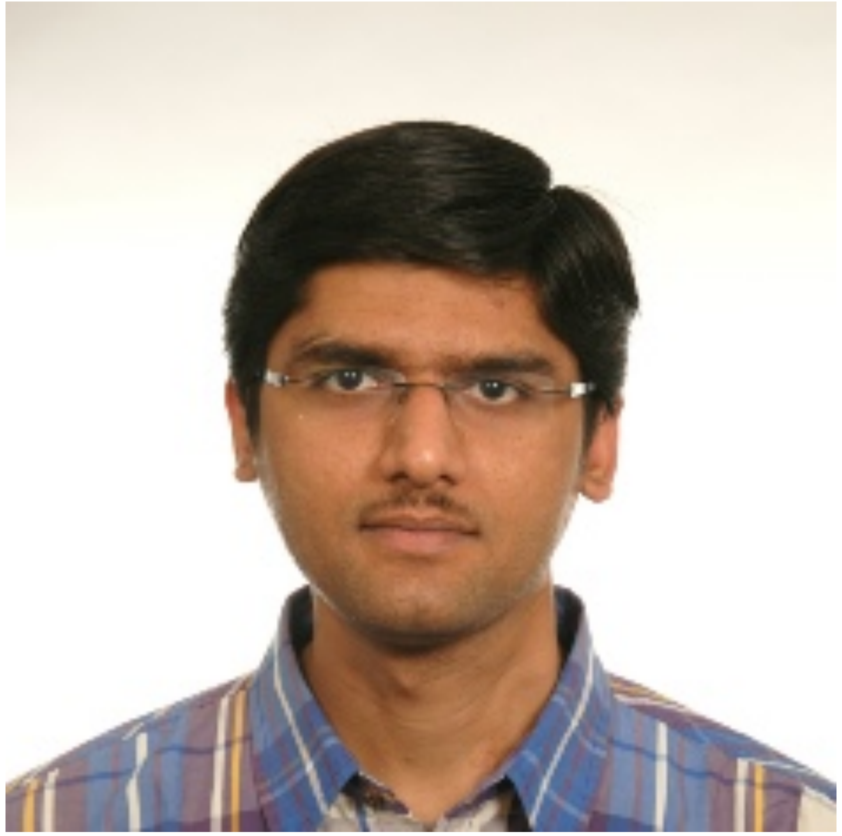
Deconfined criticality in a doped random quantum Heisenberg magnet

arXiv:1912.08822

Novel Phases of Quantum Matter
International Centre for Theoretical Sciences, Bengaluru
January 1, 2020
Subir Sachdev

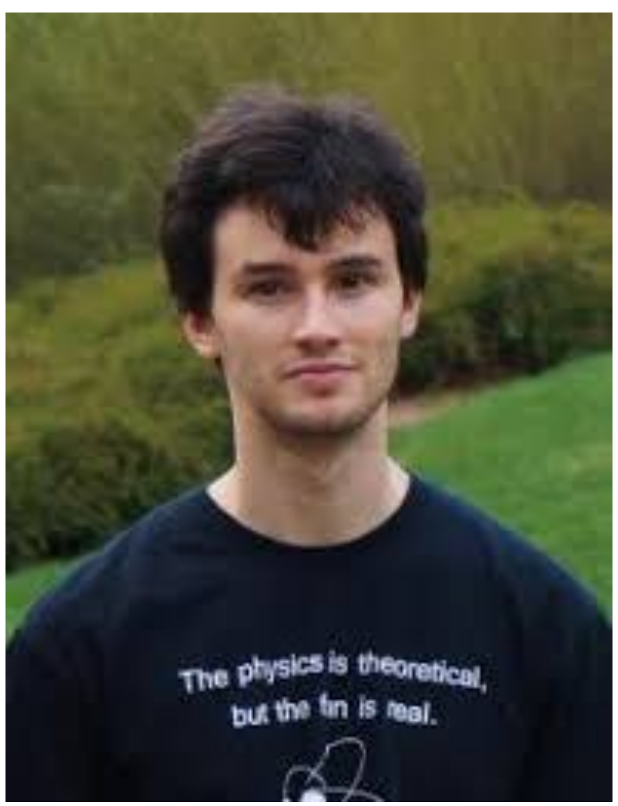
Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD



Darshan Joshi

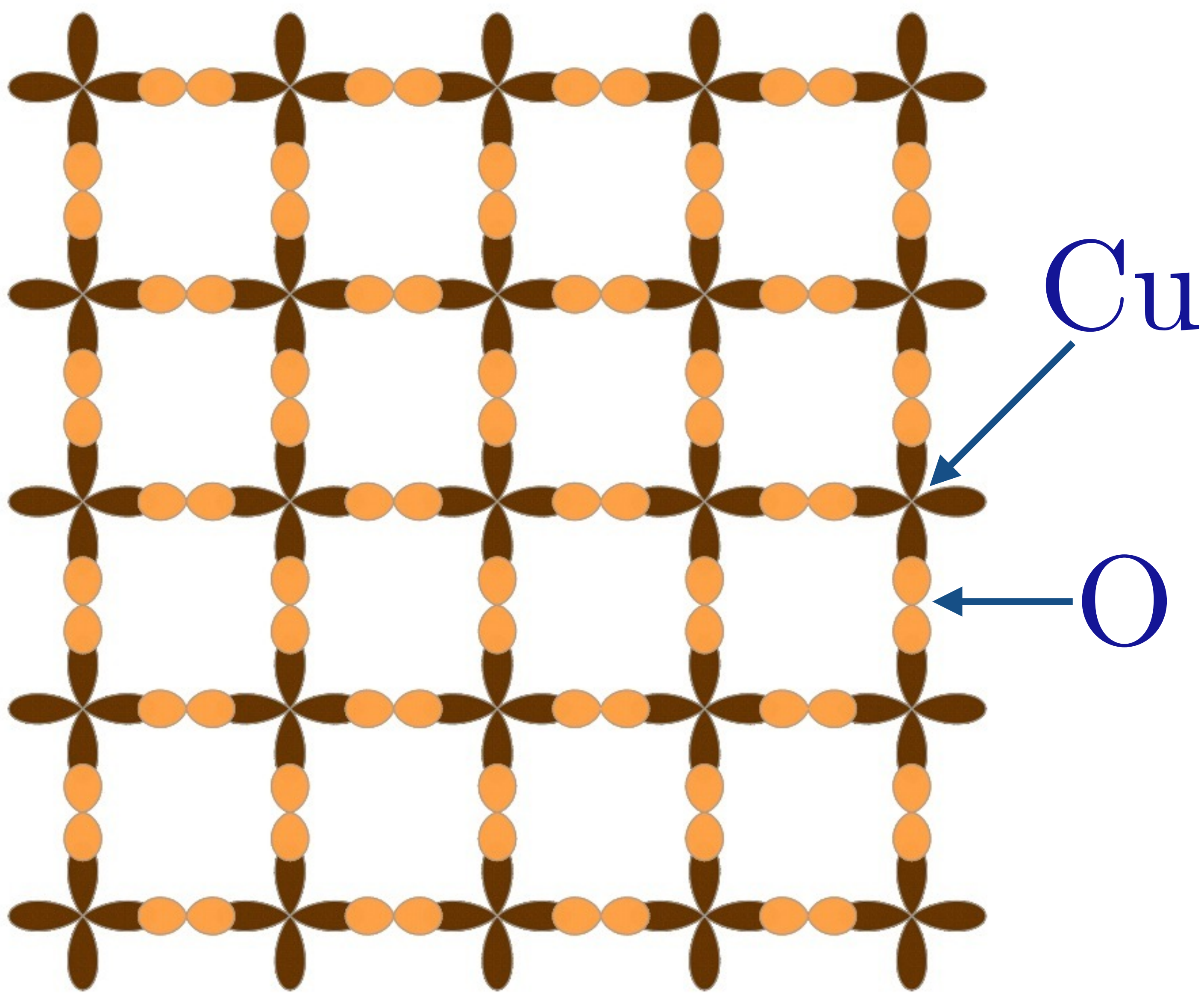
Chenyuan Li



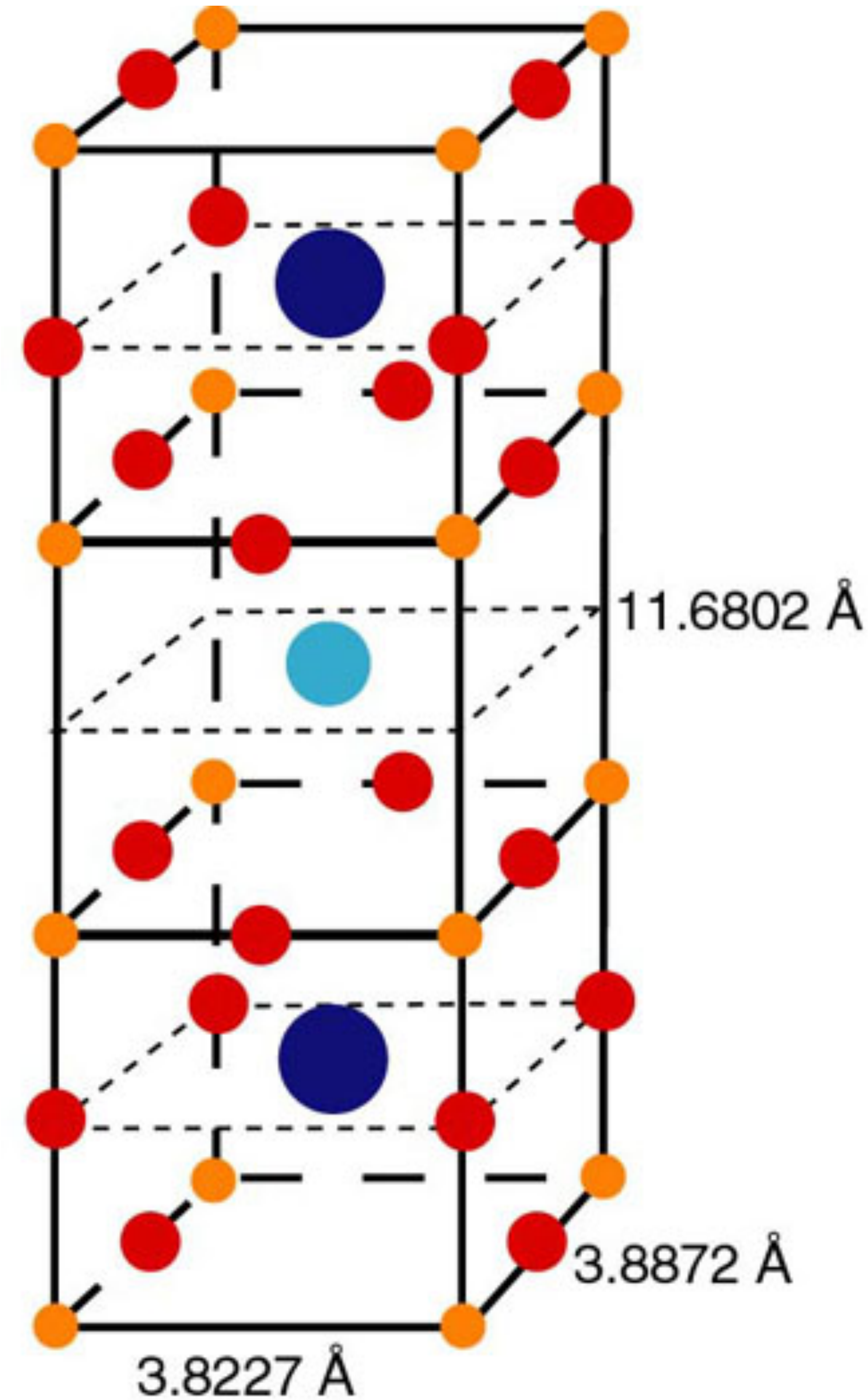
Grigory Tarnopolsky

Antoine Georges

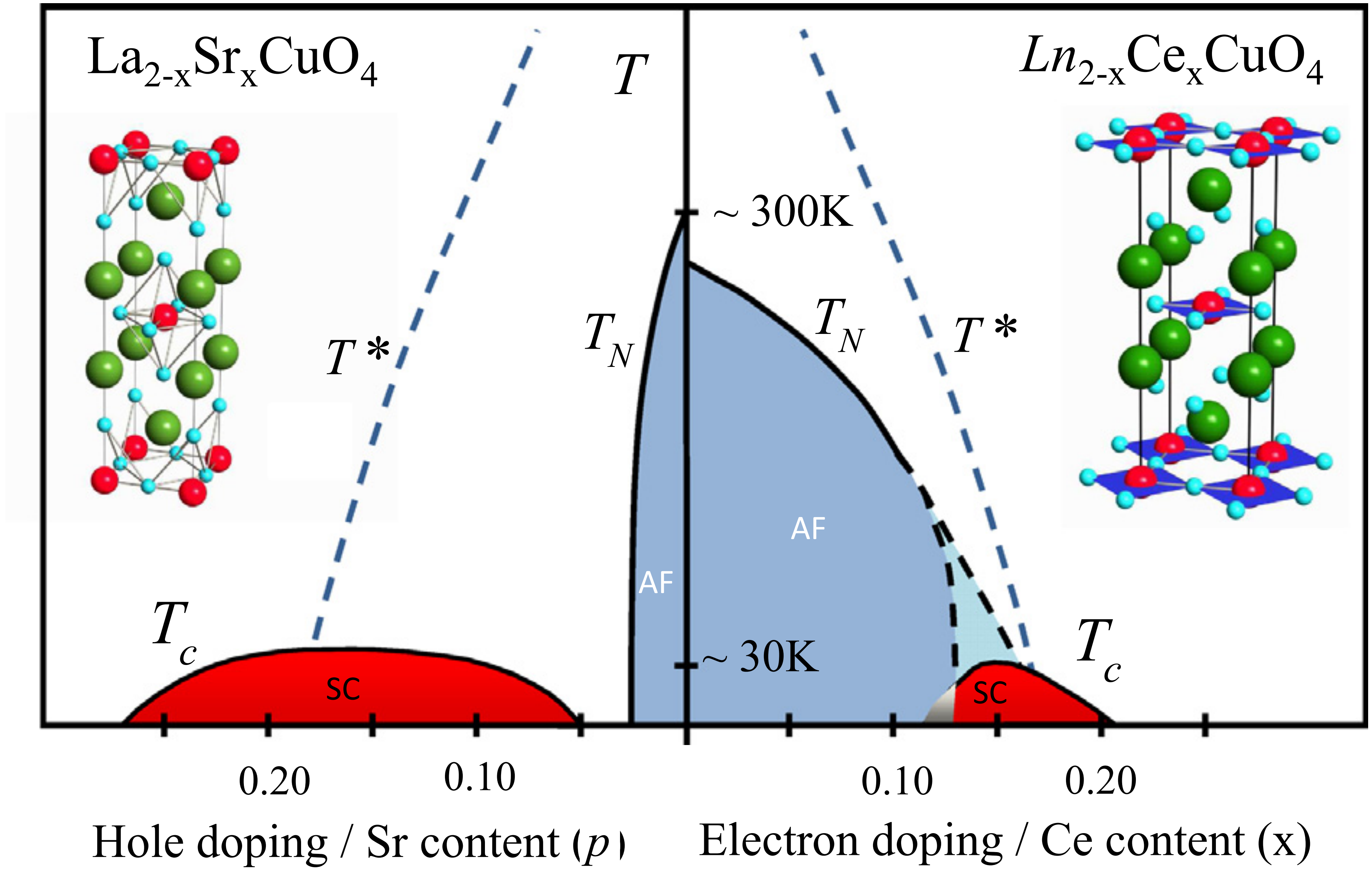
High temperature superconductors



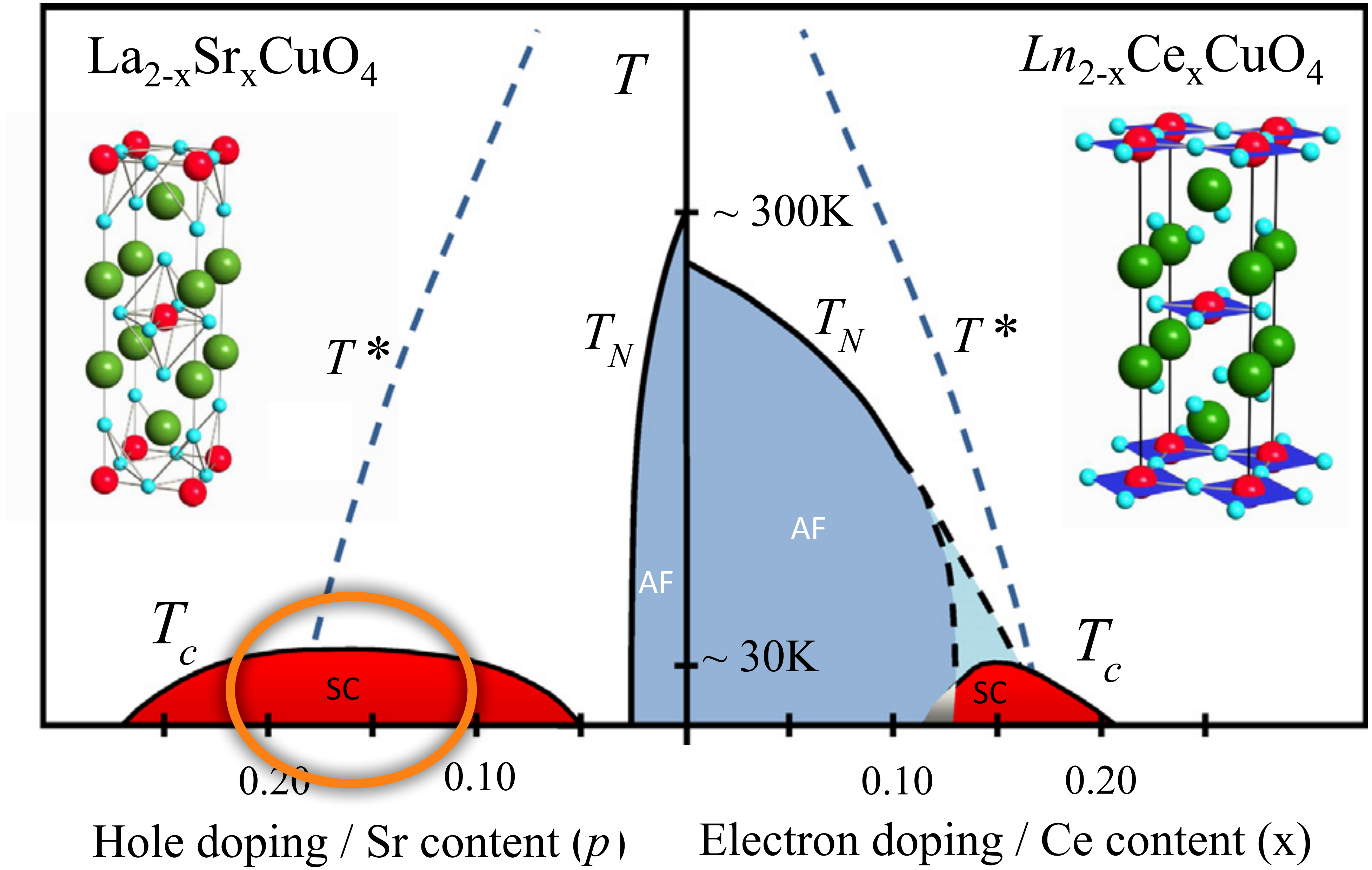
CuO_2 plane



$YBa_2Cu_3O_{6+x}$

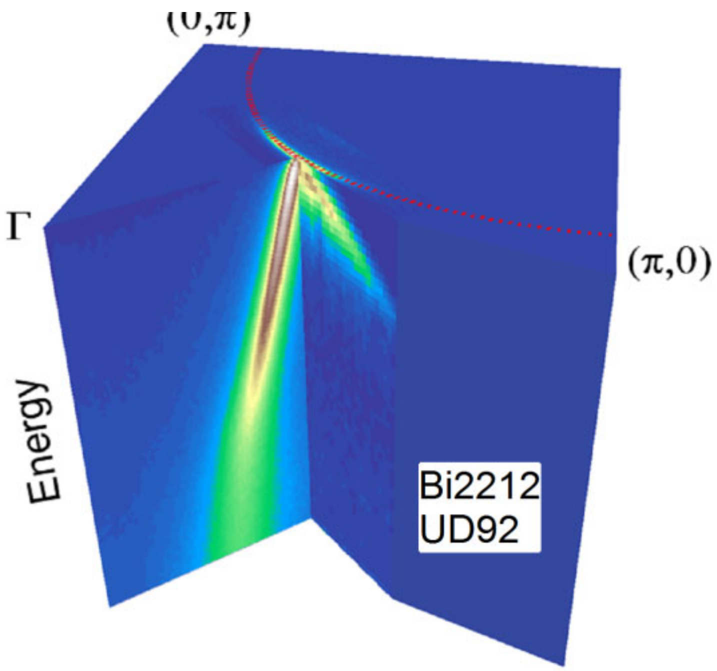


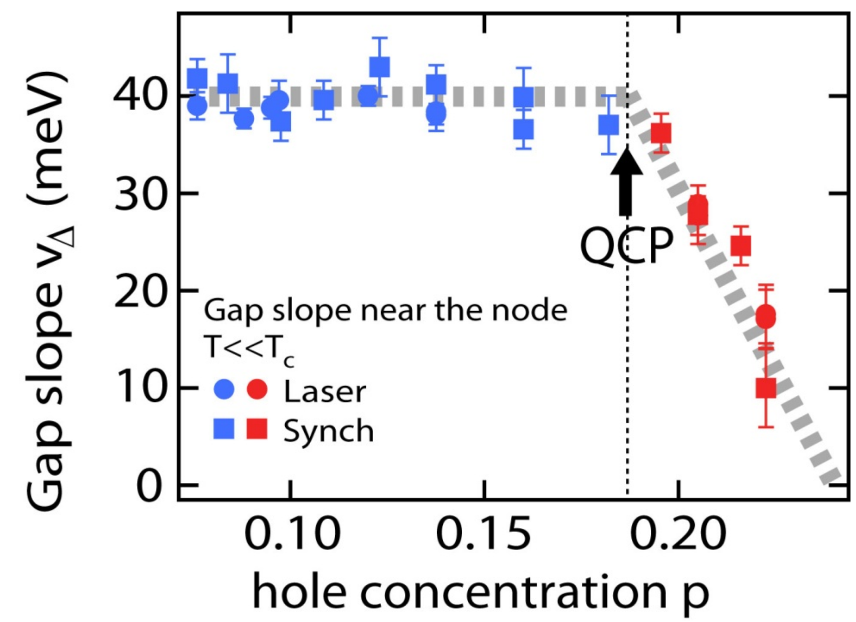
La2-xSrxCuO4
Ln2-xCexCuO4
T
~ 300K
~ 30K
T*
T*
TN
TN
Tc
Tc
AF
AF
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

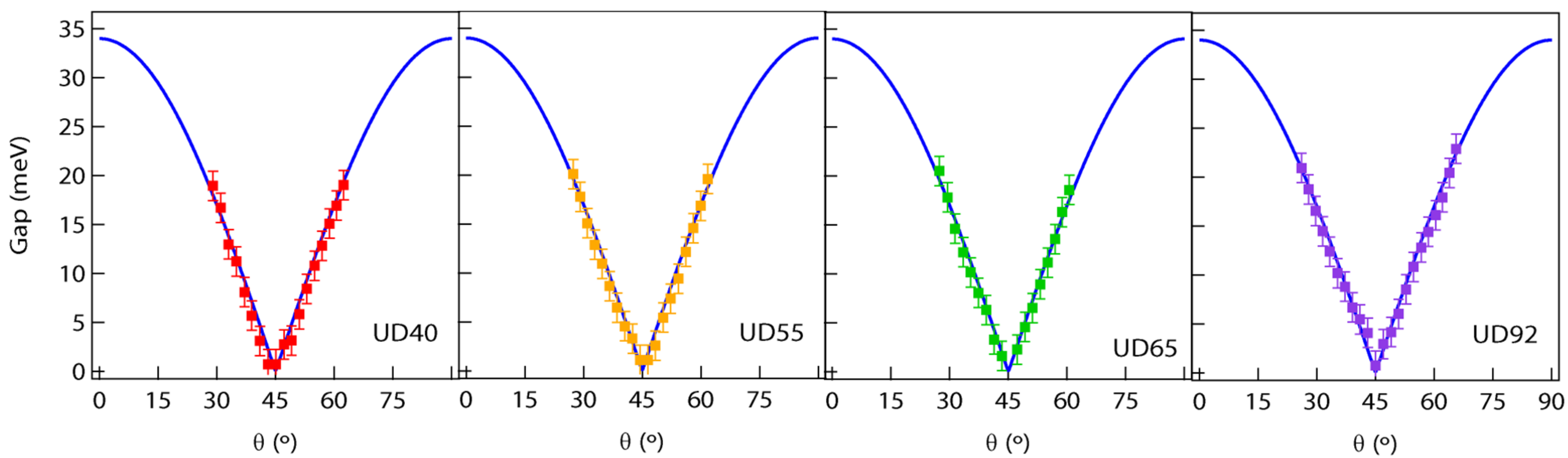


$La_{2-x}Sr_xCuO_4$
$Ln_{2-x}Ce_xCuO_4$
T
T^*
T^*
T_N
T_N
T_c
T_c
AF
AF
SC
SC
~ 300K
~ 30K
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Precision Measurement of the Node





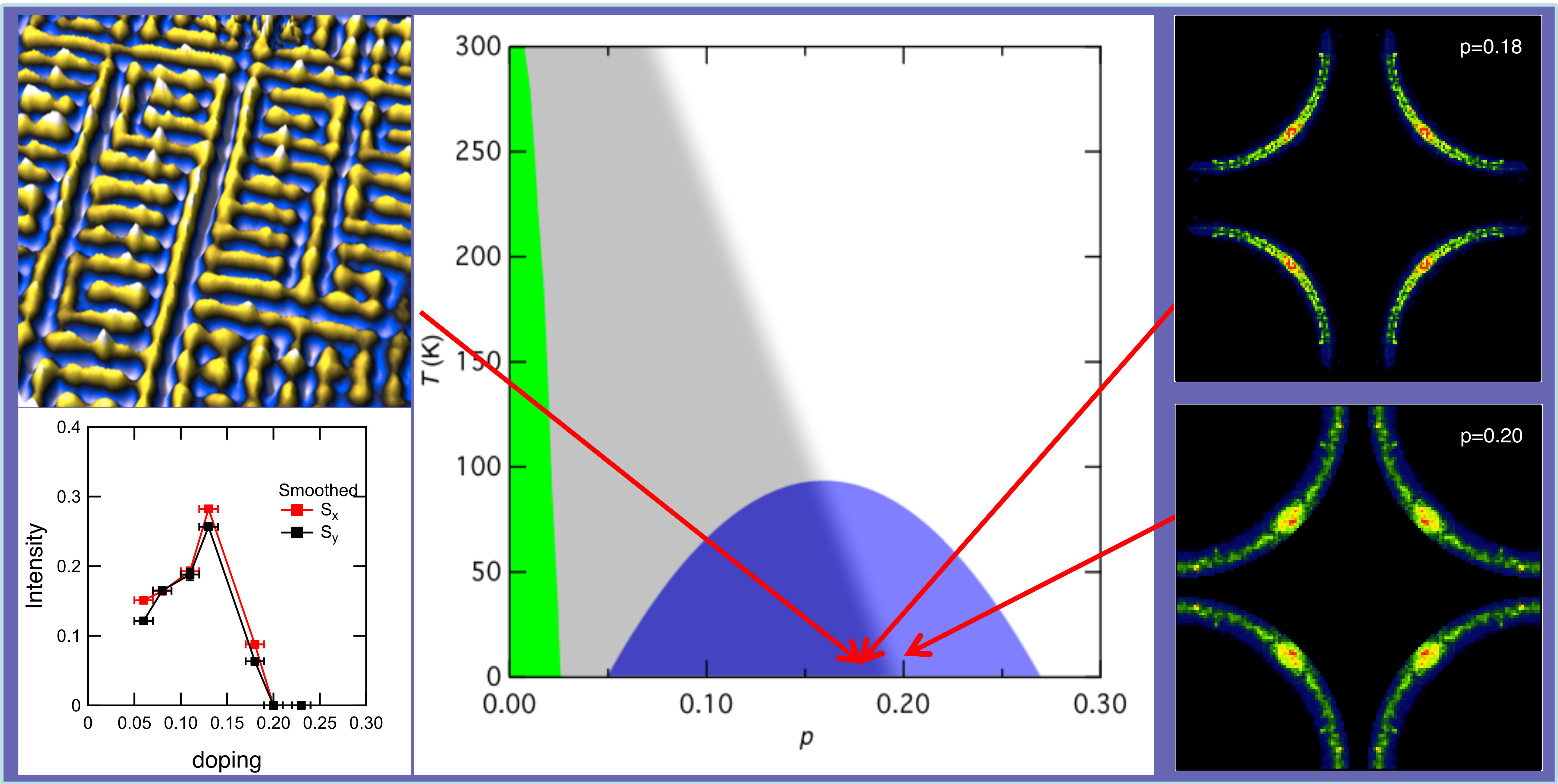


I. M. Vishik, M. Hashimoto, Rui-Hua He, Wei-Sheng Lee, Felix Schmitt, Donghui Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa, S. Uchida, Kazuhiro Fujita, S. Ishida, M. Ishikado, Yoshiyuki Yoshida, Hiroshi Eisaki, Zahid Hussain, Thomas P. Devereaux, and Zhi-Xun Shen, PNAS **109**, 18332 (2012)

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

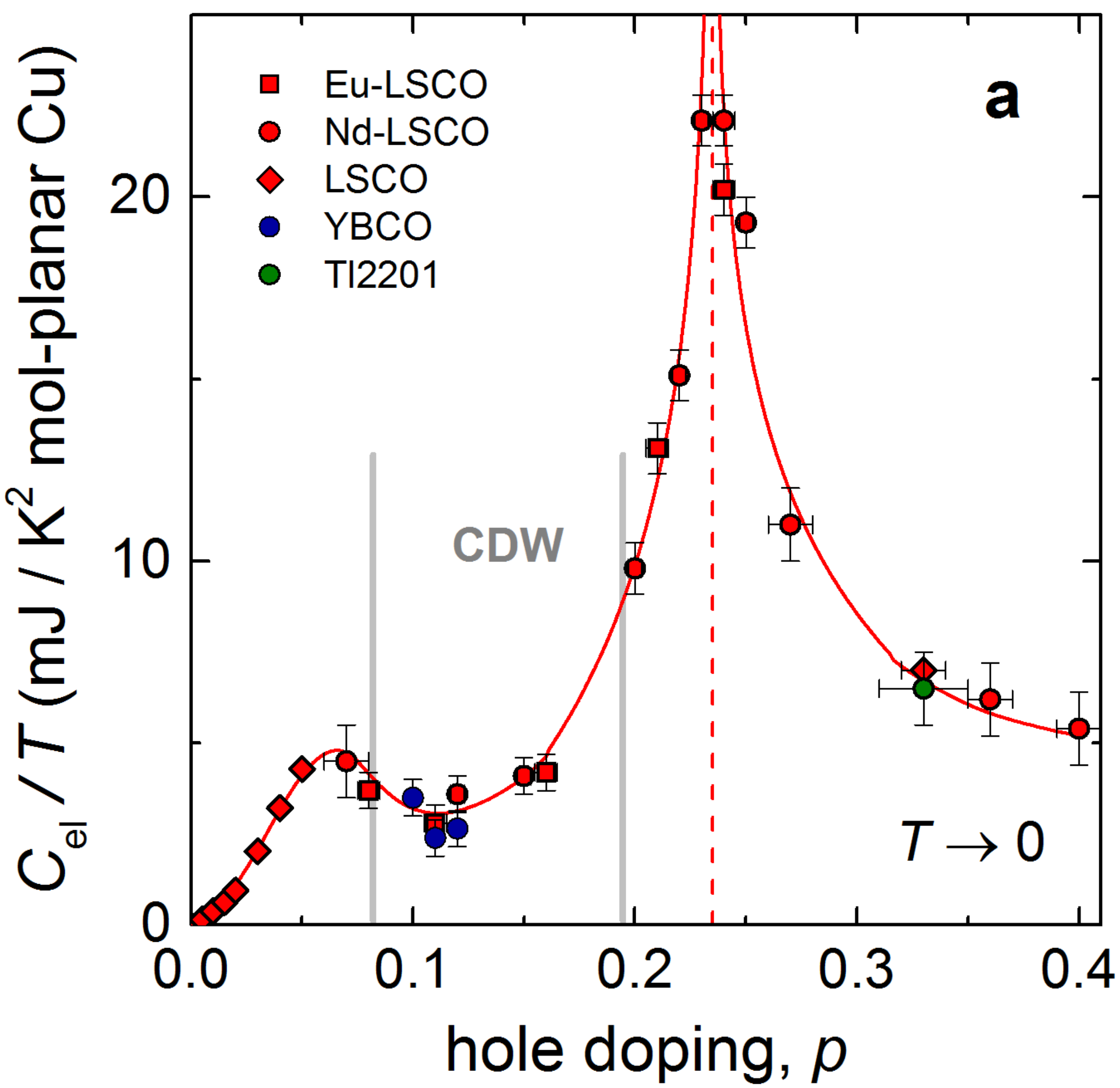
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)

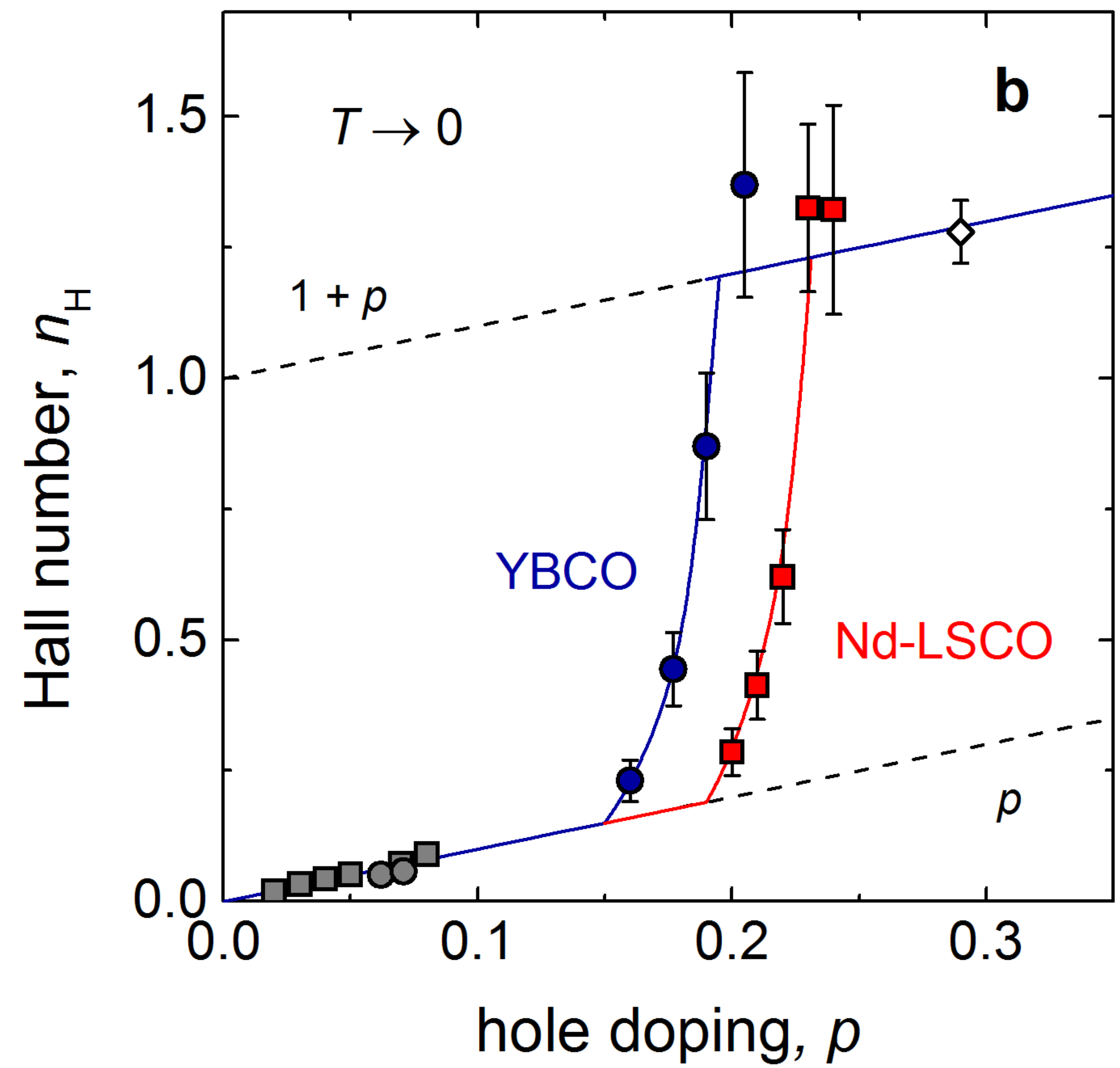


Hole doped cuprates

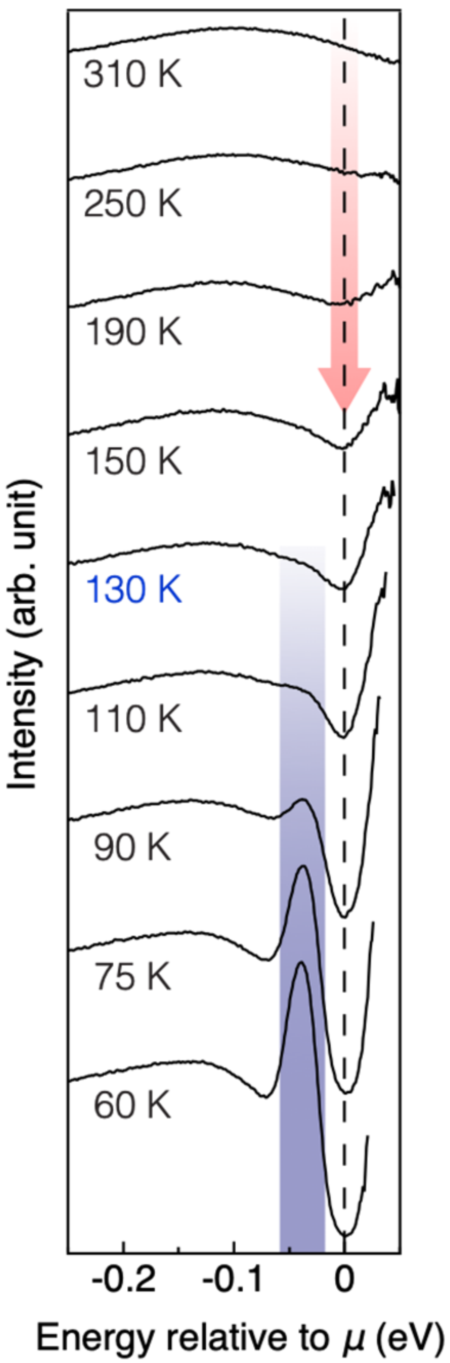
The remarkable underlying ground states of cuprate superconductors

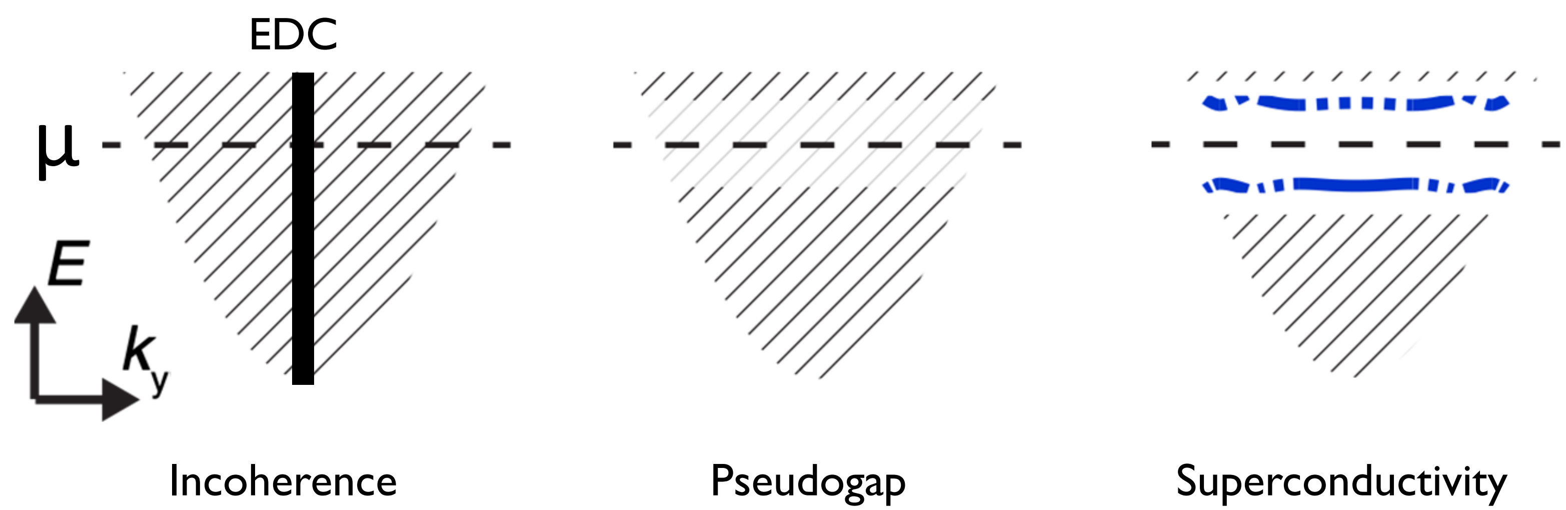
Cyril Proust and Louis Taillefer, arXiv:1807.0507

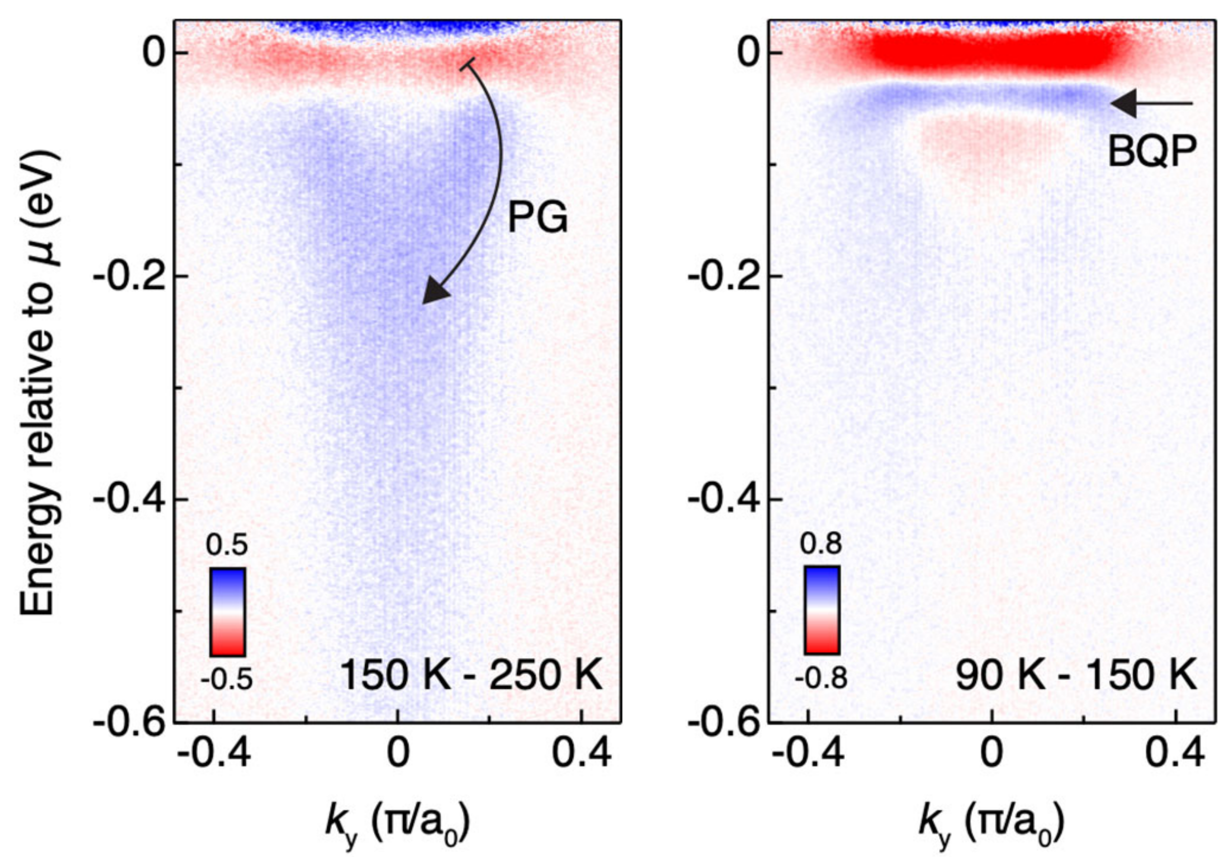




Two “gaps” for p < 0.19 (T_c~ 86 K)

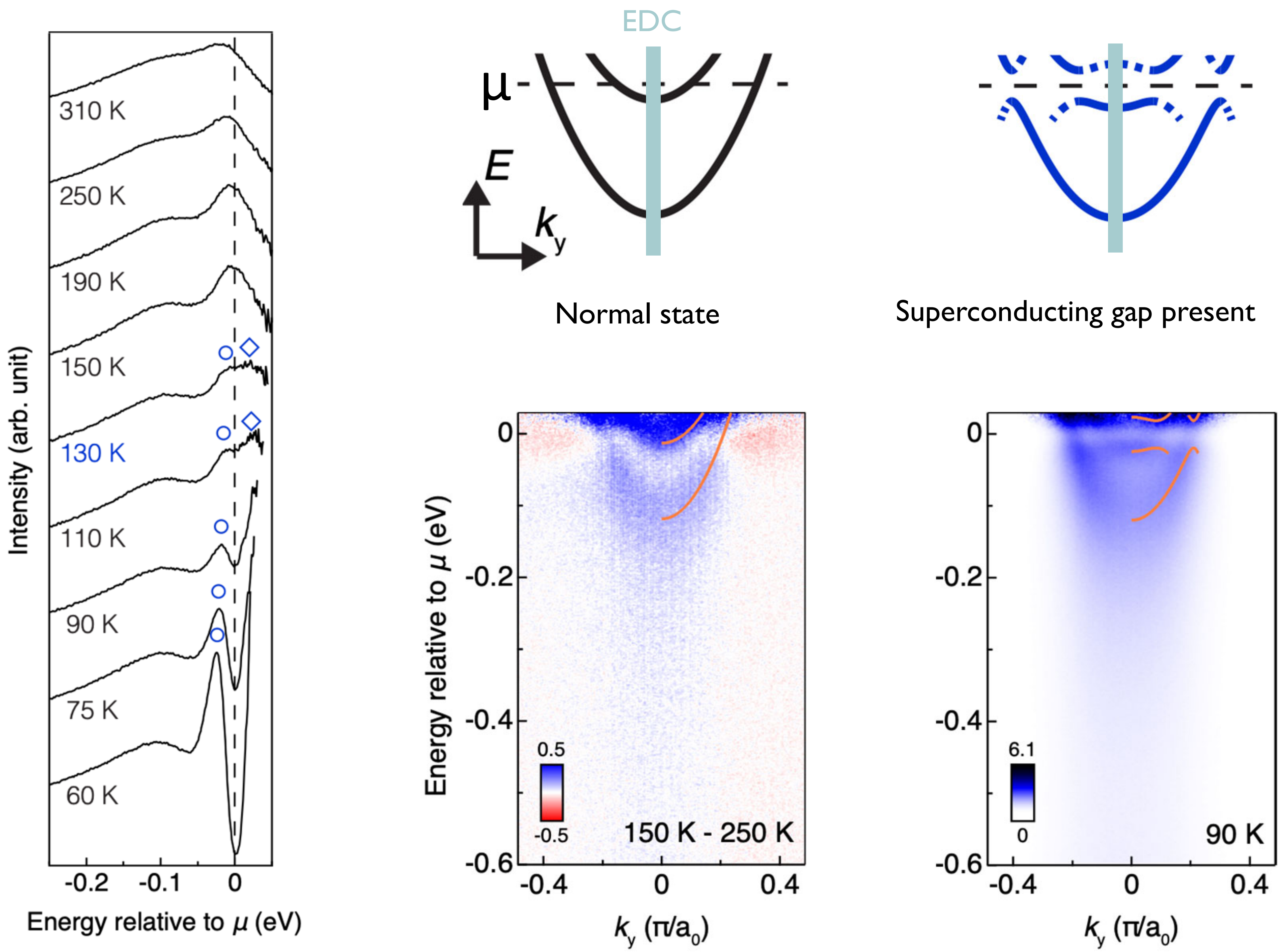






Su-Di Chen, Makoto Hashimoto, Yu He, Dongjoon Song, Ke-Jun Xu, Jun-Feng He, T. P. Devereaux, Hiroshi Eisaki, Dong-Hui Lu, J. Zaanen, Zhi-Xun Shen, *Science* **366**, 6469 (2019)

One gap for $p > 0.19$ ($T_c \sim 81$ K)



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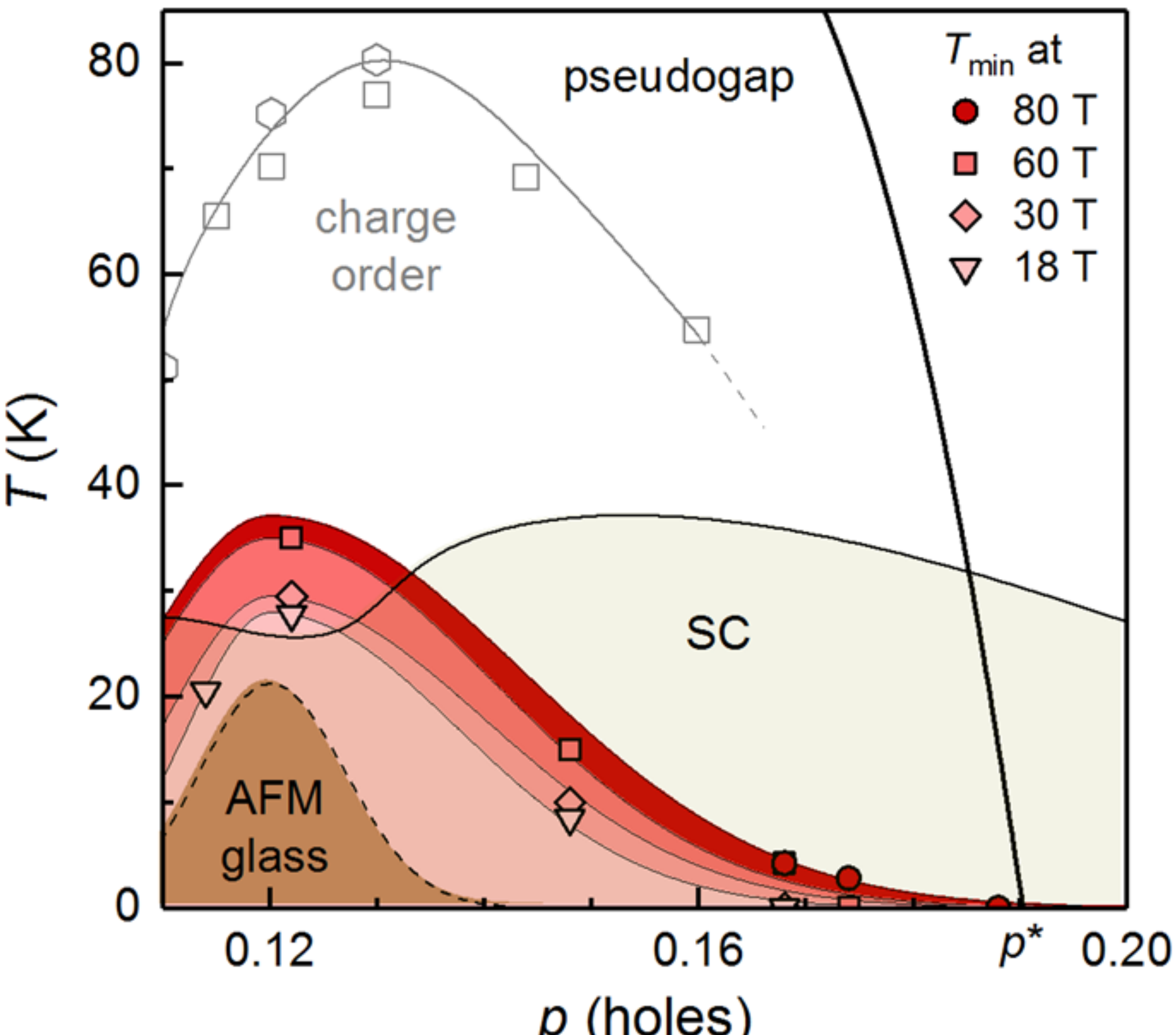
Hidden magnetism at the pseudogap critical point of a high temperature superconductor

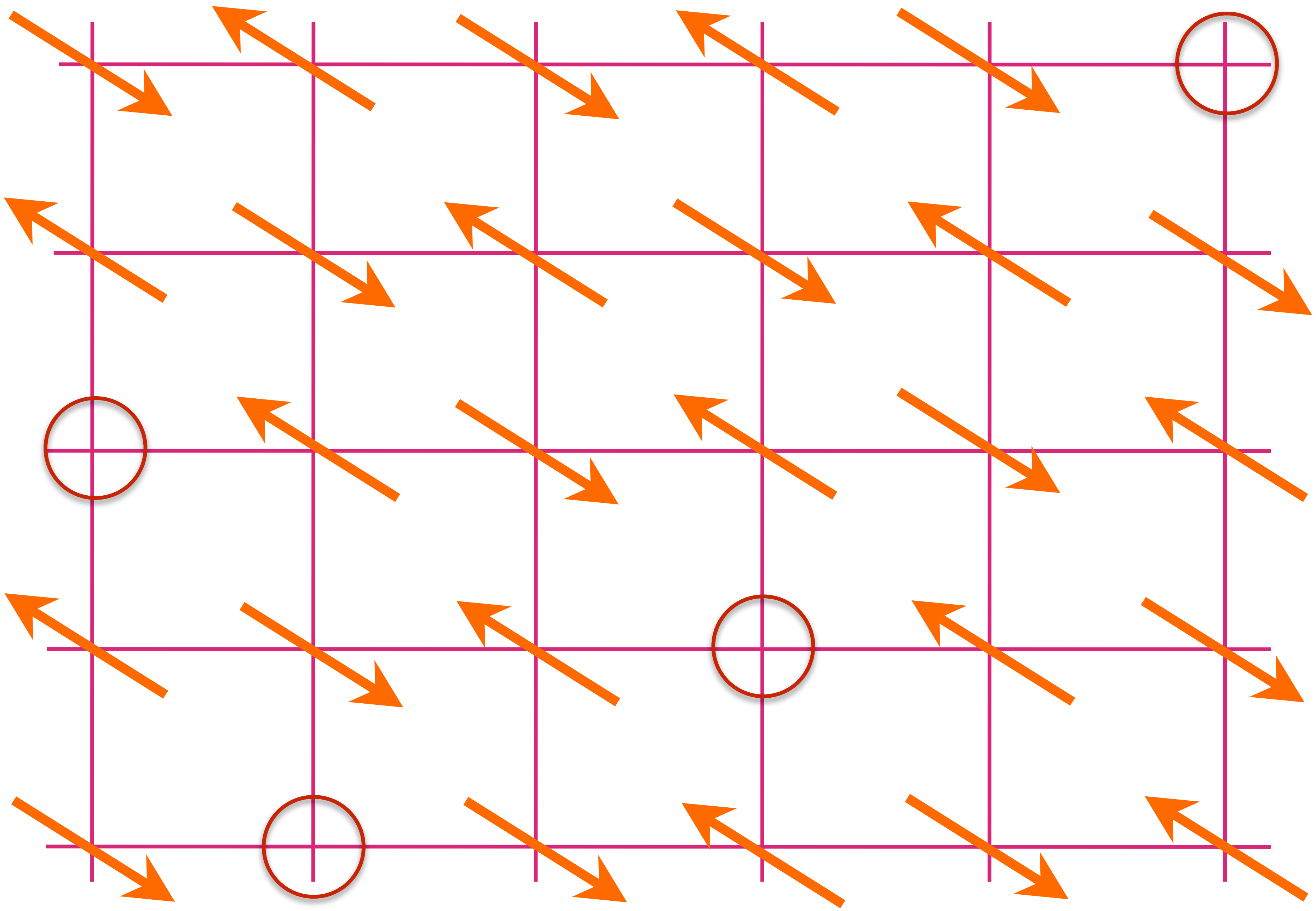
Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



arXiv:1909.10258

Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

Real-space view at small p



p mobile holes in a background of fluctuating spins

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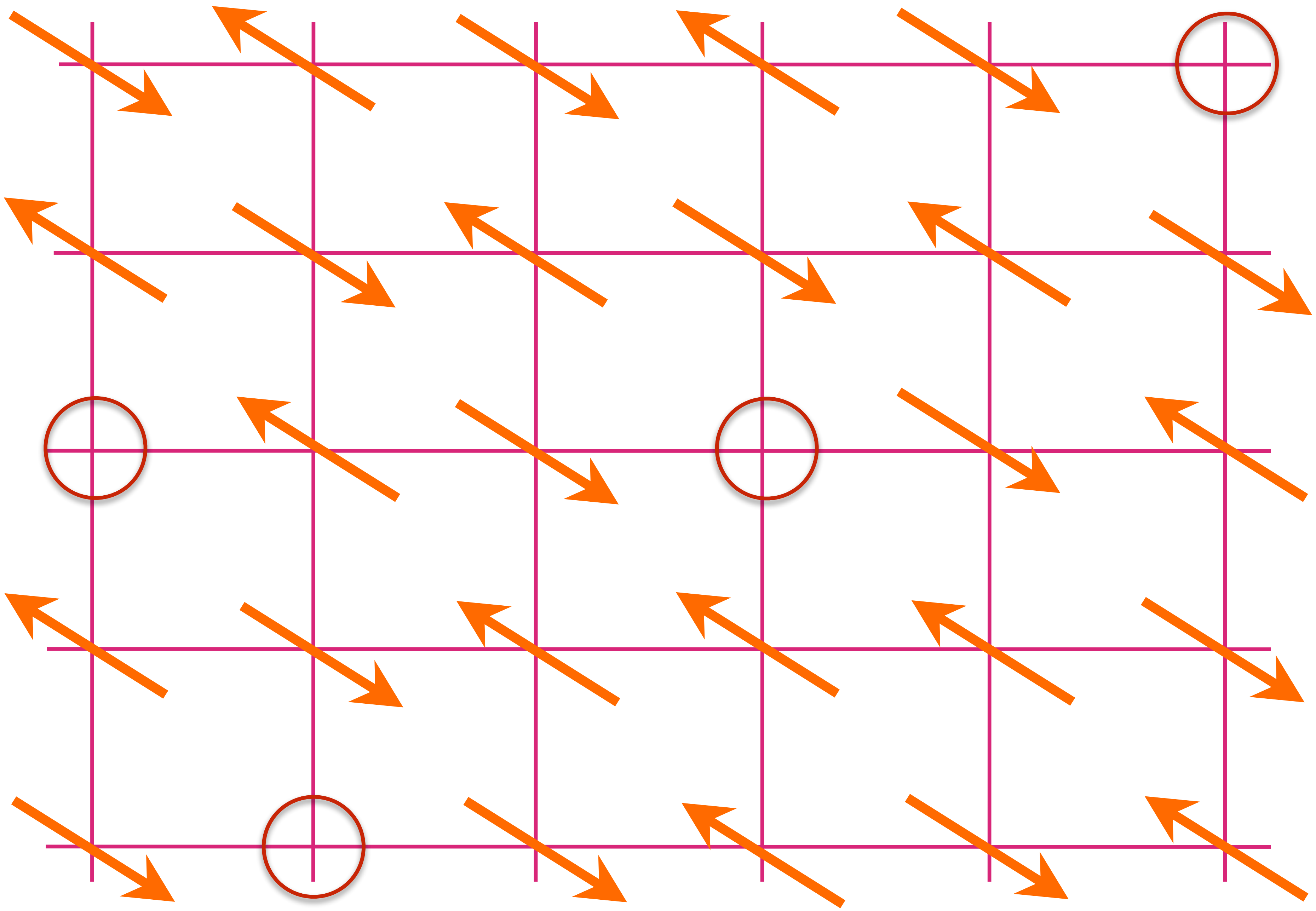

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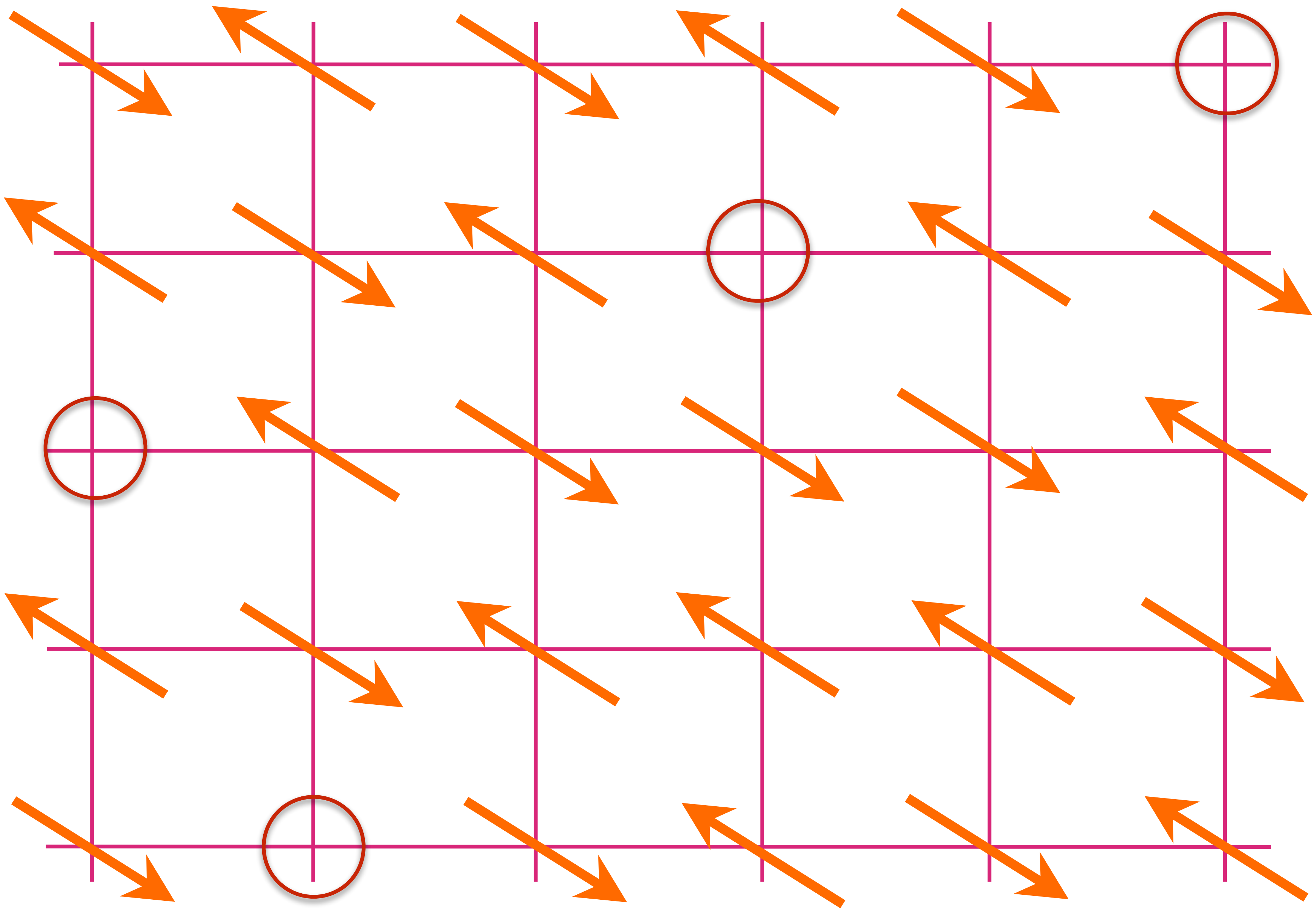

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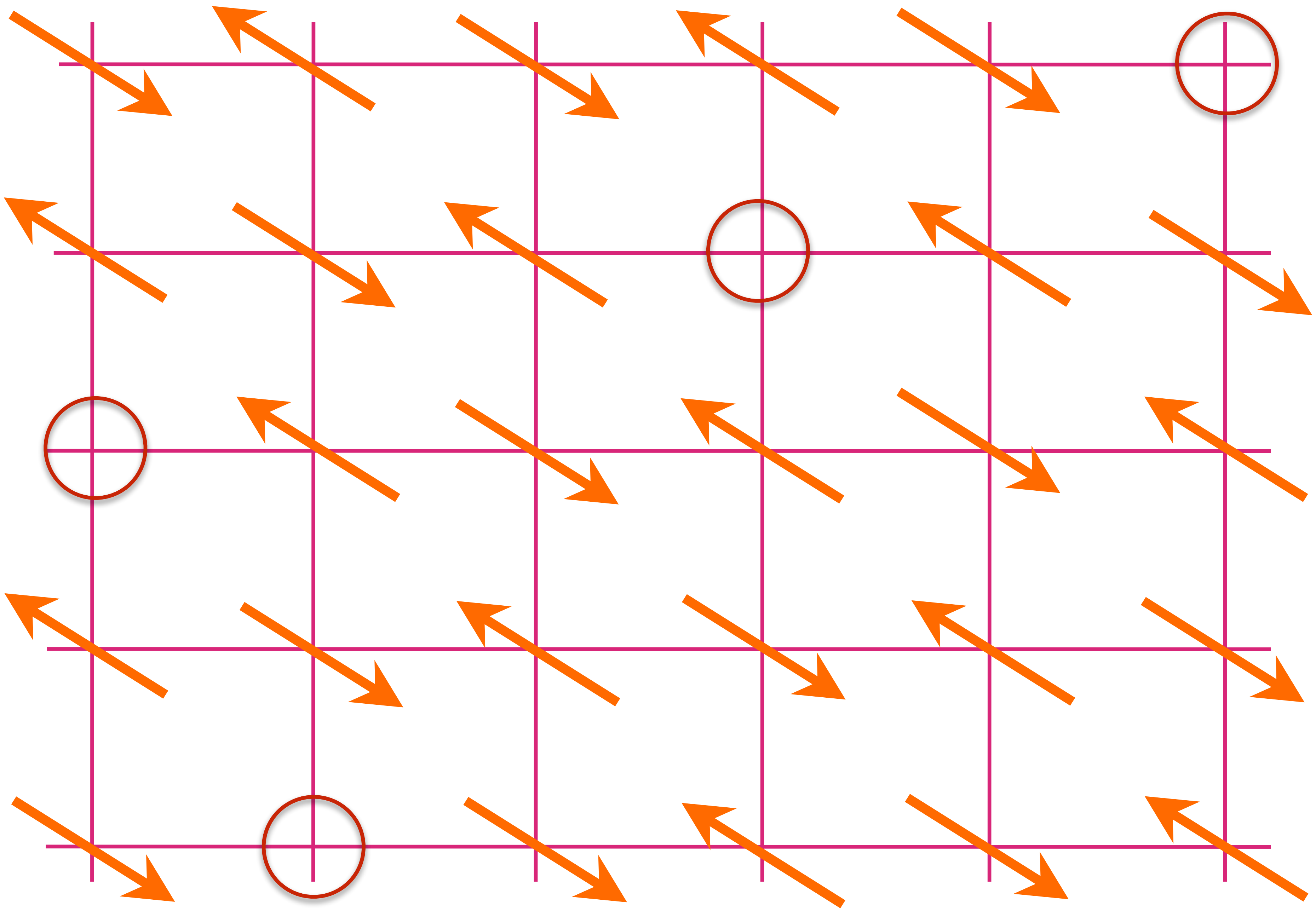

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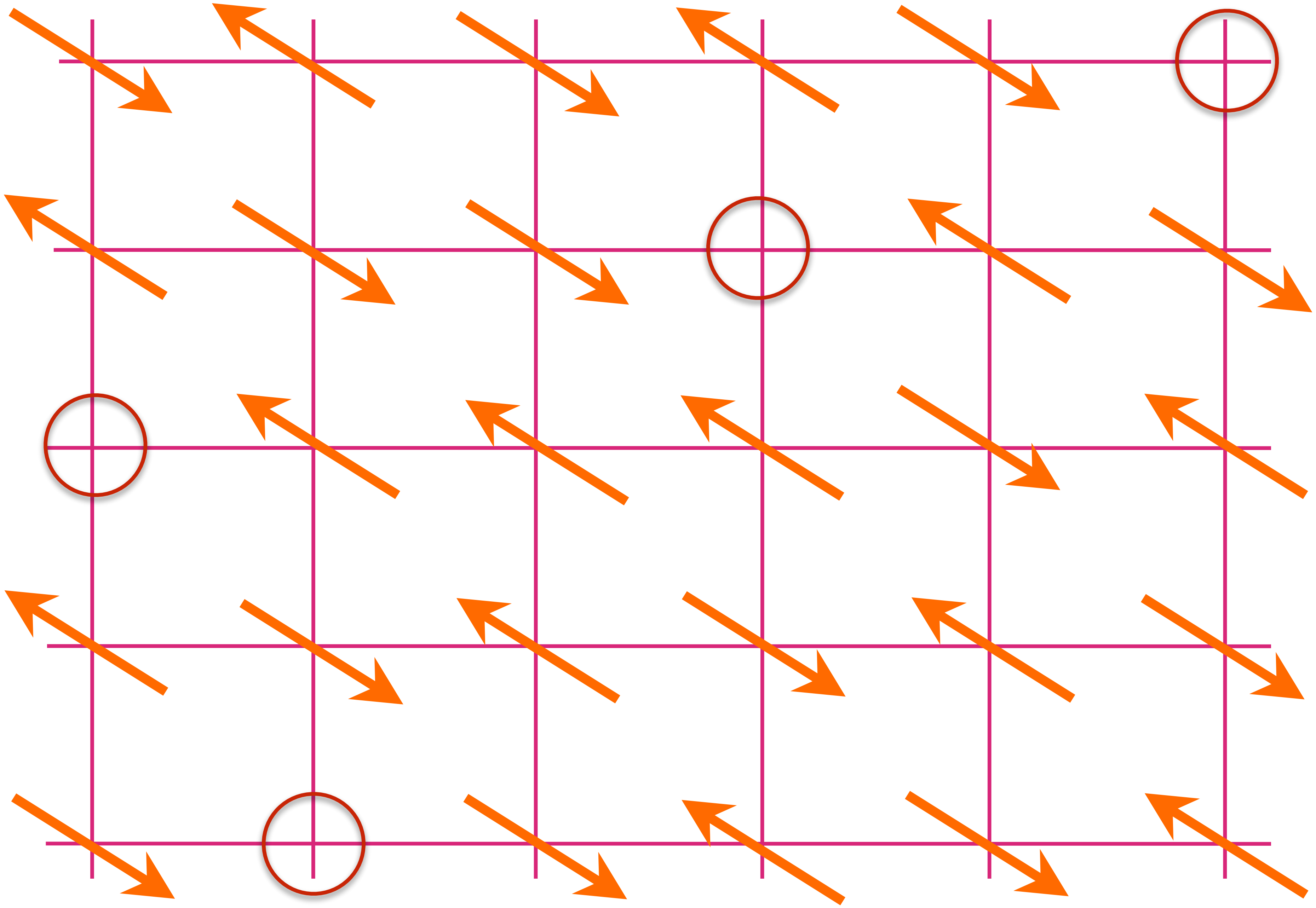

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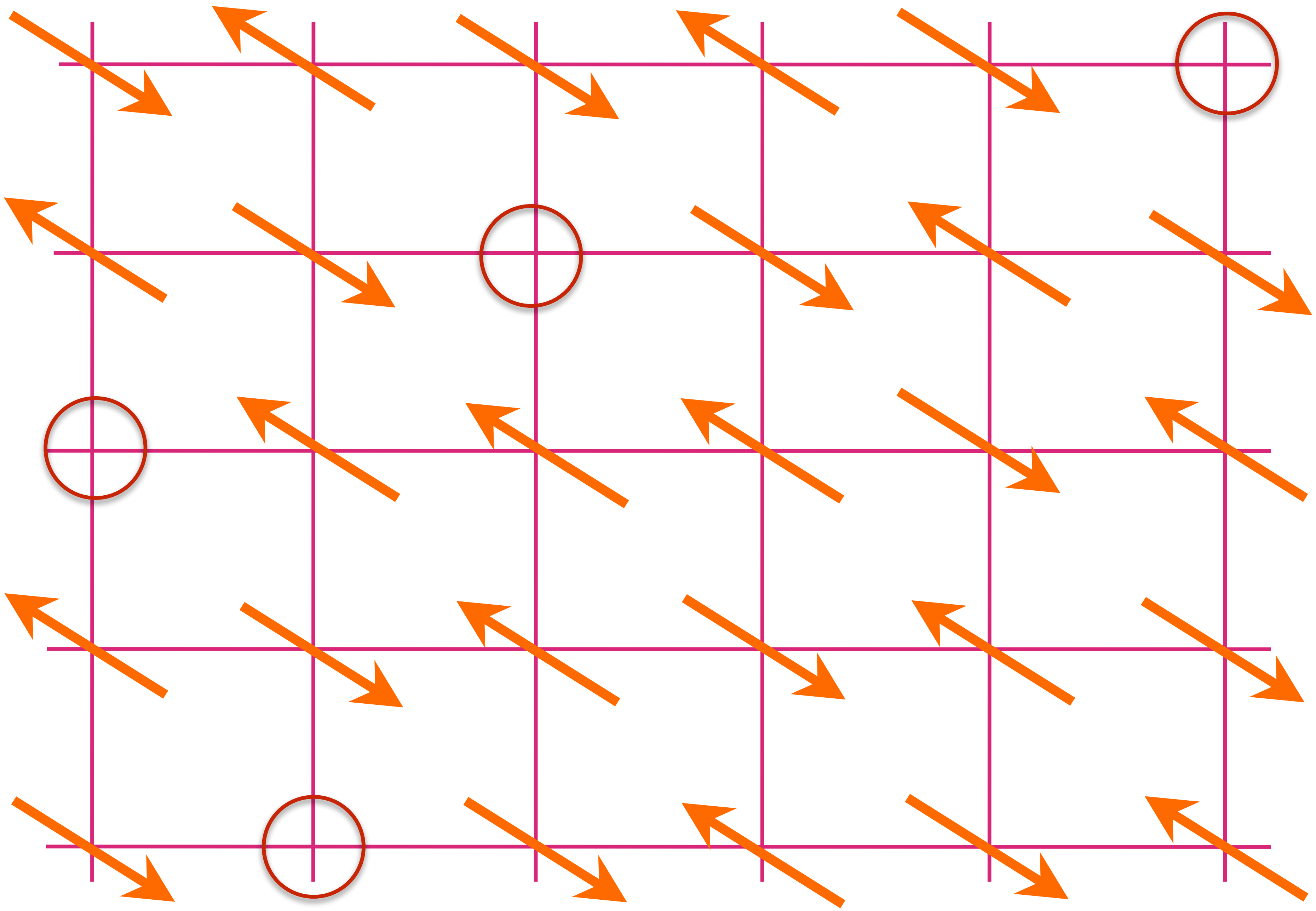

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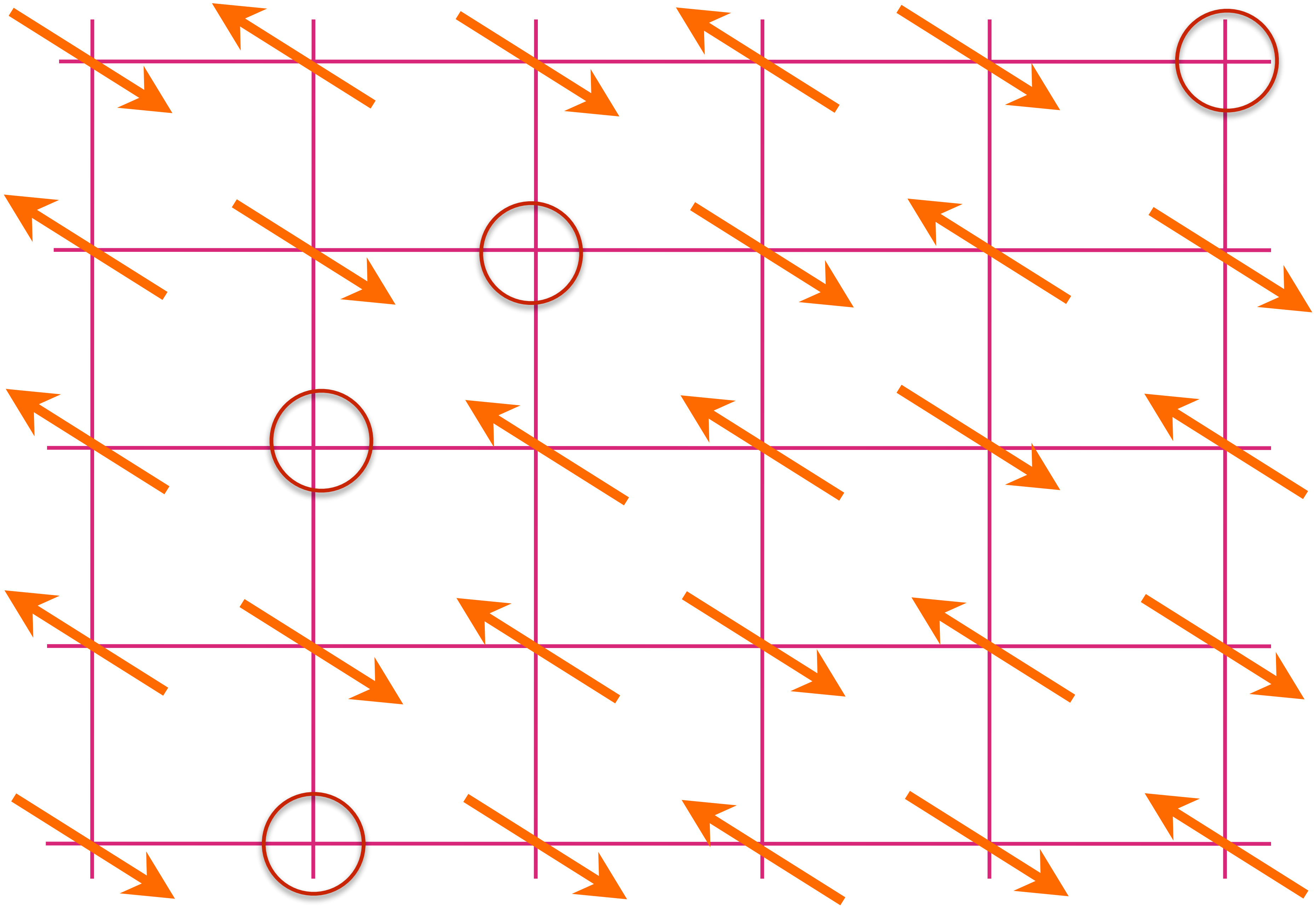

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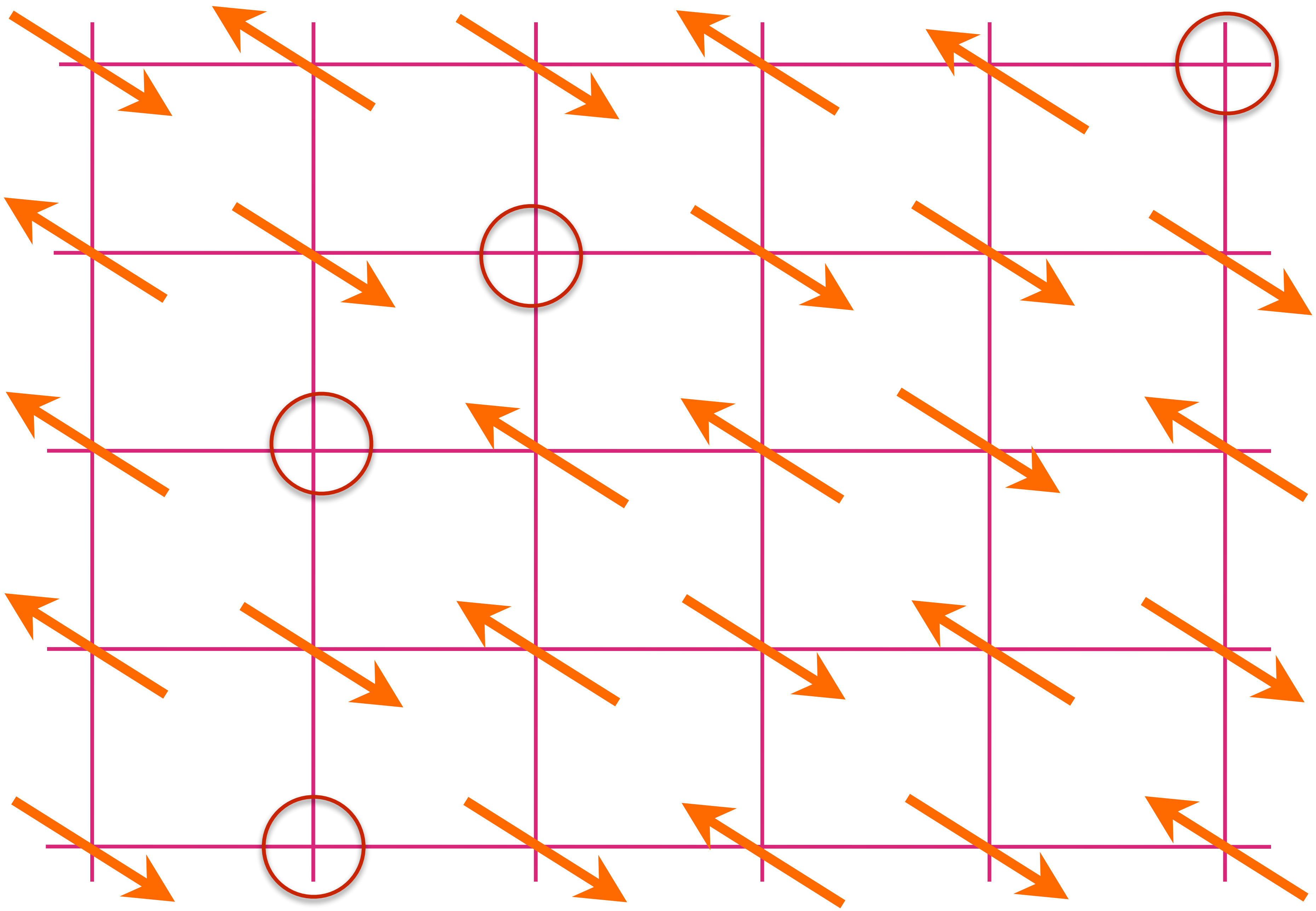

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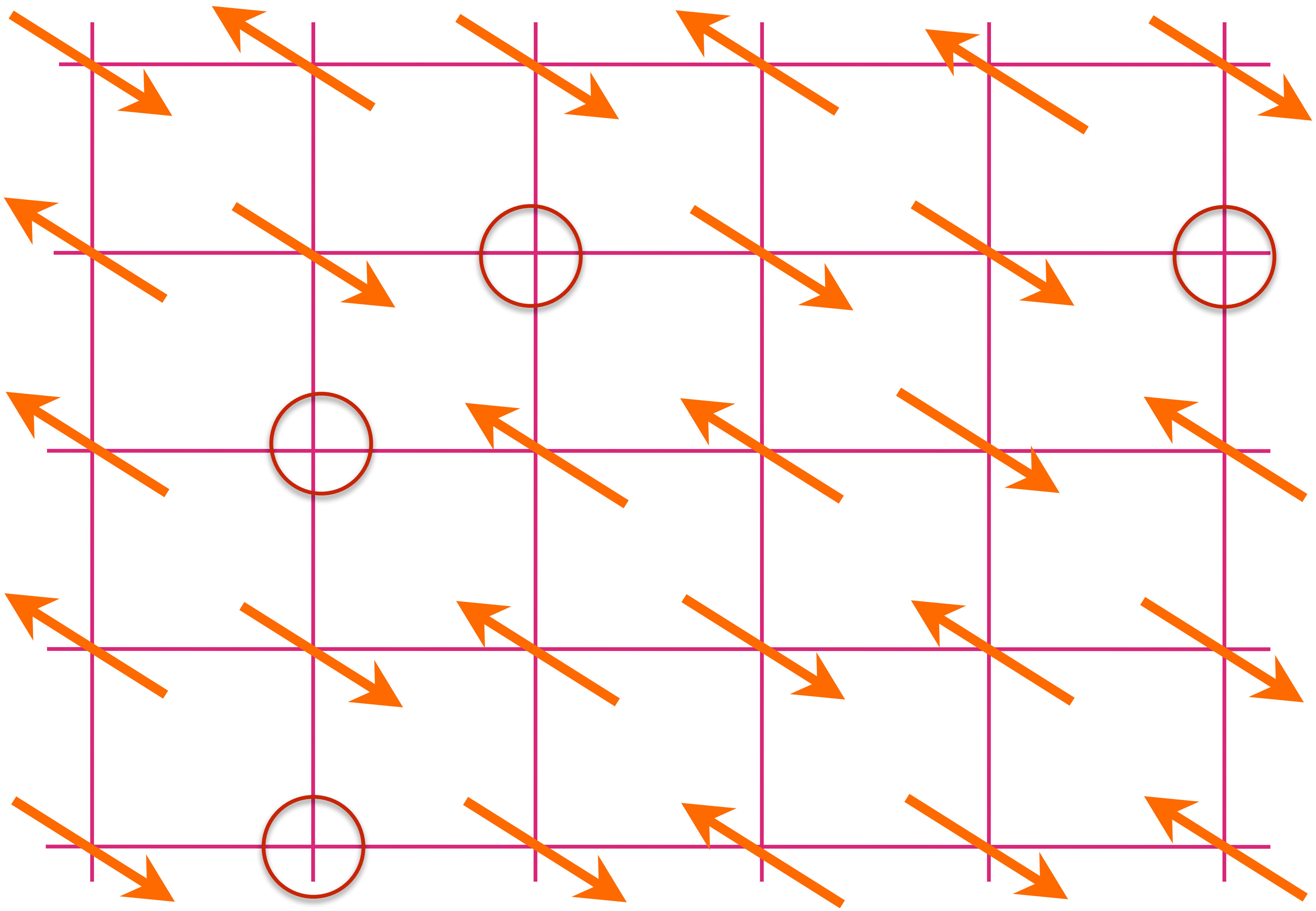
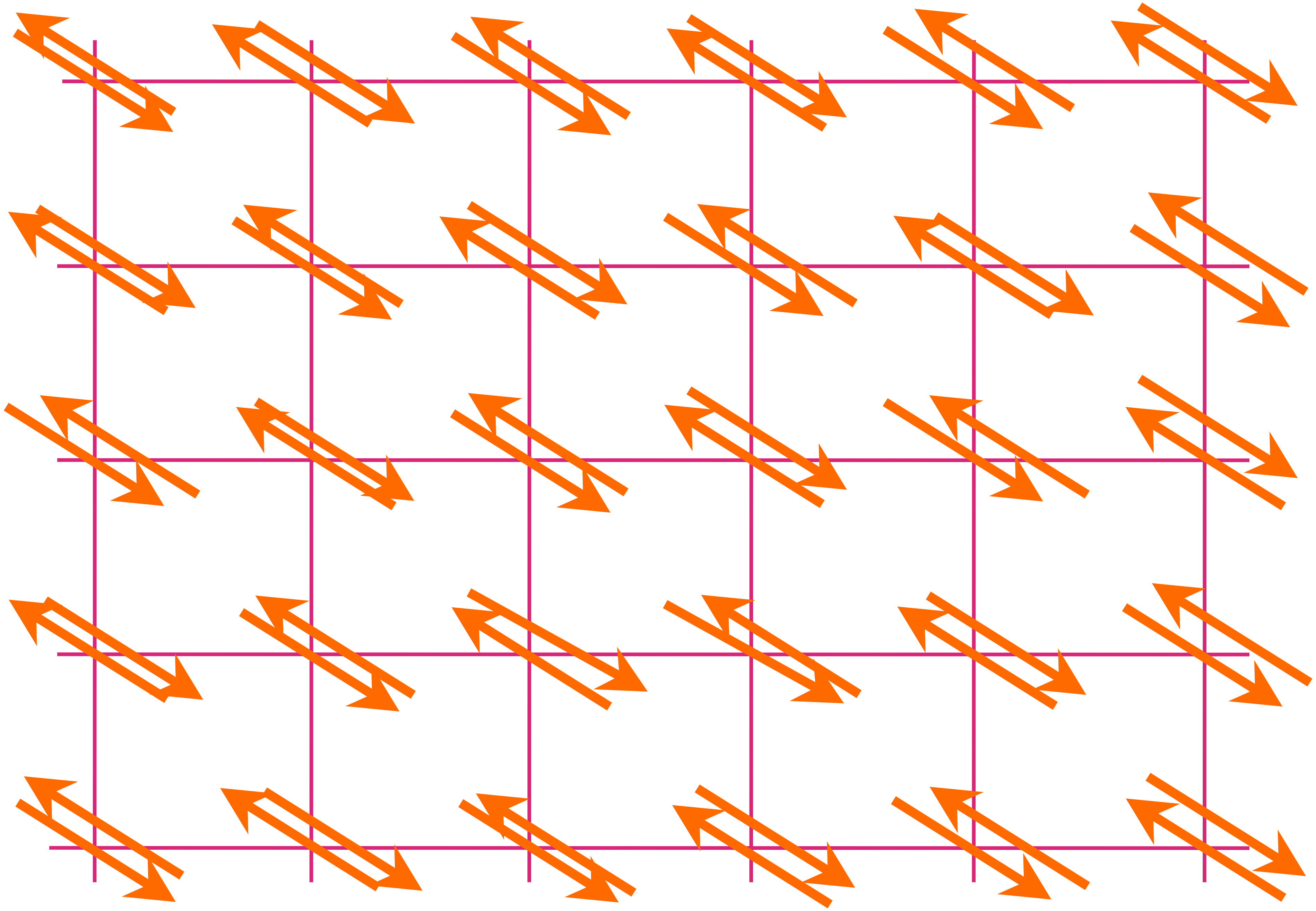


p mobile holes in a background of fluctuating spins

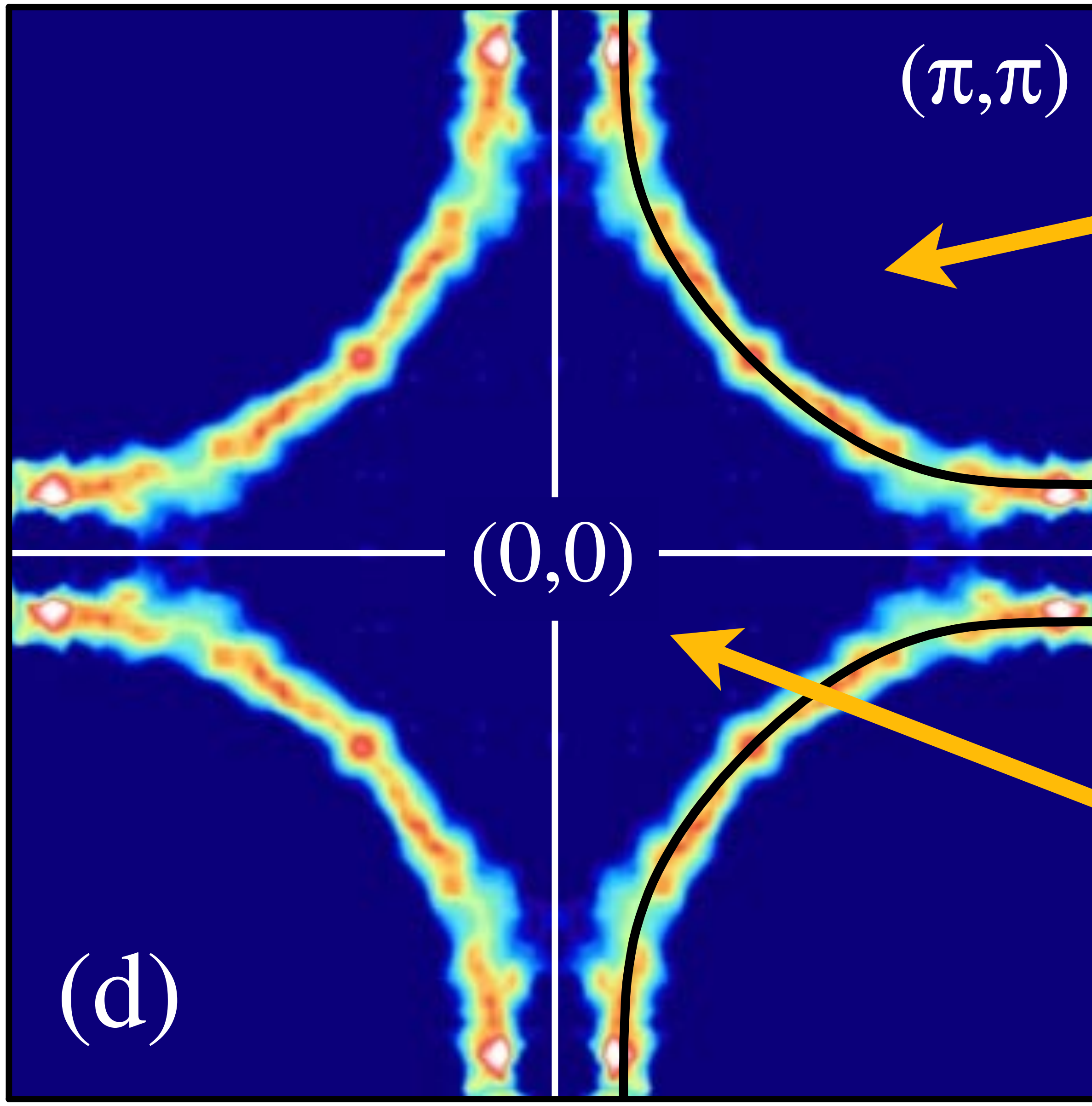
Momentum-space view at large p



Filled Band

$1+p$ mobile holes in a filled band

Momentum-space view at large p



$1+p$ holes

1-p electrons

$1+p$ mobile holes in a filled band

Questions

- Is there a sharp quantum phase transition at $p = p_c$ between the p and $1 + p$ carrier density regimes?

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- Are fractionalization and emergent gauge fields present for $p < p_c$ with or without disorder?

- Can there be a DQCP in a random system without fractionalization or broken symmetry in the $p < p_c$ state? *i.e.* an 'unnecessary' critical theory?

Questions and Answers

- Is there a sharp quantum phase transition at $p = p_c$ between the p and $1 + p$ carrier density regimes? **Yes**

- Does the sharp QPT survive in the presence of disorder? **Yes**

- If there is a broken symmetry for $p < p_c$, is the QPT described by a Landau-Ginzburg-Wilson-Hertz-Millis theory of a fluctuating order parameter damped by Fermi surface excitations? **No**

- Or is the QPT described by a *deconfined quantum critical point* with fractionalization and emergent gauge fields? **Yes**

- Are fractionalization and emergent gauge fields present for $p < p_c$ with or without disorder? **Maybe**

- Can there be a DQCP in a random system without fractionalization or broken symmetry in the $p < p_c$ state? *i.e.* an 'unnecessary' critical theory? **????**

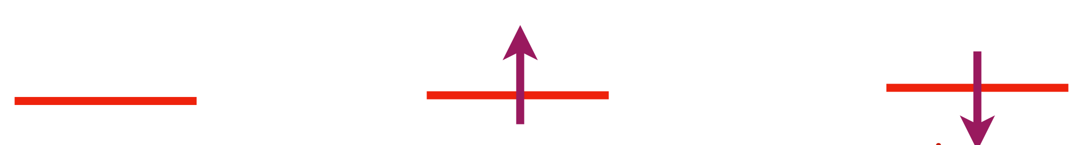
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1$$

$$J_{ij} \text{ random}, \; \overline{J_{ij}} = 0, \; \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \; \overline{t_{ij}} = 0, \; \overline{t_{ij}^2} = t^2$$



$|0\rangle \qquad c_{\uparrow}^{\dagger} |0\rangle \qquad c_{\downarrow}^{\dagger} |0\rangle$

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow b^{\dagger} |v\rangle \quad , \quad c_\alpha^{\dagger} |0\rangle \Rightarrow f_\alpha^{\dagger} |v\rangle$$

$$\begin{aligned} c_\alpha &= f_\alpha b^{\dagger} \\ \vec{S} &= \frac{1}{2} f_\alpha^{\dagger} \sigma_{\alpha\beta} f_\beta \\ f_\alpha^{\dagger} f_\alpha + b^{\dagger} b &= 1 \\ \text{U(1) gauge invariance,} \quad & b \to b e^{i\phi} \,, \quad f_\alpha \to f_\alpha e^{i\phi} \end{aligned}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

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$$\text{U(1) gauge invariance,} \qquad \mathfrak{f} \to \mathfrak{f} e^{i\phi} \, , \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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$$\vec{S} = \frac{1}{2} \mathfrak{b}_{\alpha}^{\dagger} \sigma_{\alpha\beta} \mathfrak{b}_{\beta}$$

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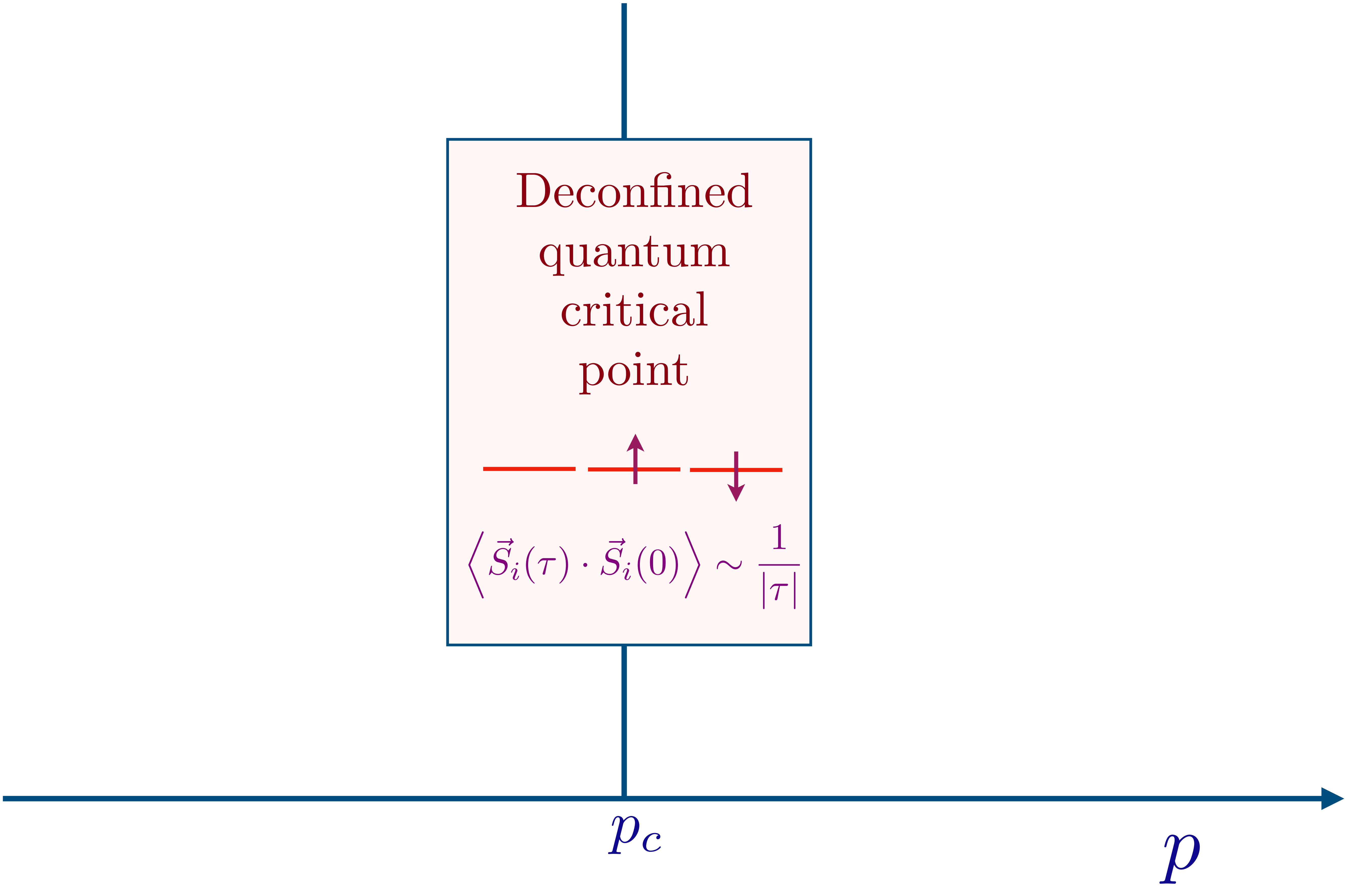
U(1) gauge invariance, $\quad \mathfrak{f} \rightarrow \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \rightarrow \mathfrak{b}_\alpha e^{i\phi}$

$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model phase diagram



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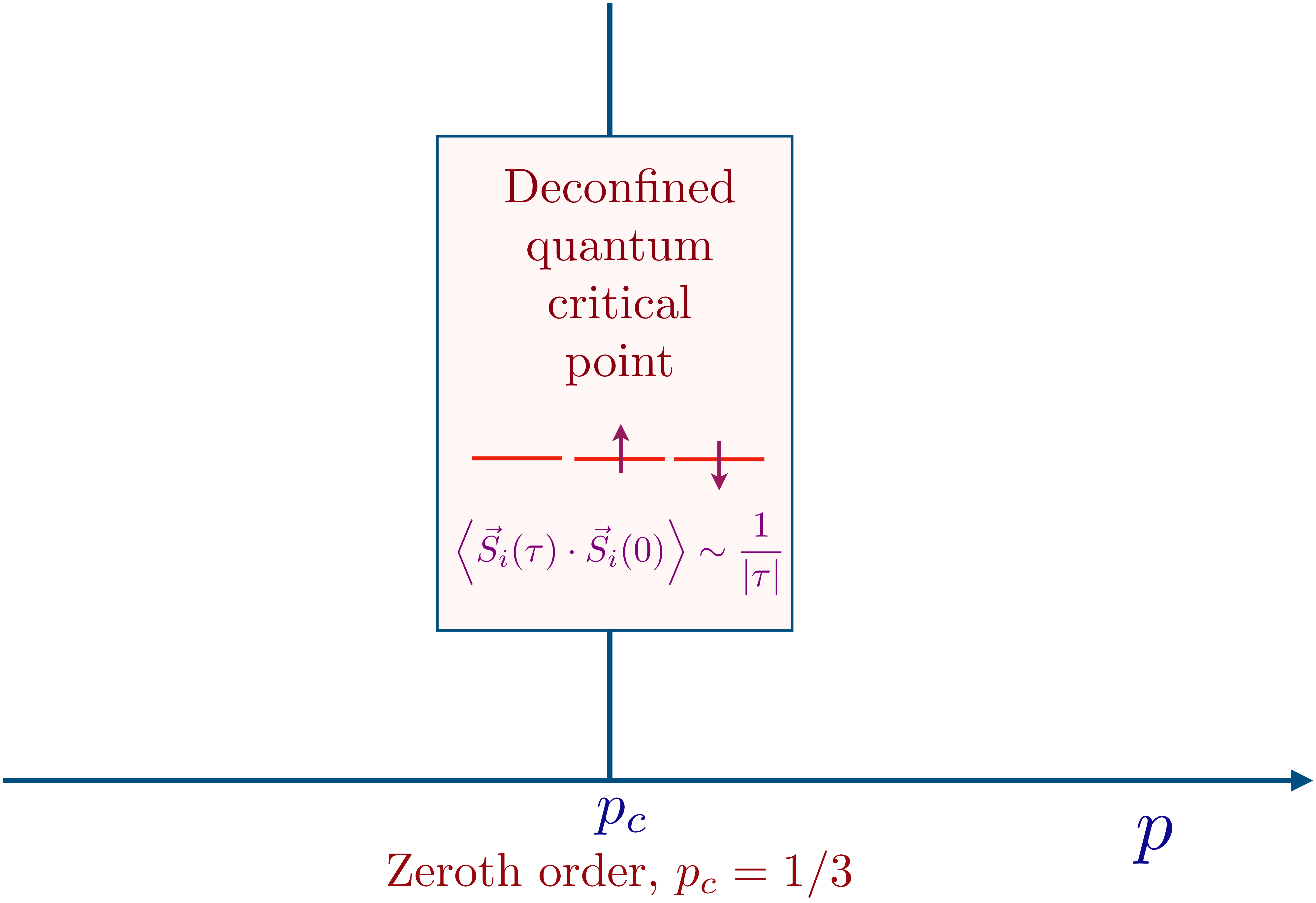

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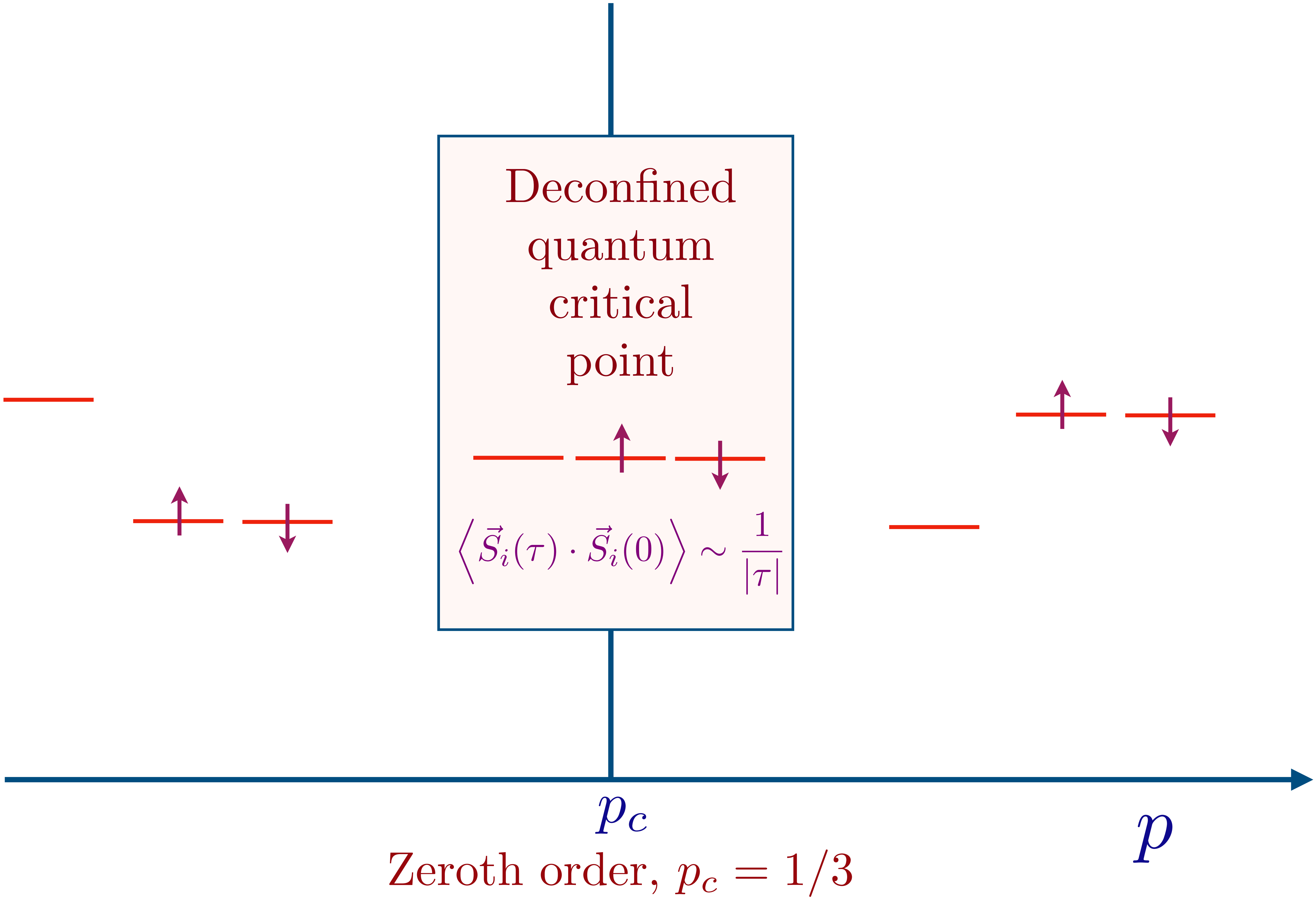

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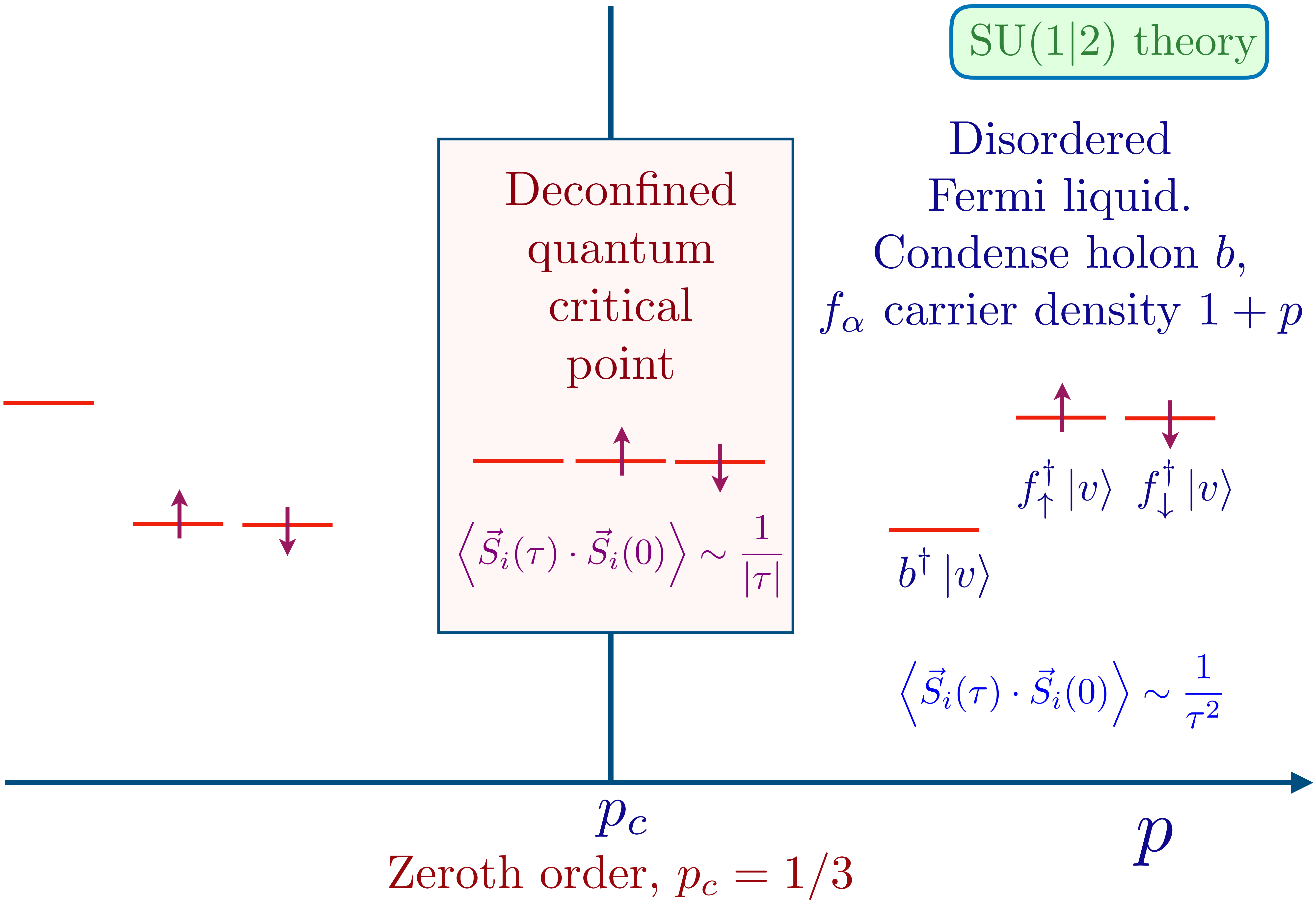

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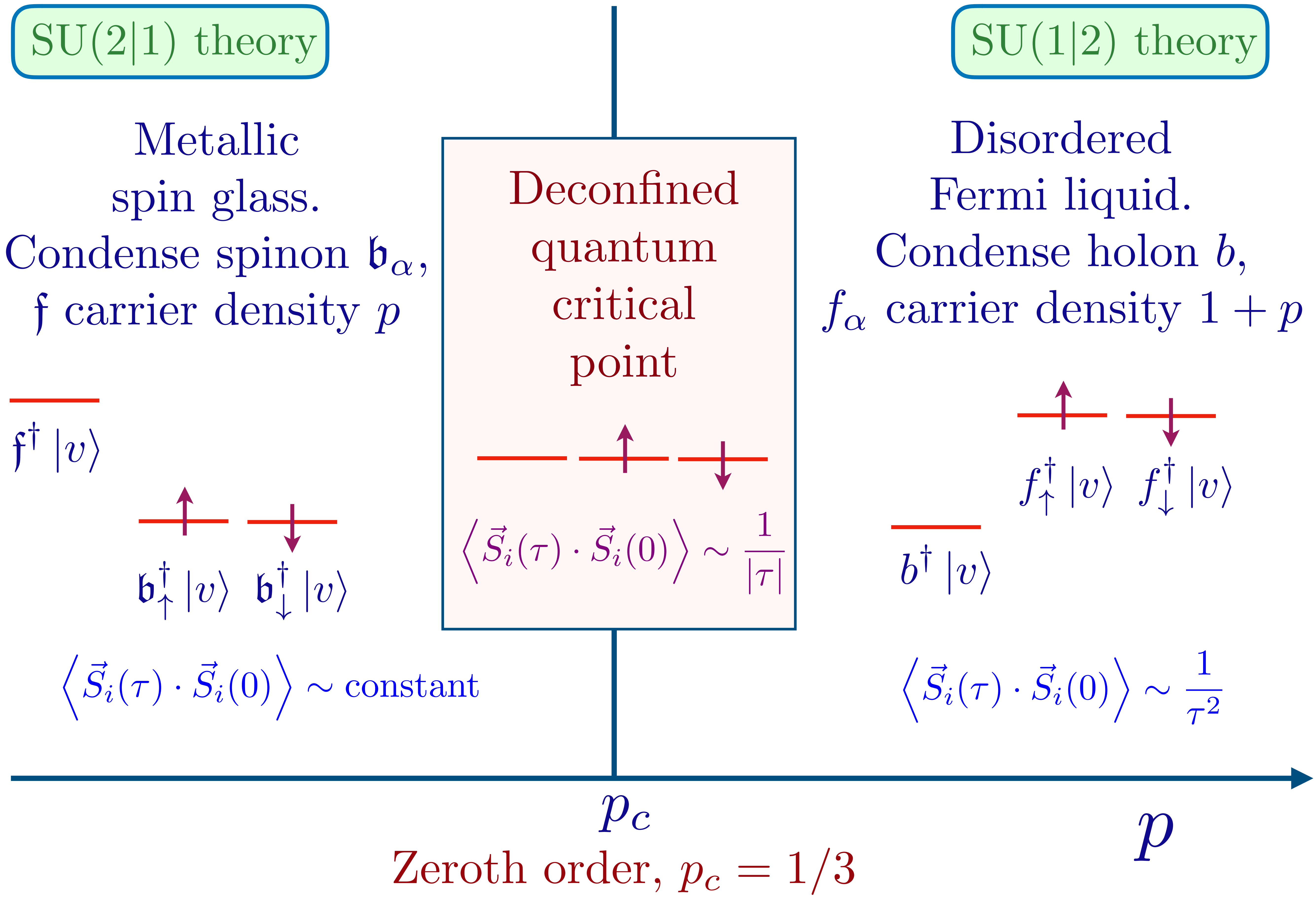

1. Insulating random magnet

2. Deconfined criticality at non-zero doping

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Insulating J model

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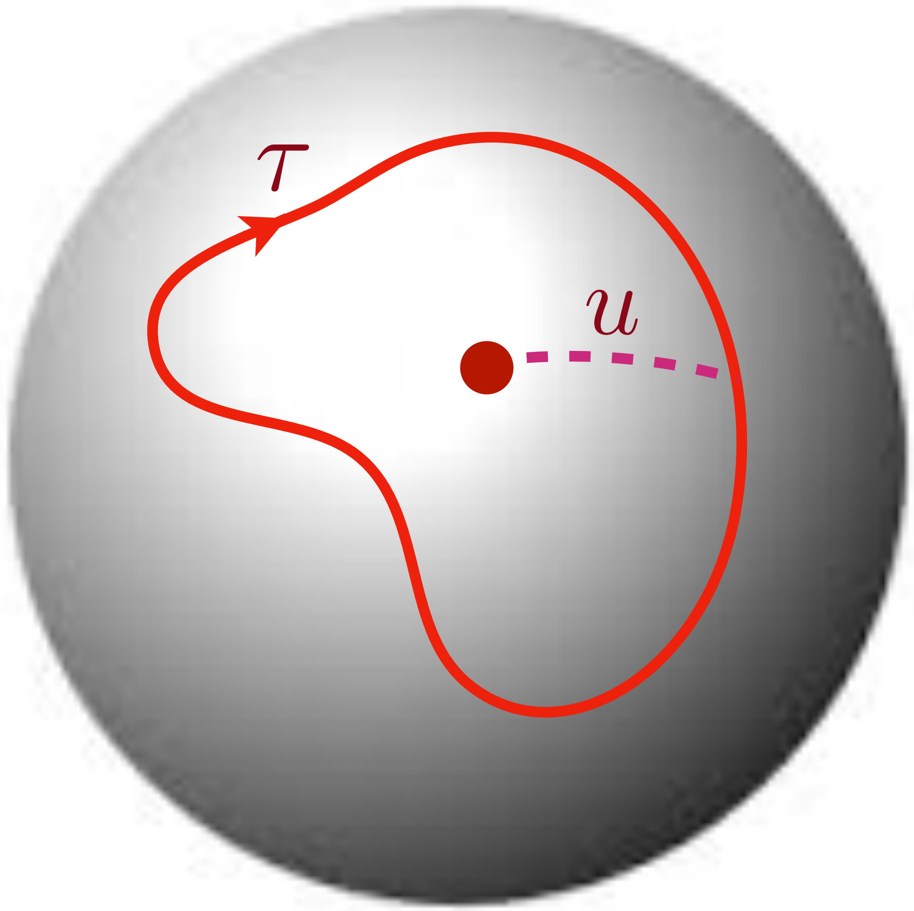
$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J^2_{ij}} = J^2$$

Insulating J model

$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}_B - \mathcal{S}_J}$$

$$\mathcal{S}_B = \frac{i}{2}\int_0^1 du \int d\tau \vec{S}\cdot\left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u}\right)$$

$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau)\cdot\vec{S}(\tau').$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Insulating J model

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$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau').$$

From this action we compute

$$\overline{Q}(\tau-\tau') = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$Q(\tau) = \overline{Q}(\tau).$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Insulating J model: RG

We assume a power-law decay

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \,.$$

Ignore the self-consistency condition for now. We decouple the $\vec{S}(\tau) \cdot \vec{S}(0)$ interaction by introducing a bosonic (ϕ_a, $a = 1 \ldots 3$) bath.

M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
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$$H_{\rm imp} \;\; = \;\; \gamma_0 f_\alpha^\dagger \frac{\sigma^a_{\alpha\beta}}{2} f_\beta \, \phi_a(0) + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right]$$

where π_a is canonically conjugate to the field ϕ_a, $\phi_a(0) \equiv \phi_a(x=0)$, and we have the constraint

$$f_\alpha^\dagger f_\alpha = 1 \,.$$

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M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
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Insulating J model: RG

Schwinger fermions

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We can perform a RG analysis in a $\epsilon = 3-d$ expansion, while imposing the fermion constraint *exactly*. The two-loop β function is

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 - \gamma^5 + \ldots$$

This has a stable fixed point at $\gamma^{*2} = \epsilon/2 + \epsilon^2/4 + \ldots$.

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The scaling dimension of the spin operator is $\dim[\vec{S}] = \epsilon/2$, exact to *all* orders in ϵ. This implies the correlator

$$\overline{Q}(\tau) = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|^{3-d}}\,.$$

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The scaling dimension of the spin operator is $\dim[\vec{S}] = \epsilon/2$, exact to *all* orders in ϵ. This implies the correlator

$$\overline{Q}(\tau) = \frac{1}{3}\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|^{3-d}}.$$

Finally, we impose the self-consistency condition $Q(\tau) = \overline{Q}(\tau)$, and obtain the same self-consistent result as in the large M expansion

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}.$$

M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
S. Sachdev, Physica C **357**, 78 (2001)

Insulating *J* model: large *M* limit

Express the spin operator in terms of fermions $\vec{S} = (1/2) f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1 \ldots M$. The fermions obey the constraint

$$\sum_{\alpha=1}^{M} f_\alpha^\dagger f_\alpha = \frac{M}{2}$$

In the large M limit we obtain for the fermion Green's function G and self energy Σ (same as the SYK equations)

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution is

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{\tau}} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Insulating J model

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Numerical studies for SU(2) spin-1/2 show spin-glass order!

L. Arrachea and M. J. Rozenberg, PRB **65**, 224430 (2002)

1. Insulating random magnet

2. Deconfined criticality at non-zero doping

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$\begin{aligned} |0\rangle \Rightarrow b^\dagger |v\rangle \quad &, \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle \\ c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \\ \text{U(1) gauge invariance,} \quad & b \to b e^{i\phi} \,, \quad f_\alpha \to f_\alpha e^{i\phi} \end{aligned}$$

SU(1|2) theory

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow \mathfrak{f}^{\dagger}\,|v\rangle \qquad , \qquad c_{\alpha}^{\dagger}\,|0\rangle \Rightarrow \mathfrak{b}_{\alpha}^{\dagger}\,|v\rangle$$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

SU(2|1) theory

U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}} \\
\mathcal{S}_B &= i \int_0^1 du \int d\tau \, \mathrm{Tr}\left(\mathcal{P}\partial_\tau \mathcal{P} \partial_u \mathcal{P}\right) \\
\mathcal{S}_{tJ} &= \int d\tau d\tau' \, \mathrm{Tr}\left(\mathcal{P}(\tau)\mathcal{Q}(\tau - \tau')\mathcal{P}(\tau')\right) \\
&\quad + \int d\tau \mathrm{Tr}\left(s_0 \, \mathcal{P}(\tau)\right) .
\end{aligned}
$$

Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$ and a 'Zeeman superfield' s_0.

t-J model

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\begin{aligned}
\mathcal{Z} &= \int \mathcal{D} f_\alpha(\tau) \mathcal{D} b(\tau) \mathcal{D}\lambda(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}} \\
\mathcal{S}_B &= \int d\tau \left[f_\alpha^\dagger(\tau) \left(\frac{\partial}{\partial \tau} + i\lambda \right) f_\alpha(\tau) + b^\dagger(\tau) \left(\frac{\partial}{\partial \tau} + i\lambda \right) b(\tau) - i\lambda \right] \\
\mathcal{S}_{tJ} &= \int d\tau \, s_0 f_\alpha^\dagger(\tau) f_\alpha(\tau) + t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') \\
&\quad - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') \, .
\end{aligned}
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SU(1|2) theory

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\mathcal{Z} &= \int \mathcal{D}f_\alpha(\tau)\mathcal{D}b(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}} \\
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\end{aligned}$$

From this action we determined the correlators

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$$\begin{aligned}
\overline{R}(\tau-\tau') &= -\left\langle c_\alpha(\tau)c_\alpha^\dagger(\tau')\right\rangle_{\mathcal{Z}} \\
\overline{Q}(\tau-\tau') &= \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}
\end{aligned}$$

and finally impose the self-consistency conditions

$$R(\tau)=\overline{R}(\tau)\quad,\quad Q(\tau)=\overline{Q}(\tau).$$

t-J model

$$\mathcal{Z} = \int \mathcal{D}\mathfrak{b}_\alpha(\tau)\mathcal{D}\mathfrak{f}(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}}$$

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We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{r+1}} \, .$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a, $a = 1 \ldots 3$) and fermionic (ψ_α) baths.

S. Sachdev, Physica C **357**, 78 (2001)
M. Vojta and L. Fritz, PRB **70**, 094502 (2004)

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The impurity superspin is coupled to
a fermionic bath by g_0,
and to a bosonic bath by γ_0,
and s_0 acts as a local field on the superspin -
a superKondo problem!

t-J model RG

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We can perform a RG analysis for small $\epsilon = 3 - d$ and $\bar{r} = (1-r)/2$, while imposing the local constraint *exactly*. The one-loop β functions are

$$\beta(g) = -\bar{r}g + \frac{3}{2}g^3 + \frac{3}{8}g\gamma^2\,,$$

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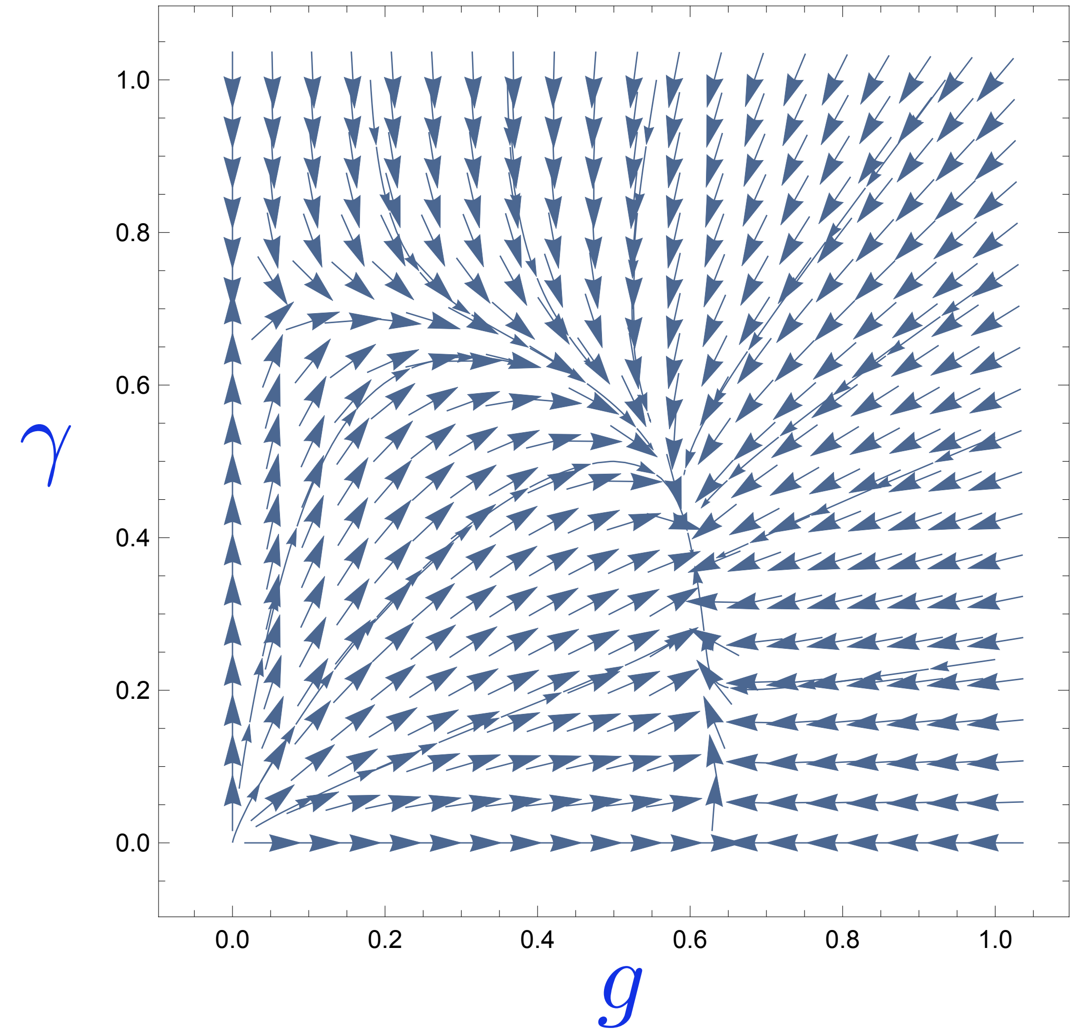
$$\beta(s) = -s + 3g^2 s - g^2 + \frac{3}{4}\gamma^2\,.$$



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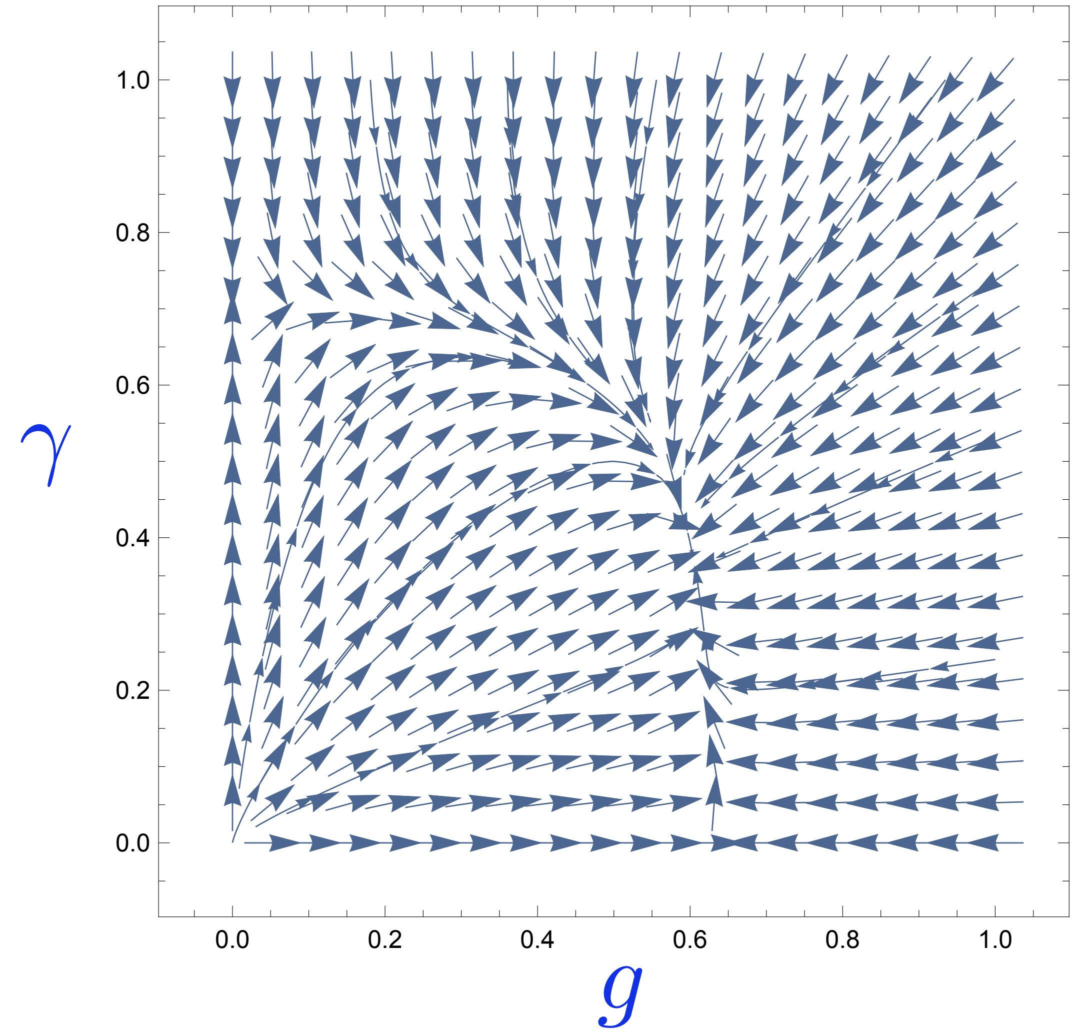

These equations have a fixed point with $s \approx 0$ with only one relevant direction, corresponding to the flow of s to $\pm\infty$. The 3 states of the superspin are nearly degenerate at the fixed point, and the flows away from the fixed point correspond to different orientations of the field on the superspin: one side (overdoped) favors the holon, and the other side (underdoped) favors the spinon.

t-J model RG

The scaling dimensions of the electron and spin operators can be determined to all orders in ϵ and $\bar{r}$ and these imply

$$\overline{R}(\tau) = -\frac{1}{2}\left\langle c_\alpha(\tau) c_\alpha^\dagger(0)\right\rangle \sim \frac{\text{sgn}(\tau)}{|\tau|^{1-r}} \quad , \quad \overline{Q}(\tau) = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|^{3-d}}\,.$$

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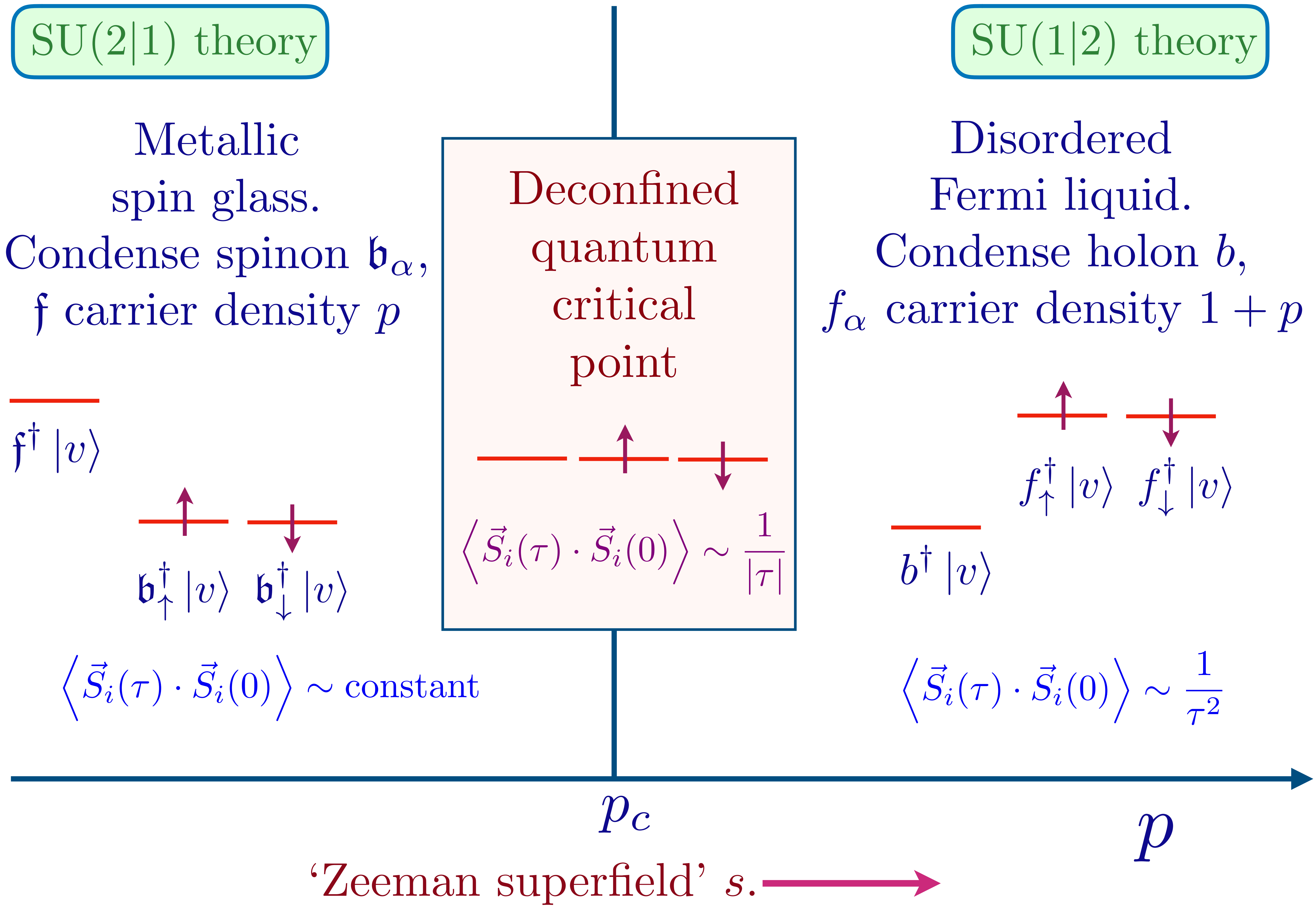
Finally, we impose the self-consistency conditions $R(\tau) = \overline{R}(\tau)$, $Q(\tau) = \overline{Q}(\tau)$ and obtain $r = 0$ ($\bar{r} = 1/2$) and $d = 2$ ($\epsilon = 1$), so that at the critical point we have

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0)\right\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|} \quad , \quad \left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}\,.$$

t-J model phase diagram



t-J model large M

Each site has 3 states which we map to the space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \quad , \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$
$$c_\alpha = f_\alpha b^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

To obtain a large M limit, let $\alpha = 1 \ldots M$, endow the boson with an 'orbital' index $a = 1 \ldots M'$ and send $M \to \infty$ at fixed $k = M'/M$. Then

$$c_{a\alpha} = f_\alpha b_a^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b_a^\dagger b_a = \frac{M}{2}$$

t-J model large M

The critical solution which is self-consistent in both the t and J terms has $\Delta_b = \Delta_f = 1/2$, implying

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \begin{cases} \dfrac{A_+}{|\tau|} & , \quad \tau > 0 \\ \\ -\dfrac{A_-}{|\tau|} & , \quad \tau < 0 \end{cases} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \, .$$

The same exponents are obtained to all orders in the ϵ, $\overline{r}$ expansion, but with $A_+ = A_-$.

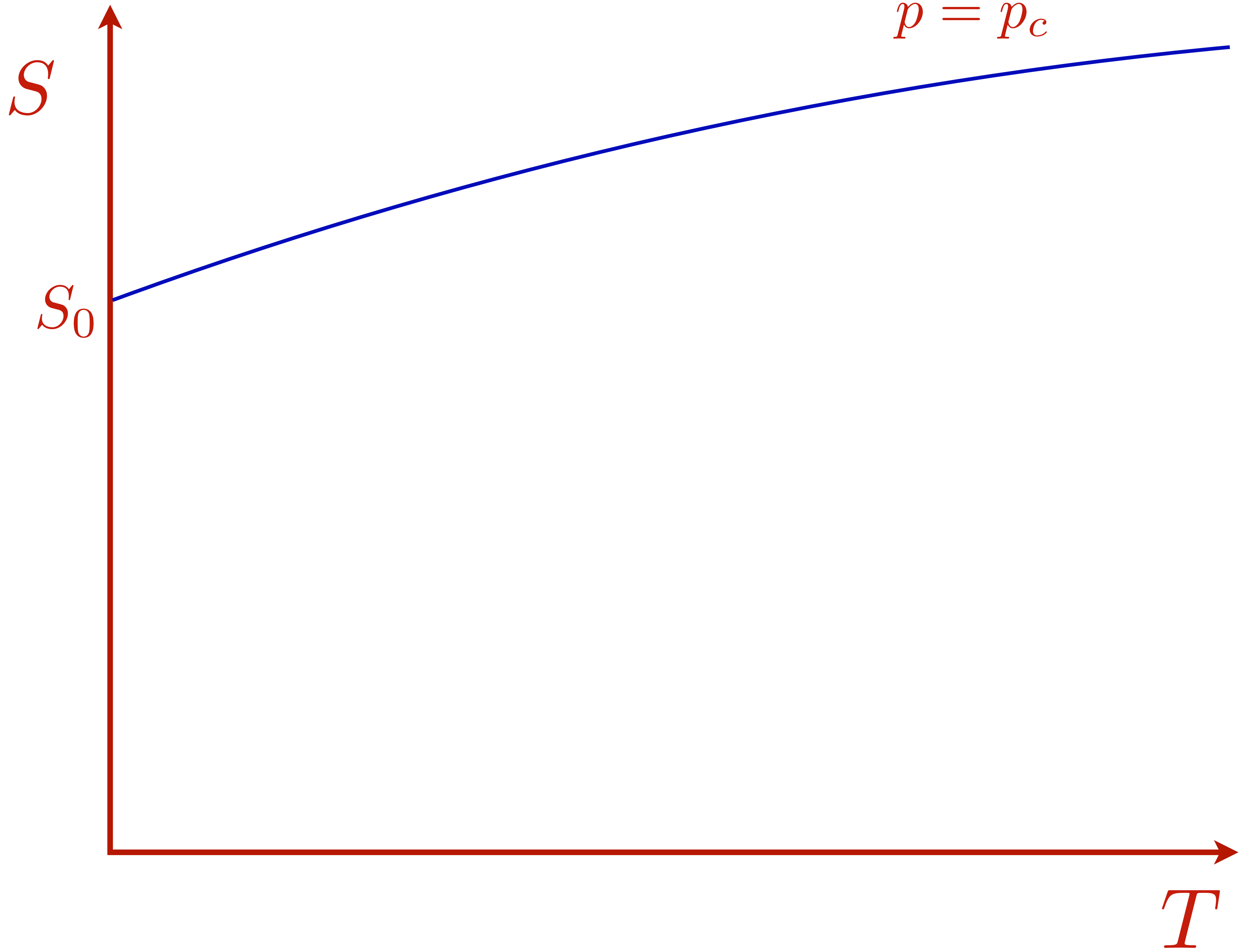
t-J model phase diagram

SU(2|1) theory		SU(1|2) theory
Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p	Deconfined quantum critical point	Disordered Fermi liquid. Condense holon b, f_α carrier density $1+p$
$\mathfrak{f}^\dagger \lvert v\rangle$		$f_\uparrow^\dagger \lvert v\rangle$ $\quad f_\downarrow^\dagger \lvert v\rangle$
$\mathfrak{b}_\uparrow^\dagger \lvert v\rangle$ $\quad \mathfrak{b}_\downarrow^\dagger \lvert v\rangle$	$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \dfrac{1}{\lvert\tau\rvert}$	$b^\dagger \lvert v\rangle$
$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \text{constant}$		$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \dfrac{1}{\tau^2}$

p_c $\qquad$ p

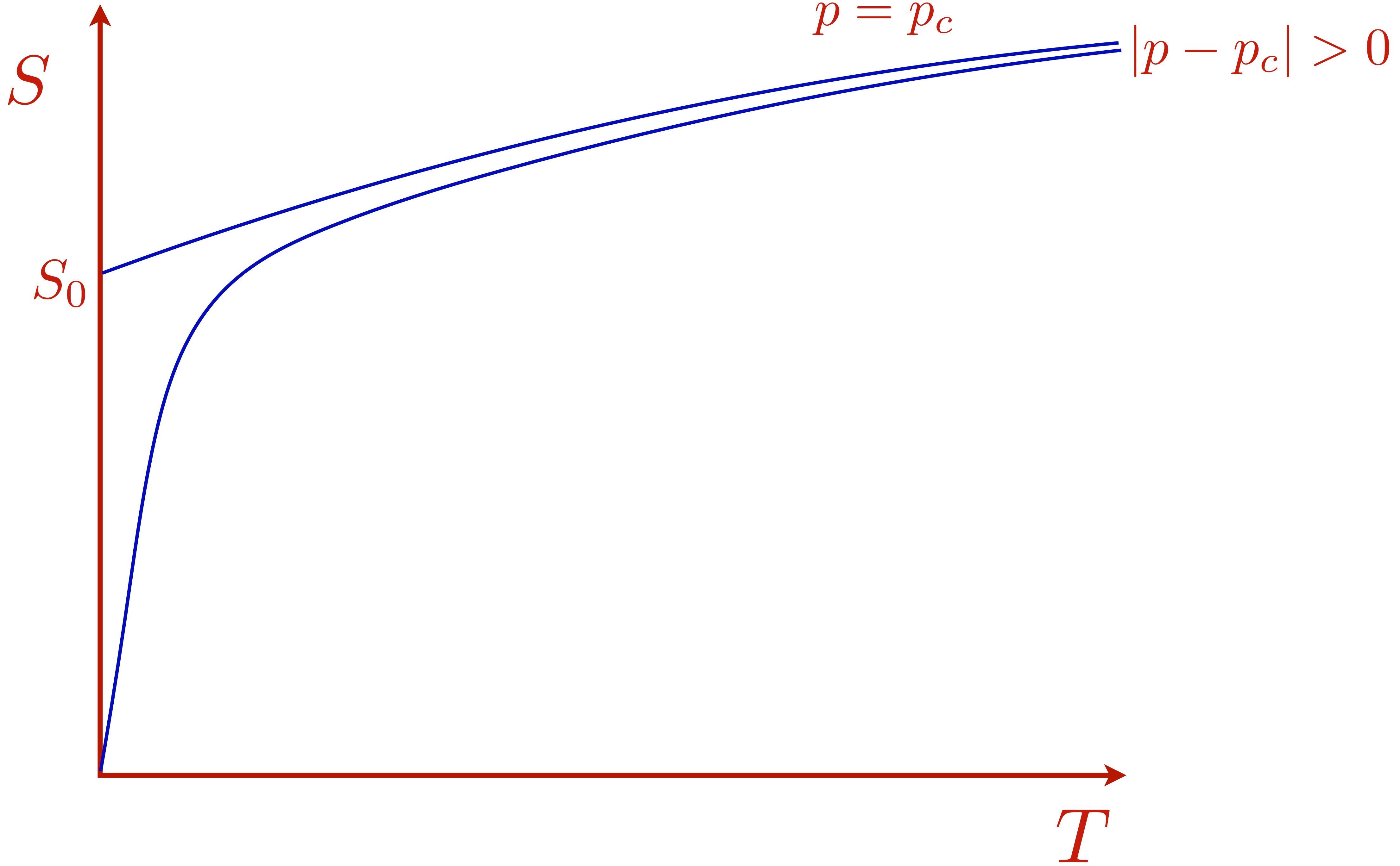
'Zeeman superfield' s. →

t-J model entropy



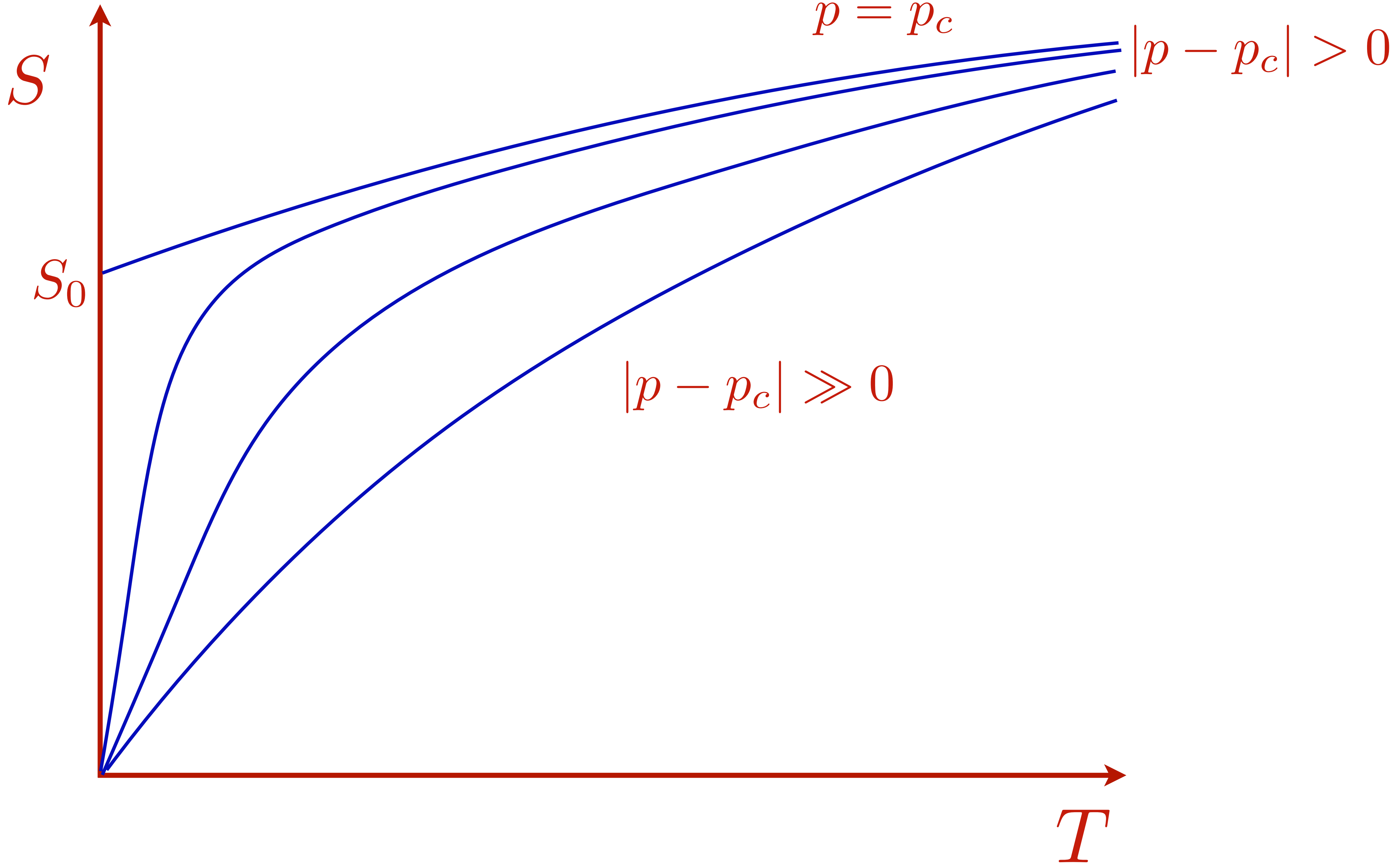
$$\frac{C}{T} = \frac{dS}{dT}$$

t-J model entropy



$$\frac{C}{T} = \frac{dS}{dT}$$

t-J model entropy

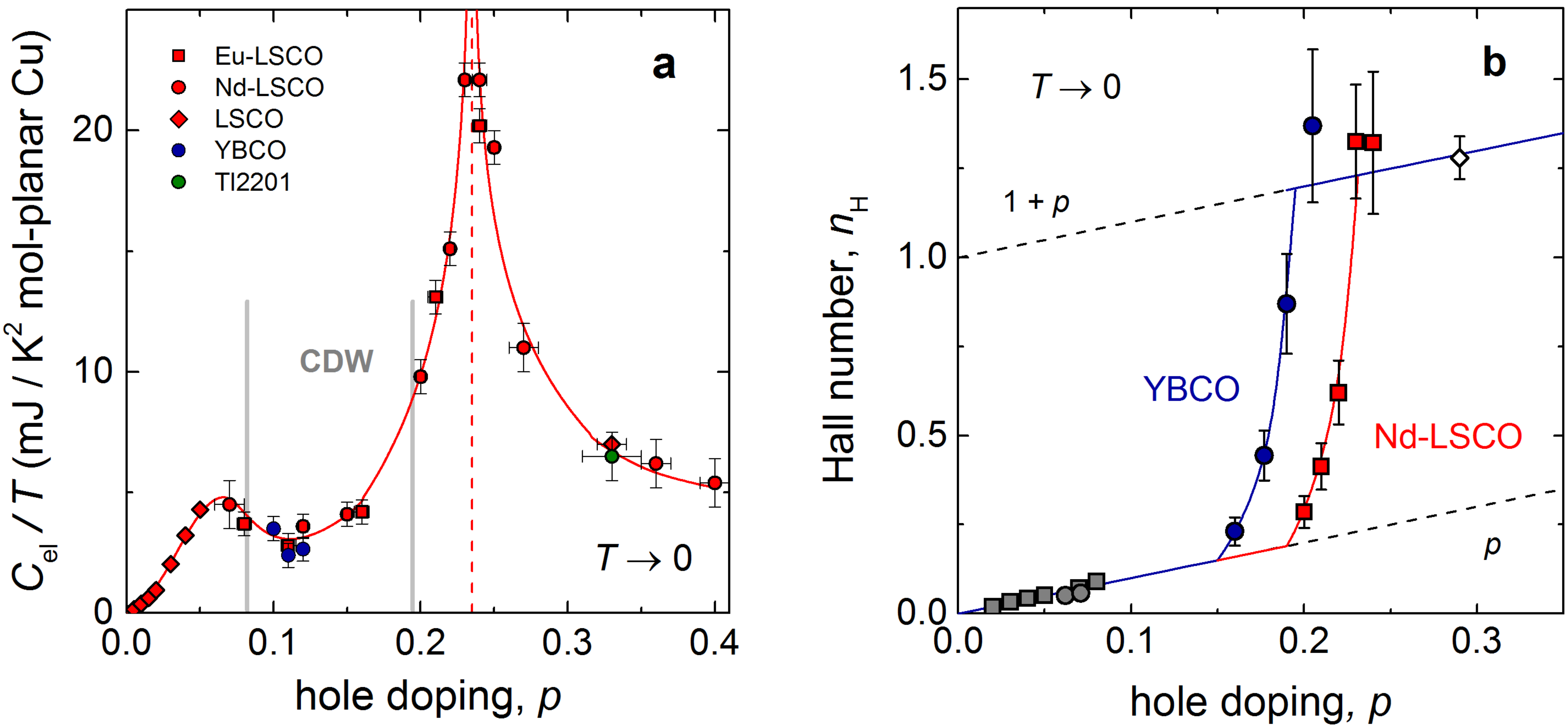


$$\frac{C}{T} = \frac{dS}{dT}$$

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507



t-J model phase diagram