Random t-J model theory of the cuprate phase diagram

ICTS String Seminar
ICTS Bengaluru
June 9 , 2021

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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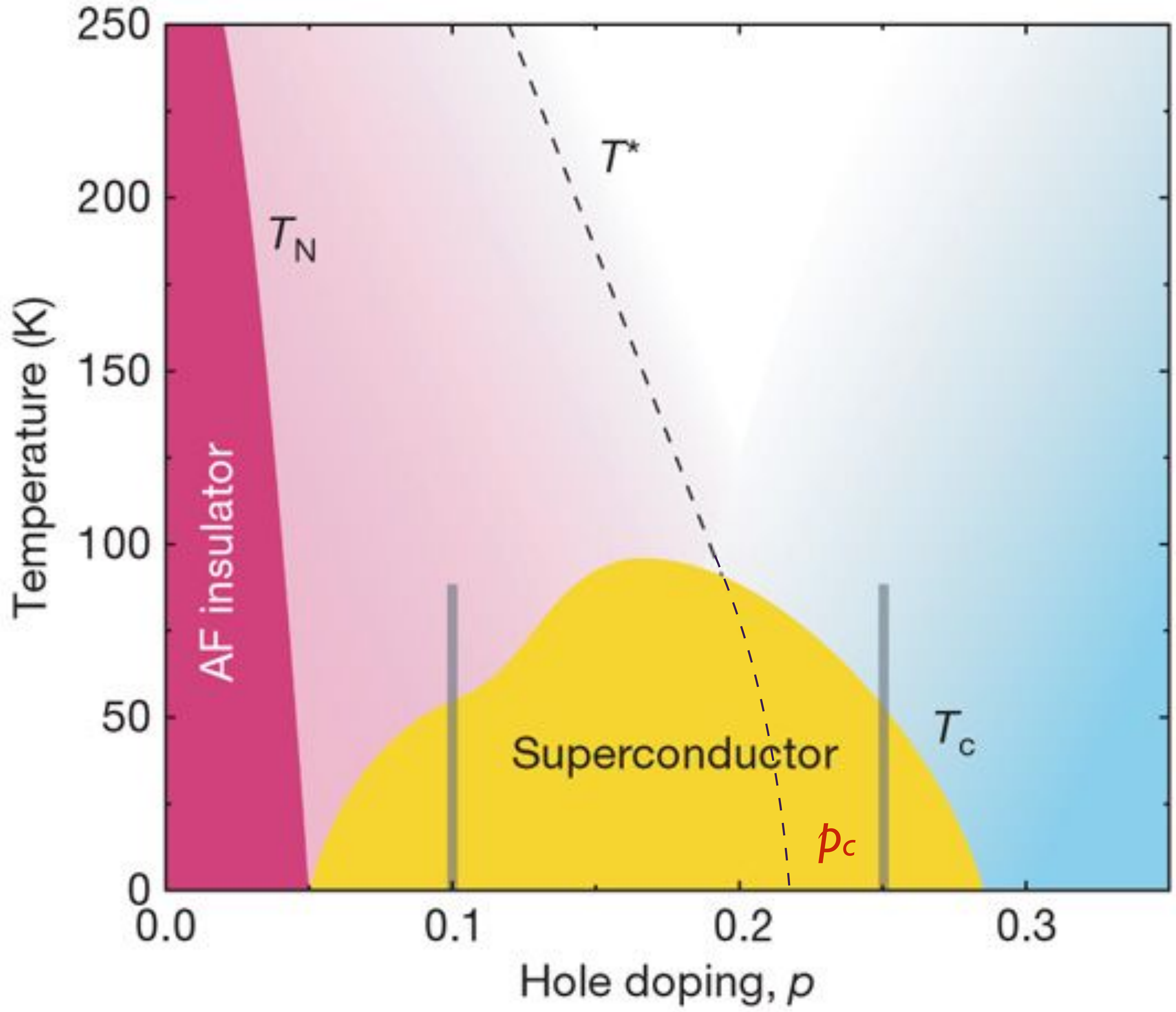

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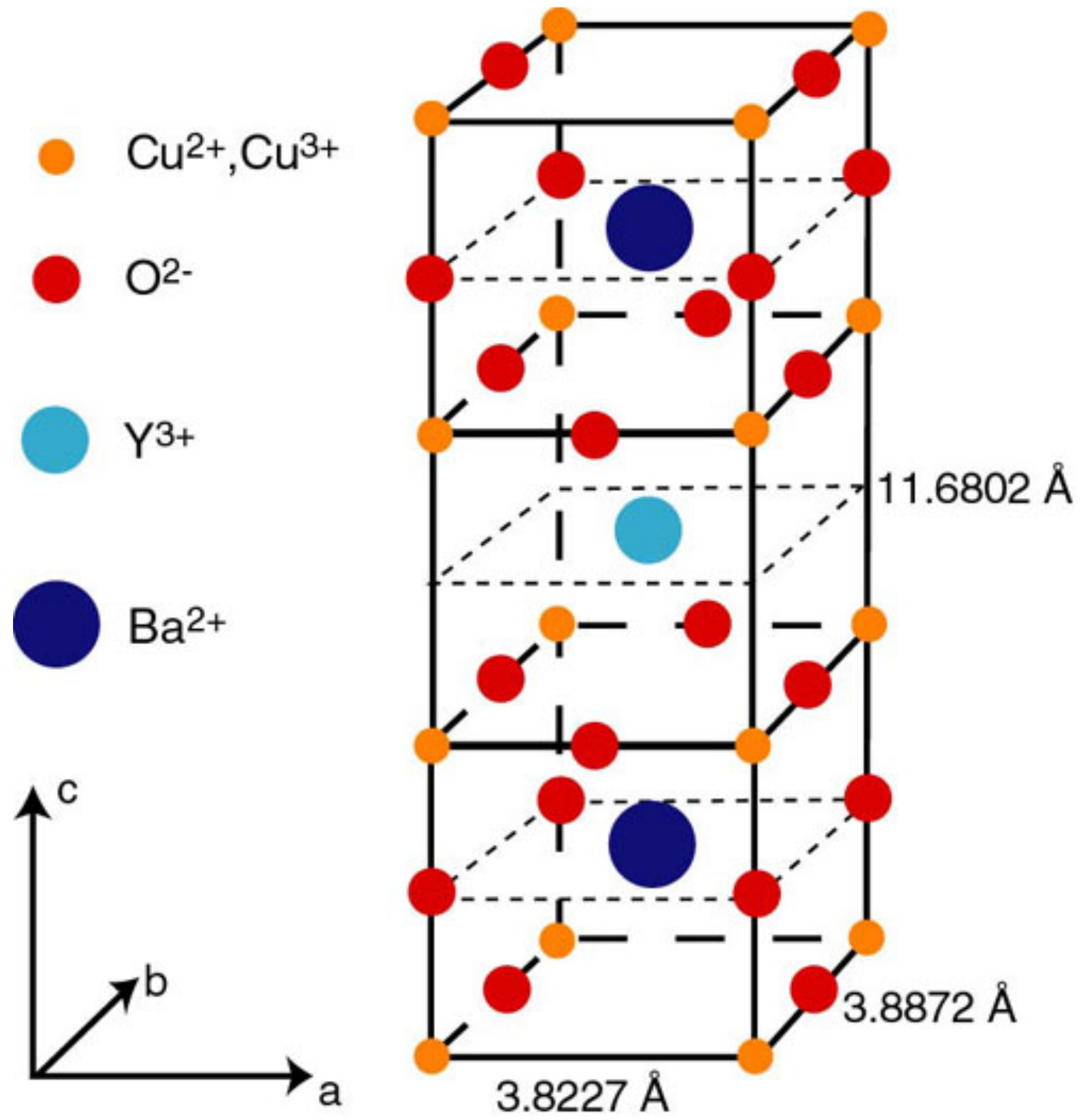
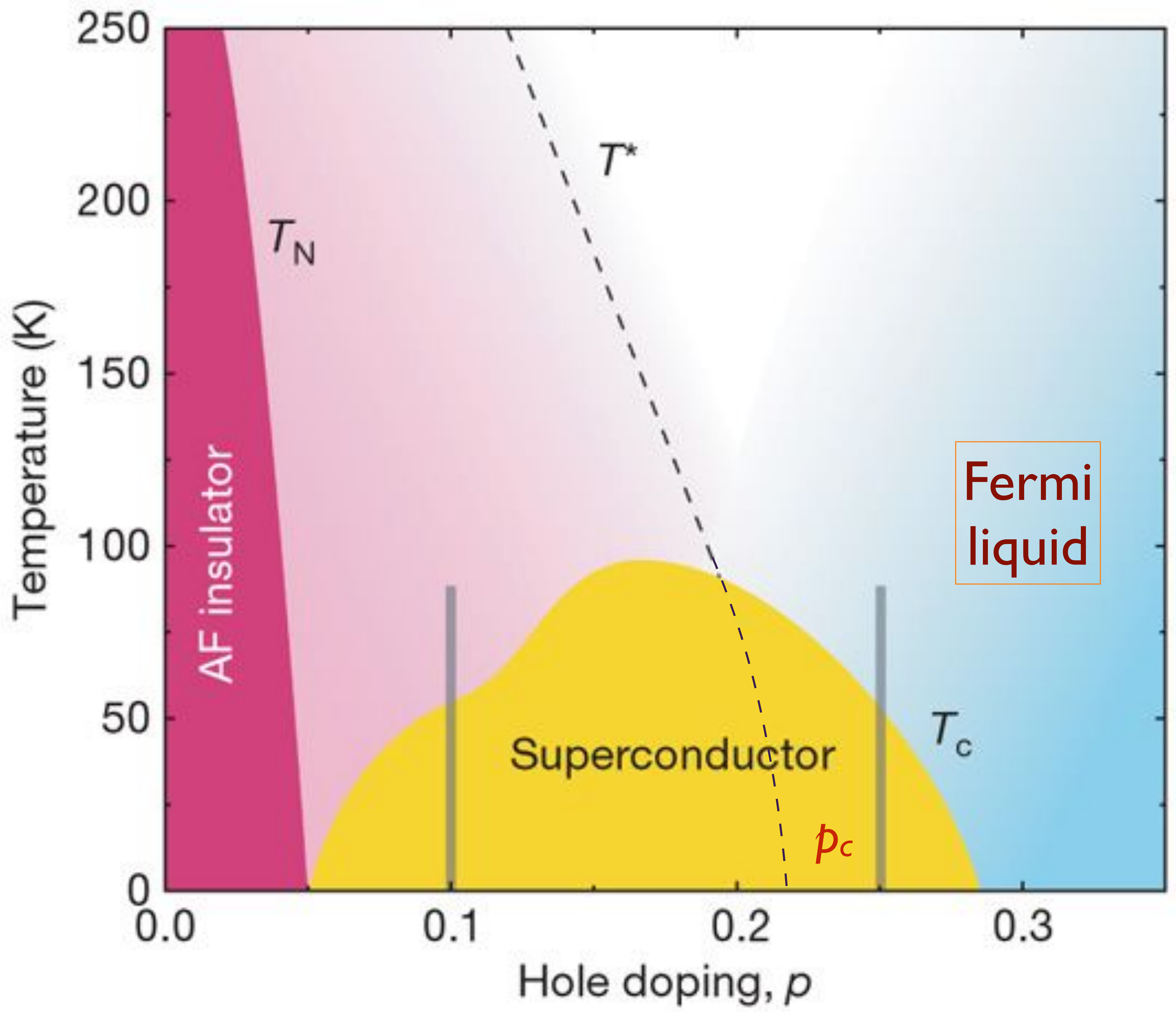
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å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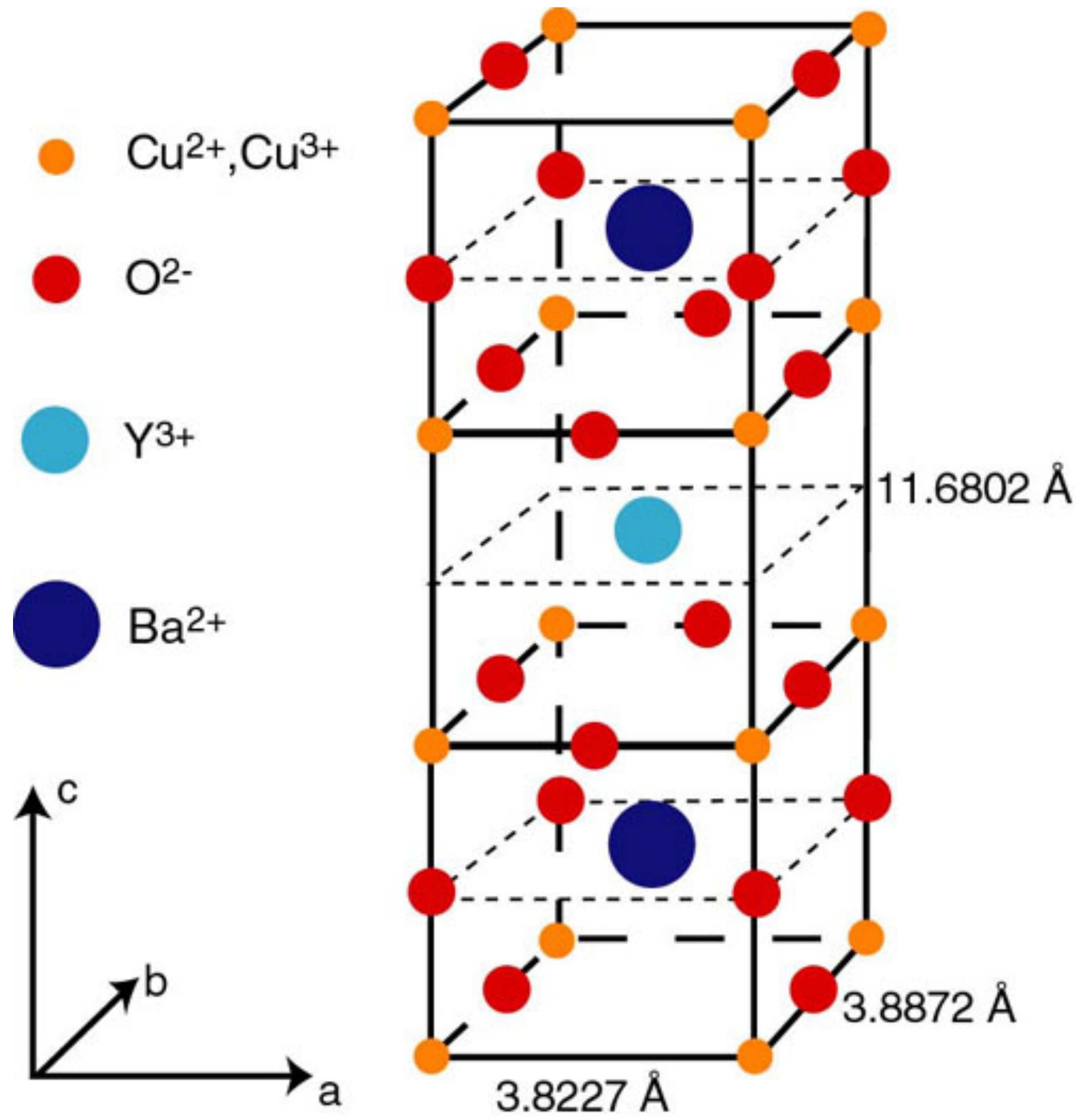
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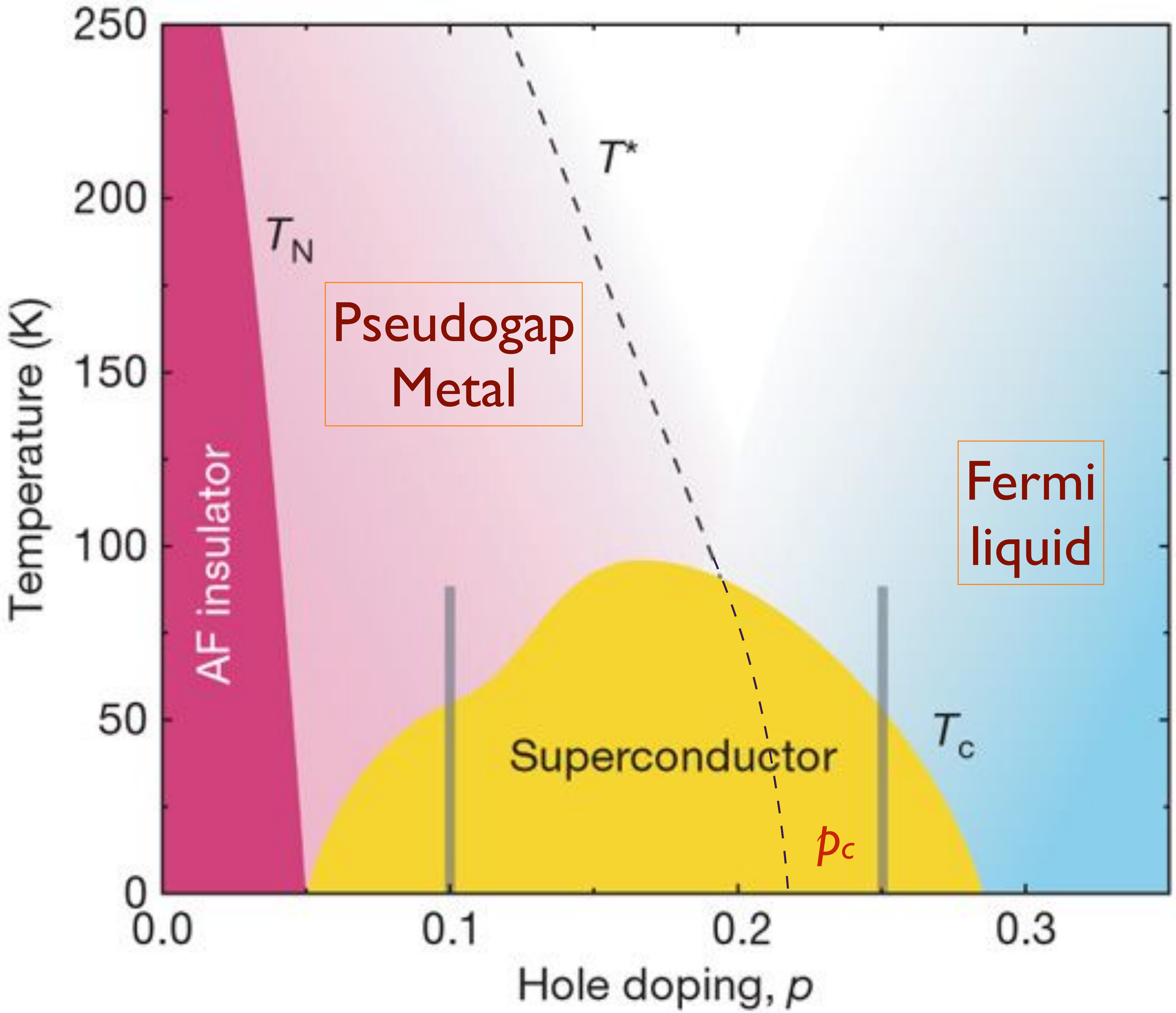
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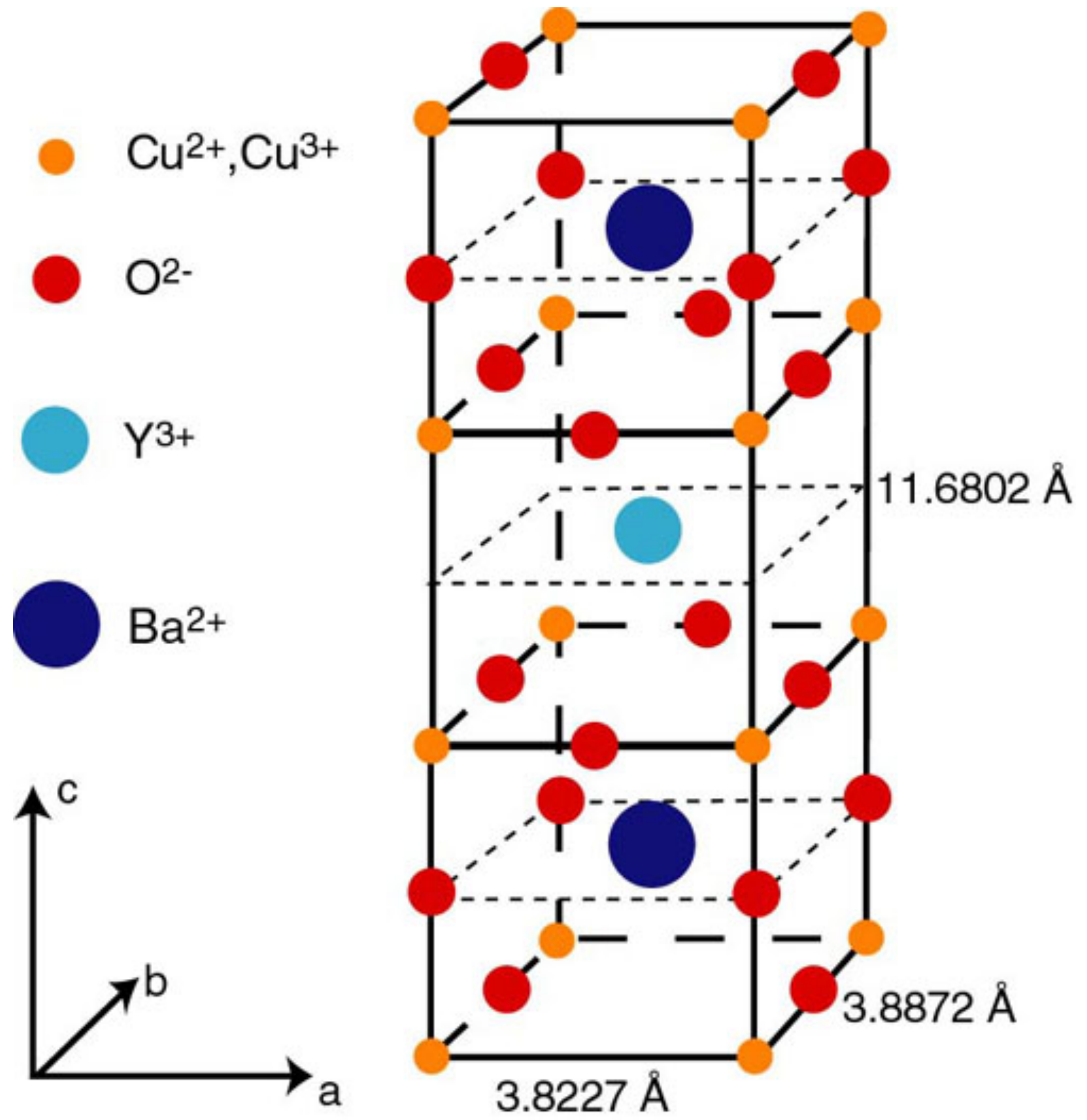
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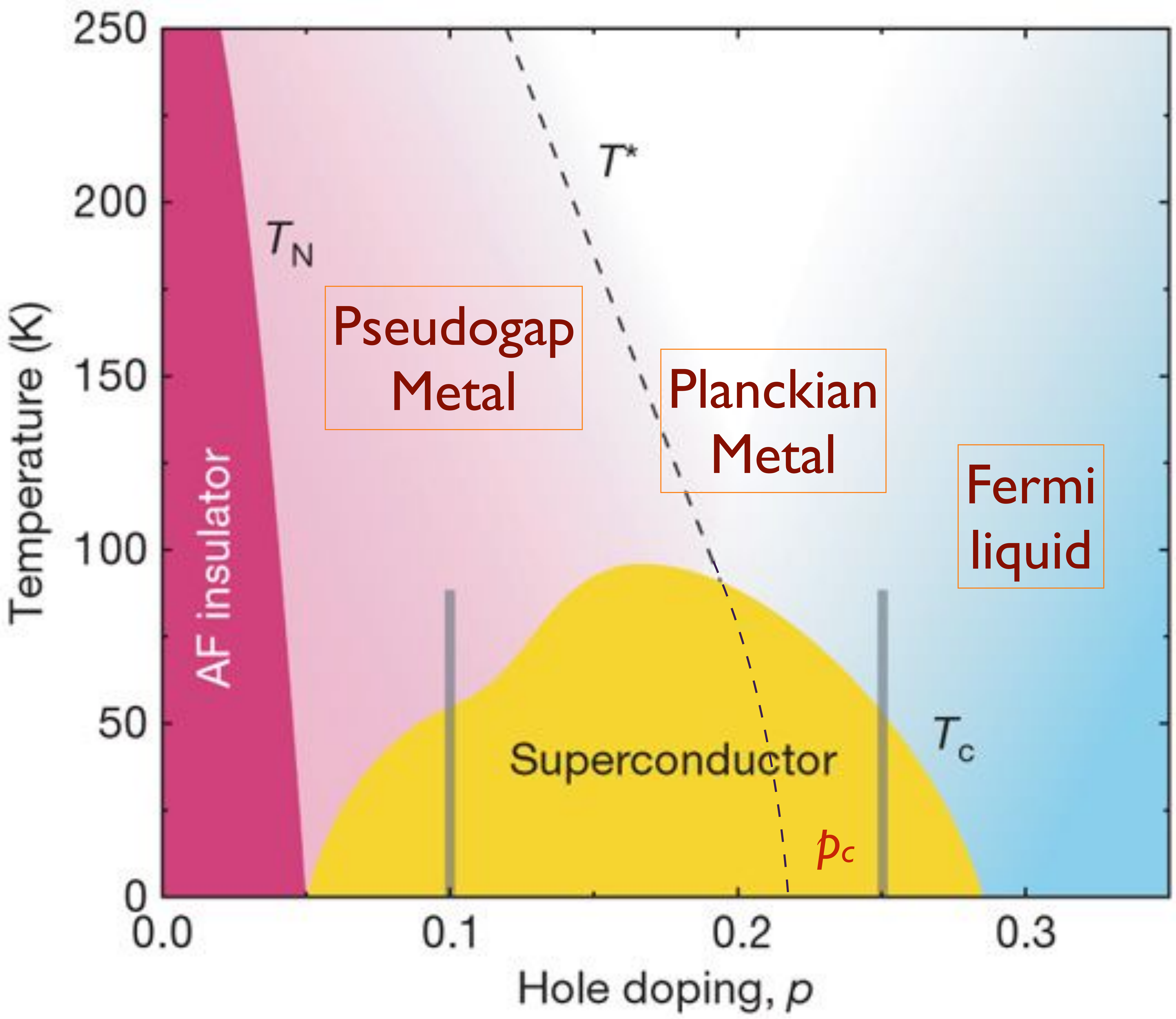

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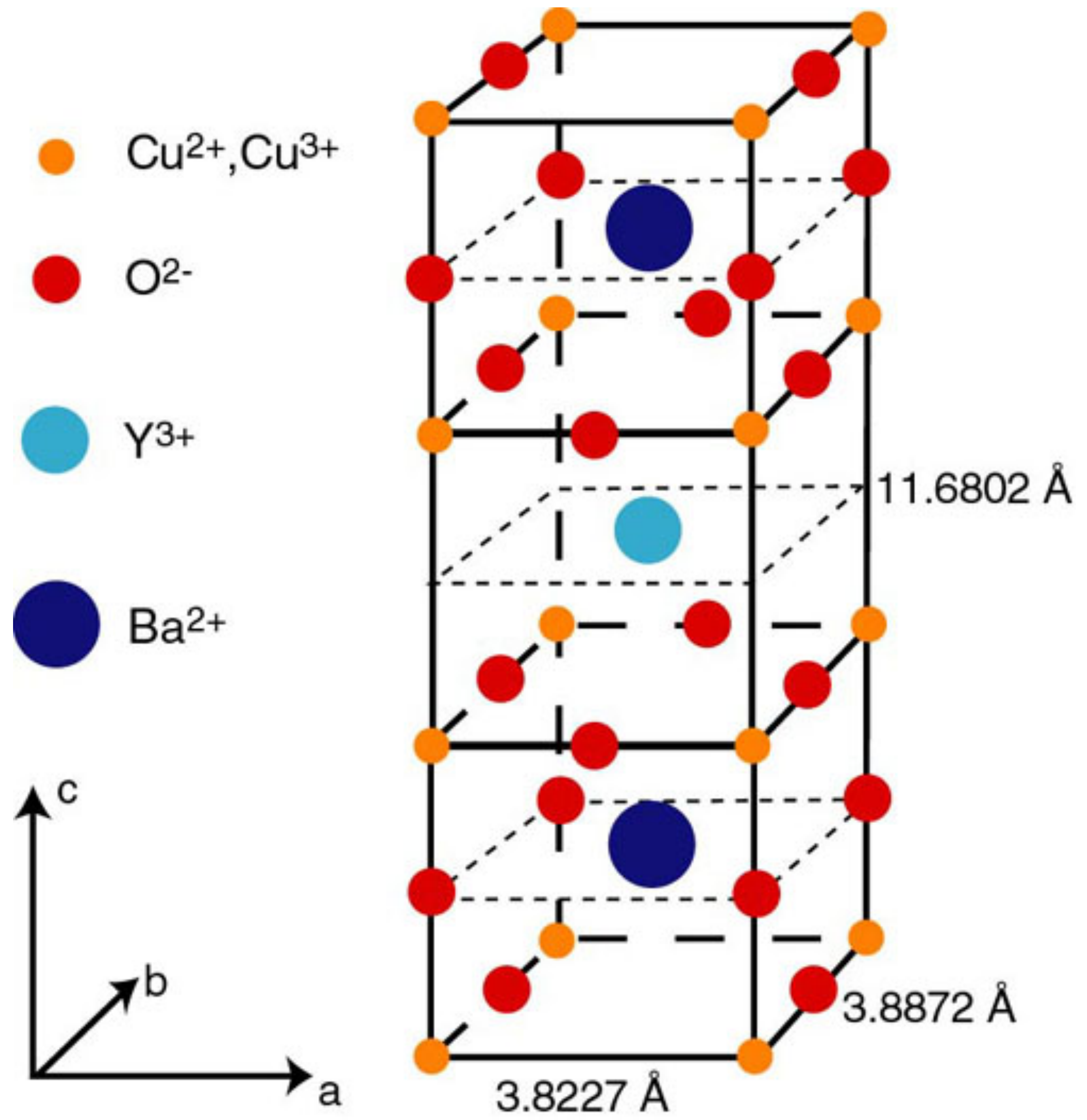

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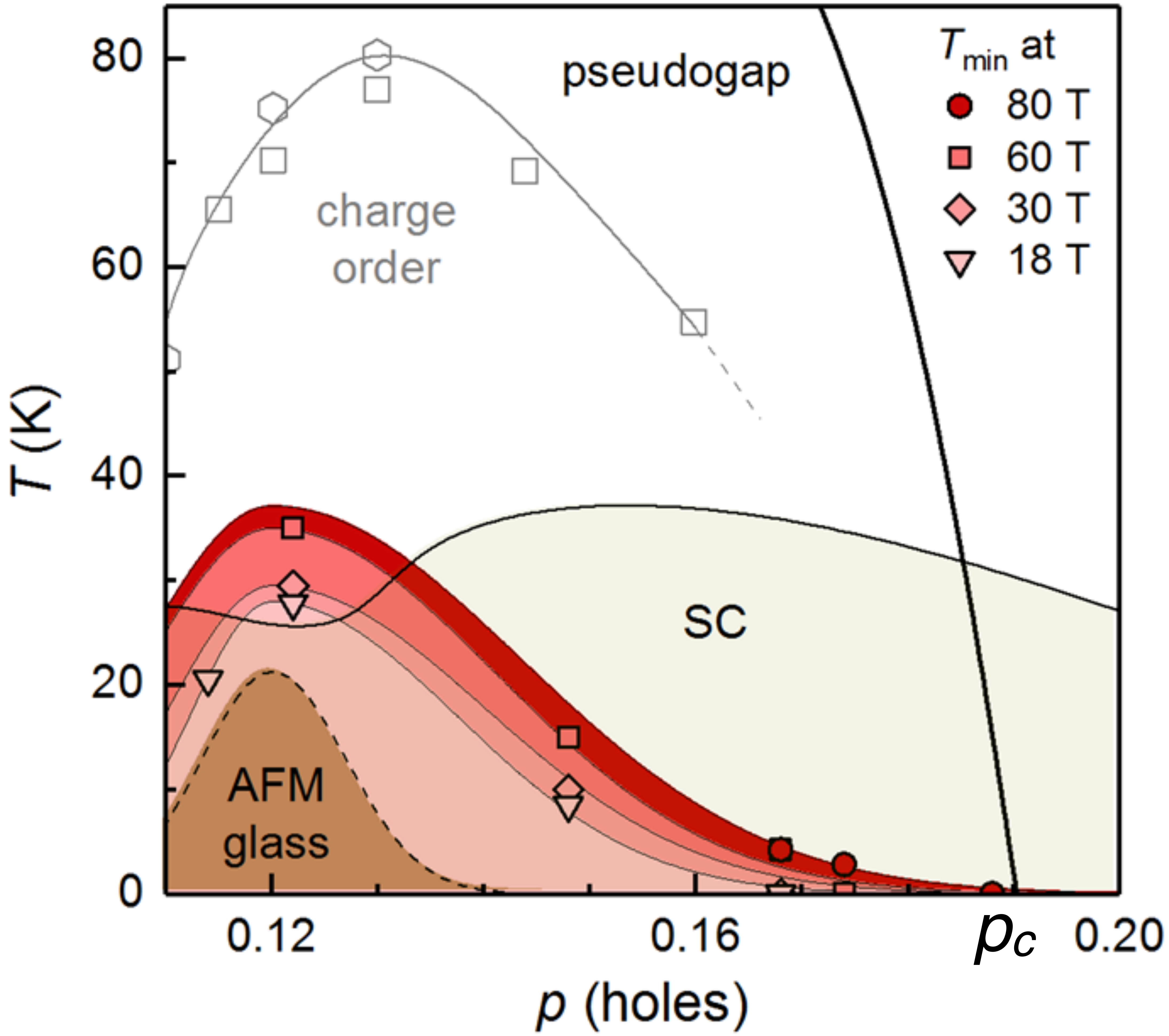
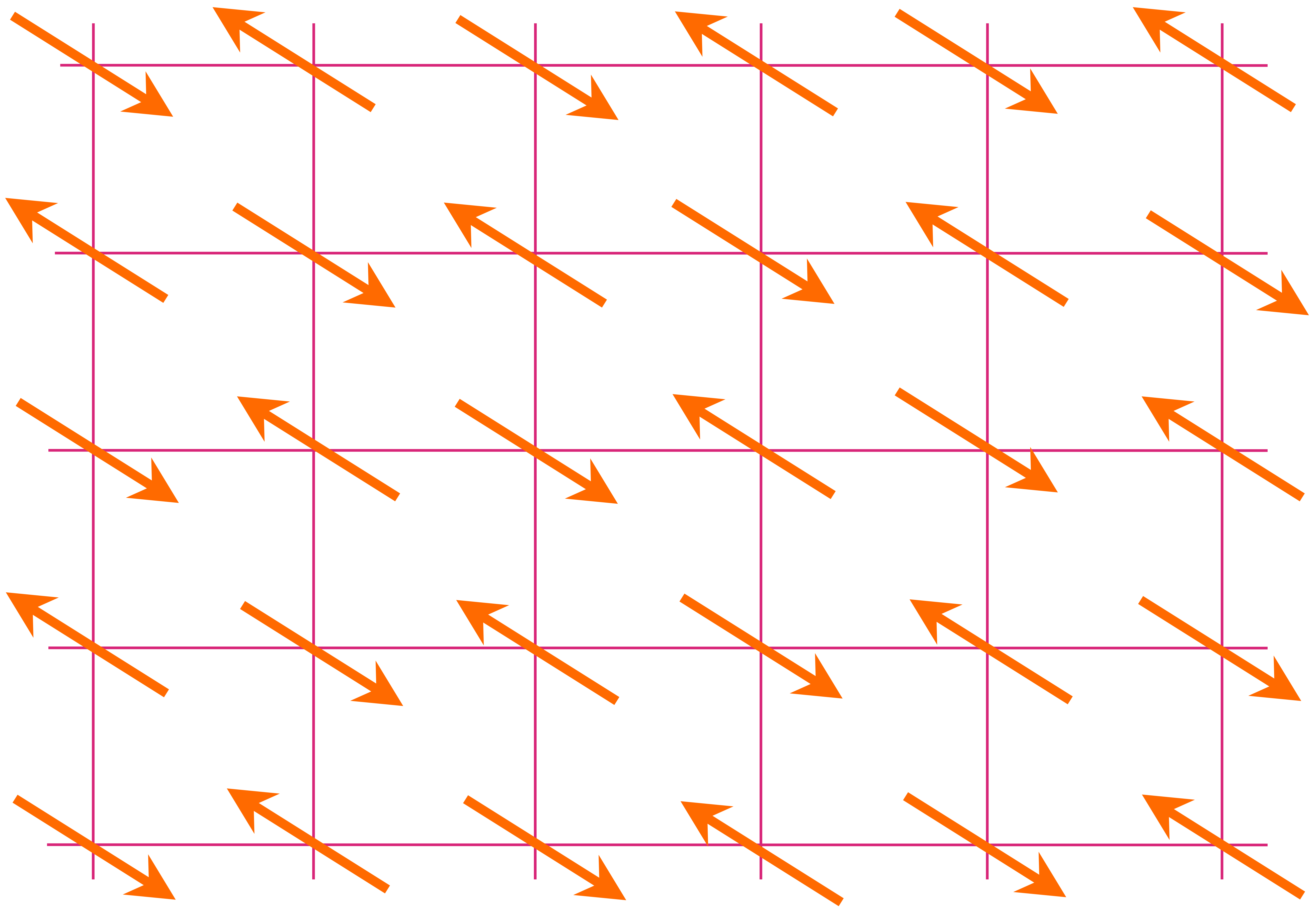
Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics **16**, 1064 (2020)

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



Insulating antiferromagnet



$p=0$

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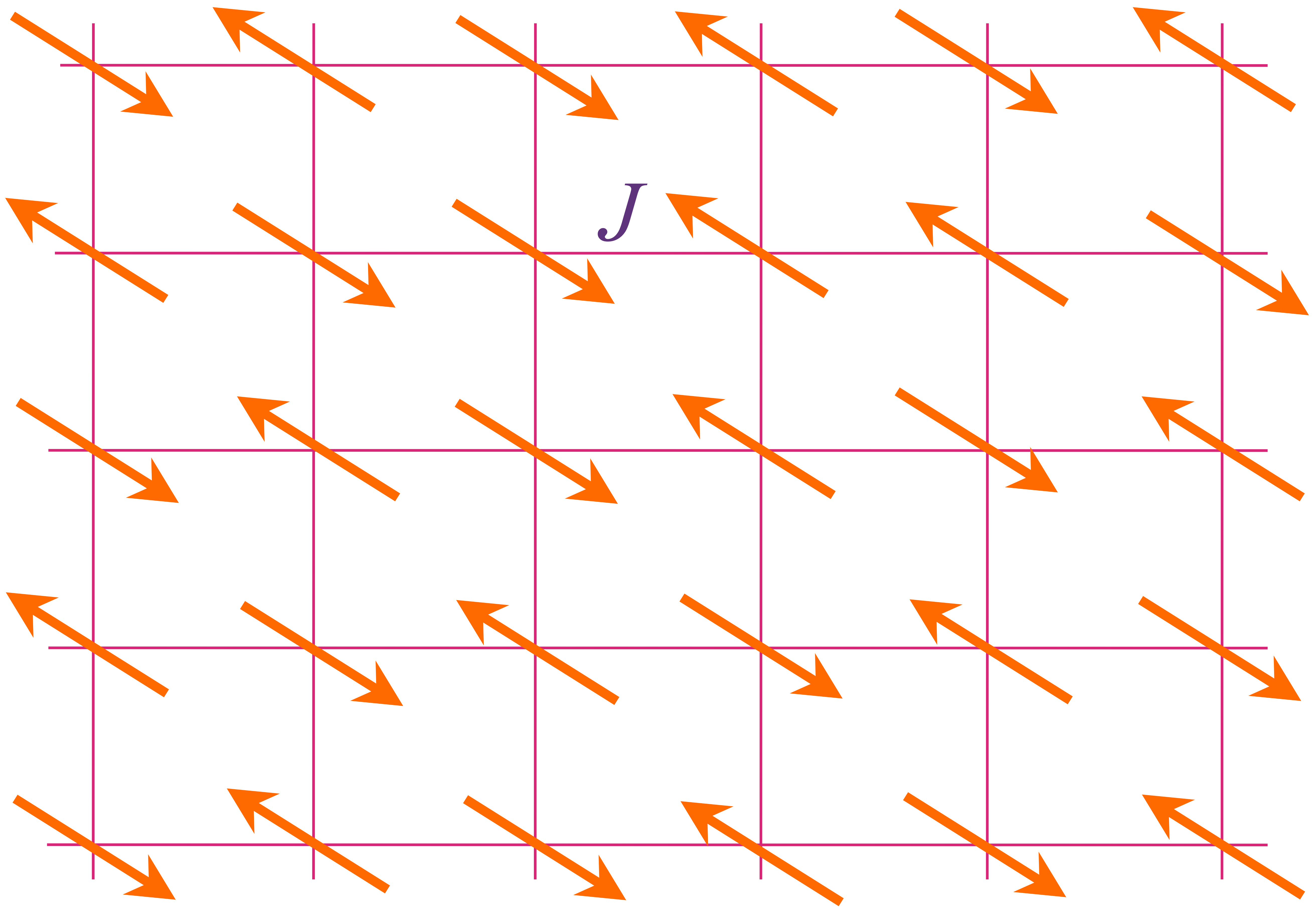

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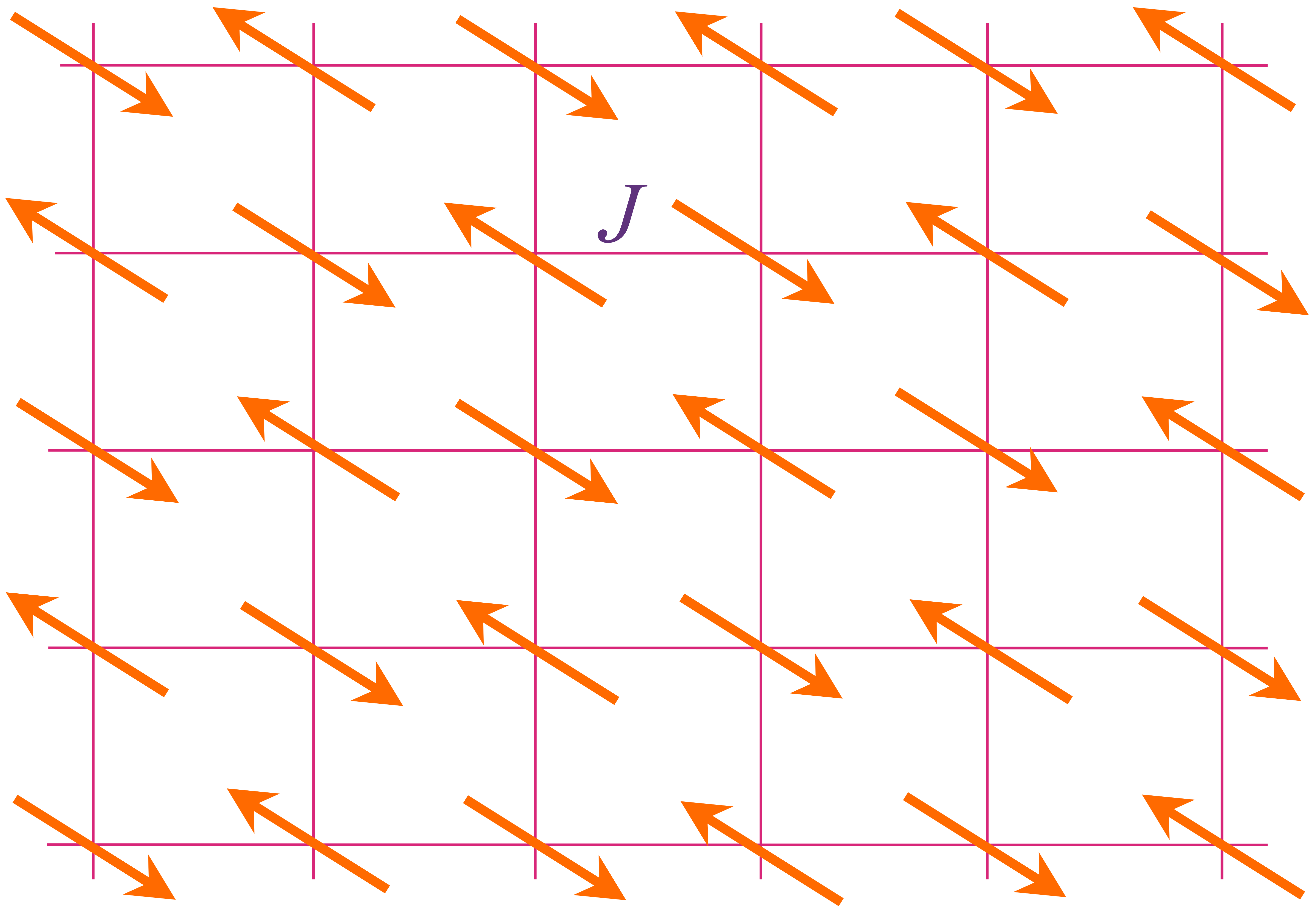

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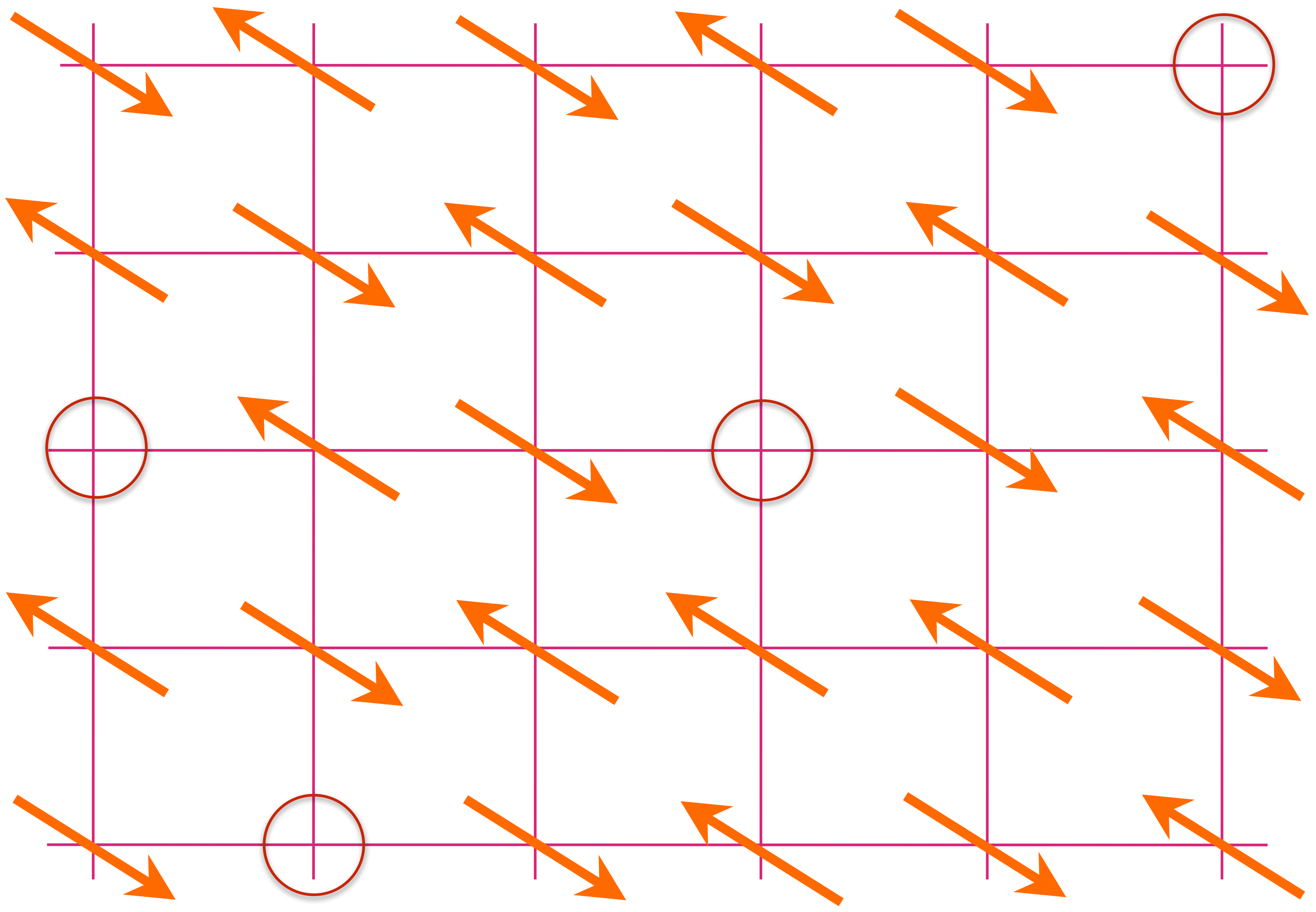
Antiferromagnet doped with hole density p



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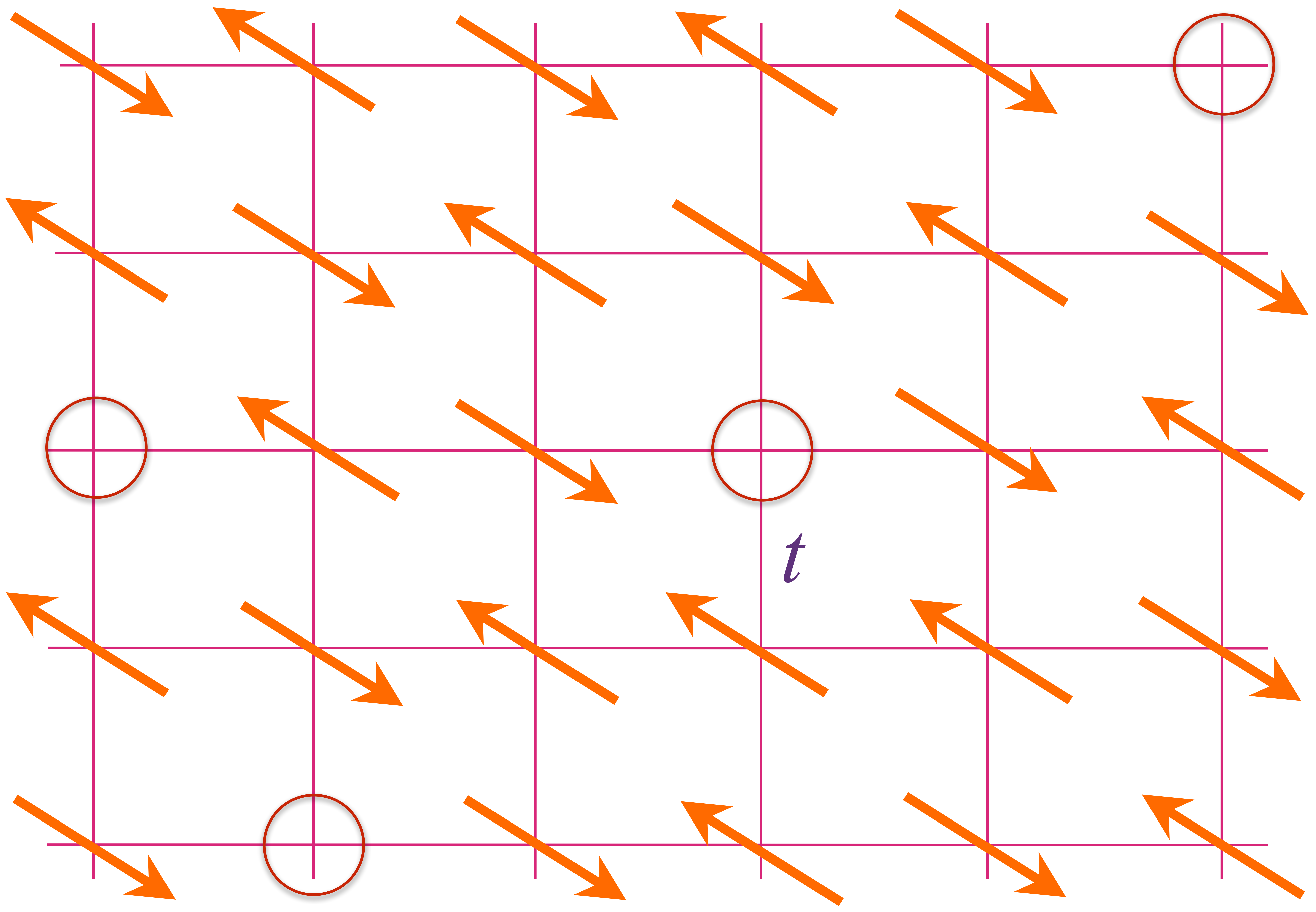

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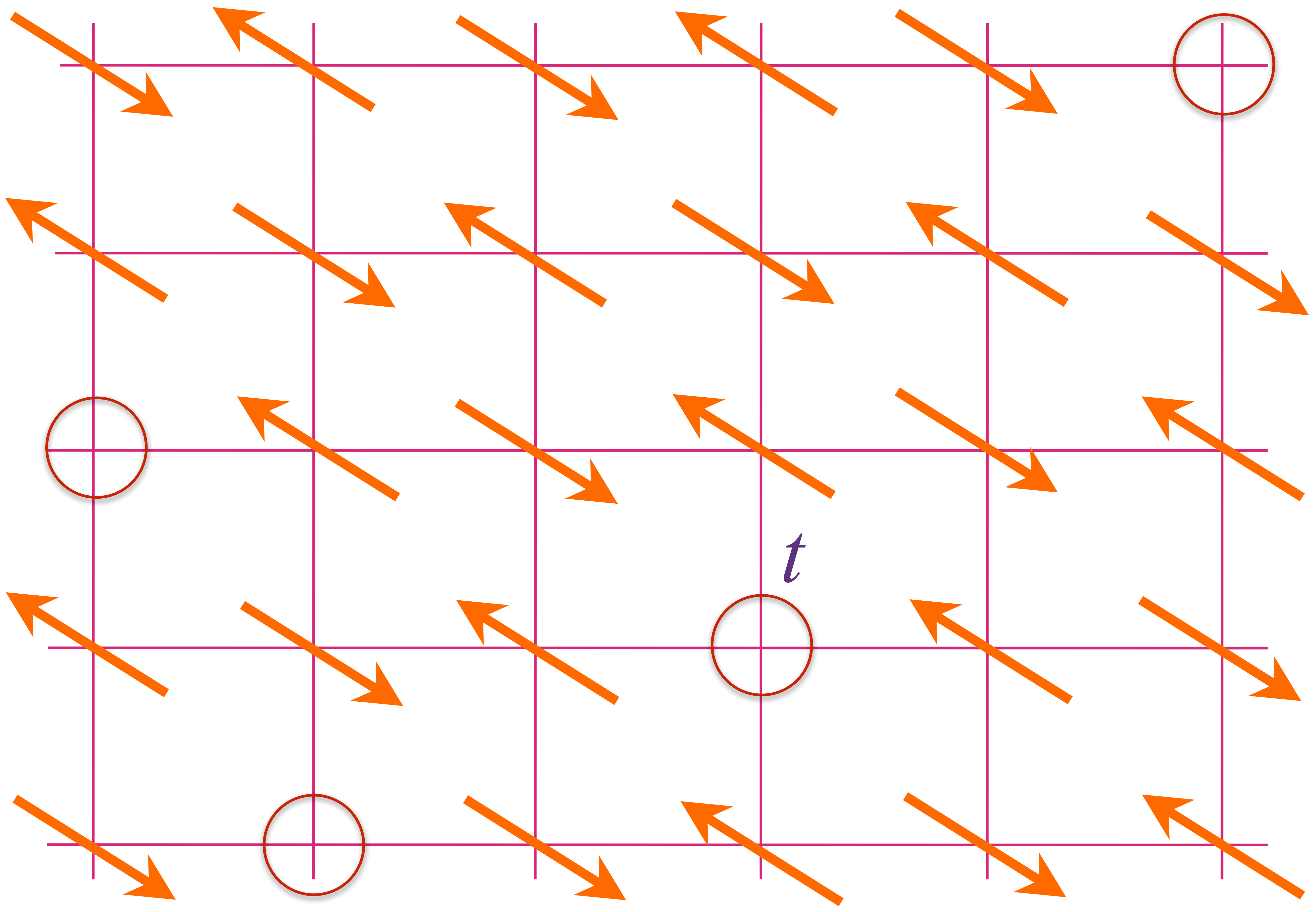

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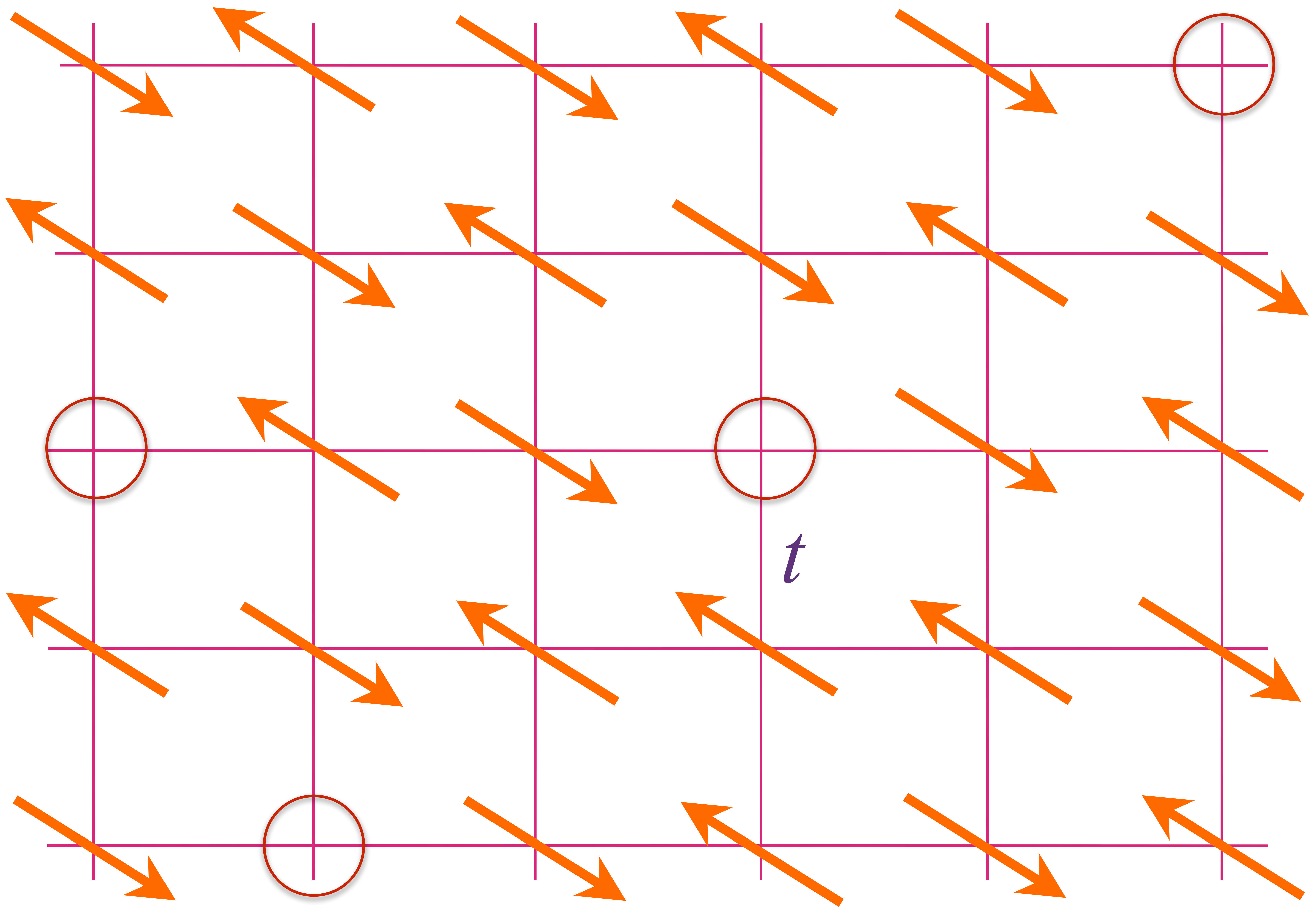

p mobile holes in a background of fluctuating spins

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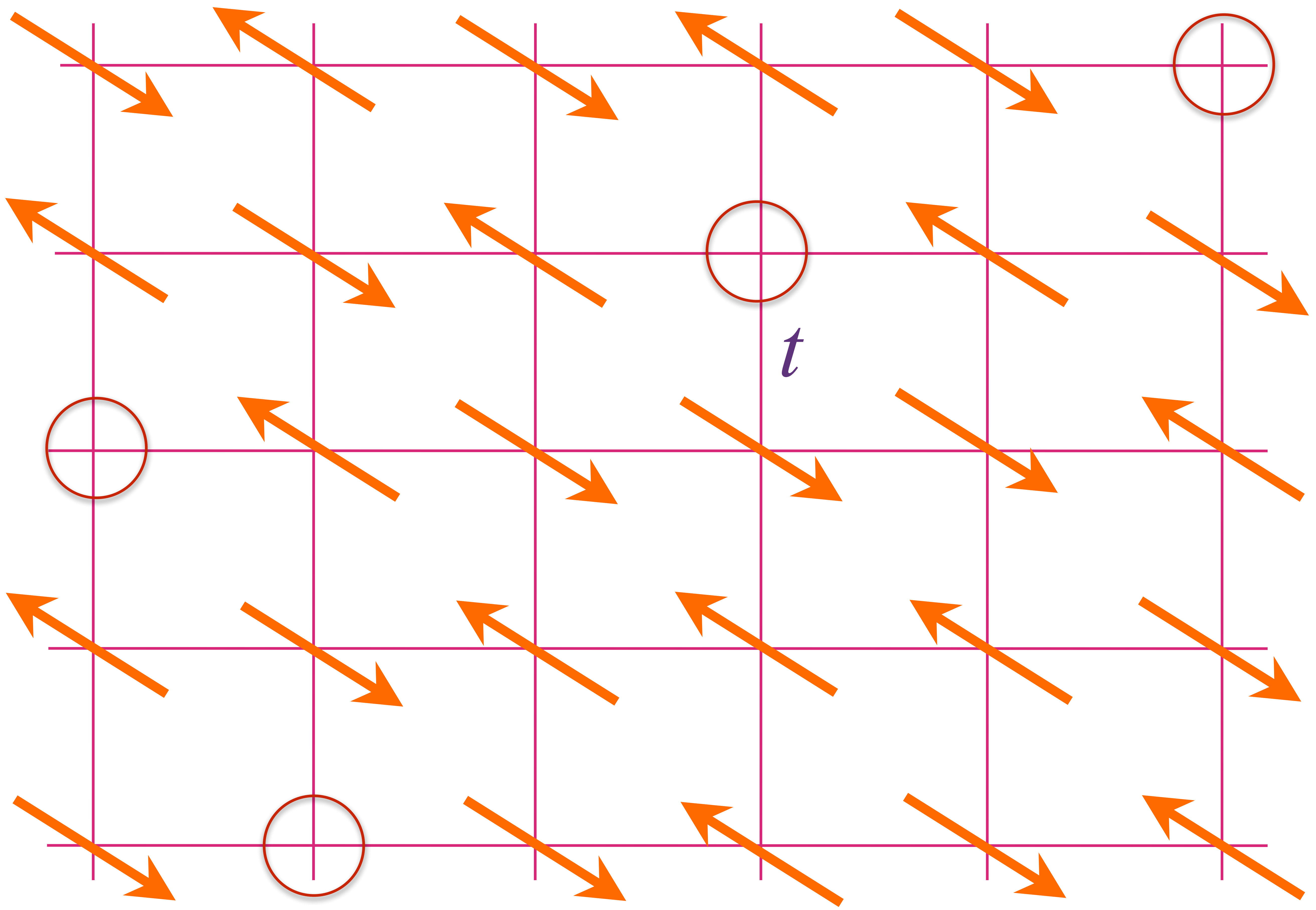

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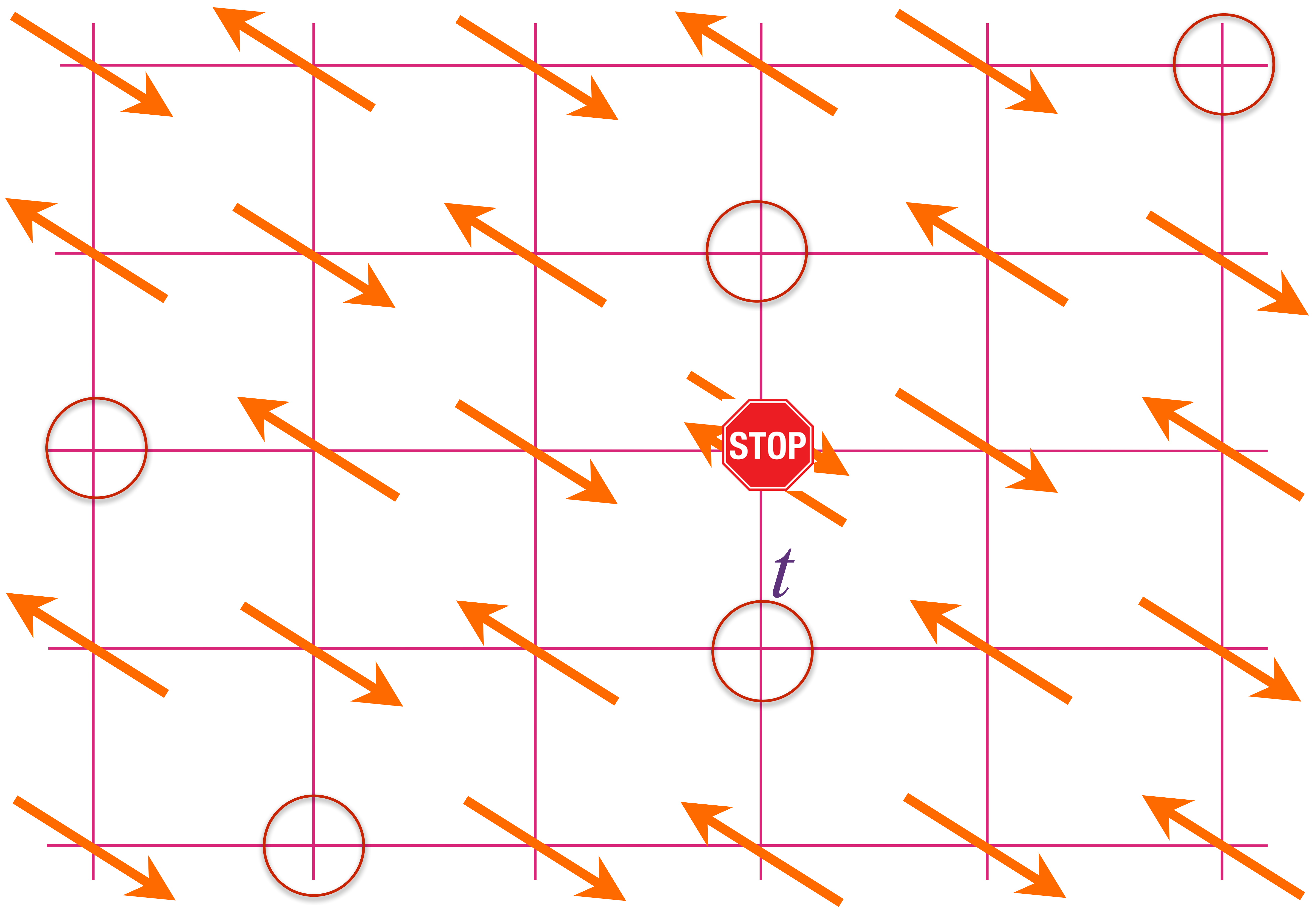

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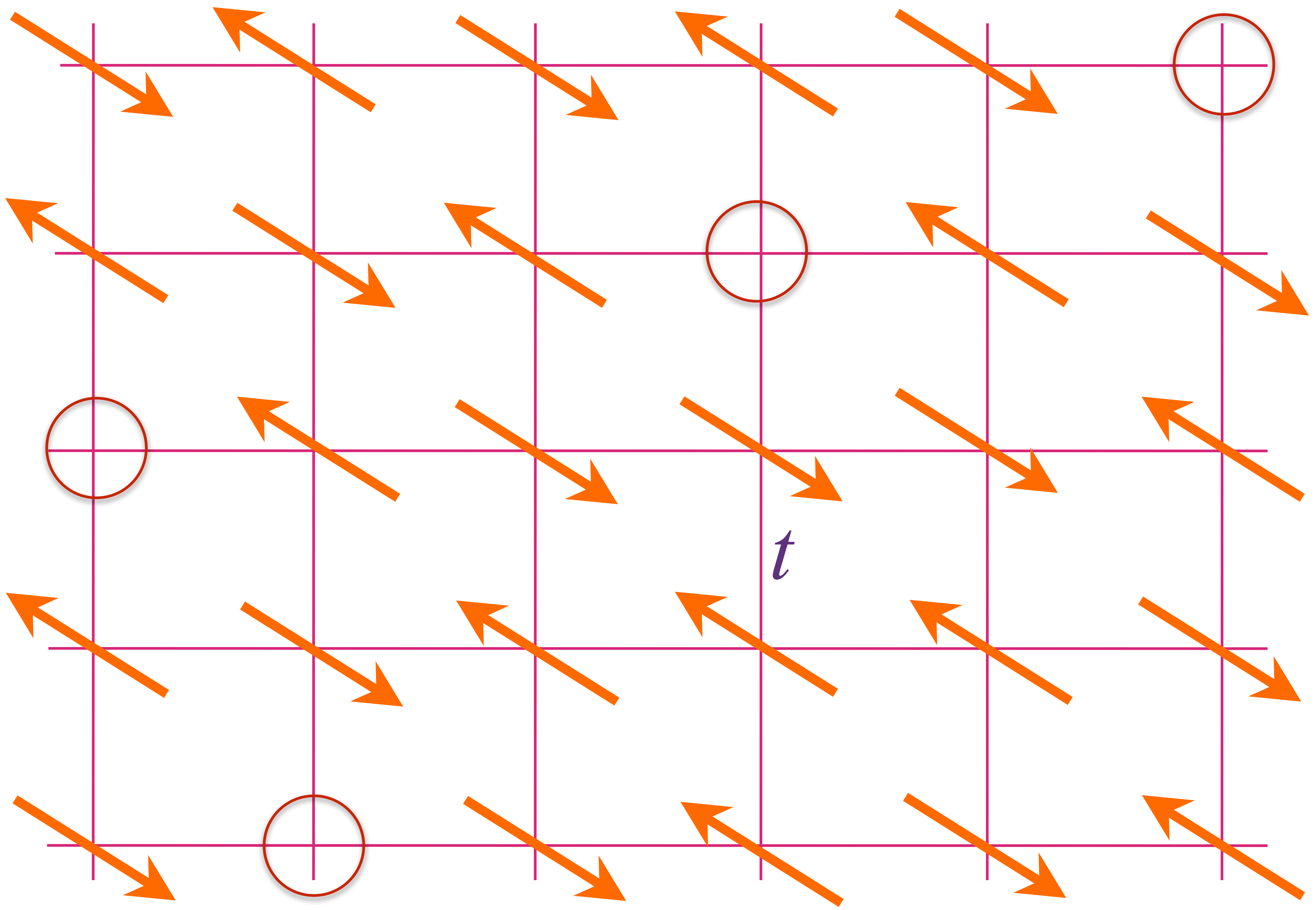

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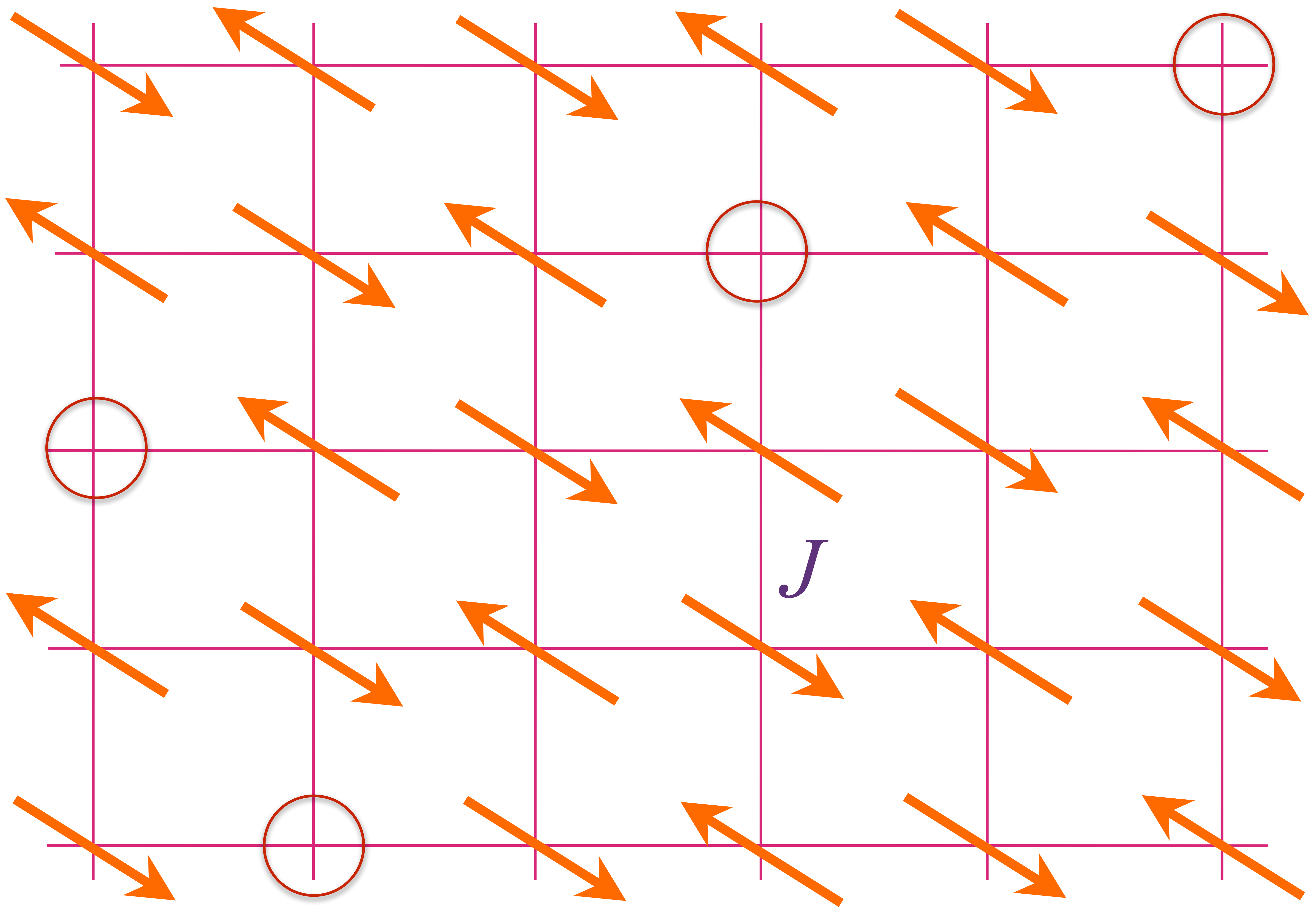

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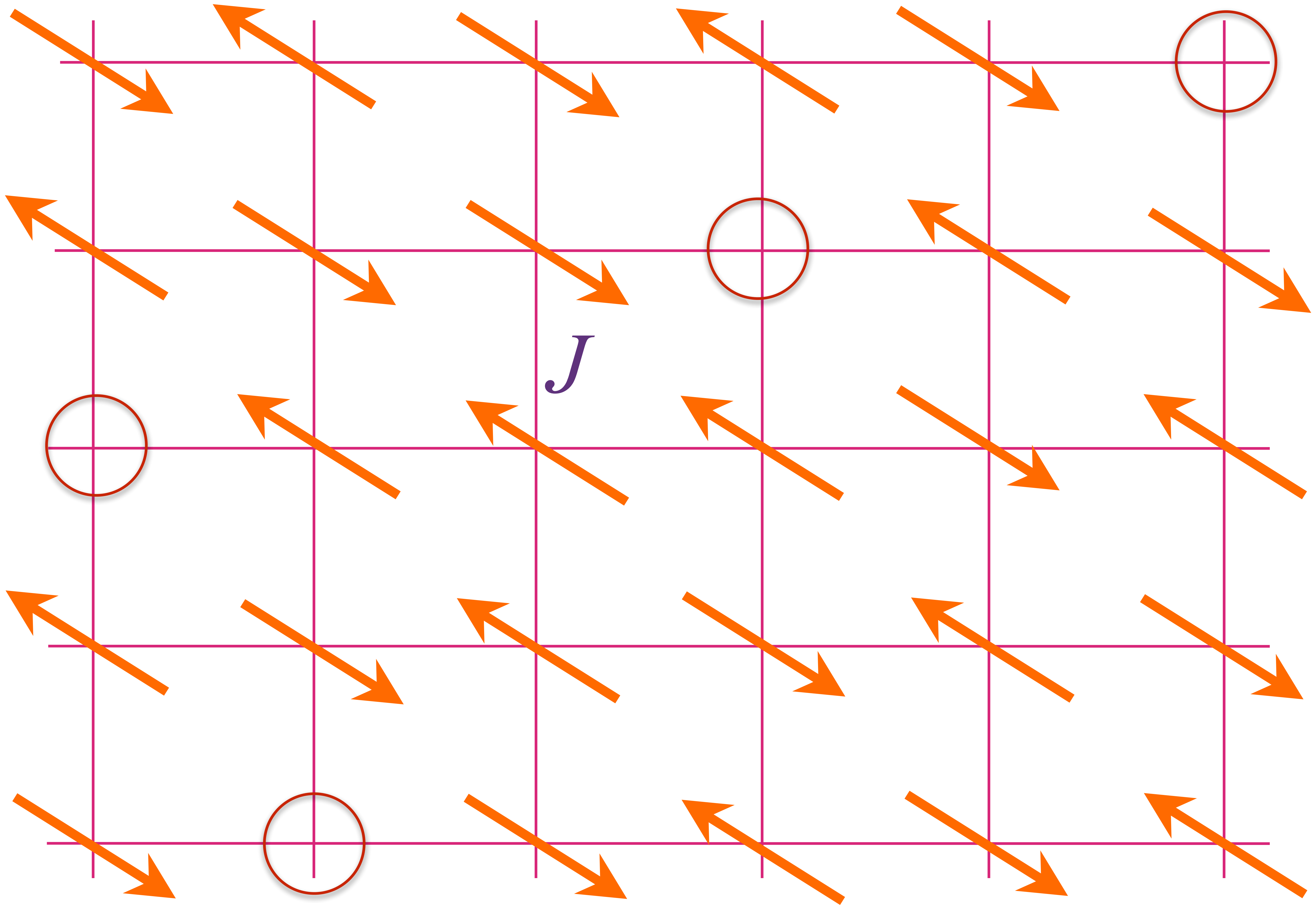


p mobile holes in a background of fluctuating spins

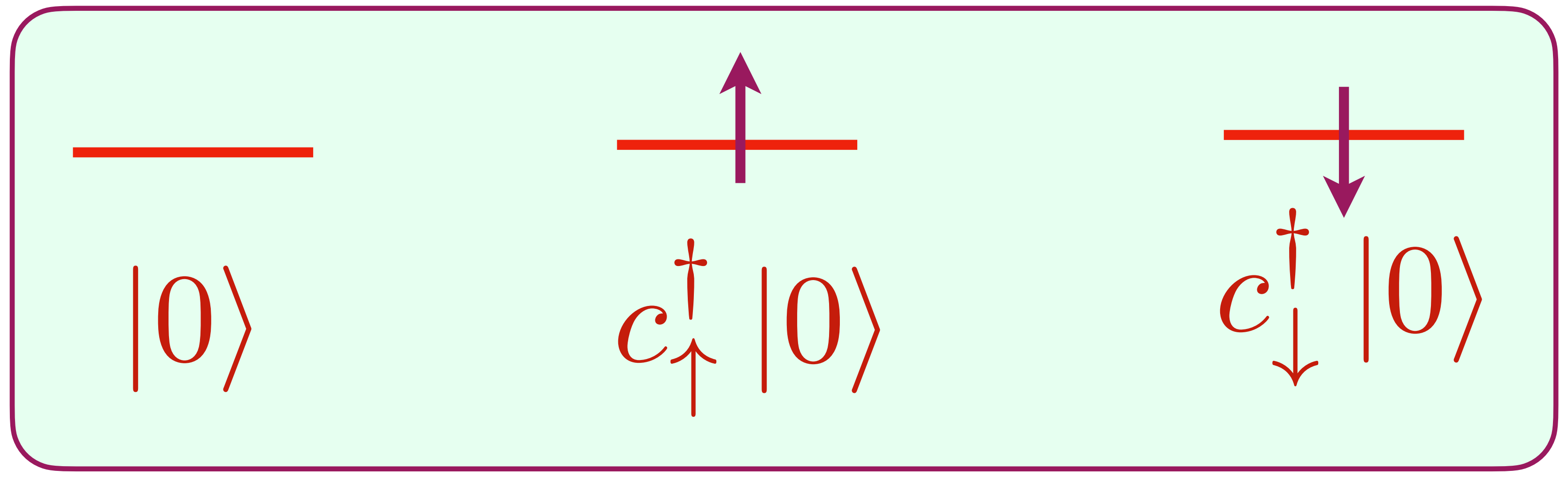
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \boxed{\sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1,} \quad \frac{1}{N}\sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$



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$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$
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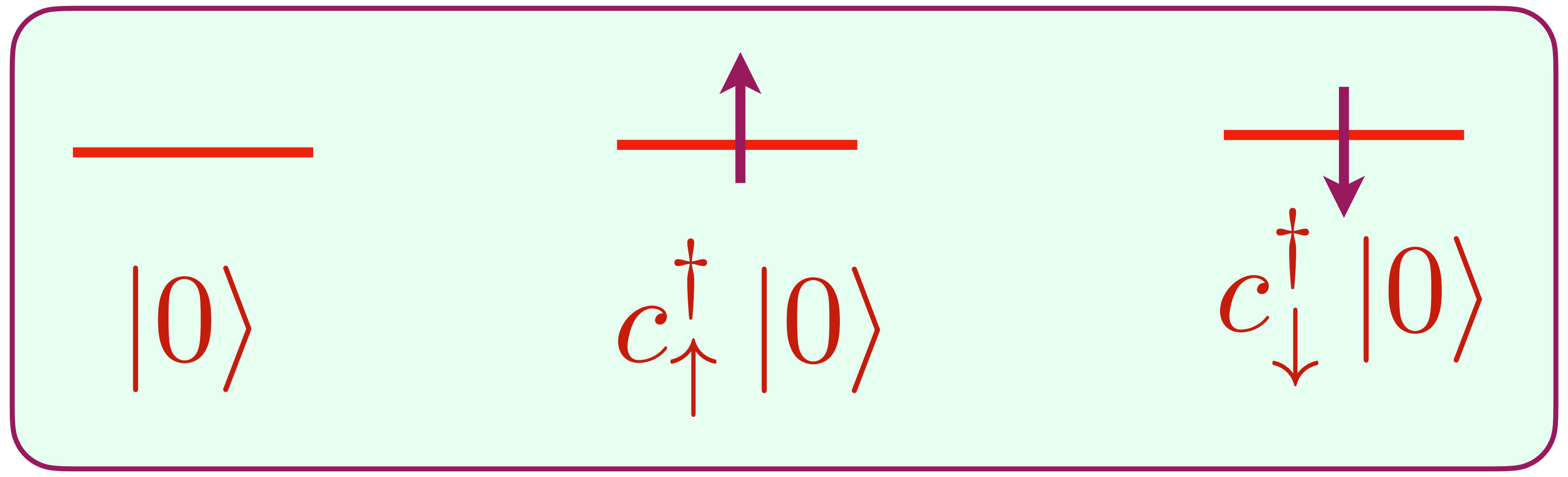

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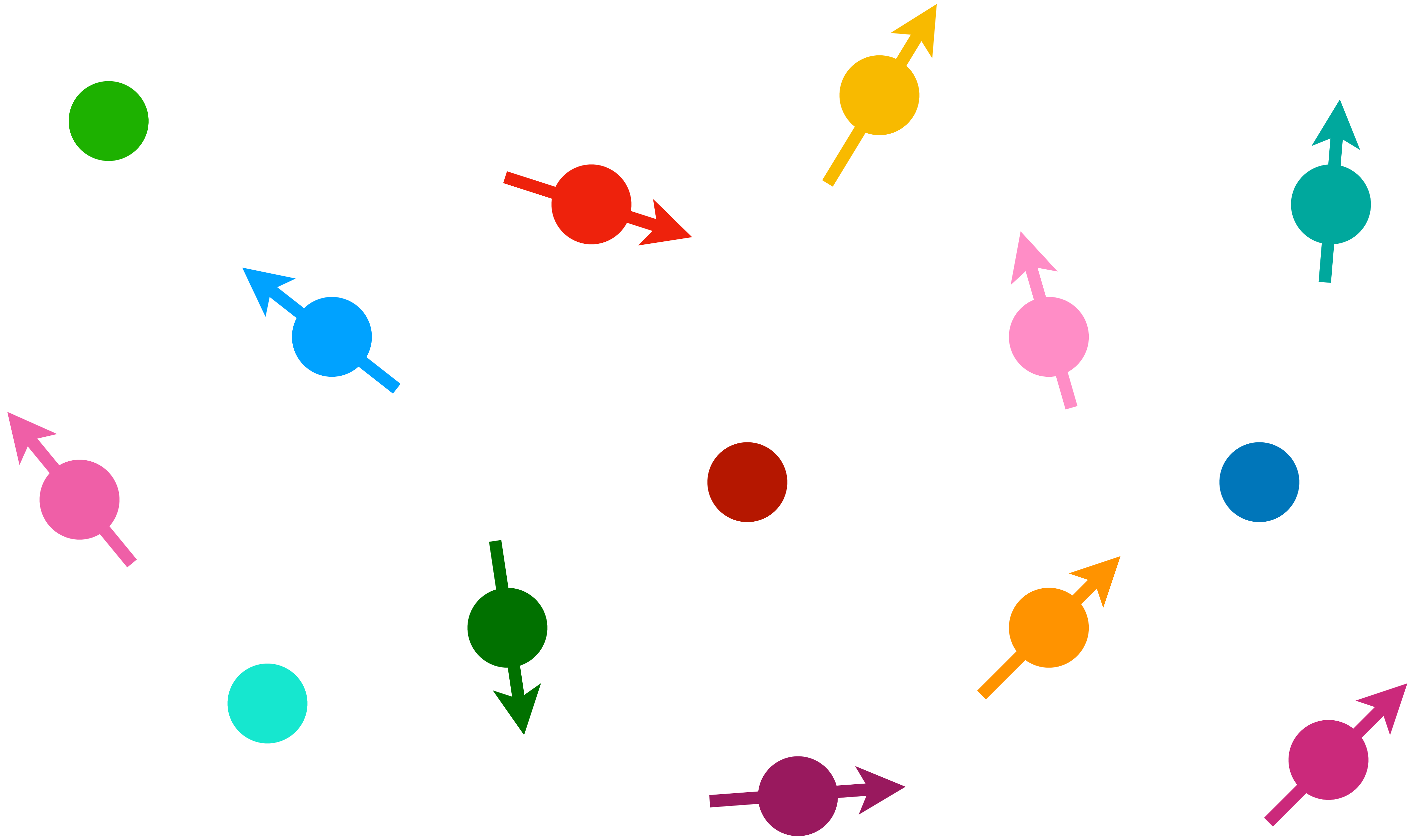

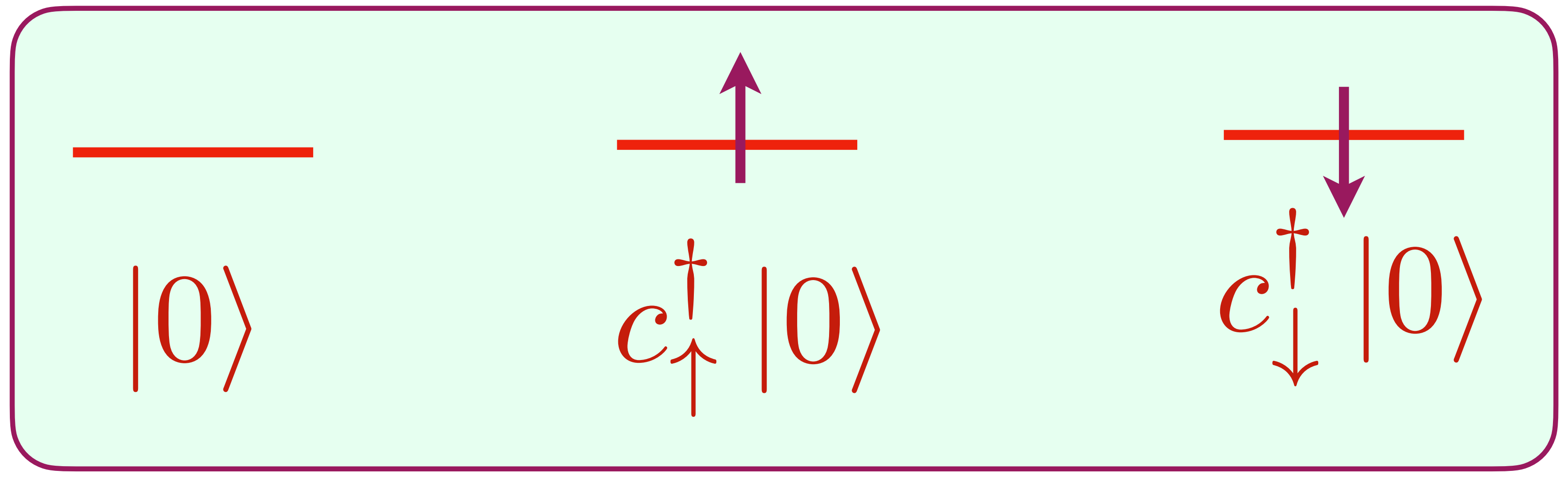

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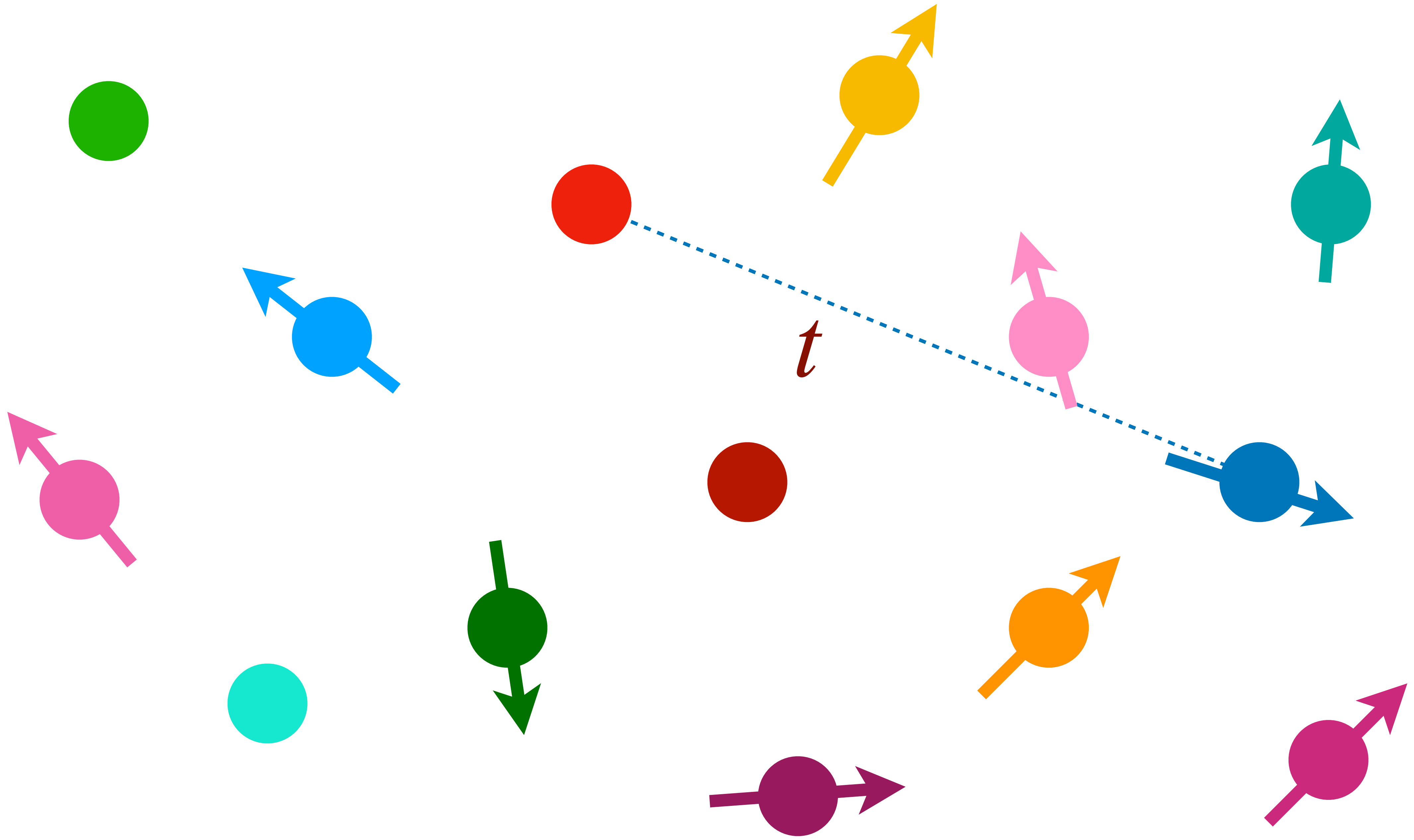

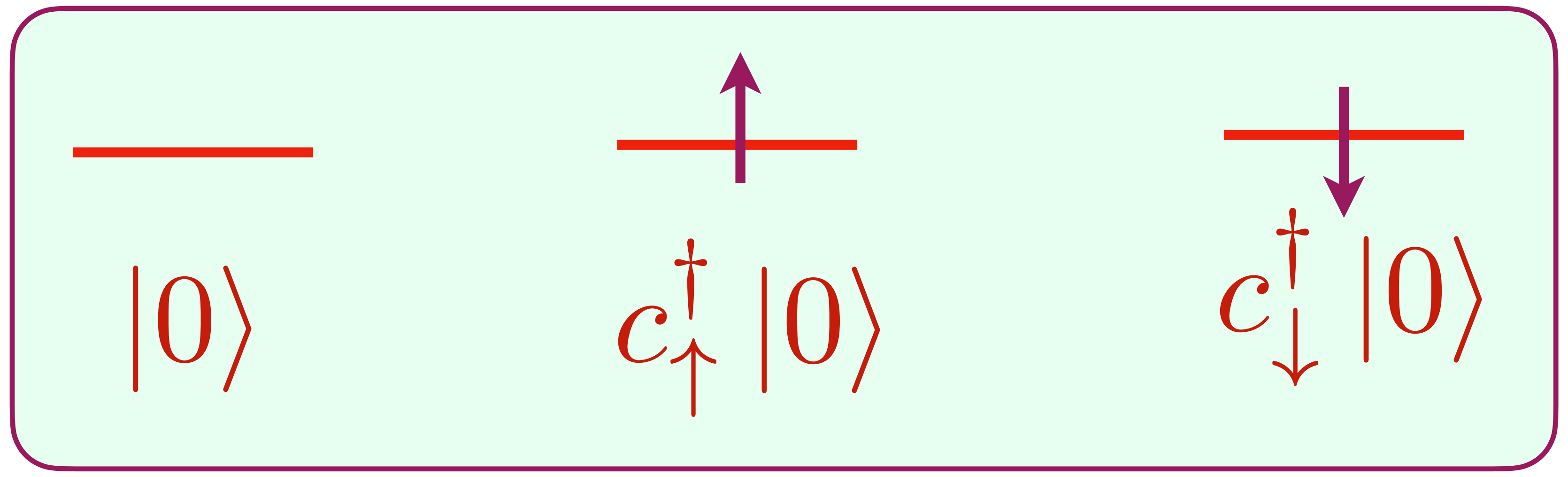

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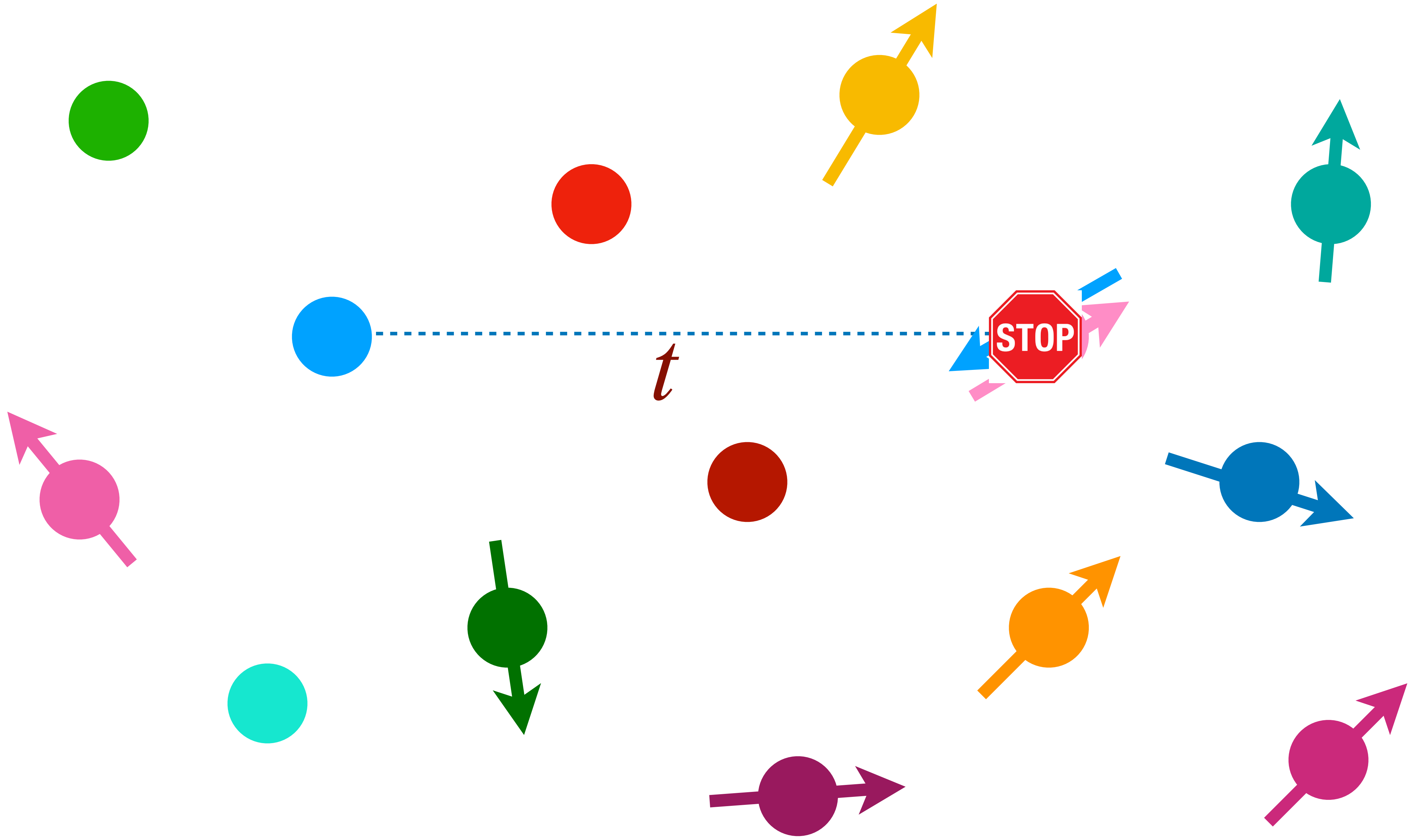

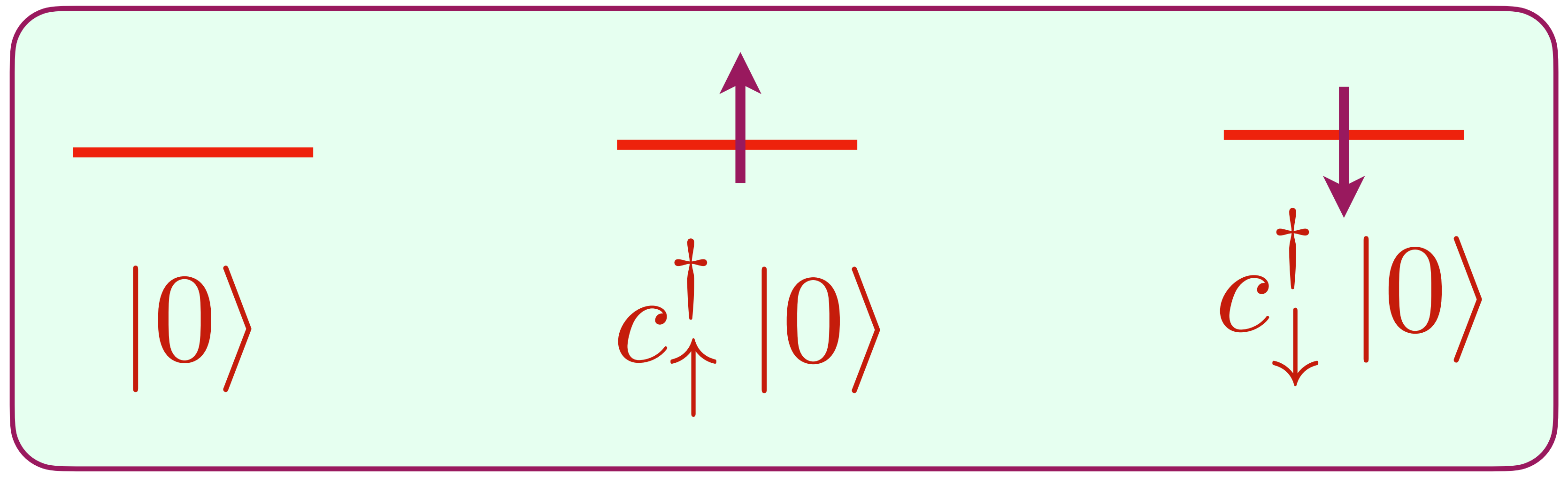

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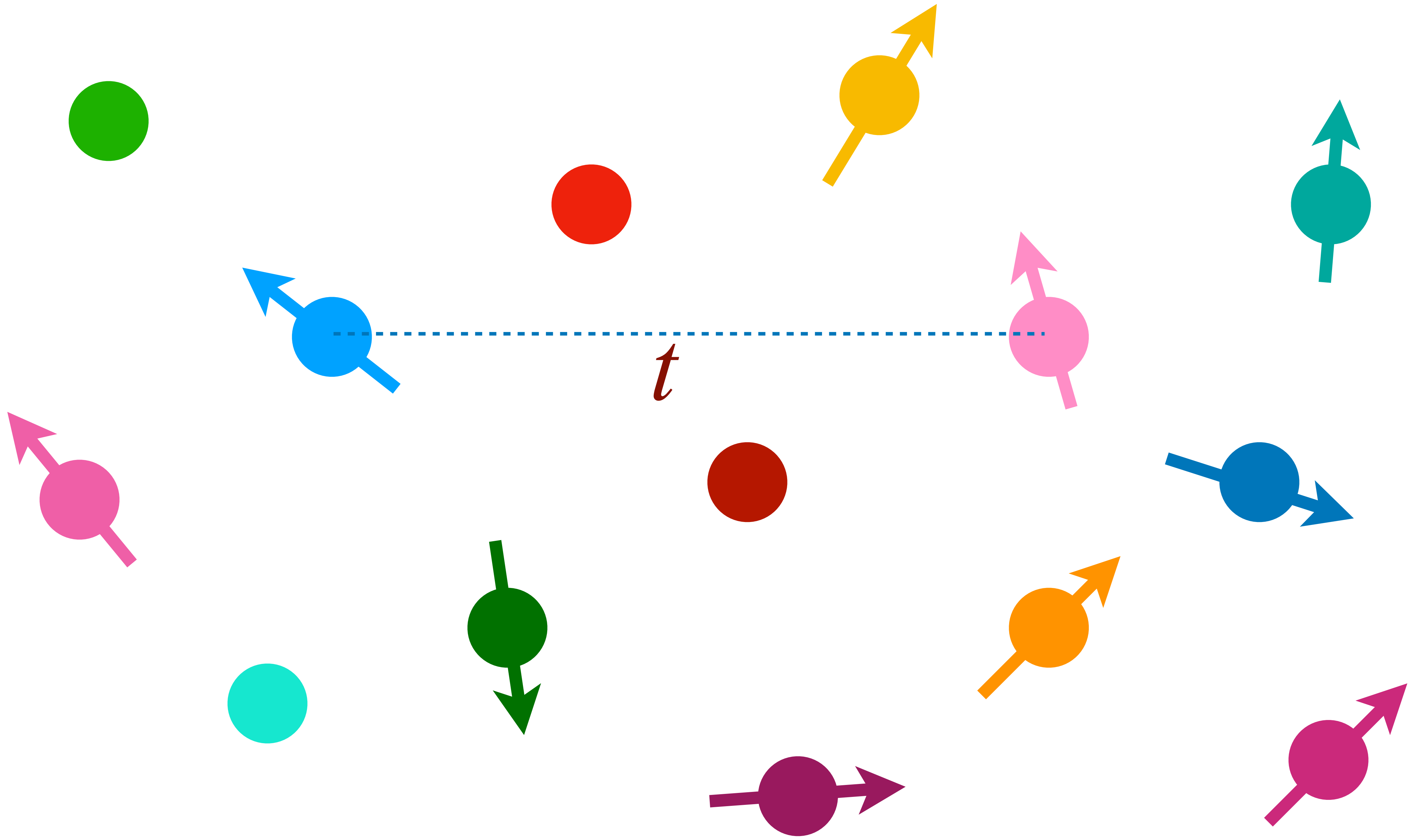

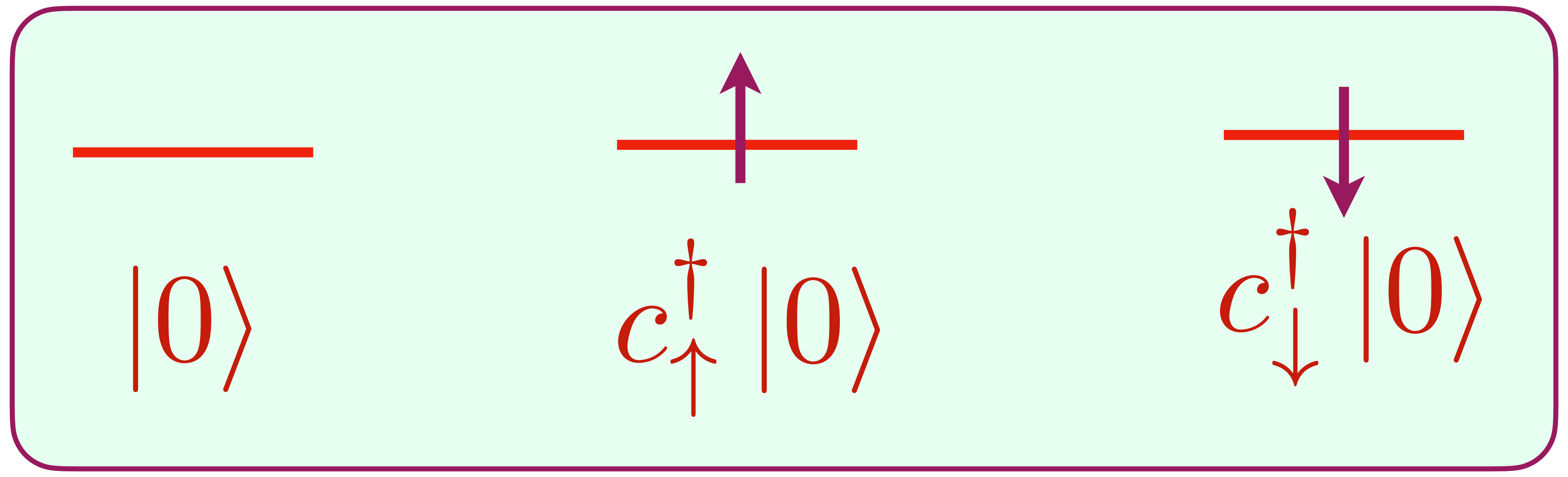

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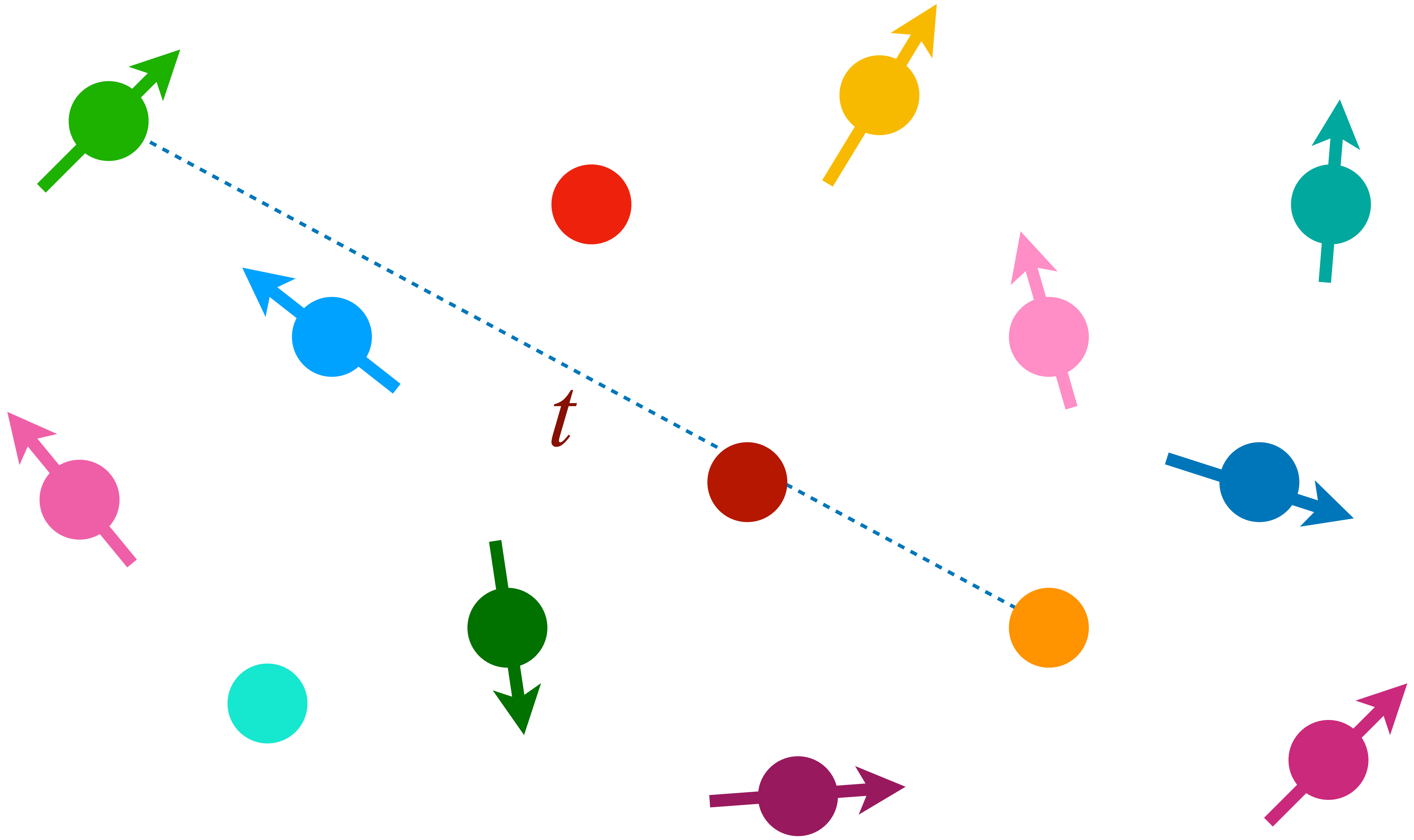

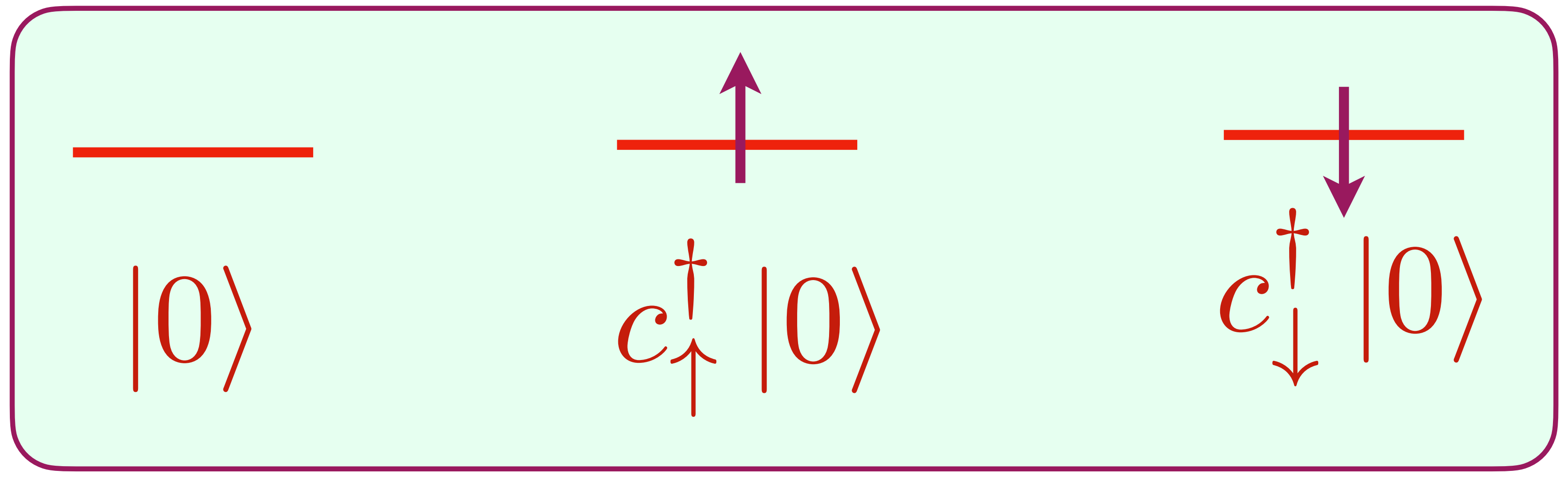

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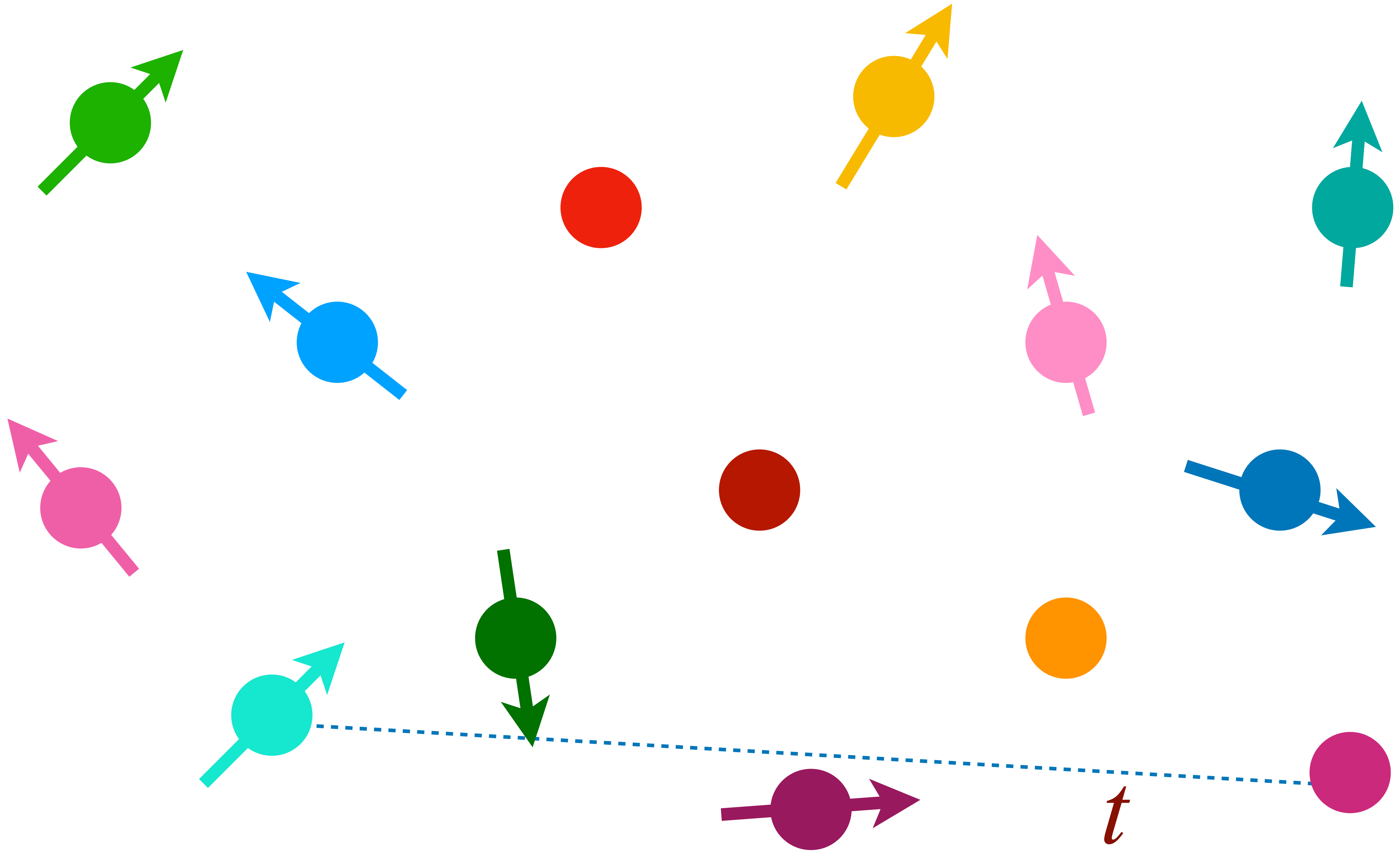

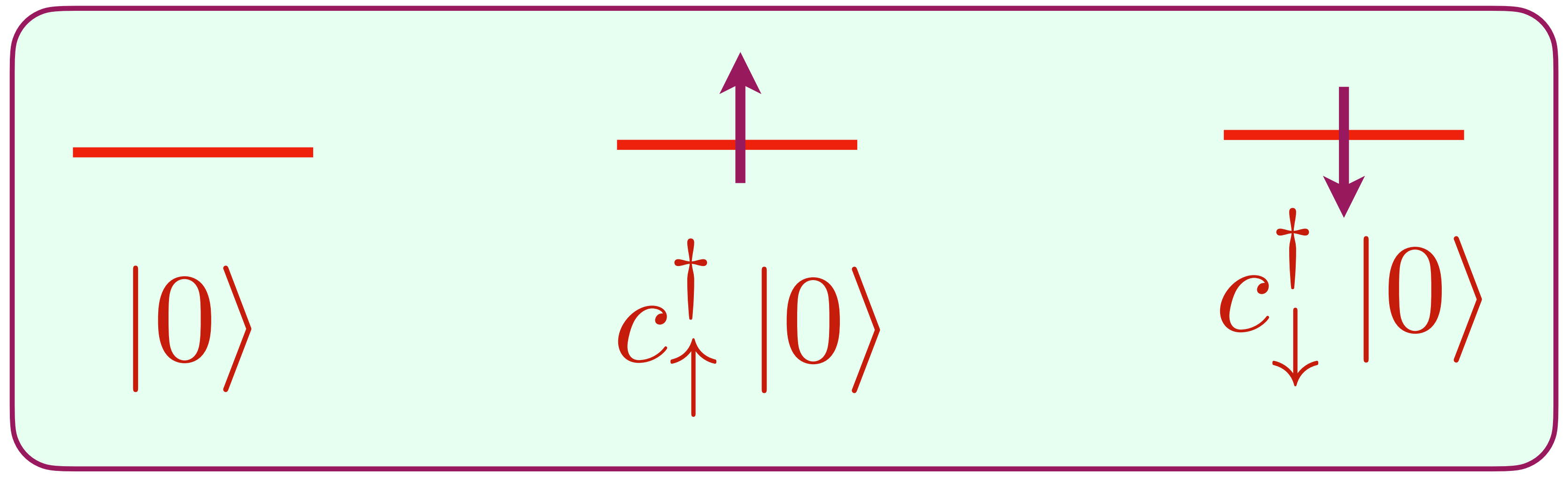

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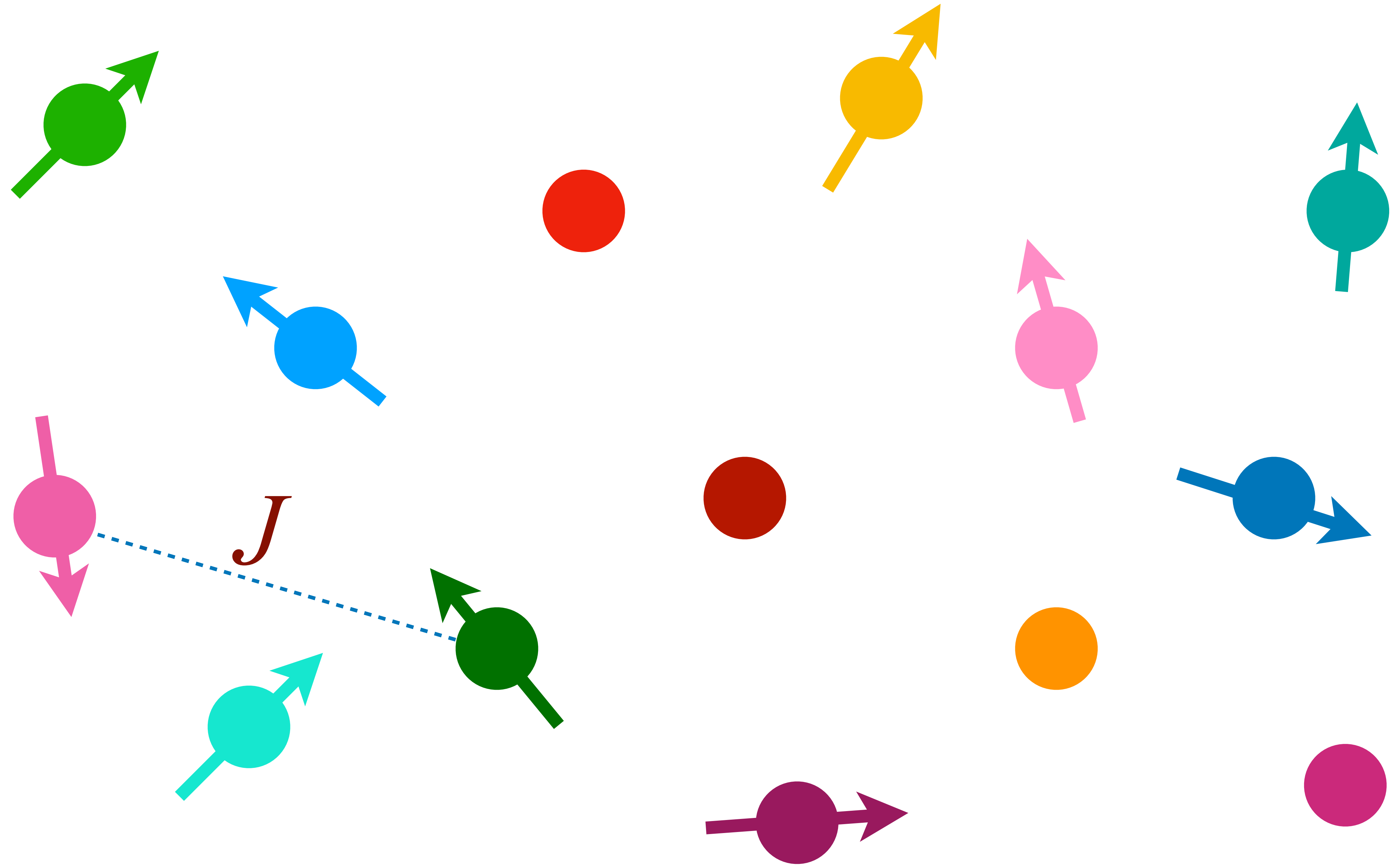

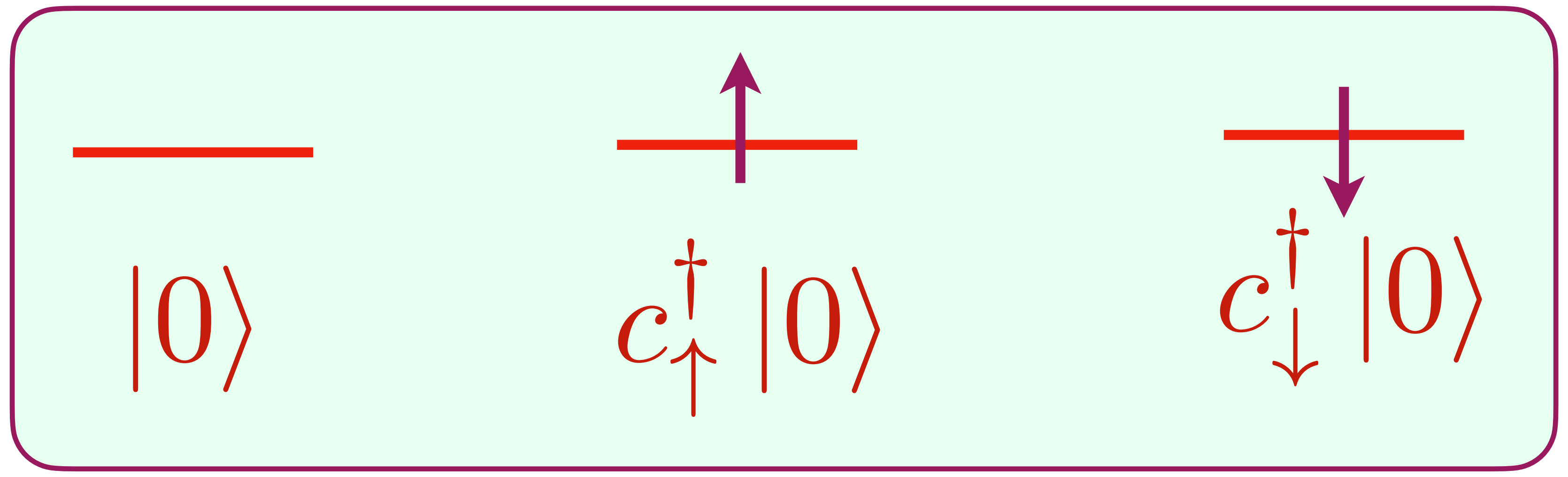

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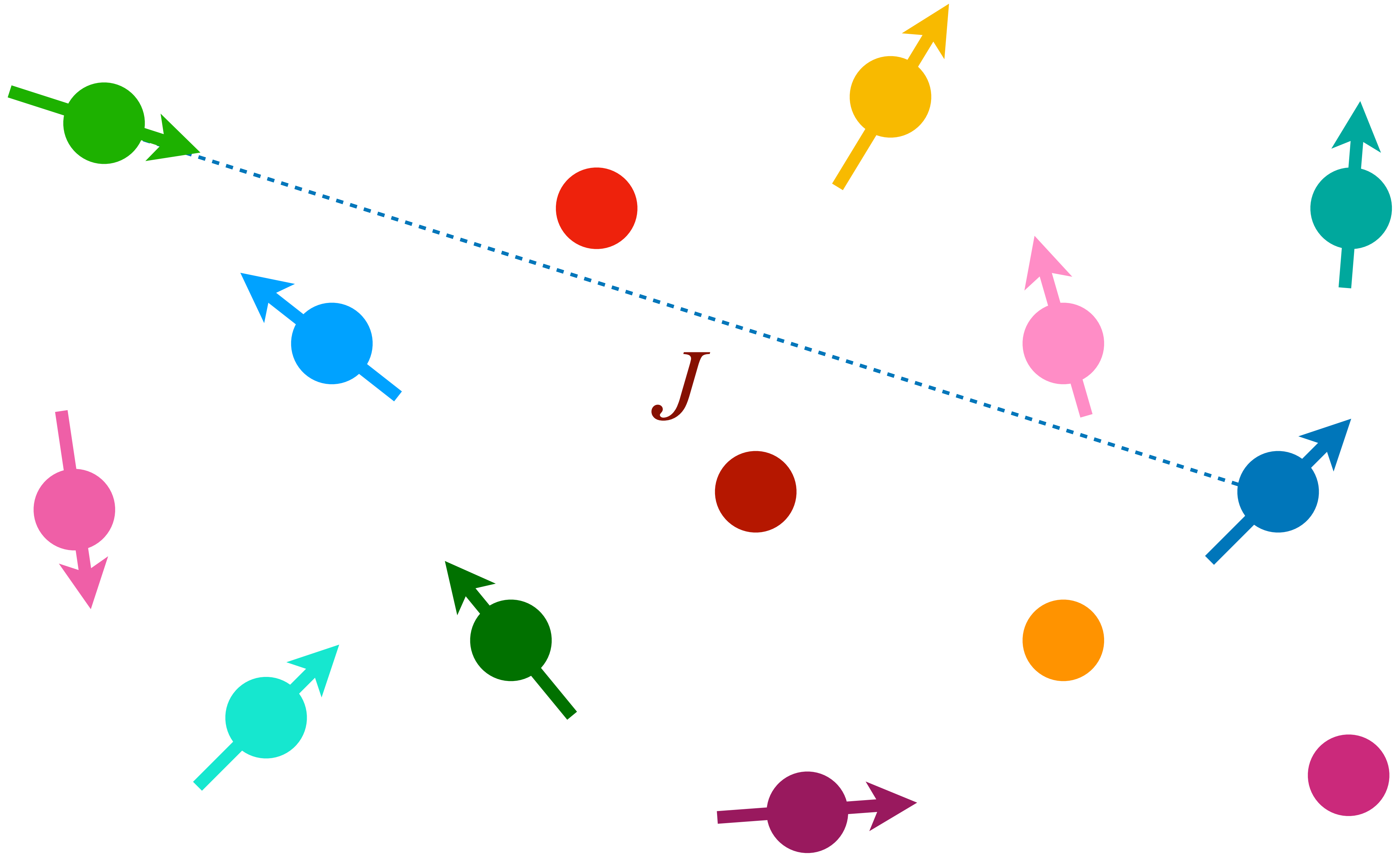

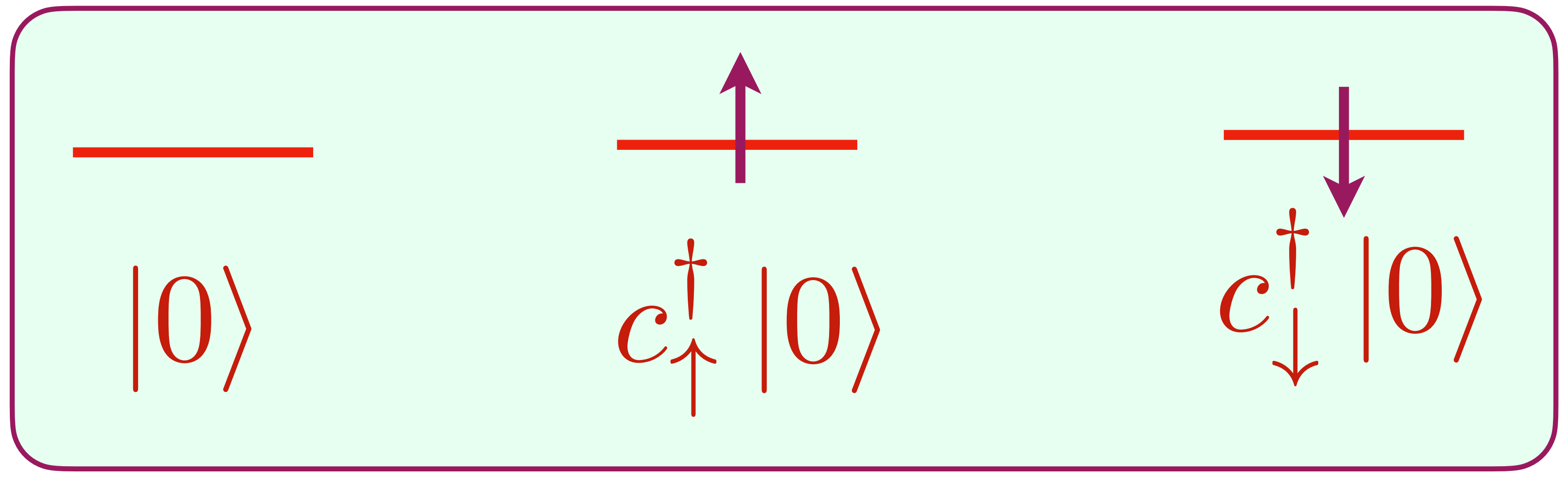

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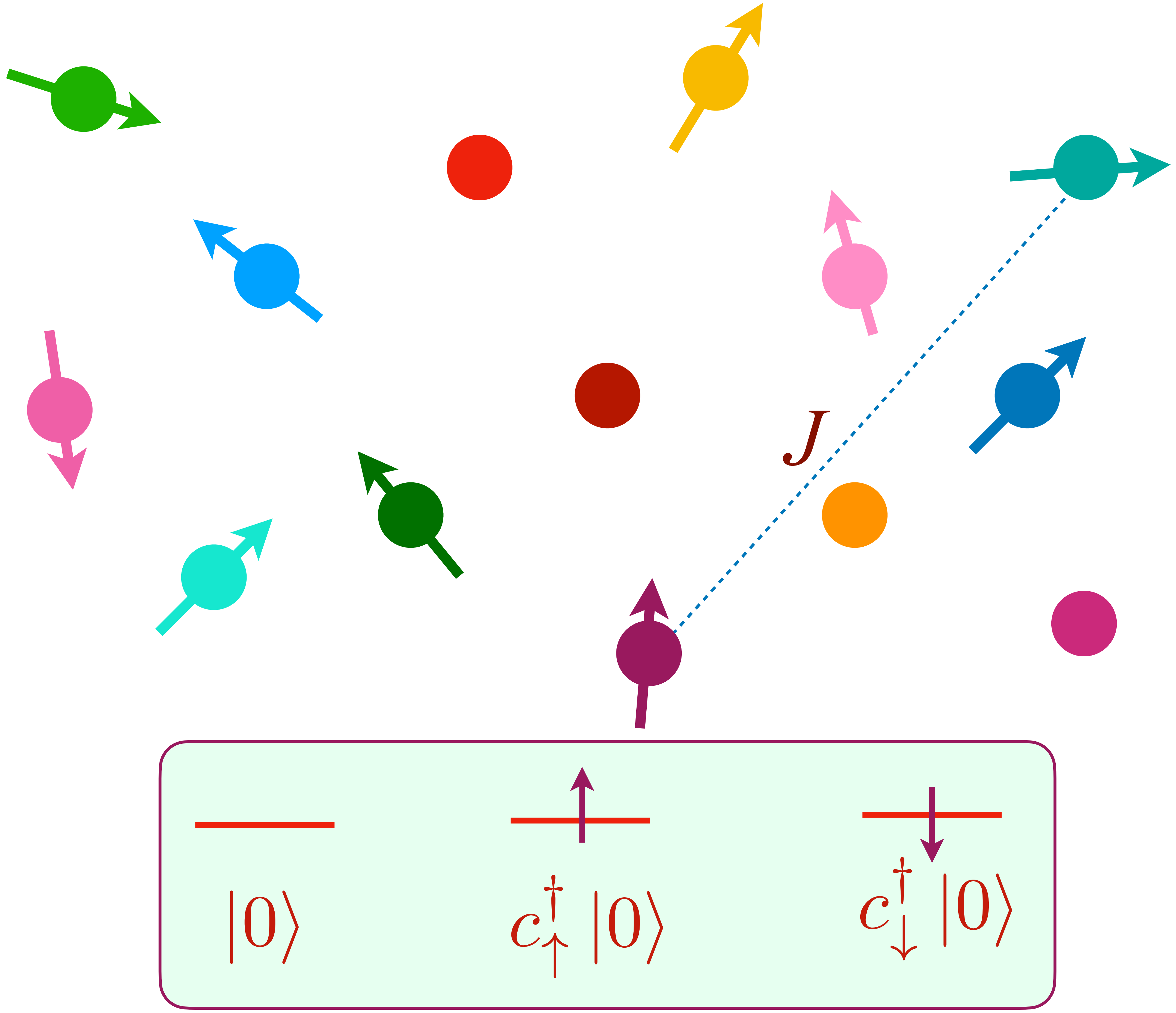

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- We will show that the random *t-J* model retains the key features of the phase diagram: the Fermi liquid and the pseudogap metal, and Planckian behavior between them.

1. Numerical results

Exact diagonalization and DMFT+Monte Carlo

2. Parton representations

The pseudogap metal and the Fermi liquid

3. SYK criticality of partons

The Planckian metal

Random *t-J* model: phase diagram

Metallic spin glass.

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

SYK criticality

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

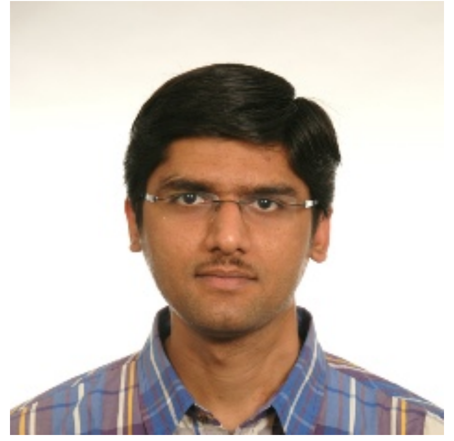
Disordered Fermi liquid.

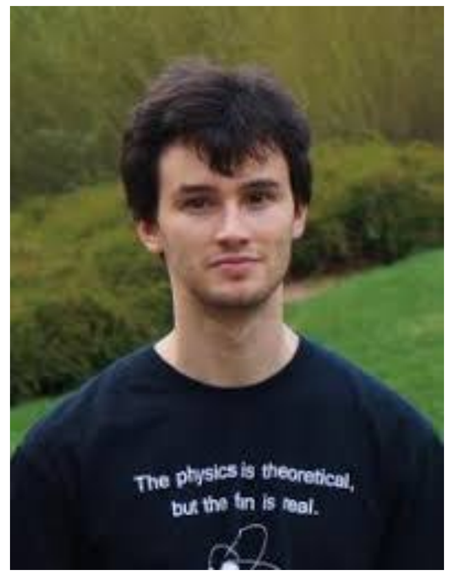
Luttinger Fermi energy

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

p_c $\qquad p$



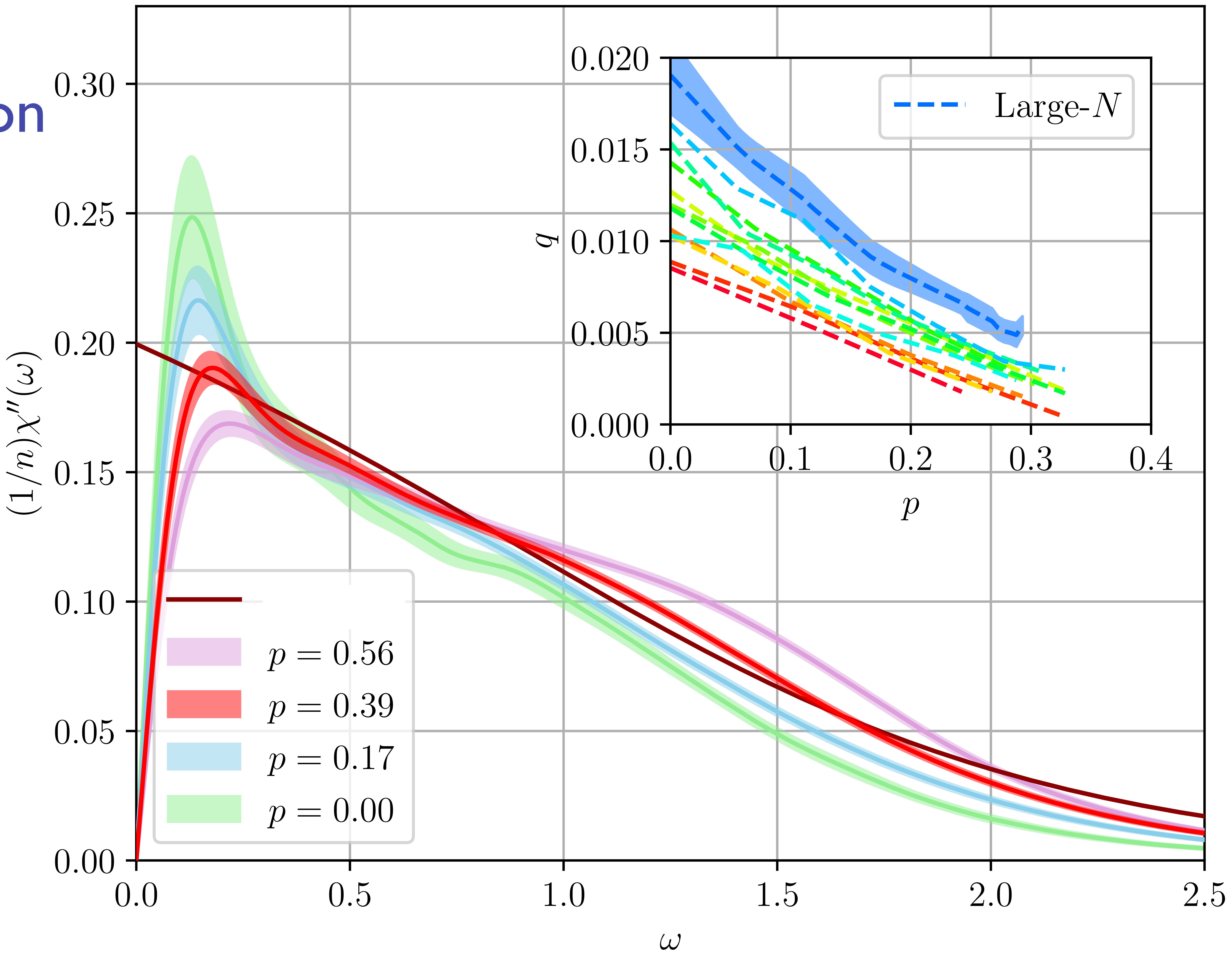


D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

$$\chi = \int_0^\beta d\tau \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle$$

Dynamic spin susceptibility

Exact diagonalization



H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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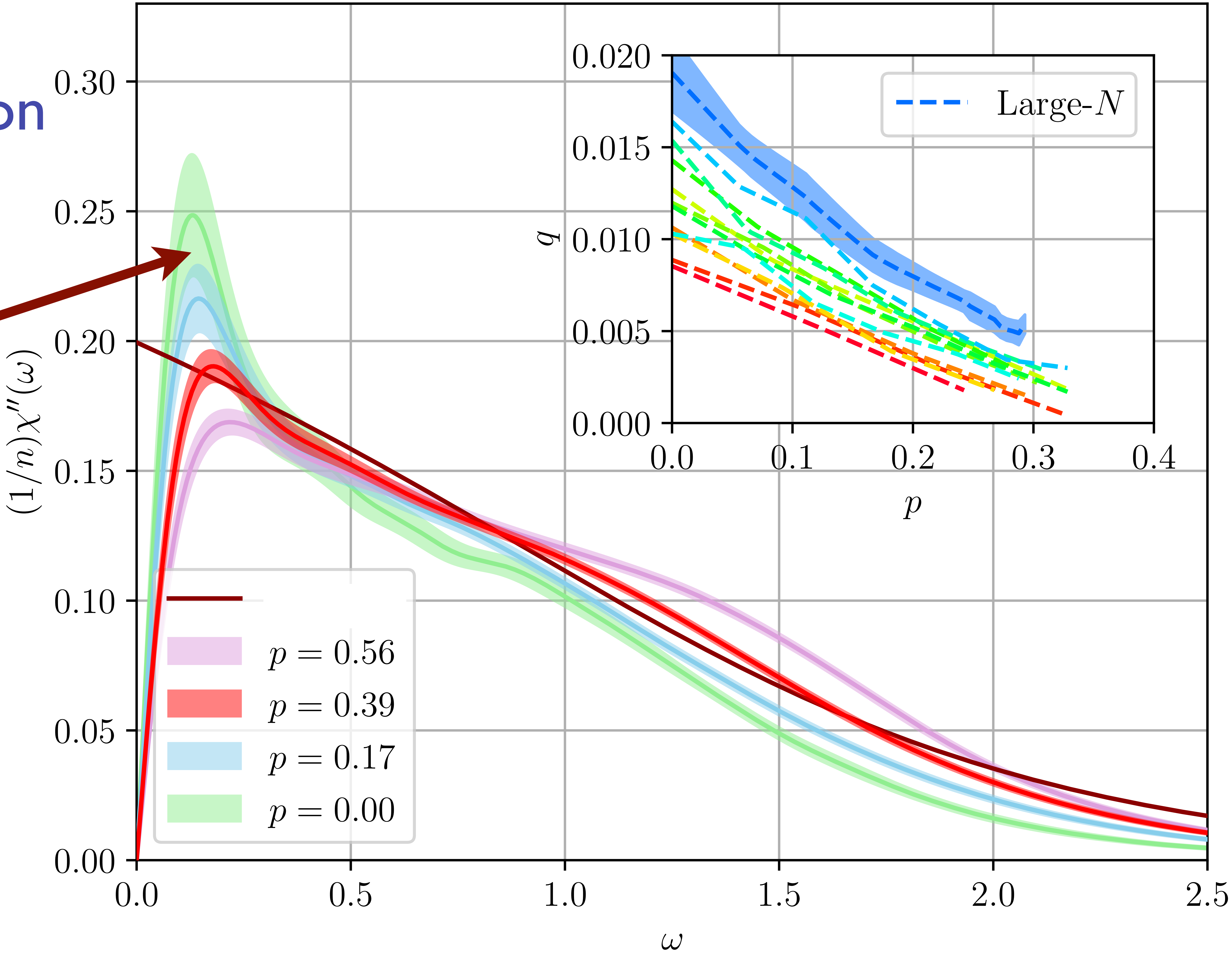
Dynamic spin susceptibility

Exact diagonalization

Spin glass order at small p



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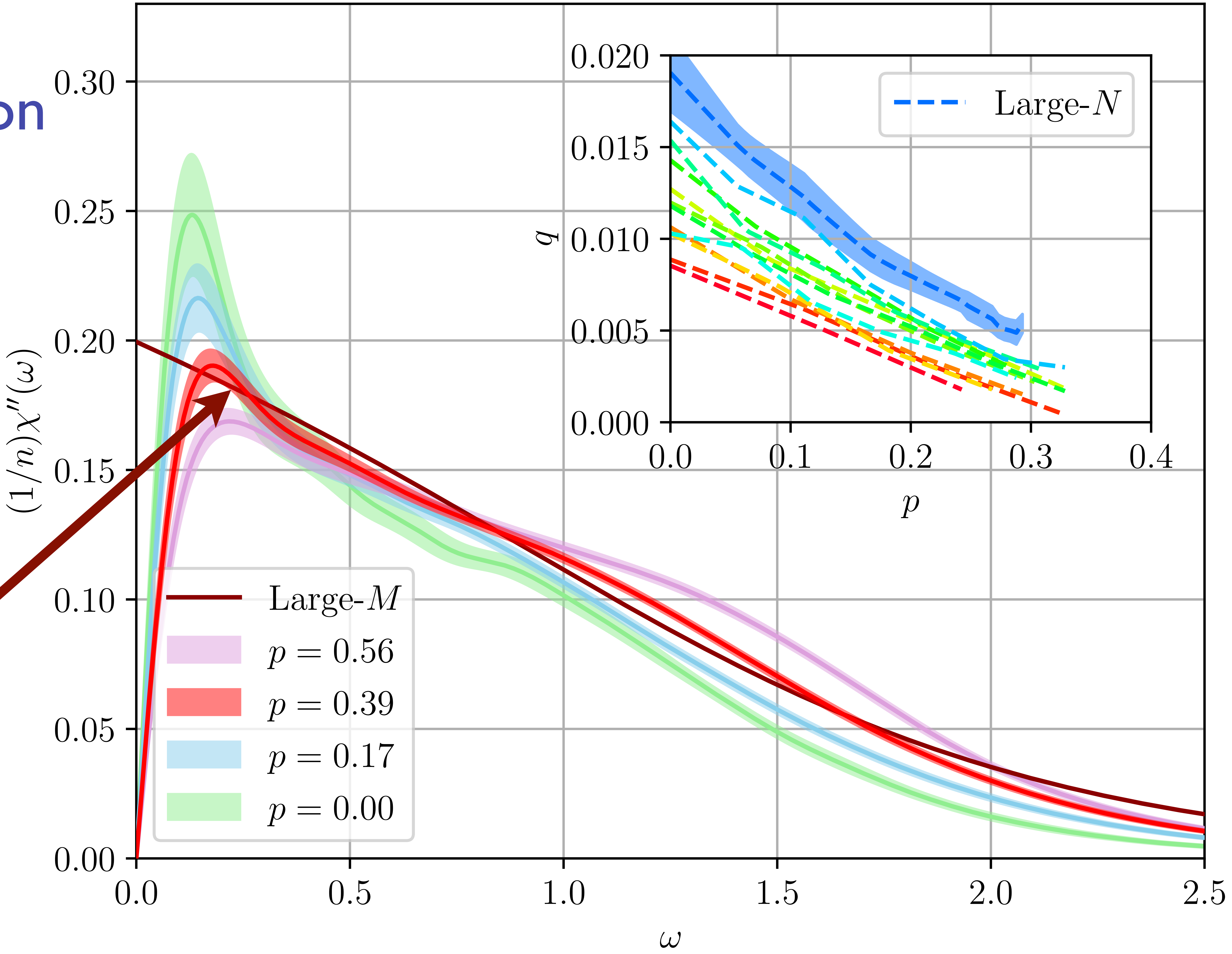
Dynamic spin susceptibility

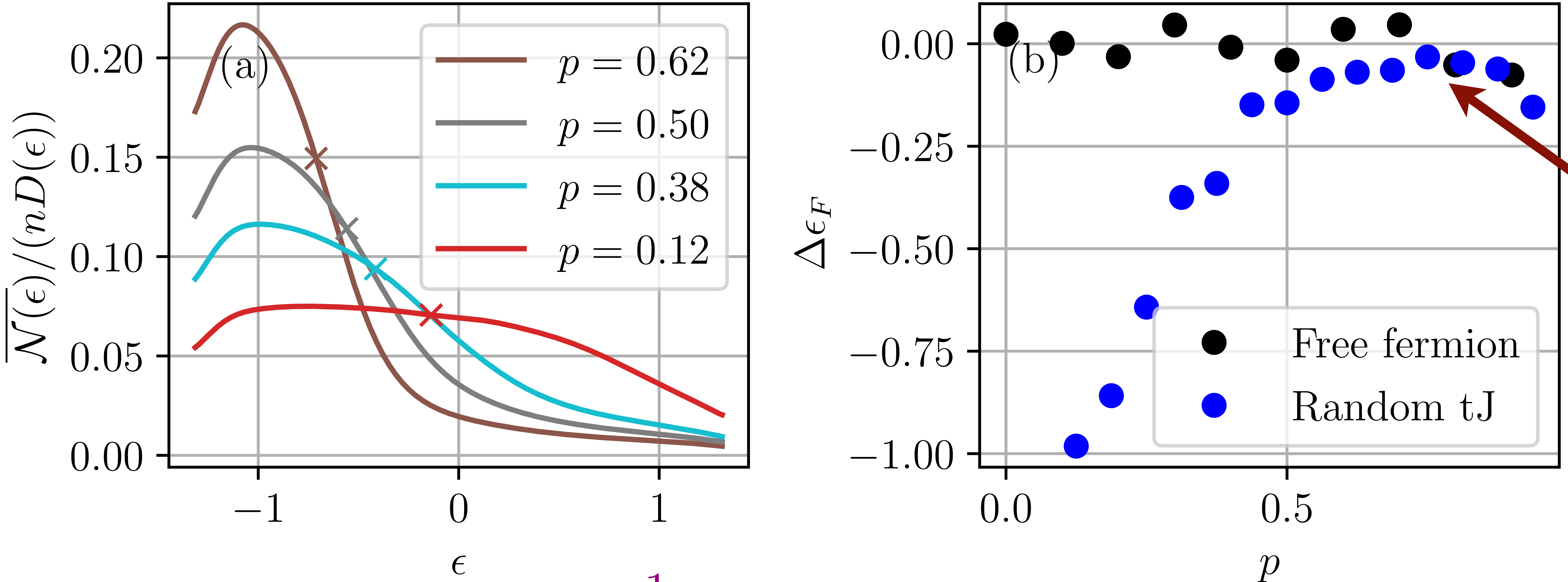
Exact diagonalization



SYK theory near p_c

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

One particle energy distribution function



Luttinger Fermi energy at large p

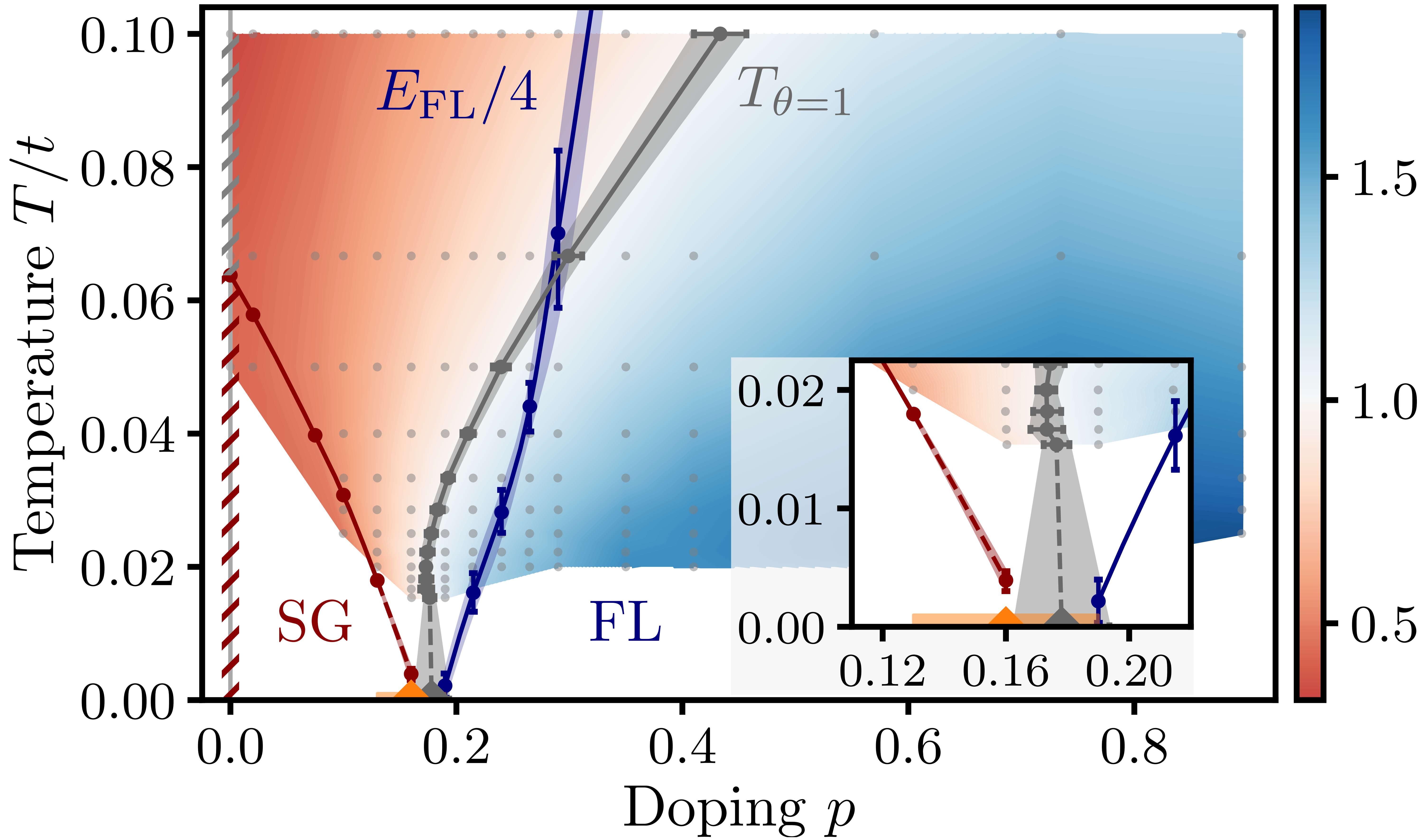
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

$$\mathcal{N}(\epsilon) = \frac{1}{N} \sum_{\lambda} \delta(\epsilon - \epsilon_\lambda) \sum_{ij\sigma} \langle \lambda | i \rangle \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle \langle j | \lambda \rangle$$

where $|\lambda\rangle$ are one-particle eigenstates of the t_{ij}. In a Fermi liquid, the Luttinger identity implies that $\mathcal{N}(\epsilon)$ has a discontinuity at the free particle Fermi energy ϵ_F. ($D(\epsilon)$ is the Wigner semi-circle density of states.)

Phase diagram (doping driven QCP)

- J = 0.5t, U = 4t



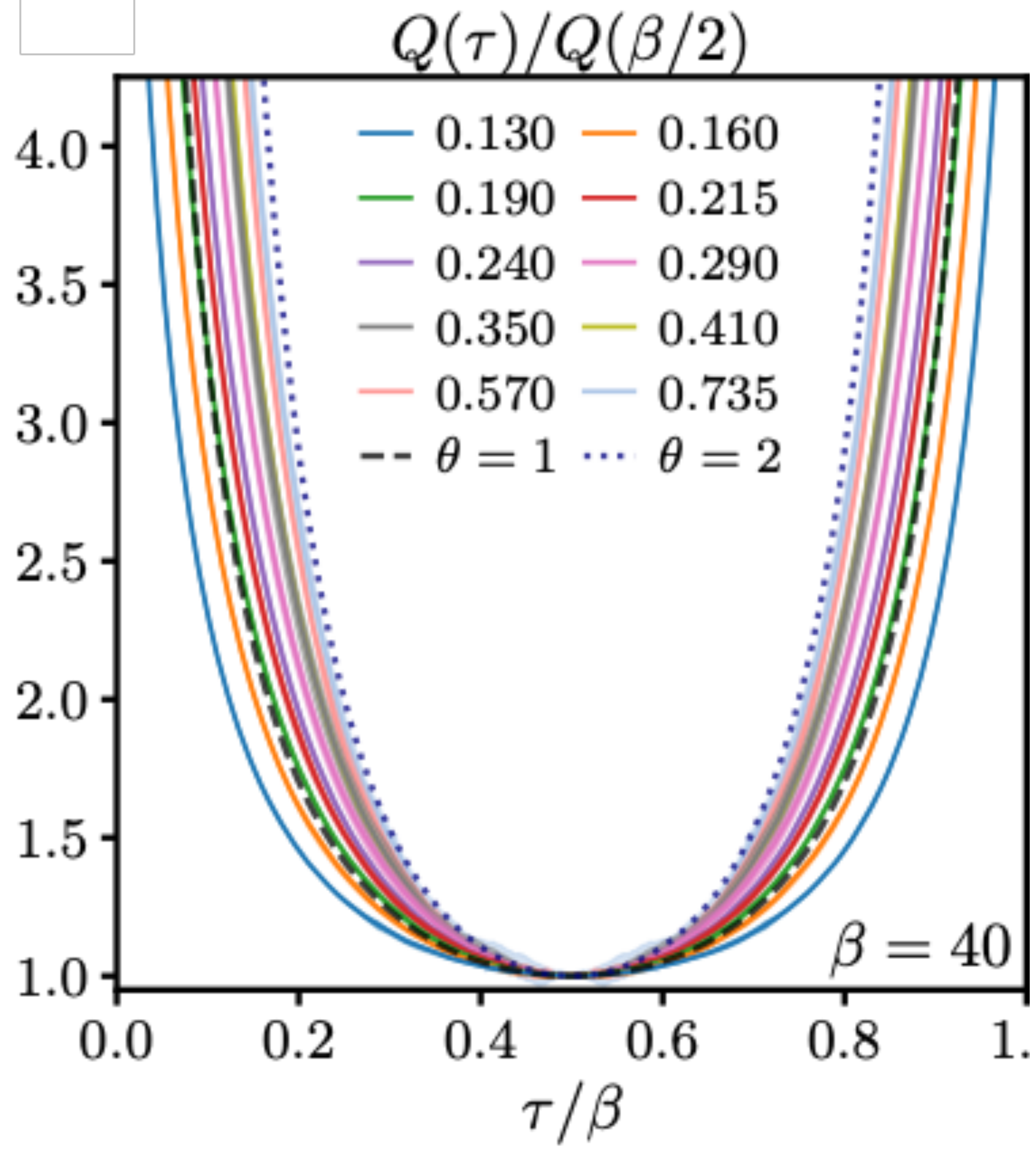
P. T. Dumitrescu,
N. Wentzell,
A. Georges,
O. Parcollet
arXiv:2103.08607

Critical scaling : spin dynamics

- Match conformal invariant form

$$Q(\tau-\tau')=\frac{1}{3}\langle \boldsymbol{S}(\tau)\cdot\boldsymbol{S}(\tau')\rangle$$

$$Q(\tau)\sim\frac{1}{[\sin(\pi\tau/\beta)]^{\theta}}$$



- θ=2 (Fermi liquid), θ=1(QCP)
- Phase diagram color map : θ

P. T. Dumitrescu,
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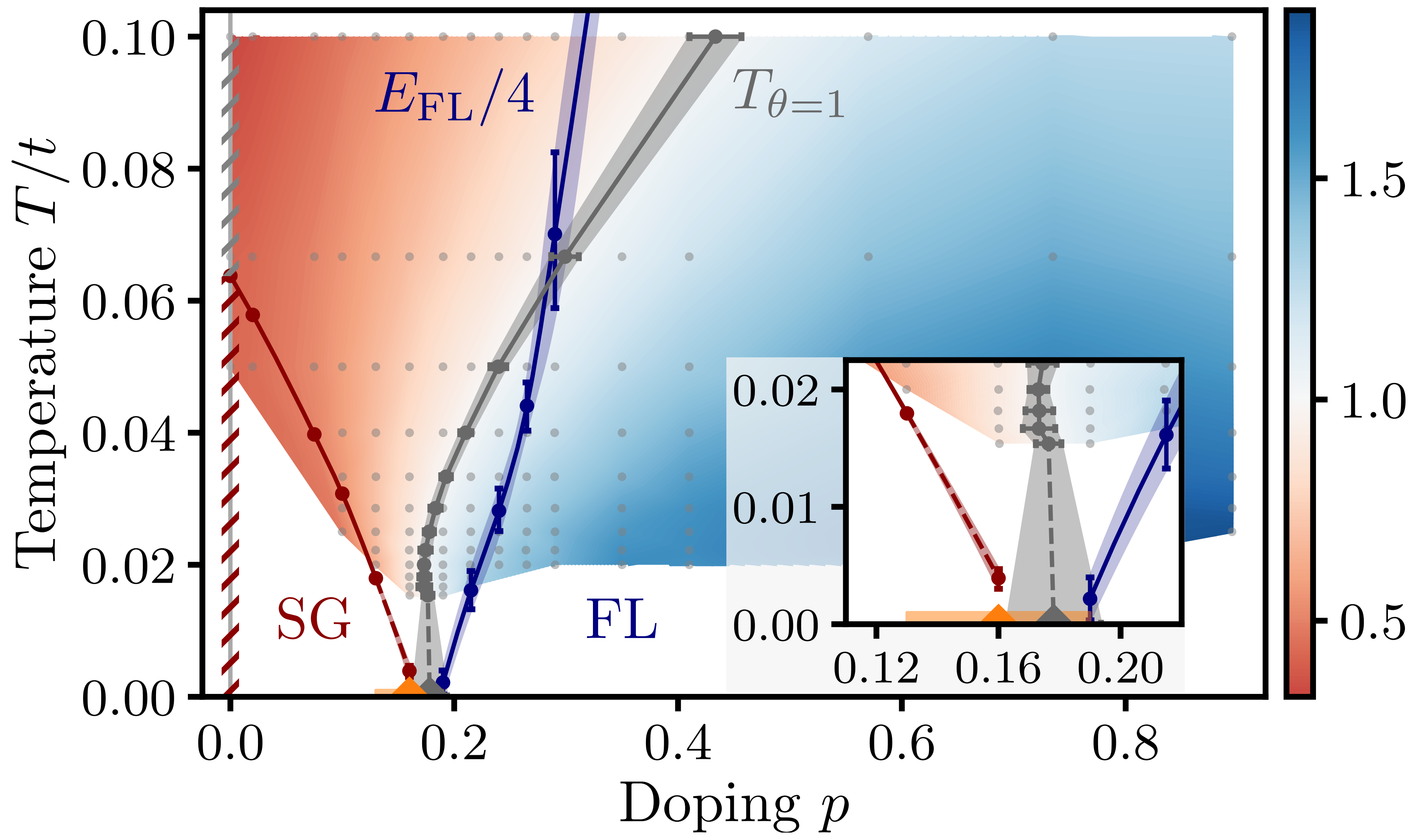
arXiv:2103.08607

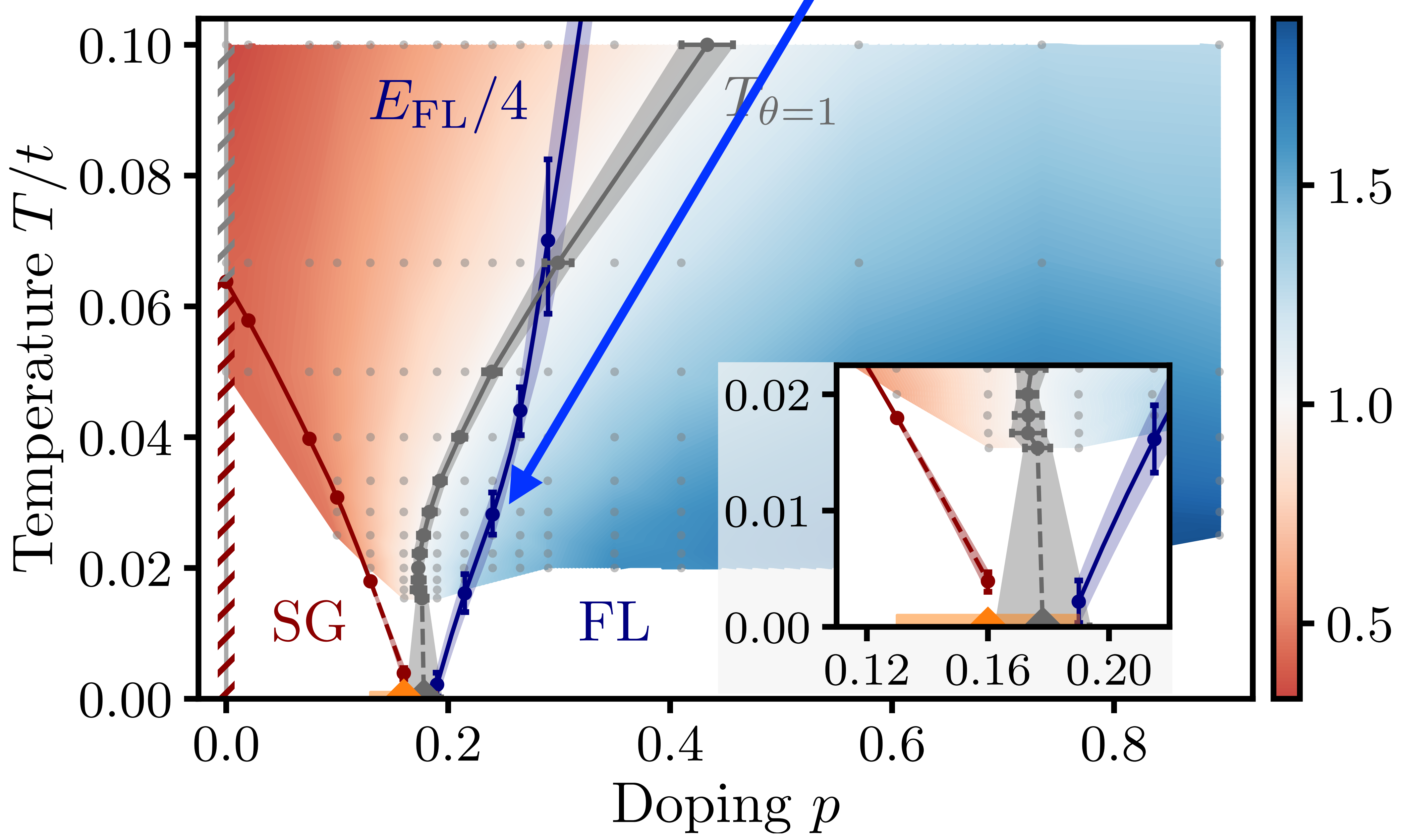


Fermi liquid collapse

- Characteristic energy scale E_{FL} vanishes at the QCP.

- Low T, low frequency Fermi liquid expansion

$$Im\Sigma(i\omega_n) = \left(1 - \frac{1}{Z}\right)\omega_n + \frac{\omega_n^2 - (\pi T)^2}{E_{\rm FL}} + O(T^3)$$



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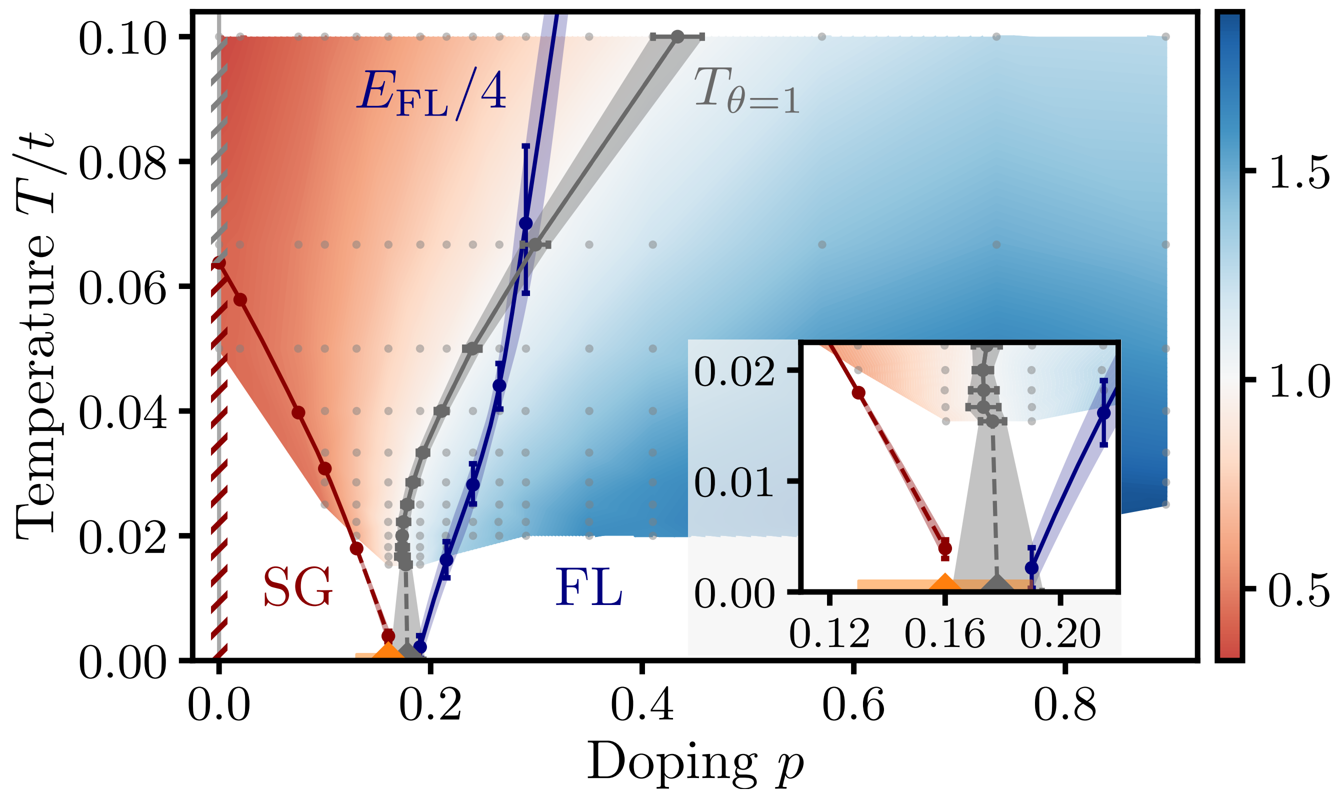
Fermi surface reconstruction at the QCP

See also Otzuki, Vollhardt

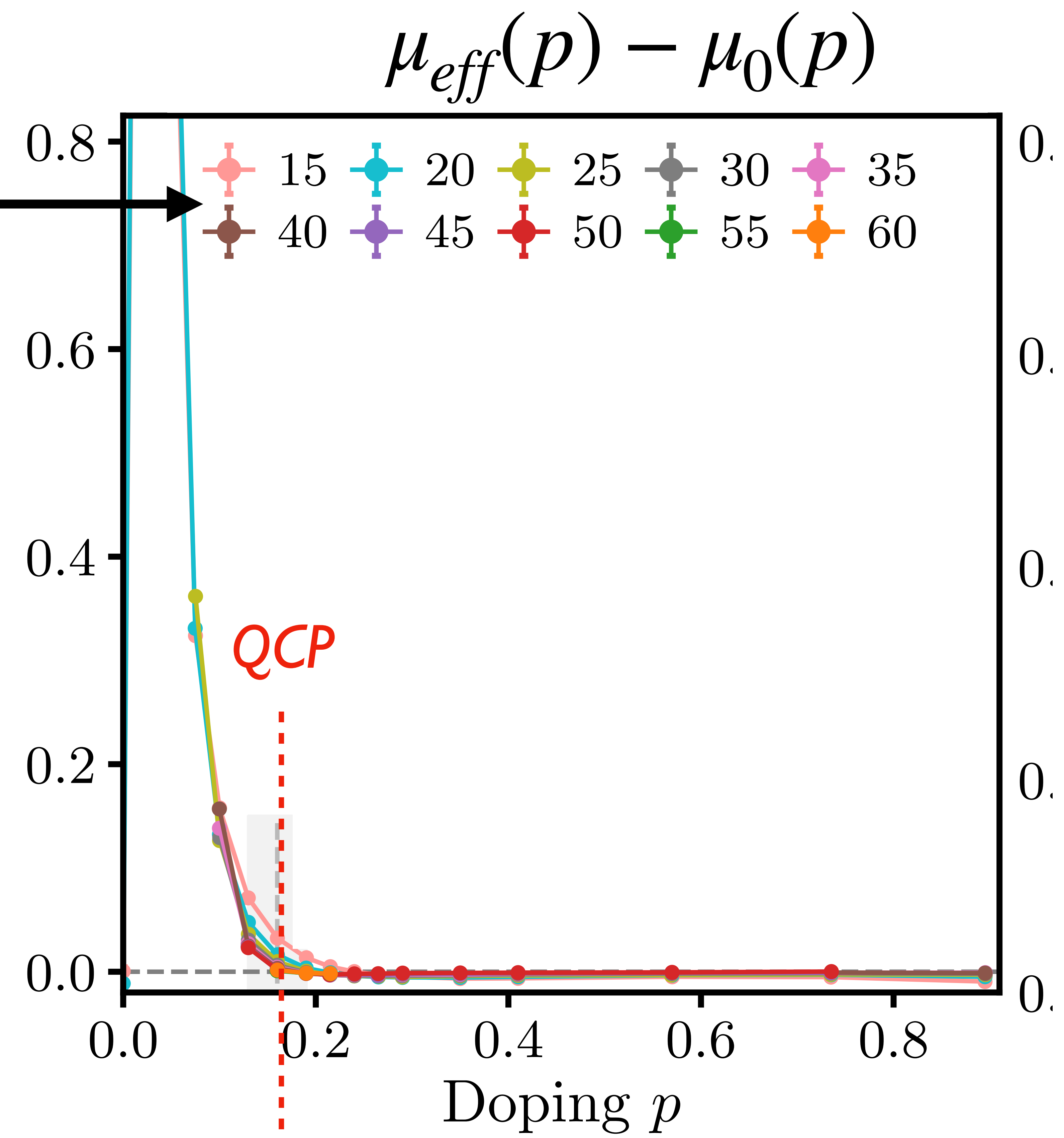
- Luttinger theorem : volume of Fermi surface independent of interaction
- Takes a simple form here, as Σ is local

$$\mu_{\text{eff}}(p) \equiv \mu - Re\Sigma(\omega = 0, T = 0)$$
$$= \mu_0(p)$$

Chemical potential of non interacting model



β



Luttinger theorem violated
Reconstruction of the Fermi surface

Luttinger theorem ok

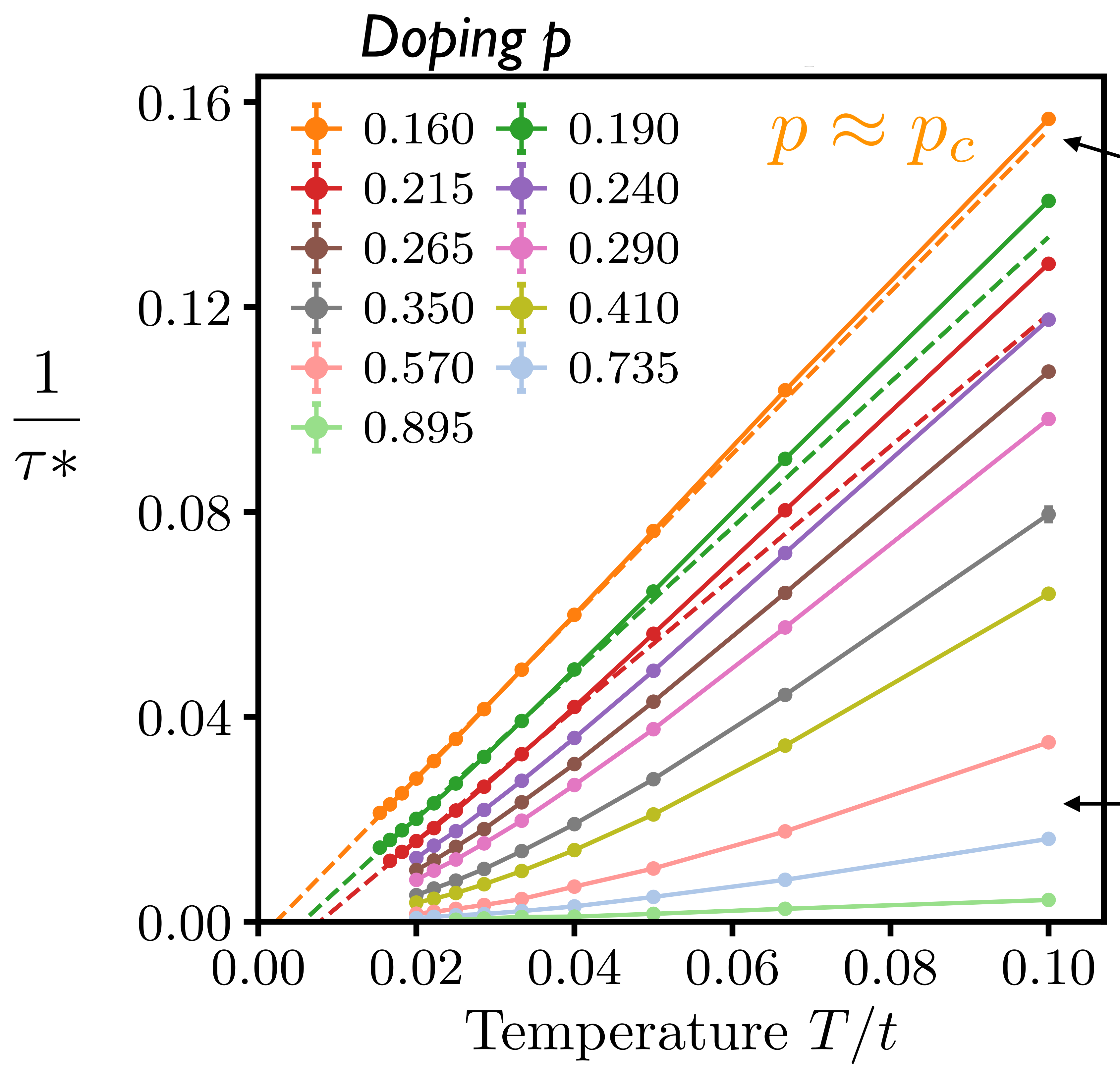
P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet arXiv:2103.08607

Single particle lifetime

- Quasiparticle lifetime in the Fermi liquid

$$\frac{1}{\tau*} = -Z Im\Sigma(\omega = 0)$$

Extrapolated to ω=0



$$\frac{1}{\tau*} \simeq c \frac{k_B T}{\hbar}$$

p = p_c
Planckian

$$\frac{1}{\tau*} \propto T^2$$

p>>p_c
Fermi Liquid

P. T. Dumitrescu,
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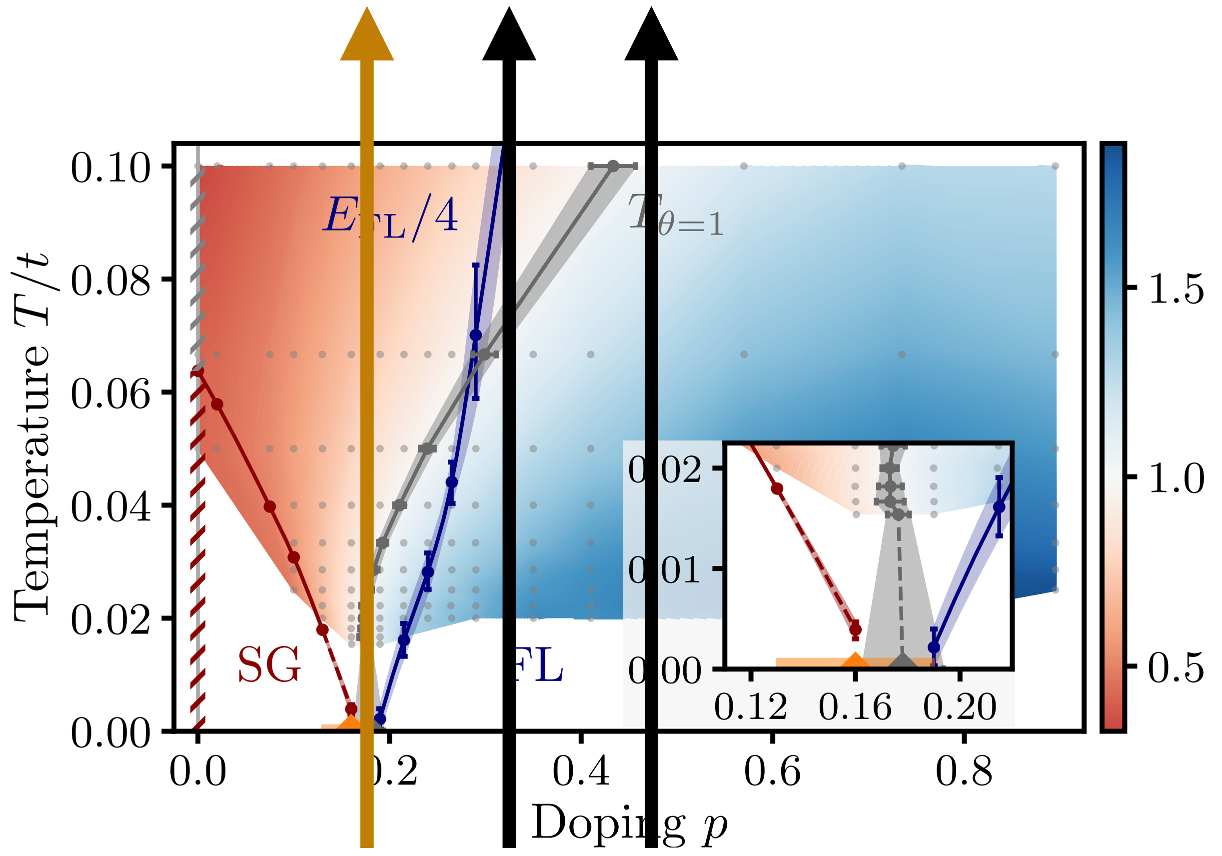
arXiv:2103.08607



- NB : Z factor is important to get the constant *c* of order 1

1. Numerical results

Exact diagonalization and DMFT+Monte Carlo

2. Parton representations

The pseudogap metal and the Fermi liquid

3. SYK criticality of partons

The Planckian metal

Random *t-J* model: phase diagram

Metallic spin glass.

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

SYK criticality

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

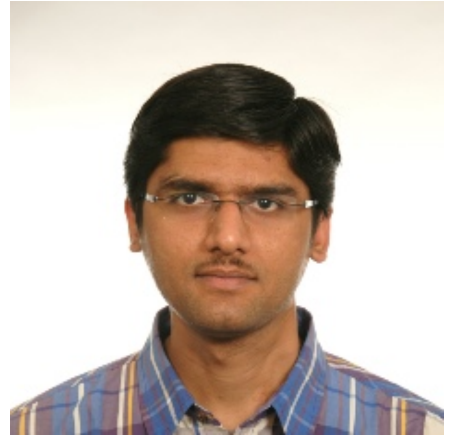
Disordered Fermi liquid.

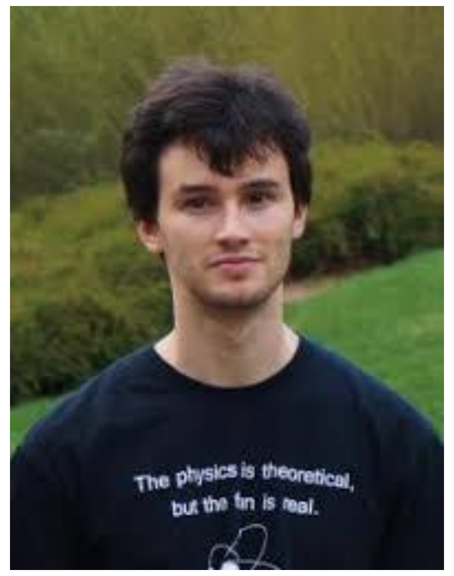
Luttinger Fermi energy

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

p_c $\quad$ p



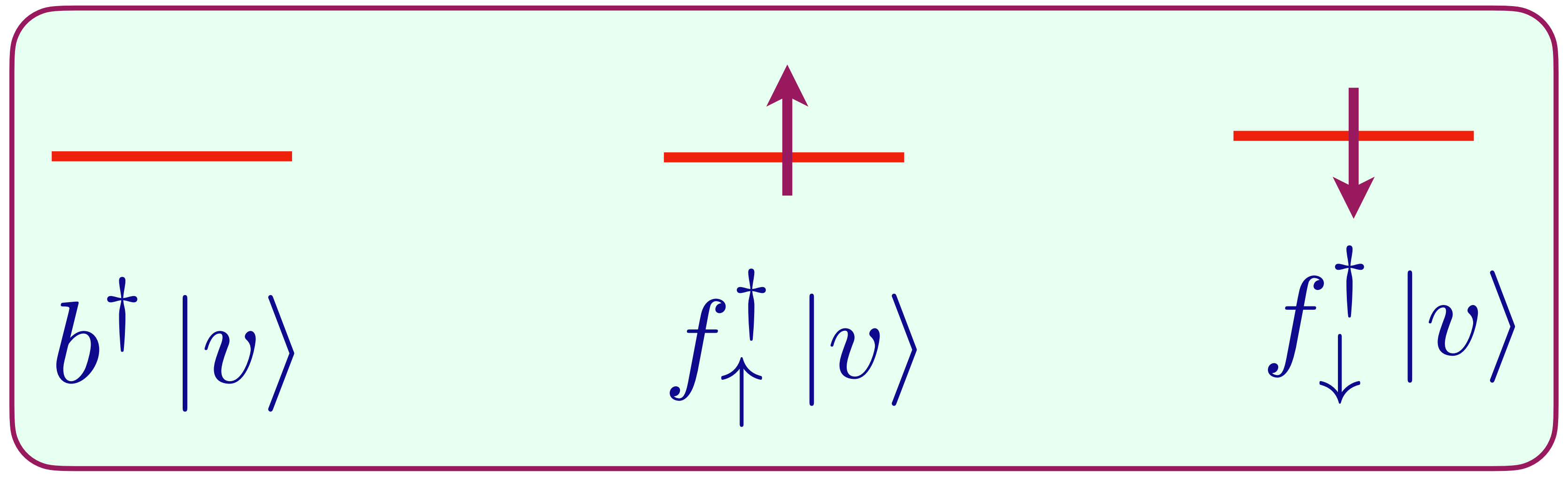


D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Parton theory I

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad b \to b e^{i\phi}\,, \quad f_\alpha \to f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

Parton theory I

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, f_{i\alpha}^\dagger f_{j\alpha} b_j^\dagger b_i + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} \frac{J_{ij}}{4}\, f_{i\alpha}^\dagger \sigma_{\alpha\beta} f_{i\beta} \cdot f_{j\gamma}^\dagger \sigma_{\gamma\delta} f_{j\delta}$$

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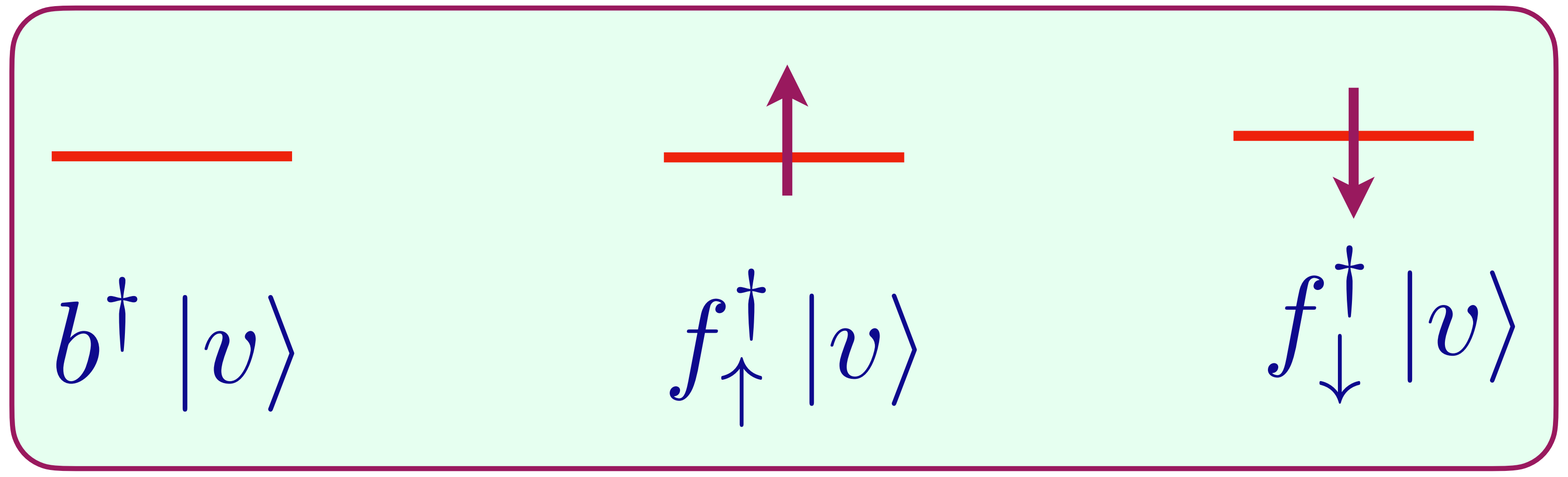


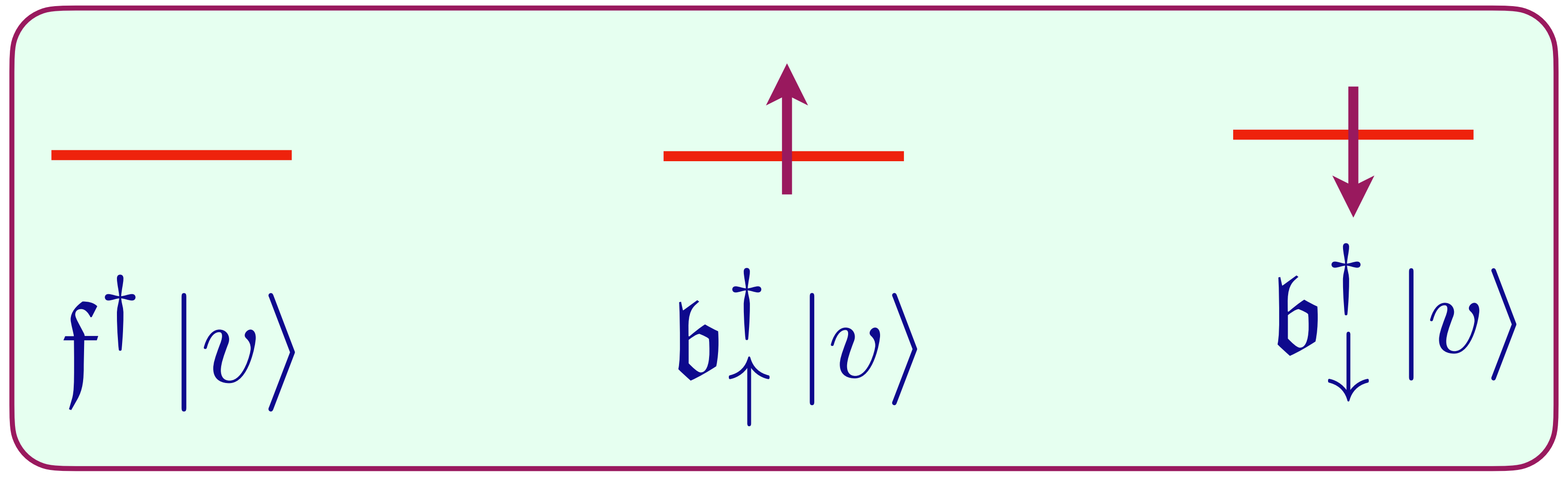
$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \\ \text{U(1) gauge invariance,} \quad & \quad b \rightarrow b e^{i\phi}\,, \quad f_\alpha \rightarrow f_\alpha e^{i\phi} \end{aligned}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

Parton theory II

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{b}_\alpha$ (the spinon):



$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Parton theory II

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, \mathfrak{b}^\dagger_{i\alpha} \mathfrak{b}_{j\alpha} \mathfrak{f}_i \mathfrak{f}^\dagger_j + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} \frac{J_{ij}}{4}\, \mathfrak{b}^\dagger_{i\alpha} \sigma_{\alpha\beta} \mathfrak{b}_{i\beta} \cdot \mathfrak{b}^\dagger_{j\gamma} \sigma_{\gamma\delta} \mathfrak{b}_{j\delta}$$

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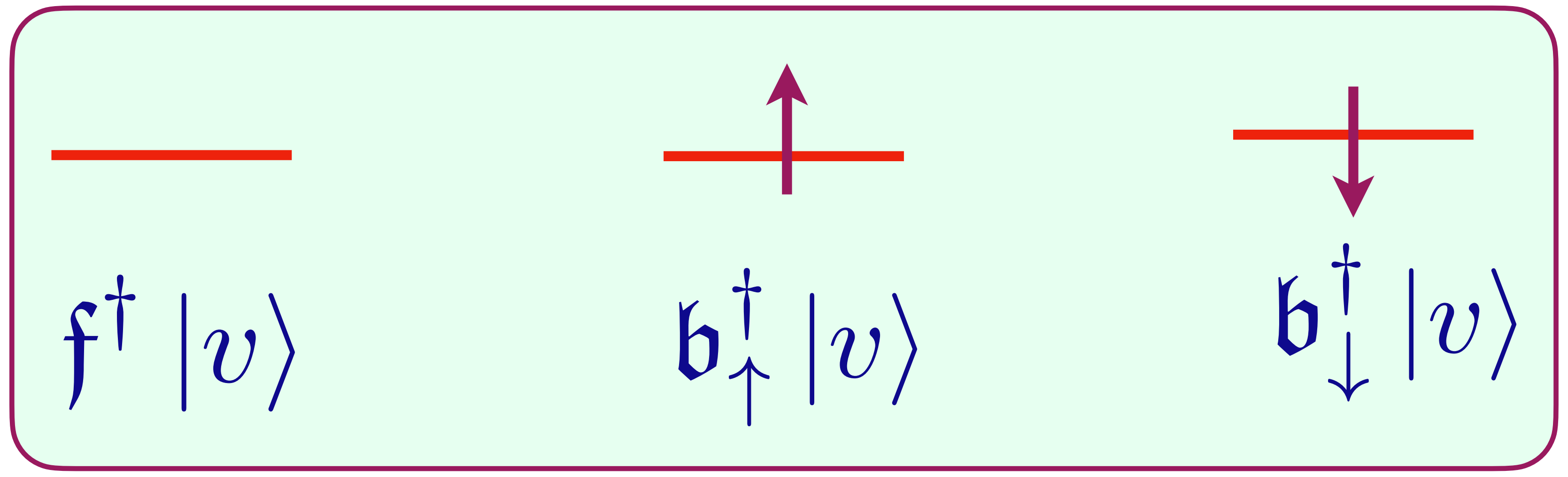

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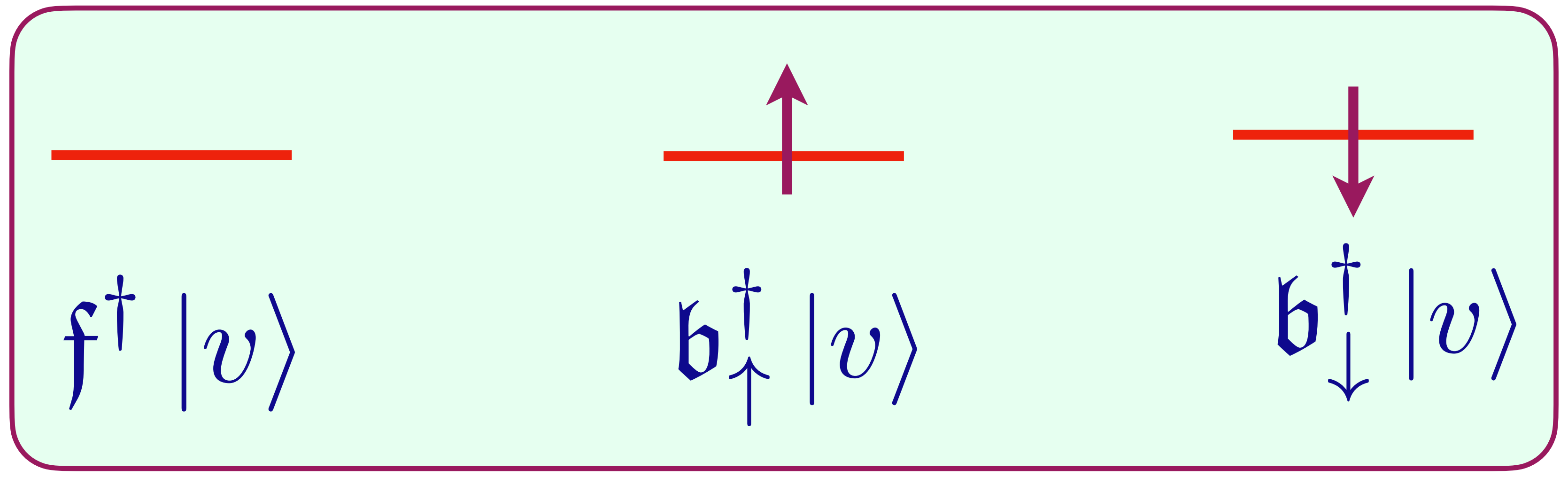

$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}^\dagger_\alpha \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}^\dagger_\alpha \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

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The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random *t-J* model: phase diagram

Metallic spin glass.

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SYK criticality

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Disordered Fermi liquid.

Luttinger Fermi energy

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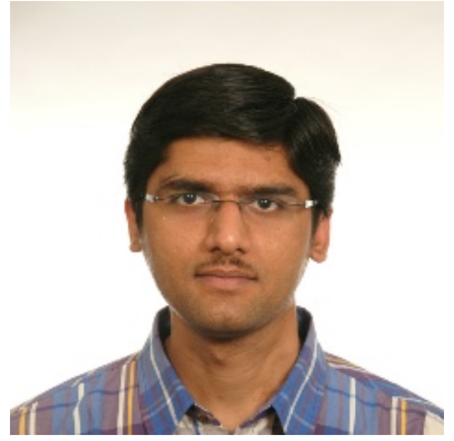
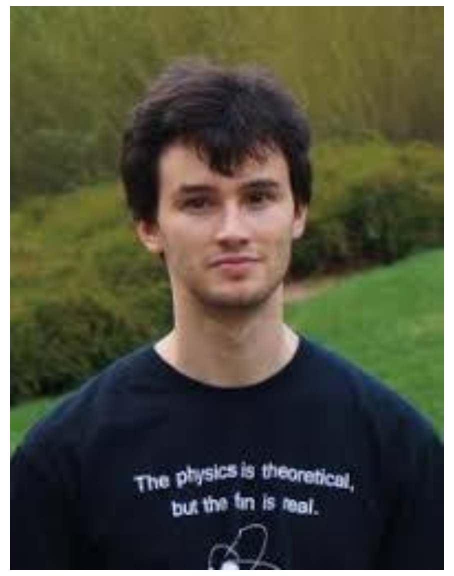
p_c $\qquad$ p





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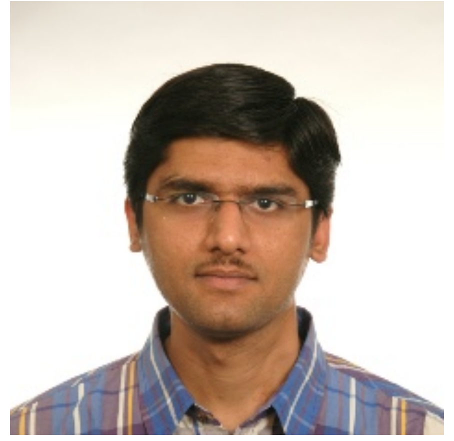
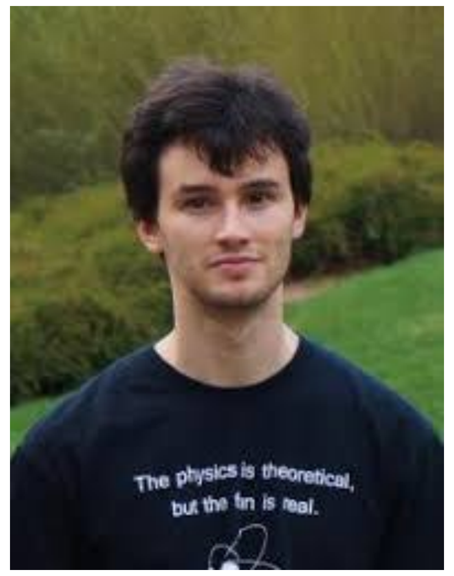
Random *t-J* model: phase diagram





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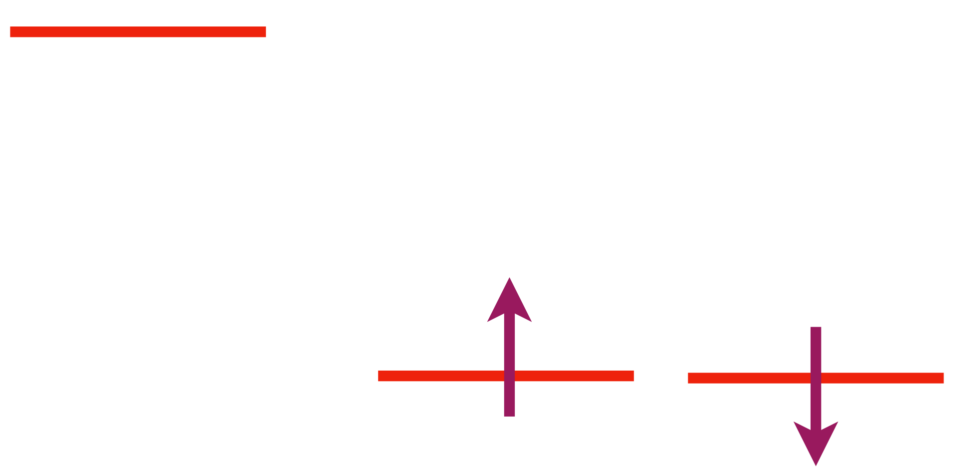

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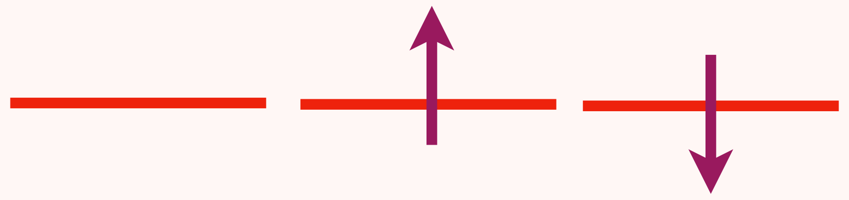
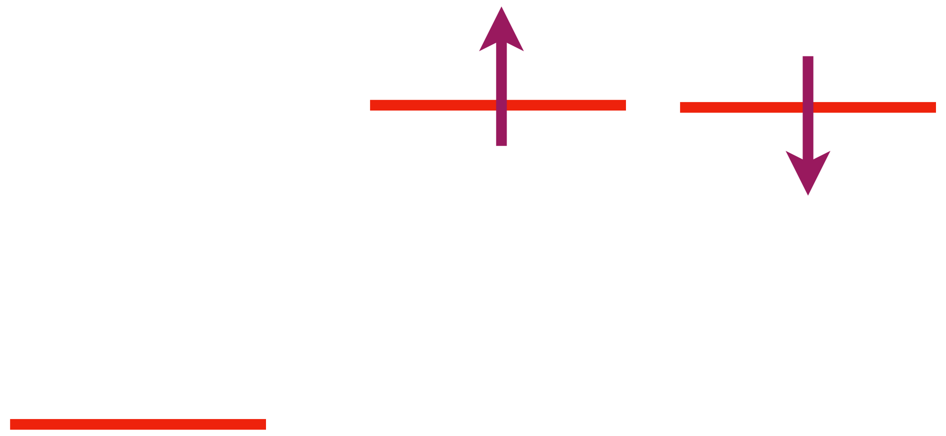
SYK criticality



$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Disordered Fermi liquid.



$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

p_c $\quad$ p

Zeroth order, $p_c = 1/3$

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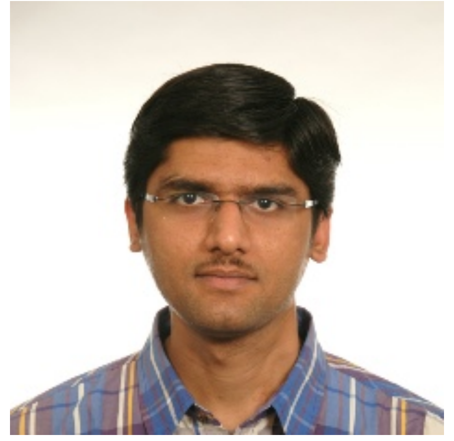
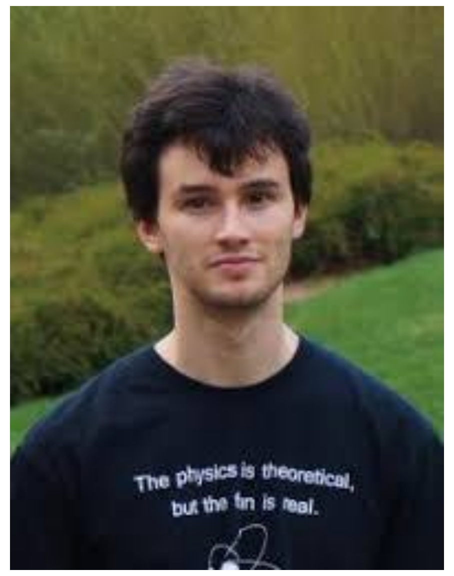
Random *t-J* model: phase diagram





SU(1|2) theory

Metallic spin glass.

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SYK criticality

$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \frac{1}{|\tau|}$$

Disordered Fermi liquid. Condense holon b, f_α carrier density $1+p$

$f^\dagger_\uparrow|v\rangle$ $f^\dagger_\downarrow|v\rangle$

$b^\dagger|v\rangle$

$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau)c^\dagger_{i\alpha}(0)\right\rangle \sim \frac{1}{\tau}$$

Zeroth order, $p_c = 1/3$ p_c p

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random *t-J* model: phase diagram

SU(2|1) theory

Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger \left| v \right\rangle$

$\mathfrak{b}_\uparrow^\dagger \left| v \right\rangle$ $\mathfrak{b}_\downarrow^\dagger \left| v \right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SYK criticality

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

SU(1|2) theory

Disordered Fermi liquid. Condense holon b, f_α carrier density $1+p$

$f_\uparrow^\dagger \left| v \right\rangle$ $f_\downarrow^\dagger \left| v \right\rangle$

$b^\dagger \left| v \right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

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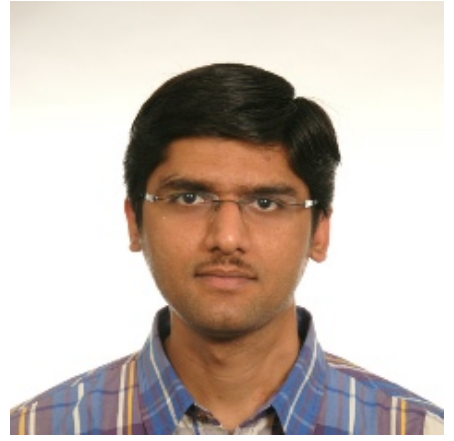
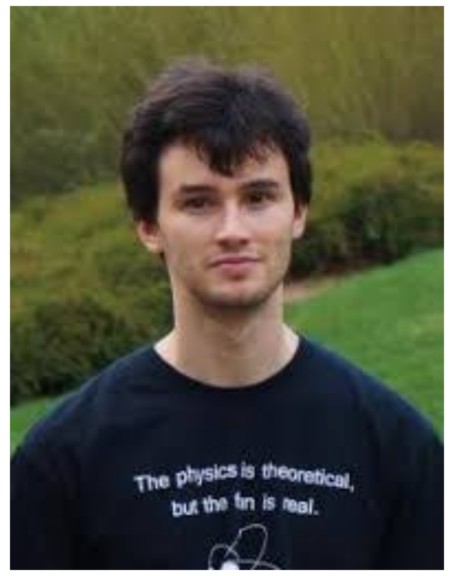
Zeroth order, $p_c = 1/3$ p_c p





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Random *t-J* model: phase diagram

SU(2|1) theory

Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SYK criticality of fractionalized excitations

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SU(1|2) theory

Disordered Fermi liquid. Condense holon b, f_α carrier density $1 + p$

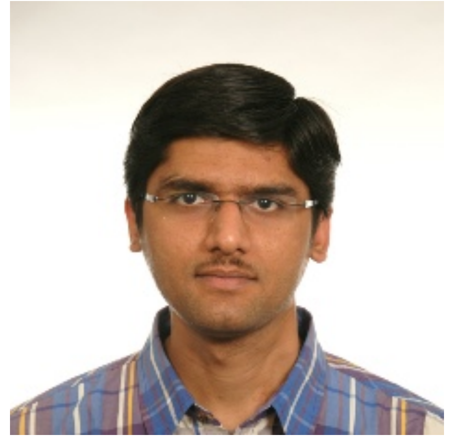
$f_\uparrow^\dagger |v\rangle$ $f_\downarrow^\dagger |v\rangle$

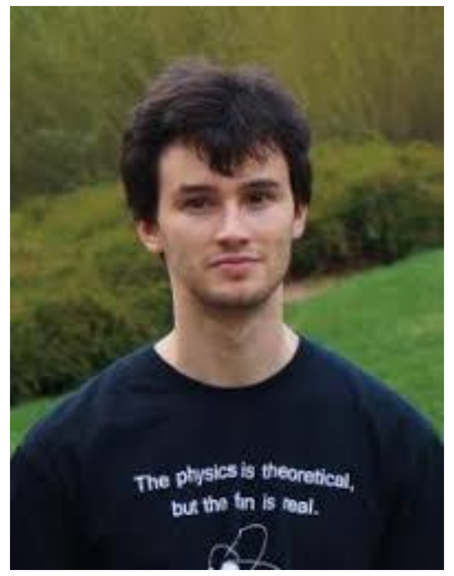
$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

Zeroth order, $p_c = 1/3$ p_c p



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Random *t-J* model: large *M* limit

Both the t and the J terms involve four single-particle operators. Consequently, in a large M limit (where the spin symmetry SU(2)→SU(M)), the saddle-point equations are very similar to the $q = 4$ SYK equations. These equations realize a critical phase with SYK criticality, provided none of the bosons condense.

$$\begin{aligned} G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\ \Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\ G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\ \Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k\, t^2 G_f(\tau) G_b(\tau) G_b(-\tau) \end{aligned}$$

Here μ_f and μ_b are chemical potentials chosen to satisfy

$$\langle f^\dagger f \rangle = \frac{1}{2} - k\delta \quad , \quad \langle b^\dagger b \rangle = \delta \, .$$

Random *t-J* model: large *M* limit

Both the t and the J terms involve four single-particle operators. Consequently, in a large M limit (where the spin symmetry SU(2)→SU(M)), the saddle-point equations are very similar to the $q = 4$ SYK equations. These equations realize a critical phase with SYK criticality, provided none of the bosons condense.

This leads to solutions with

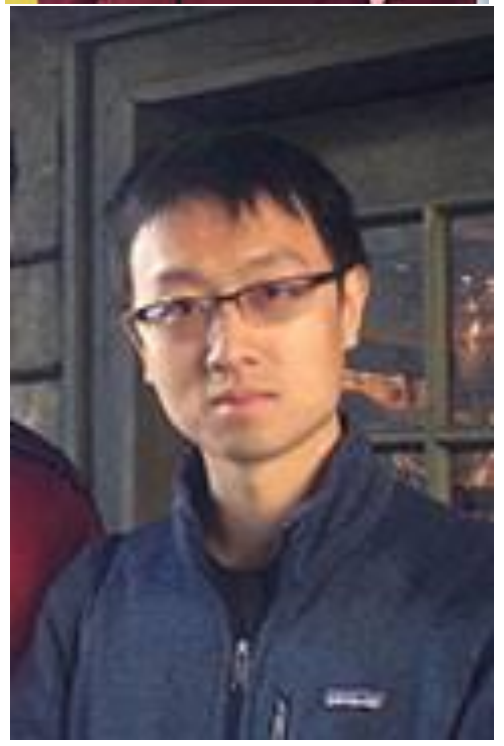
$$G_f(\tau) \sim G_b(\tau) \sim \frac{1}{\sqrt{\tau}}$$

and so

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

Time reparameterization soft mode and linear-T resistivity



SYK-type models have a time reparameterization soft-mode, which is the boundary graviton in the JT gravity low energy theory. This now leads to corrections to the Green's functions of the partons

$$G_{f,b}(\tau) \sim \frac{\pm 1}{\sqrt{|\tau|}} \left(1 + \frac{\alpha_{f,b}}{|\tau|} + \ldots \right)$$

We can compute the resistivity from this in a large-d model, and find

$$\rho(T) = \rho(0) \left(1 + 8\alpha_G \frac{T}{J} + \ldots \right) .$$

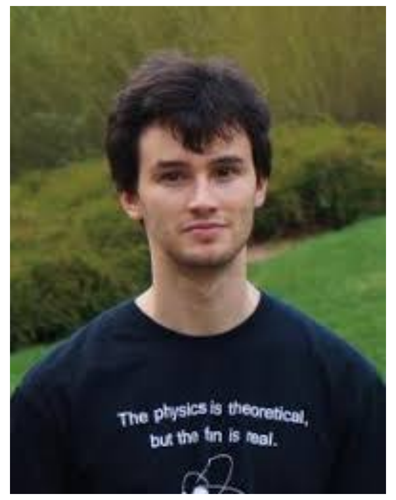
The α_G term arises from the contribution of the boundary graviton!

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)
Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)
Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

Corrections to the dynamic spin susceptibility of SYK model

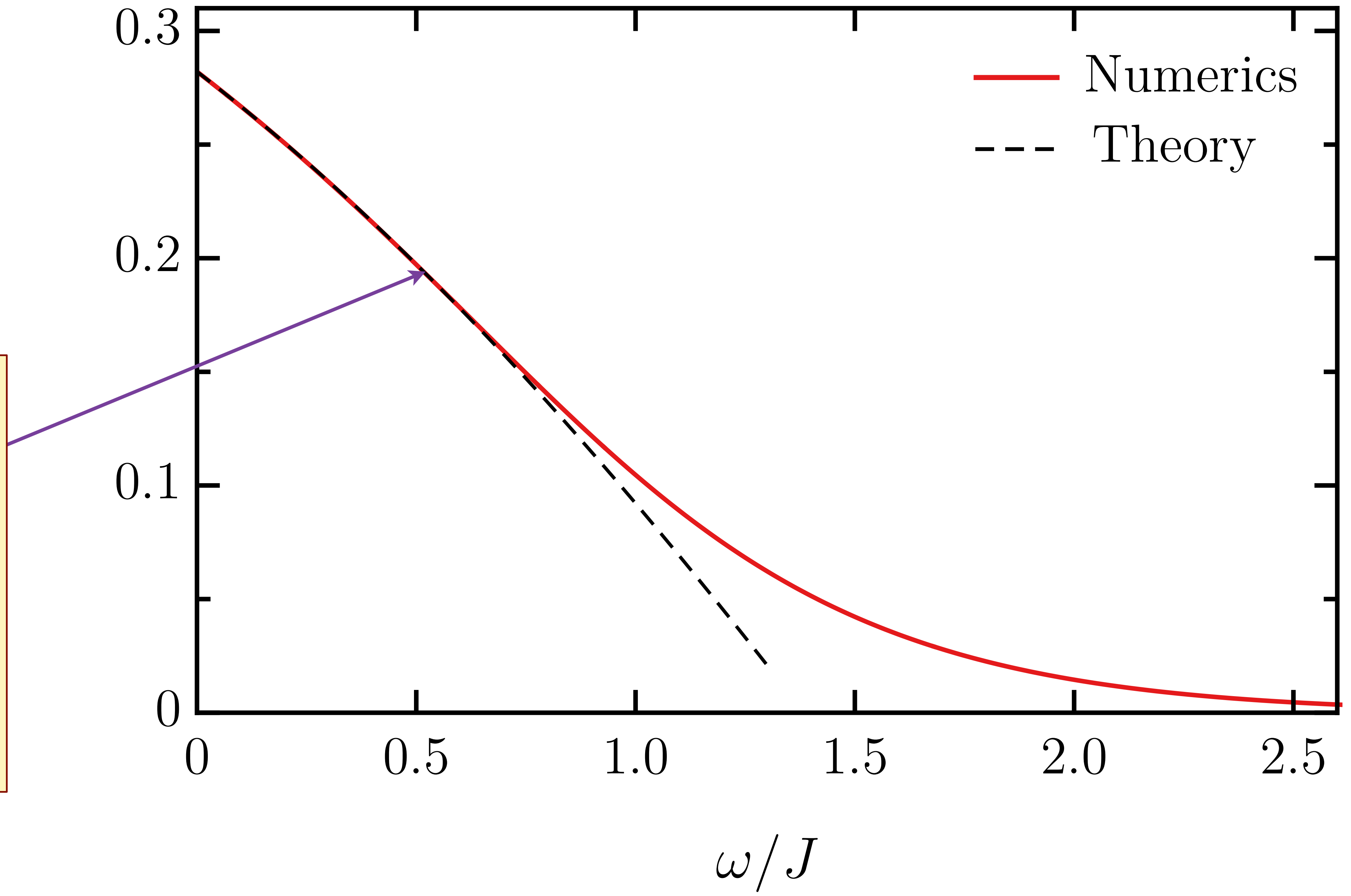
$$\chi_L(\tau) = G(\tau)G(-\tau)$$

$$\text{Im}\chi_L(\omega) \sim \text{sgn}(\omega)\left[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\ldots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \ldots\right]$$



Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory. $\mathcal{C}$ is a known number, and γ is the co-efficient of the action for the 'boundary graviton' in holographic dual.

Correction from the boundary graviton



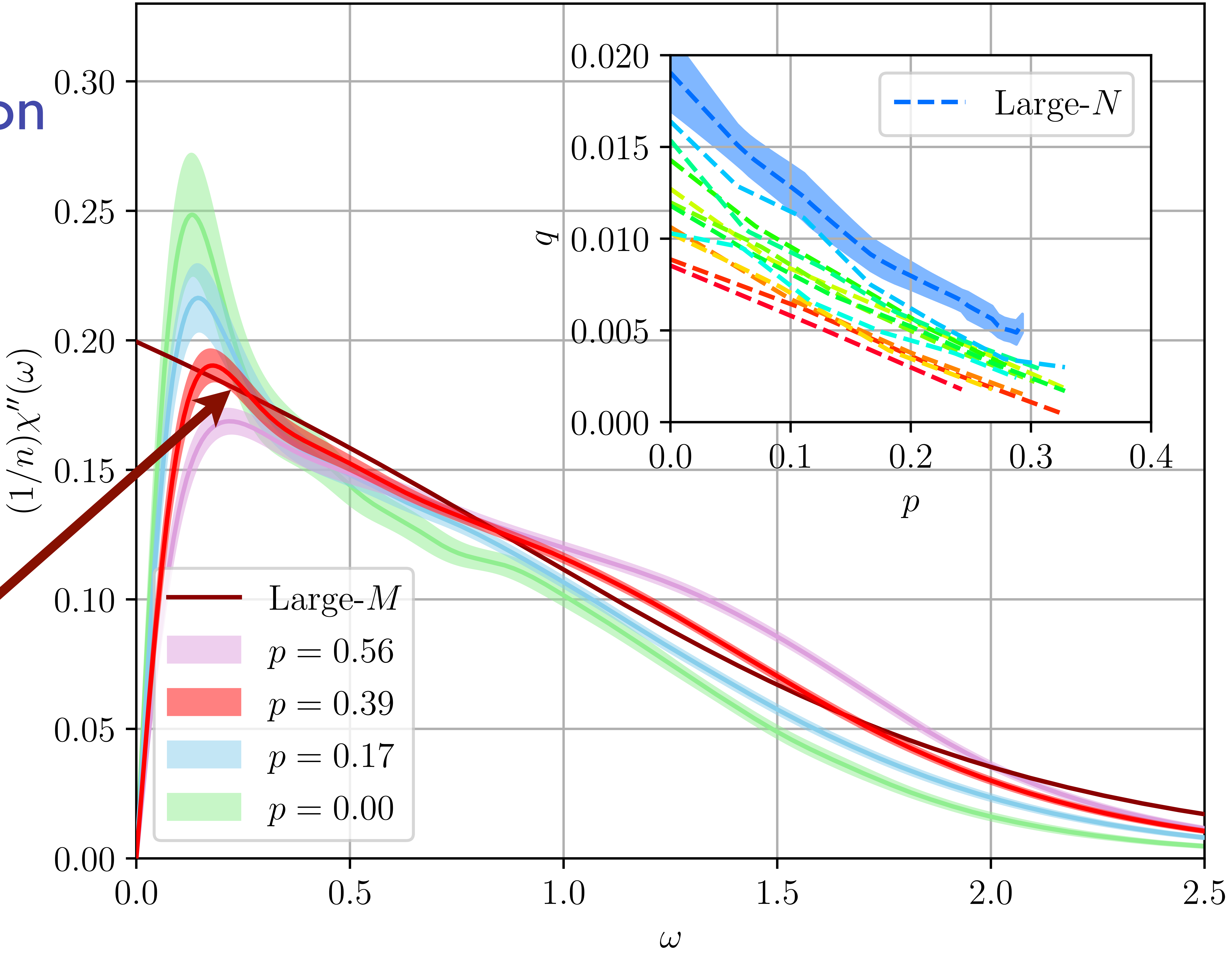
Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

$$\chi = \int_0^\beta d\tau \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle$$

Dynamic spin susceptibility

Exact diagonalization

SYK theory near p_c

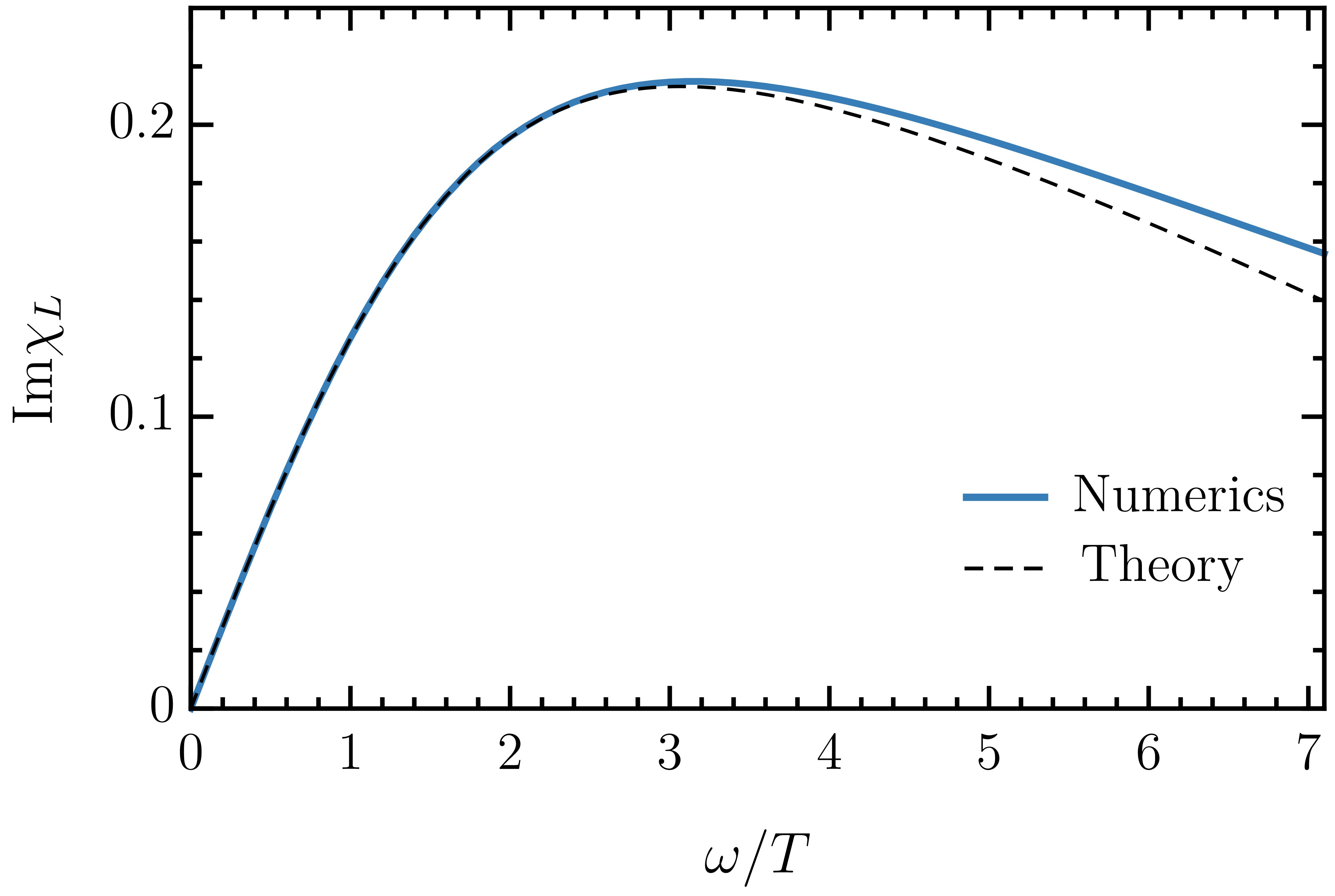


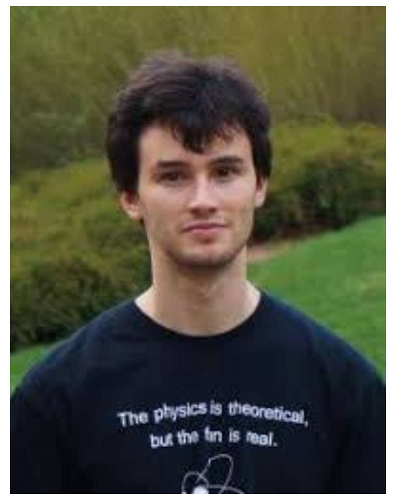
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Corrections to the dynamic spin susceptibility of SYK model

$$\chi_L(\tau) = G(\tau)G(-\tau)$$

$$\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_BT}\right)\left[1 - \mathcal{C}\gamma\,\omega\tanh\left(\frac{\hbar\omega}{2k_BT}\right) - \ldots\right]$$





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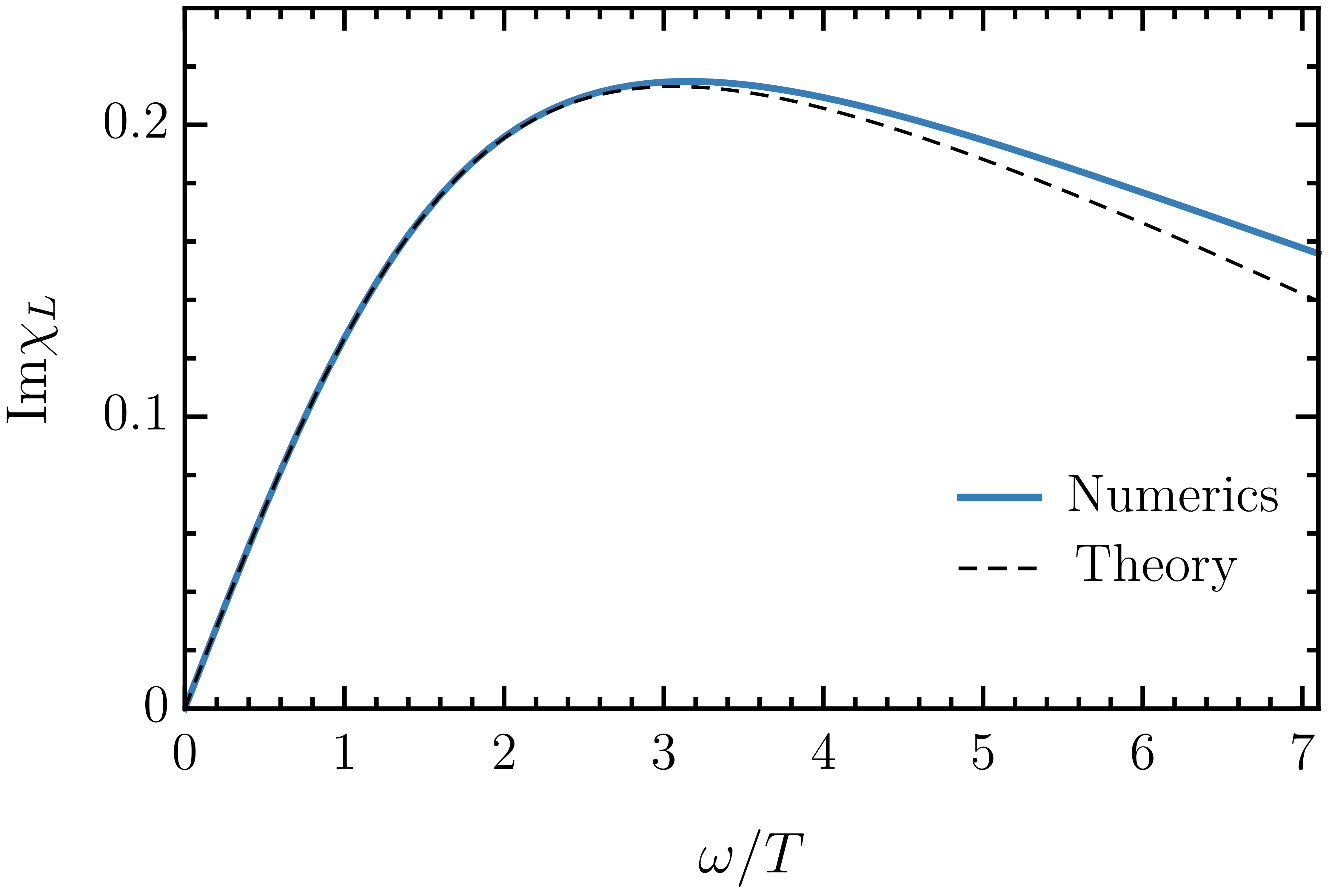
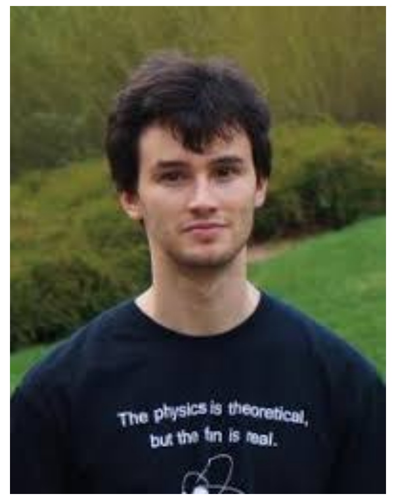
Conformally (SL(2,R)) invariant form with 'Planckian' dissipative time $\sim \hbar/(k_BT)$, *independent* of J.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)





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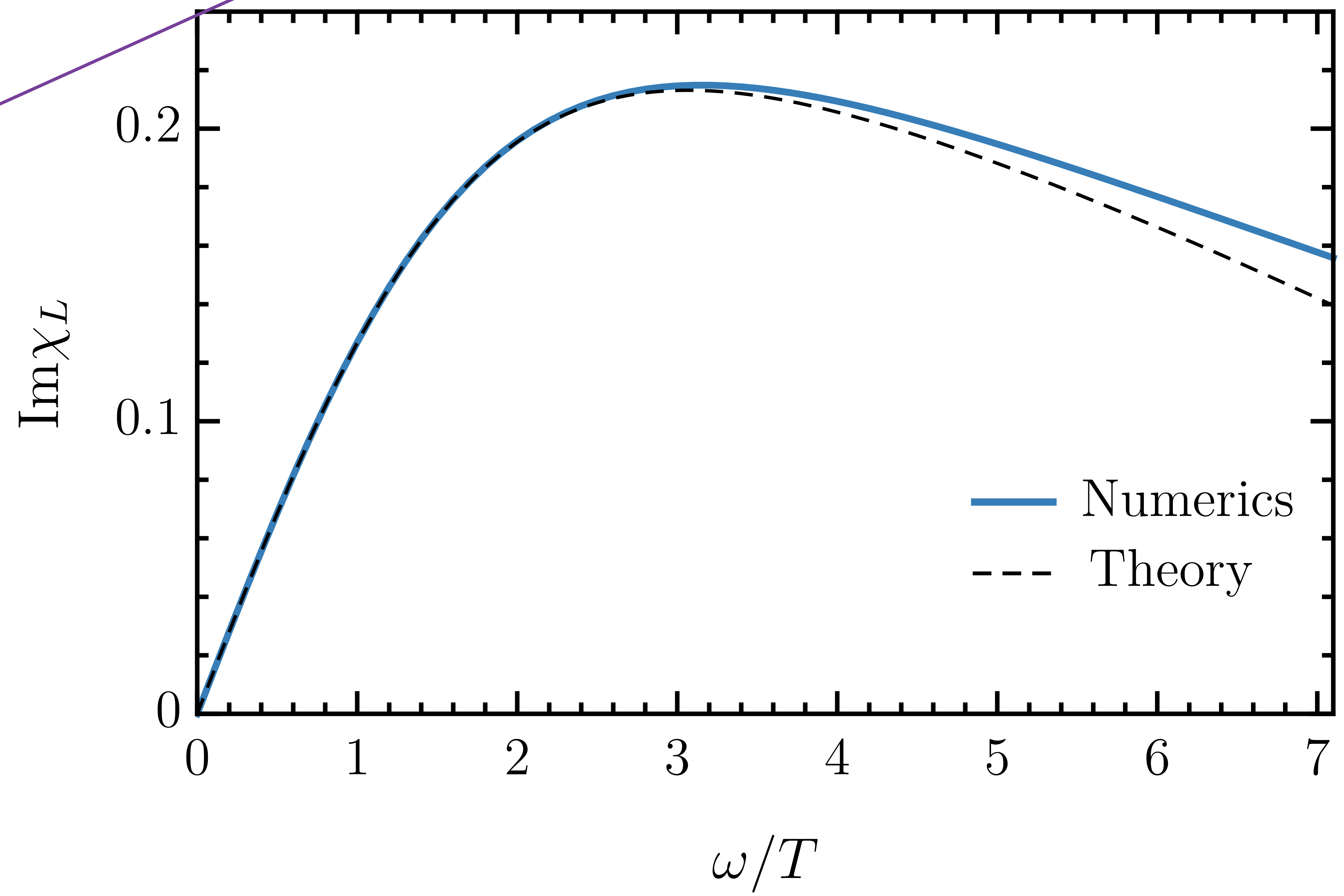
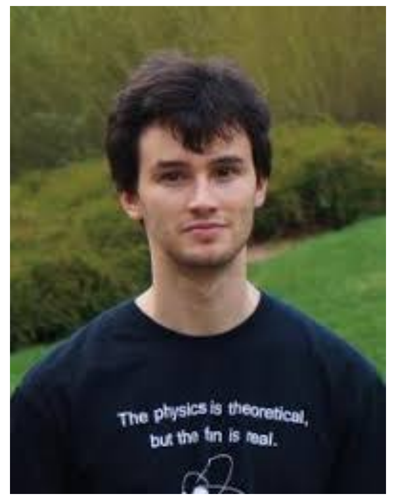
Correction from the boundary graviton





Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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- SYK criticality: thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

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