Quantum entanglement and the phases of matter

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PHYSICS
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Outline

1. Conformal quantum matter

   Entanglement, emergent dimensions and string theory

2. Compressible quantum matter

   Holography of strange metals

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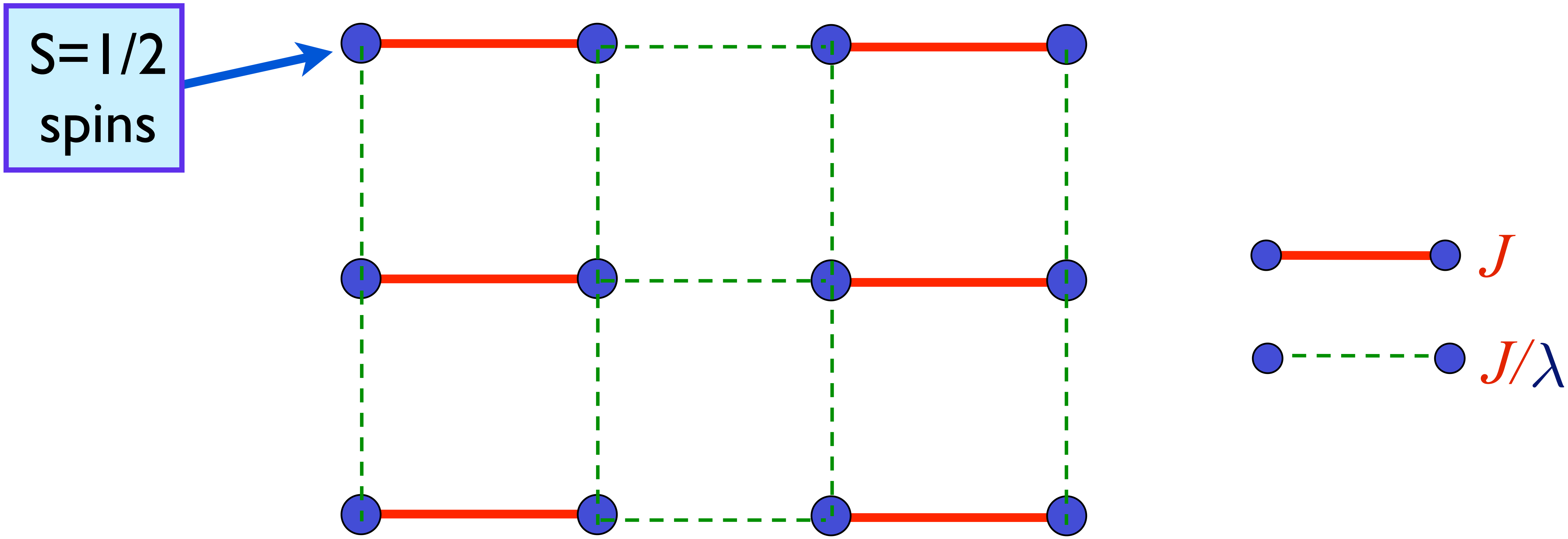
2. Compressible quantum matter

   Holography of strange metals

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

Square lattice antiferromagnet

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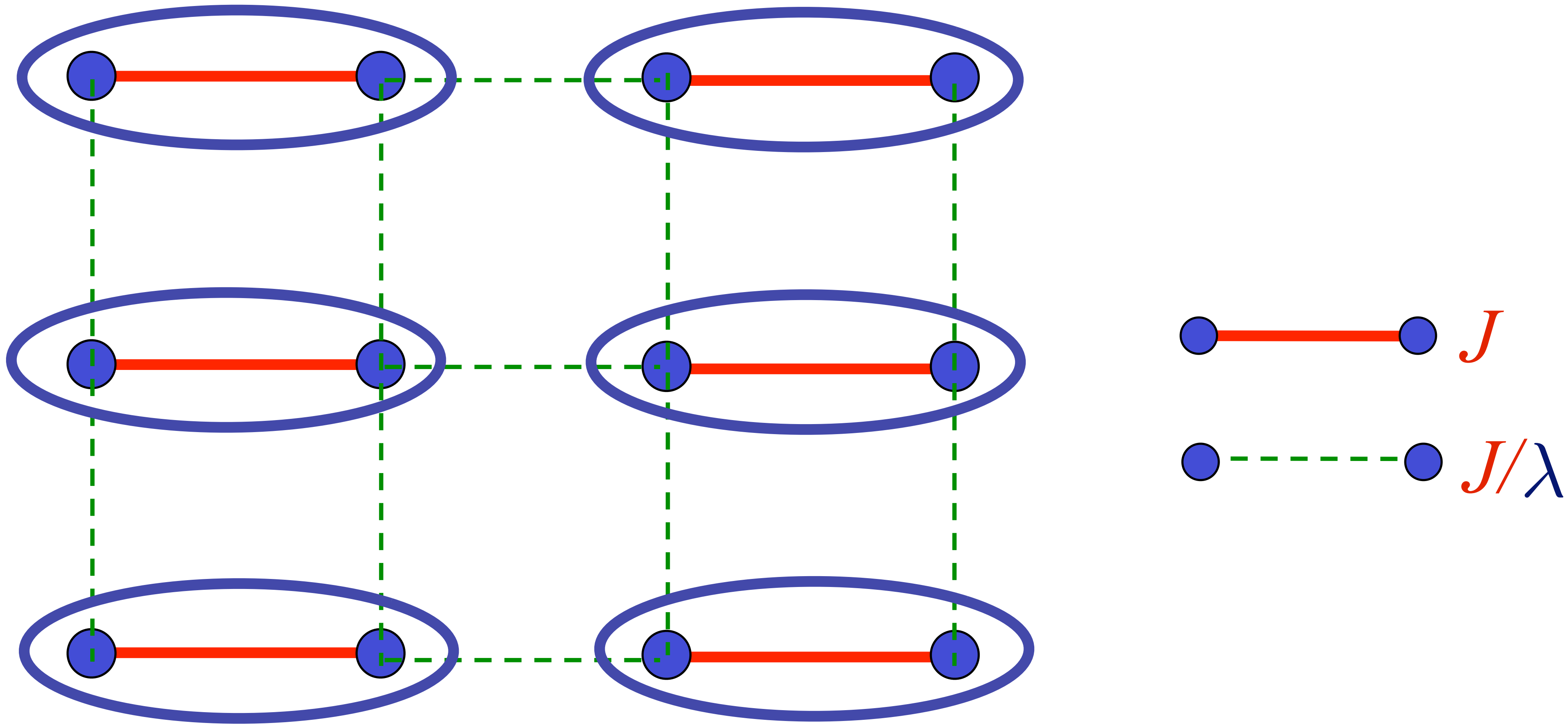

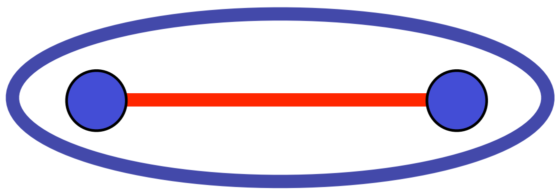 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

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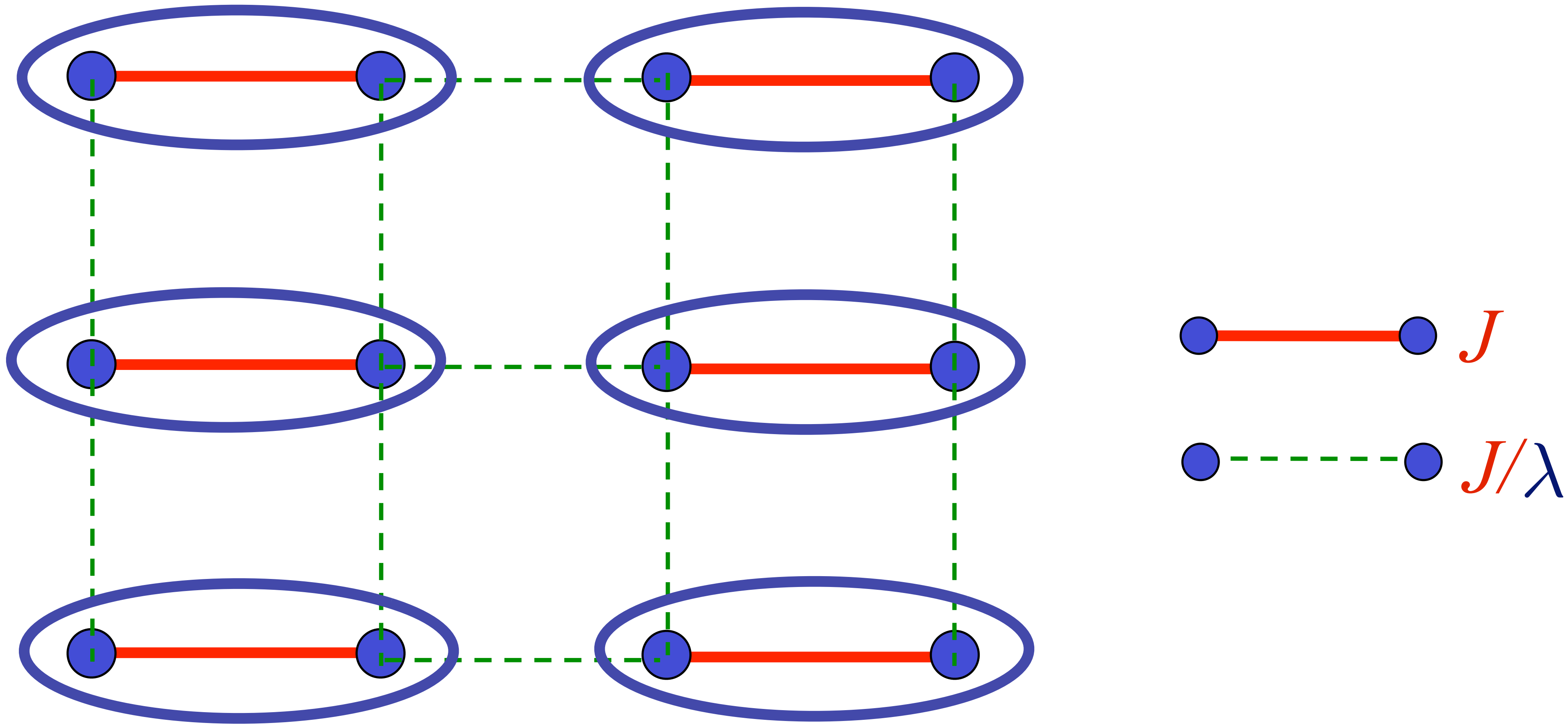

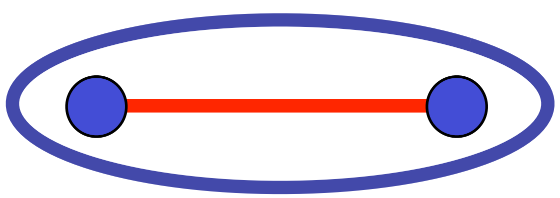

$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Square lattice antiferromagnet

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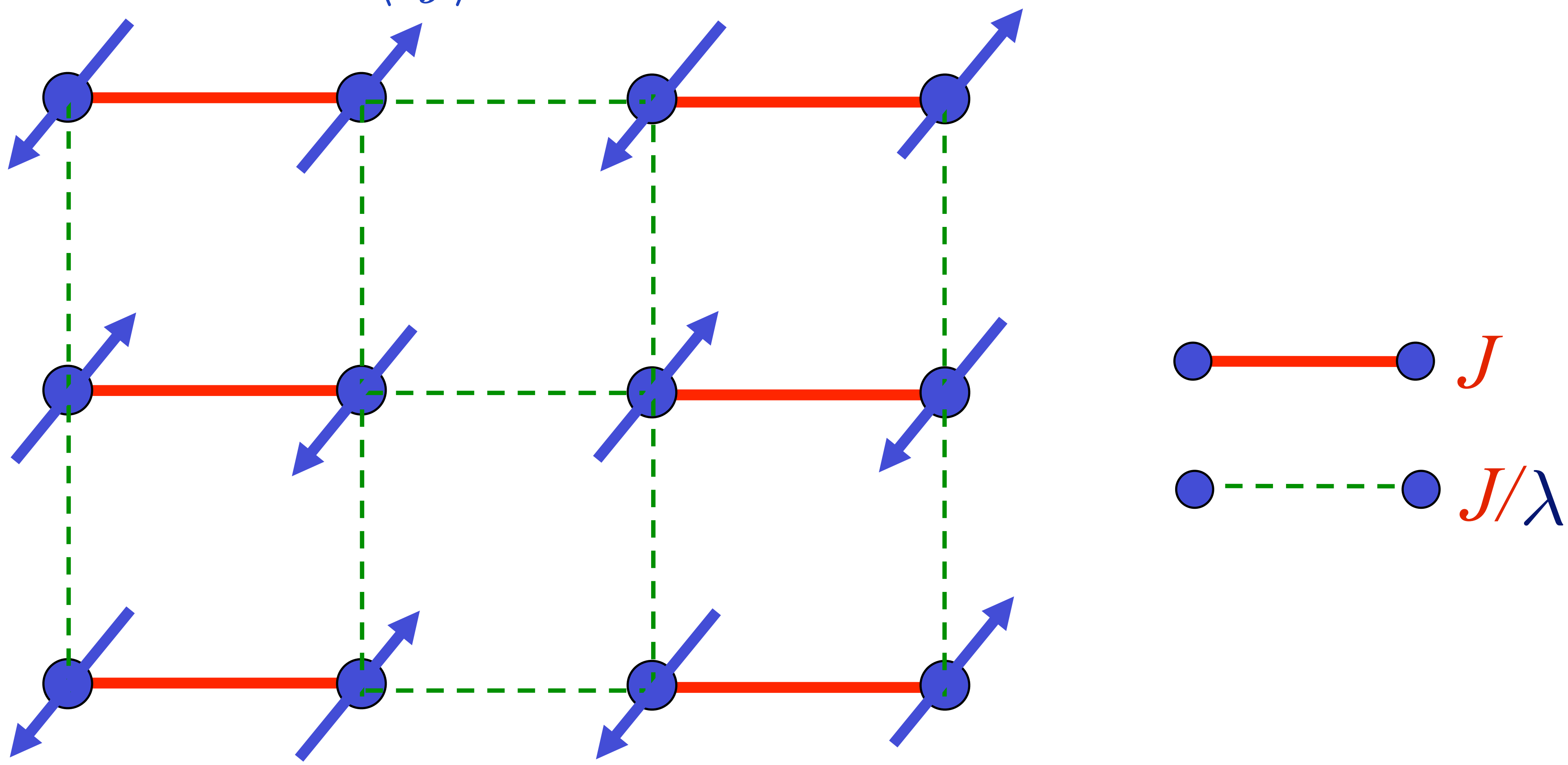

For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

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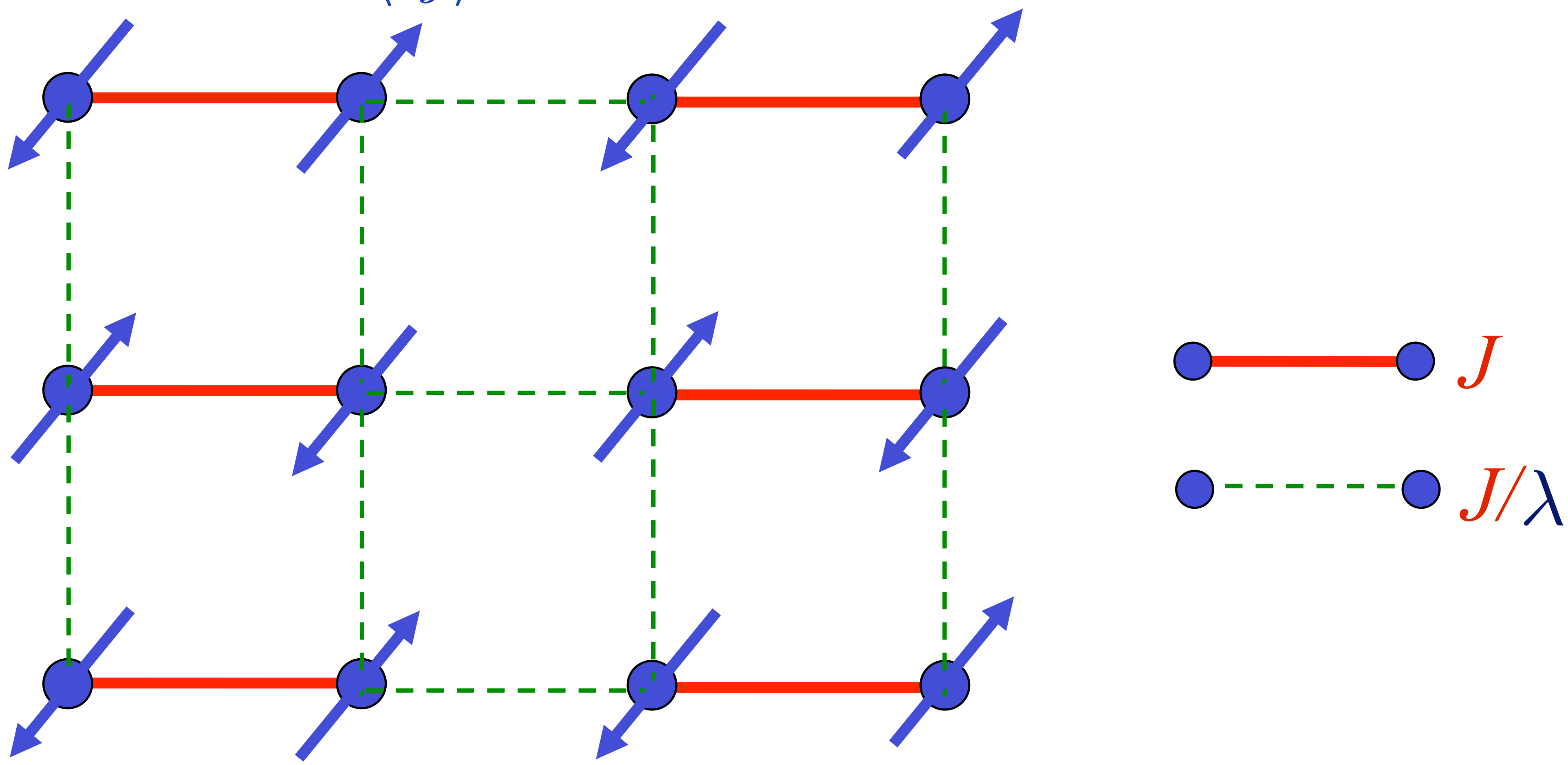

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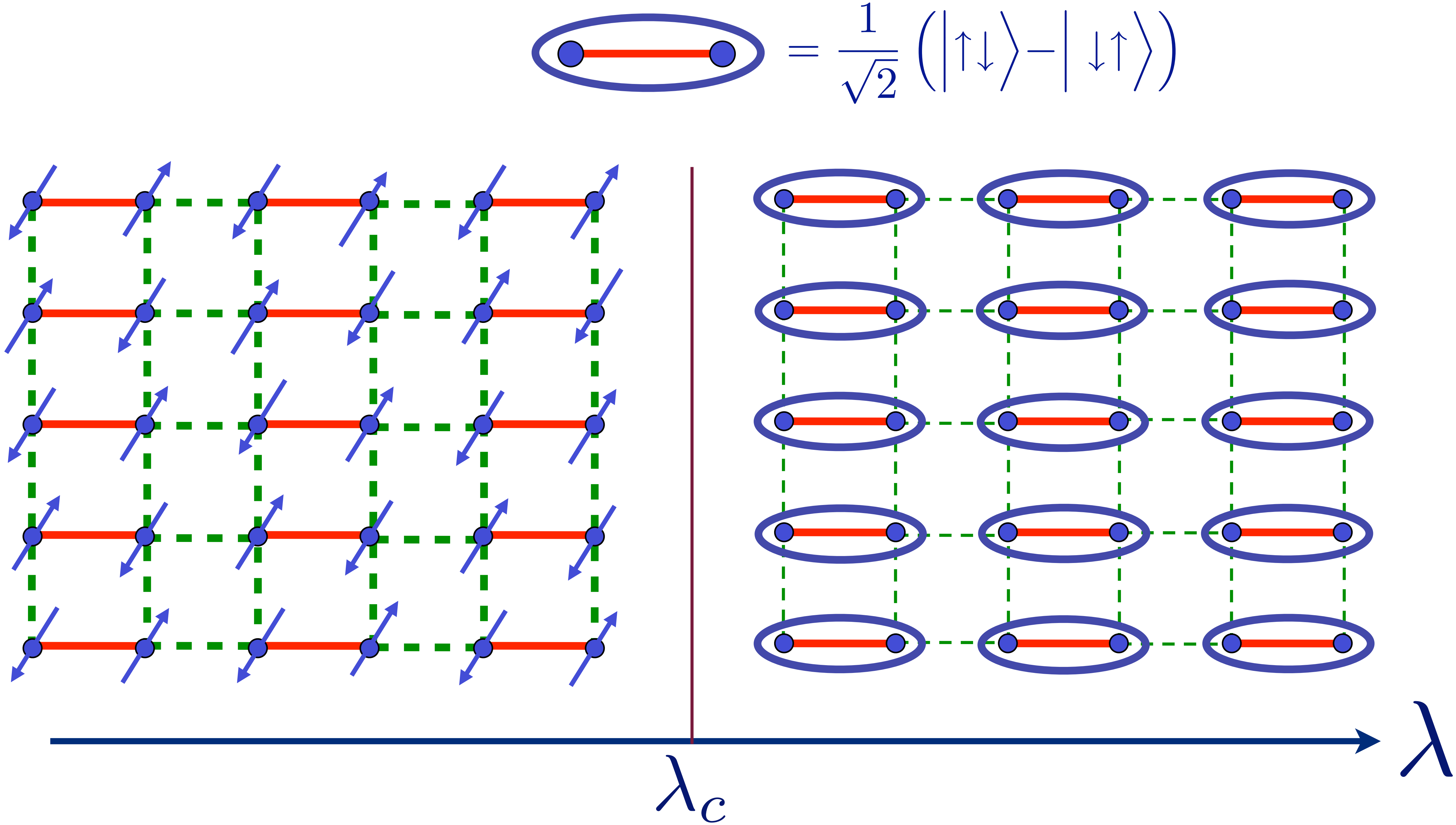
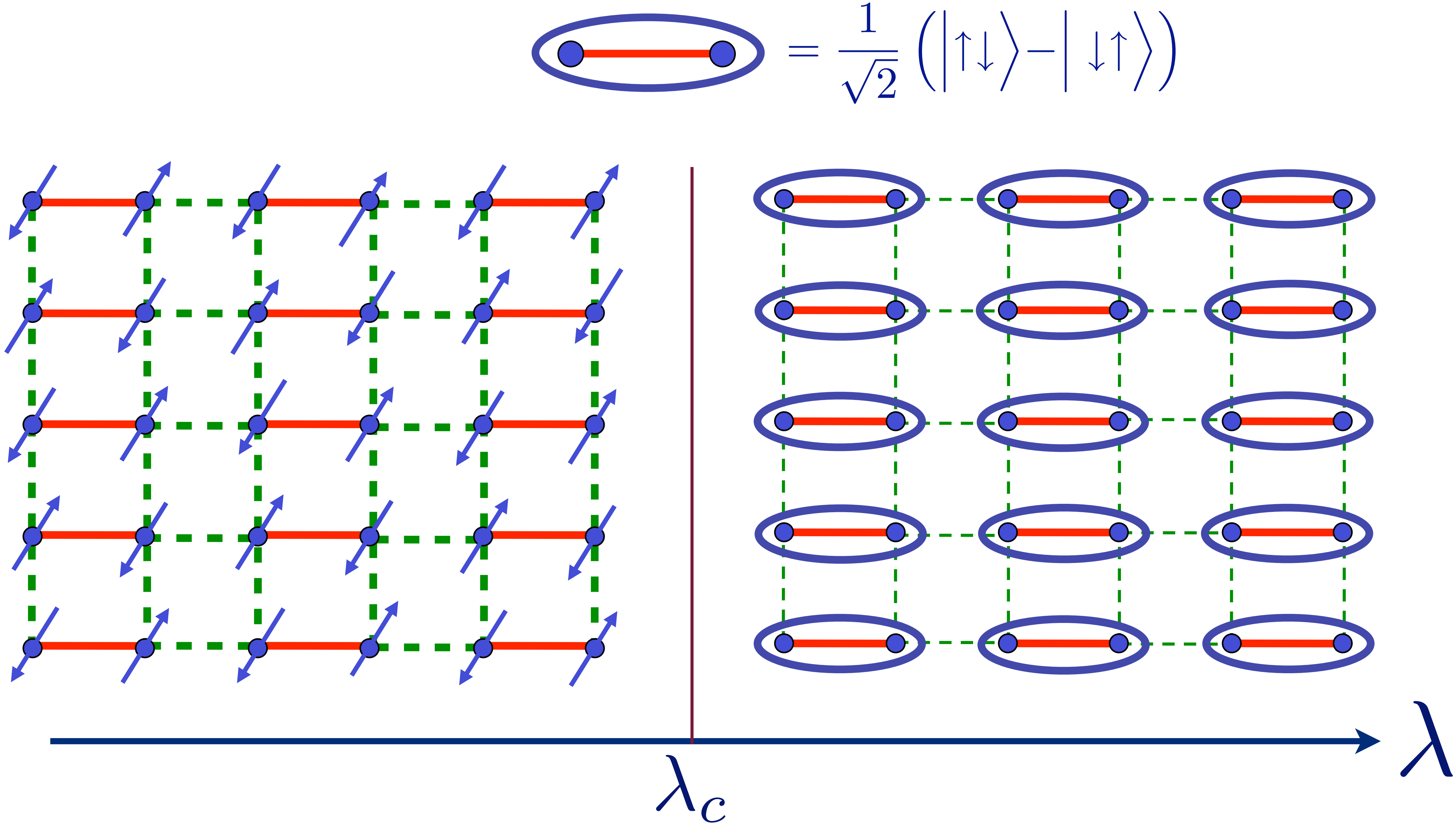
No EPR pairs

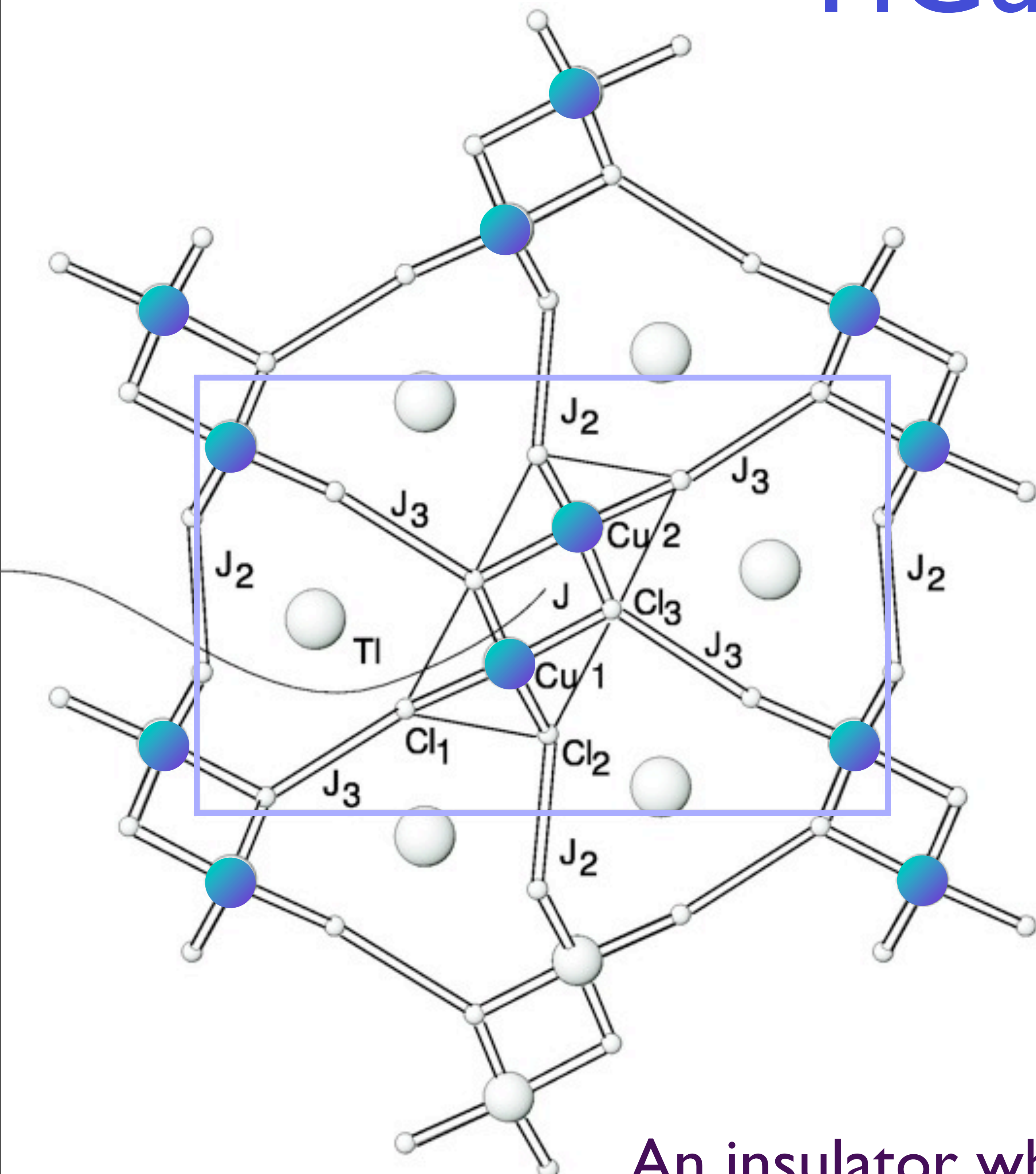

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ
λ_c

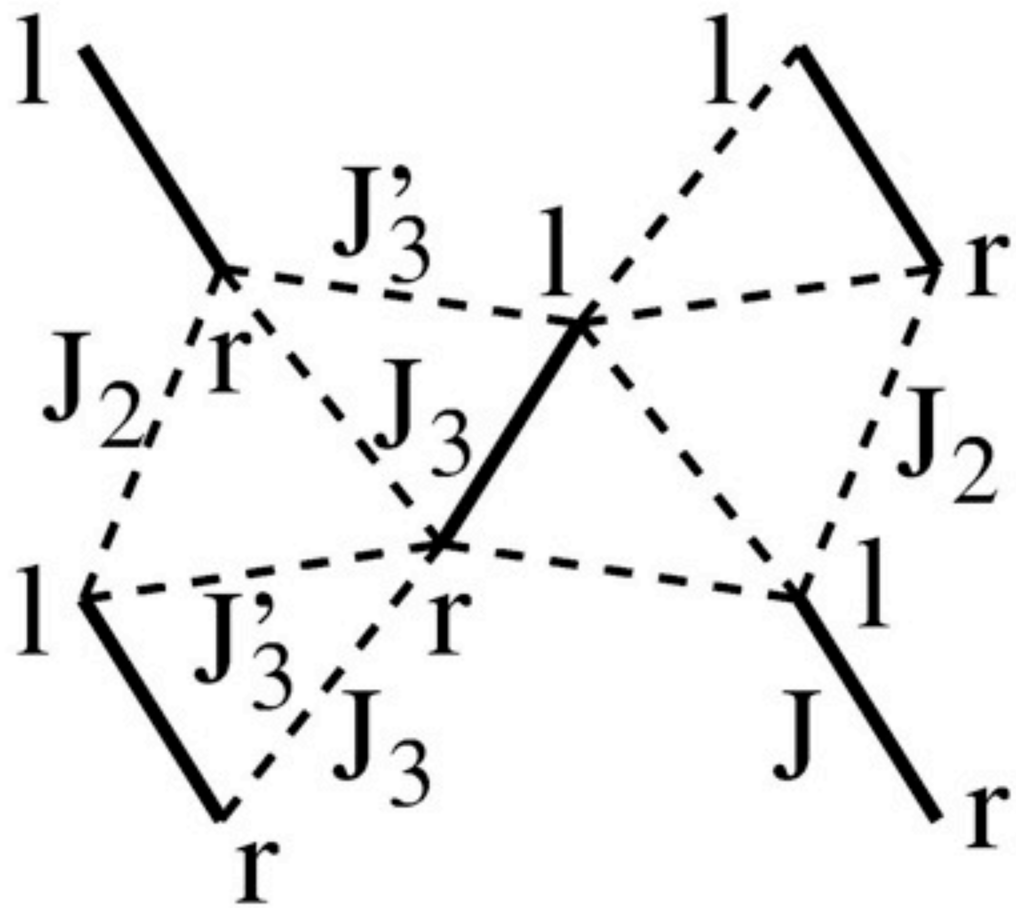


Pressure in $TlCuCl_3$

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

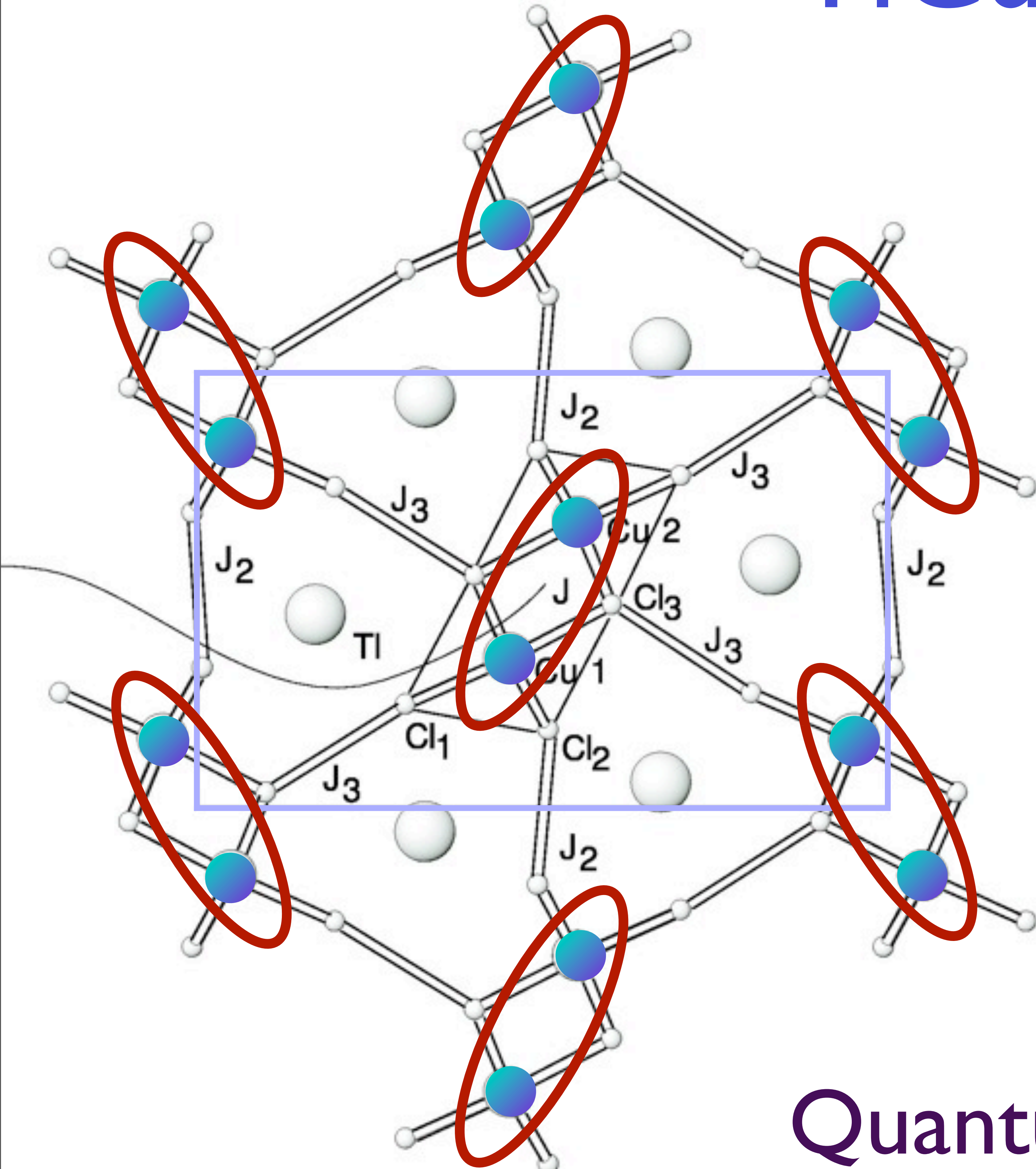
$TlCuCl_3$

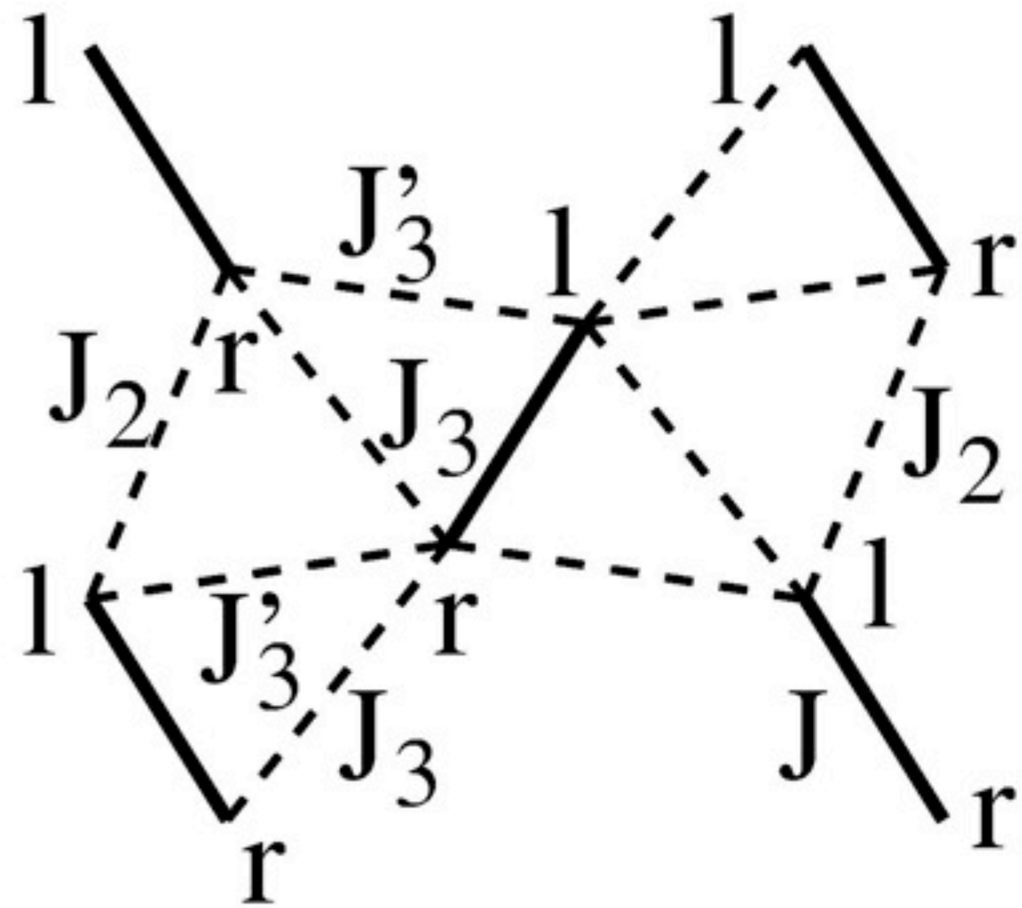




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

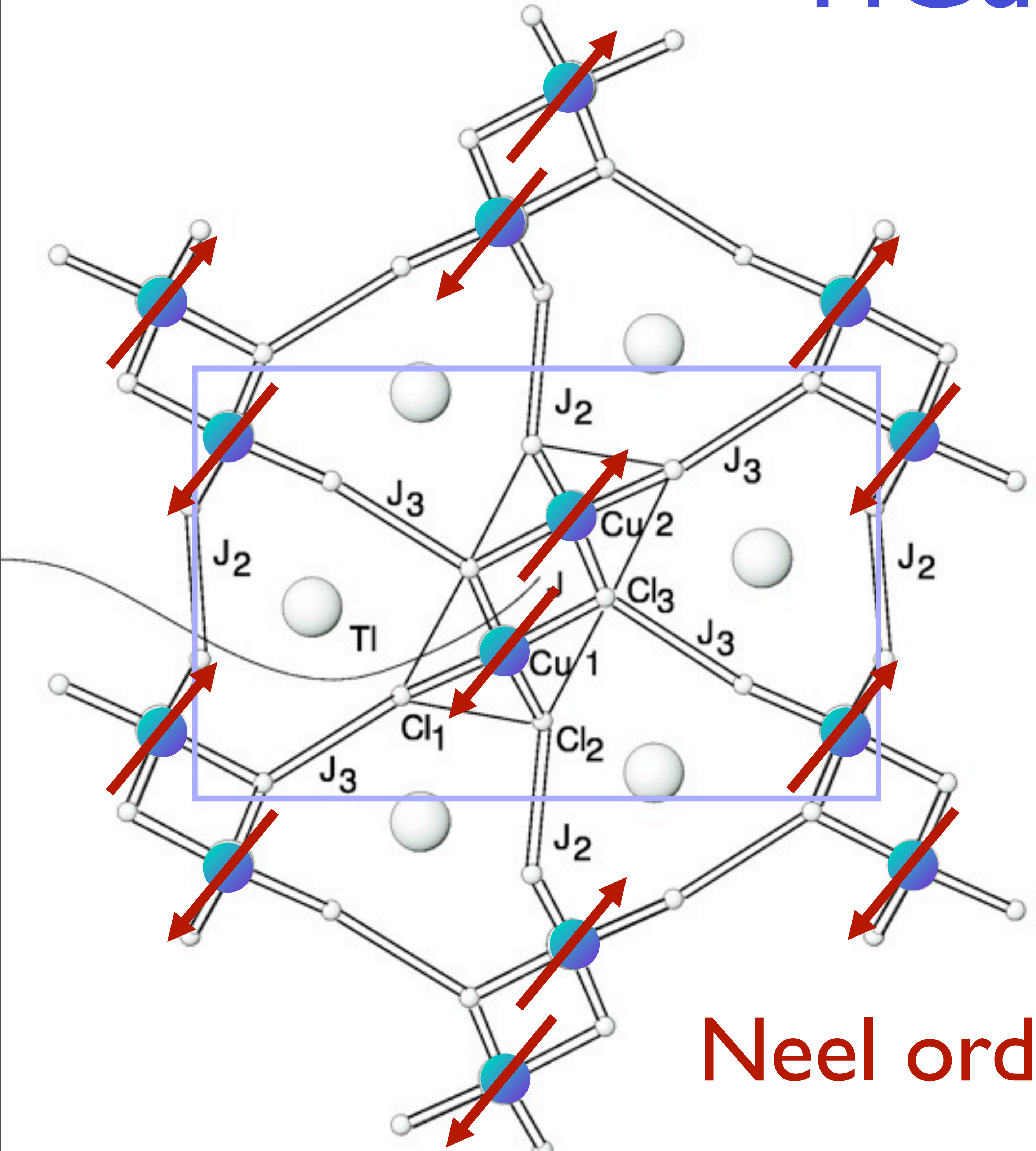
$TlCuCl_3$

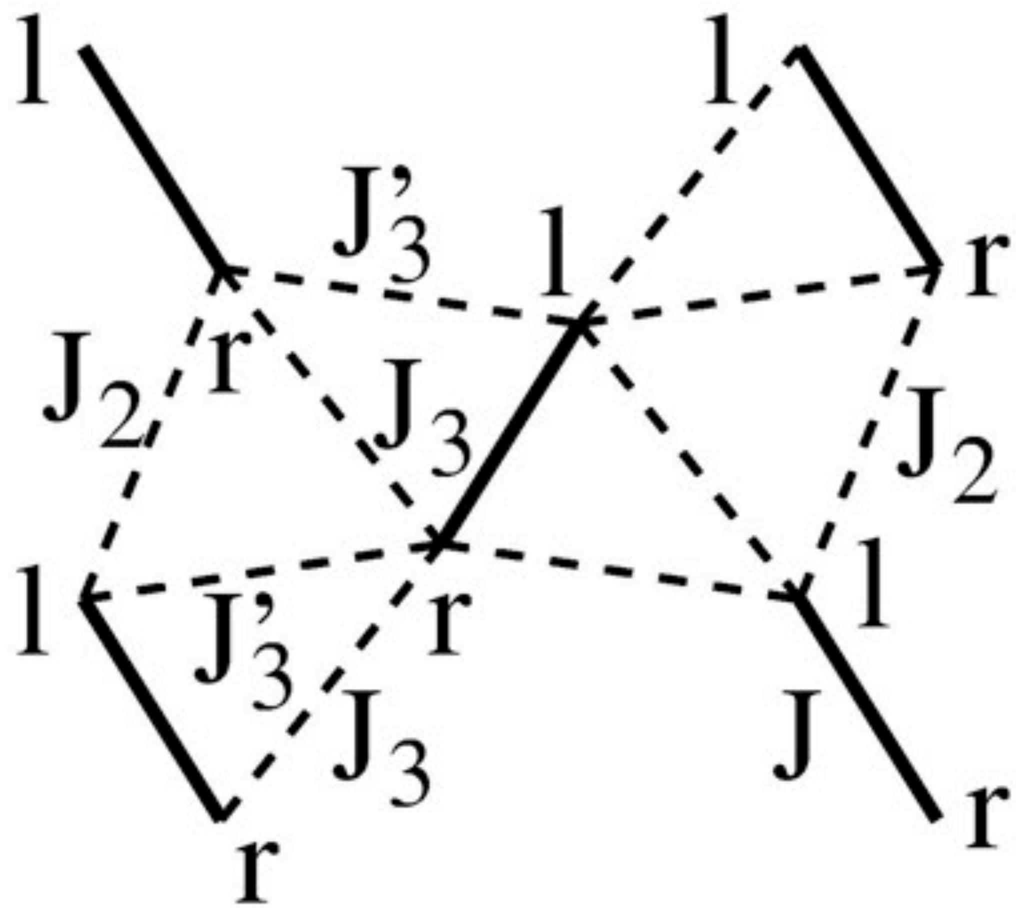




Quantum paramagnet at ambient pressure

$TlCuCl_3$

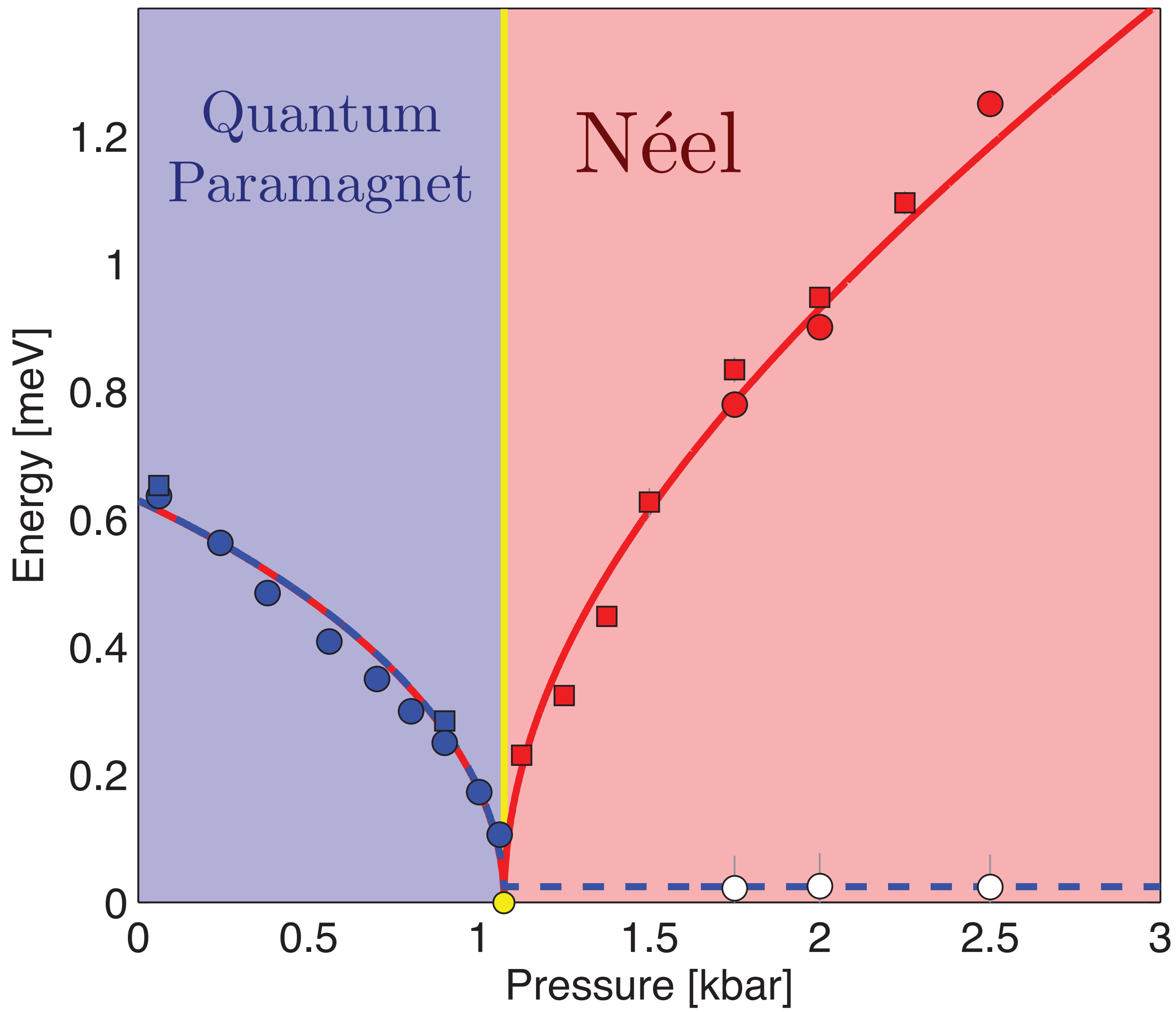




Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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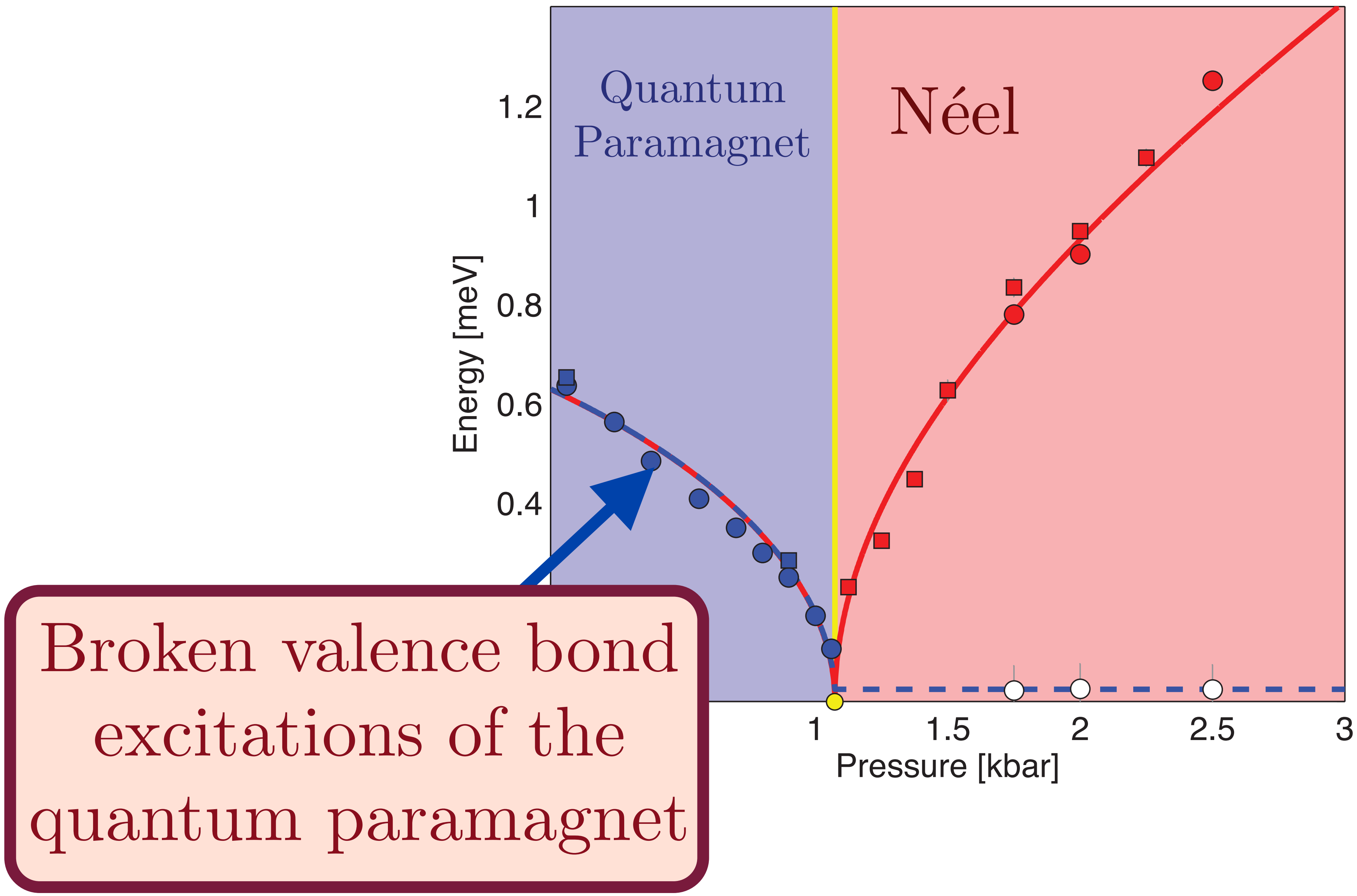

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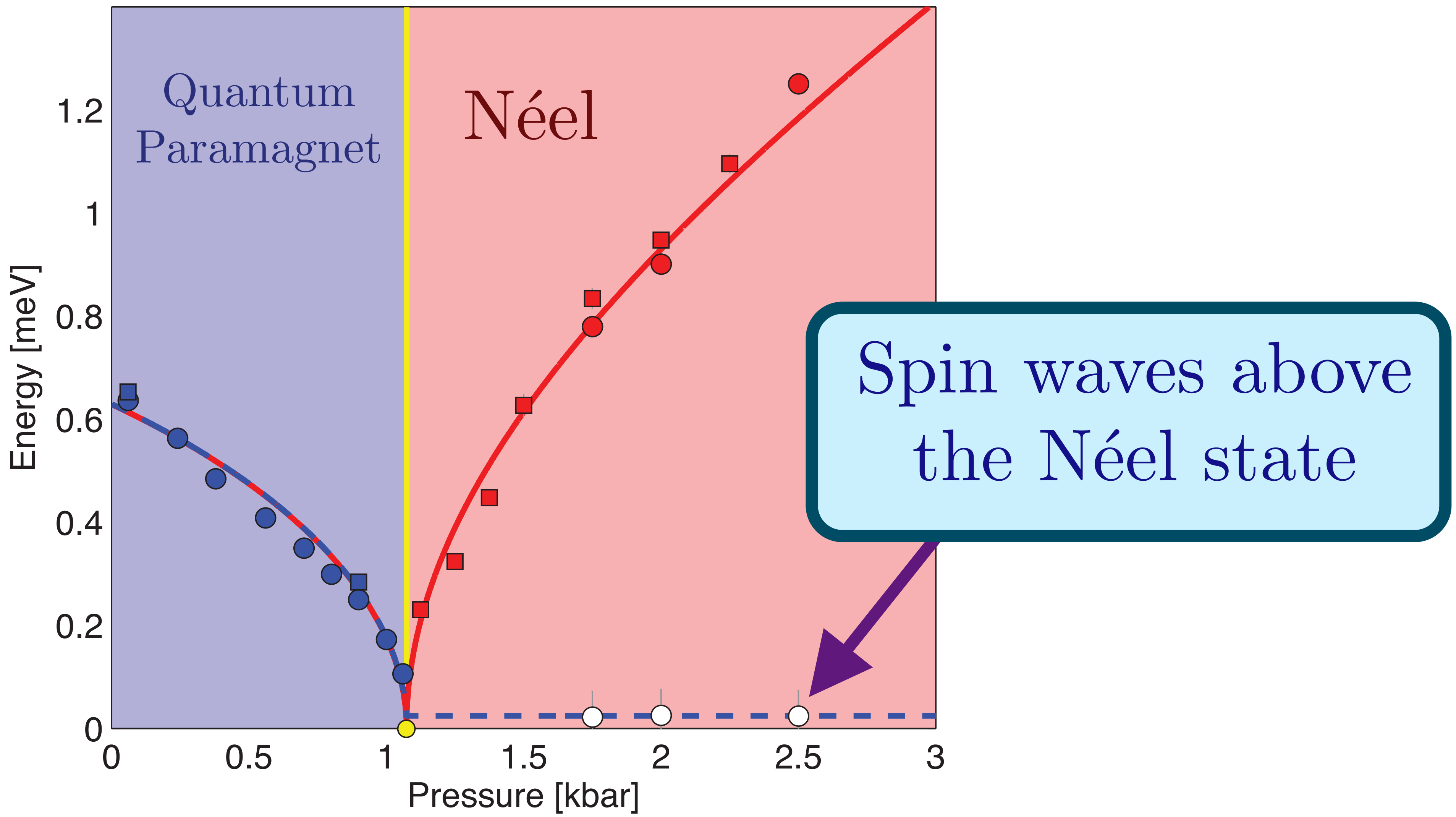

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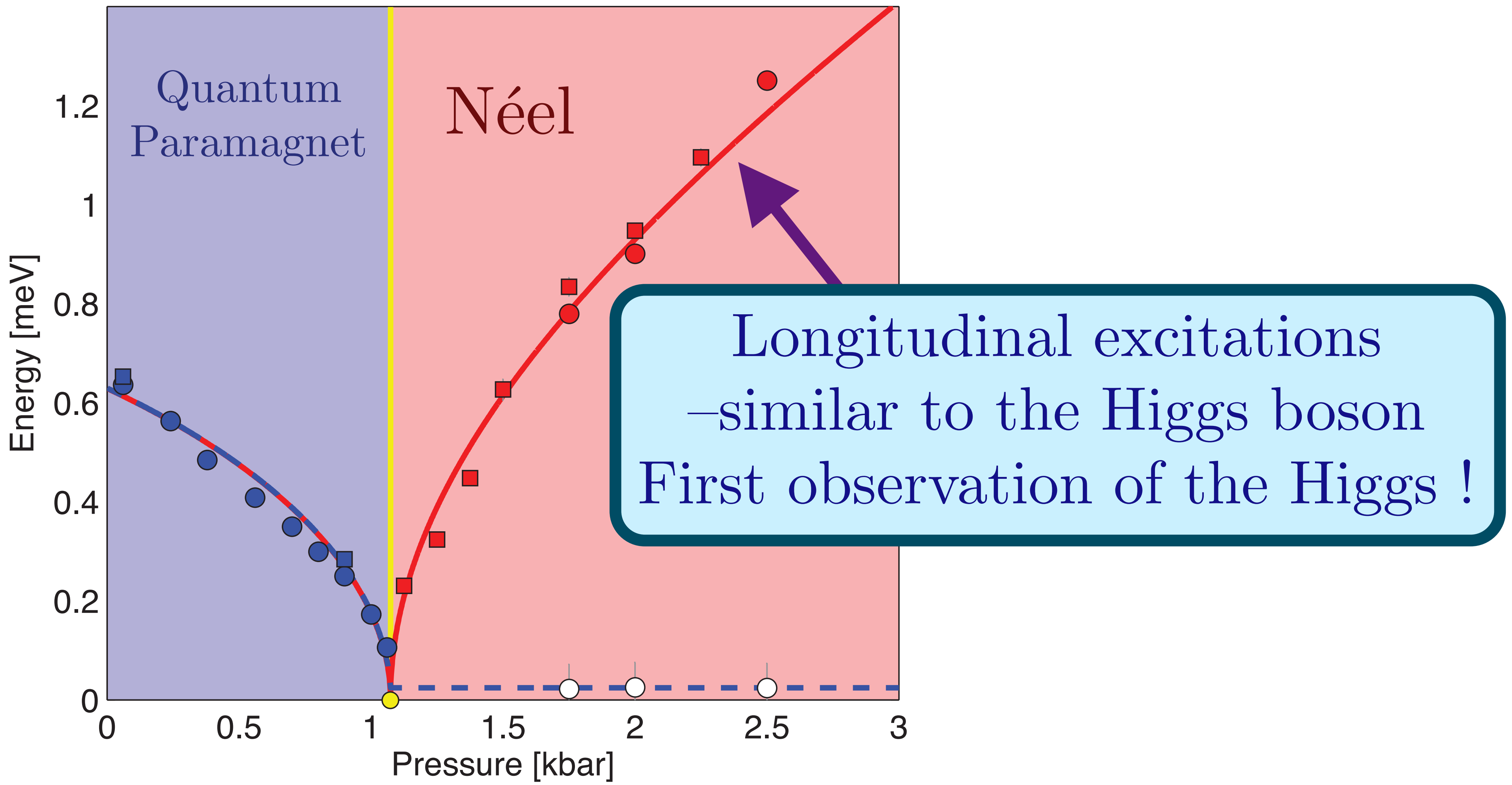
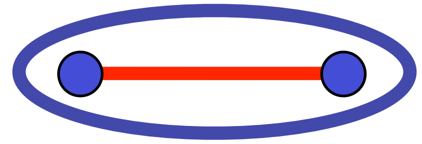


"Higgs" particle appears at theoretically predicted energy

S. Sachdev, arXiv:0901.4103

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)



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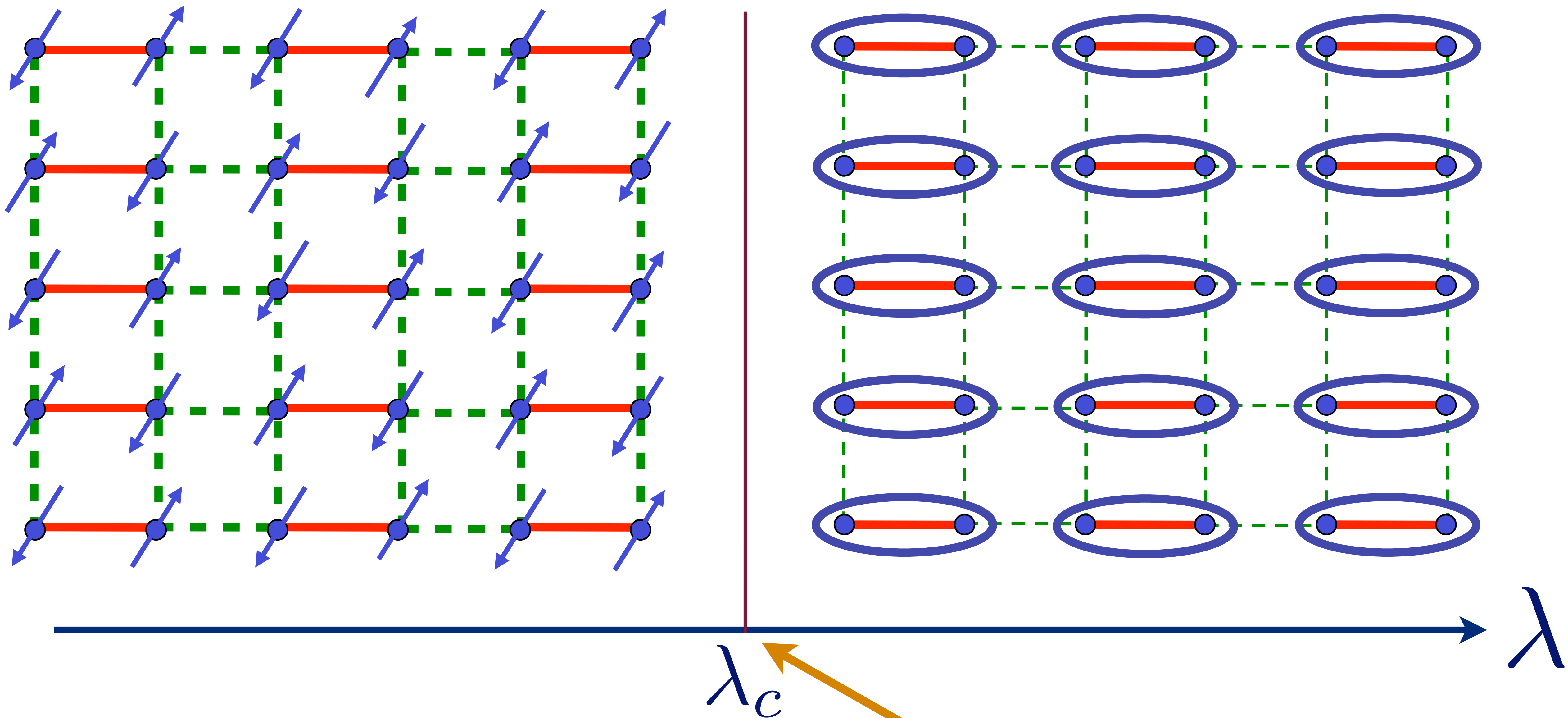
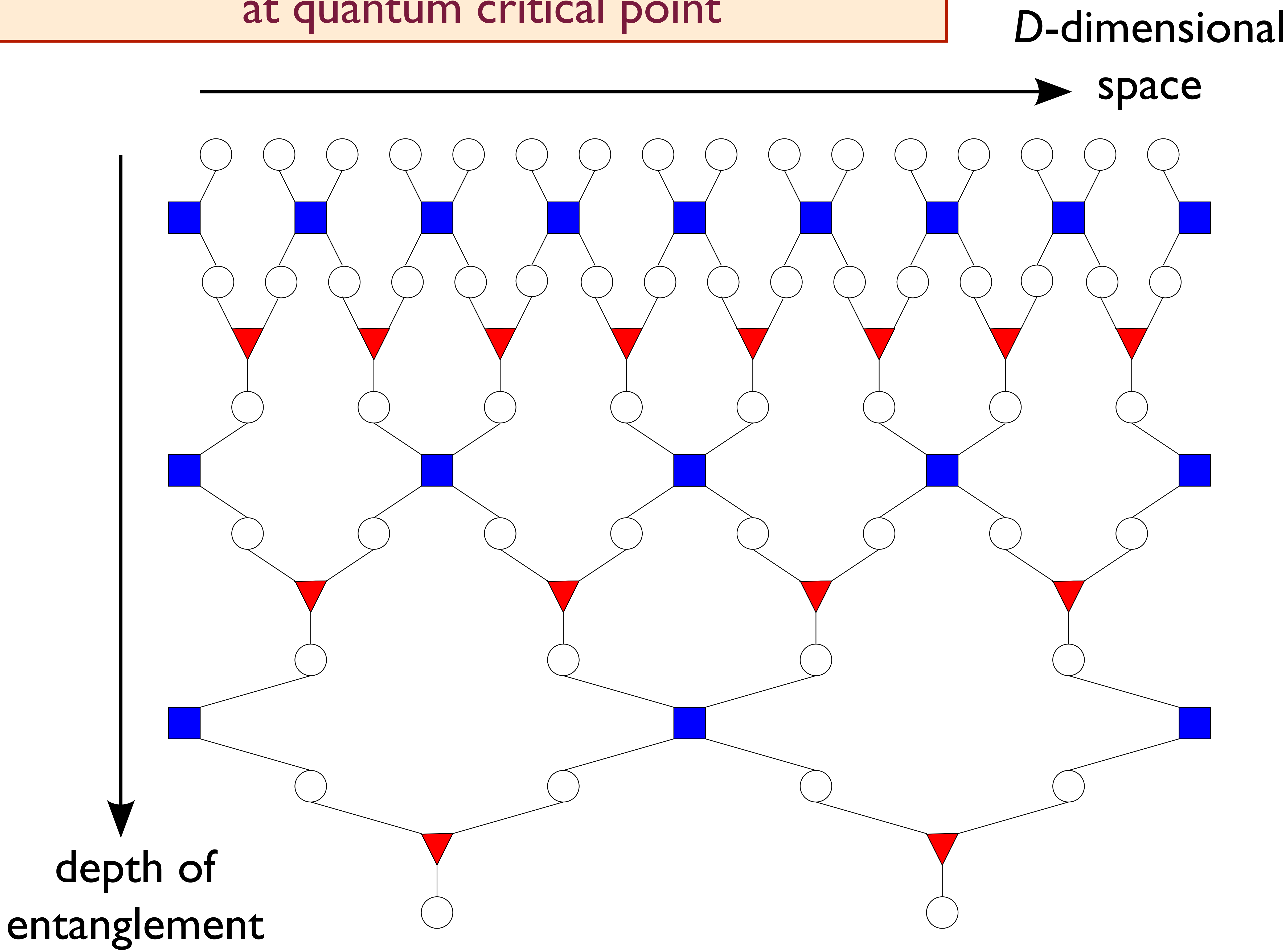


Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point



Characteristics of quantum critical point

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- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").

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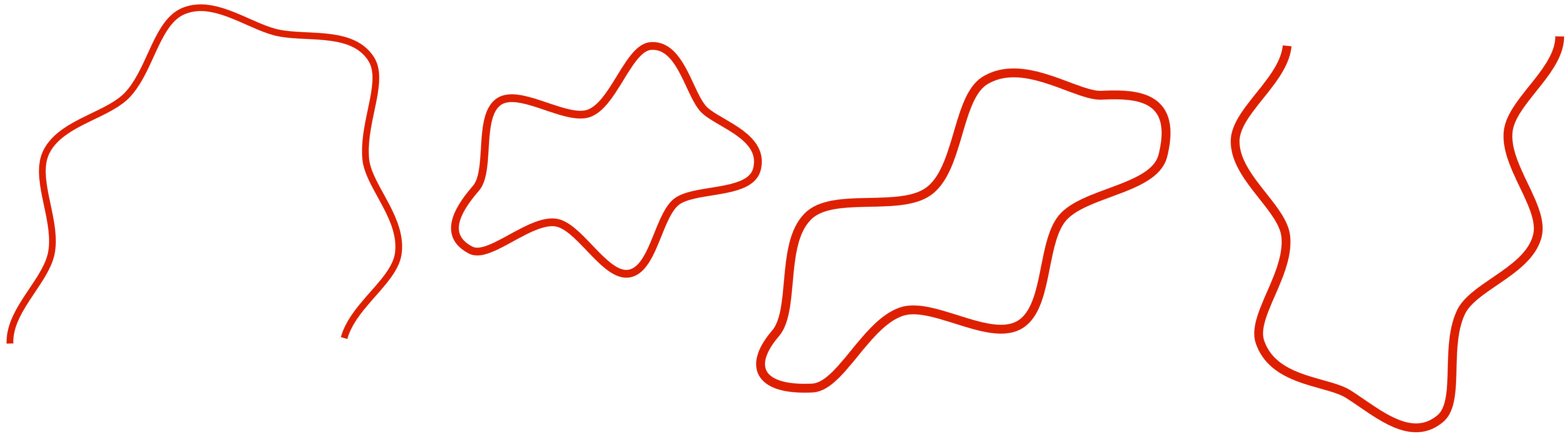
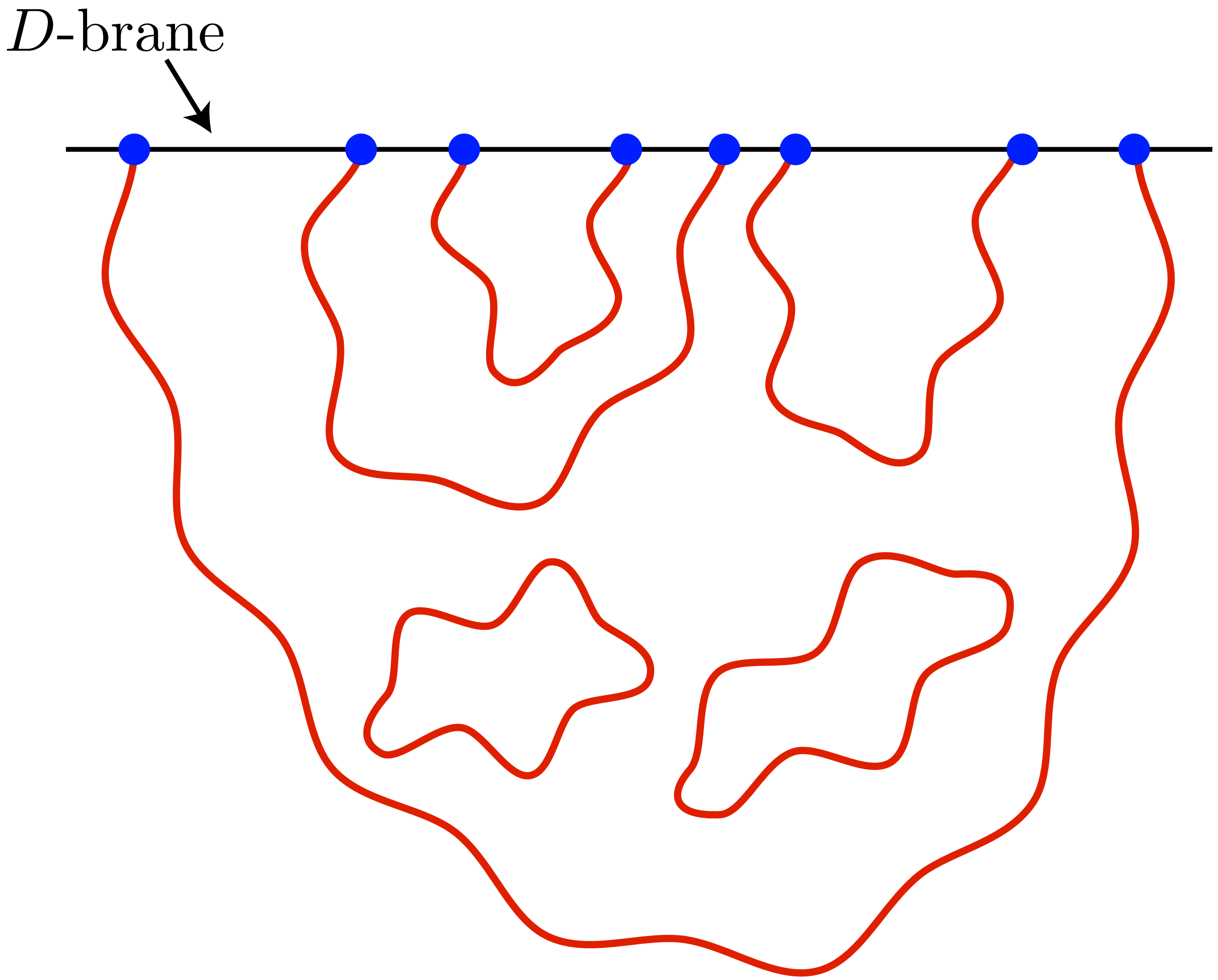
- Long-range entanglement
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").
- The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a **CFT3**

String theory

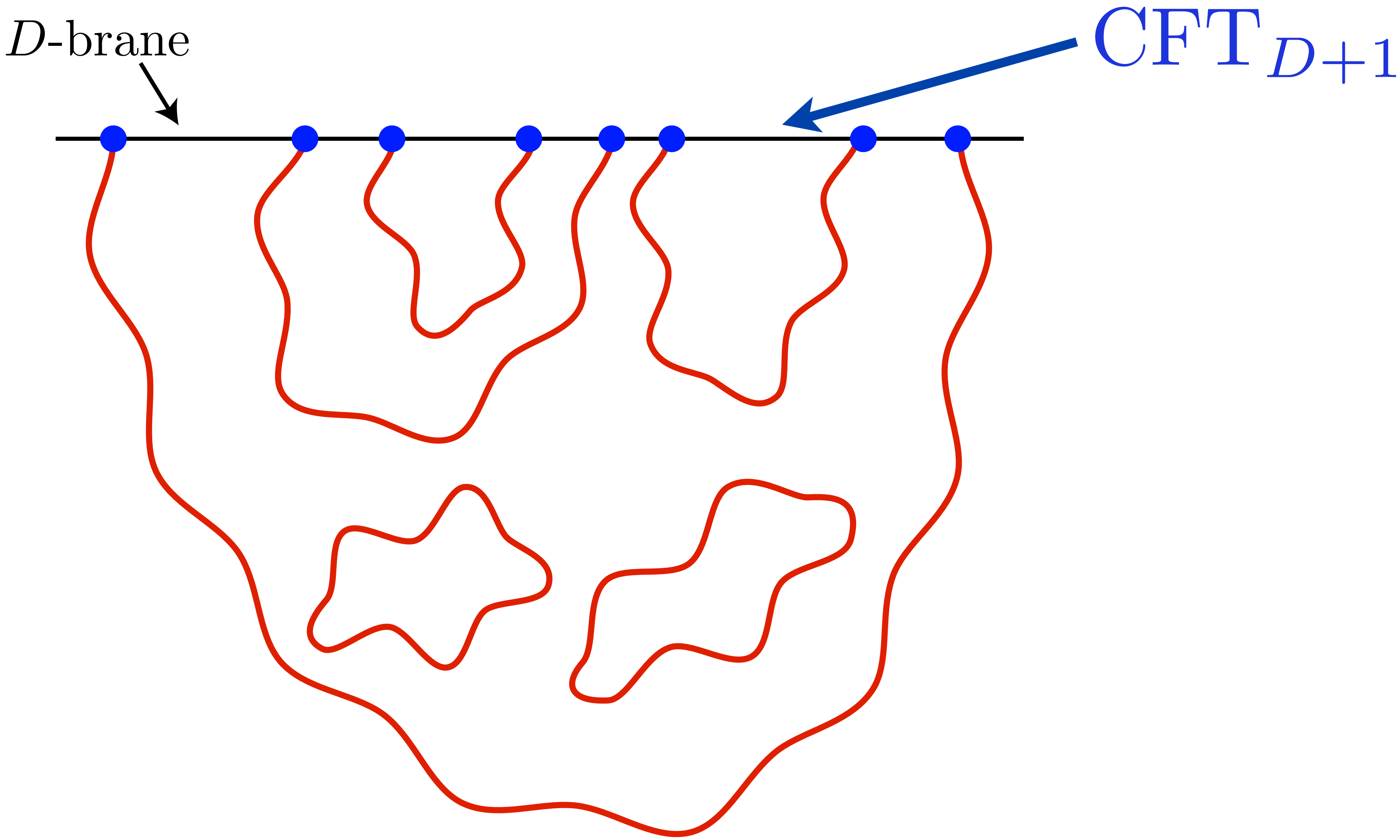


- Allows unification of the standard model of particle physics with gravity.

- Low-lying string modes correspond to gauge fields, gravitons, quarks ...

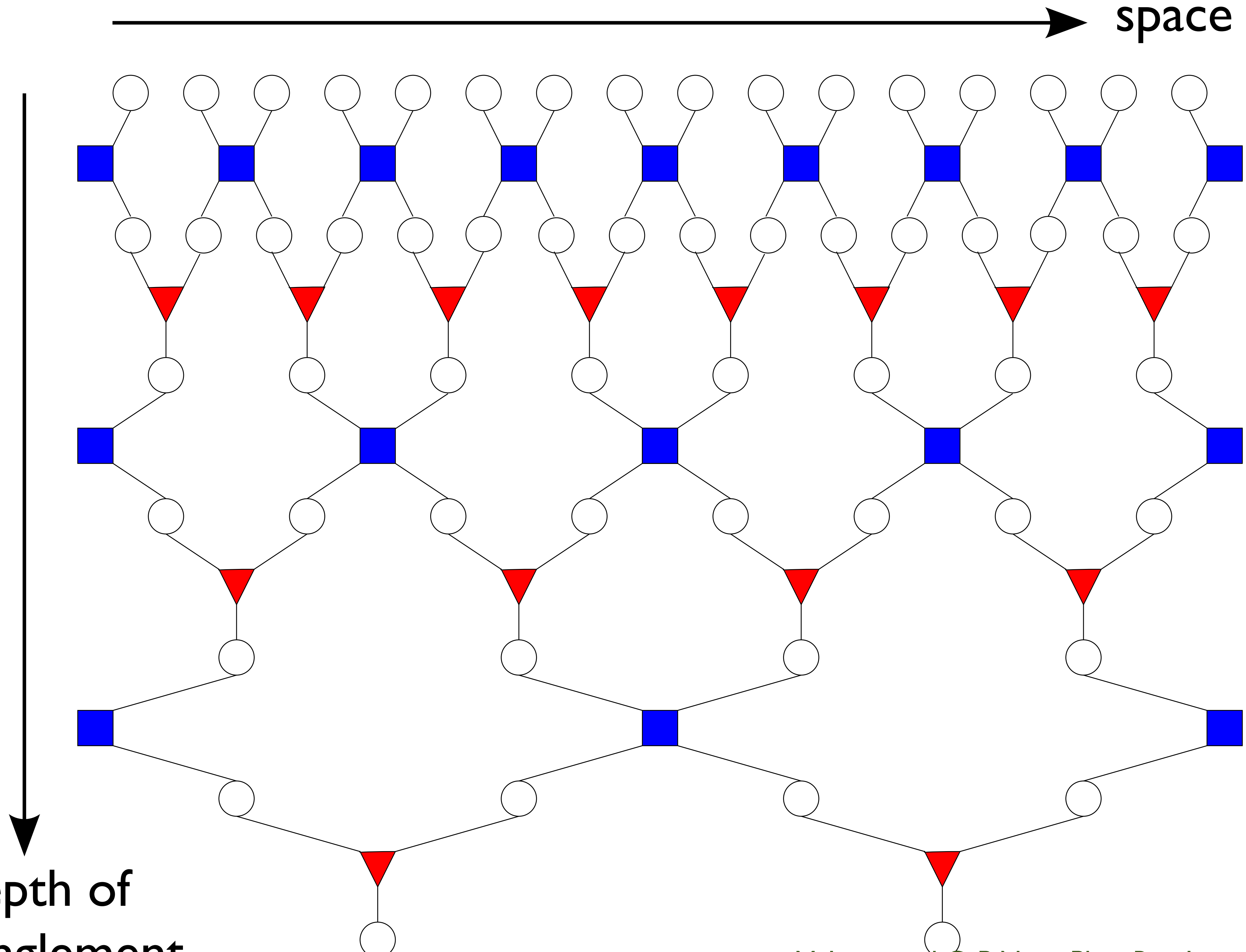


- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.

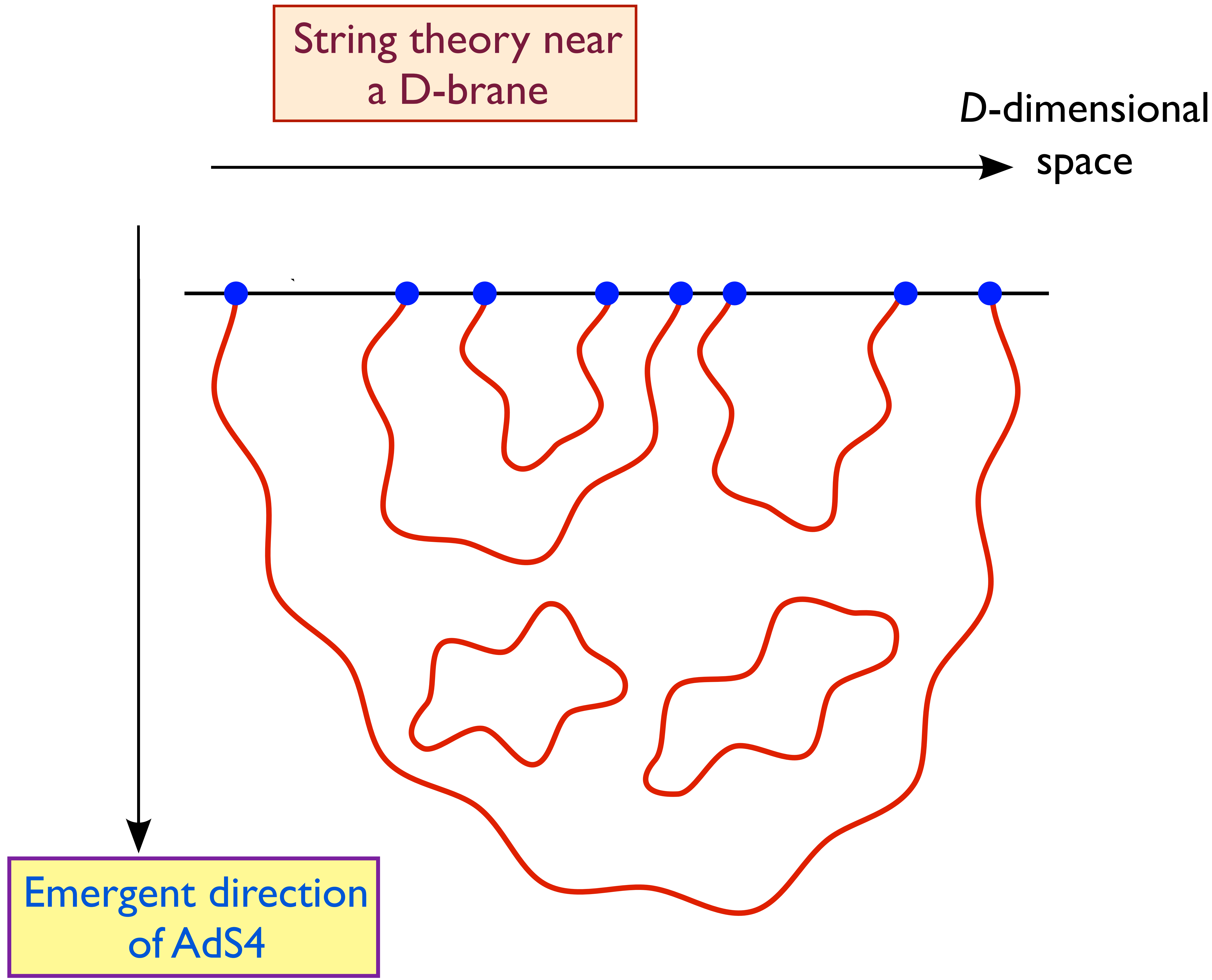


- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.
- In $D = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)


String theory near a D-brane
D-dimensional space
Emergent direction of AdS4

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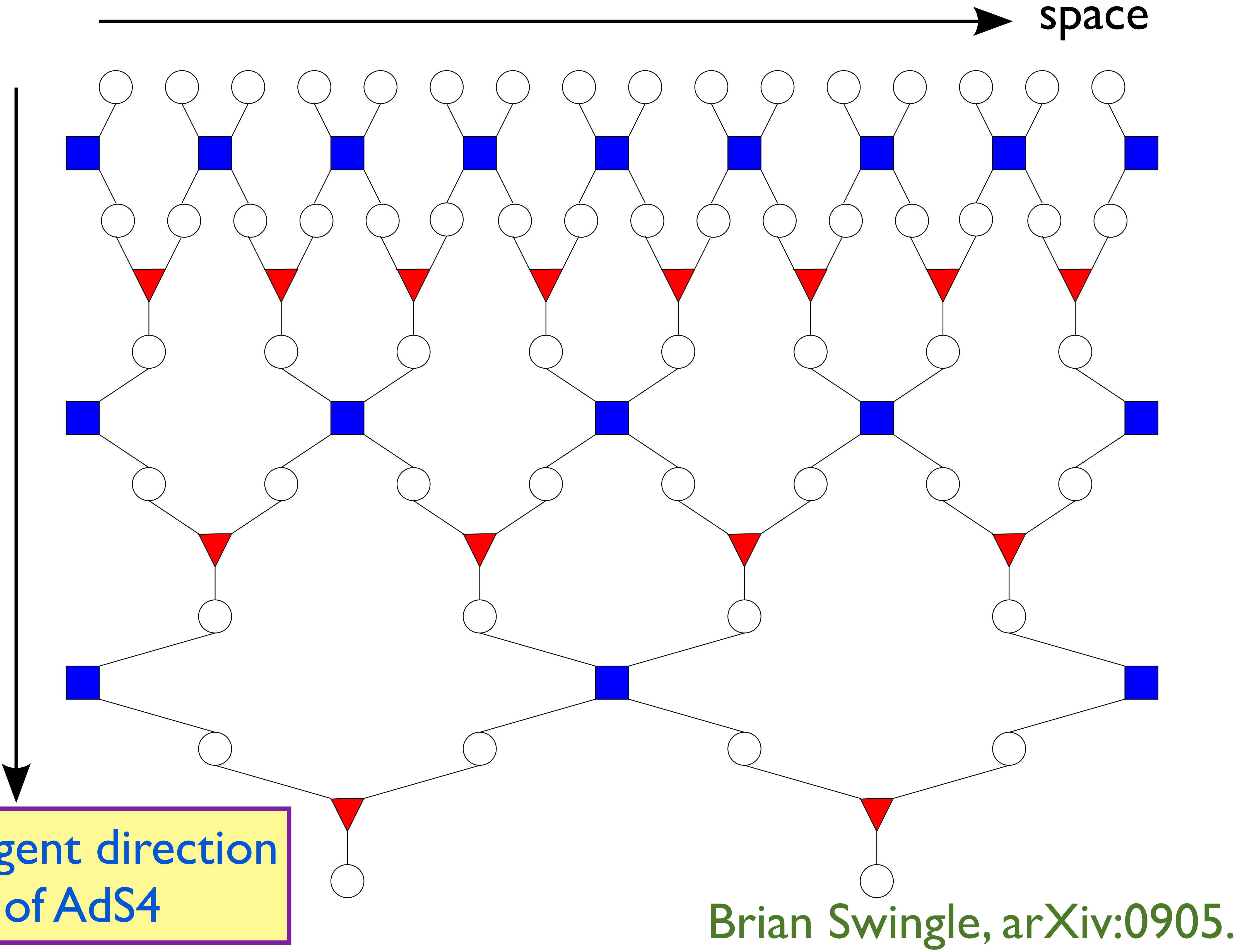
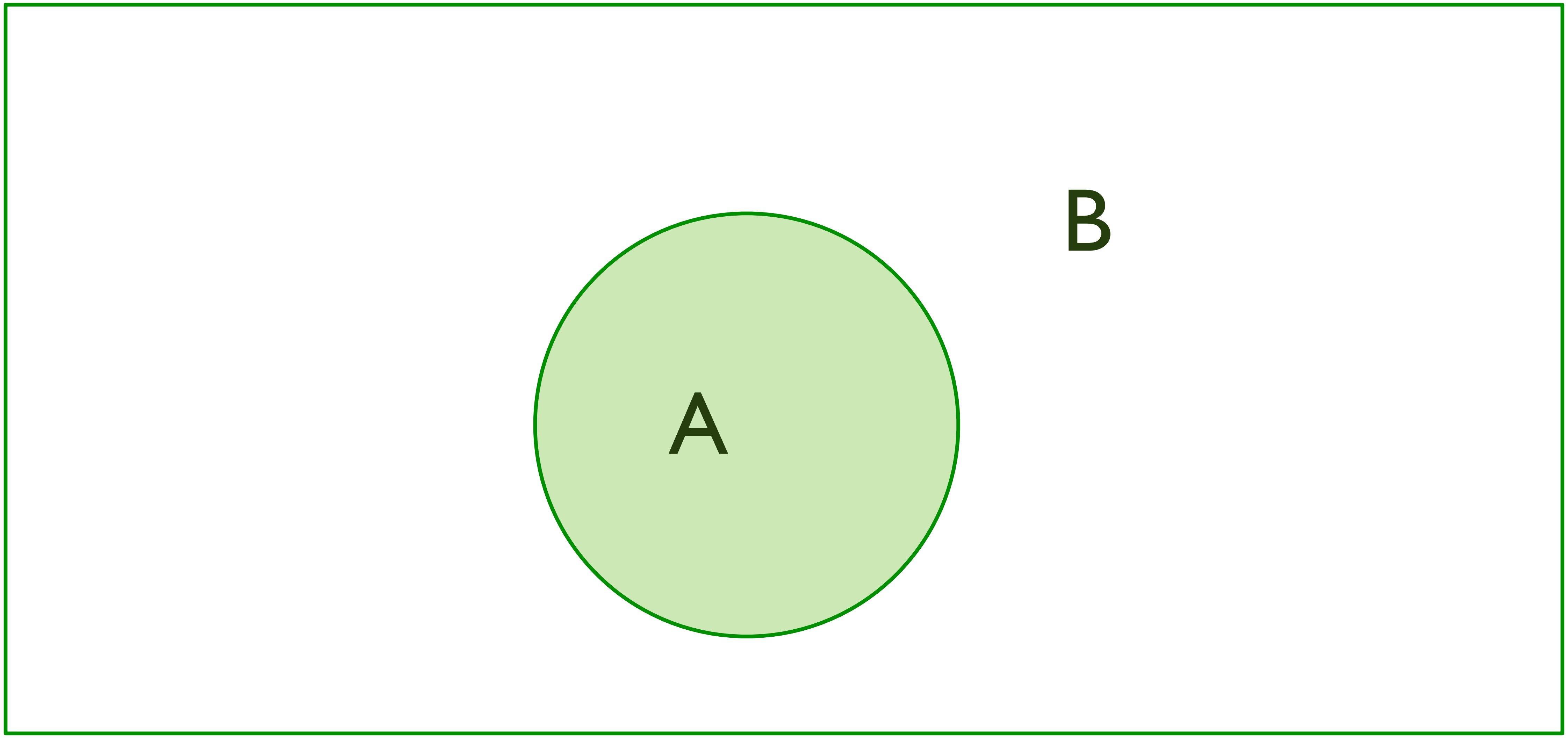


Brian Swingle, arXiv:0905.1317

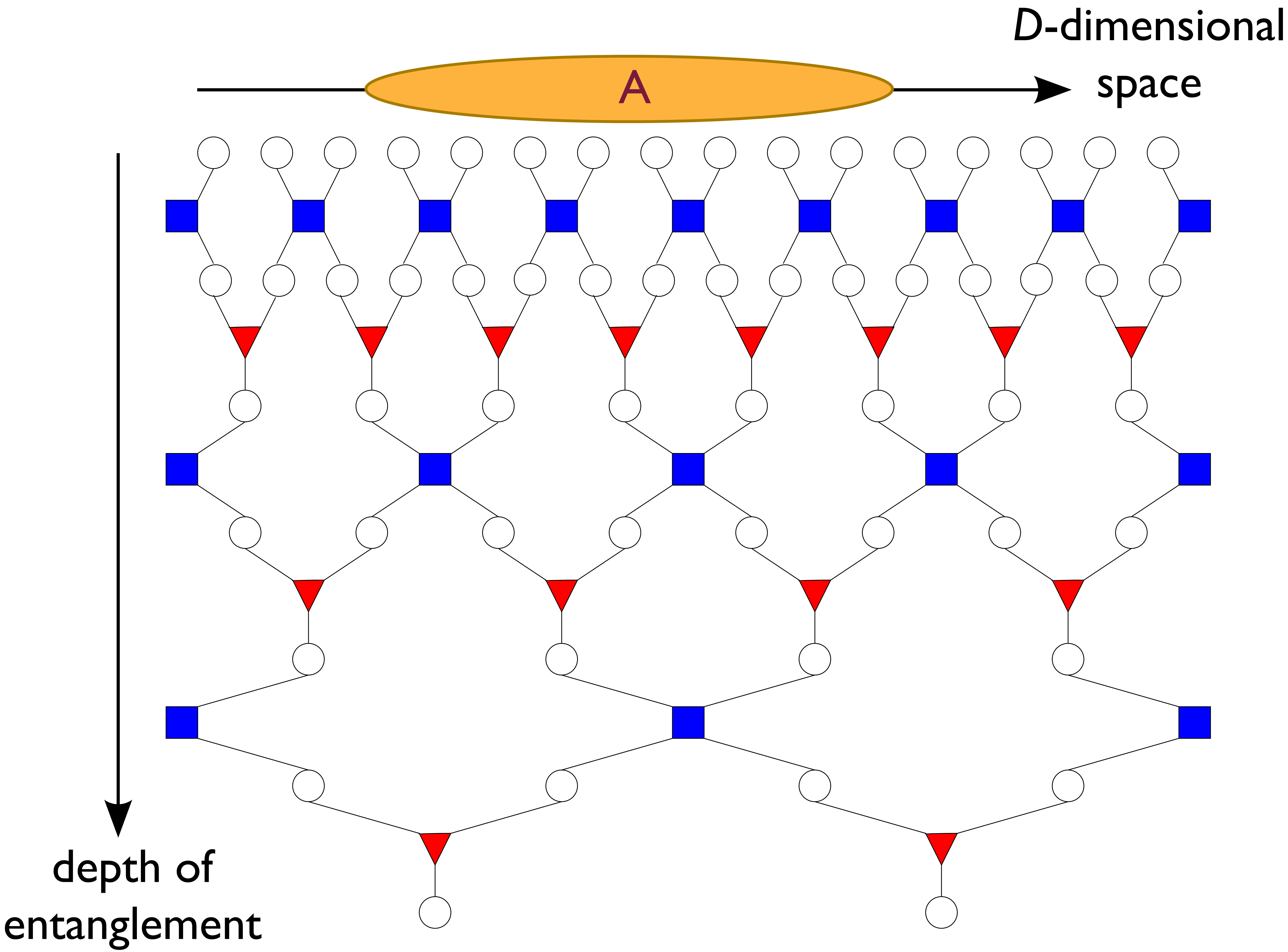
Entanglement entropy



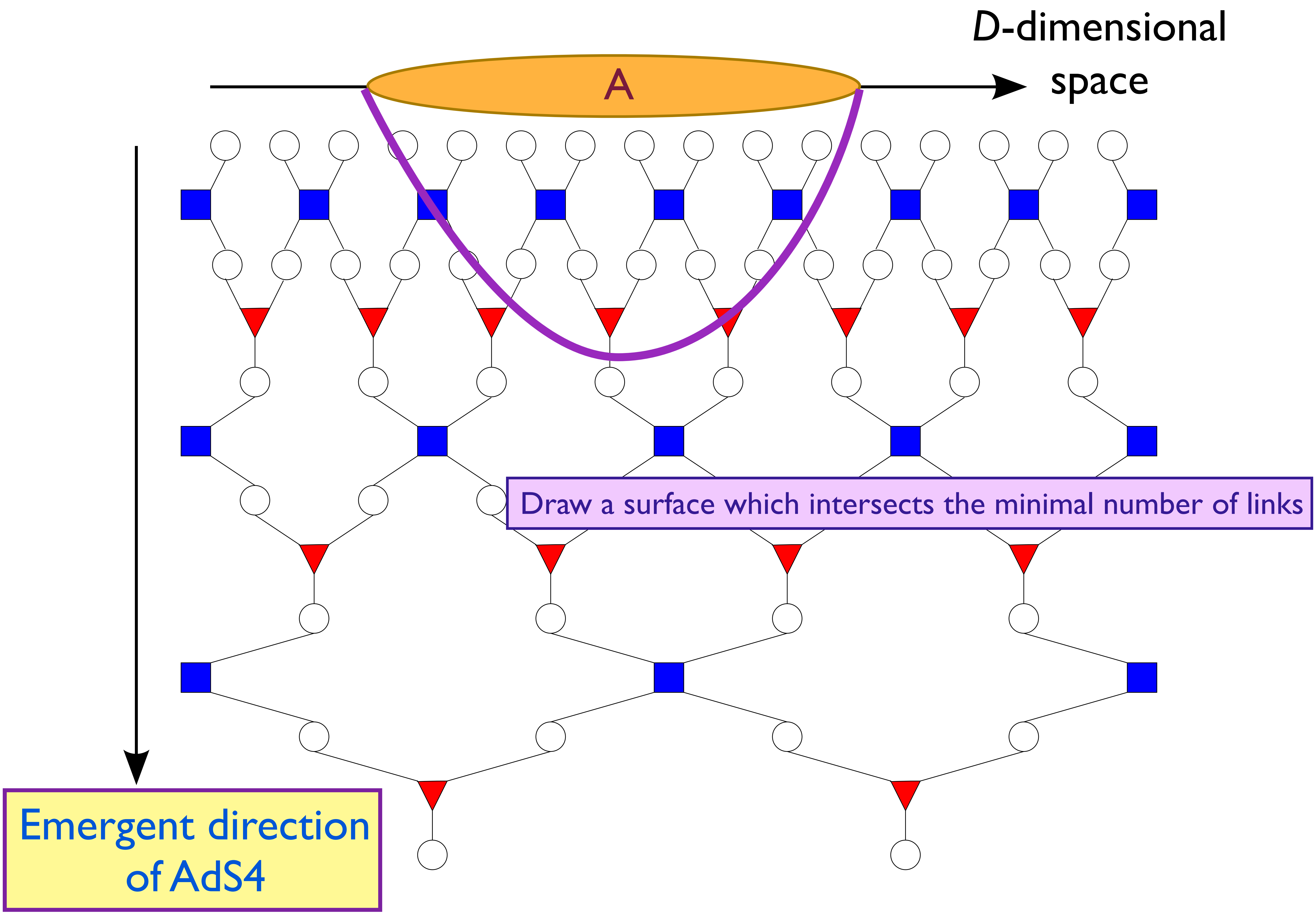
$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy



Entanglement entropy

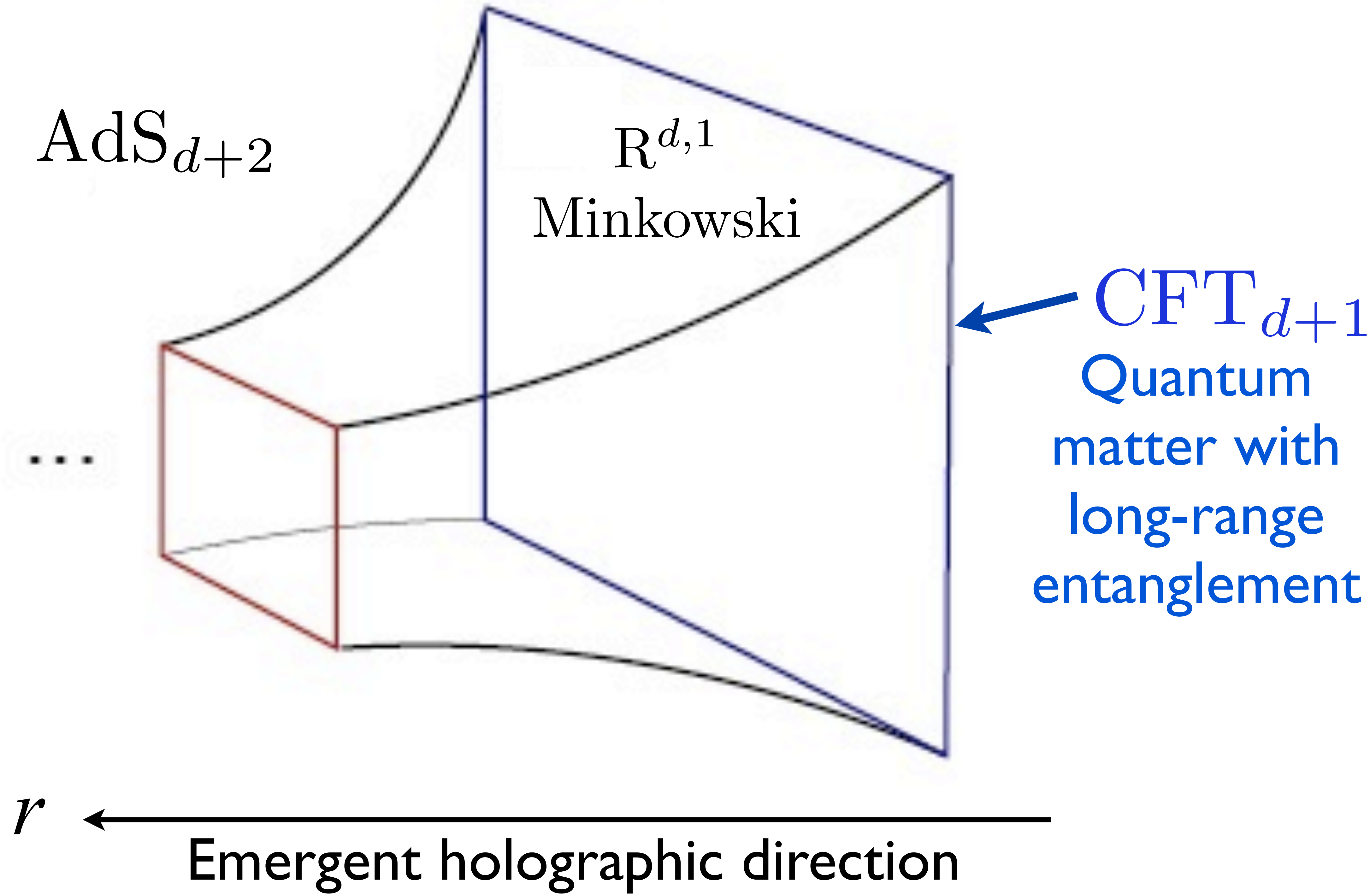


Entanglement entropy

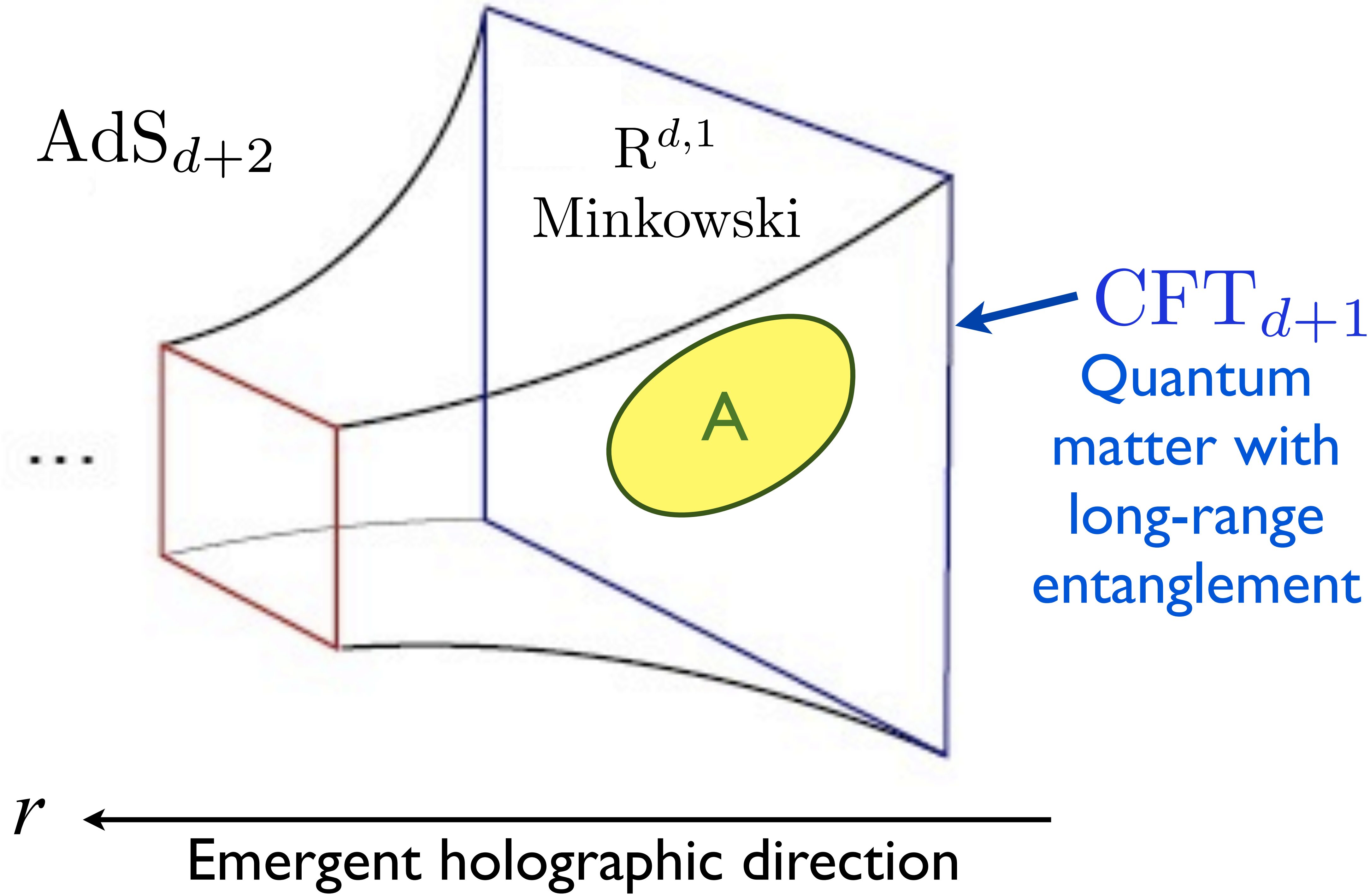
The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

This can be seen both the string and tensor-network pictures

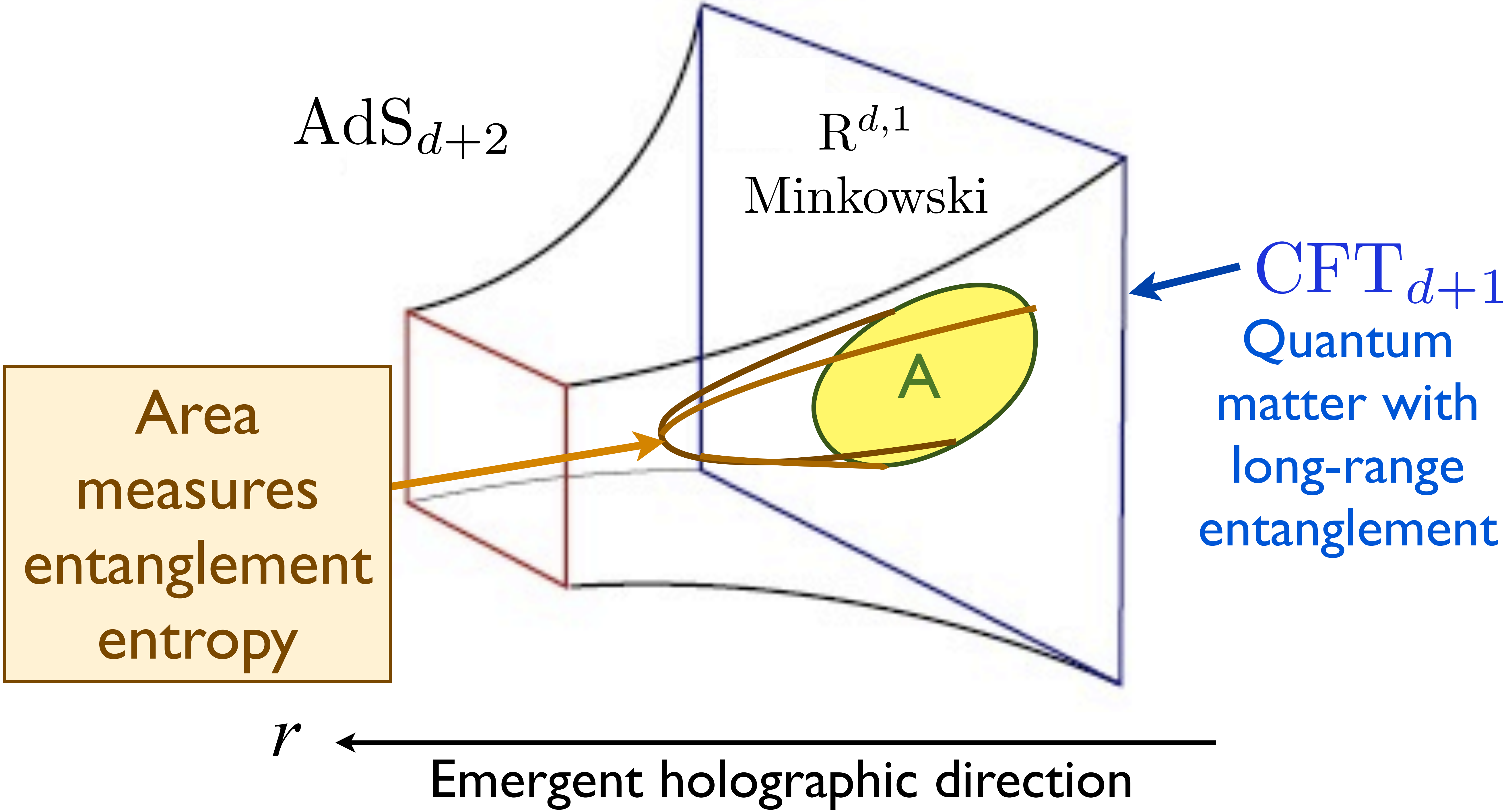
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317



J. McGreevy, arXiv0909.0518



AdS_{d+2}
$R^{d,1}$
Minkowski
CFT_{d+1}
Quantum matter with long-range entanglement
A
...
r
Emergent holographic direction


AdS_{d+2}
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Quantum matter with long-range entanglement
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Area measures entanglement entropy
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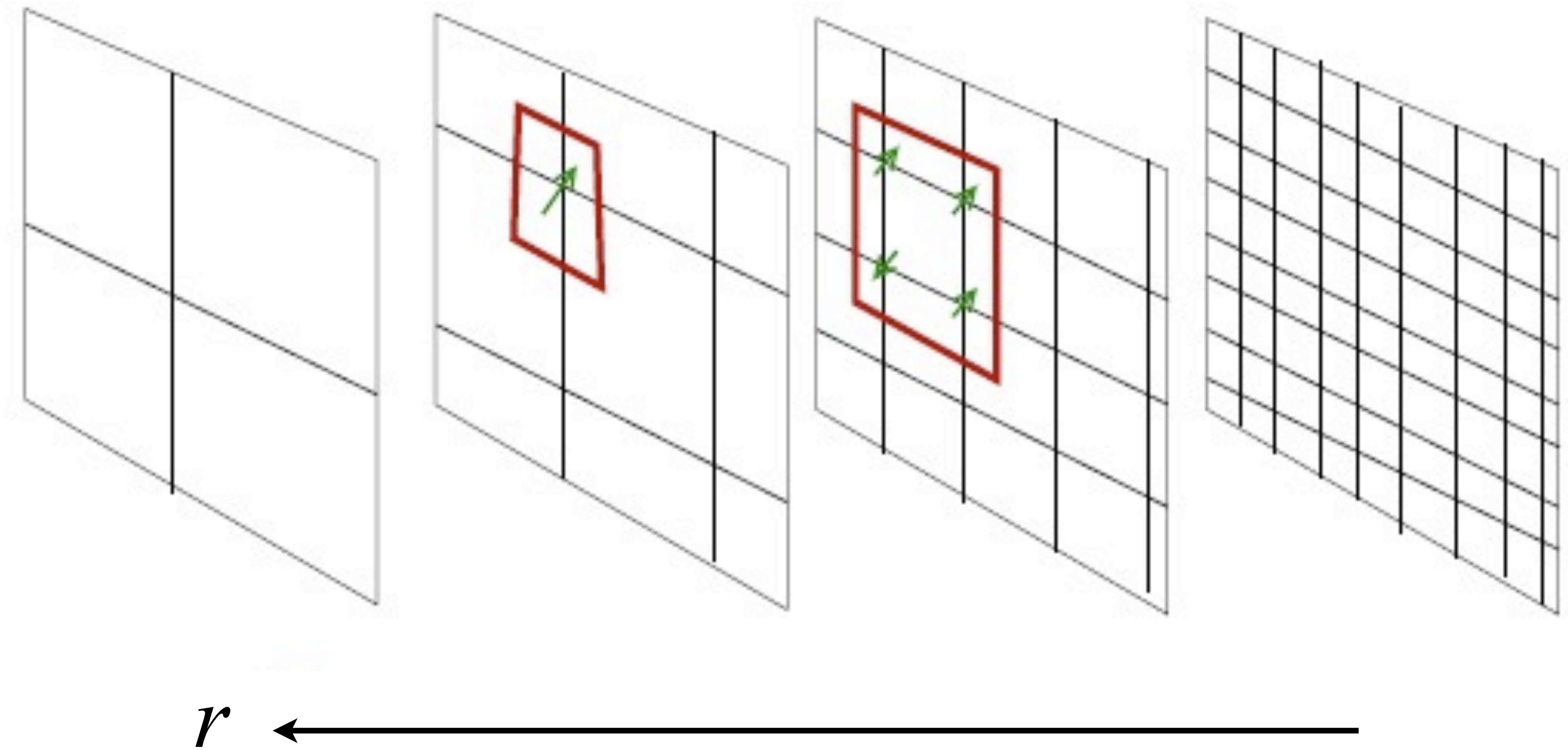
Emergent holographic direction

Why AdS$_{d+2}$?



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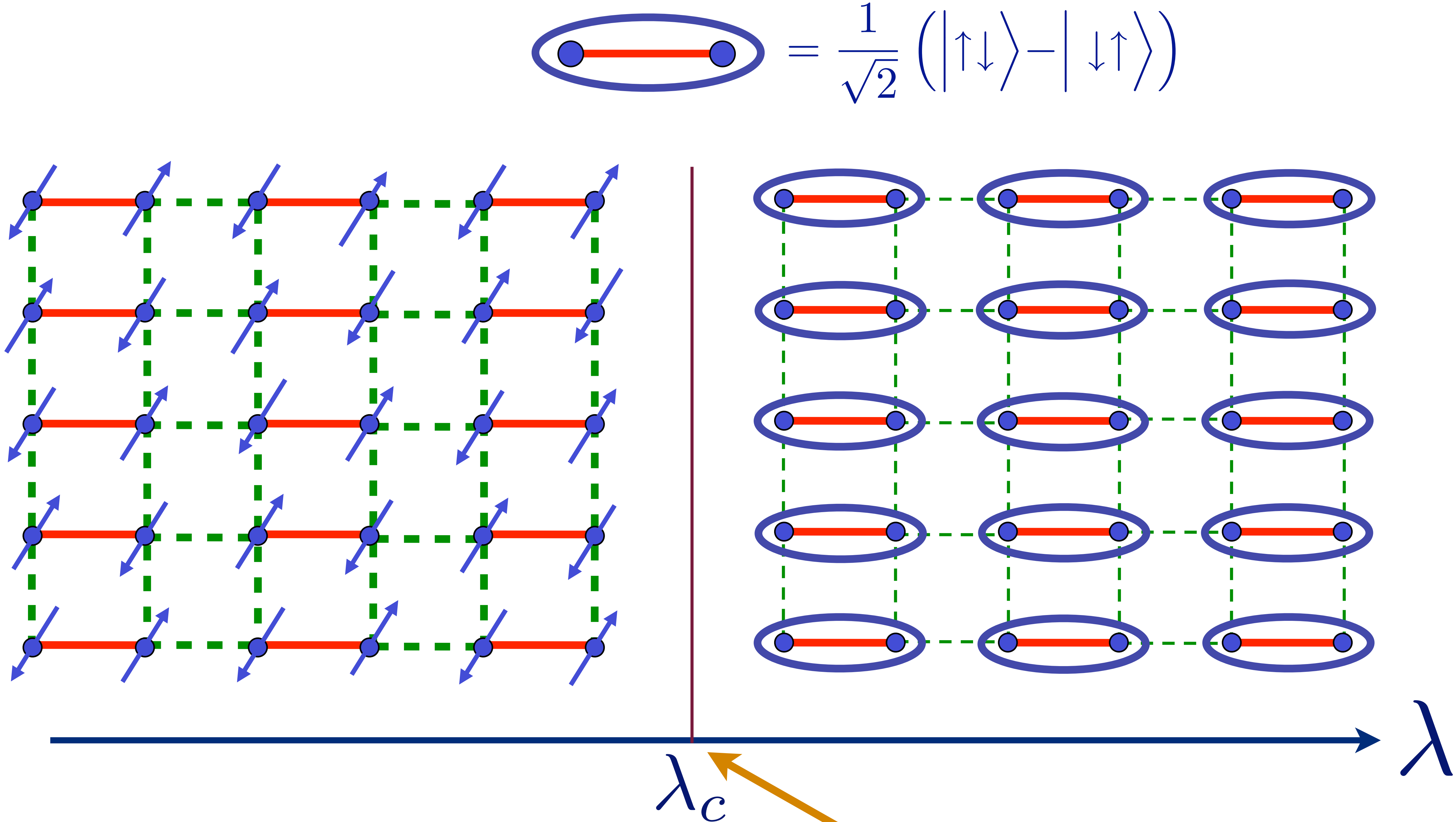
For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS$_{d+2}$.



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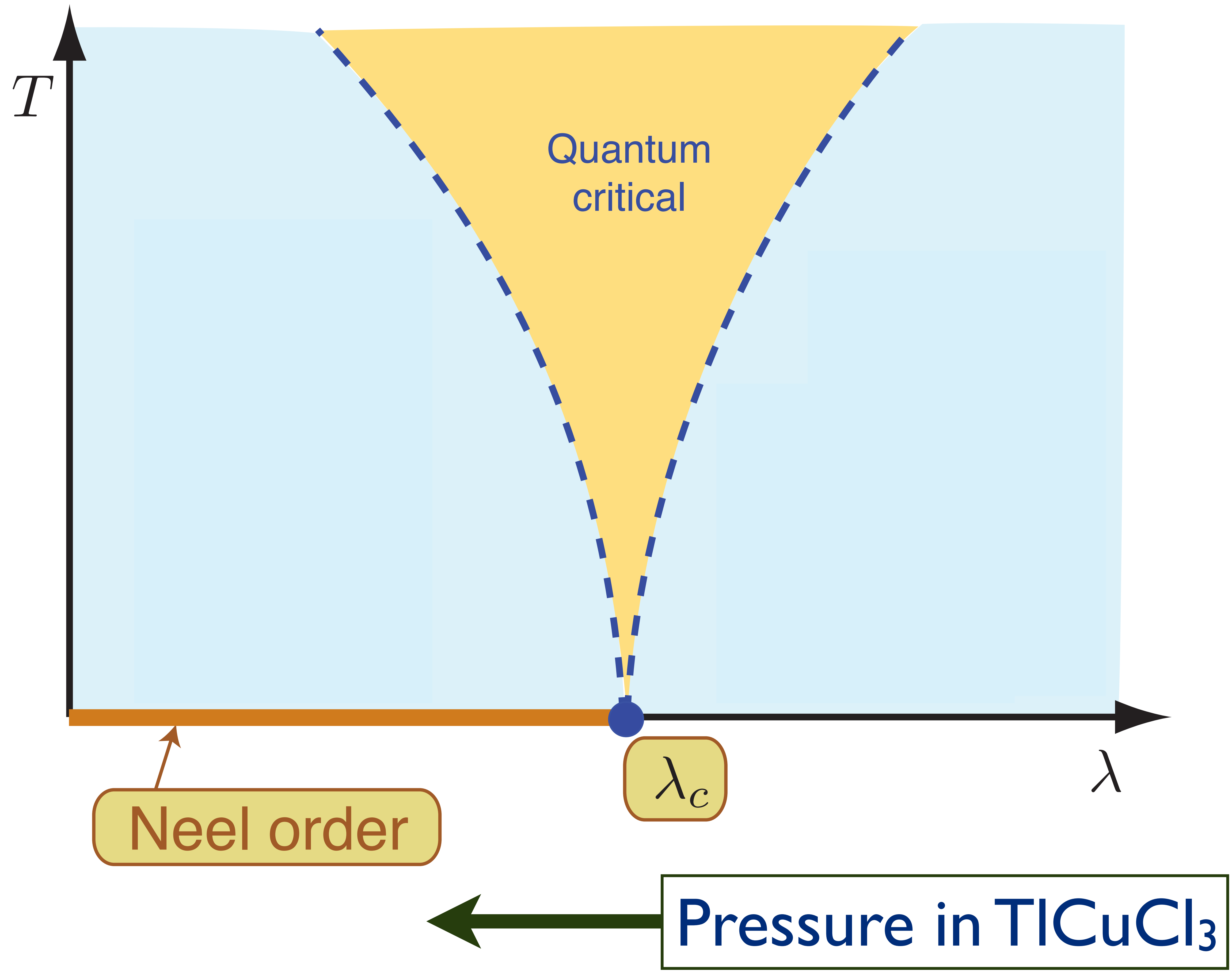

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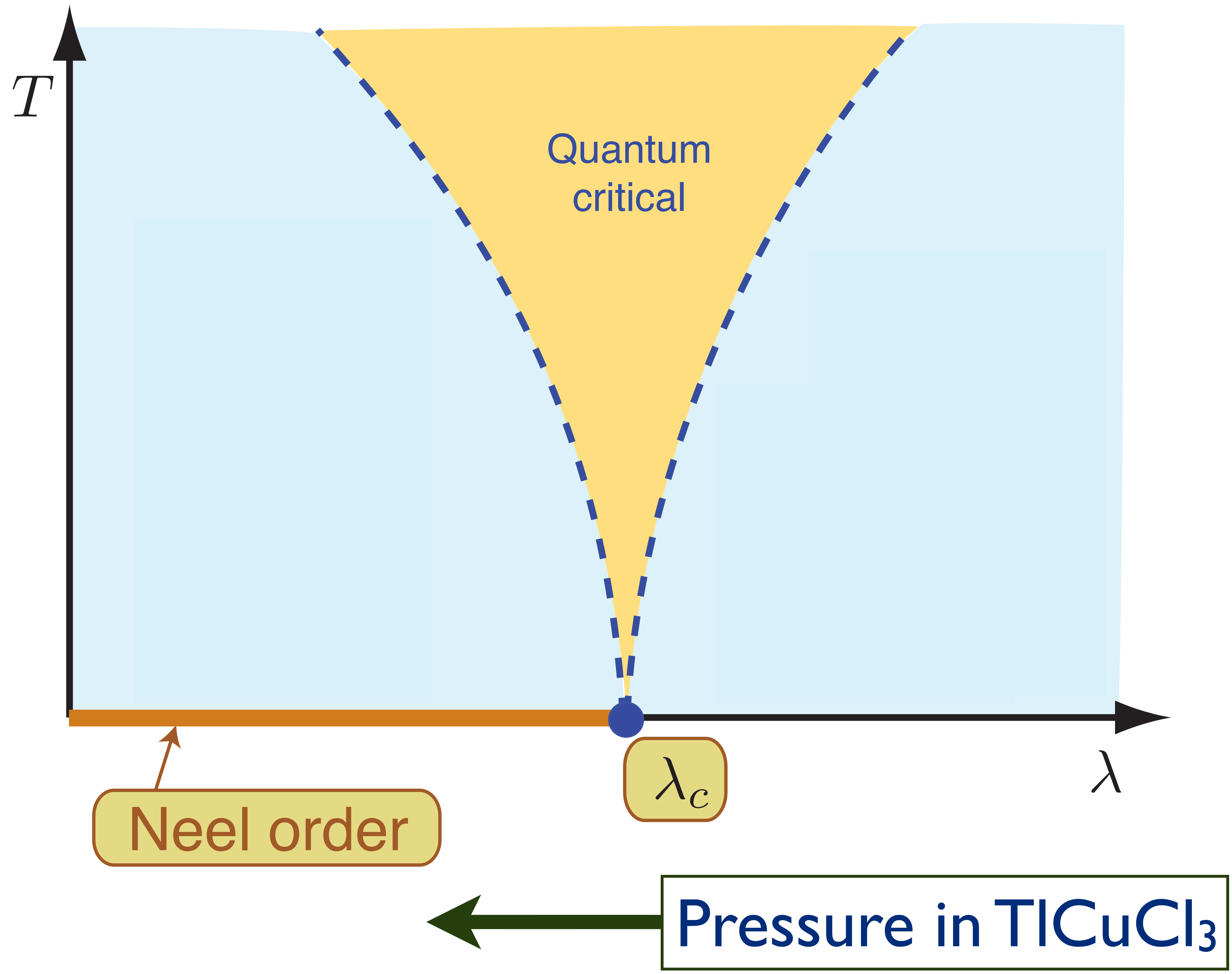

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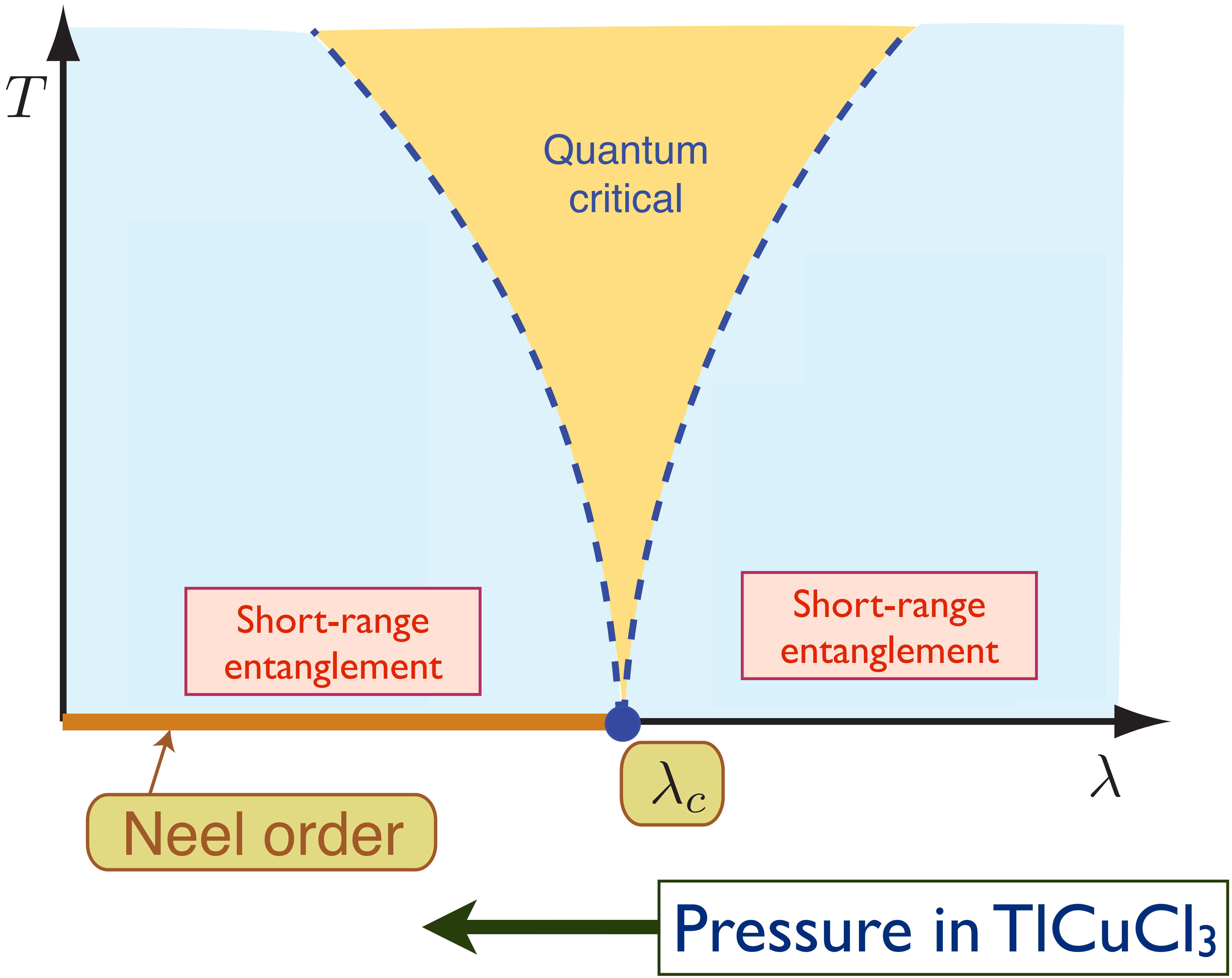

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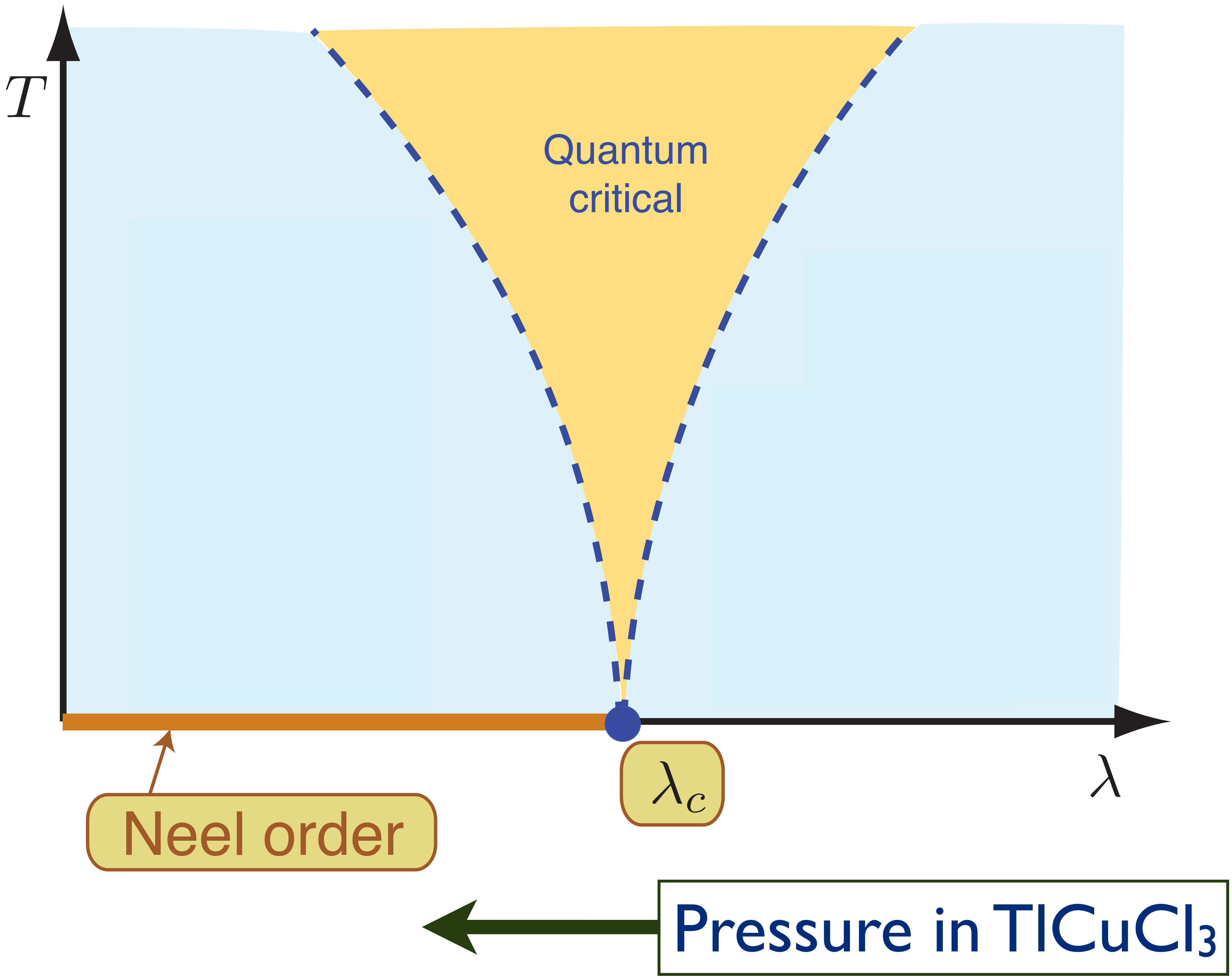

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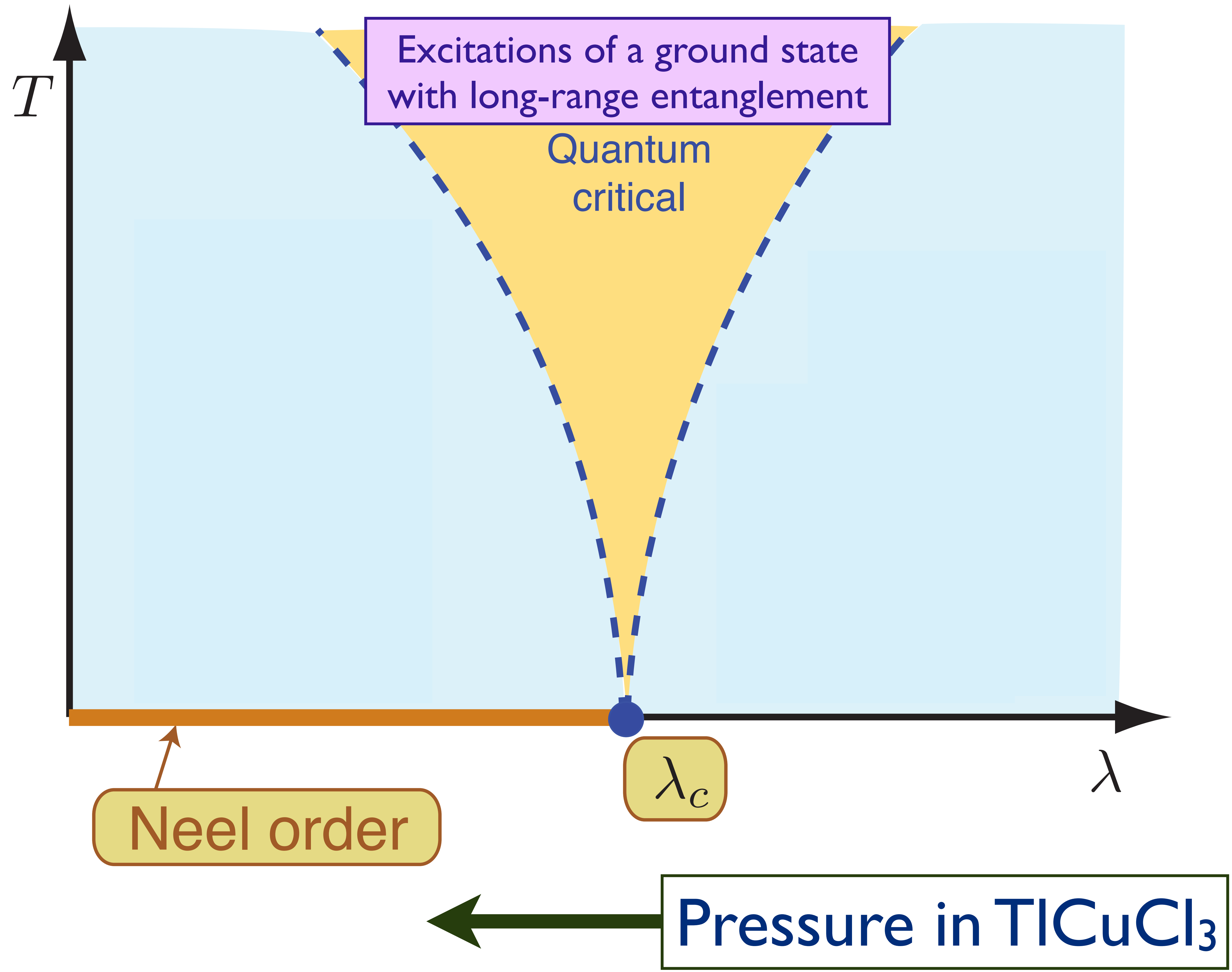
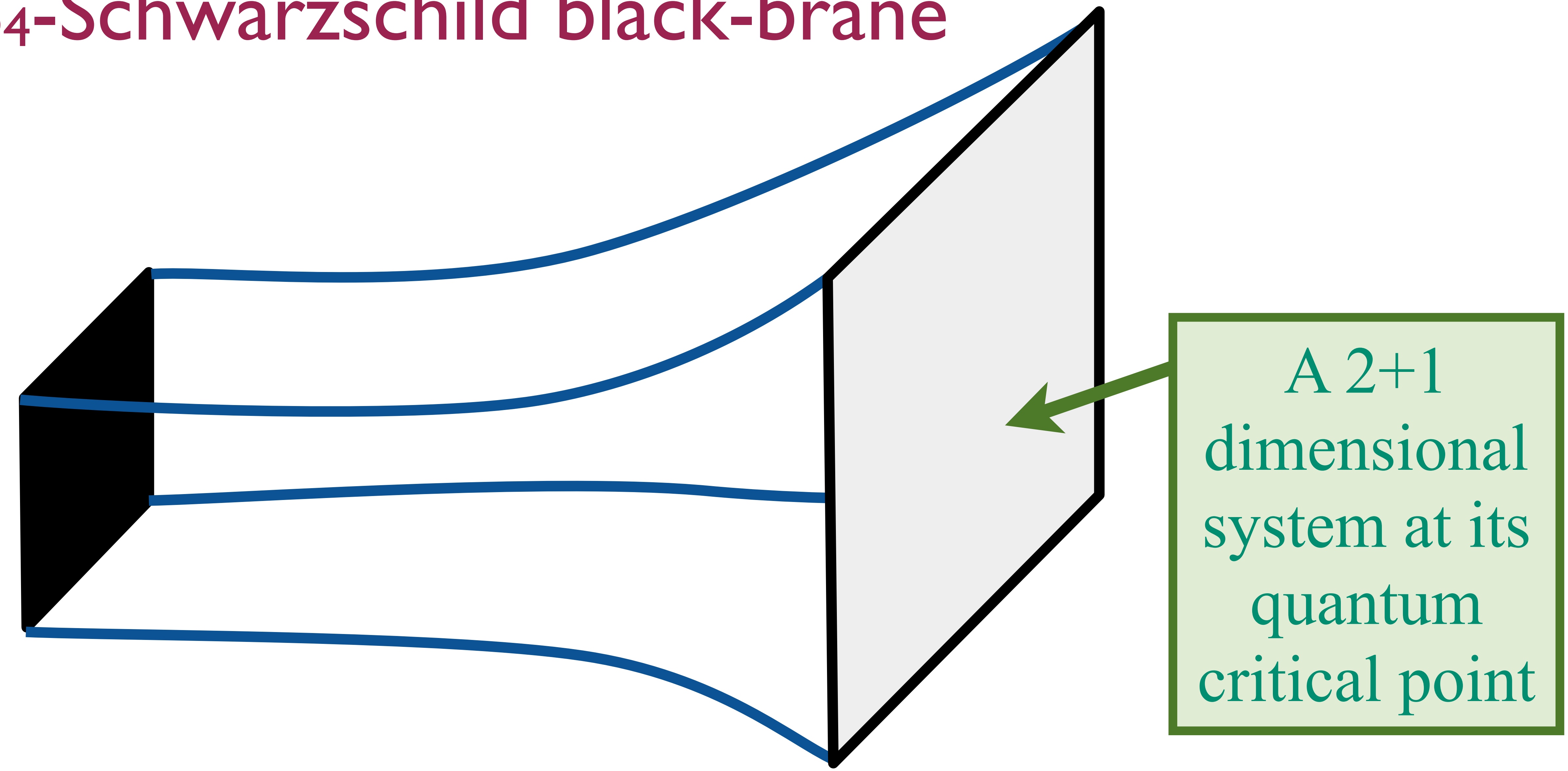


AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



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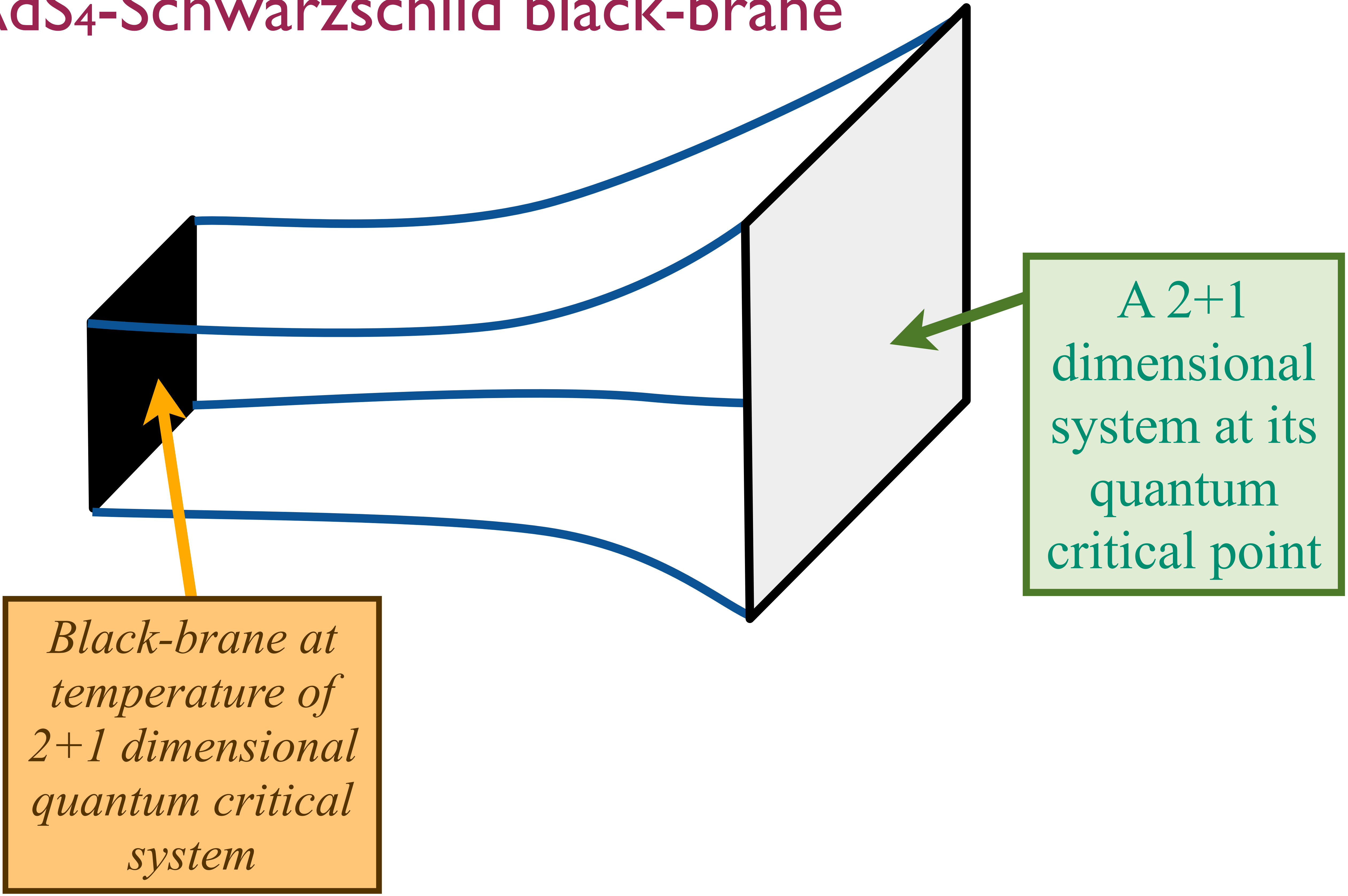

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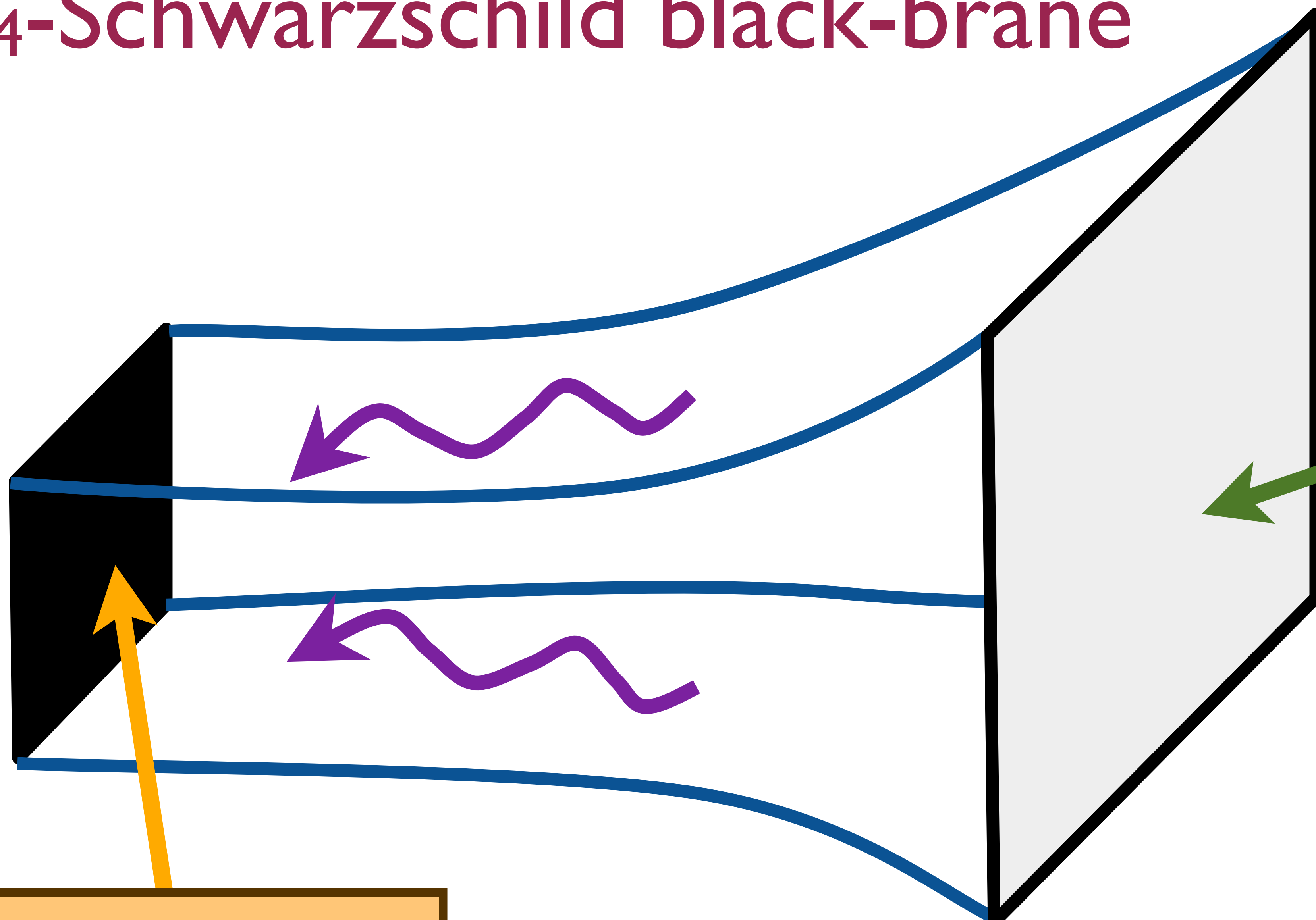


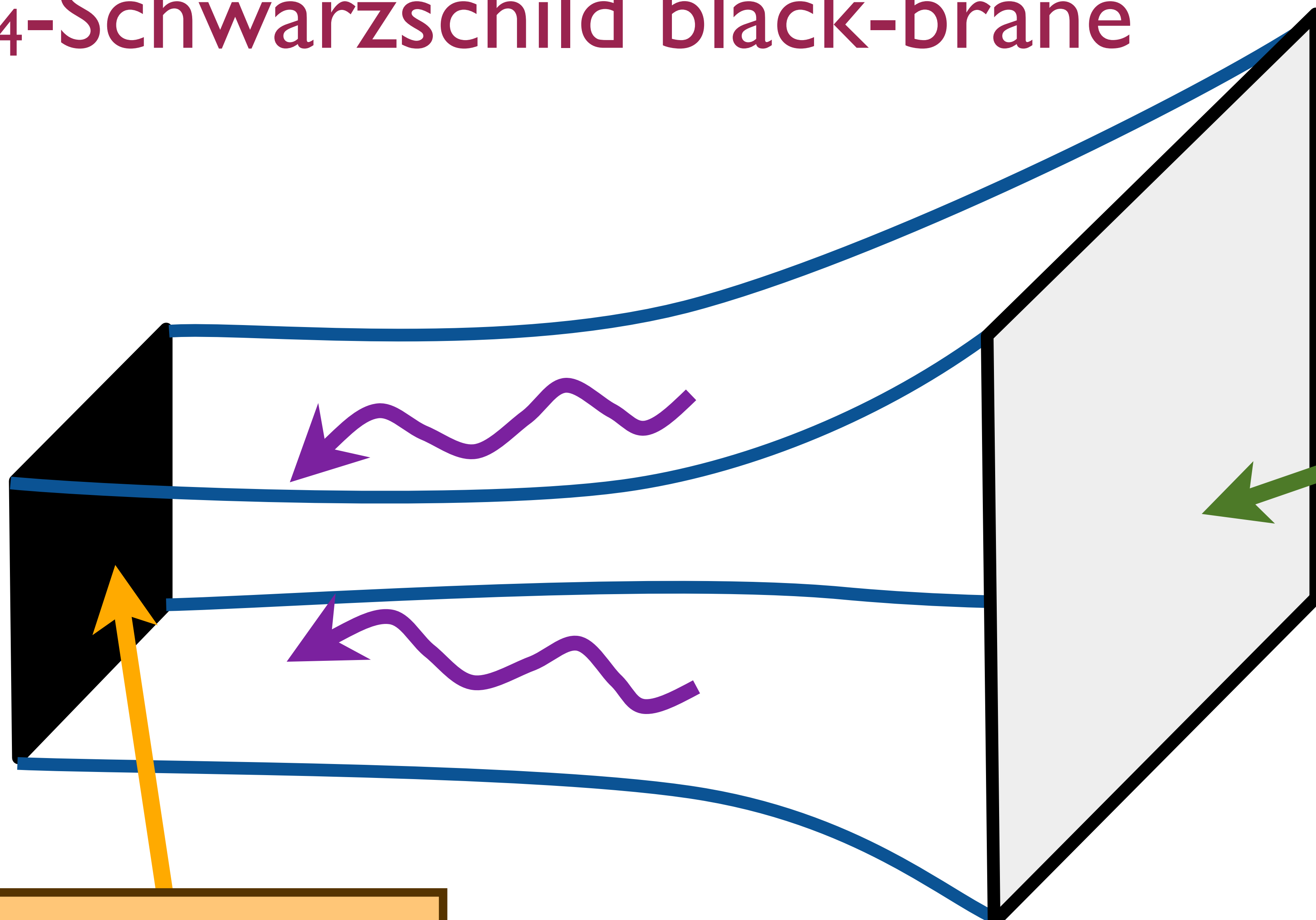
Black-brane at temperature of 2+1 dimensional quantum critical system

A 2+1 dimensional system at its quantum critical point

Friction of quantum criticality = waves falling into black brane

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane



Black-brane at temperature of 2+1 dimensional quantum critical system

A 2+1 dimensional system at its quantum critical point

Provides successful description of many properties of quantum critical points at non-zero temperatures

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

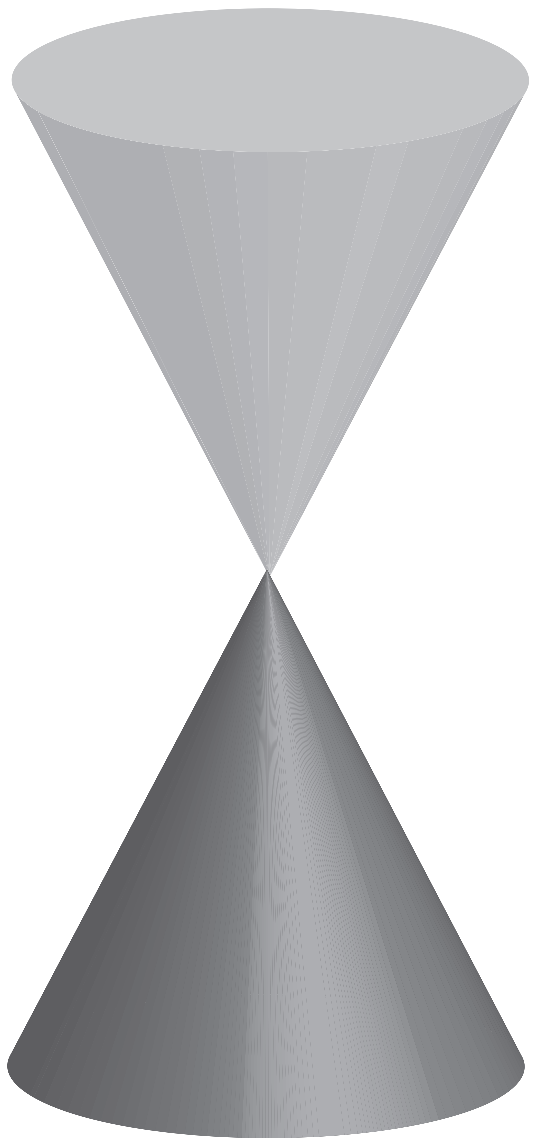
Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe *zero temperature* phases where $d\langle \mathcal{Q} \rangle / d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu\mathcal{Q}$.

The only compressible phase of traditional condensed matter physics which does not break the translational or $U(1)$ symmetries is the Landau Fermi liquid

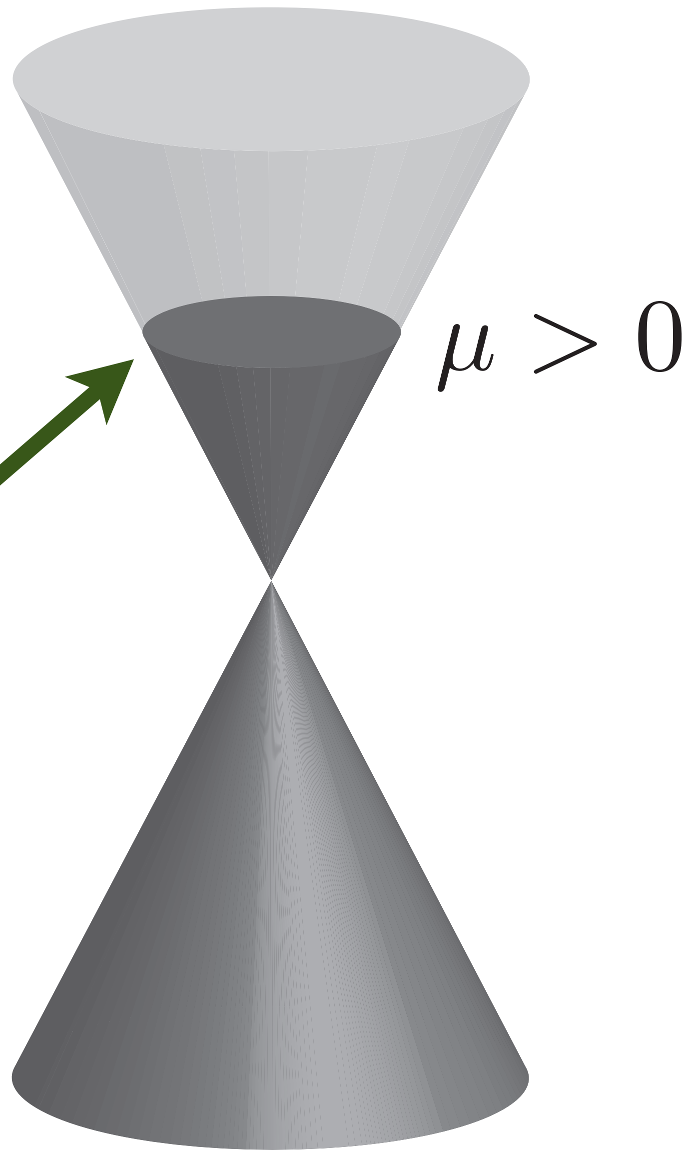
Conformal quantum matter



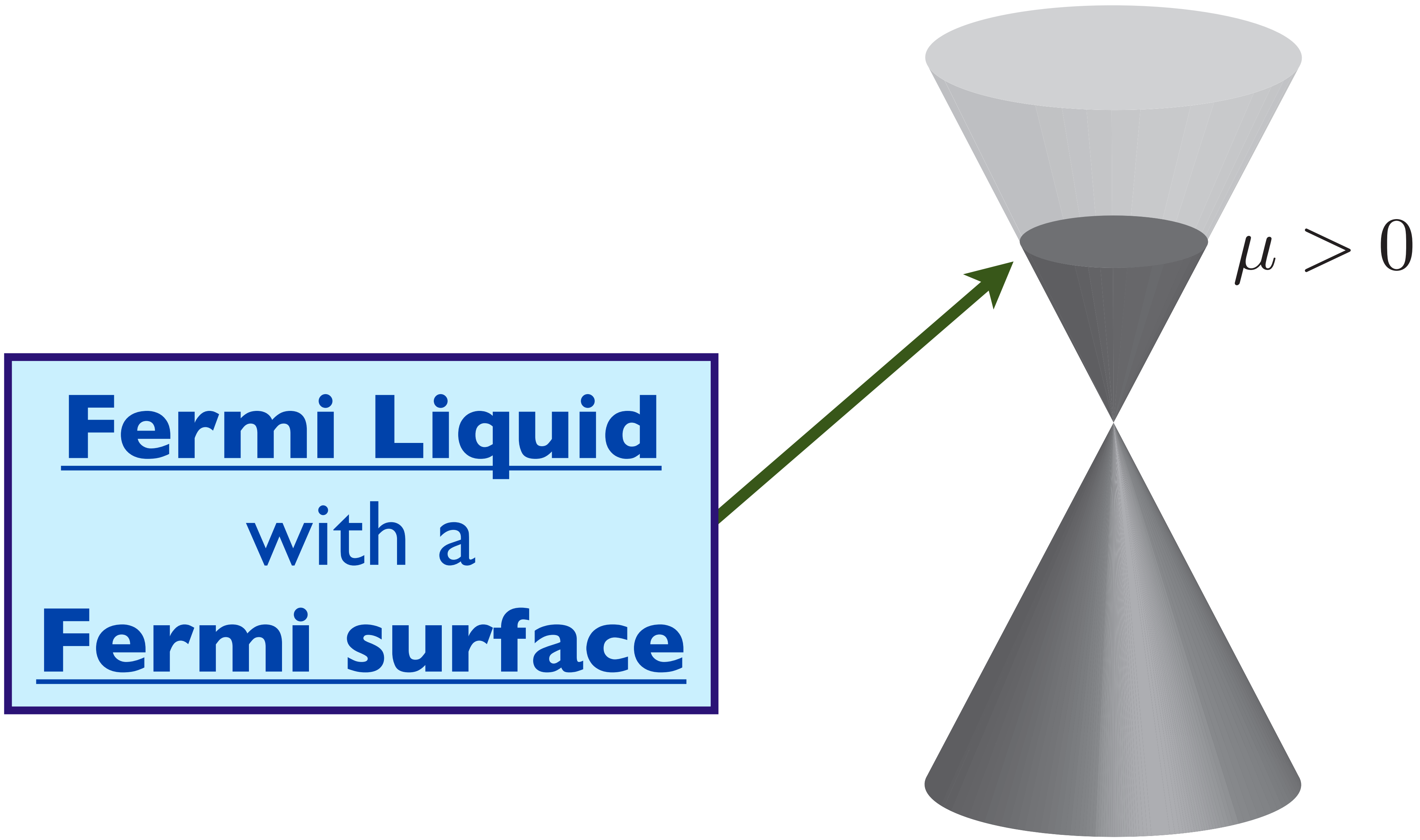
Graphene

Compressible quantum matter

Fermi Liquid
with a
Fermi surface

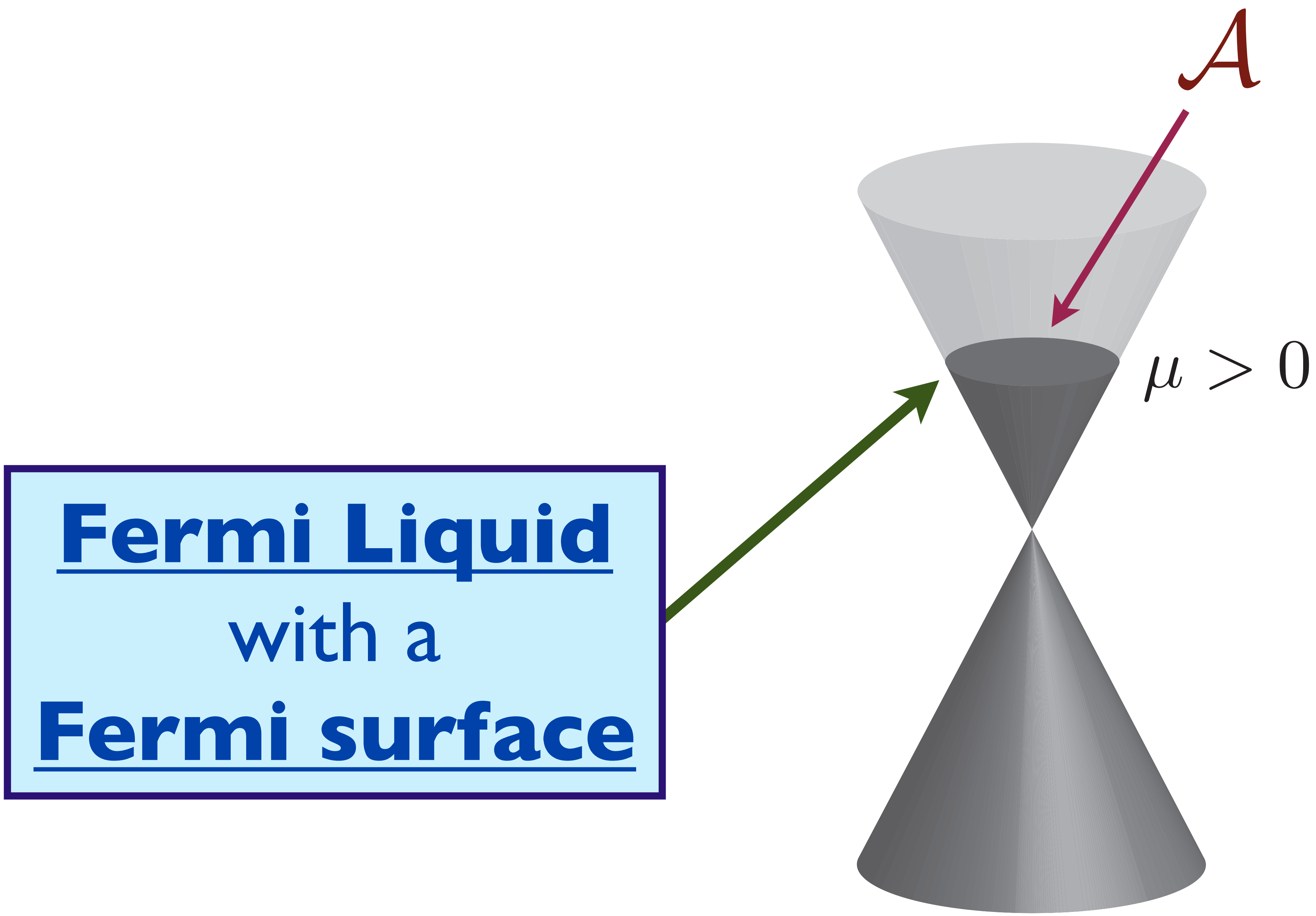


Compressible quantum matter



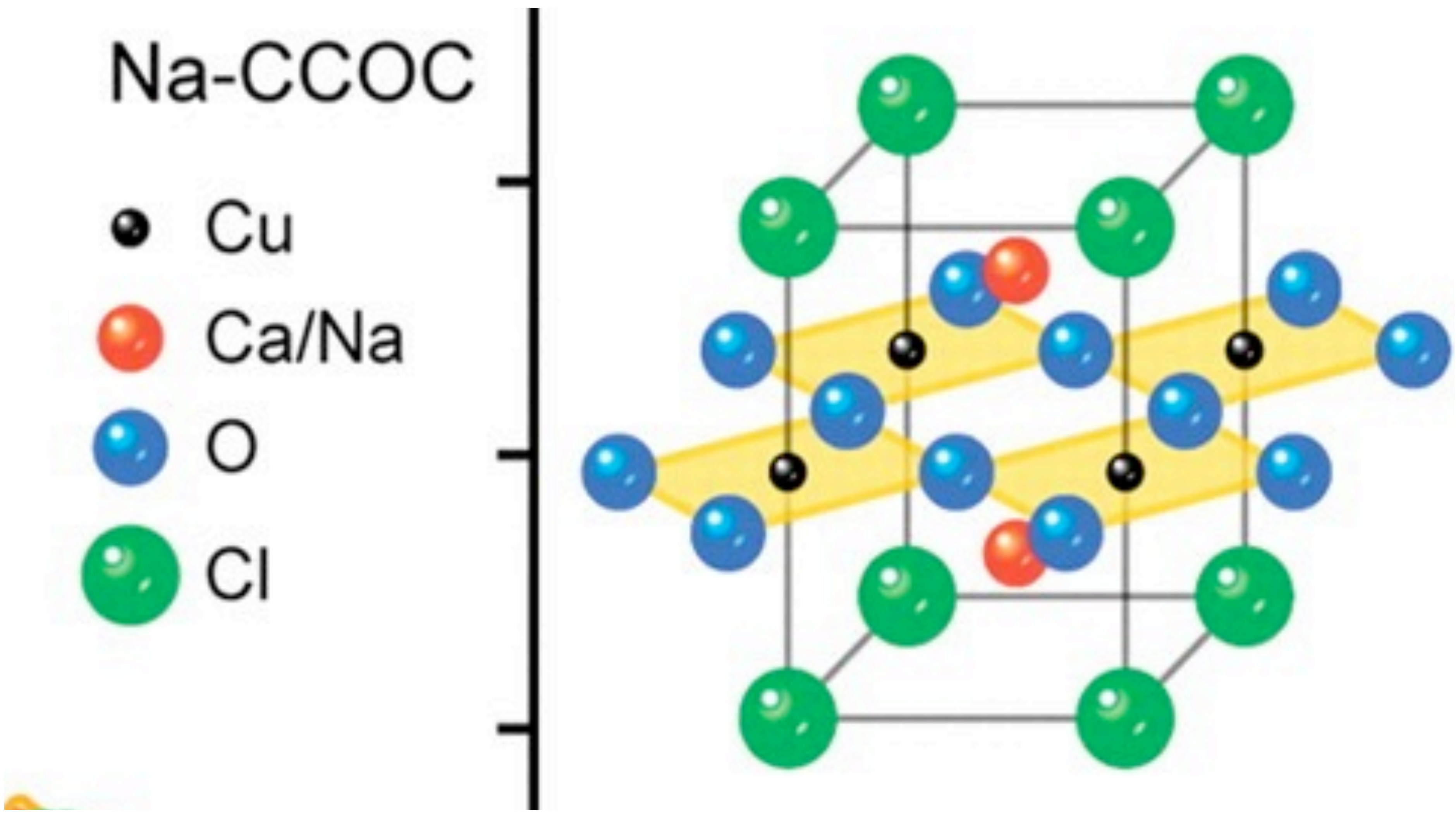
- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

Compressible quantum matter



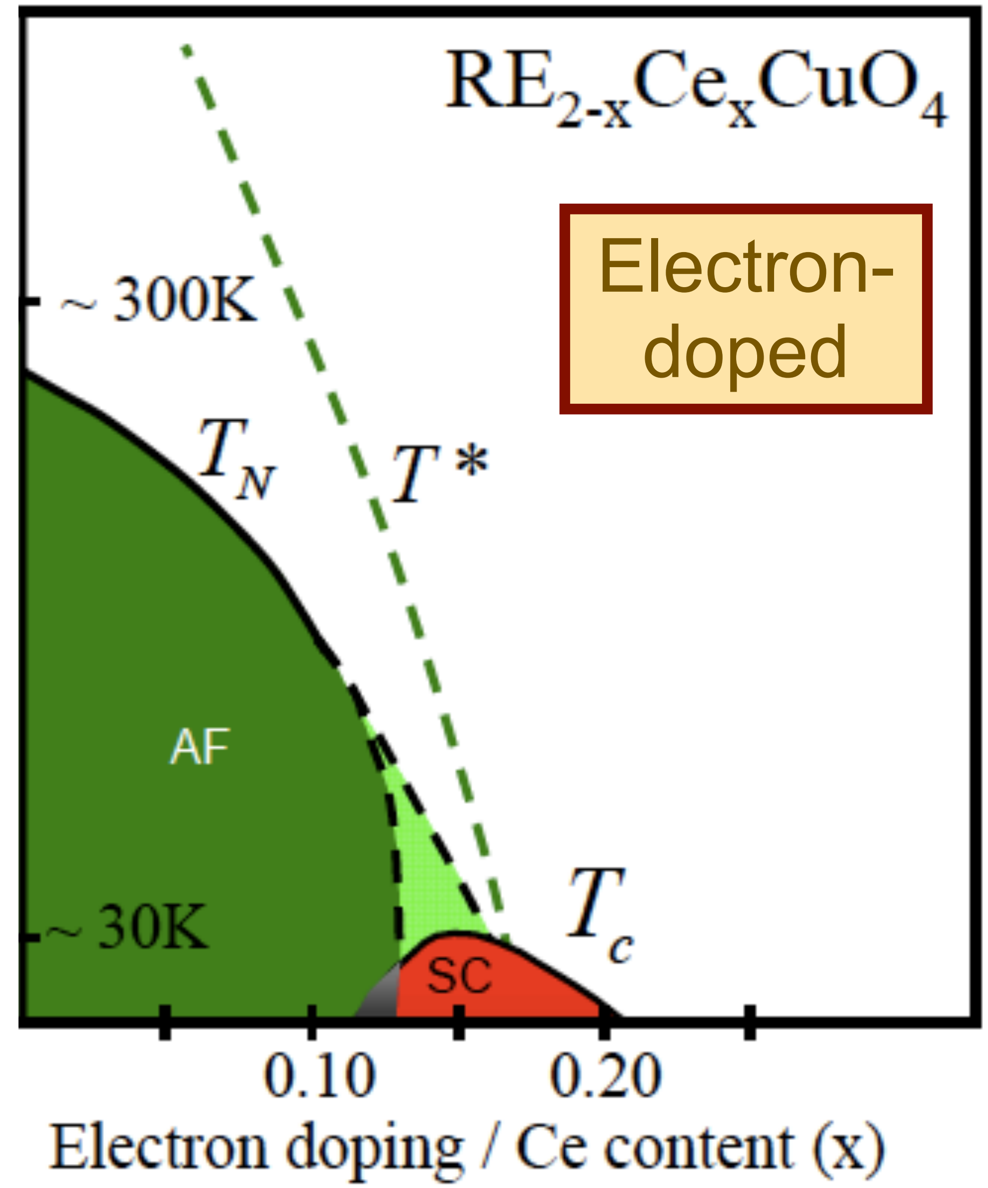
- **Luttinger relation:** The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$.

The cuprate superconductors



Davis

Electron-doped cuprate superconductors



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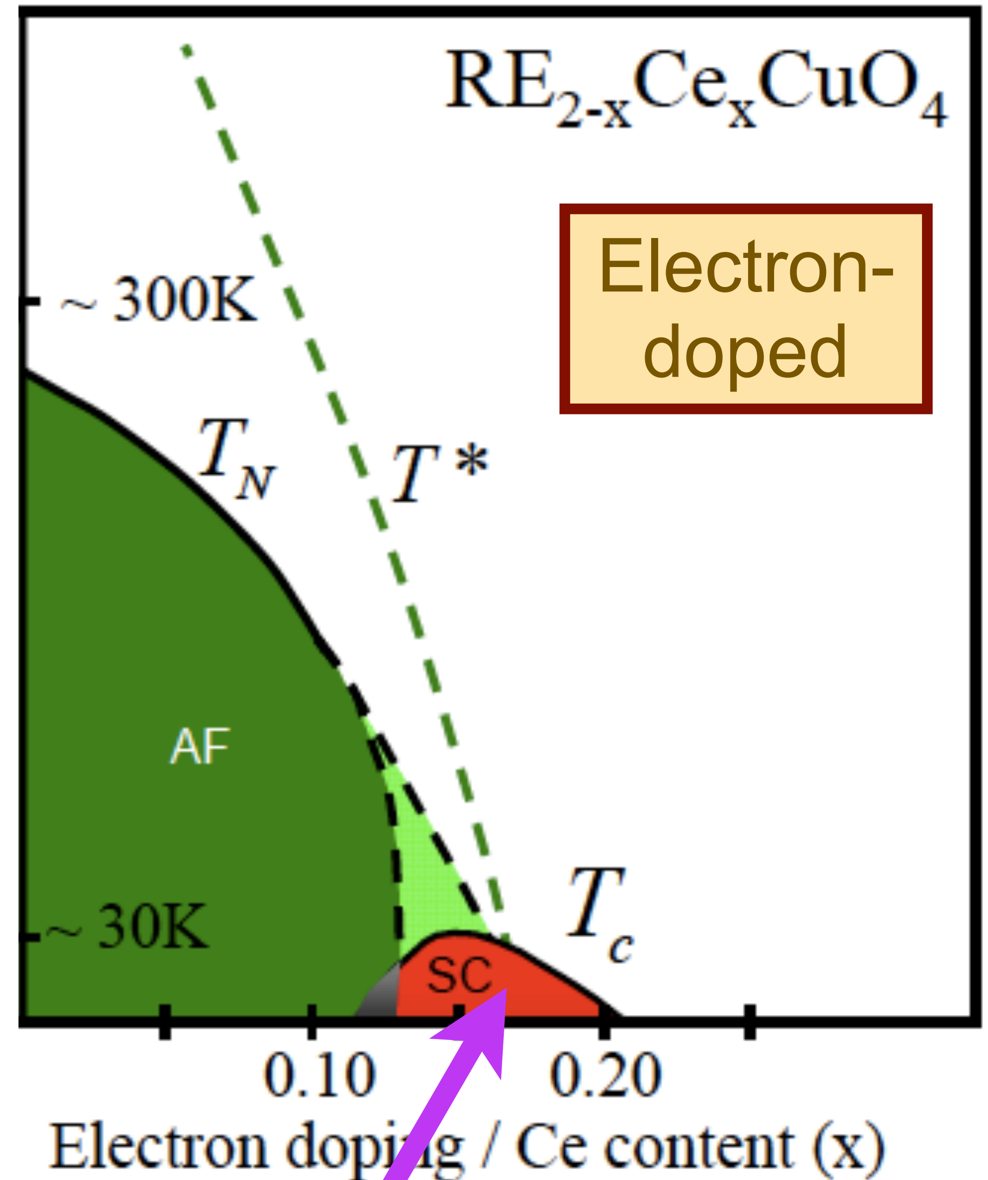


Superconductor
Bose condensate of pairs of electrons
Short-range entanglement

Electron-doped cuprate superconductors

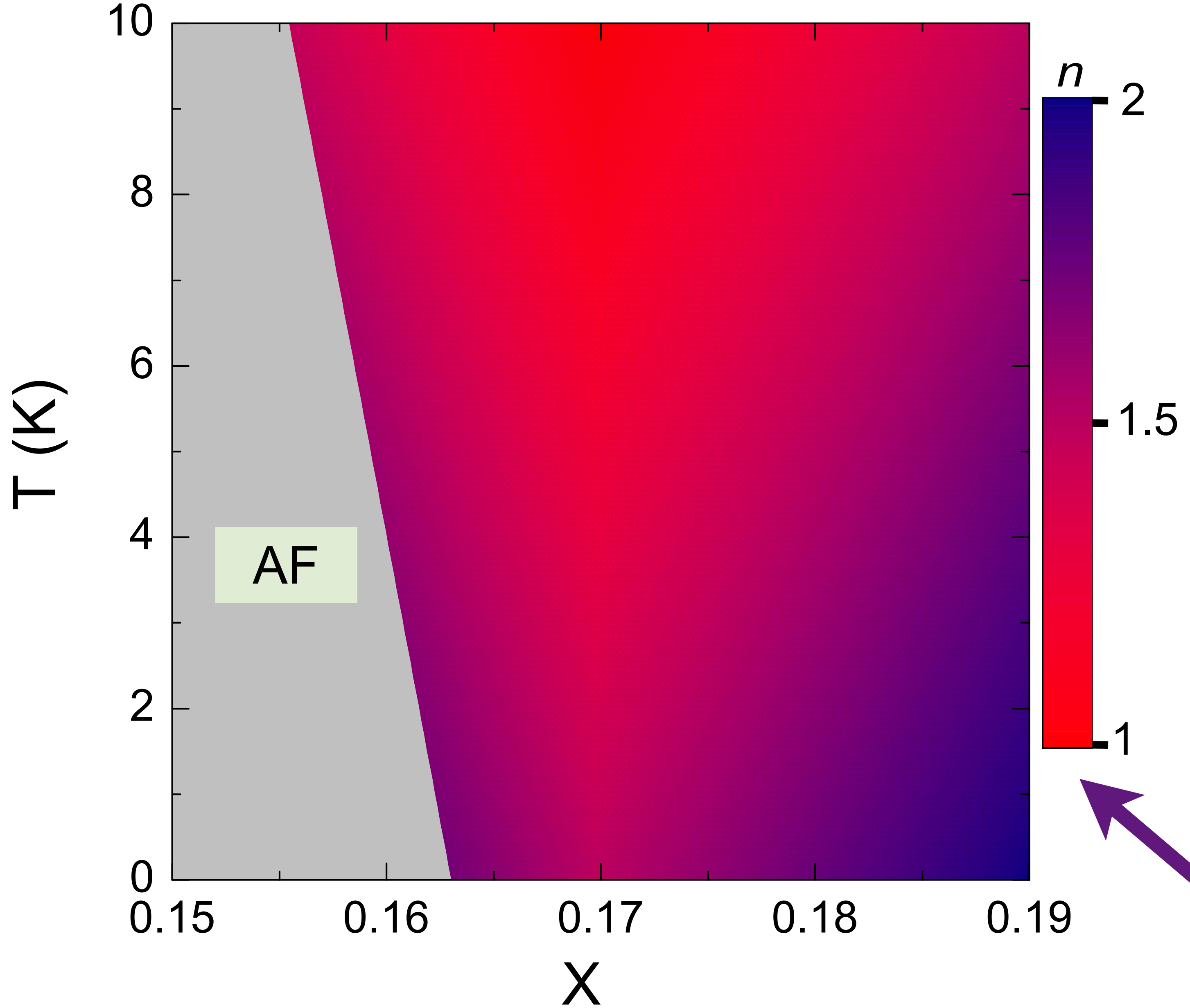
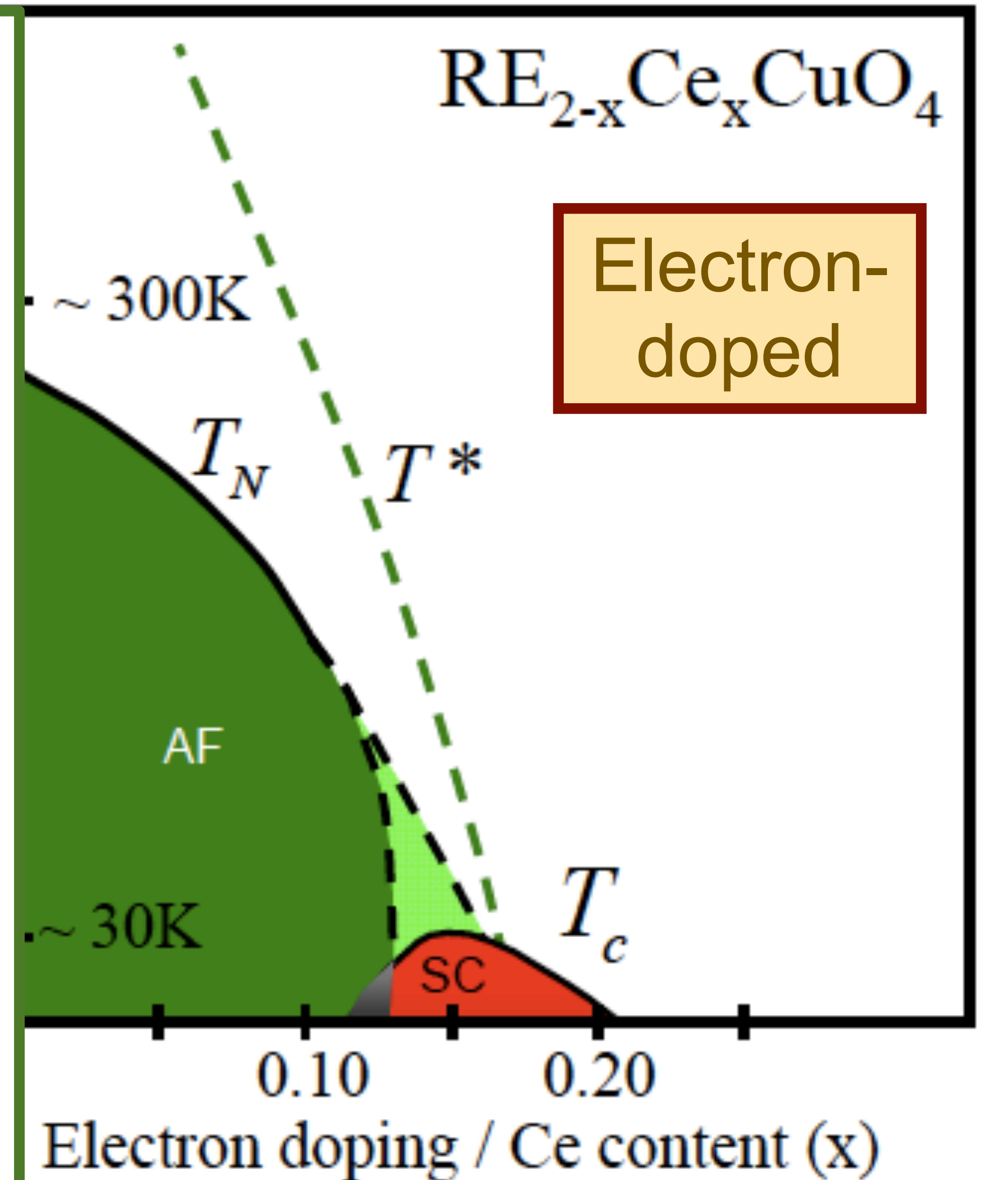
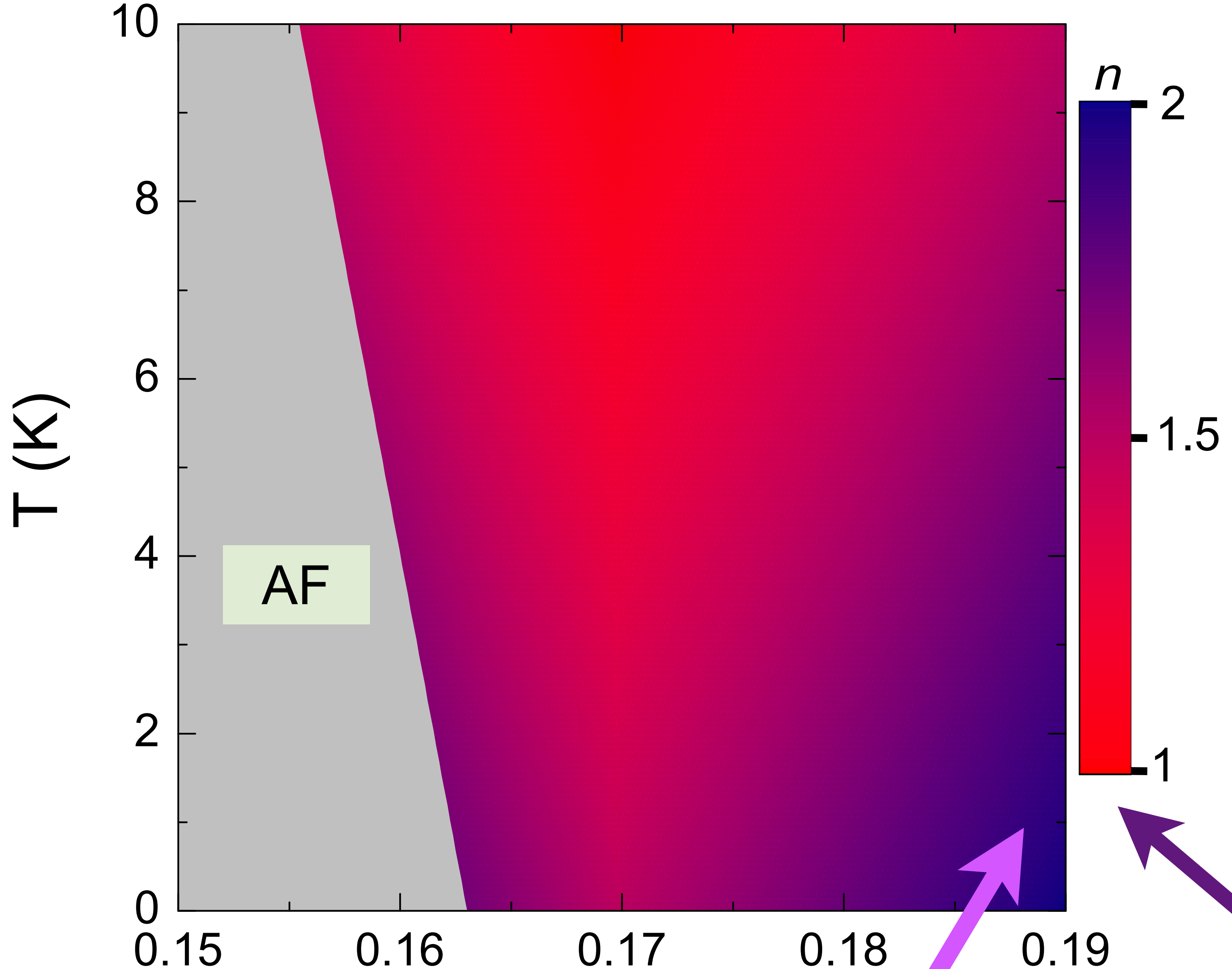


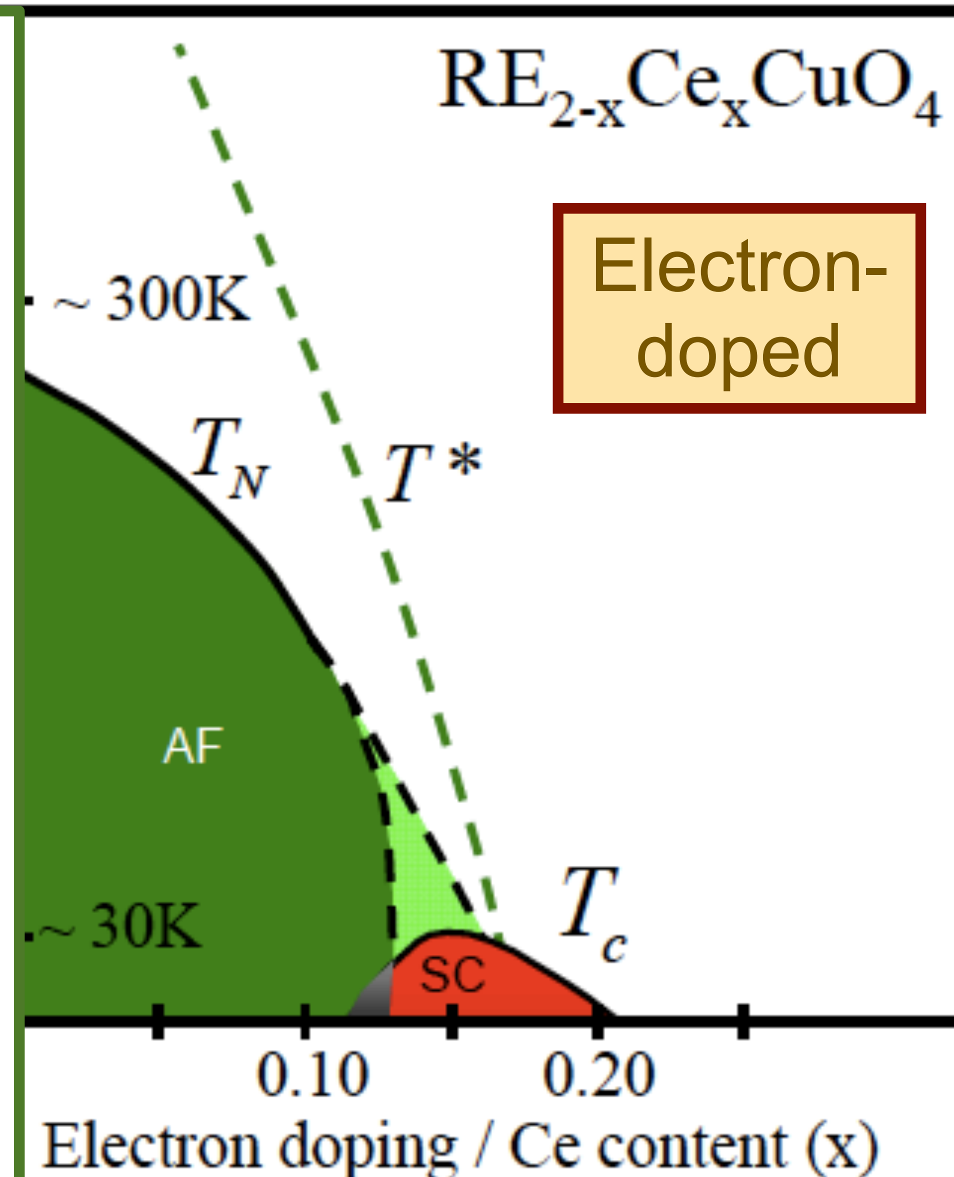
Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors





Ordinary metal (Fermi liquid)

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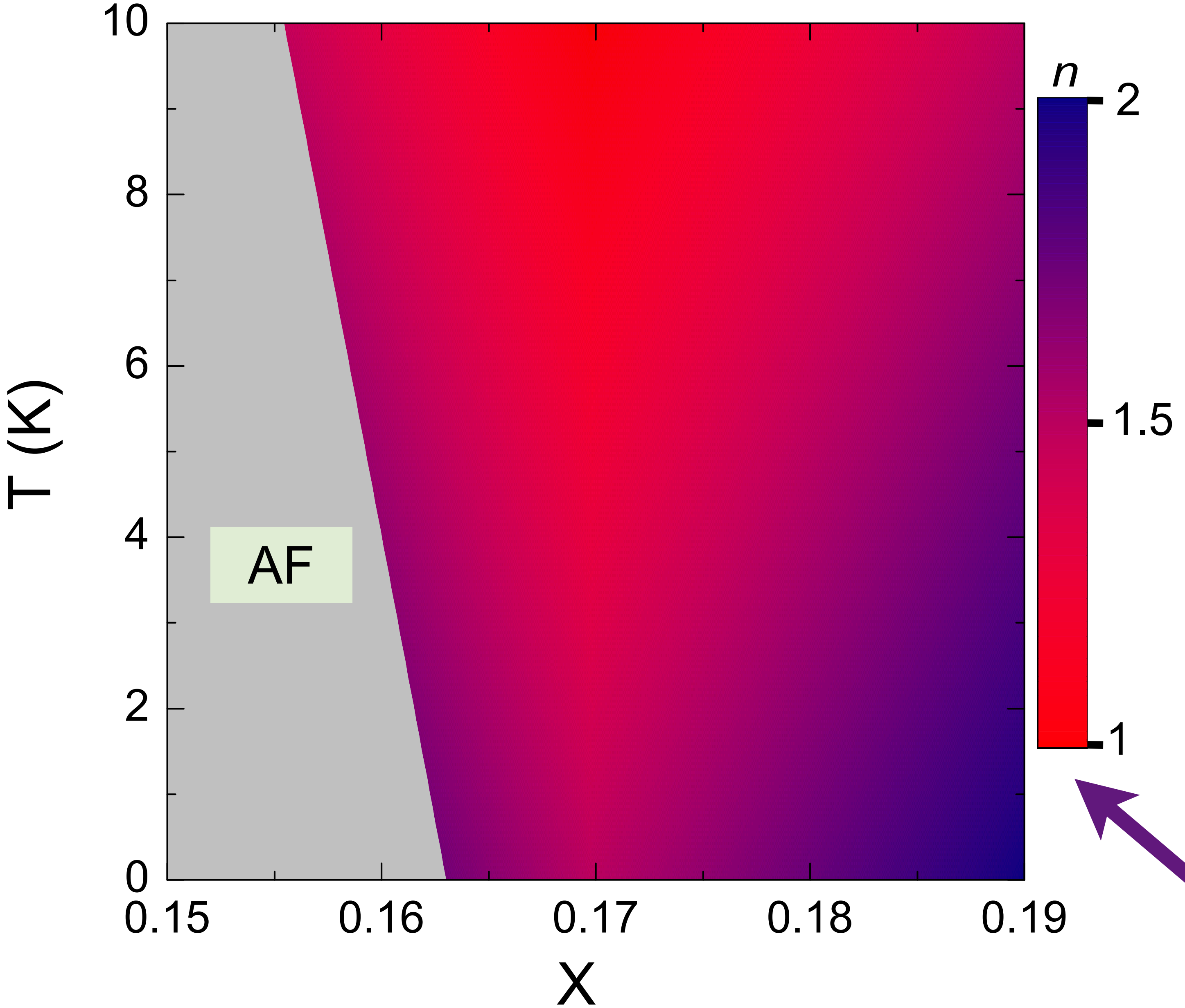
Electron-doped cuprate superconductors



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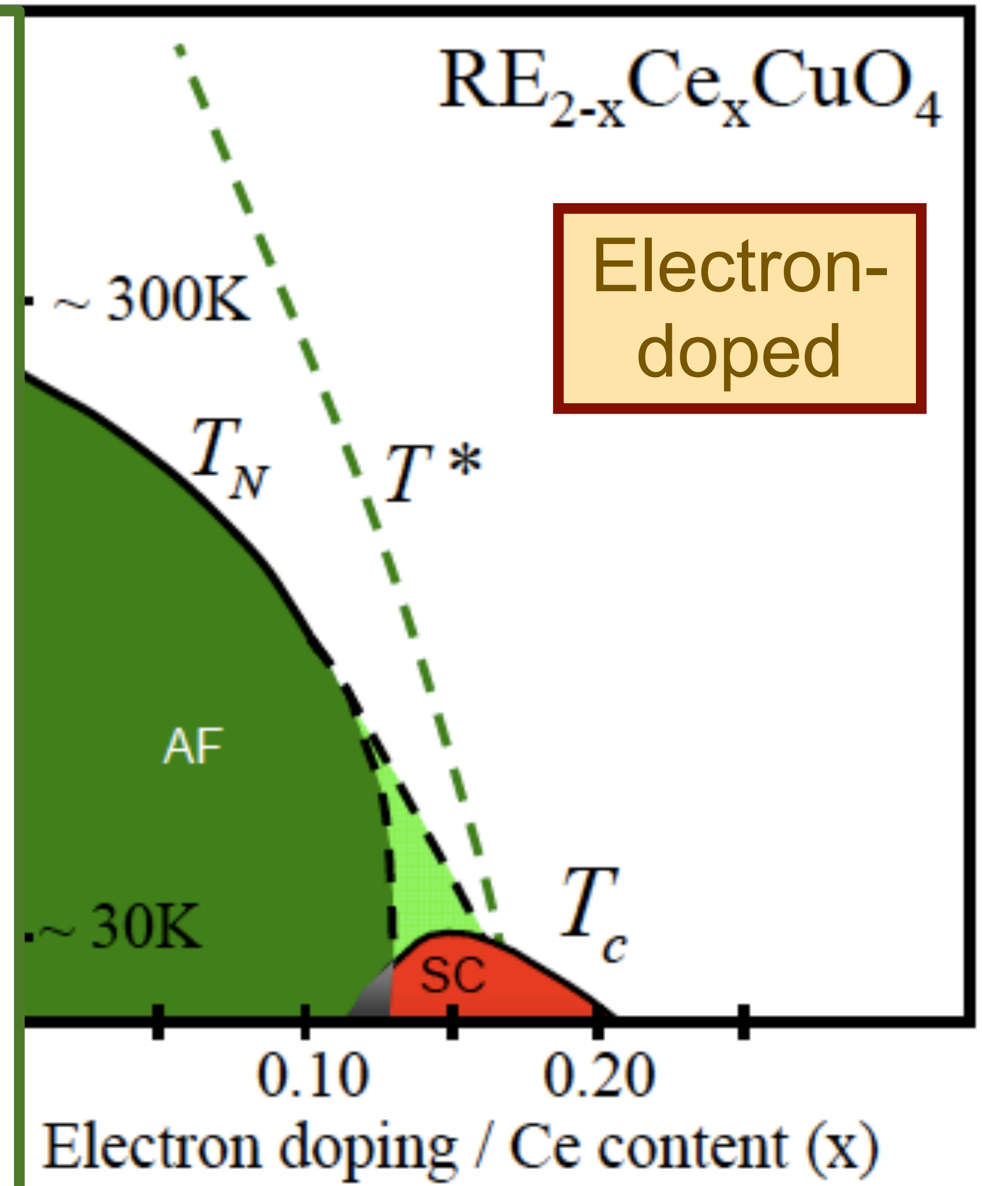

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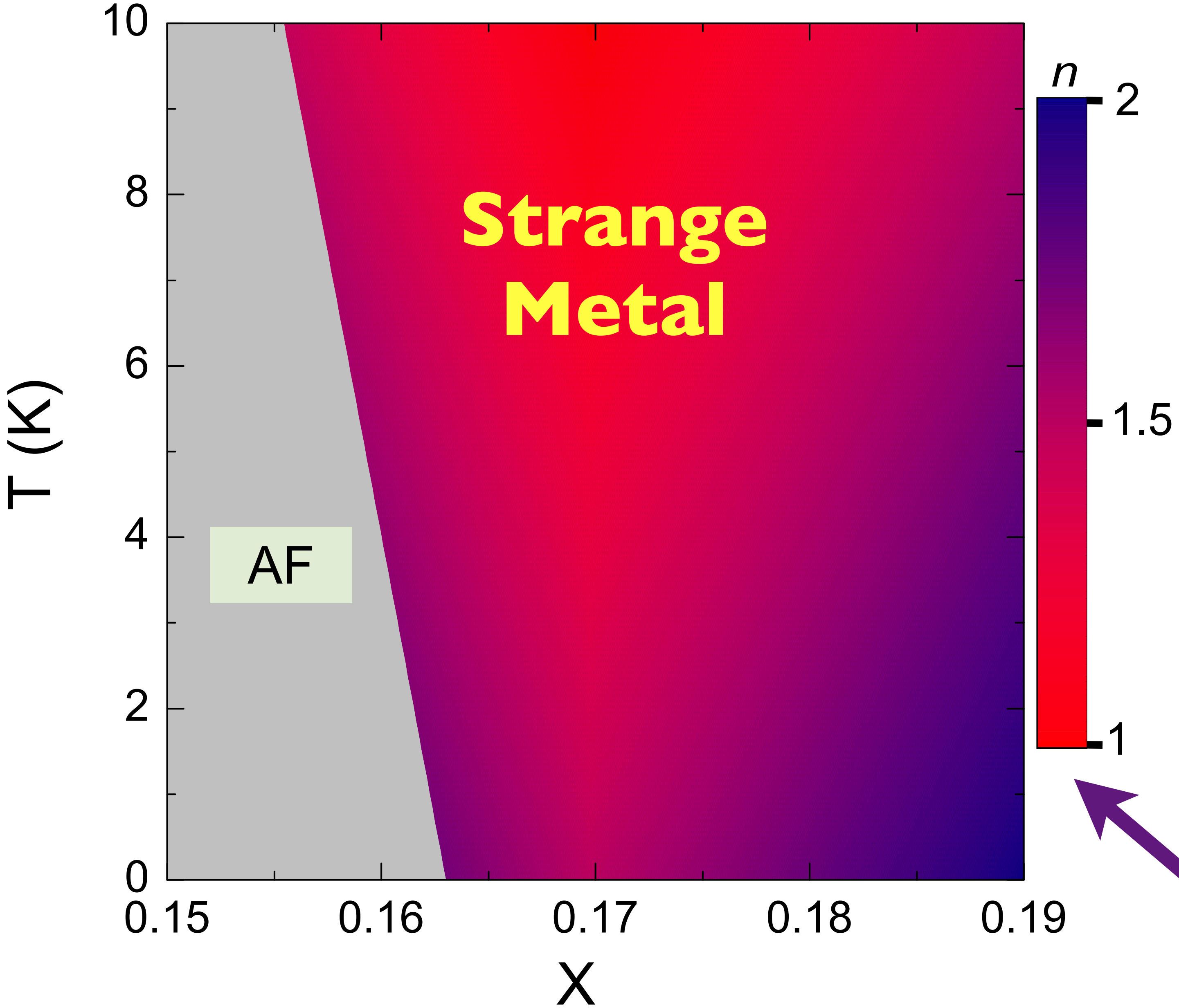

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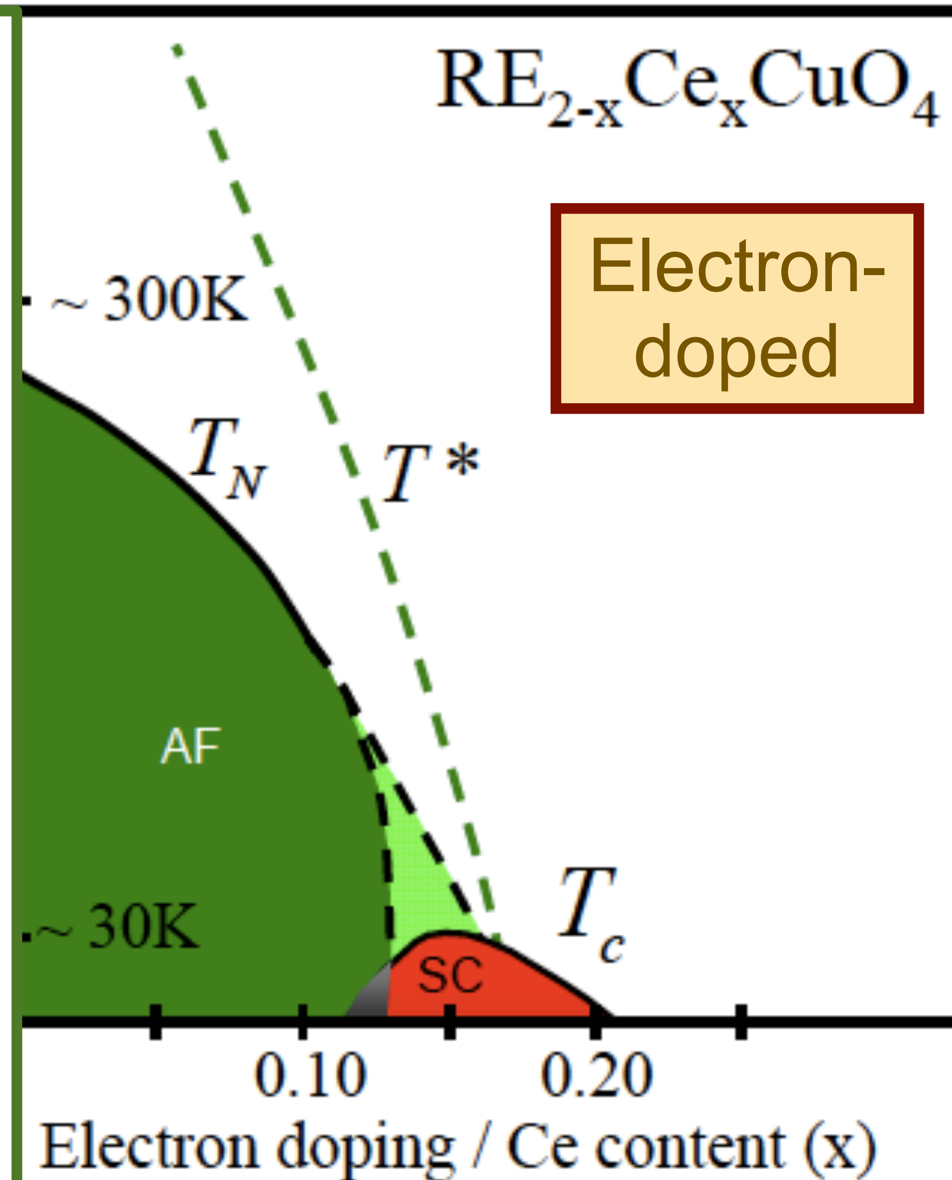

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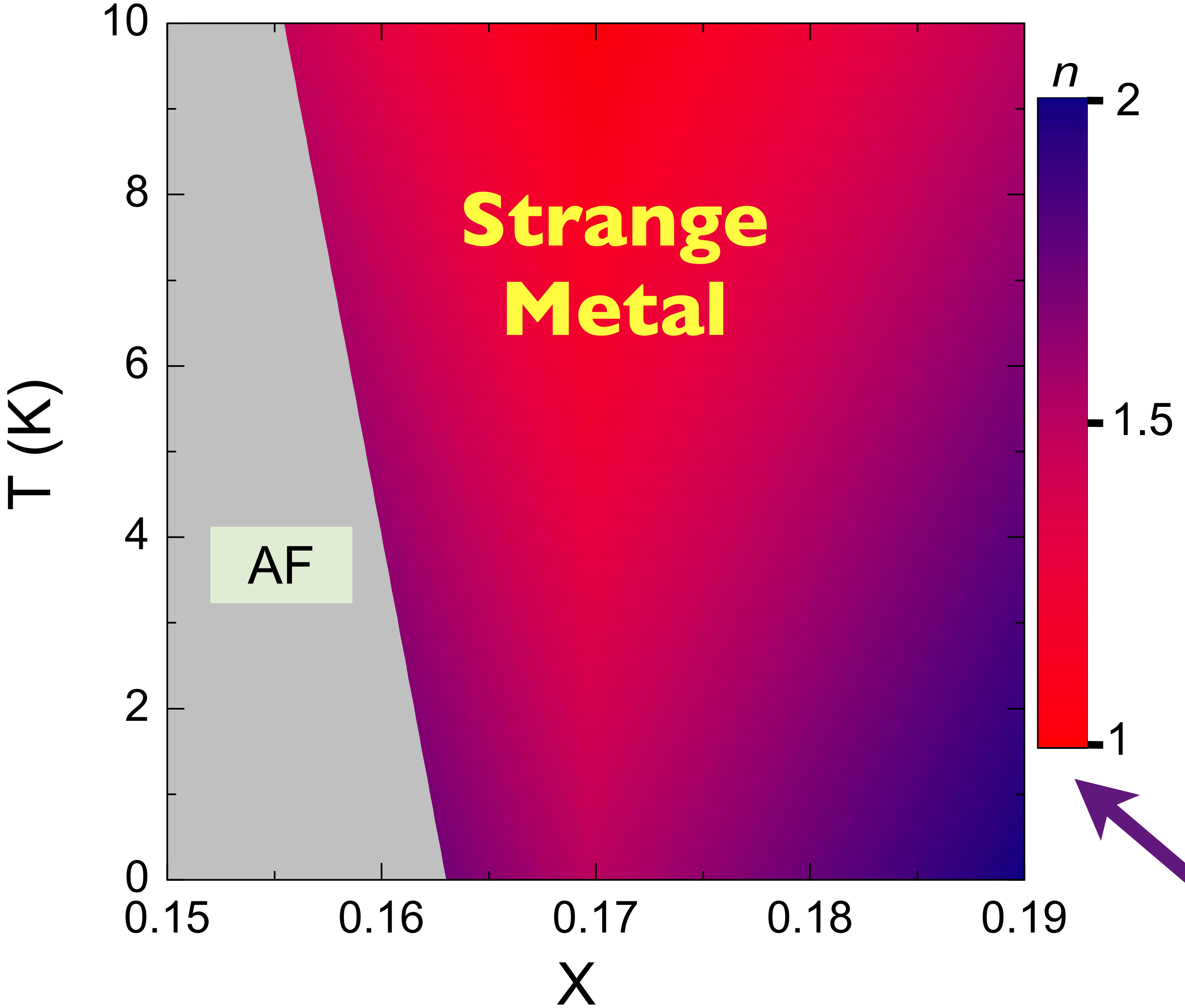

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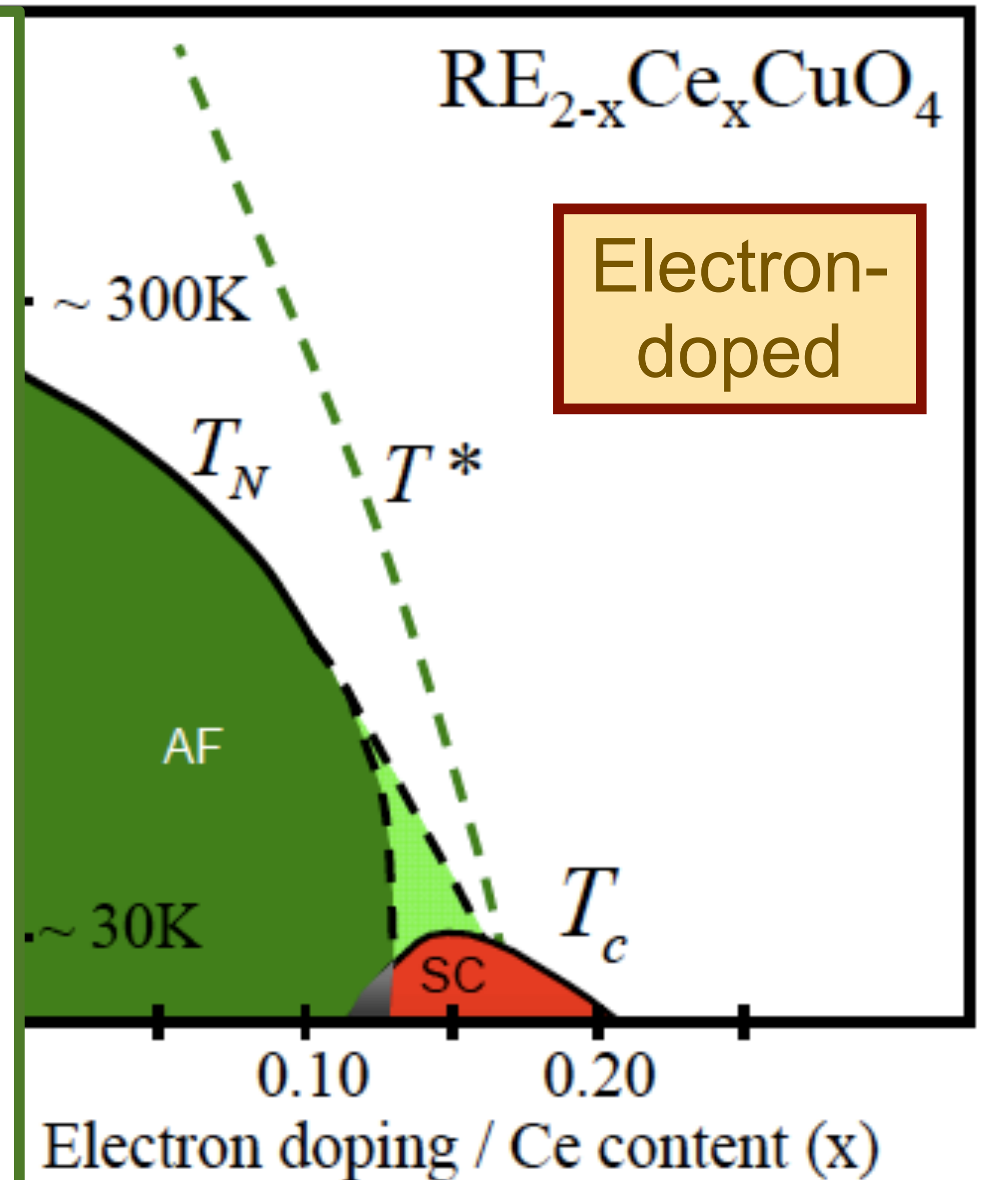

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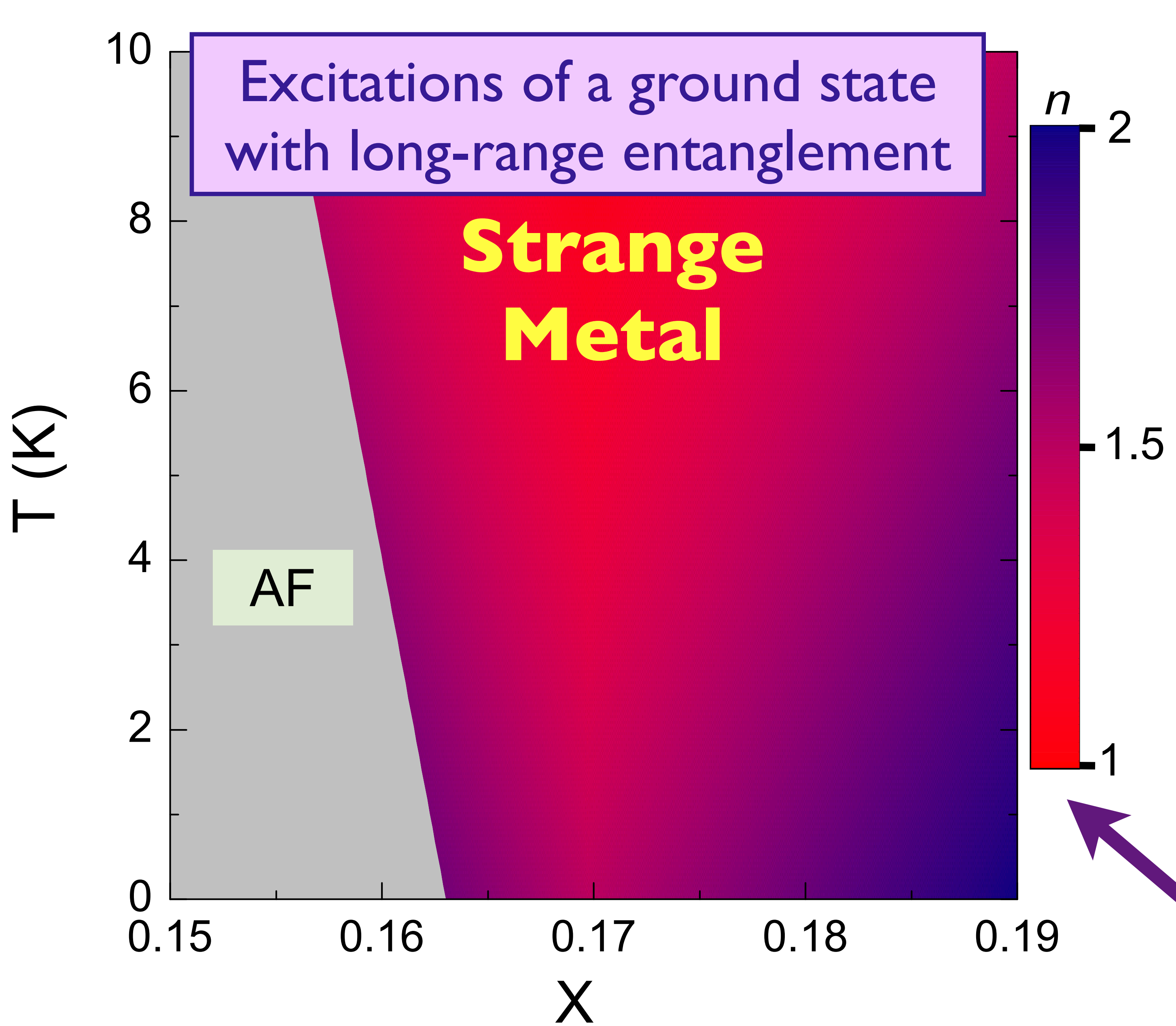

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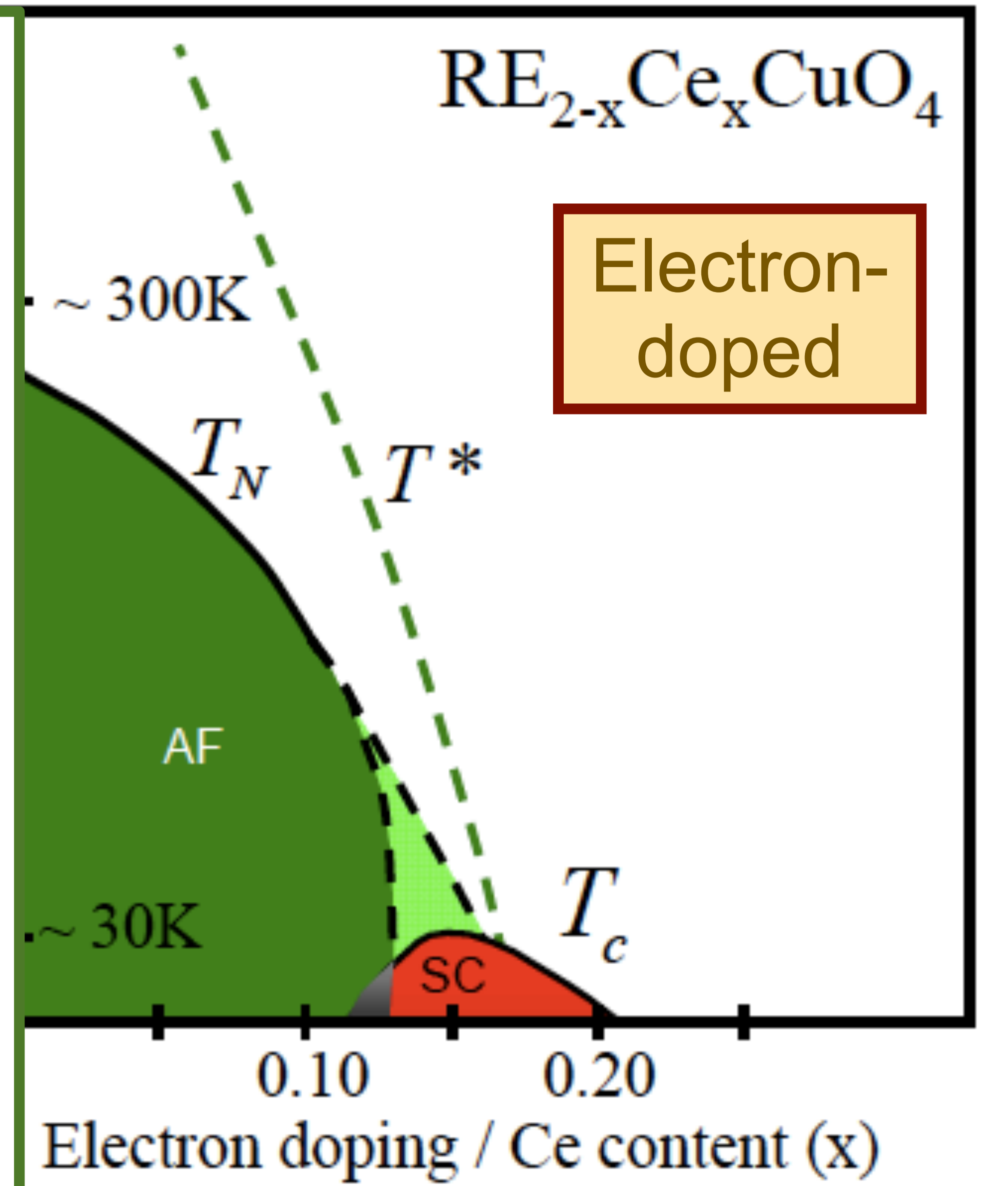

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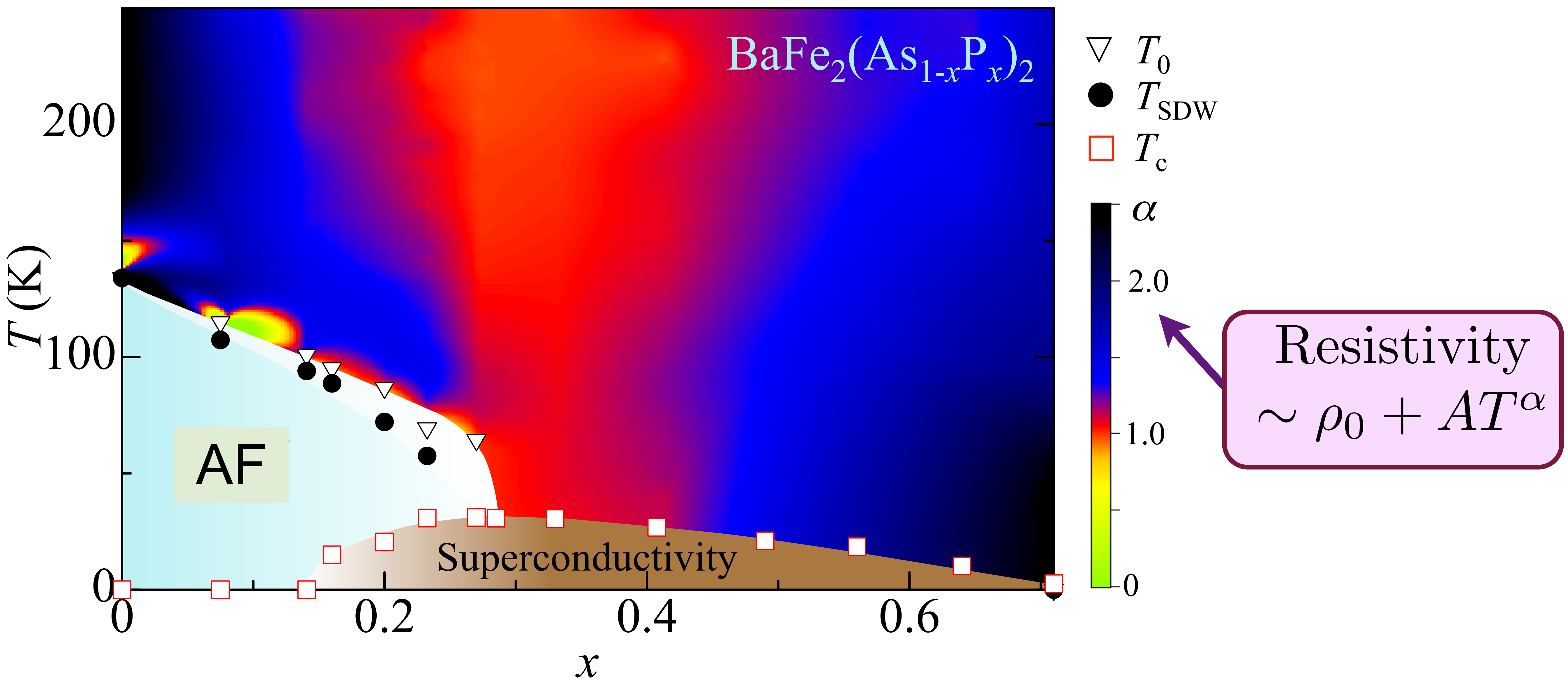
Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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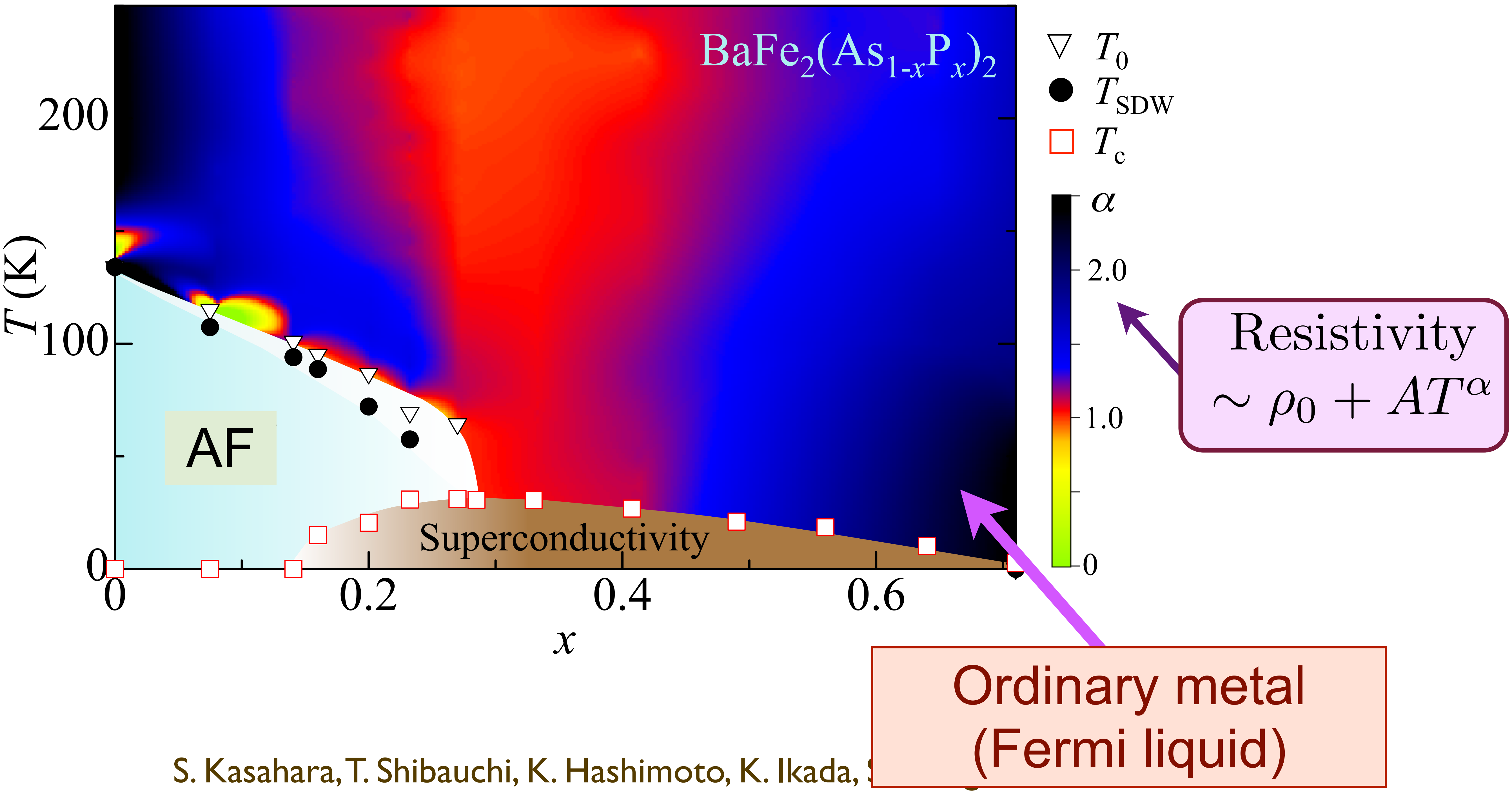

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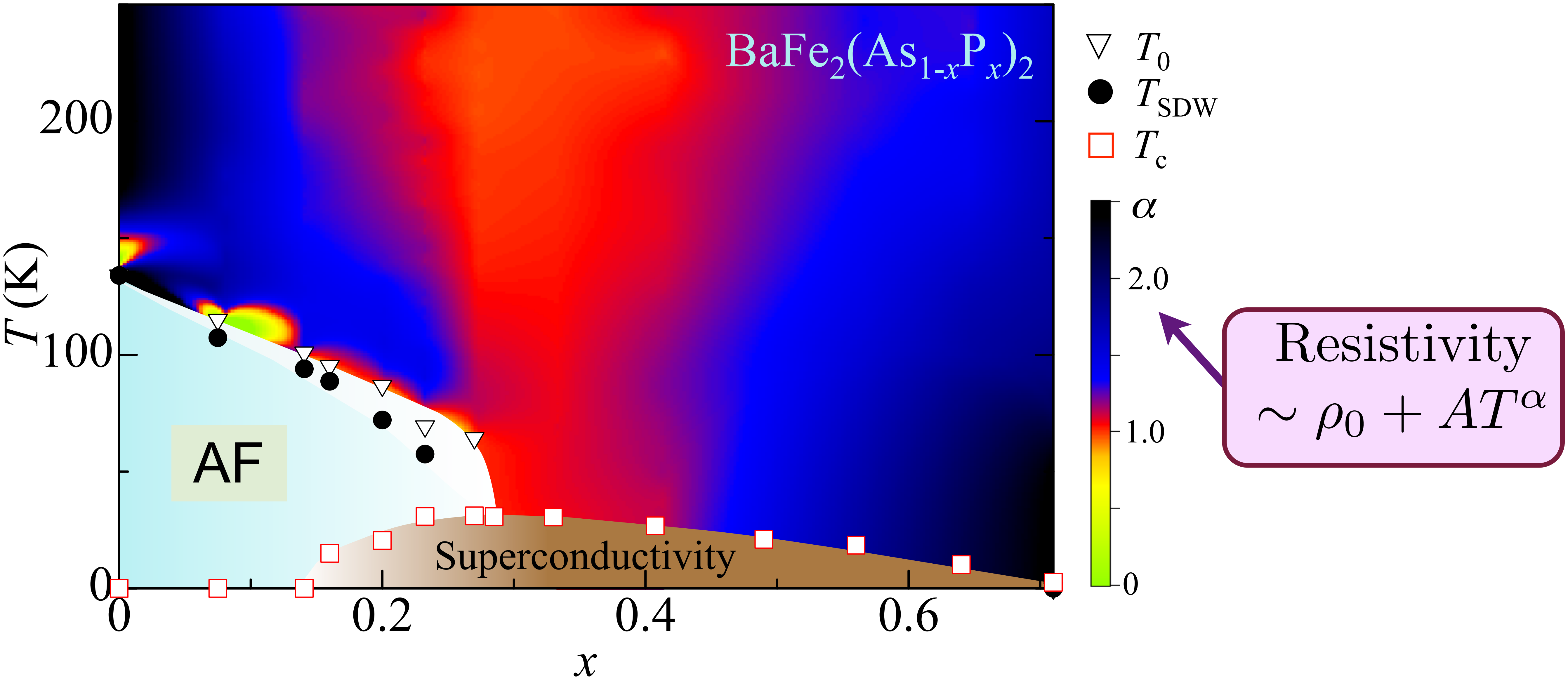

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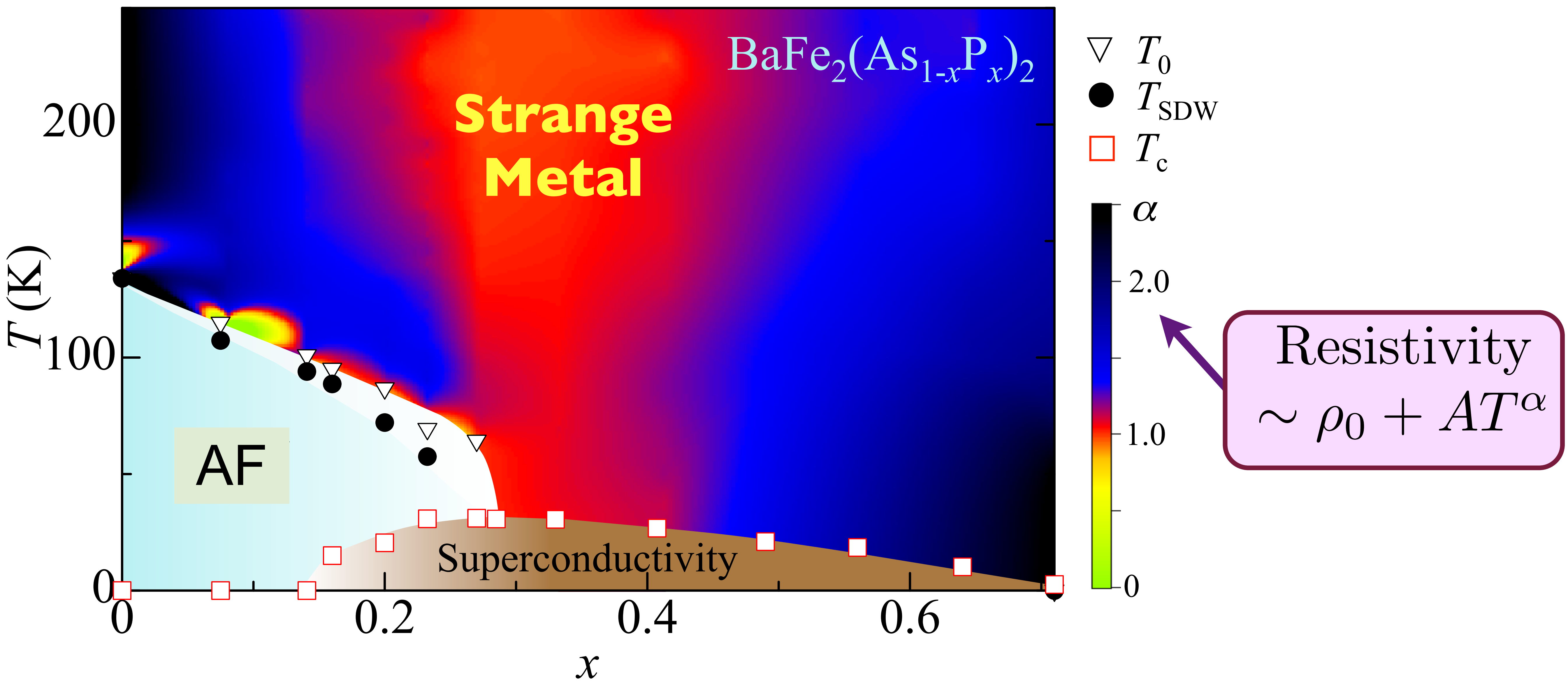

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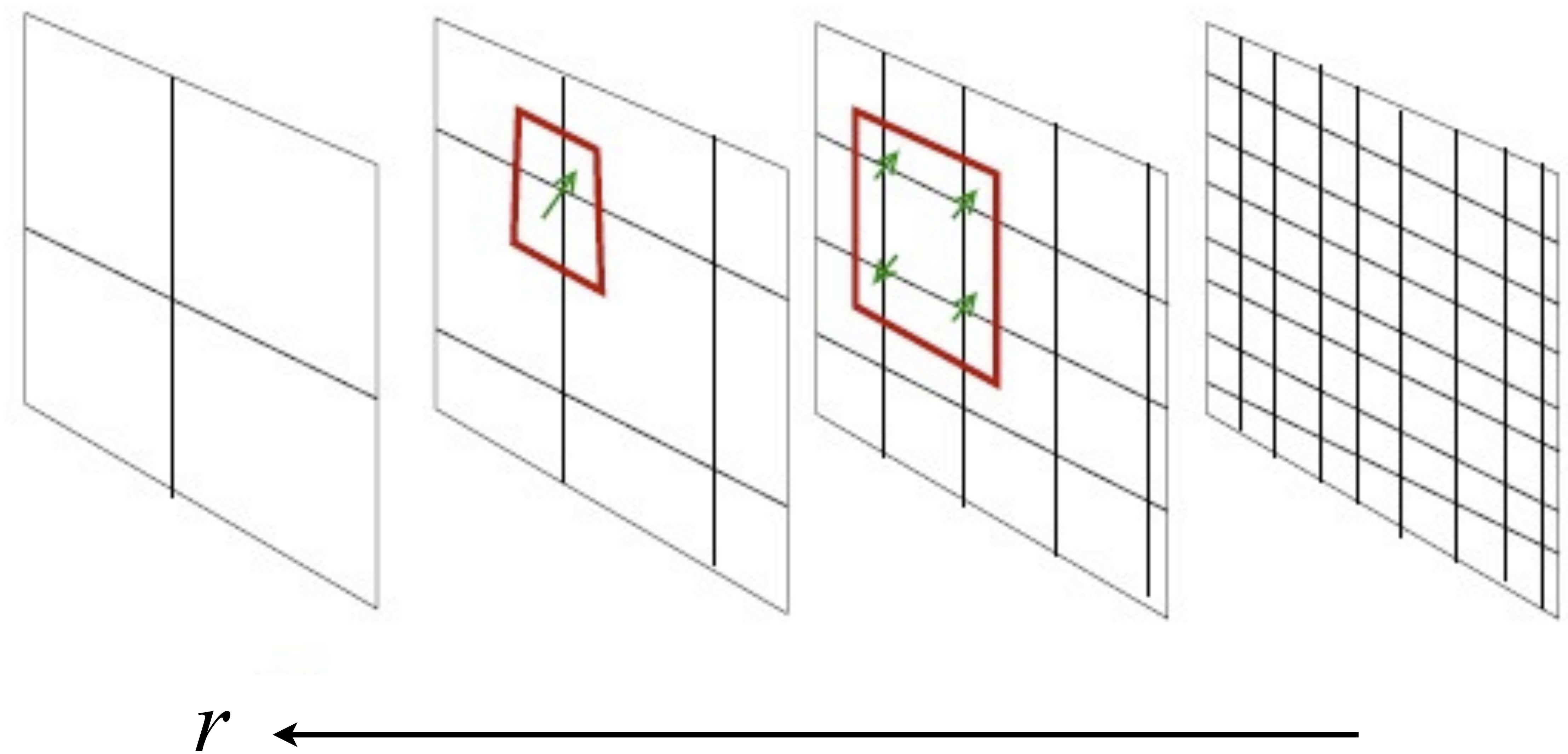
The theory of this strange metal is strongly coupled in two spatial dimensions, and the traditional field-theoretic expansion methods break down.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Study the large N limit of a SU(N) gauge field coupled to adjoint (matrix) fermions at a non-zero chemical potential

Holography of non-Fermi liquids



J. McGreevy, arXiv0909.0518

Holography of non-Fermi liquids

Consider the following (most) general metric for the holographic theory

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

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This metric transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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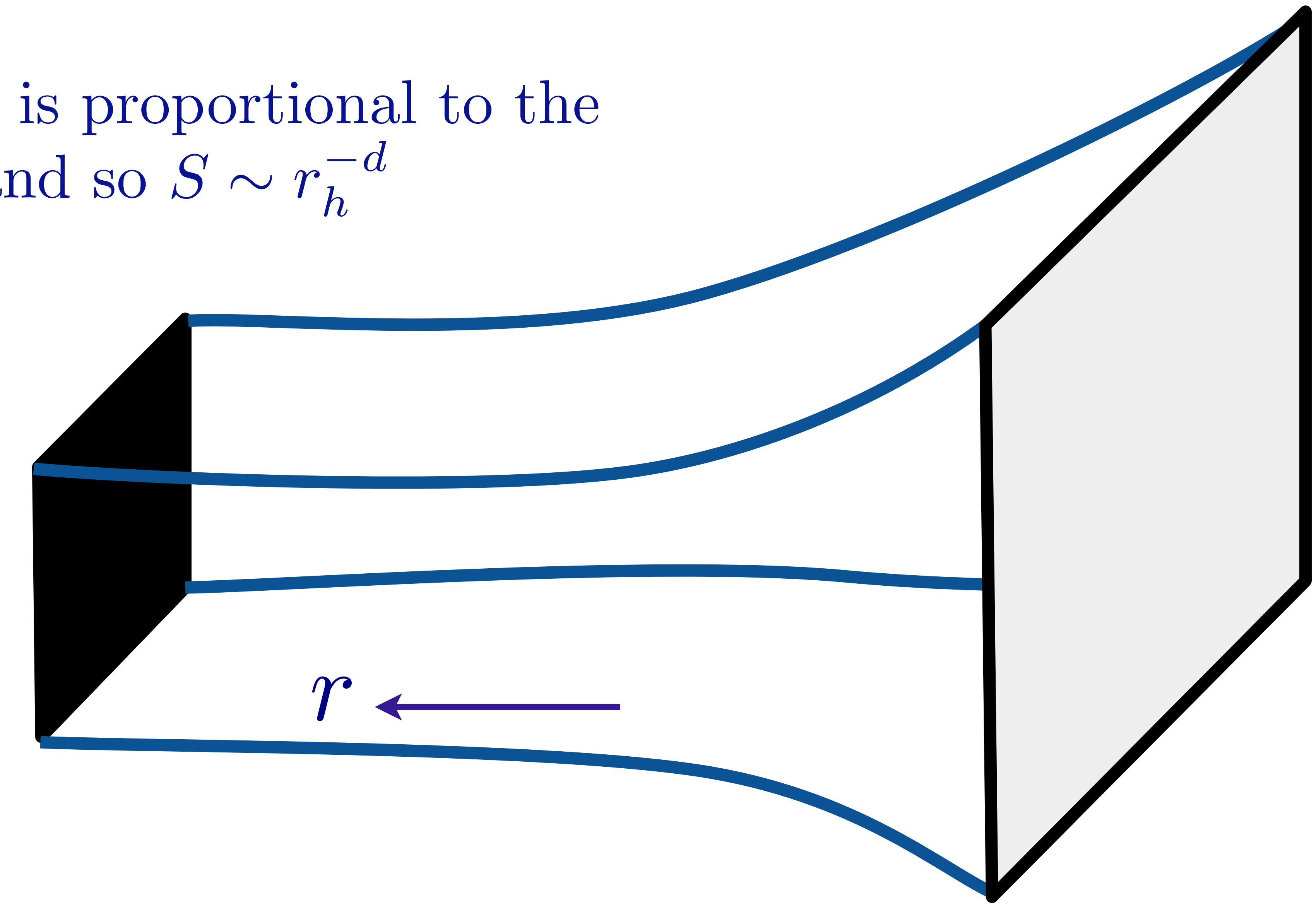
What is θ ? ($\theta = 0$ for "relativistic" quantum critical points).

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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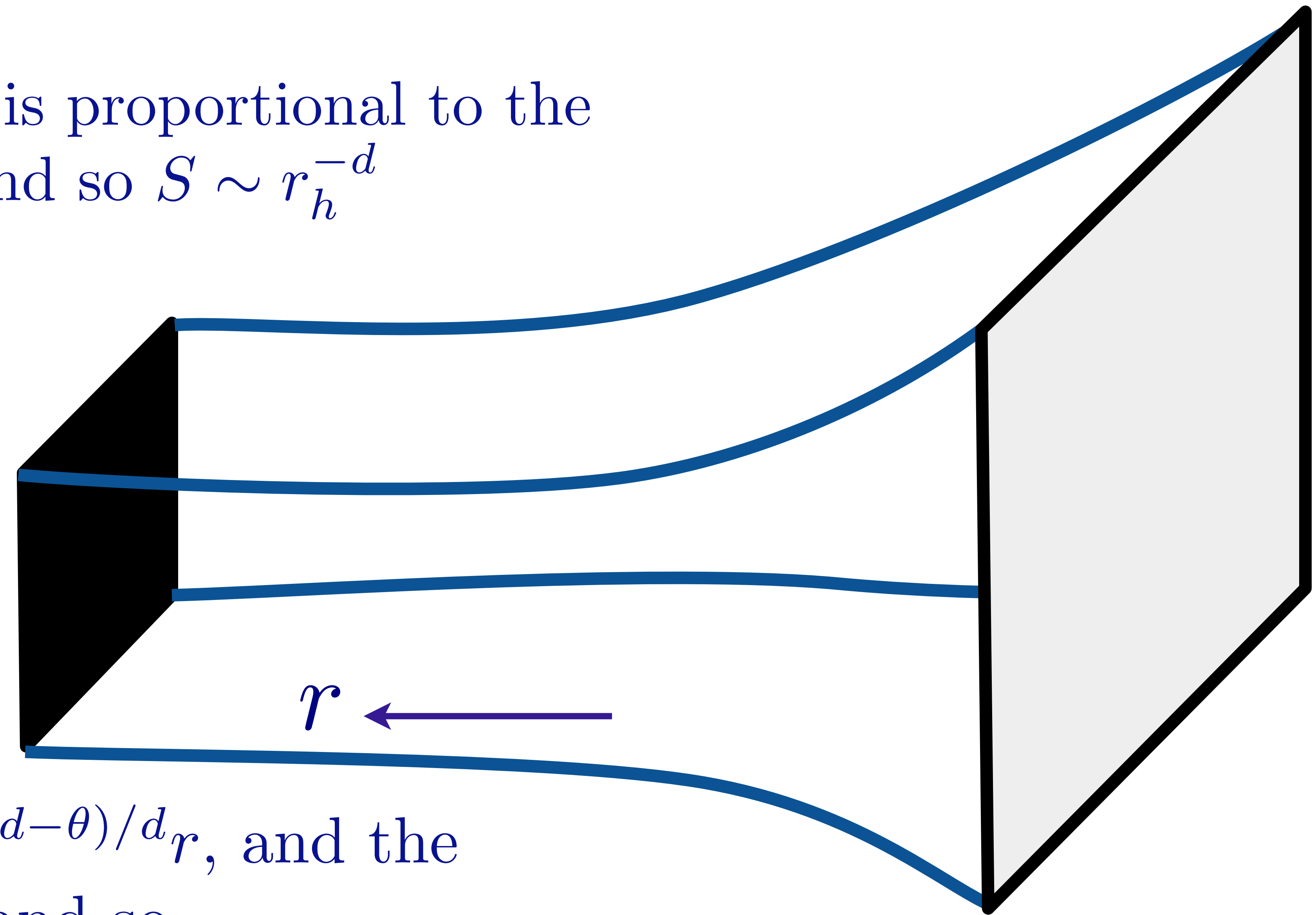


Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z}$$

So θ is the "violation of hyperscaling" exponent.

Holography of non-Fermi liquids

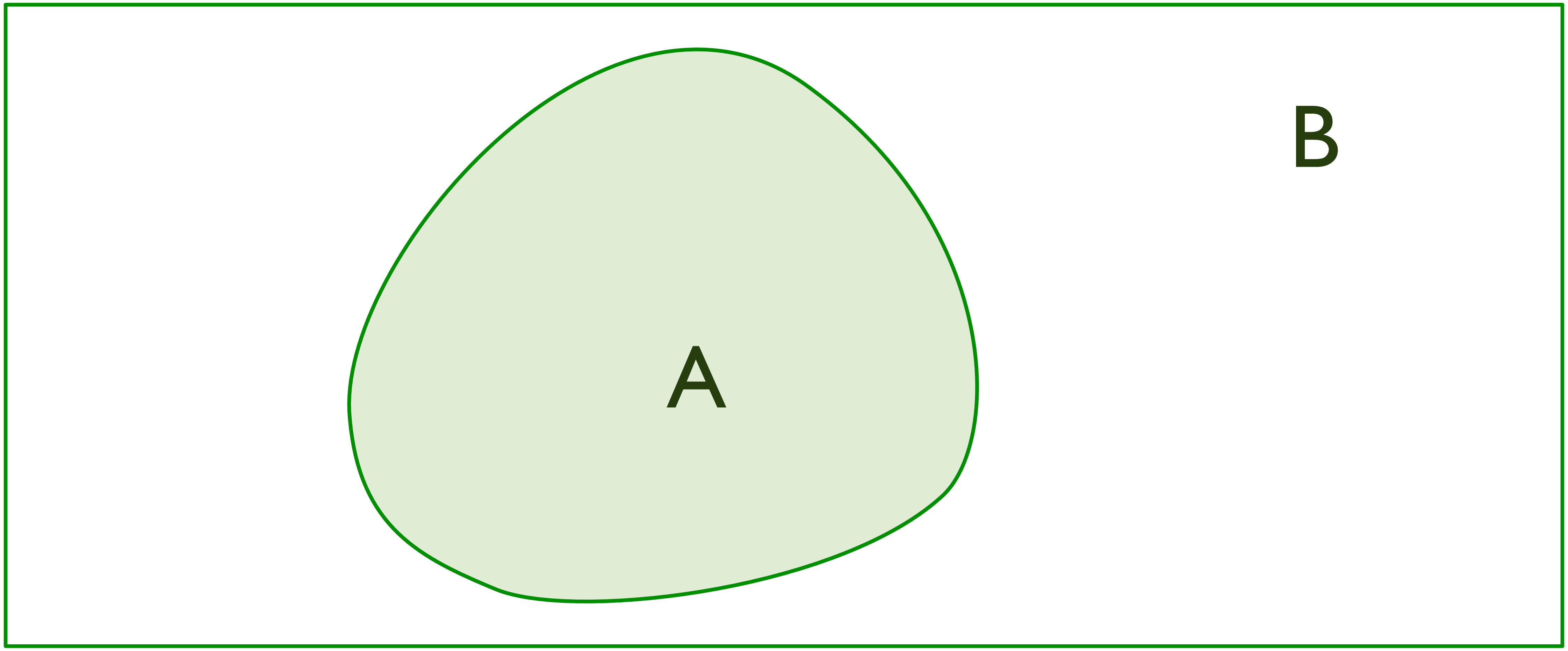
$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

A non-Fermi liquid has gapless fermionic excitations on the Fermi surface, which disperse in the single transverse direction with dynamic critical exponent z. So we expect compressible quantum states to have an effective dimension $d-\theta$ with

$$\theta = d - 1$$

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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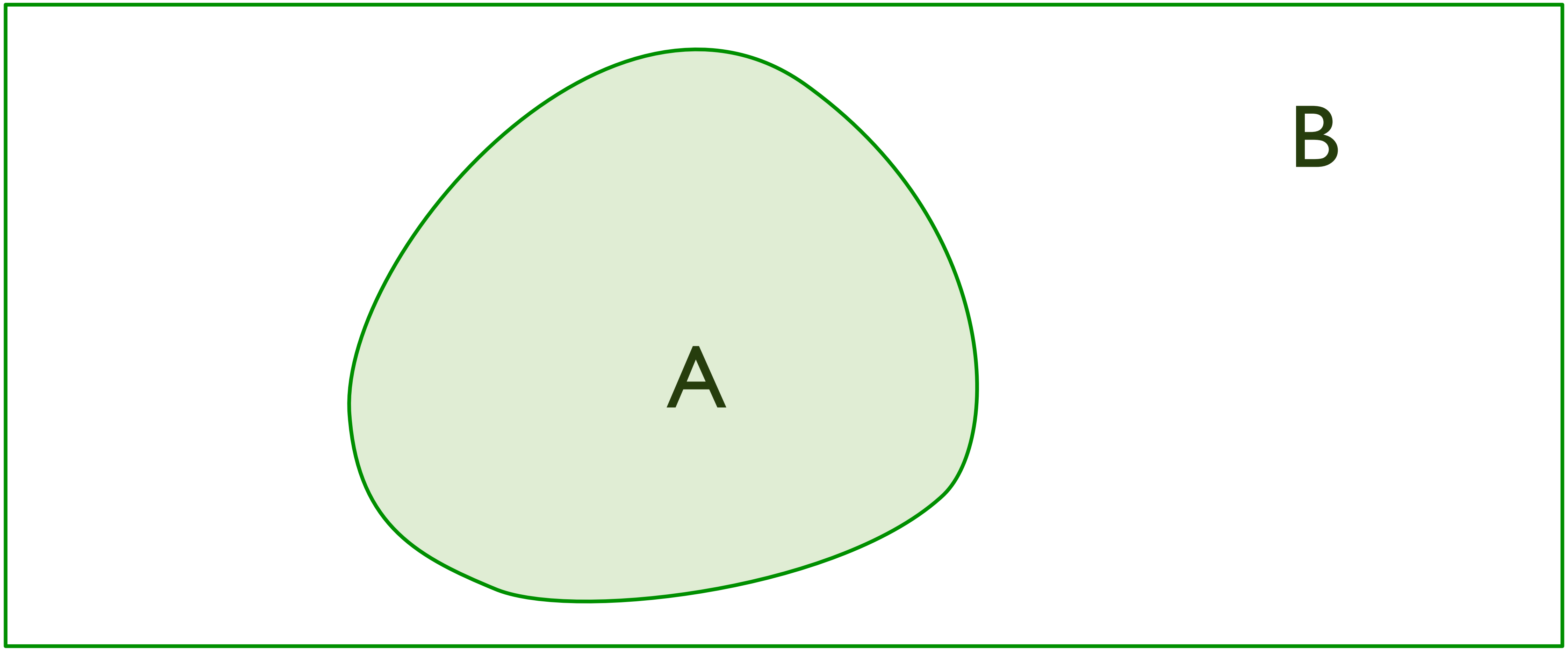

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Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023; L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !
- The co-efficient of the logarithmic term is consistent with the Luttinger relation.
- Many other features of the holographic theory are consistent with a boundary theory which has "hidden" Fermi surfaces of gauge-charged fermions.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.

Conclusions

Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

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String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.

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Much recent progress offers hope of a holographic description of "strange metals"