Disordered metals without quasiparticles, and charged black holes

String Theory: Past and Present
(SpentaFest)
International Center for Theoretical Sciences, Bengaluru
January 11-13, 2017

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

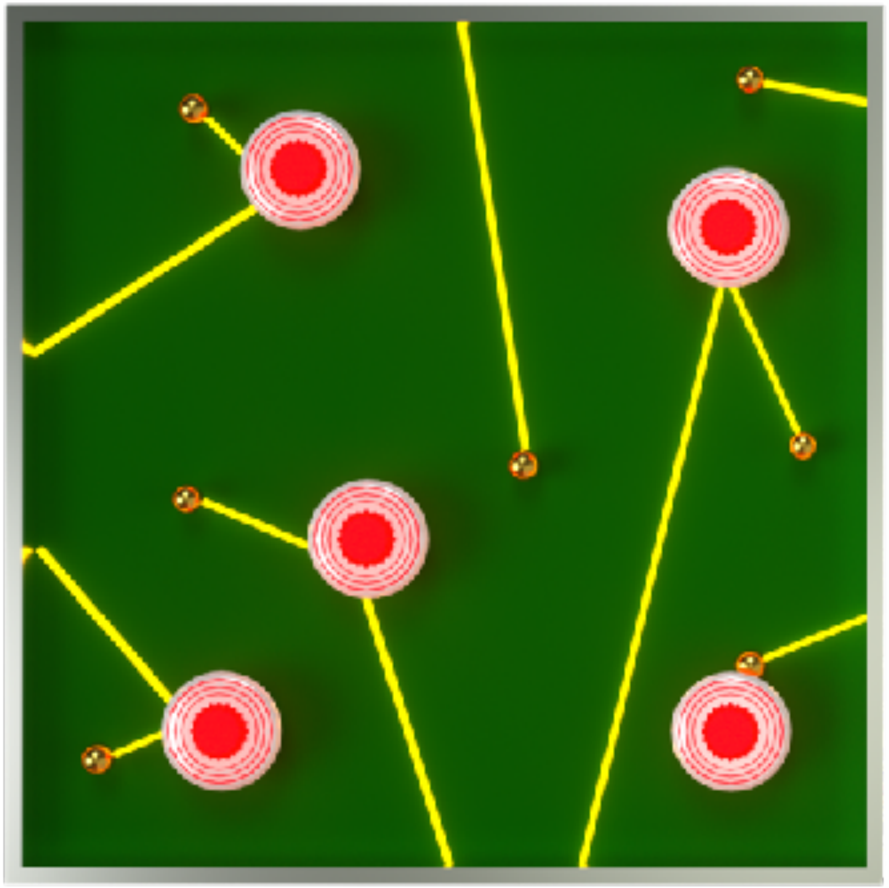
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

NSF

PHYSICS
VE RI TAS
HARVARD

Quantum matter with quasiparticles:

- *Landau quasi-particles & holes*
- *Phonon*
- *Magnon*
- *Roton*
- *Plasmon*
- *Polaron*
- *Exciton*
- *Laughlin quasiparticle*
- *Bogoliubovon*
- *Anderson-Higgs mode*
- *Massless Dirac Fermions*
- *Weyl fermions*
- ….



Quantum matter <u>with</u> quasiparticles:

Most generally, a quasiparticle is an "additive" excitation:

Quasiparticles can be combined to yield additional excitations, with energy determined by the energies and densities of the constituents. Such a procedure yields all the low-lying excitations. Then we can apply the Boltzmann-Landau theory to make predictions for dynamics.

Quantum matter without quasiparticles:

No quasiparticle structure to excitations.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some exotic quasiparticles inaccessible to current experiments……..

Consider how rapidly the system loses "phase coherence", reaches local thermal equilibrium, or becomes "chaotic"

Local thermal equilibration or phase coherence time, τ_φ:

- There is an *lower bound* on τ_φ in all many-body quantum systems as $T \to 0$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

where C is a T-independent constant. Systems *without* quasiparticles have $\tau_\varphi \sim \hbar/(k_B T)$.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T;
e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A bound on quantum chaos:

- In classical chaos, we measure the sensitivity of the position at time t, $q(t)$, to variations in the initial position, $q(0)$, *i.e.* we measure

$$\left(\frac{\partial q(t)}{\partial q(0)}\right)^2 = \left(\{q(t), p(0)\}_{\text{P.B.}}\right)^2$$

- By analogy, we define τ_L as the Lyapunov time over which the wave-function of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$\left\langle \left| [\hat{A}(x,t), \hat{B}(0,0)] \right|^2 \right\rangle \sim \exp\left(\frac{1}{\tau_L} \left[t - \frac{|x|}{v_B} \right] \right),$$

where v_B is the 'butterfly velocity'. This time τ_L was argued to obey lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}.$$

There is no analogous bound in classical mechanics.

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

A bound on quantum chaos:

- In classical chaos, we measure the sensitivity of the position at time t, $q(t)$, to variations in the initial position, $q(0)$, *i.e.* we measure

$$\left(\frac{\partial q(t)}{\partial q(0)}\right)^2 = \left(\{q(t), p(0)\}_{\text{P.B.}}\right)^2$$

- By analogy, we define τ_L as the <u>Lyapunov time</u> over which the wave-function of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$\left\langle \left| [\hat{A}(x,t), \hat{B}(0,0)] \right|^2 \right\rangle \sim \exp\left(\frac{1}{\tau_L}\left[t - \frac{|x|}{v_B}\right]\right),$$

where v_B is the 'butterfly velocity'. This time τ_L was argued to obey lower bound

$$\tau_L \geq \frac{1}{2\pi}\frac{\hbar}{k_B T}.$$

Quantum matter without quasiparticles
$\approx$ fastest possible many-body quantum chaos

P **28**, 6 (1969)

iv:1503.01409

Disordered metals

There is an extensive theory of metals in the presence of disorder and interactions
(Altshuler, Aronov, Lee, Ramakrishnan, Finkelstein…)

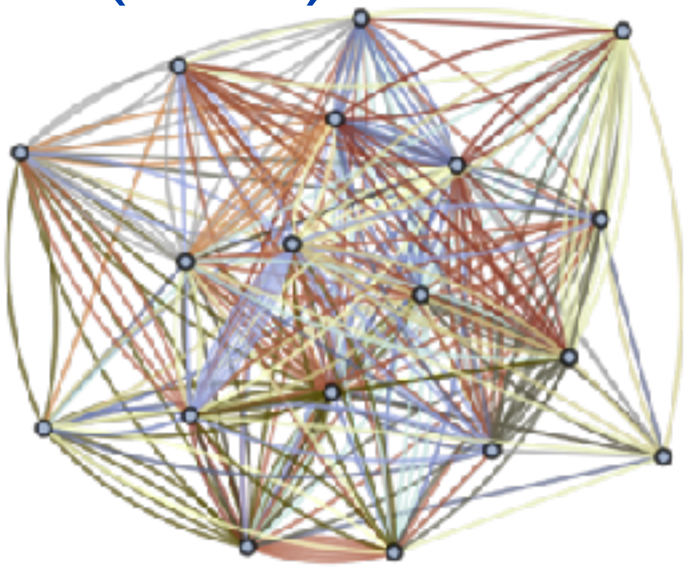
However the theory is only reliable in states in which quasiparticles are present.

Often there is a flow to strong coupling, but it is invariably assumed that this leads to disordered insulators.

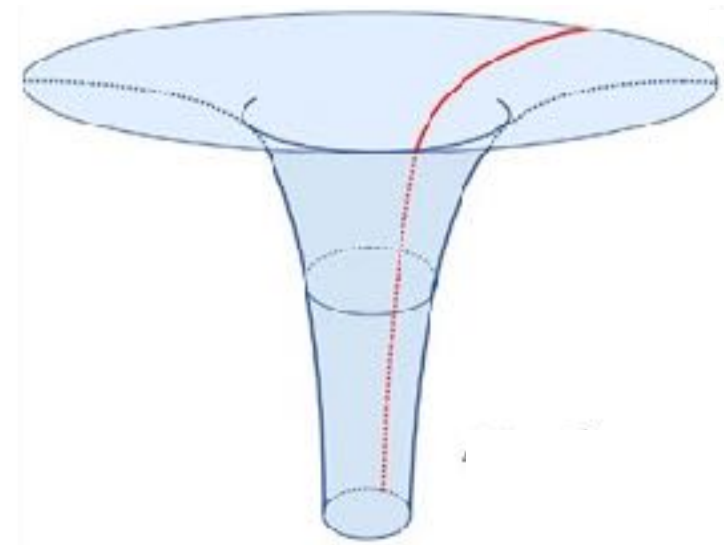
But perhaps the strong coupling state is a metallic state without quasiparticles, in which the influence of disorder largely self averages….(there are many experimental indications this may be the case)

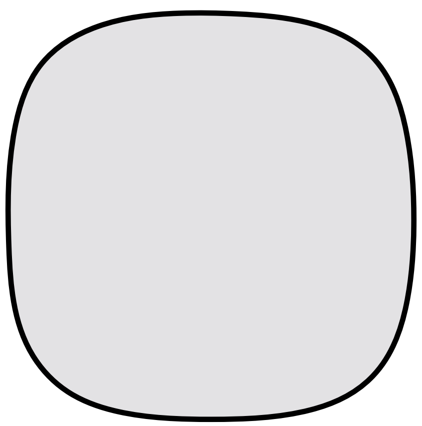
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons



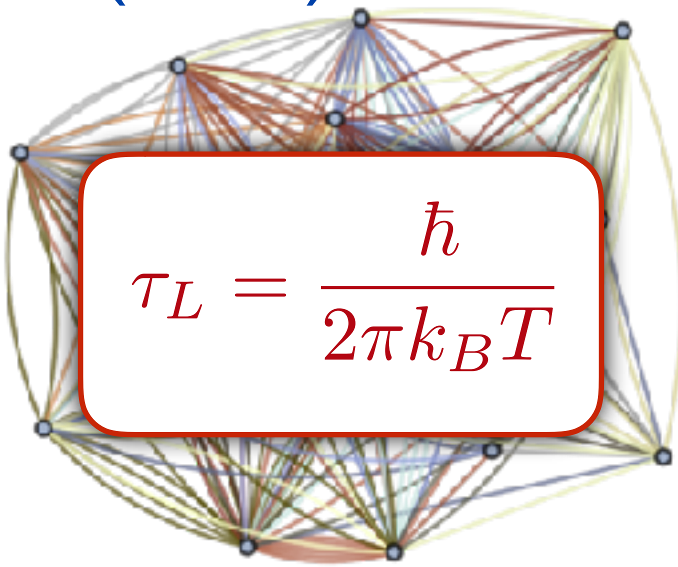


Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

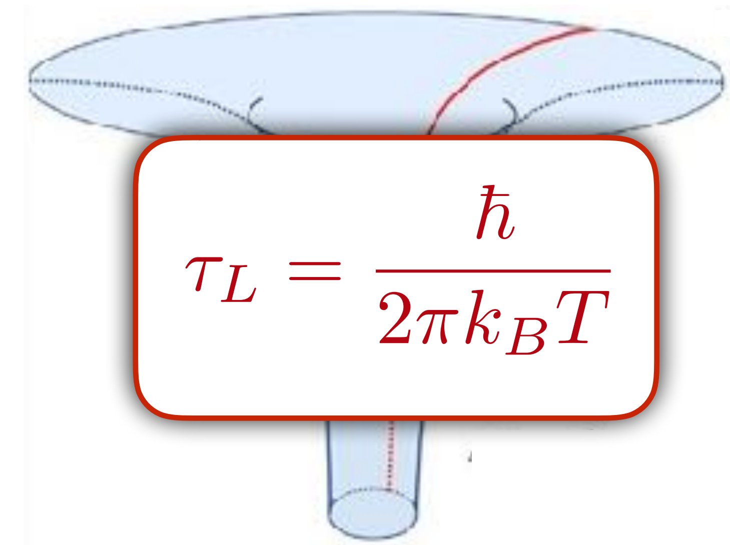
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

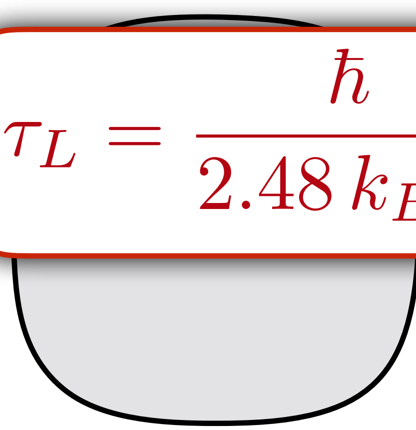


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Black holes with AdS_2 horizons



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$



$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

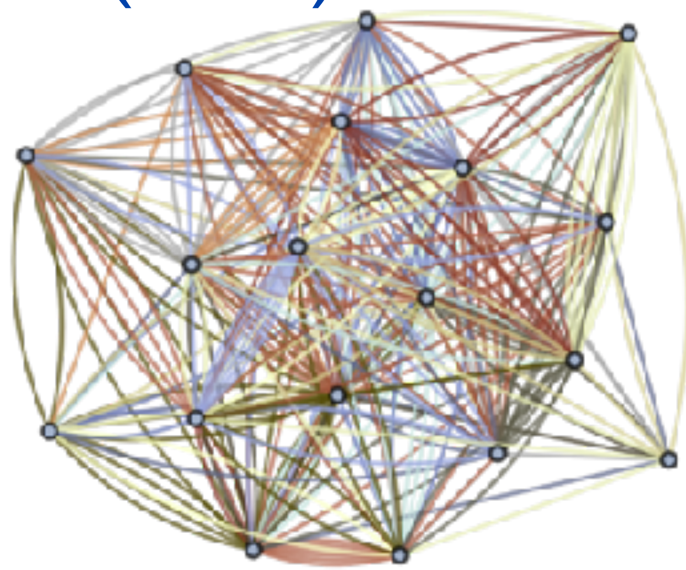
Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

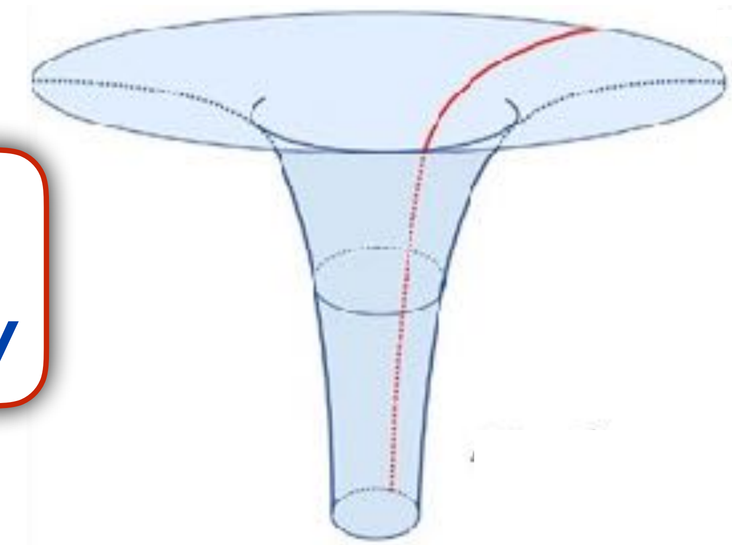
Quantum matter without quasiparticles:

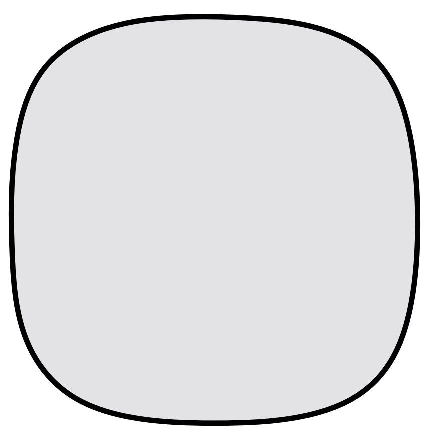
The Sachdev-Ye-Kitaev (SYK) models



Same low energy theory

Black holes with AdS_2 horizons





Fermi surface coupled to a gauge field

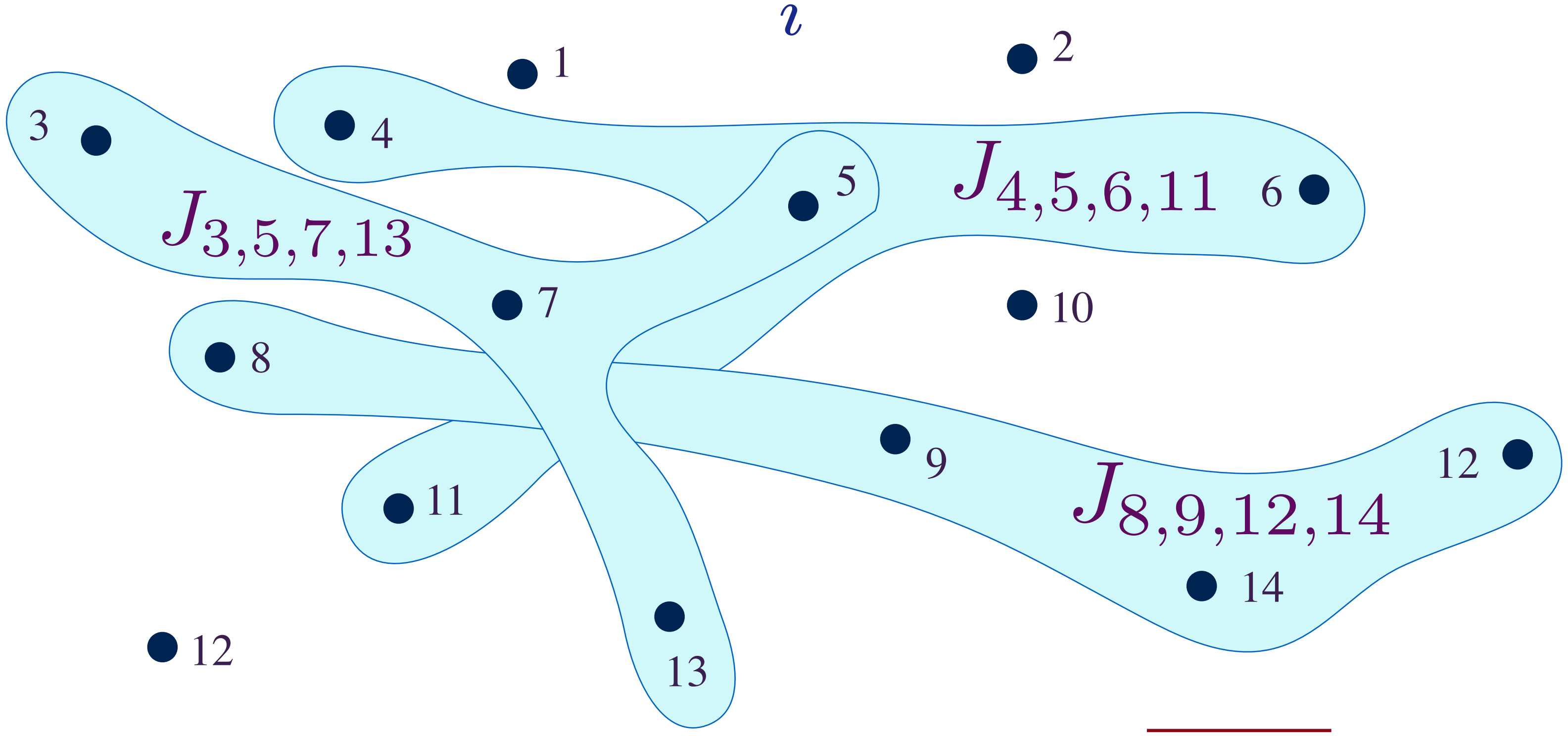
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK and AdS_2

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = NS_0 + \ldots$ **Not a ground state degeneracy**: due to an exponentially small (in N) many-body level spacing at all energies down to the ground state energy.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

)

SYK and AdS_2

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = NS_0 + \ldots$ **Not a ground state degeneracy**: due to an exponentially small (in N) many-body level spacing at all energies down to the ground state energy.

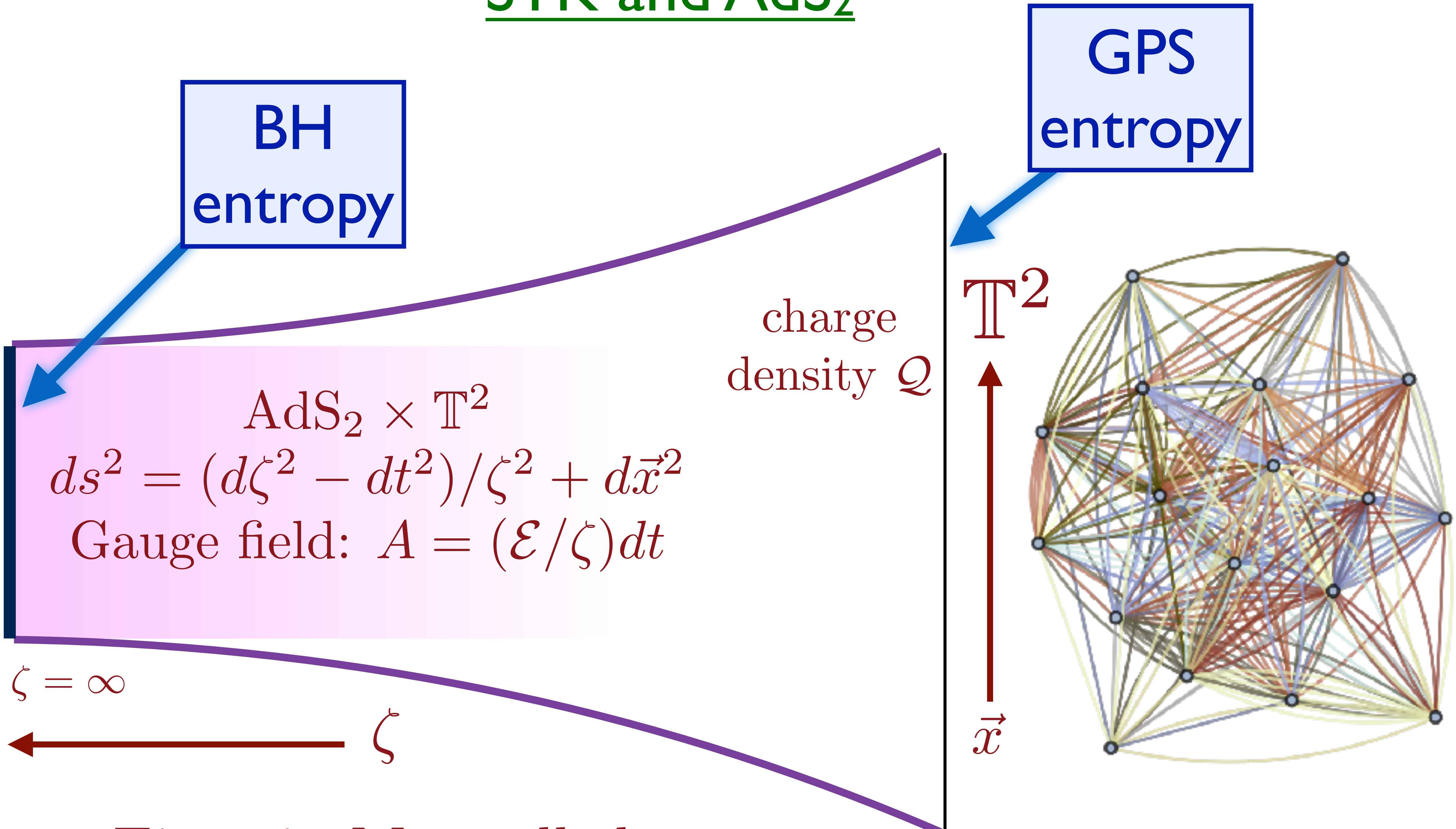
A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

- This entropy, and other dynamic correlators of the SYK models, imply that the SYK model is holographically dual to black holes with an AdS_2 horizon. The Bekenstein-Hawking entropy of the black hole equals NS_0:

$$\mathbf{GPS} = \mathbf{BH}.$$

S. Sachdev, PRL **105**, 151602 (2010)

SYK and AdS_2



Einstein-Maxwell theory
+ cosmological constant

S. Sachdev, PRL **105**, 151602 (2010)

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

SYK and AdS$_2$

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK and AdS$_2$

$$G(i\omega) = \frac{1}{\cancel{i\omega} + \cancel{\mu} - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \cancel{\mu} - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished

S. Sachdev, PRX **5,** 041025 (2015)

SYK and AdS_2

Let us write the large N saddle point solutions of S as

$$
\begin{aligned}
G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\
\Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}.
\end{aligned}
$$

The saddle point will be invariant under a reperamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\widetilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the SL(2, R) transformations under which

$$
f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.
$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

A. Kitaev, unpublished

SYK and AdS_2

Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism

- SL(2,R) is the isometry group of AdS_2.

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}$$

with $ad - bc = 1$.

A. Kitaev, unpublished

SYK and AdS_2

Same long-time effective action
for energy and number fluctuations,
involving Schwarzian derivatives of $f(\tau)$.

$\mathrm{AdS}_2 \times \mathbb{T}^2$

$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

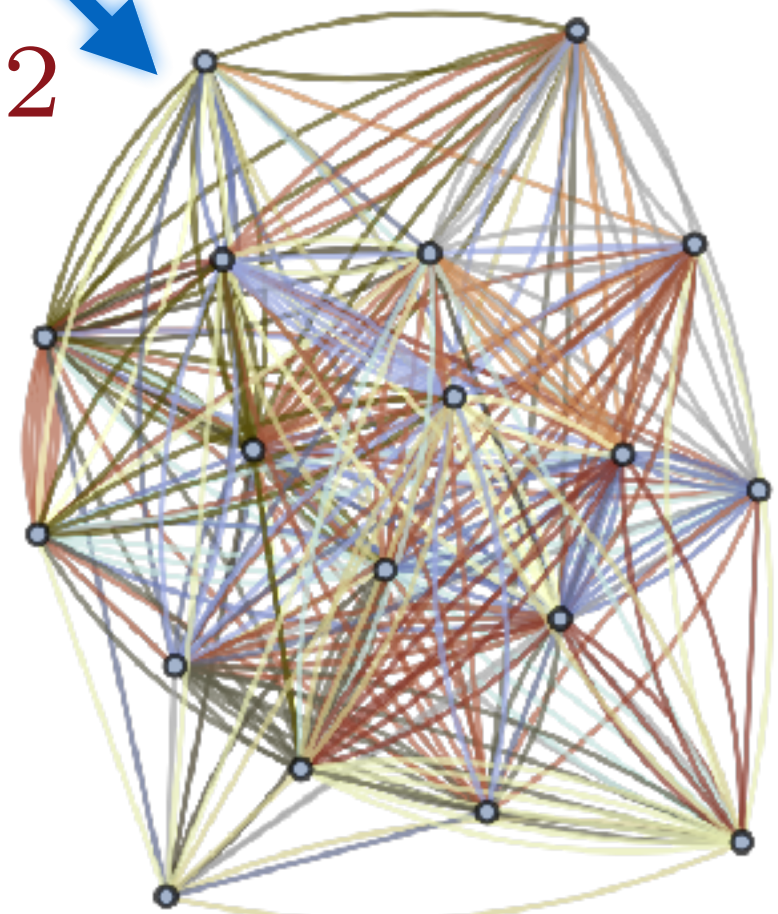
$\zeta = \infty$

ζ

charge
density $\mathcal{Q}$

$\mathbb{T}^2$

$\vec{x}$

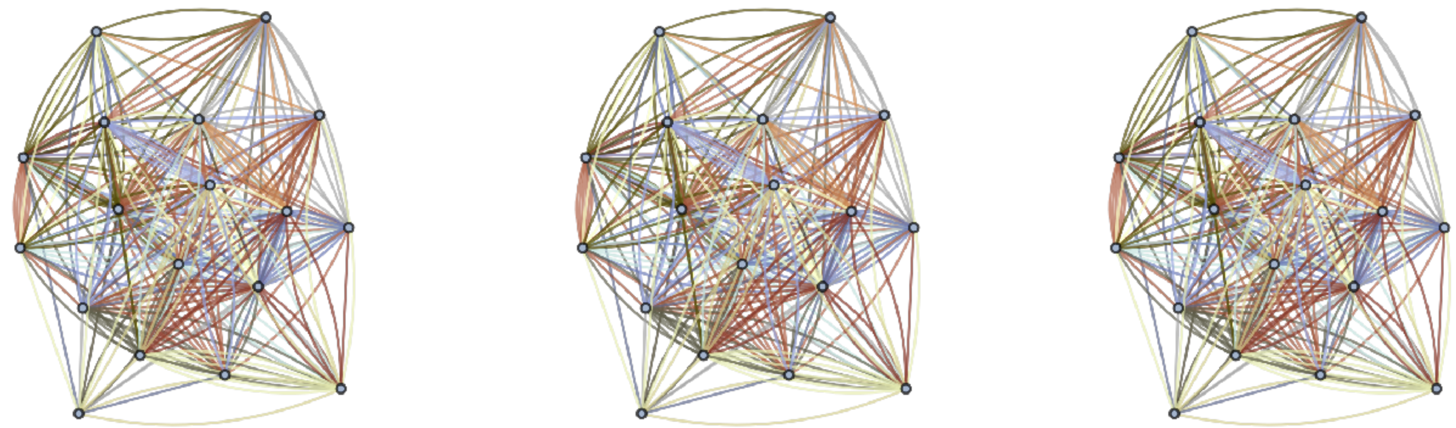


Einstein-Maxwell theory
+ cosmological constant

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Coupled SYK models



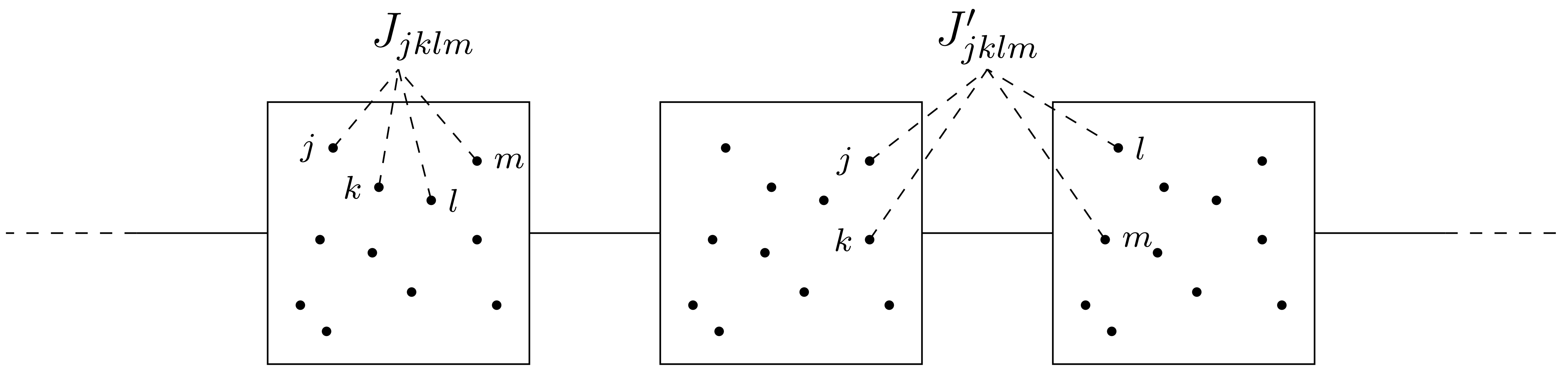
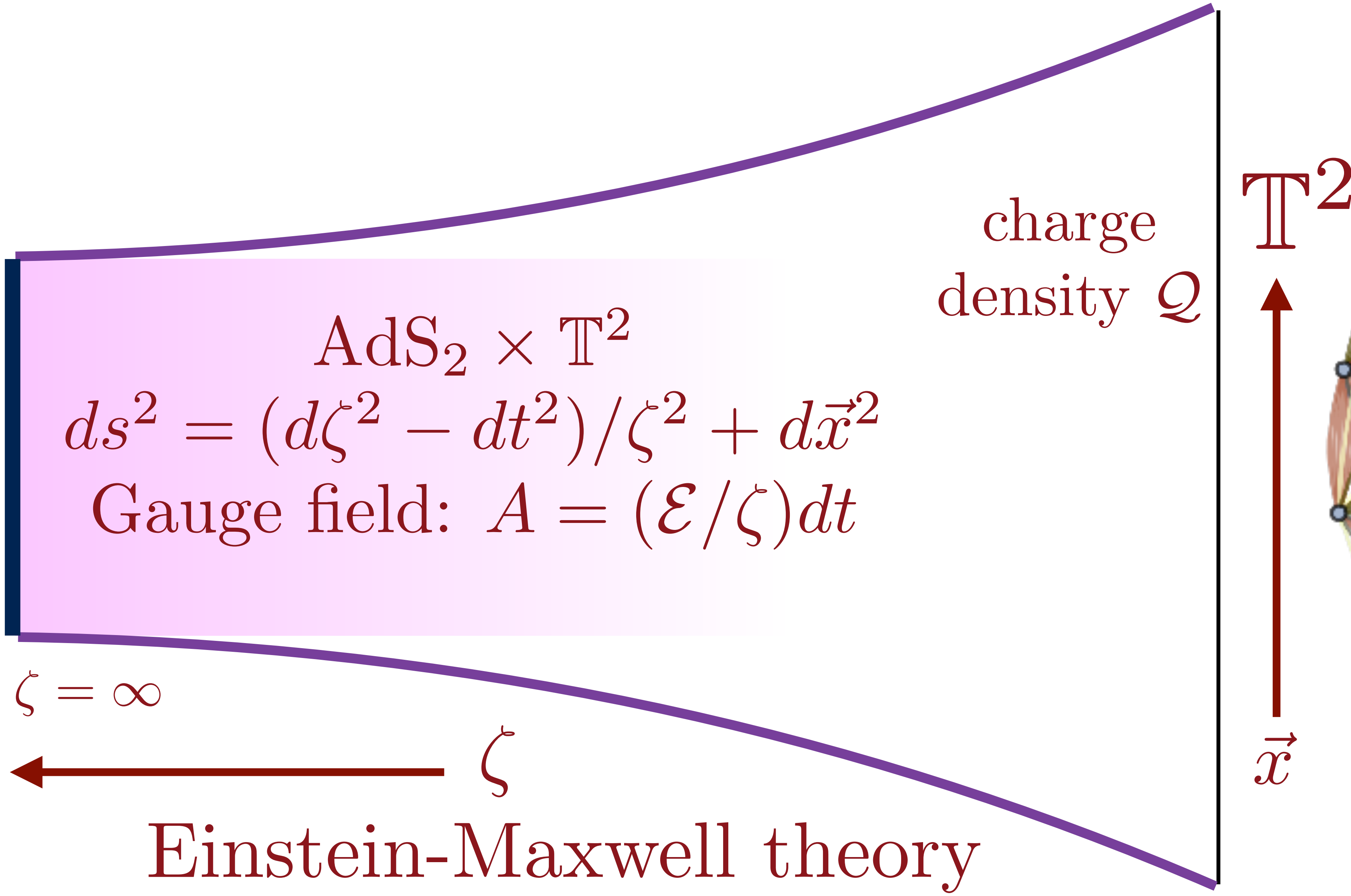
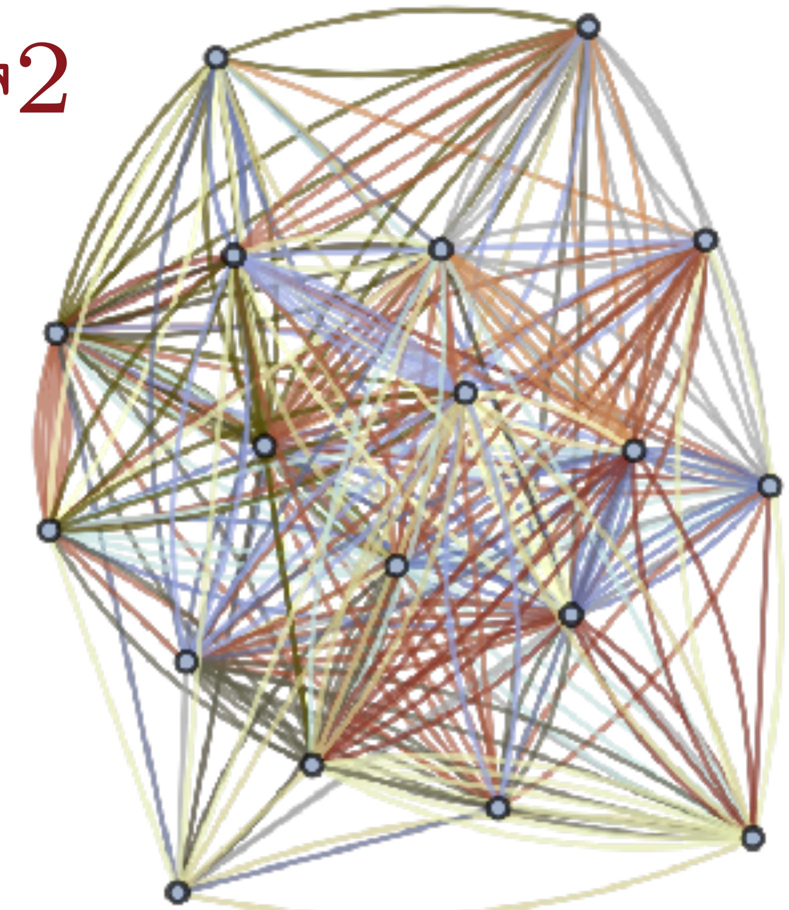


Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

SYK and AdS_2





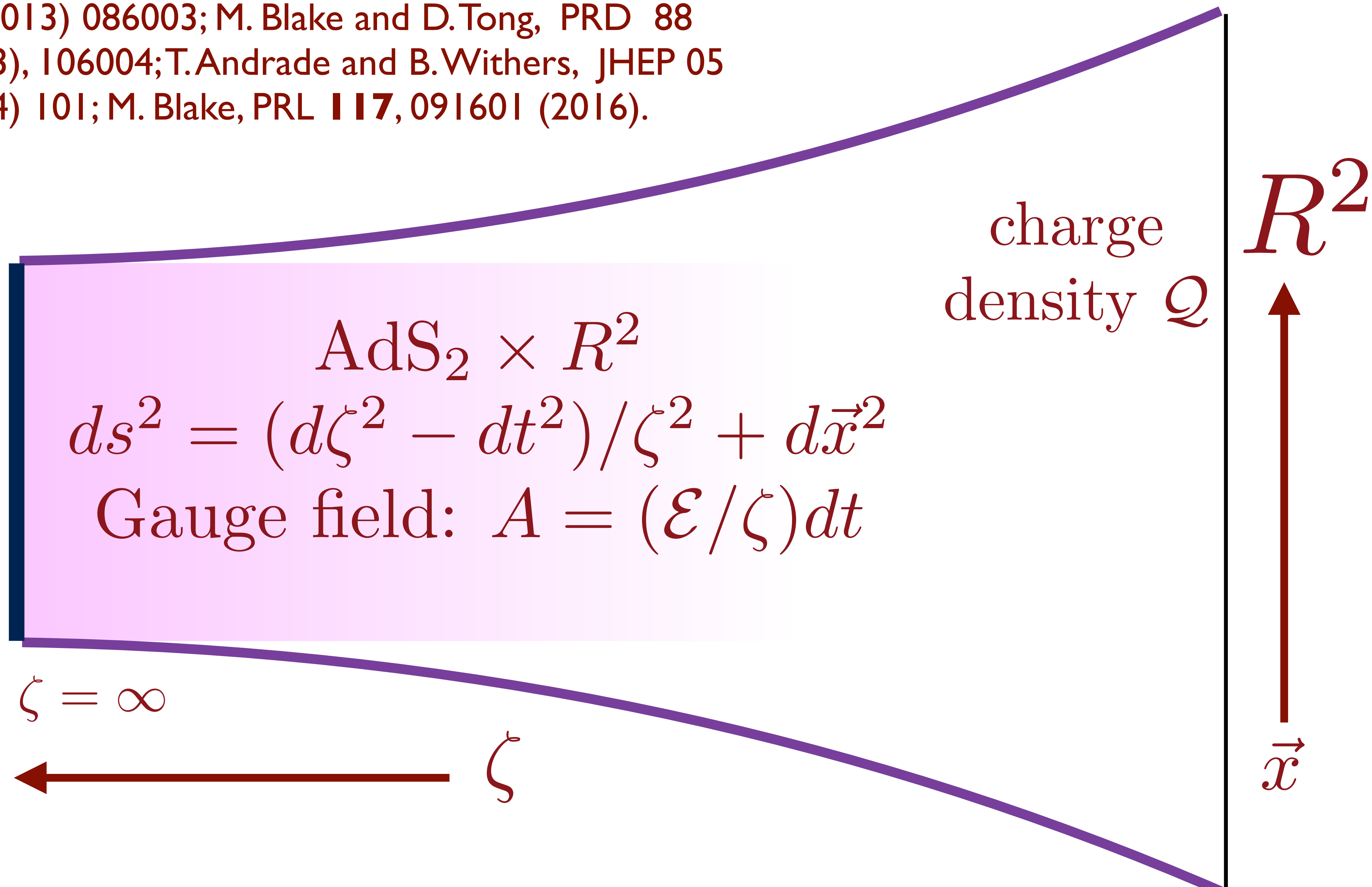
Einstein-Maxwell theory
$+$ cosmological constant

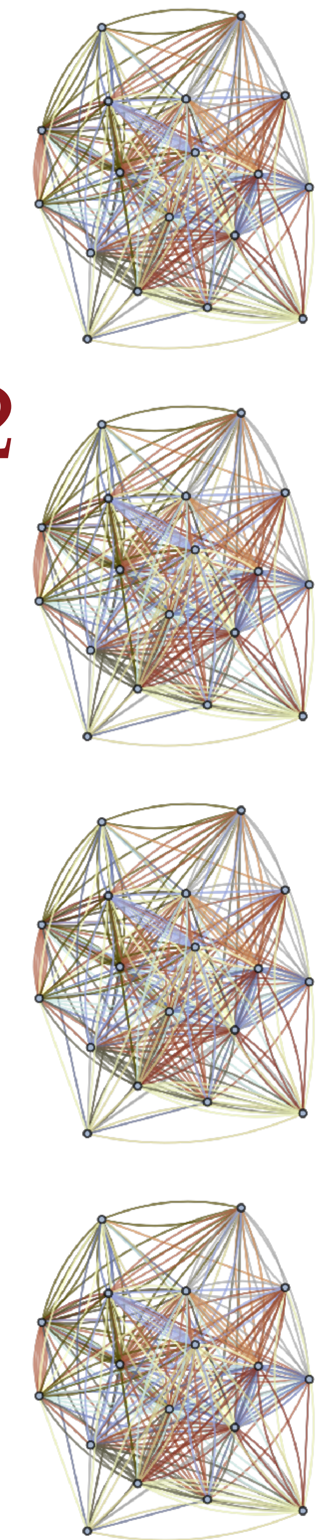
Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

S. Sachdev, PRL **105**, 151602 (2010)

Coupled SYK and AdS_4

Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054; D. Vegh, arXiv:1301.0537; R. A. Davison, PRD 88 (2013) 086003; M. Blake and D. Tong, PRD 88 (2013), 106004; T. Andrade and B. Withers, JHEP 05 (2014) 101; M. Blake, PRL **117**, 091601 (2016).





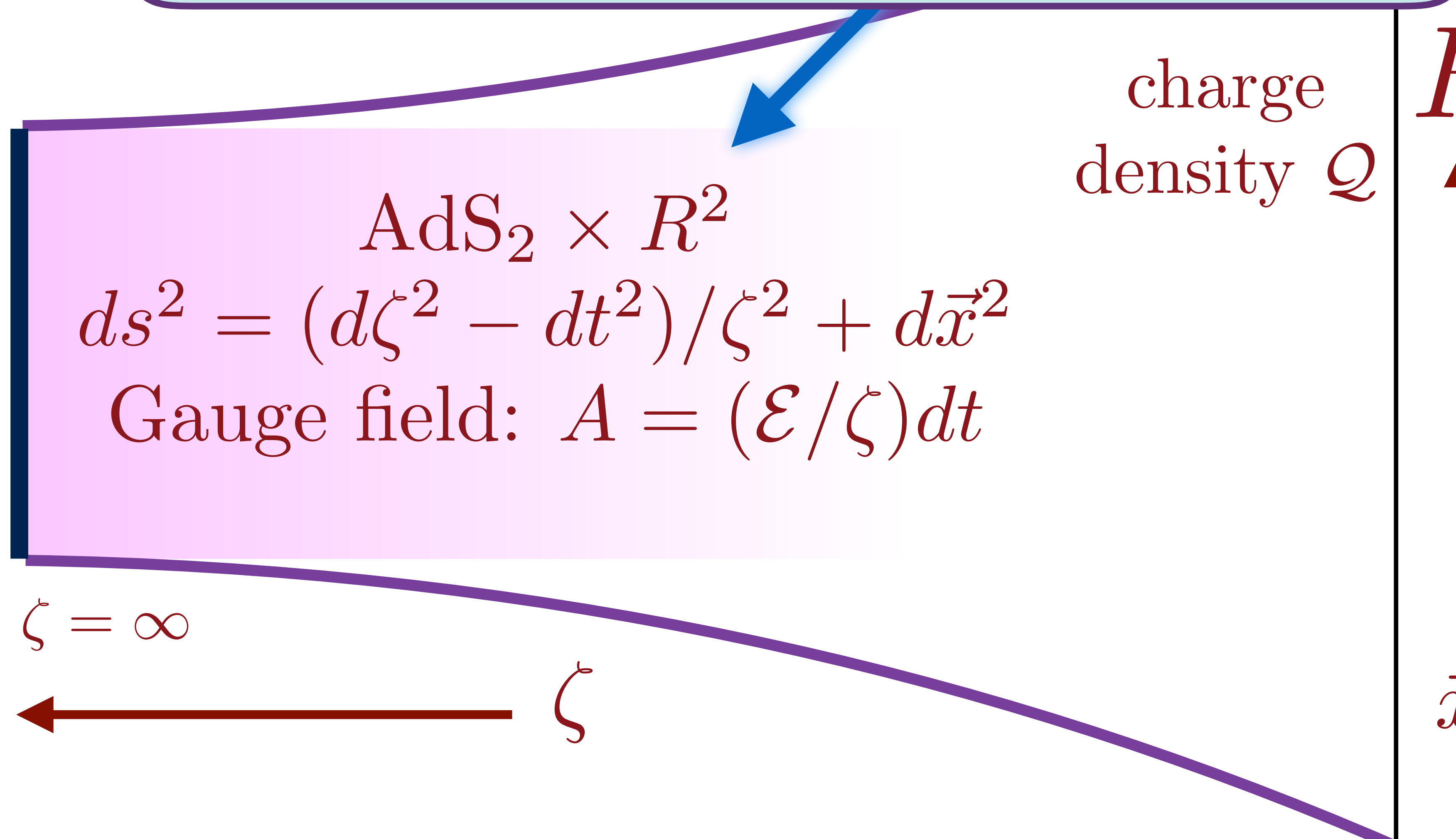
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv. 1612.00849

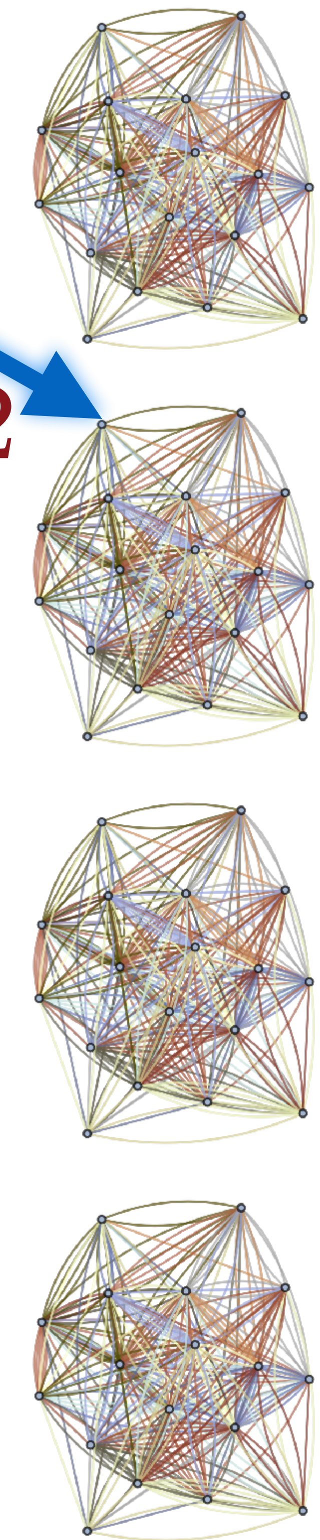
$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2}\sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} \right),$$

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$ leading to momentum disspation

Coupled SYK and AdS$_4$

Matching correlators for thermoelectric diffusion, and quantum chaos
$\tau_L = \hbar/(2\pi k_B T)$, $v_B \sim T^{1/2}$,
and thermal diffusivity $D_E = v_B^2 \tau_L$





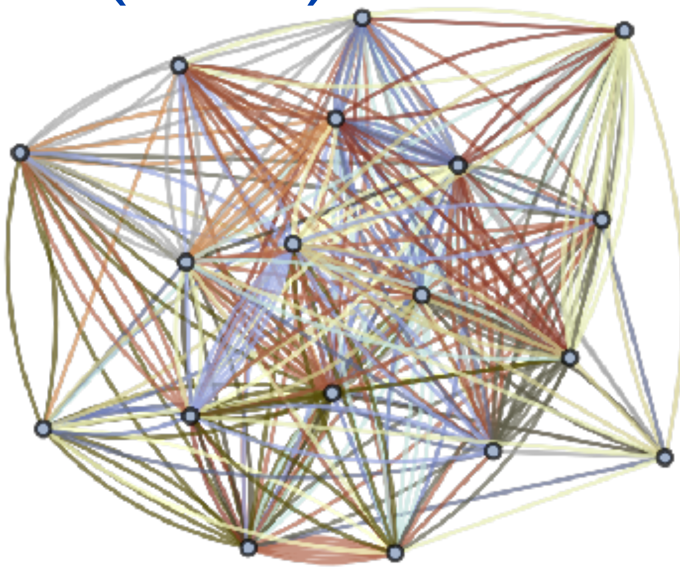
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv. 1612.00849

$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2}\sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} \right),$$

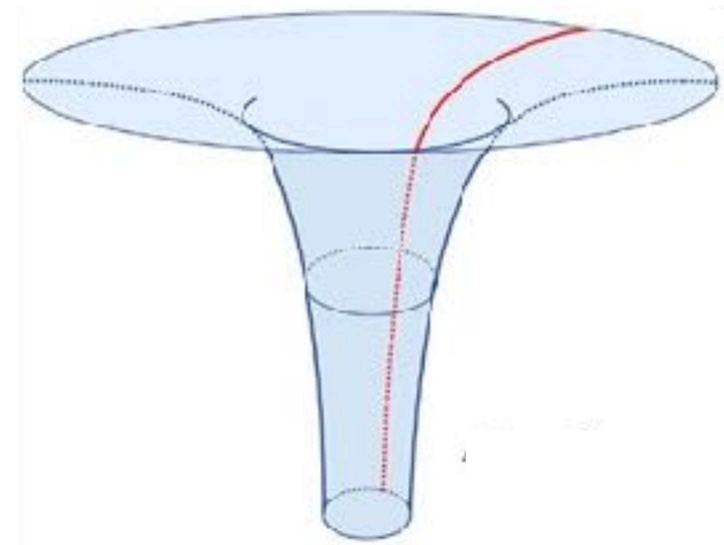
Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$ leading to momentum disspation

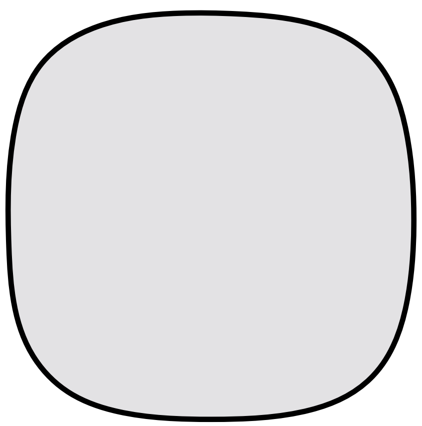
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons



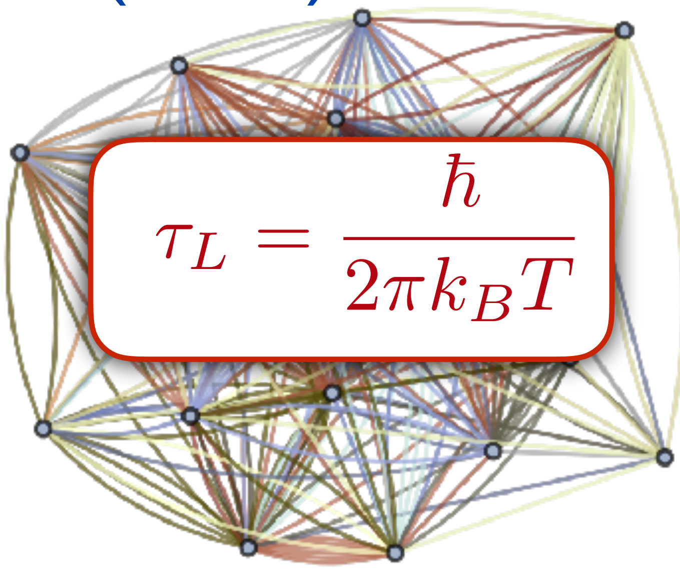


Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

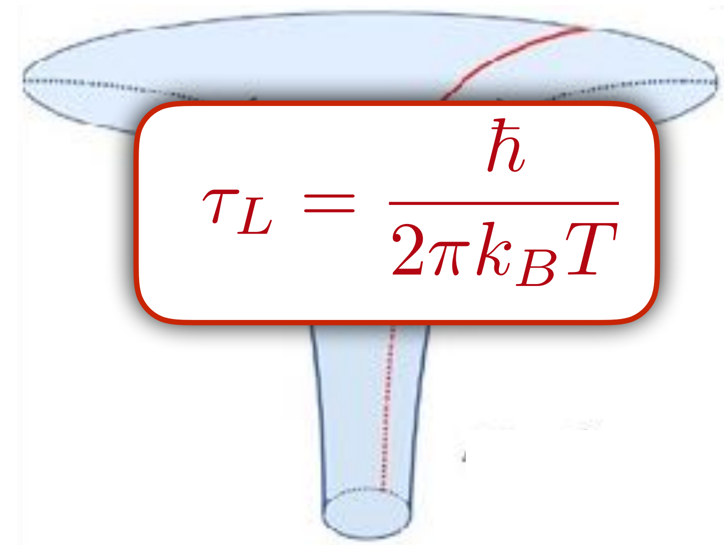
Quantum matter <u>without</u> quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

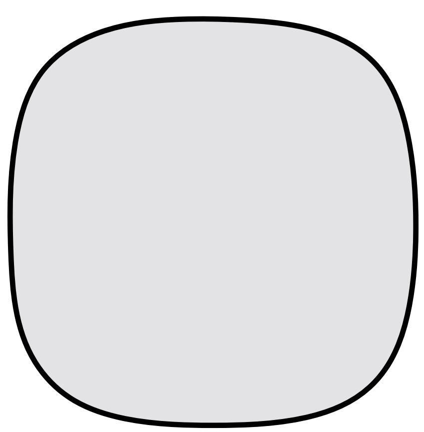


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Black holes with AdS_2 horizons



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$



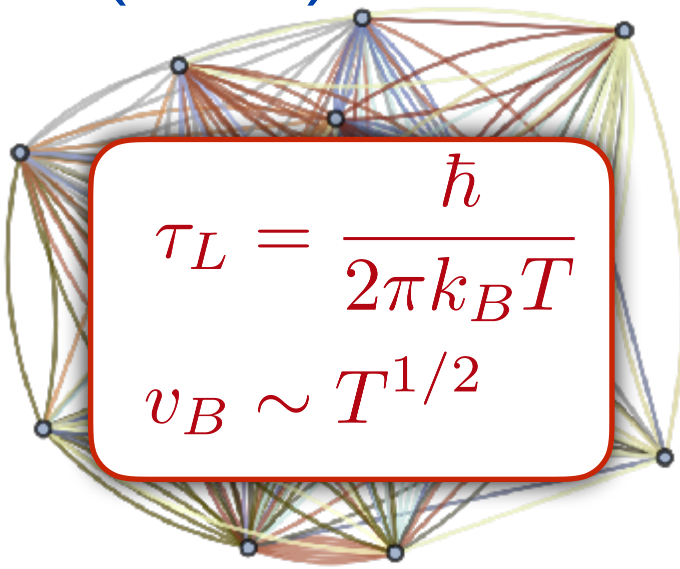
Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

Quantum matter without quasiparticles:

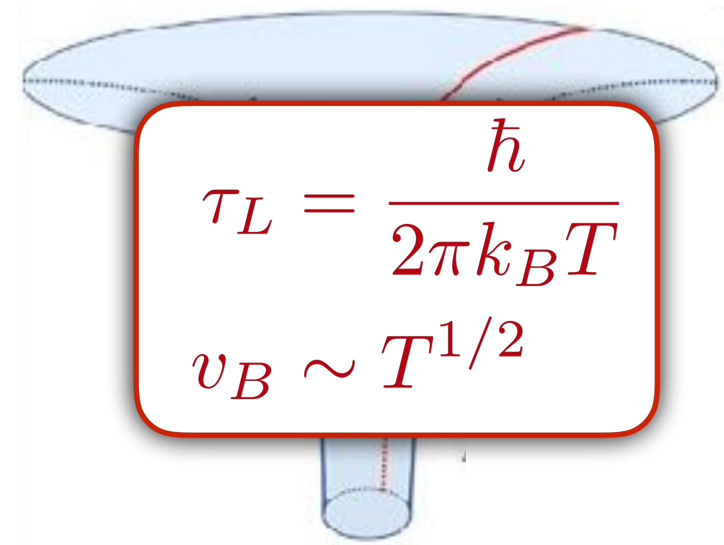
The Sachdev-Ye-Kitaev (SYK) models



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

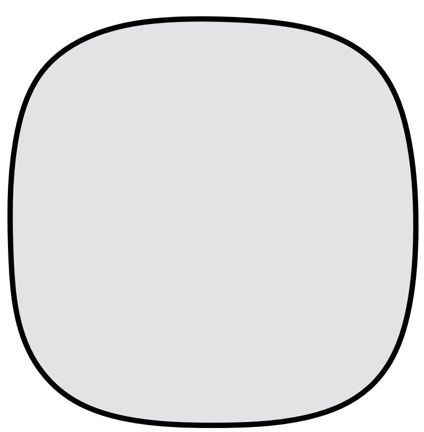
Black holes with AdS_2 horizons



$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

Fermi surface coupled to a gauge field



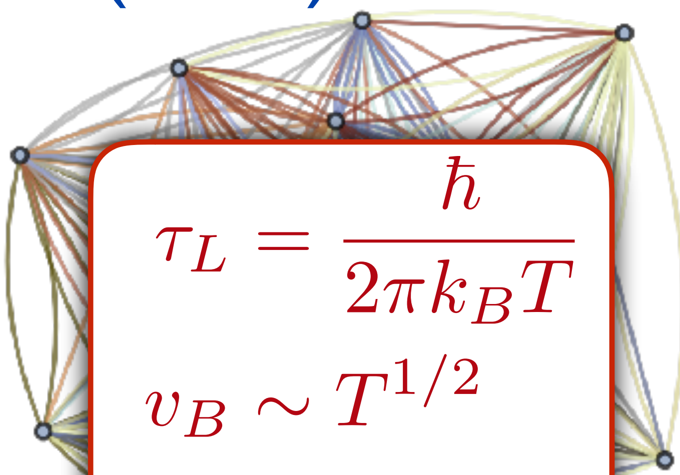
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

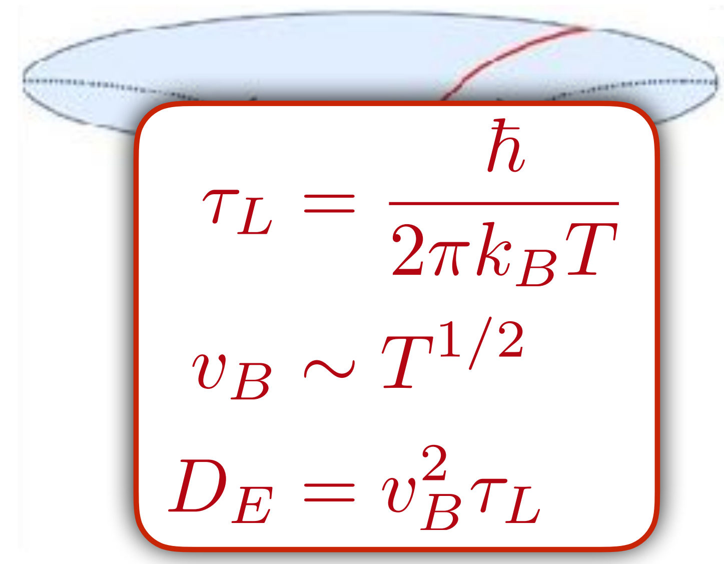


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

Black holes with AdS_2 horizons

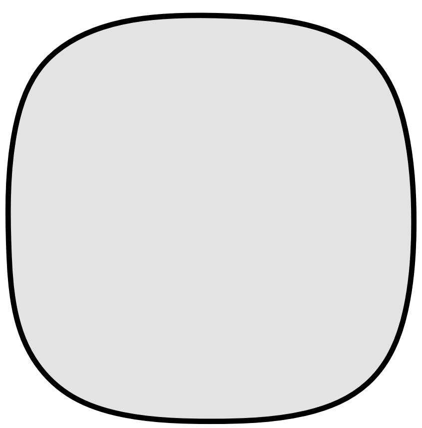


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

Fermi surface coupled to a gauge field



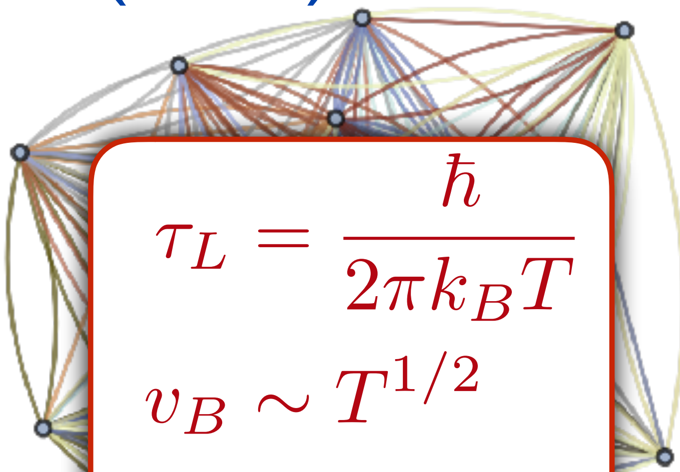
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

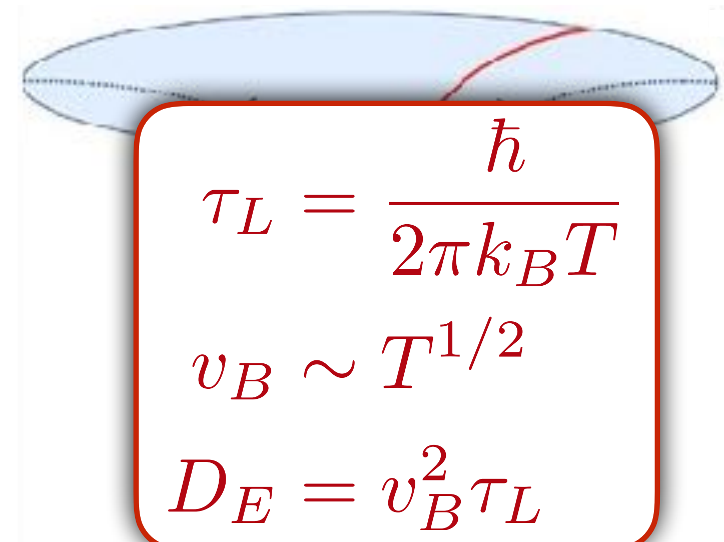


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

Black holes with AdS_2 horizons

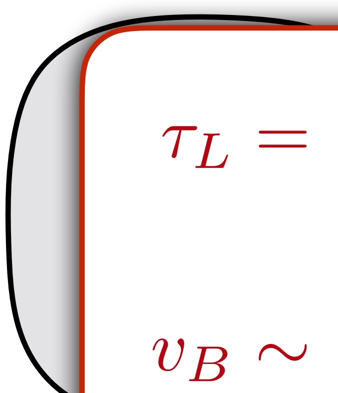


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

A. A. Patel and S. Sachdev, arXiv: 1611.00003



rmi surface coupled to a gauge field

$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$v_B \sim \frac{N v_F^{5/3}}{e^{4/3}\gamma^{1/3}} T^{1/3}$$

$$D_E = 0.42 v_B^2 \tau_L$$

$$\mathcal{L}[\Psi \ldots \frac{(\nabla - i\vec{a})^2}{2m} - \mu \Big) \Psi + \frac{1}{2g^2}(\nabla \times \vec{a})^2$$

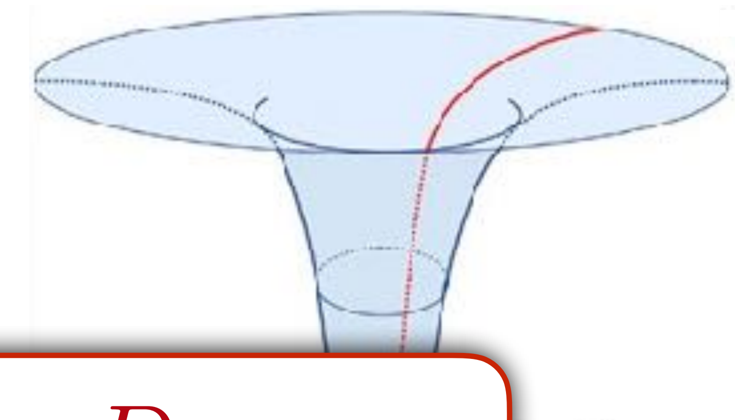
τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

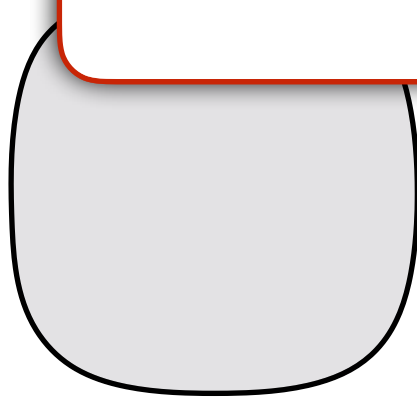
Quantum matter <u>without</u> quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

Black holes with AdS_2 horizons



Thermal diffusivity, D_E:
$D_E = (\text{universal number}) \times v_B^2 \tau_L$
in all three models



Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

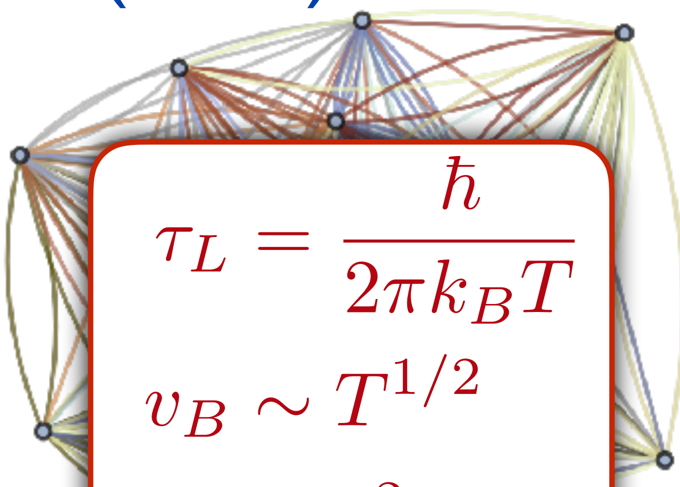
τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

- Quantum chaos is intimately linked to the loss of phase coherence from electron-electron interactions. As the time derivative of the local phase is determined by the local energy, phase fluctuations and chaos are linked to interaction-induced energy fluctuations, and hence thermal diffusivity.

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

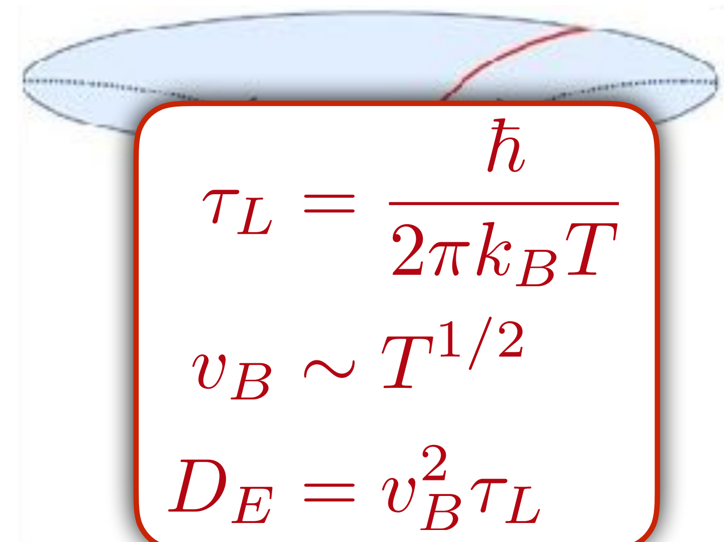


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

Black holes with AdS_2 horizons

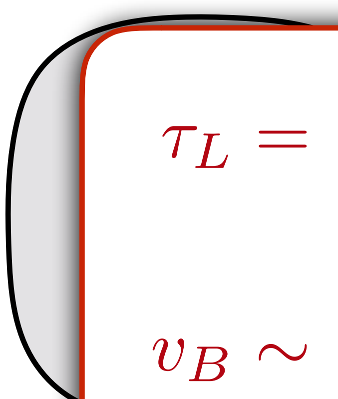


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

...ermi surface coupled to a gauge field



$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$v_B \sim \frac{N v_F^{5/3}}{e^{4/3}\gamma^{1/3}} T^{1/3}$$

$$D_E = 0.42 v_B^2 \tau_L$$

$$\mathcal{L}[\Psi \ldots \left(\ldots - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos