Quantum phase transitions in condensed matter

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Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. The holographic view: charged black-branes

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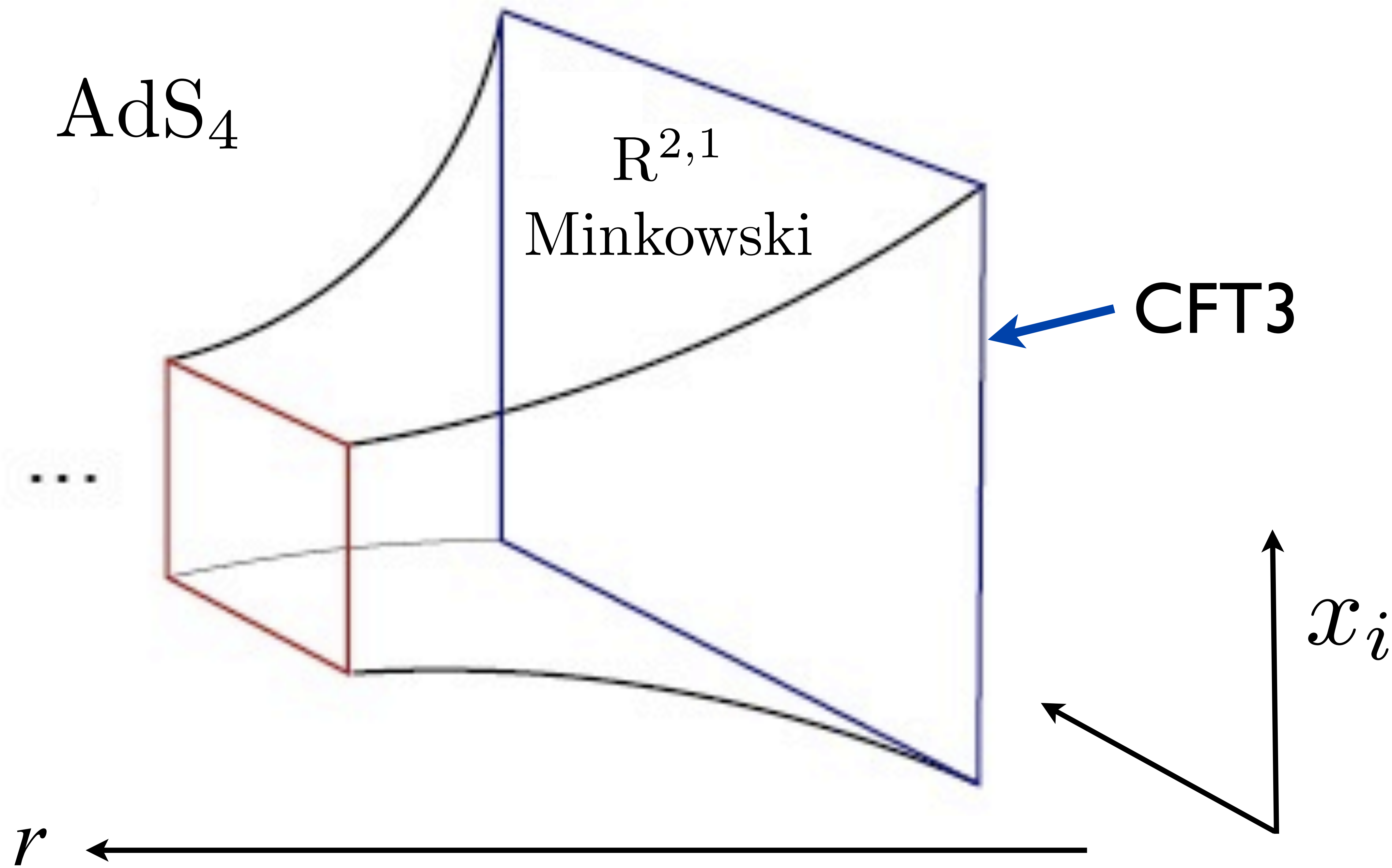
AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence at zero temperature

Suvrat Raju

Consider a CFT in D space-time dimensions with a scalar operator $O(\boldsymbol{x})$ with scaling dimension Δ. This is presumed to be equivalent to a dual gravity theory on AdS$_{D+1}$ with action $\mathcal{S}_{\text{bulk}}$. The CFT and the bulk theory are related by the GKPW ansatz

$$\int \mathcal{D}\phi \exp\left(-\mathcal{S}_{\text{bulk}}\right)\Big|_{\text{bdy}} = \left\langle \exp\left(\int d^D x\, \phi_0(\boldsymbol{x}) O(\boldsymbol{x})\right)\right\rangle_{\text{CFT}}$$

where the boundary condition is

$$\lim_{r\to 0} \phi(\boldsymbol{x}, r) = r^{D-\Delta}\phi_0(\boldsymbol{x}).$$

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We consider the simplest case of a single scalar field, where the bulk action is

$$\mathcal{S}_{\text{bulk}} = \frac{1}{2}\int d^{D+1}x\sqrt{g}\left[g^{ab}\partial_a\phi\partial_b\phi + m^2\phi^2\right]$$

where g_{ab} is the AdS_{D+1} metric (we are working with a Euclidean signature, and a, b extend over $D+1$ dimensions) and $g = \det(g_{ab})$. After Fourier transforming space-time co-ordinates to momenta $\boldsymbol{k}$, the saddle-point equation for $\phi(\boldsymbol{k}, r)$ is

$$-r^{D-1}\frac{d}{dr}\left(\frac{1}{r^{D-1}}\frac{d\phi}{dr}\right) + \left(k^2 + \frac{m^2}{r^2}\right)\phi = 0.$$

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This equation has two solutions as $r \to 0$, with $\phi \sim r^{\Delta}$ or $\phi \sim r^{D-\Delta}$ where

$$\Delta = \frac{D}{2} \pm \sqrt{\frac{D^2}{4} + m^2}.$$

We will choose the positive sign in the manipulations below, but the final results hold for both signs. The complete solution of the saddle-point equation with the needed boundary condition can be written as

$$\phi(\boldsymbol{k}, r) = G_{\text{bulk}-\text{bdy}}(k, r)\phi_0(\boldsymbol{k})$$

where

$$G_{\text{bulk}-\text{bdy}}(k, r) = \frac{2^{1-\Delta+D/2}}{\Gamma(\Delta - D/2)} k^{\Delta - D/2} r^{D/2} K_{\Delta - D/2}(kr)$$

where $K_{\Delta-D/2}$ is a modified Bessel function.

Next, it is useful to obtain the bulk-bulk Green's function by inverting the operator in the equation of motion. A standard computation yields

$$G_{\text{bulk}-\text{bulk}}(k, r_1, r_2) = (r_1 r_2)^{D/2} I_{\Delta-D/2}(kr_<) K_{\Delta-D/2}(kr_>)$$

where $r_<$ ($r_>$) is the smaller (larger) or $r_{1,2}$. This bulk-bulk Green's function is evaluated in the absence of any sources on the boundary, and so we have to impose

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the boundary condition $\phi(\boldsymbol{k}, r) \sim r^{\Delta}$ as $r \to 0$ in solving the saddle-point equation. The utility of this bulk-bulk Green's function is that it now allows us to extend our results to include interactions in $\mathcal{S}_{\rm bulk}$ by the usual Feynman graph expansion. We can account for the presence of the boundary source $\phi_0(\boldsymbol{k})$ in the CFT by imagining there is a corresponding bulk source field $J_0(\boldsymbol{k}, r)$ which is localized at very small values of r. Then this bulk source field will generate a bulk $\phi(\boldsymbol{k}, r)$ via the propagator $G_{\rm bulk-bulk}$. We now note that

$$\lim_{r_2 \to 0} G_{\rm bulk-bulk}(k, r_1, r_2) = \frac{r_2^{\Delta}}{(2\Delta - D)} G_{\rm bulk-bdy}(k, r_1).$$

This is a key relation which shows us that functional derivatives of the full action w.r.t. $J_0(\boldsymbol{k}, r)$ (which yield bulk-bulk correlation functions) are the *same* as functional derivatives w.r.t. $\phi_0(\boldsymbol{k})$ (which yield correlators of the CFT). This yields the second statement of the equivalence between the bulk and boundary theories

$$\langle O(\boldsymbol{x}_1) \ldots O(\boldsymbol{x}_n) \rangle_{\rm CFT} = Z^n \lim_{r \to 0} r_1^{-\Delta} \ldots r_n^{-\Delta} \langle \phi(\boldsymbol{x}_1, r_1) \ldots \phi(\boldsymbol{x}_n, r_n) \rangle_{\rm bulk}$$

where the "wave function renormalization" factor $Z = (2\Delta - D)$. Note that this relationship holds for arbitrary bulk actions, and permits full quantum fluctuations in the bulk theory. Also, both correlators are evaluated in the absence of external sources; for the bulk theory this means that we have the boundary condition

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$\phi(\boldsymbol{k}, r) \sim r^{\Delta}$ as $r \to 0$. From this general relation we can evaluate the two-point correlator of the CFT for the case of a bulk Gaussian action:

$$\begin{aligned} \langle O(\boldsymbol{k}) O(-\boldsymbol{k}) \rangle_{\mathrm{CFT}} &= Z^2 \lim_{r \to 0} (r)^{-2\Delta} G_{\mathrm{bulk-bulk}}(k, r, r) \\ &= \lim_{r \to 0} (2\Delta - D) r^{-(2\Delta - D)} - (2\Delta - D) \left(\frac{k}{2}\right)^{2\Delta - D} \frac{\Gamma(1 - \Delta + D/2)}{\Gamma(1 + \Delta - D/2)} \end{aligned}$$

The first term is divergent, but it is independent of k: so it does not contribute to the long-distance correlations of the CFT, and can be dropped. The second term has the singular dependence $\sim k^{2\Delta - D}$, which is just as expected for a field with scaling dimension Δ, for the Fourier transformation yields

$$\langle O(\boldsymbol{x}_1) O(\boldsymbol{x}_2) \rangle_{\mathrm{CFT}} \sim |\boldsymbol{x}_1 - \boldsymbol{x}_2|^{-2\Delta}.$$

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The final formulation of the bulk-boundary correspondence appears by using the above relations for arbitrary multi-point correlators in the absence of a source, to make a statement for the one-point function in the presence of a source, working to all orders in the source and all bulk interactions. As we noted earlier, the CFT source $\phi_0(\boldsymbol{k})$ can be simulated by a source $J_0(\boldsymbol{k}, r)$ which is localized near the boundary but acts on the bulk theory. Because of the identity above between the source-free correlators, we can conclude that $\langle O(\boldsymbol{x})\rangle$ equals $Z \lim_{r\to 0} r^{-\Delta}\langle\phi(\boldsymbol{x}, r)\rangle$. However, we have to remember that the source $J_0(\boldsymbol{k}, r)$ is actually realized by a boundary condition on $\phi(\boldsymbol{x}, r)$, and so the complete statement is

$$\lim_{r\to 0}\langle\phi(\boldsymbol{x}, r)\rangle = r^{D-\Delta}\phi_0(\boldsymbol{x}) + \frac{r^{\Delta}}{Z}\langle O(\boldsymbol{x})\rangle,$$

in the presence of the source $\phi_0(\boldsymbol{x})$. Note that this result is not just linear response, and holds to all orders in the source; it also allows for arbitrary bulk interactions and quantum fluctuations. It can be checked that it is indeed obeyed by the correlators above of the Gaussian theory. This relationship is frequently used in practice, because it is often straightforward to implement, especially when we are using the classical saddle-point approximation for the bulk theory. Then we simply have to extract the subleading behavior in $\phi(\boldsymbol{x}, r)$ as $r \to 0$ to extract the full non-linear response function to the perturbation $\phi_0(\boldsymbol{x})$ to the CFT.

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A similar analysis can be applied to operators of the CFT with non-zero Lorentz spin. Of particular interest to are correlators of a conserved current, J_μ, associated with a global 'flavor' symmetry, and the conserved stress energy tensor $T_{\mu\nu}$.

We couple the conserved current to a source a_μ and so are interested in evaluating

$$\mathcal{Z}(a_\mu) = \left\langle \exp\left(\int d^D x\, a_\mu(\boldsymbol{x}) J_\mu(\boldsymbol{x})\right)\right\rangle_{\text{CFT}}.$$

The conservation law $\partial_\mu J_\mu = 0$ now implies that this partition function is invariant under the gauge transformation $a_\mu \rightarrow a_\mu + \partial_\mu \alpha$. So the bulk field dual to a (say) U(1) conserved current J_μ is a U(1) gauge field, which we denote $A_a(\boldsymbol{x}, r)$. We assume the gauge field has a Maxwell action

$$\mathcal{S}_M = \frac{1}{4 g_M^2} \int d^{D+1}x\, \sqrt{g} F_{ab} F^{ab}$$

plus other possible gauge couplings to the bulk fields. By an analysis very similar to the scalar field, we can establish the following bulk-boundary correspondences

$$\begin{aligned}
\lim_{r\to 0} \langle A_\mu(\boldsymbol{x}, r)\rangle &= a_\mu(\boldsymbol{x}) + \frac{r^{D-2}}{Z g_M^{-2}} \langle J_\mu(\boldsymbol{x})\rangle \\
\lim_{r\to 0} \langle A_r(\boldsymbol{x}, r)\rangle &= 0 \\
\langle J_\mu(\boldsymbol{x}_1) \ldots J_\nu(\boldsymbol{x}_n)\rangle_{\text{CFT}} &= (Z g_M^{-2})^n \lim_{r\to 0} r_1^{2-D} \ldots r_n^{2-D} \langle A_\mu(\boldsymbol{x}_1, r_1) \ldots A_\nu(\boldsymbol{x}_n, r_n)\rangle_{\text{bulk}}
\end{aligned}$$

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with $Z = D - 2$. Working with only the Maxwell action these relations yield

$$\langle J_\mu(\boldsymbol{k}) J_\nu(\boldsymbol{k}) \rangle_{\text{CFT}} = \frac{(D-2)}{g_M^2} \frac{\Gamma(2 - D/2)}{\Gamma(D/2)} \left(\frac{k}{2}\right)^{D-2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

This is precisely the expected form for the correlator of a conserved current in a CFT in D space-time dimensions. For the case $D = 3$ it has the expected form

$$\langle J_\mu(\boldsymbol{k}) J_\nu(\boldsymbol{k}) \rangle_{\text{CFT}} = \mathcal{K}\, k \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

with

$$\mathcal{K} = \frac{1}{g_M^2}.$$

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A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the metric of the bulk theory. Now the needed partition function is

$$\mathcal{Z}(\chi_{\mu\nu}) = \left\langle \exp\left(\int d^D x \, \chi_{\mu\nu}(\boldsymbol{x}) T_{\mu\nu}(\boldsymbol{x}) \right) \right\rangle_{\rm CFT}$$

and the source is related to the metric g_{ab} via

$$\begin{aligned} \lim_{r\to 0} g_{rr} &= \frac{L^2}{r^2} \\ \lim_{r\to 0} g_{r\mu} &= 0 \\ \lim_{r\to 0} g_{\mu\nu} &= \frac{L^2}{r^2}\left(\delta_{\mu\nu} + \chi_{\mu\nu}\right) \end{aligned}$$

The bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(\boldsymbol{x}_1)\ldots T_{\rho\sigma}(\boldsymbol{x}_n)\rangle_{\rm CFT} = \left(\frac{ZL^2}{\kappa^2}\right)^n \lim_{r\to 0} r_1^{-D}\ldots r_n^{-D}\,\langle \chi_{\mu\nu}(\boldsymbol{x}_1, r_1)\ldots \chi_{\rho\sigma}(\boldsymbol{x}_n, r_n)\rangle_{\rm bul}$$

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with $Z = D$. Applying this prescription to the Einstein action, we obtain in $D = 3$

$$\langle T_{\mu\nu}(\boldsymbol{k}) T_{\rho\sigma}(-\boldsymbol{k})\rangle_{\rm CFT} = \mathcal{C}_T |k|^3 \Big(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\nu\rho}\delta_{\mu\sigma} - \delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\nu}\frac{k_\rho k_\sigma}{k^2} + \delta_{\rho\sigma}\frac{k_\mu k_\nu}{k^2} - \delta_{\mu\rho}\frac{k_\nu k_\sigma}{k^2} - \delta_{\nu\rho}\frac{k_\mu k_\sigma}{k^2} - \delta_{\mu\sigma}\frac{k_\nu k_\rho}{k^2} - \delta_{\nu\sigma}\frac{k_\mu k_\rho}{k^2} + \frac{k_\mu k_\nu k_\rho k_\sigma}{k^4} \Big)$$

This is the most-general form expected for any CFT, and the "central charge" is related to a dimensionless combination of the gravitational constant and the AdS radius

$$\mathcal{C}_T \propto \frac{L^2}{2\kappa^2}.$$

AdS/CFT correspondence at zero temperature

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So, to recapitulate, we have equated the correlators of the CFT3 to a bulk theory on AdS_4 with the Einstein-Hilbert action

$$\mathcal{S} = \frac{1}{4g_M^2}\int d^4x\,\sqrt{g}F_{ab}F^{ab} + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right].$$

This action is characterized by two dimensionless parameters: g_M and L^2/κ^2. These parameters determine, respectively, the two-point correlators of a conserved U(1) current J_μ and the stress-energy tensor $T_{\mu\nu}$.

However, this action is non-linear, and it also implies non-zero multipoint correlators of these operators, even at tree-level in the bulk theory. For the simplest 3-point correlator, a lengthy computation from the bulk theory yields

$$\langle J_\mu(\boldsymbol{k}_1)J_\nu(\boldsymbol{k}_2)T_{\rho\sigma}(-\boldsymbol{k}_1-\boldsymbol{k}_2)\rangle \sim \frac{k_1k_2}{(k_1+k_2)^5}k_{1\mu}k_{1\nu}k_{1\rho}k_{1\sigma} + 175\text{ terms}$$

with co-efficients determined by g_M and L^2/κ^2.

We can now compare this 3-point correlator with that obtained by direct computation on a CFT3. A general analysis of the constraints from conformal invariance (Osborn and Petkou, 1993) shows that this 3-point correlator is fully determined by the values $\mathcal{K}$, C_T, and *one* additional dimensionless constant which is characteristic of the CFT3.

AdS/CFT correspondence at zero temperature

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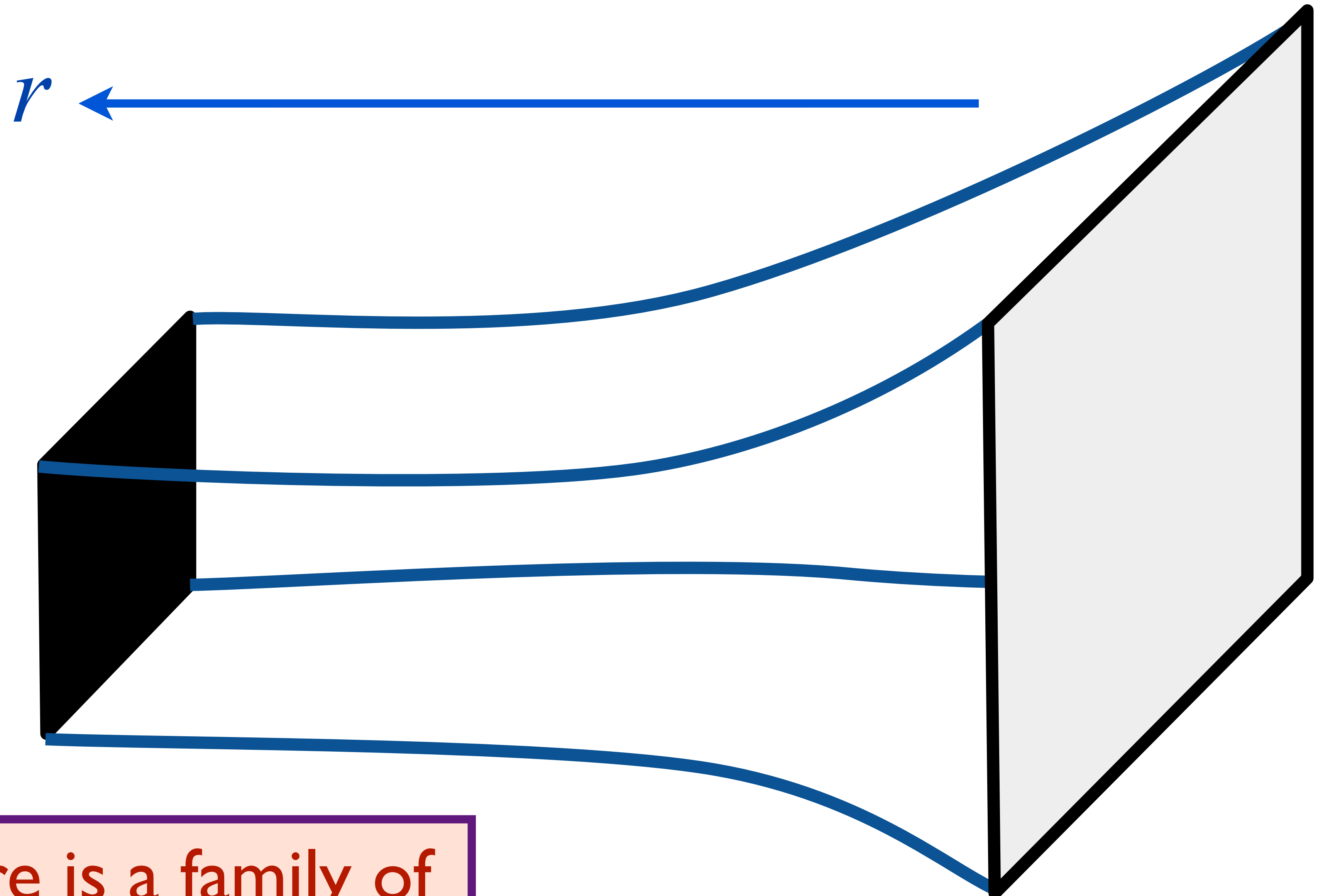
To fix this additional constant by the bulk theory, we have to go beyond the Einstein-Maxwell action. This action is the simplest action with up to 2 derivatives of the bulk fields. So, in the spirit of effectively field theory, let us now include all terms up to 4 derivatives. We want to work in linear response for the conserved current, and so we exclude terms which have more than 2 powers of F_{ab}. Then, up to some field redefinitions, there turns out to be a unique 4 derivative term, and the extend action of the bulk theory now becomes

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

where C_{abcd} is the Weyl tensor. Now we have a new dimensionless parameter, γ; stability constraints on this action restrict $|\gamma| < 1/12$. The Weyl tensor vanishes on the AdS metric, and consequently γ does not modify the previous results on the 2-point correlators of J_μ and $T_{\mu\nu}$. However, γ does change the structure of the 3-point correlator. We computed γ in a large-flavor-number expansion for a class of CFT3s (D. Chowdhury, S. Raju, S. Sachdev, A Singh, and P. Strack, arXiv:1210.5247), and found $\gamma = 1/12$ for free fermions, $\gamma = -1/12$ for free scalars, and $\gamma = 1/12$ for the topological current of the U(1) gauge field in the Abelian Higgs model.

It is clear that similar results apply at higher orders: matching higher multipoint correlators of the CFT requires higher derivative terms in the effective bulk theory.

AdS_4-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence at non-zero temperatures

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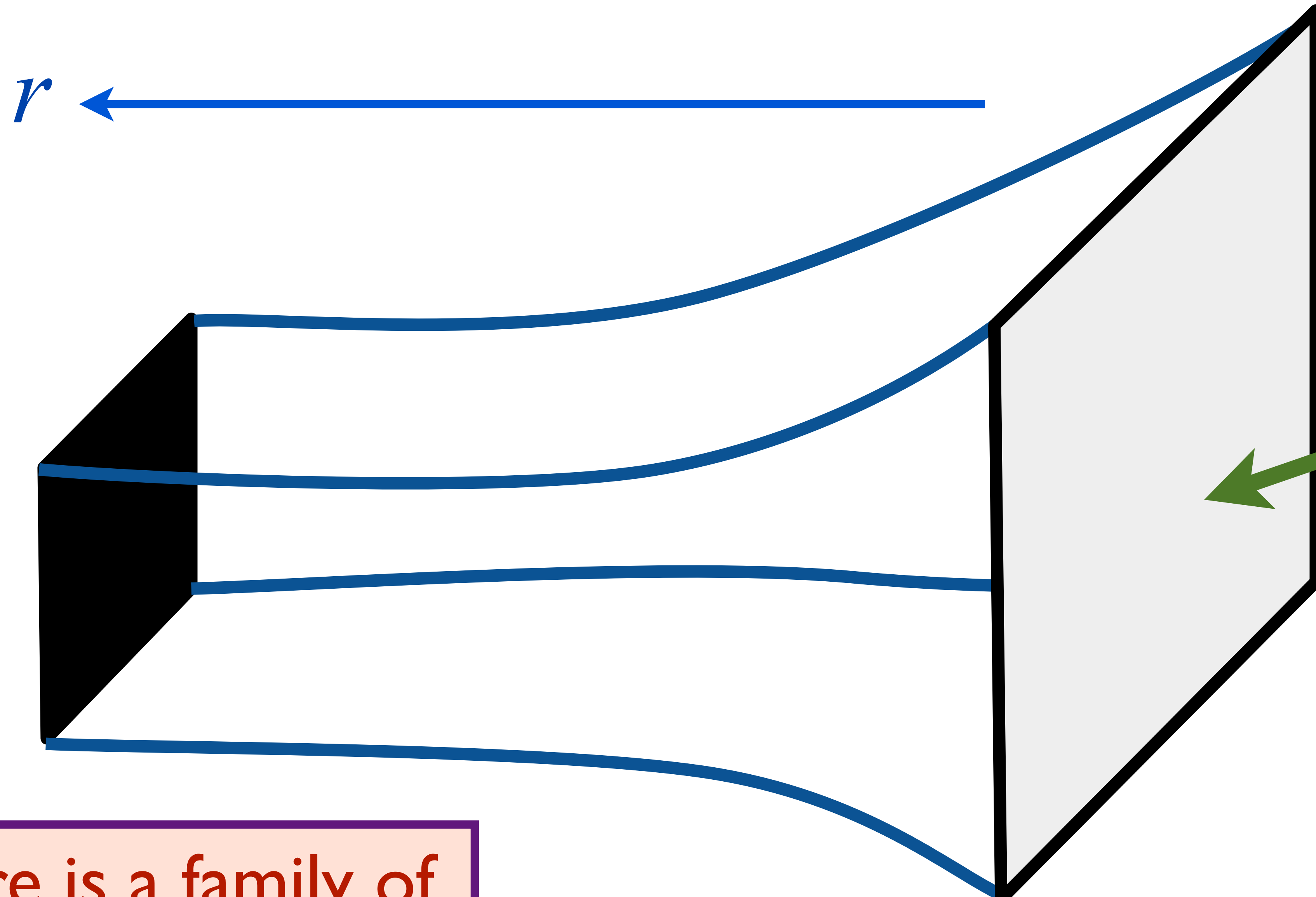

A 2+1 dimensional system at its quantum critical point: $k_B T = \frac{3\hbar}{4\pi R}$.

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$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

AdS/CFT correspondence at non-zero temperatures

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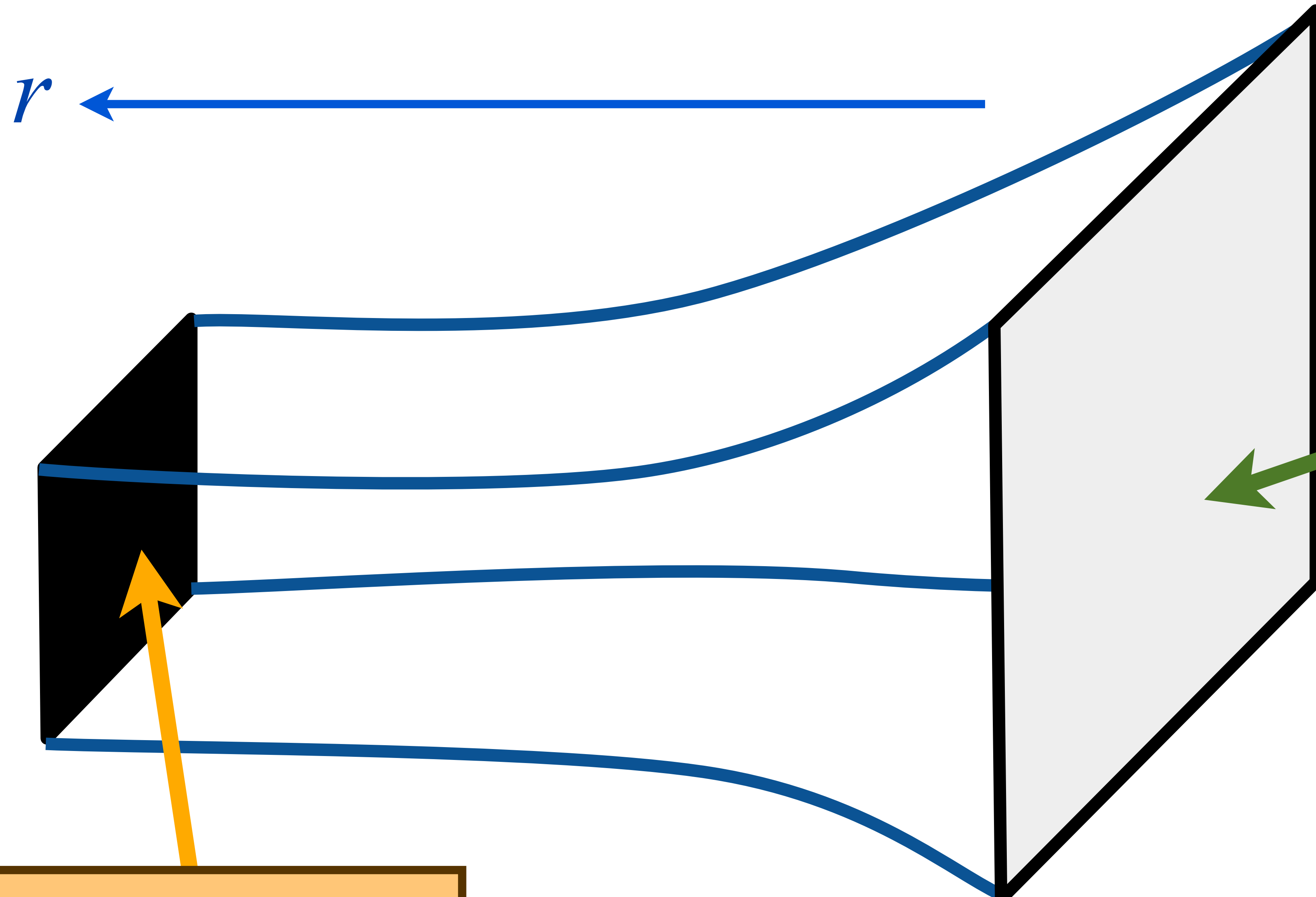

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Black-brane at temperature of 2+1 dimensional quantum critical system

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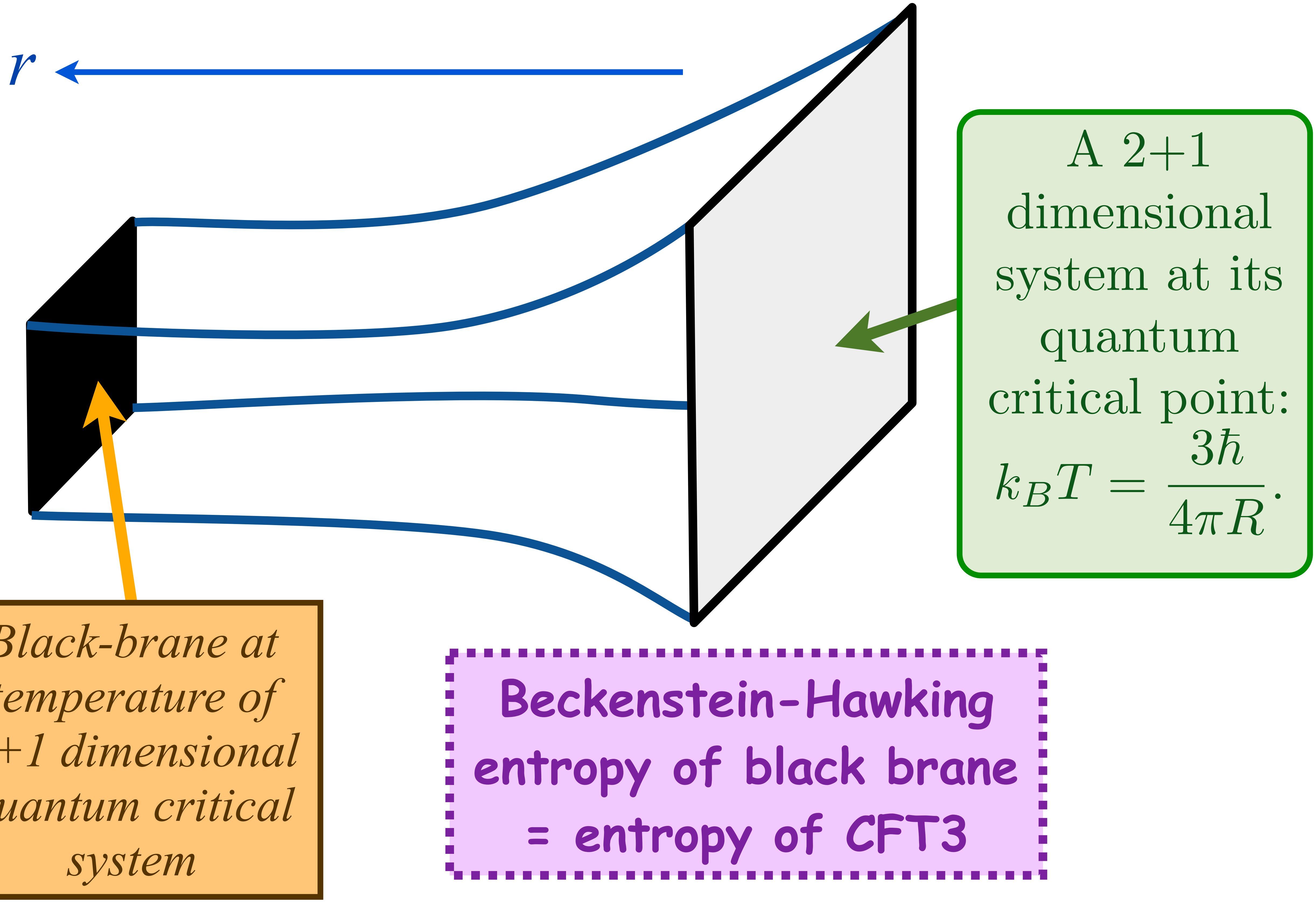

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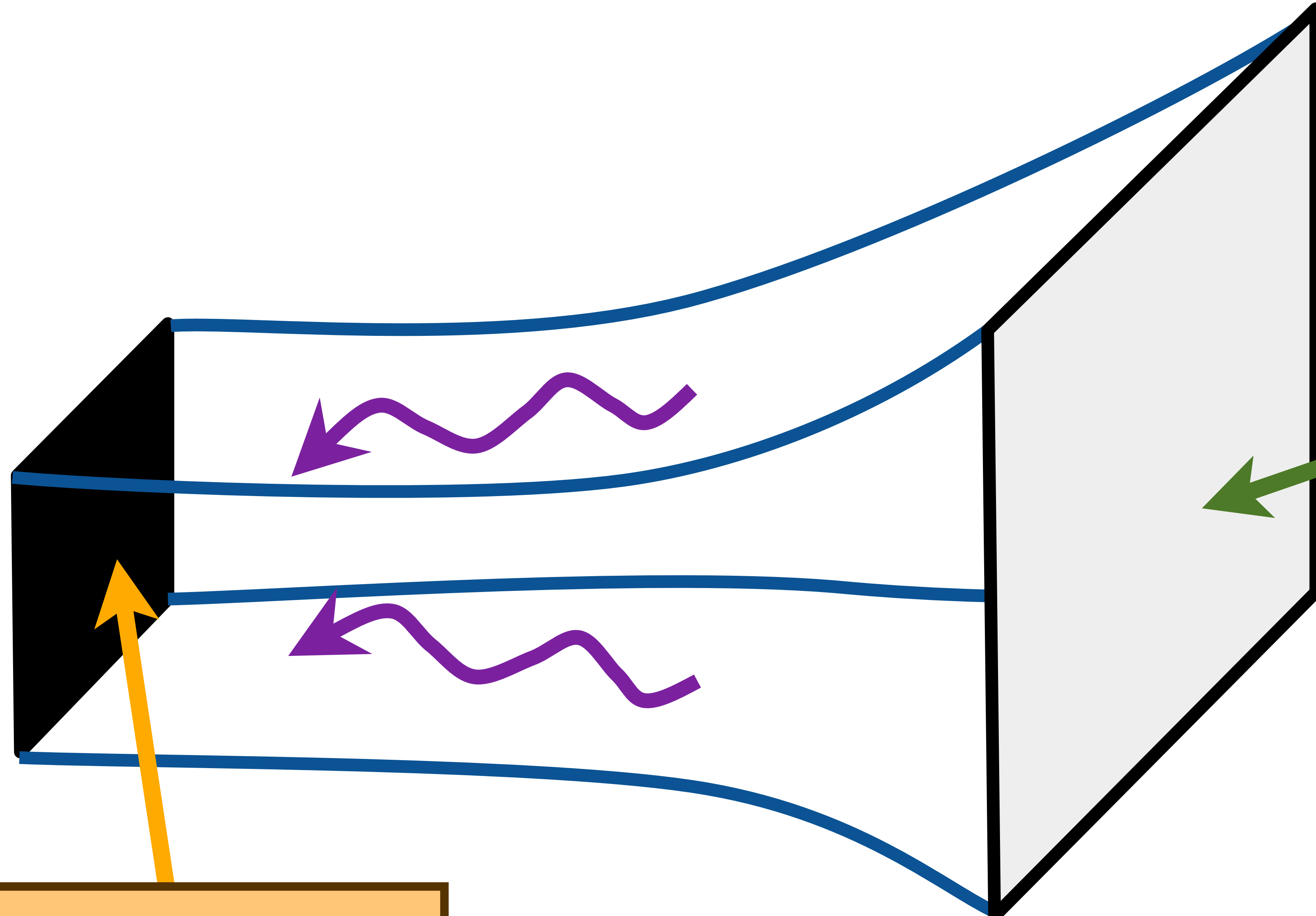
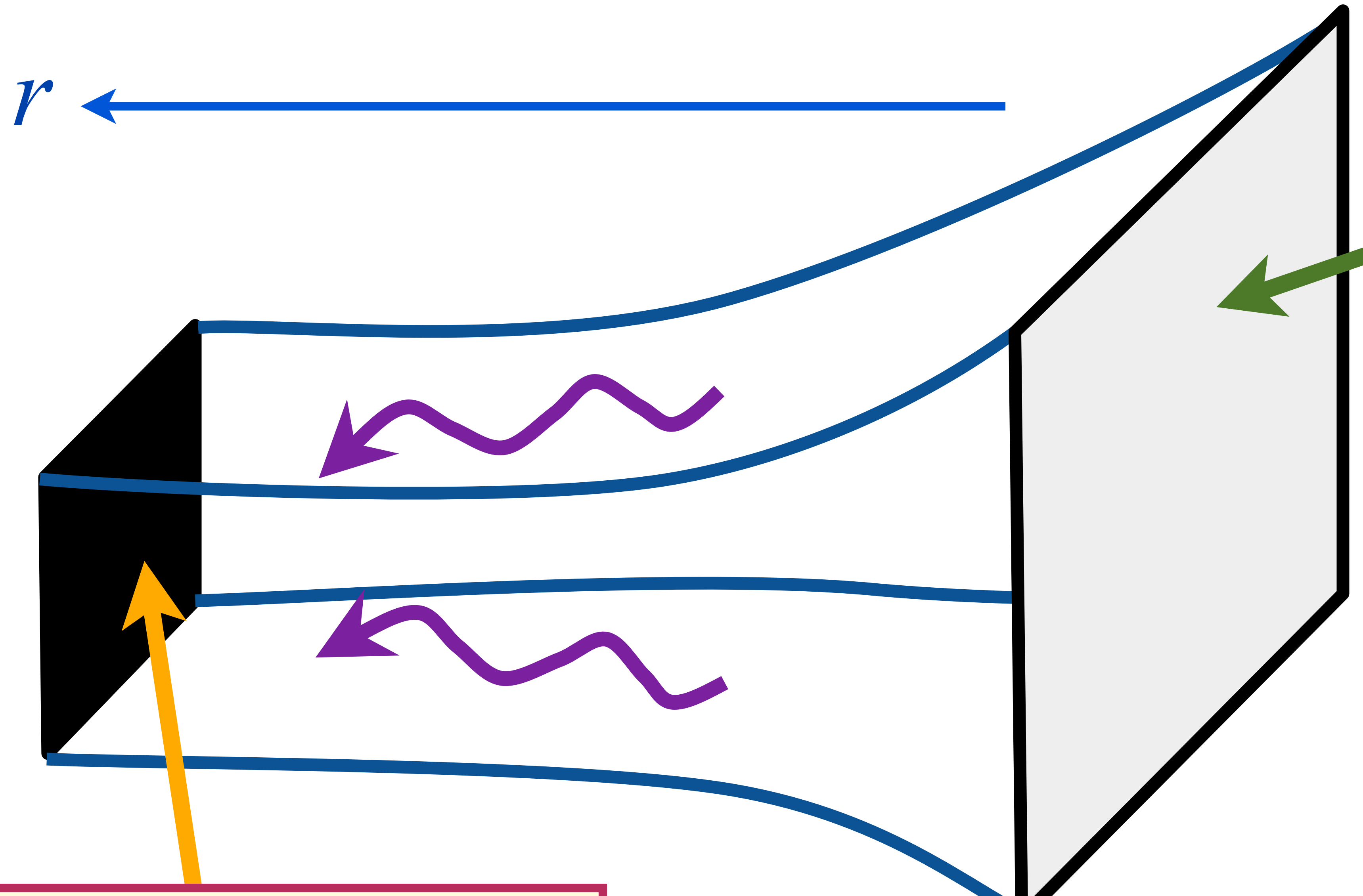


A 2+1 dimensional system at its quantum critical point:
$$k_B T = \frac{3\hbar}{4\pi R}.$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

Gauge-gravity duality at non-zero temperatures



A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Characteristic damping time of quasi-normal modes: $(k_B/\hbar)\times$ Hawking temperature

AdS/CFT correspondence at non-zero temperatures

At non-zero temperatures, we consider a Euclidean metric with a horizon at $r = R$:

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} + f(r)d\tau^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$; note $f(R) = 0$. In the near horizon region we define $z = R - r$ and write this metric as

$$ds^2 = \left(\frac{L}{R}\right)^2 \left[\frac{dz^2}{|f'(R)|z} + |f'(R)|z\, d\tau^2 + dx^2 + dy^2\right]$$

Now we introduce co-ordinates $\rho = 2\sqrt{z/|f'(R)|}$ and $\theta = 2\pi T\tau$, and then the metric is

$$ds^2 = \left(\frac{L}{R}\right)^2 \left[d\rho^2 + \left(\frac{f'(R)}{4\pi T}\right)^2 \rho^2 d\theta^2 + dx^2 + dy^2\right]$$

Now if we choose the Hawking temperature

$$T = \frac{|f'(R)|}{4\pi}$$

then the spacetime is periodic under $\tau \to \tau + 1/T$, and there is no singularity at the horizon.

Computing conductivity at non-zero temperatures

In Euclidean signature, all the correspondences between the bulk and boundary correlators remain exactly the same as before. We need only add the additional requirement that the bulk solutions remain integrable at the horizon.

However, it is often convenient to work directly in real time and frequencies, and obtain the corresponding response functions directly, rather than by analytic continuation. It can be shown that the process of analytic continuation translates into the requirement of *in-going waves* at the horizon. The only other change in the equations is due to the change in the metric from AdS_4 to AdS_4-Schwarzschild, via the factor $f(r)$.

In terms of the co-ordinate $u = r/R$, the equation for $A_x(u)$ in the presence of a probe oscillating at frequency ω is

$$A_x'' + \frac{f'(3 - 2u^2\gamma f'') - 2u\gamma f(2f'' + uf''')}{f(3 - 2u^2\gamma f'')} A_x' + \frac{L^4}{R^2}\frac{\omega^2}{f^2} A_x = 0,$$

where the primes are derivatives w.r.t u. Solution of this equation, subject to the boundary conditions discussed earlier yields the conductivity.

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

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Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations

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- Relate OPE co-efficients to couplings of an effective graviational theory on AdS
- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

AdS$_4$ theory of quantum criticality

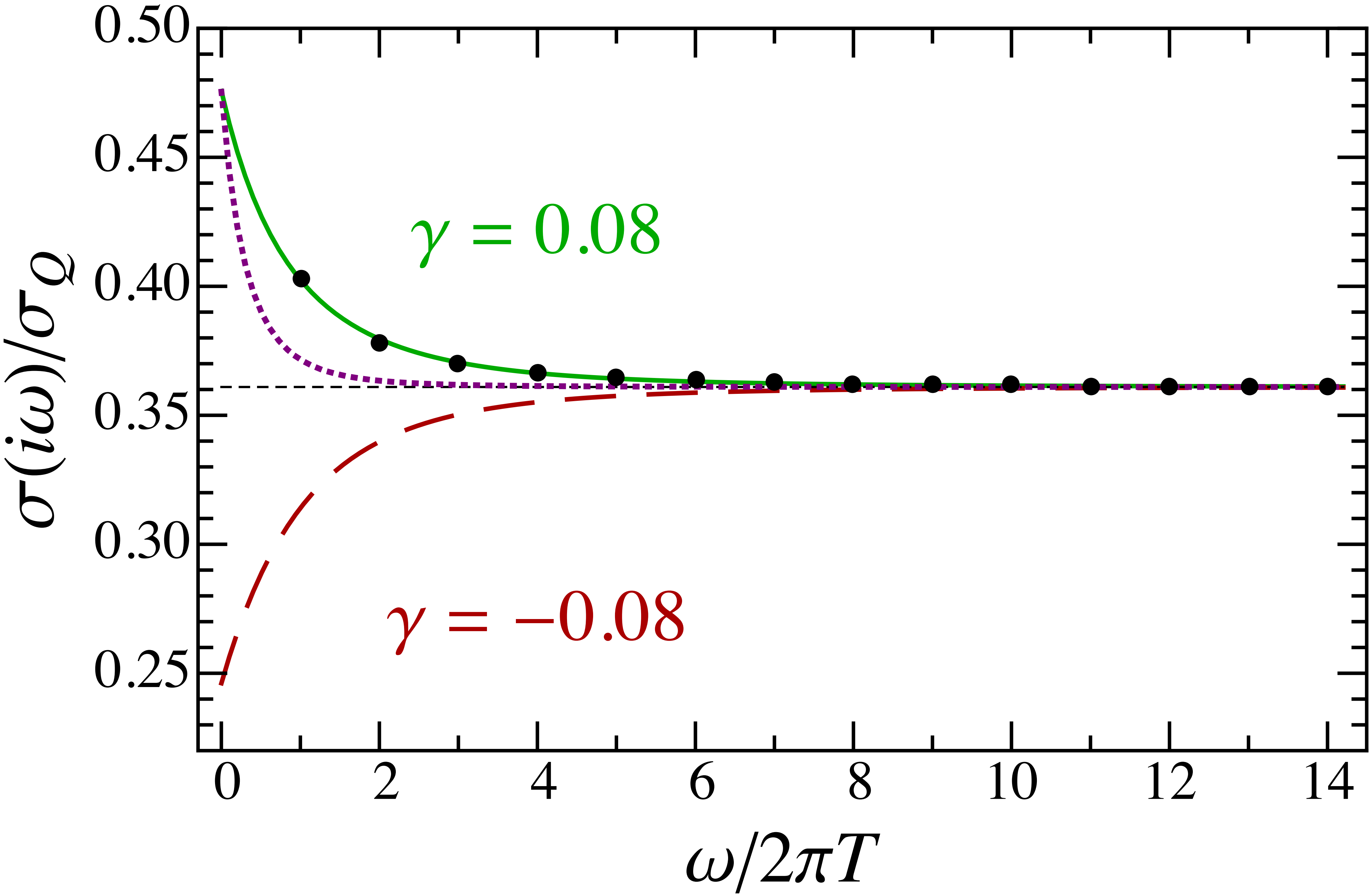
Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator. Constraints from both the CFT and the gravitational theory bound $|\gamma| \leq 1/12 = 0.0833$..

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

AdS$_4$ theory of quantum criticality



Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective T and taking $\sigma_Q = 1/g_M^2$.

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

arXiv.org > cond-mat > arXiv:1309.2941

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Condensed Matter > Strongly Correlated Electrons

The dynamics of quantum criticality via Quantum Monte Carlo and holography

William Witczak-Krempa, Erik Sorensen, Subir Sachdev

(Submitted on 11 Sep 2013 (v1), last revised 29 Nov 2013 (this version, v2))

Understanding the real time dynamics of quantum systems without quasiparticles constitutes an important yet challenging problem. We study the superfluid-insulator quantum-critical point of bosons on a two-dimensional lattice, a system whose excitations cannot be described in a quasiparticle basis. We present detailed quantum Monte Carlo results for two separate lattice realizations: their low-frequency conductivities are found to have the same universal dependence on imaginary frequency and temperature. We then use the structure of the real time dynamics of conformal field theories described by the holographic gauge/gravity duality to make progress on the difficult problem of analytically continuing the Monte Carlo data to real time. Our method yields quantitative and experimentally testable results on the frequency-dependent conductivity near the quantum critical point, and on the spectrum of quasinormal modes in the vicinity of the superfluid-insulator quantum phase transition. Extensions to other observables and universality classes are discussed.

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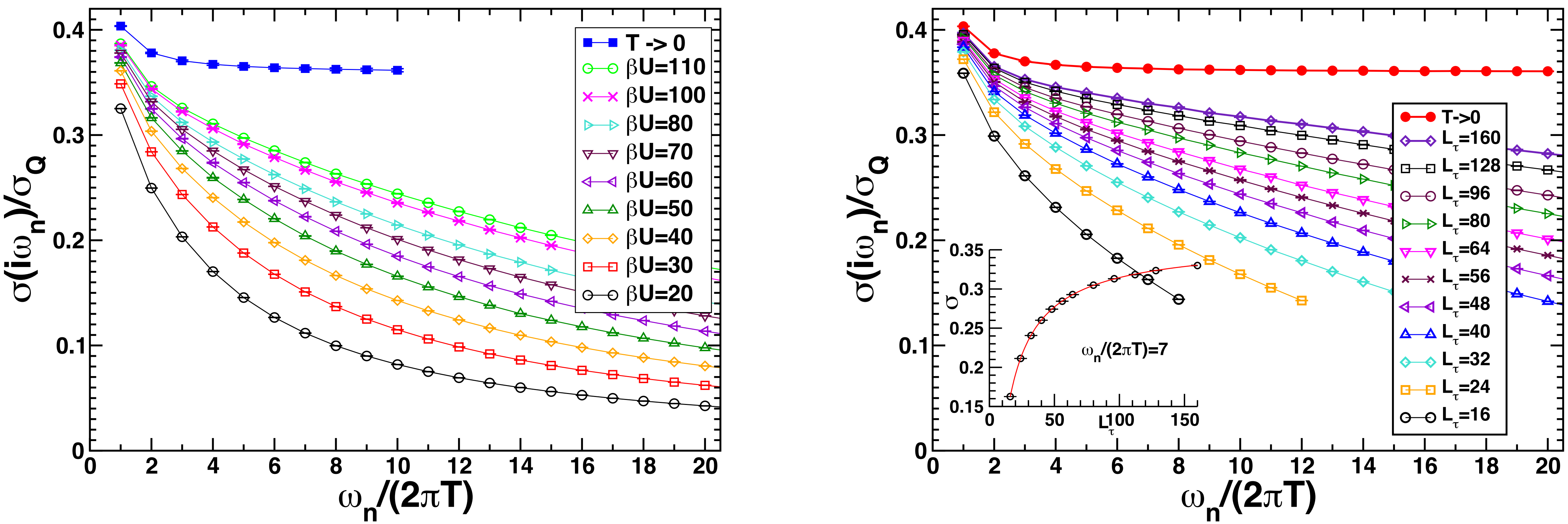
Universal Conductivity in a Two-dimensional Superfluid-to-Insulator Quantum Critical System

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, Nikolay Prokof'ev

(Submitted on 22 Sep 2013)

We compute the universal conductivity of the (2+1)-dimensional XY universality class, which is realized for a superfluid-to-Mott insulator quantum phase transition at constant density. Based on large-scale Monte Carlo simulations of the classical (2+1)-dimensional J-current model and the two-dimensional Bose-Hubbard model, we can precisely determine the conductivity on the quantum critical plateau, $\sigma(\infty) = 0.359(4)\sigma_Q$ with σ_Q the conductivity quantum. The universal conductivity is the schoolbook example of where the AdS/CFT correspondence from string theory can be tested and made to use. The shape of our $\sigma(i\omega_n) - \sigma(\infty)$ function in the Matsubara representation is accurate enough for a conclusive comparison and establishes the particle-like nature of charge transport. We find that the holographic gauge/gravity duality theory for transport properties can be made compatible with the data if temperature of the horizon of the black brane is different from the temperature of the conformal field theory. The requirements for measuring the universal conductivity in a cold gas experiment are also determined by our calculation.

Quantum Monte Carlo for lattice bosons



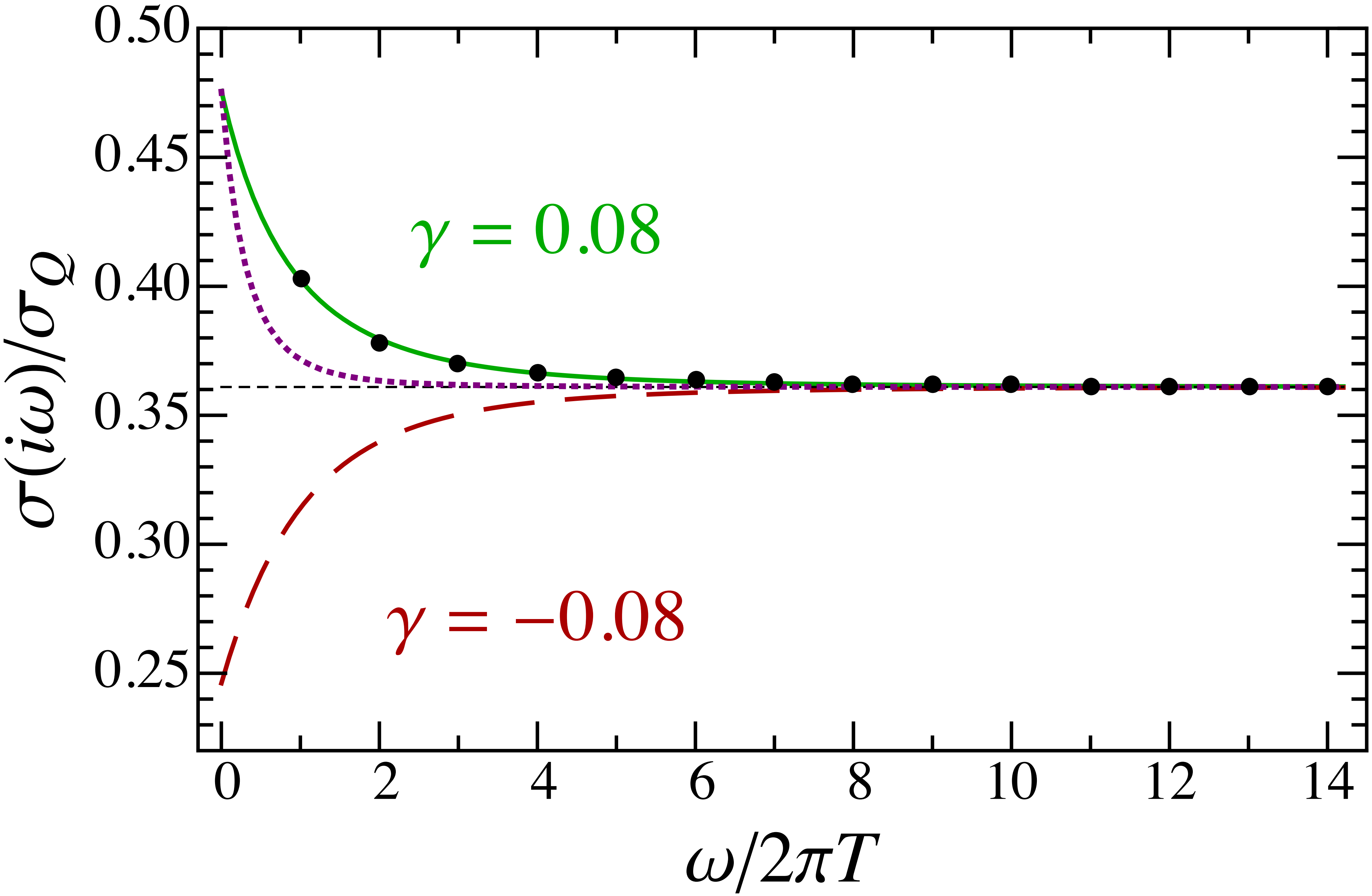
(a) (b)

FIG. 2. **Quantum Monte Carlo data** (a) Finite-temperature conductivity for a range of βU in the $L \to \infty$ limit for the quantum rotor model at $(t/U)_c$. The solid blue squares indicate the final $T \to 0$ extrapolated data. (b) Finite-temperature conductivity in the $L \to \infty$ limit for a range of L_τ for the Villain model at the QCP. The solid red circles indicate the final $T \to 0$ extrapolated data. The inset illustrates the extrapolation to $T = 0$ for $\omega_n/(2\pi T) = 7$. The error bars are statistical for both a) and b).

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

AdS$_4$ theory of quantum criticality

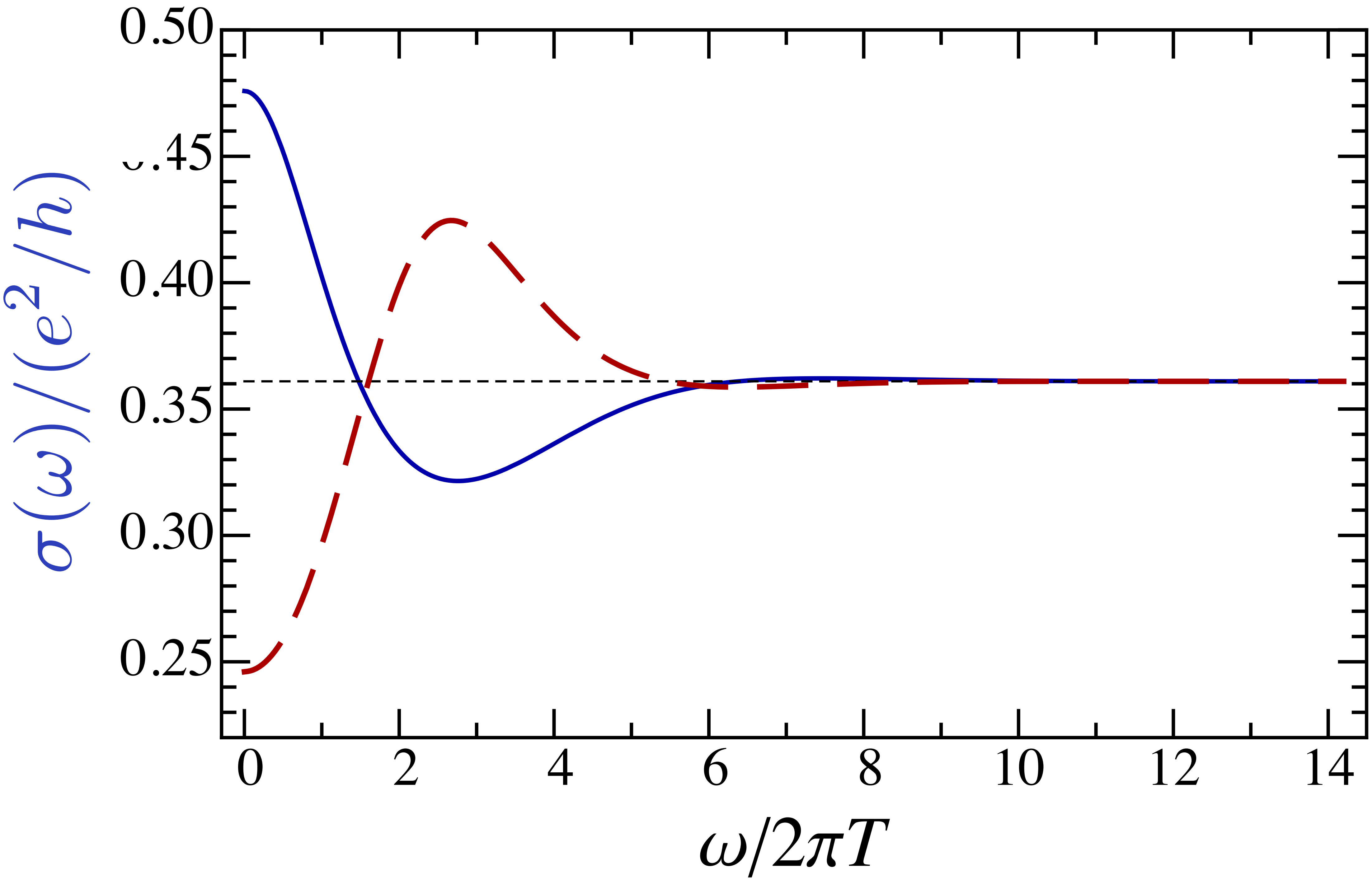


Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective T and taking $\sigma_Q = 1/g_M^2$.

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

AdS_4 theory of quantum criticality



Predictions of holographic theory,
after analytic continuation to real frequencies

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941
See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635