Quantum phase transitions in condensed matter

The 8th Asian Winter School on
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Talk online: sachdev.physics.harvard.edu

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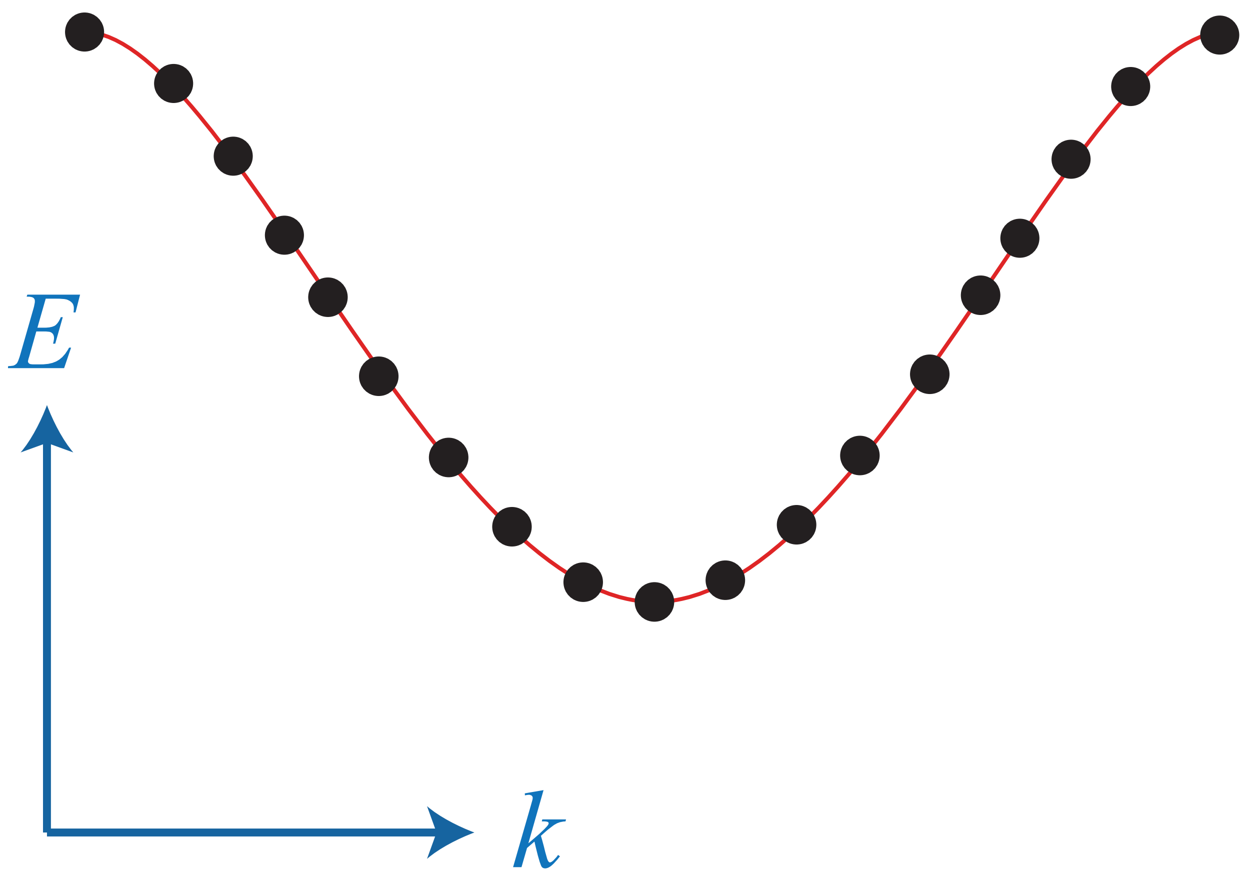
Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically connected to independent electron states

Band insulators

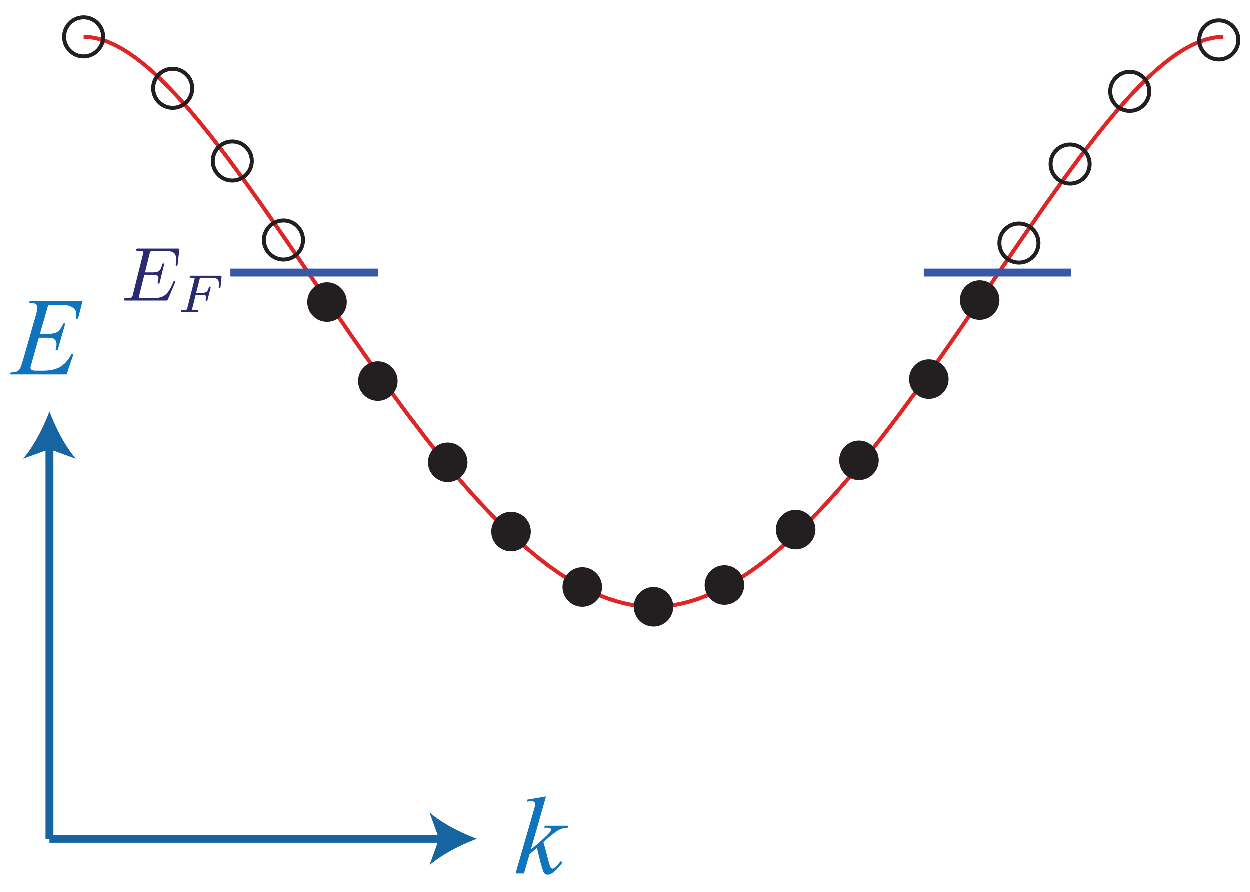


An even number of electrons per unit cell

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

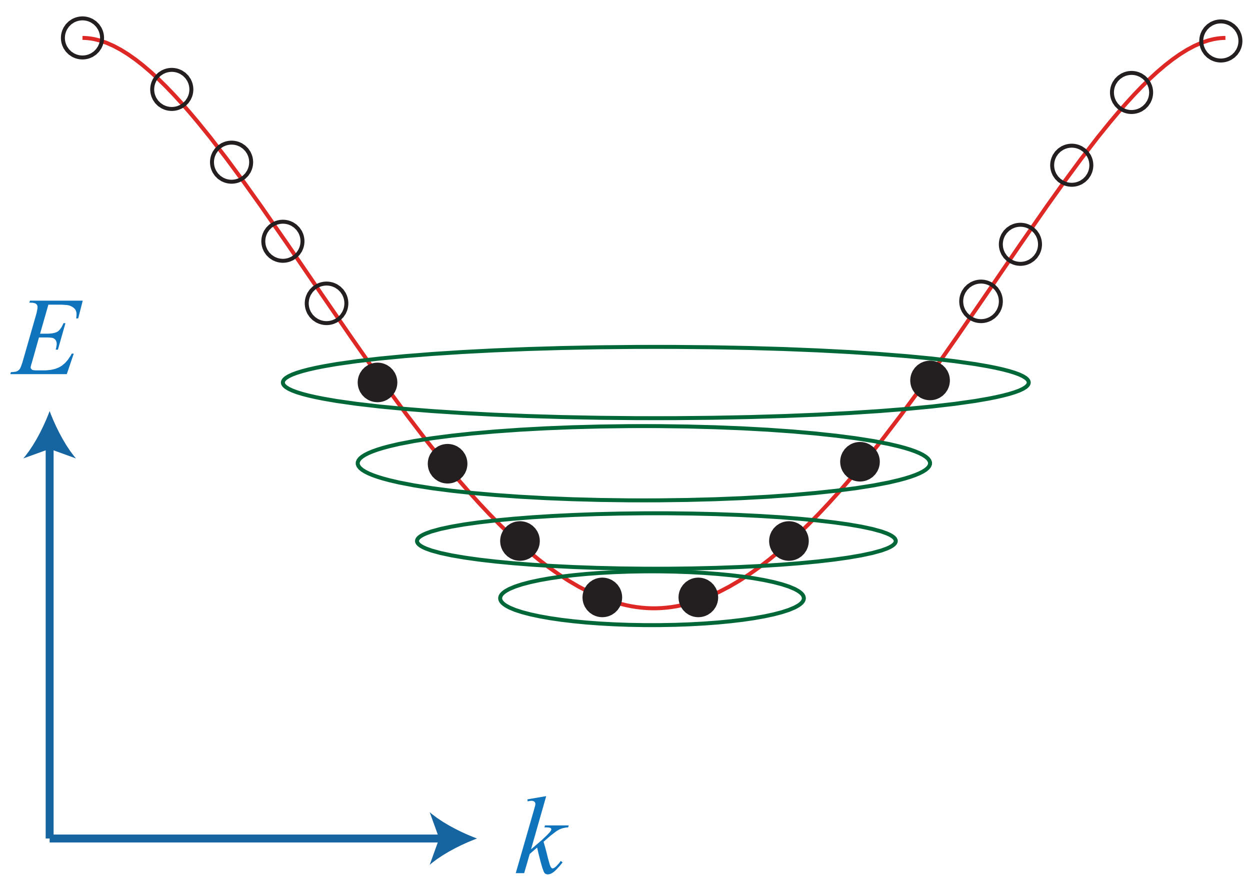
Metals



Sommerfeld-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

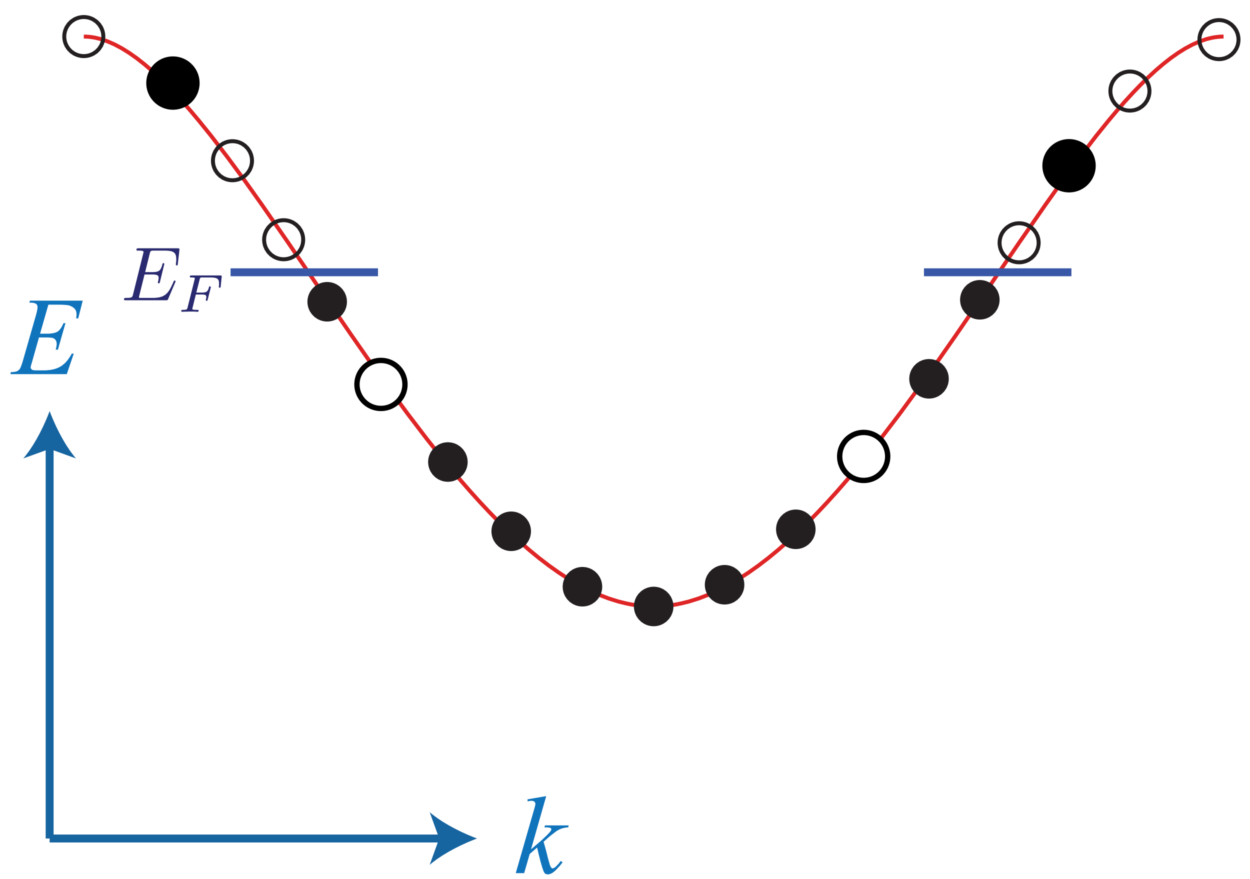
Superconductors



Boltzmann-Landau theory of dynamics of metals:

Long-lived ***quasiparticles*** (and ***quasiholes***) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter

Not adiabatically connected
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many-particle

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Modern phases of quantum matter

Not adiabatically connected
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Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,
and no quasiparticles

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. The holographic view: charged black-branes

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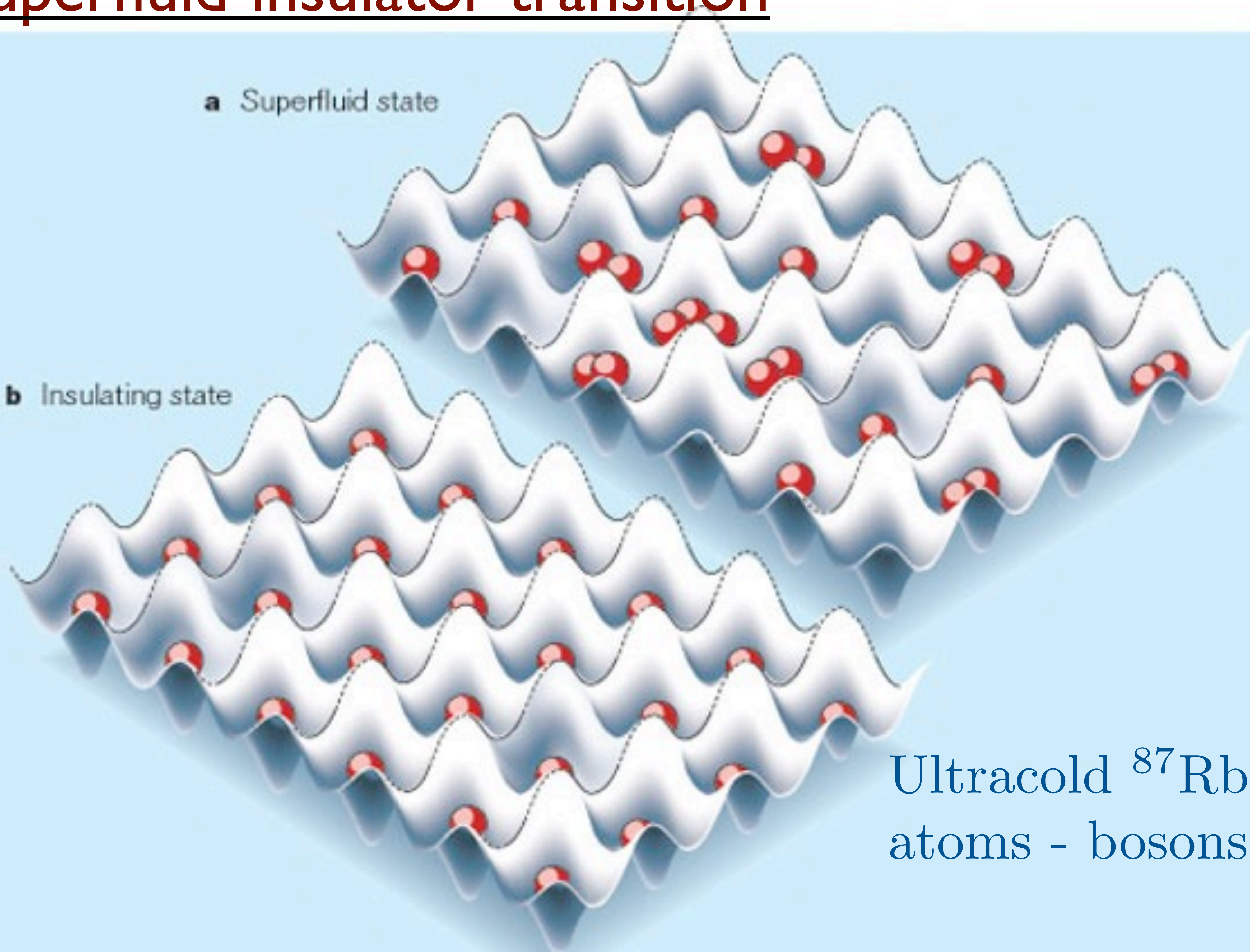
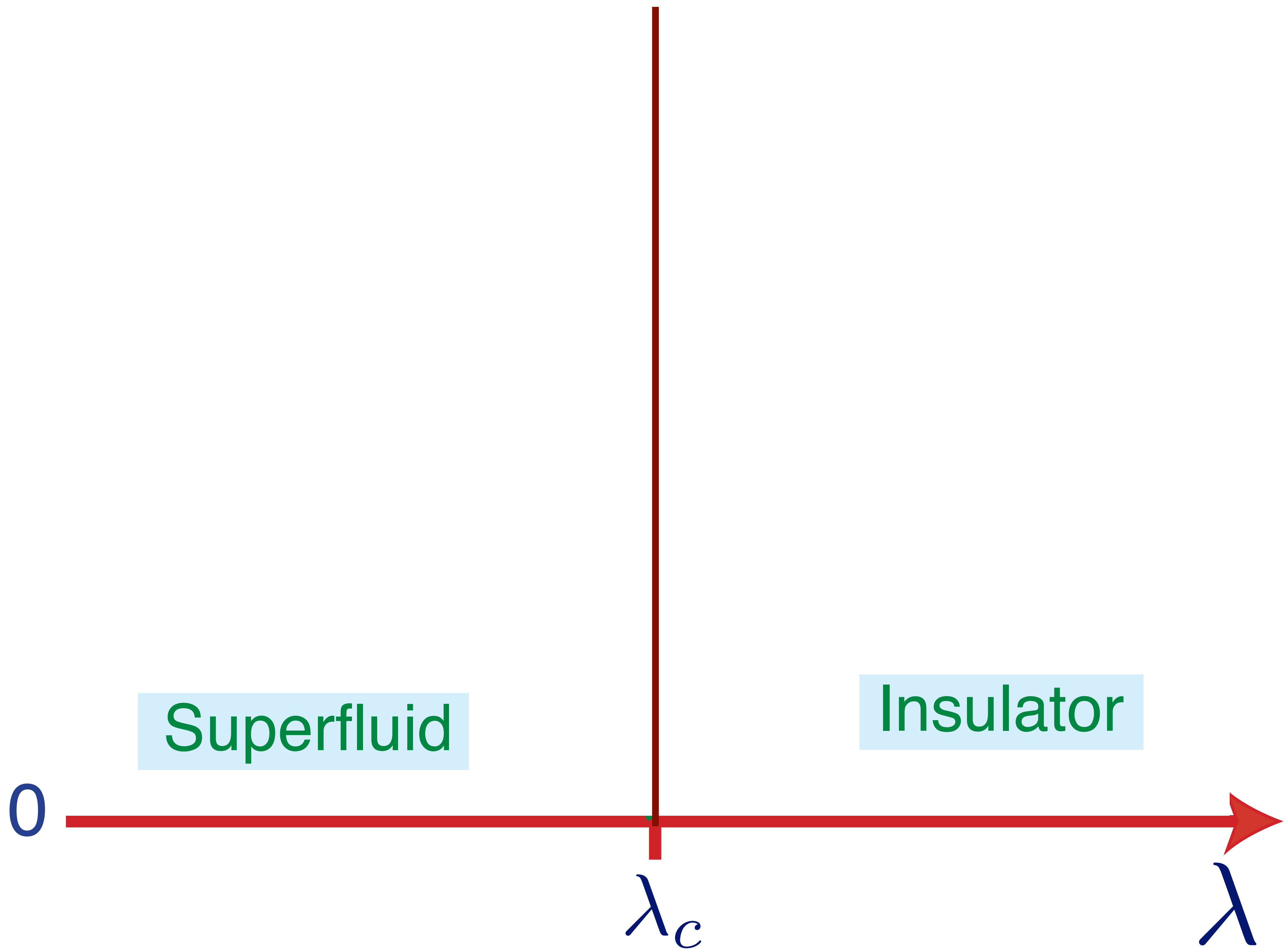
Superfluid-insulator transition

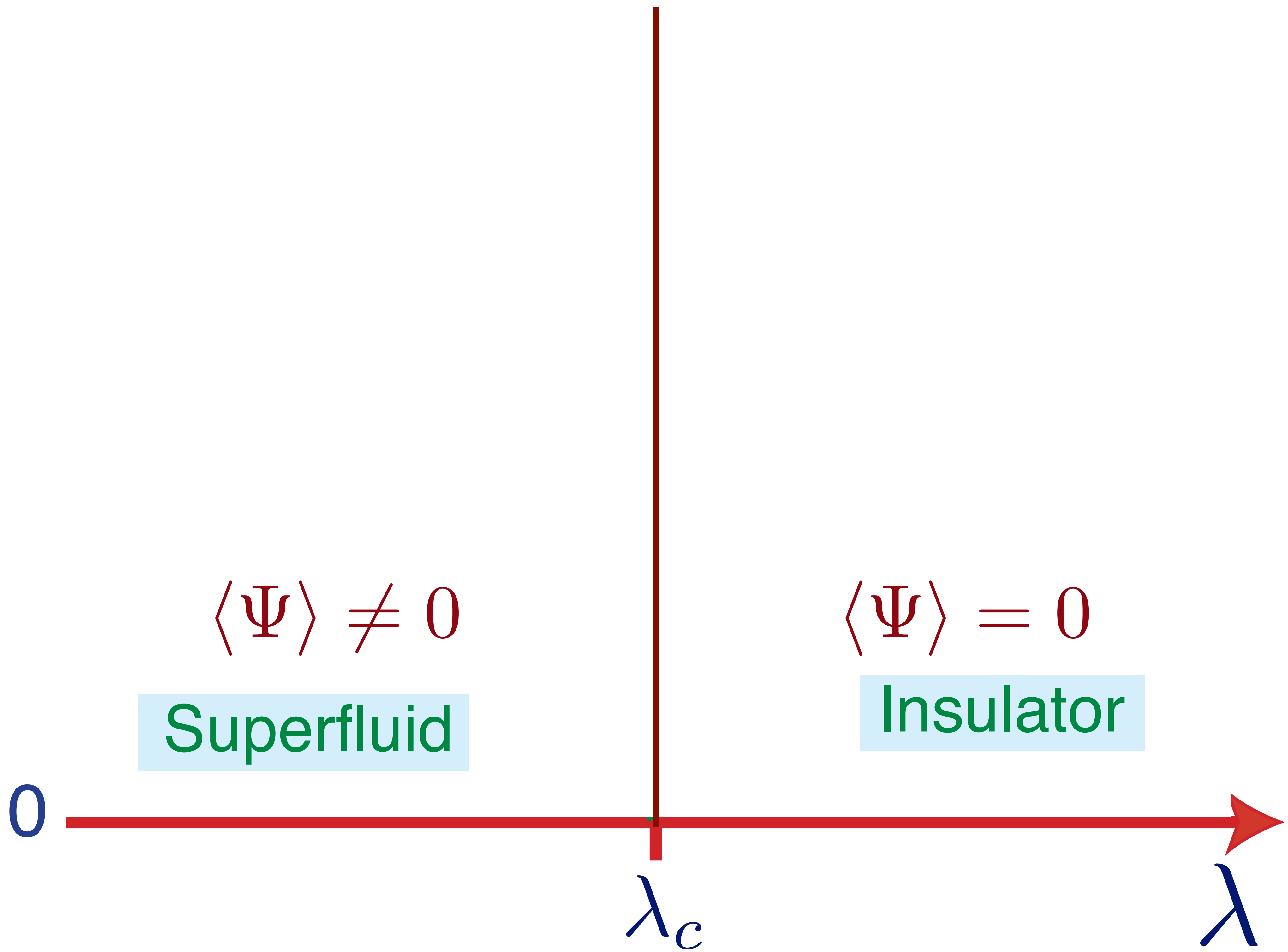


Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

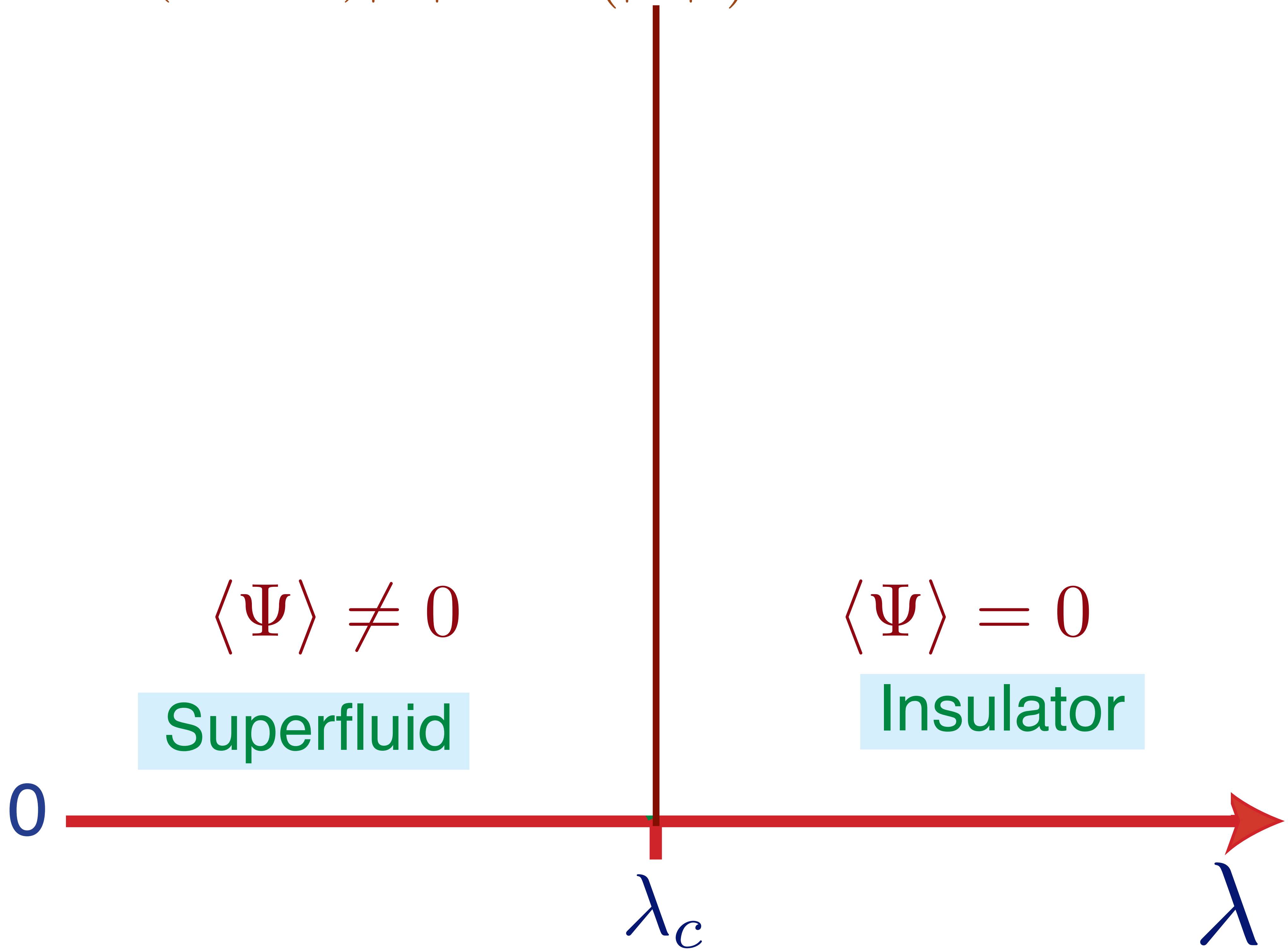

Superfluid
Insulator
0
λ_c
λ

$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2 r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



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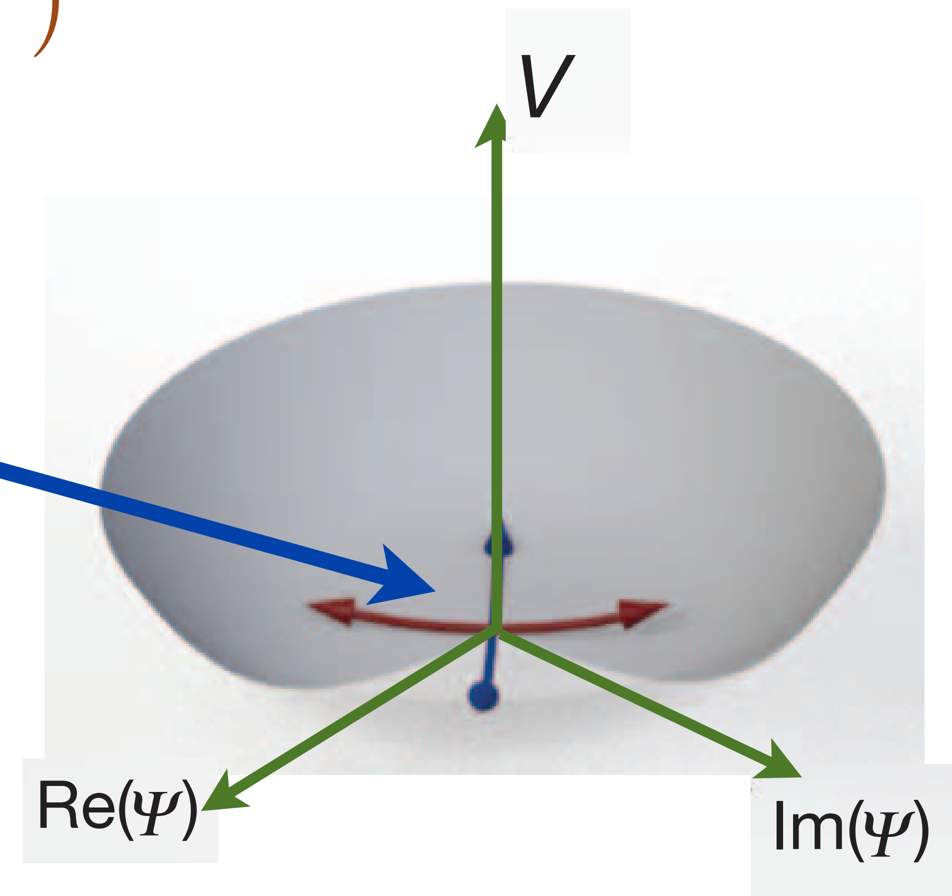
Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



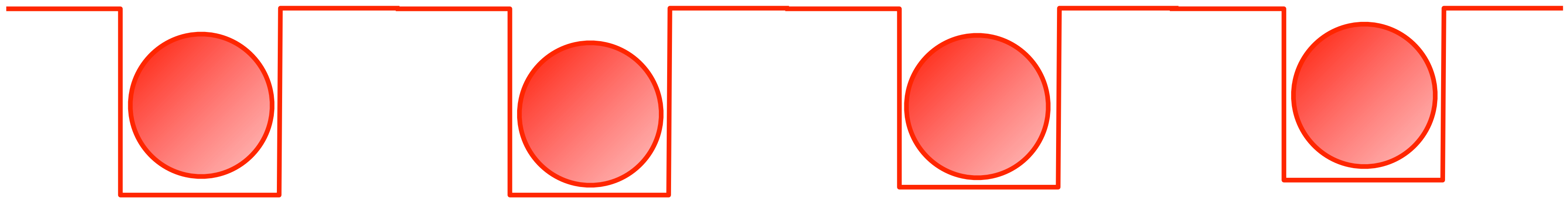
$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

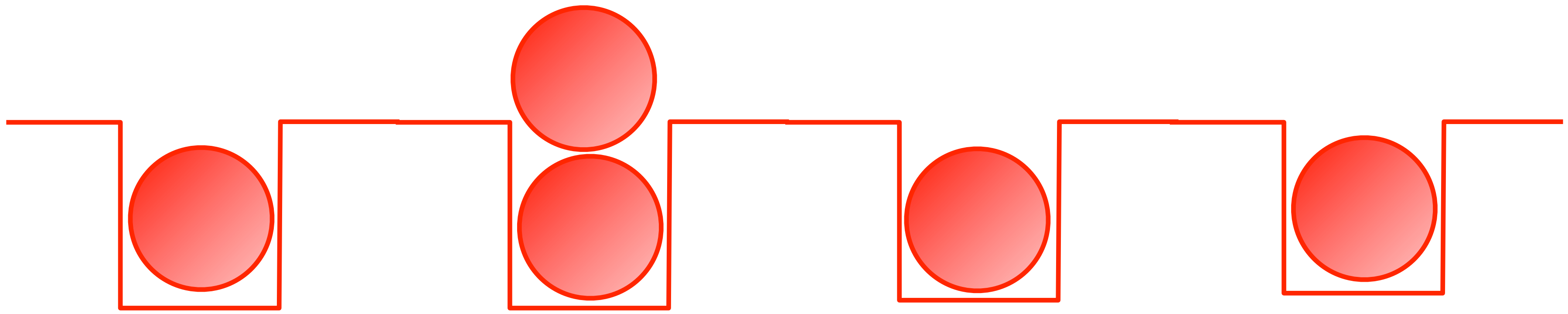
Insulator

0 λ_c λ



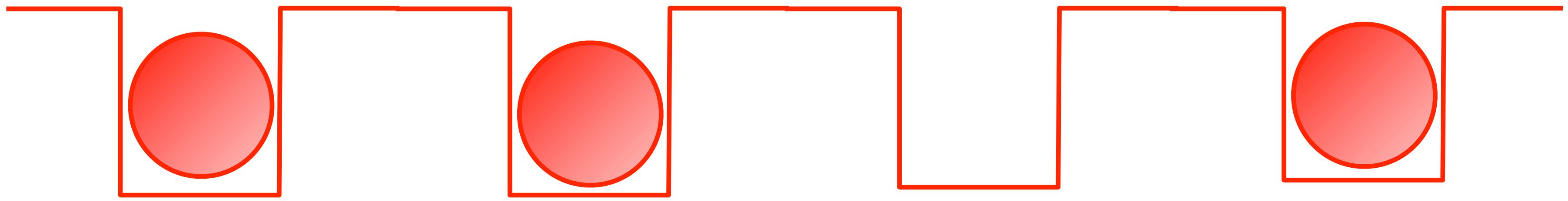
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



$$\text{Particles} \sim \Psi^\dagger$$

Excitations of the insulator:



$$\text{Holes} \sim \Psi$$

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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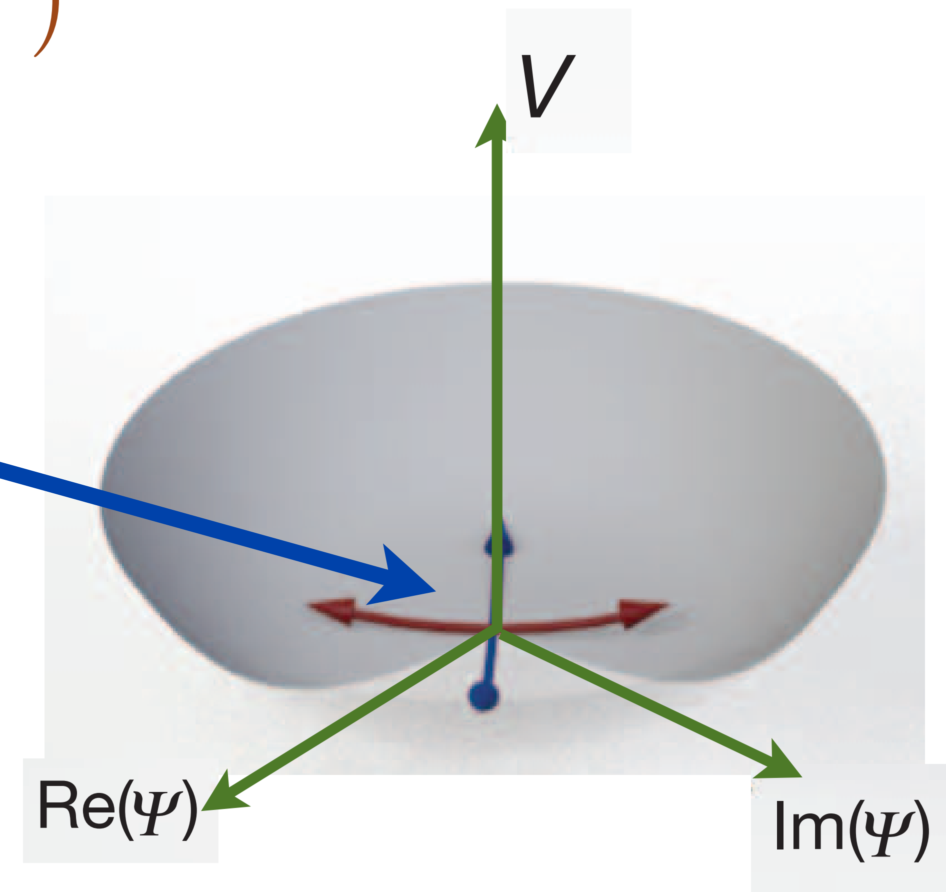

$\langle \Psi \rangle \neq 0$ — Superfluid

$\langle \Psi \rangle = 0$ — Insulator

0 — λ_c — λ

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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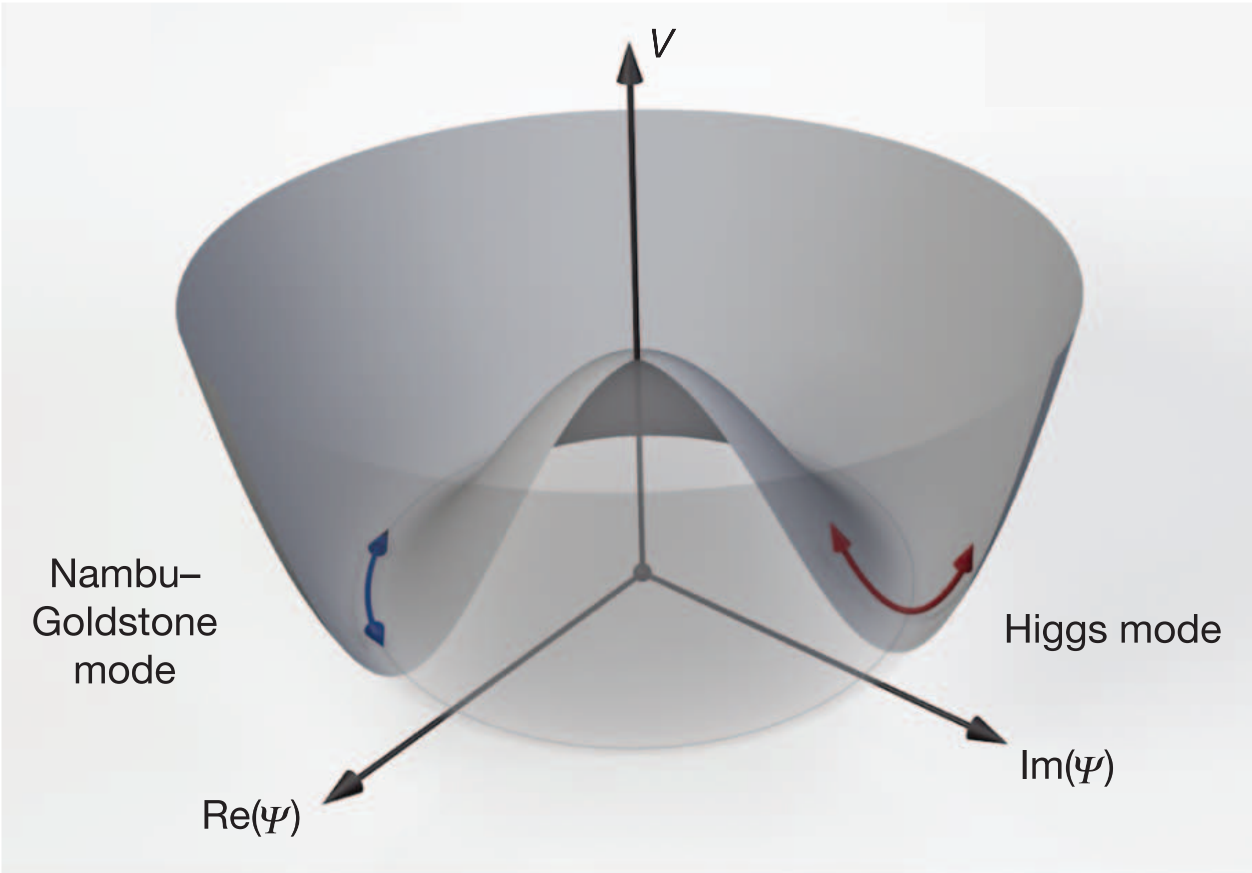


$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

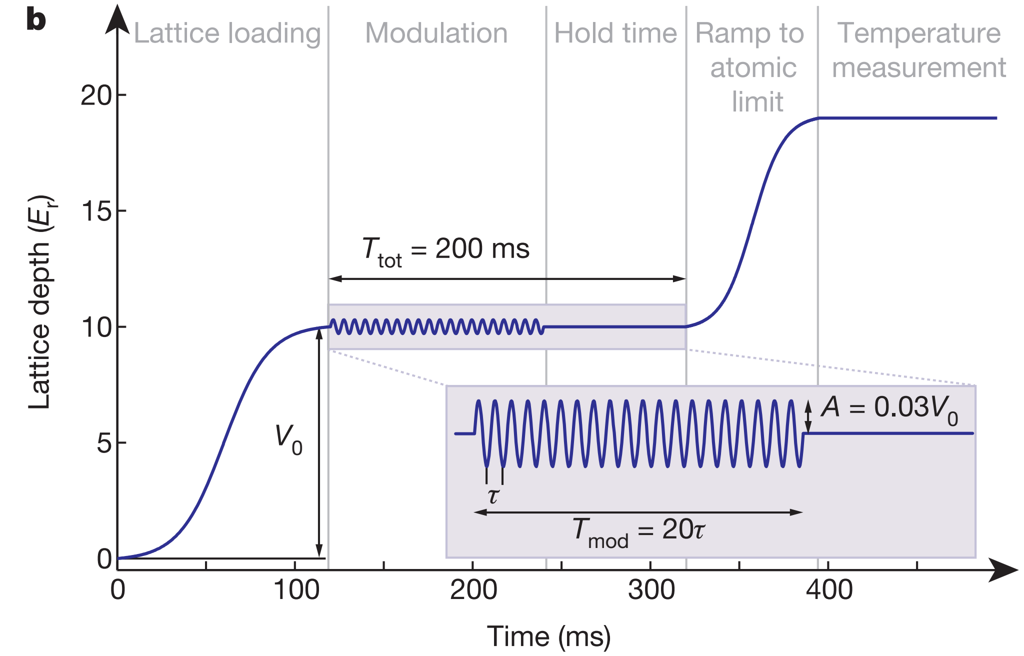
0

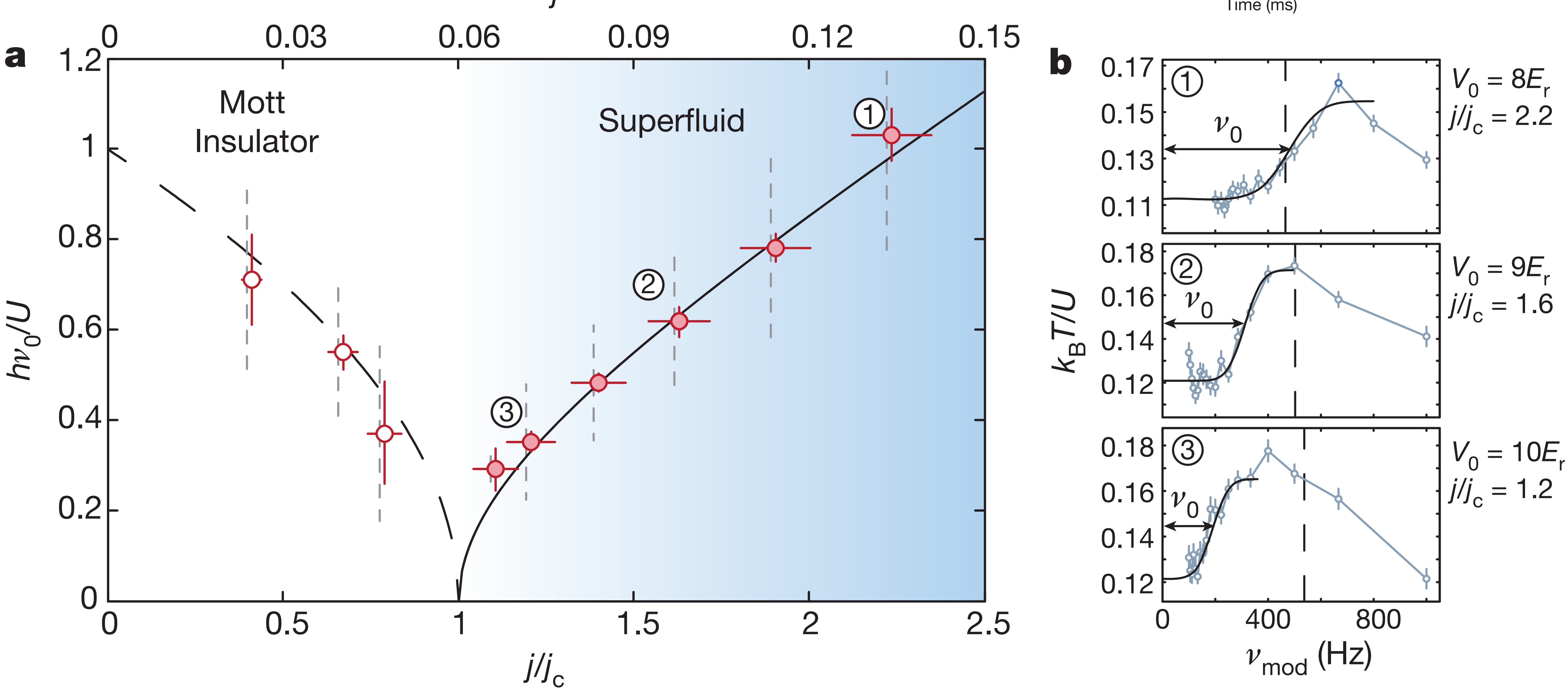
λ_c

λ

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

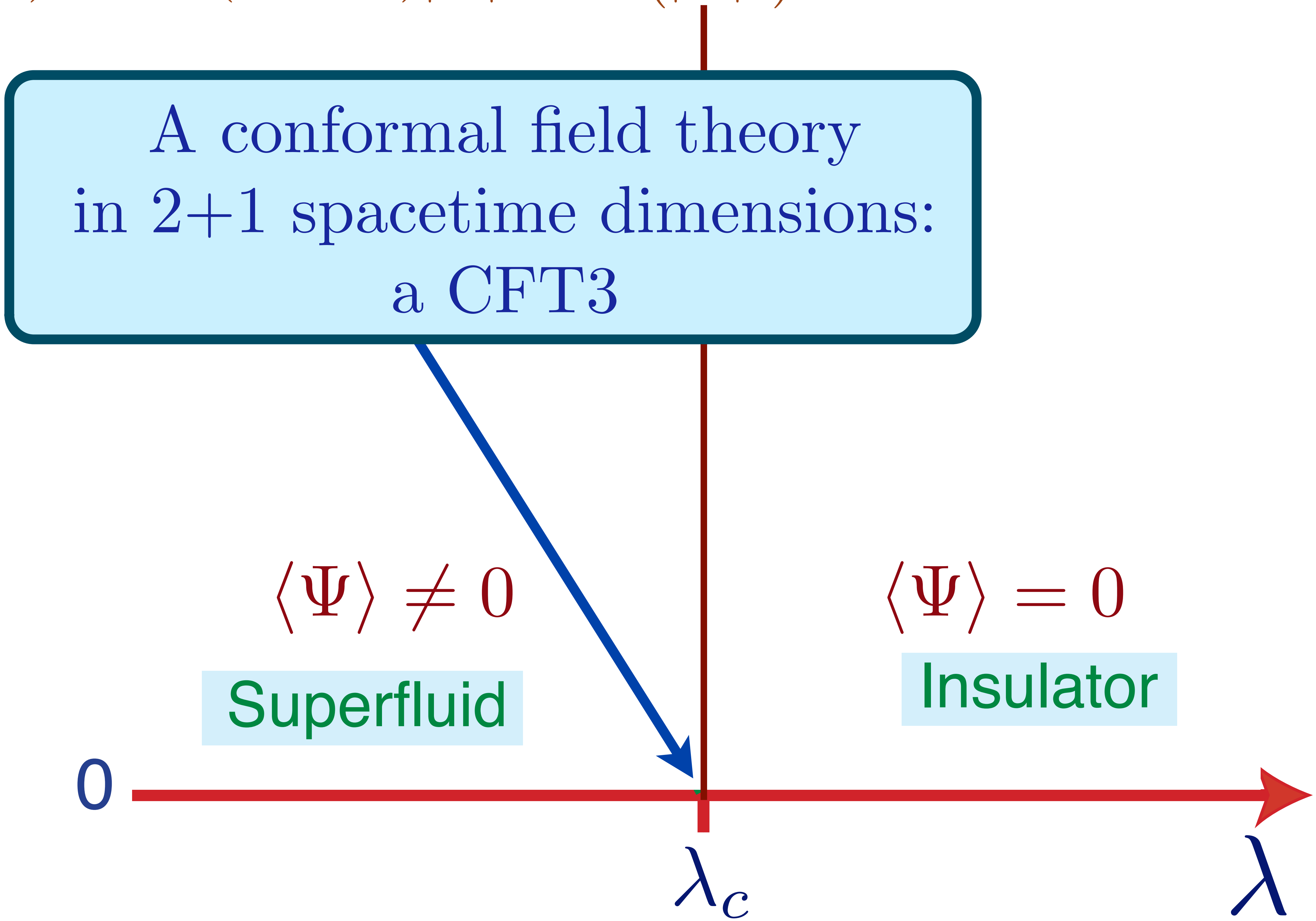
Response to modulation of lattice depth scales as expected from the LHP pole





Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

$$
\begin{aligned}
\mathcal{S} &= \int d^2 r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \\
V(\Psi) &= (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2
\end{aligned}
$$



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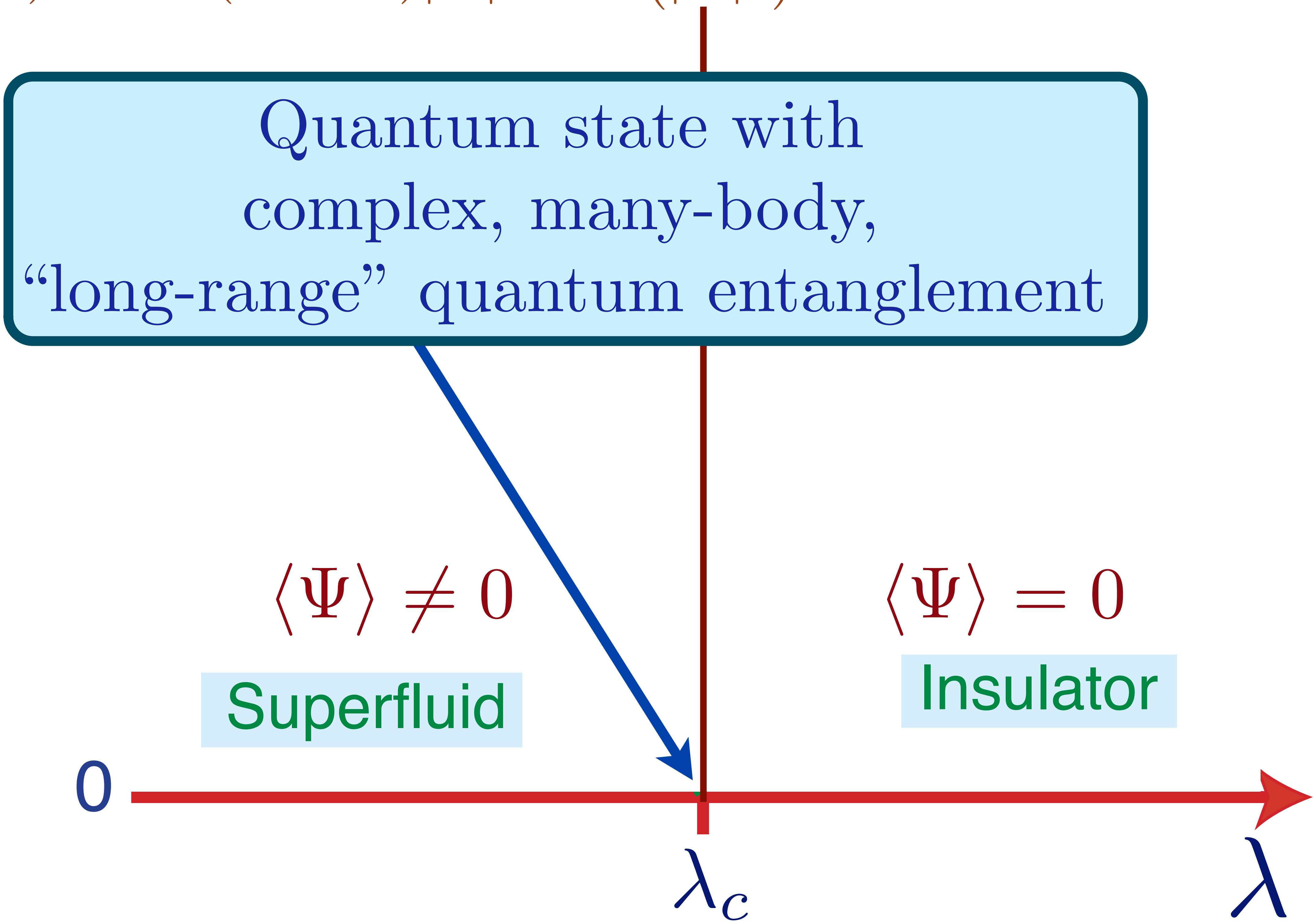

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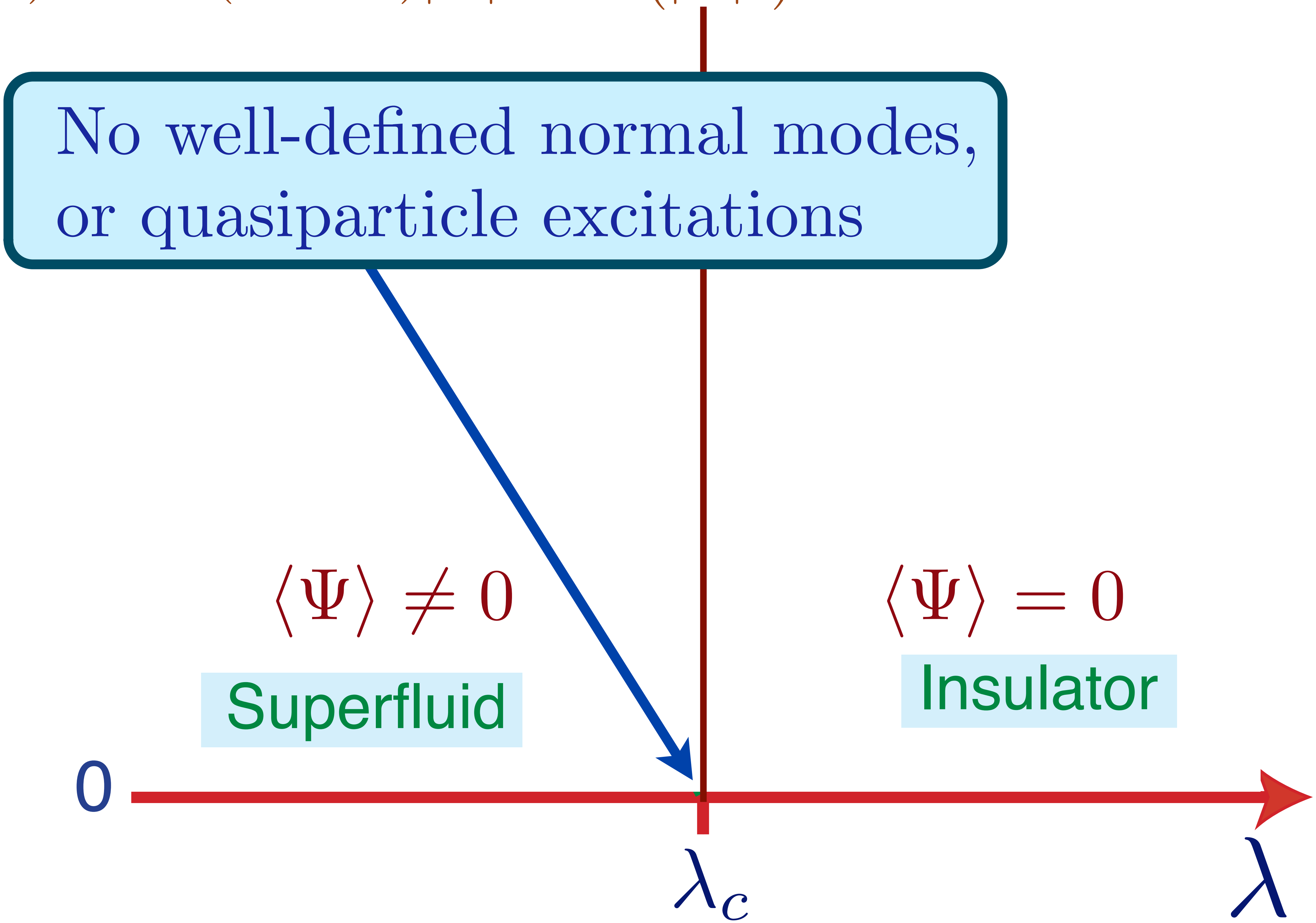
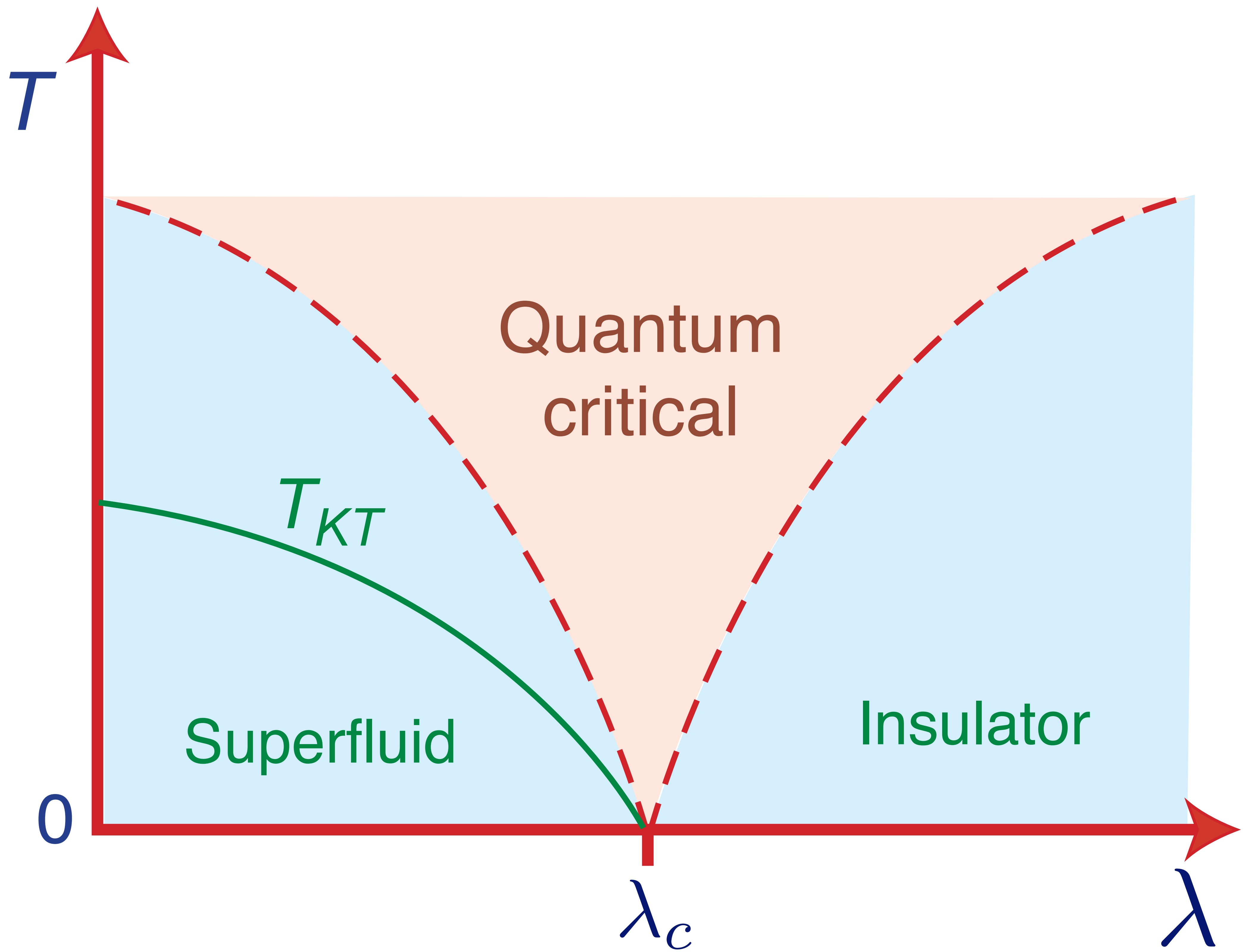
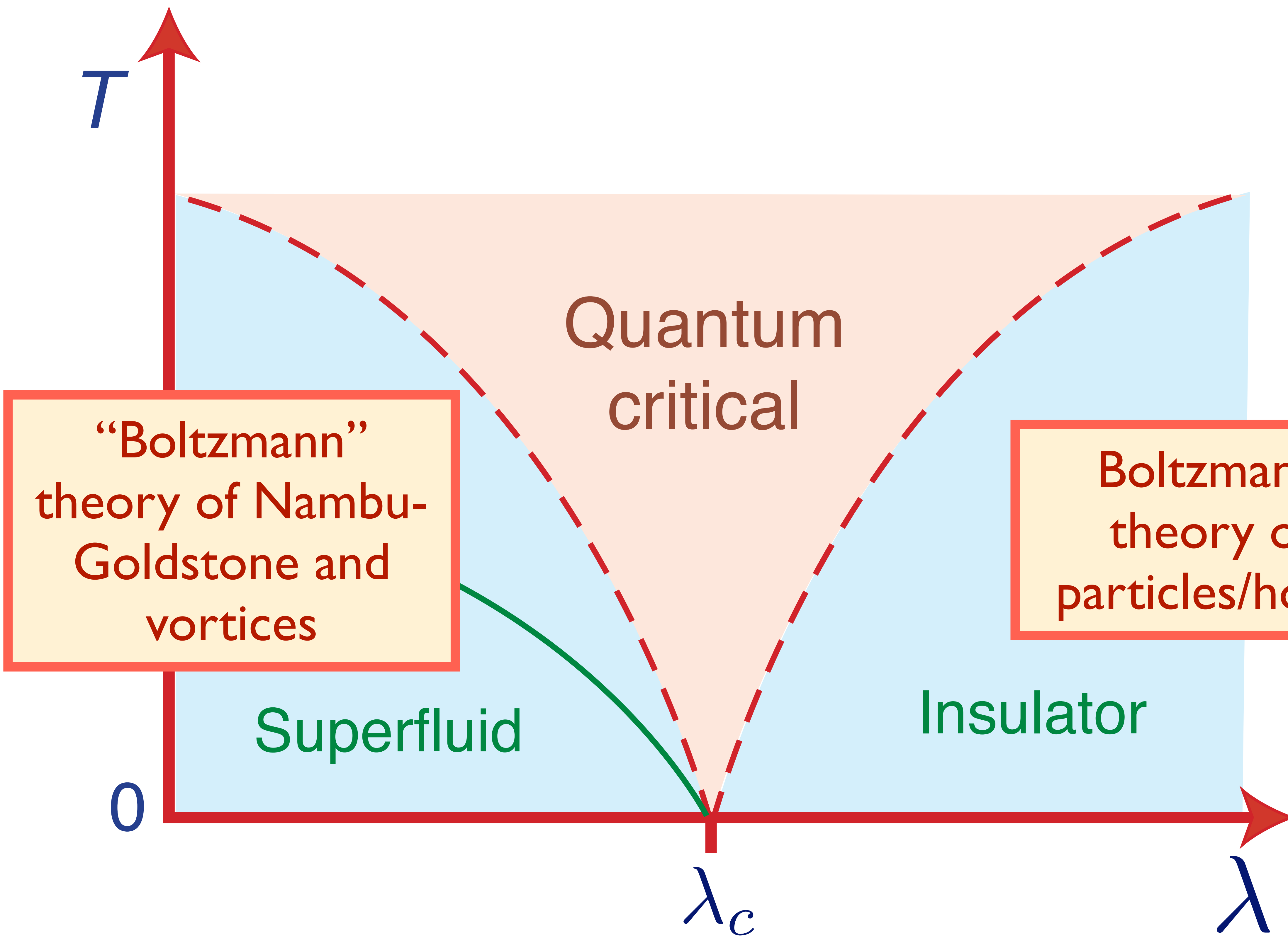
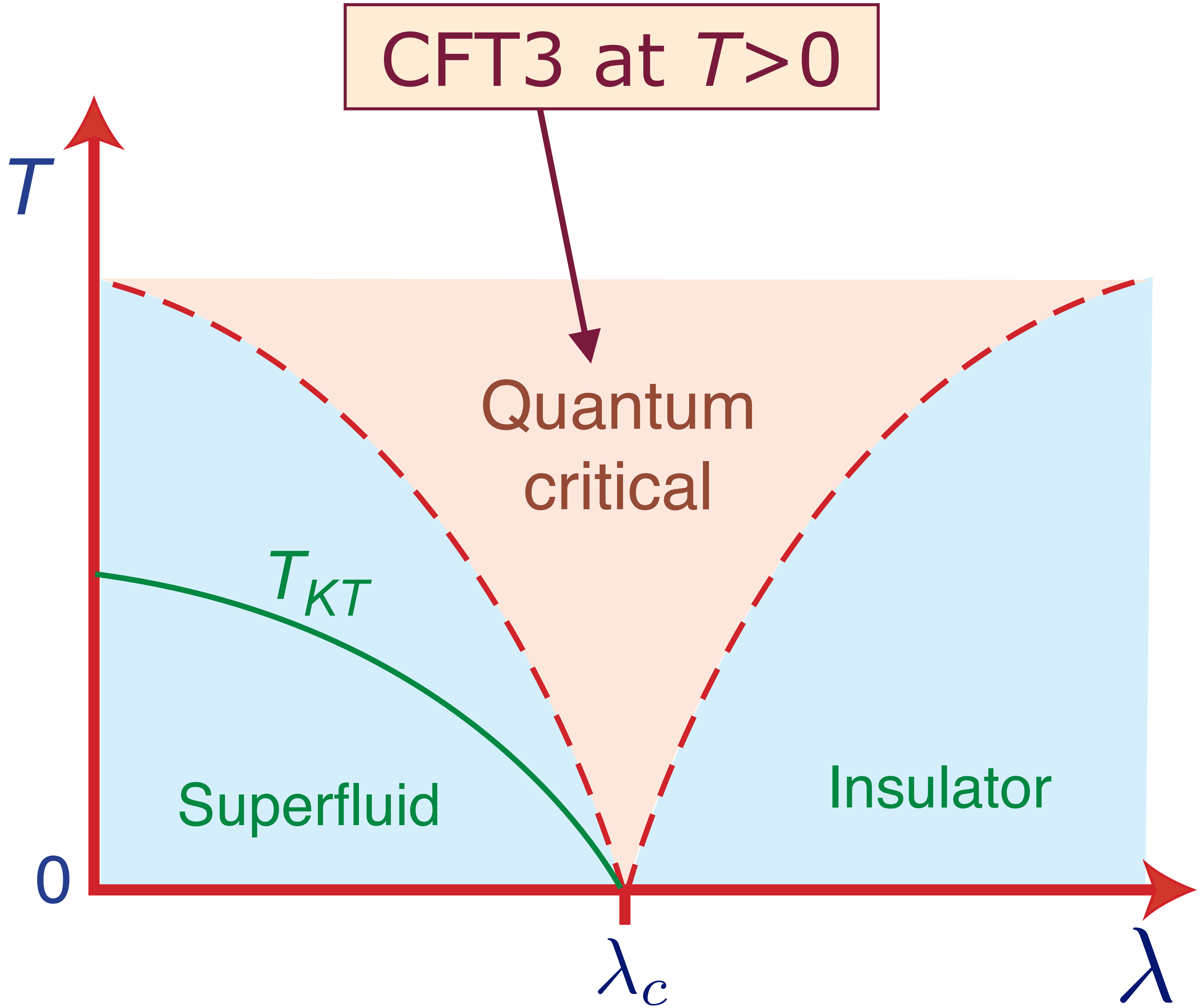
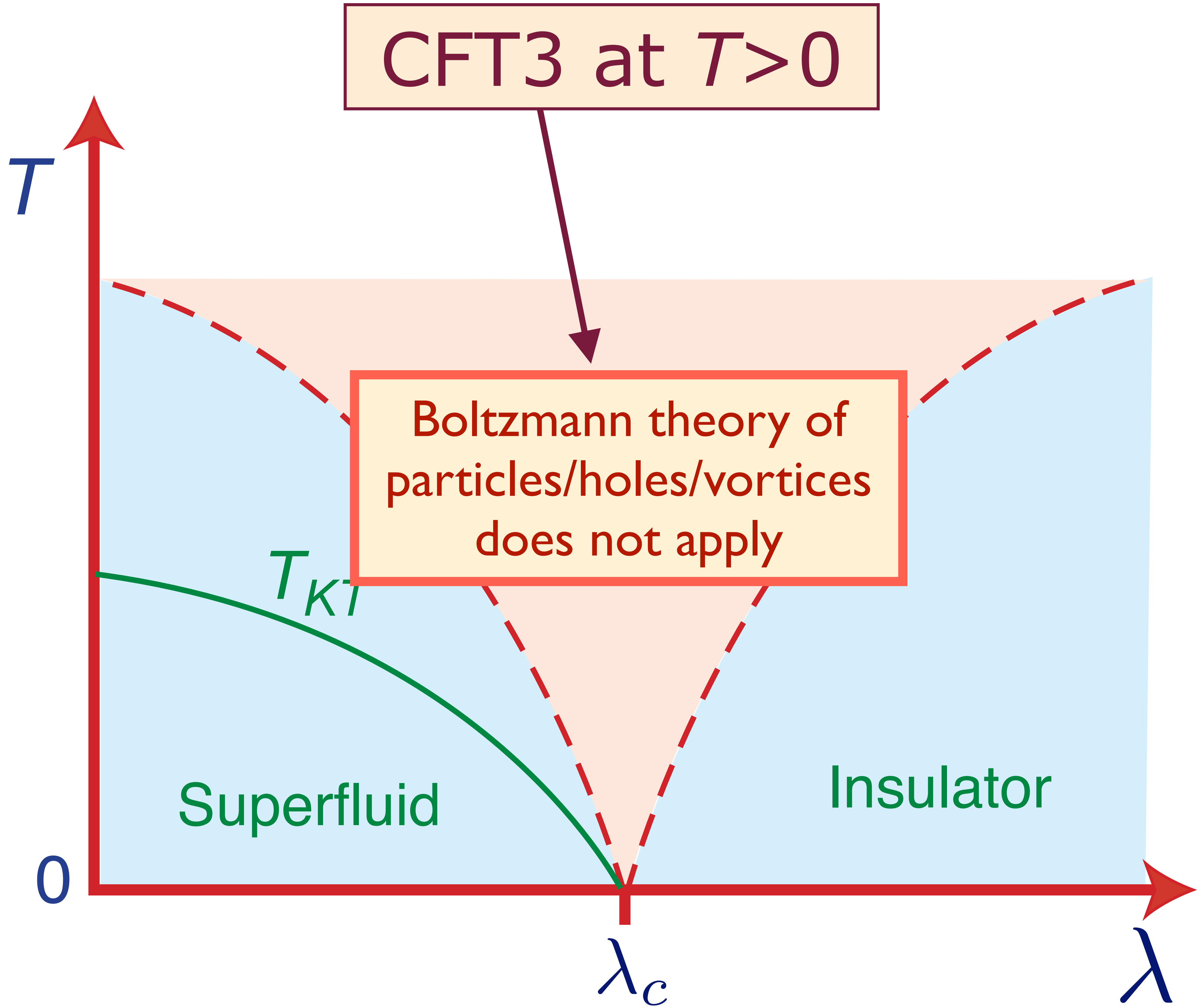


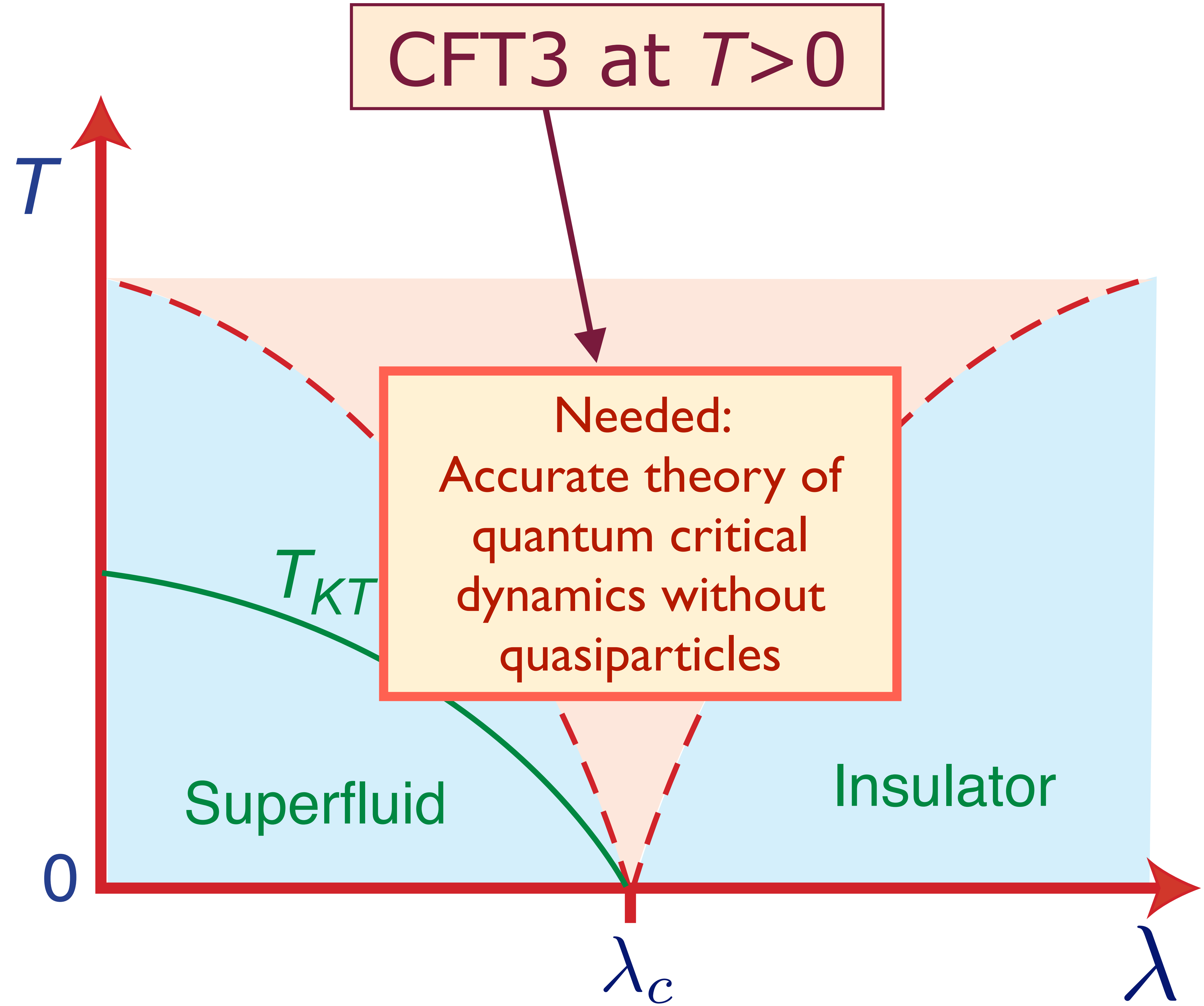


T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


T
Quantum critical
"Boltzmann" theory of Nambu-Goldstone and vortices
Boltzmann theory of particles/holes
Superfluid
Insulator
0
λ_c
λ

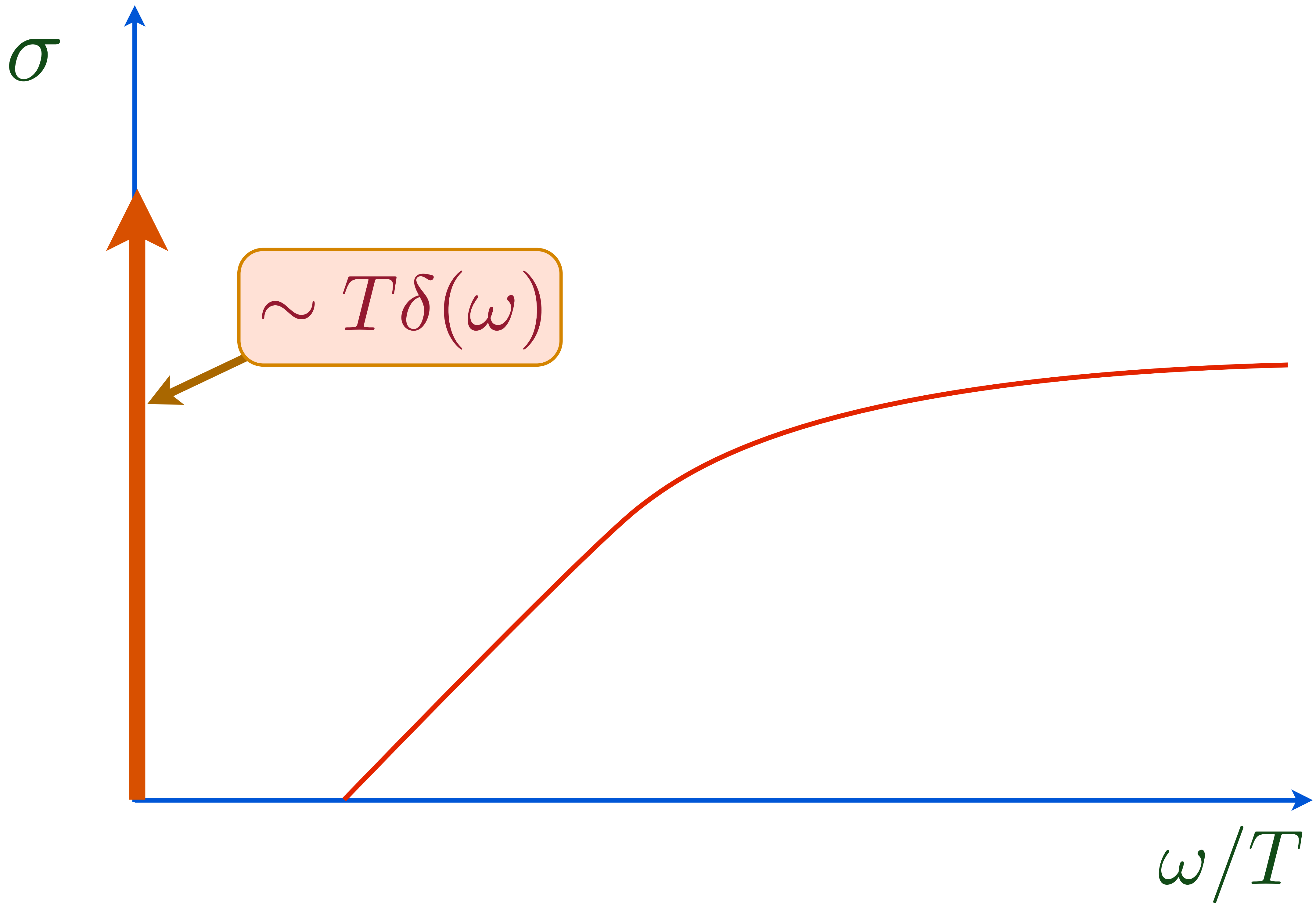

CFT3 at $T>0$
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


CFT3 at T>0
T
Boltzmann theory of
particles/holes/vortices
does not apply
T_KT
Superfluid
Insulator
0
λ_c
λ



CFT3 at $T>0$
T
T_{KT}
Needed:
Accurate theory of
quantum critical
dynamics without
quasiparticles
Superfluid
Insulator
0
λ_c
λ

Electrical transport in a free quasiparticle CFT3 for *T > 0*

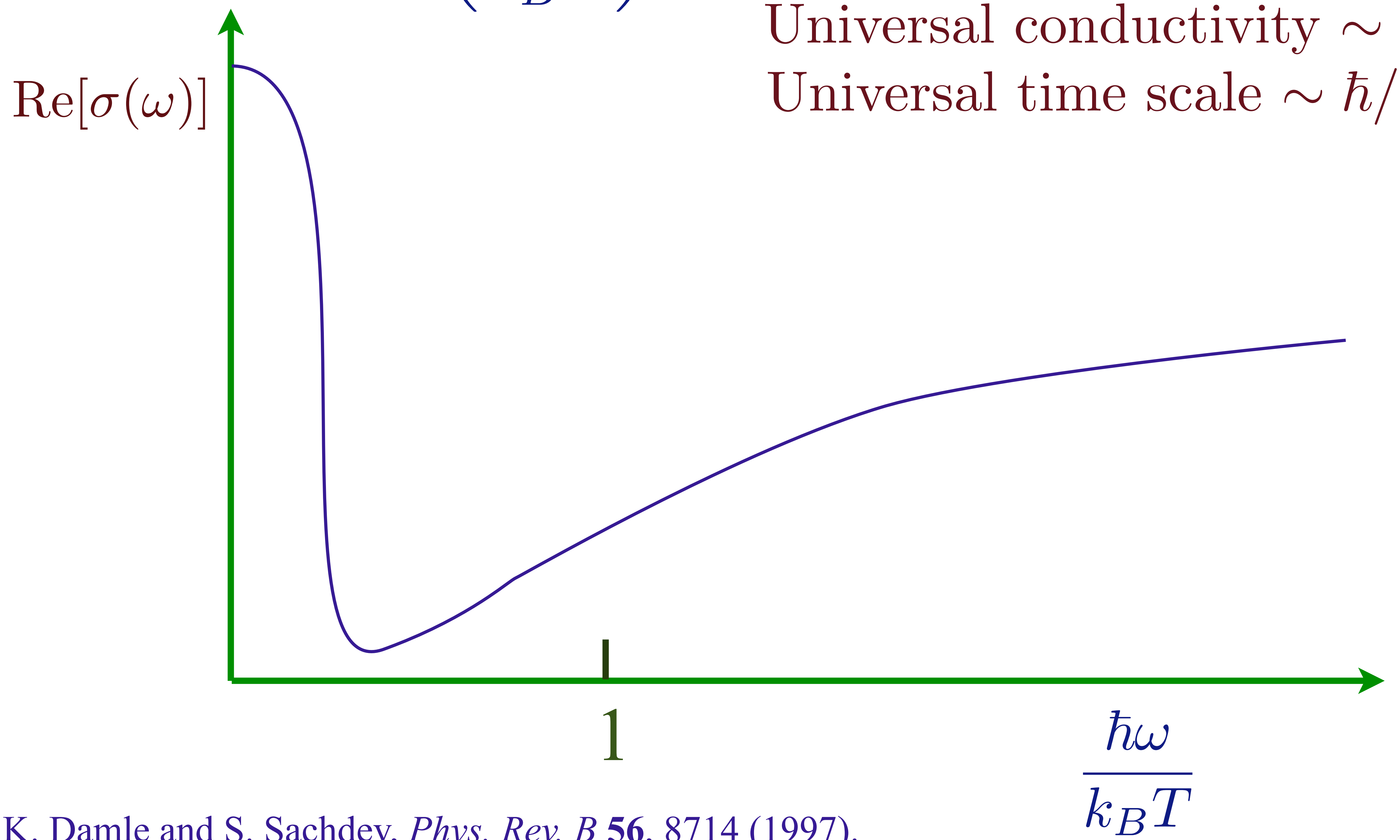


Quasiparticle view of quantum criticality: Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad ; \quad \Sigma \to \text{a universal function}$$

Universal conductivity $\sim e^2/h$

Universal time scale $\sim \hbar/k_B T$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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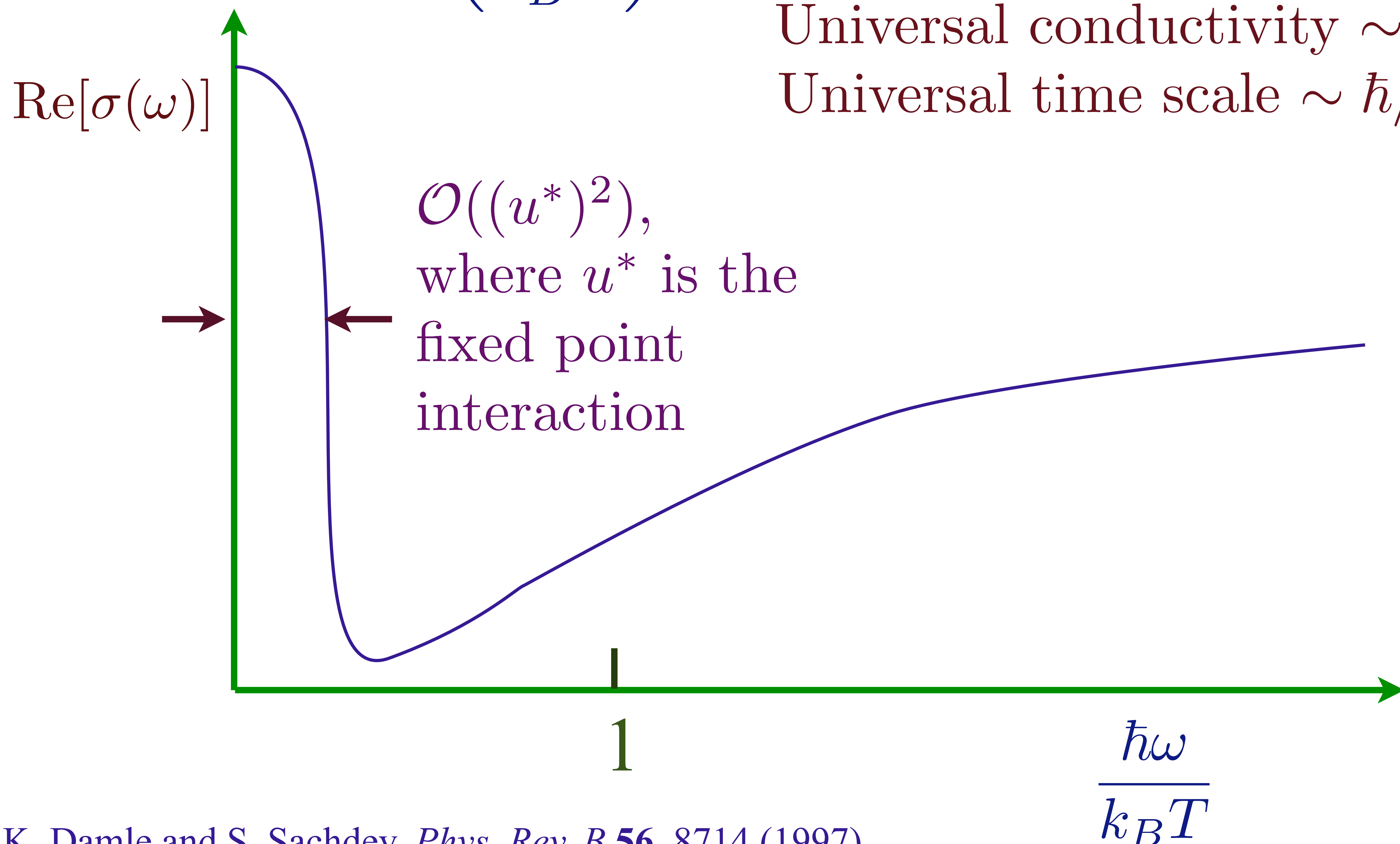

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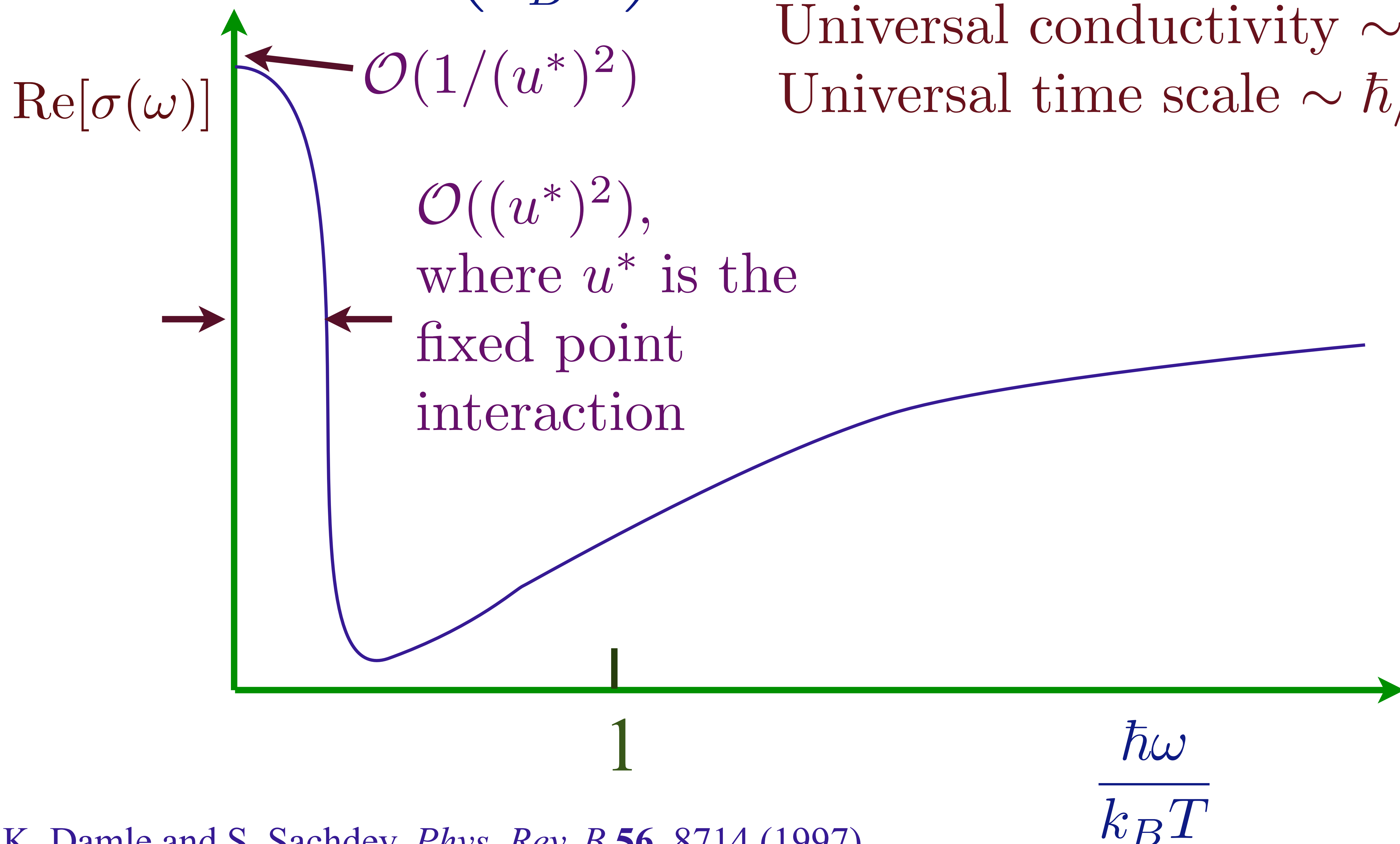

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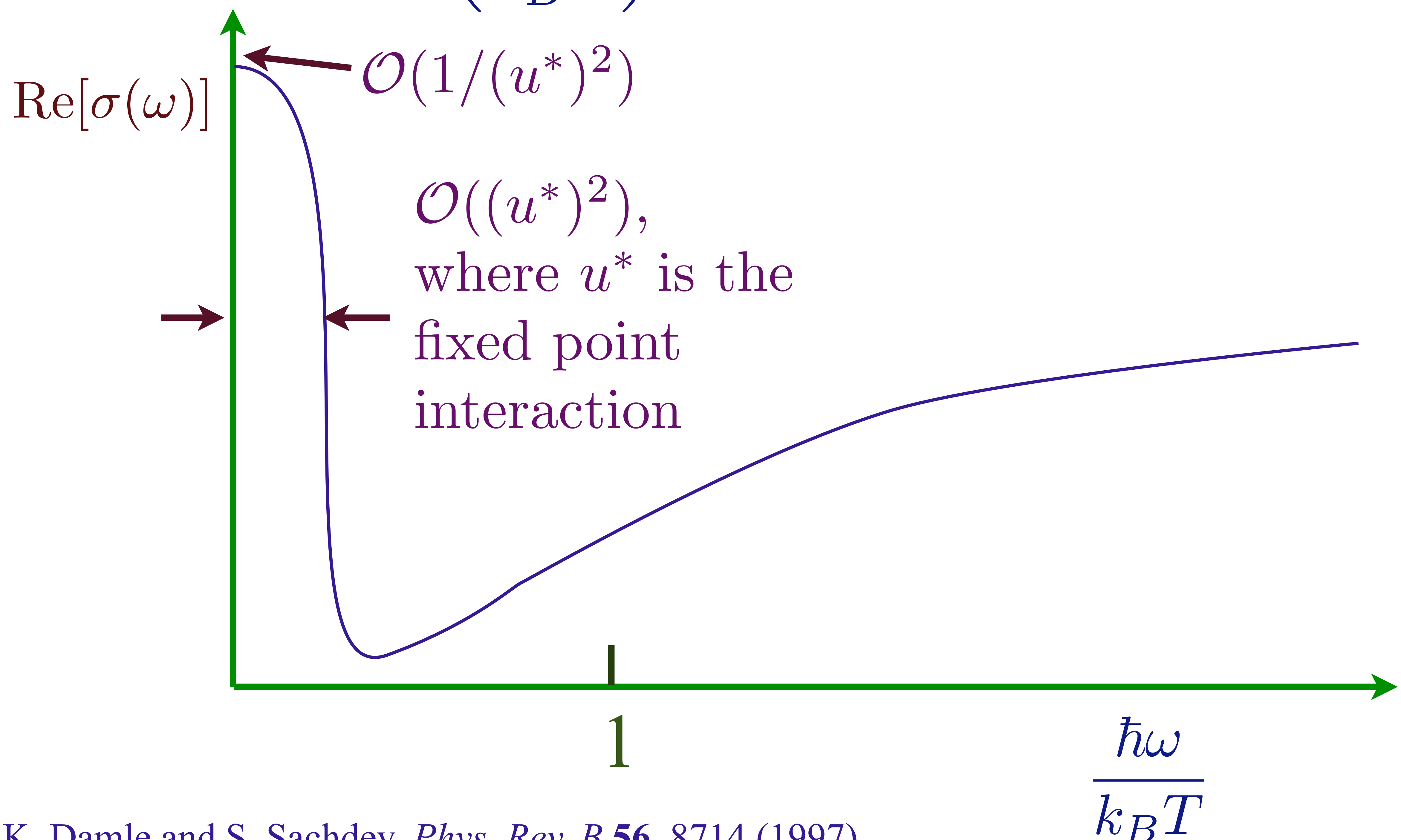

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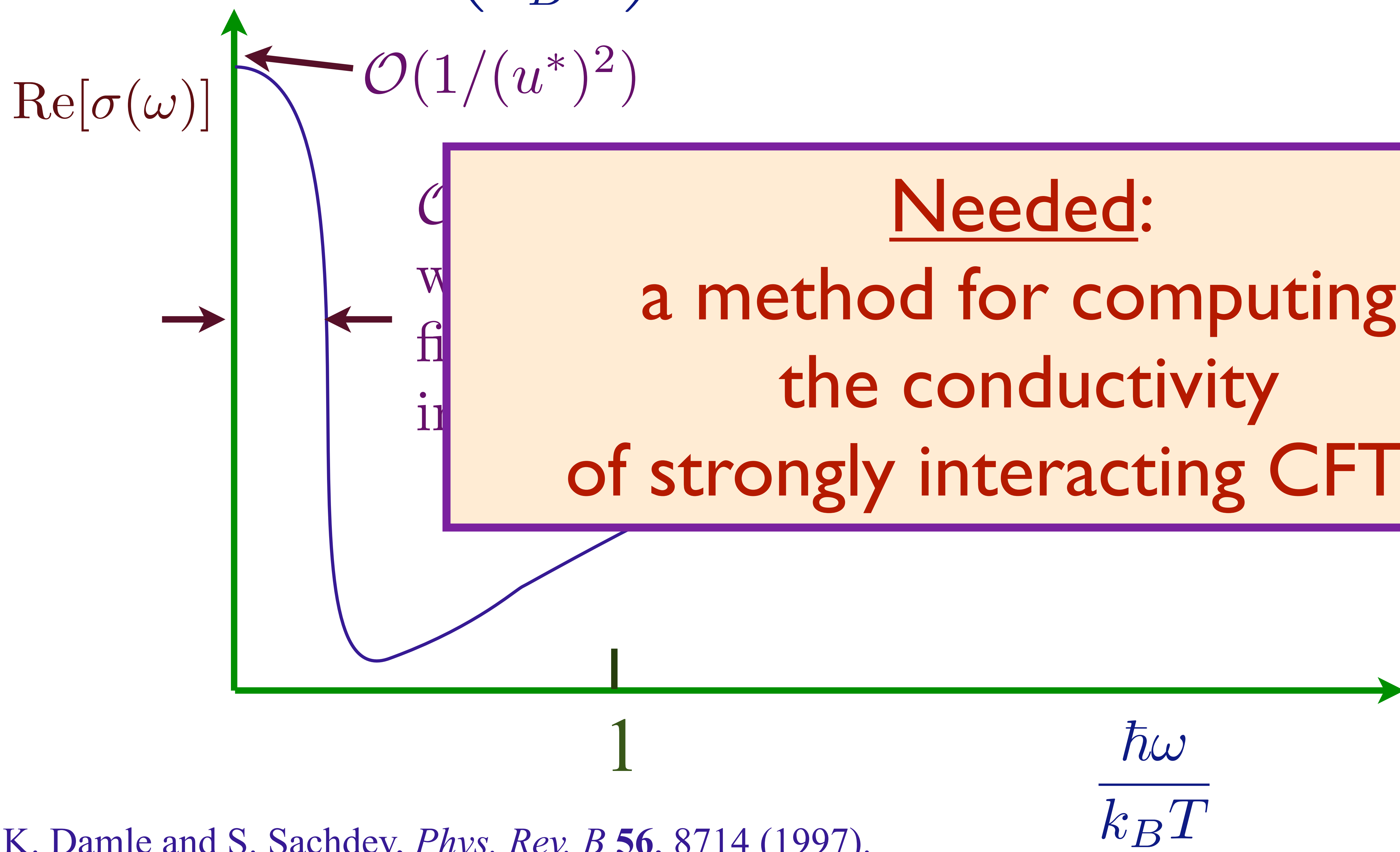


Needed:
a method for computing
the conductivity
of strongly interacting CFT3s

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

arXiv.org > cond-mat > arXiv:1309.2941

Search or Articl

Condensed Matter > Strongly Correlated Electrons

The dynamics of quantum criticality via Quantum Monte Carlo and holography

William Witczak-Krempa, Erik Sorensen, Subir Sachdev

(Submitted on 11 Sep 2013 (v1), last revised 29 Nov 2013 (this version, v2))

Understanding the real time dynamics of quantum systems without quasiparticles constitutes an important yet challenging problem. We study the superfluid-insulator quantum-critical point of bosons on a two-dimensional lattice, a system whose excitations cannot be described in a quasiparticle basis. We present detailed quantum Monte Carlo results for two separate lattice realizations: their low-frequency conductivities are found to have the same universal dependence on imaginary frequency and temperature. We then use the structure of the real time dynamics of conformal field theories described by the holographic gauge/gravity duality to make progress on the difficult problem of analytically continuing the Monte Carlo data to real time. Our method yields quantitative and experimentally testable results on the frequency-dependent conductivity near the quantum critical point, and on the spectrum of quasinormal modes in the vicinity of the superfluid-insulator quantum phase transition. Extensions to other observables and universality classes are discussed.

arXiv.org > cond-mat > arXiv:1309.5635

Search or

Condensed Matter > Strongly Correlated Electrons

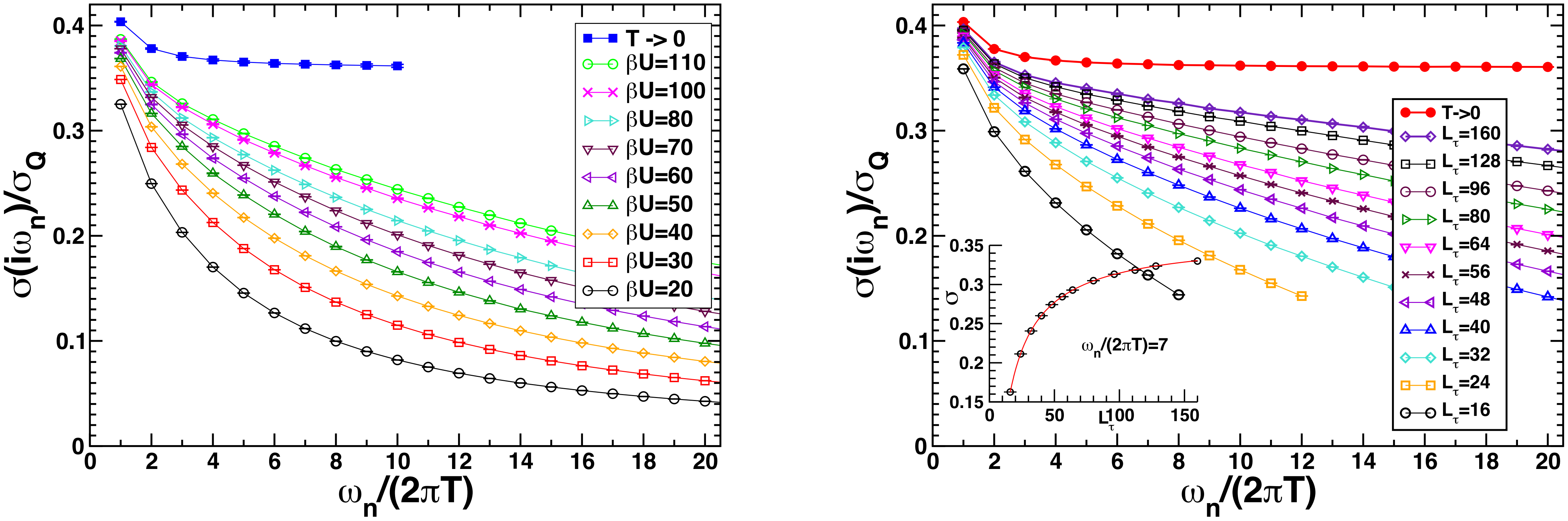
Universal Conductivity in a Two-dimensional Superfluid-to-Insulator Quantum Critical System

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, Nikolay Prokof'ev

(Submitted on 22 Sep 2013)

We compute the universal conductivity of the (2+1)-dimensional XY universality class, which is realized for a superfluid-to-Mott insulator quantum phase transition at constant density. Based on large-scale Monte Carlo simulations of the classical (2+1)-dimensional J-current model and the two-dimensional Bose-Hubbard model, we can precisely determine the conductivity on the quantum critical plateau, $\sigma(\infty) = 0.359(4)\sigma_Q$ with σ_Q the conductivity quantum. The universal conductivity is the schoolbook example of where the AdS/CFT correspondence from string theory can be tested and made to use. The shape of our $\sigma(i\omega_n) - \sigma(\infty)$ function in the Matsubara representation is accurate enough for a conclusive comparison and establishes the particle-like nature of charge transport. We find that the holographic gauge/gravity duality theory for transport properties can be made compatible with the data if temperature of the horizon of the black brane is different from the temperature of the conformal field theory. The requirements for measuring the universal conductivity in a cold gas experiment are also determined by our calculation.

Quantum Monte Carlo for lattice bosons



(a) (b)

FIG. 2. **Quantum Monte Carlo data** (a) Finite-temperature conductivity for a range of βU in the $L \to \infty$ limit for the quantum rotor model at $(t/U)_c$. The solid blue squares indicate the final $T \to 0$ extrapolated data. (b) Finite-temperature conductivity in the $L \to \infty$ limit for a range of L_τ for the Villain model at the QCP. The solid red circles indicate the final $T \to 0$ extrapolated data. The inset illustrates the extrapolation to $T = 0$ for $\omega_n/(2\pi T) = 7$. The error bars are statistical for both a) and b).

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635