

Quantum matter and gauge-gravity duality

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Outline

1. Conformal quantum matter
2. Compressible quantum matter

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The boson Hubbard model

and the superfluid-insulator transition

2. Compressible quantum matter

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The fermion Hubbard model

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The AdS_4 - Schwarzschild black brane

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Non-Fermi liquid phases map to holographic spaces with logarithmic violation of entanglement entropy

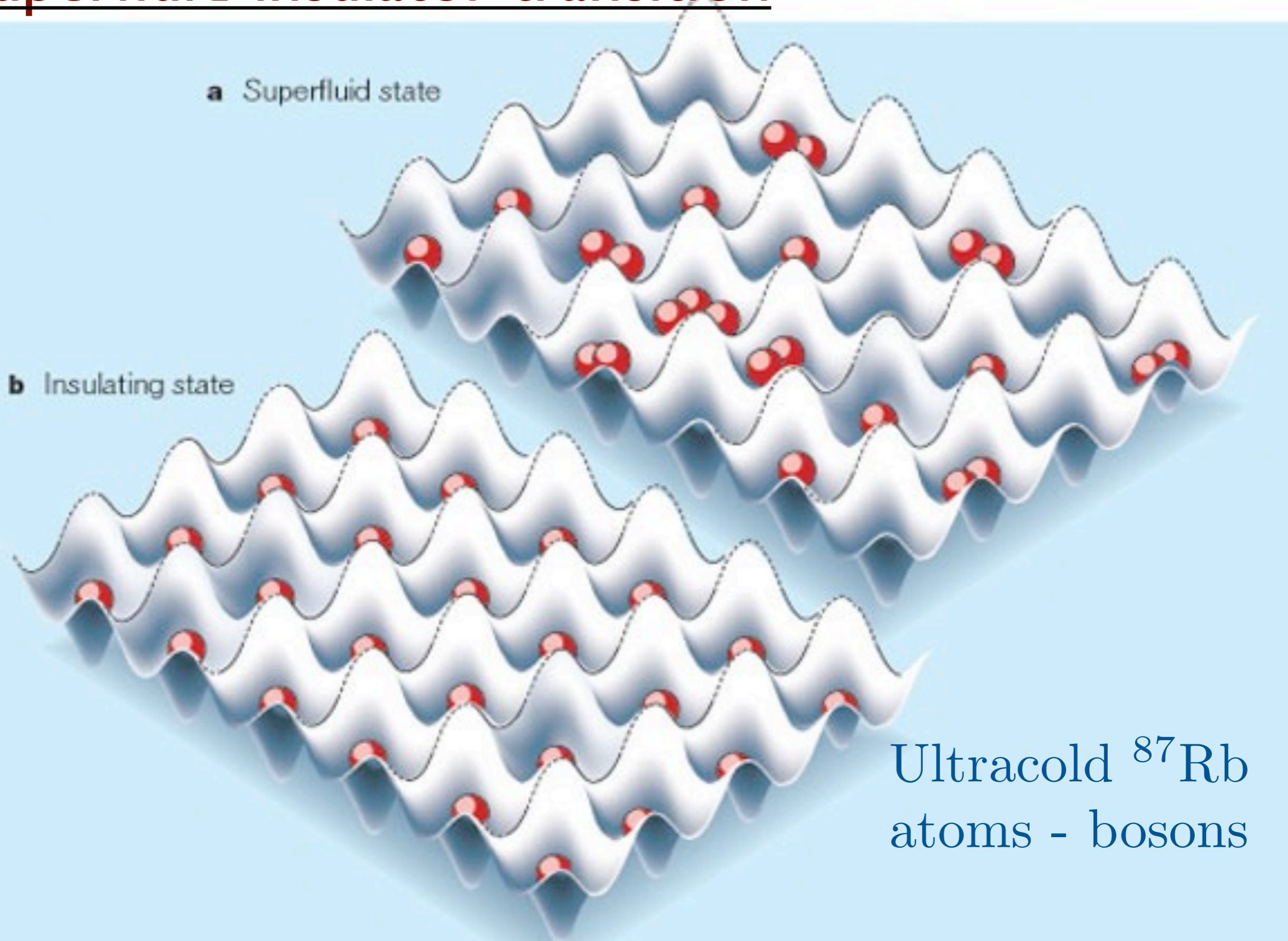
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*The boson Hubbard model
and the superfluid-insulator transition*

2. Compressible quantum matter

Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, b_j^\dagger , hopping between the sites, j , of a lattice, with short-range repulsive interactions.

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \dots$$

$$n_j \equiv b_j^\dagger b_j$$

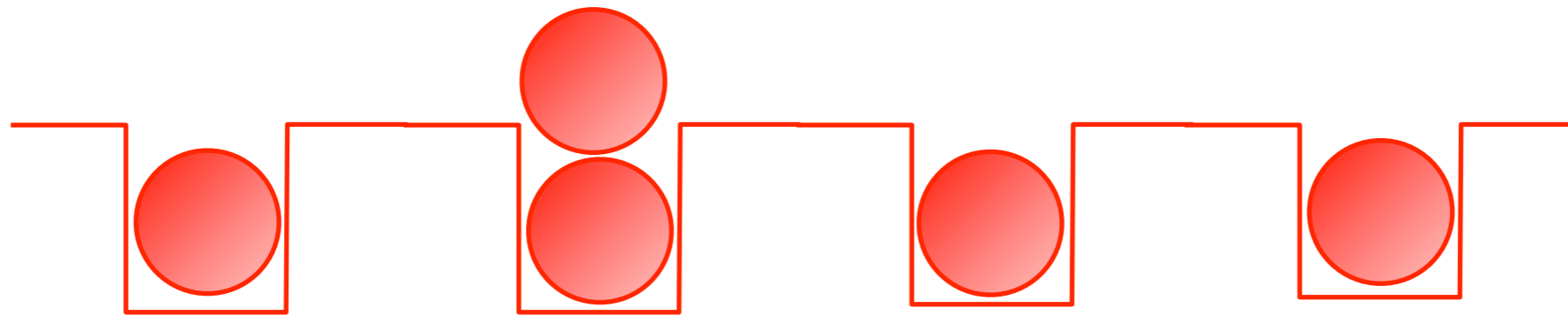
$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).



Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



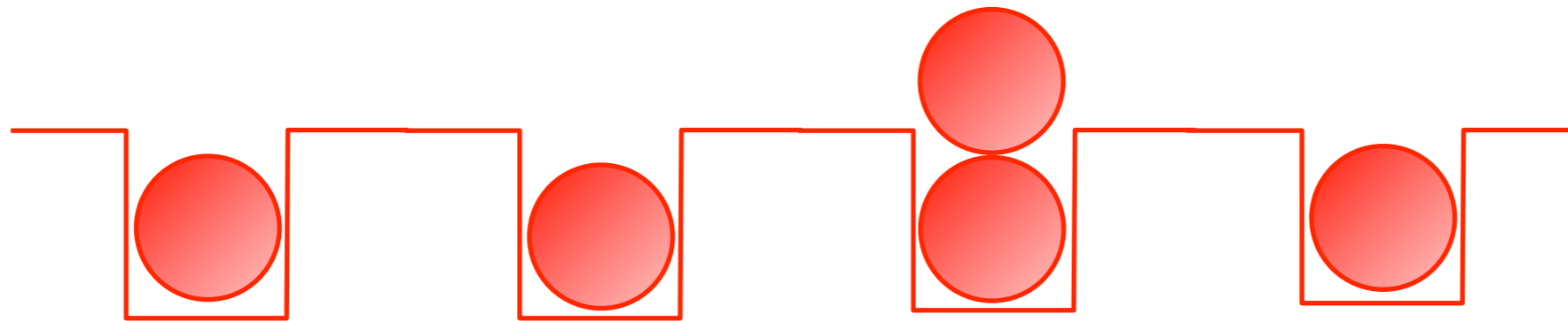
Particles $\sim \psi^\dagger$

Excitations of the insulator:

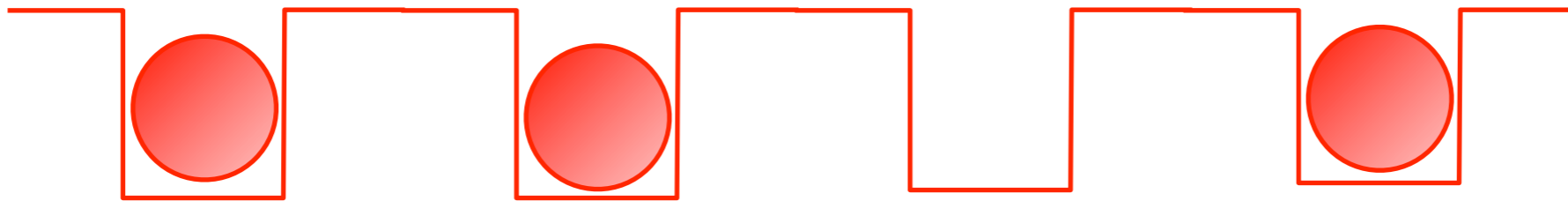


Holes $\sim \psi$

Excitations of the insulator:



Particles $\sim \psi^\dagger$



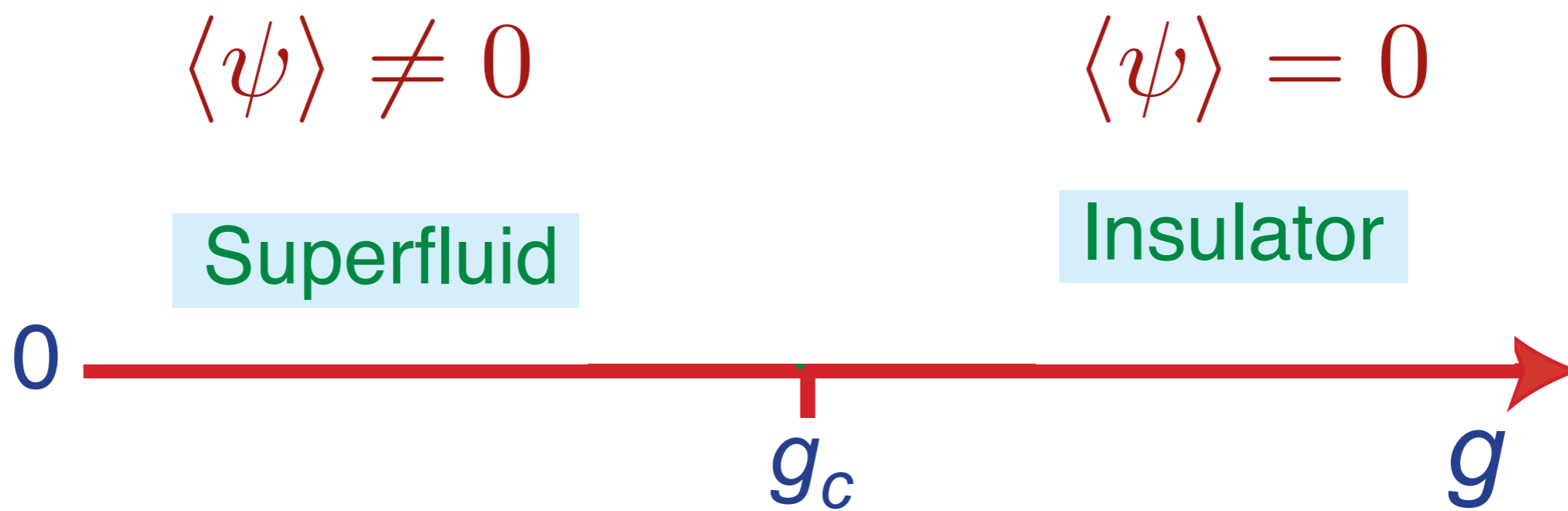
Holes $\sim \psi$

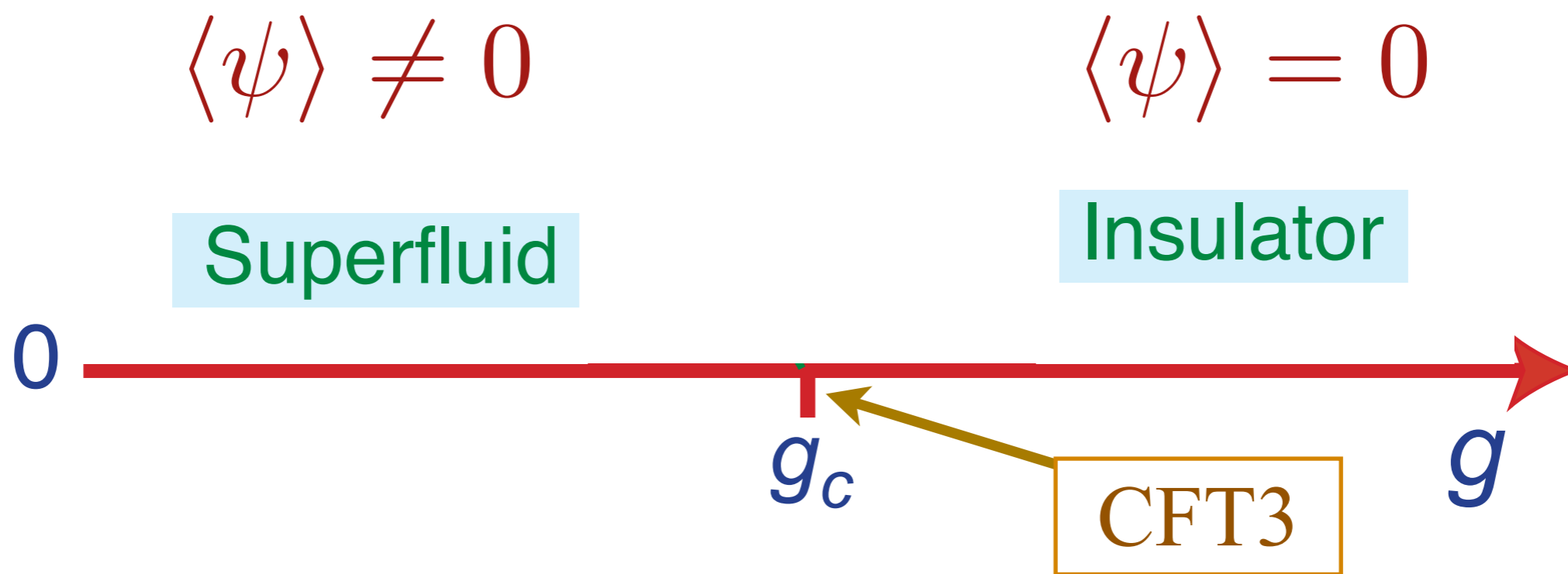
Density of particles = density of holes \Rightarrow

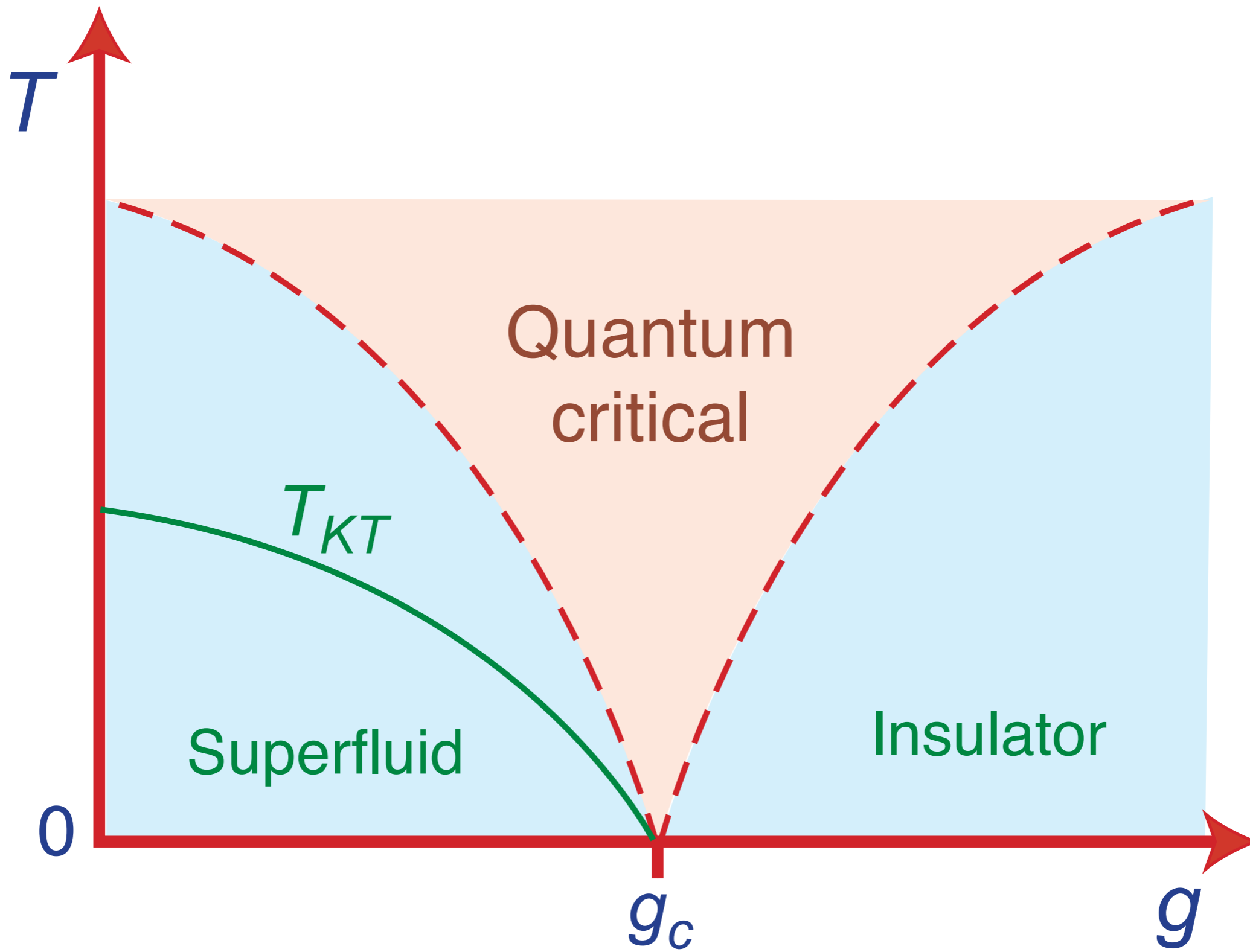
“relativistic” field theory for ψ :

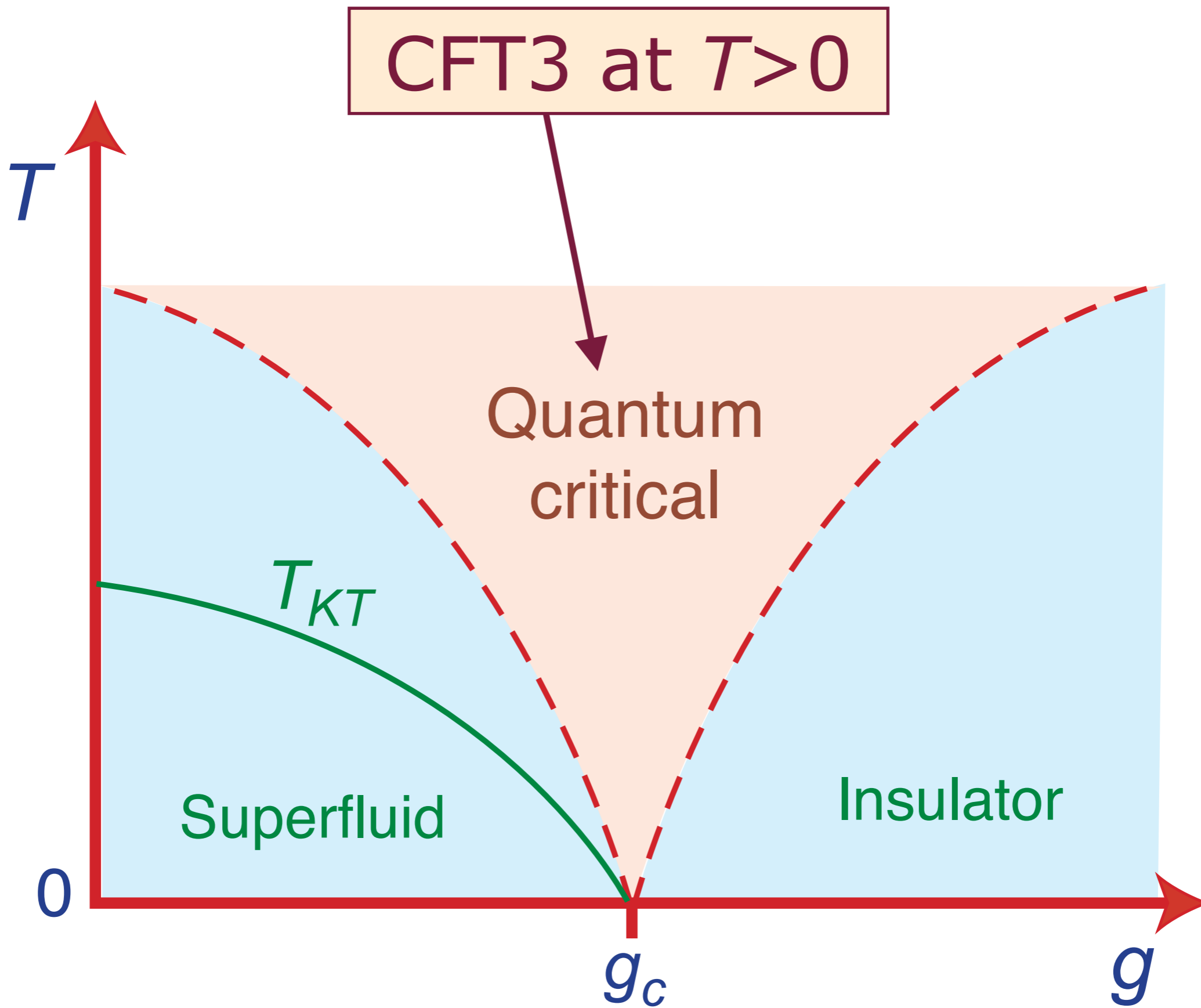
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c) |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).









Quantum critical transport

Quantum “*nearly perfect fluid*”
with shortest possible
equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C} \frac{\hbar}{k_B T}$$

where \mathcal{C} is a *universal* constant

Quantum critical transport

Transport co-efficients not determined
by collision rate, but by
universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)]$$

(Q is the “charge” of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-efficients not determined
by collision rate, but by
universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94**, 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Quantum critical transport

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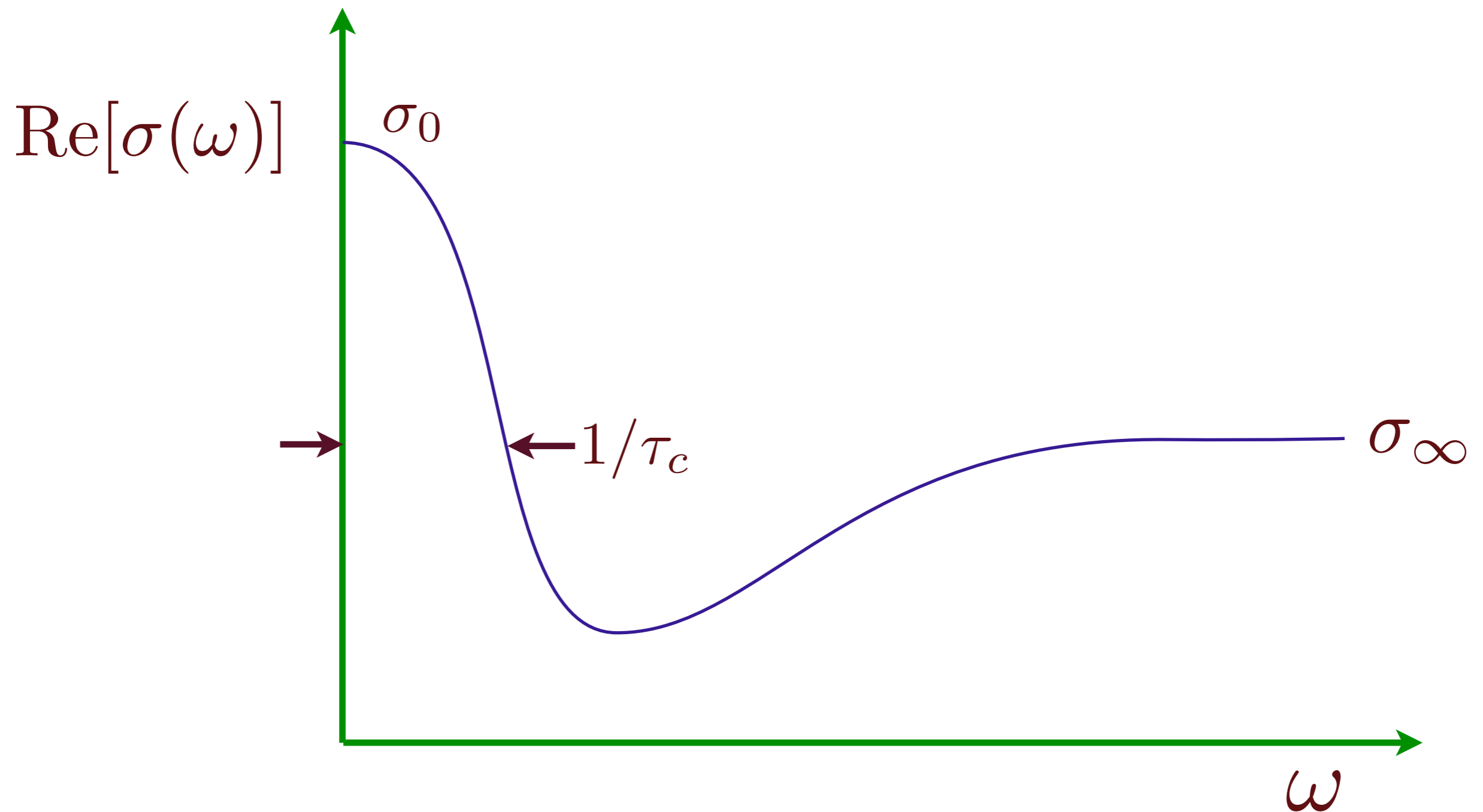
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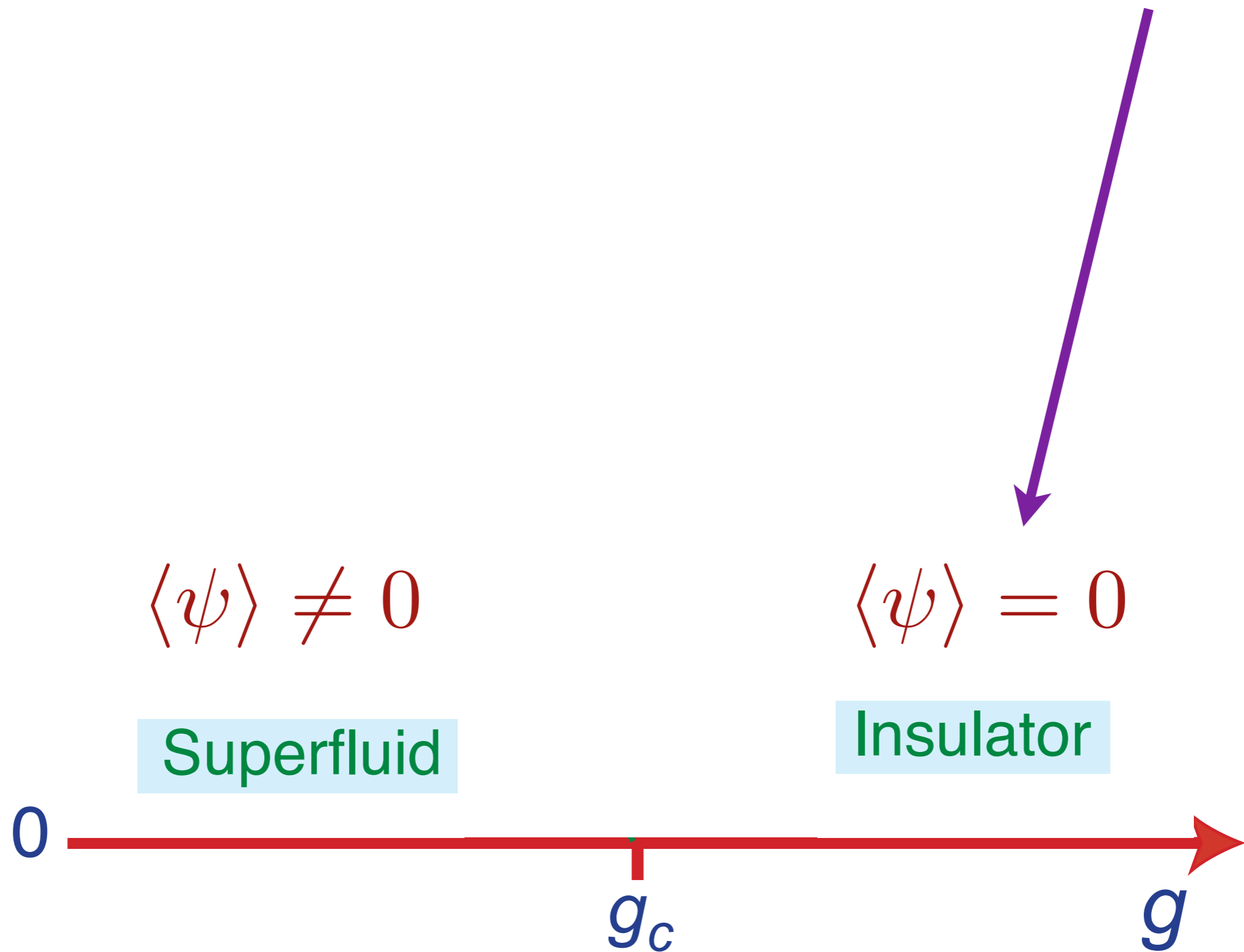
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \rightarrow \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the “optical conductivity”) in the CFT3.

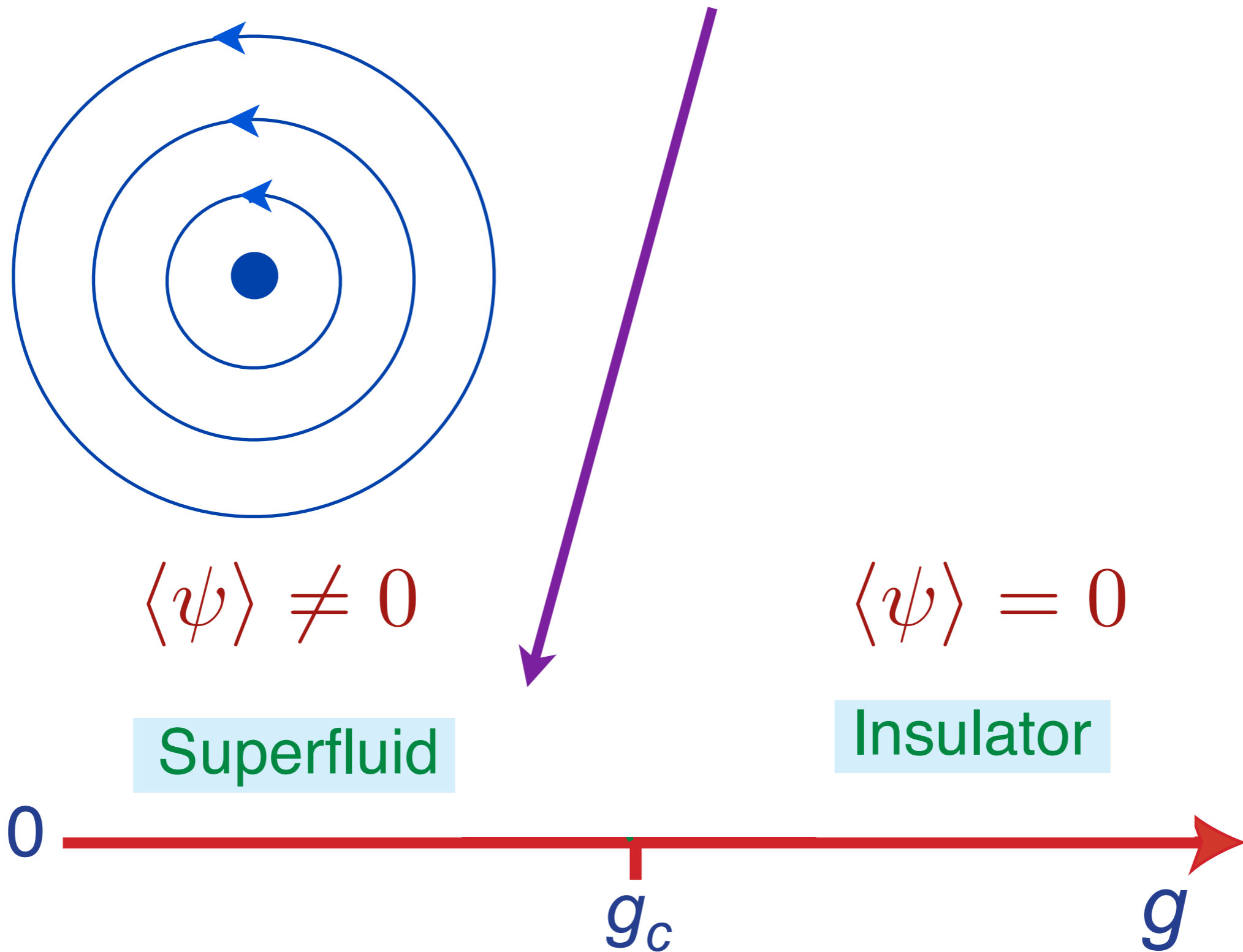
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



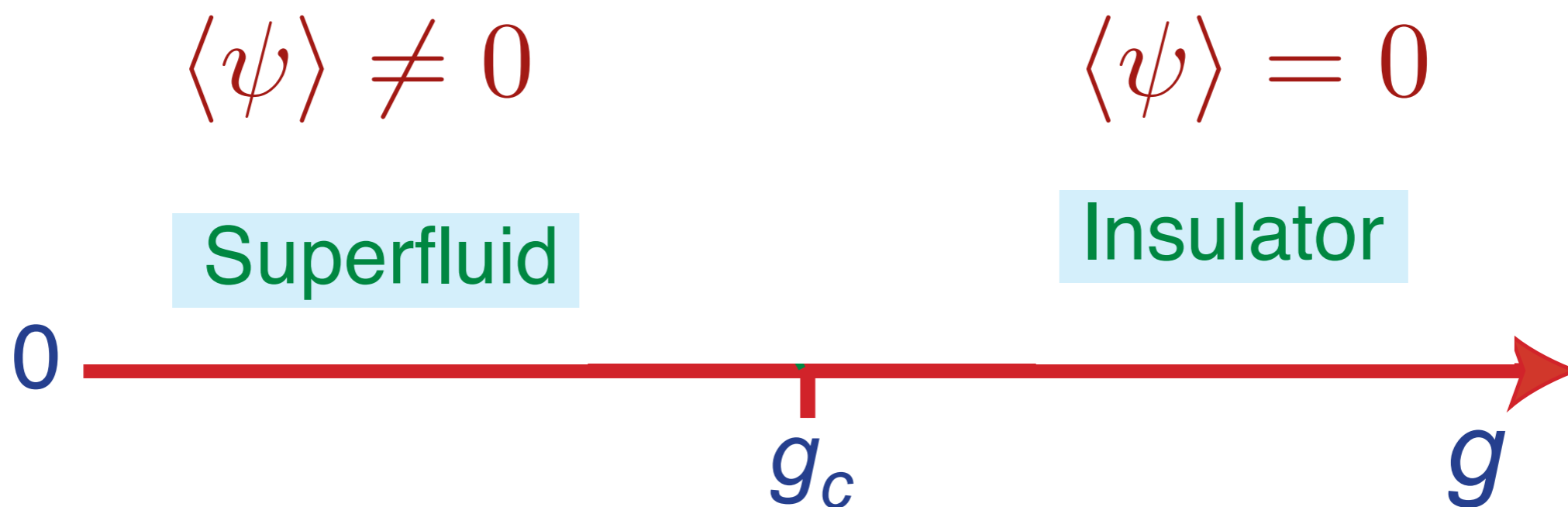
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



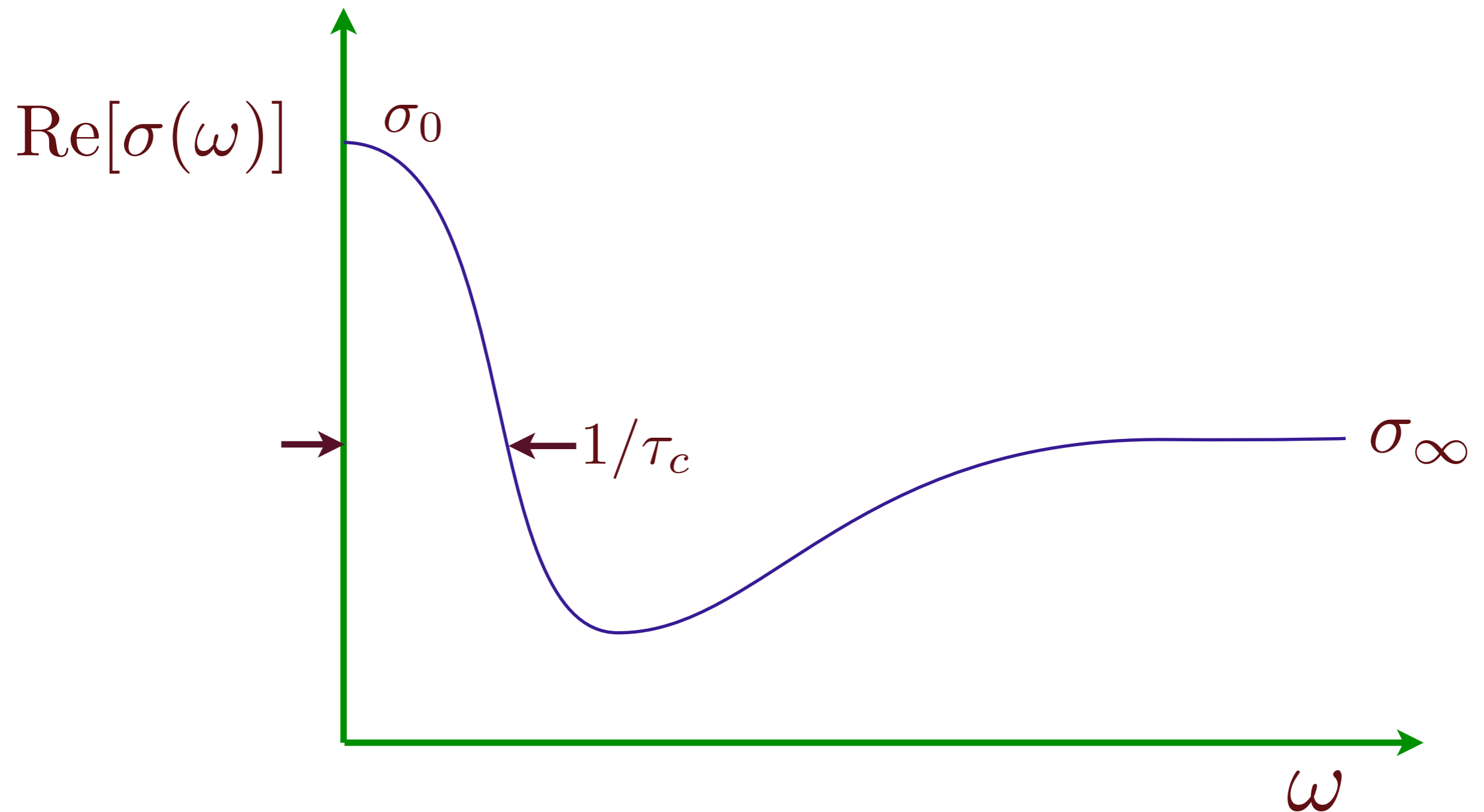
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These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

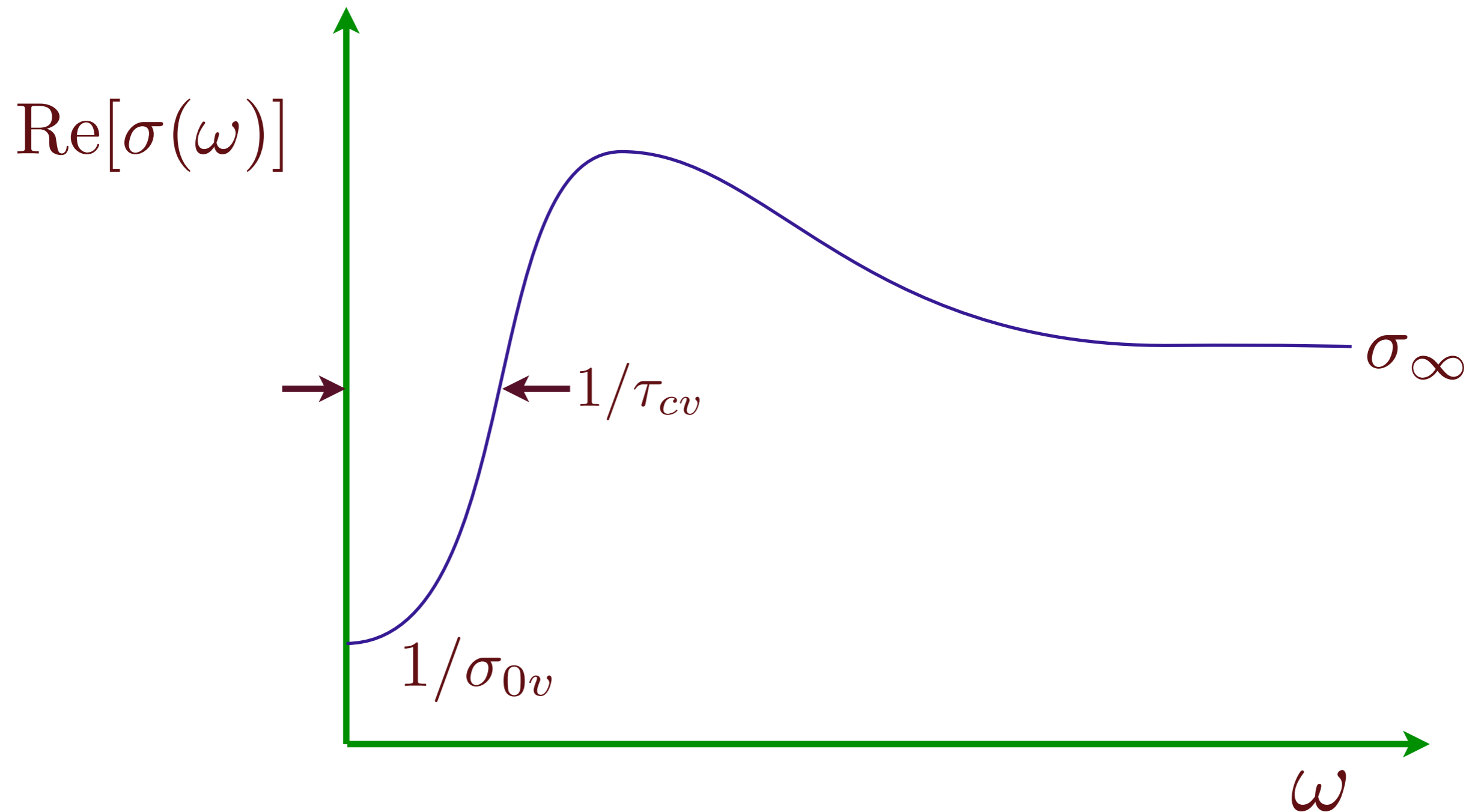
Conductivity = Resistivity of vortices



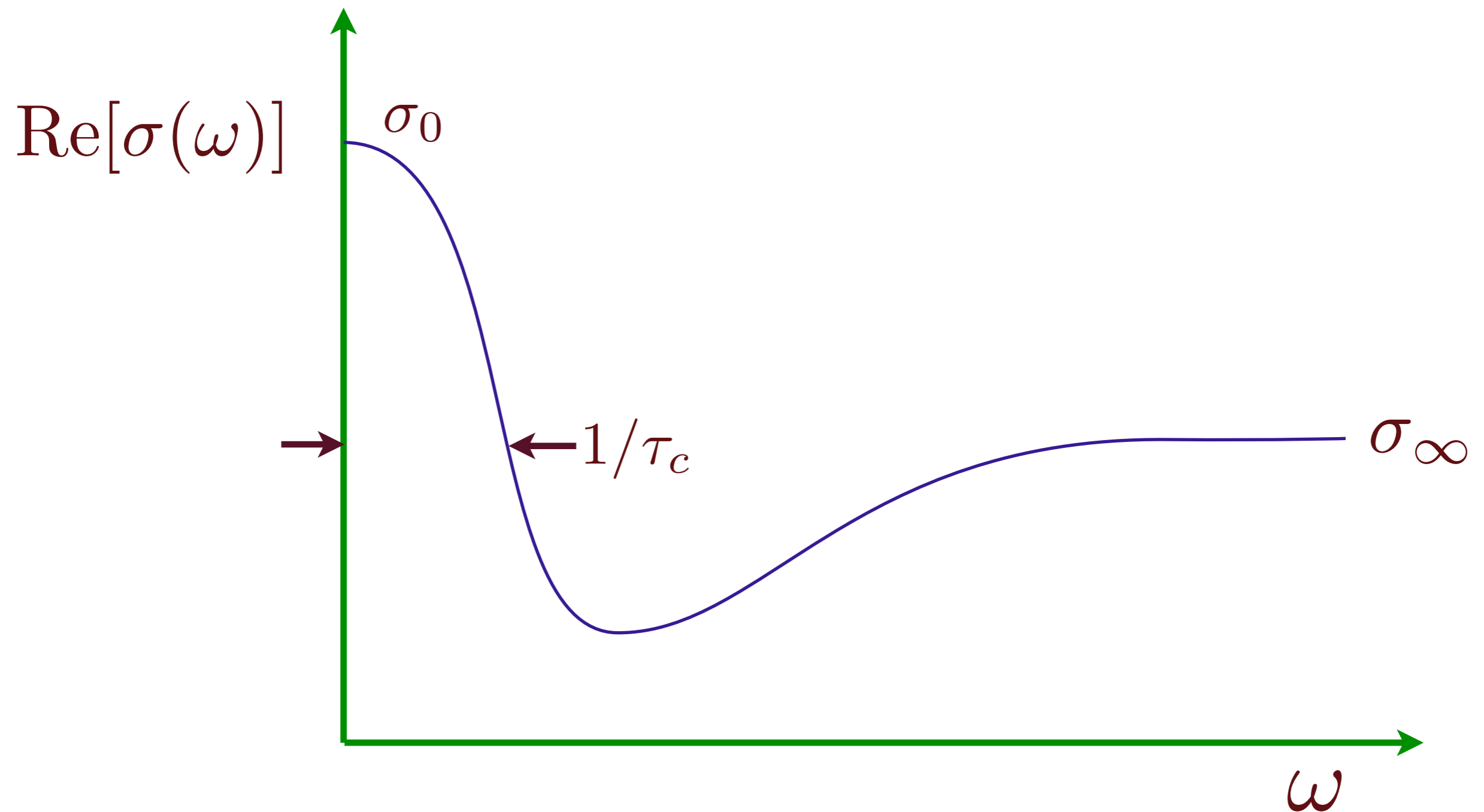
Boltzmann theory of bosons



Boltzmann theory of vortices

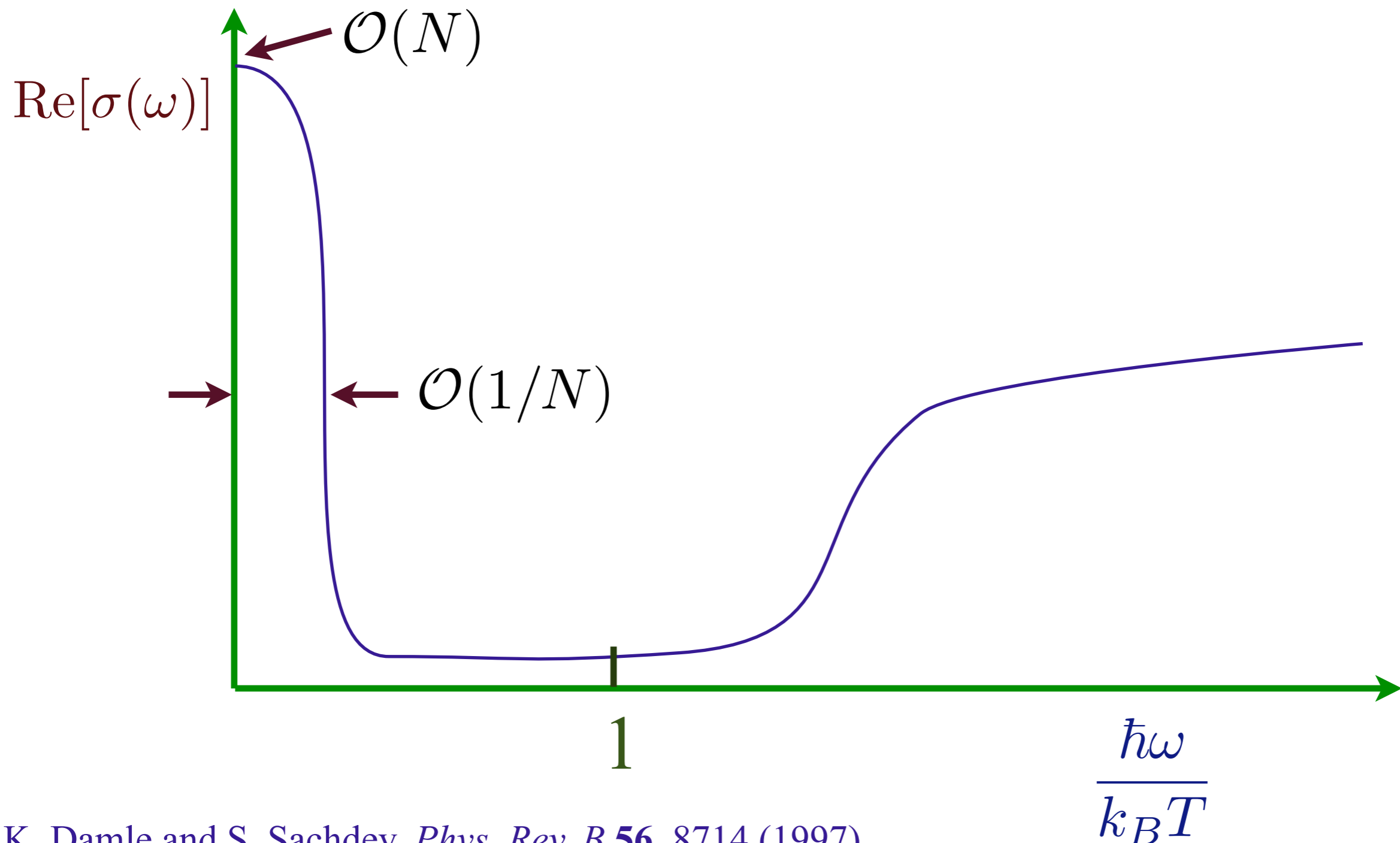


Boltzmann theory of bosons



Vector large N expansion for CFT3

$$\sigma = \frac{Q^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right); \quad \Sigma \rightarrow \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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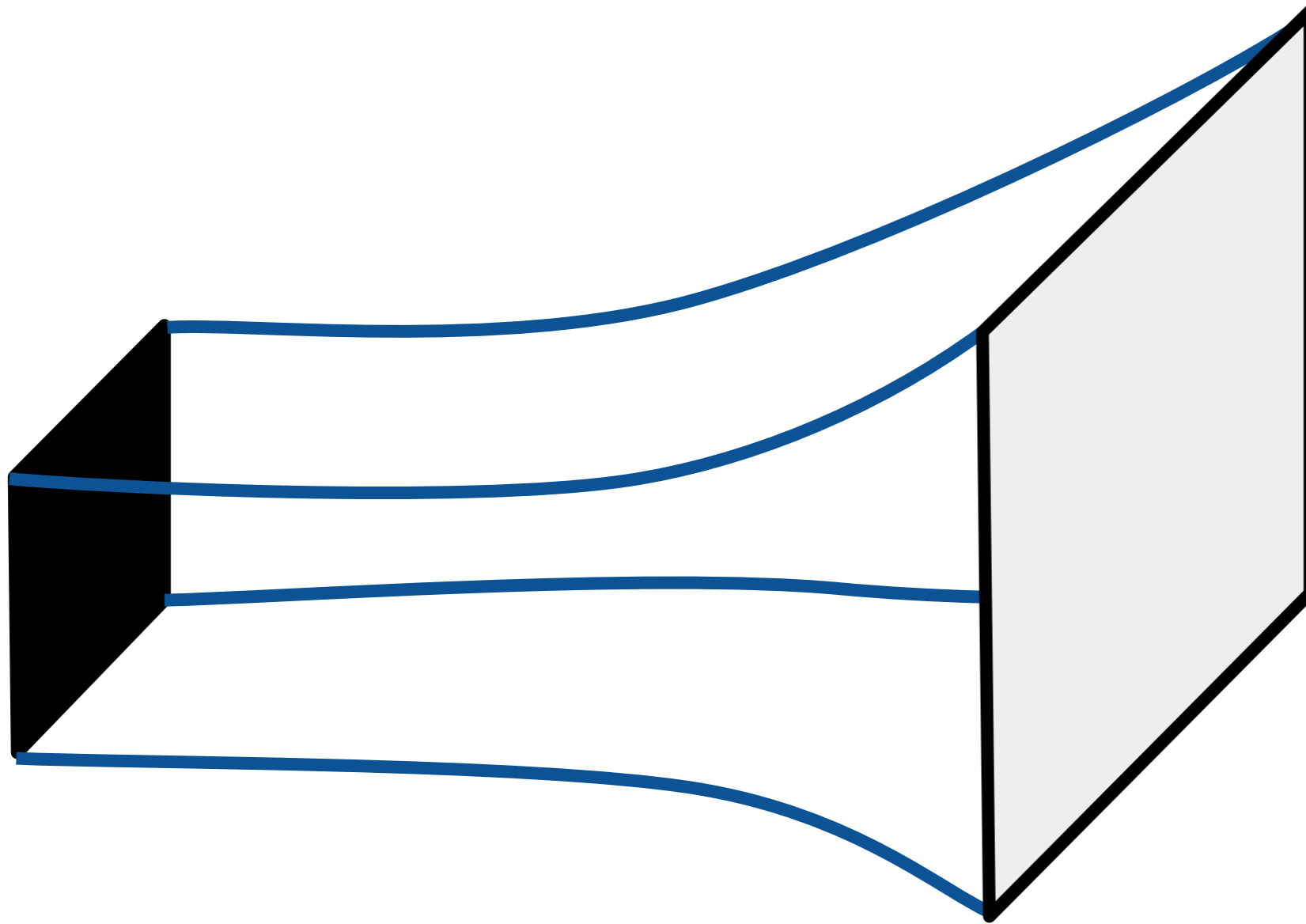
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The AdS_4 - Schwarzschild black brane

2. Compressible quantum matter

AdS/CFT correspondence

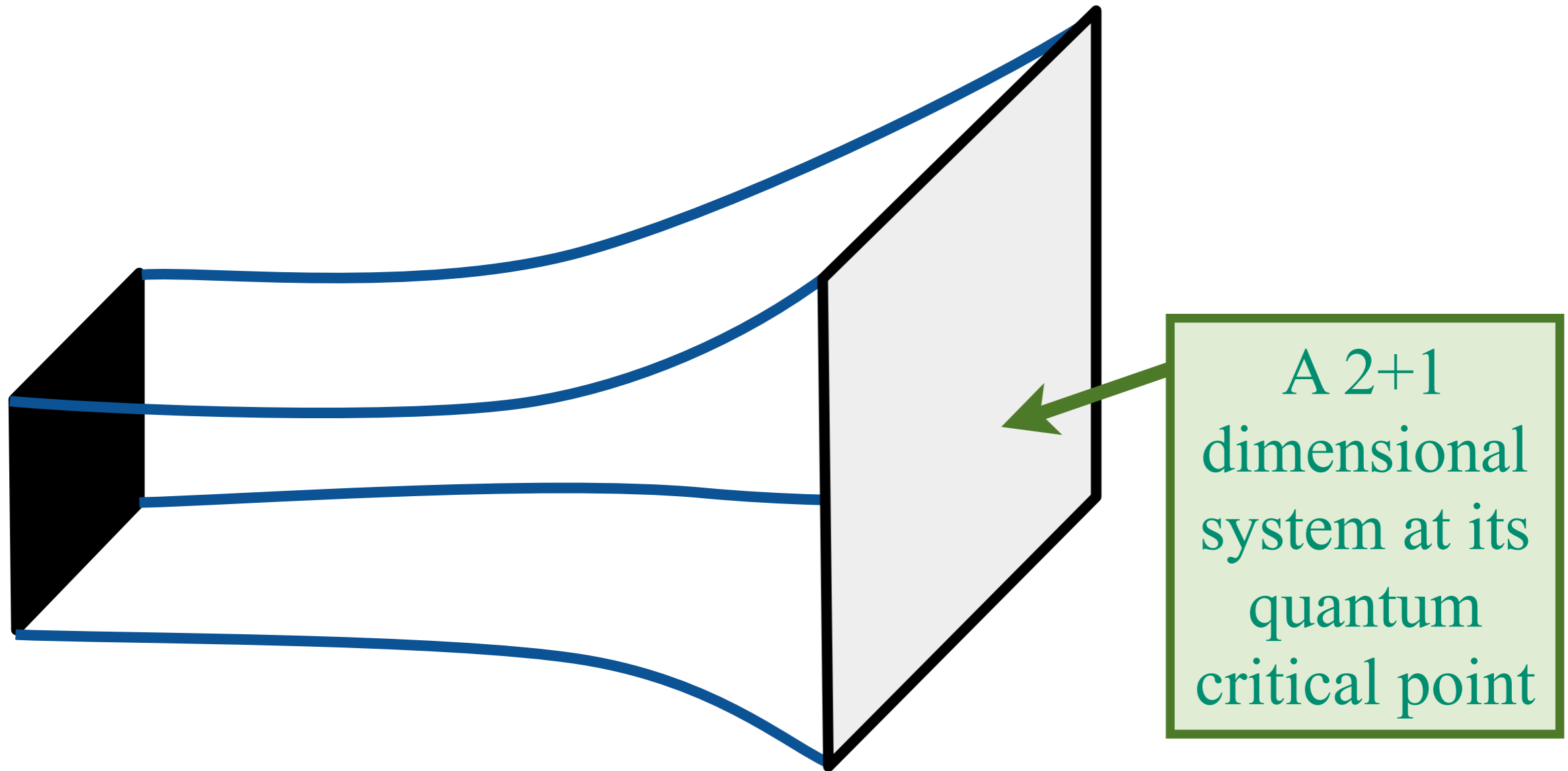
AdS₄-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence

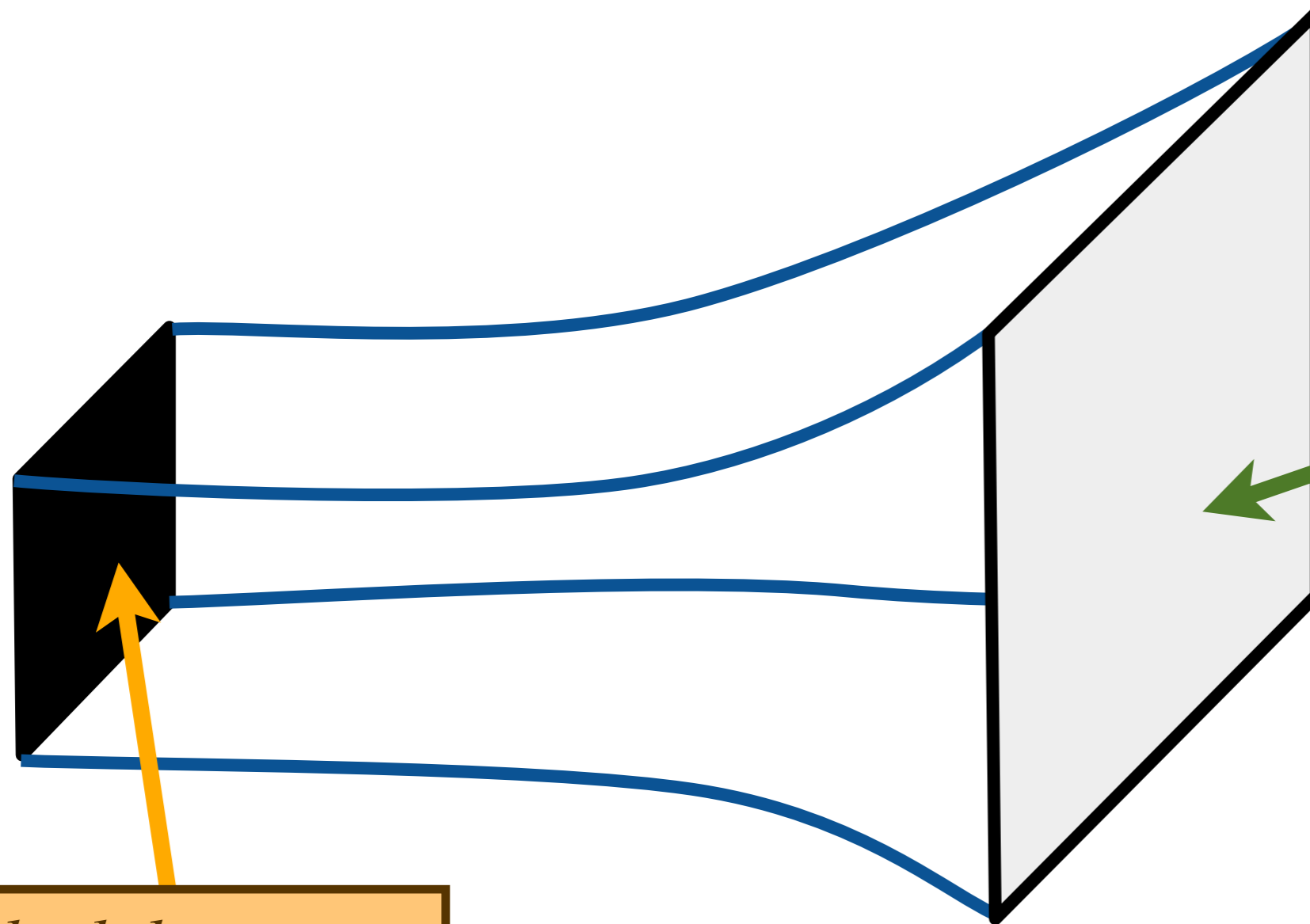
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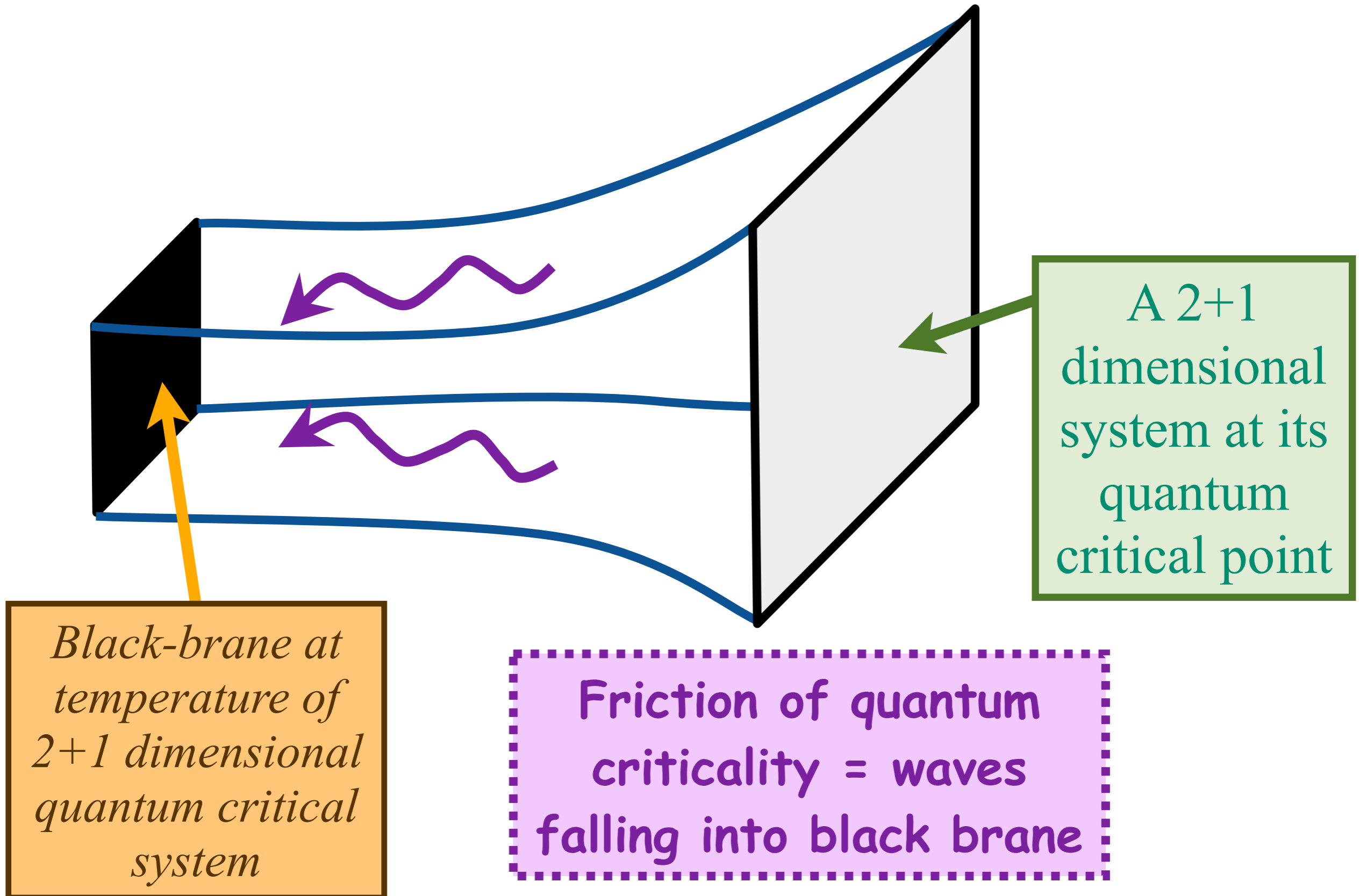
Black-brane at temperature of 2+1 dimensional quantum critical system

A 2+1 dimensional system at its quantum critical point

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence

AdS₄-Schwarzschild black-brane



AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right].$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS₄):

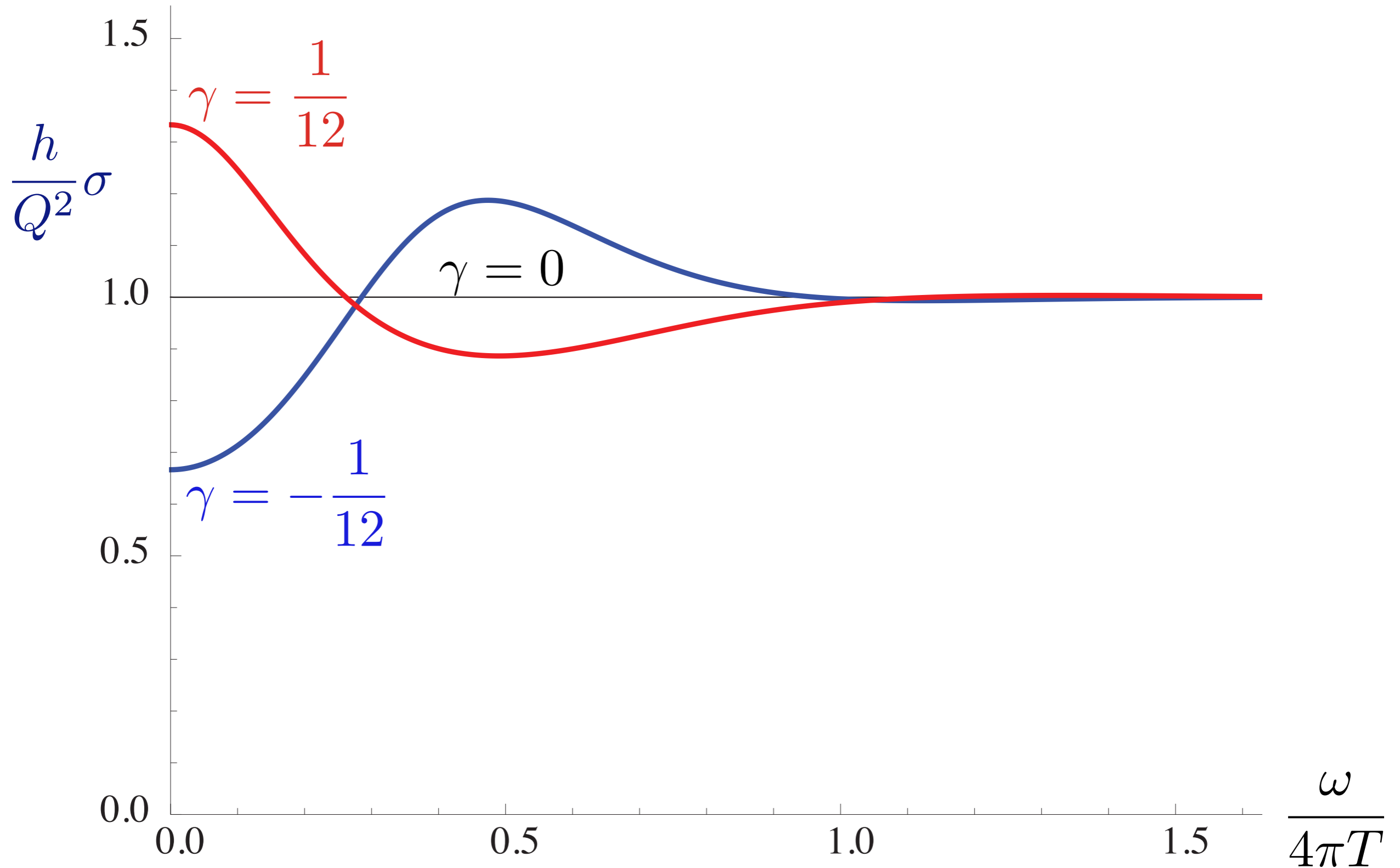
$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right],$$

where C_{abcd} is the Weyl curvature tensor.

Stability and causality constraints restrict $|\gamma| < 1/12$.

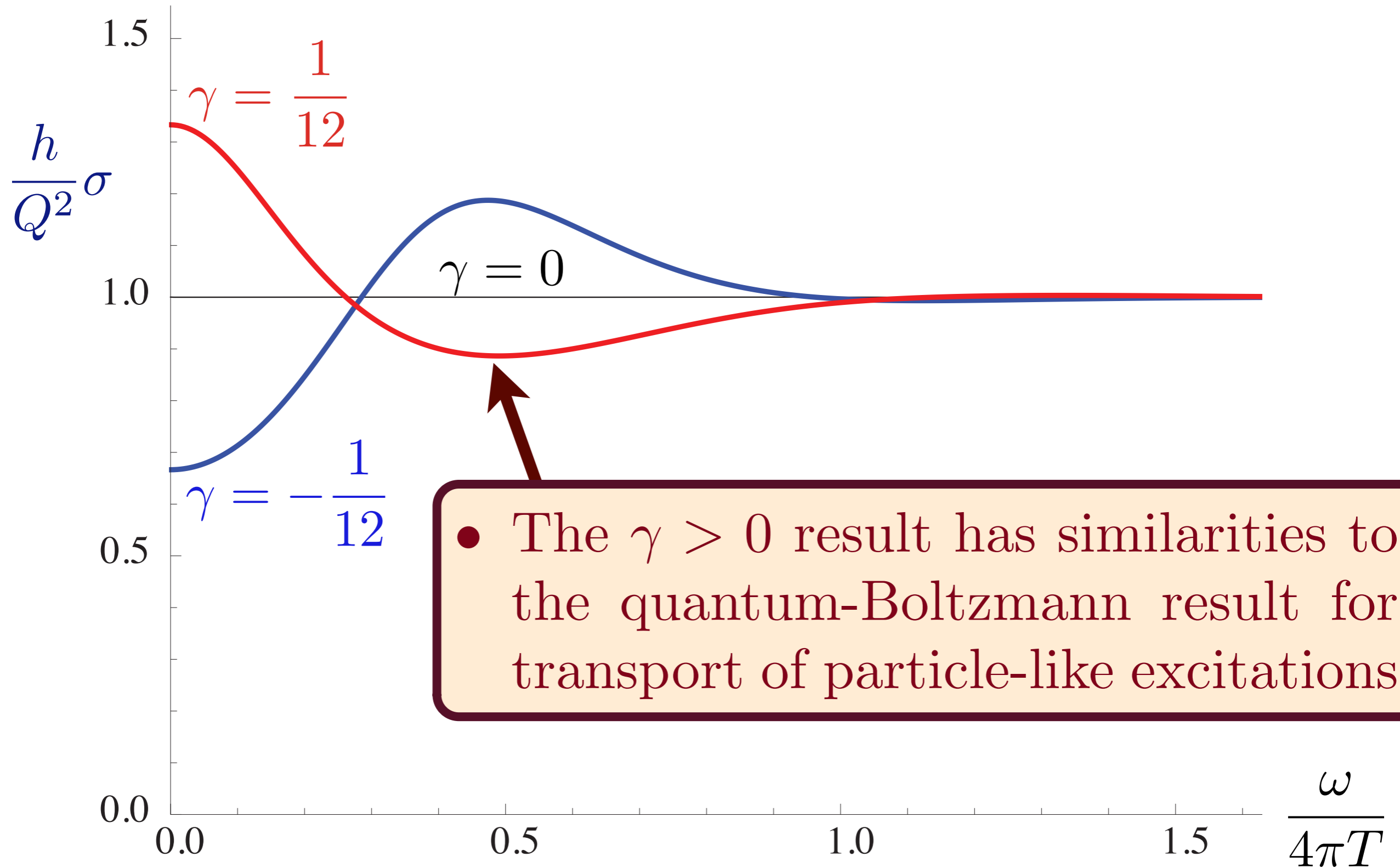
R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS₄ theory of strongly interacting “perfect fluids”



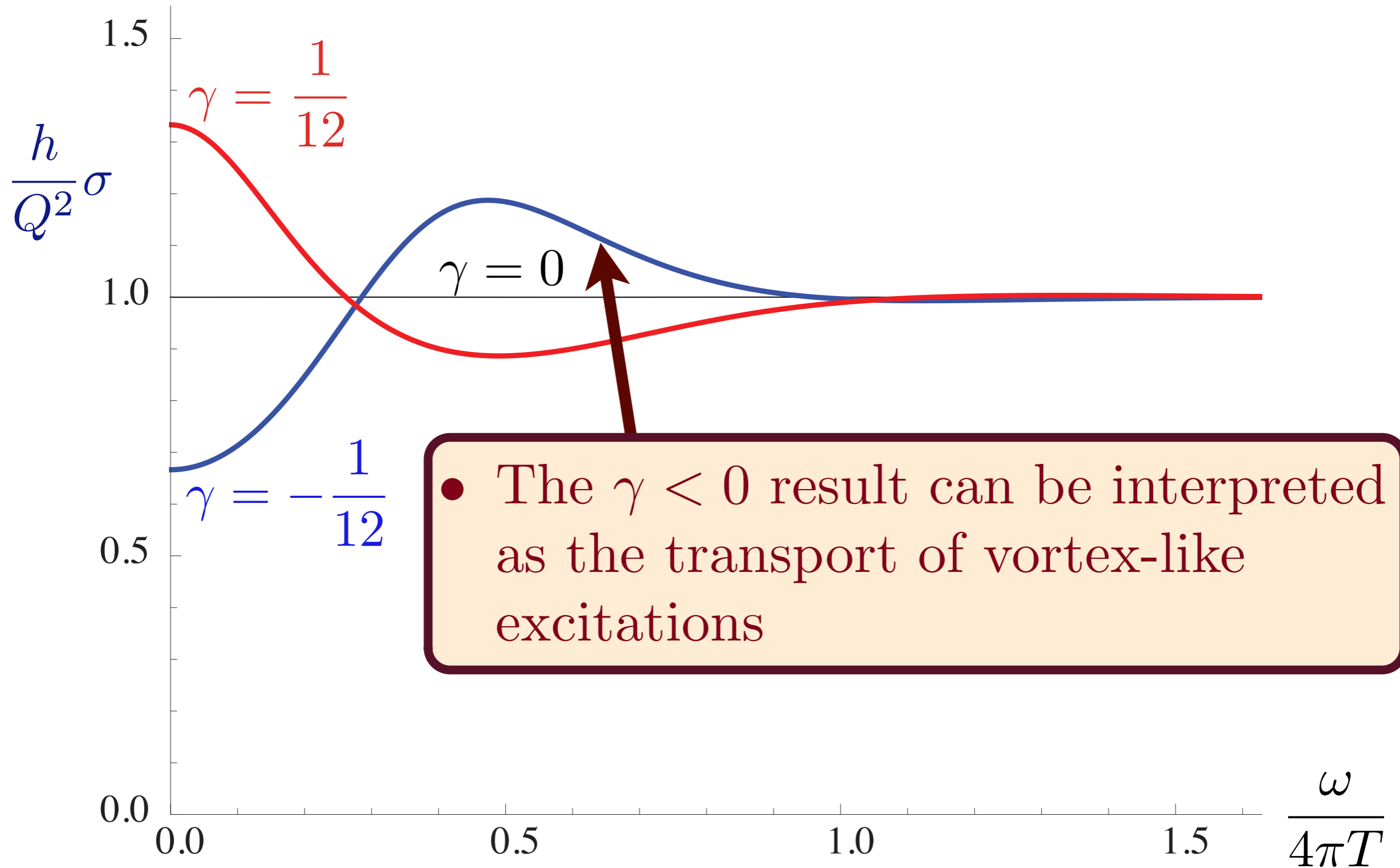
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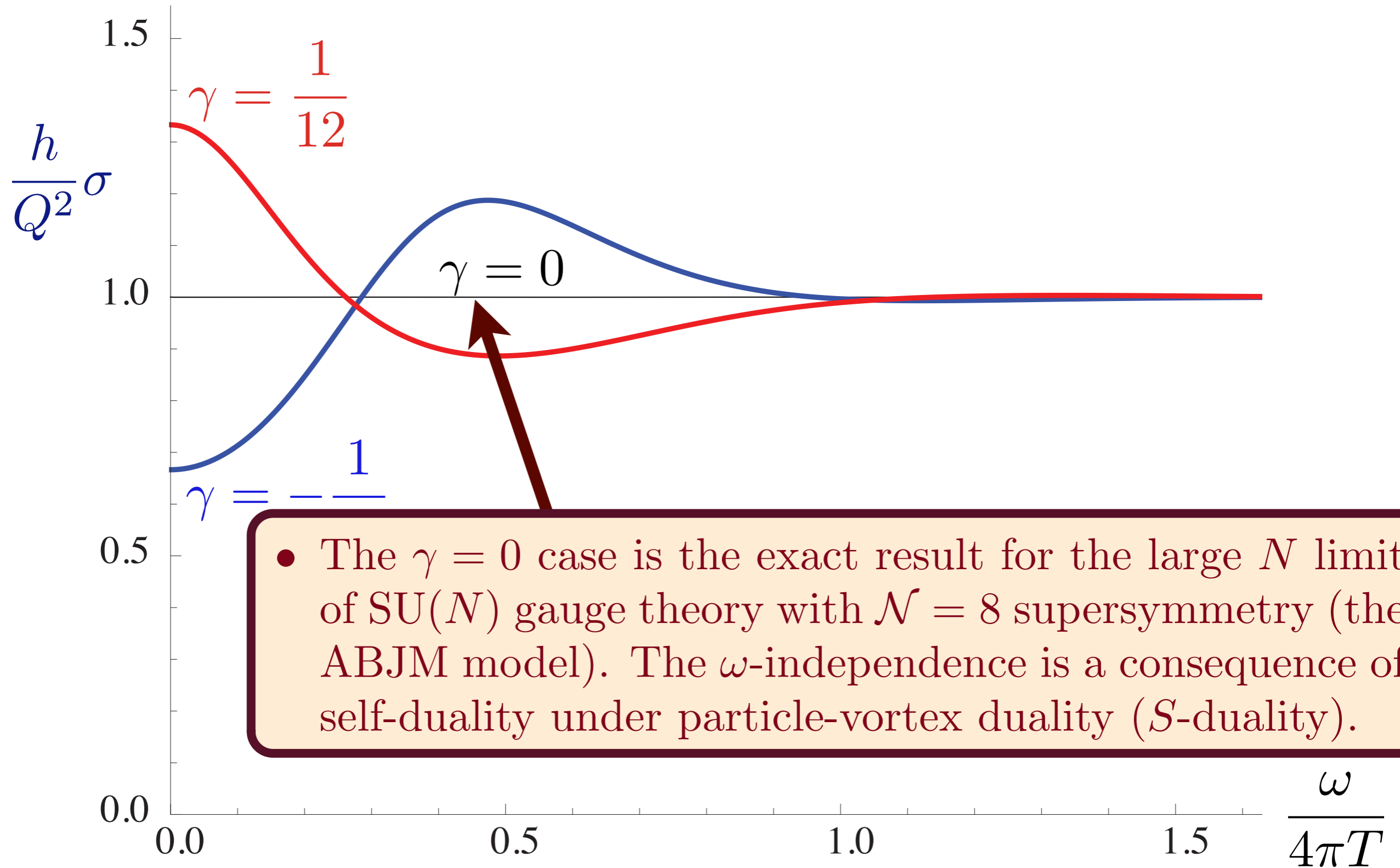
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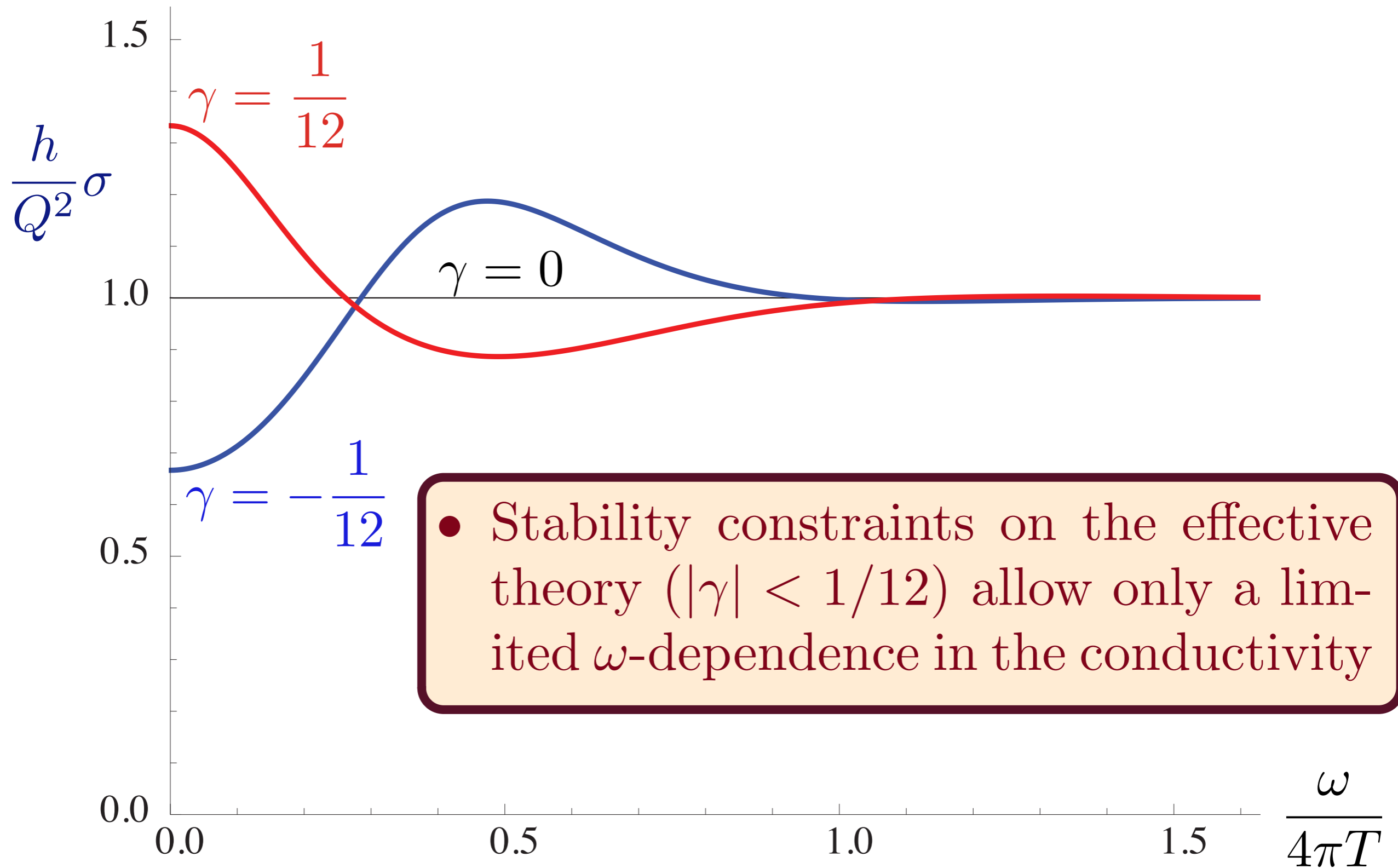
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