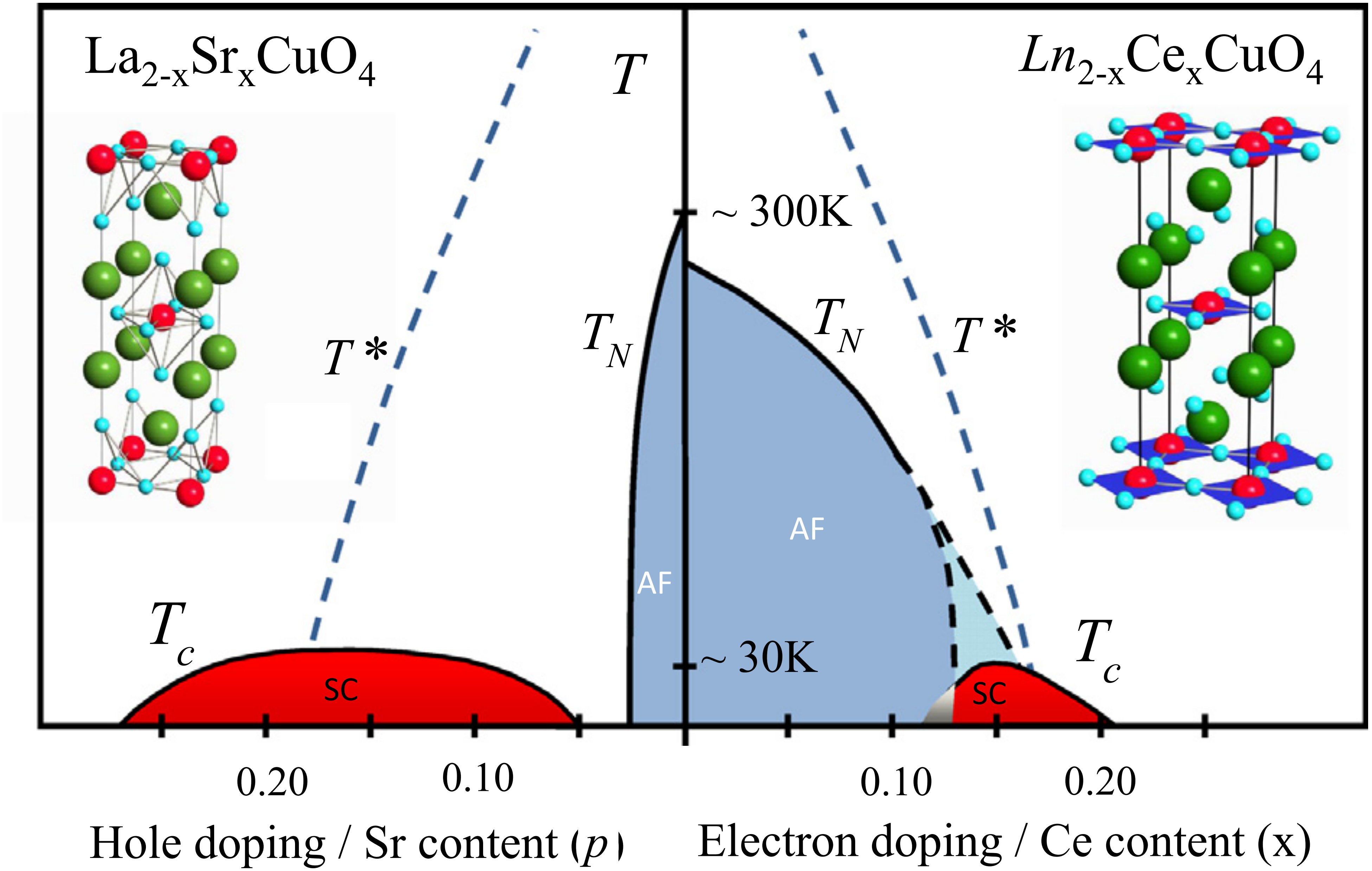
$\mathrm{YBa_2Cu_3O_{6+x}}$

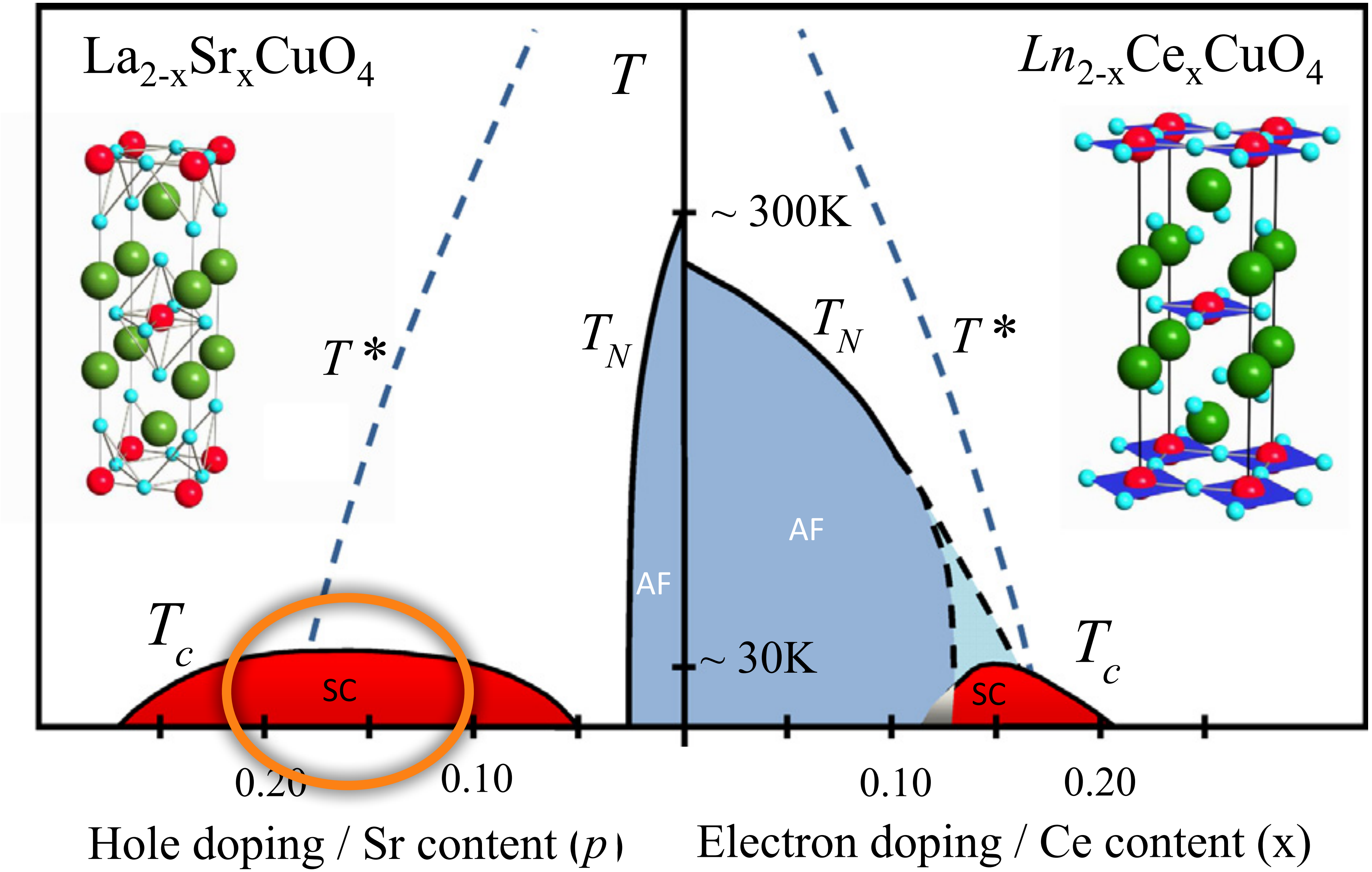
Insulating antiferromagnet



Antiferromagnet doped with hole density p

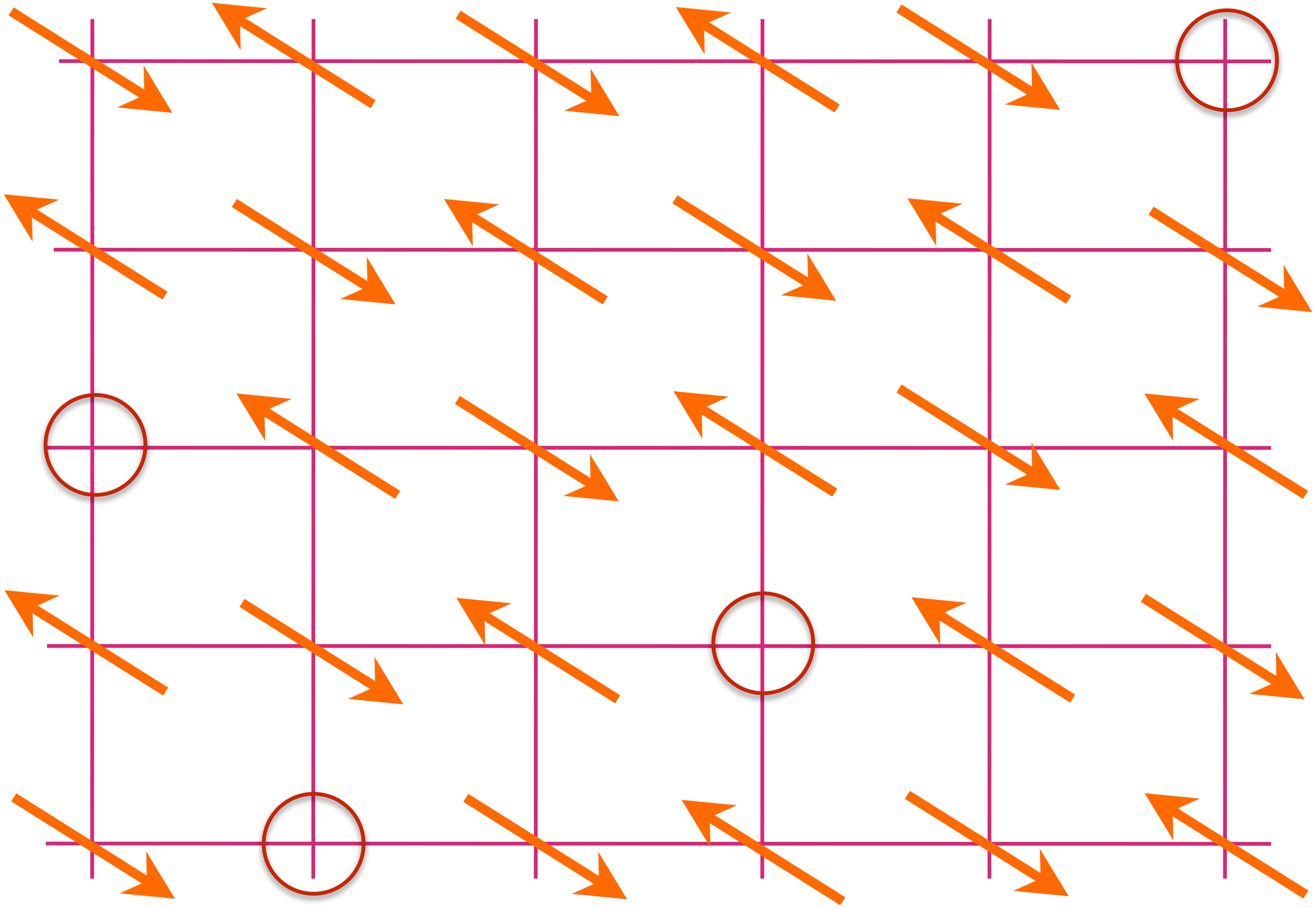



$La_{2-x}Sr_xCuO_4$
T
$Ln_{2-x}Ce_xCuO_4$
~ 300K
T^*
T_N
T_N
T^*
AF
AF
T_c
~ 30K
T_c
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)



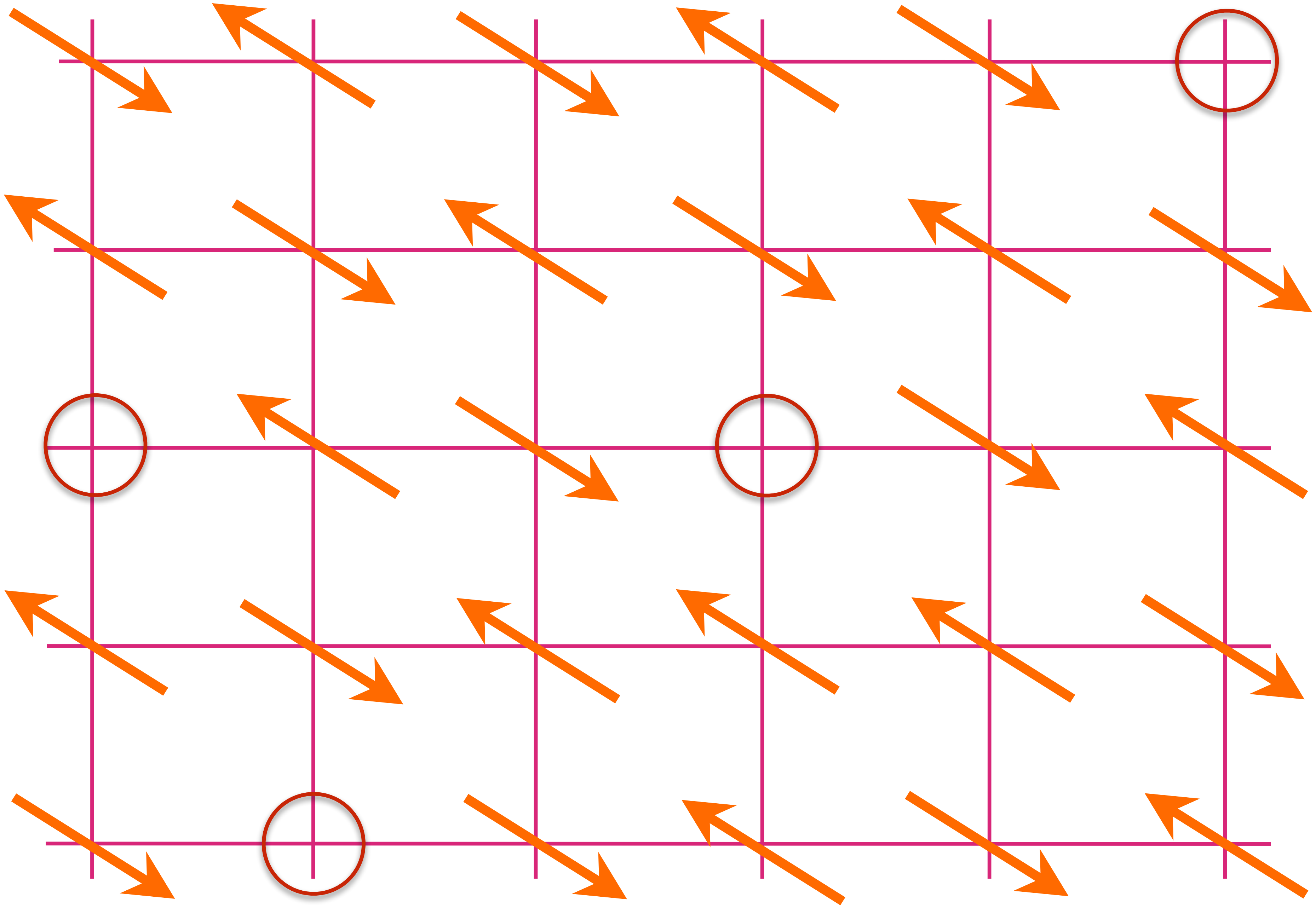
Is there a quantum phase transition at a critical $p=p_c$?

Real-space view at small p



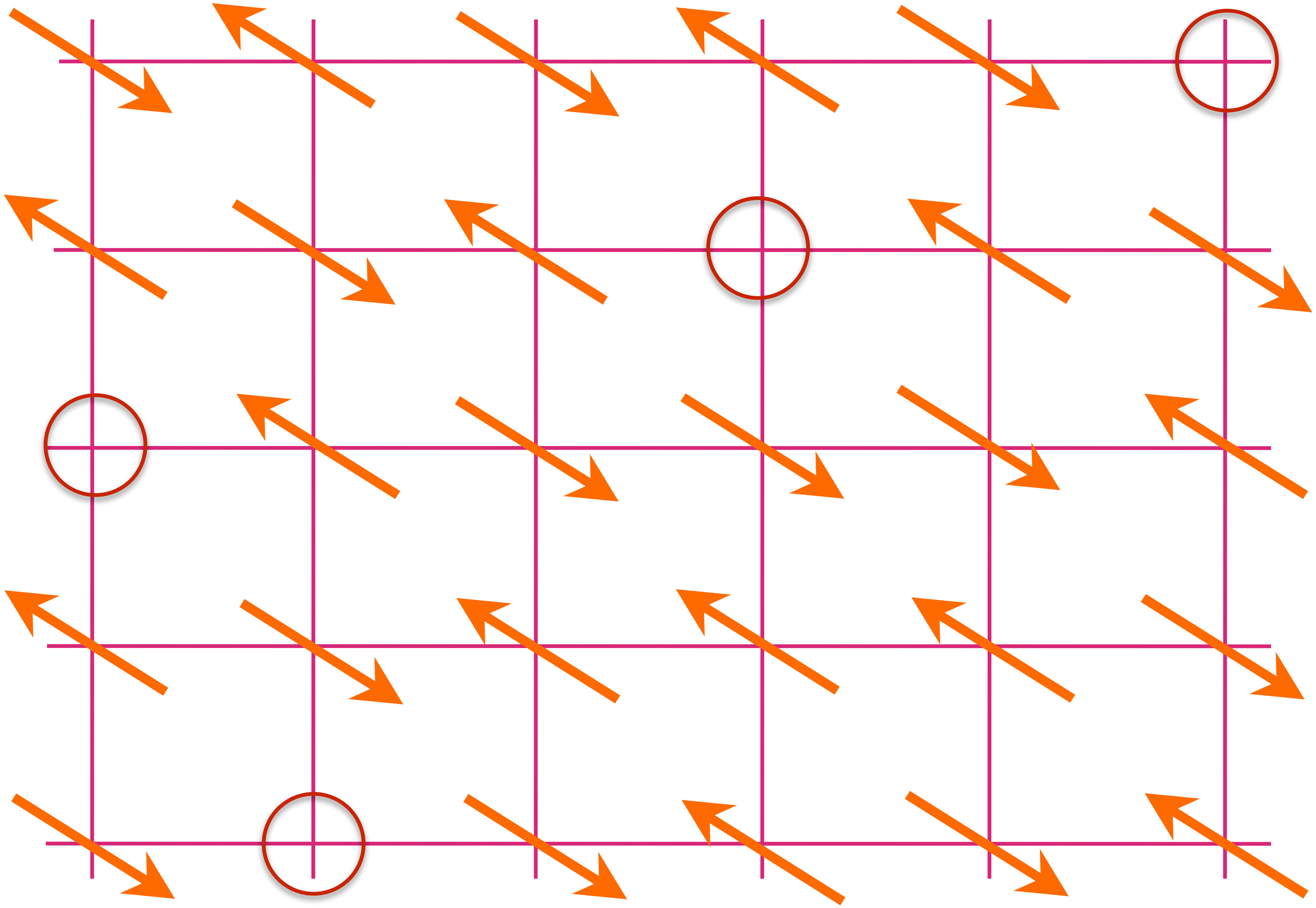
p mobile holes in a background of fluctuating spins

Real-space view at small p



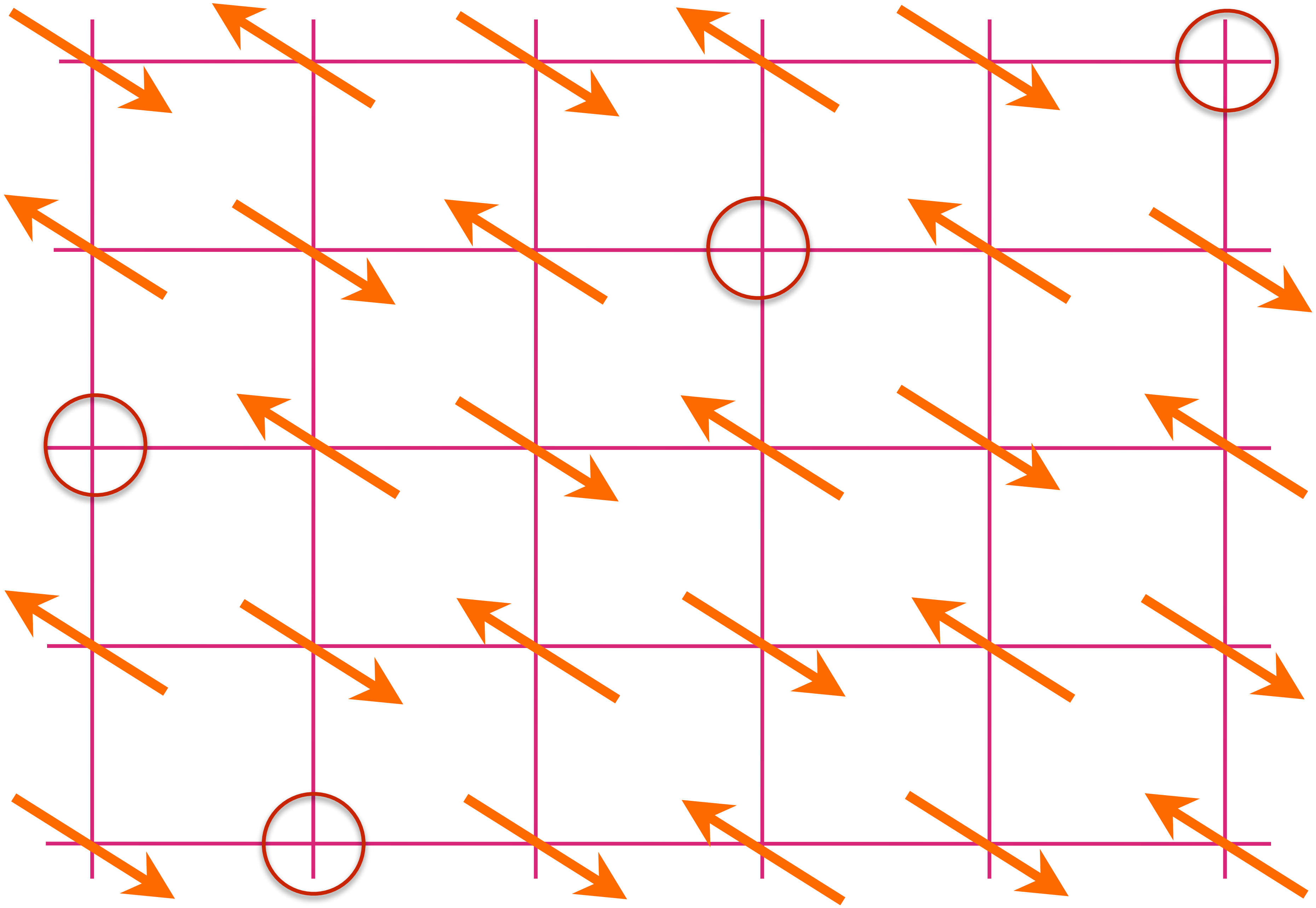
p mobile holes in a background of fluctuating spins

Real-space view at small p



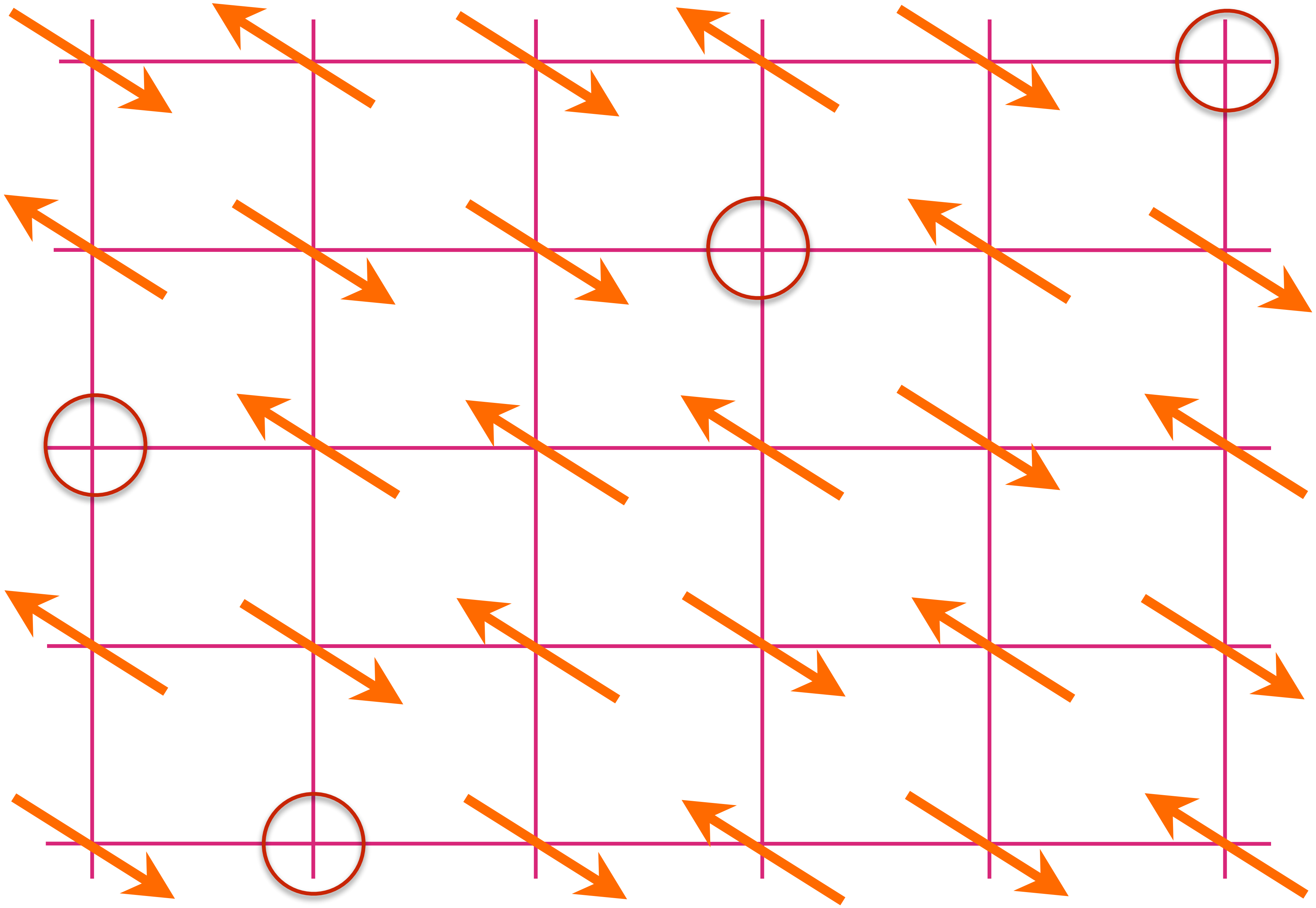
p mobile holes in a background of fluctuating spins

Real-space view at small p



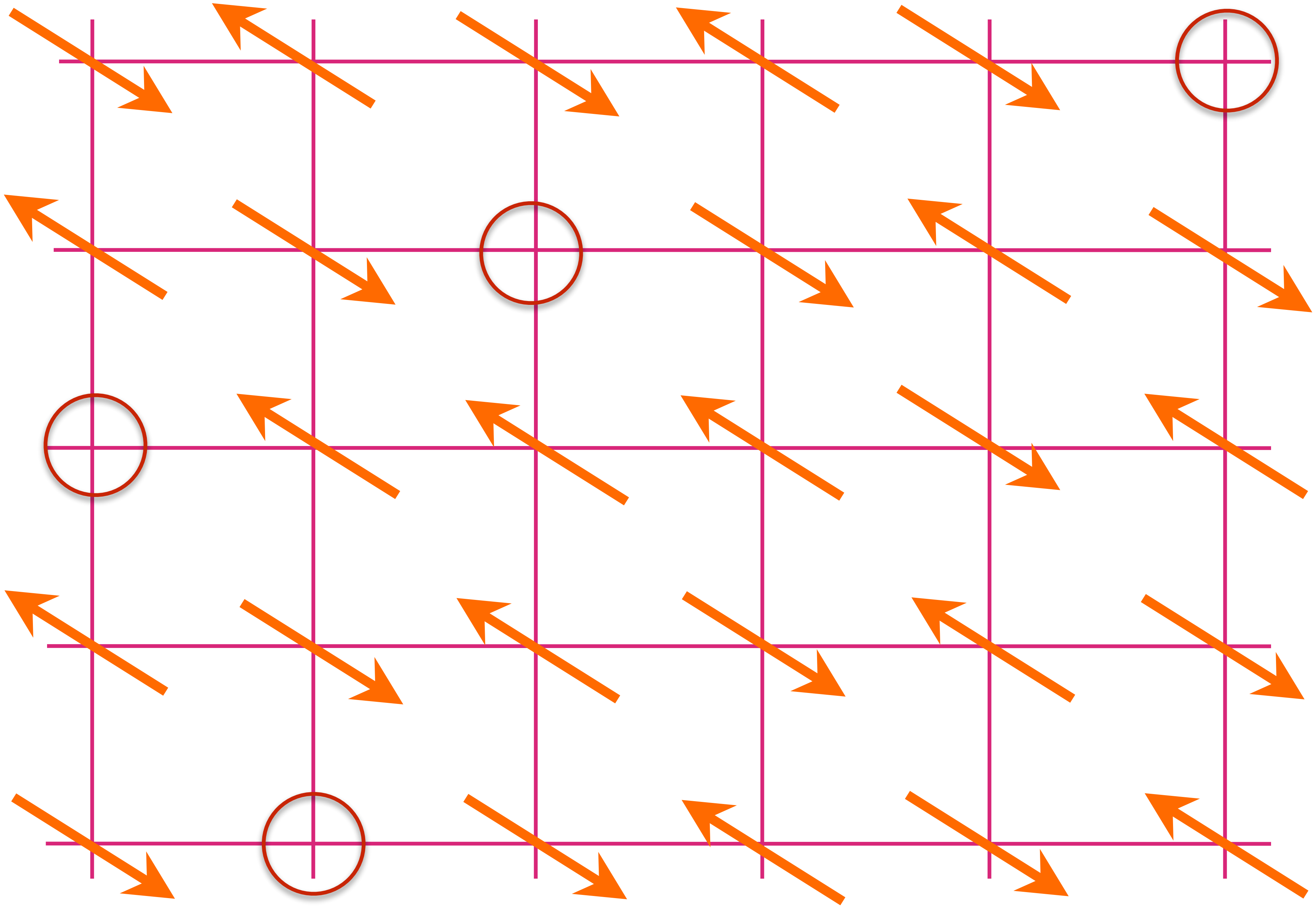
p mobile holes in a background of fluctuating spins

Real-space view at small p



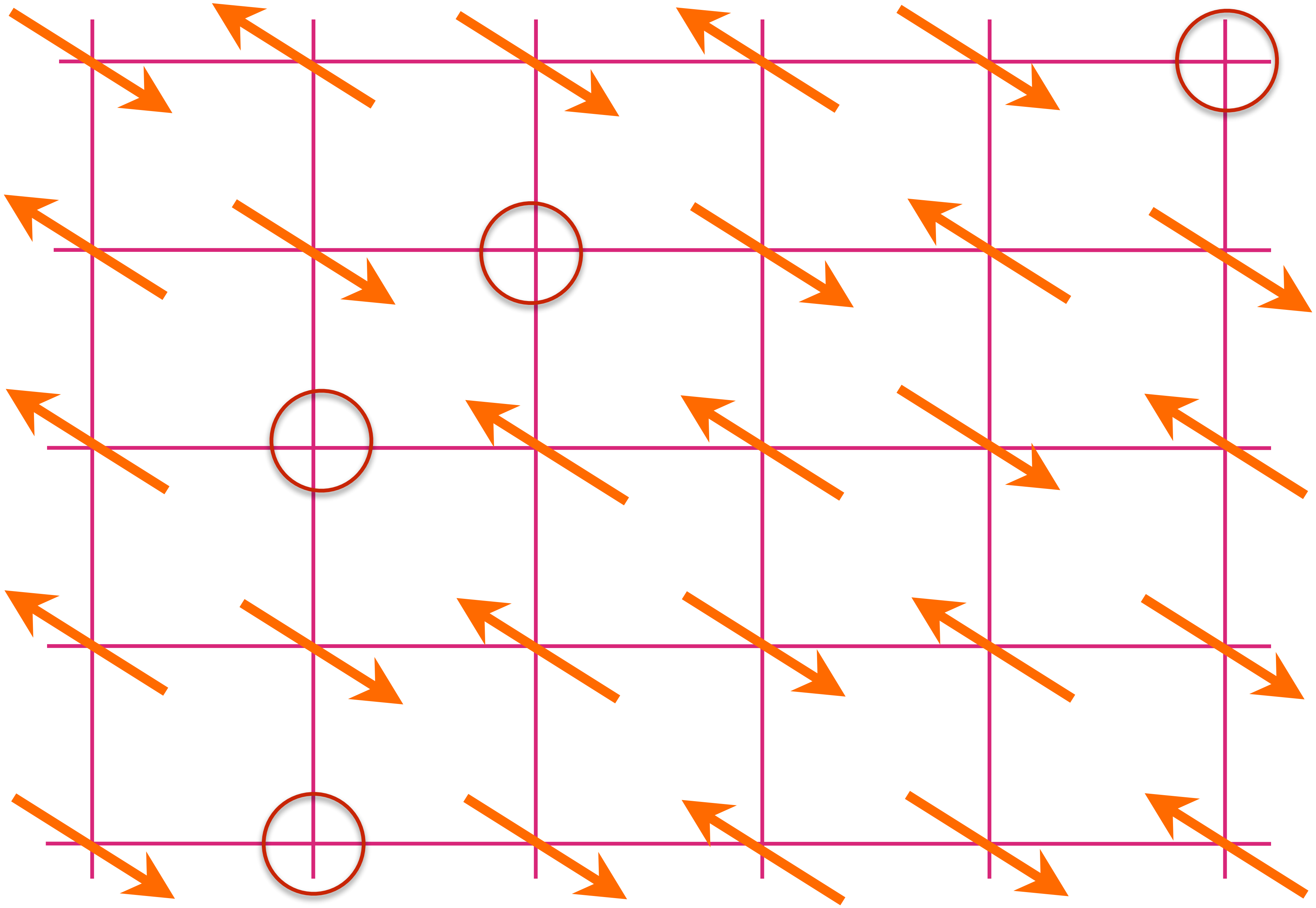
p mobile holes in a background of fluctuating spins

Real-space view at small p



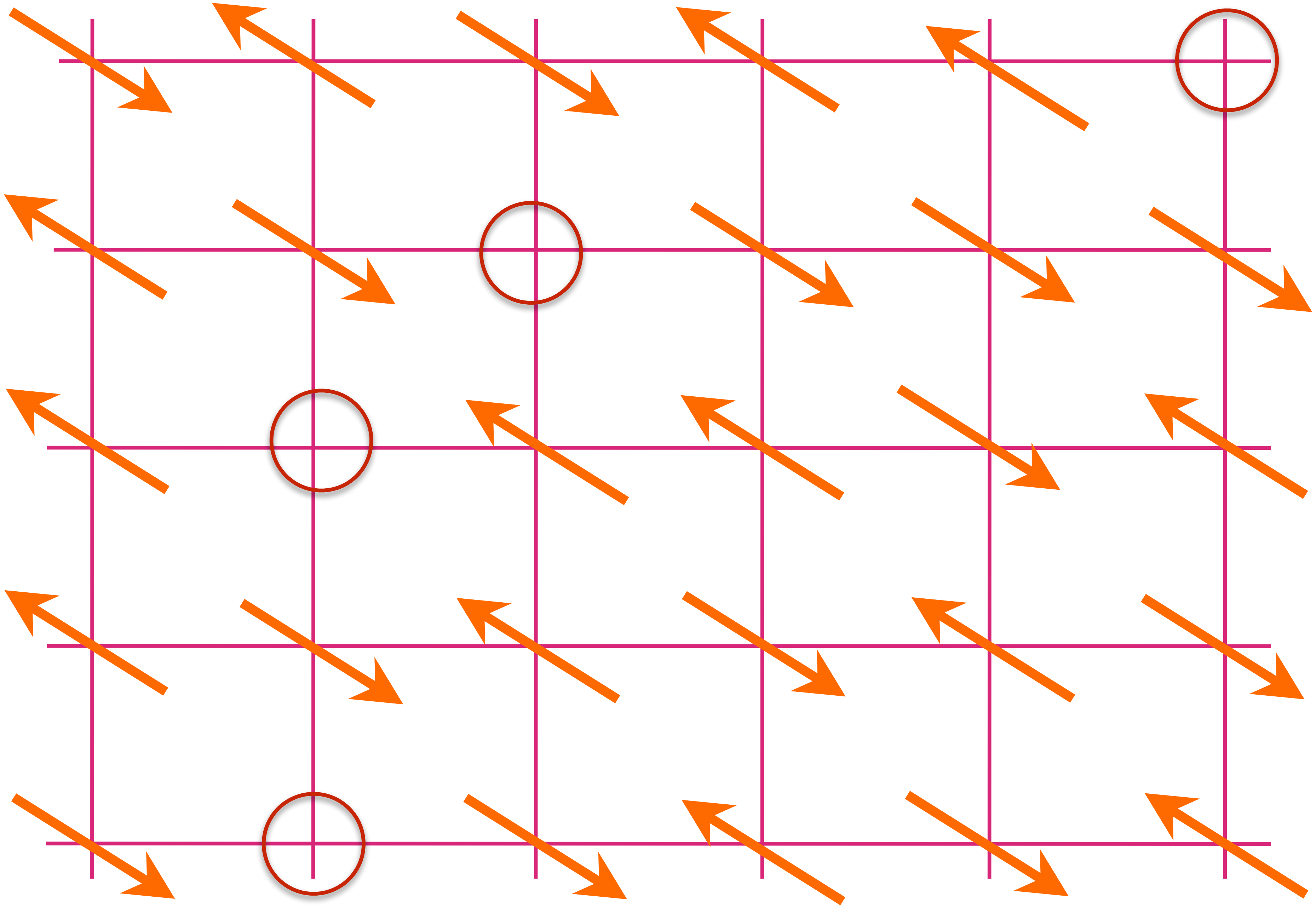
p mobile holes in a background of fluctuating spins

Real-space view at small p



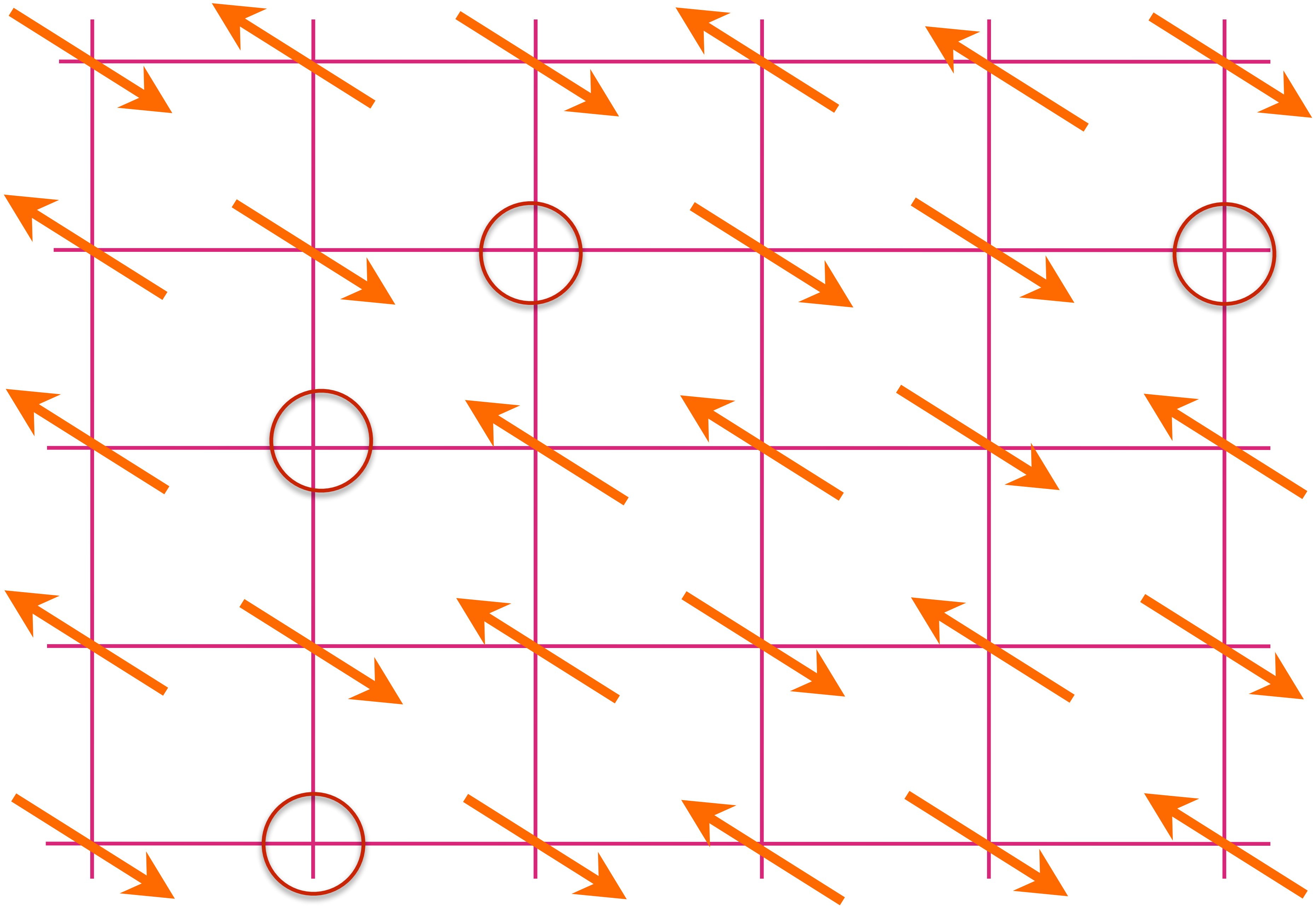
p mobile holes in a background of fluctuating spins

Real-space view at small p



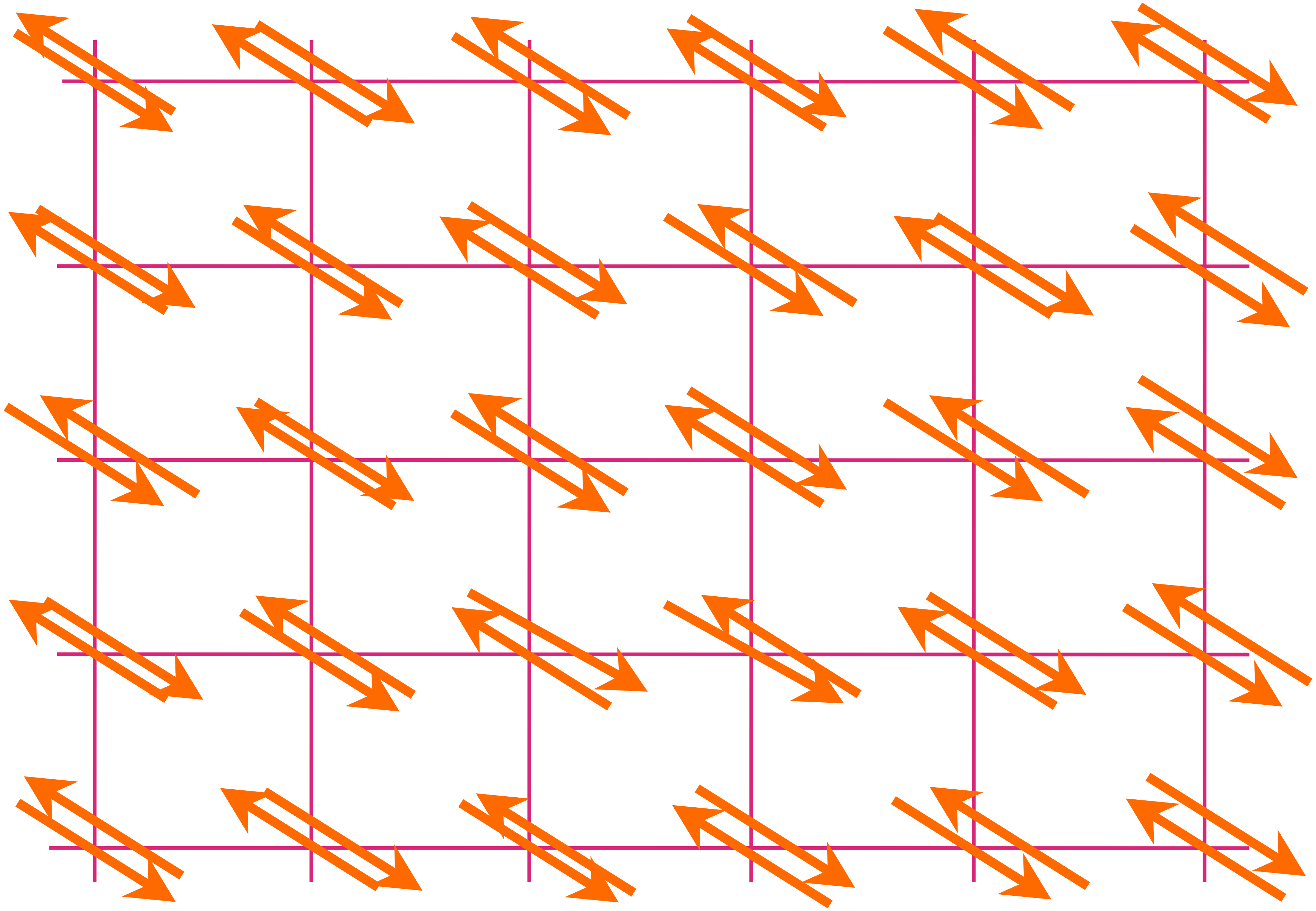
p mobile holes in a background of fluctuating spins

Real-space view at small p



p mobile holes in a background of fluctuating spins

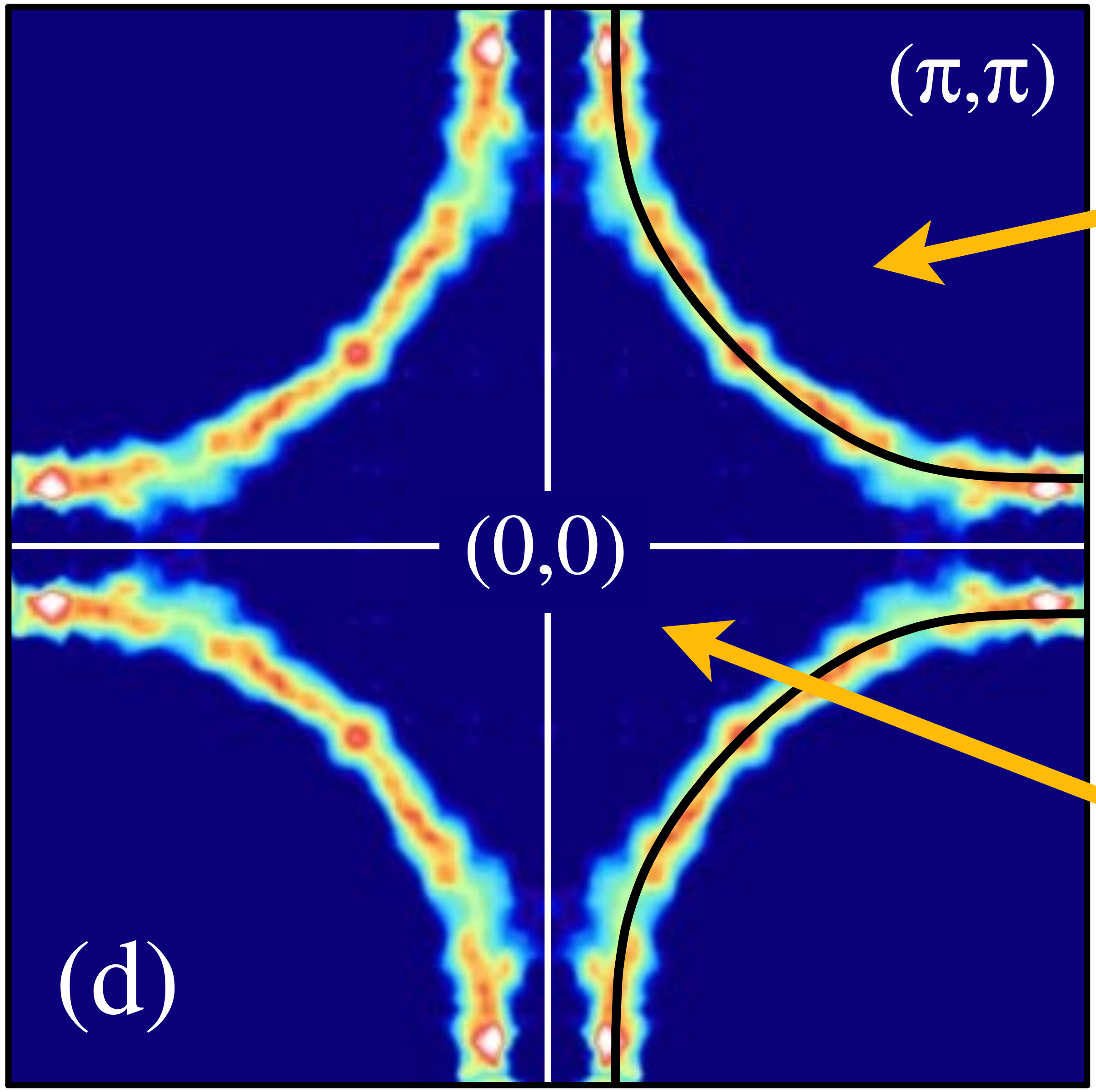
Momentum-space view at large p



Filled Band

$1+p$ mobile holes in a filled band

Momentum-space view at large p



$1+p$ holes

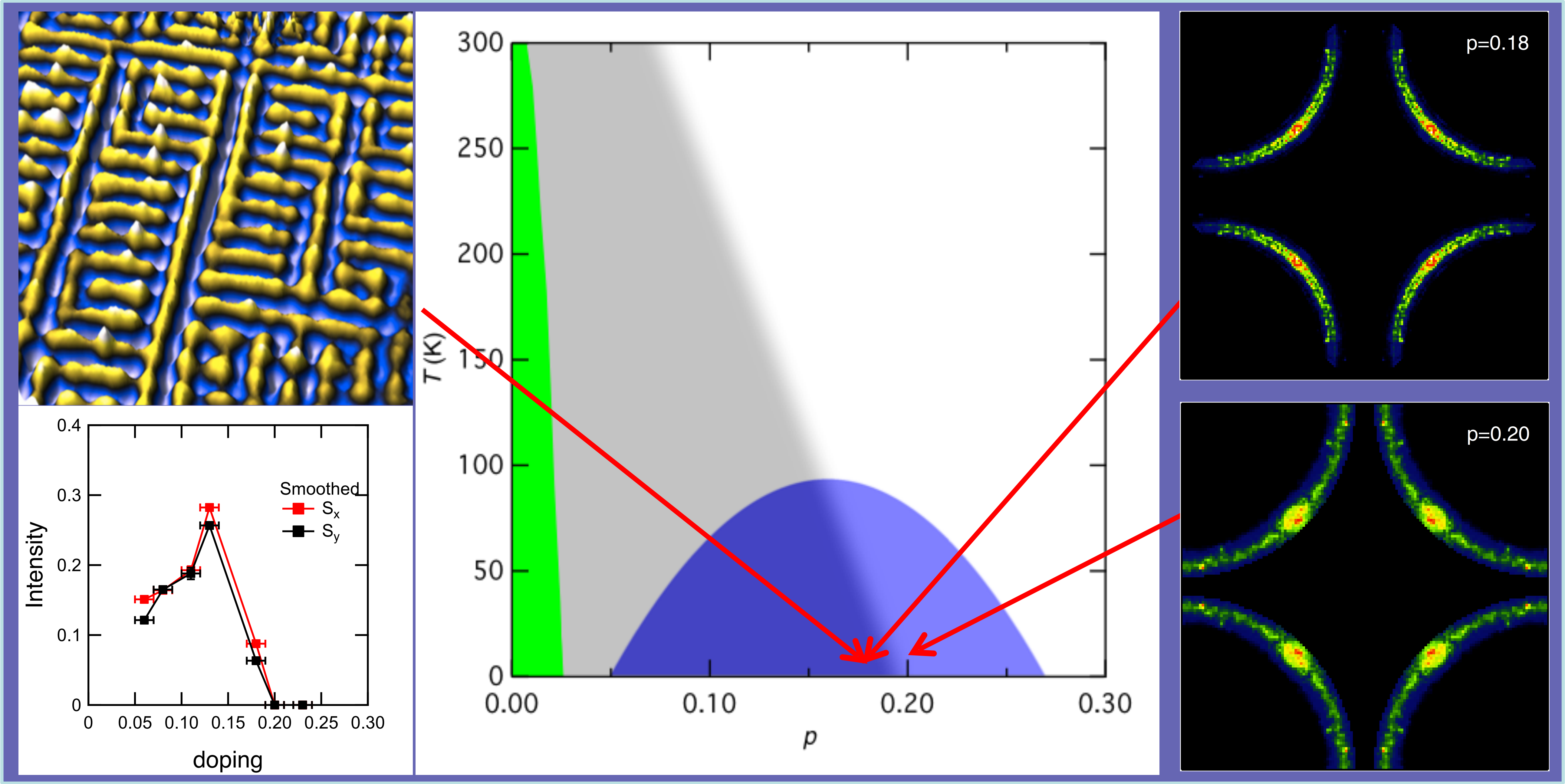
1-p electrons

$1+p$ mobile holes in a filled band

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

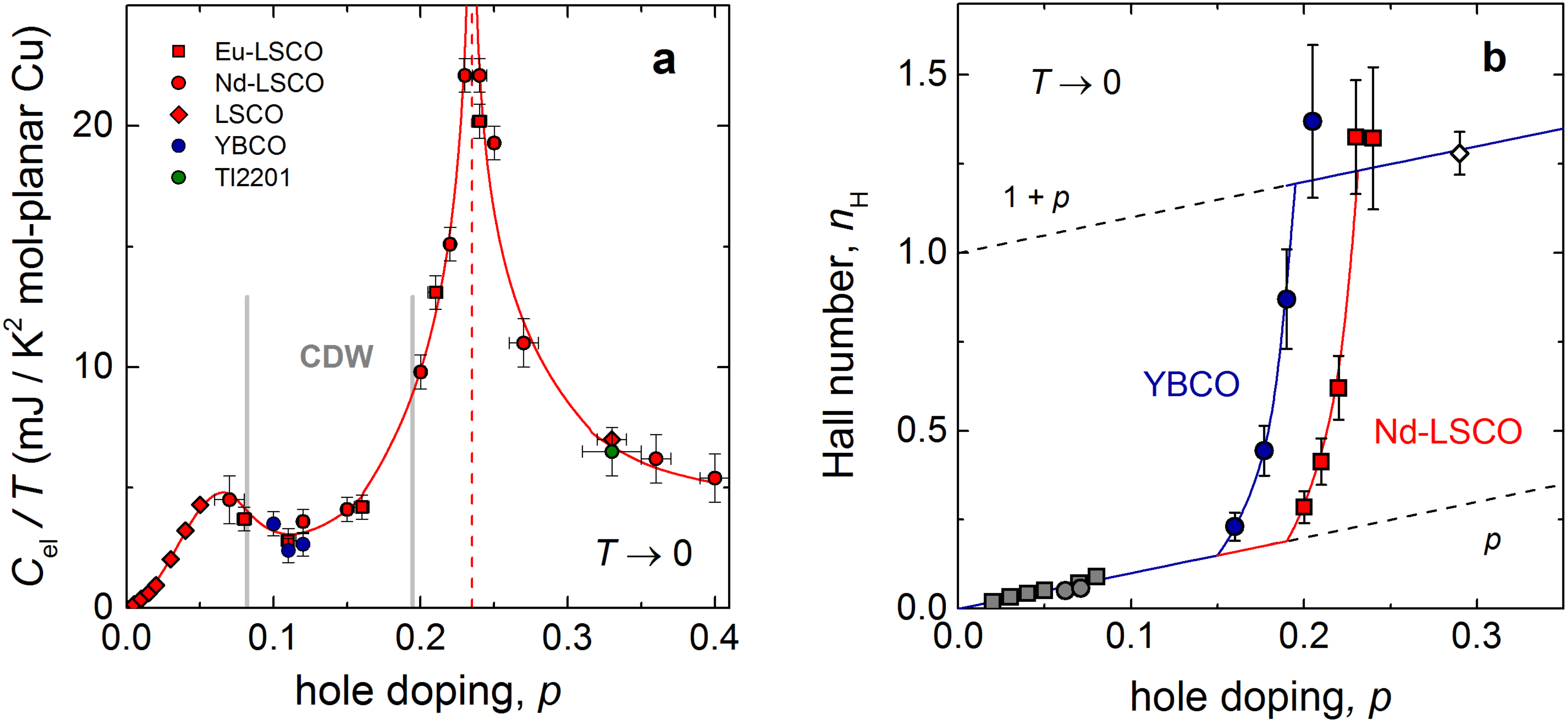
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)



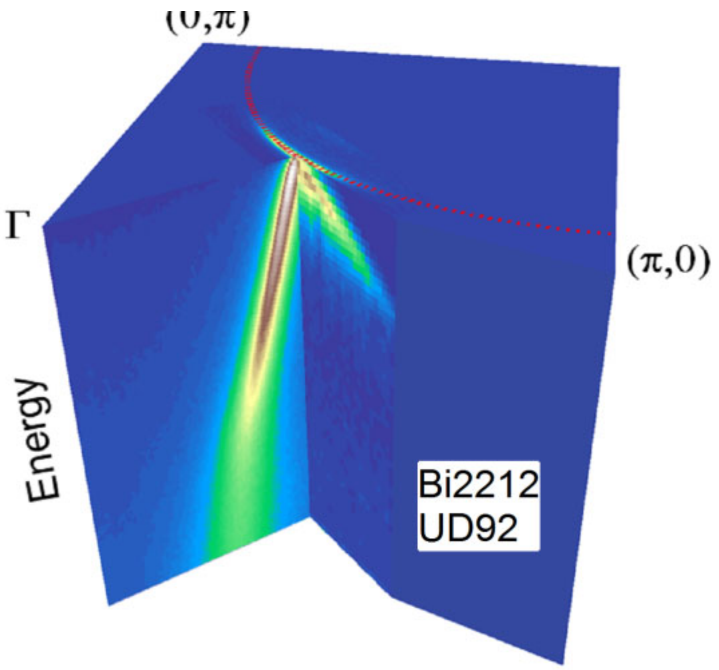
Hole doped cuprates

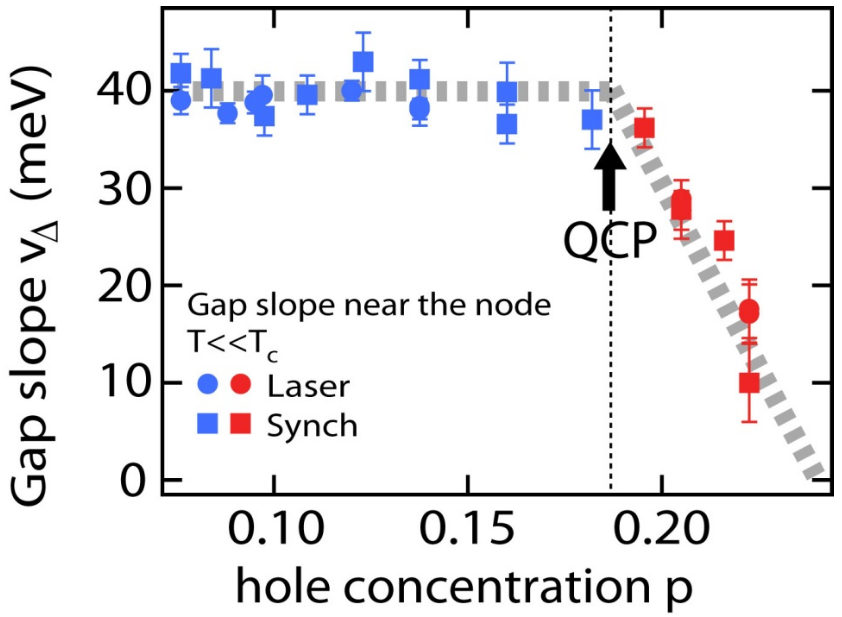
The remarkable underlying ground states of cuprate superconductors

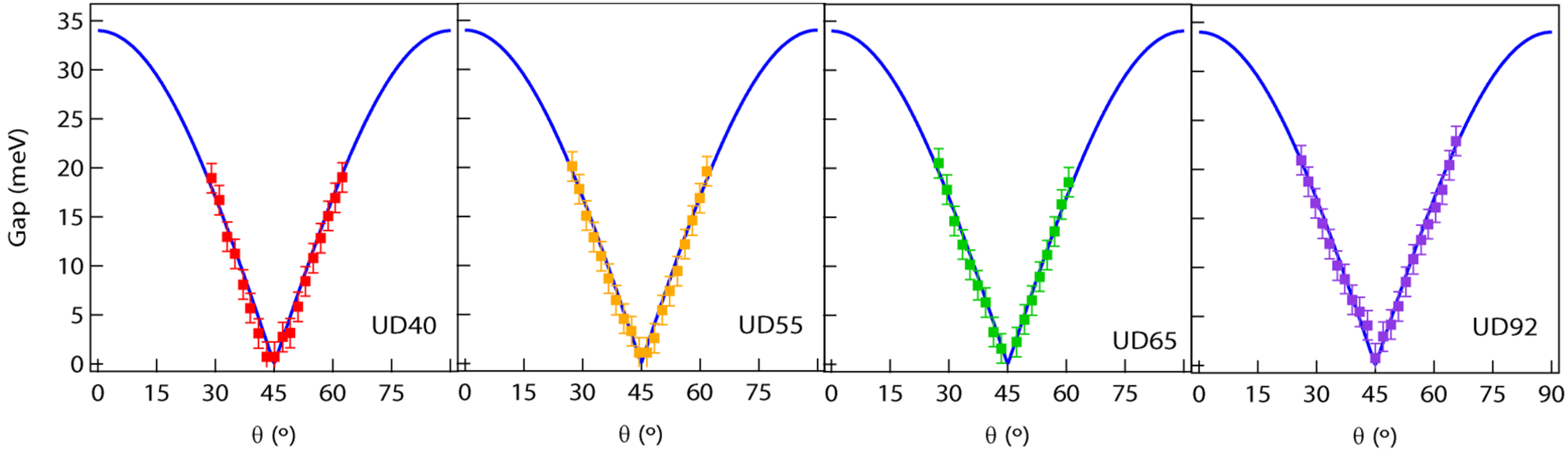
Cyril Proust and Louis Taillefer, arXiv:1807.0507



Precision Measurement of the Node

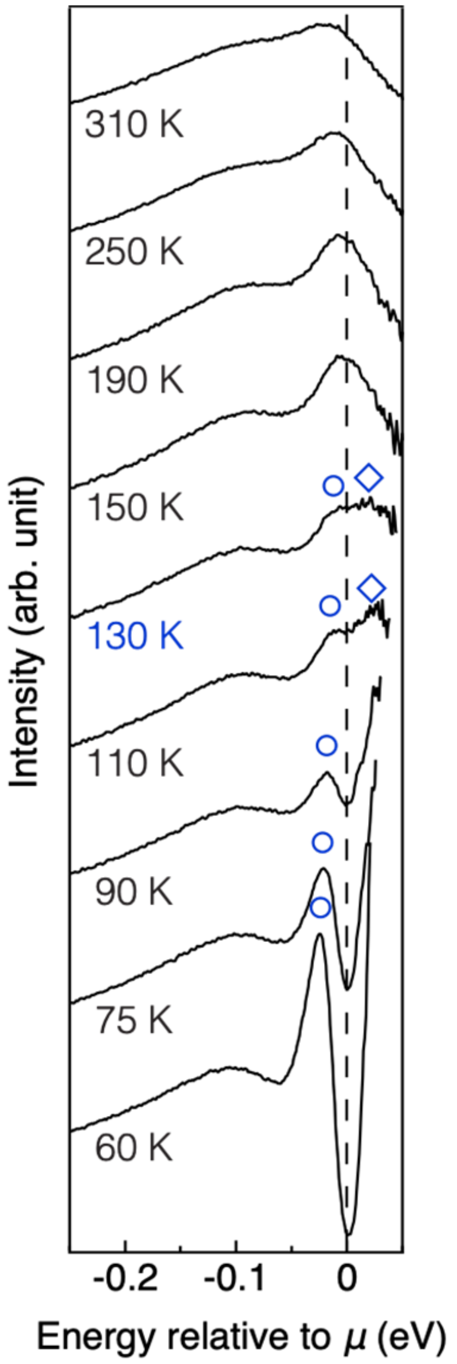


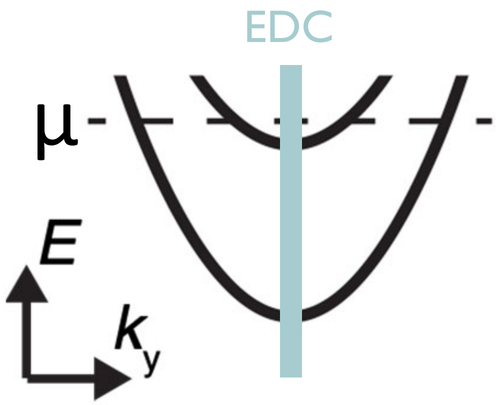




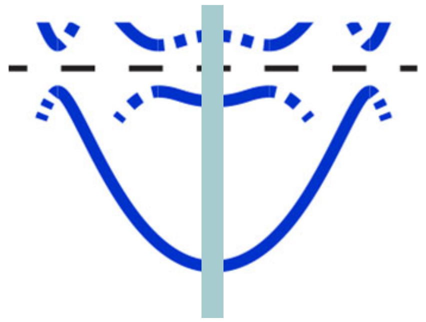
I. M. Vishik, M. Hashimoto, Rui-Hua He, Wei-Sheng Lee, Felix Schmitt, Donghui Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa, S. Uchida, Kazuhiro Fujita, S. Ishida, M. Ishikado, Yoshiyuki Yoshida, Hiroshi Eisaki, Zahid Hussain, Thomas P. Devereaux, and Zhi-Xun Shen, PNAS **109**, 18332 (2012)

One gap for p > 0.19 (T_c ~ 81 K)

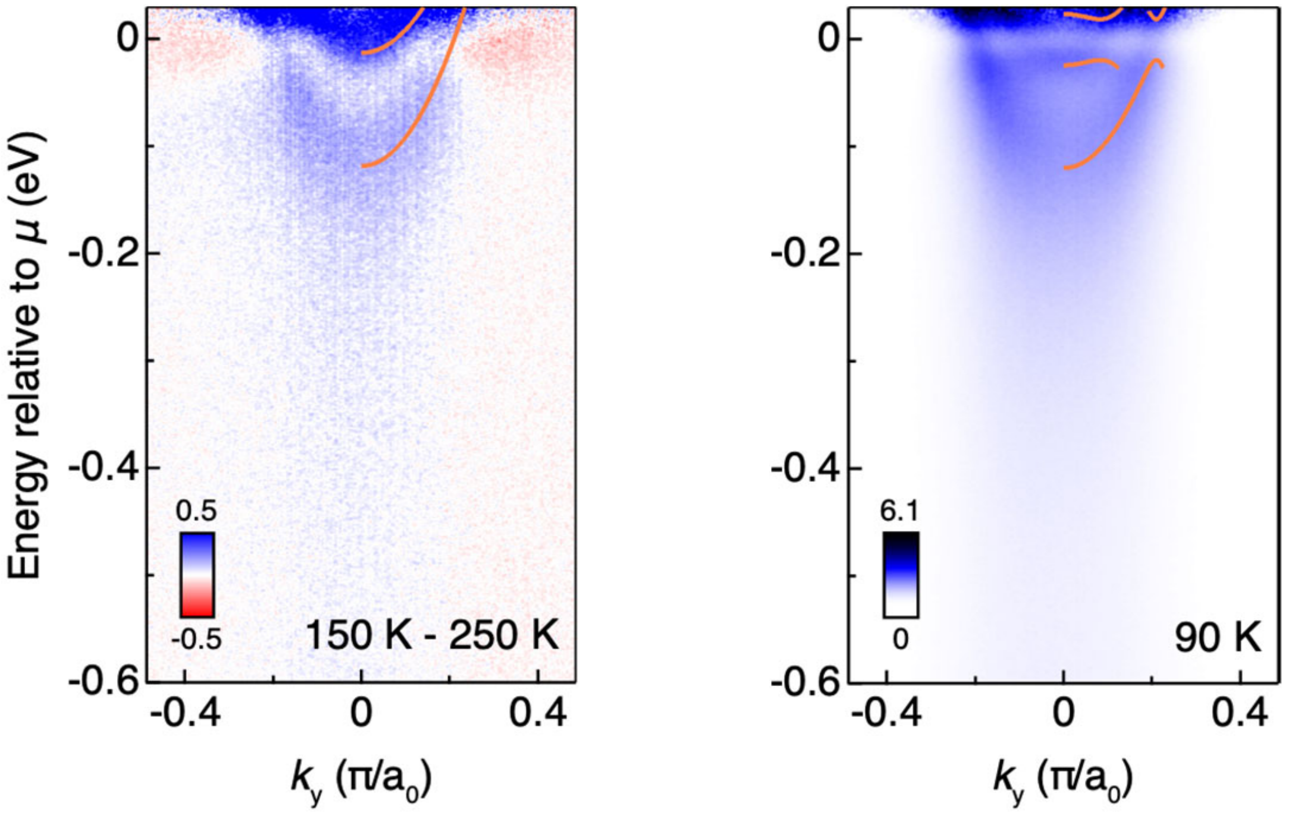




Normal state

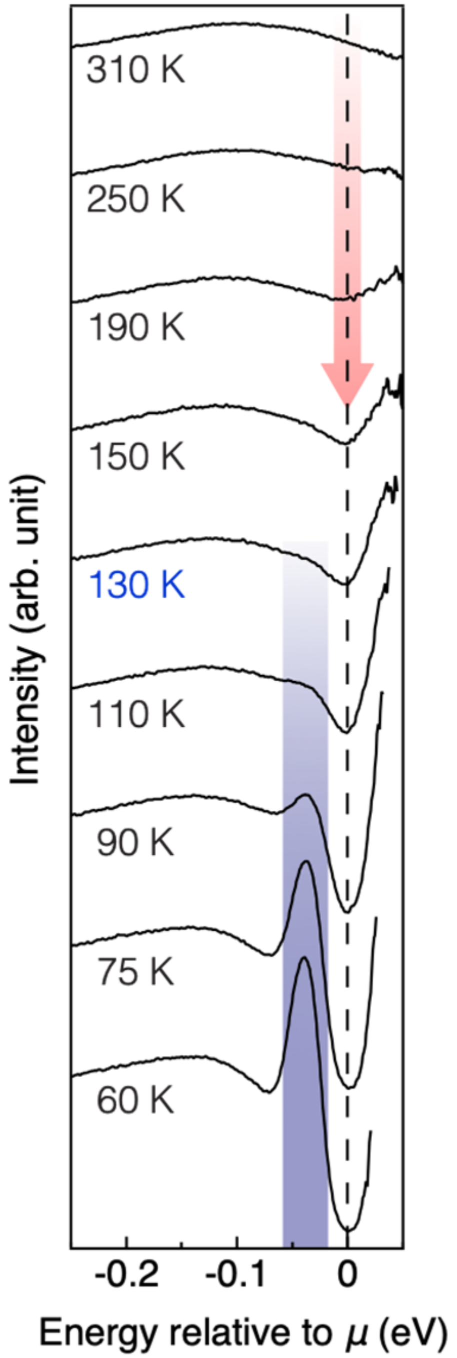


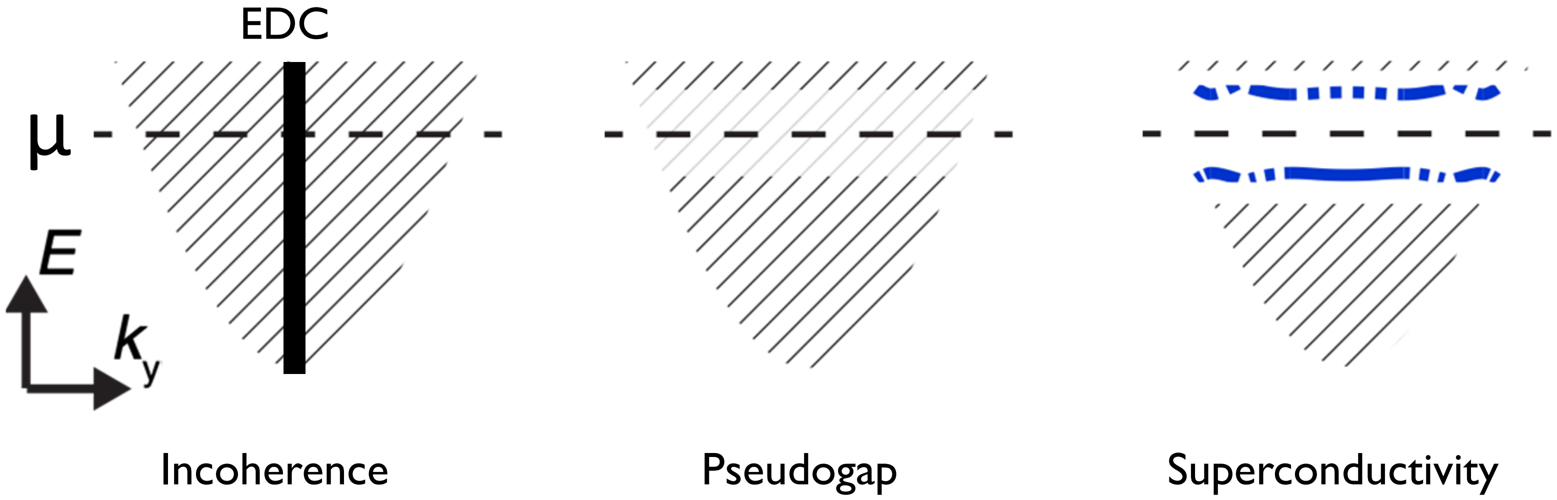
Superconducting gap present

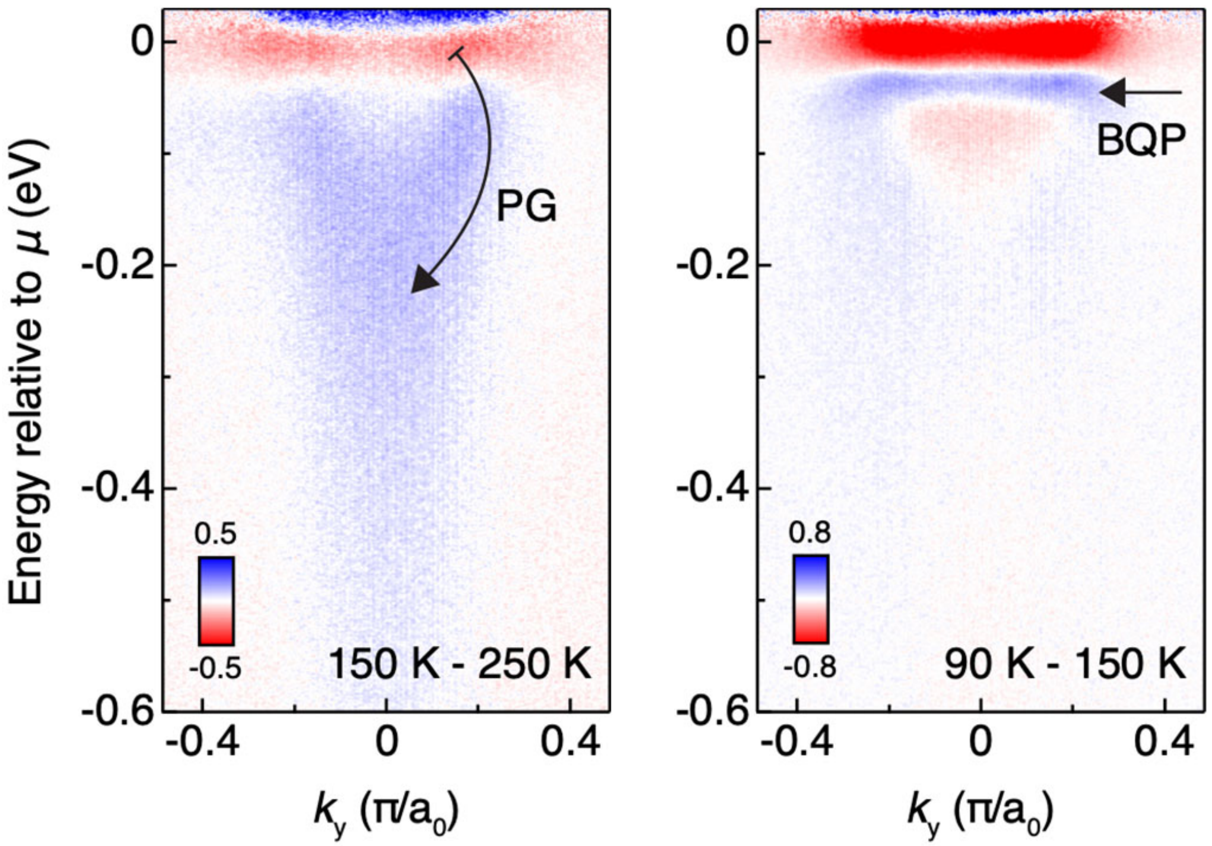


Su-Di Chen, Makoto Hashimoto, Yu He, Dongjoon Song, Ke-Jun Xu, Jun-Feng He, T. P. Devereaux, Hiroshi Eisaki, Dong-Hui Lu, J. Zaanen, Zhi-Xun Shen, *Science* **366**, 6469 (2019)

Two “gaps” for $p < 0.19$ ($T_c \sim 86$ K)



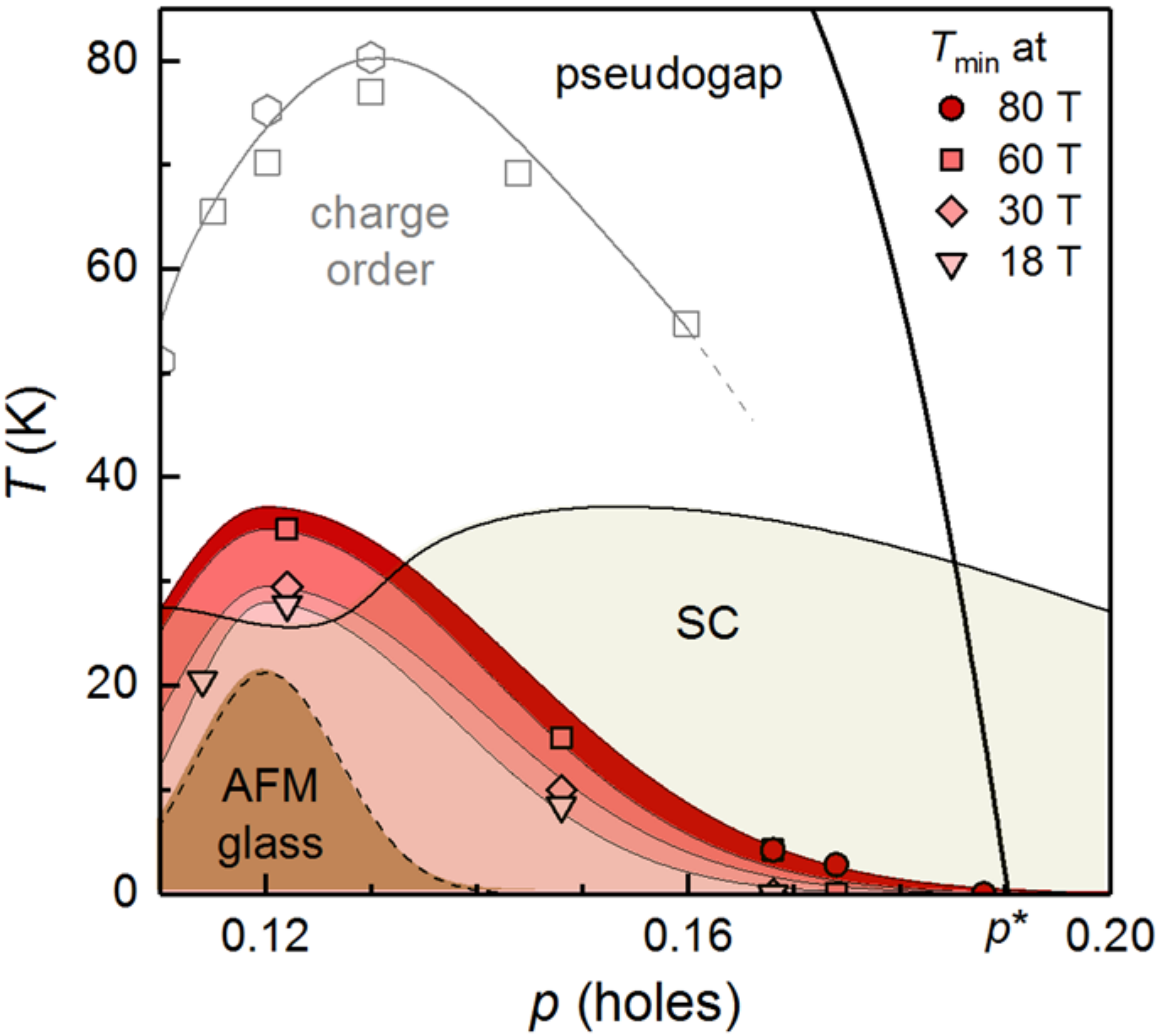




Su-Di Chen, Makoto Hashimoto, Yu He, Dongjoon Song, Ke-Jun Xu, Jun-Feng He, T. P. Devereaux, Hiroshi Eisaki, Dong-Hui Lu, J. Zaanen, Zhi-Xun Shen, *Science* **366**, 6469 (2019)

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1]*, Marc-Henri Julien[1]*



arXiv:1909.10258

Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

1. Deconfined quantum criticality of random *t*-*J* models

2. RG analysis

3. Transport…

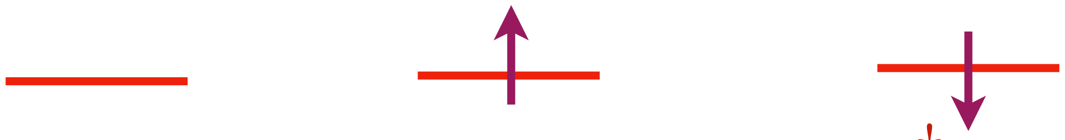
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t_{ij}^2} = t^2$$



$|0\rangle \qquad c_{\uparrow}^{\dagger}\,|0\rangle \qquad c_{\downarrow}^{\dagger}\,|0\rangle$

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \qquad , \qquad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad b \to b e^{i\phi} \, , \quad f_\alpha \to f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow \mathfrak{f}^{\dagger}\,|v\rangle \qquad , \qquad c_{\alpha}^{\dagger}\,|0\rangle \Rightarrow \mathfrak{b}_{\alpha}^{\dagger}\,|v\rangle$$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^{\dagger} \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_{\alpha}^{\dagger} \sigma_{\alpha\beta} \mathfrak{b}_{\beta} \\ \mathfrak{b}_{\alpha}^{\dagger} \mathfrak{b}_{\alpha} + \mathfrak{f}^{\dagger}\mathfrak{f} &= 1 \end{aligned}$$

$$\text{U(1) gauge invariance,} \qquad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow \mathfrak{f}^{\dagger}\, |v\rangle \qquad , \qquad c_{\alpha}^{\dagger}\, |0\rangle \Rightarrow \mathfrak{b}_{\alpha}^{\dagger}\, |v\rangle$$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^{\dagger} \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_{\alpha}^{\dagger} \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_{\alpha}^{\dagger} \mathfrak{b}_\alpha + \mathfrak{f}^{\dagger}\mathfrak{f} &= 1 \end{aligned}$$

$$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$$

$$\text{U(1) gauge invariance,} \qquad \mathfrak{f} \rightarrow \mathfrak{f} e^{i\phi}\, , \quad \mathfrak{b}_\alpha \rightarrow \mathfrak{b}_\alpha e^{i\phi}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

Differences from SYK models:

- The interaction term J_{ij} has 2-index randomness, in contrast to the 4-index randomness in the SYK models

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Differences from SYK models:

- The interaction term J_{ij} has 2-index randomness, in contrast to the 4-index randomness in the SYK models
- There is a *local* constraint. Consequently *both* the t_{ij} and J_{ij} are 4-particle terms, when expressed in terms of SU(1|2) particles f_α, b, or the SU(2|1) particles $\mathfrak{b}_\alpha$, $\mathfrak{f}$.

t-*J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Differences from SYK models:

- The interaction term J_{ij} has 2-index randomness, in contrast to the 4-index randomness in the SYK models
- There is a *local* constraint. Consequently *both* the t_{ij} and J_{ij} are 4-particle terms, when expressed in terms of SU$(1|2)$ particles f_α, b, or the SU$(2|1)$ particles $\mathfrak{b}_\alpha$, $\mathfrak{f}$.
- There is competition between the hopping and exchange terms, controlled by the values J/t and p, leading to a deconfined quantum critical point (DQCP) separating two distinct phases with Fermi-liquid-like behavior as $T \to 0$.

t-J model

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}} \\
\mathcal{S}_B &= i \int_0^1 du \int d\tau \, \mathrm{Tr}\left(\mathcal{P}\partial_\tau \mathcal{P} \partial_u \mathcal{P}\right) \\
\mathcal{S}_{tJ} &= \int d\tau d\tau' \, \mathrm{Tr}\left(\mathcal{P}(\tau)\mathcal{Q}(\tau-\tau')\mathcal{P}(\tau')\right) \\
&\qquad + \int d\tau \mathrm{Tr}\left(s_0 \, \mathcal{P}(\tau)\right) \, .
\end{aligned}
$$

Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$ and a ‘Zeeman superfield’ s_0.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}f_\alpha(\tau)\mathcal{D}b(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}} \\
\mathcal{S}_B &= \int d\tau \left[f_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)f_\alpha(\tau) + b^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)b(\tau) - i\lambda \right] \\
\mathcal{S}_{tJ} &= \int d\tau\, s_0 f_\alpha^\dagger(\tau) f_\alpha(\tau) + t^2 \int d\tau d\tau' R(\tau-\tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') \\
&\quad - \frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.
\end{aligned}
$$

SU(1|2) theory

From this action we determined the correlators

$$
\begin{aligned}
\overline{R}(\tau-\tau') &= -\left\langle c_\alpha(\tau) c_\alpha^\dagger(\tau')\right\rangle_{\mathcal{Z}} \\
\overline{Q}(\tau-\tau') &= \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}
\end{aligned}
$$

and finally impose the self-consistency conditions

$$
R(\tau) = \overline{R}(\tau) \quad , \quad Q(\tau) = \overline{Q}(\tau).
$$

D. Joshi,
Chenyuan Li,
G. Tarnopolsky,
A. Georges,
S. Sachdev,
arXiv:1912.08822

t-J model

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\mathfrak{b}_\alpha(\tau)\mathcal{D}\mathfrak{f}(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}} \\
\mathcal{S}_B &= \int d\tau \left[\mathfrak{b}_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)\mathfrak{b}_\alpha(\tau)+\mathfrak{f}^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)\mathfrak{f}(\tau)-i\lambda\right] \\
\mathcal{S}_{tJ} &= \int d\tau\, s_0\mathfrak{b}_\alpha^\dagger(\tau)\mathfrak{b}_\alpha(\tau)+t^2\int d\tau d\tau' R(\tau-\tau')c_\alpha^\dagger(\tau)c_\alpha(\tau') \\
&\quad -\frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.
\end{aligned}
$$

From this action we determined the correlators

$\mathrm{SU}(2|1)$ theory

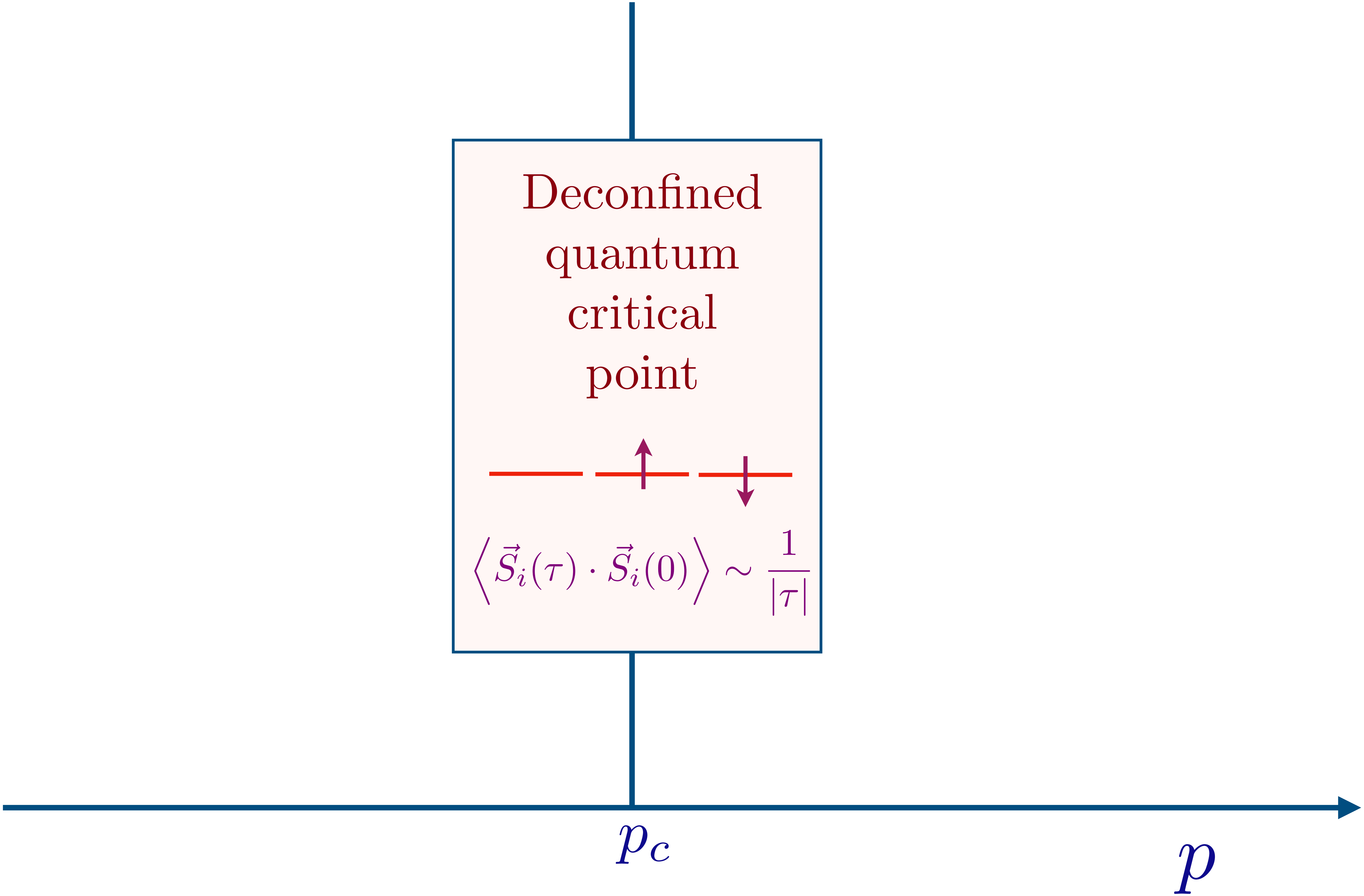
$$
\begin{aligned}
\overline{R}(\tau-\tau') &= -\left\langle c_\alpha(\tau)c_\alpha^\dagger(\tau')\right\rangle_{\mathcal{Z}} \\
\overline{Q}(\tau-\tau') &= \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}
\end{aligned}
$$

and finally impose the self-consistency conditions

$$
\boxed{R(\tau)=\overline{R}(\tau)\quad,\quad Q(\tau)=\overline{Q}(\tau).}
$$

D. Joshi,
Chenyuan Li,
G. Tarnopolsky,
A. Georges,
S. Sachdev,
arXiv:1912.08822

t-J model phase diagram



Deconfined
quantum
critical
point

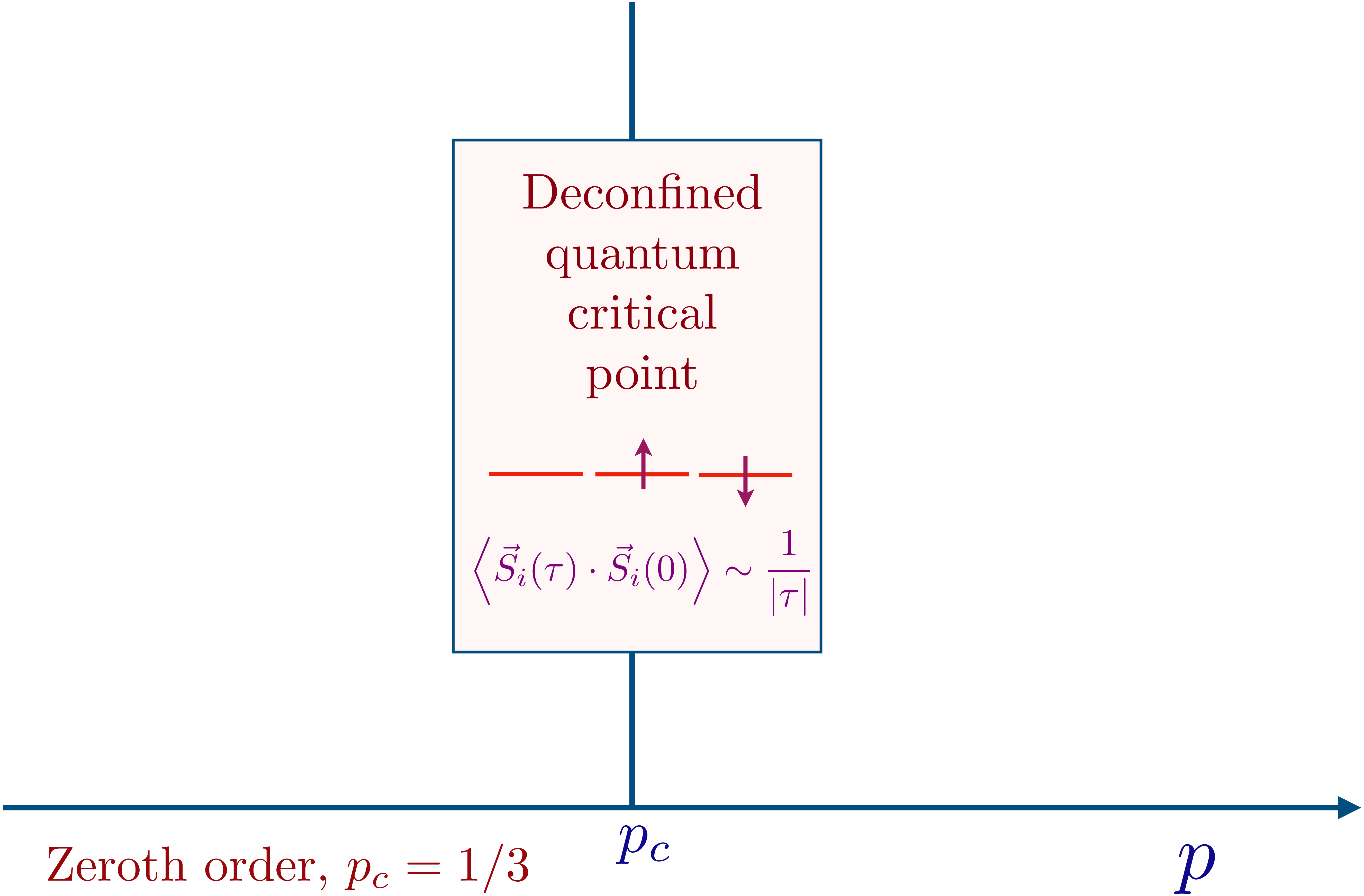
$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

p_c

p

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



Deconfined
quantum
critical
point

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

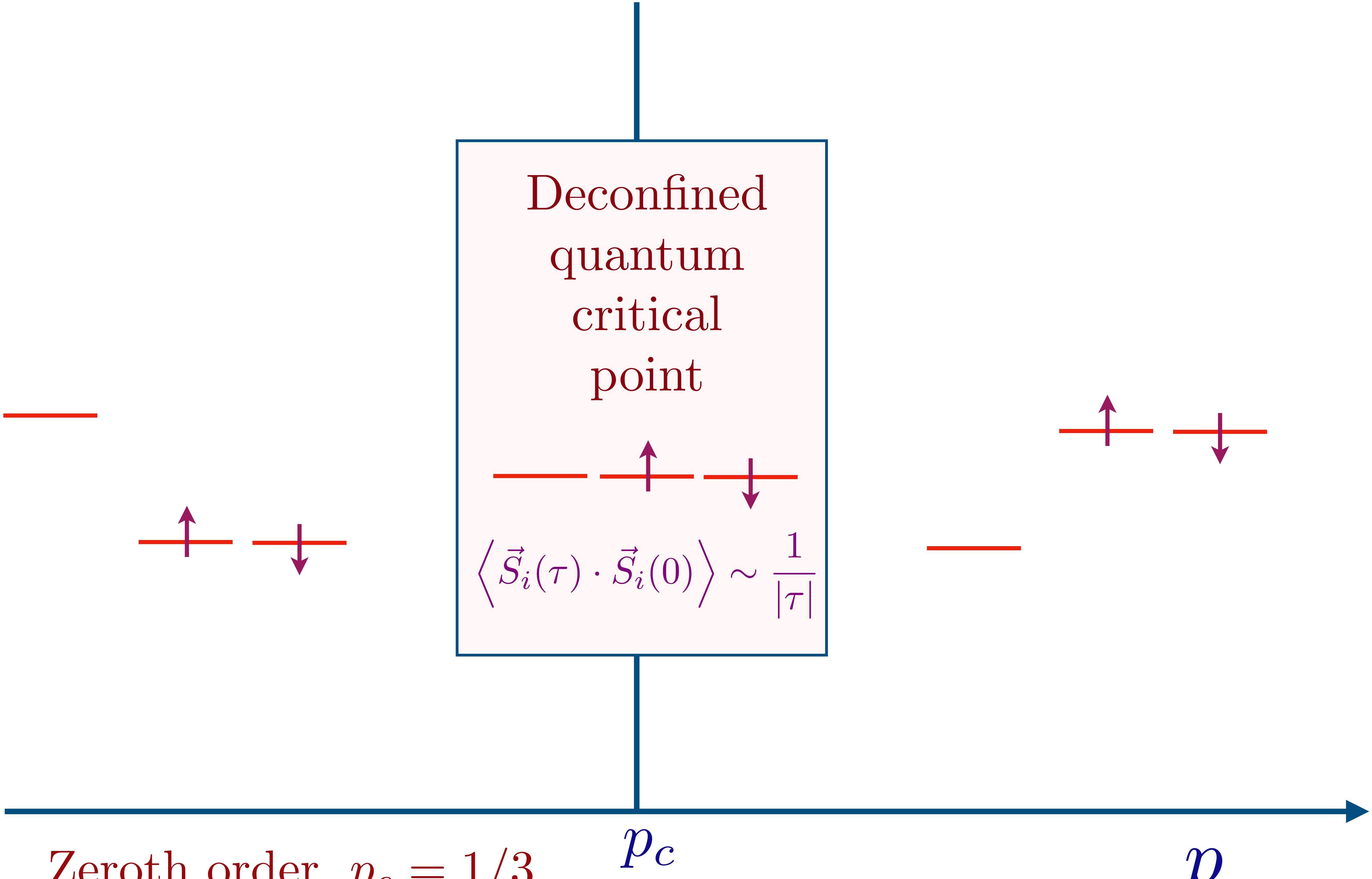
p_c

Zeroth order, $p_c = 1/3$

p

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



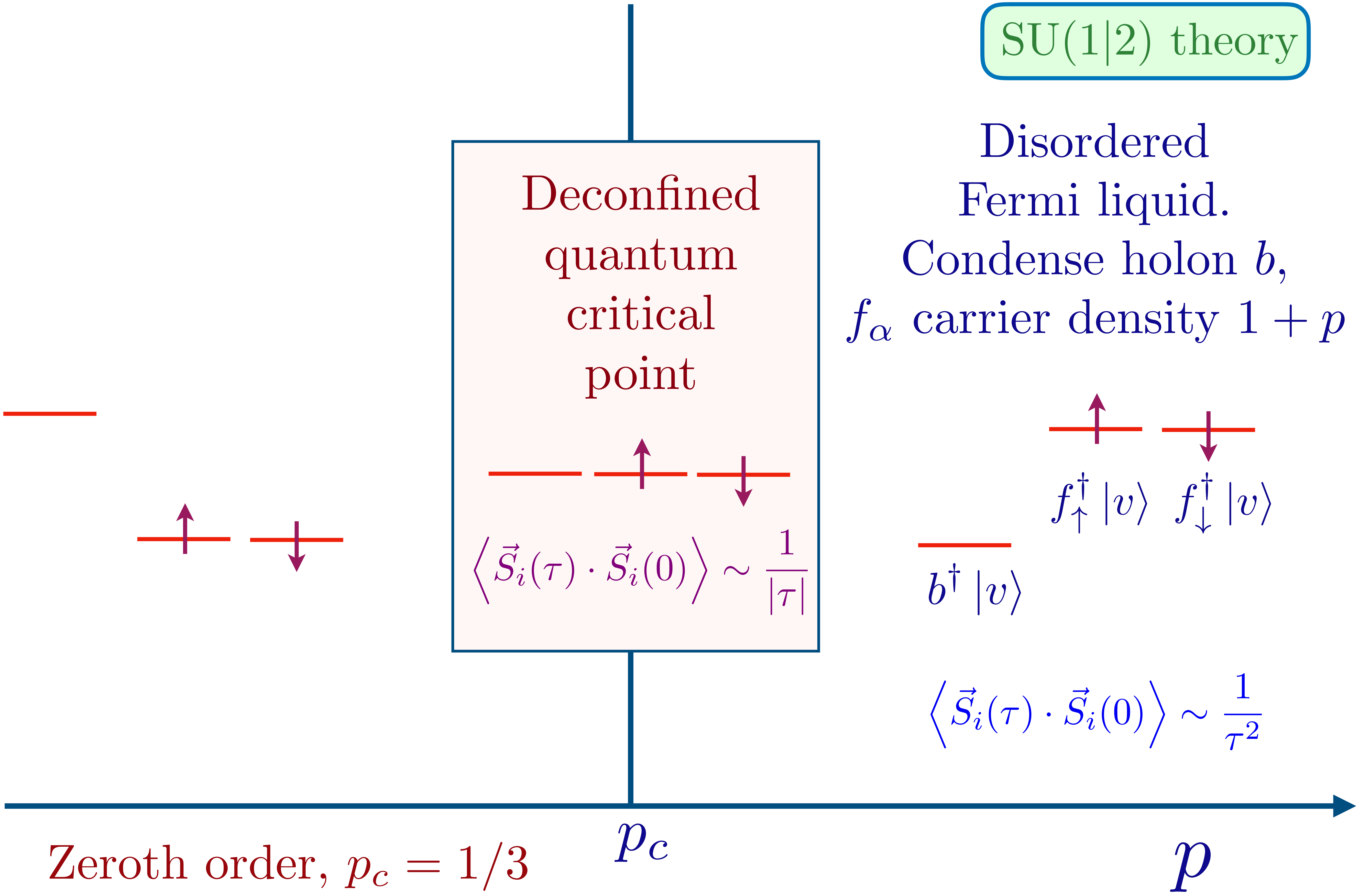
Deconfined
quantum
critical
point

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



SU(1|2) theory

Deconfined
quantum
critical
point

$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \frac{1}{|\tau|}$

Disordered
Fermi liquid.
Condense holon b,
f_α carrier density $1+p$

$f_\uparrow^\dagger\,|v\rangle$ $f_\downarrow^\dagger\,|v\rangle$

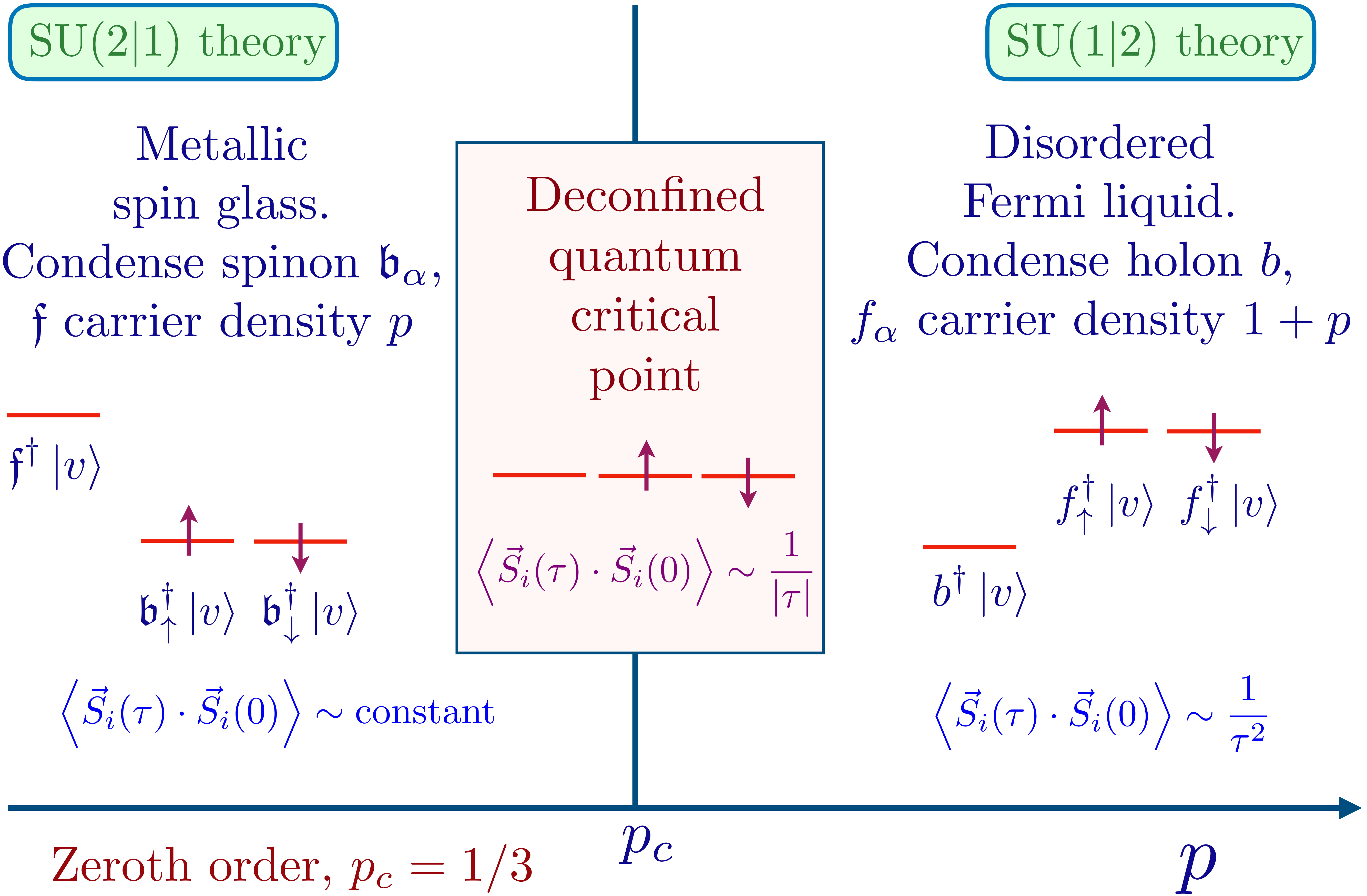
$b^\dagger\,|v\rangle$

$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \frac{1}{\tau^2}$

Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



SU(2|1) theory

Metallic
spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}^\dagger_\uparrow |v\rangle$ $\mathfrak{b}^\dagger_\downarrow |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

Deconfined
quantum
critical
point

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

SU(1|2) theory

Disordered
Fermi liquid.
Condense holon b,
f_α carrier density $1 + p$

$f^\dagger_\uparrow |v\rangle$ $f^\dagger_\downarrow |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

p_c

p

Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

- t-J models with random and all-to-all interactions exhibit a deconfined critical point at a non-zero doping $p = p_c$, flanked by conventional confining phases with carrier densities p and $1 + p$.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

- t-J models with random and all-to-all interactions exhibit a deconfined critical point at a non-zero doping $p = p_c$, flanked by conventional confining phases with carrier densities p and $1 + p$.
- Closely related saddle-point equations were obtained in an "extended dynamical mean-field theory (EDMFT)" of non-random lattice t-J models.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

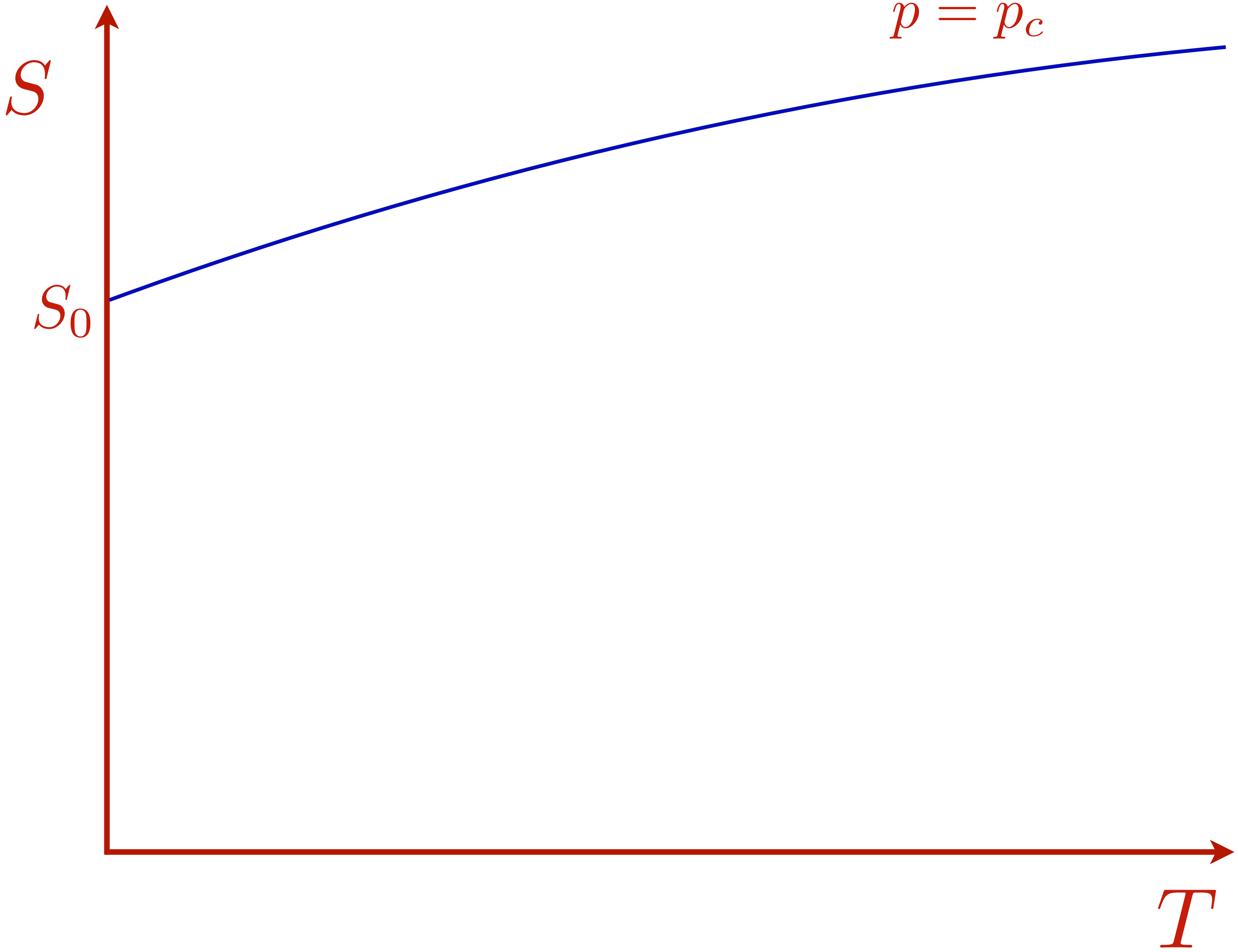
- t-J models with random and all-to-all interactions exhibit a deconfined critical point at a non-zero doping $p = p_c$, flanked by conventional confining phases with carrier densities p and $1 + p$.

- Closely related saddle-point equations were obtained in an "extended dynamical mean-field theory (EDMFT)" of non-random lattice t-J models.

- The structure of the DQCP is similar to the SYK models: both have local spin correlations which decay as $\sim 1/|\tau|$ in imaginary time τ.

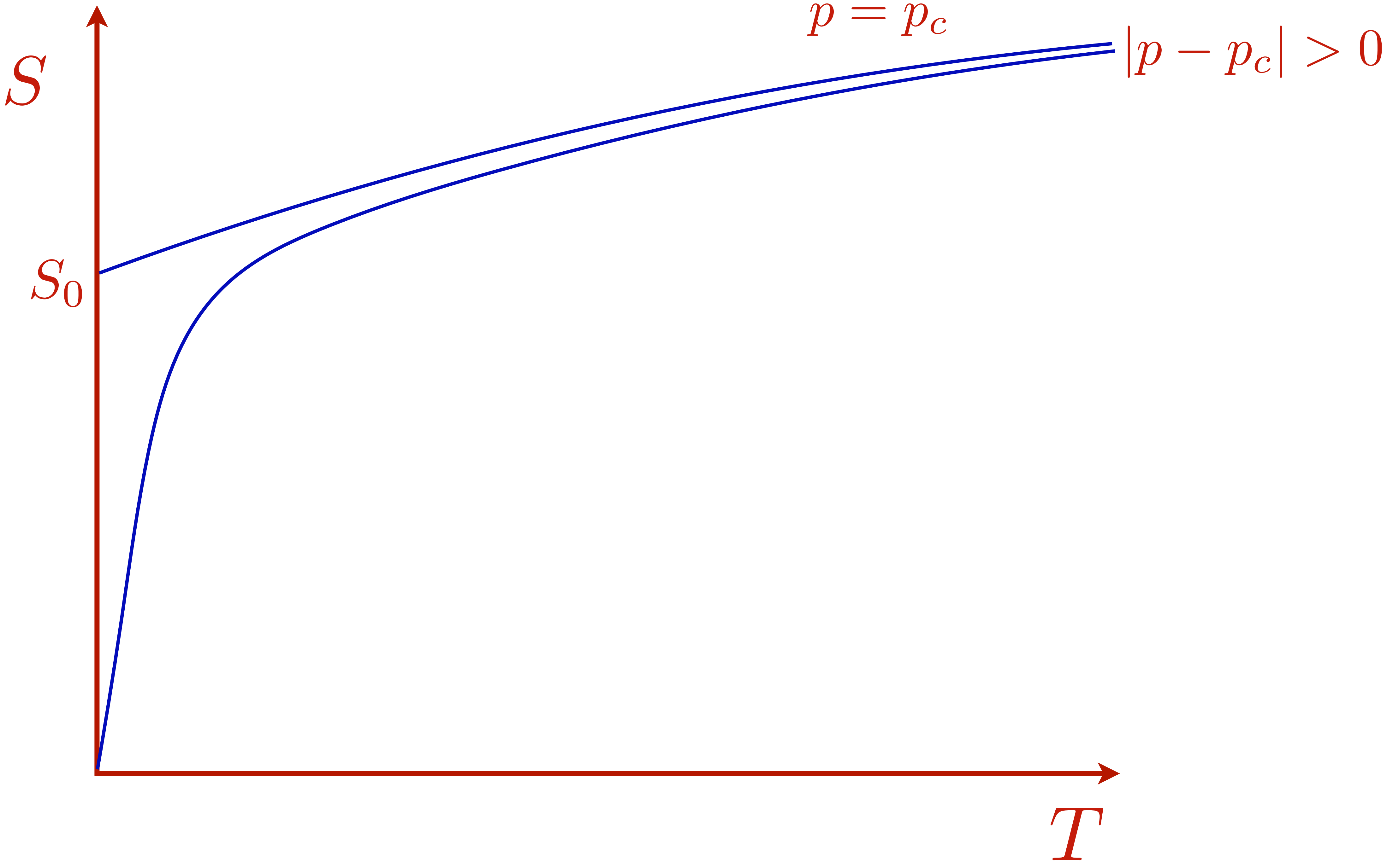
D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-*J* model entropy



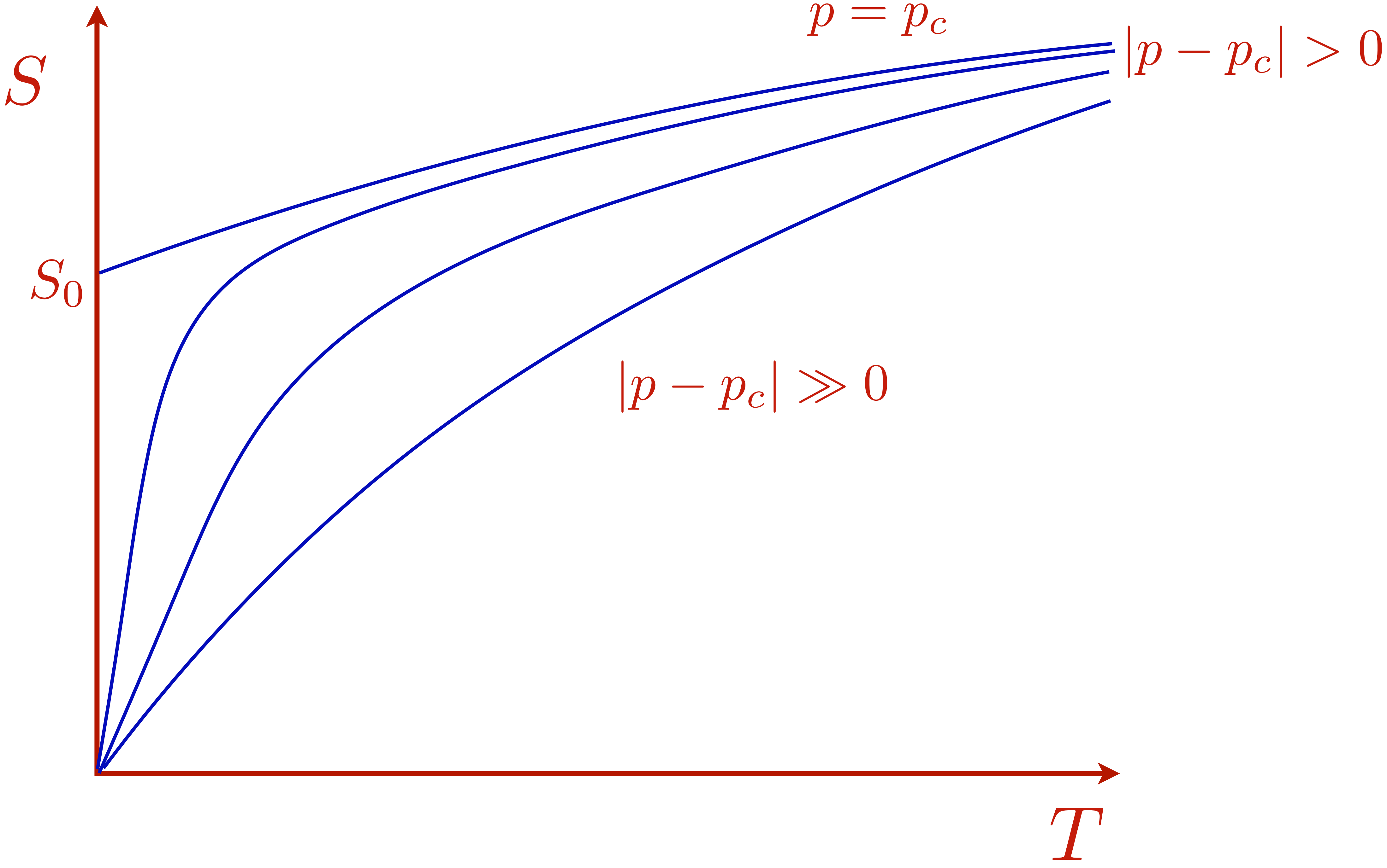
$$\frac{C}{T} = \frac{dS}{dT}$$

t-*J* model entropy



$$\frac{C}{T} = \frac{dS}{dT}$$

t-J model entropy

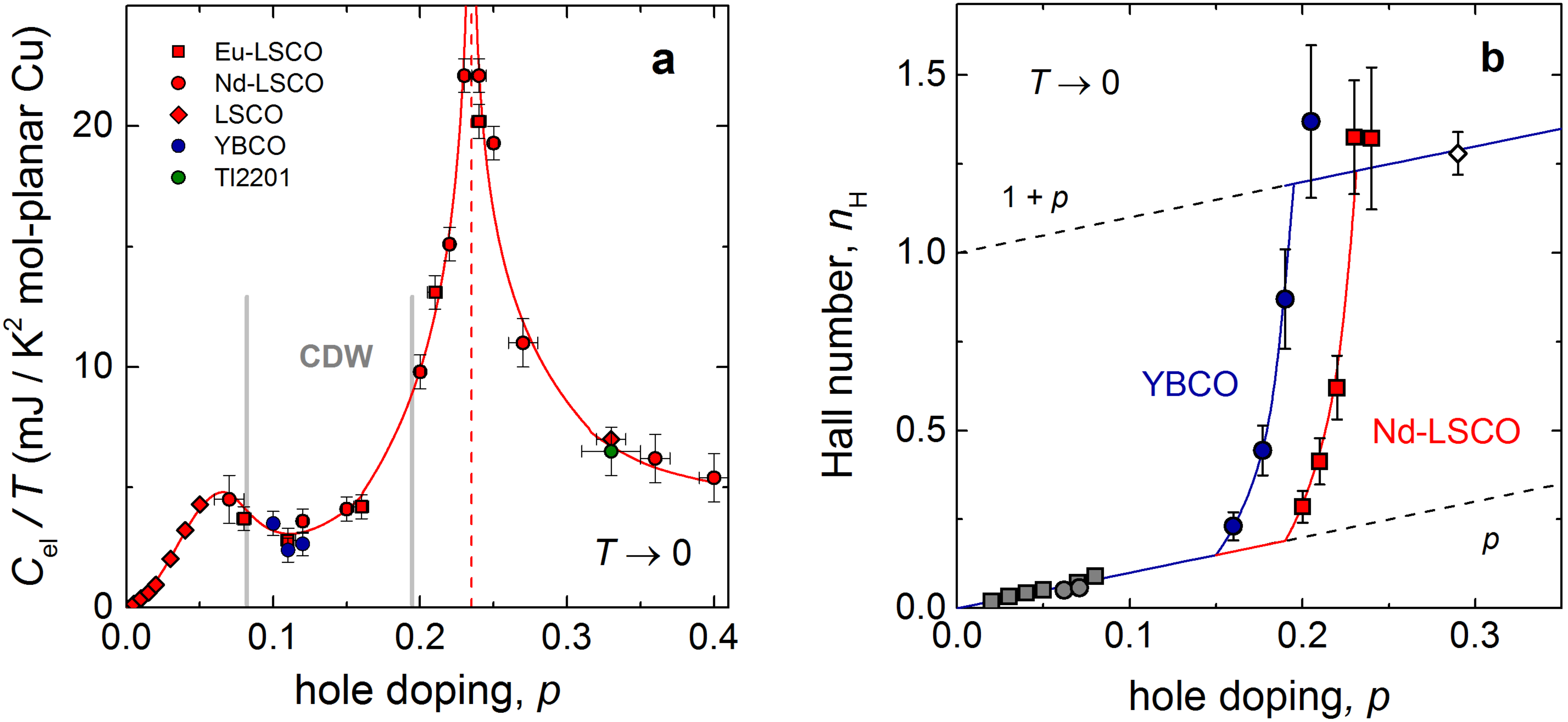


$$\frac{C}{T} = \frac{dS}{dT}$$

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507



1. Deconfined quantum criticality of random *t-J* models

2. RG analysis

3. Transport…

t-J model

$$\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}f_\alpha(\tau)\mathcal{D}b(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}} \\
\mathcal{S}_B &= \int d\tau \left[f_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)f_\alpha(\tau)+b^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)b(\tau)-i\lambda\right] \\
\mathcal{S}_{tJ} &= \int d\tau\, s_0 f_\alpha^\dagger(\tau)f_\alpha(\tau)+t^2\int d\tau d\tau' R(\tau-\tau')c_\alpha^\dagger(\tau)c_\alpha(\tau') \\
&\quad -\frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.
\end{aligned}$$

From this action we determined the correlators

SU(1|2) theory

$$\begin{aligned}
\overline{R}(\tau-\tau') &= -\left\langle c_\alpha(\tau)c_\alpha^\dagger(\tau')\right\rangle_{\mathcal{Z}} \\
\overline{Q}(\tau-\tau') &= \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}
\end{aligned}$$

and finally impose the self-consistency conditions

$$R(\tau)=\overline{R}(\tau) \quad , \quad Q(\tau)=\overline{Q}(\tau).$$

D. Joshi,
Chenyuan Li,
G. Tarnopolsky,
A. Georges,
S. Sachdev,
arXiv:1912.08822

t-J model RG

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} \, .$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a, $a = 1 \ldots 3$) and fermionic (ψ_α) baths.

S. Sachdev, Physica C **357**, 78 (2001)
M. Vojta and L. Fritz, PRB **70**, 094502 (2004)

t-J model RG

SU(1|2) theory

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} \, .$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a, $a = 1 \ldots 3$) and fermionic (ψ_α) baths. Then the problem reduces to the Hamiltonian

$$\begin{aligned} H \quad &= (s_0 + \lambda) f_\alpha^\dagger f_\alpha + \lambda\, b^\dagger b + g_0 \left(f_\alpha^\dagger b\, \psi_\alpha(0) + \text{H.c.} \right) + \gamma_0 f_\alpha^\dagger \frac{\sigma^a_{\alpha\beta}}{2} f_\beta\, \phi_a(0) \\ &+ \int |k|^r dk\, k\, \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right] \end{aligned}$$

where $a = (x, y, z)$, σ^a are Pauli matrices, π_a is canonically conjugate to the field ϕ_a, and $\phi_a(0) \equiv \phi_a(x=0)$, $\psi_\alpha(0) \equiv \int |k|^r dk\, \psi_{k\alpha}$.

S. Sachdev, Physica C **357**, 78 (2001)
M. Vojta and L. Fritz, PRB **70**, 094502 (2004)

t-J model RG

SU(1|2) theory

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{r+1}} \,.$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a, $a = 1 \ldots 3$) and fermionic (ψ_α) baths. Then the problem reduces to the Hamiltonian

$$\begin{aligned} H \quad &= (s_0 + \lambda) f_\alpha^\dagger f_\alpha + \lambda\, b^\dagger b + g_0 \left(f_\alpha^\dagger b\, \psi_\alpha(0) + \mathrm{H.c.} \right) + \gamma_0 f_\alpha^\dagger \frac{\sigma^a_{\alpha\beta}}{2} f_\beta\, \phi_a(0) \\ &+ \int |k|^r dk\, k\, \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right] \end{aligned}$$

where $a = (x, y, z)$, σ^a are Pauli matrices, π_a is canonically conjugate to the field ϕ_a, and $\phi_a(0) \equiv \phi_a(x = 0)$, $\psi_\alpha(0) \equiv \int |k|^r dk\, \psi_{k\alpha}$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and $R(\tau)$ with the temporal correlator of $\psi_\alpha(0)$, and it can be verified that these correlators decay as above.

S. Sachdev, Physica C **357**, 78 (2001)
M. Vojta and L. Fritz, PRB **70**, 094502 (2004)

t-J model RG

SU(1|2) theory

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\operatorname{sgn}(\tau)}{|\tau|^{r+1}} \, .$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a, $a = 1 \ldots 3$) and fermionic (ψ_α) baths. Then the problem reduces to the Hamiltonian

$$\begin{aligned} H \quad &= (s_0 + \lambda) f_\alpha^\dagger f_\alpha + \lambda\, b^\dagger b + g_0 \left(f_\alpha^\dagger b\, \psi_\alpha(0) + \text{H.c.} \right) + \gamma_0 f_\alpha^\dagger \frac{\sigma^a_{\alpha\beta}}{2} f_\beta\, \phi_a(0) \\ &+ \int |k|^r dk\, k\, \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right] \end{aligned}$$

The impurity superspin is coupled to
a fermionic bath by g_0,
and to a bosonic bath by γ_0,
and s_0 acts as a local field on the superspin -
a superKondo problem!

t-J model RG

SU(2|1) theory

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} \, .$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a, $a = 1 \ldots 3$) and fermionic (ψ_α) baths. Then the problem reduces to the Hamiltonian

$$\begin{aligned} H &= (s_0 + \lambda) \mathfrak{b}^\dagger_\alpha \mathfrak{b}_\alpha + \lambda \, \mathfrak{f}^\dagger \mathfrak{f} + g_0 \left(\mathfrak{b}^\dagger_\alpha \mathfrak{f} \, \psi_\alpha(0) + \text{H.c.} \right) + \gamma_0 \mathfrak{b}^\dagger_\alpha \frac{\sigma^a_{\alpha\beta}}{2} \mathfrak{b}_\beta \, \phi_a(0) \\ &\quad + \int |k|^r dk \, k \, \psi^\dagger_{k\alpha} \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right] \end{aligned}$$

The impurity superspin is coupled to
a fermionic bath by g_0,
and to a bosonic bath by γ_0,
and s_0 acts as a local field on the superspin -
a superKondo problem!

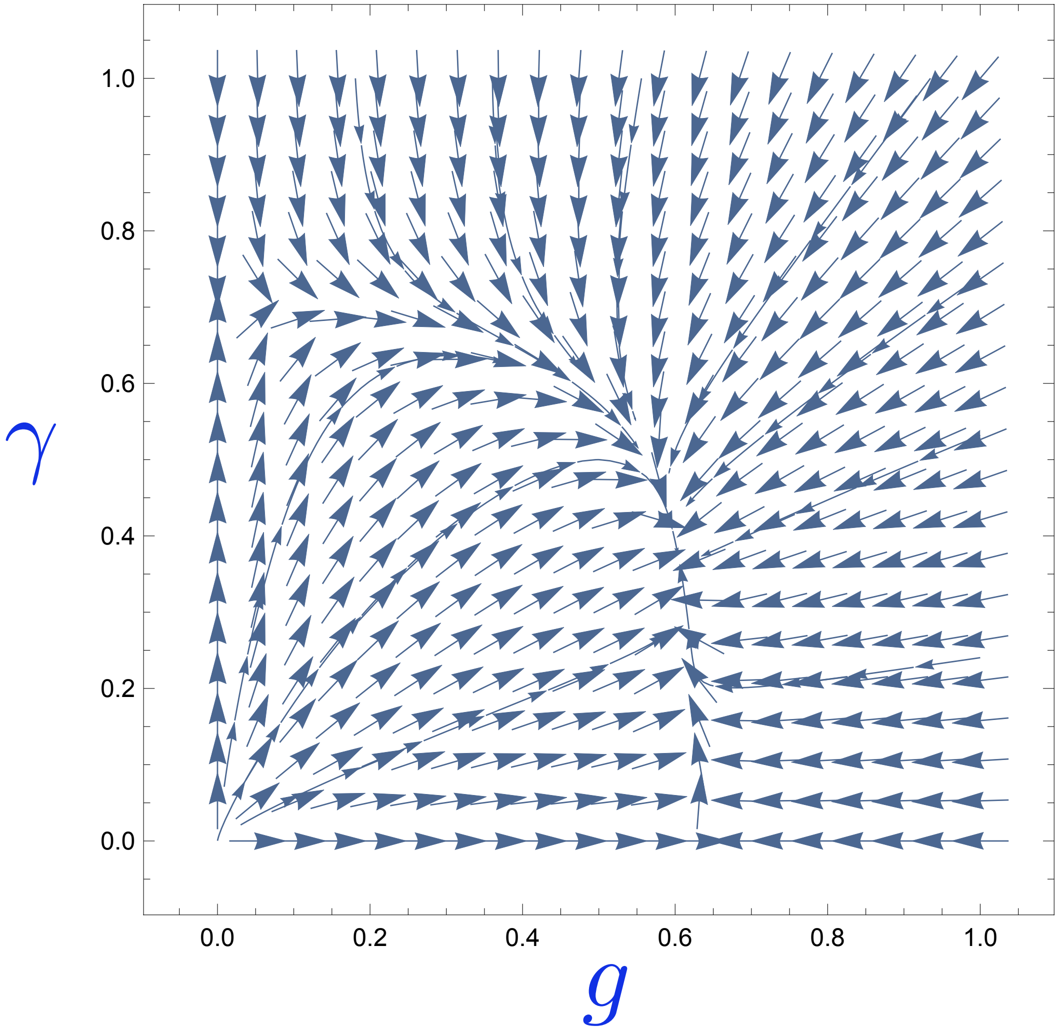
t-J model RG

We can perform a RG analysis for small $\epsilon = 3 - d$ and $\bar{r} = (1-r)/2$, while imposing the local constraint *exactly*. The one-loop β functions are

$$\beta(g) = -\bar{r}g + \frac{3}{2}g^3 + \frac{3}{8}g\gamma^2\,,$$

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 + g^2\gamma\,.$$

$$\beta(s) = -s + 3g^2 s - g^2 + \frac{3}{4}\gamma^2\,.$$



These equations have a fixed point with $s \approx 0$ with only one relevant direction, corresponding to the flow of s to $\pm\infty$.

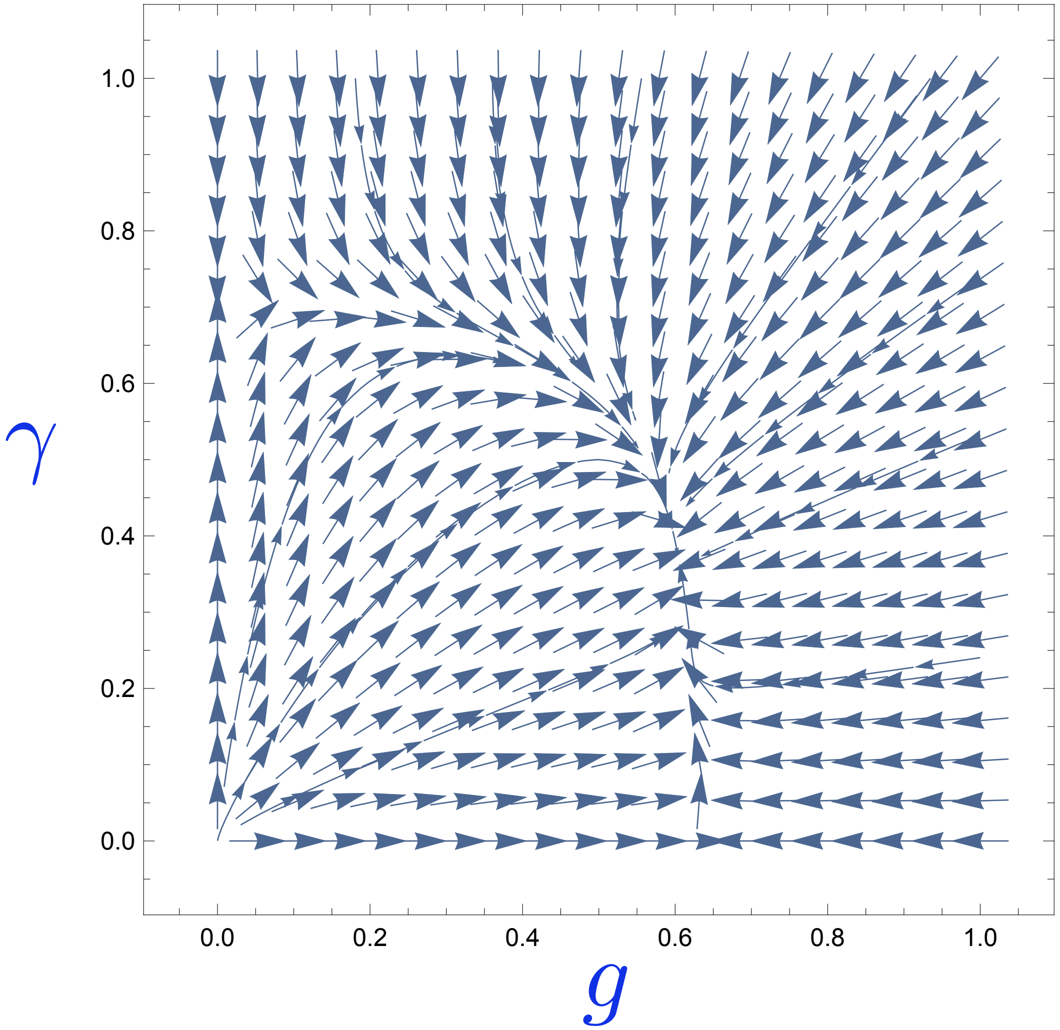
t-J model RG

We can perform a RG analysis for small $\epsilon = 3 - d$ and $\bar{r} = (1-r)/2$, while imposing the local constraint *exactly*. The one-loop β functions are

$$\beta(g) = -\bar{r}g + \frac{3}{2}g^3 + \frac{3}{8}g\gamma^2 \, ,$$

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 + g^2\gamma \, .$$

$$\beta(s) = -s + 3g^2 s - g^2 + \frac{3}{4}\gamma^2 \, .$$



These equations have a fixed point with $s \approx 0$ with only one relevant direction, corresponding to the flow of s to $\pm\infty$. The 3 states of the superspin are nearly degenerate at the fixed point, and the flows away from the fixed point correspond to different orientations of the field on the superspin: one side (overdoped) favors the holon, and the other side (underdoped) favors the spinon.

t-J model RG

The scaling dimensions of the electron and spin operators can be determined to all orders in ϵ and $\bar{r}$ and these imply

$$\overline{R}(\tau) = -\frac{1}{2}\left\langle c_\alpha(\tau) c_\alpha^\dagger(0)\right\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{1-r}} \quad , \quad \overline{Q}(\tau) = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|^{3-d}}\,.$$

t-J model RG

The scaling dimensions of the electron and spin operators can be determined to all orders in ϵ and $\bar{r}$ and these imply

$$\overline{R}(\tau) = -\frac{1}{2}\left\langle c_\alpha(\tau)c_\alpha^\dagger(0)\right\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{1-r}} \quad , \quad \overline{Q}(\tau) = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|^{3-d}}\,.$$

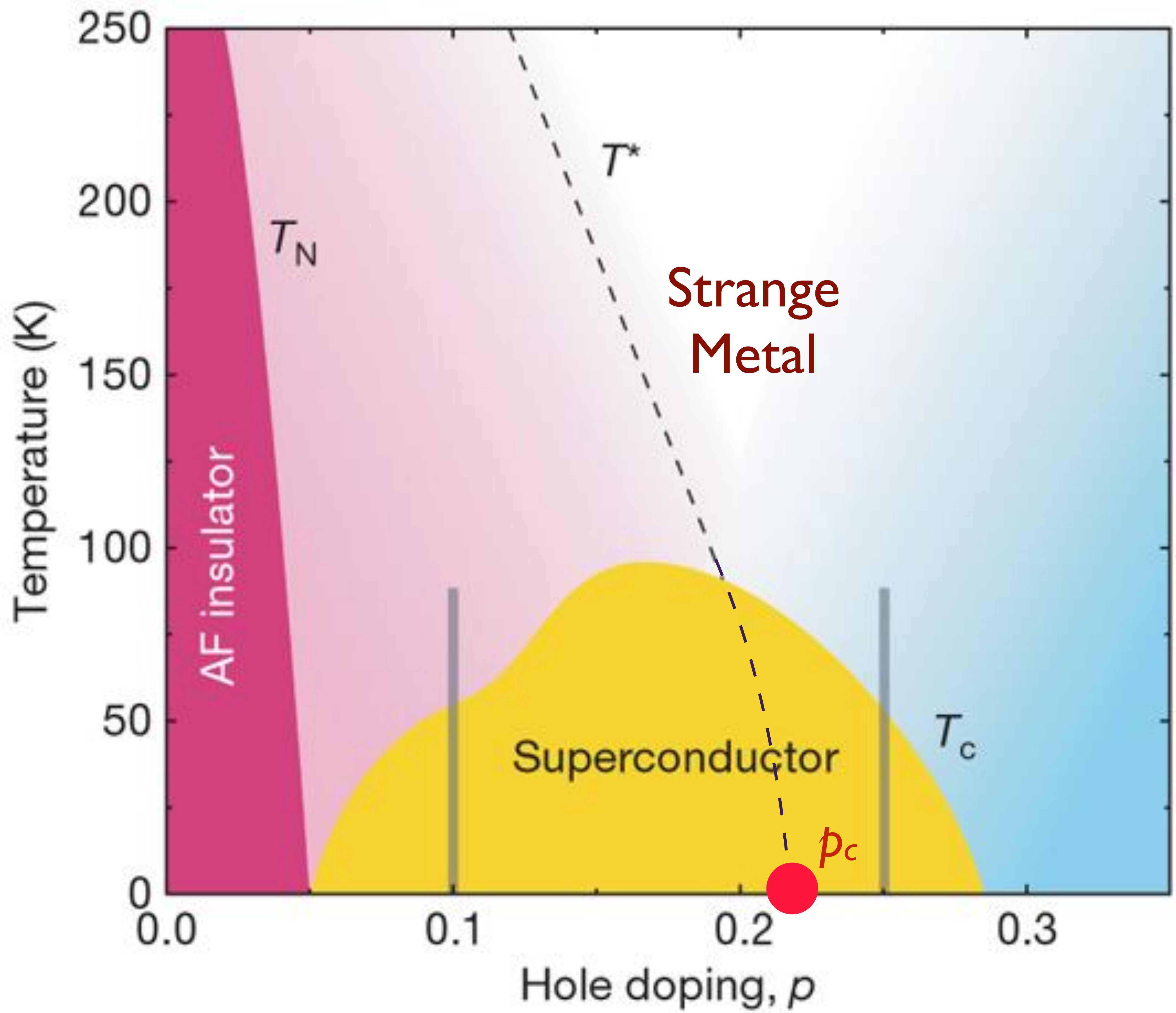
Finally, we impose the self-consistency conditions $R(\tau) = \overline{R}(\tau)$, $Q(\tau) = \overline{Q}(\tau)$ and obtain $r = 0$ ($\bar{r} = 1/2$) and $d = 2$ ($\epsilon = 1$), so that at the critical point we have

$$\left\langle c_\alpha(\tau)c_\alpha^\dagger(0)\right\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|} \quad , \quad \left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}\,.$$

1. Deconfined quantum criticality of random *t*-*J* models

2. RG analysis

3. Transport…


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Strange Metal
Superconductor
T_c
p_c

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau}$$

with a universal scattering rate

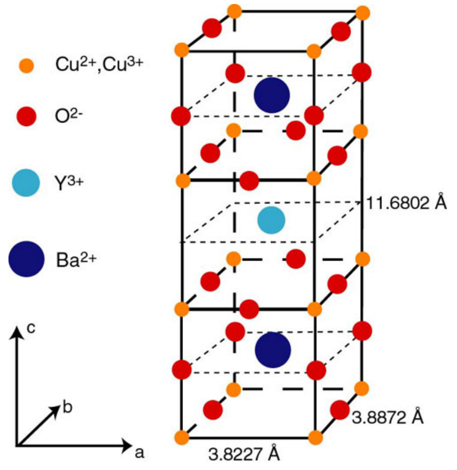
$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

independent of the strength of interactions!

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

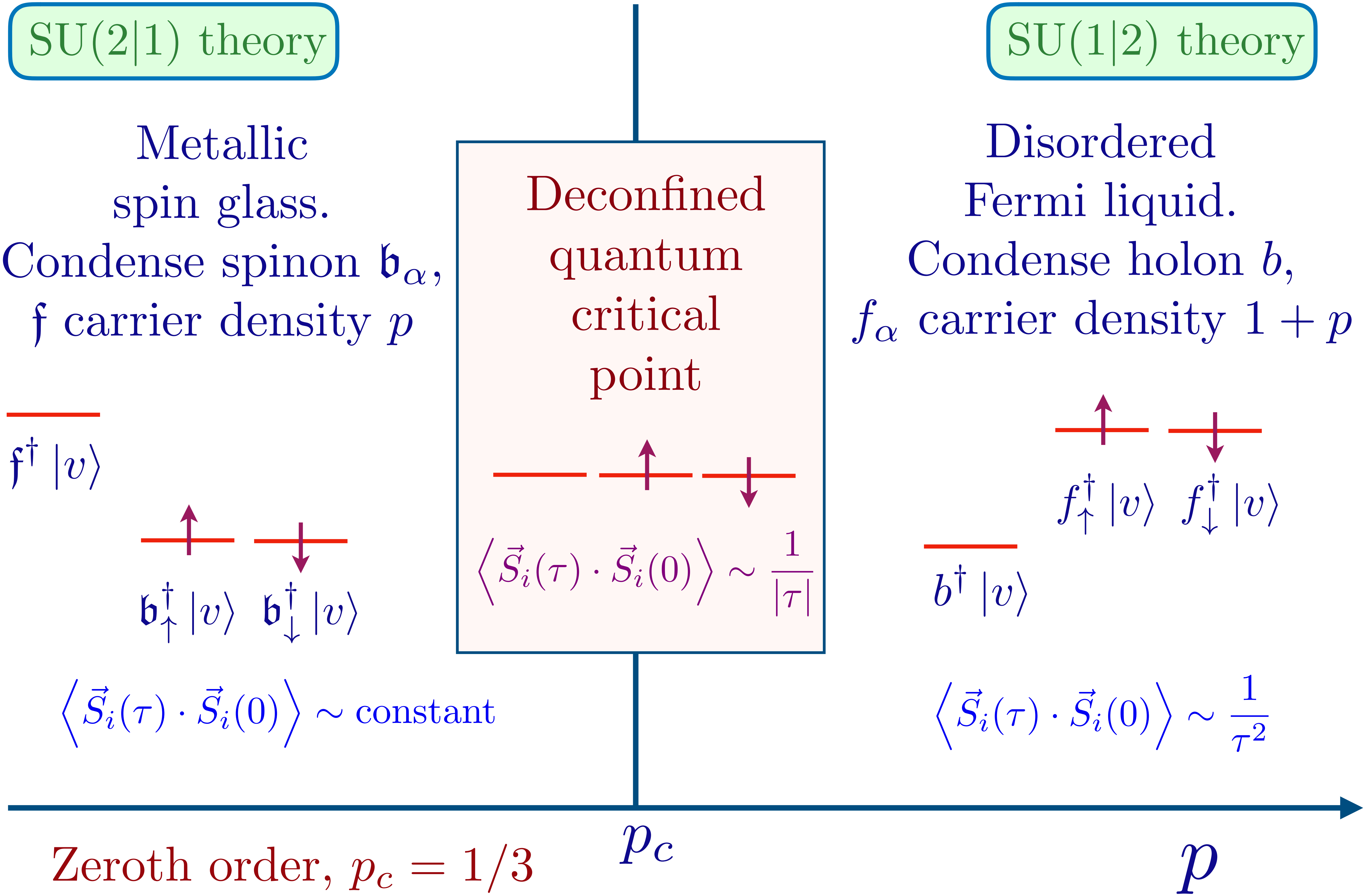
Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

t-J model phase diagram



SU(2|1) theory

Metallic
spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger \left|v\right\rangle$

$\mathfrak{b}^\dagger_\uparrow \left|v\right\rangle$ $\mathfrak{b}^\dagger_\downarrow \left|v\right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

Deconfined
quantum
critical
point

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

SU(1|2) theory

Disordered
Fermi liquid.
Condense holon b,
f_α carrier density $1 + p$

$f^\dagger_\uparrow \left|v\right\rangle$ $f^\dagger_\downarrow \left|v\right\rangle$

$b^\dagger \left|v\right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

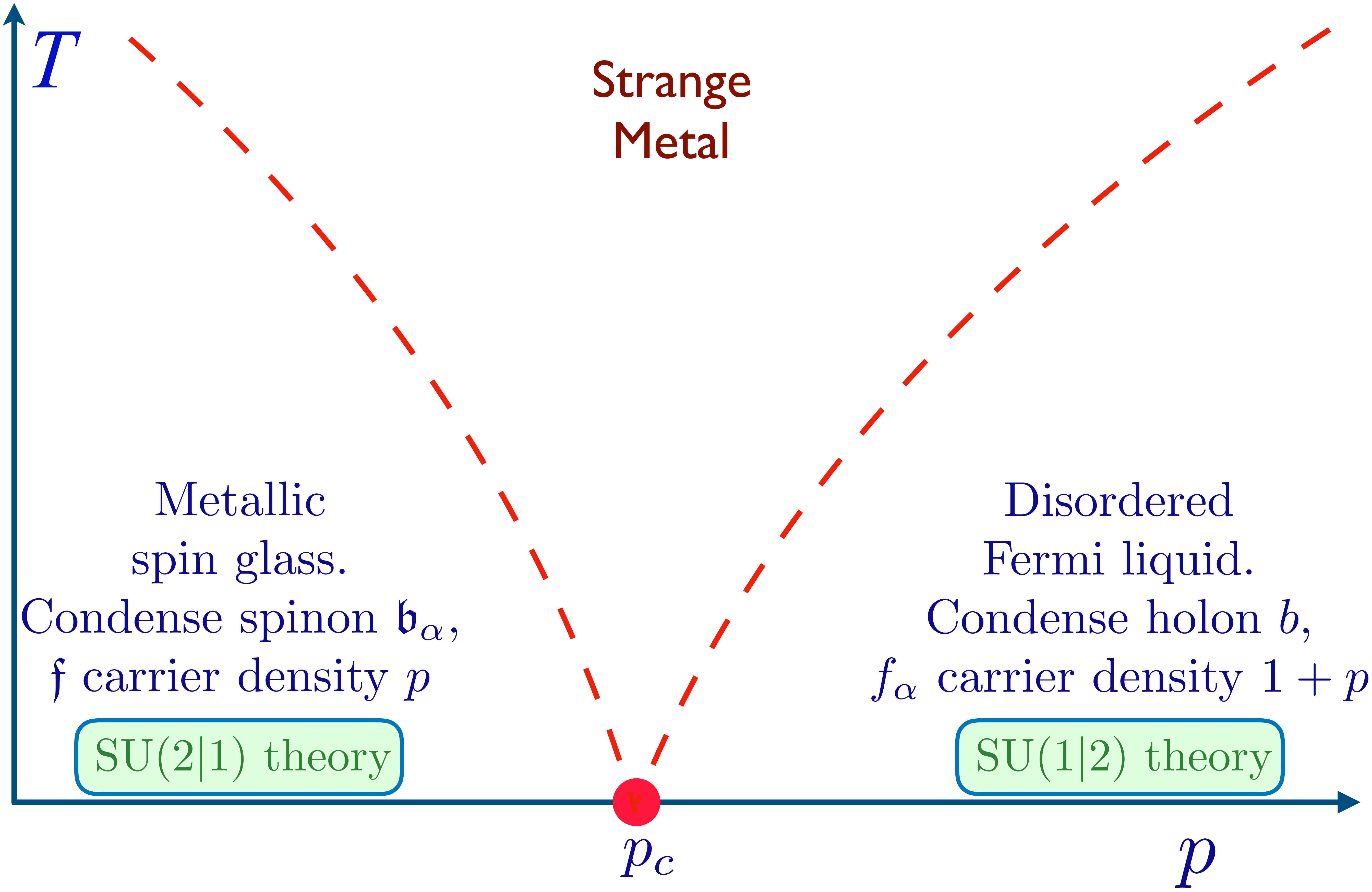
p_c

p

Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



Strange
Metal

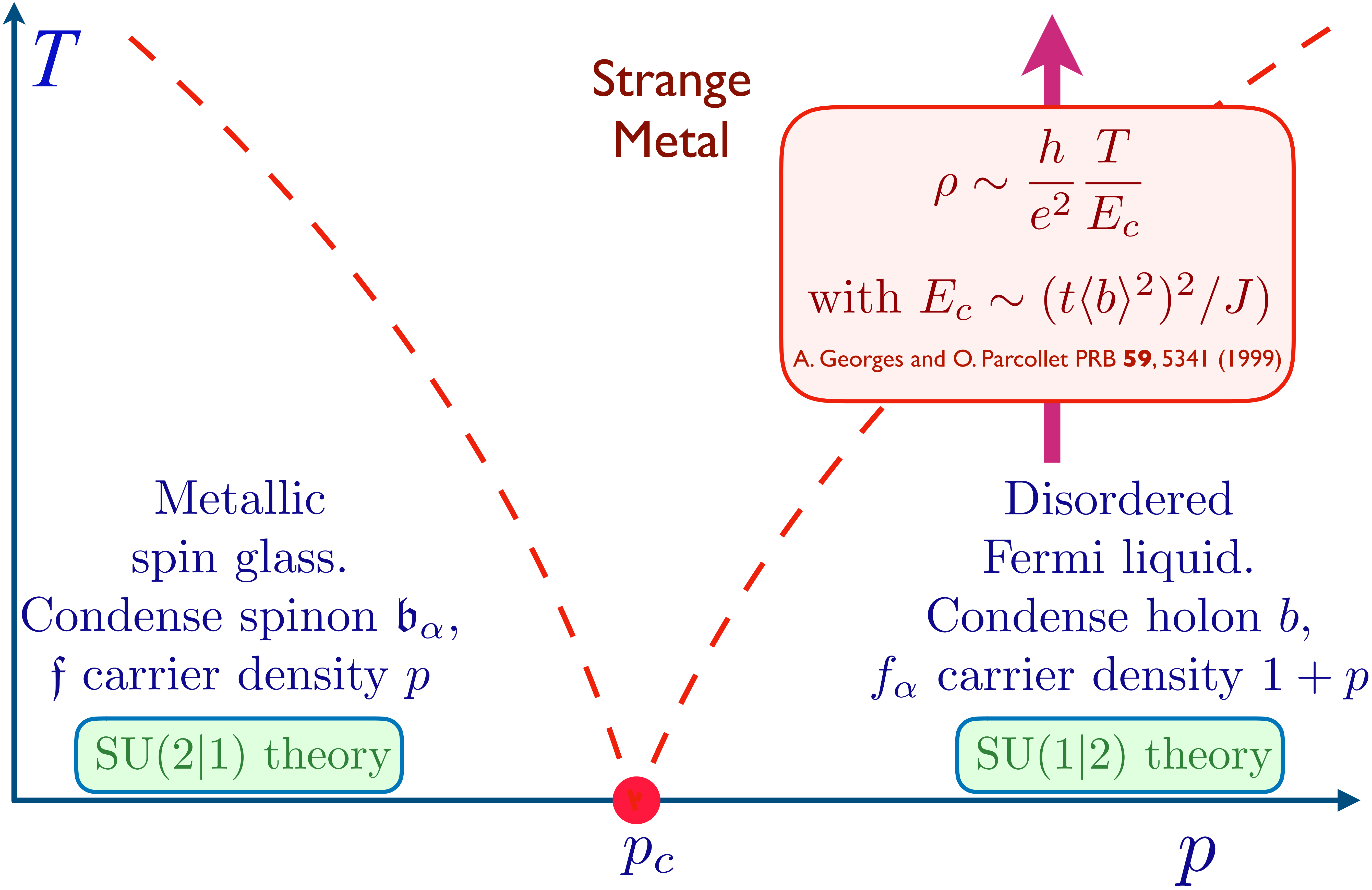
Metallic
spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

SU$(2|1)$ theory

Disordered
Fermi liquid.
Condense holon b,
f_α carrier density $1+p$

SU$(1|2)$ theory

t-J model phase diagram



Strange Metal

$$\rho \sim \frac{h}{e^2}\frac{T}{E_c}$$

with $E_c \sim (t\langle b\rangle^2)^2/J)$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

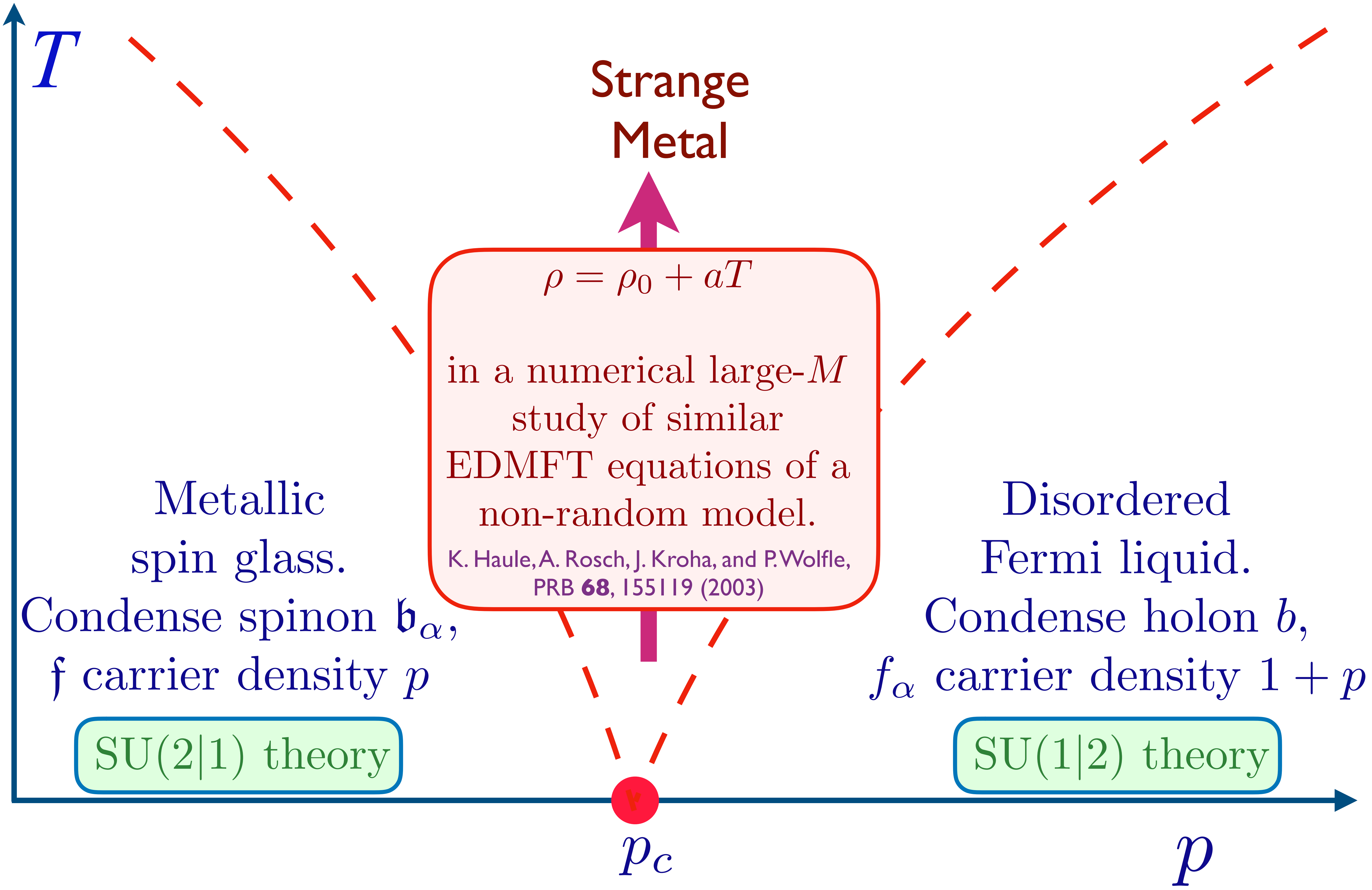
Metallic
spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

SU(2|1) theory

Disordered
Fermi liquid.
Condense holon b,
f_α carrier density $1+p$

SU(1|2) theory

t-J model phase diagram



Strange Metal

$\rho = \rho_0 + aT$

in a numerical large-M study of similar EDMFT equations of a non-random model.

K. Haule, A. Rosch, J. Kroha, and P. Wolfle, PRB **68**, 155119 (2003)

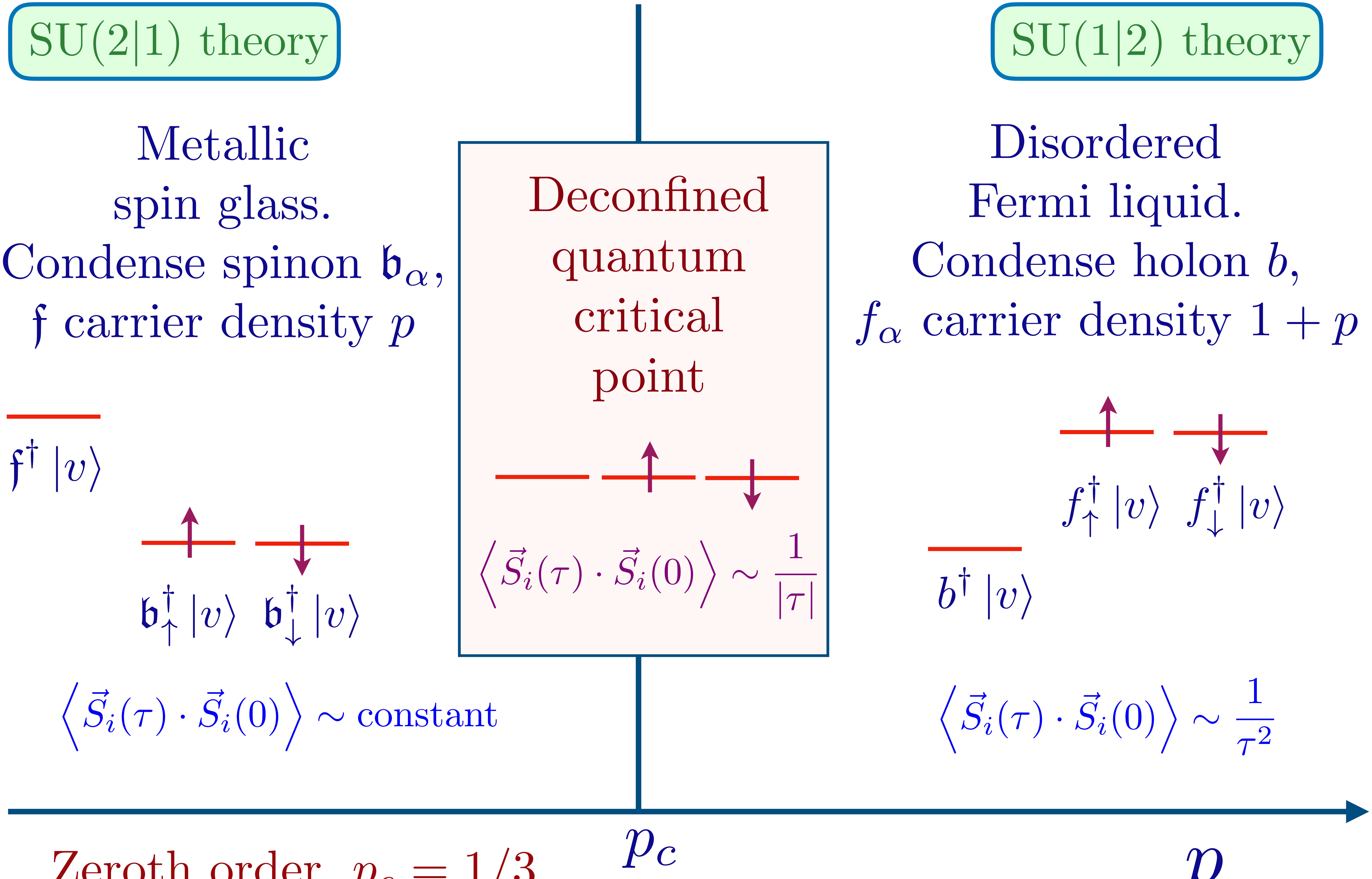
Metallic spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

SU(2|1) theory

Disordered Fermi liquid.
Condense holon b,
f_α carrier density $1 + p$

SU(1|2) theory

t-J model phase diagram



SU(2|1) theory

SU(1|2) theory

Metallic
spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger \left|v\right\rangle$

$\mathfrak{b}_\uparrow^\dagger \left|v\right\rangle$ $\mathfrak{b}_\downarrow^\dagger \left|v\right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

Deconfined
quantum
critical
point

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Disordered
Fermi liquid.
Condense holon b,
f_α carrier density $1 + p$

$f_\uparrow^\dagger \left|v\right\rangle$ $f_\downarrow^\dagger \left|v\right\rangle$

$b^\dagger \left|v\right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822