Gauge-gravity duality and its applications

Subir Sachdev

Iguaçu, Brasil, June 7, 2011

Talk online: sachdev.physics.harvard.edu

PHYSICS

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Outline

1. Quantum criticality and conformal field theories in condensed matter

2. Quantum transport and Einstein-Maxwell theory on AdS_4

3. Compressible quantum matter
   Fermi surfaces

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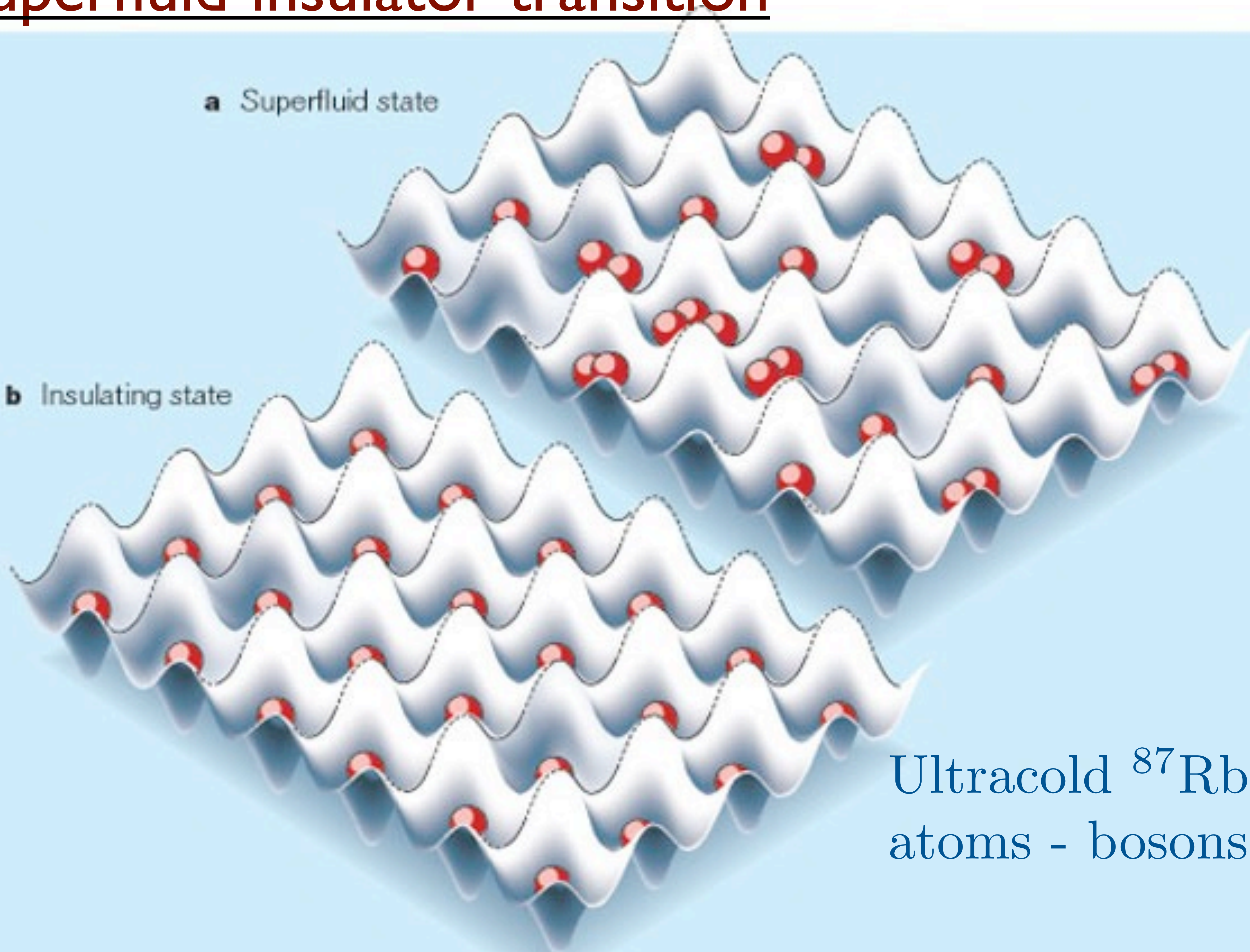
Superfluid-insulator transition

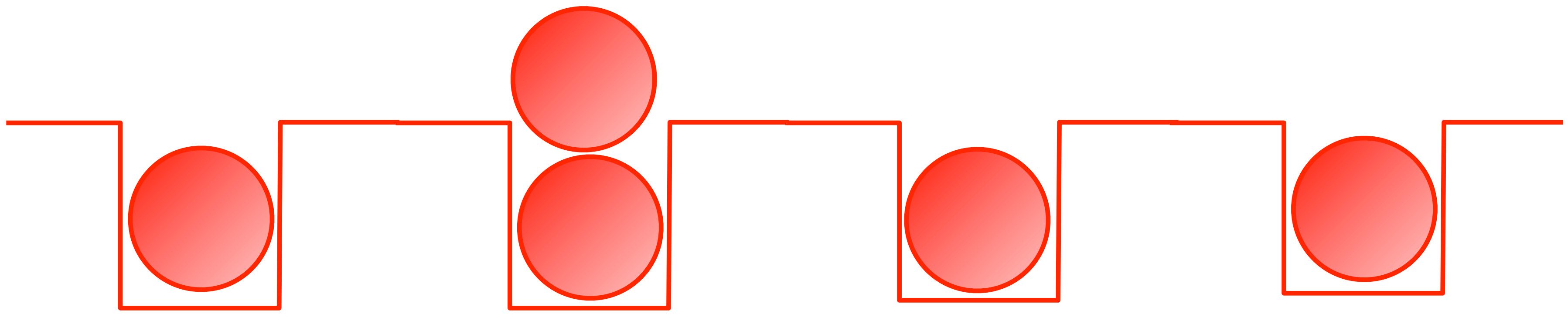


Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

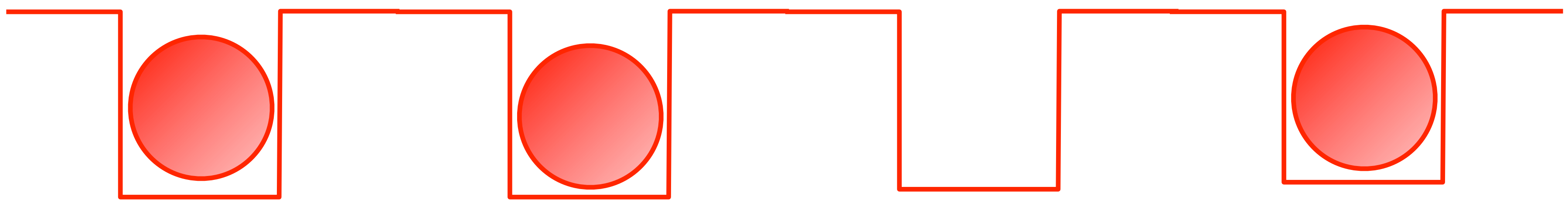
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



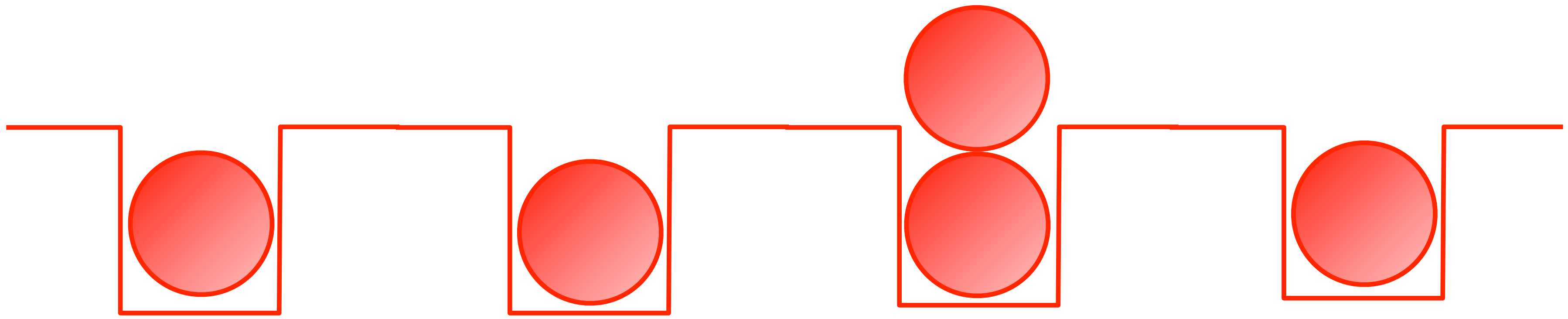
Particles $\sim \psi^\dagger$

Excitations of the insulator:



Holes ~ ψ

Excitations of the insulator:



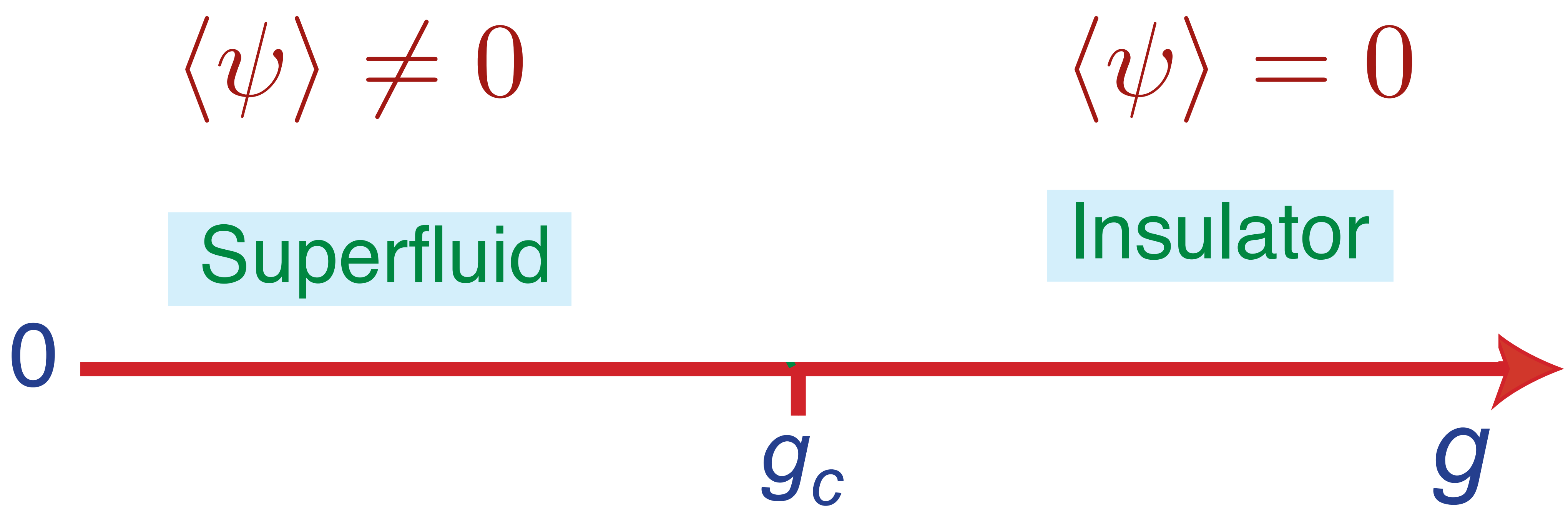
Particles ~ $\psi^\dagger$

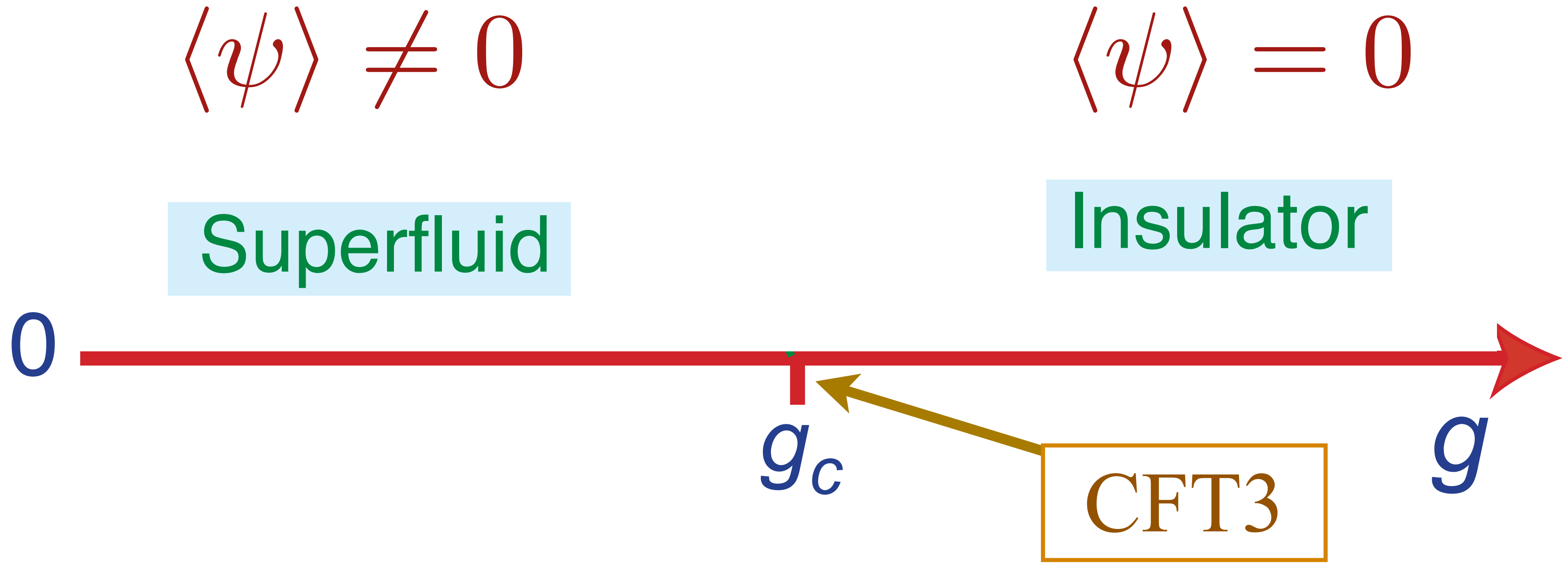
Holes ~ ψ

Density of particles = density of holes ⇒
"relativistic" field theory for ψ:

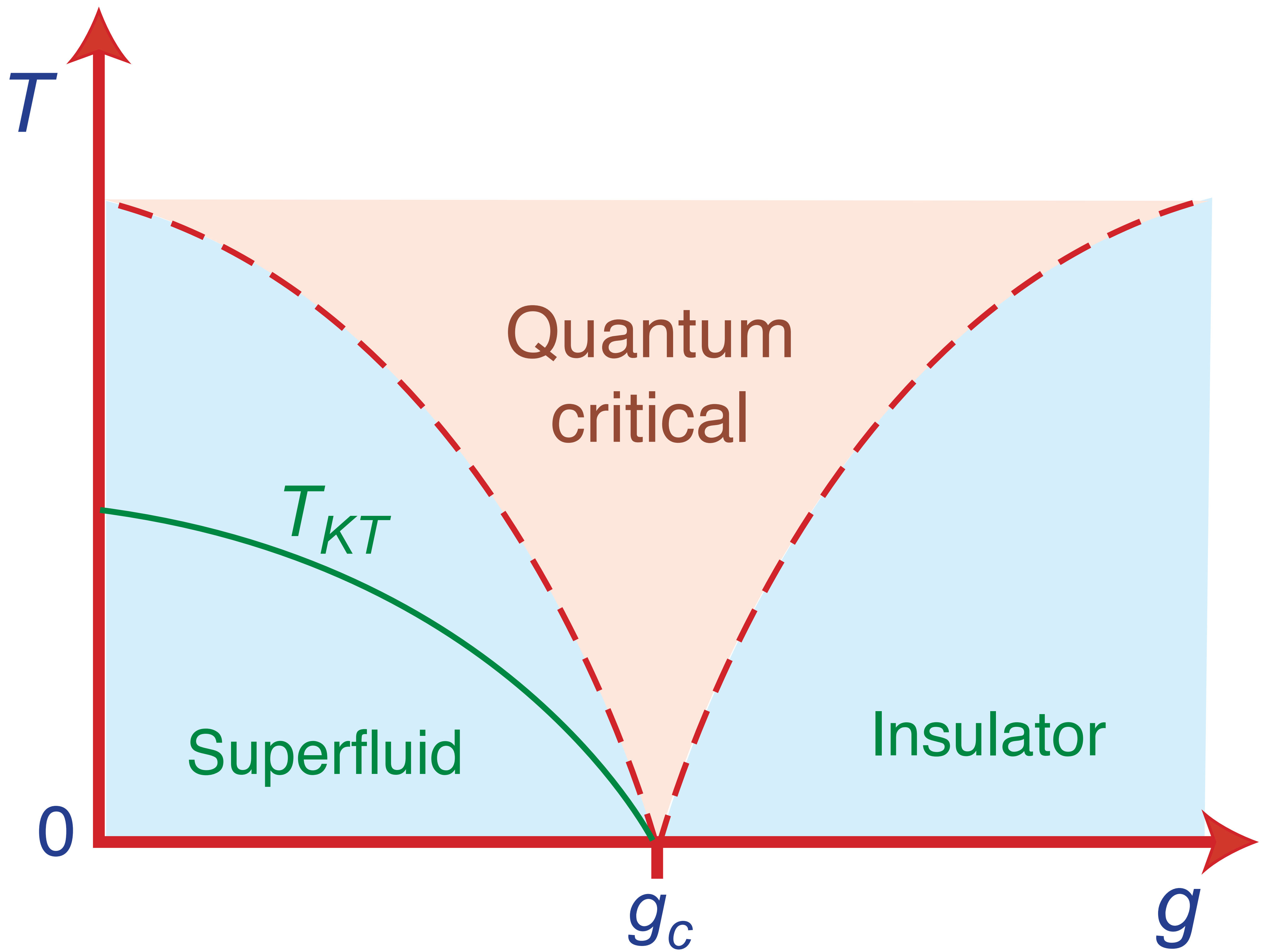
.

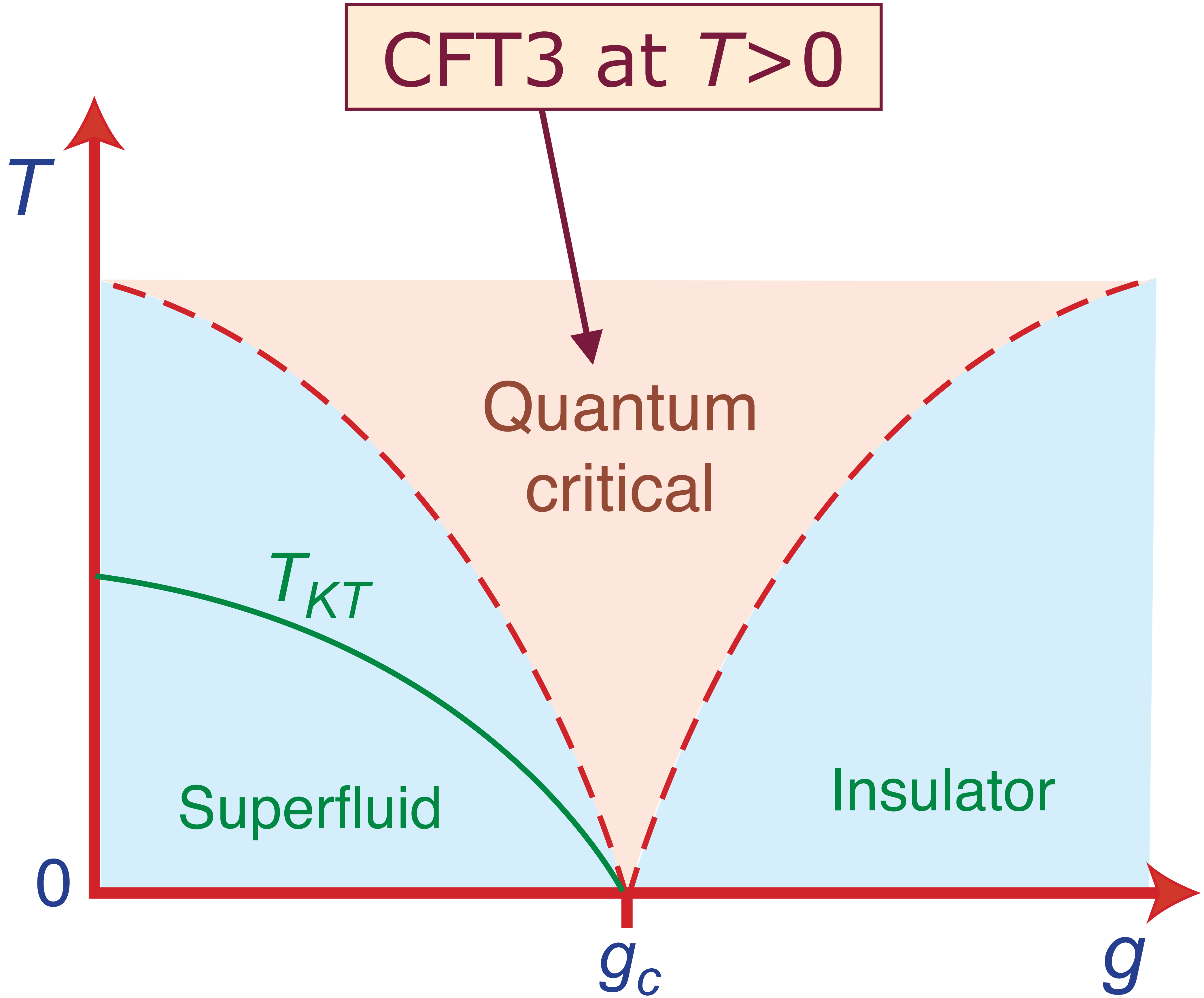
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$


$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g



$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
CFT3
g


T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g



CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, $\tau_{\rm eq}$

$$\tau_{\rm eq} = \mathcal{C}\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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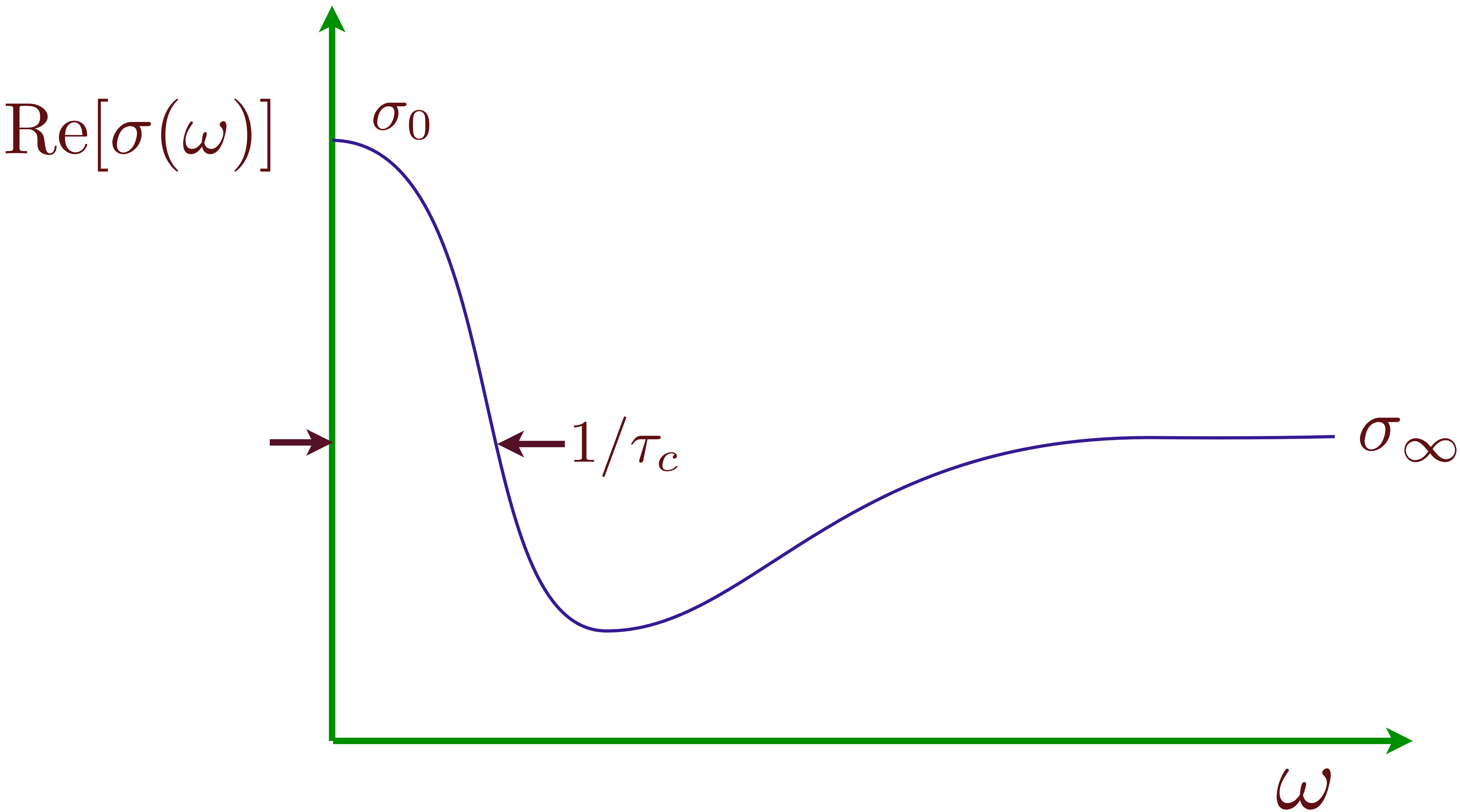
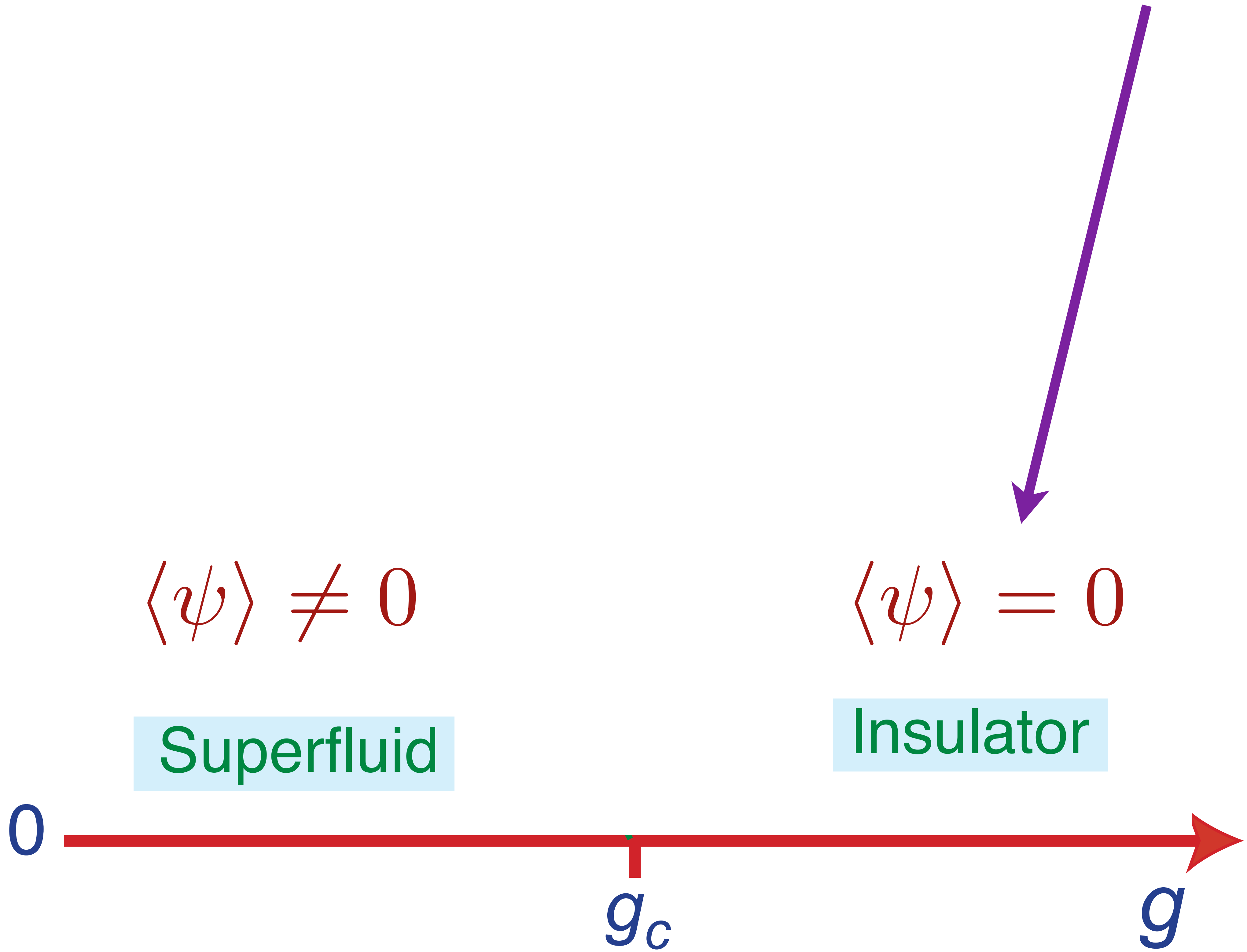
Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

Boltzmann theory of bosons

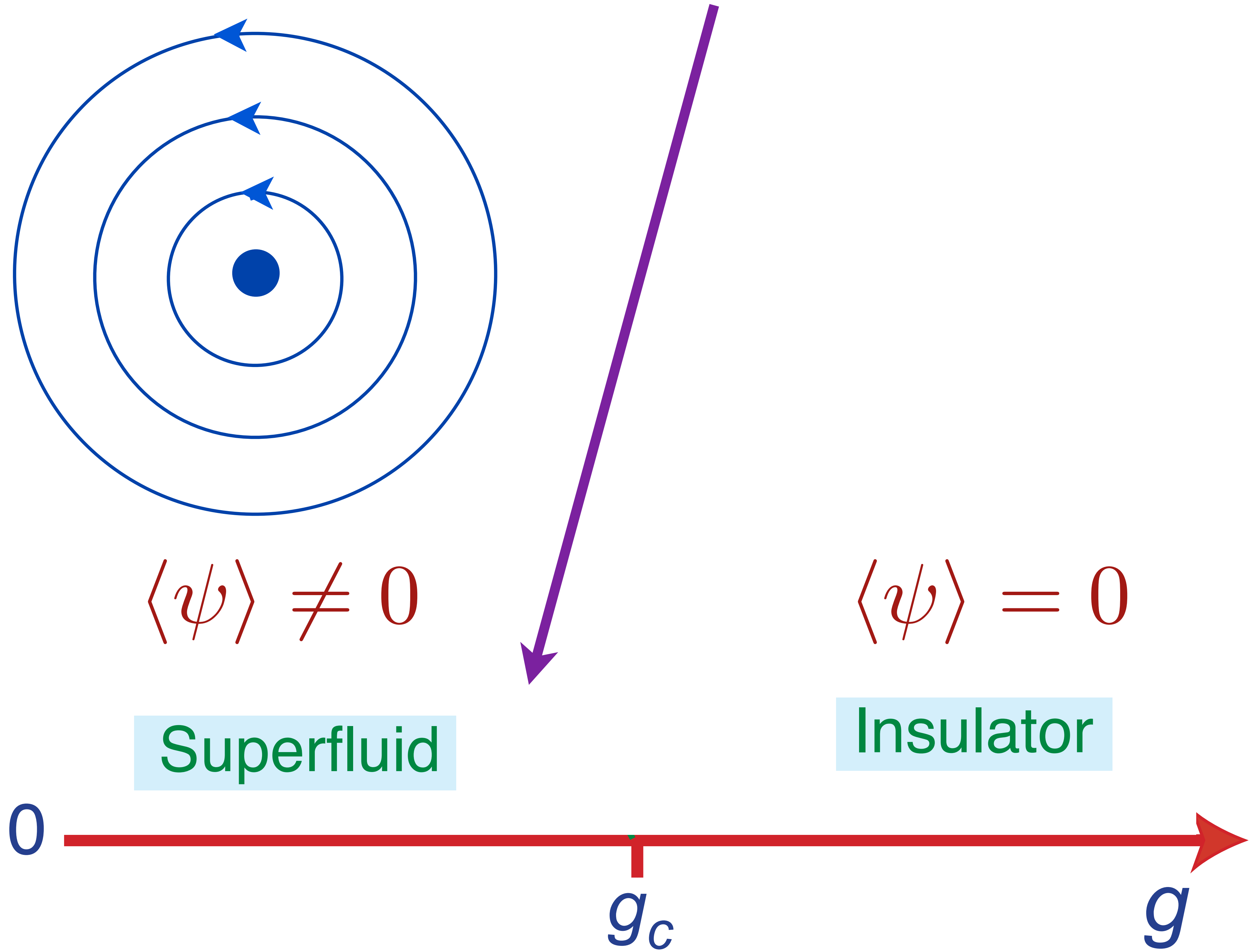


K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



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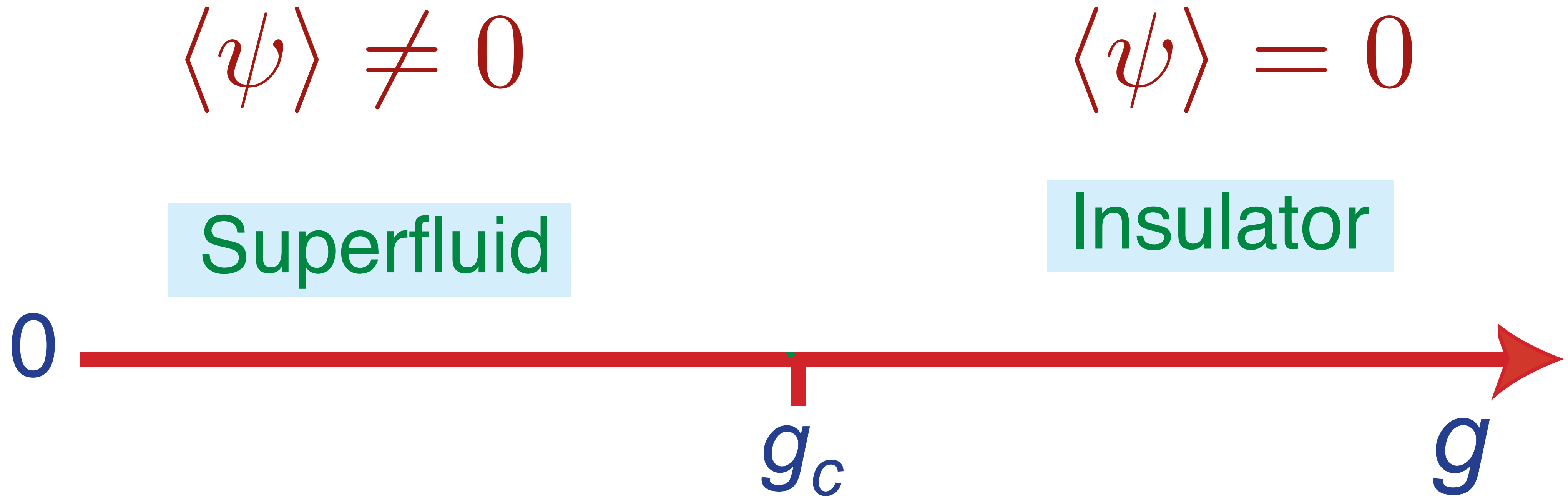
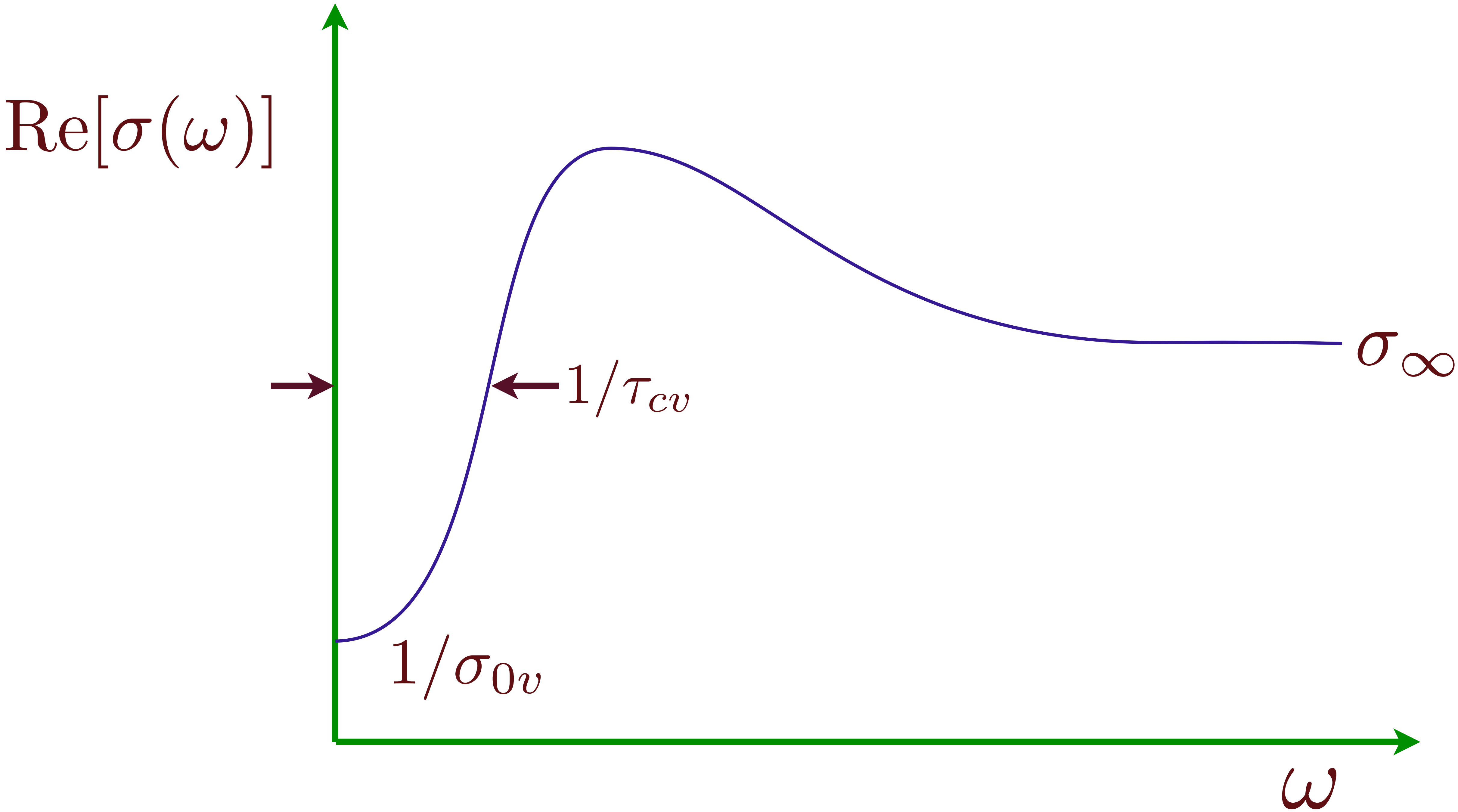
These are quantum particles (in 2+1 dimensions) which described by a "dual" CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

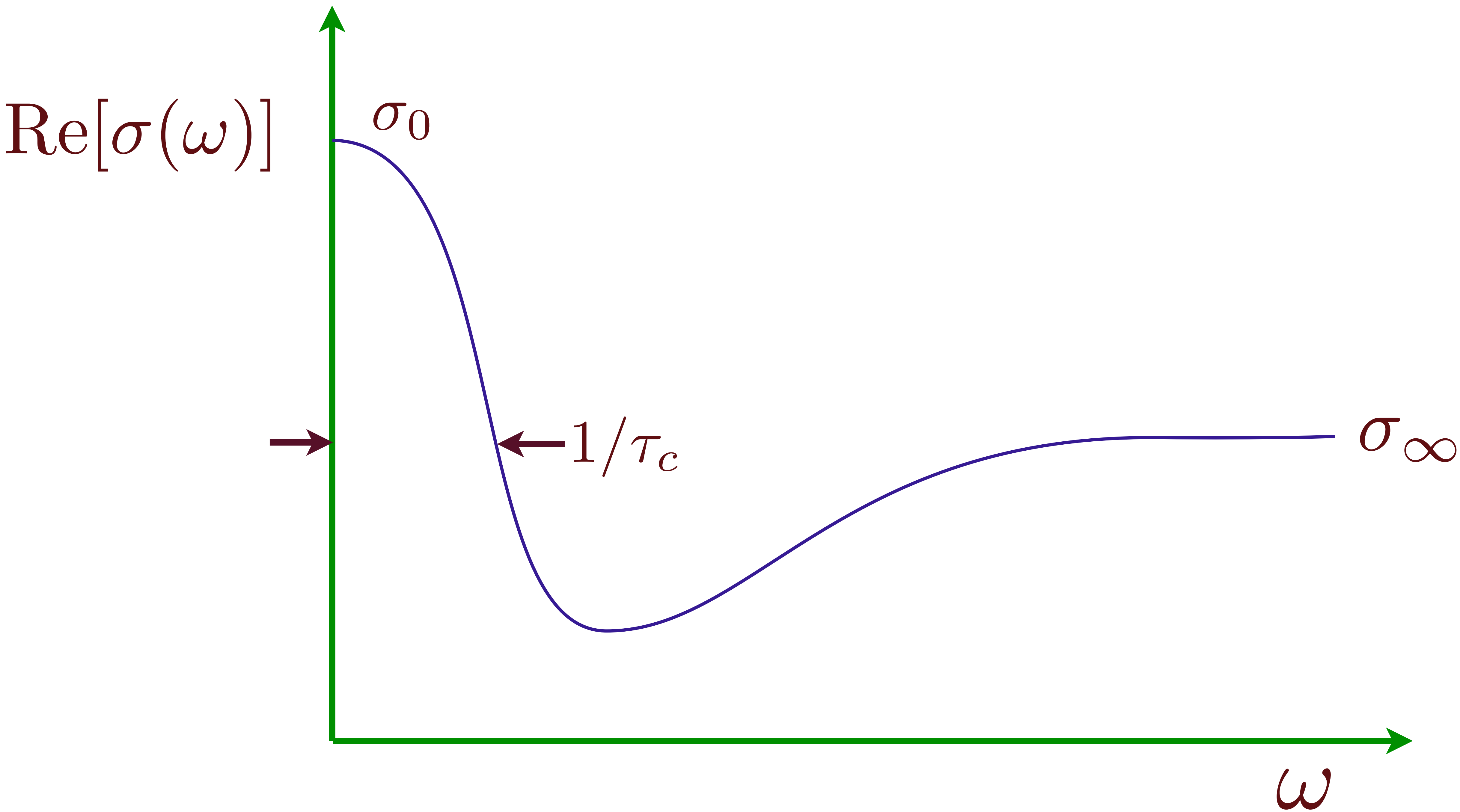
$$\text{Conductivity} = \text{Resistivity of vortices}$$



Boltzmann theory of vortices



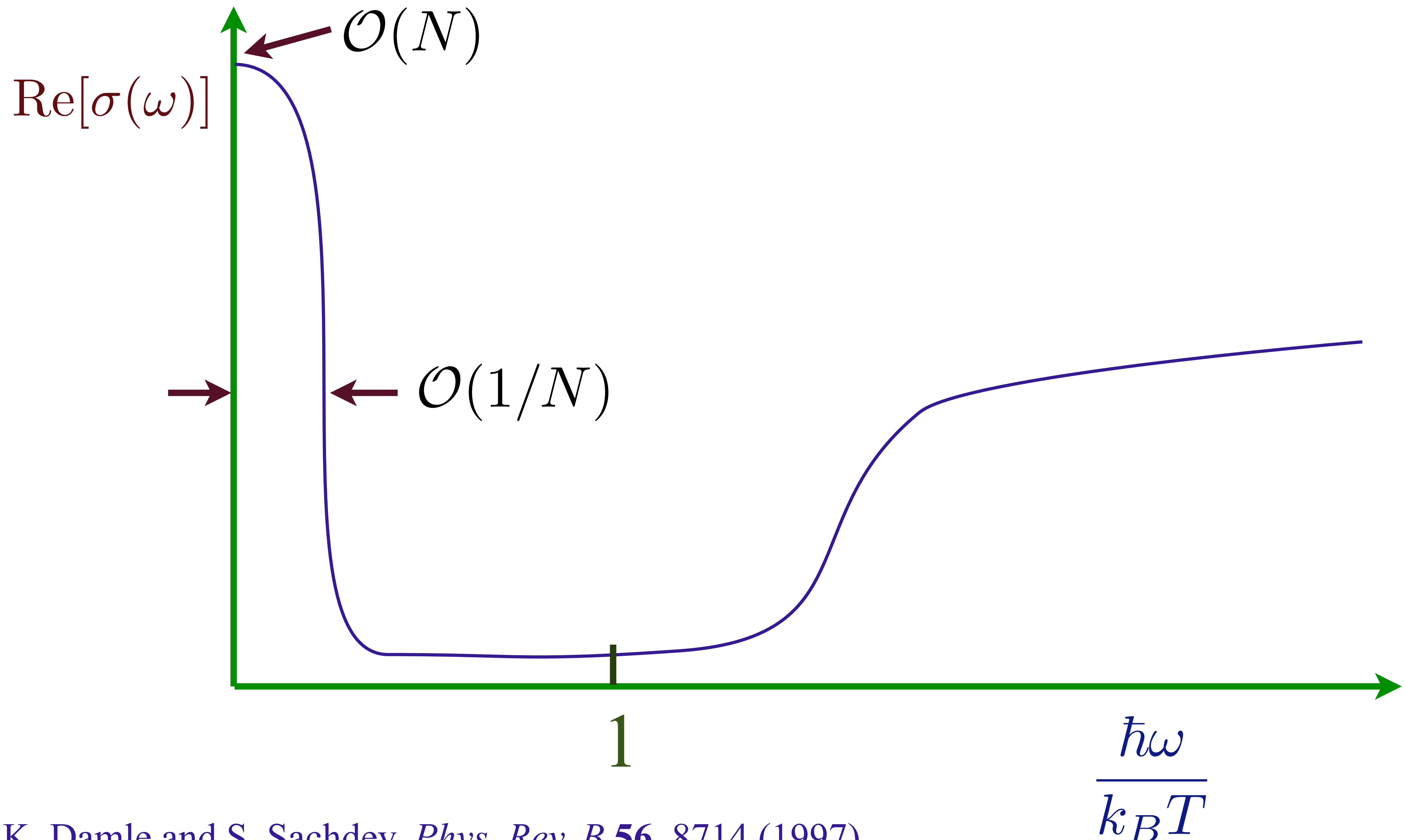
Boltzmann theory of bosons



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Vector large *N* expansion for CFT3

$$\sigma = \frac{Q^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right); \quad \Sigma \to \text{a universal function}$$



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Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value

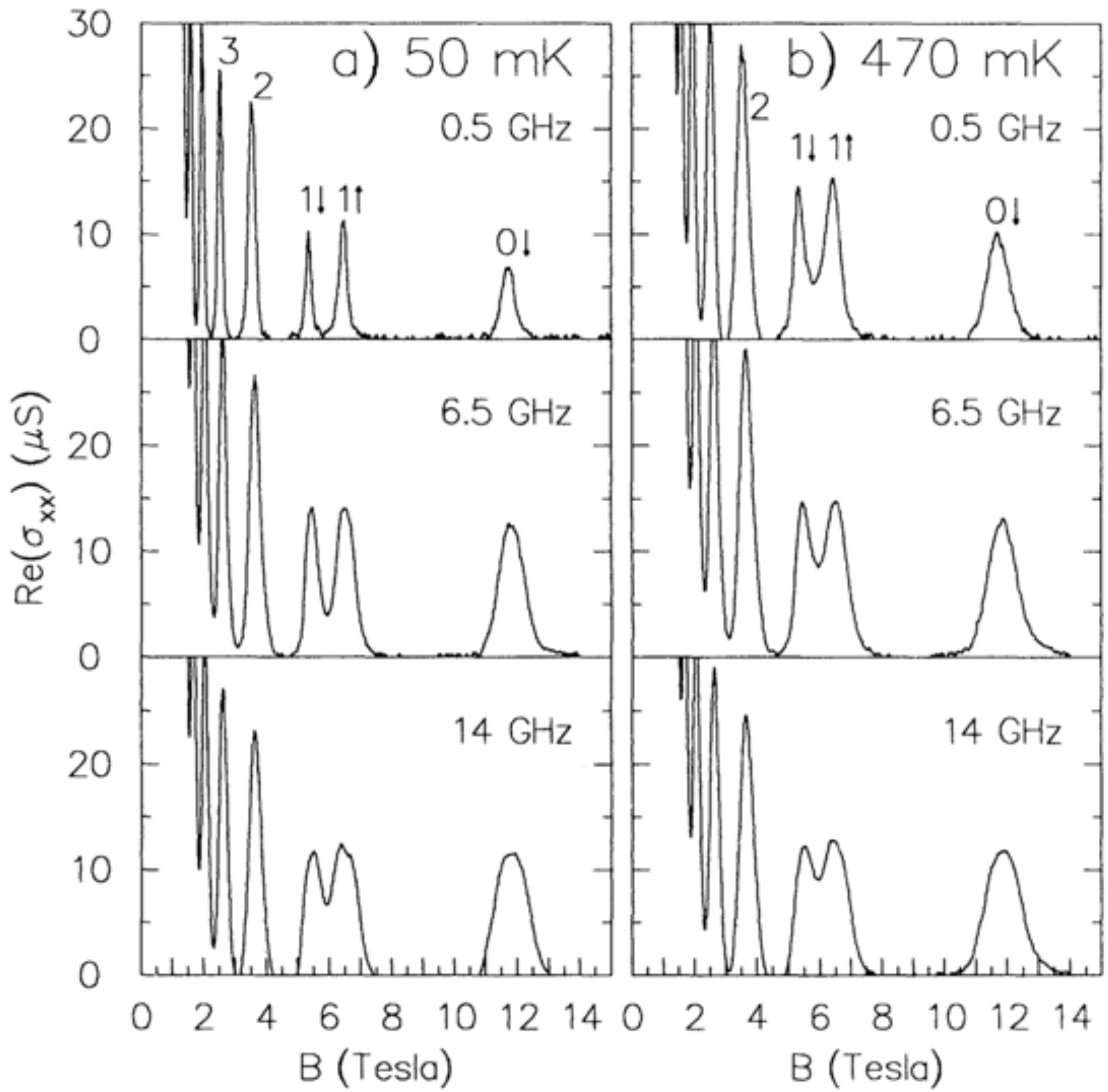

FIG. 3. Re(σ_{xx}) vs B at three frequencies and two temperatures. Peaks are marked with Landau level index N and spin.

L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui, *Physical Review Letters* **71**, 2638 (1993).

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AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

3+1 dimensional AdS space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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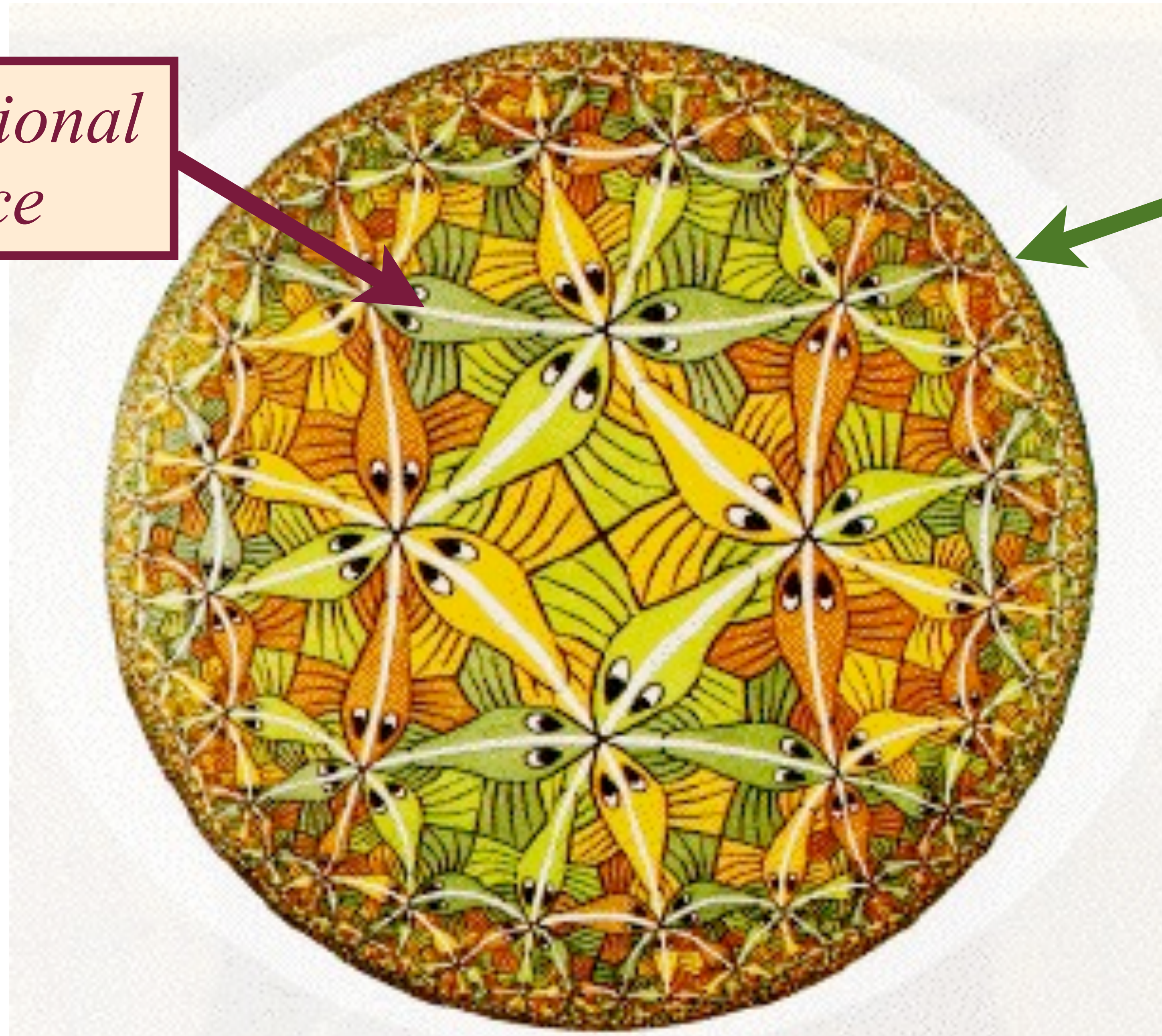

A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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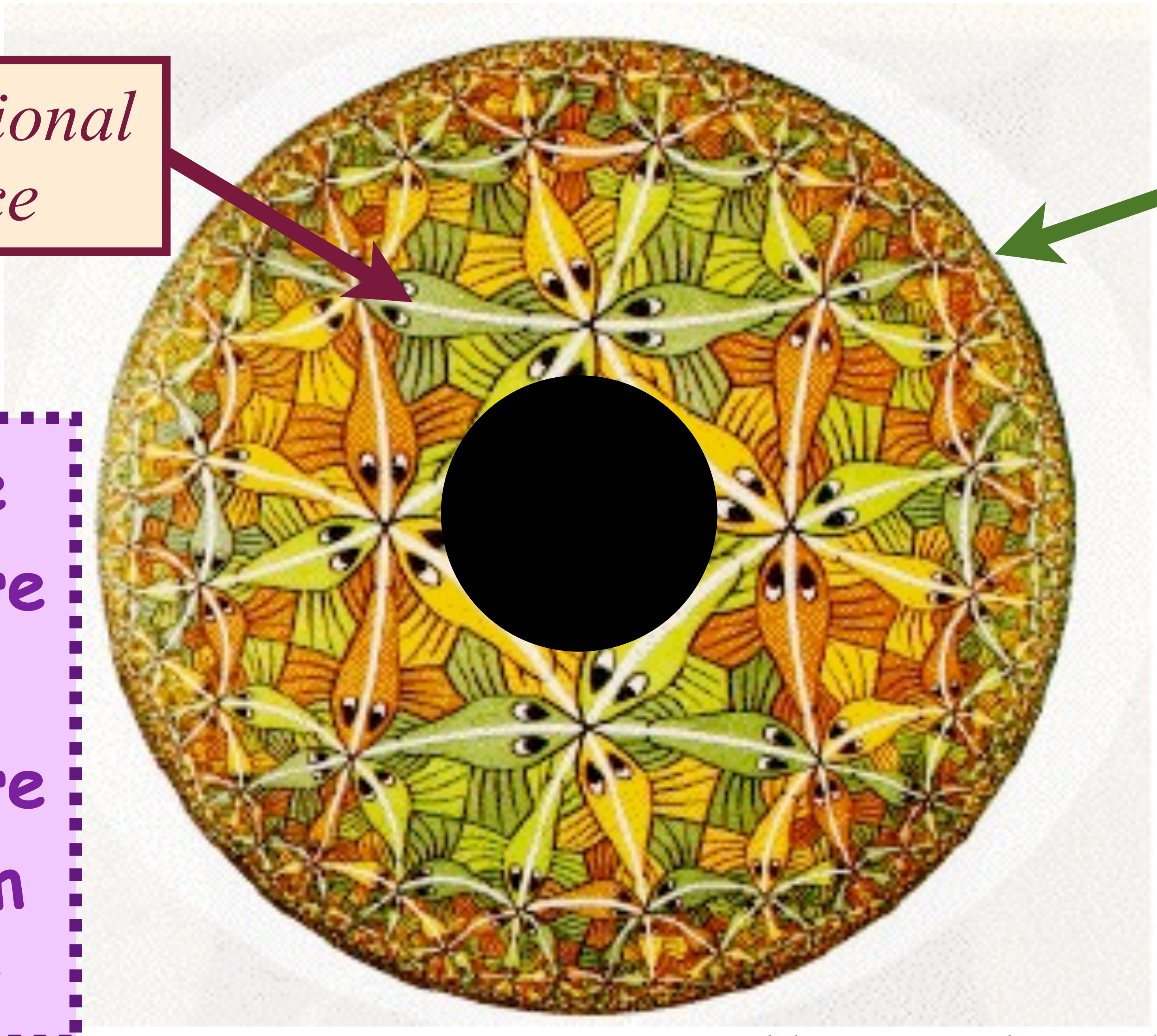

Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

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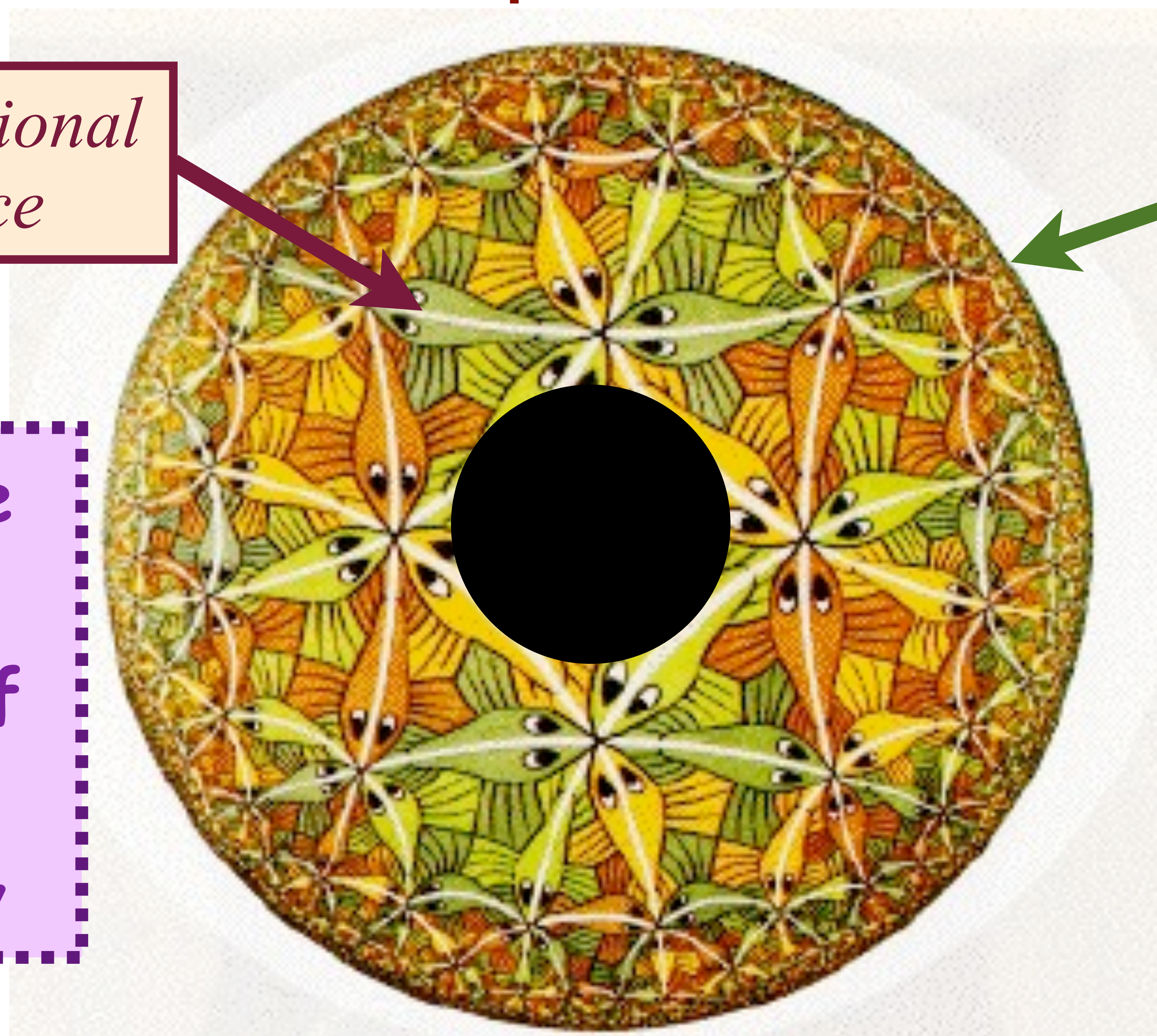

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

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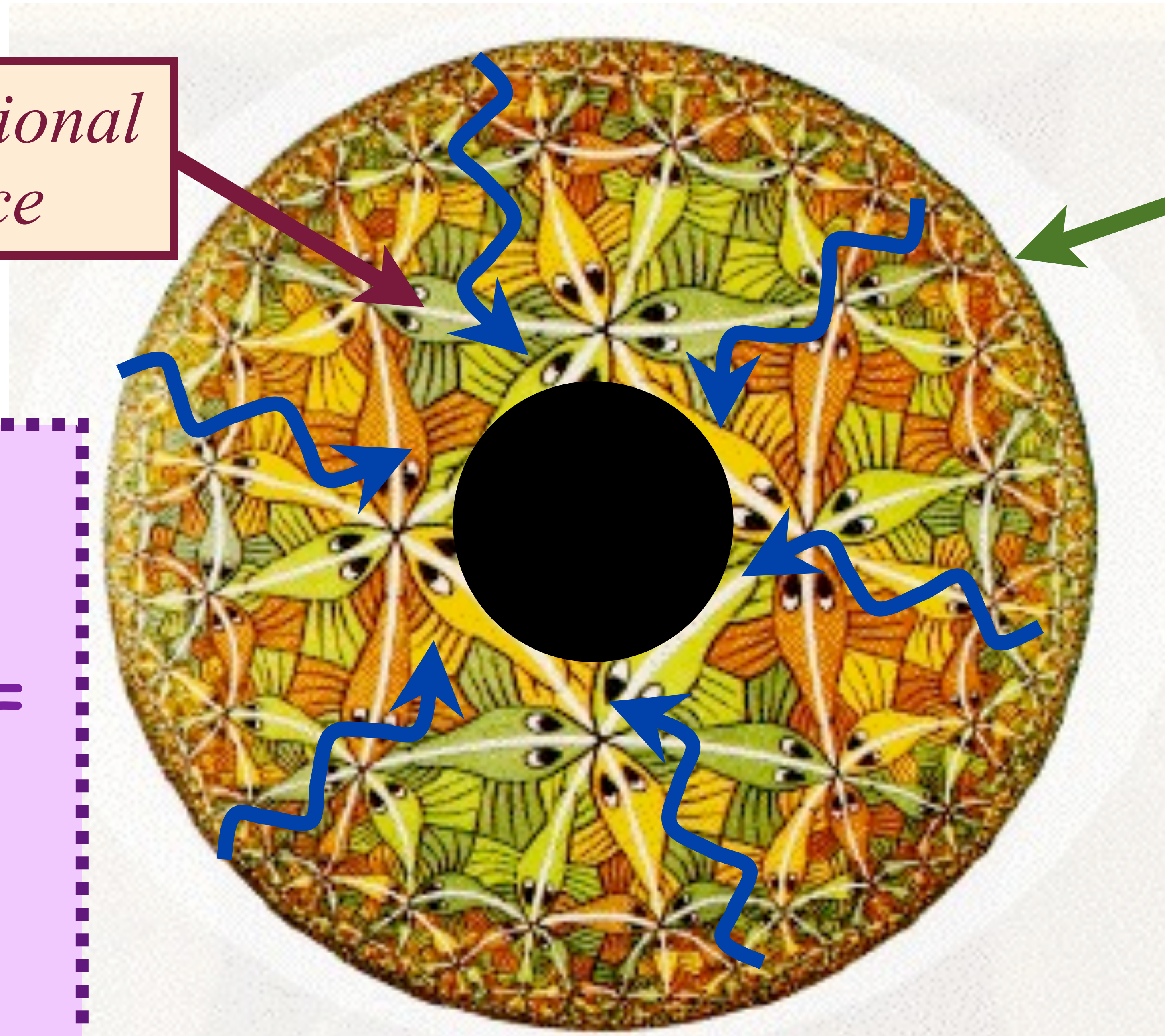

Quantum criticality in 2+1 dimensions

Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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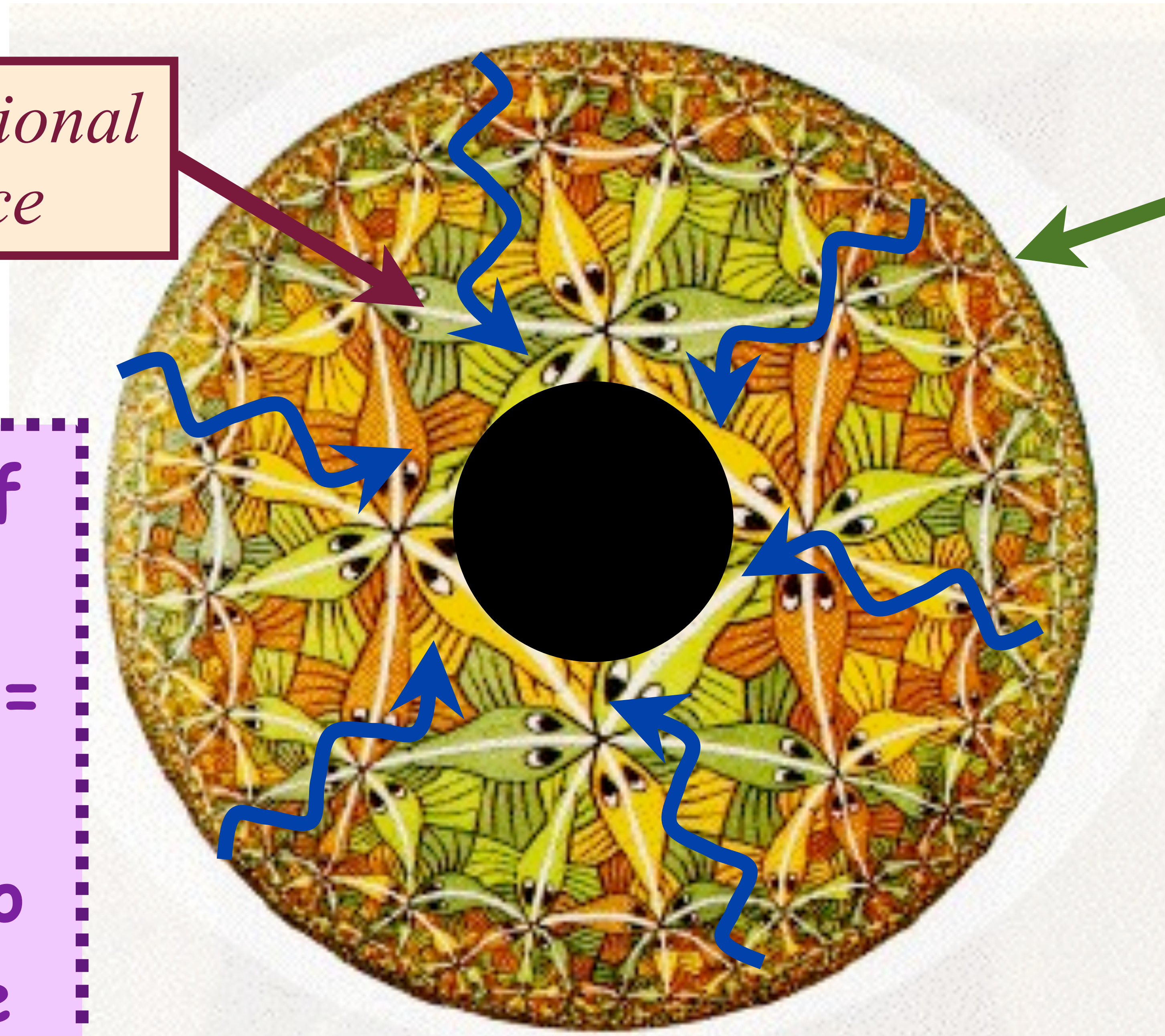


Quantum criticality in 2+1 dimensions

Friction of quantum criticality = waves falling into black hole

Kovtun, Policastro, Son

AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$

$$\mathcal{S}_{EM} = \frac{1}{g_4^2} \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4

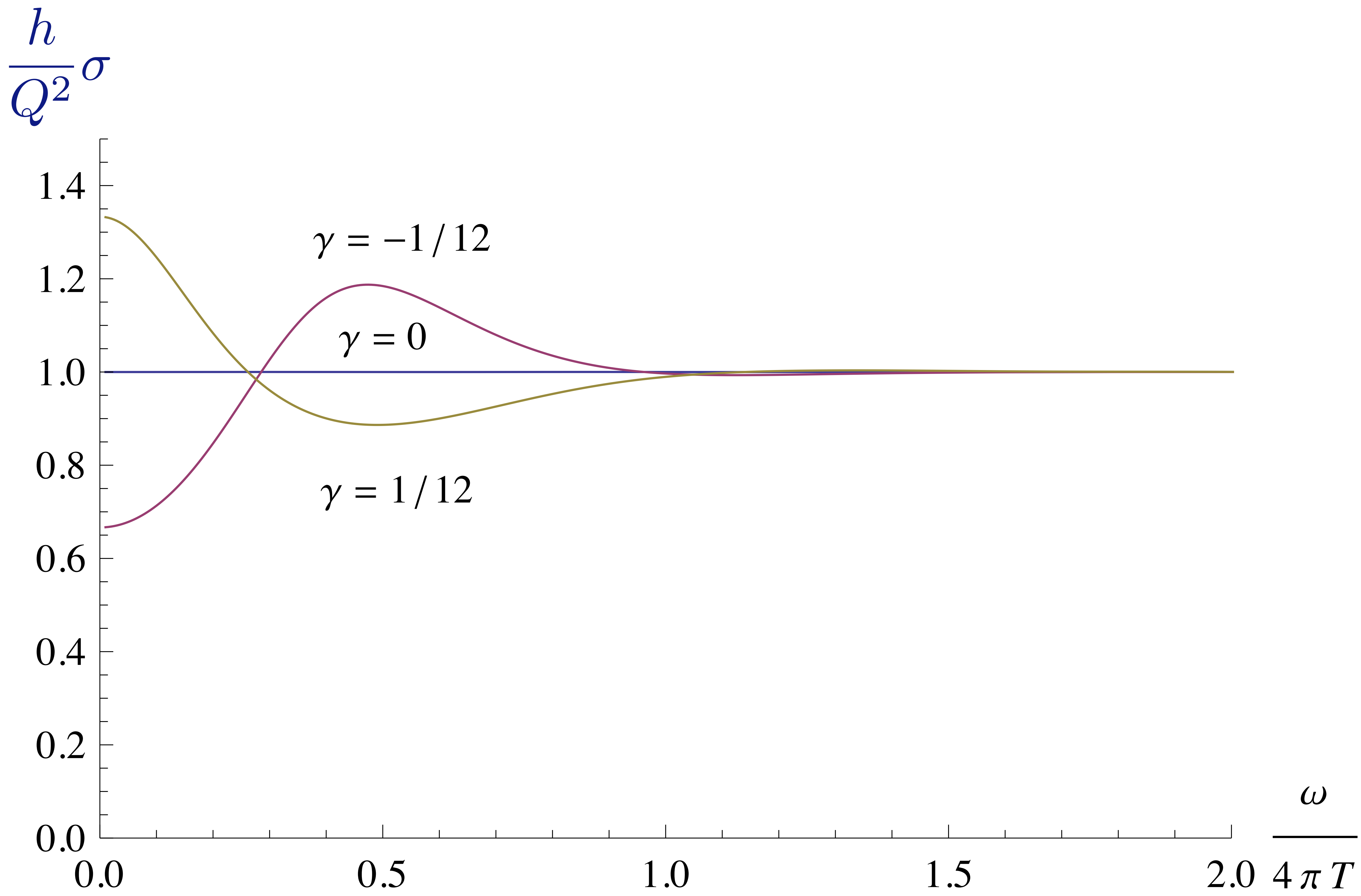
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

$$\mathcal{S} = \frac{1}{g_4^2} \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{ab} F^{ab} + \gamma \, L^2 C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"



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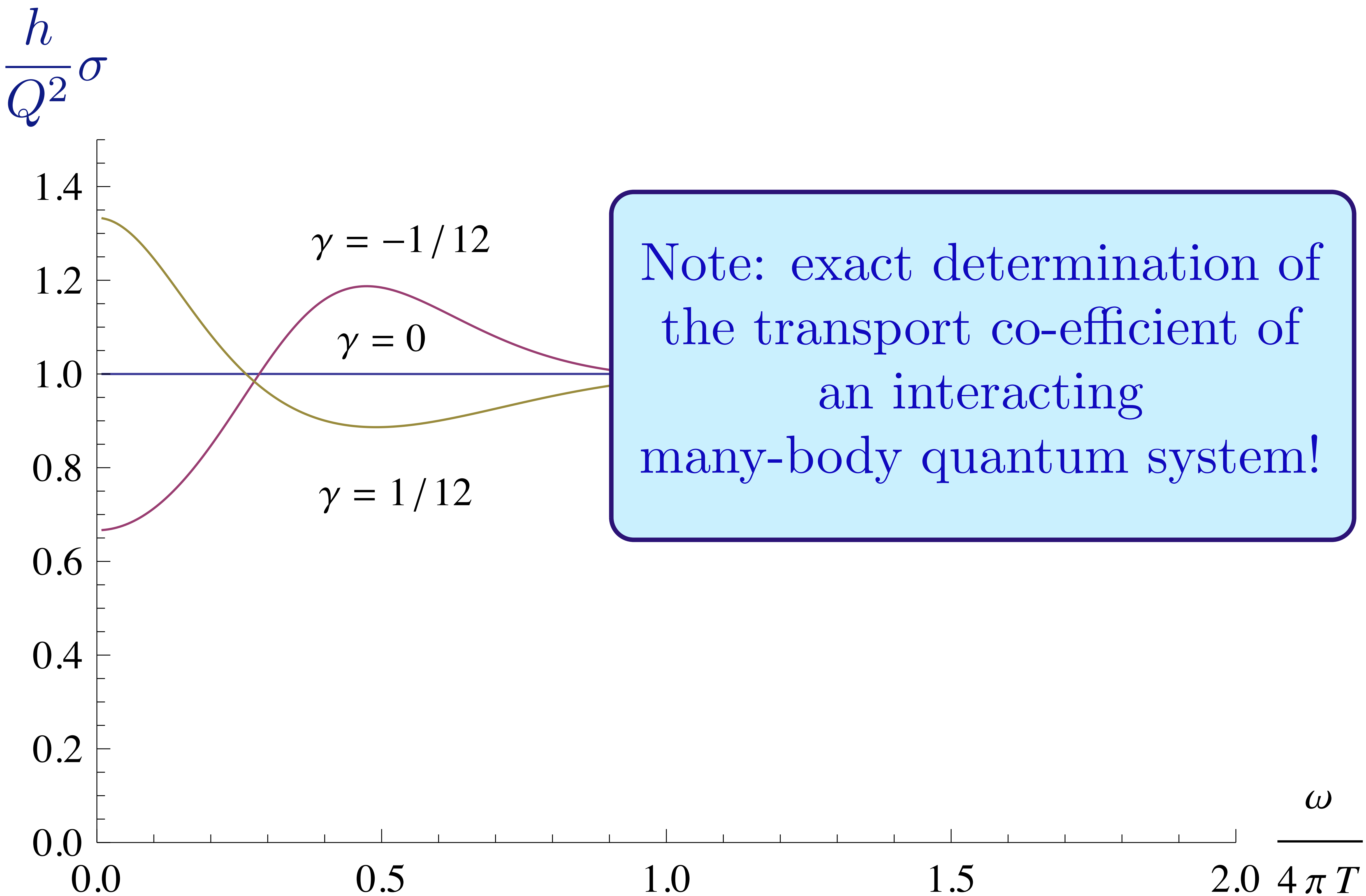

Note: exact determination of the transport co-efficient of an interacting many-body quantum system!

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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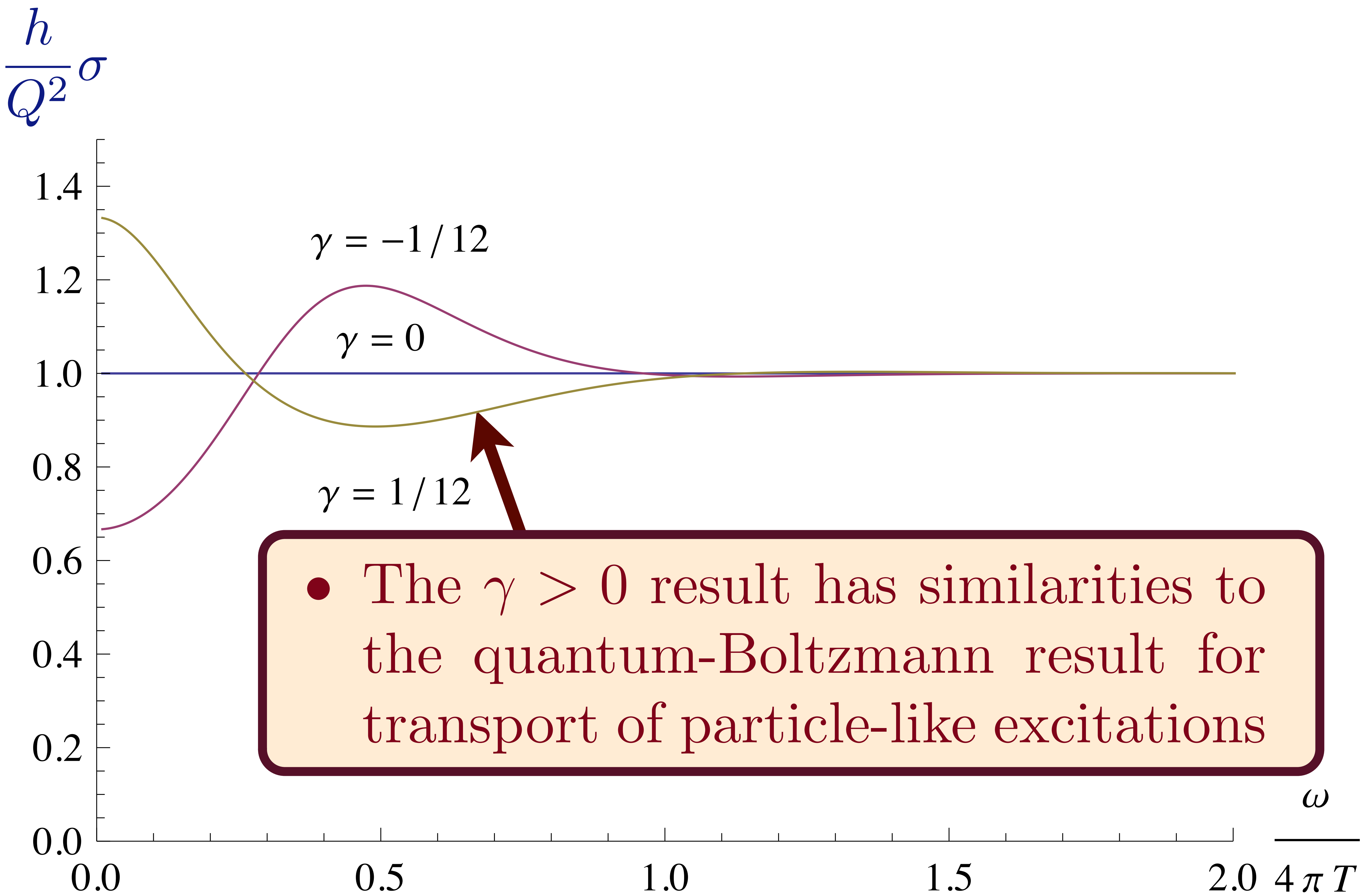

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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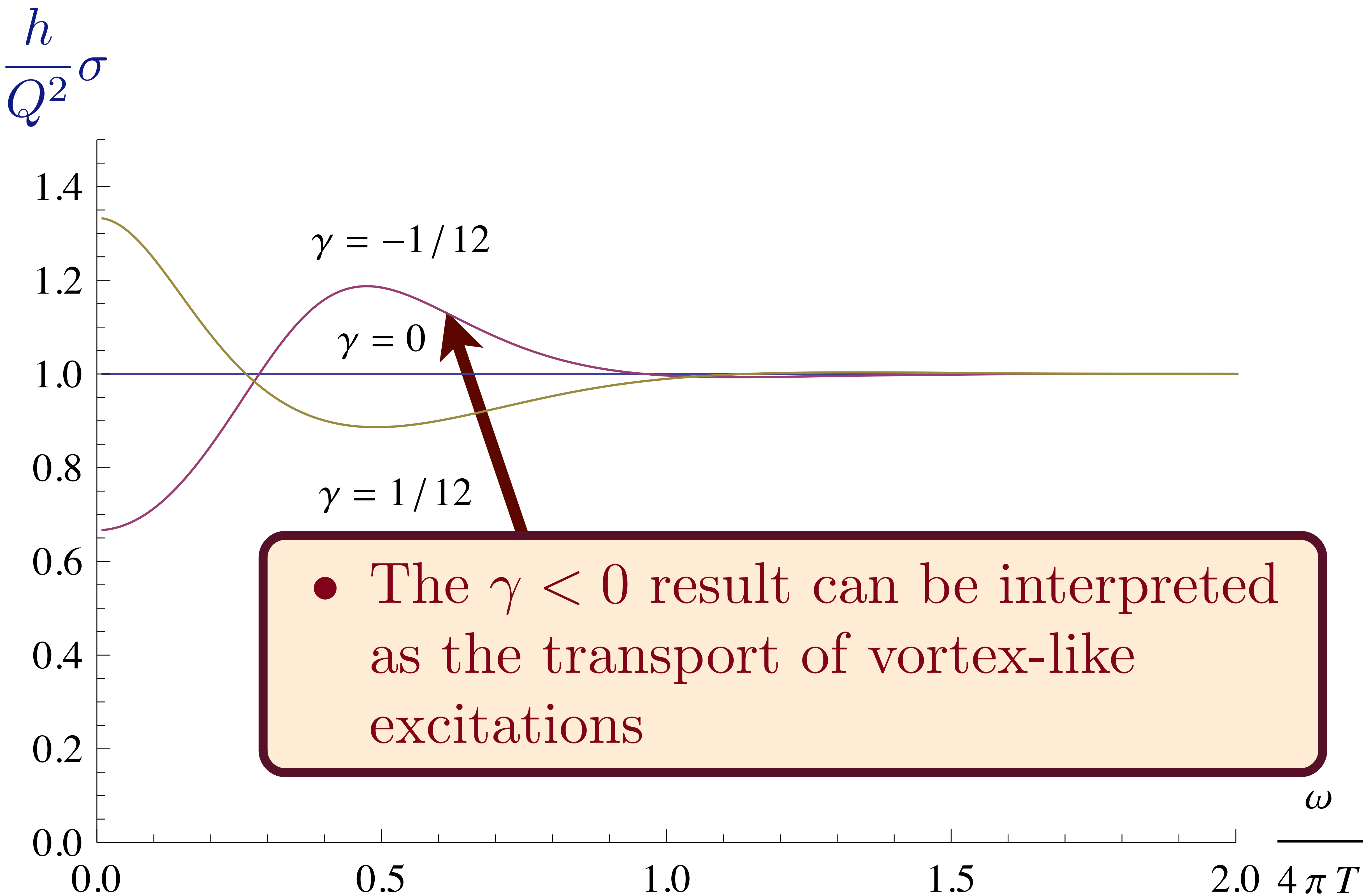

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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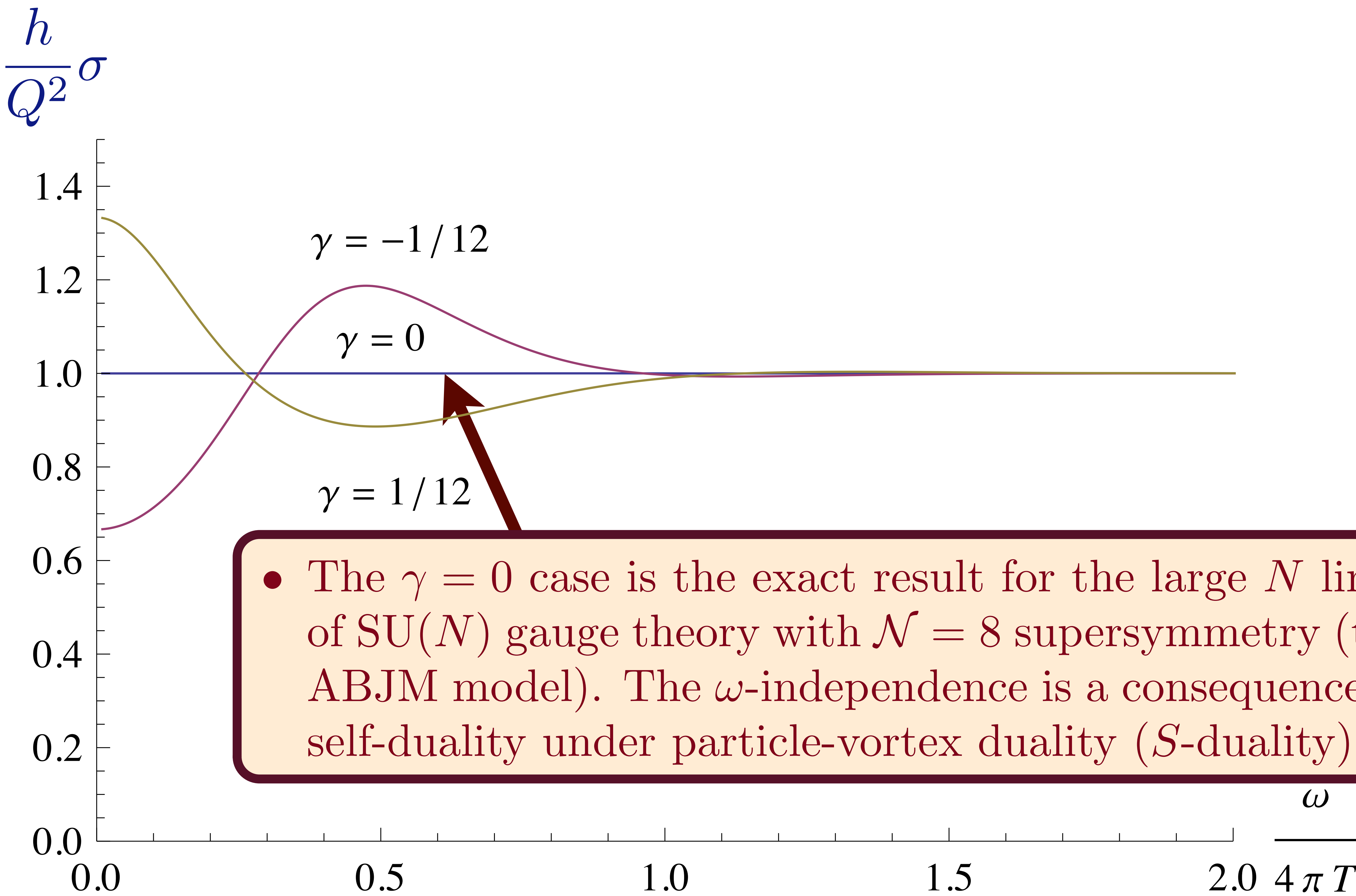
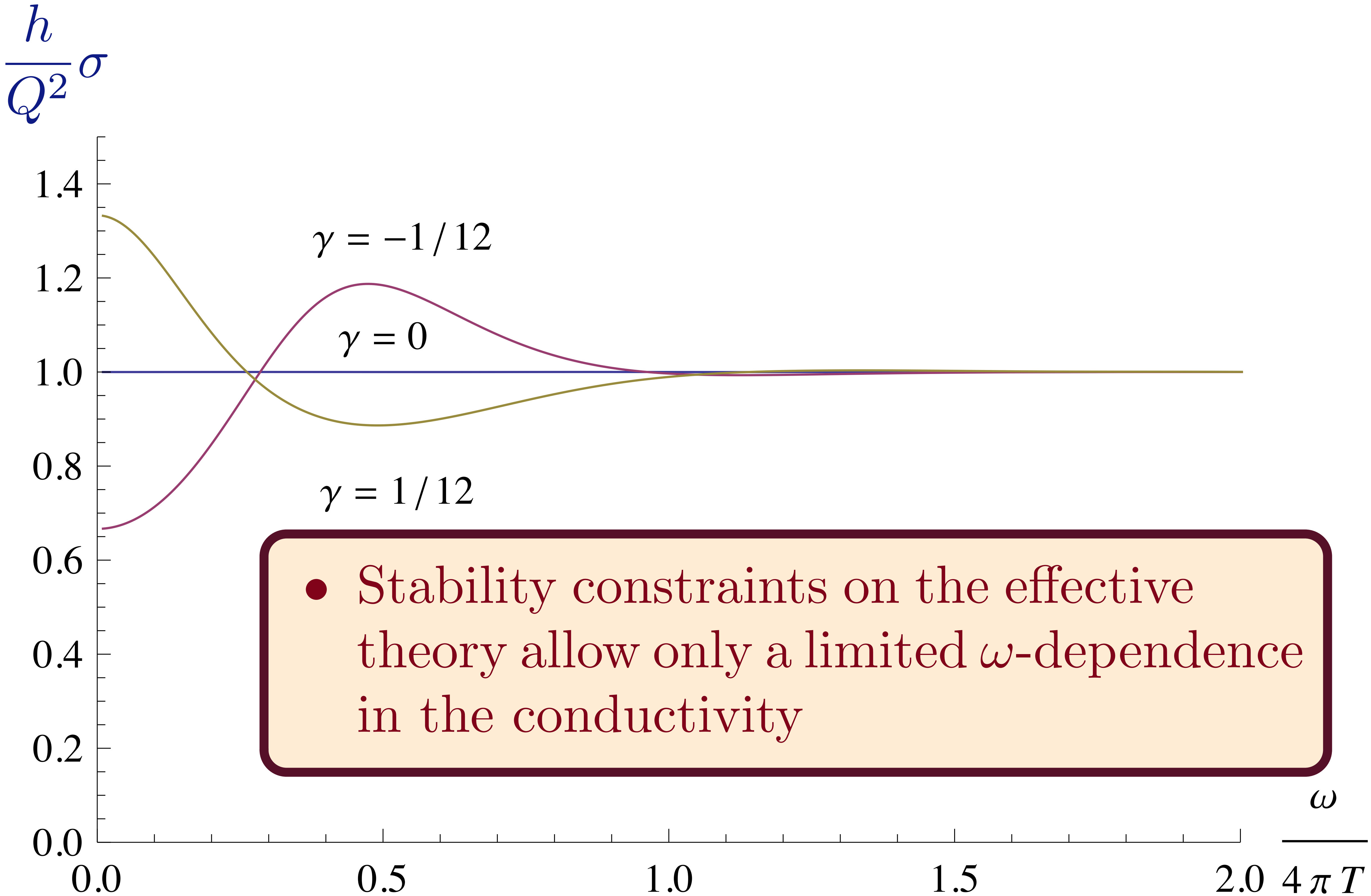


- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"

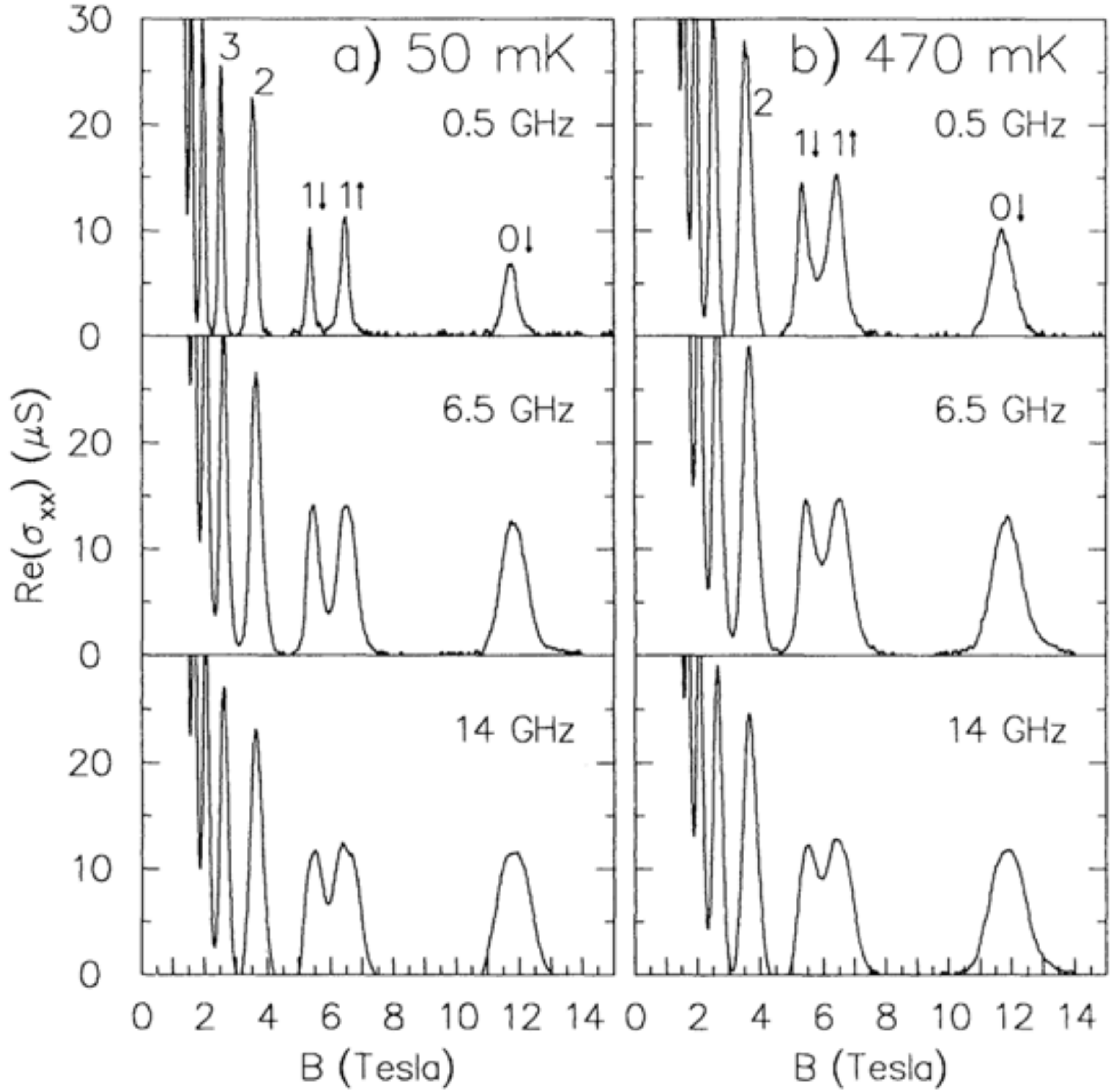


- Stability constraints on the effective theory allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value



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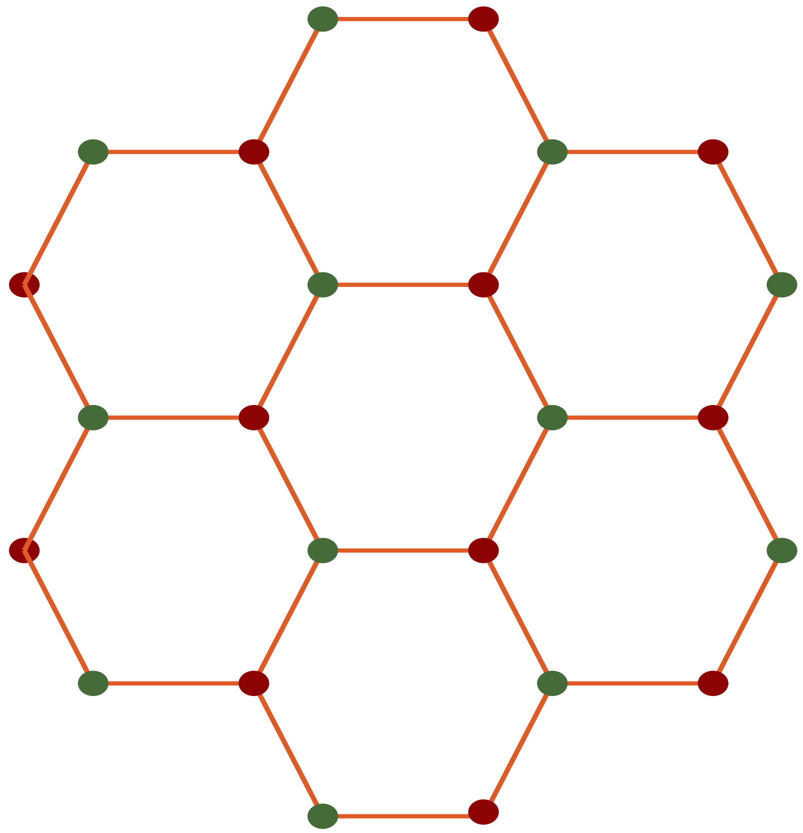
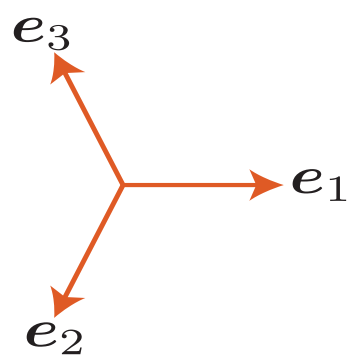
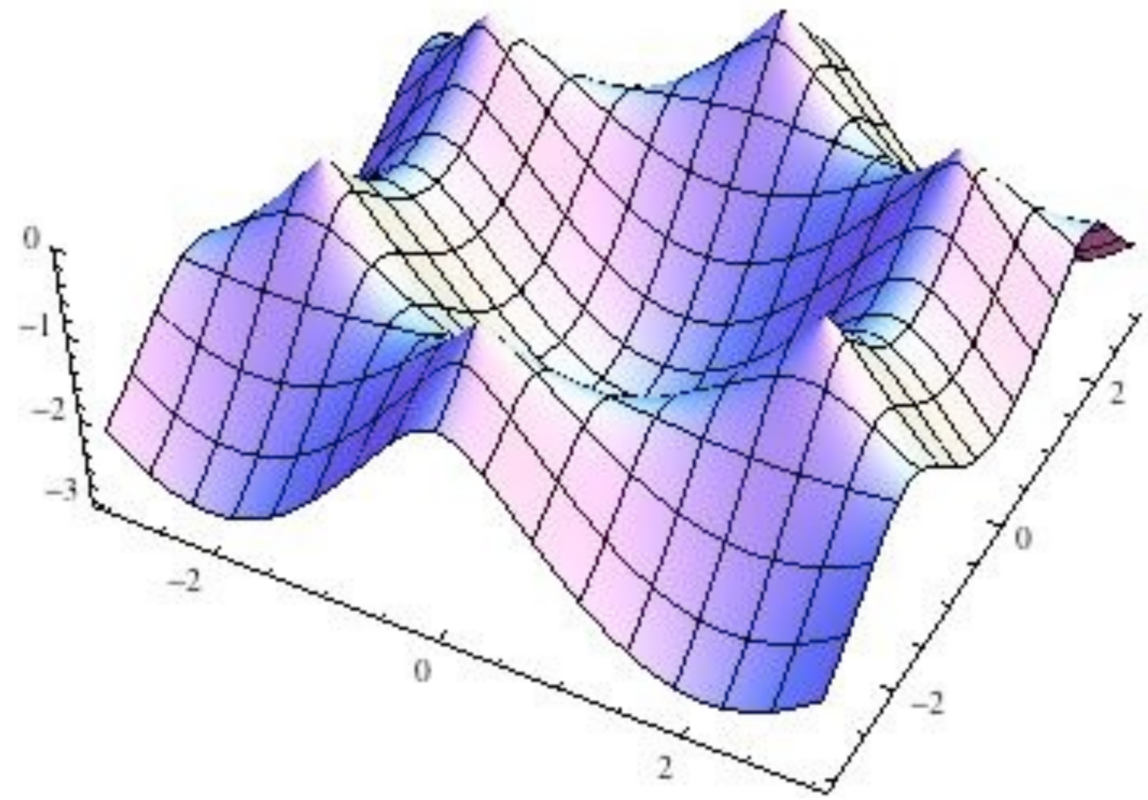
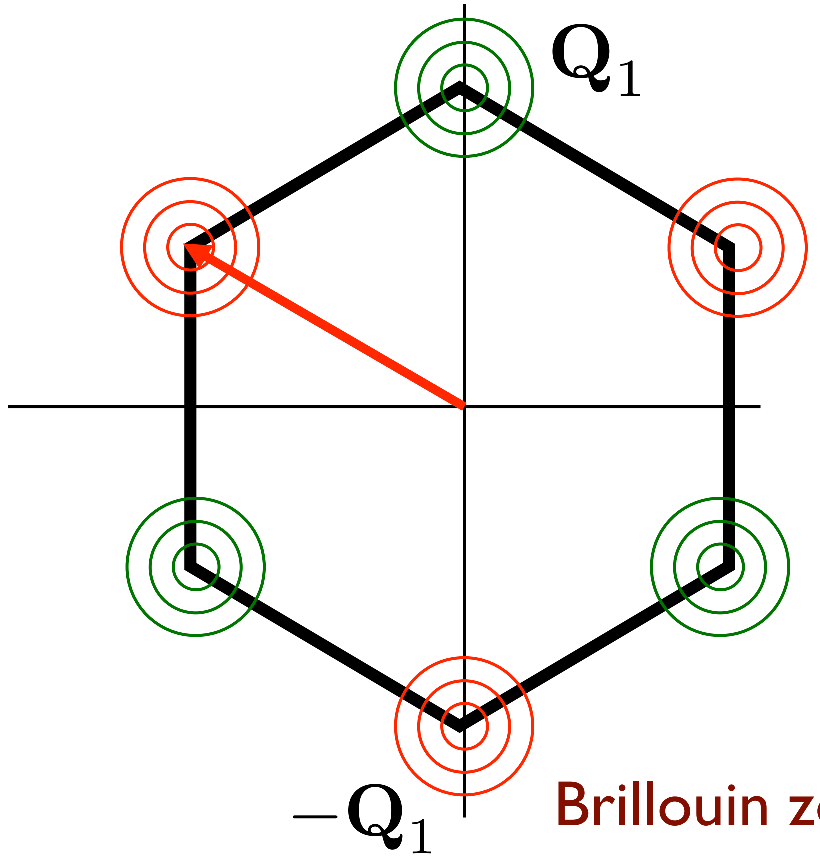
Graphene





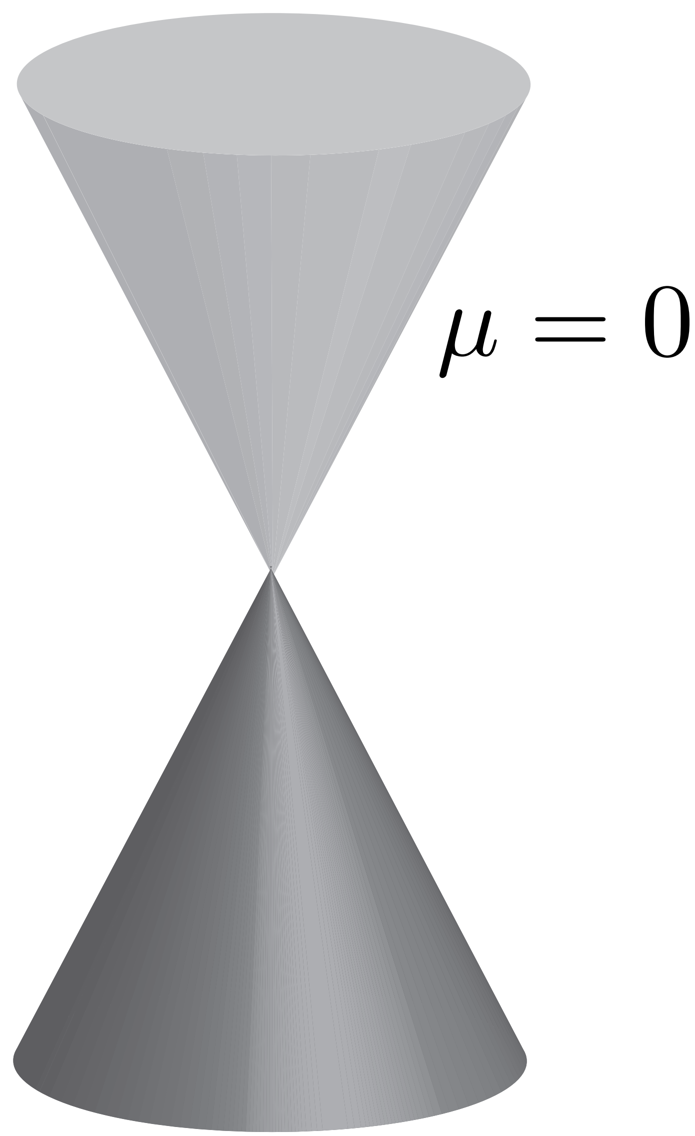




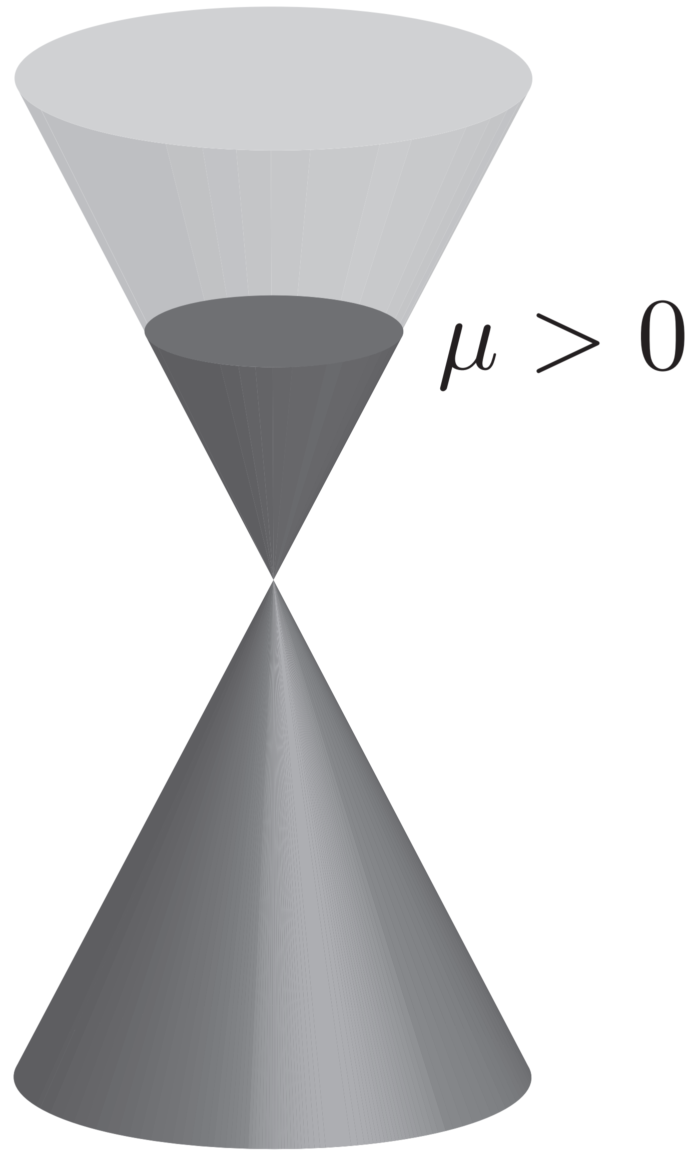
Brillouin zone

Semi-metal with
massless Dirac fermions

Turn on a chemical potential on a CFT

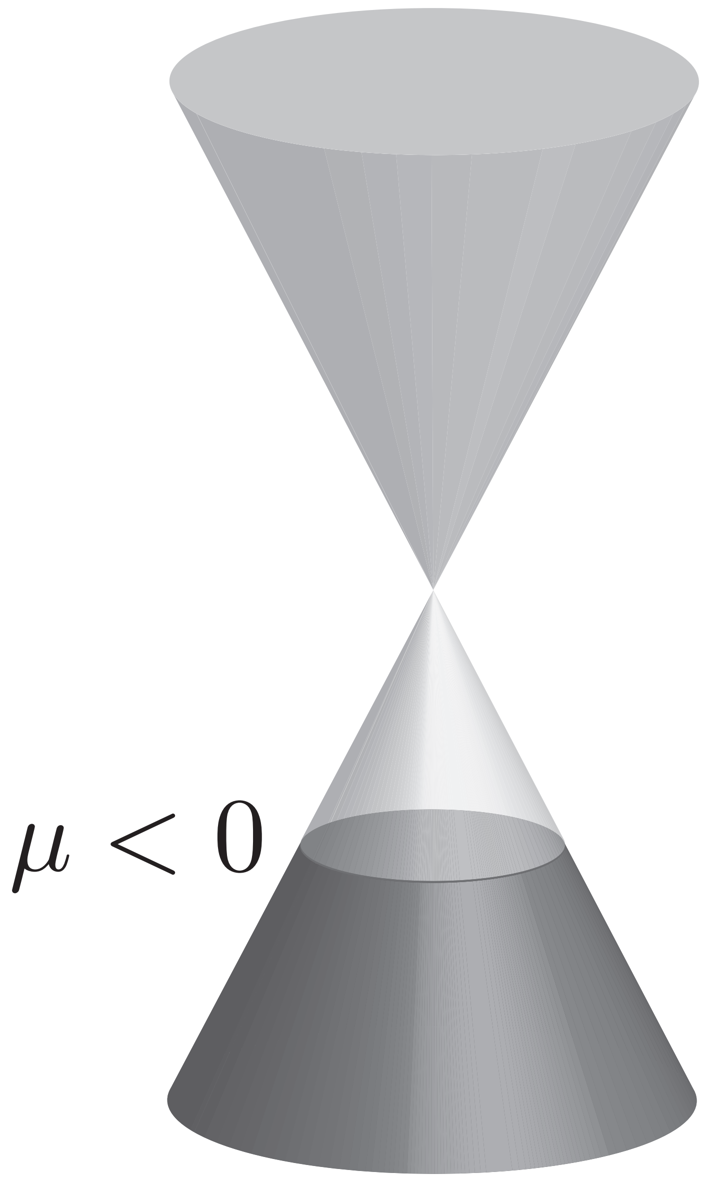


Turn on a chemical potential on a CFT



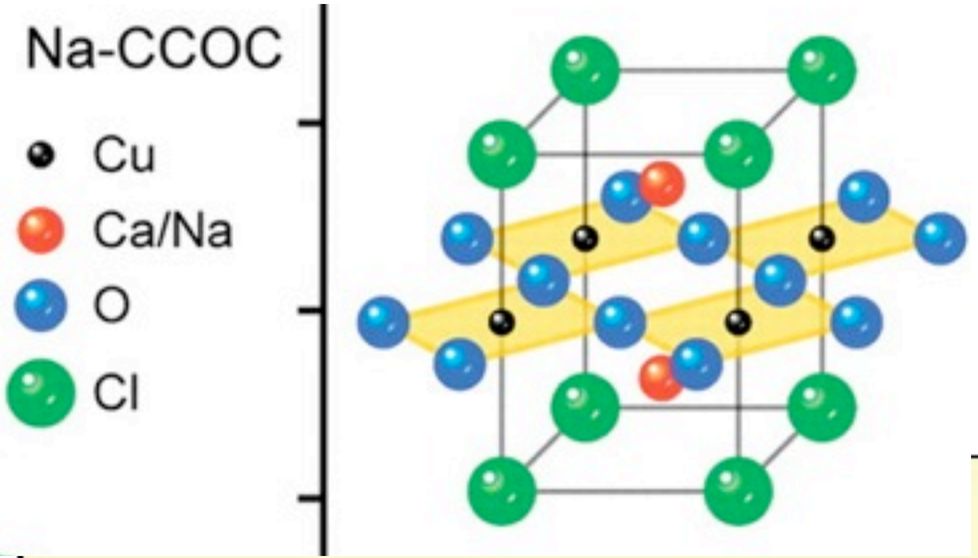
Electron
Fermi surface

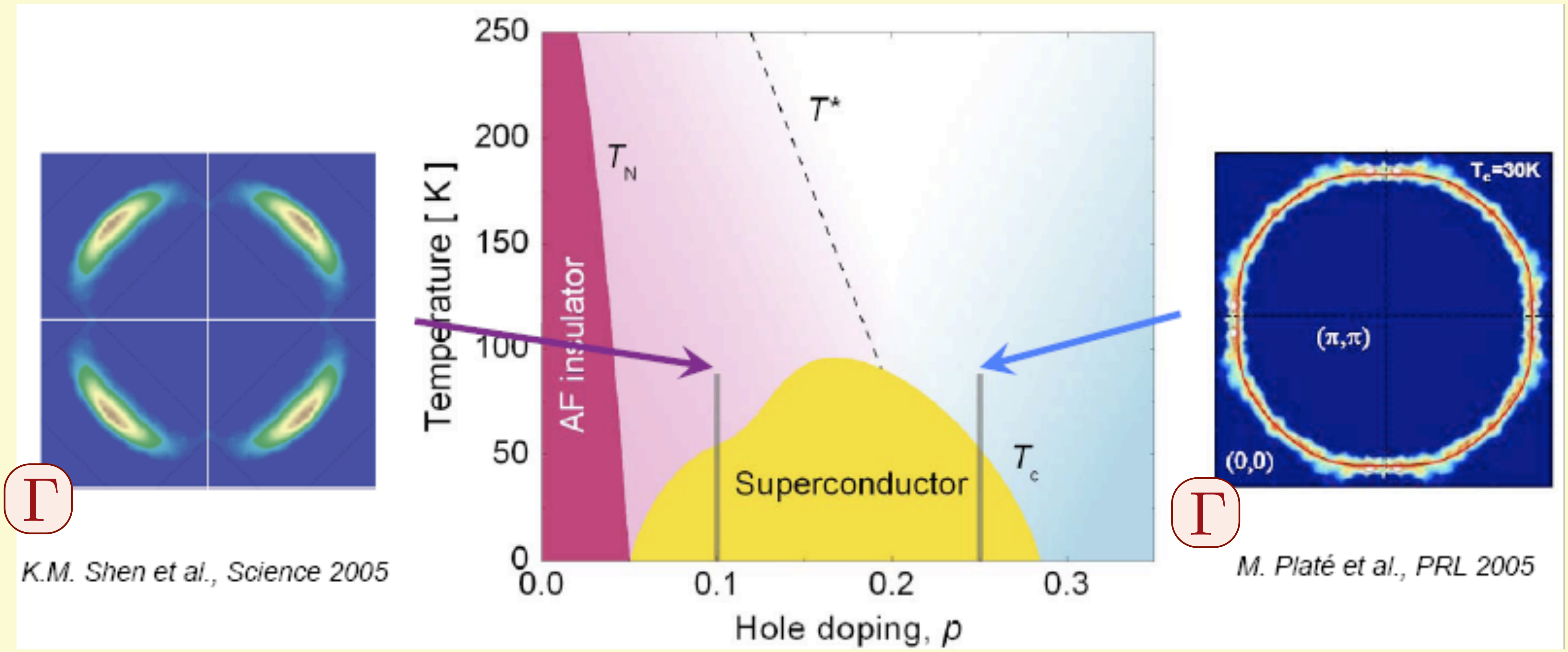
Turn on a chemical potential on a CFT



Hole
Fermi surface

The cuprate superconductors

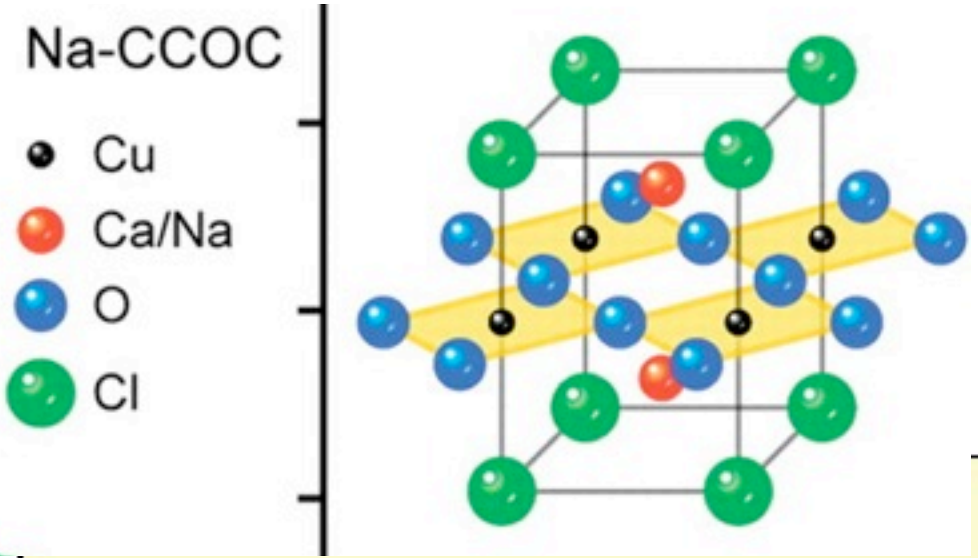


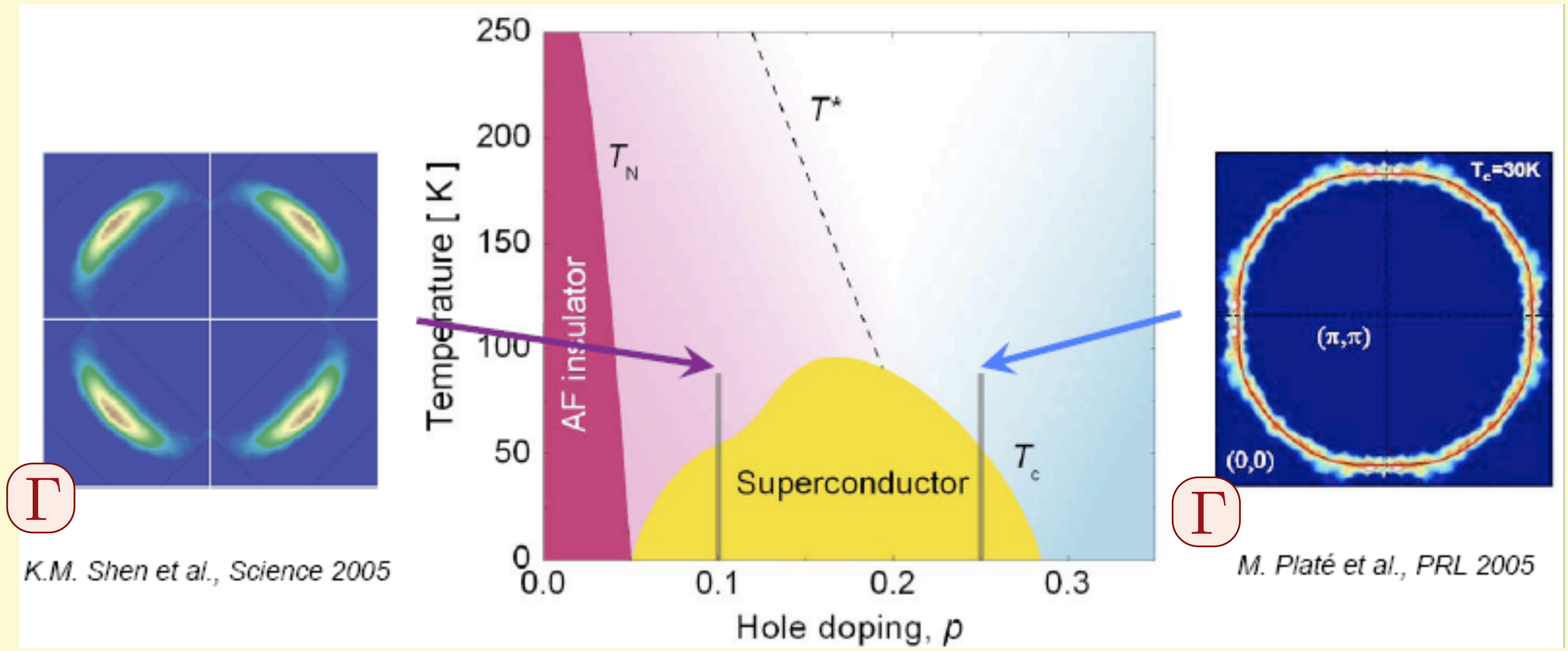


Smaller hole Fermi-pockets

Large hole Fermi surface

The cuprate superconductors





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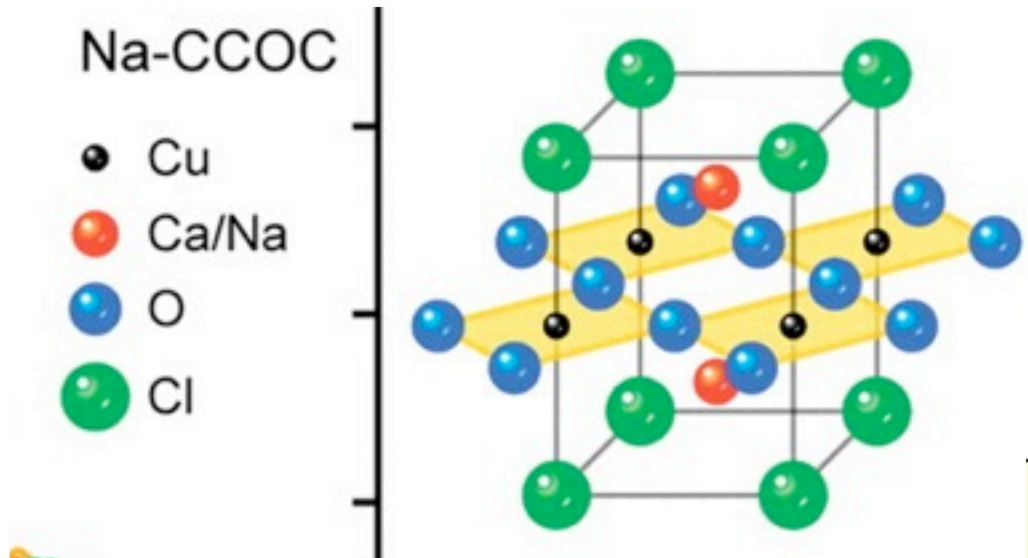
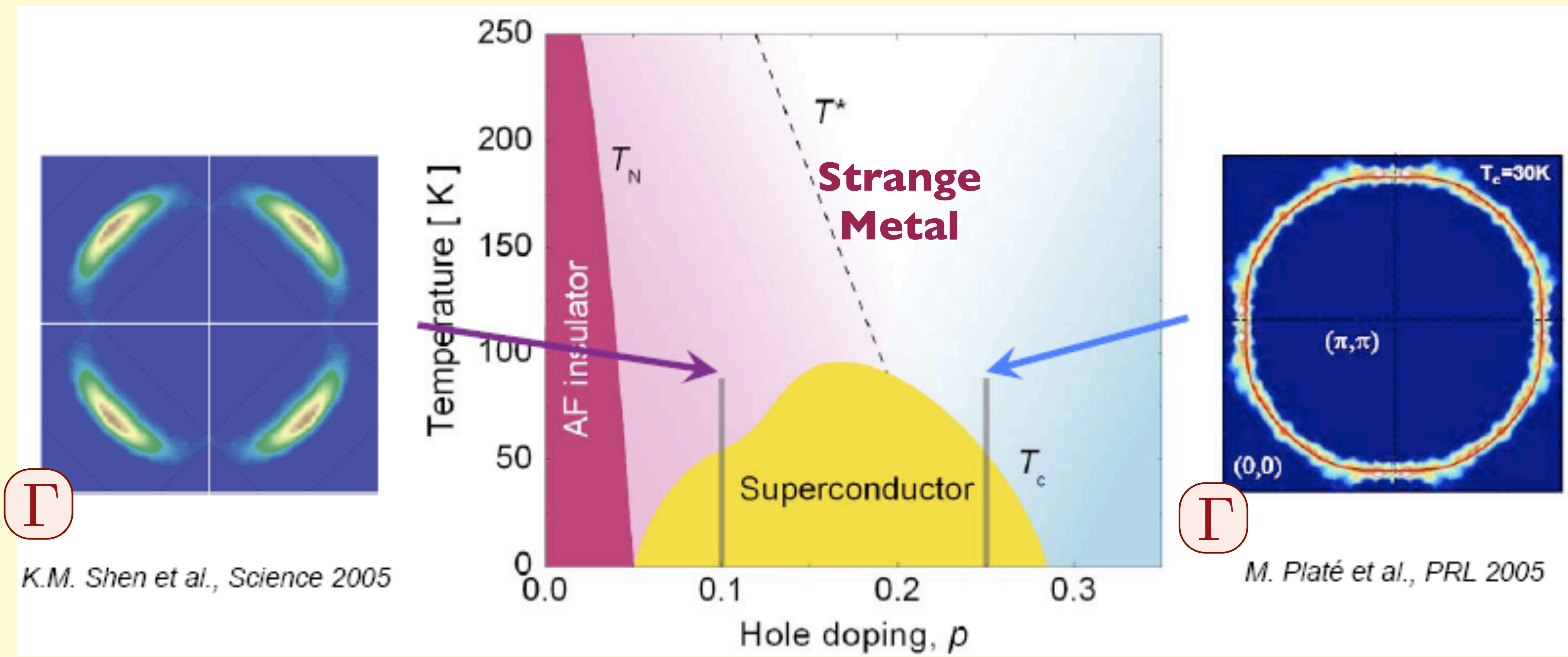




Smaller hole Fermi-pockets

Large hole Fermi surface

Electron-doped cuprate superconductors

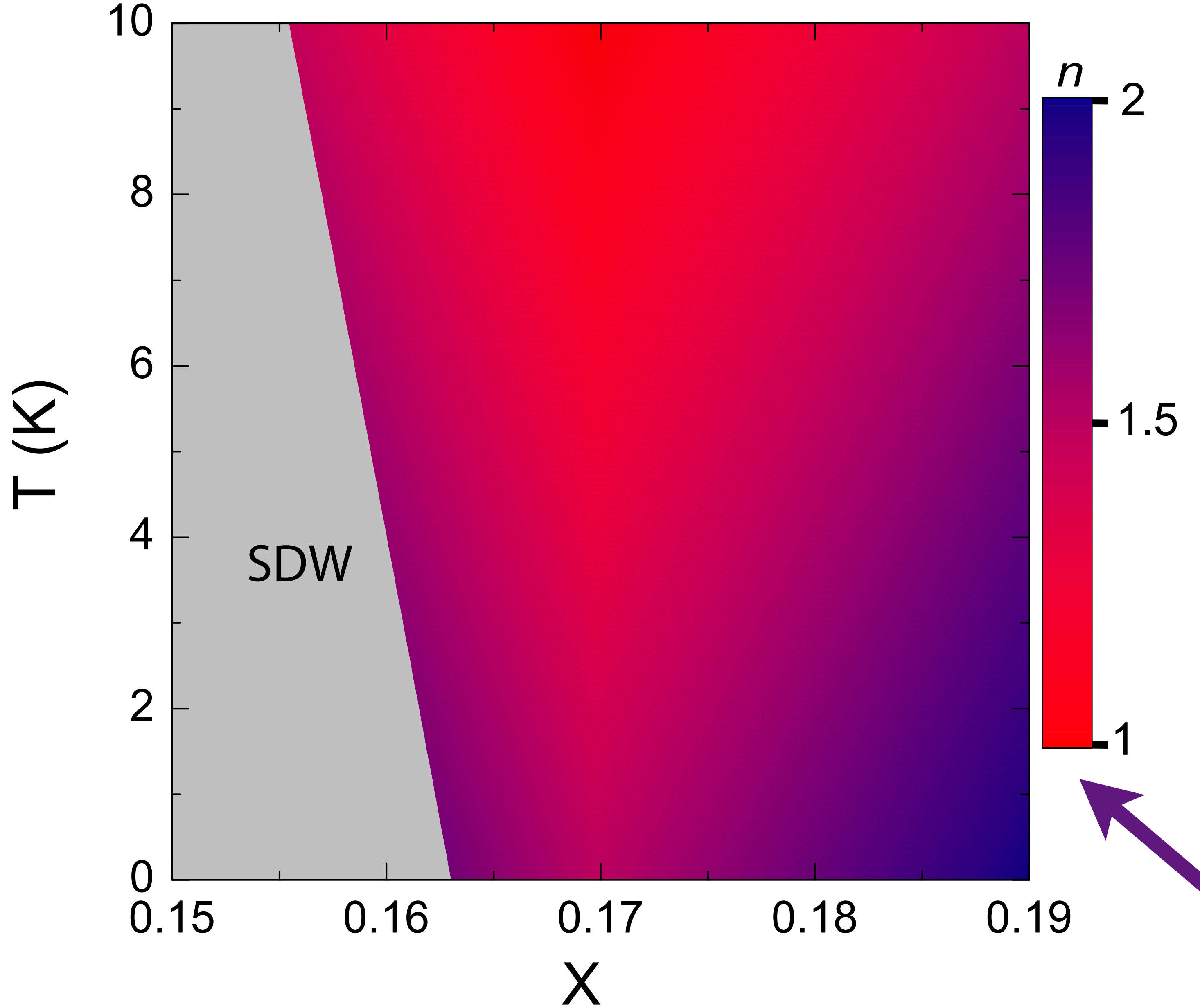
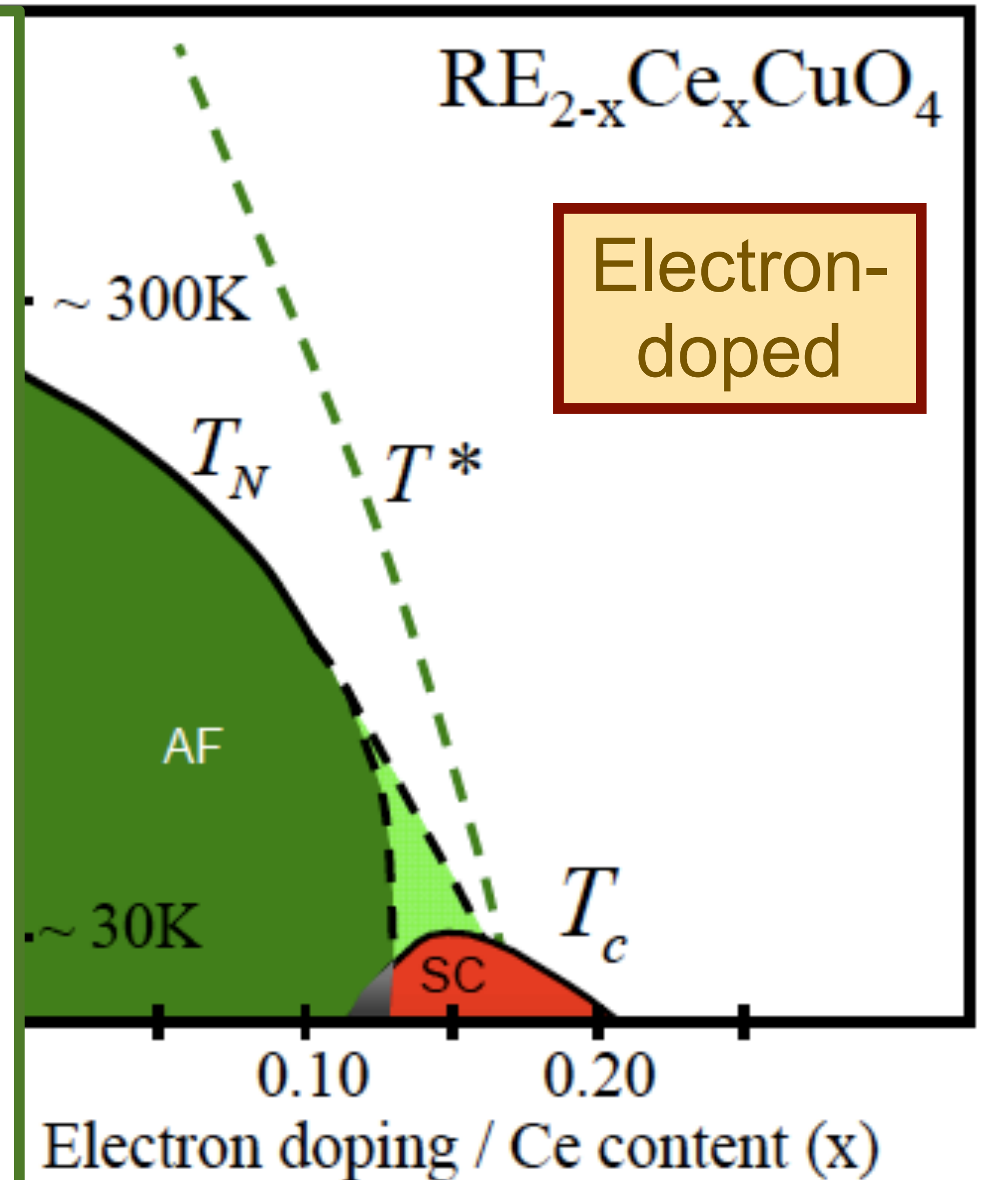
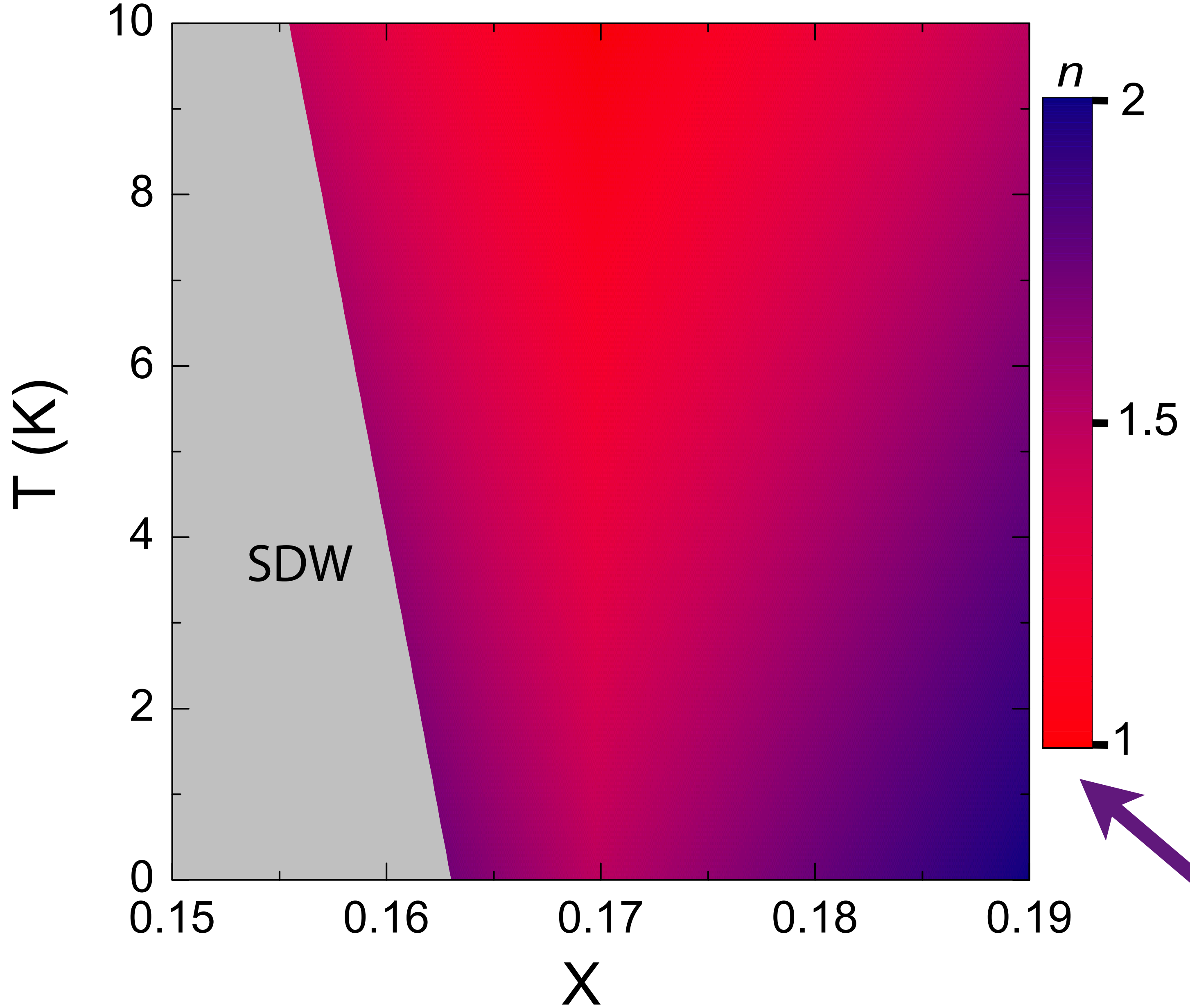


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors



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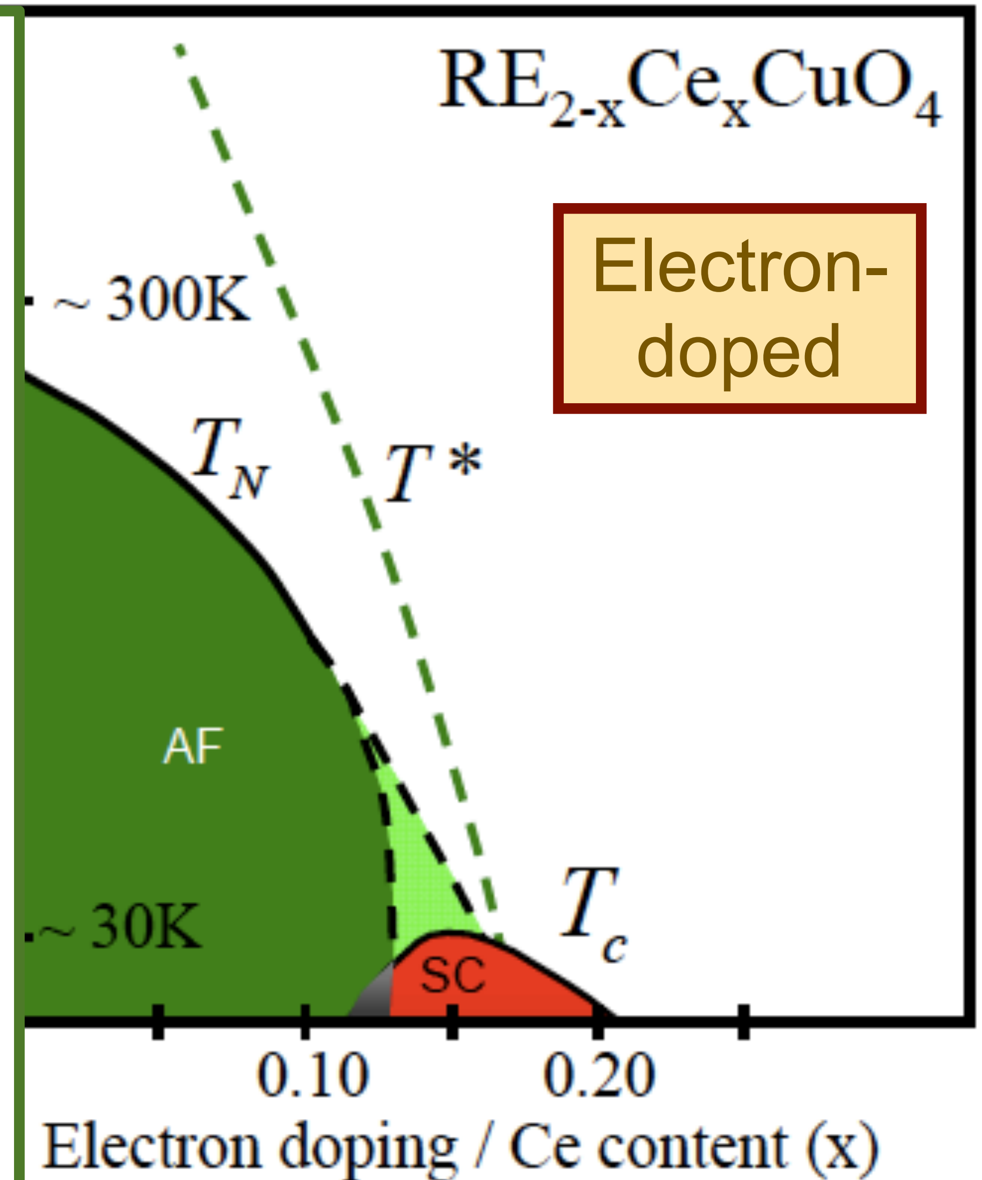
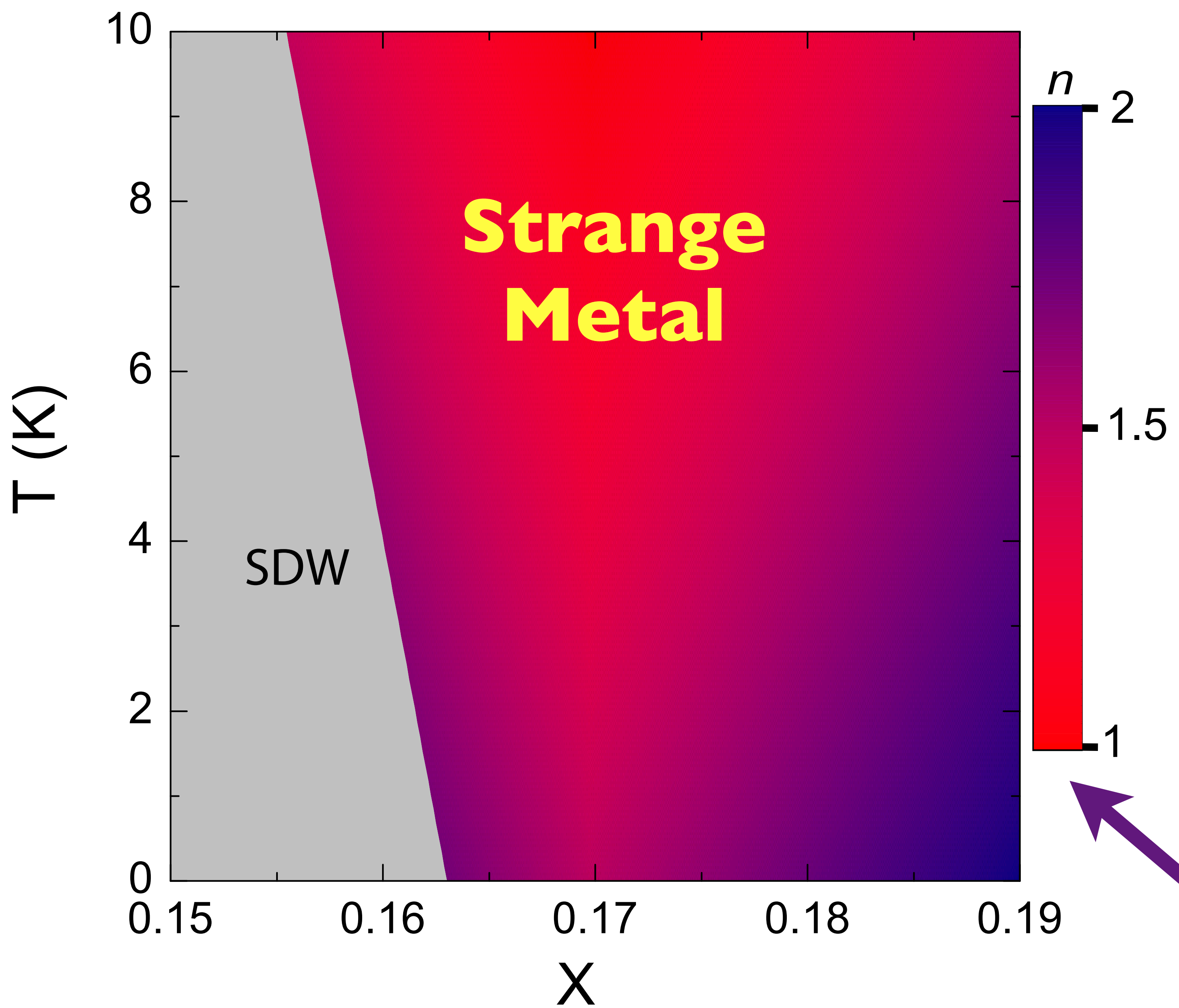


Resistivity $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors



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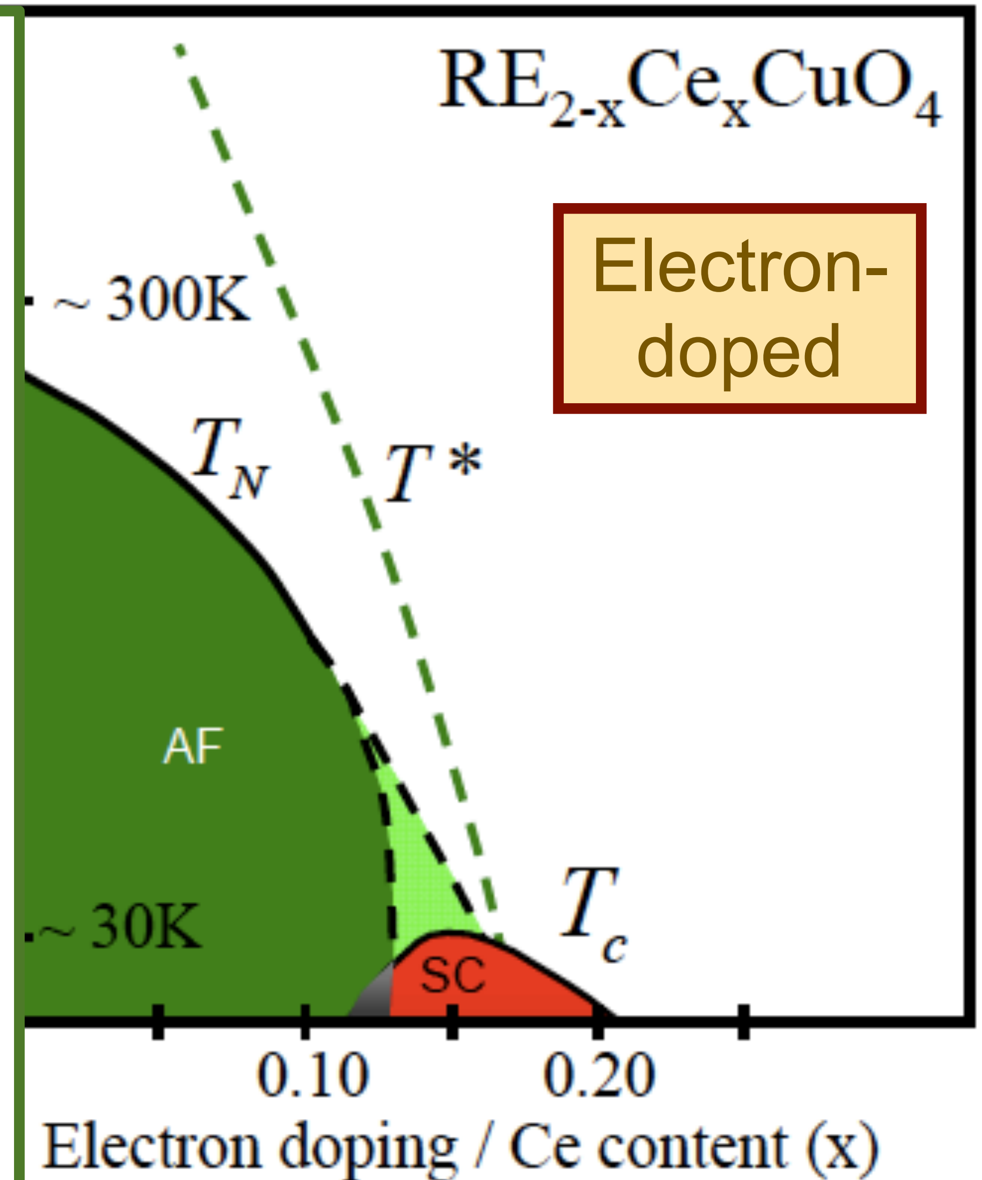
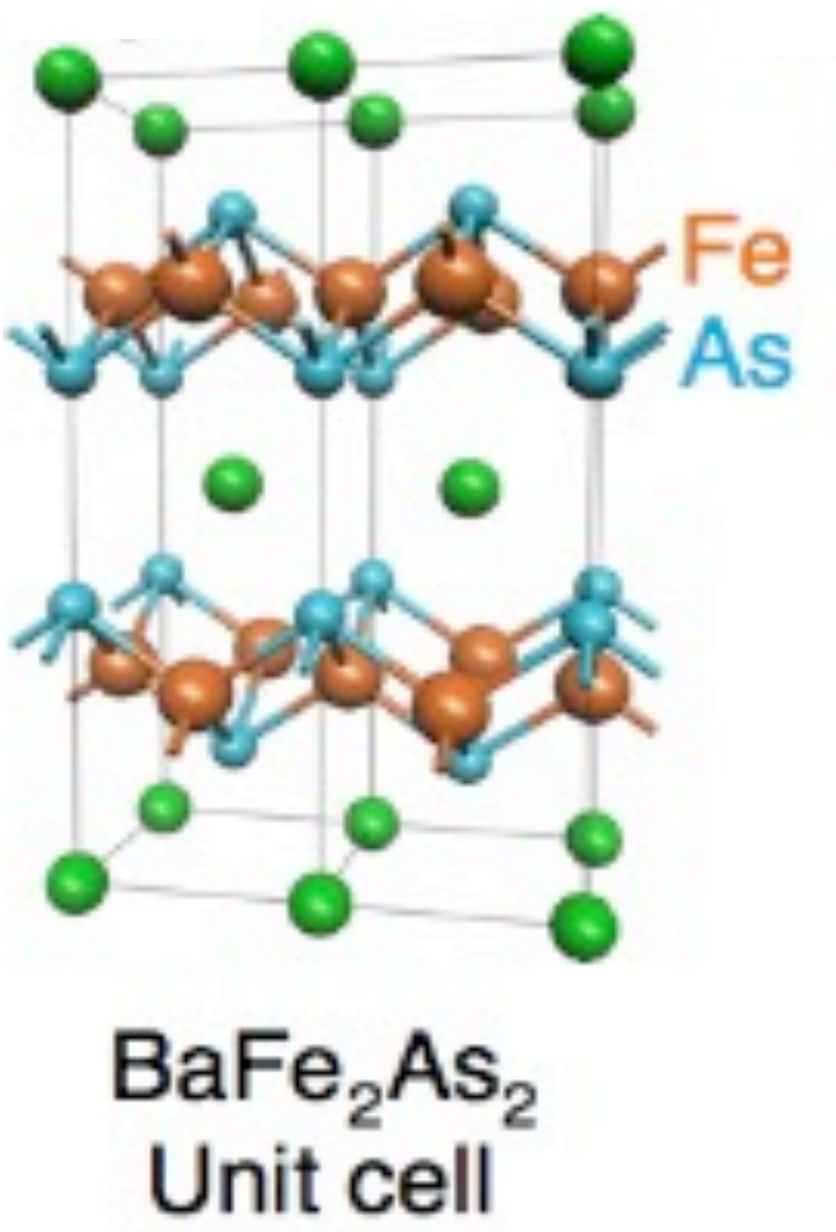
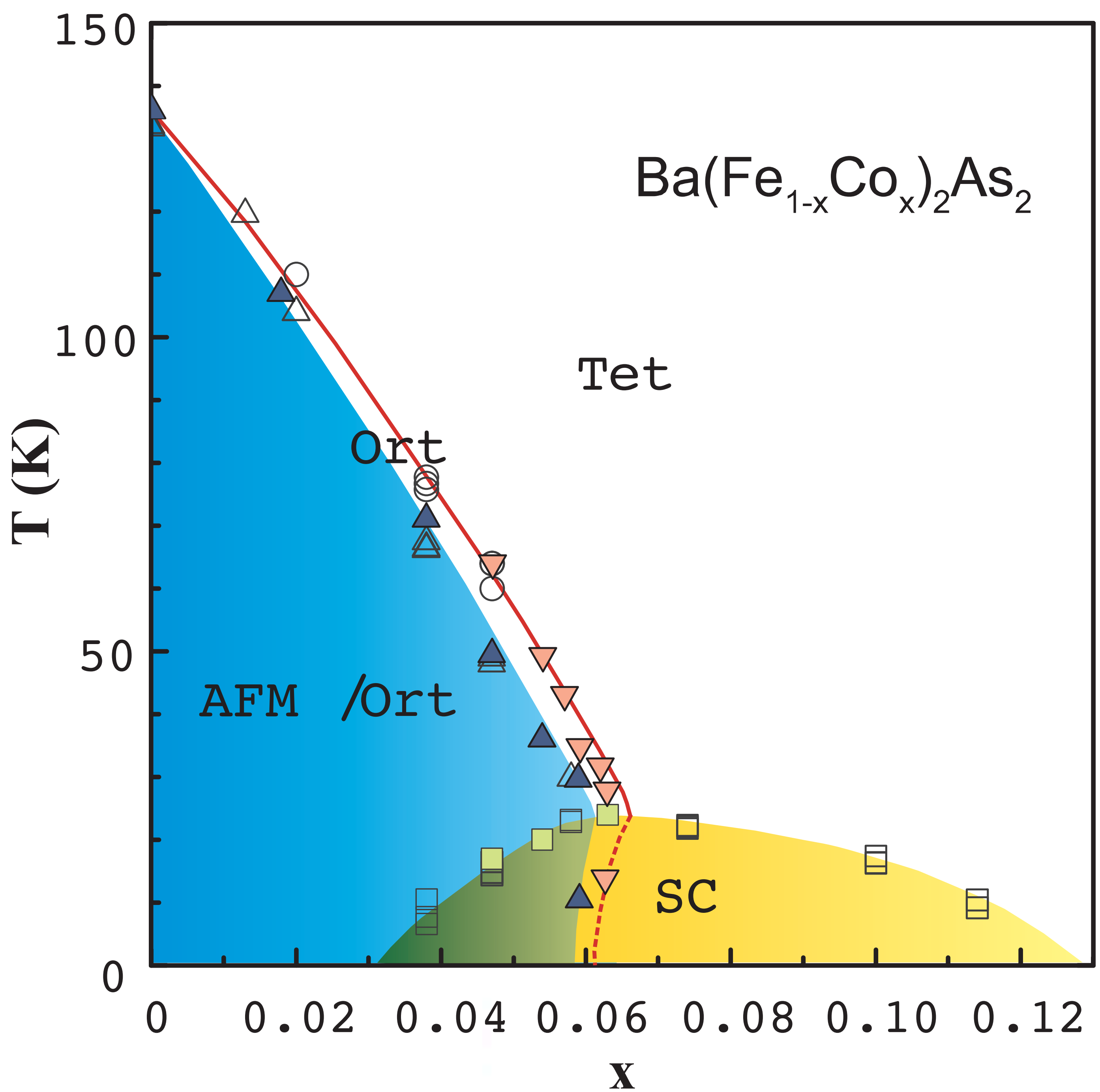


Resistivity $\sim \rho_0 + AT^n$

Iron pnictides:

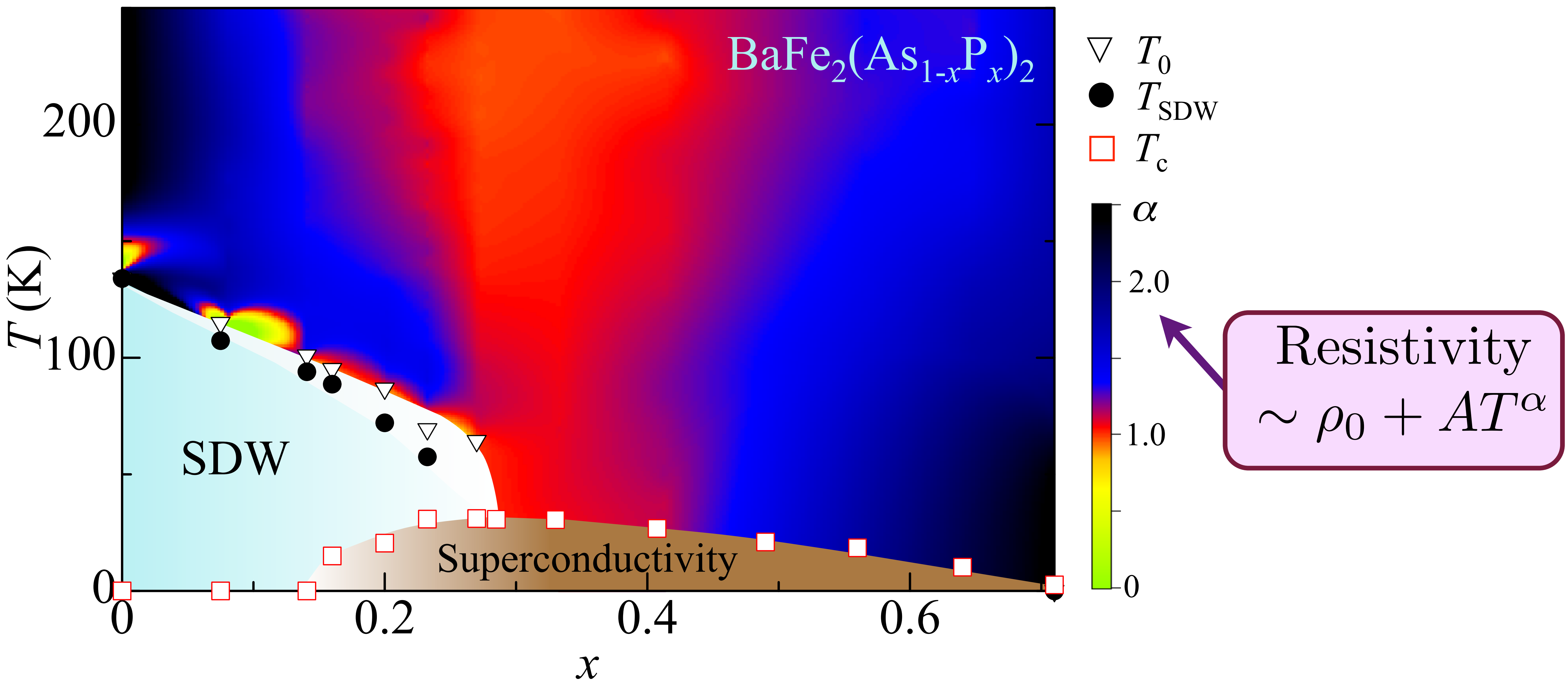
a new class of high temperature superconductors





S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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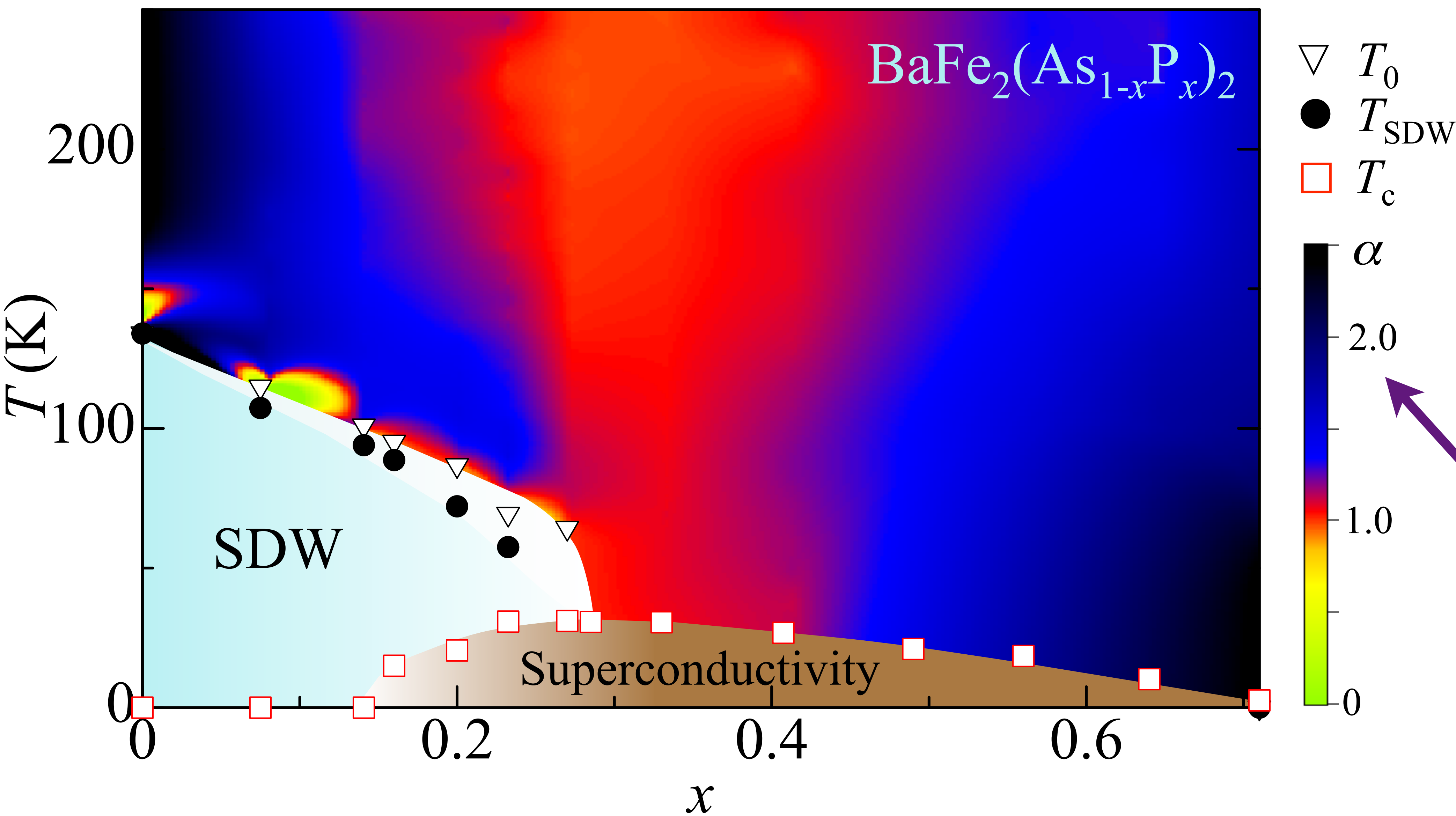

Resistivity

$\sim \rho_0 + AT^{\alpha}$

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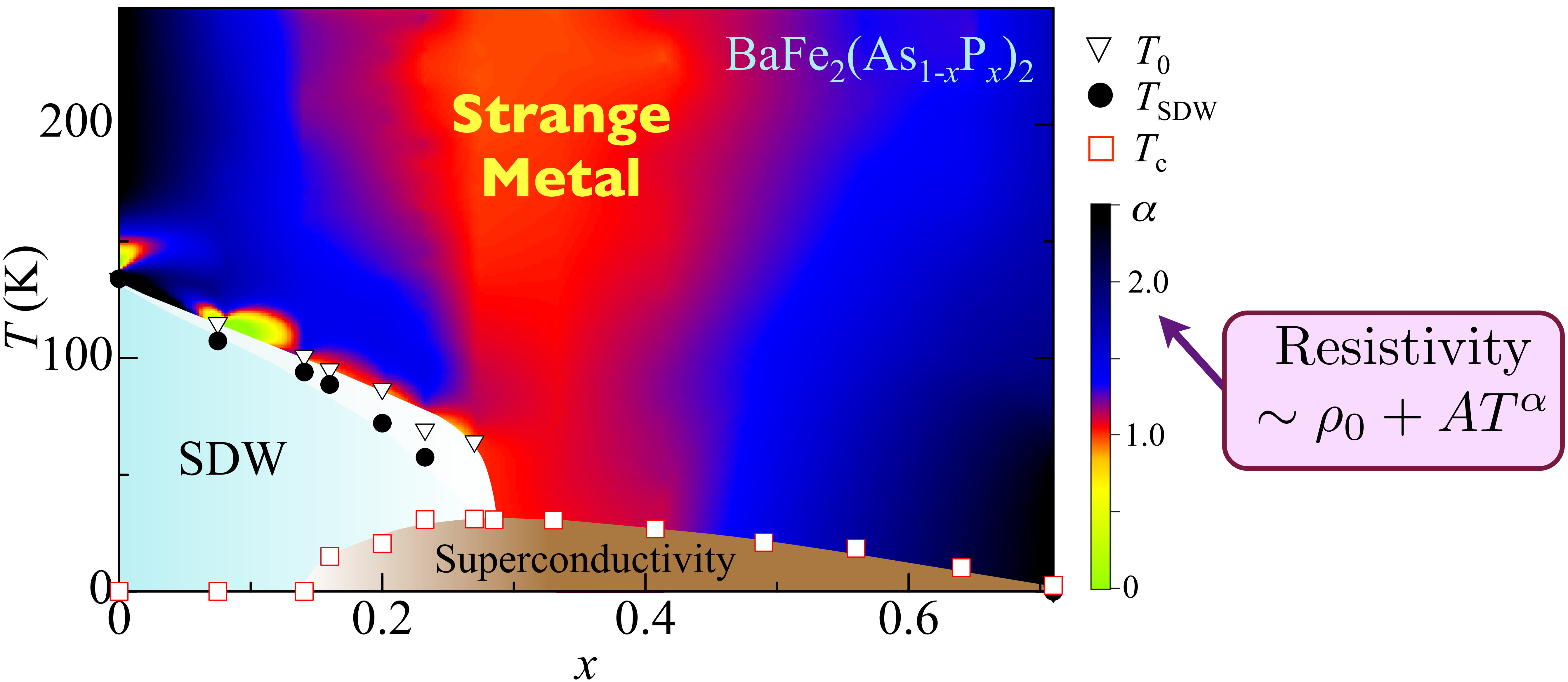

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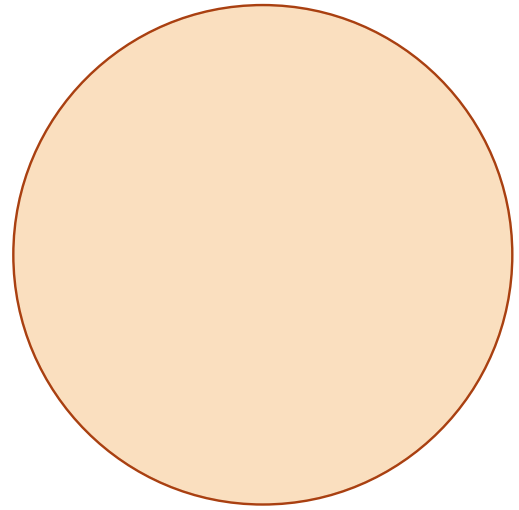
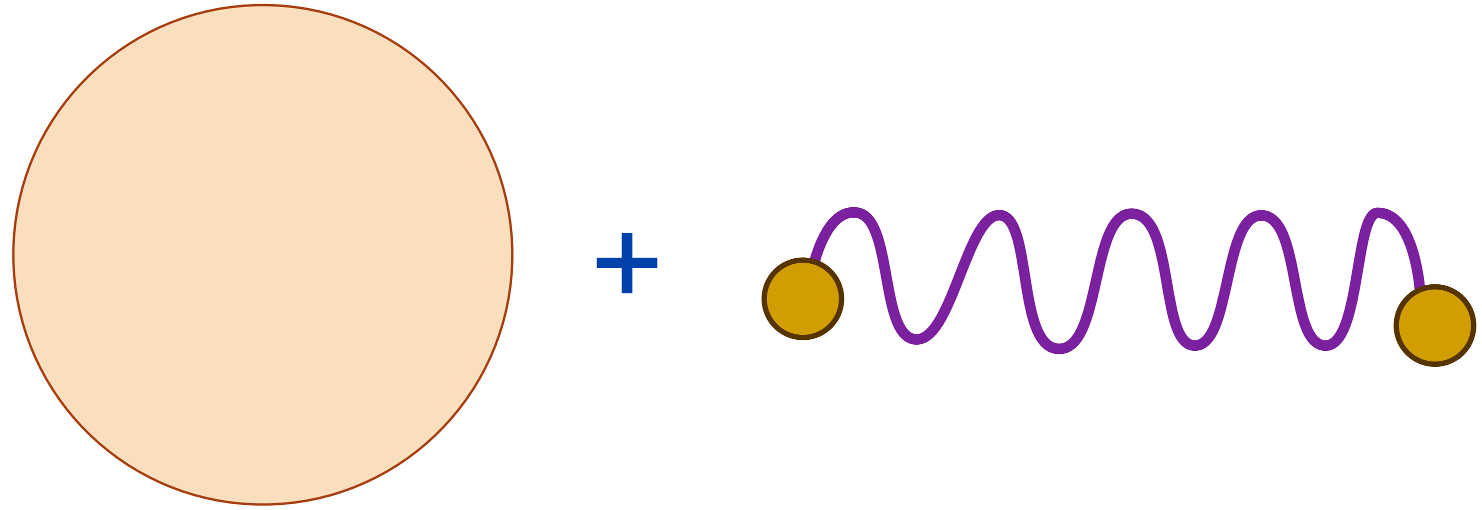
Basic ingredient in many models of strange metals:



- Begin with fermions with short-range interactions. This leads to a Fermi liquid, with sharp fermionic quasiparticles near the Fermi surface.

Basic ingredient in many models of strange metals:



- Couple fermions to a gauge field (physics turns out to be similar for abelian or non-Abelian gauge fields). This is an "emergent" gauge field, found in many analyses of Hubbard or Kondo models of correlated electrons.

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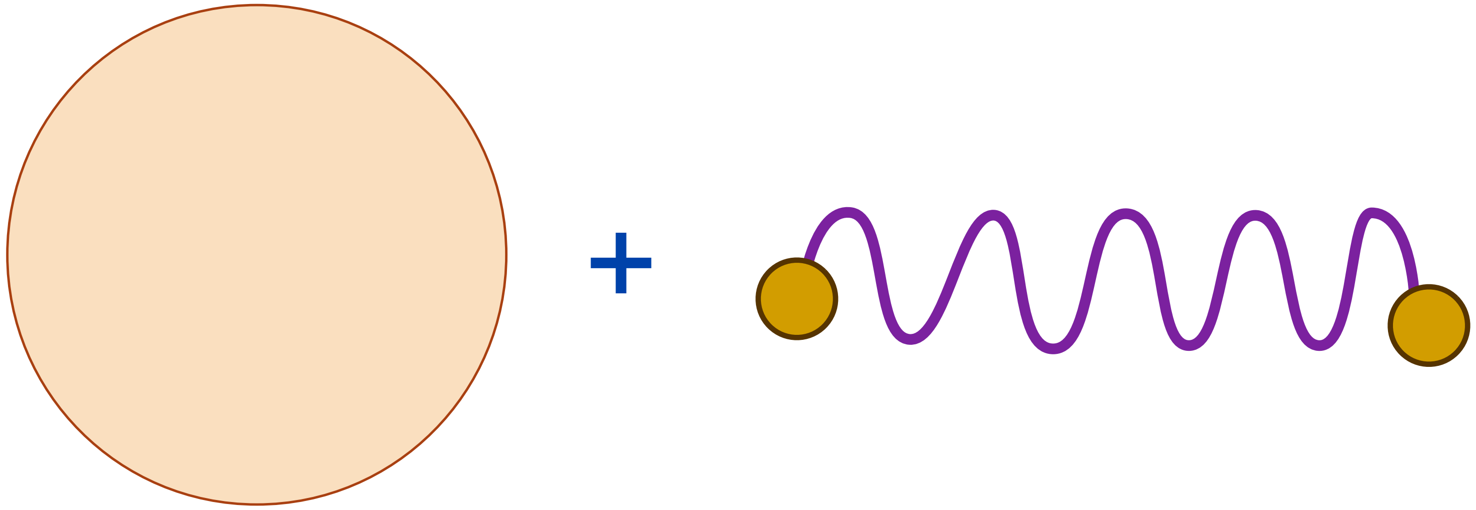

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- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions*.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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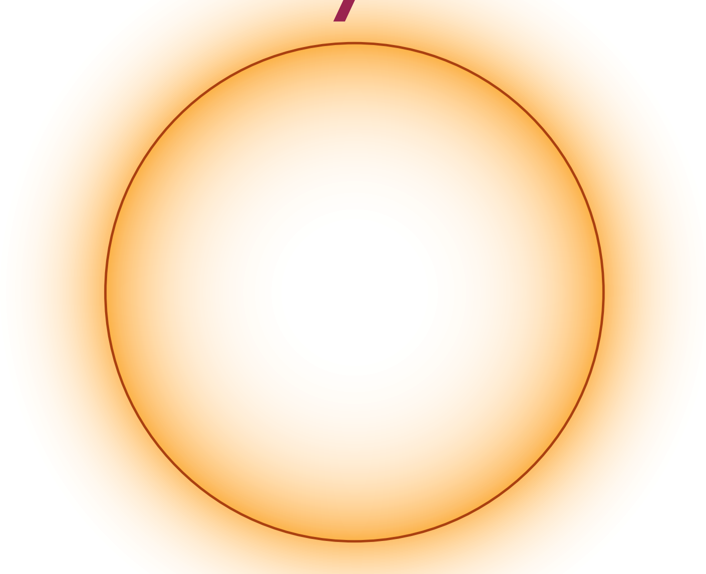

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- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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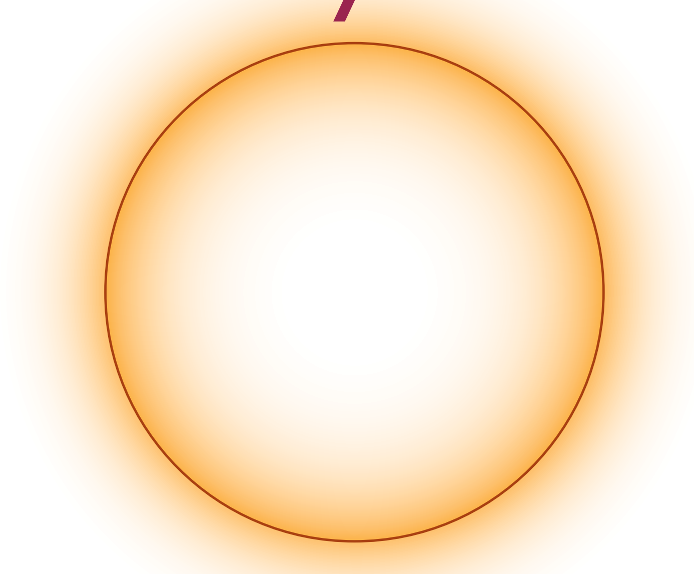

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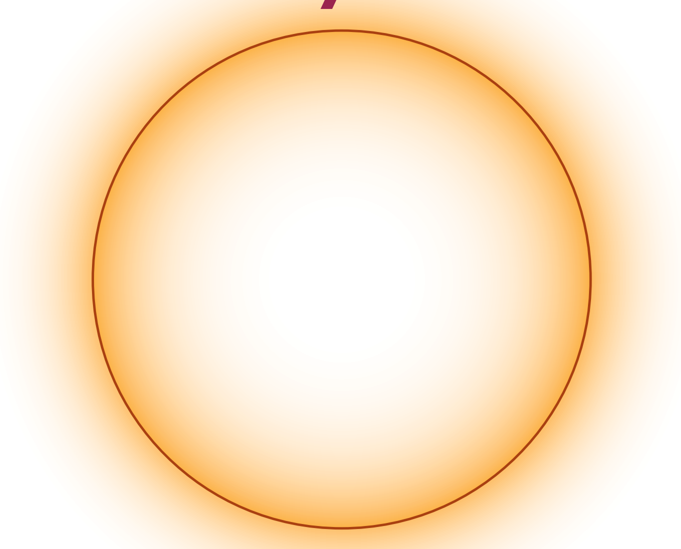

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Needed: a complete theory of this non-Fermi liquid state

- Long ... mions. But ... nd are Lanc ... e. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions*.

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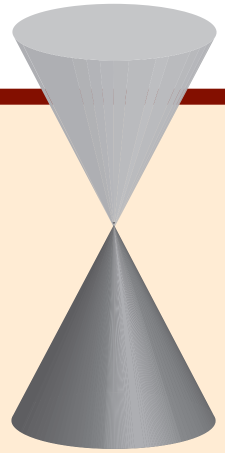
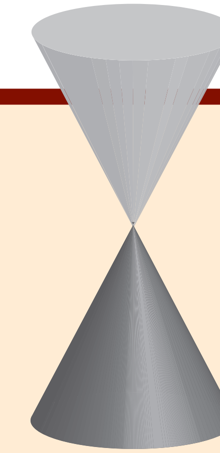
Gauge-gravity duality



- Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry

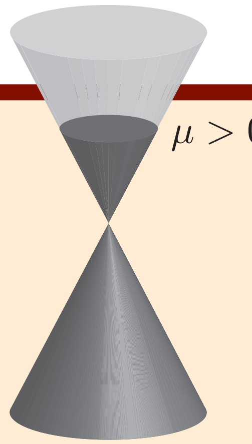
Gauge-gravity duality



- Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry

- The CFT is dual to a gravity theory on $AdS_4 \times S^7$

Gauge-gravity duality



- Begin with a CFT e.g. the ABJM theory with a SU(4) global symmetry
- Add some SU(4) charge by turning on a chemical potential (this breaks the SU(4) symmetry): we obtain Fermi surfaces coupled to non-Abelian gauge fields, similar to many correlated electron models

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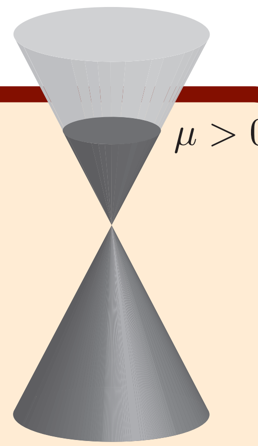
L. Huijse and S. Sachdev, arXiv:1104.5022

Gauge-gravity duality



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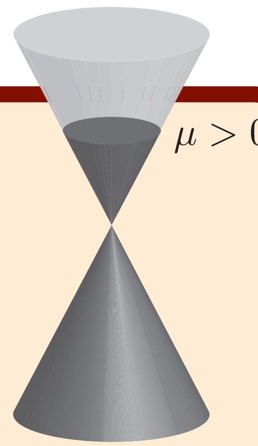
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Gauge-gravity duality



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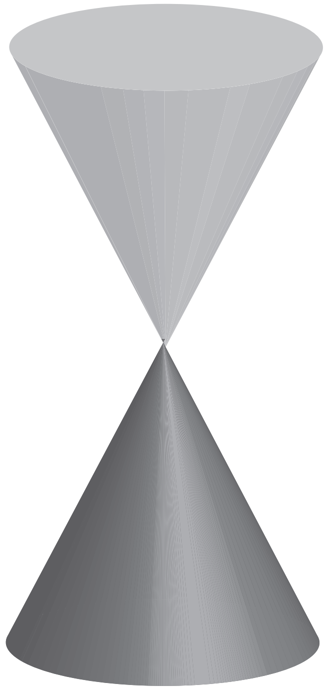
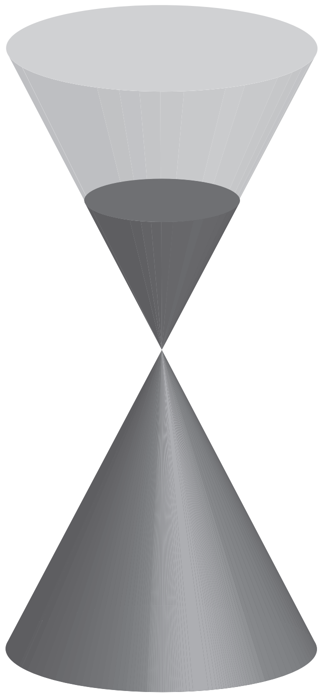
- The RN black hole describes a non-Fermi liquid, but with infinite range hopping.

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).




3+1 dimensional
AdS space



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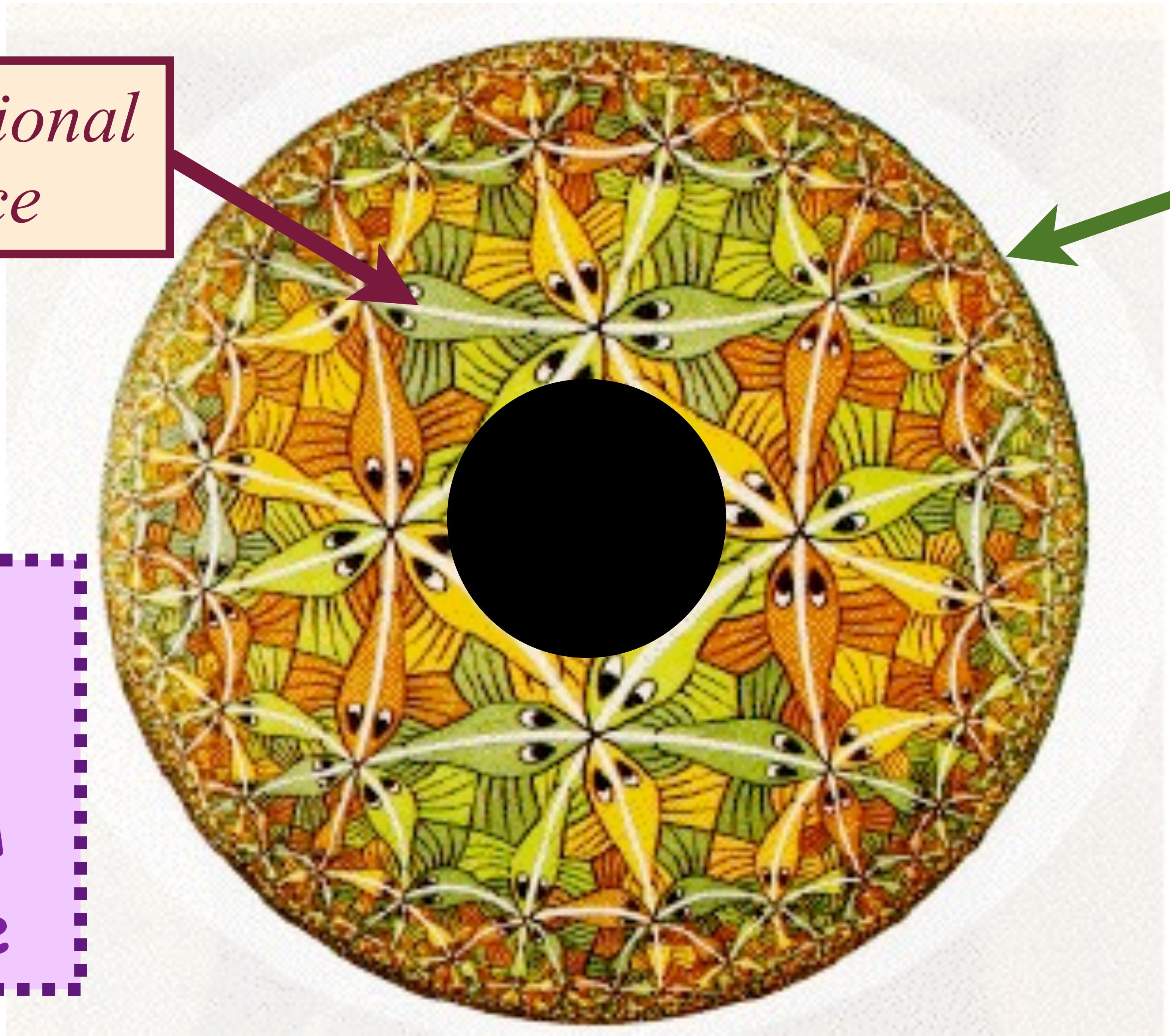
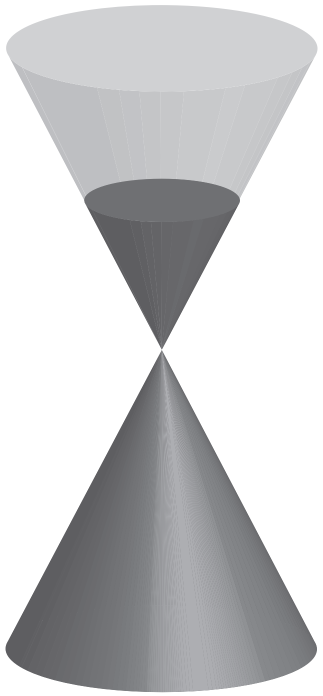


Finite density matter in 2+1 dimensions

Extremal Reissner-Nordtrom black hole

Examine the free energy and Green's function of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



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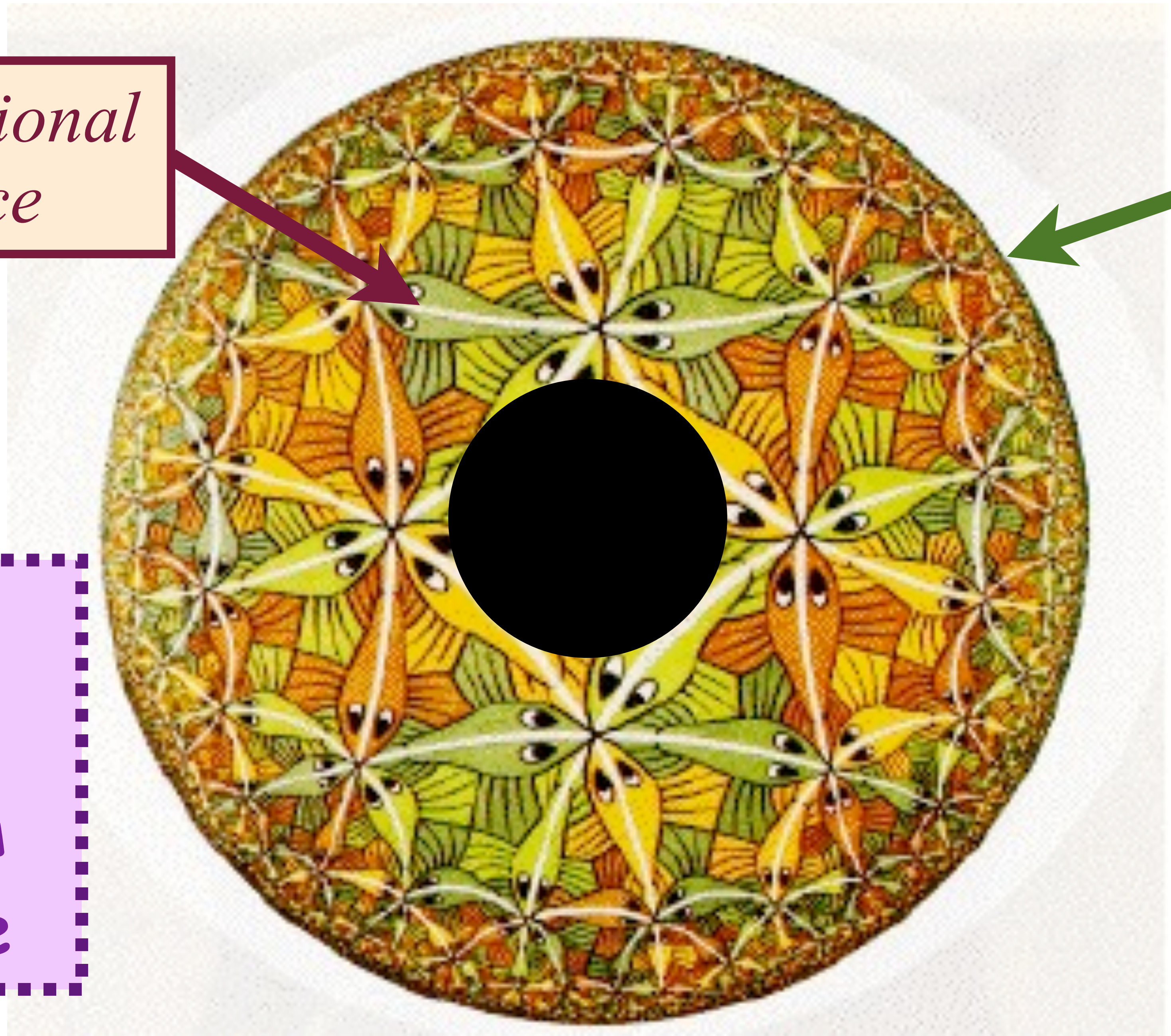

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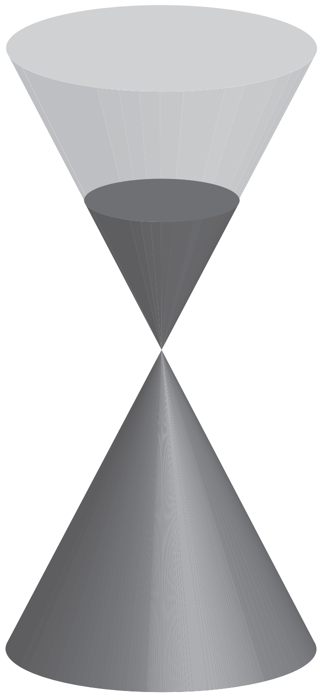

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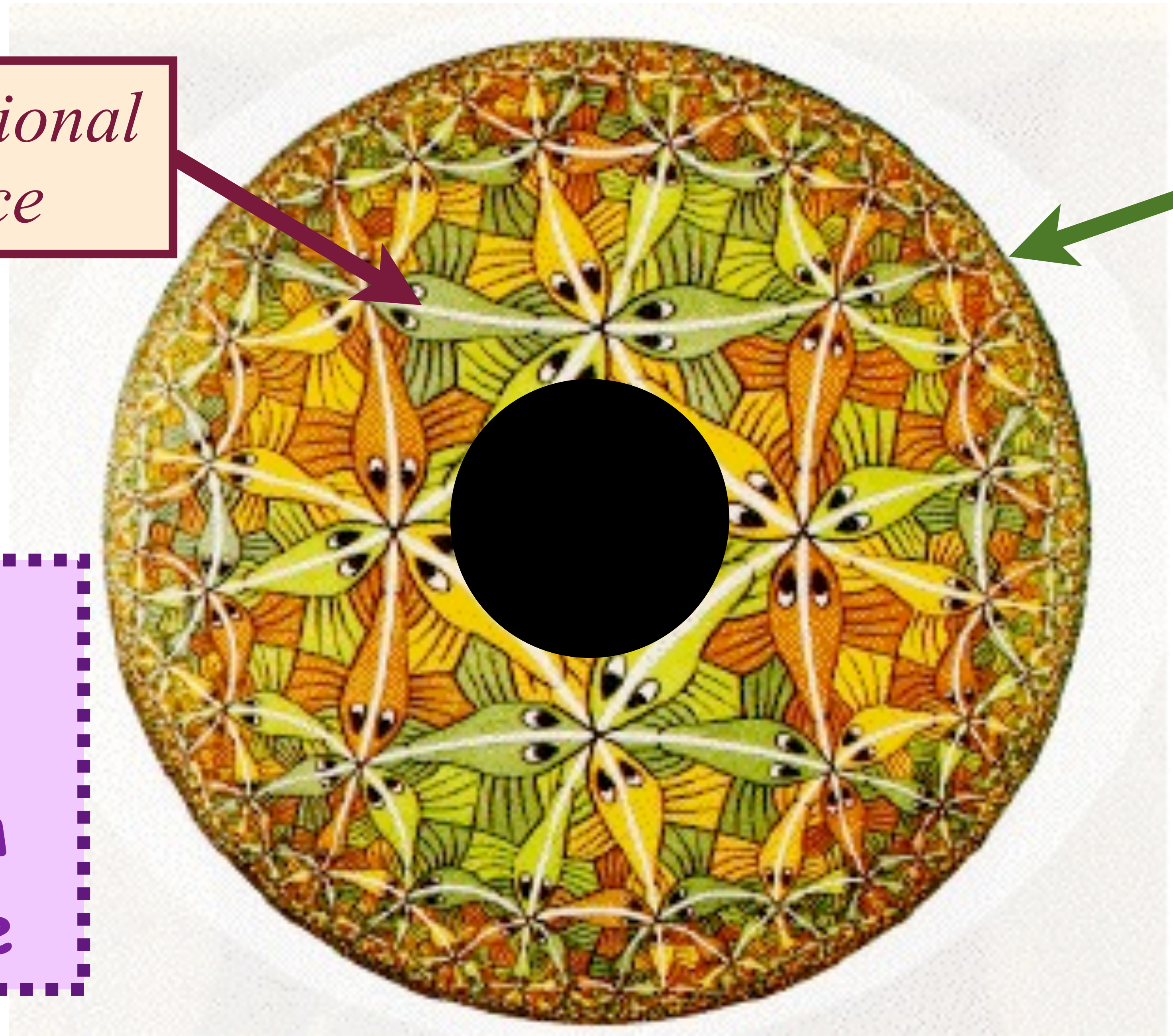

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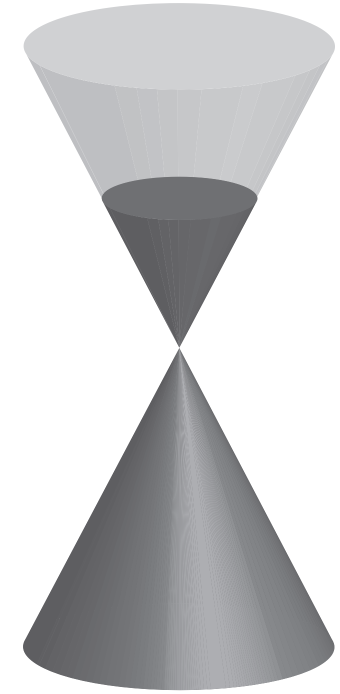

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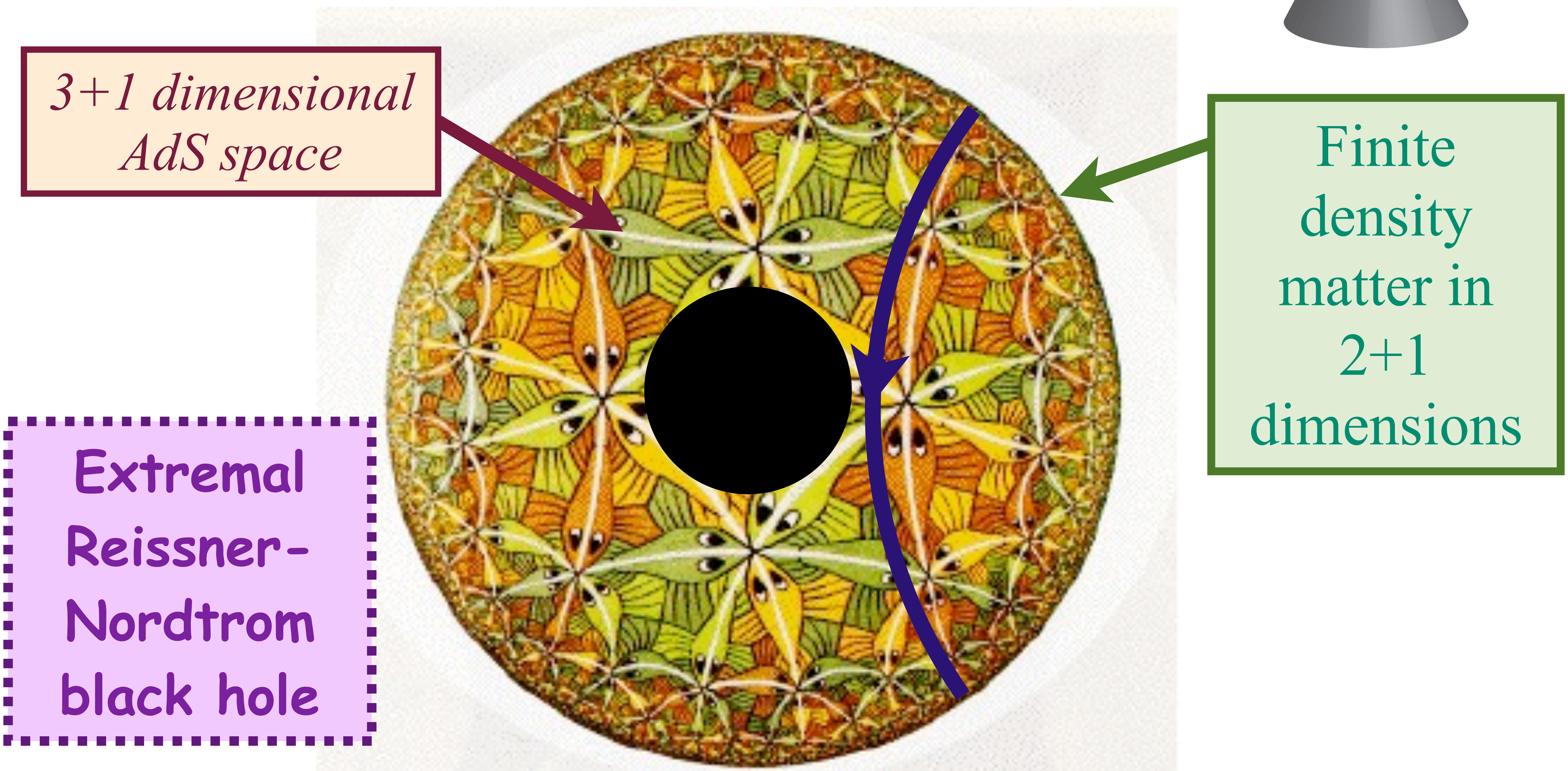
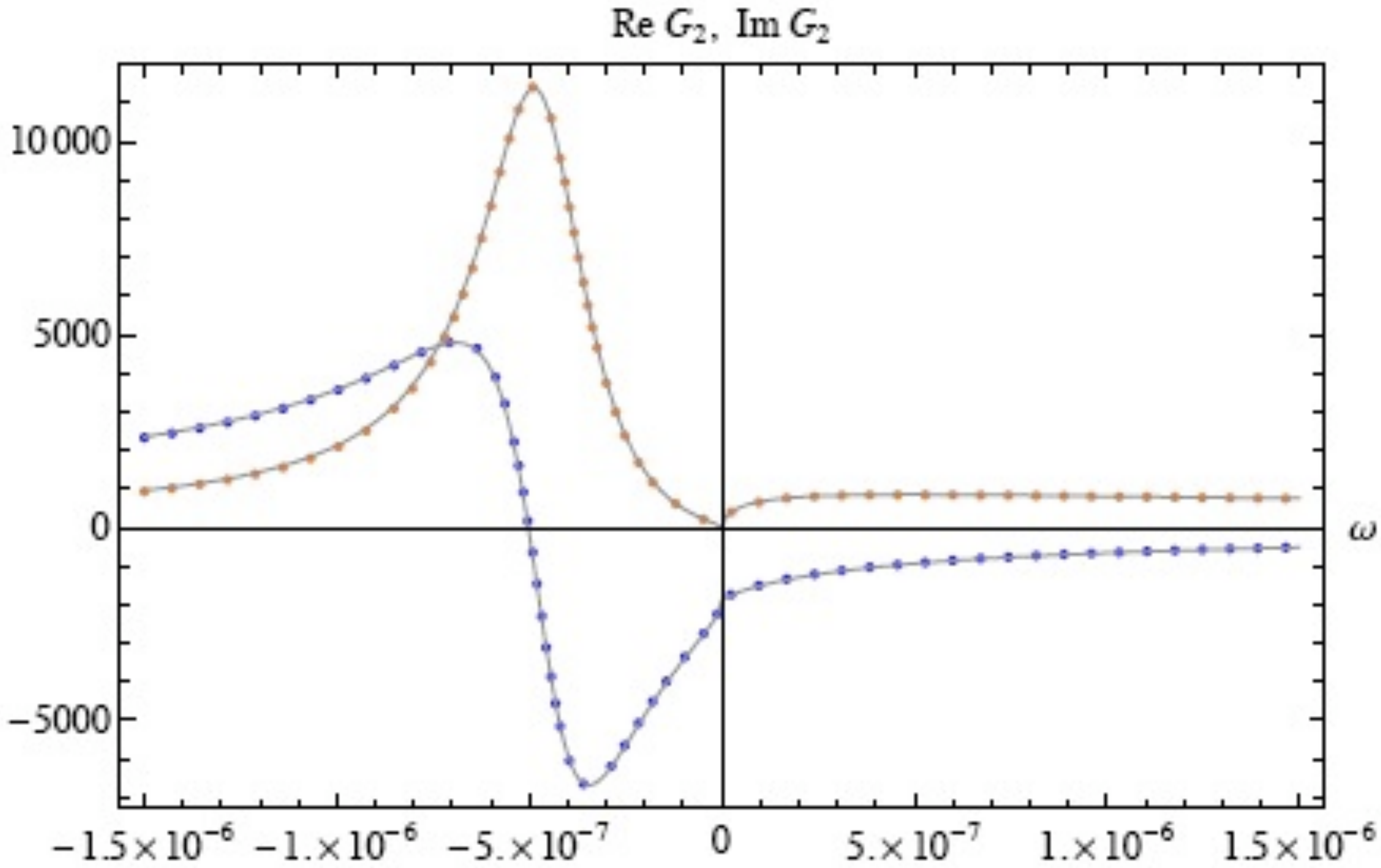
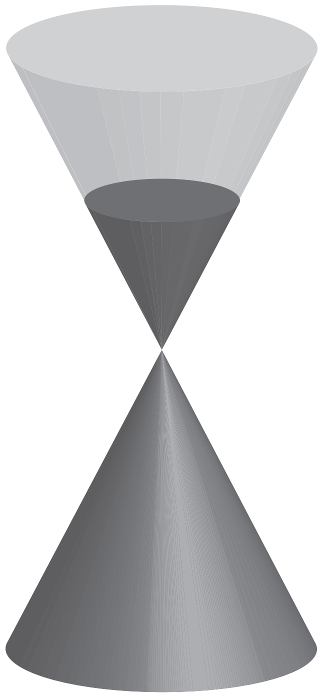


Green's function of a fermion





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$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);
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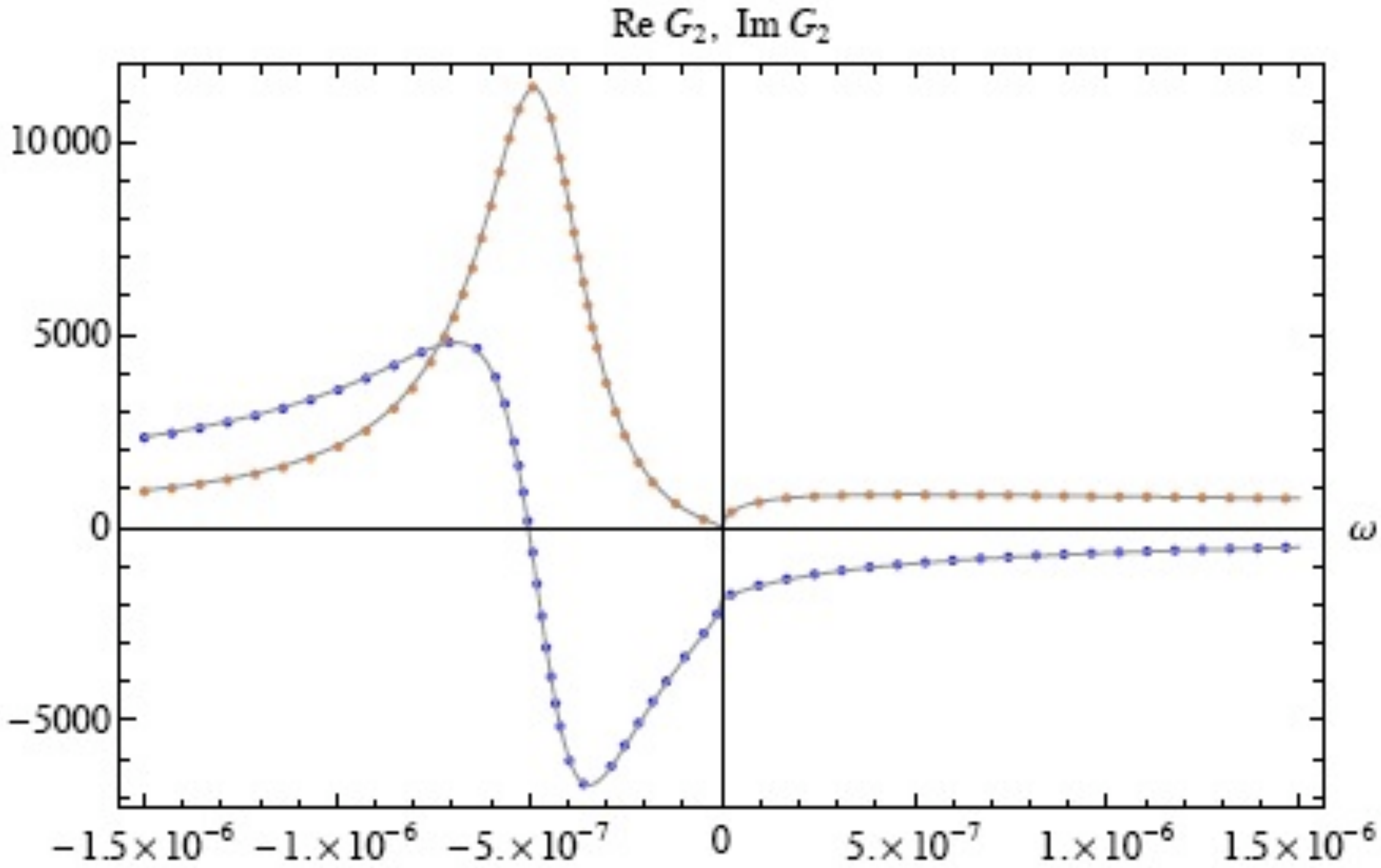

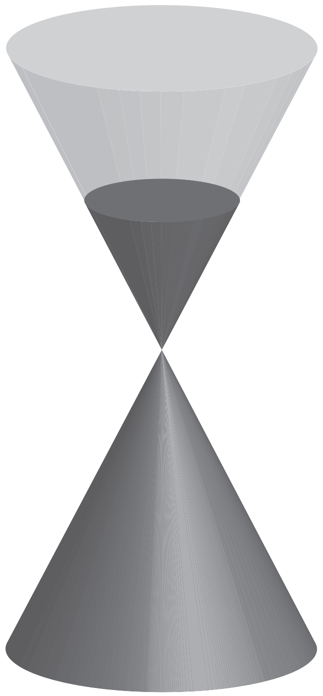

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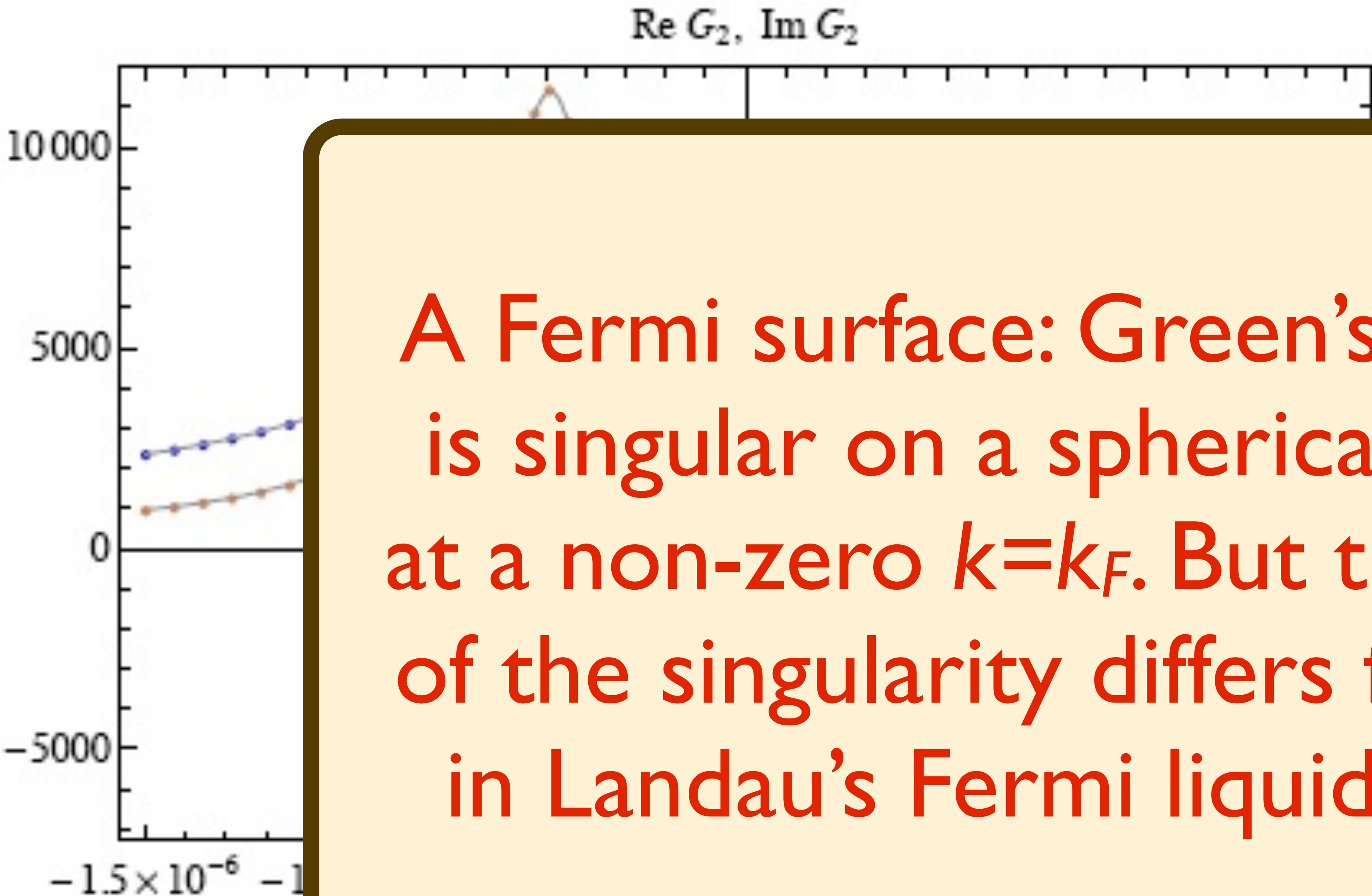

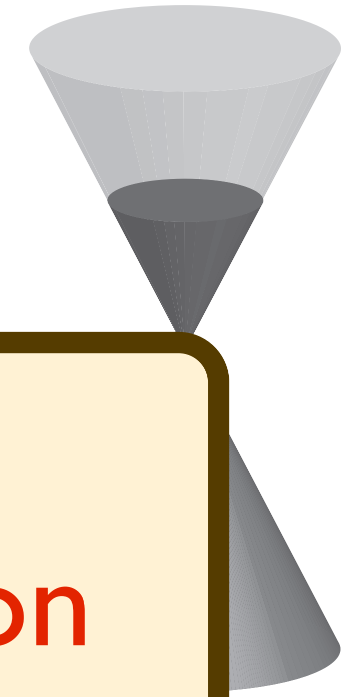

A Fermi surface: Green's function is singular on a spherical surface at a non-zero $k=k_F$. But the nature of the singularity differs from that in Landau's Fermi liquid theory.

H. Liu,
y, and
n,
.2694

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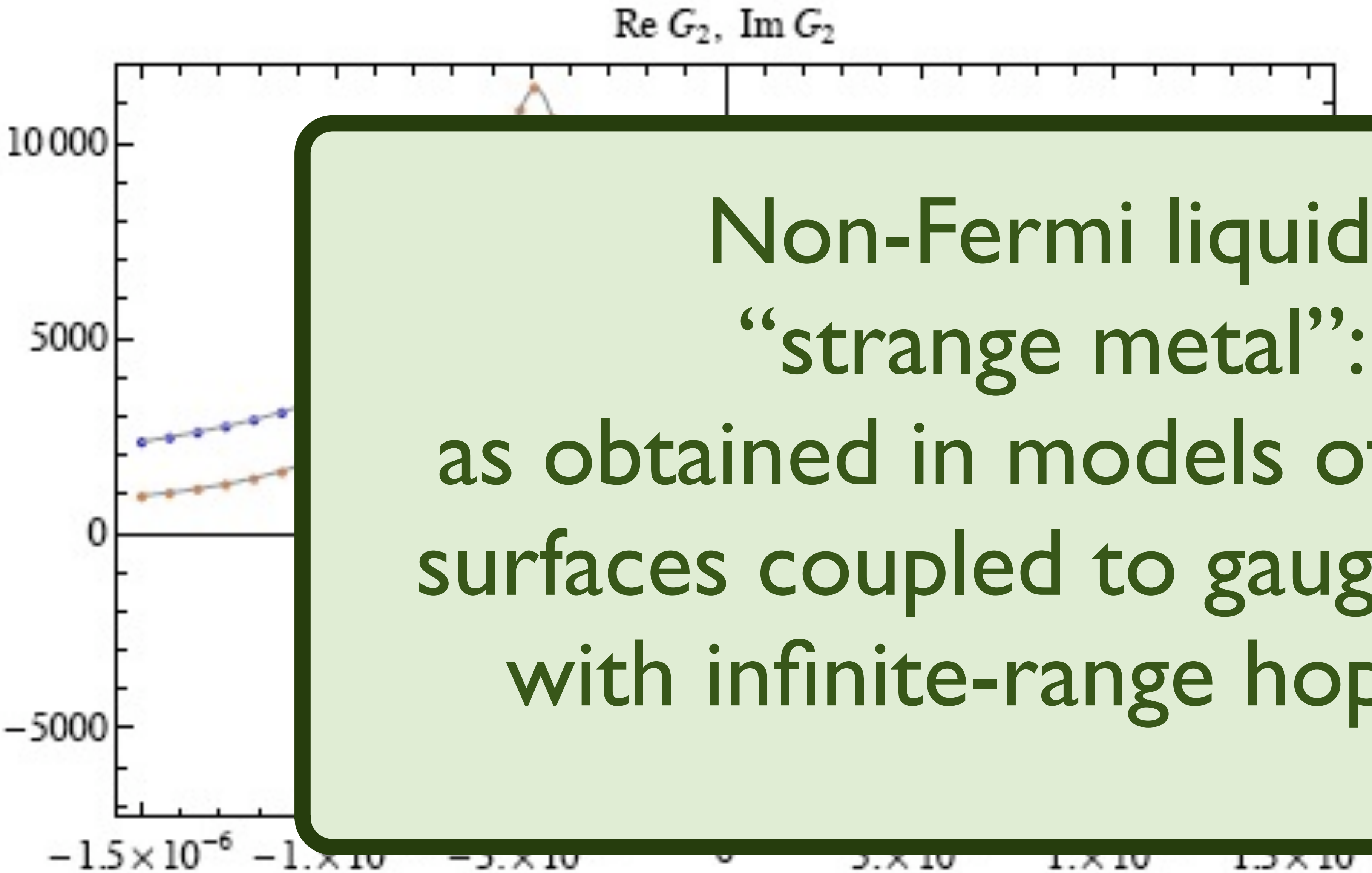

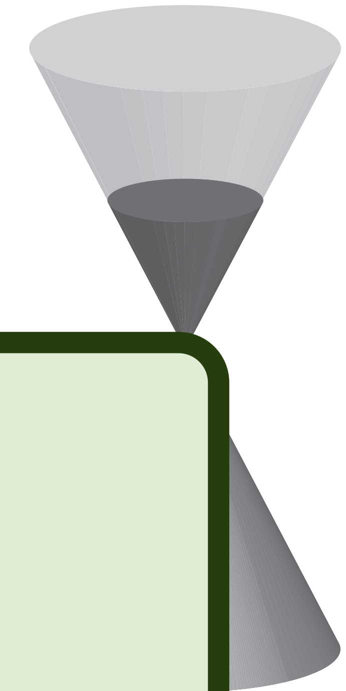

Non-Fermi liquid
"strange metal":
as obtained in models of Fermi
surfaces coupled to gauge fields,
with infinite-range hopping.

H. Liu,
y, and
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.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

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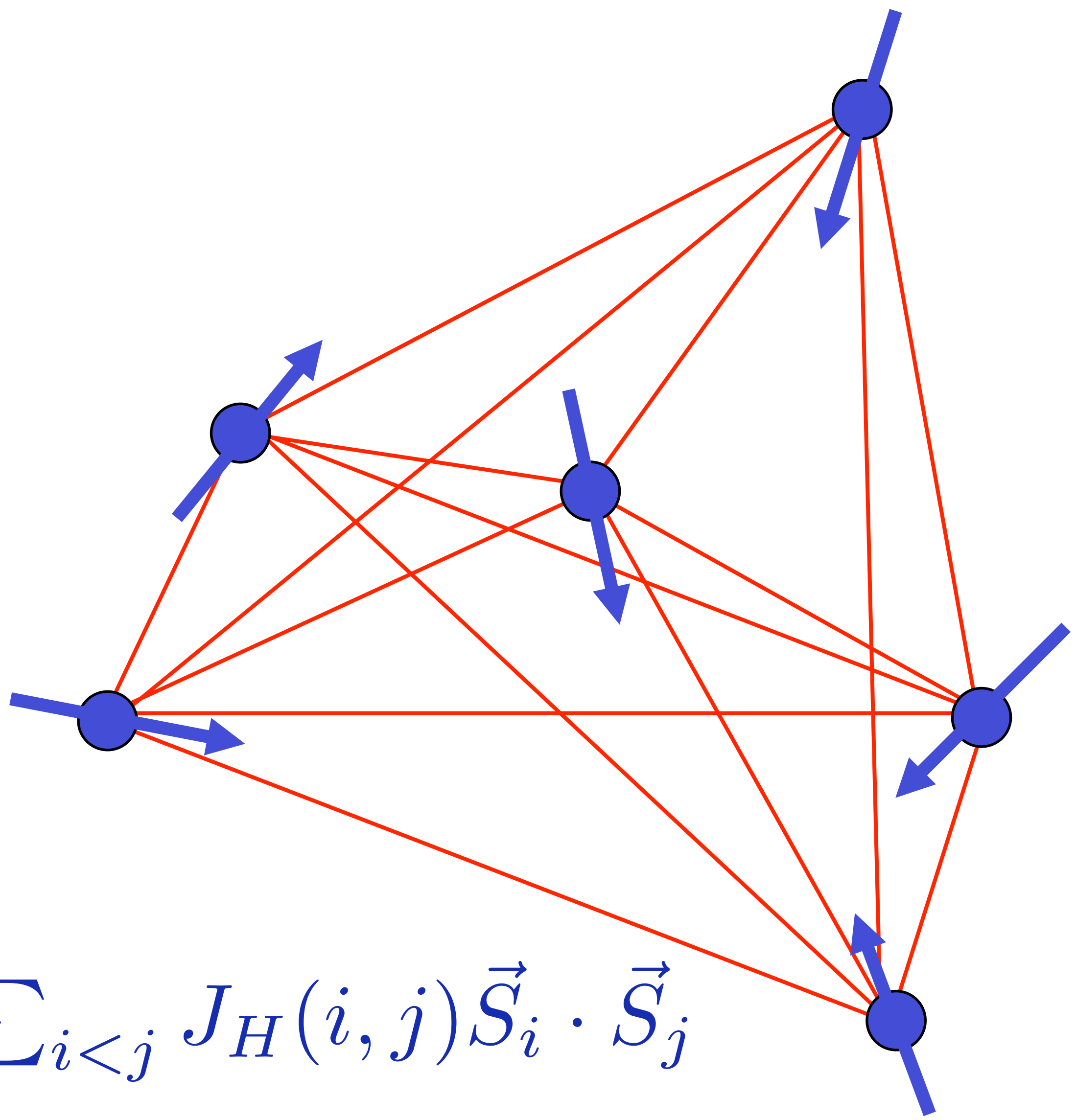
A Kondo lattice model for the $AdS_2 \times R^d$ region of an extremal Reissner-Nordstrom black hole



$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$

$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick model of SU(N) spins.

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

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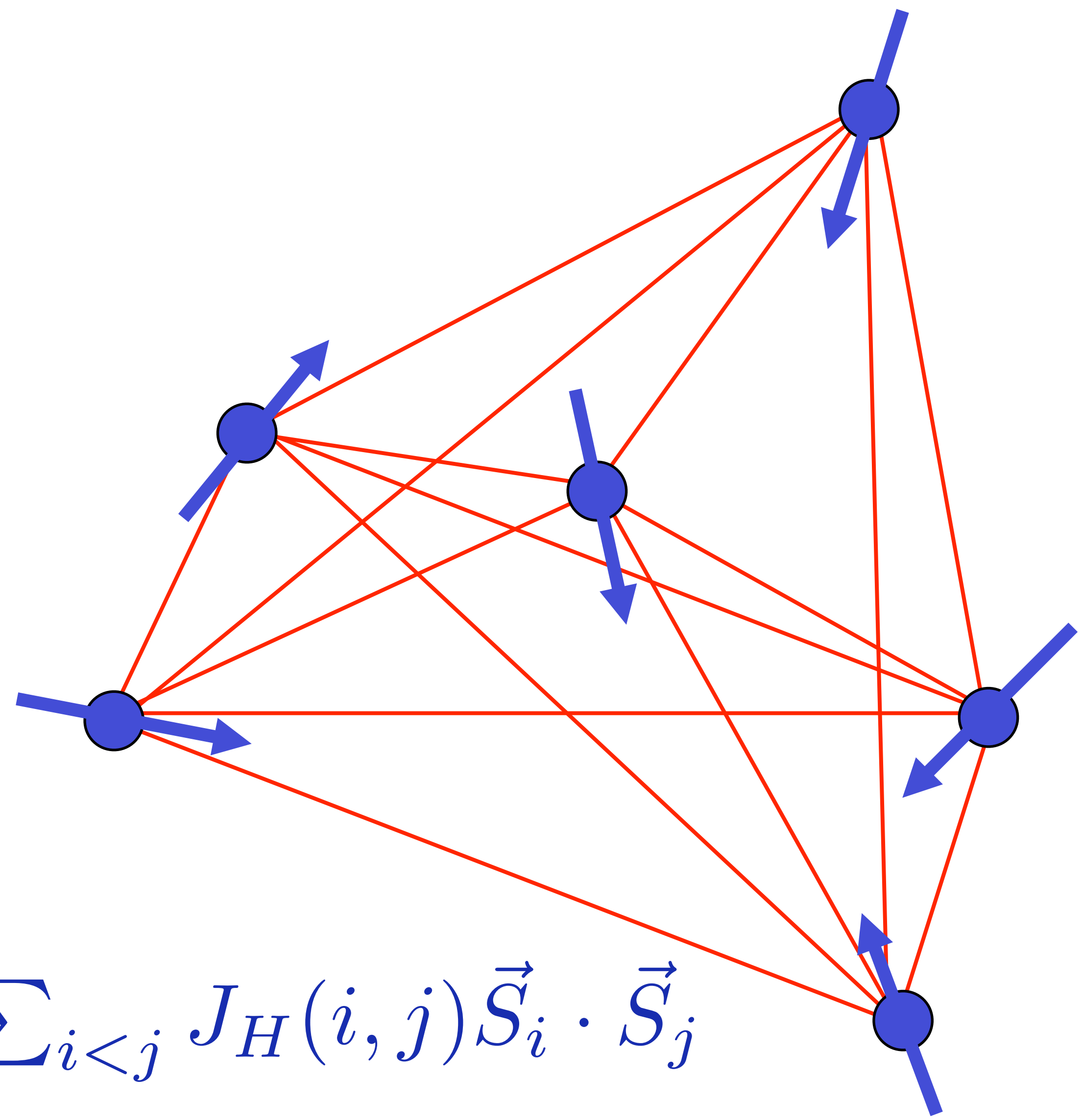

$\sum_{i<j} J_H(i,j) \vec{S}_i \cdot \vec{S}_j$

Described by the conformal quantum mechanics of a quantum spin fluctuating in a self-consistent time-dependent magnetic field: a realization the finite entropy density $\mathrm{AdS}_2 \times R^d$ state

$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick model of $\mathrm{SU}(N)$ spins.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001).

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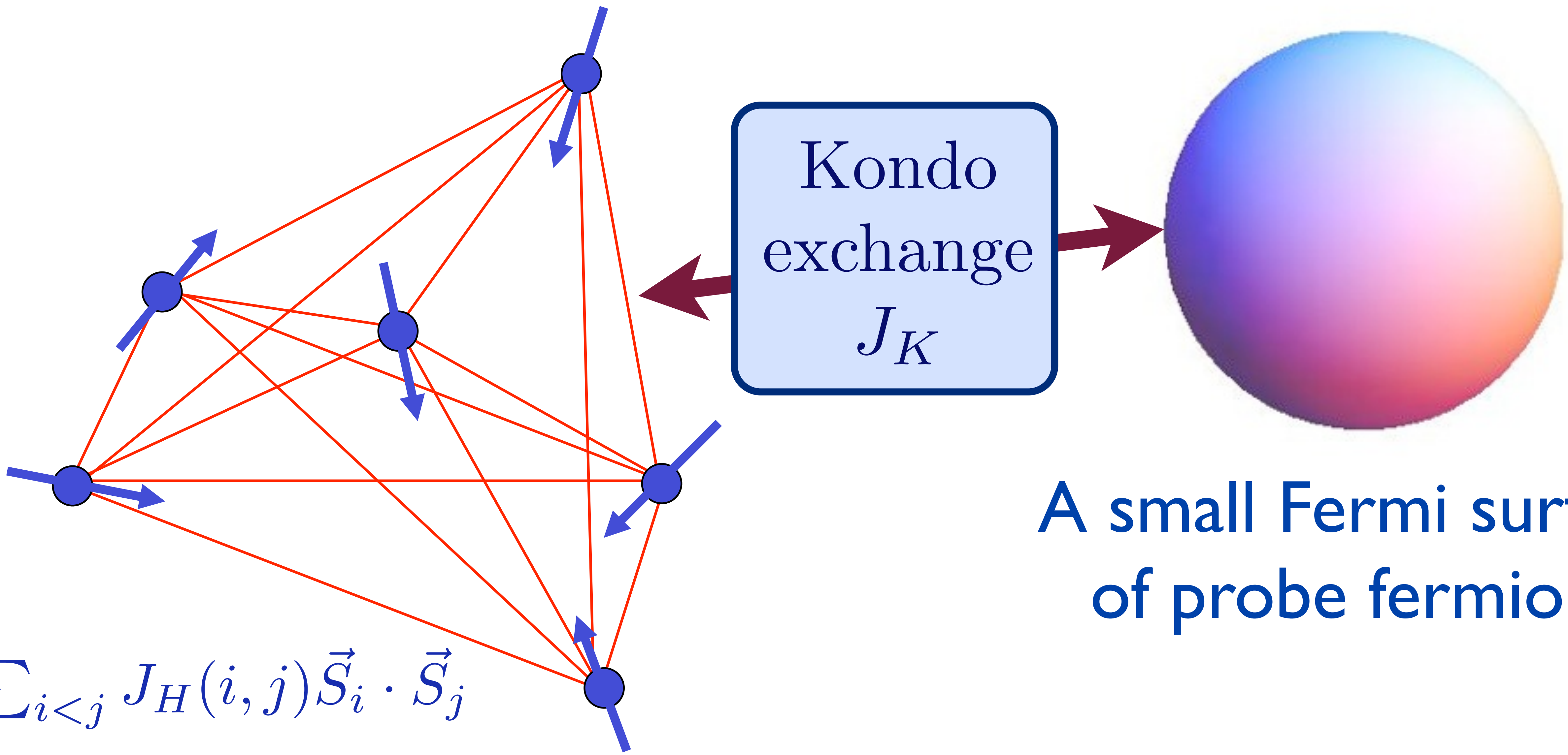

A small Fermi surface of probe fermions

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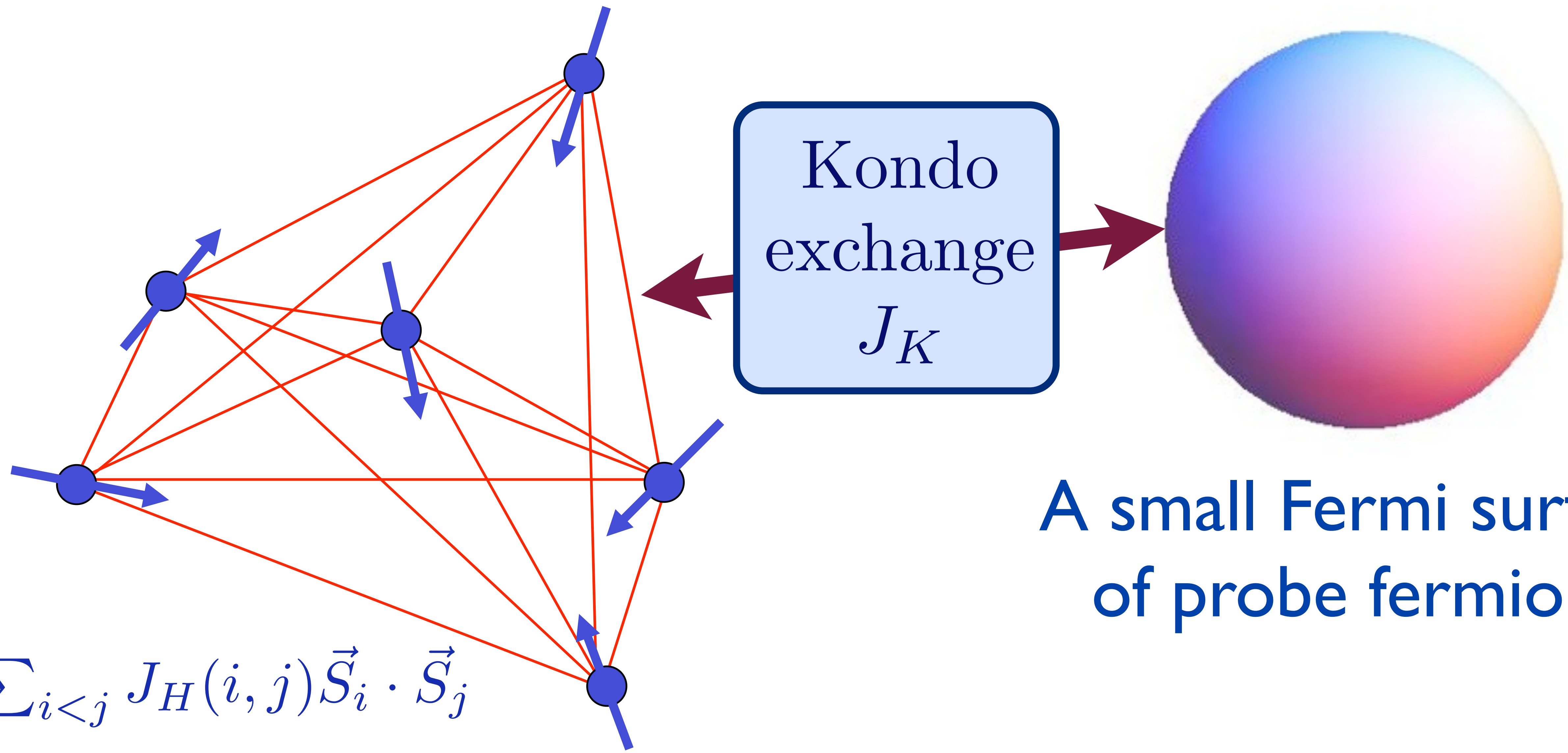

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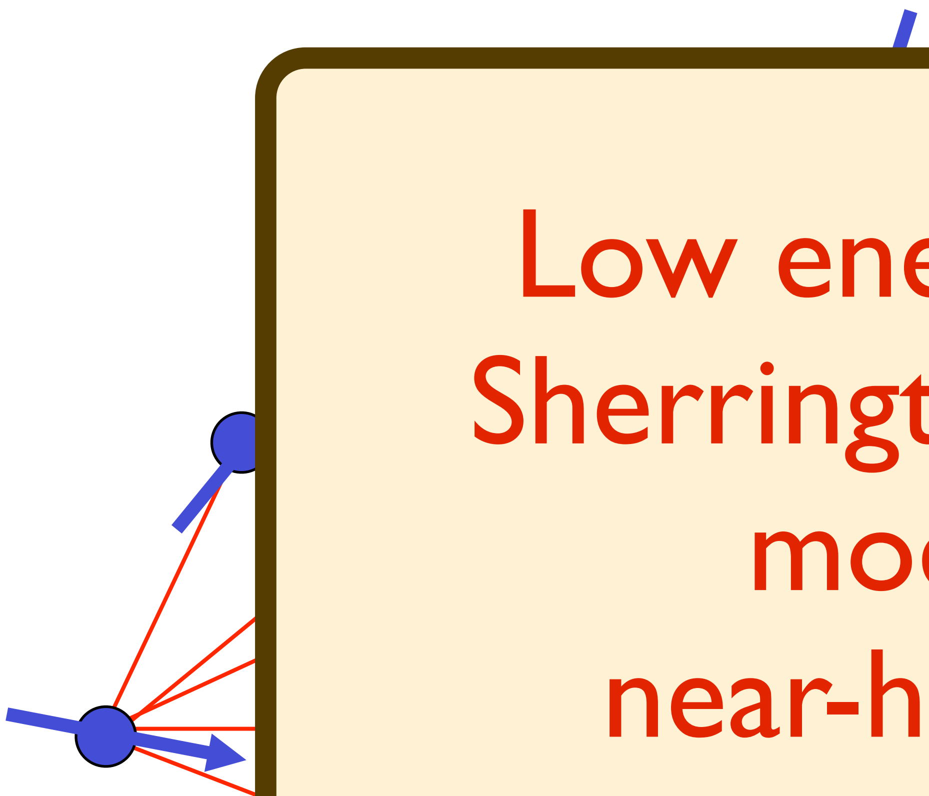


Low energy properties of the Sherrington-Kirkpatrick-Kondo model map onto the near-horizon physics of an extremal Reissner-Nordstrom black hole

surface

nions

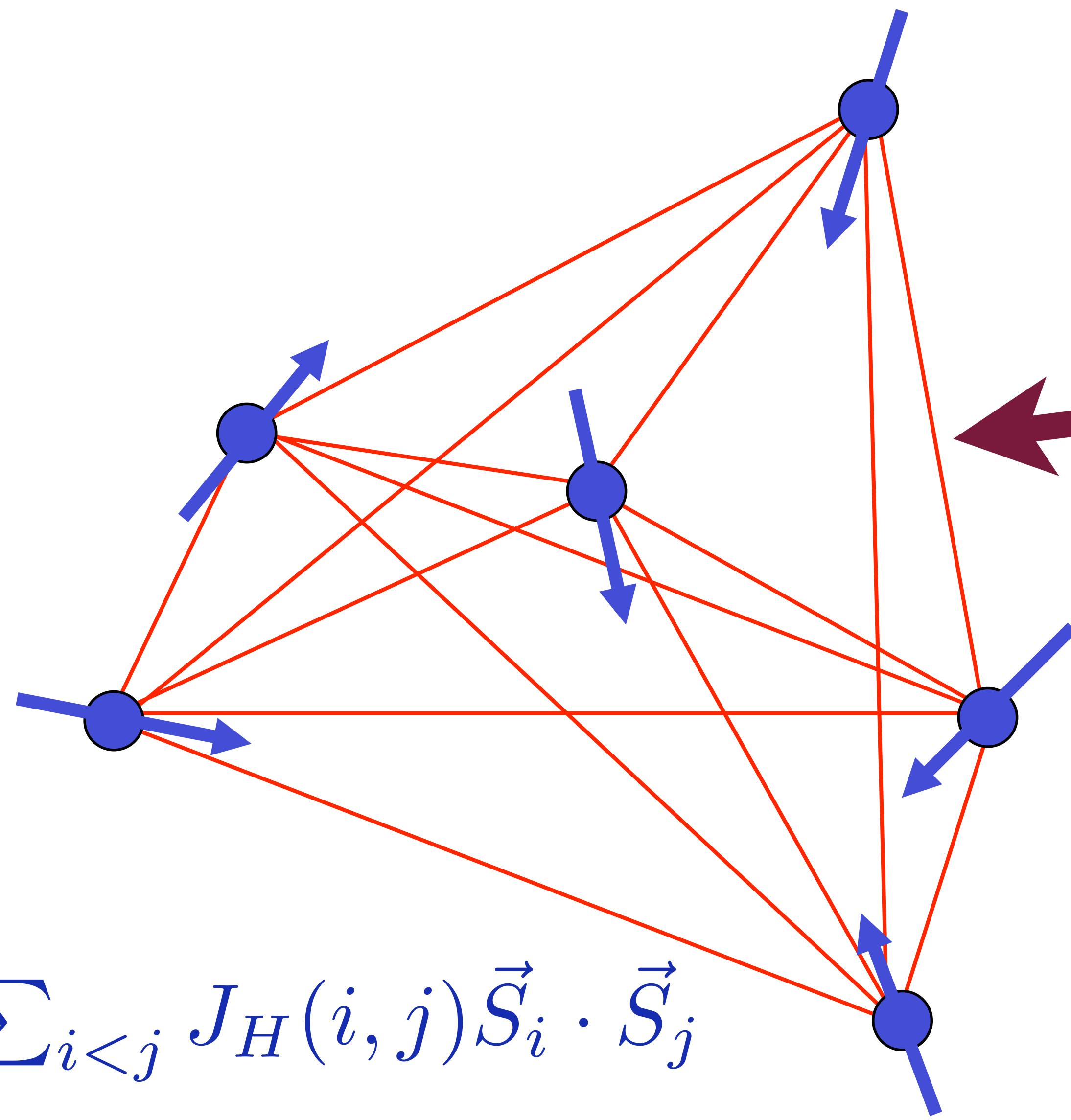
$\sum_{i<j} J_H$

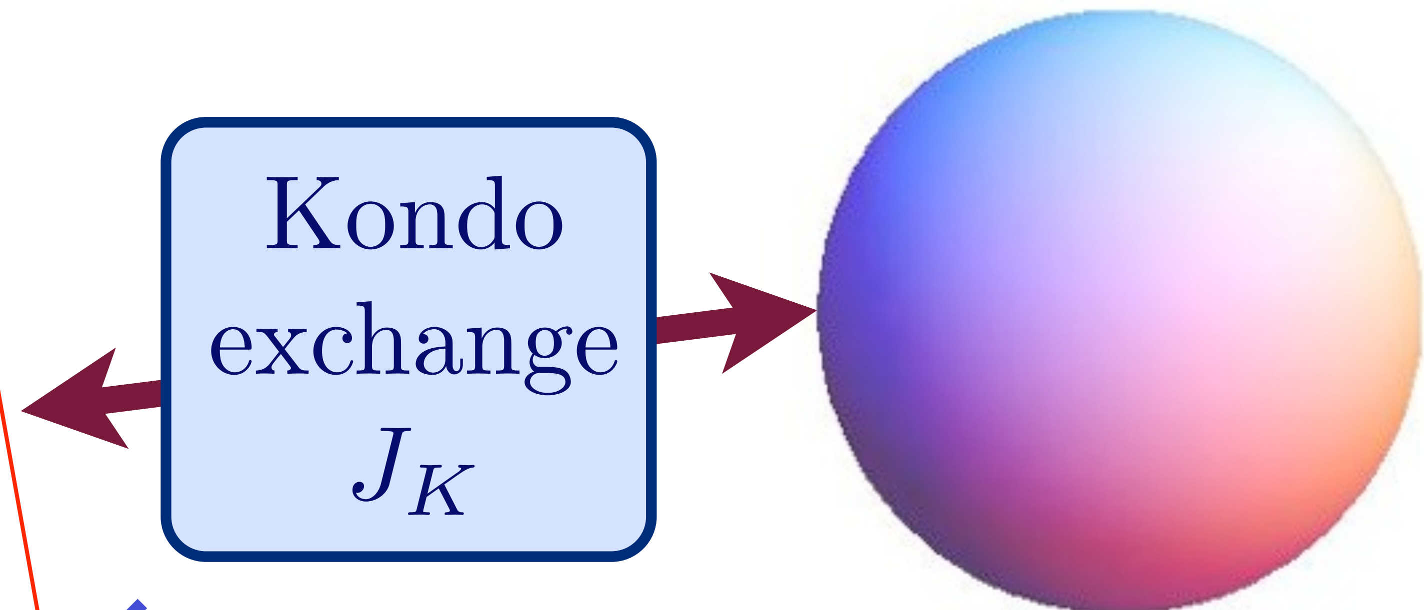
$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick model of SU(N) spins.

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

- The infinite-range Sherrington-Kirkpatrick-Kondo model has properties which match with those of the $AdS_2 \times R^2$ holographic solutions:
  - A non-zero ground state entropy.
  - Single fermion self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
  - A marginal Fermi liquid spectrum for the conduction electrons (for the holographic solution, this requires tuning a free parameter).
  - The low energy sector has conformally invariant correlations which are consistent with the AdS_2 geometry.

A Kondo lattice model for the $AdS_2 \times R^d$ region of an extremal Reissner-Nordstrom black hole





A small Fermi surface of probe fermions

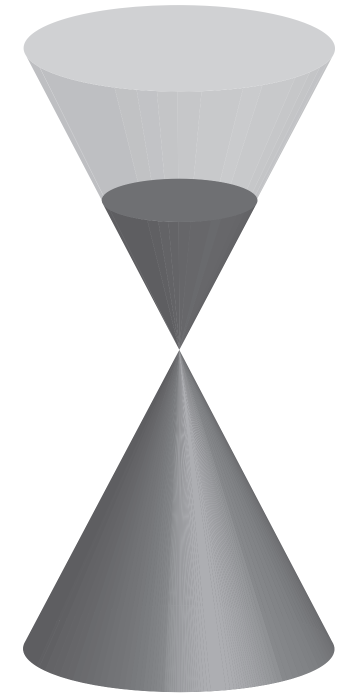
$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$

$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick model of SU(N) spins.

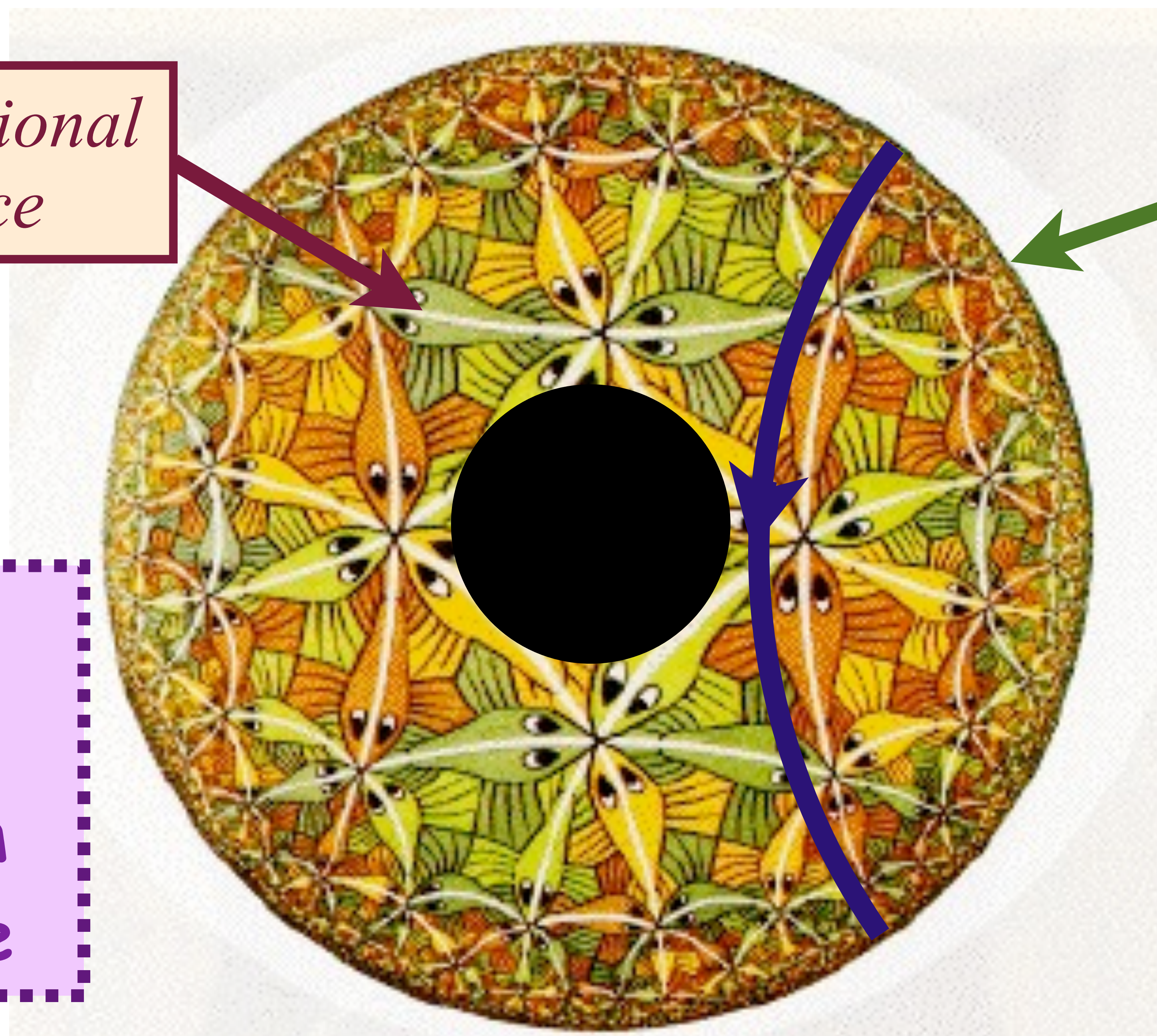
S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

Examine the free energy and Green's function of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



3+1 dimensional AdS space



Finite density matter in 2+1 dimensions

Extremal Reissner-Nordtrom black hole

Much work remains in extending these solvable models to a realistic theory of the cuprate superconductors. Considerable recent progress in gravitational theories which include back-reaction of Fermi surfaces on the AdS metric

Conclusions

New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

Conclusions

The AdS/CFT correspondence offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density