Entangled phases of quantum matter

School on Current Frontiers in Condensed Matter Research
International Center for Theoretical Sciences, Bengaluru
June 24, 2016

Subir Sachdev

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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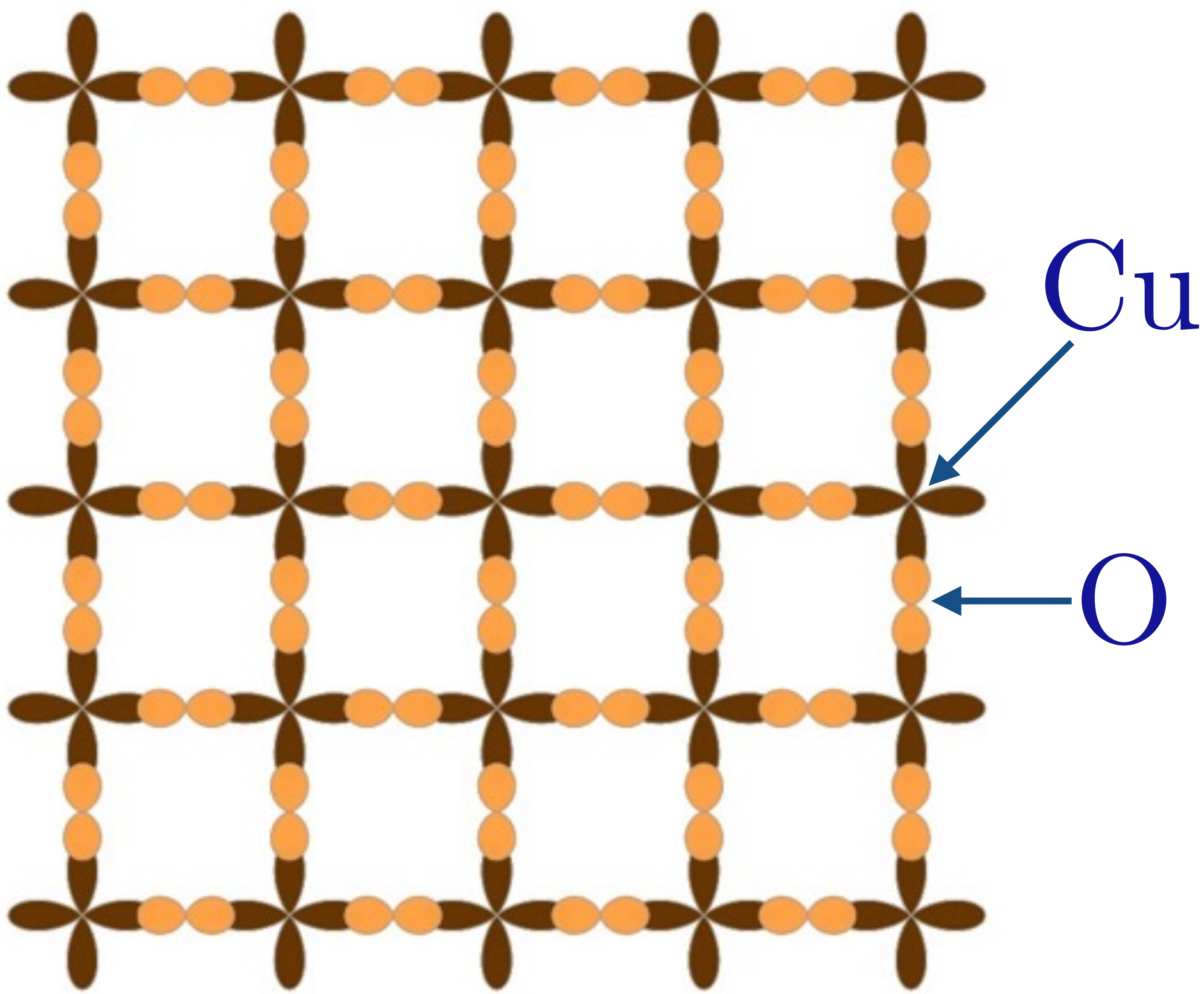
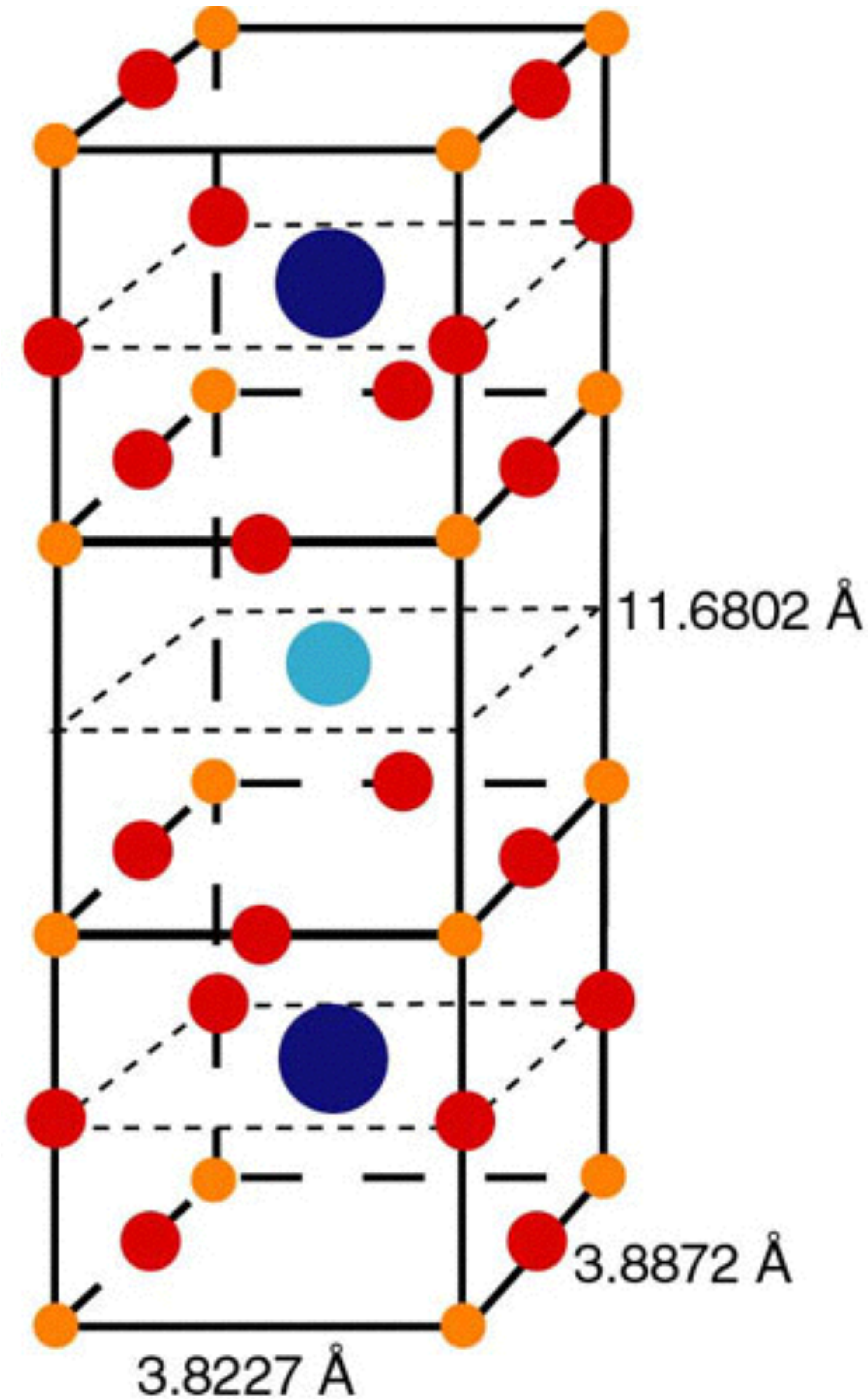
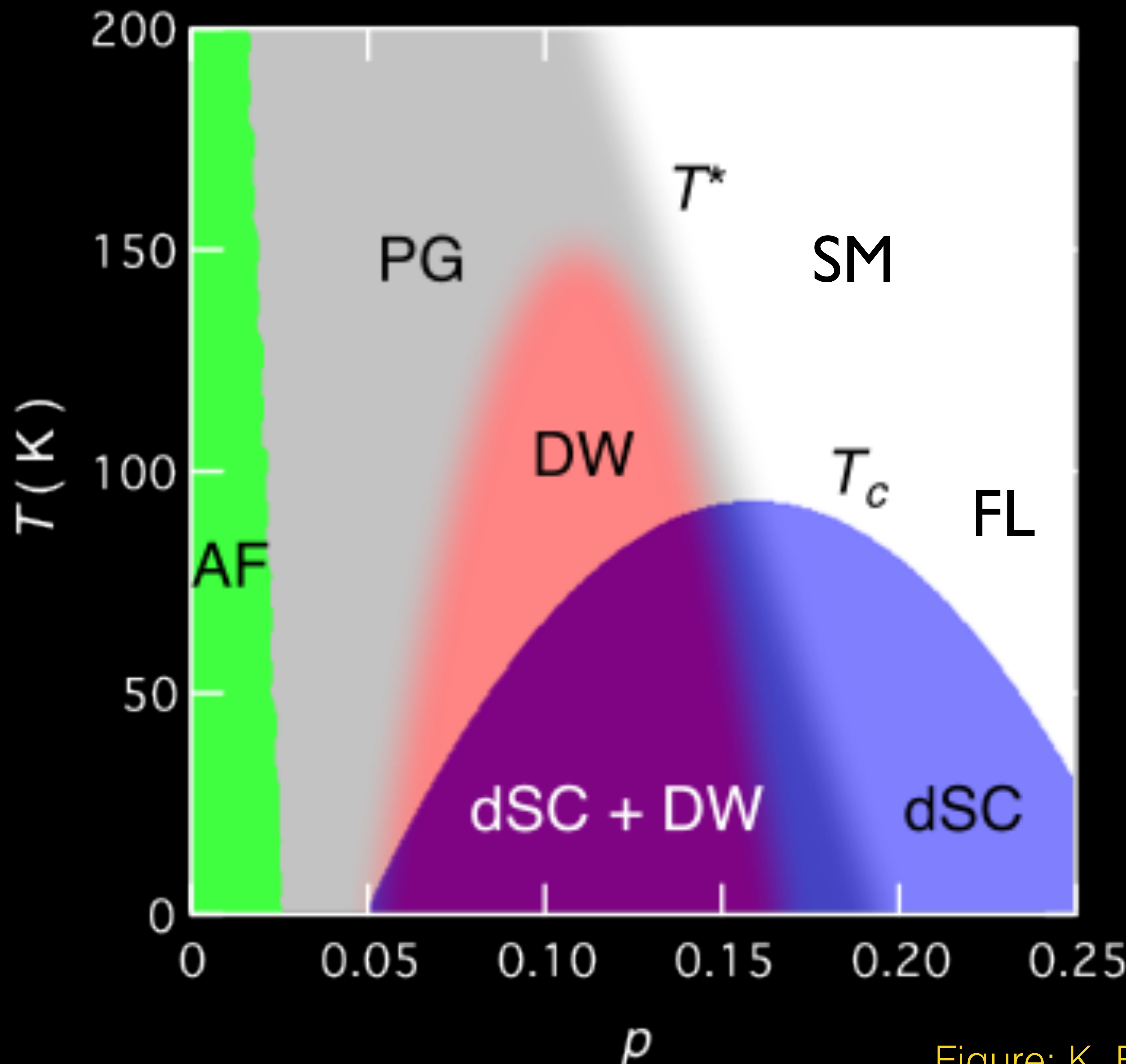
High temperature superconductors

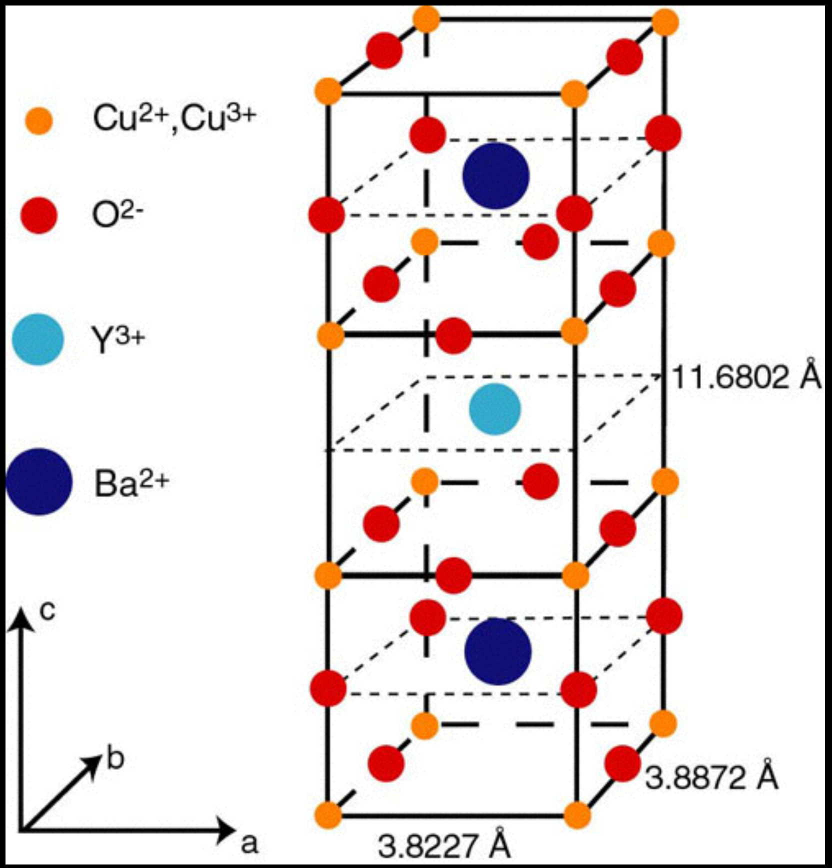


CuO_2 plane



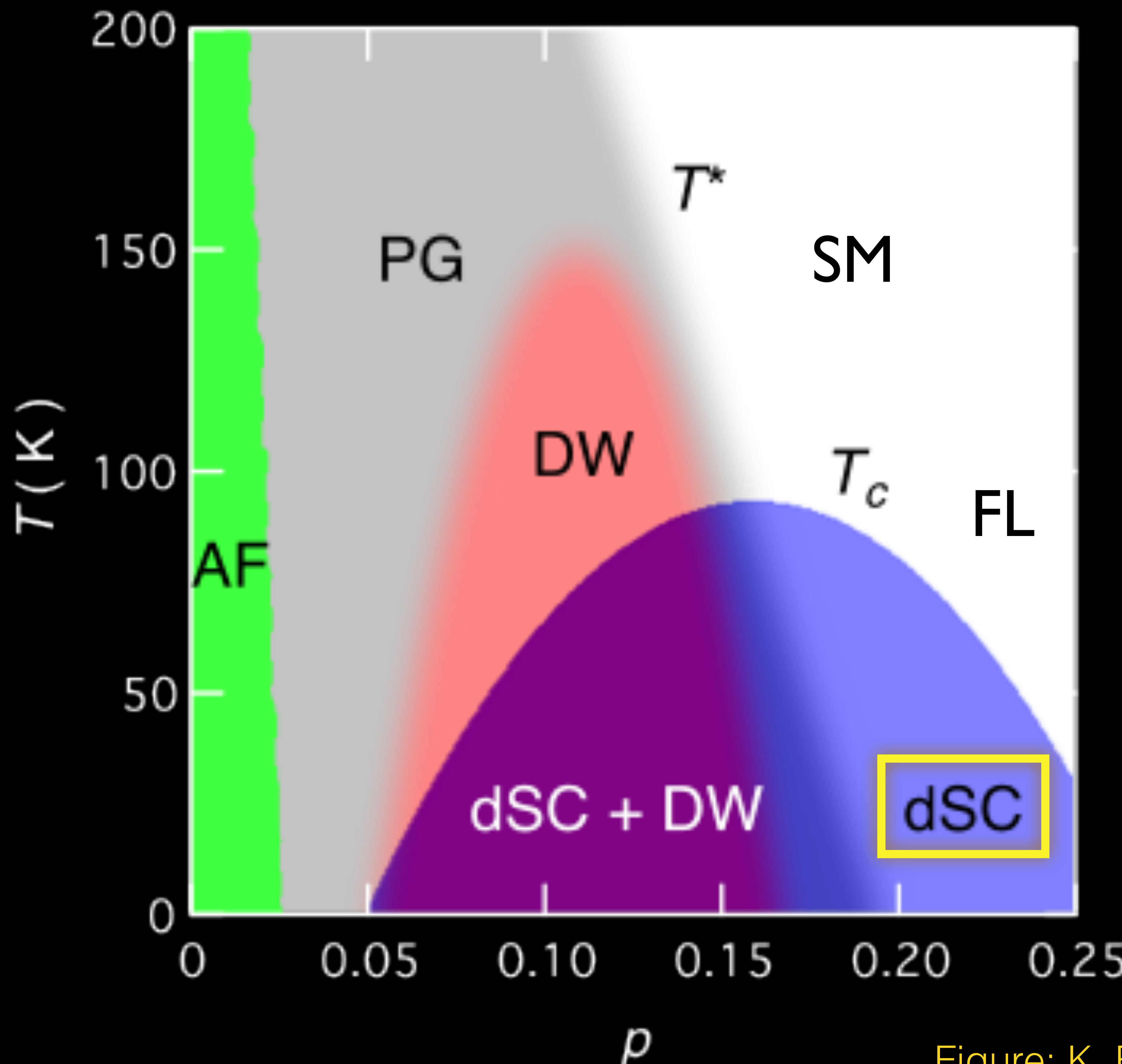
$YBa_2Cu_3O_{6+x}$

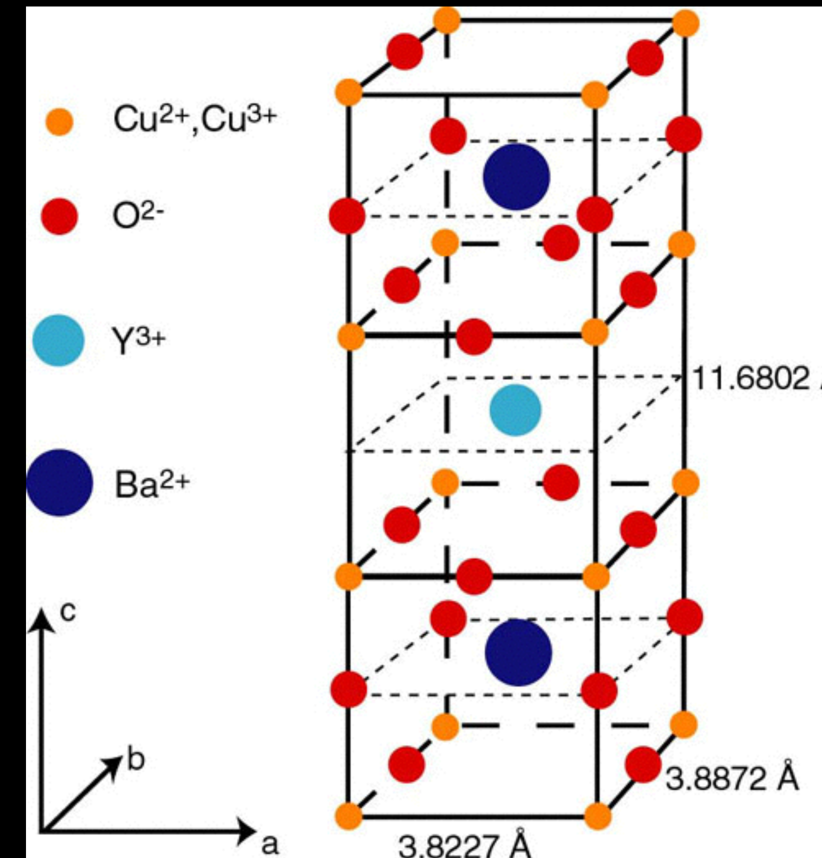




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis





$YBa_2Cu_3O_{6+x}$

d-wave superconductor

Figure: K. Fujita and J. C. Seamus Davis

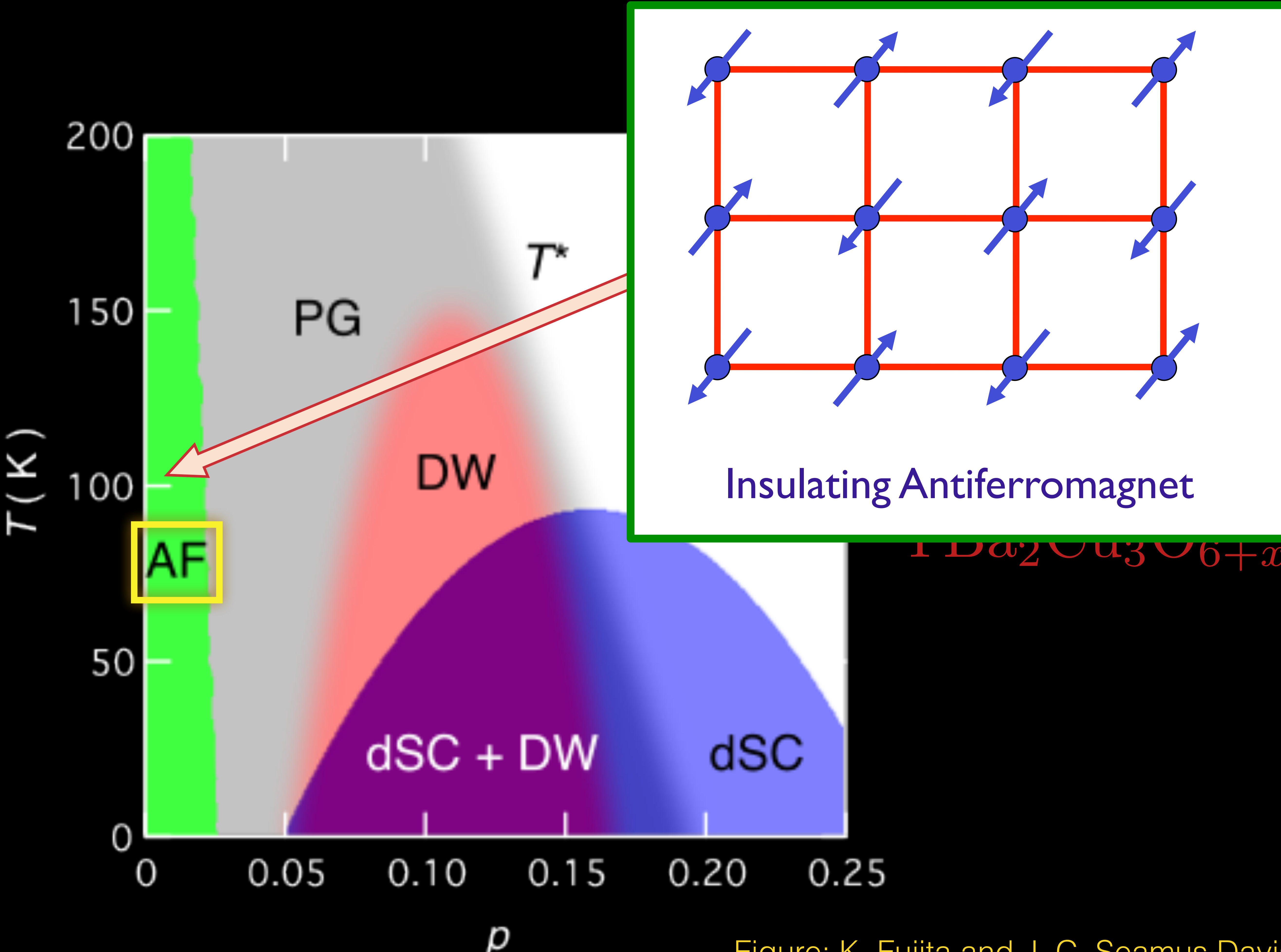
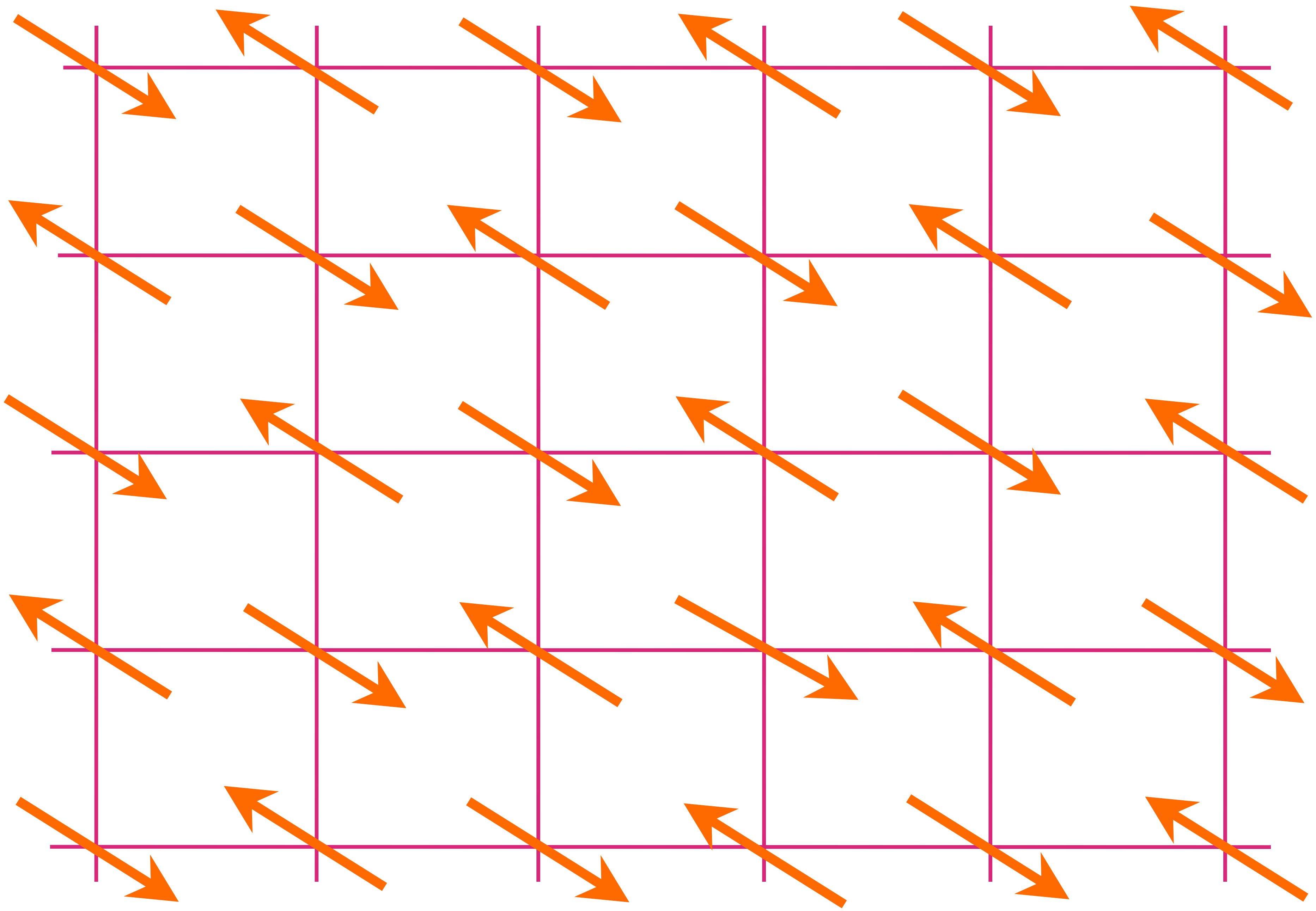
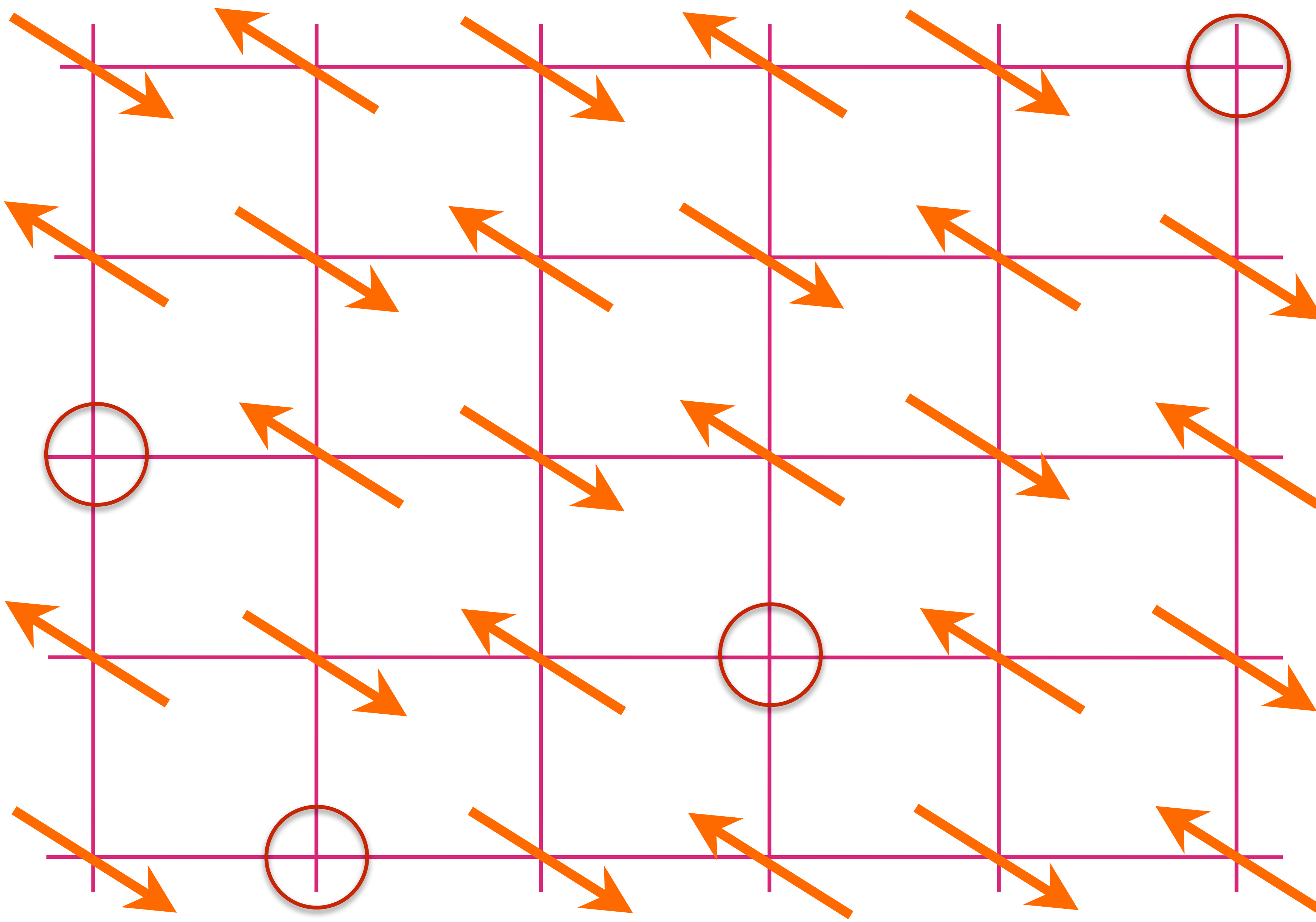


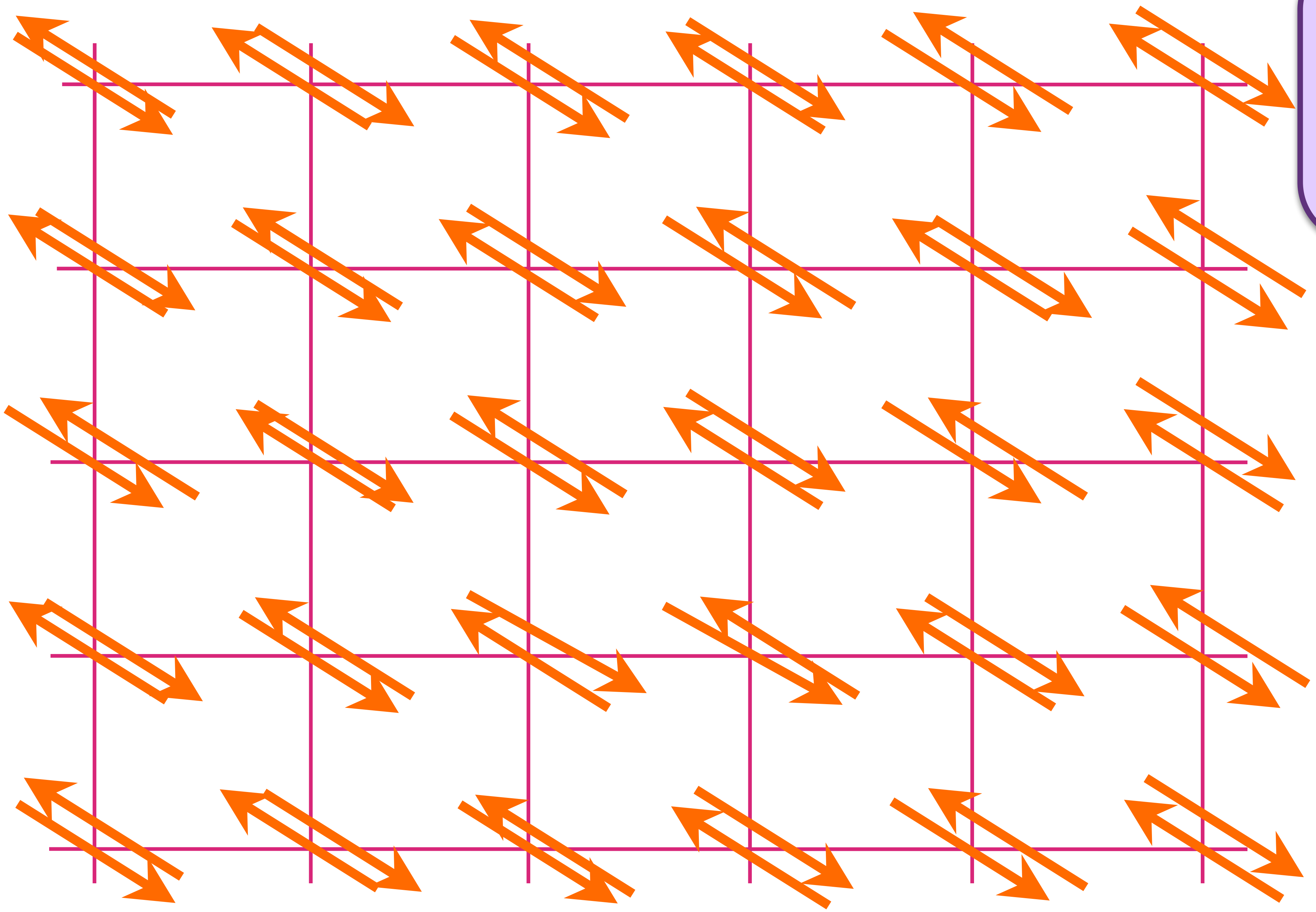
Figure: K. Fujita and J. C. Seamus Davis



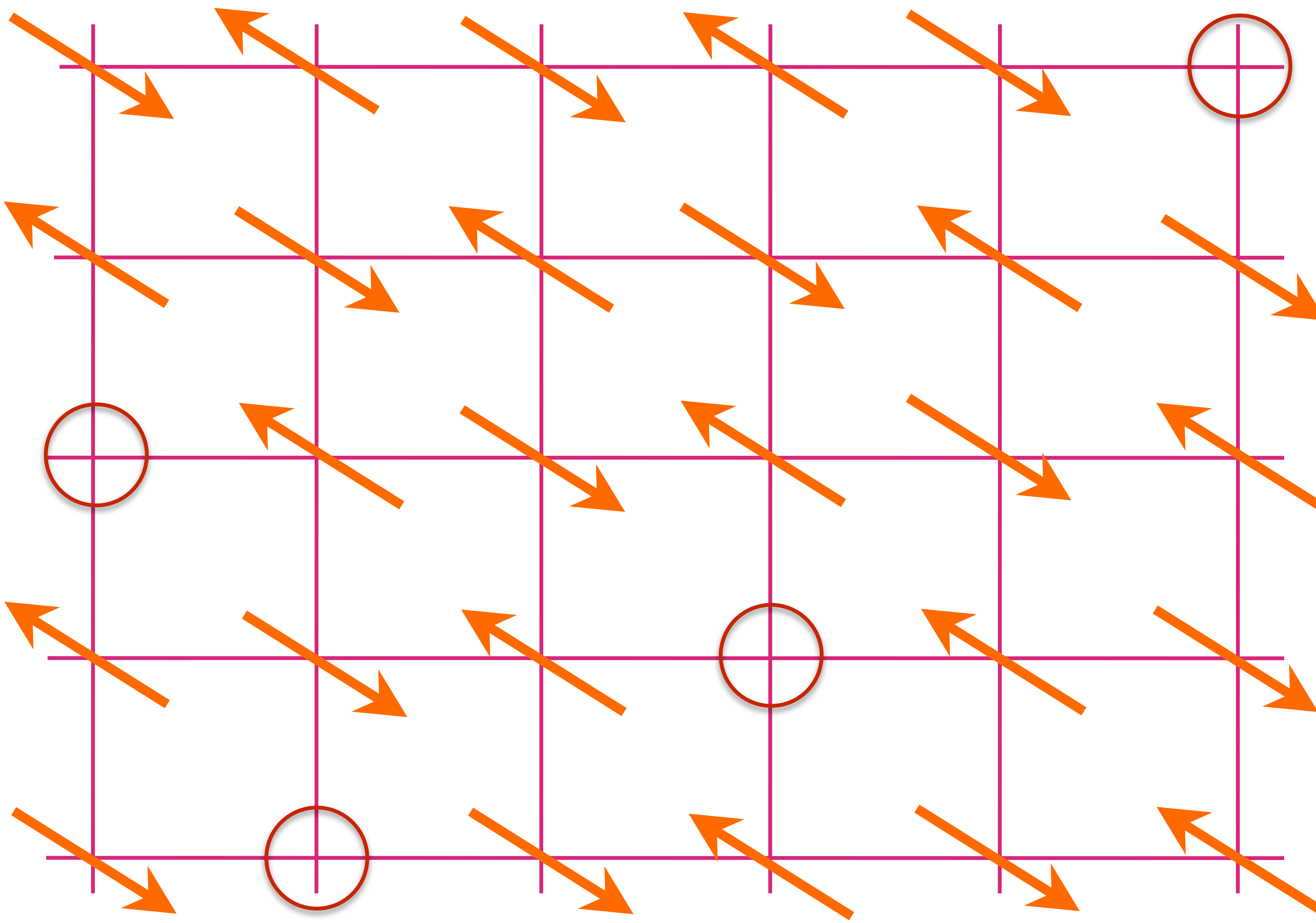
“Undoped” insulating anti-ferromagnet



Anti-ferromagnet with p mobile holes per square

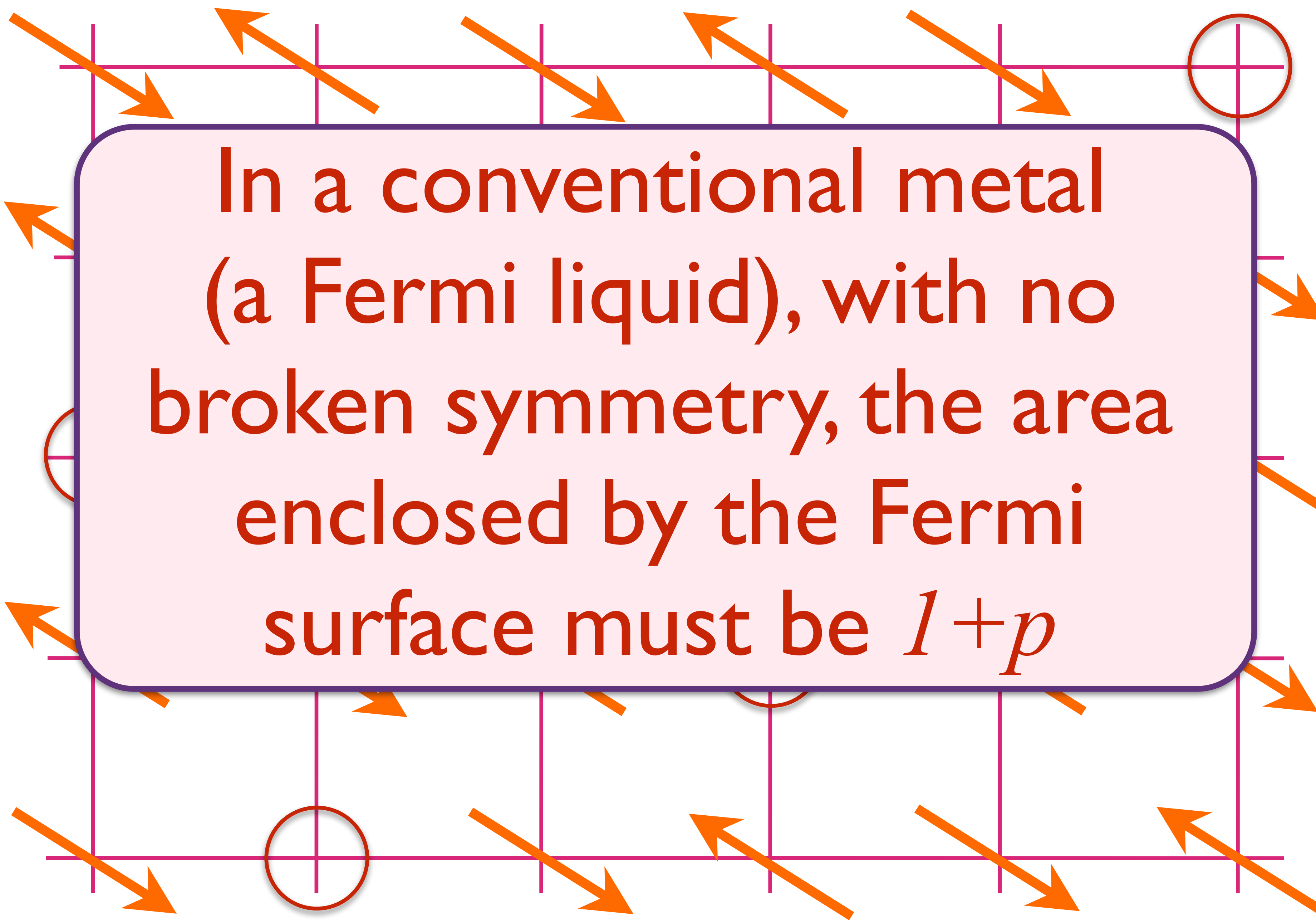


Filled Band



Anti-ferromagnet with p mobile holes per square

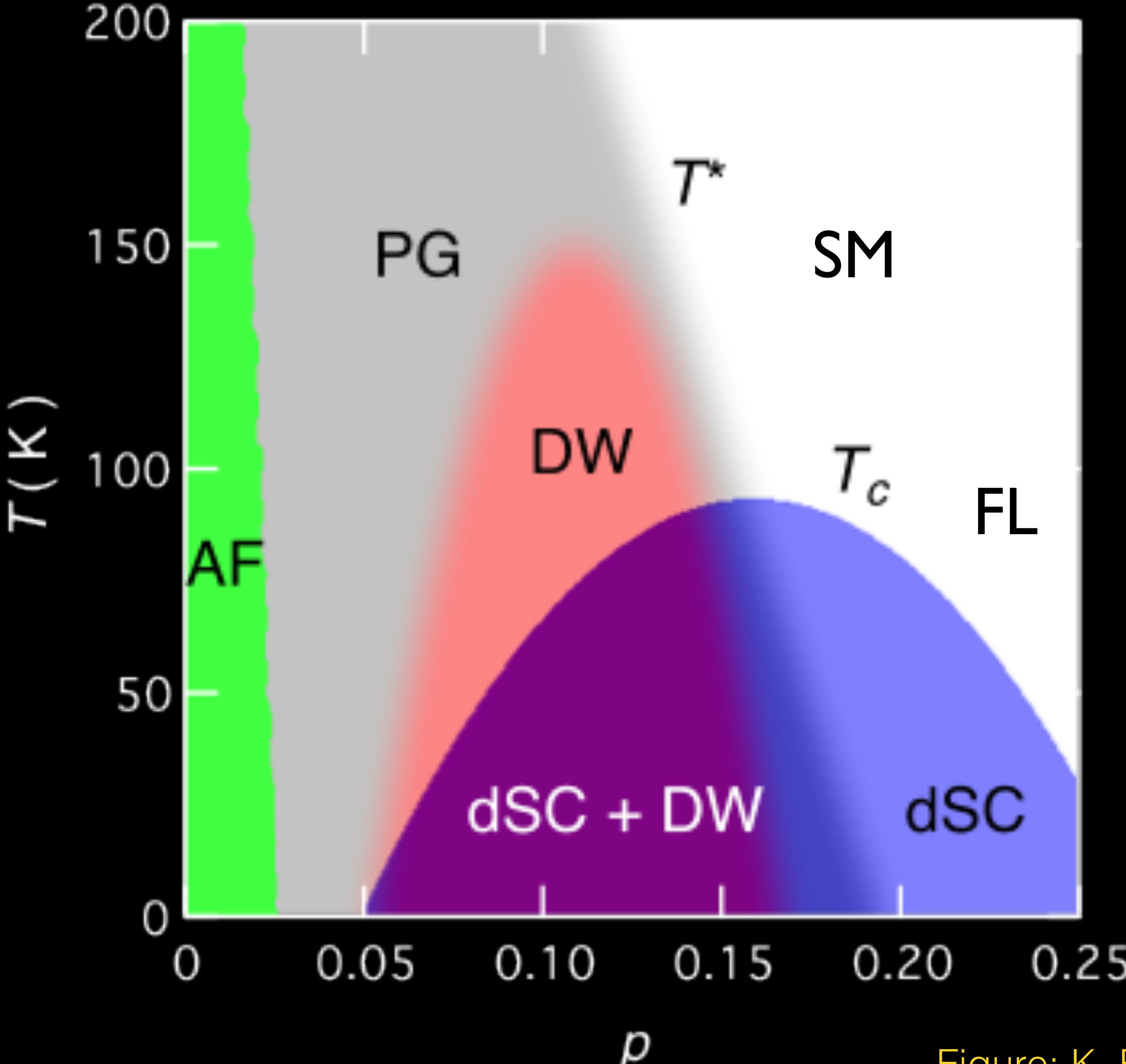
But relative to the band insulator, there are $1+p$ holes per square

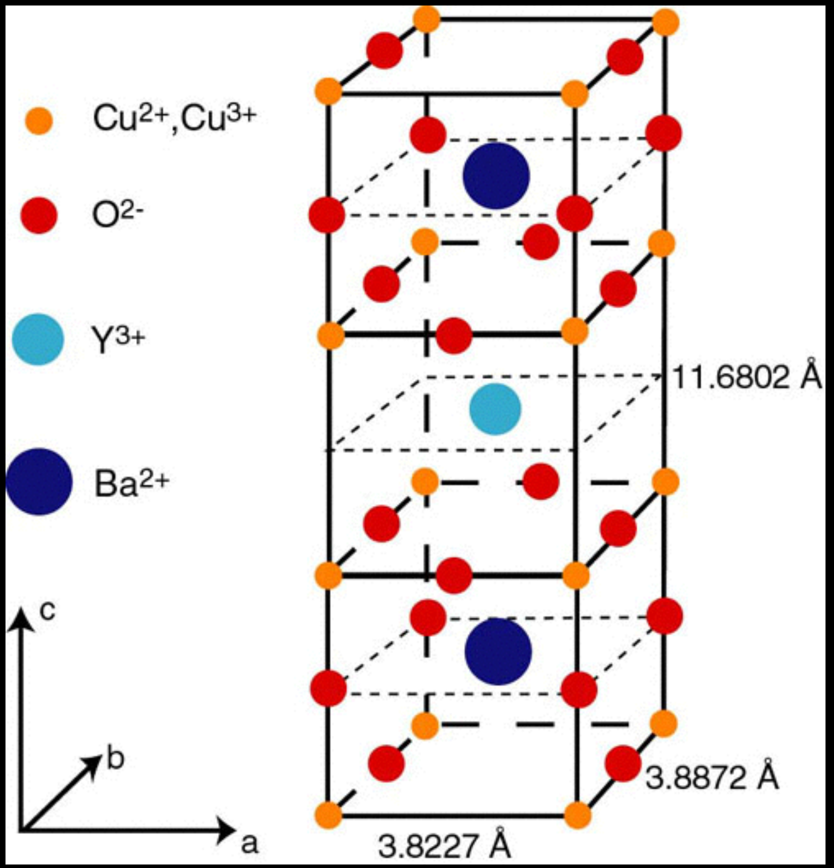


Anti-ferromagnet with p mobile holes per square

In a conventional metal (a Fermi liquid), with no broken symmetry, the area enclosed by the Fermi surface must be $1+p$

But relative to the band insulator, there are $1+p$ holes per square

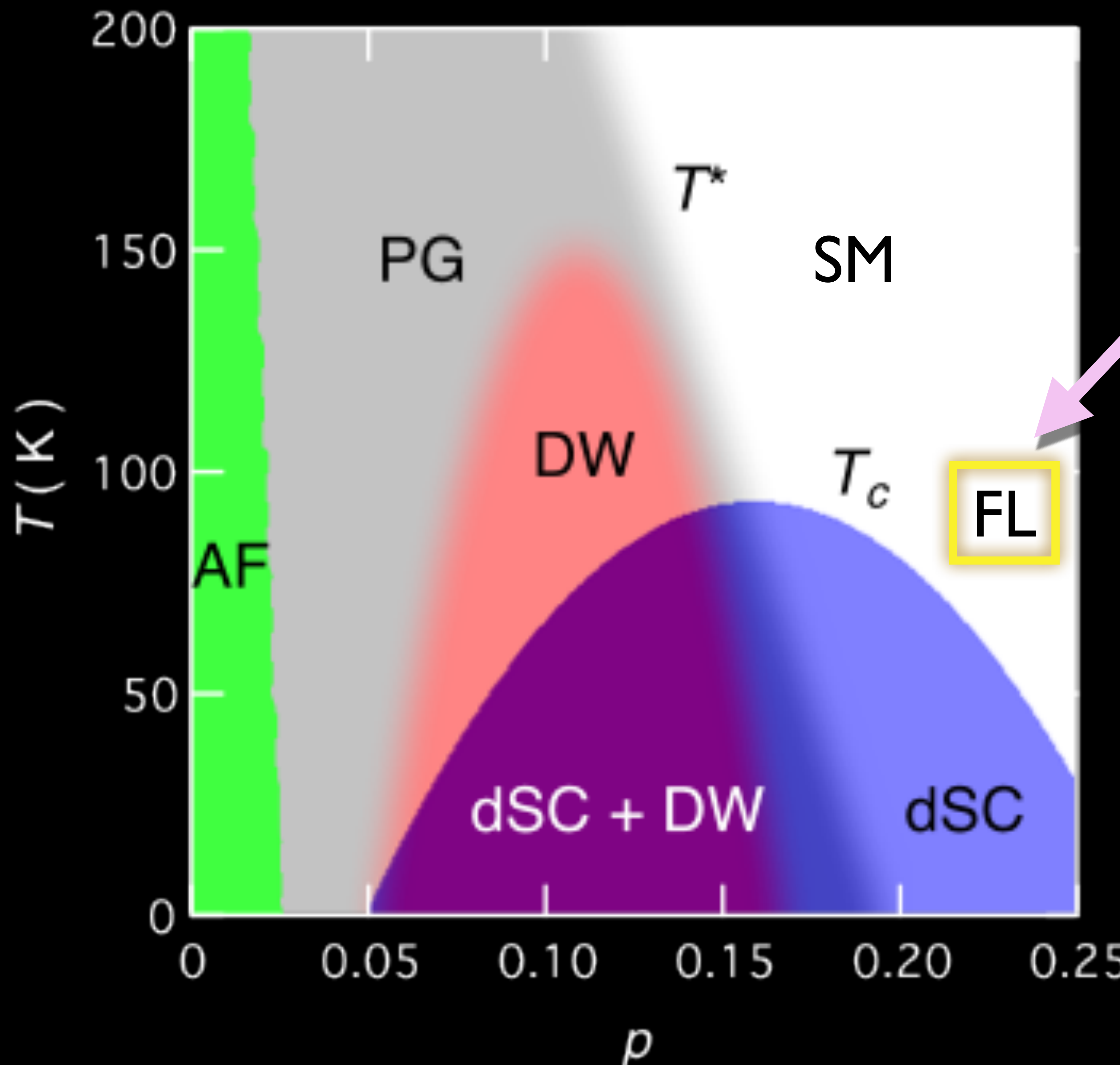


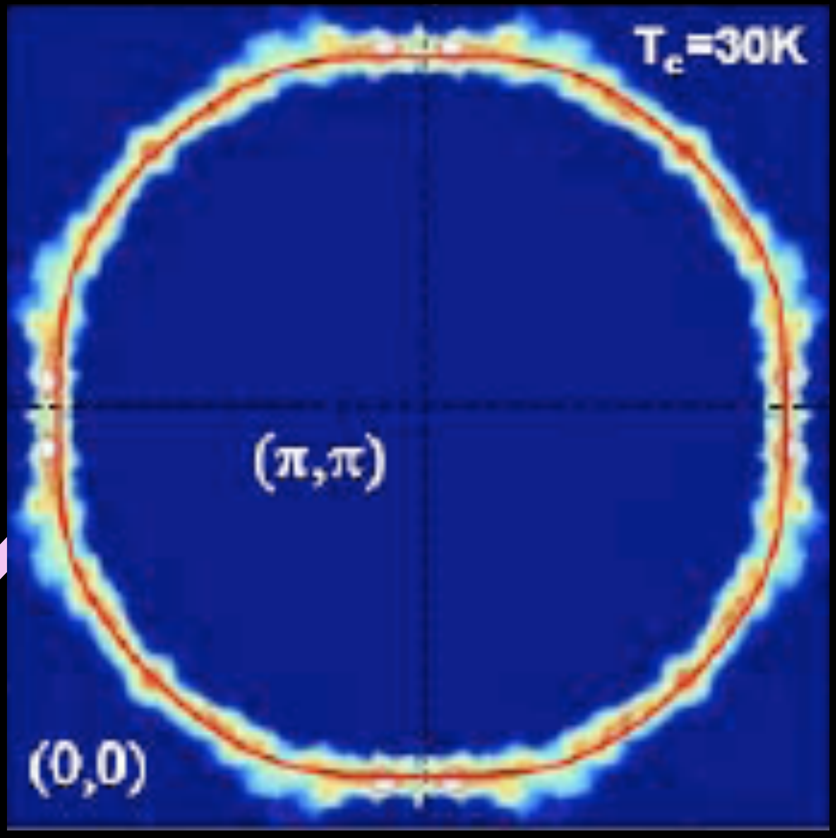


$YBa_2Cu_3O_{6+x}$

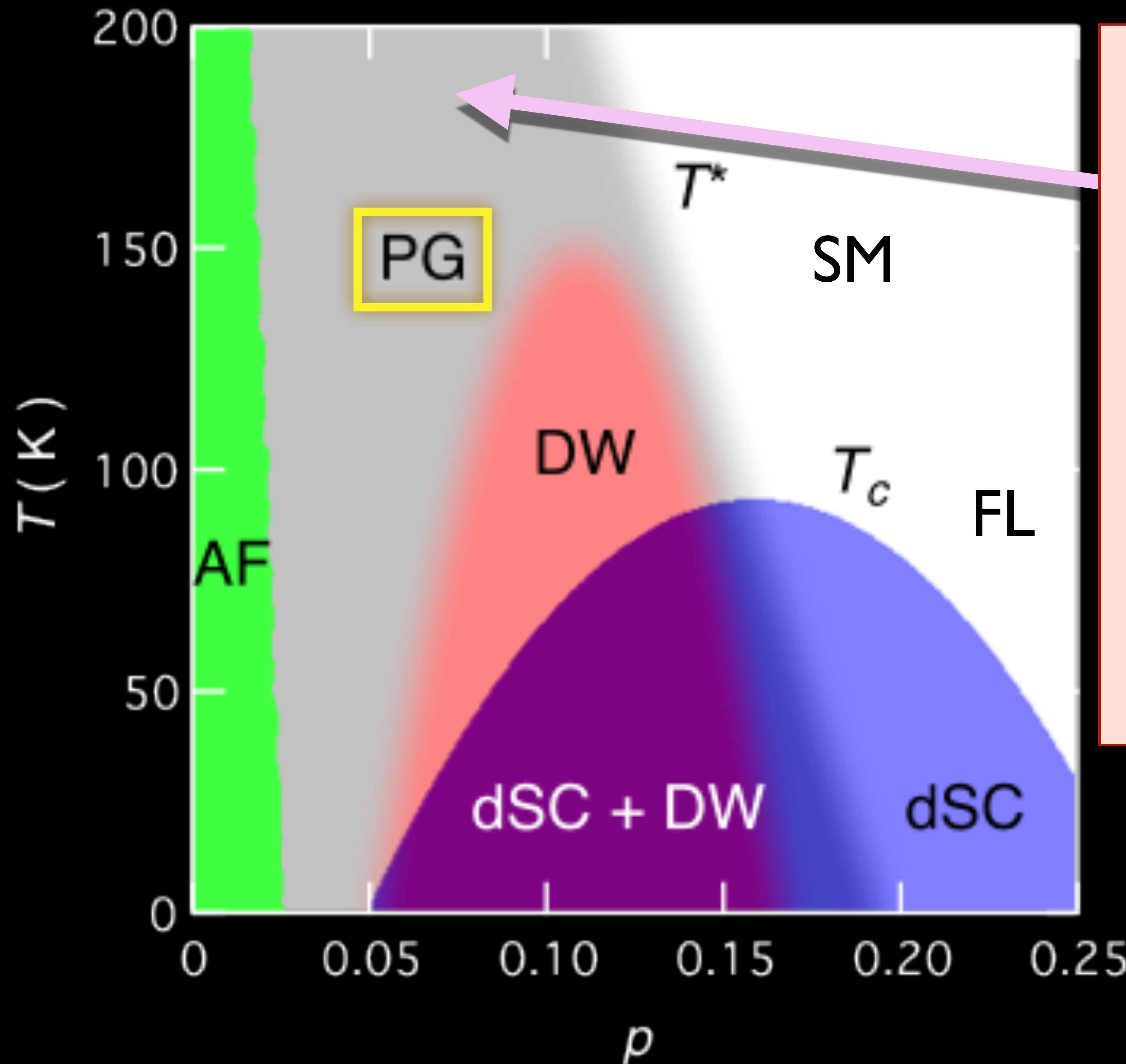
Figure: K. Fujita and J. C. Seamus Davis

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





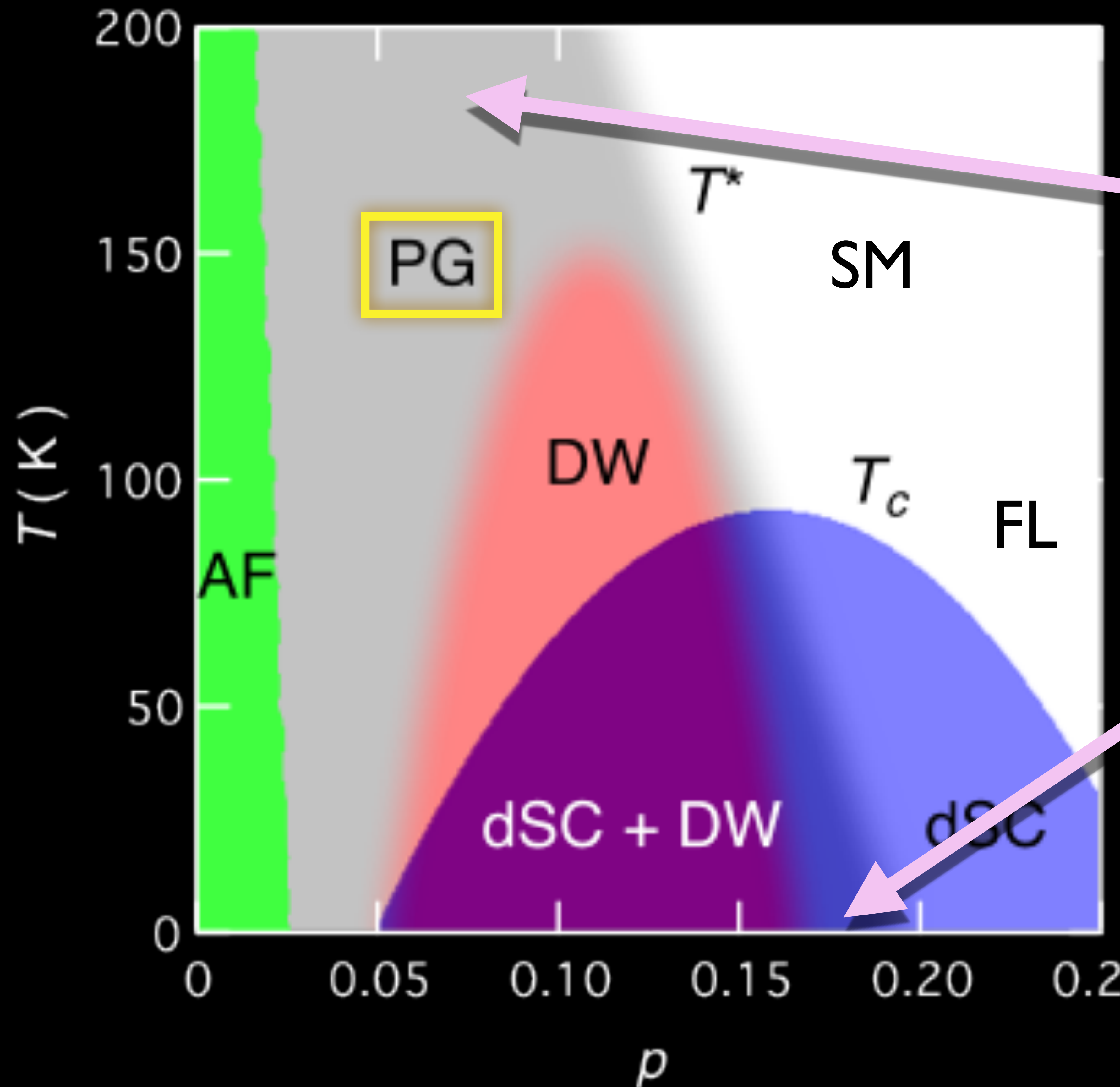
A conventional metal:
the Fermi liquid with Fermi surface of size $1+p$



Pseudogap metal at low p

Many experimental indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

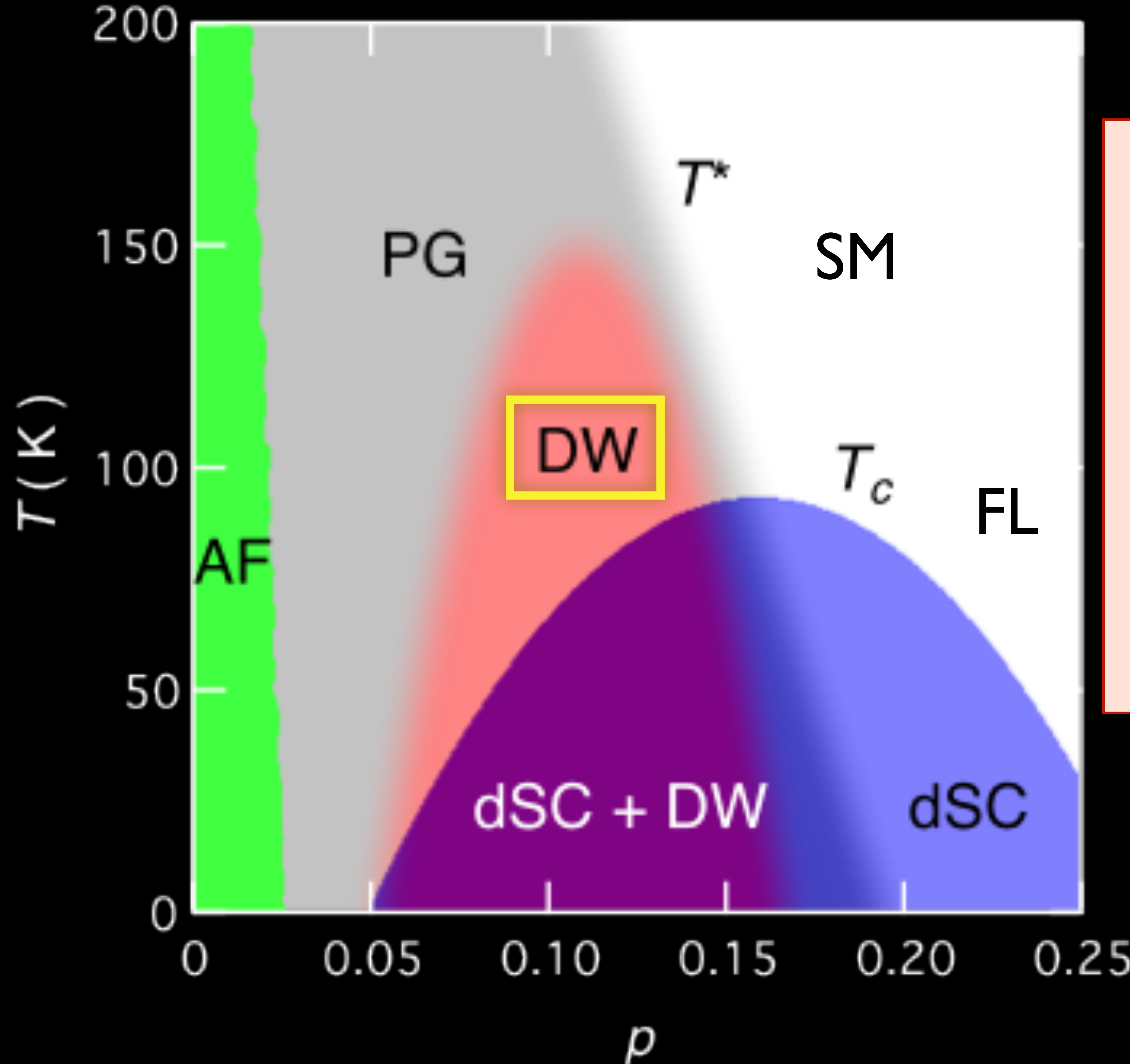
S. Badoux, W. Tabis, F. Laliberté, G. Grissonnanche, B. Vignolle, D. Vignolles, J. Béard, D.A. Bonn, W.N. Hardy, R. Liang, N. Doiron-Leyraud, L. Taillefer, and C. Proust, Nature **531**, 210 (2016).



Pseudogap metal at low p

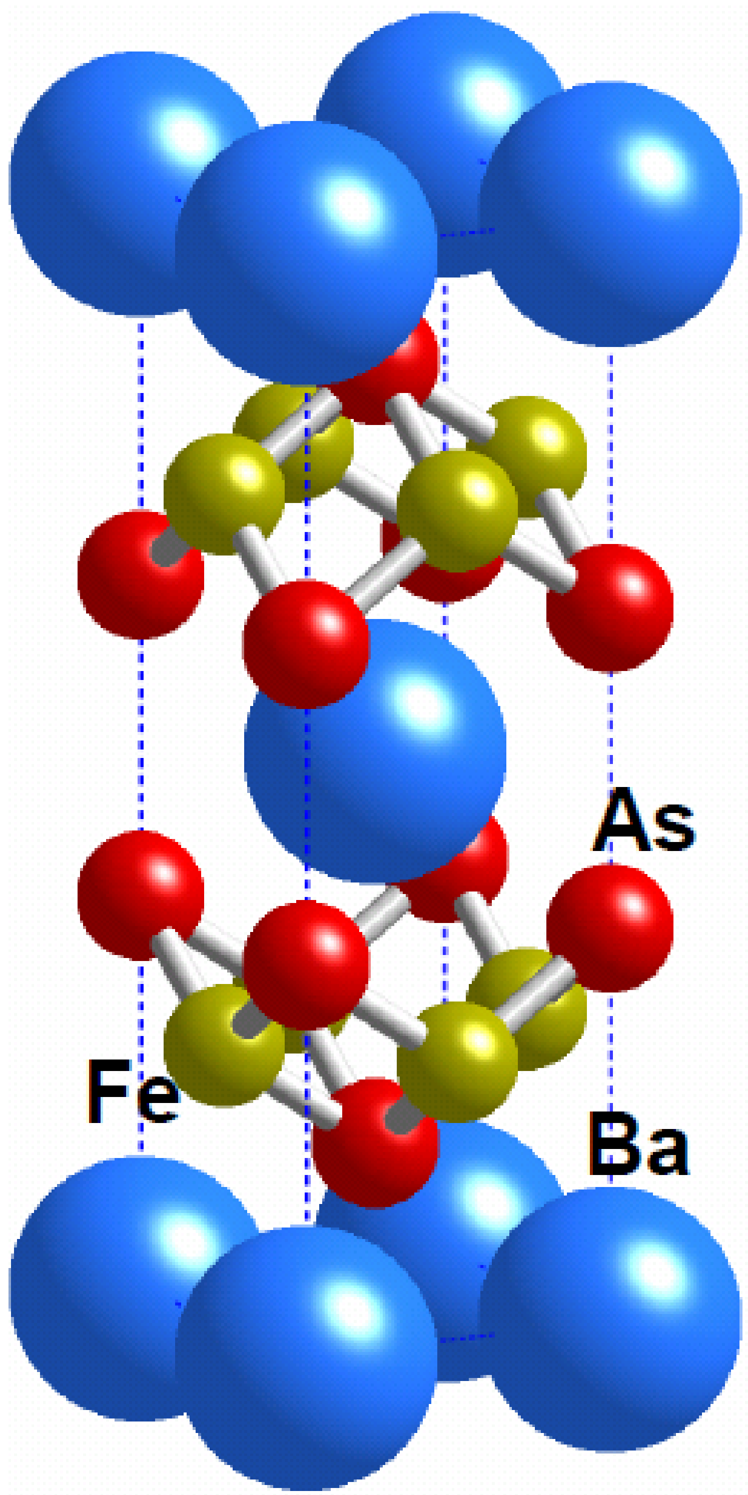
Many experimental indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

Recent experiments show the PG metal is also present at low T in high magnetic field

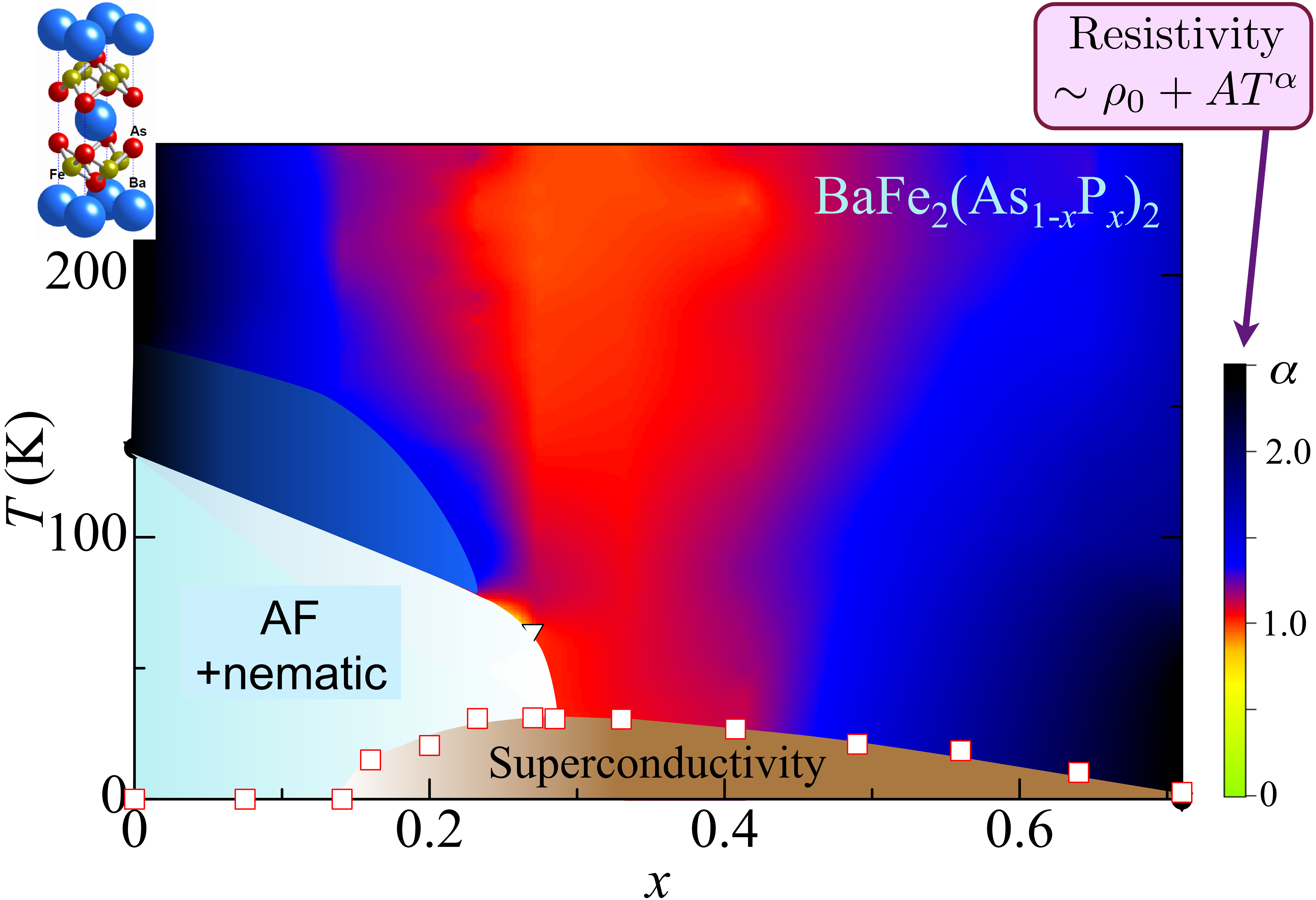


DW is "(charge) density wave" order, which is a low T instability of the PG metal. It yields important clues on the nature of the PG metal, and will be discussed later.

Onset of antiferromagnetism in metals, and d-wave superconductivity



$BaFe_2(As_{1-x}P_x)_2$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

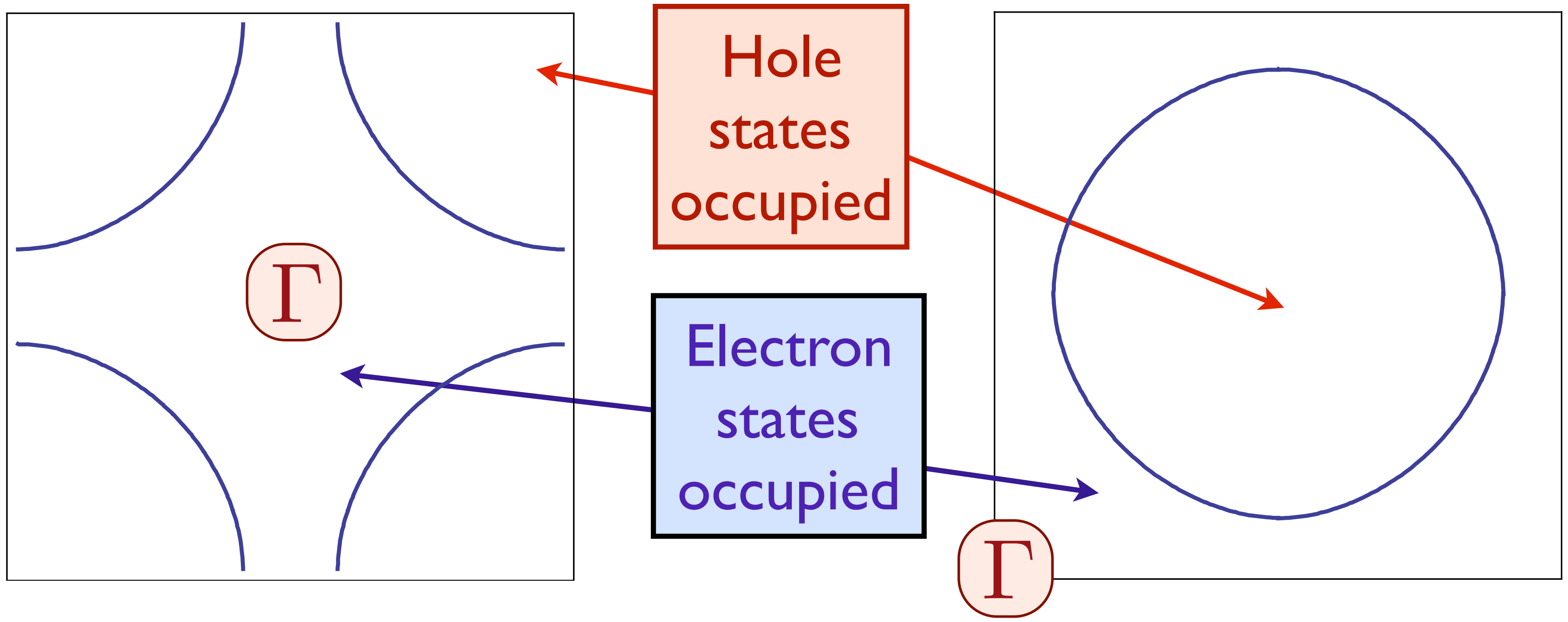
Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}$$

$$c_{i\alpha}^\dagger c_{j\beta} + c_{j\beta} c_{i\alpha}^\dagger = \delta_{ij}\delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

Will study on the square lattice

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

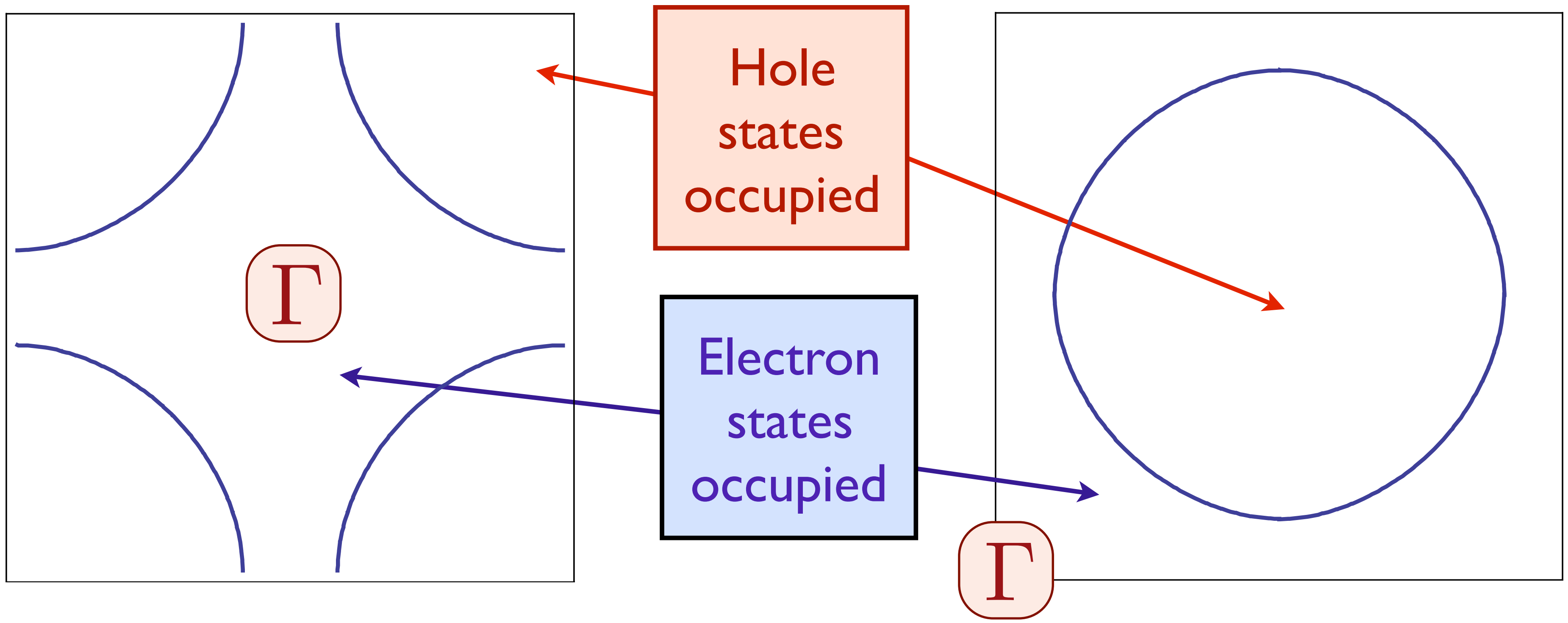
$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Fermi surface+antiferromagnetism



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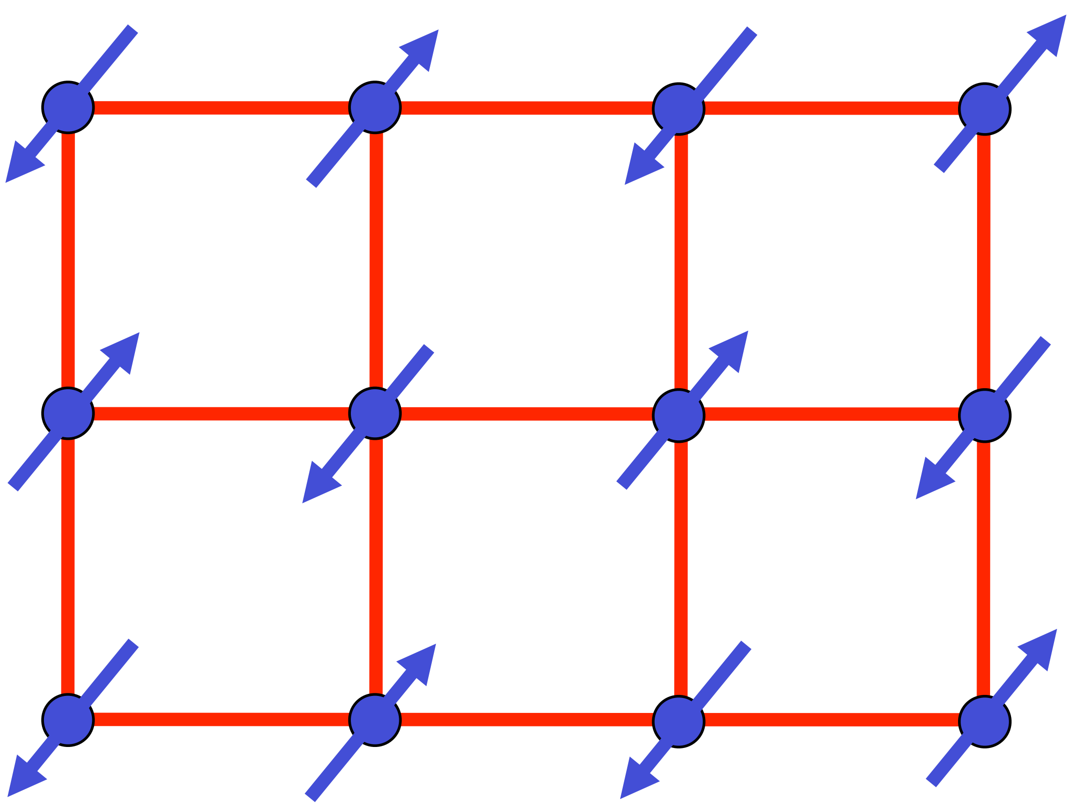


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism

We use the operator equation (valid on each site i):

$$U\left(n_{\uparrow}-\frac{1}{2}\right)\left(n_{\downarrow}-\frac{1}{2}\right)=-\frac{2U}{3}\vec{S}^{2}+\frac{U}{4} \tag{1}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i\int d\tau\vec{S}_i^2\right)=\int\mathcal{D}\vec{J}_i(\tau)\exp\left(-\sum_i\int d\tau\left[\frac{3}{8U}\vec{J}_i^2-\vec{J}_i\vec{S}_i\right]\right) \tag{2}$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for $\vec{J}_i$. At the saddle-point we find that the lowest energy is achieved when the vector has opposite orientations on the A and B sublattices. Anticipating this, we look for a continuum limit in terms of a field $\vec{\varphi}_i$ where

$$\vec{J}_i=\vec{\varphi}_i\,e^{i\mathbf{K}\cdot\mathbf{r}_i} \tag{3}$$

Fermi surface+antiferromagnetism

In this manner, we obtain the “spin-fermion” model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2r \left[\frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{1}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4 \right]$$

Fermi surface+antiferromagnetism

In the Hamiltonian form (ignoring, for now, the time dependence of $\vec{\varphi}$), the coupling between $\vec{\varphi}$ and the electrons takes the form

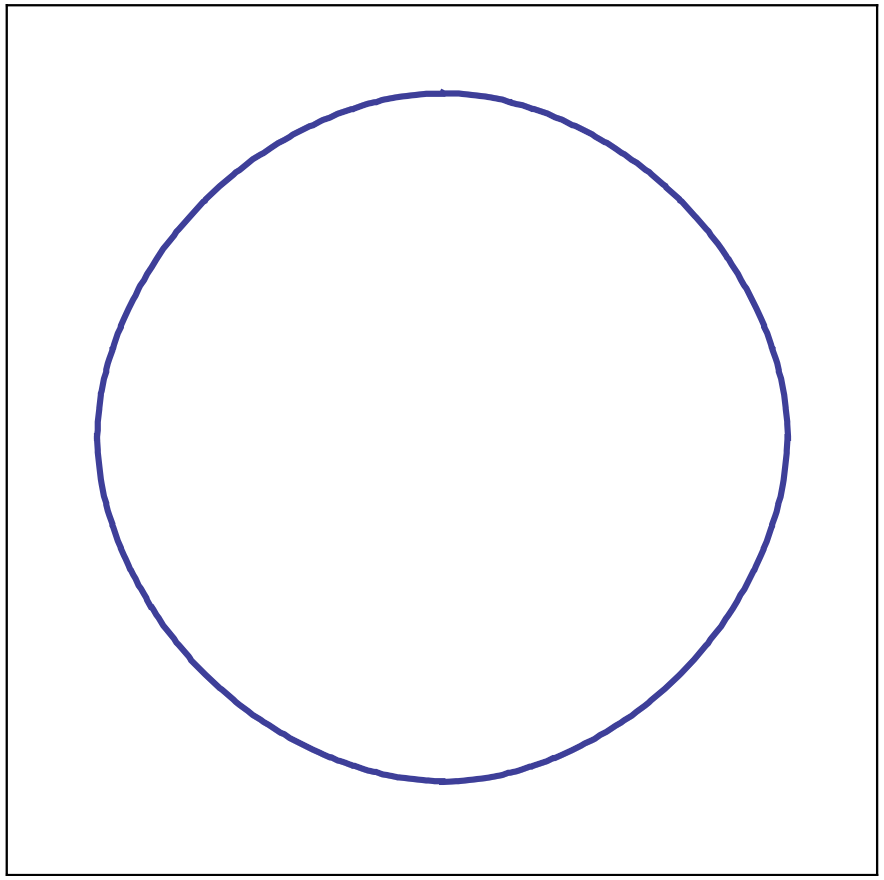
$$H_{\rm sdw} = \lambda \sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^\dagger_{\mathbf{k}+\mathbf{q},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices, the boson momentum $\mathbf{q}$ is small, while the fermion momenum $\mathbf{k}$ extends over the entire Brillouin zone. In the antiferromagnetically ordered state, we may take $\vec{\varphi} \propto (0,0,1)$, and the electron dispersions obtained by diagonalizing $H_0 + H_{\rm sdw}$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \lambda^2 |\vec{\varphi}|^2}$$

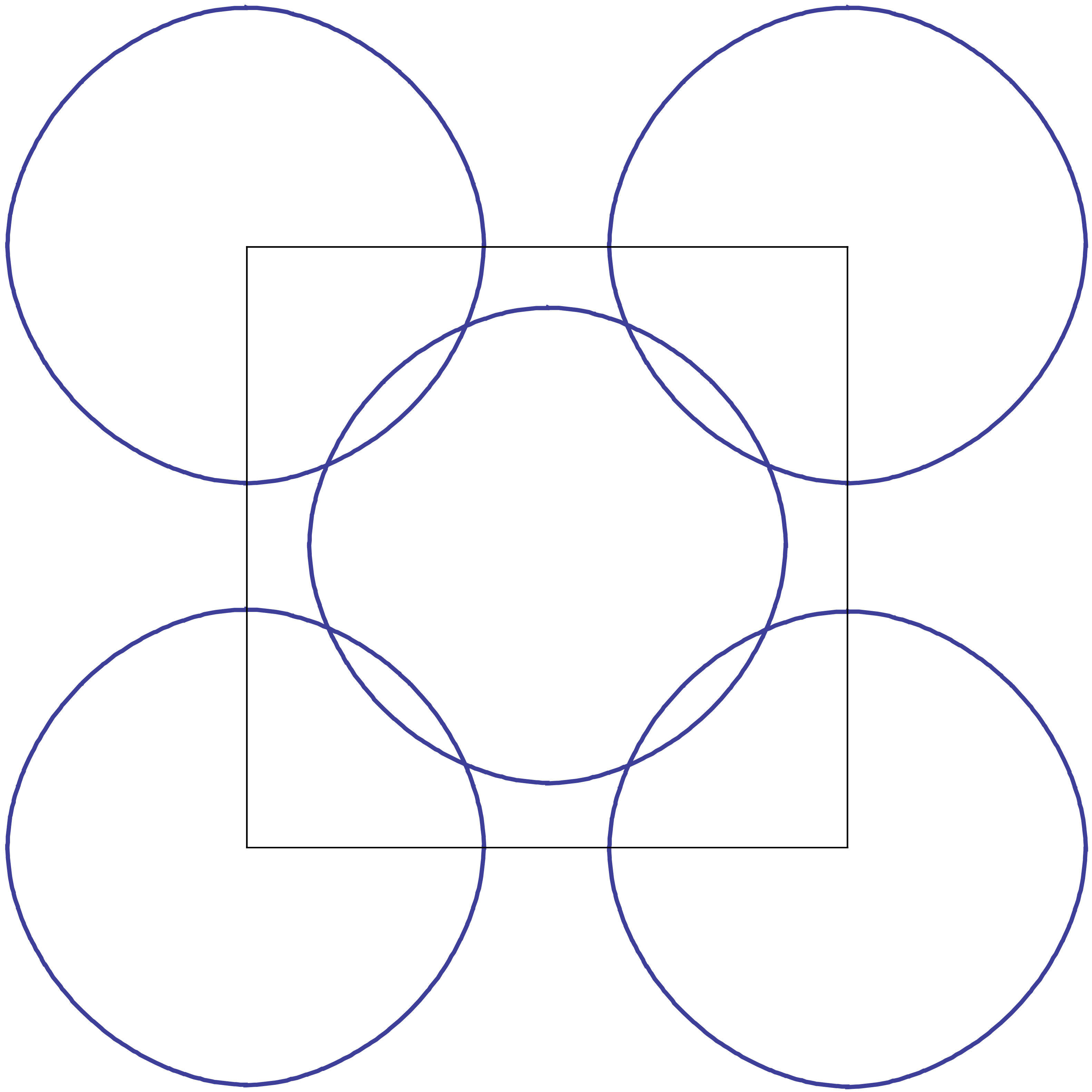
This leads to the Fermi surfaces shown in the following slides as a function of increasing $|\vec{\varphi}|$.

Fermi surface+antiferromagnetism



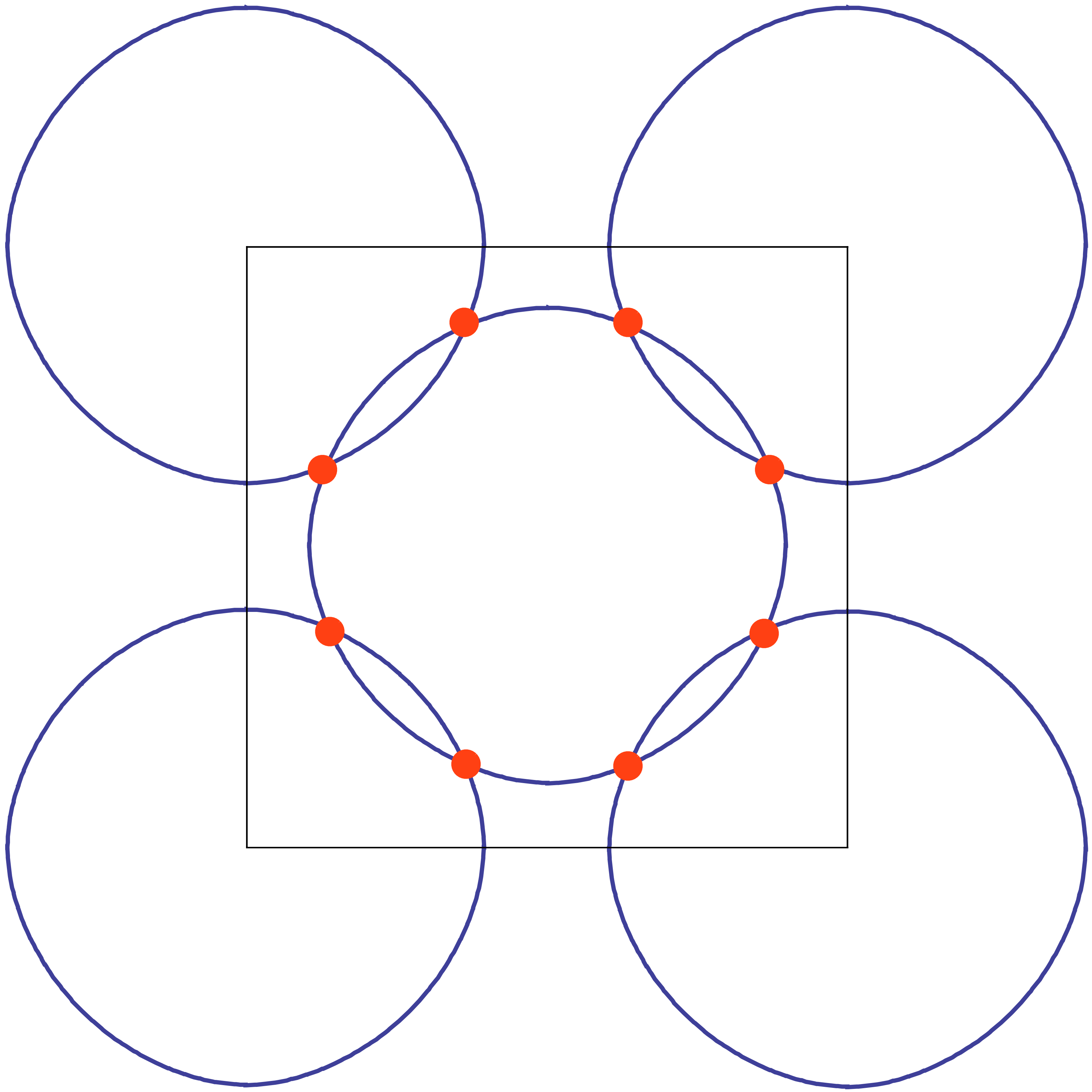
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



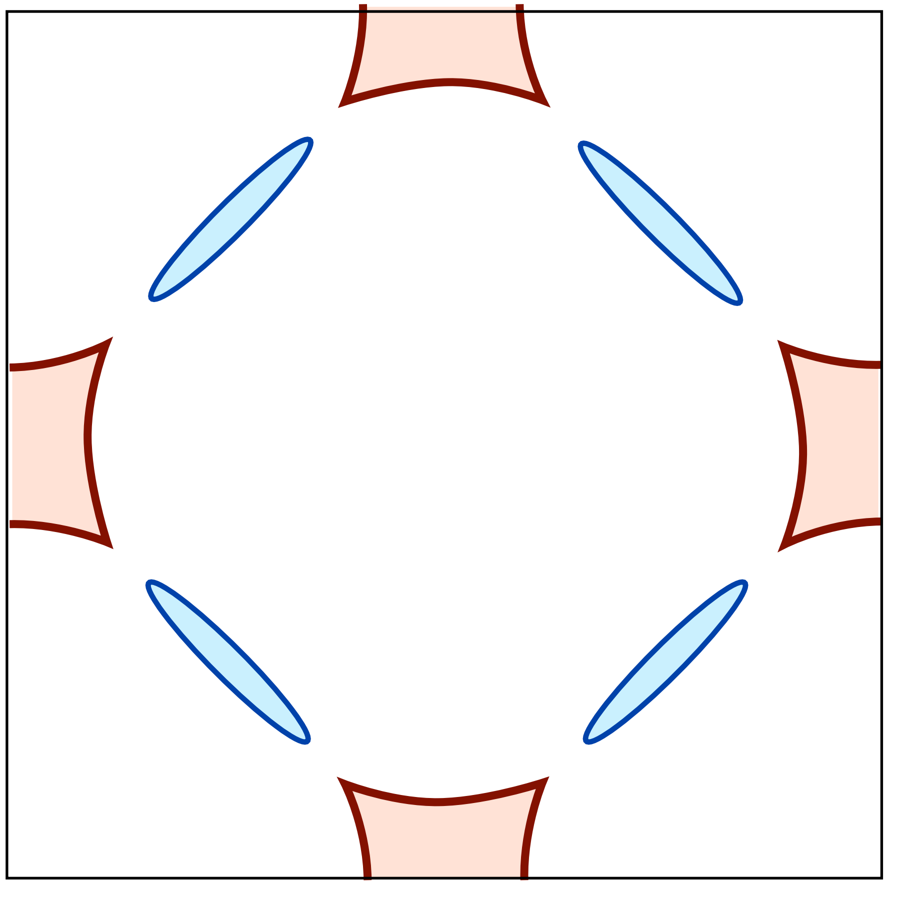
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



“Hot” spots

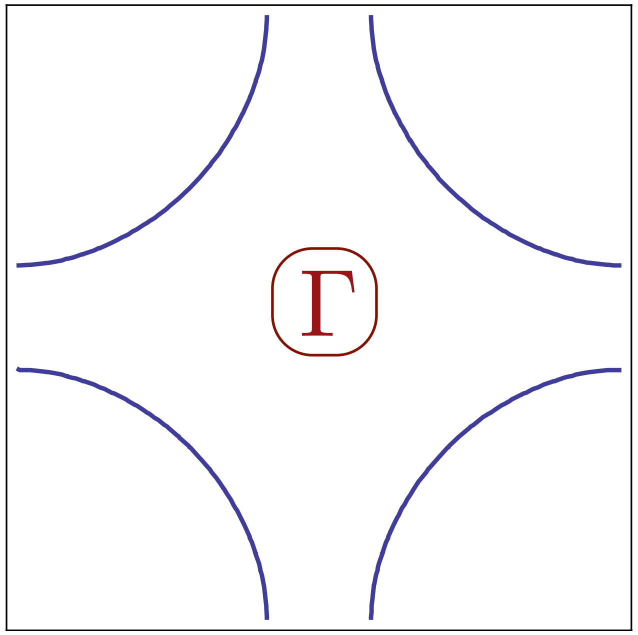
Fermi surface+antiferromagnetism



Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Square lattice Hubbard model with hole doping

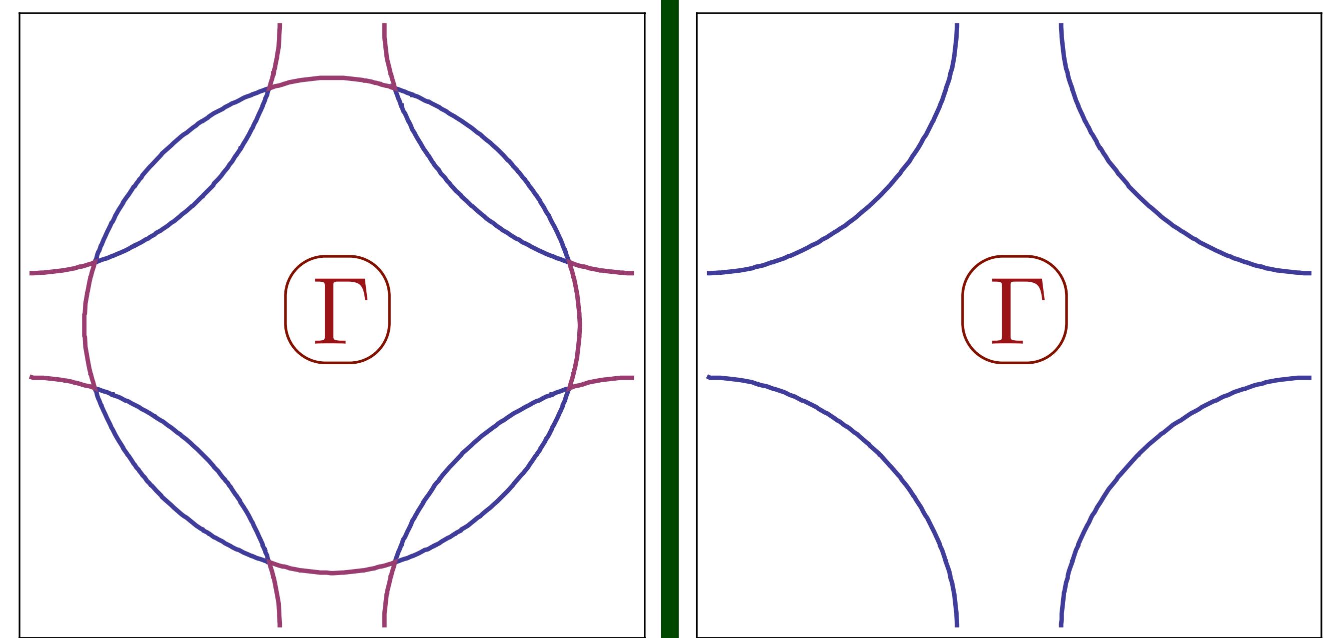
⟵ Increasing SDW order ⟶



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Square lattice Hubbard model with hole doping

Increasing SDW order



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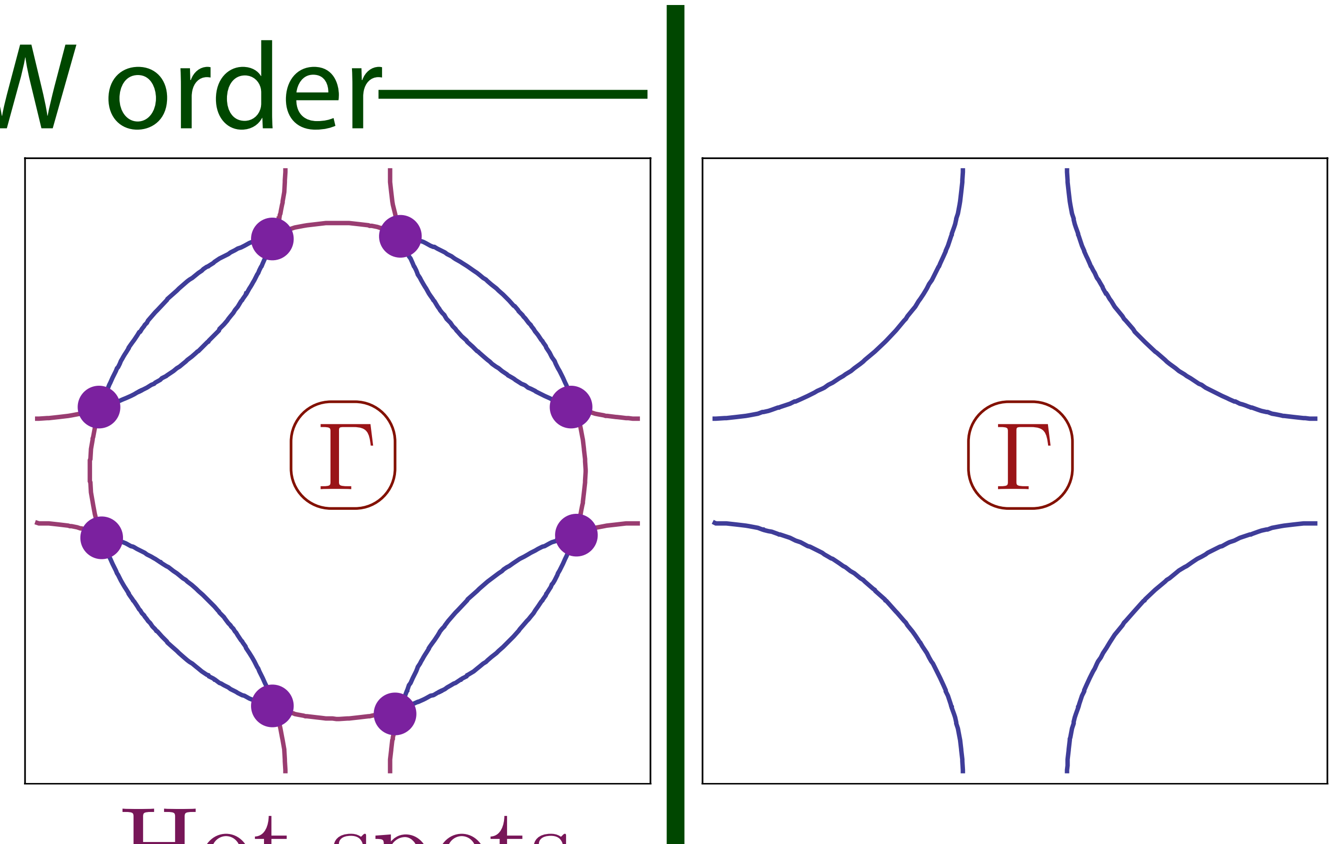
Square lattice Hubbard model with hole doping

← Increasing SDW order



Hot spots

where $\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}+\mathbf{K}}$

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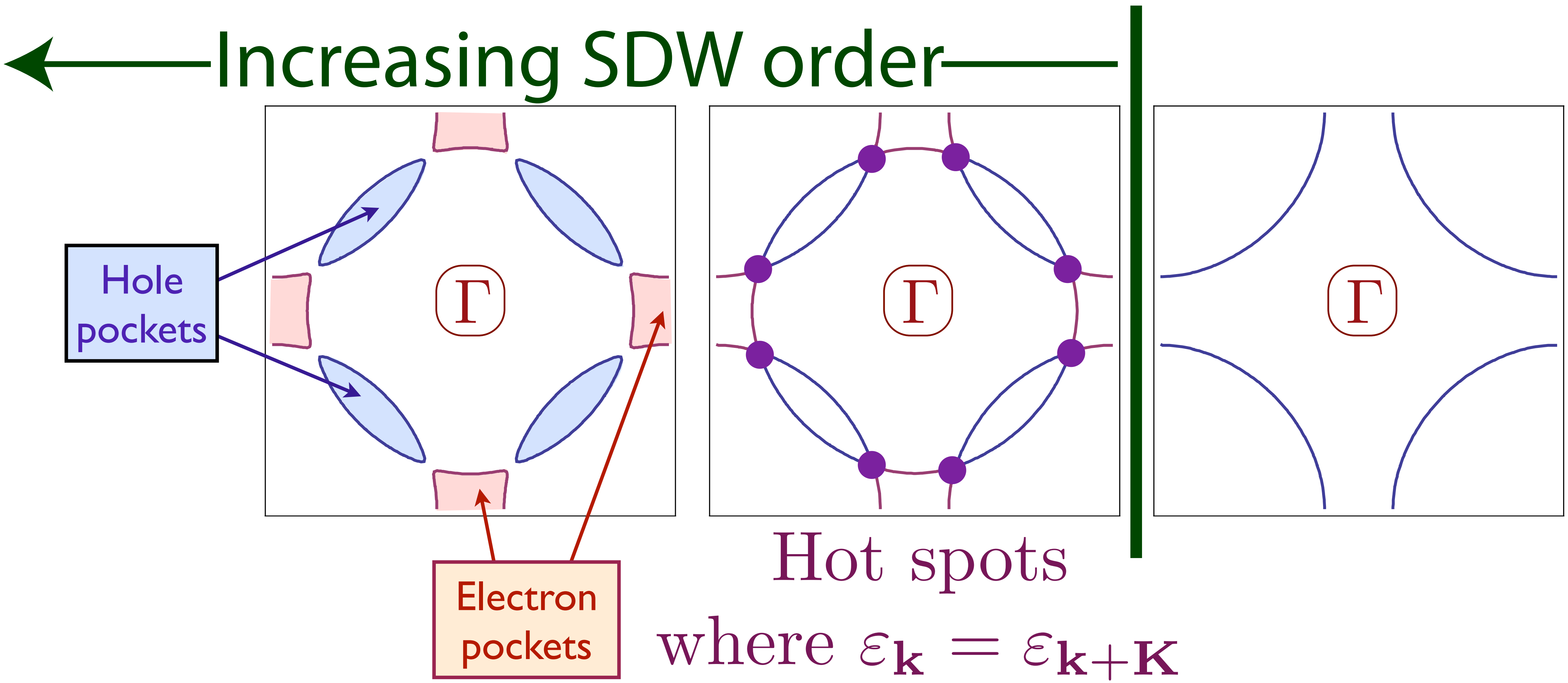
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Square lattice Hubbard model with hole doping



Fermi surface breaks up at hot spots into electron and hole "pockets"

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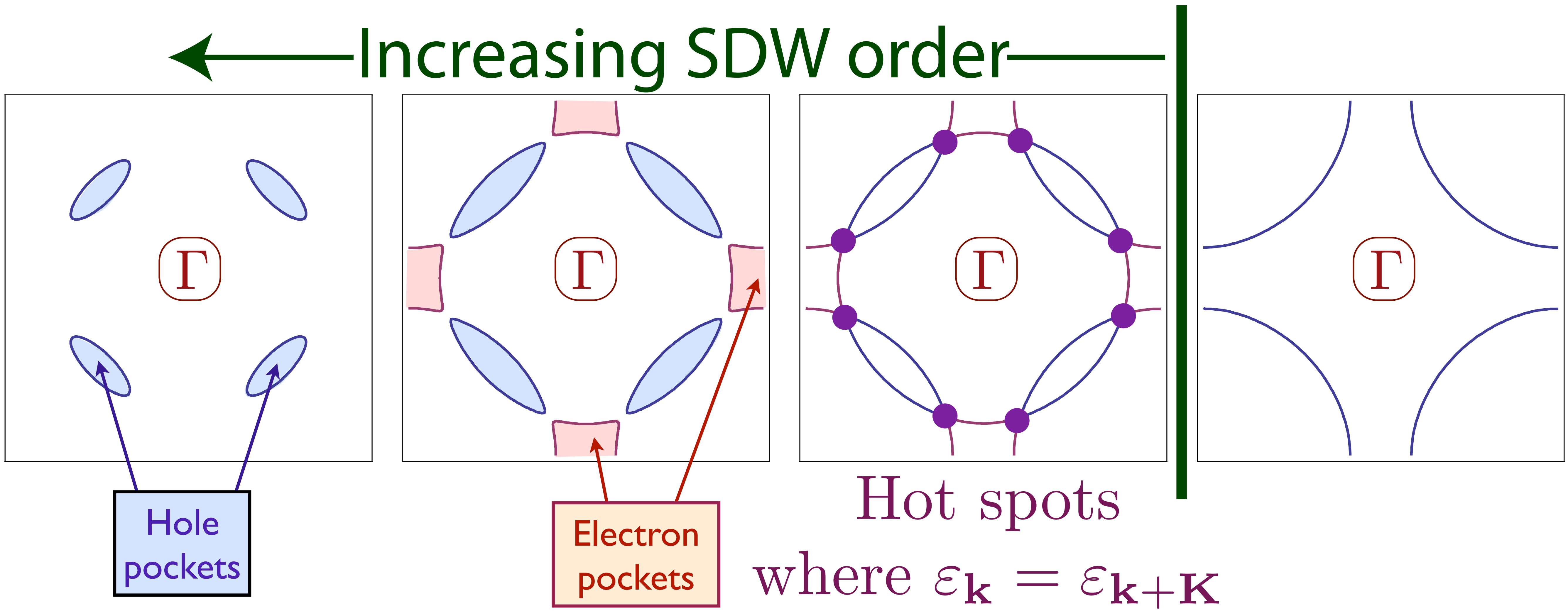
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Square lattice Hubbard model with hole doping



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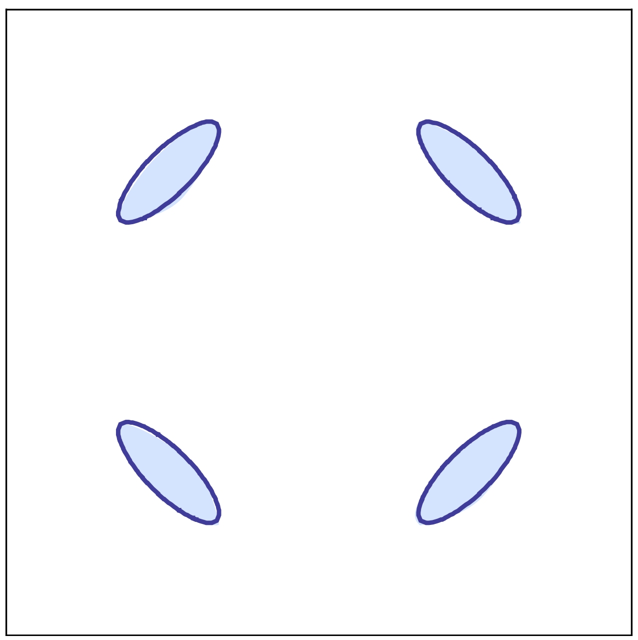
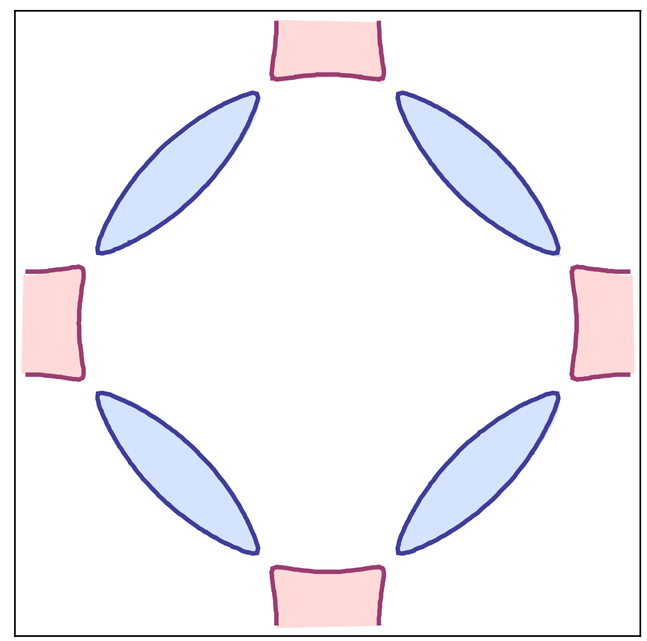
Square lattice Hubbard model with hole doping

$\langle \vec{\varphi} \rangle \neq 0$ and large



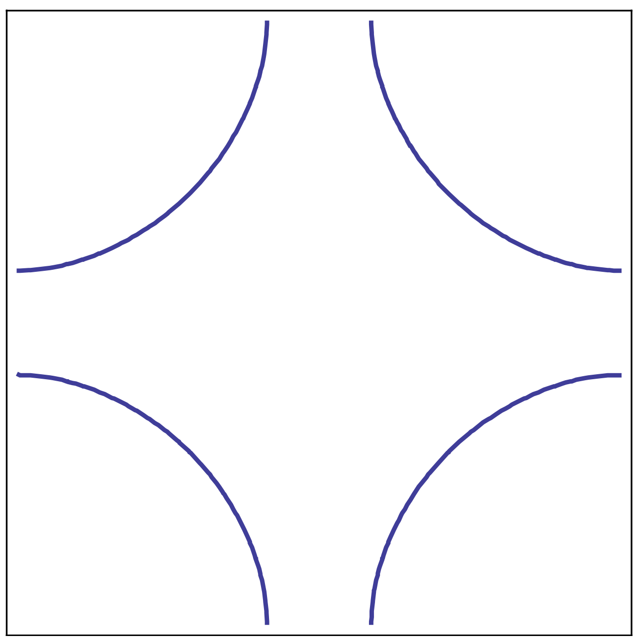
Metal with hole pockets

$\langle \vec{\varphi} \rangle \neq 0$ and small



Metal with electron and hole pockets

$\langle \vec{\varphi} \rangle = 0$

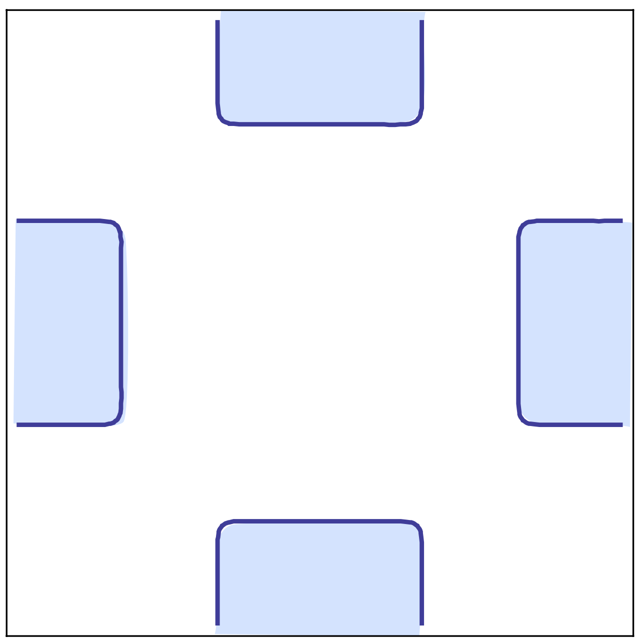


Metal with "large" Fermi surface

s

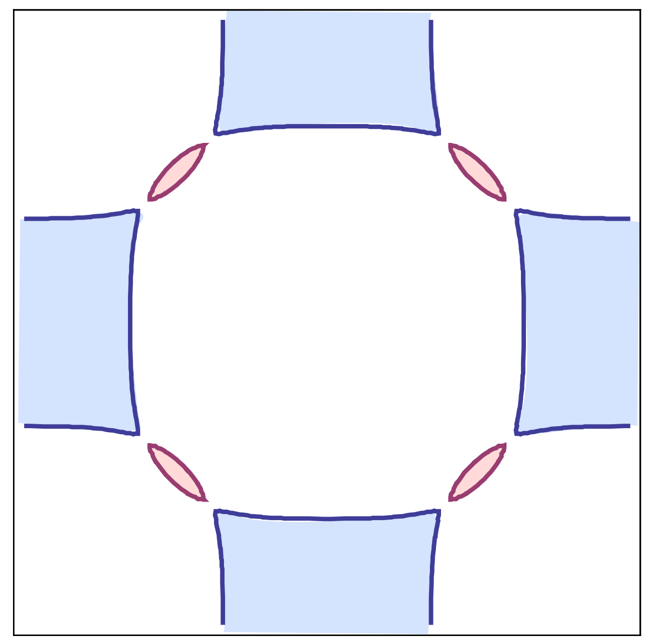
Square lattice Hubbard model with electron doping

$\langle \vec{\varphi} \rangle \neq 0$ and large



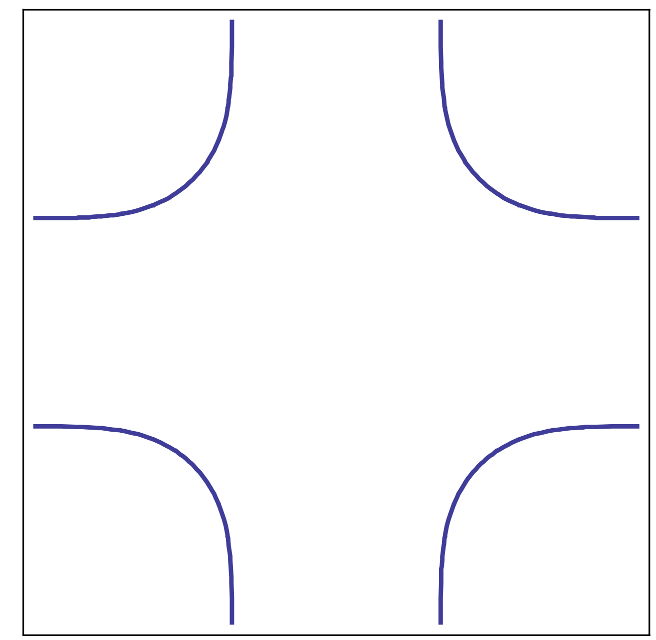
Metal with electron pockets

$\langle \vec{\varphi} \rangle \neq 0$ and small



Metal with electron and hole pockets

$\langle \vec{\varphi} \rangle = 0$



Metal with "large" Fermi surface

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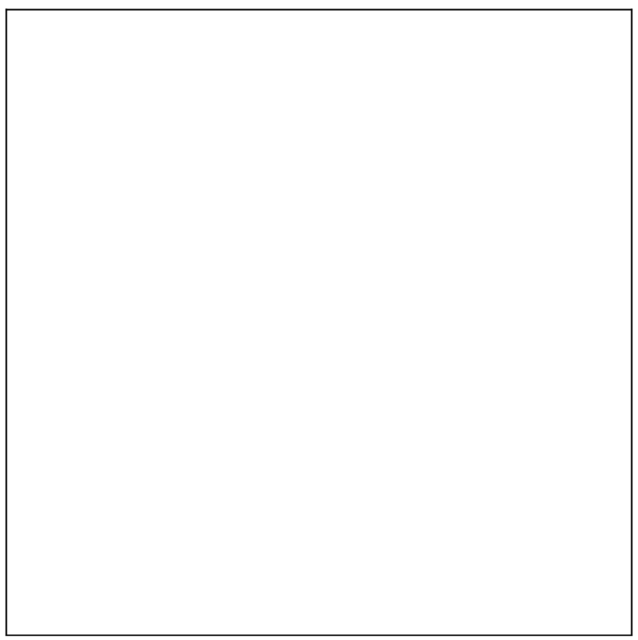
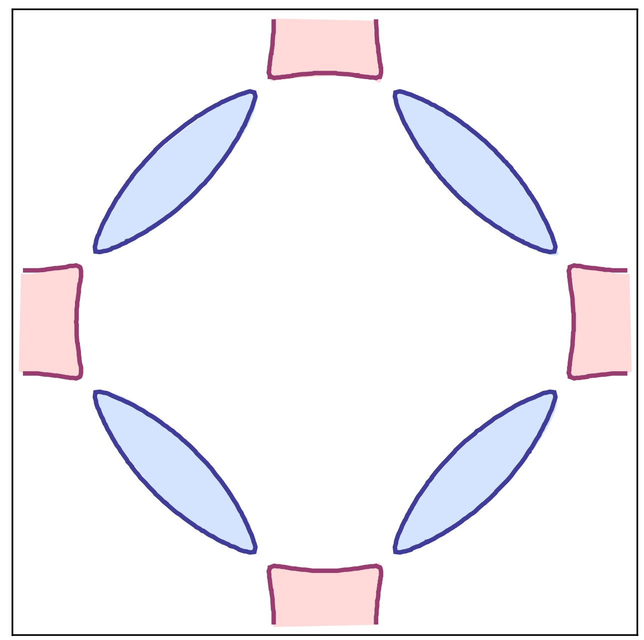
Square lattice Hubbard model with no doping

$\langle \vec{\varphi} \rangle \neq 0$ and large



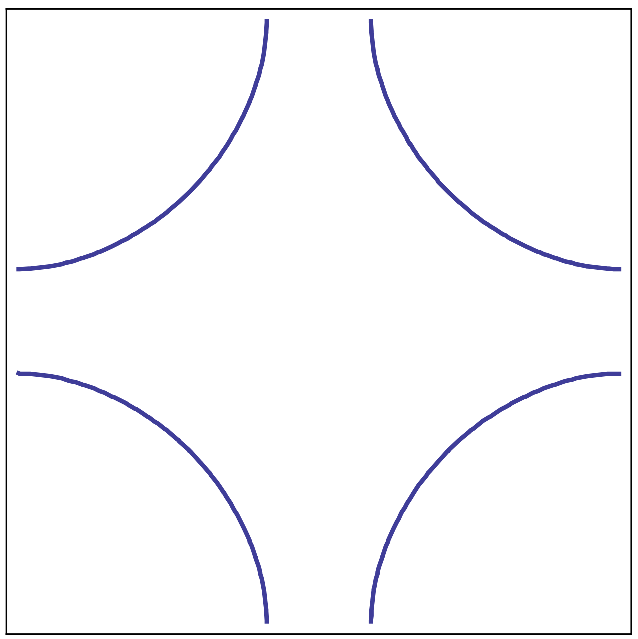
Insulator

$\langle \vec{\varphi} \rangle \neq 0$ and small



Metal with electron and hole pockets

$\langle \vec{\varphi} \rangle = 0$



Metal with "large" Fermi surface

s

Spin-fluctuation exchange theory of d-wave superconductivity

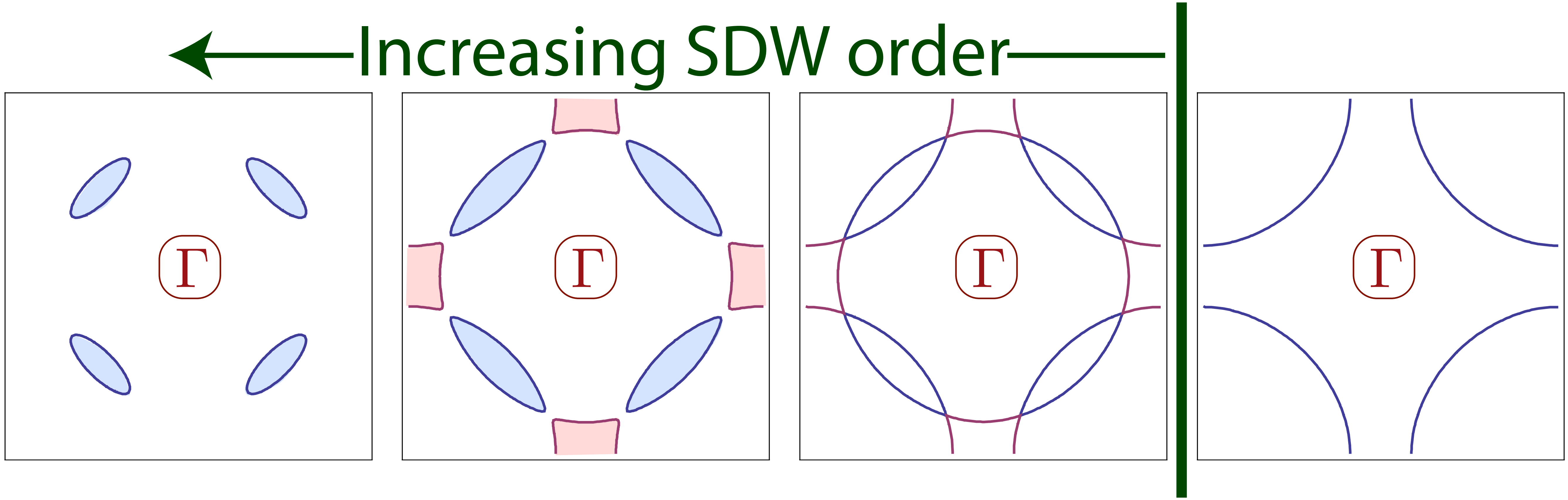
d-wave pairing near a spin-density-wave instability

D. J. Scalapino, E. Loh, Jr.,* and J. E. Hirsch†
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 23 June 1986)

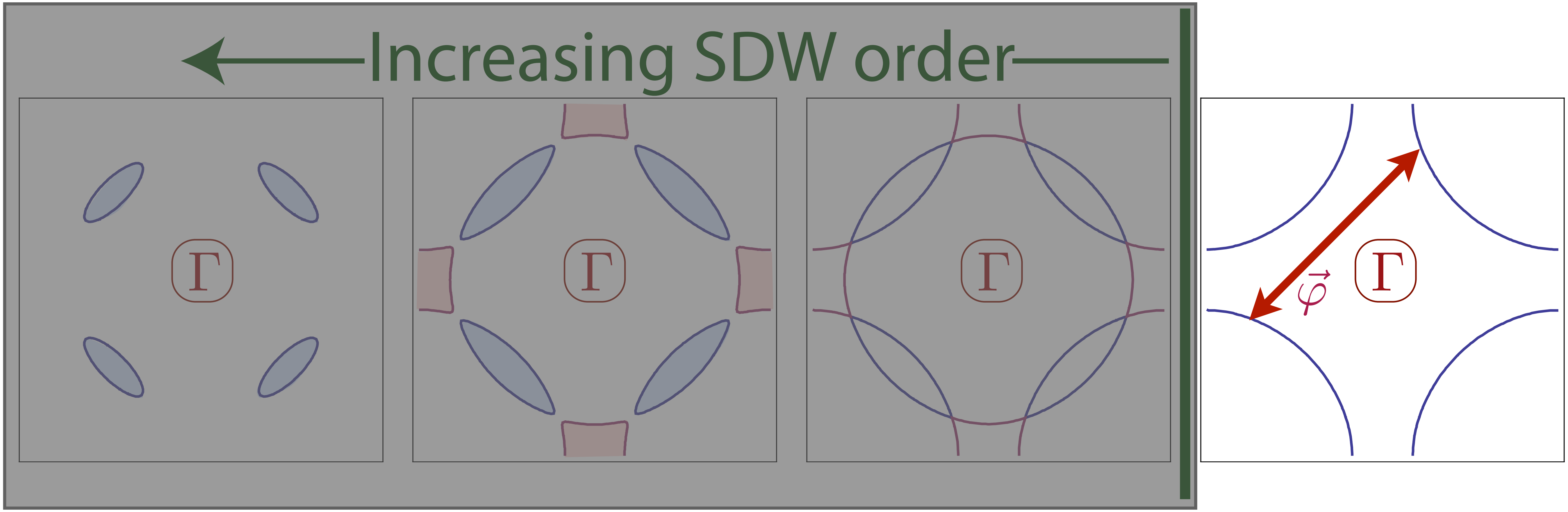
We investigate the three-dimensional Hubbard model and show that paramagnon exchange near a spin-density-wave instability gives rise to a strong singlet d-wave pairing interaction. For a cubic band the singlet ($d_{x^2-y^2}$ and $d_{3z^2-r^2}$) channels are enhanced while the singlet (d_{xy}, d_{xz}, d_{yz}) and triplet p-wave channels are suppressed. A unique feature of this pairing mechanism is its sensitivity to band structure and band filling.

Physical Review B **34**, 8190 (1986)

Spin-fluctuation exchange theory of d-wave superconductivity



Spin-fluctuation exchange theory of d-wave superconductivity



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

Spin-fluctuation exchange theory of d-wave superconductivity

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^\dagger_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2}\sum_{\mathbf{q}}\sum_{\mathbf{p},\gamma,\delta}\sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^\dagger_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^\dagger_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q} - \mathbf{K})^2},$$

with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

Spin-fluctuation exchange theory of d-wave superconductivity

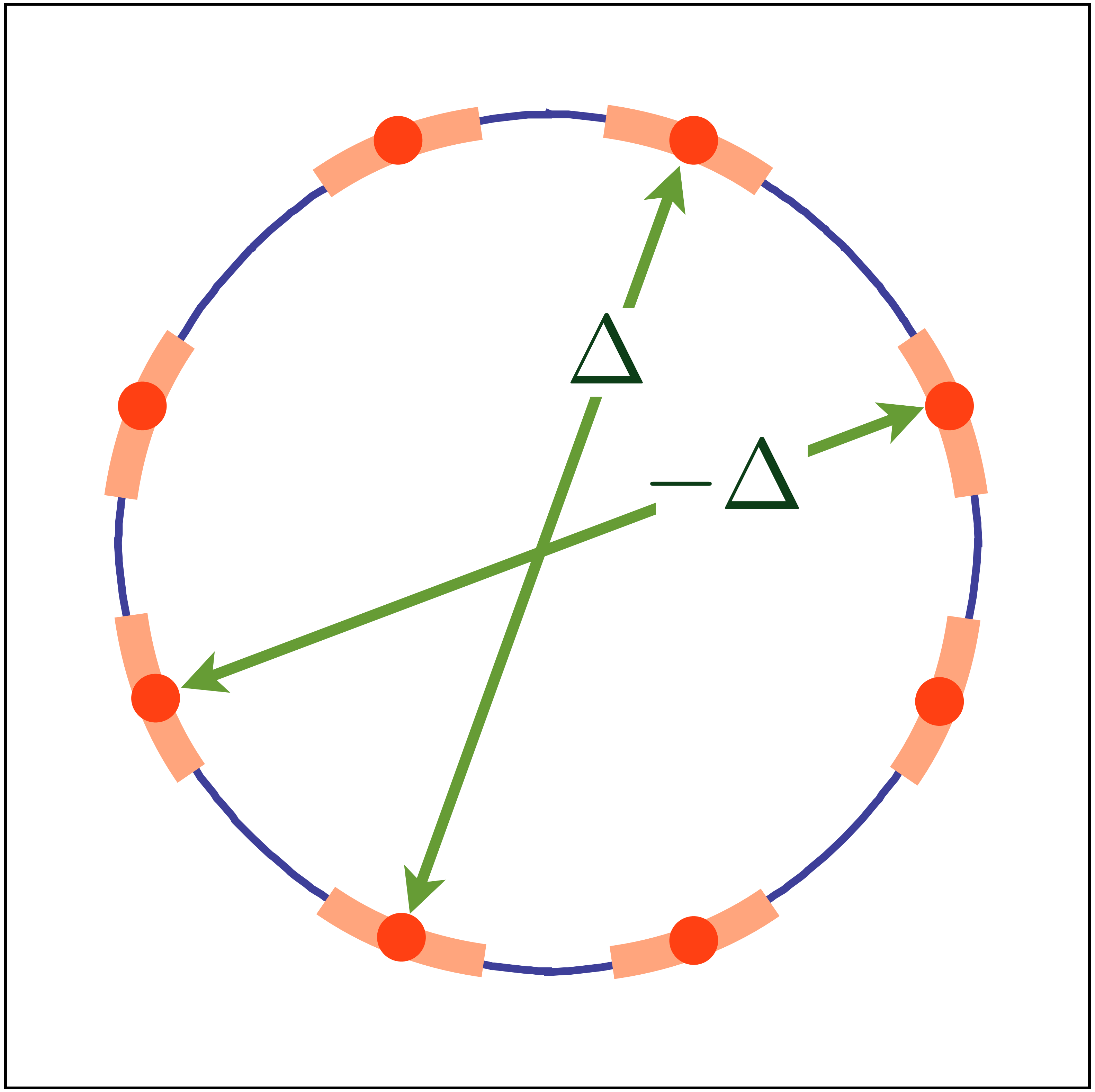
BCS Gap equation

In BCS theory, this interaction leads to the 'gap equation' for the pairing gap $\Delta_{\mathbf{k}} \propto \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$.

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{p}} \left(\frac{3\chi_0}{\xi^{-2} + (\mathbf{p} - \mathbf{k} - \mathbf{K})^2} \right) \frac{\Delta_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_{\mathbf{p}}^2}}$$

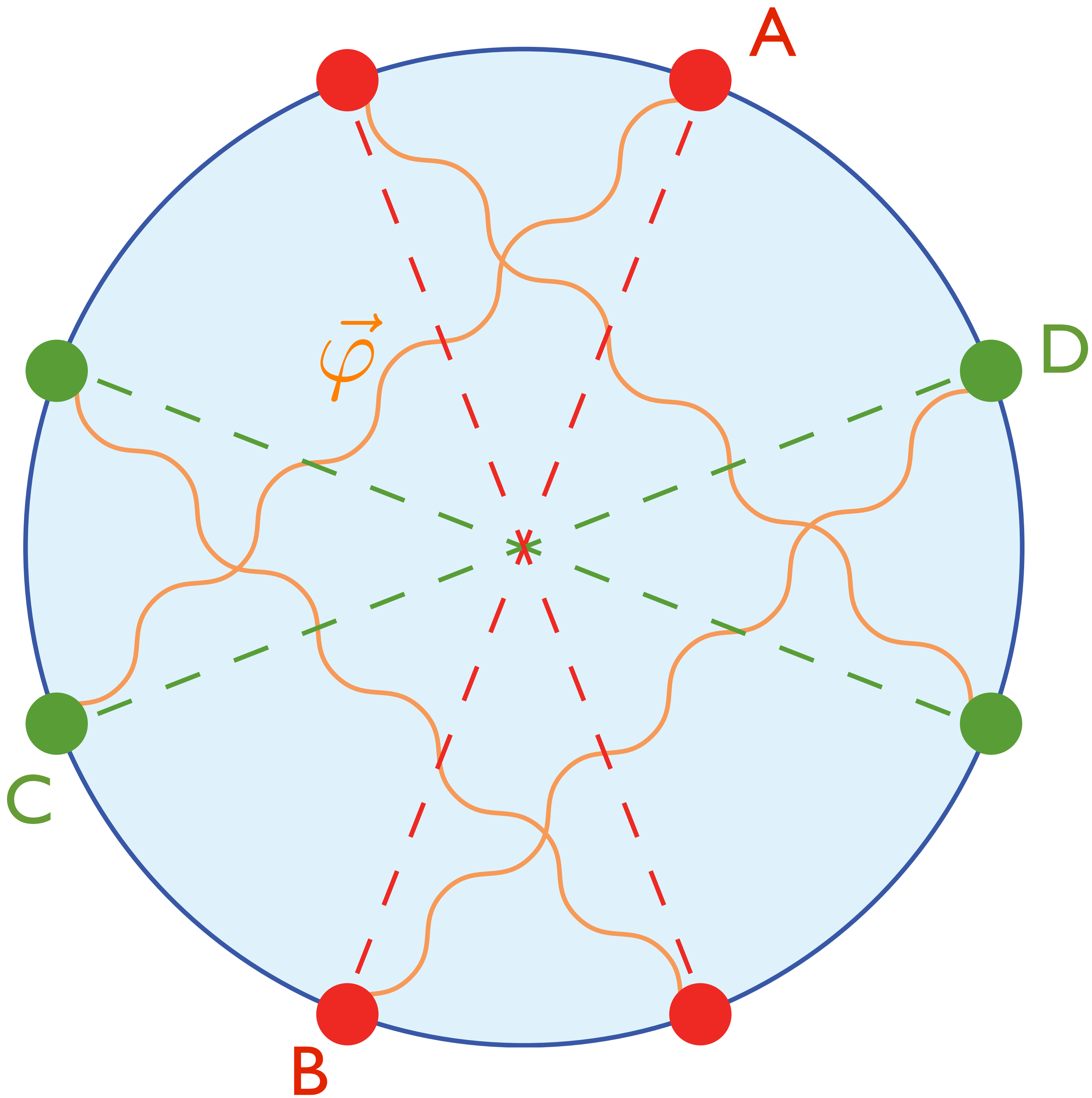
Non-zero solutions of this equation require that $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{p}}$ have opposite signs when $\mathbf{p} - \mathbf{k} \approx \mathbf{K}$.

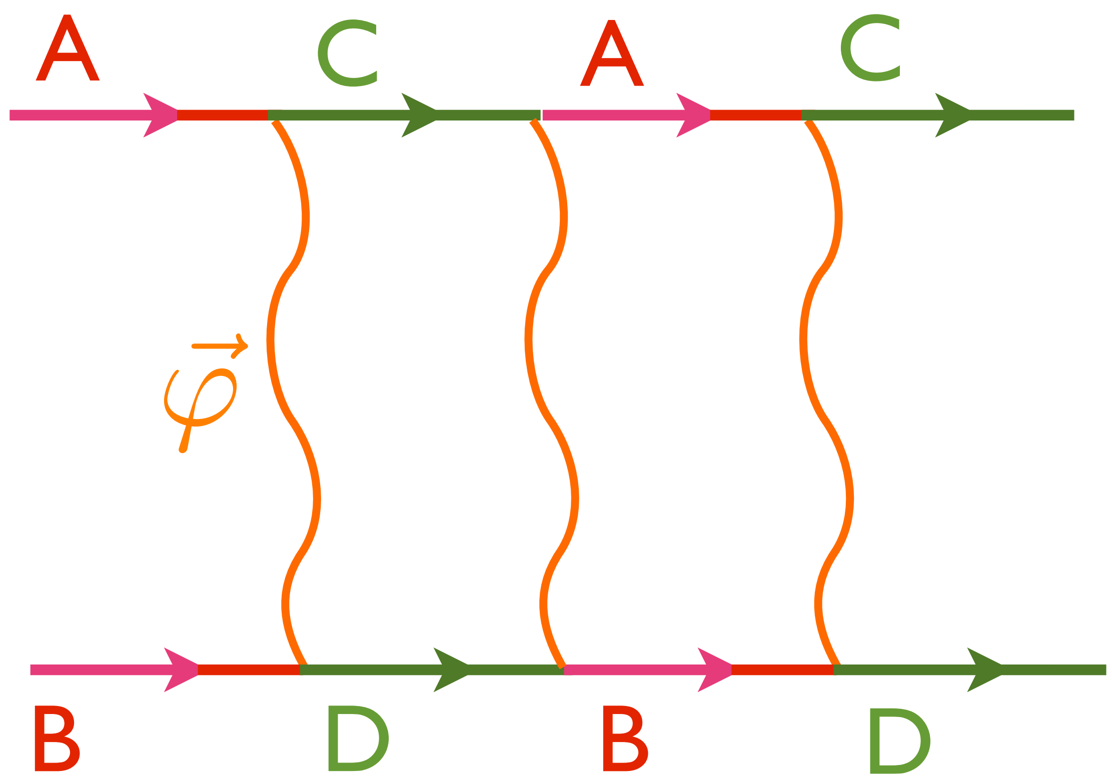
$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

Pairing "glue" from antiferromagnetic fluctuations





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D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
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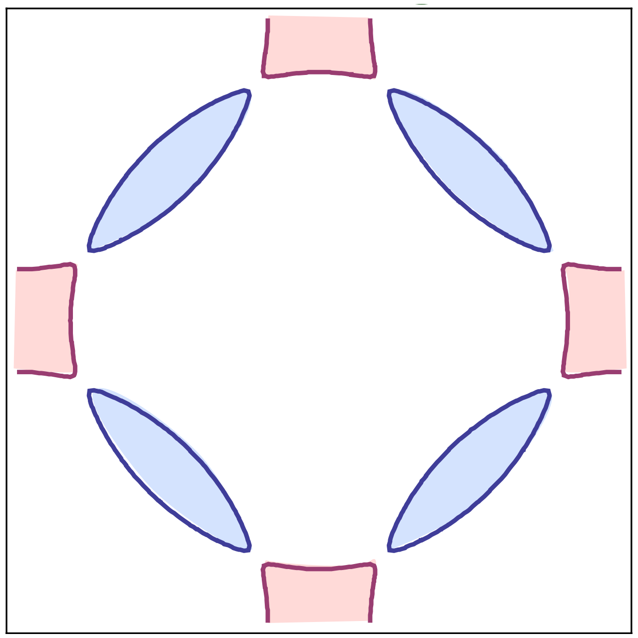
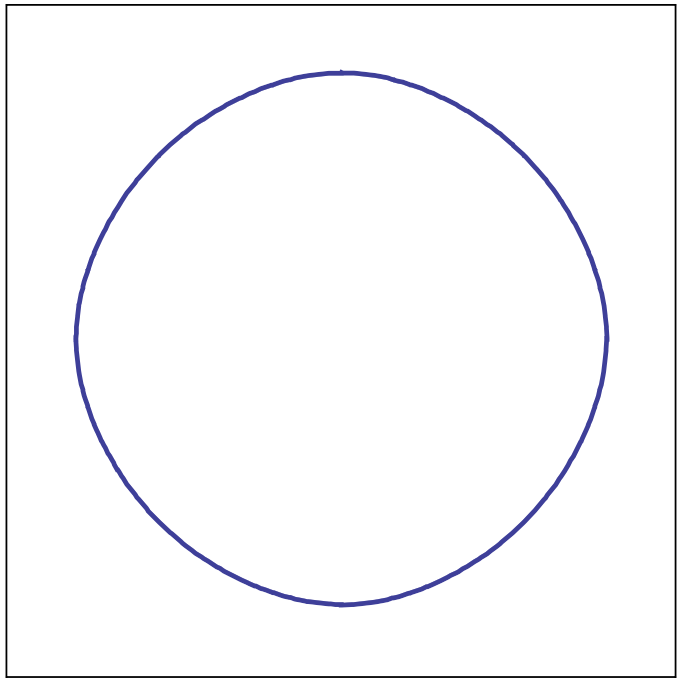
Quantum phase transition with onset of antiferromagnetism in a metal



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

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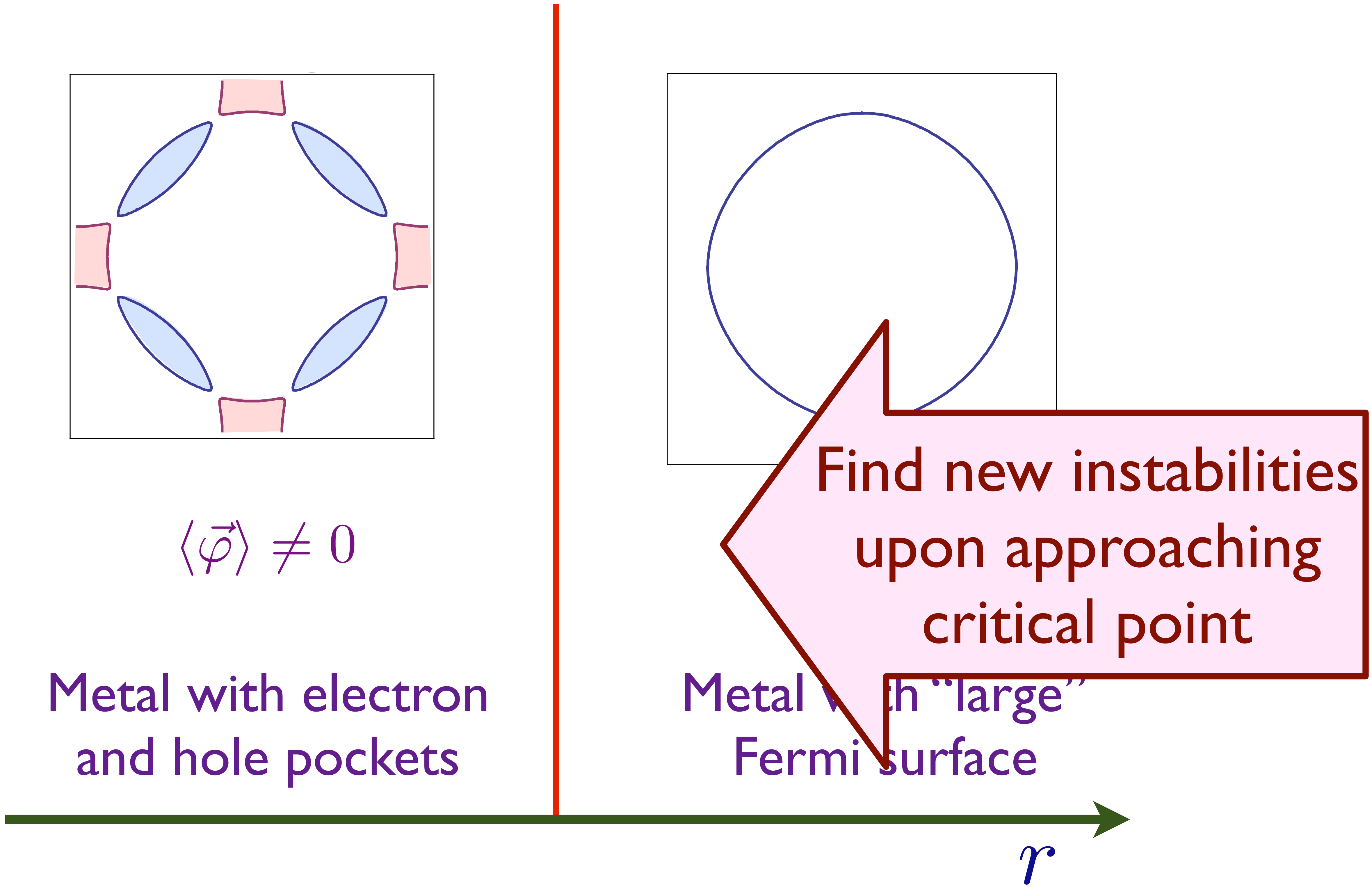
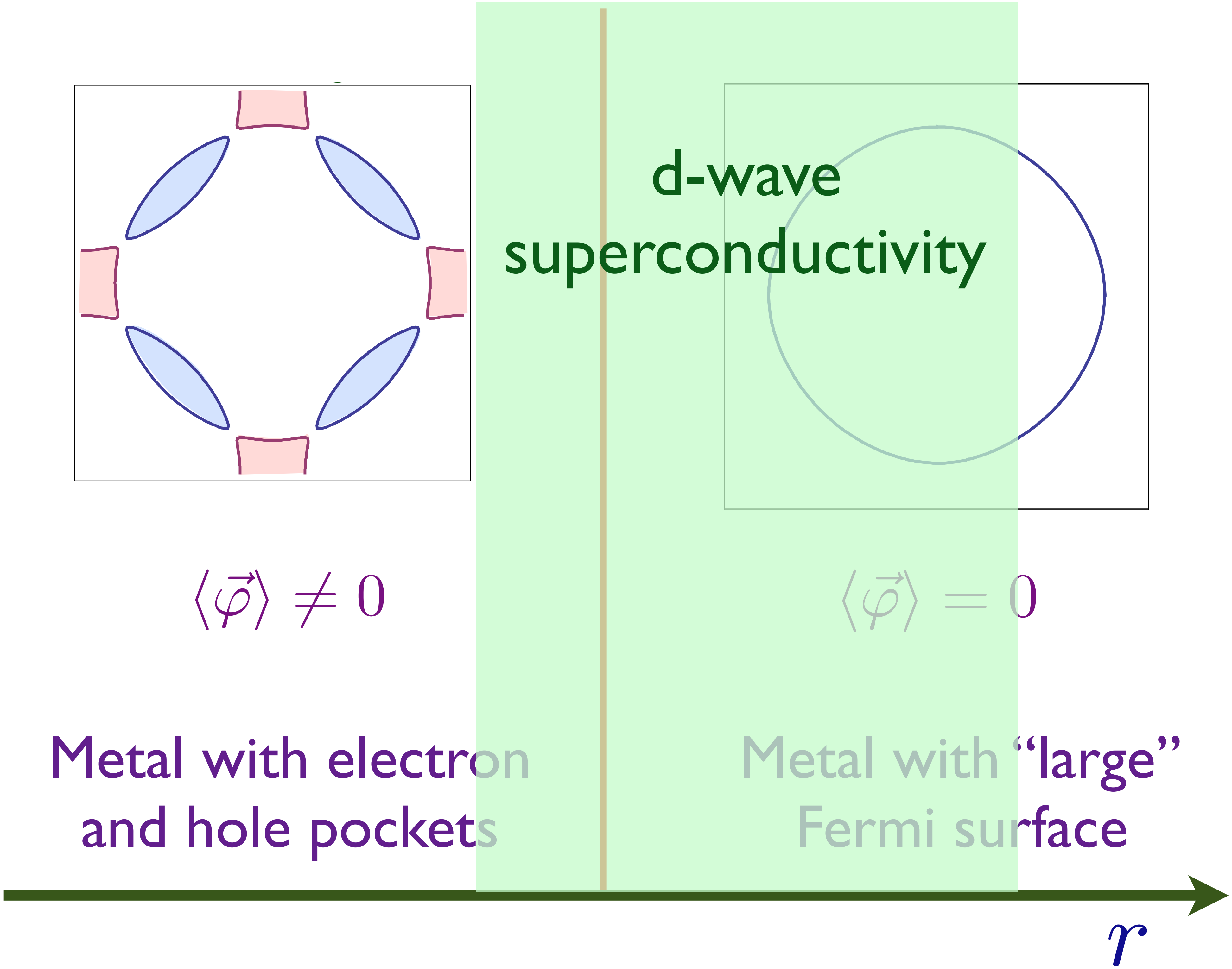
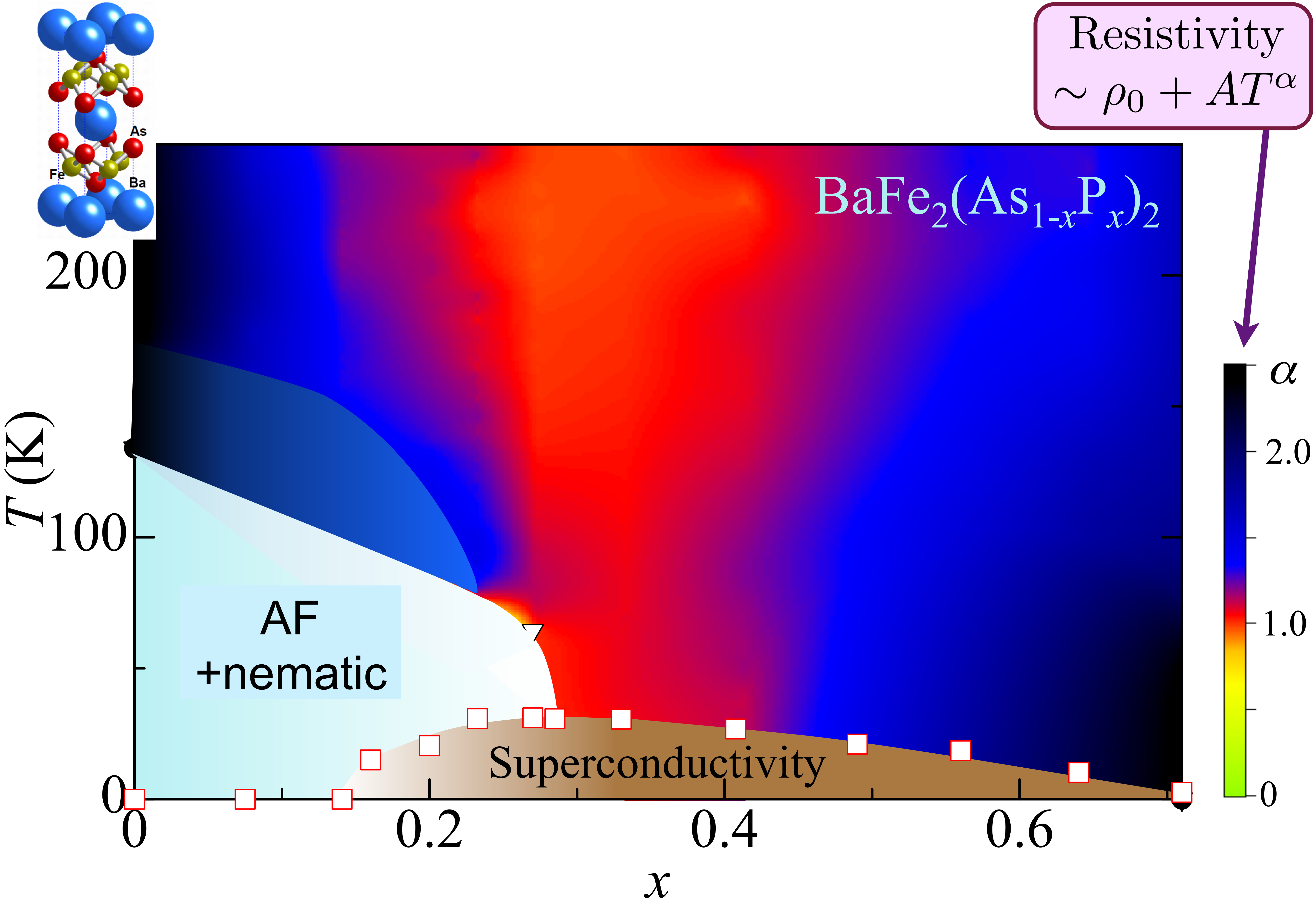
Quantum phase transition with onset of antiferromagnetism in a metal



Quantum phase transition with onset of antiferromagnetism in a metal

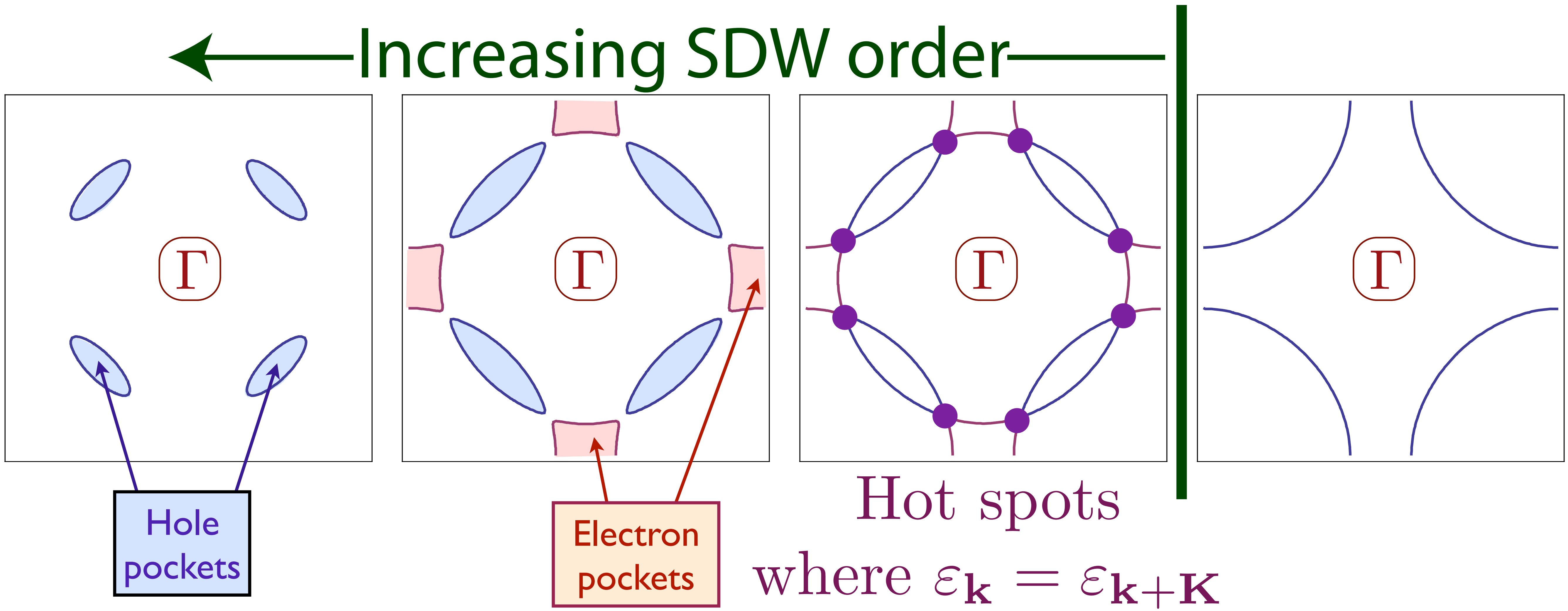




S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Sign-problem free quantum Monte Carlo for the onset of antiferromagnetism in metals

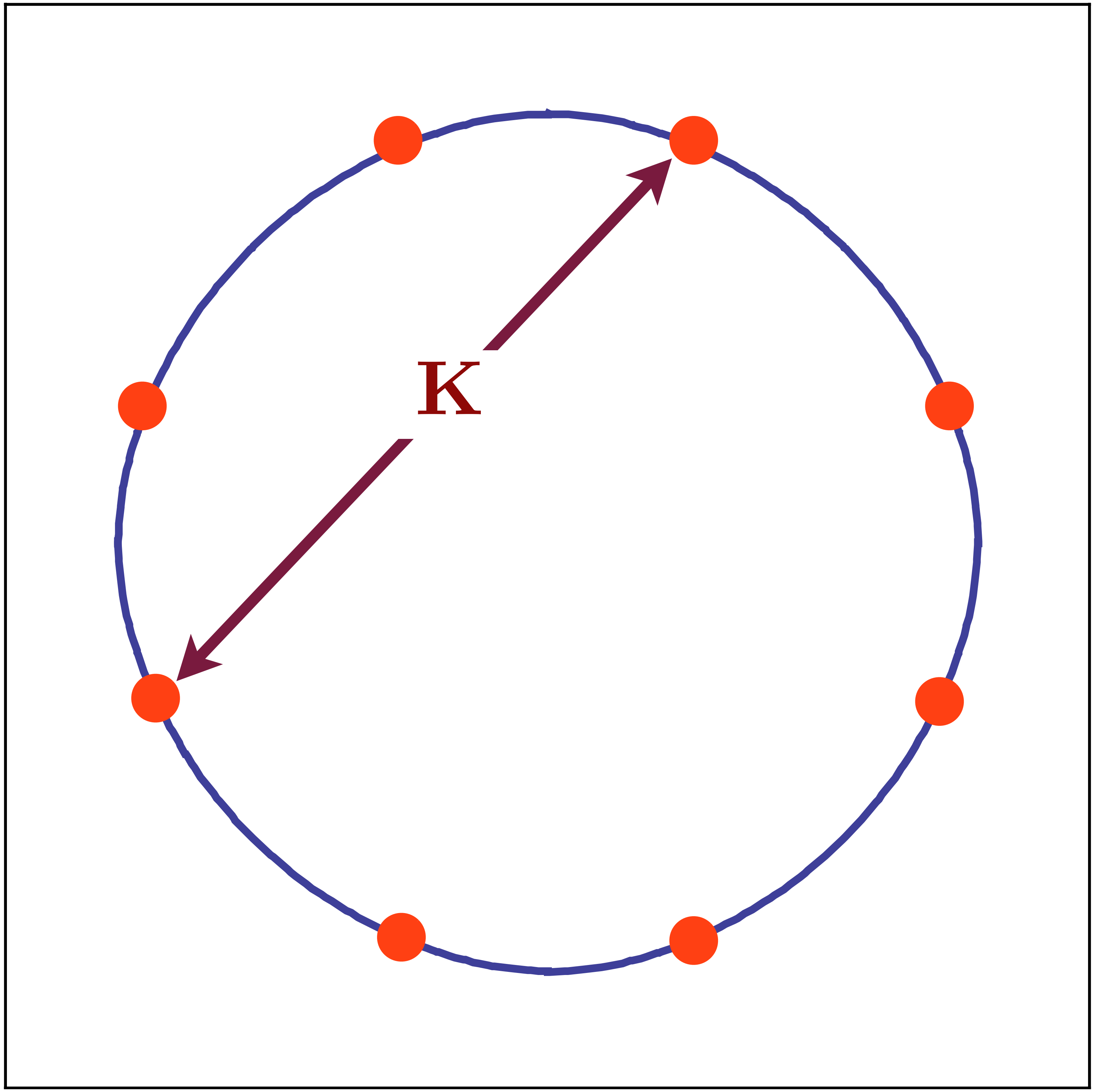
Square lattice Hubbard model with hole doping



Fermi surface breaks up at hot spots into electron and hole "pockets"

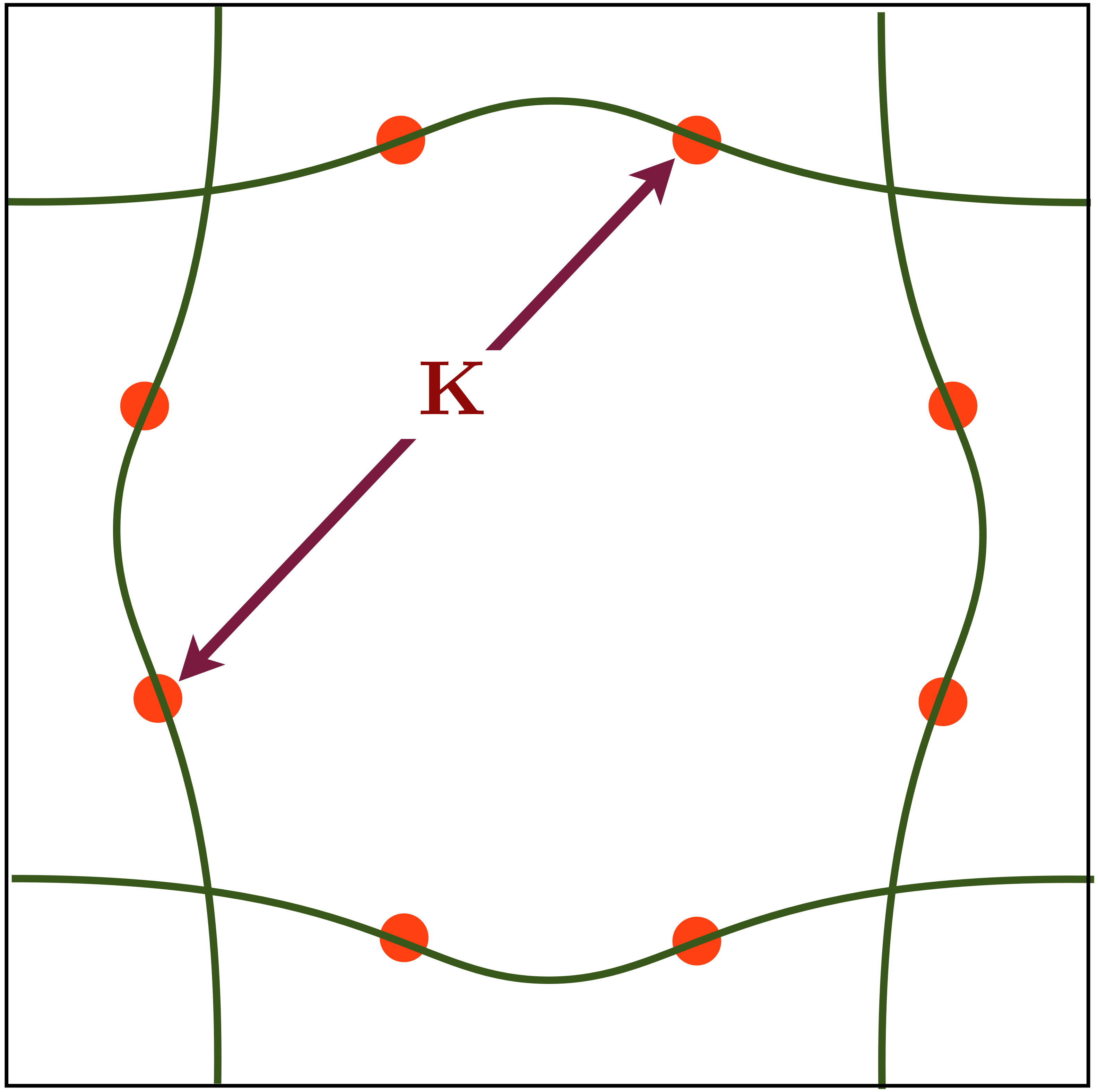
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

QMC for the onset of antiferromagnetism



Hot spots in a single band model

QMC for the onset of antiferromagnetism

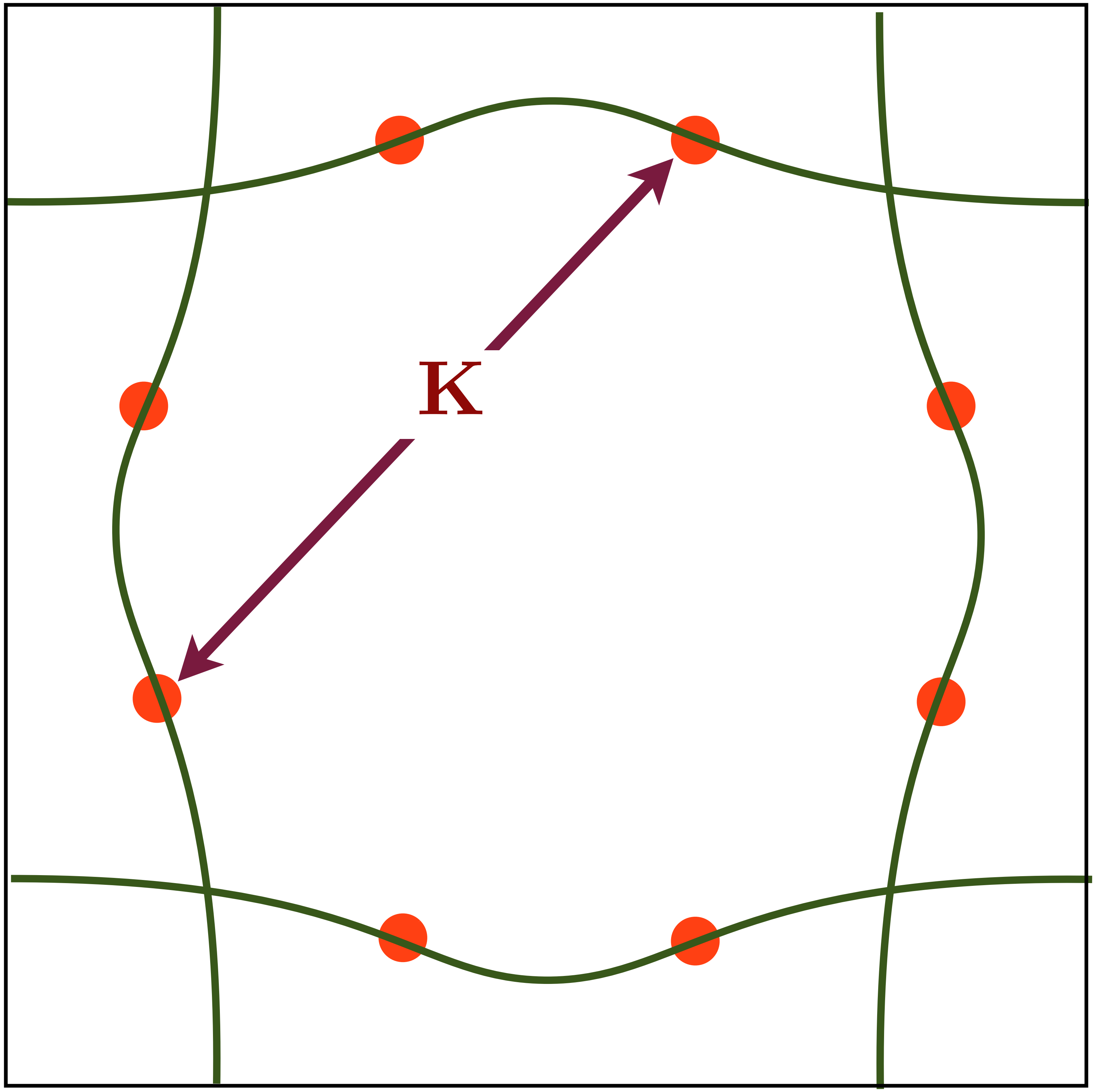


Hot spots in a two band model

E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
(2012).

QMC for the onset of antiferromagnetism

Faithful realization of the *generic* universal low energy theory for the onset of antiferromagnetism.

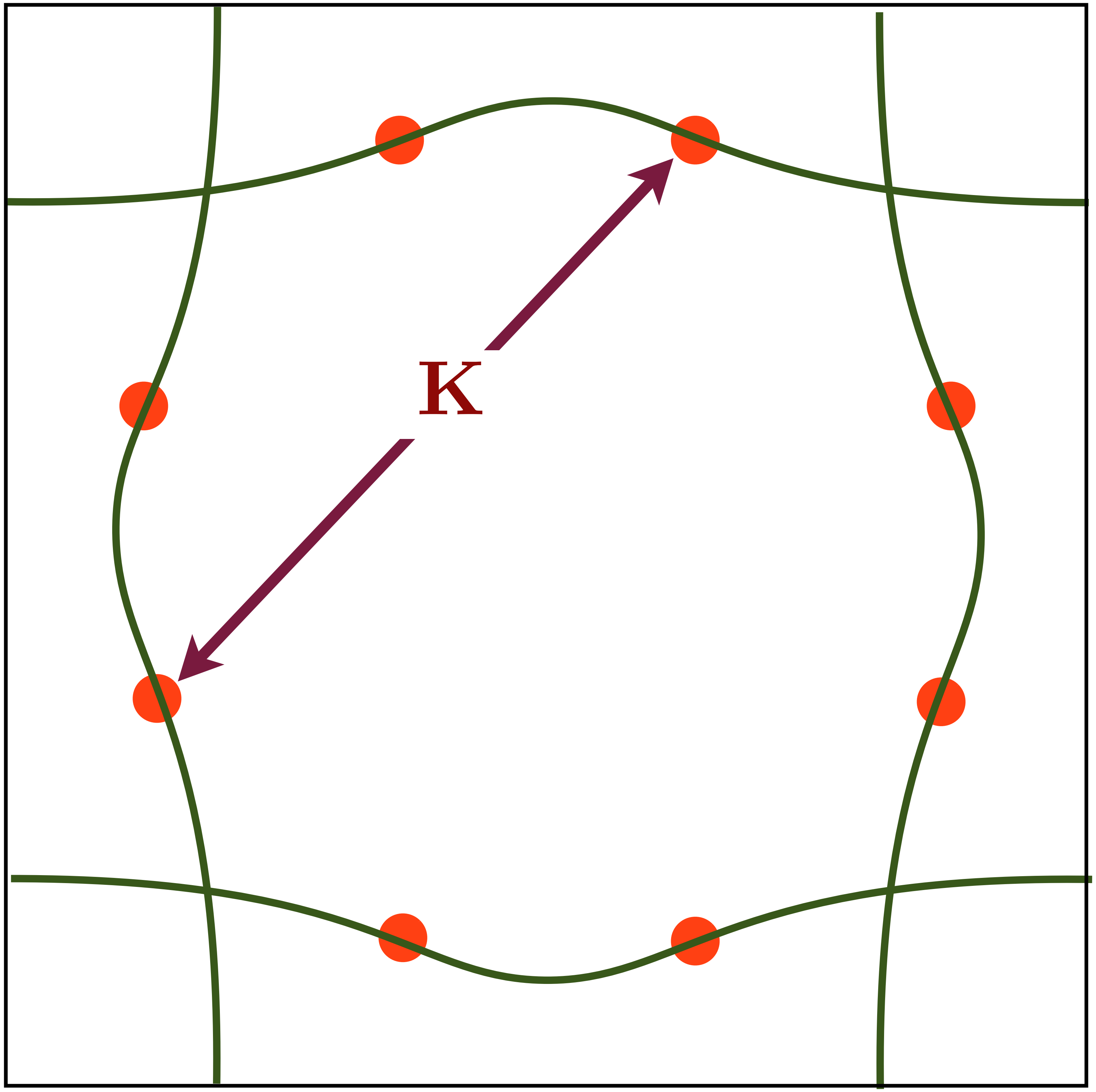


Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism

Sign problem is absent as long as K connects hotspots in distinct bands



Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.}$$

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

No sign problem !

QMC for the onset of antiferromagnetism

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$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

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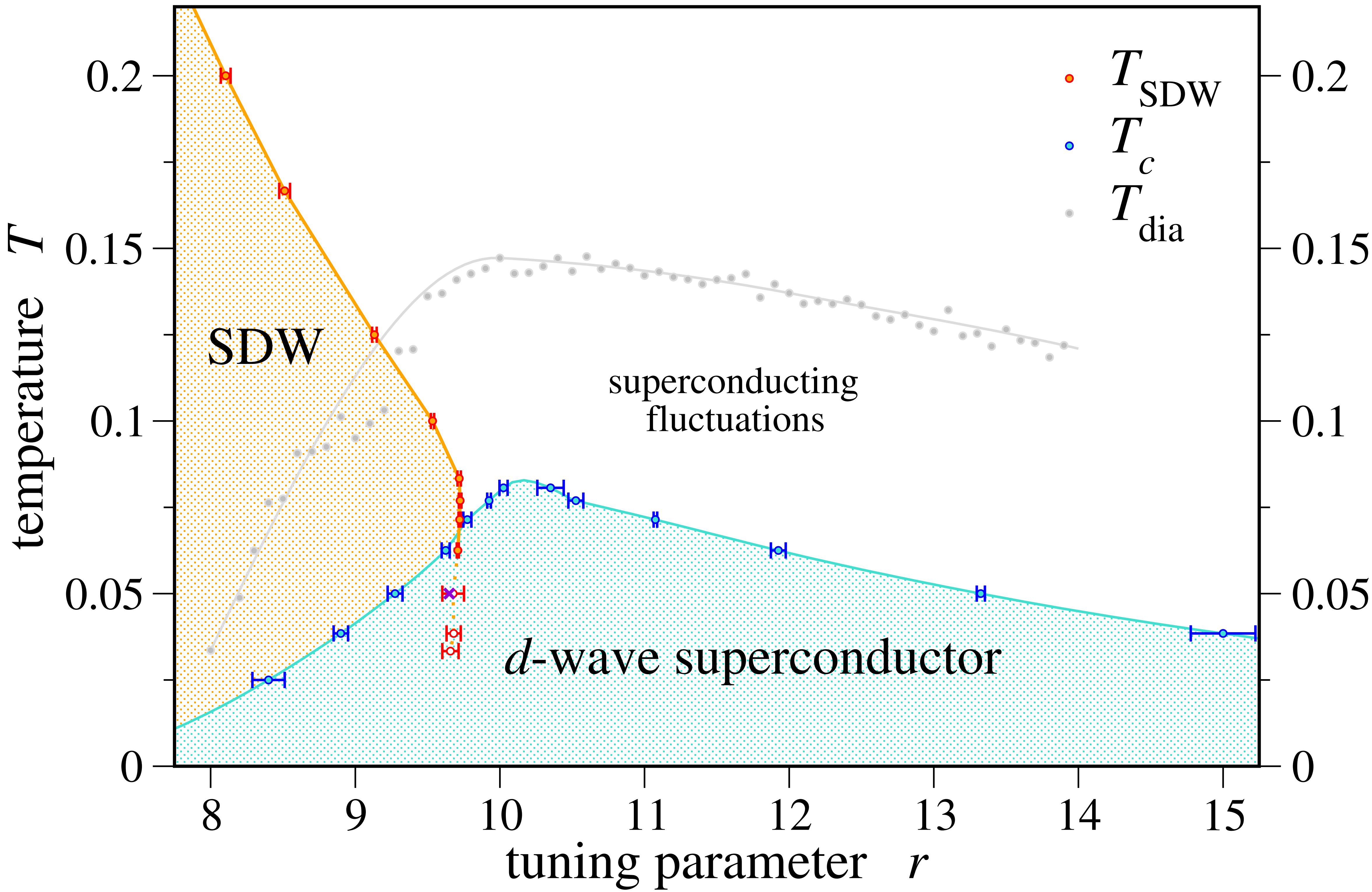
$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.}$$

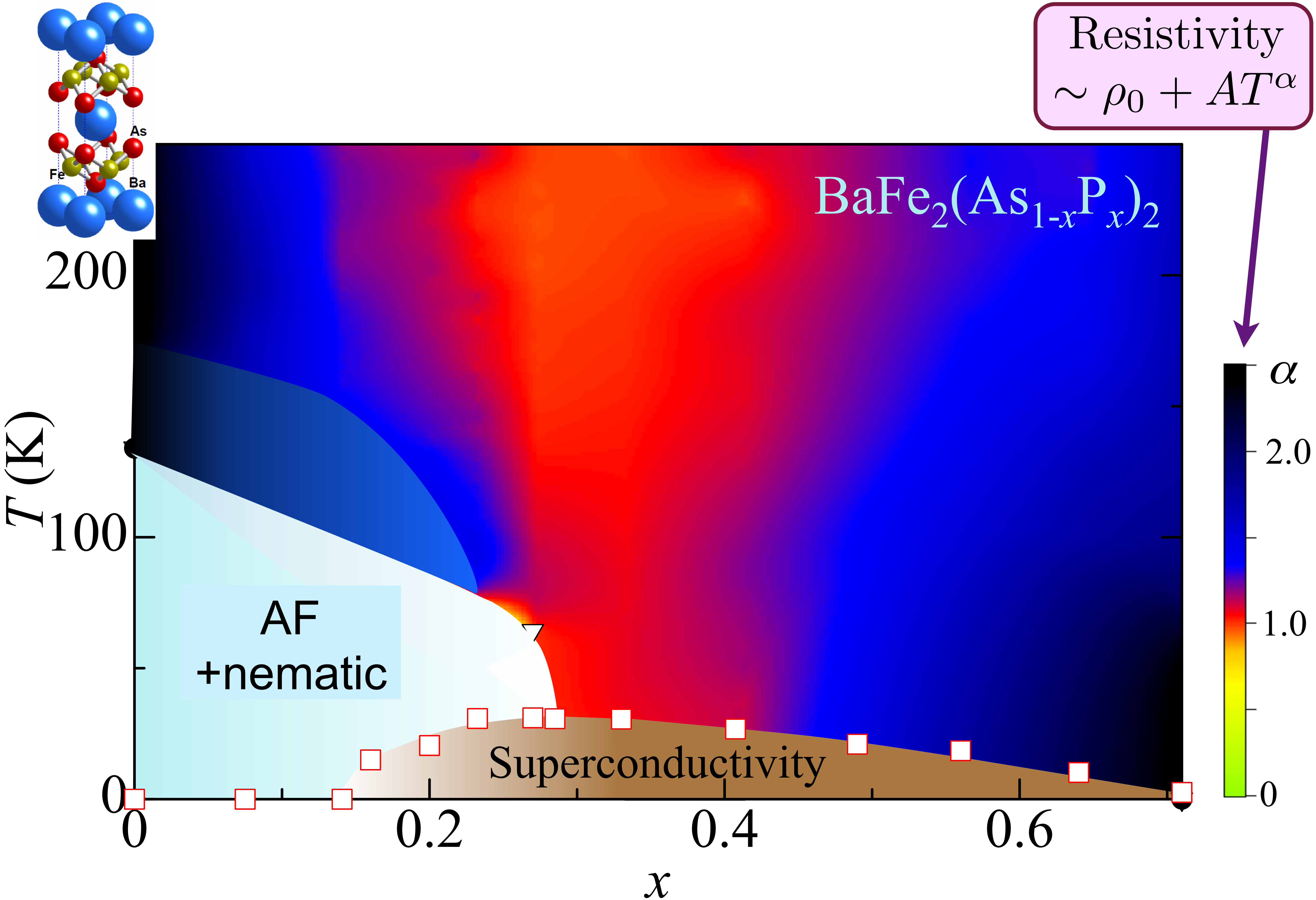
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Applies without changes to the microscopic band structure in the iron-based superconductors

QMC for the onset of antiferromagnetism



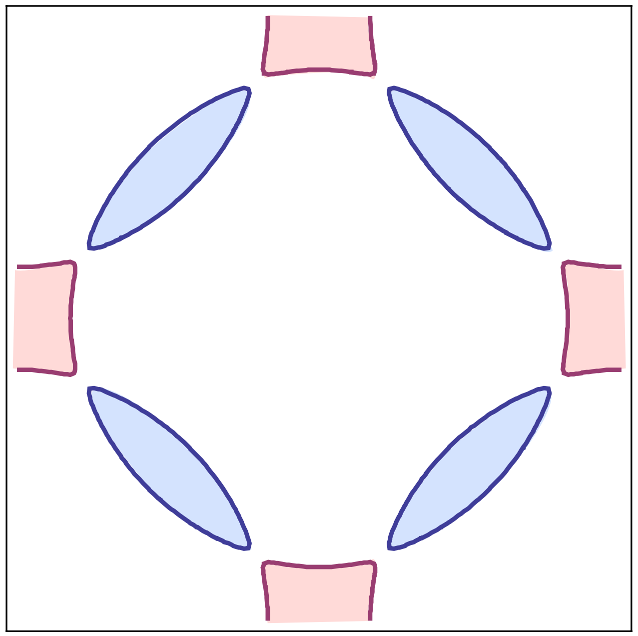
Yoni Schattner, Max H. Gerlach, Simon Trebst, and Erez Berg, arXiv:1512.07257

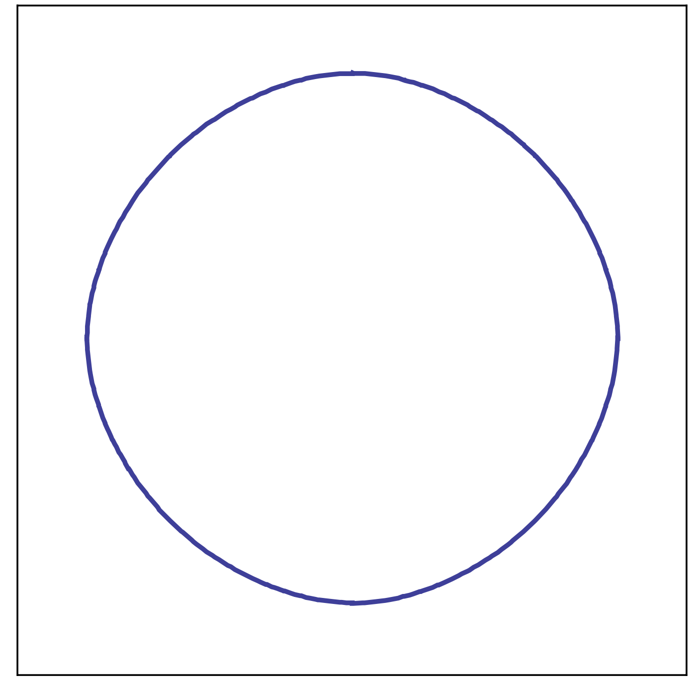


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Spin density wave order,
topological order,
and Fermi surface reconstruction

Quantum phase transition with Fermi surface reconstruction





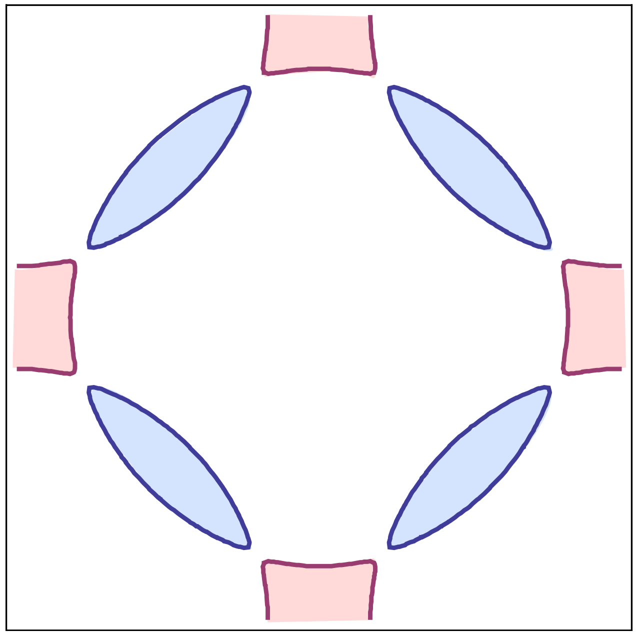
$\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

Separating onset of SDW order and Fermi surface reconstruction



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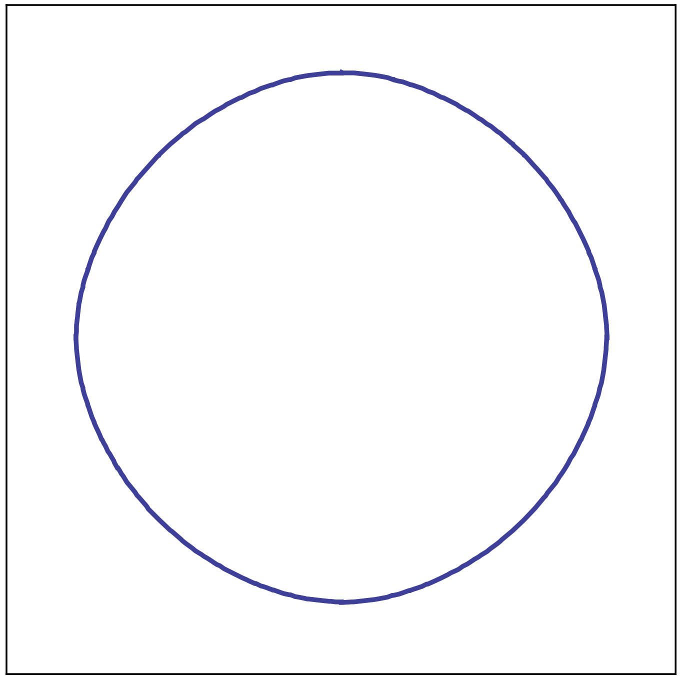

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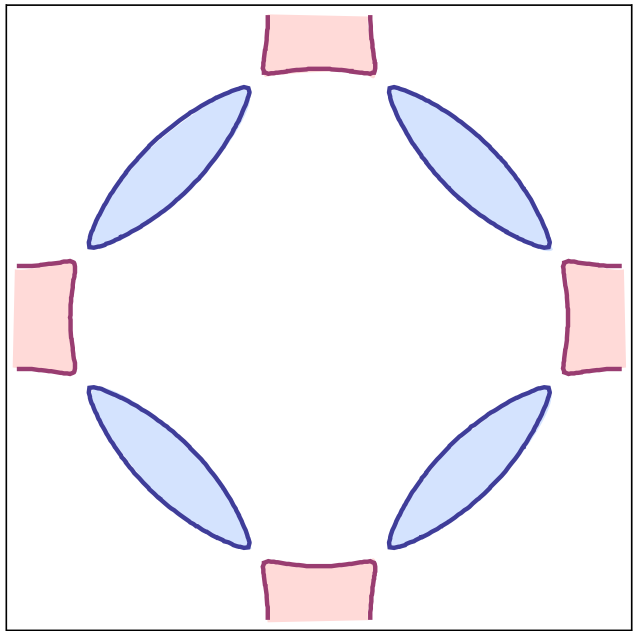
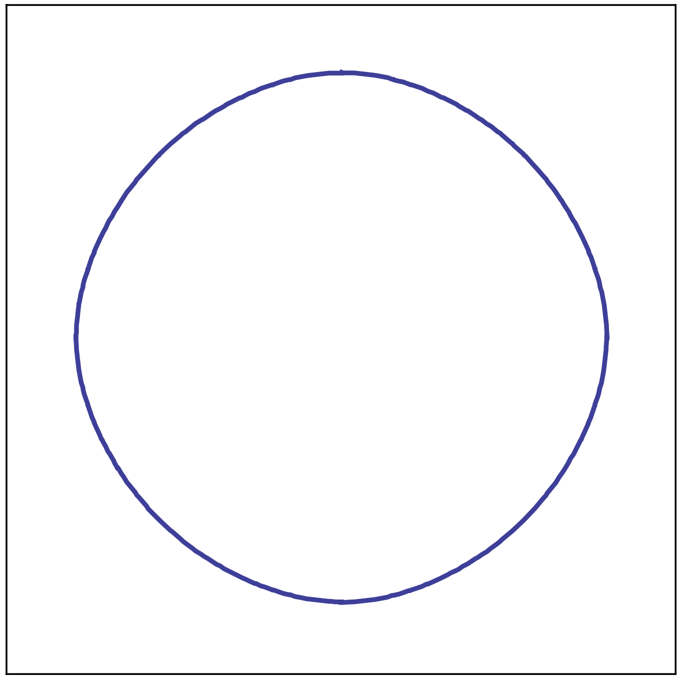


$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order

$\langle \vec{\varphi} \rangle = 0$

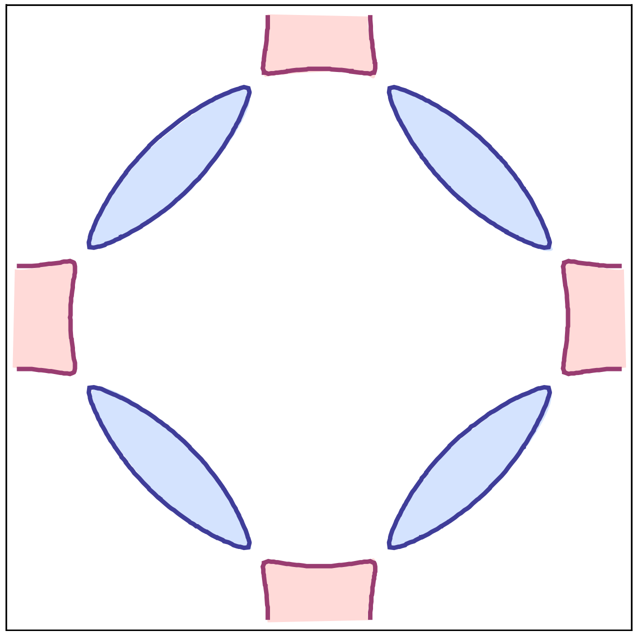


$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Separating onset of SDW order and Fermi surface reconstruction



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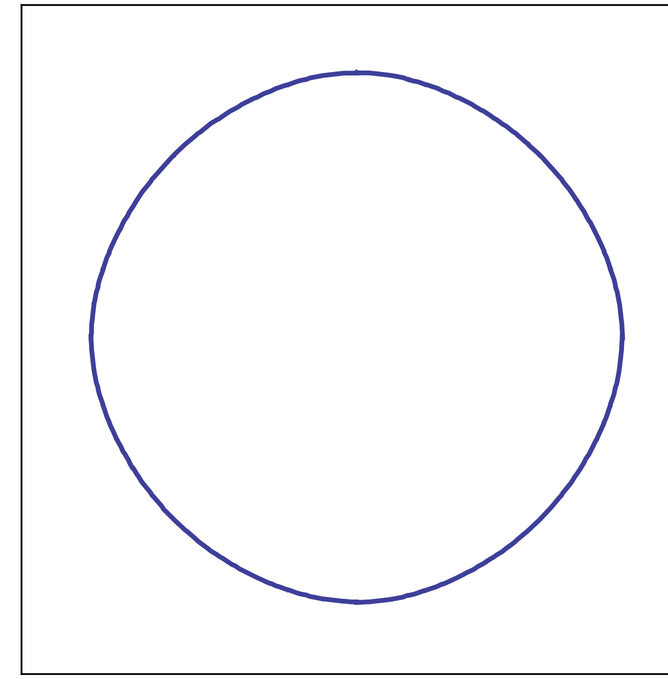
Algebraic Charge liquid (ACL) or Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and pocket Fermi surfaces



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Metal with "large" Fermi surface

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

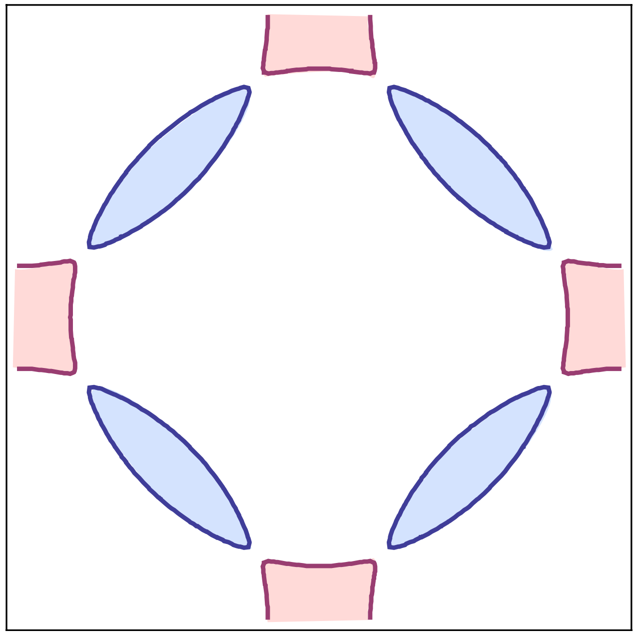
Spin density wave order, topological order, and Fermi surface reconstruction

Subir Sachdev,[1,2] Erez Berg,[3] Shubhayu Chatterjee,[1] and Yoni Schattner[3]

arXiv:1606.xxxxx

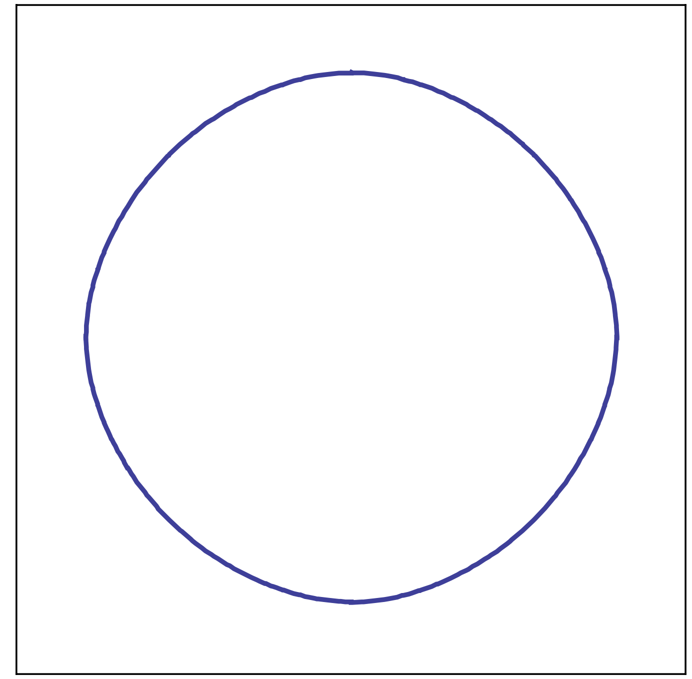
http://qpt.physics.harvard.edu/p300.pdf

Quantum phase transition with Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large" Fermi surface

Hertz theory for XY SDW order

The Hertz theory for the onset of SDW order can be described by the following Hamiltonian

$$H_{\rm sdw} = H_c + H_\theta + H_Y, \tag{1.1}$$

where H_c describes electrons (of density $(1-p)$) hopping on the sites of a square lattice

$$H_c = -\sum_{i,j} (t_{ij} + \mu \delta_{ij})\, c_{i\alpha}^\dagger c_{j\alpha} \tag{1.2}$$

with $c_{i\alpha}$ the electron annihilation operator on site i with spin $\alpha = \uparrow, \downarrow$. We represent the SDW order by a lattice XY rotor model, described by an angle θ_i, and its canonically conjugate number operator N_i, obeying

$$H_\theta = -\sum_{i<j} J_{ij} \cos(\theta_i - \theta_j) + 4\Delta \sum_i N_i^2 \quad ; \quad [\theta_i, N_j] = i\delta_{ij}, \tag{1.3}$$

where J_{ij} positive exchange constants, and Δ is proportional to the bare spin-wave gap (the 4 is for future convenience). A term linear in N_i is also allowed in H_θ, but we ignore it for simplicity; such a linear term will not be allowed when we consider models with SU(2) symmetry in Section IV.

Hertz theory for XY SDW order

Finally, there is a 'Yukawa' coupling between the XY order parameter, $e^{i\theta}$, and the fermions

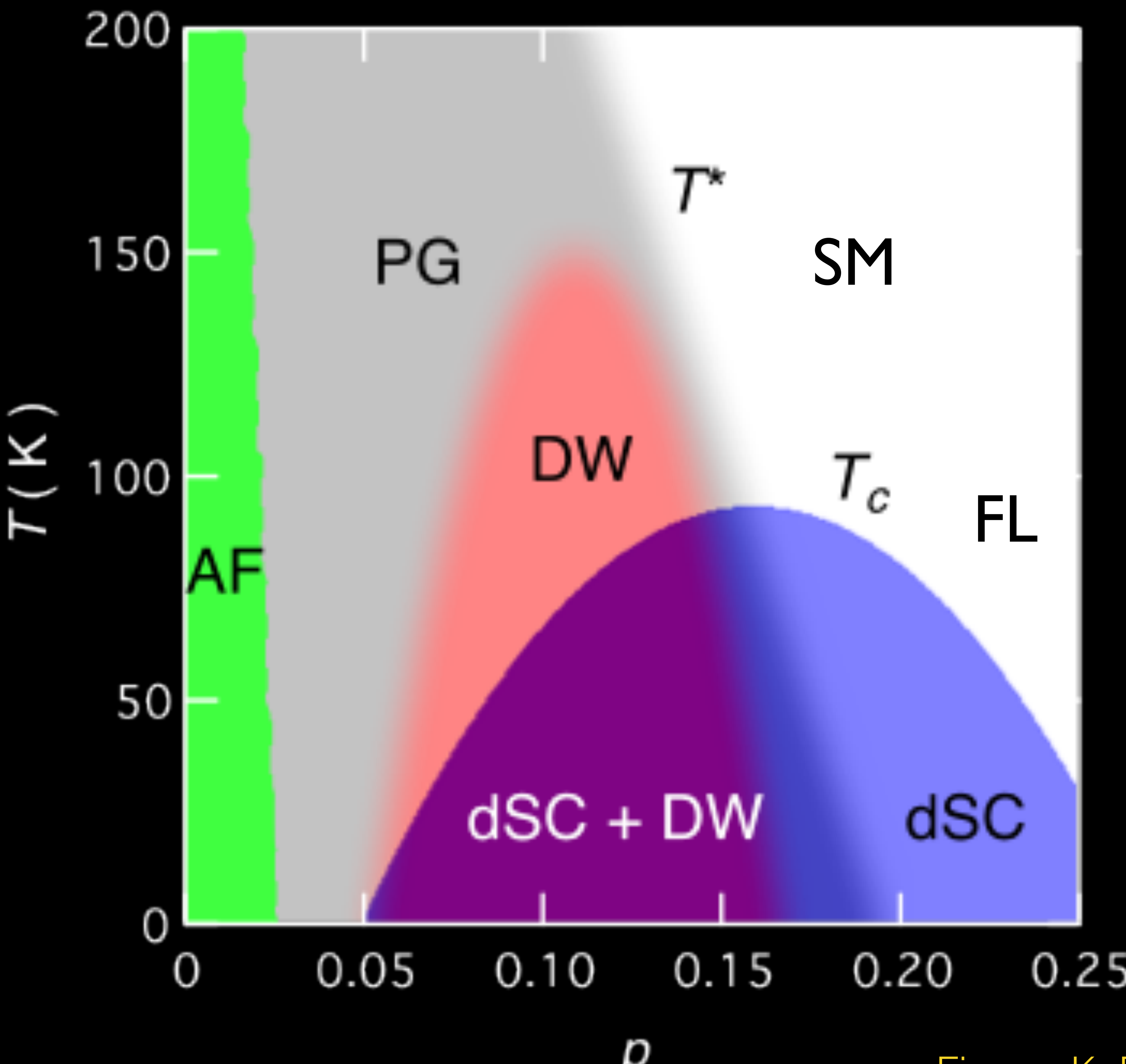
$$H_Y = -\lambda \sum_i \eta_i \left[e^{-i\theta_i} c_{i\uparrow}^\dagger c_{i\downarrow} + e^{i\theta_i} c_{i\downarrow}^\dagger c_{i\uparrow} \right], \tag{1.4}$$

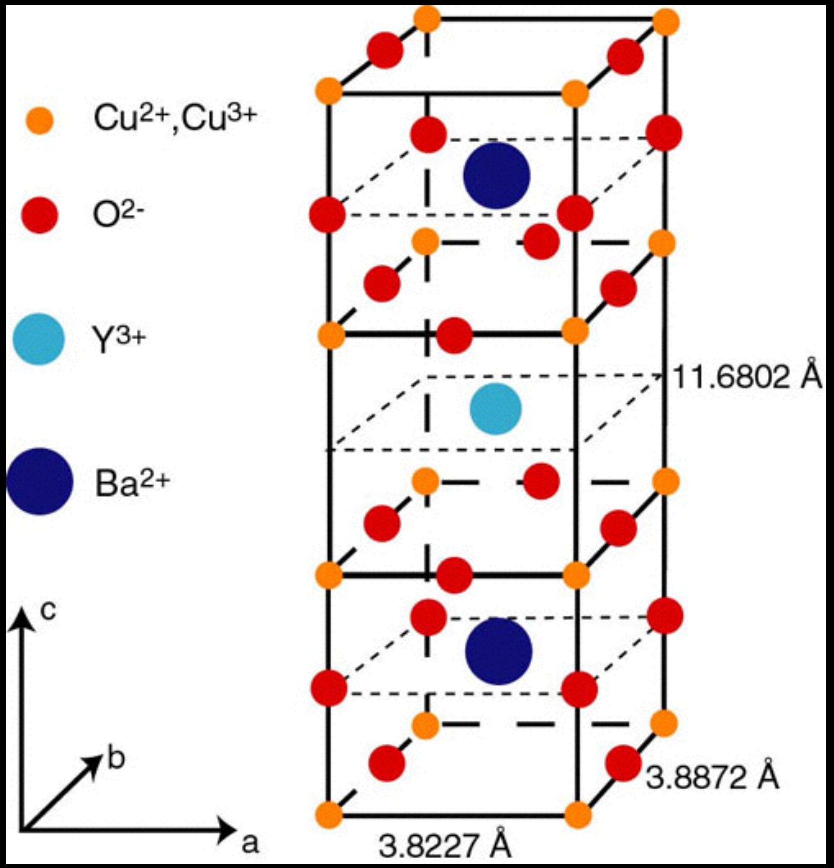
where

$$\eta_i \equiv (-1)^{x_i + y_i} \tag{1.5}$$

is the staggering factor representing the opposite spin orientations on the two sublattices. Note that the Yukawa coupling, and the remaining Hamiltonian, commute with the total spin along the z direction

$$S_z = \sum_i \left(N_i + \frac{1}{2} c_{i\uparrow}^\dagger c_{i\uparrow} - \frac{1}{2} c_{i\downarrow}^\dagger c_{i\downarrow} \right). \tag{1.6}$$

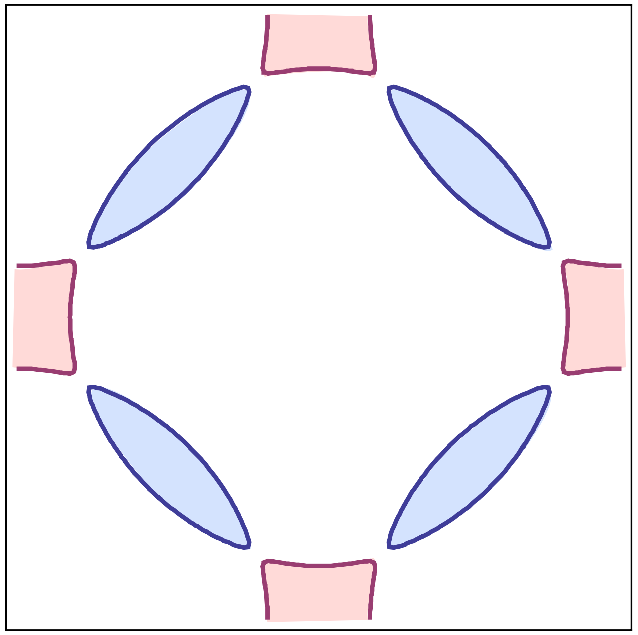




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

Separating onset of SDW order and Fermi surface reconstruction



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Metal with electron and hole pockets

Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order

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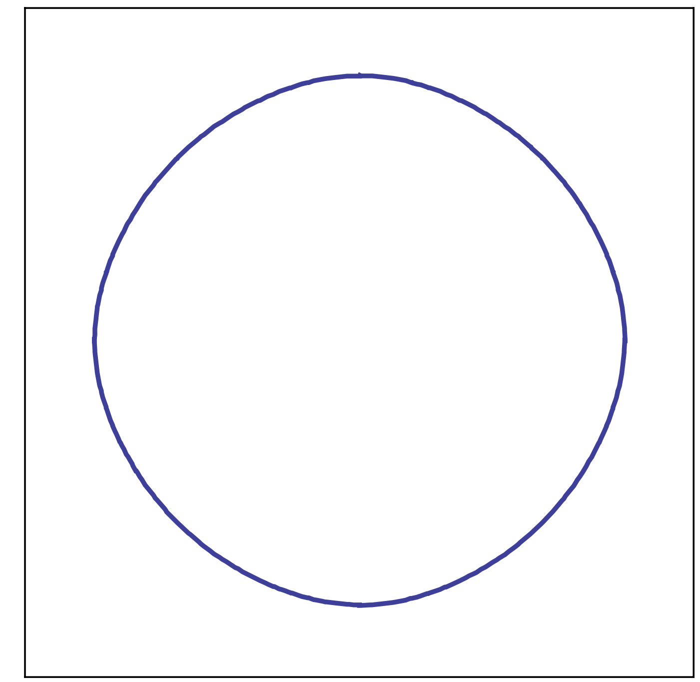

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Metal with "large" Fermi surface

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

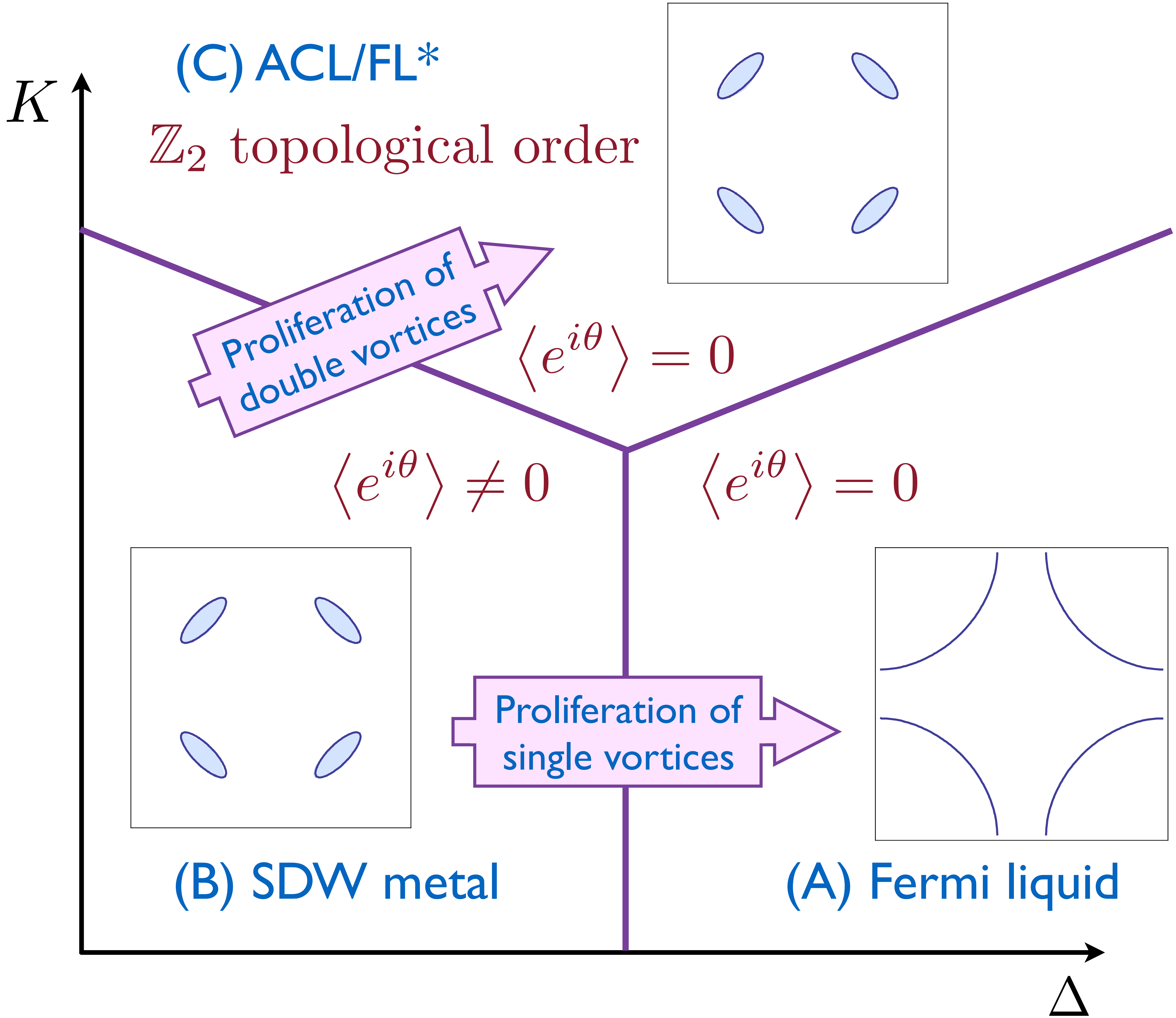
Z_2 gauge theory for fractionalized XY SDW order coupled to electrons

$$\mathcal{H}_1 = H_c + H_{\theta,\mathbb{Z}_2} + H_Y$$

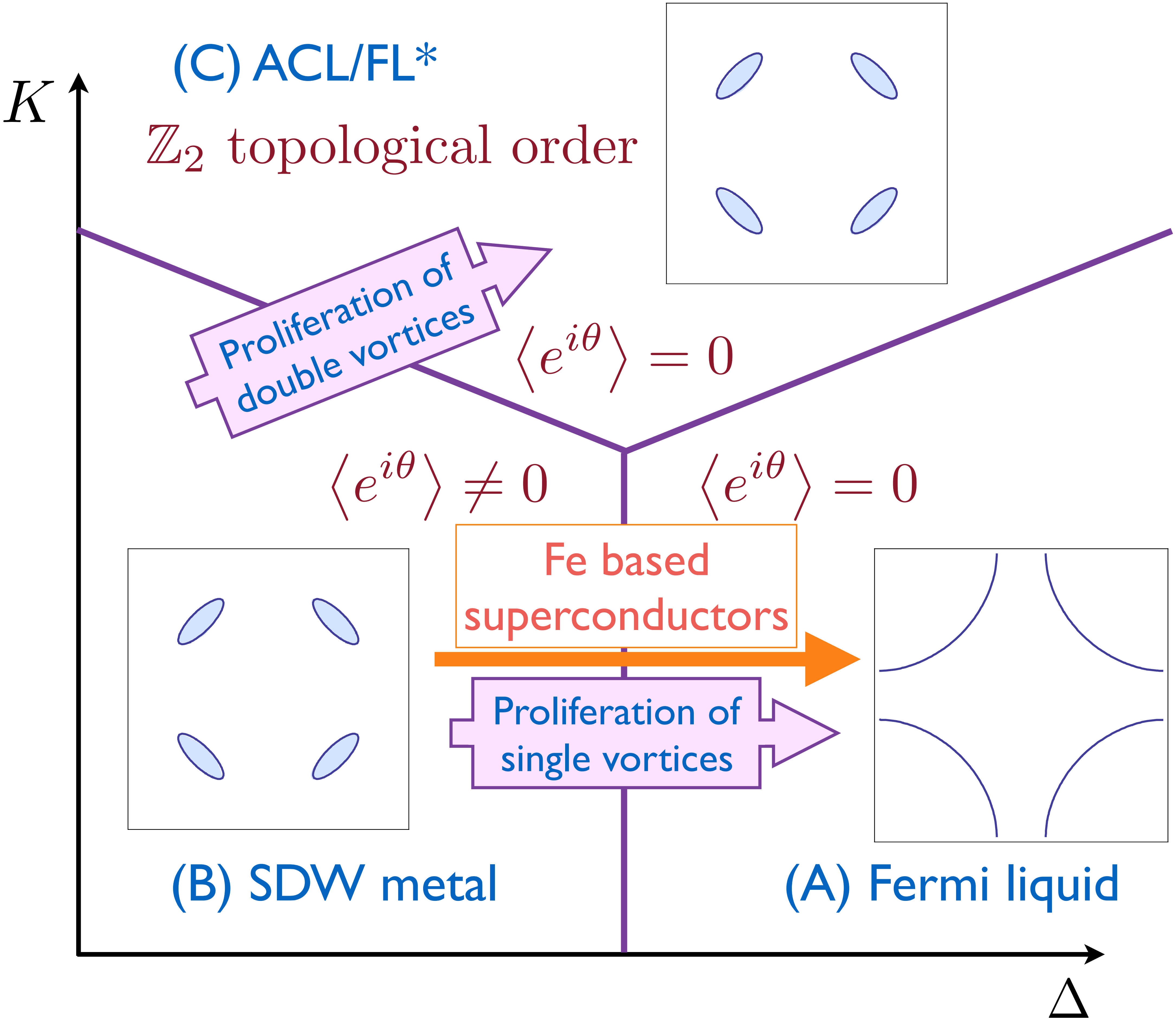
$$H_c = -\sum_{i,j} (t_{ij} + \mu\delta_{ij})\, c_{i\alpha}^\dagger c_{j\alpha}$$

$$H_Y = -\lambda \sum_i \eta_i \left[e^{-i\theta_i} c_{i\uparrow}^\dagger c_{i\downarrow} + e^{i\theta_i} c_{i\downarrow}^\dagger c_{i\uparrow} \right]$$

$$H_{\theta,\mathbb{Z}_2} = -\sum_{i<j} J_{ij} \mu_{ij}^z \cos\left((\theta_i - \theta_j)/2\right) + 4\Delta \sum_i N_i^2 - g \sum_{\langle ij \rangle} \mu_{ij}^x - K \sum_{\Box} \left[\prod_{\Box} \mu_{ij}^z \right],$$



(C) ACL/FL*
K
$\mathbb{Z}_2$ topological order
Proliferation of double vortices
$\langle e^{i\theta} \rangle = 0$
$\langle e^{i\theta} \rangle \neq 0$
$\langle e^{i\theta} \rangle = 0$
Proliferation of single vortices
(B) SDW metal
(A) Fermi liquid
Δ


(C) ACL/FL*
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K
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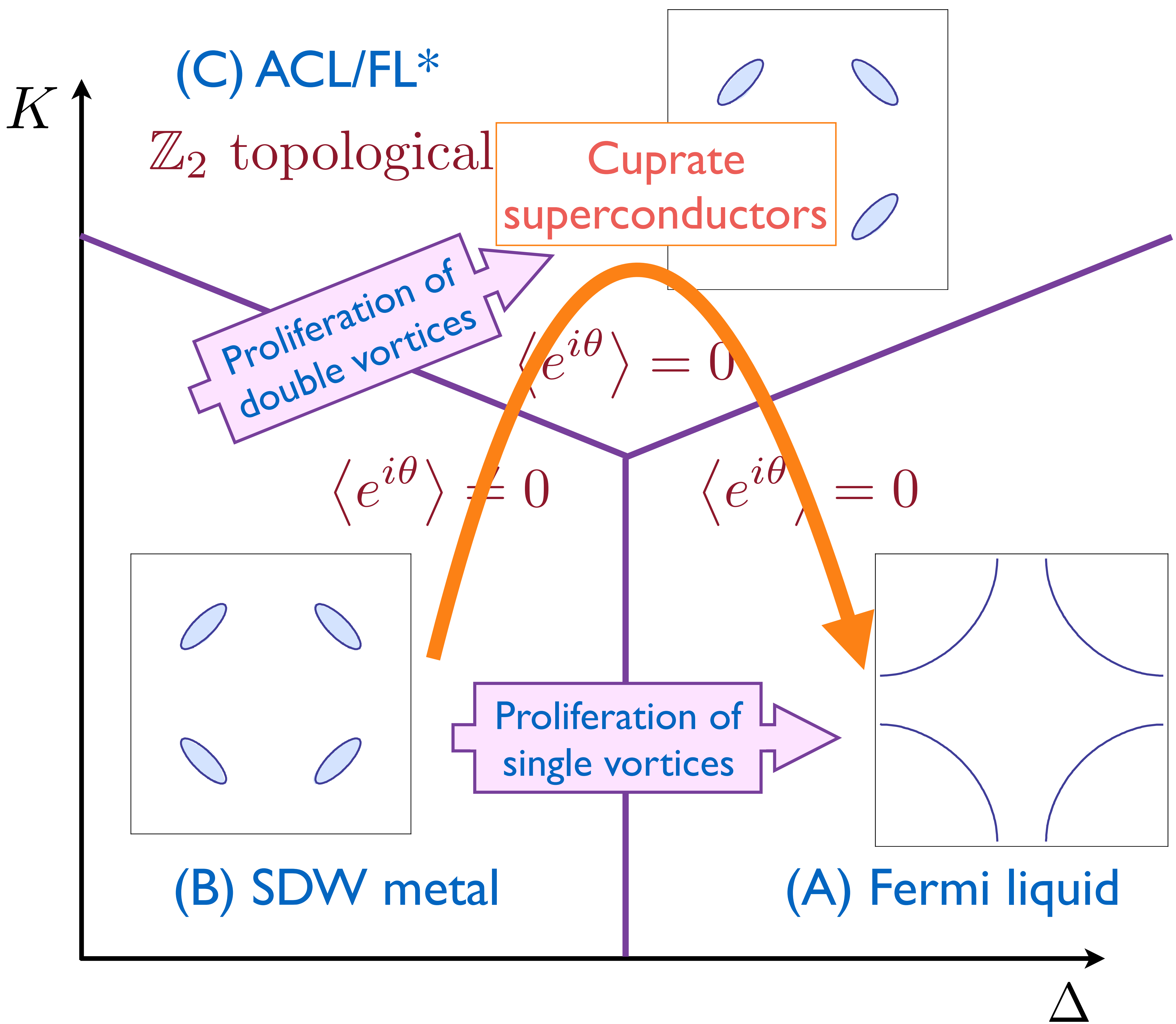
Fe based superconductors
Proliferation of single vortices
(B) SDW metal
(A) Fermi liquid
Δ



K
(C) ACL/FL*
$\mathbb{Z}_2$ topological
Cuprate superconductors
Proliferation of double vortices
$\langle e^{i\theta} \rangle = 0$
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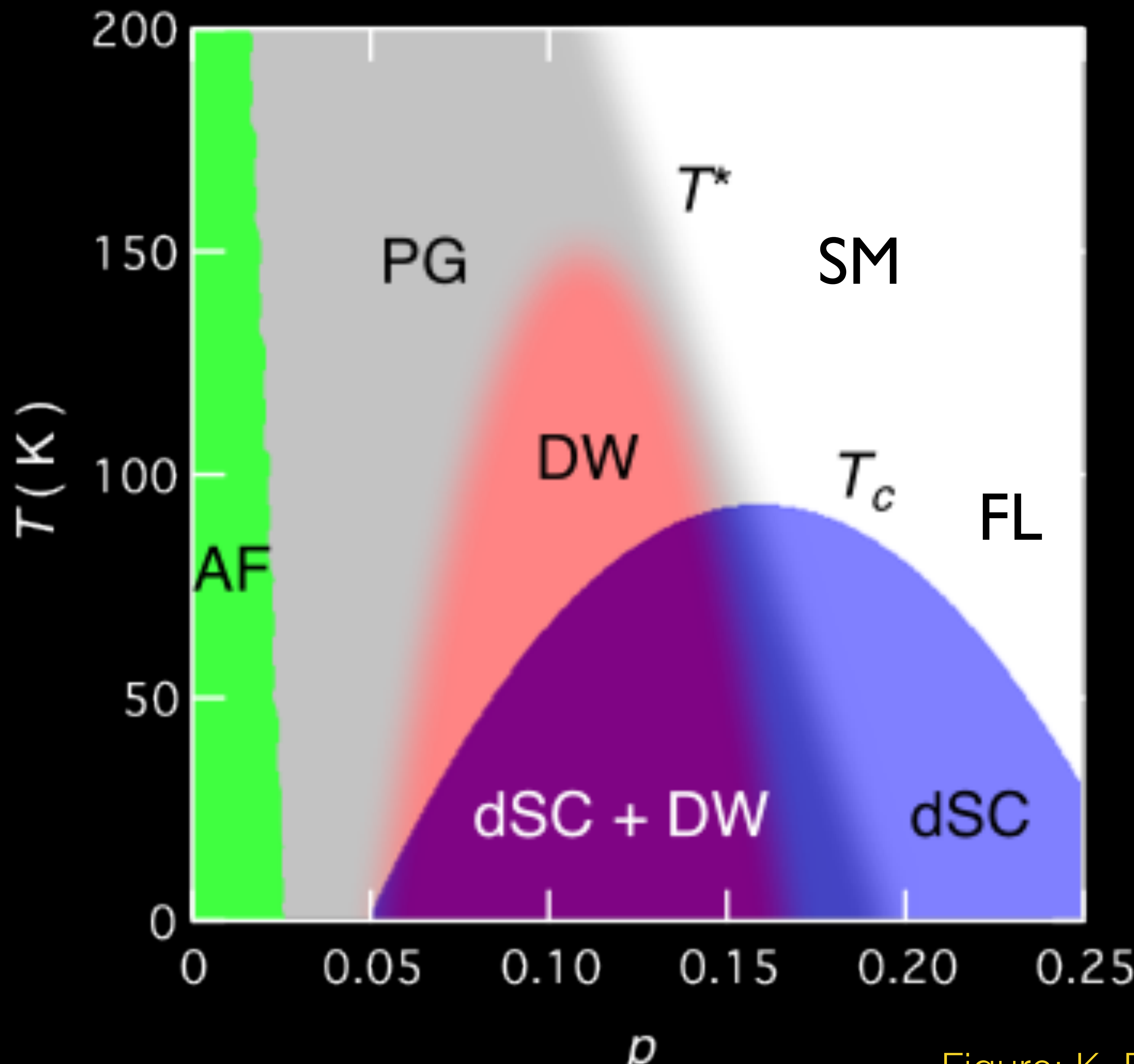
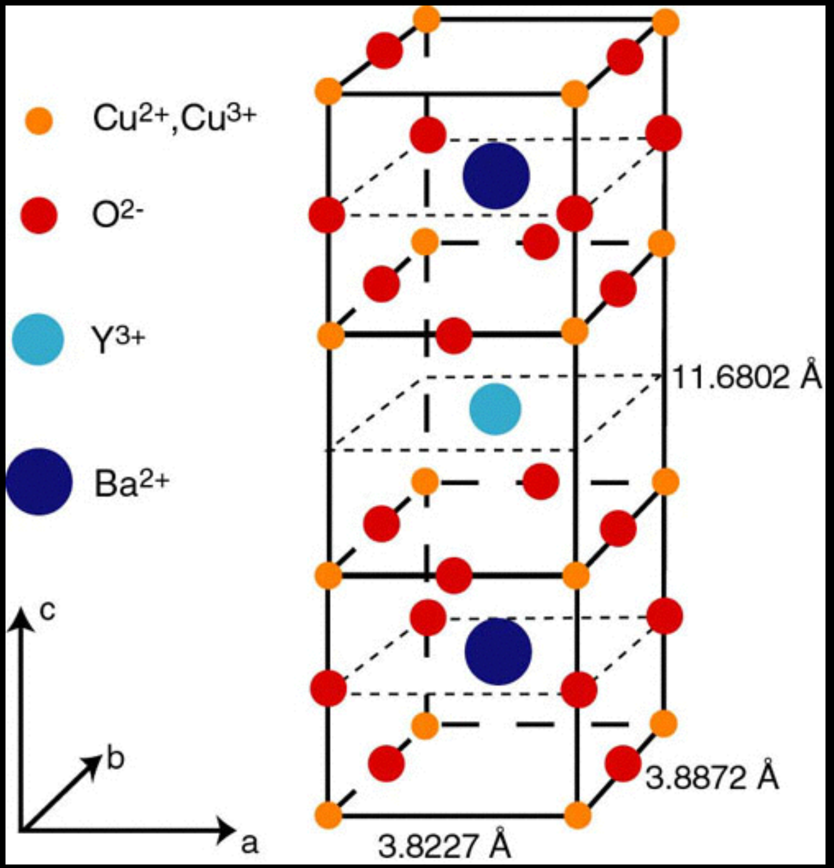
Proliferation of single vortices
(B) SDW metal
(A) Fermi liquid
Δ

Evidence for pseudogap metal as ACL/FL*





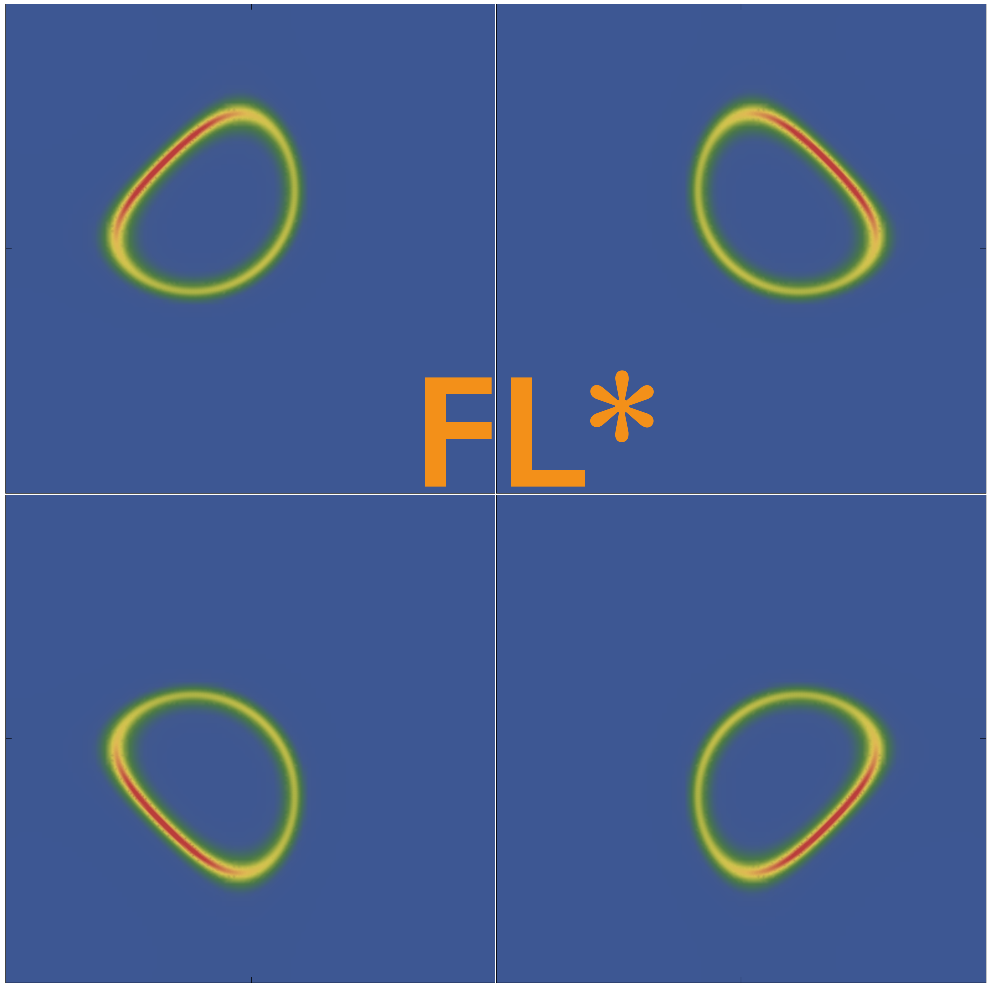
$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

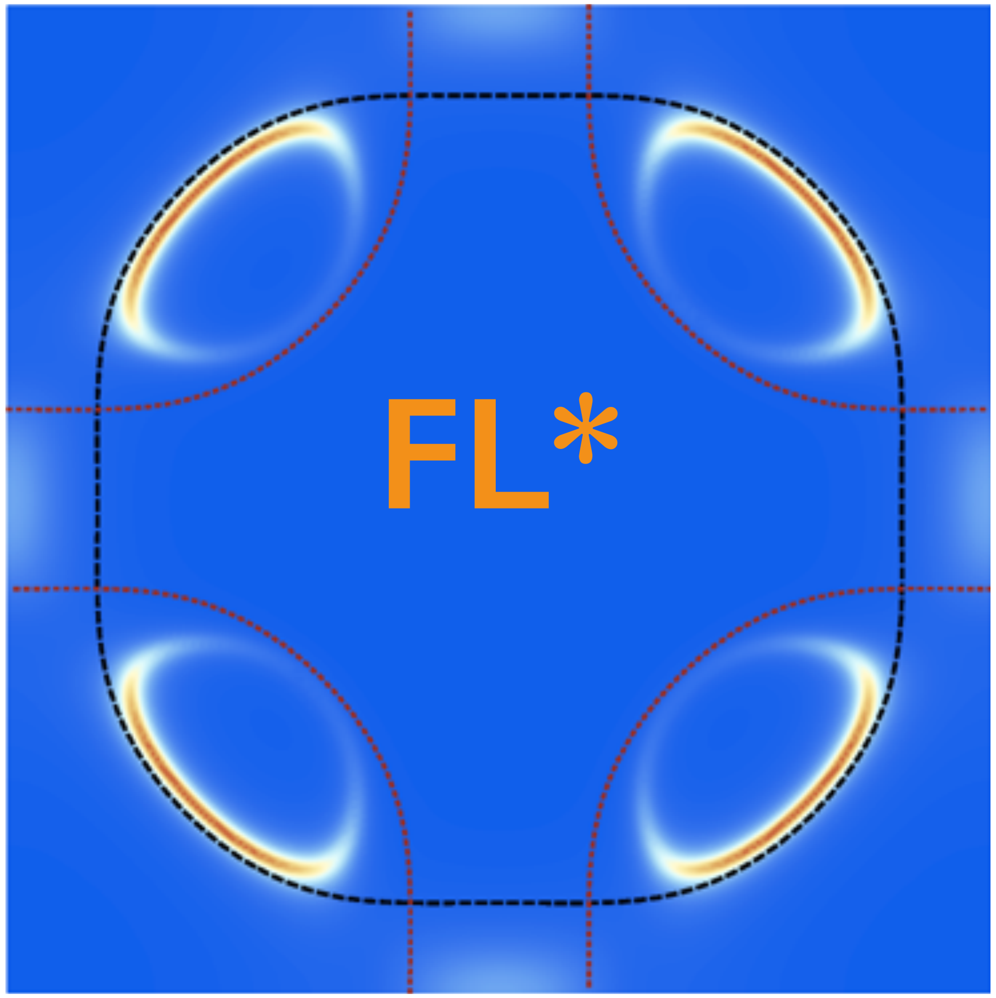
Evidence for pseudogap metal as ACL/FL*

- FL* Fermi pockets are compatible with photoemission at high T.

Fermi surfaces in one-band models of FL*



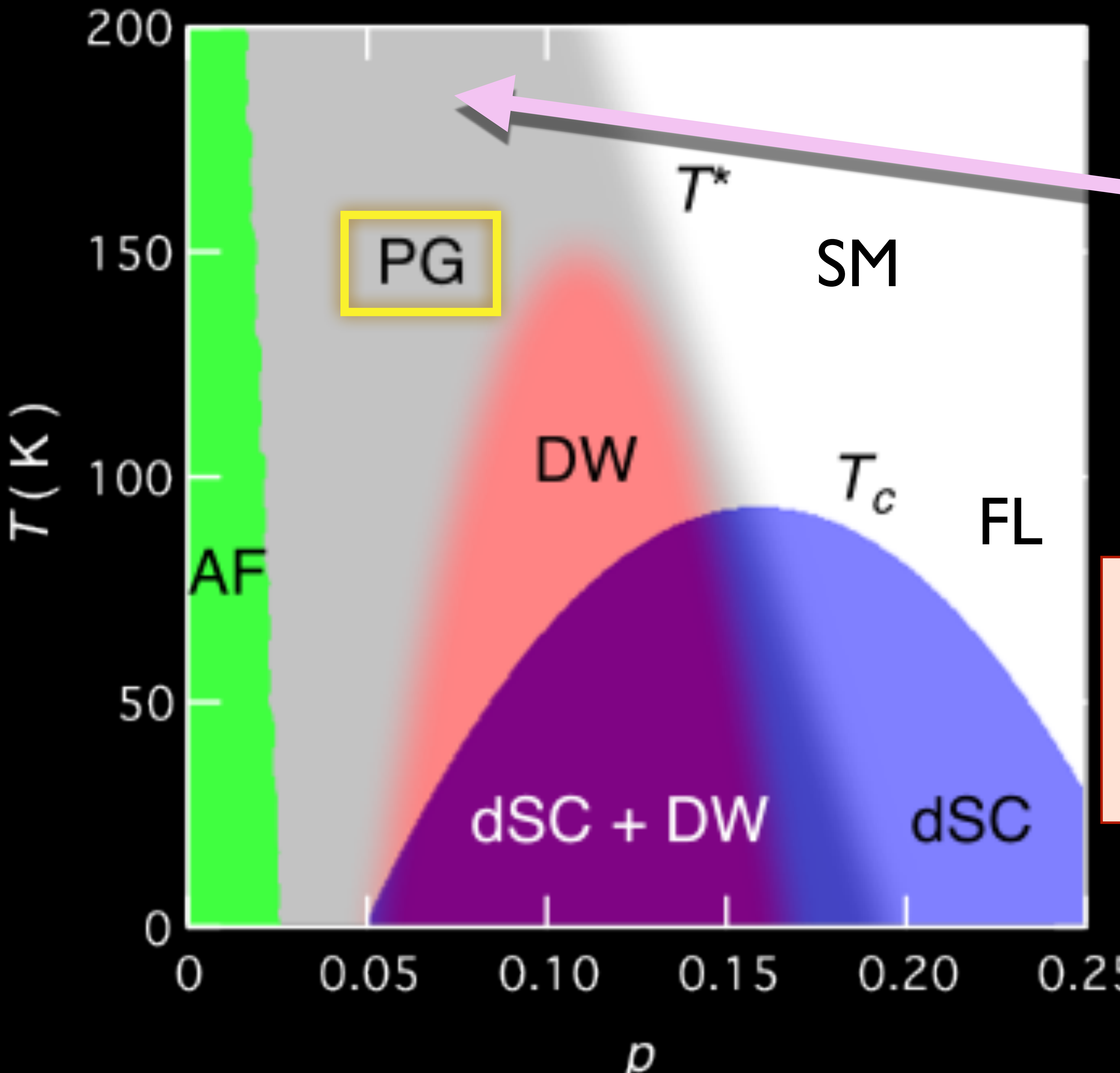
M. Punk, A. Allais, and S. Sachdev,
PNAS **112**, 9552 (2015)

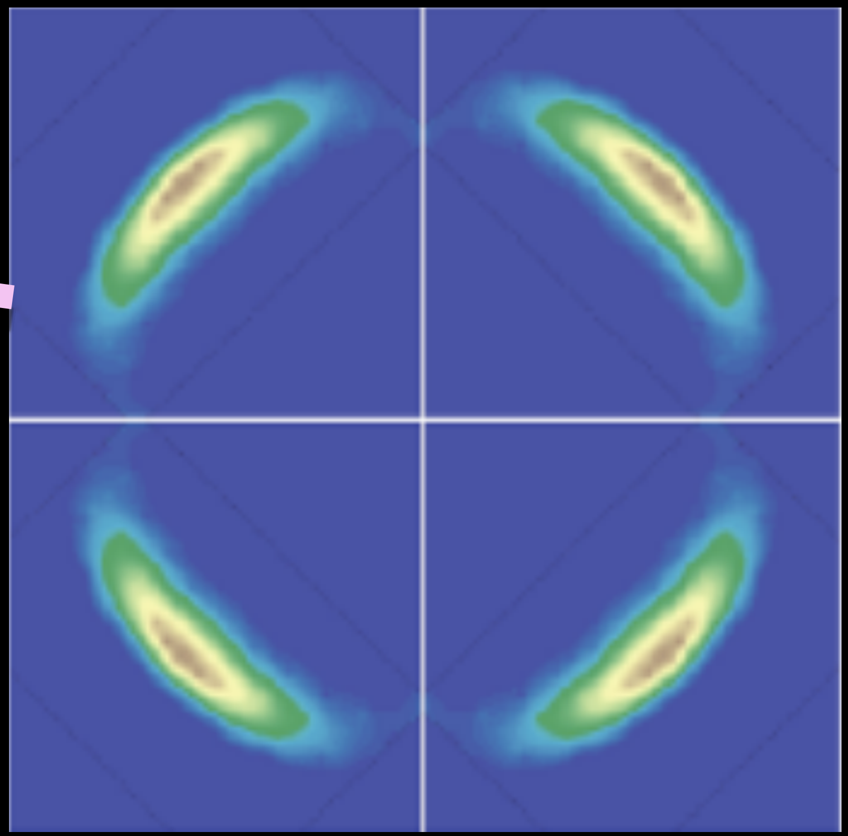


Y. Qi and S. Sachdev,
Phys. Rev. B **81**, 115129 (2010)

"Back side" of Fermi surface is suppressed for observables which change electron number in the square lattice

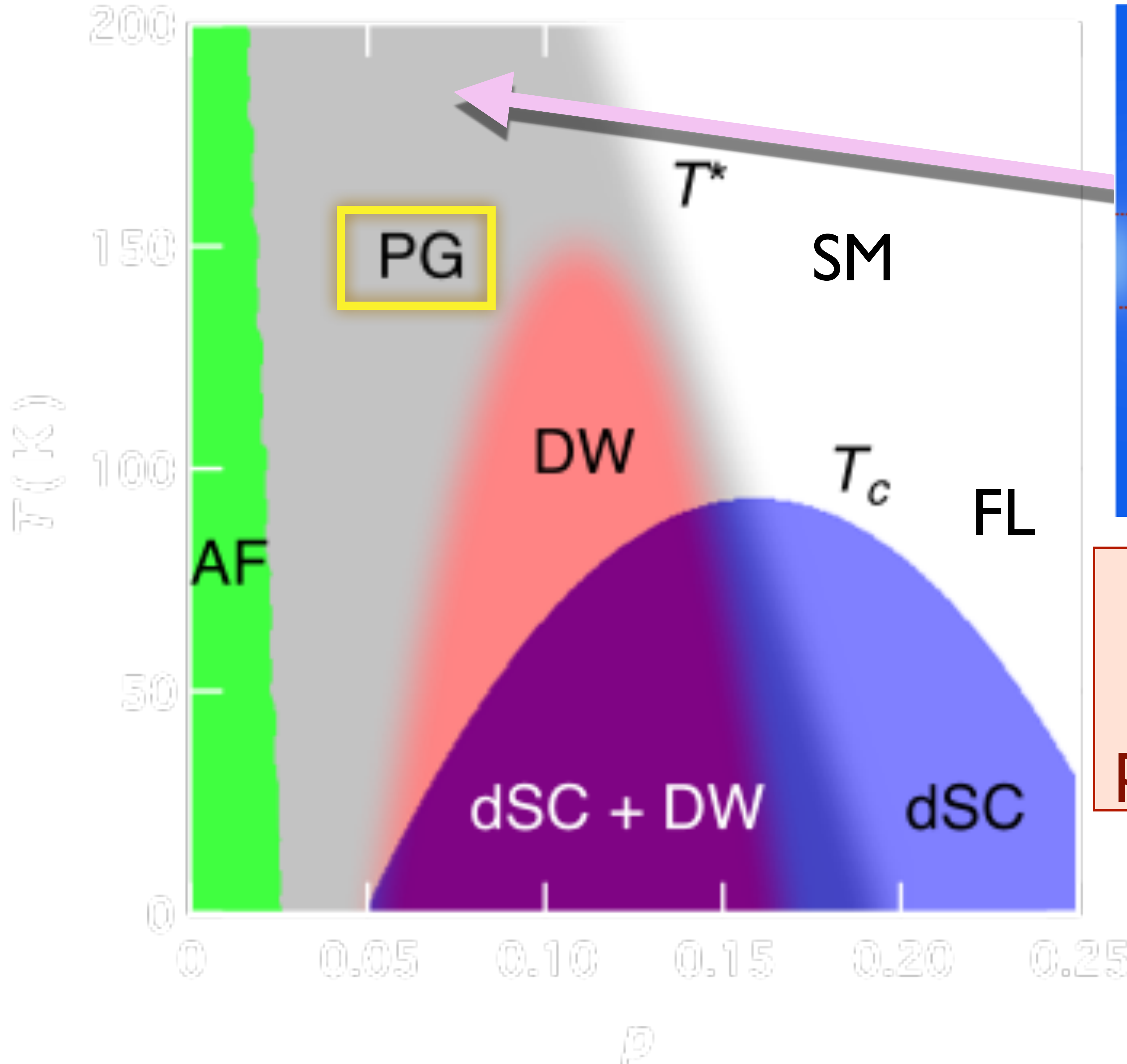
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

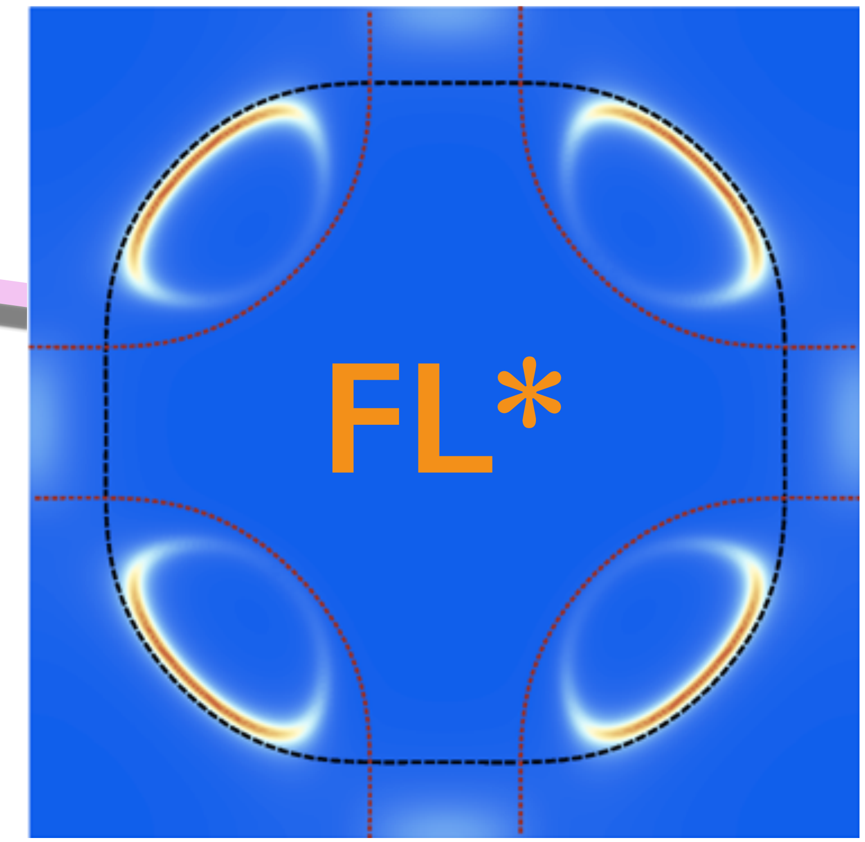




Photoemission in pseudogap metal

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)





Photoemission in FL* model of pseudogap metal

Evidence for pseudogap metal as ACL/FL*

- FL* Fermi pockets are compatible with photoemission at high T.

- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).

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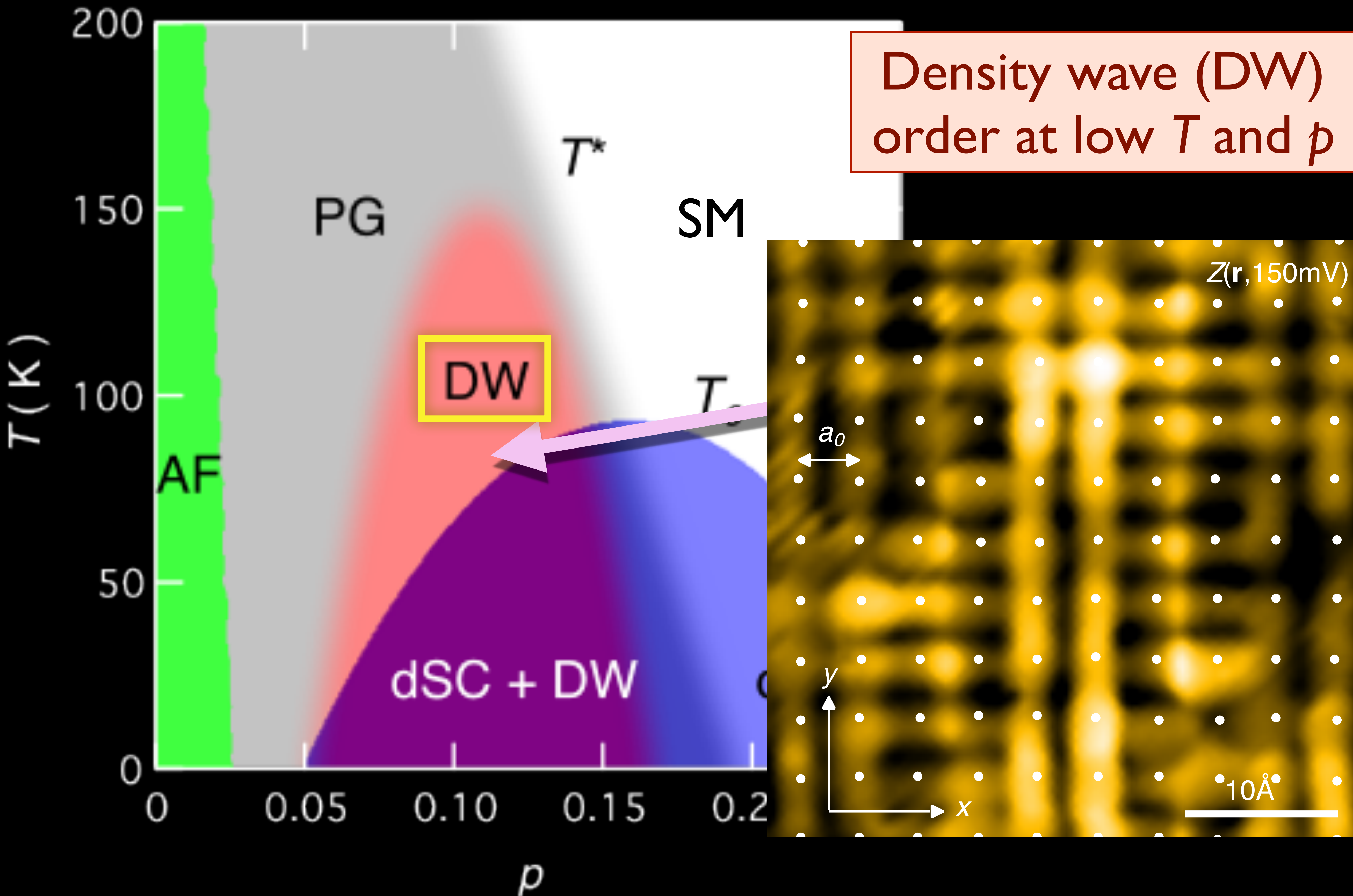
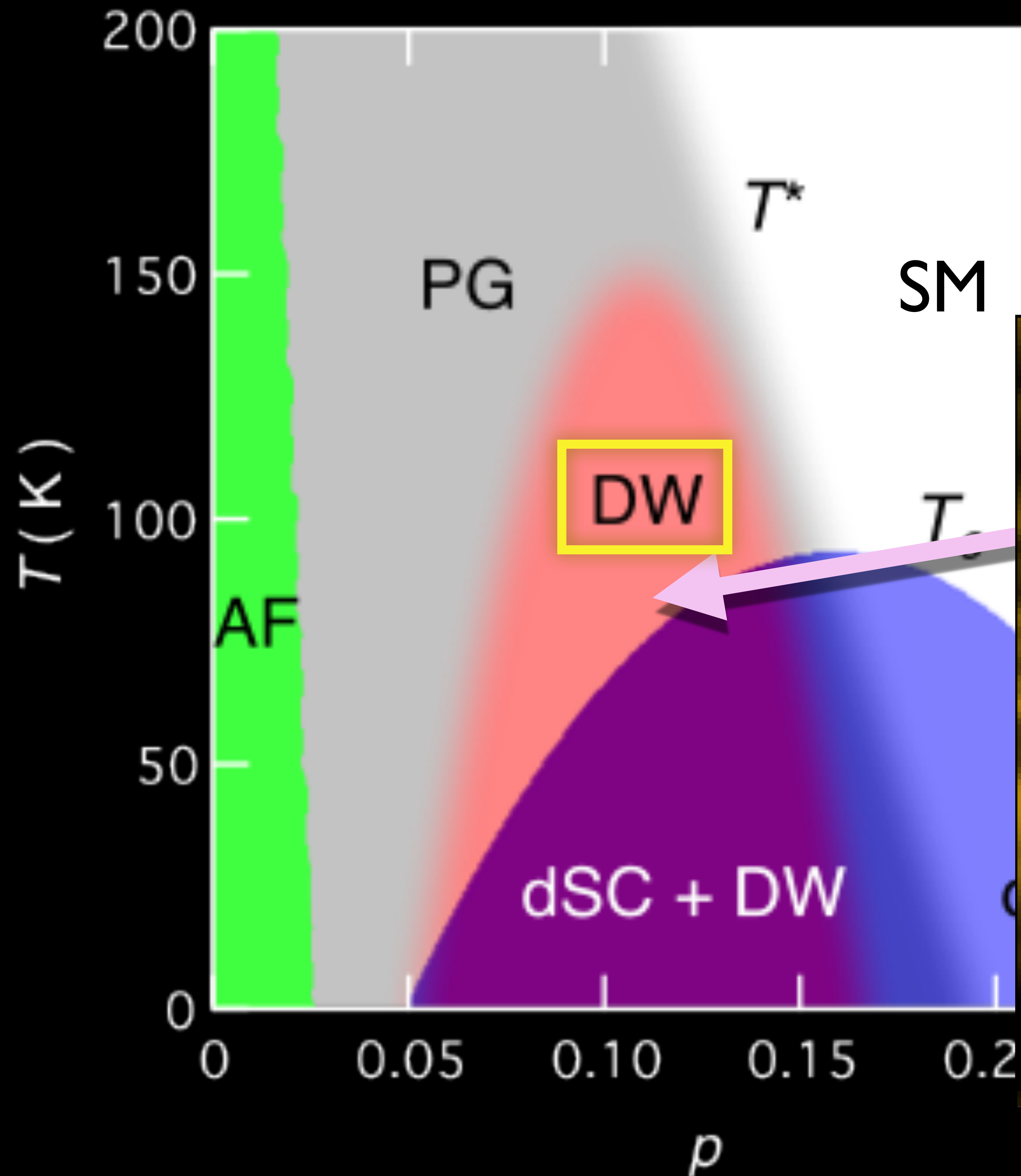
- Charge density wave instabilities of FL* have wave vector and form-factors which agree with STM/X-ray observations in DW region (D. Chowdhury and S. Sachdev, PRB **90**, 245136 (2014)).

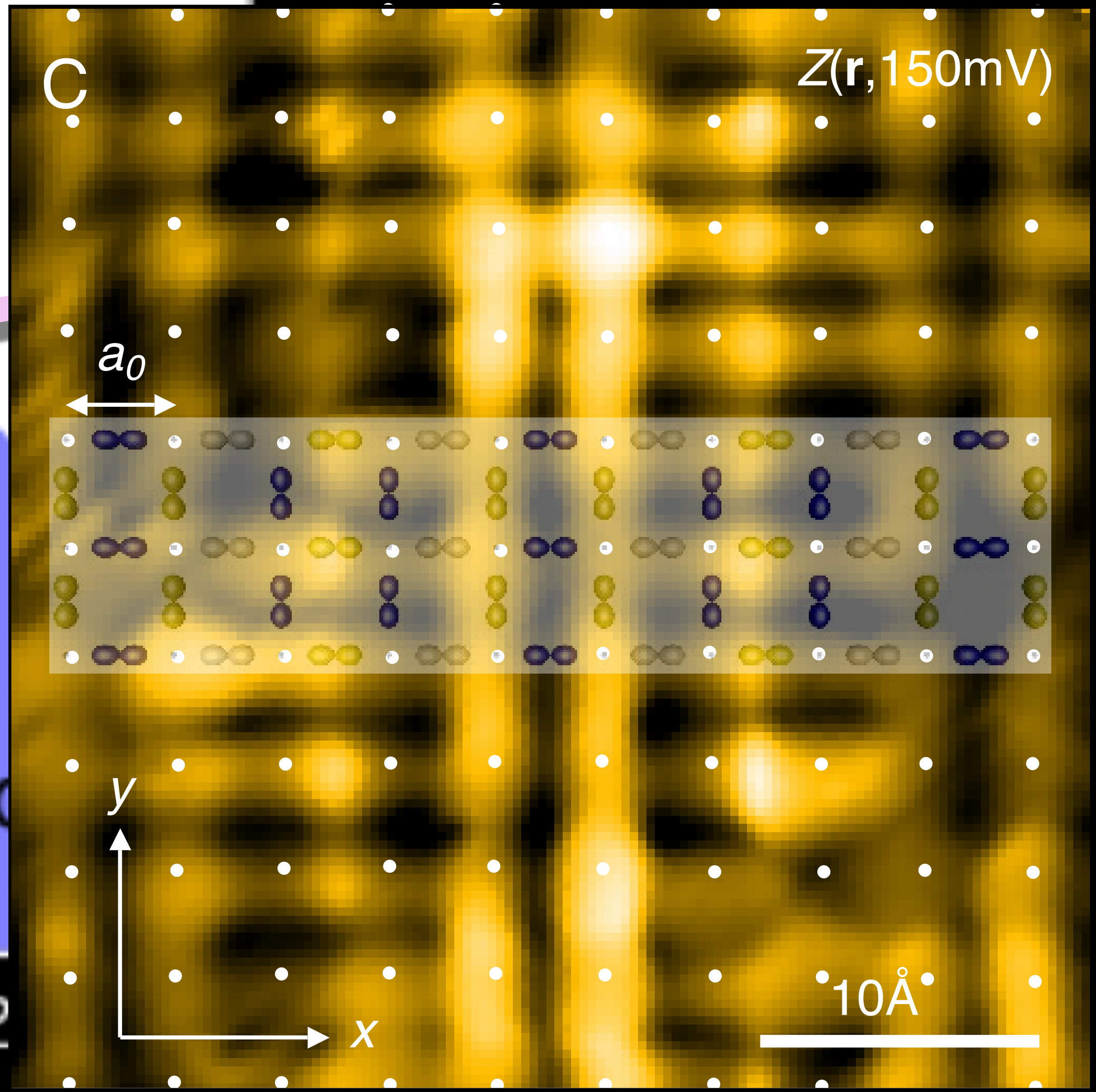
Y. Kohsaka *et al.*, Science **315**, 1380 (2007)
M. H. Hamidian *et al.*, Nature Physics **12**, 150 (2016)



M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)



Identified as a predicted "*d*-form factor density wave"



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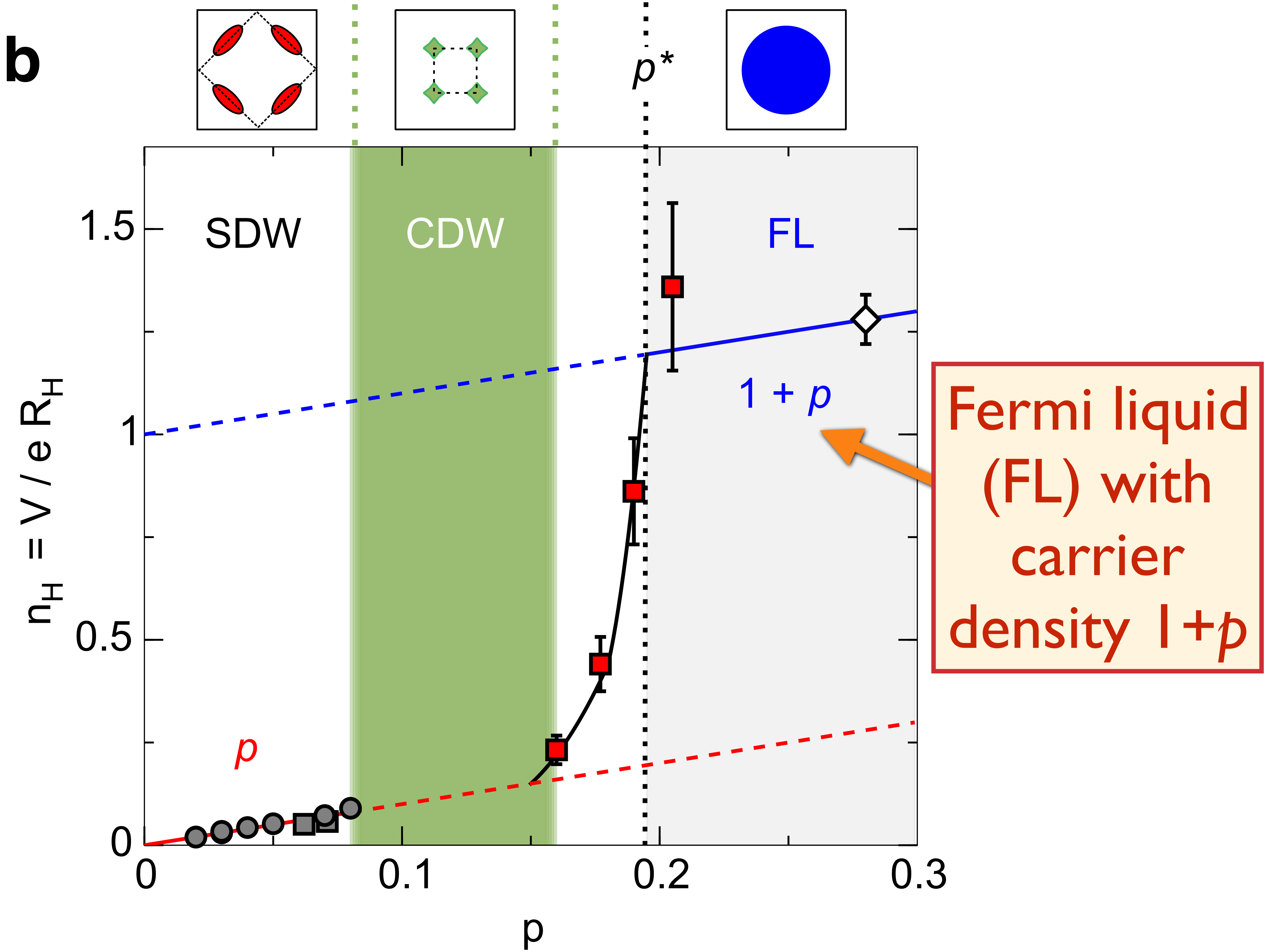
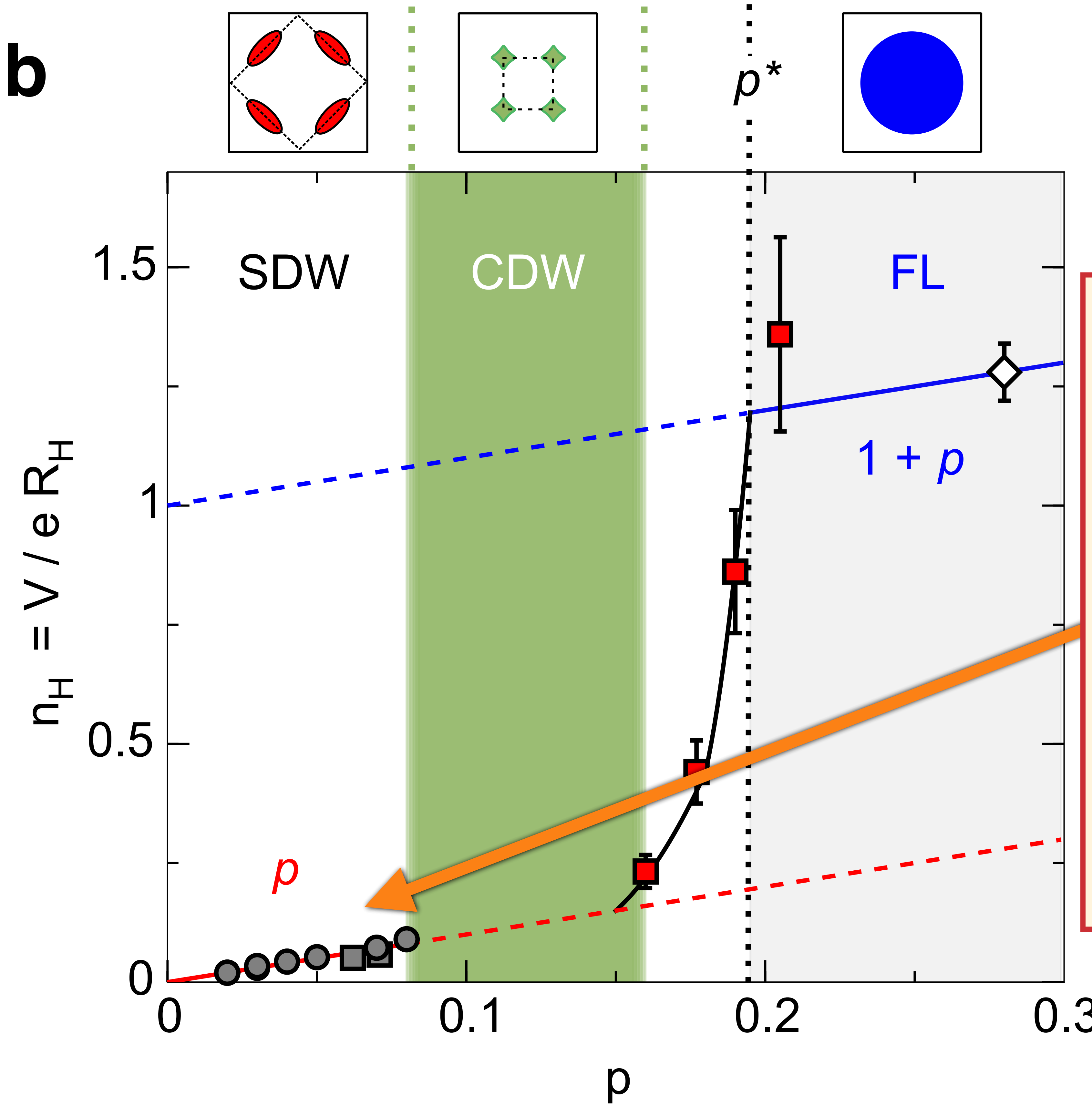
- T-independent positive Hall co-efficient, R_H, corresponding to carrier density p in the higher temperature pseudogap (Ando *et al.*, PRL **92**, 197001 (2004)) <u>*and*</u> in recent measurements at high fields, low T, and around $p \approx 0.16$ in YBCO (Badoux *et al.*, Nature **531**, 210 (2016)).

Hall effect measurements in YBCO



Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

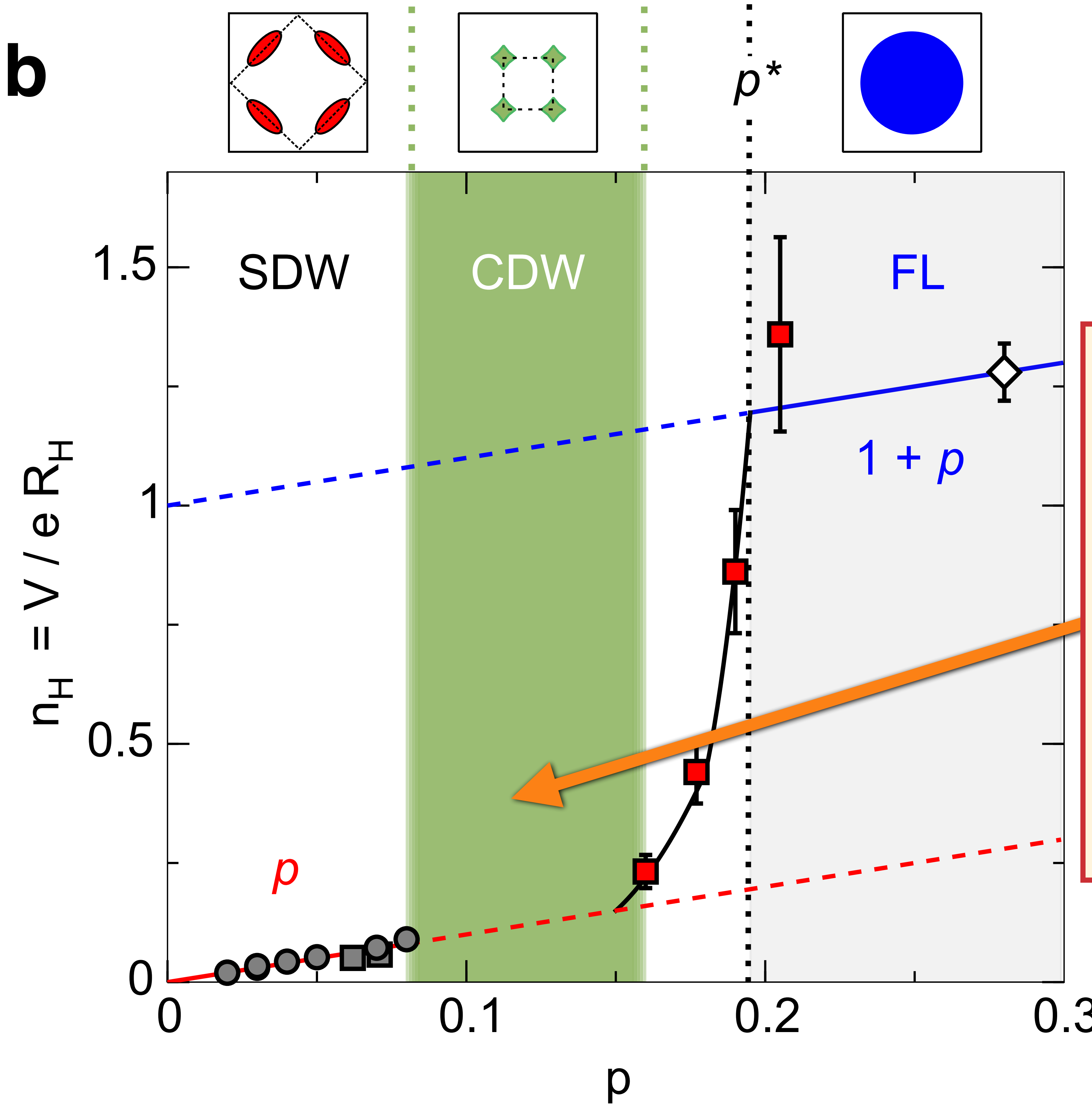
Hall effect measurements in YBCO



Spin density wave (SDW) breaks translational invariance, and the Fermi liquid then has carrier density p

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

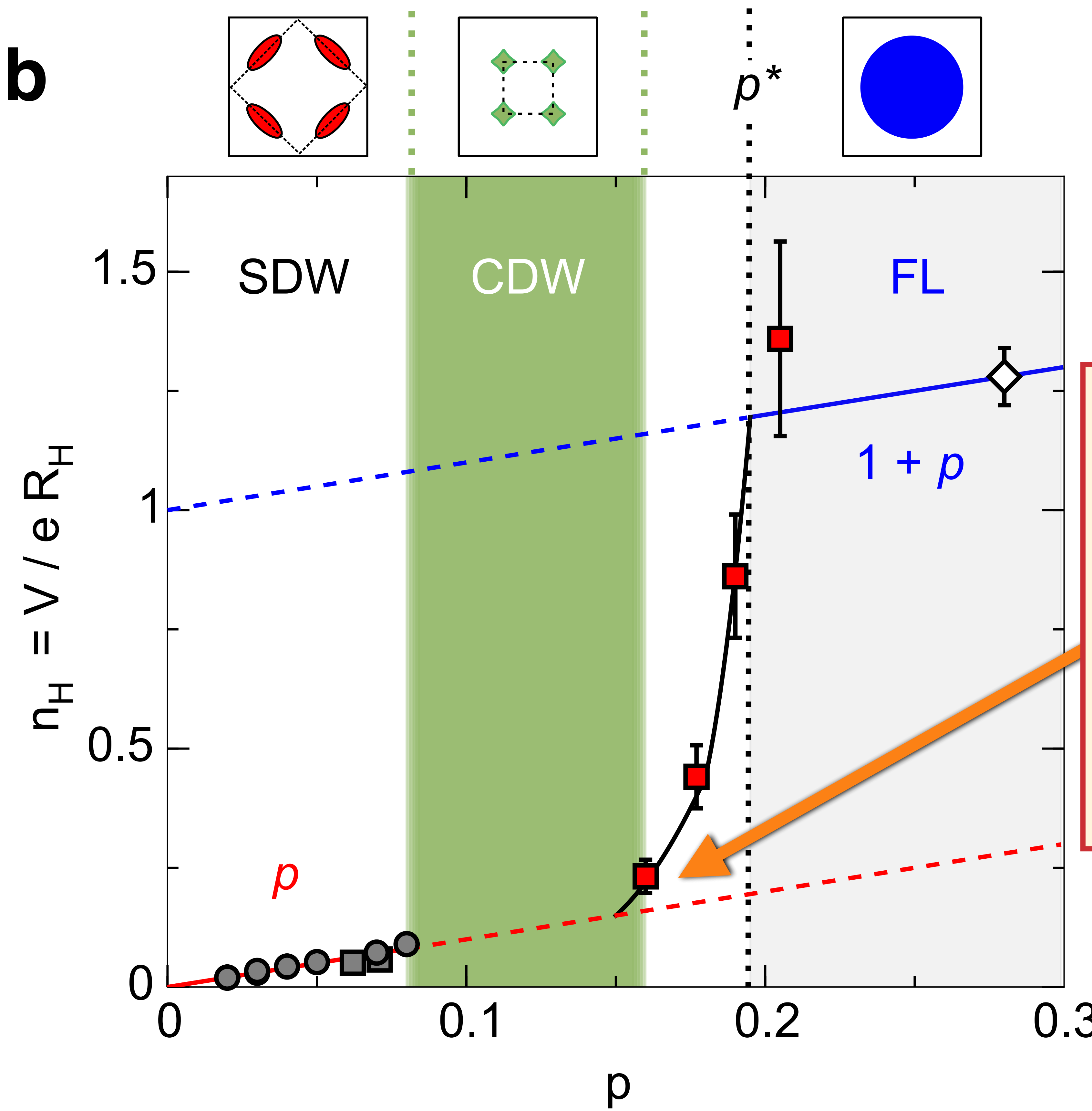
Hall effect measurements in YBCO



Charge density wave (CDW) leads to complex Fermi surface reconstruction and negative Hall resistance

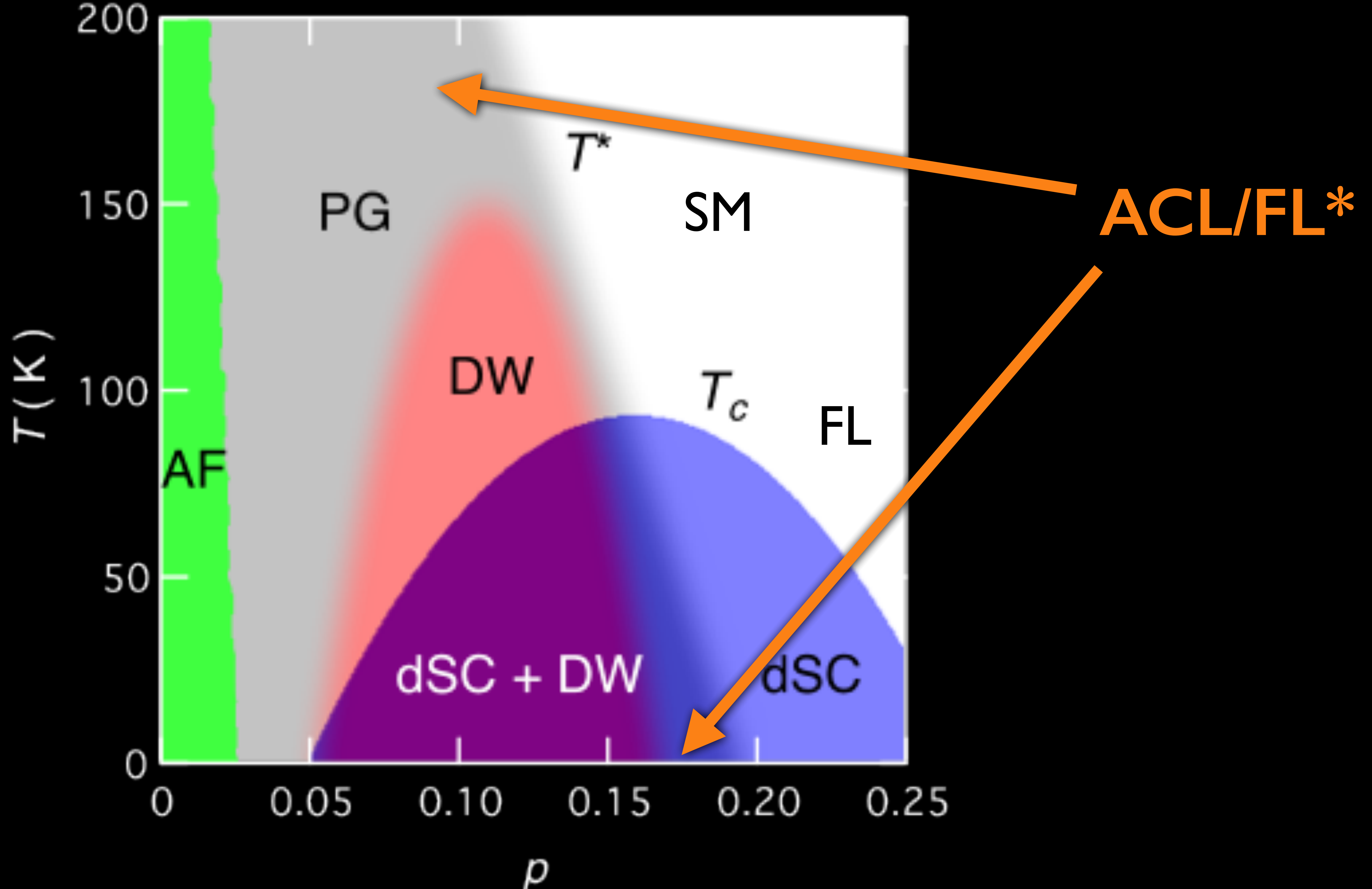
Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

Hall effect measurements in YBCO



Evidence for ACL/FL* metal with Fermi surface of size p !

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)



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dSC
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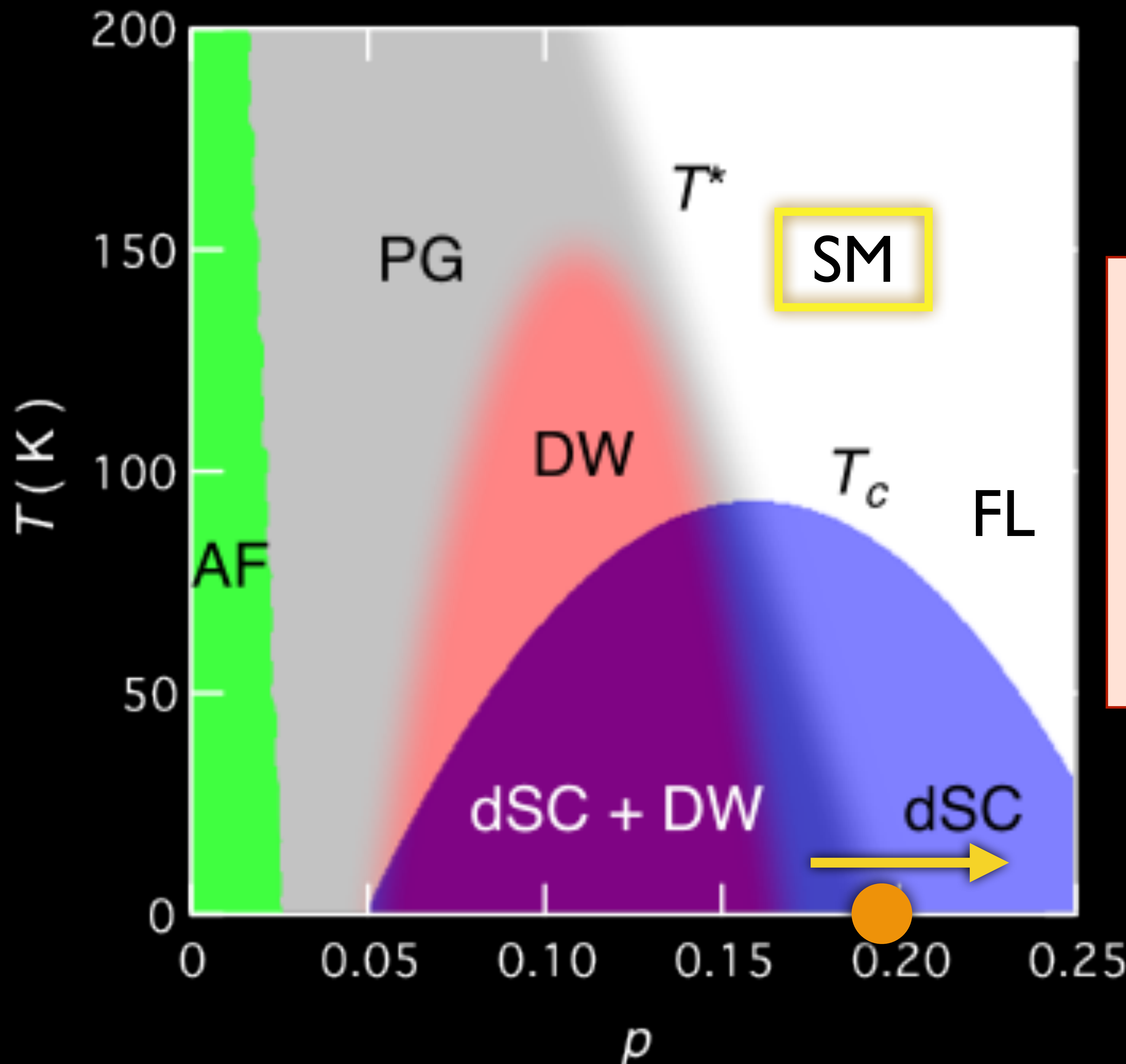

Transtion from ACL/FL* to FL as a theory of the strange metal (SM)

Quantum critical point at optimal doping

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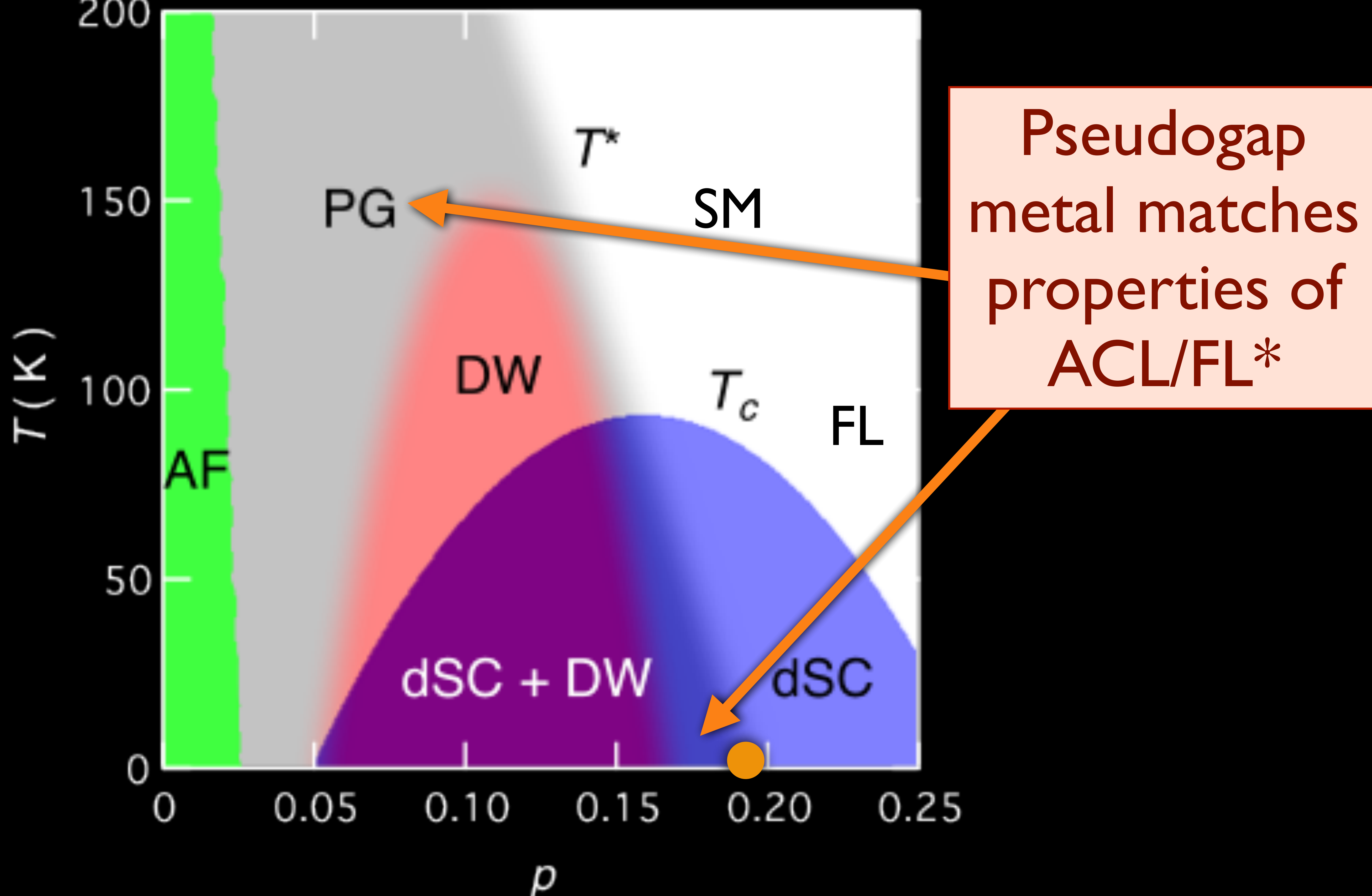
- Need a gauge theory for transition from "topological" to "confined" state.



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Mean field theory of a non-Fermi liquid ("strange metal")

Infinite-range model with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

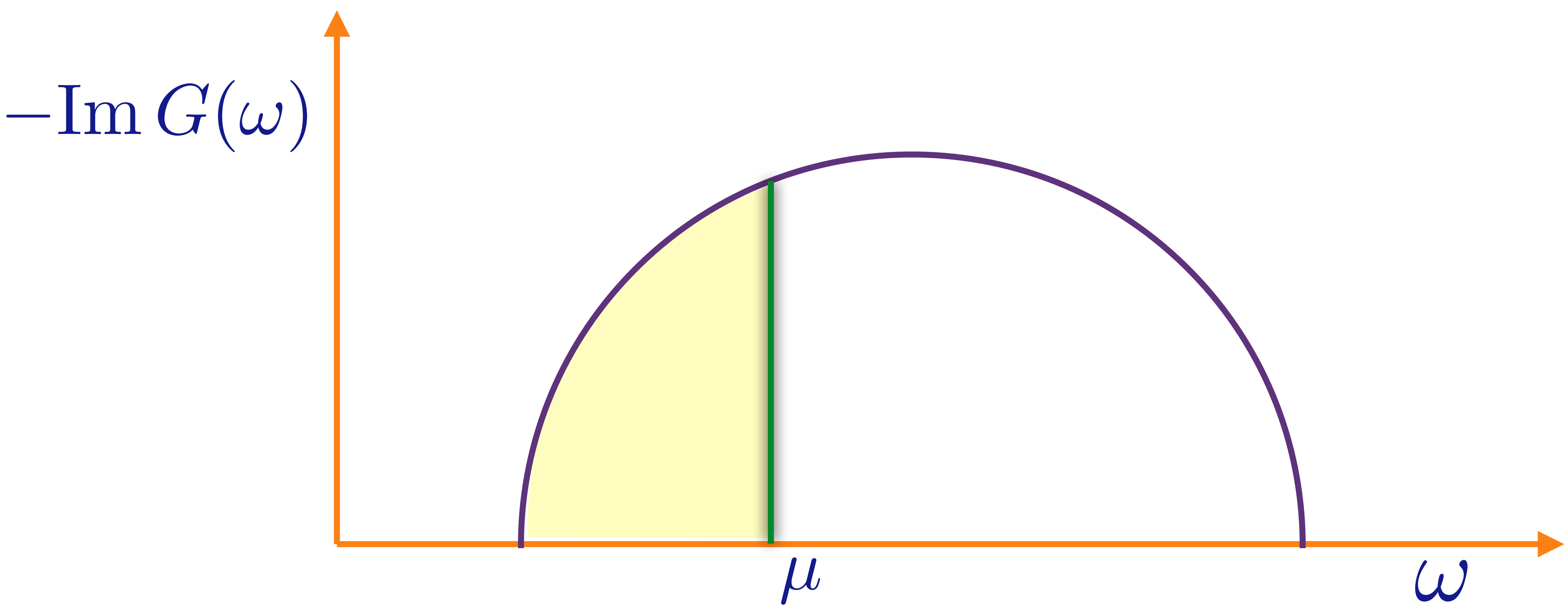
Fermions occupying the eigenstates of a
N x *N* random matrix

Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy E_α. By Fermi's Golden rule, for E_α at the Fermi energy

$$\frac{1}{\tau_\alpha} = \pi J^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta)$$
$$= \frac{\pi^3 J^2 \rho_0^2}{4} T^2$$

where ρ_0 is the density of states at the Fermi energy.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

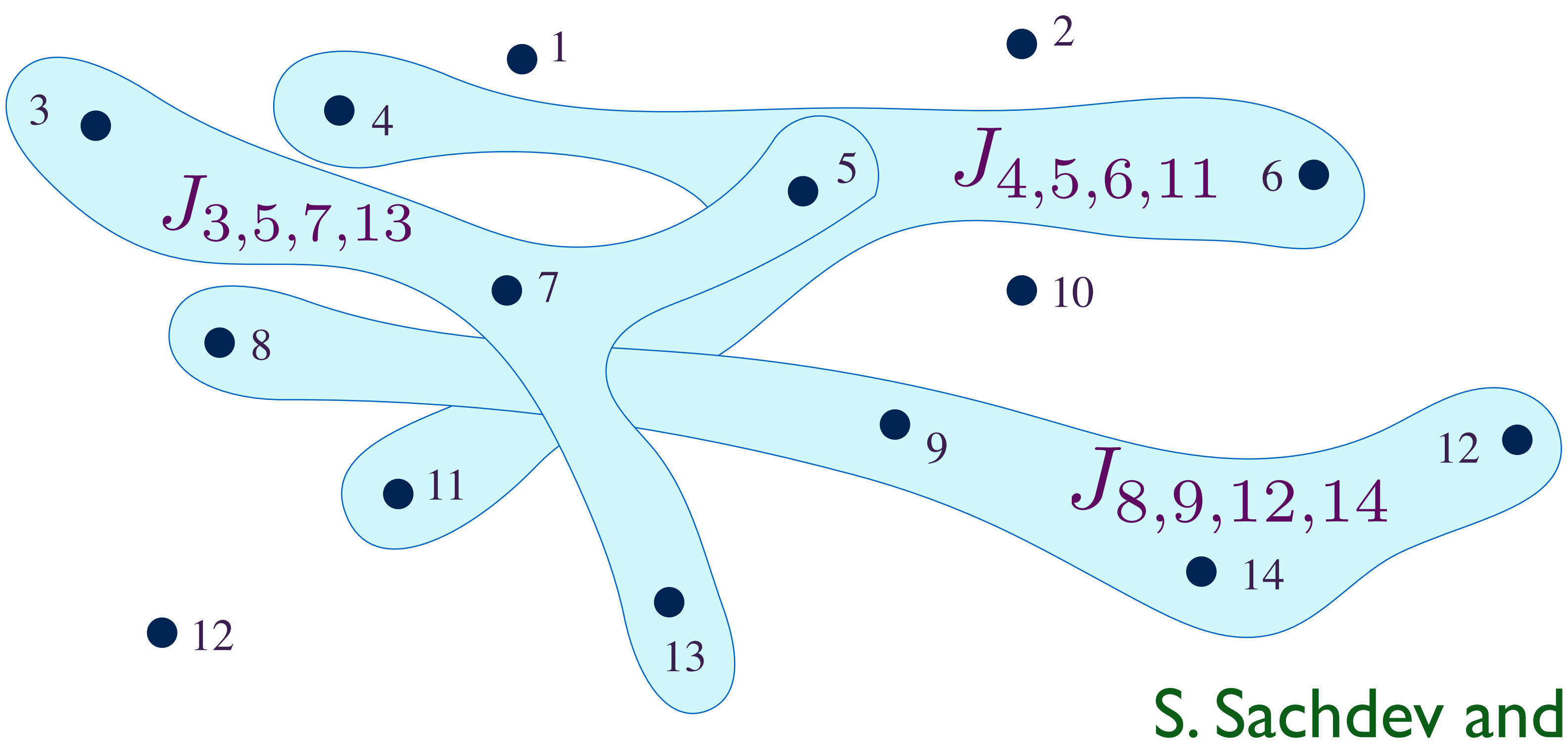
SYK model without quasiparticles

To obtain a non-Fermi liquid, we set $t_{ij} = 0$:

$$H_{\rm SYK} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$H_{\rm SYK}$ is similar, and has identical properties, to a related model proposed by SY in 1993.



S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

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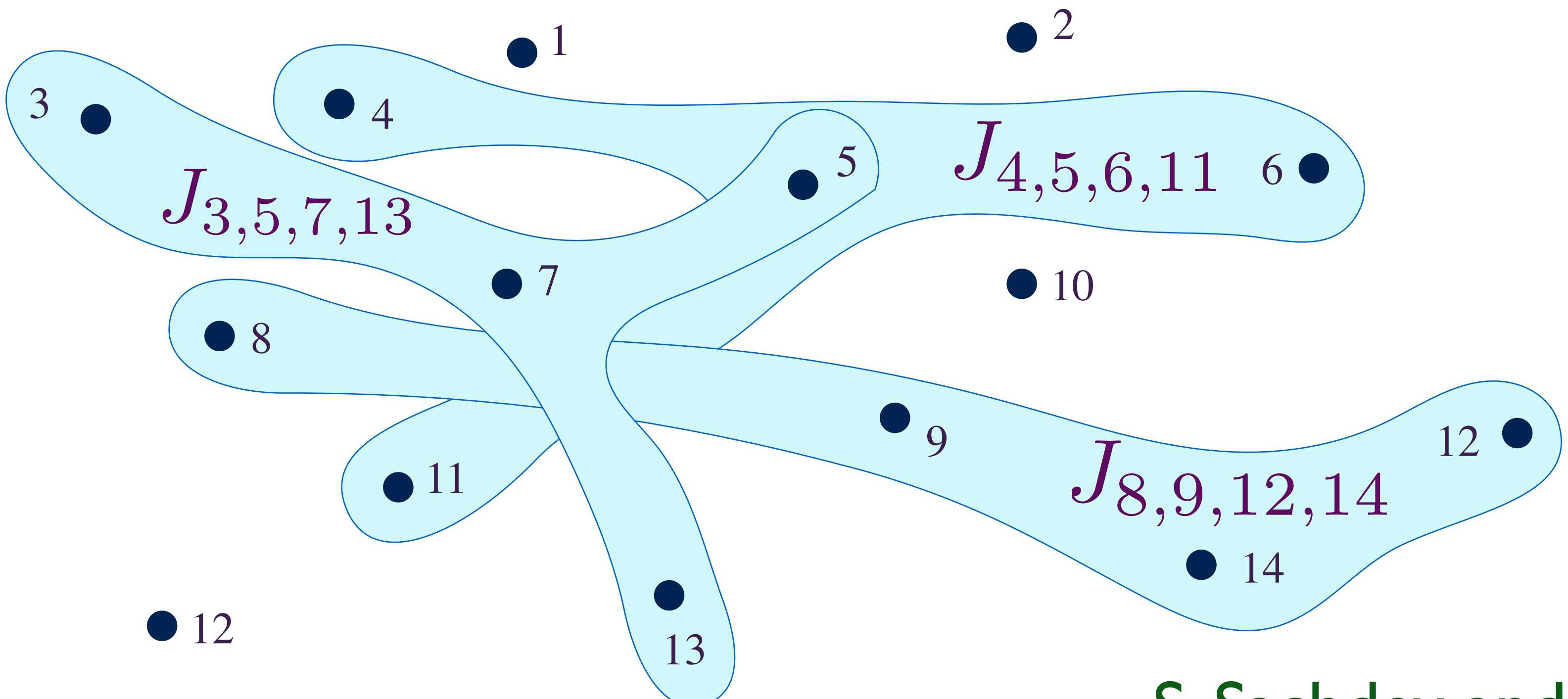

A fermion can move only by entangling with another fermion: the Hamiltonian has "nothing but entanglement".

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

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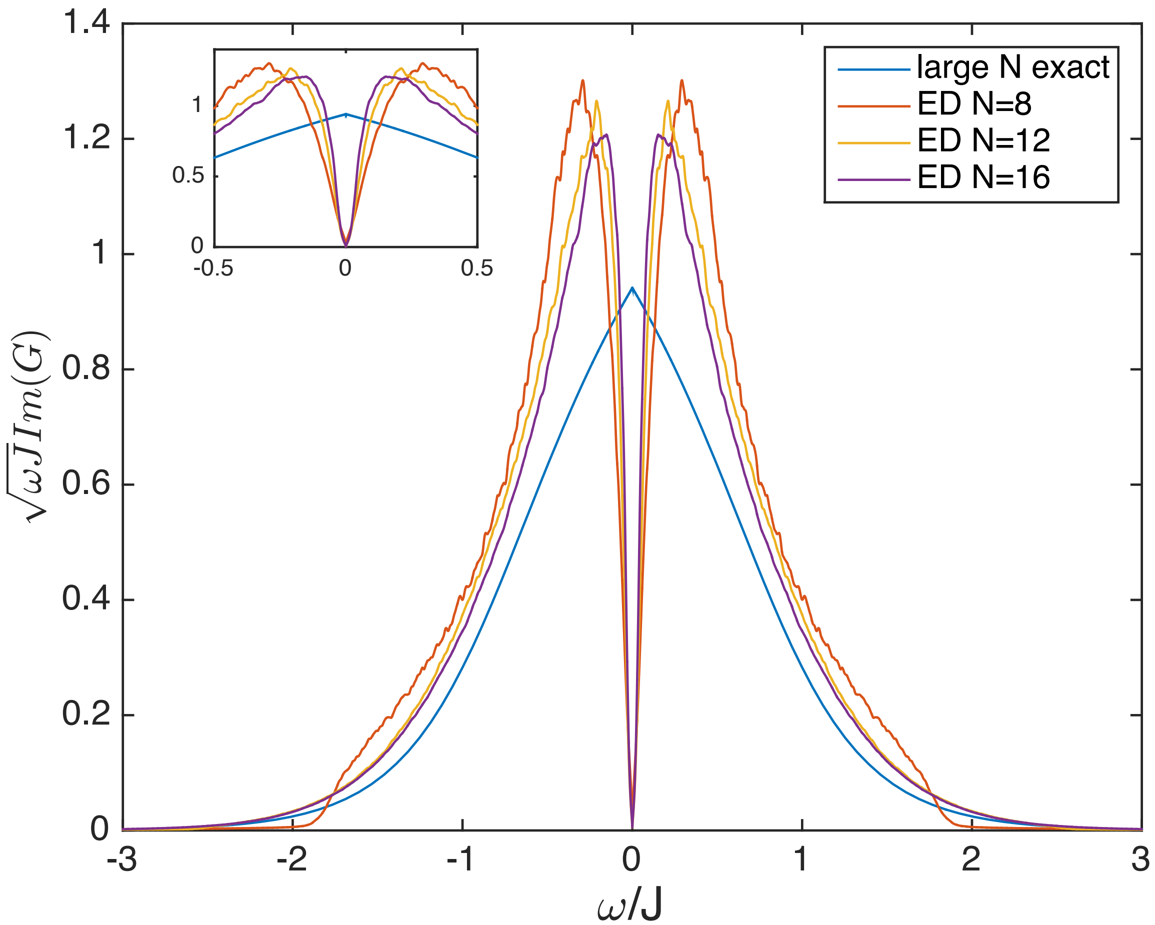
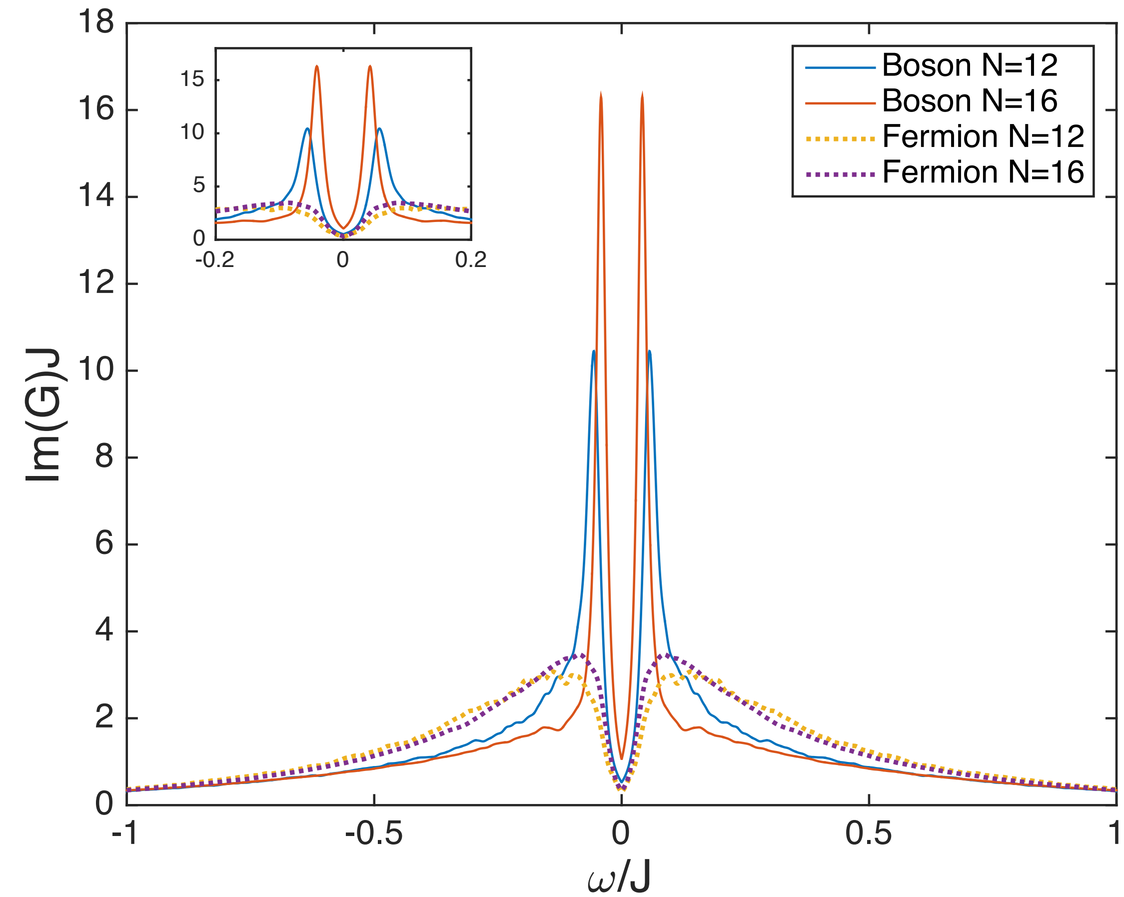




Large N solution of equations for G and Σ agree well with exact diagonalization of the finite N Hamiltonian.

However, exact diagonalization of the same model with hard-core bosons indicates the presence of spin-glass order in the ground state.

W. Fu and S. Sachdev, arXiv: 1603.05246

SYK model without quasiparticles

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$. This is *not* due to an extensive degeneracy, but due to an energy level spacing $\sim e^{-aN}$ in the entire many-body spectrum all the way down to the ground state. At low T we write

$$\mathcal{S}(T \to 0) = \mathcal{S}_0 + \gamma T + \ldots$$

where the specific heat is $\mathcal{C} = \gamma T$, and $\mathcal{S}_0$ obeys

$$\frac{d\mathcal{S}_0}{d\mathcal{Q}} = 2\pi\mathcal{E},$$

with $\mathcal{E}$ a spectral asymmetry parameter, which is a known function of $\mathcal{Q}$. $\mathcal{E}$ fully determines the Green's function at low T and ω as a ratio of Gamma functions.

Note that $\mathcal{S}_0$ and $\mathcal{E}$ involve low-lying states, while $\mathcal{Q}$ depends upon *all* states, and details of the UV structure.

A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)
J. Maldacena and D. Stanford, arXiv:1604.07818

Infinite-range (SYK) model without quasiparticles

After integrating the fermions, the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln \det \left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right]$$
$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left[G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, PRX **5**, 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

Infinite-range (SYK) model without quasiparticles

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

Reparametrization zero mode

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))$$

(and similarly for Σ) and obtain an effective action for $f(\tau)$. This action does not vanish because of the time derivative in the determinant which is not reparameterization invariant.

J. Maldacena and D. Stanford, arXiv:1604.07818

See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

Infinite-range (SYK) model without quasiparticles

However the effective action must vanish for SL(2,R) transformations because G_s, Σ_s are invariant under it. In this manner we obtain the effective action as a Schwarzian

$$N S_{\text{eff}} = -\frac{N\gamma}{4\pi^2} \int d\tau \, \{f, \tau\} \quad , \quad \{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2 ,$$

where the specific heat, $\mathcal{C} = \gamma T$.

The Schwarzian effective action implies that the SYK model *saturates* a lower bound on a Lyapunov time for many-body quantum chaos: this is time over which the quantum system loses memory of its initial state (the "butterfly effect")

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

Infinite-range (SYK) model without quasiparticles

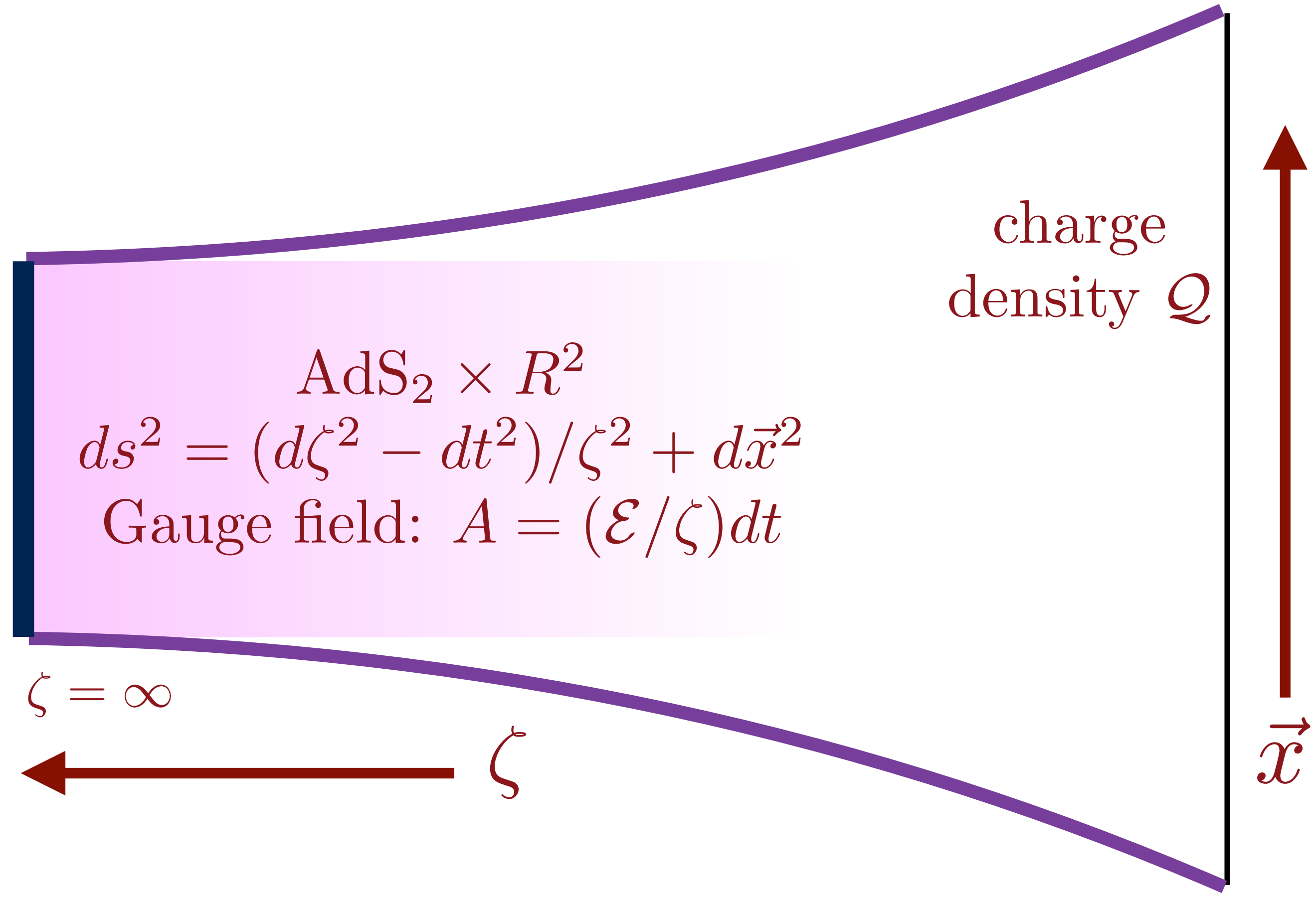
The Schwarzian describes fluctuations of the energy operator with scaling dimension $h = 2$.
Apart from the energy operator associated with the Schwarzian, there are an infinite number of other scalar operators with irrational scaling dimensions given by the roots of

$$\tan\left(\frac{\pi(2h-1)}{4}\right) = \frac{1-2h}{3}$$

$$\Rightarrow \quad h = 3.77354\ldots,\ 5.67946\ldots,\ 7.63197\ldots,\ 9.60396\ldots,\ldots$$

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

SYK and AdS_2



PHYSICAL REVIEW LETTERS 105, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

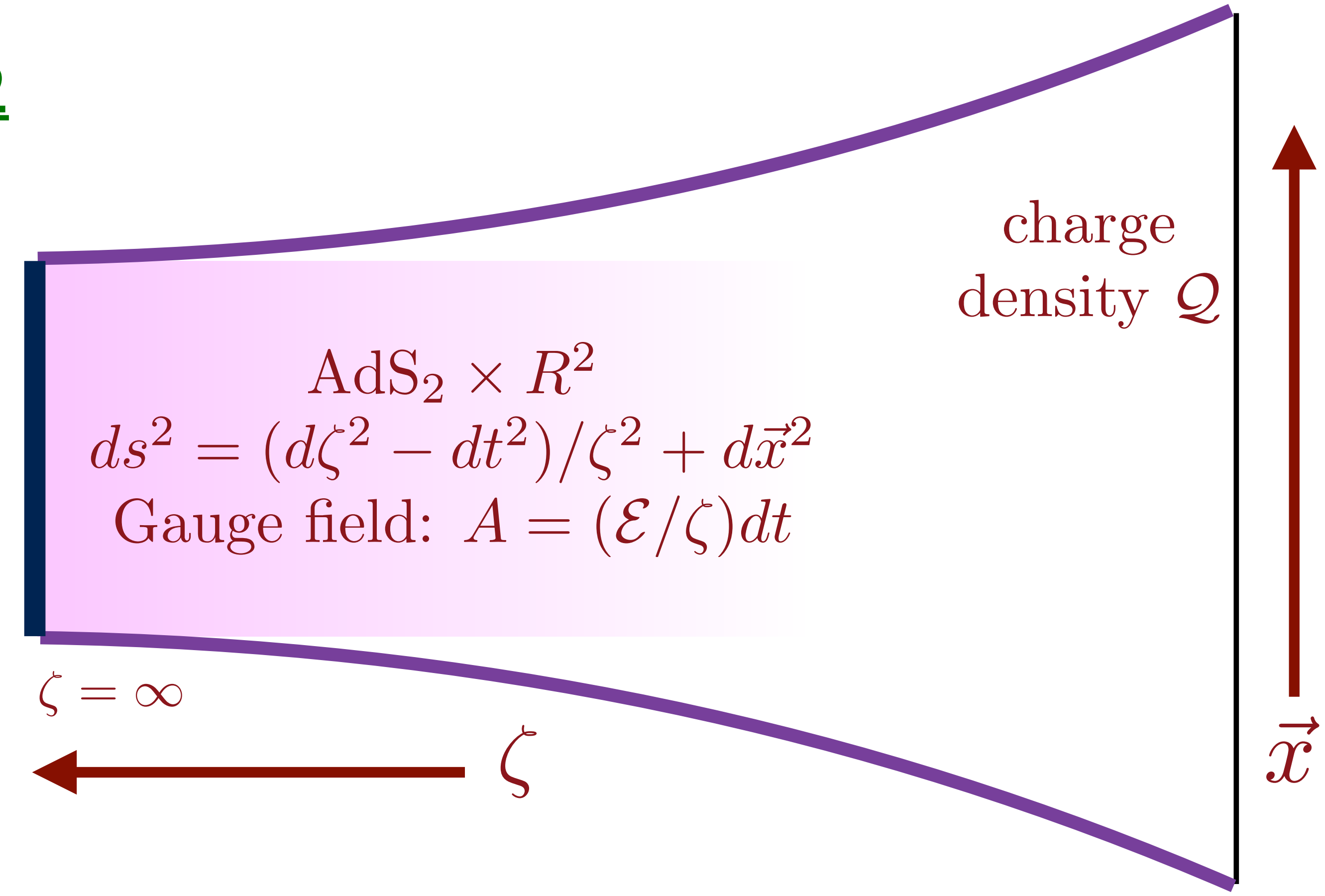
Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.

SYK and AdS_2



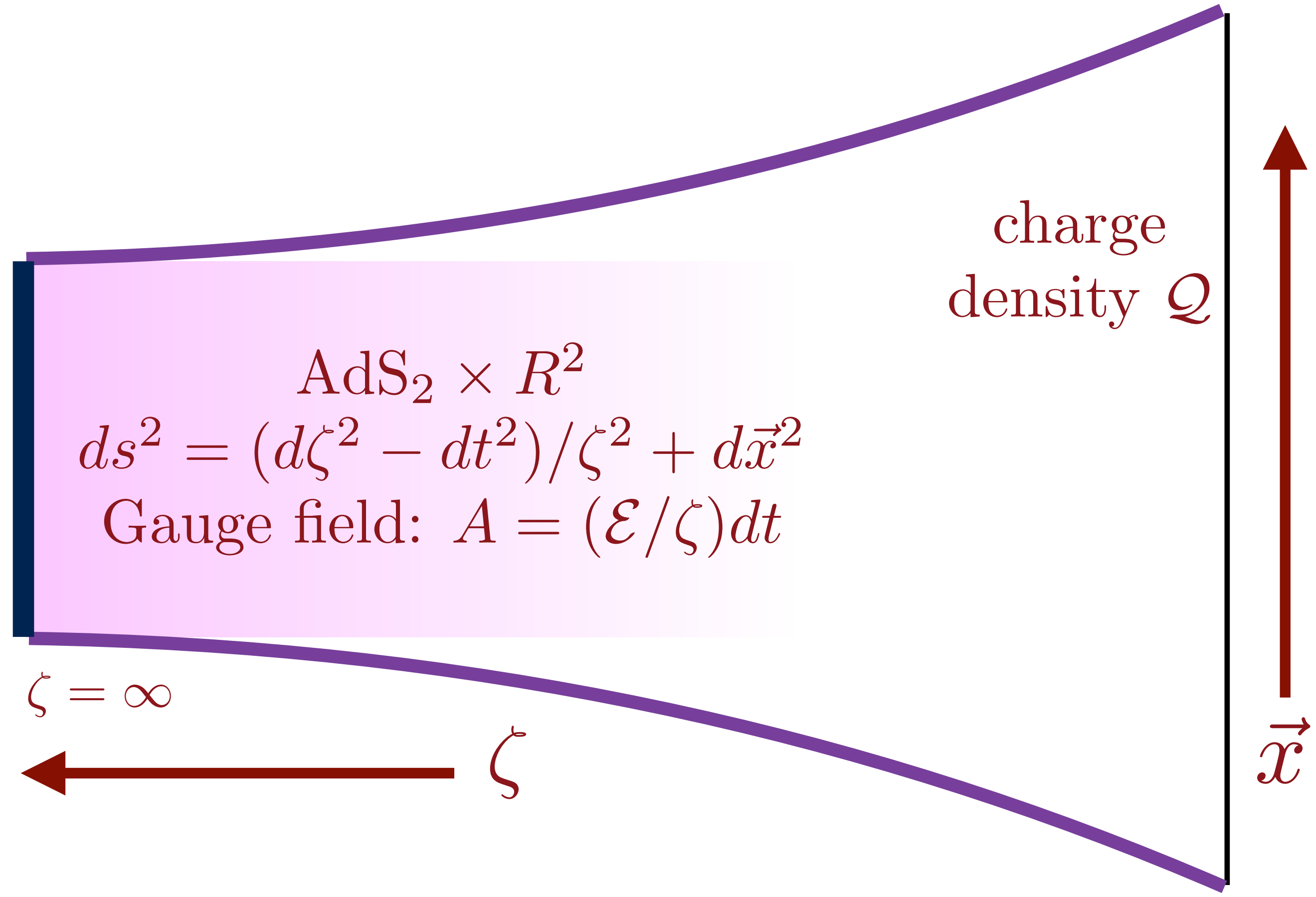
- The non-zero $T \to 0$ entropy density, $\mathcal{S}_0$, matches the Bekenstein-Hawking-Wald entropy density of extremal AdS_2 horizons, and the dependence of the fermion Green's function on ω, T, and $\mathcal{E}$, matches that of a Dirac fermion in AdS_2 (as computed by T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)).

S. Sachdev, PRL **105**, 151602 (2010)

- More recently, it was noted that the relation $d\mathcal{S}_0/d\mathcal{Q} = 2\pi\mathcal{E}$ also matches between SYK and gravity, where $\mathcal{E}$, the electric field on the horizon, also determines the spectral asymmetry of the Dirac fermion.

S. Sachdev, PRX **5**, 041025 (2015)

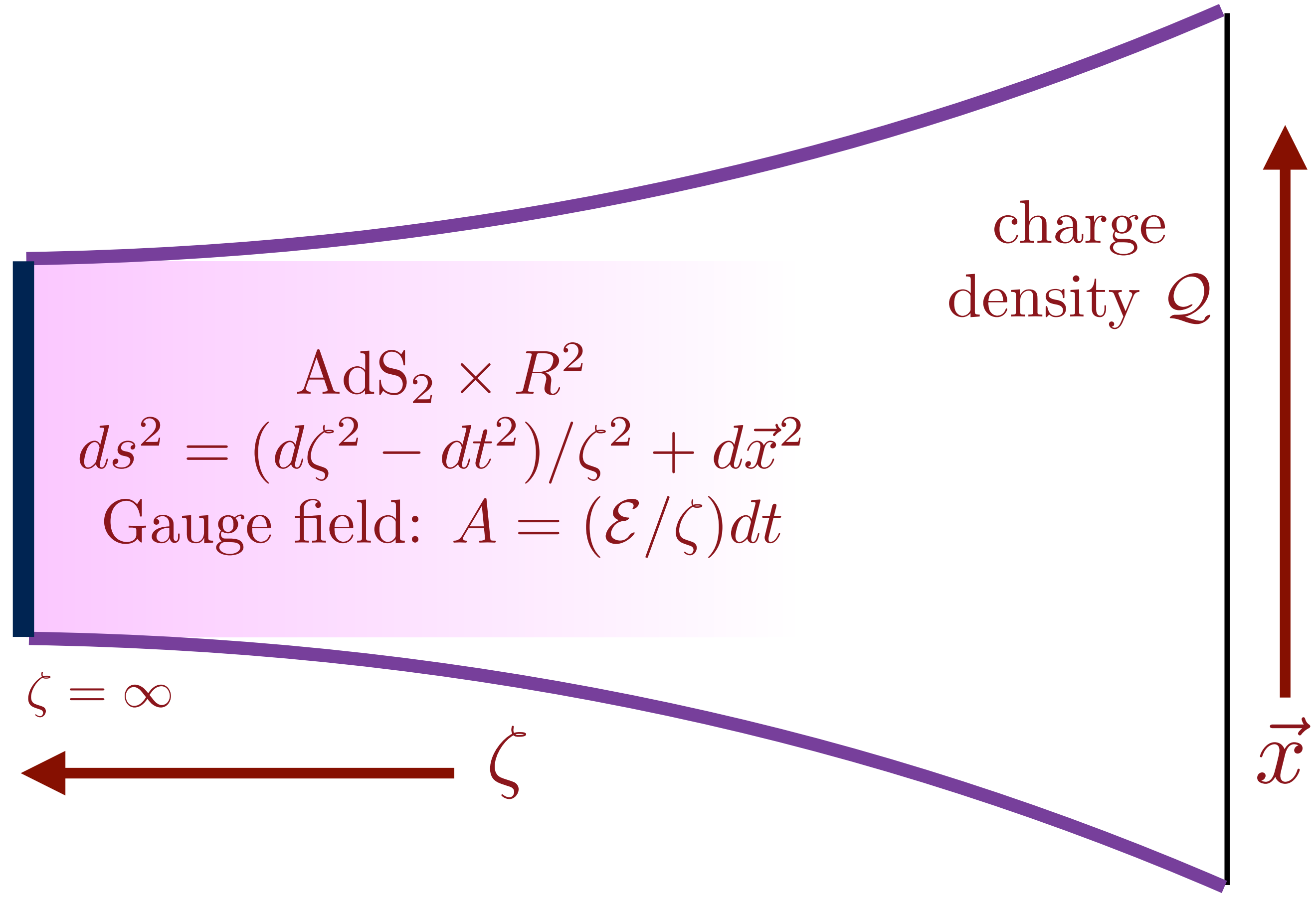
SYK and AdS_2



The same Schwarzian effective action describes low energy fluctuations on the boundary theory of gravity theories with AdS_2 near-horizon geometries (including the AdS-Reissner-Nordstrom solution of Einstein-Maxwell theory in 4 space-time dimensions). And the co-efficient of the Schwarzian, $N\gamma/4\pi^2$, determines the specific heat $\mathcal{C} = \gamma T$.

A. Kitaev, unpublished; A. Almheiri and J. Polchinski, JHEP **1511** (2015) 014; J. Polchinski and V. Rosenhaus, arXiv: 1601.06768; J. Maldacena and D. Stanford, arXiv:1604.07818; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438; A. Almheiri and B. Kang, arXiv: 1606.04108

SYK and AdS_2



The Schwarzian effective action implies that both the SYK model and the AdS_2 theories *saturate* the lower bound on the Lyapunov time

$$\tau_L = \frac{1}{2\pi}\frac{\hbar}{k_B T}.$$

A. Kitaev, unpublished; A. Almheiri and J. Polchinski, JHEP **1511** (2015) 014; J. Polchinski and V. Rosenhaus, arXiv: 1601.06768; J. Maldacena and D. Stanford, arXiv:1604.07818; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438; A. Almheiri and B. Kang, arXiv: 1606.04108