Confinement-deconfinement transitions in Z_2 gauge theories, and deconfined criticality

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1. Z_2 lattice gauge theory and topological order

2. The Ising* confinement transition

3. "Odd" Z_2 lattice gauge theory and deconfined criticality with an emergent U(1) gauge field

4. Z_2 lattice gauge theory with fermions at half-filling, and deconfined criticality with an emergent SU(2) gauge field

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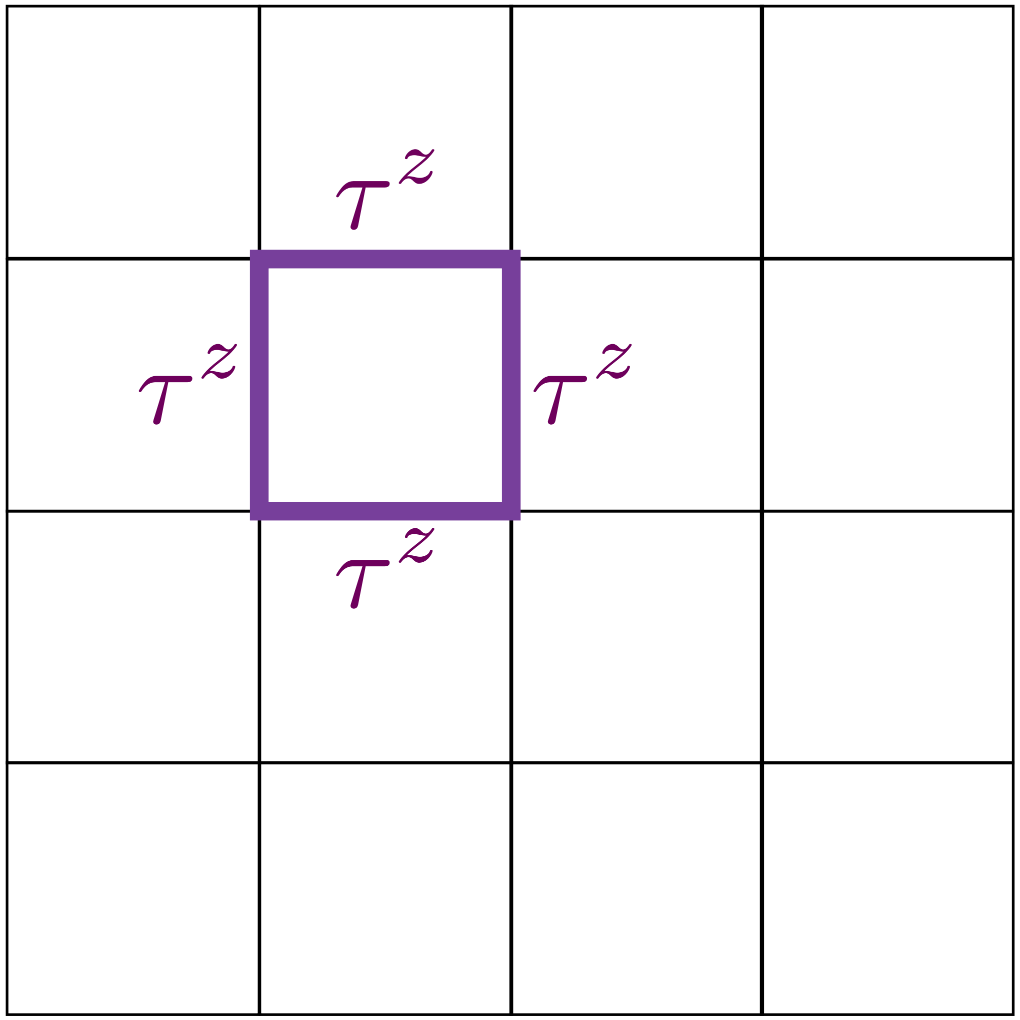
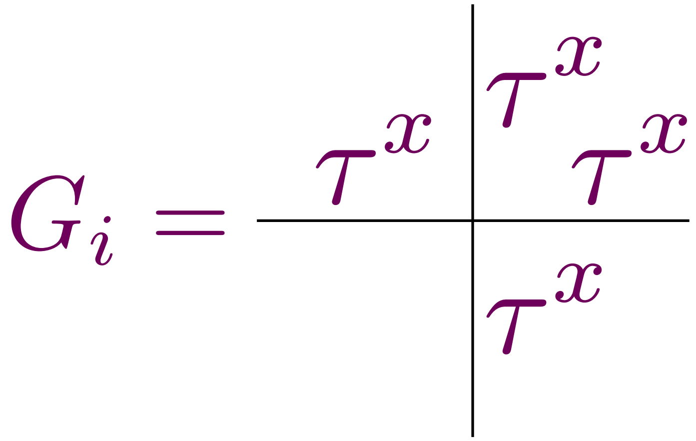
Z_2 lattice gauge theory (Wegner, 1971)



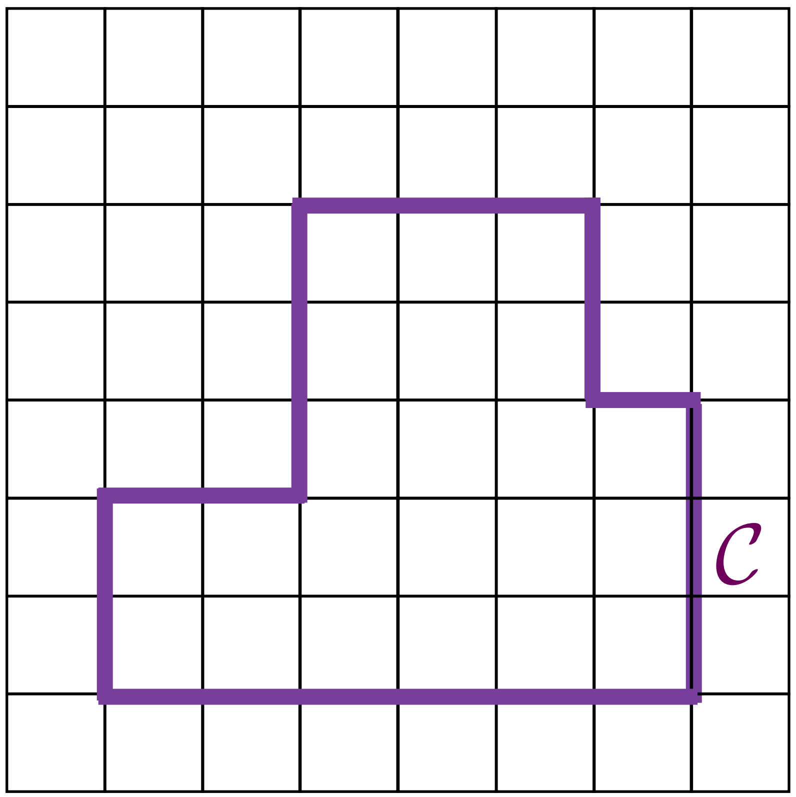
$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$



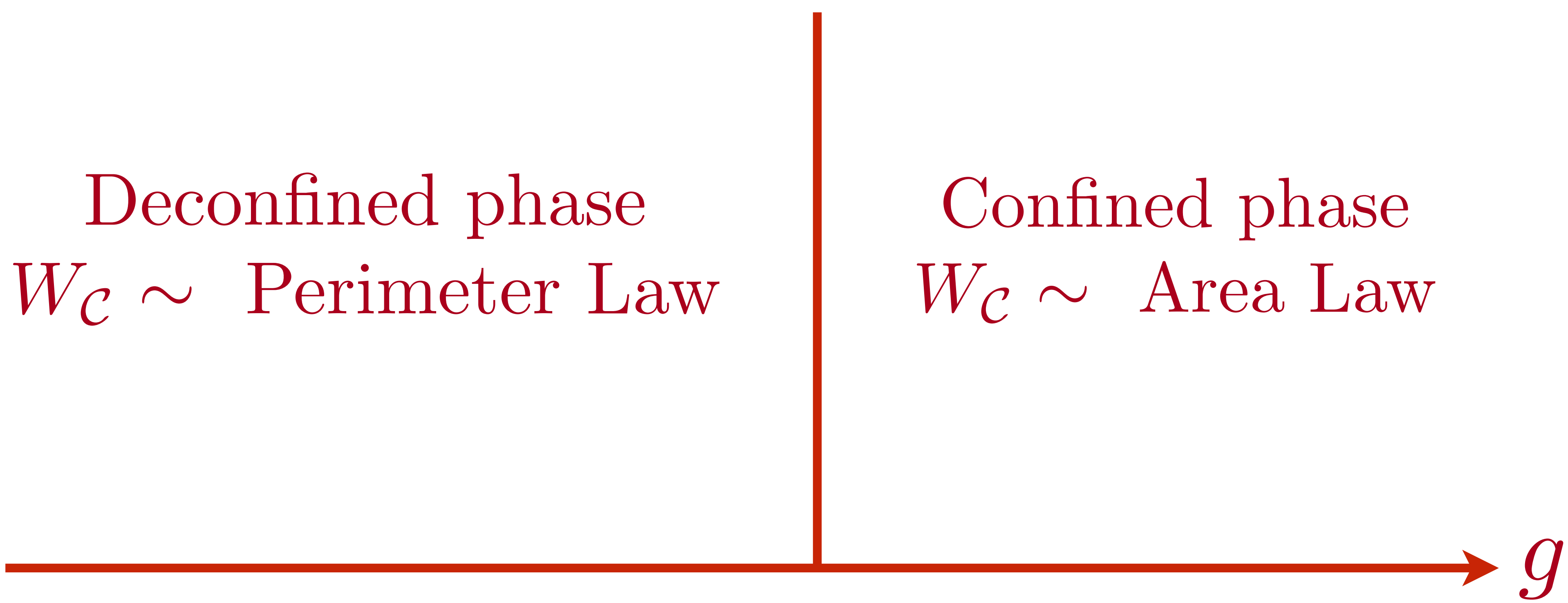
Gauss's Law: $[H, G_i] = 0 \quad , \quad G_i = 1$

Z_2 lattice gauge theory

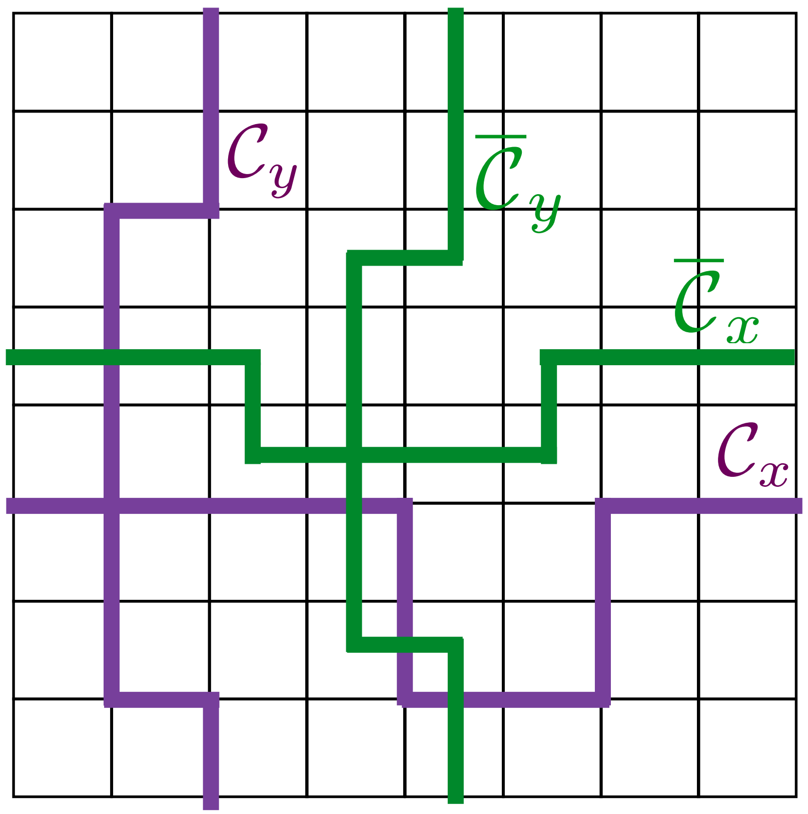
(Wegner, 1971)



$$W_{\mathcal{C}} = \prod_{\mathcal{C}} \tau^z$$



Topological order



$$V_x = \prod_{\overline{\mathcal{C}}_x} \tau^x \quad , \quad V_y = \prod_{\overline{\mathcal{C}}_y} \tau^x$$

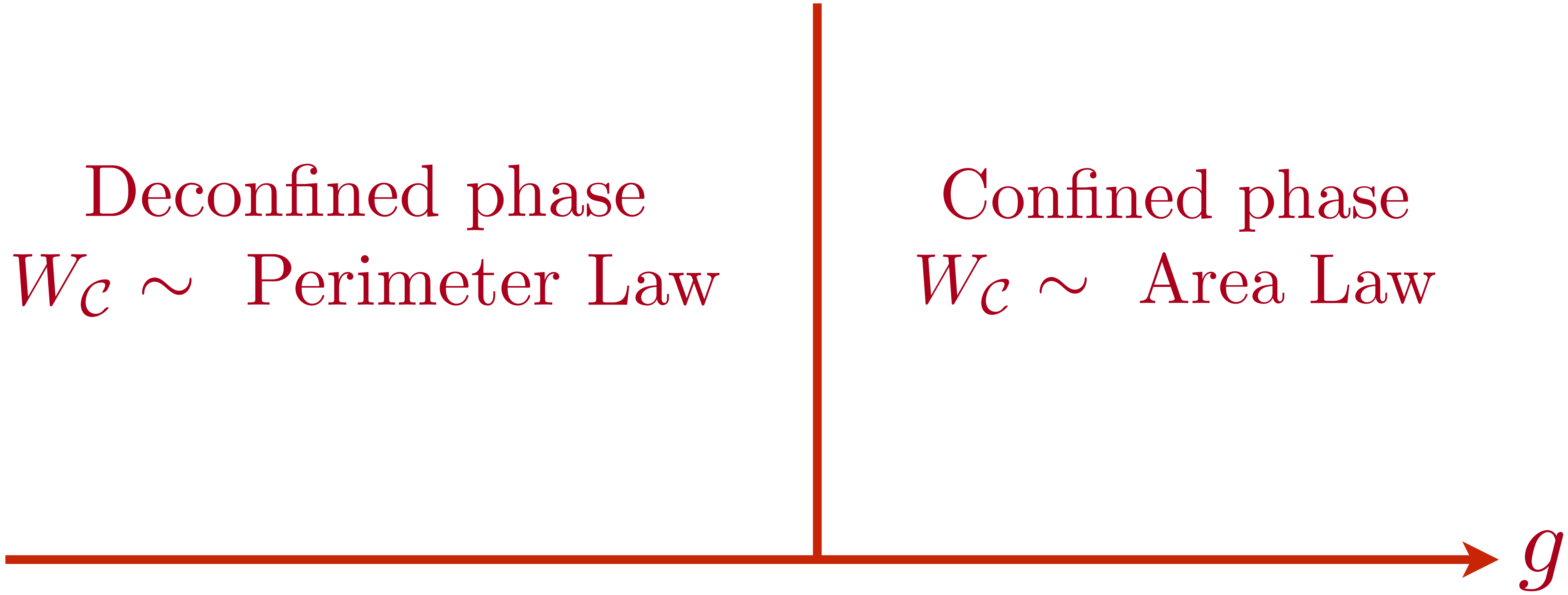
$$W_x = \prod_{\mathcal{C}_x} \tau^z \quad , \quad W_y = \prod_{\mathcal{C}_y} \tau^z$$

$$V_x W_y = - W_y V_x \quad , \quad V_y W_x = - W_x V_y$$

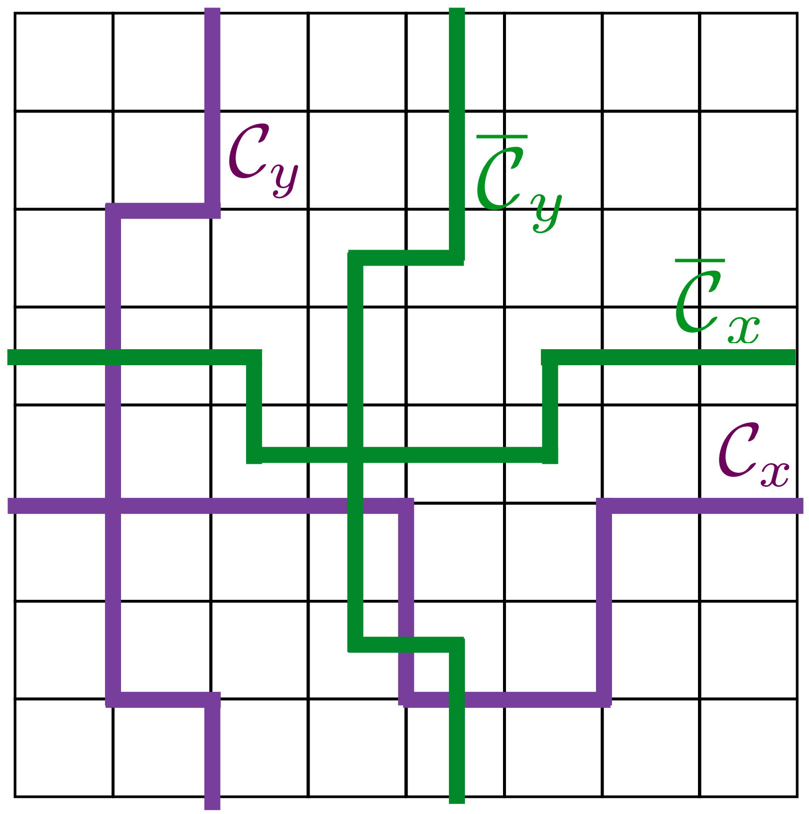
and all other pairs commute.

On a torus, there are two additional independent operators, V_x and V_y which commute with the Hamiltonian:

$$[H, V_x] = [H, V_y] = 0$$



Topological order



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Deconfined phase.
4-fold degenerate ground state: $V_x = \pm 1$, $V_y = \pm 1$.
Can take linear combinations to make eigenstates with $W_x = \pm 1$, $W_y = \pm 1$.
Topological order

Confined phase.
Unique ground state has $V_x = 1$, $V_y = 1$.
No topological order

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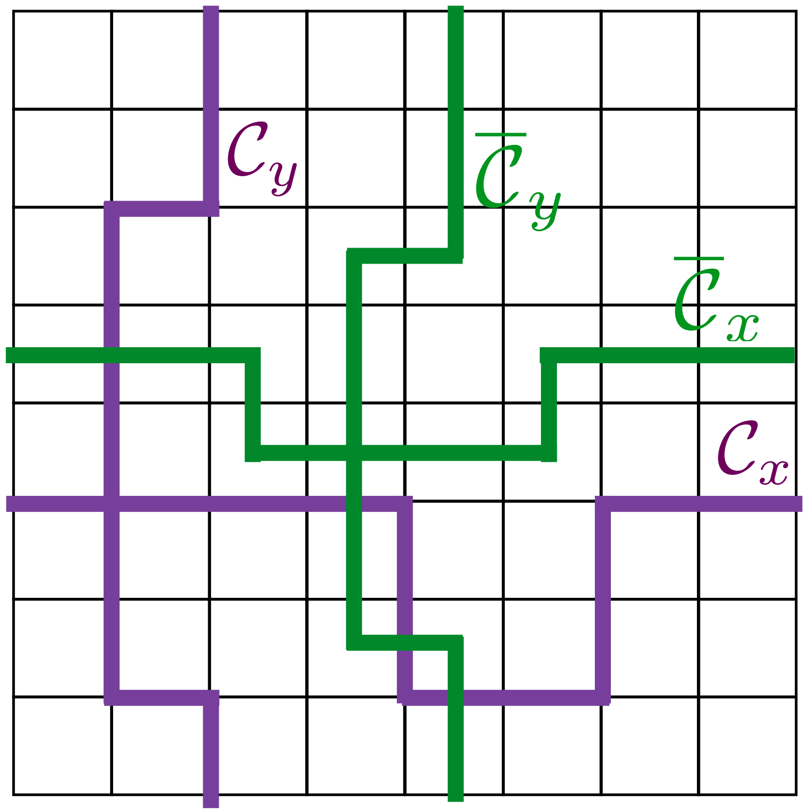

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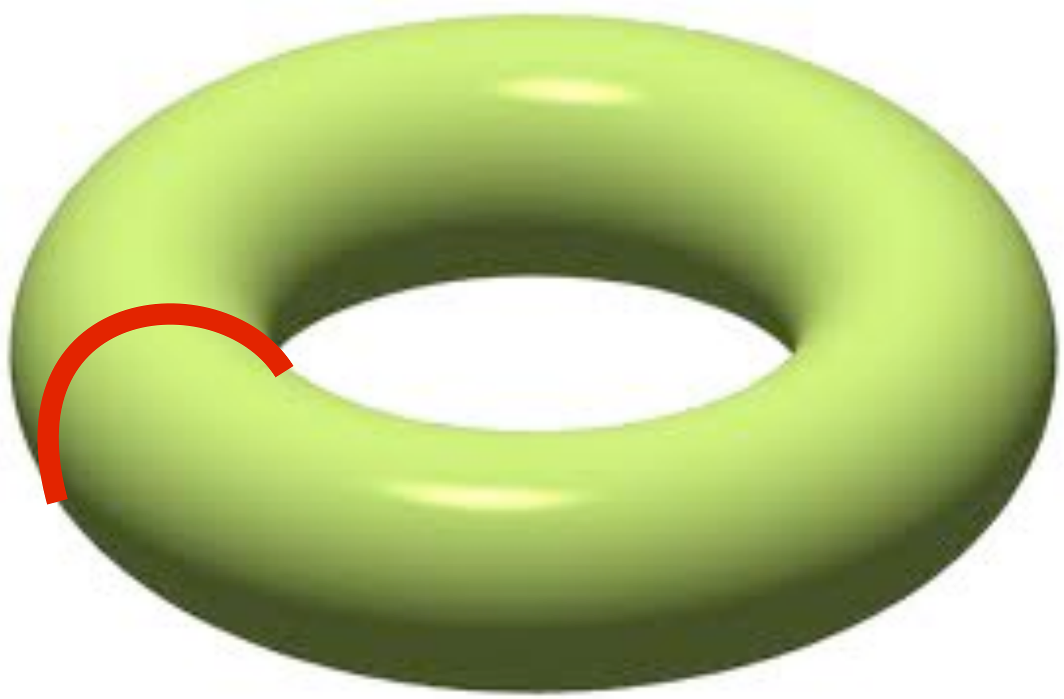
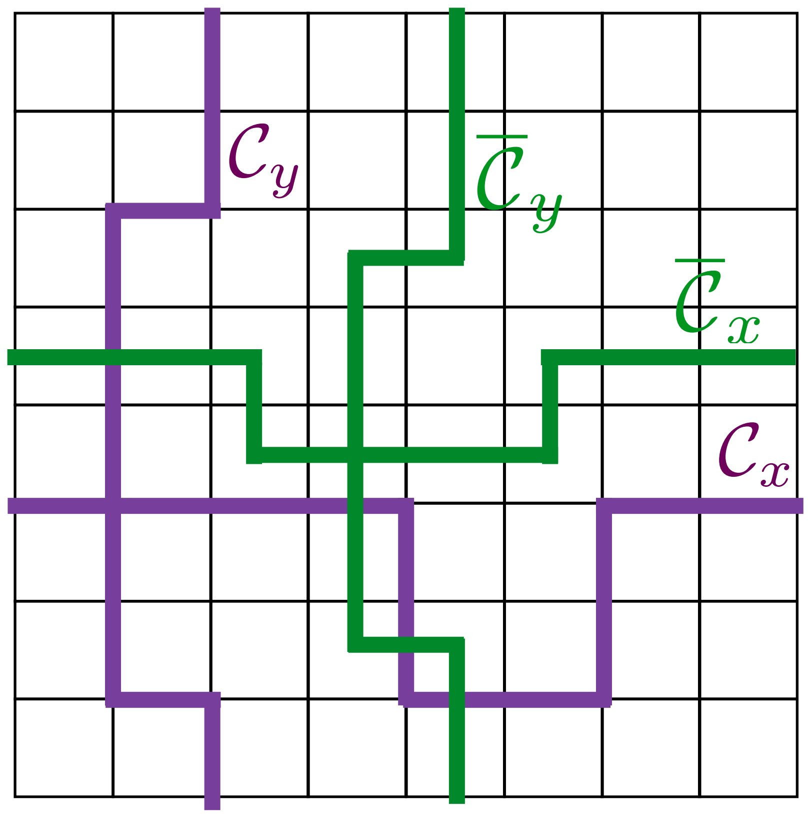
Topological phase has degenerate states with Z_2 flux $W = \pm 1$ through the holes of the torus



(N. Read and S.S., 1991)

Topological order



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This criterion can distinguish the phases when dynamical (or even gapless) matter fields are present

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$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

$G_i = 1$

Kramers-Wannier-Wegner duality,
Ising* criticality

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Embed in a compact U(1) gauge theory

Define

$$\tau^z \sim e^{iA}$$

and impose $A = 0, \pi$ by adding a potential $\sim -\cos(2A)$. Then make a gauge transformation $A_\mu \to A_\mu - \Delta_\mu \theta/2$, and make θ dynamical to make the Hamiltonian gauge invariant. In this manner the $\mathbb{Z}_2$ gauge theory becomes a compact U(1) gauge theory with a charge 2 Higgs field:

$$\mathcal{L} = \sum_i \left[\frac{1}{2g} \left(\frac{\partial \vec{A}_i}{\partial \tau} \right)^2 + \frac{1}{2U} \left(\frac{\partial \theta_i}{\partial \tau} \right)^2 \right] - \sum_{\square} \cos(\vec{\Delta} \times \vec{A}) - J \sum_i \cos(\vec{\Delta}\theta_i - 2\vec{A}_i)$$

E. Fradkin and S. Shenker, Phys Rev D **19**, 3682 (1979)

Now we take the naive continuum limit with the Higgs field $\Phi \sim e^{i\theta}$, and obtain a theory of complex scalar Φ coupled to a U(1) gauge field

$$\mathcal{L} \;=\; |(\partial_\mu - 2iA_\mu)\Phi|^2 + s|\Phi|^2 + u|\Phi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2$$

However, this turns continuum theory out to be incorrect: we cannot ignore the monopoles is the compact U(1) gauge field, A_μ near the confinement transition.

Particle-vortex duality

But we proceed anyway, and perform a Dasgupta-Halperin-Peskin particle-vortex duality on $\mathcal{L}$. This requires a complex scalar ϕ which creates a vortex with flux π, and the dual theory is

$$\widetilde{\mathcal{L}} = |\partial_\mu \phi|^2 + \widetilde{s}|\phi|^2 + \widetilde{u}|\phi|^4 \,.$$

Now we have to impose the requirement that vortices with flux π and flux $-\pi$ are the same *i.e.* allow 2π monopoles to be created from the vacuum. This modifies the Lagrangian to

$$\widetilde{\mathcal{L}} = |\partial_\mu \phi|^2 + \widetilde{s}|\phi|^2 + \widetilde{u}|\phi|^4 - \lambda(\phi^2 + (\phi^*)^2) \,.$$

Finally, we write $\phi = \sigma + i\vartheta$. The field σ has a smaller mass then ϑ, and so we can integrated out ϑ to obtain the final correct dual theory

$$\widetilde{\mathcal{L}} = (\partial_\mu \sigma)^2 + \widetilde{s}\sigma^2 + \widetilde{u}\sigma^4 \,.$$

This is the promised dual Ising* theory of the confinement-deconfinement transition. The σ field creates the 'vison' particle. The * refers to the fact that a single vison cannot be created locally, and this changes some topological properties on compact spaces.

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Symmetry-enriched topological (SET) order

$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x \quad , \quad \boxed{G_i = -1}$$

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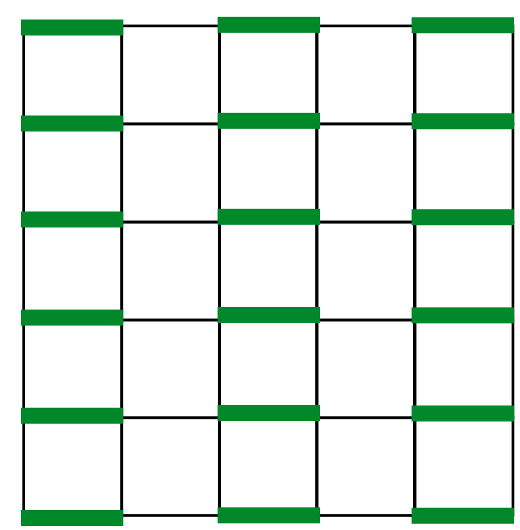
Broken symmetry and valence bond solid (VBS) order



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(R. Jalabert and S.S., 1991; T. Senthil, A. Vishwanath, L. Balents, S. S. and M.P.A. Fisher, 2004)

Symmetry-enriched topological (SET) order and deconfined criticality

$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x \quad , \quad G_i = -1$$

Deconfined quantum criticality with a U(1) gauge theory and a charge 2 complex scalar

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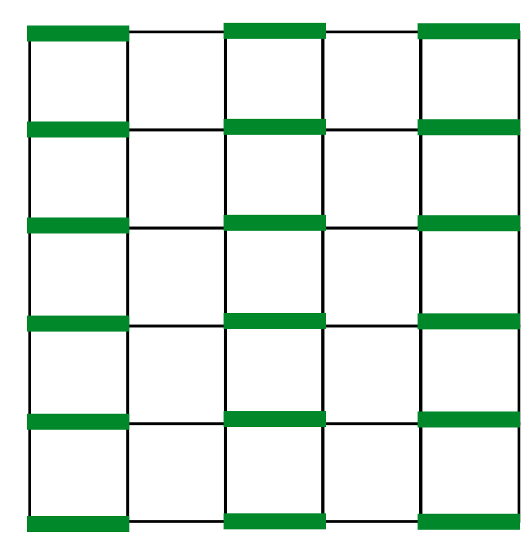

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Berry phases suppress monopoles at the critical point

Embedding the $\mathbb{Z}_2$ gauge theory in a compact U(1) gauge theory as before, the $G_i = -1$ background charges lead to a source term for A_τ (a Polyakov loop)

$$\mathcal{L} = \sum_i \left[\frac{1}{2g} \left(\frac{\partial \vec{A}_i}{\partial \tau} \right)^2 + \frac{1}{2U} \left(\frac{\partial \theta_i}{\partial \tau} \right)^2 \right] - \sum_{\square} \cos(\vec{\Delta} \times \vec{A}) - J \sum_i \cos(\vec{\Delta}\theta_i - 2\vec{A}_i)$$
$$+ i \sum_i (-1)^{i_x + i_y} A_{i\tau}$$

Performing the Dasgupta-Halperin duality transform directly on this lattice model with the source term, we now find a dual vortex theory in which only *quadrupled* monopoles are permitted.

$$\widetilde{\mathcal{L}} = |\partial_\mu \phi|^2 + \widetilde{s}|\phi|^2 + \widetilde{u}|\phi|^4 - \lambda_4(\phi^8 + (\phi^*)^8)\,.$$

The λ_4 coupling is known to be irrelevant at the (Wilson-Fisher) critical point, and so monopoles can be ignored in the critical theory! Undualizing back to the original theory, this means that it is now valid to take the naive continuum limit of $\mathcal{L}$ to obtain the deconfined critical theory with a U(1) gauge field

$$\mathcal{L} = |(\partial_\mu - 2iA_\mu)\Phi|^2 + s|\Phi|^2 + u|\Phi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2\,.$$

RG flow of $\mathcal{L}$

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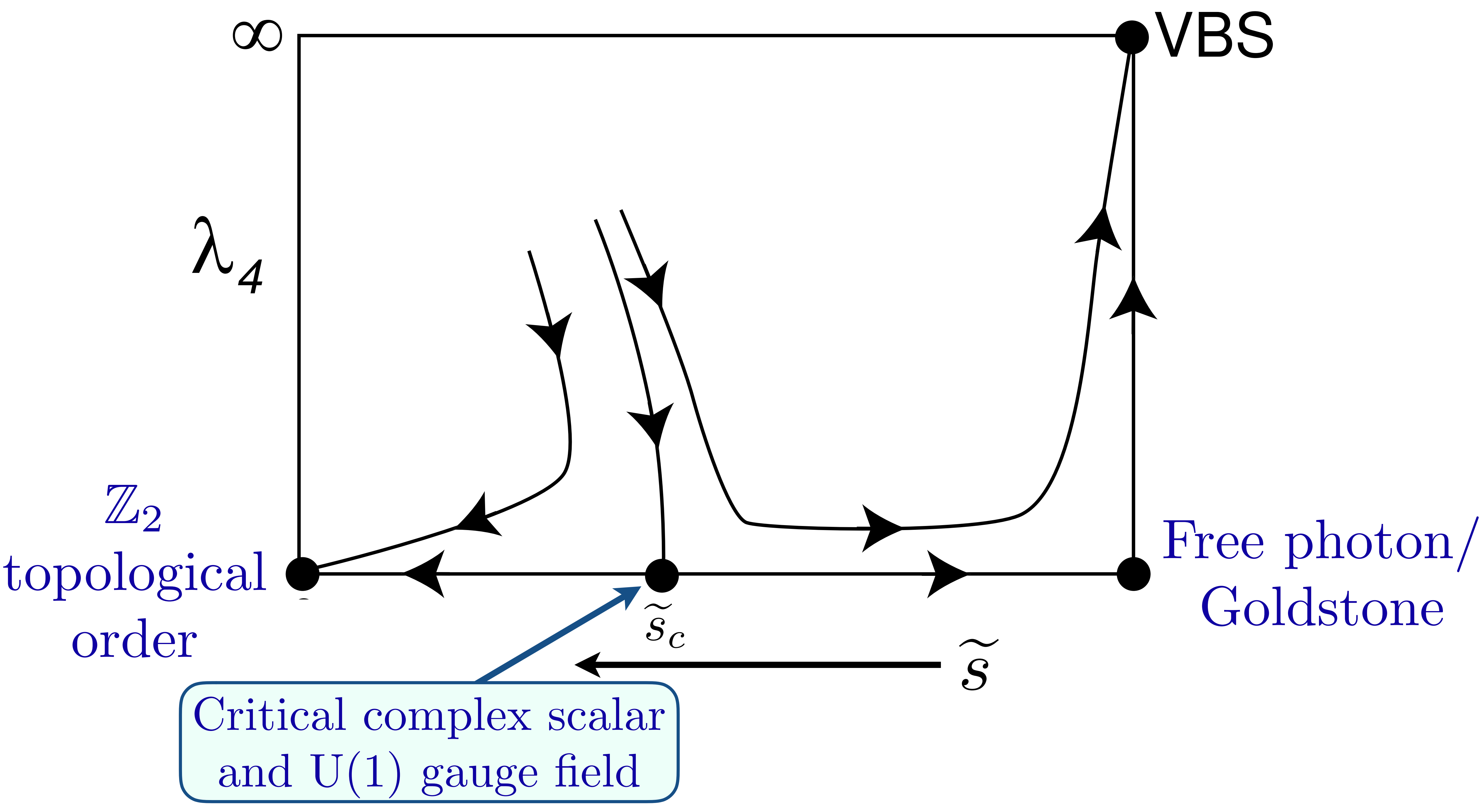

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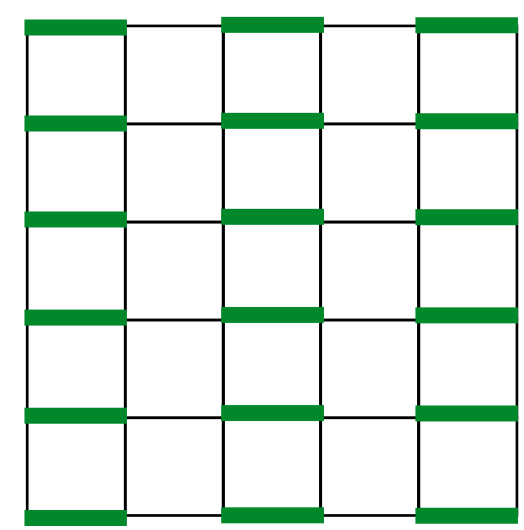

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Fermionic matter at half filling

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S. Gazit, M. Randeria, and A. Vishwanath, *Nature Physics* **13**, 484 (2017)

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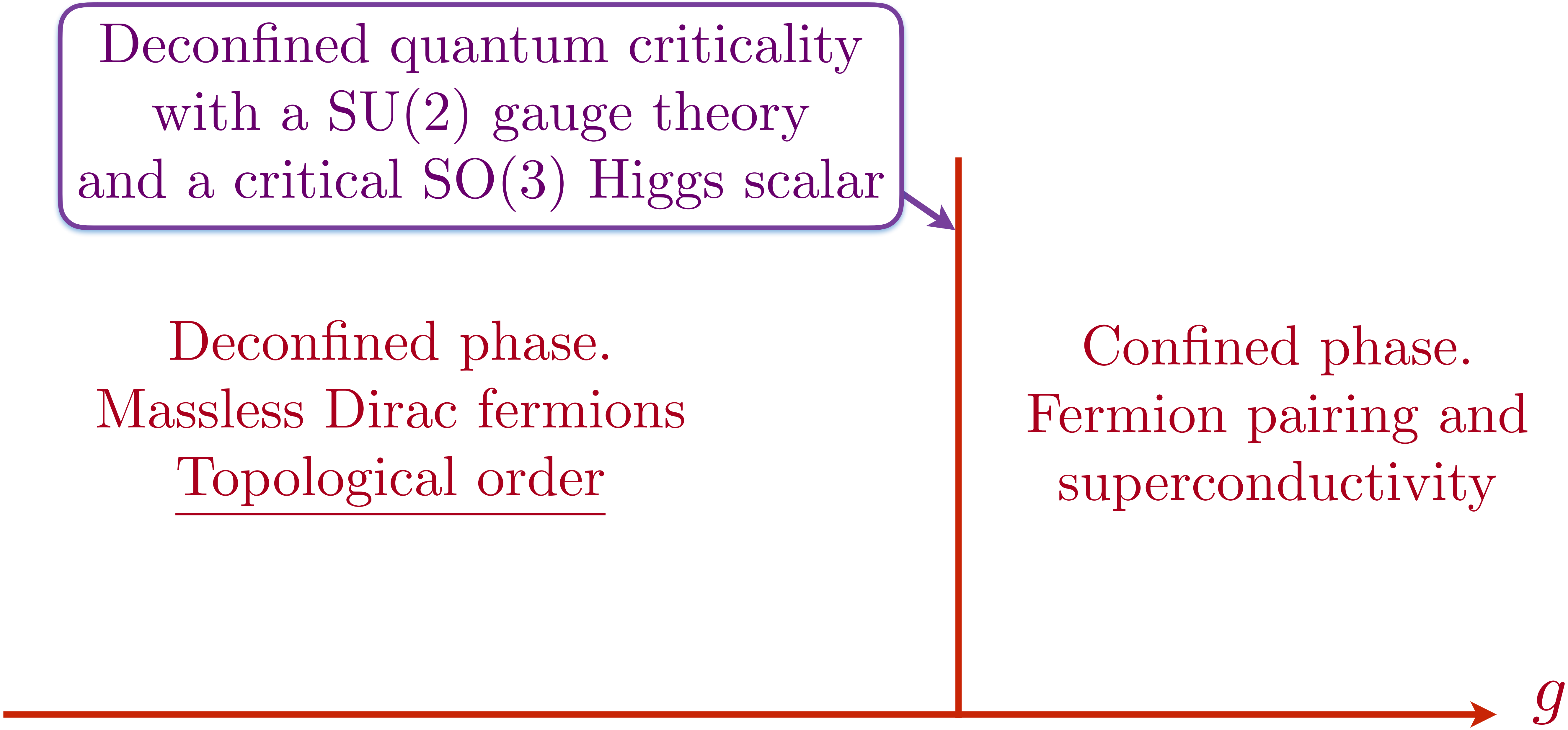

S. Gazit, M. Randeria, and A. Vishwanath, *Nature Physics* **13**, 484 (2017)
S. Gazit, F. F. Assaad, Chong Wang, S. Sachdev, and A. Vishwanath, to appear

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