

# Spin liquids on the triangular lattice

ICFCM, Sendai, Japan, Jan 11-14, 2011

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)



# Outline

## 1. Classification of spin liquids

*Quantum-disordering magnetic order vs.  
projected Fermi sea*

## 2. Quantum-disordering magnetic order

*Application to  $\kappa$ -(ET)<sub>2</sub>Cu<sub>2</sub>(CN)<sub>3</sub>*

## 3. Fermi surfaces of spinful Majorana fermions

*Candidate for EtMe<sub>3</sub>Sb[Pd(dmit)<sub>2</sub>]<sub>2</sub> ?*

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# Fractionalization of the electron spin and charge

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## A) Quantum “disordering” magnetic order

$$\begin{pmatrix} c_{\uparrow} \\ c_{\downarrow} \end{pmatrix} = \begin{pmatrix} z_{\uparrow} & -z_{\downarrow}^* \\ z_{\downarrow} & z_{\uparrow}^* \end{pmatrix} \begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix}$$

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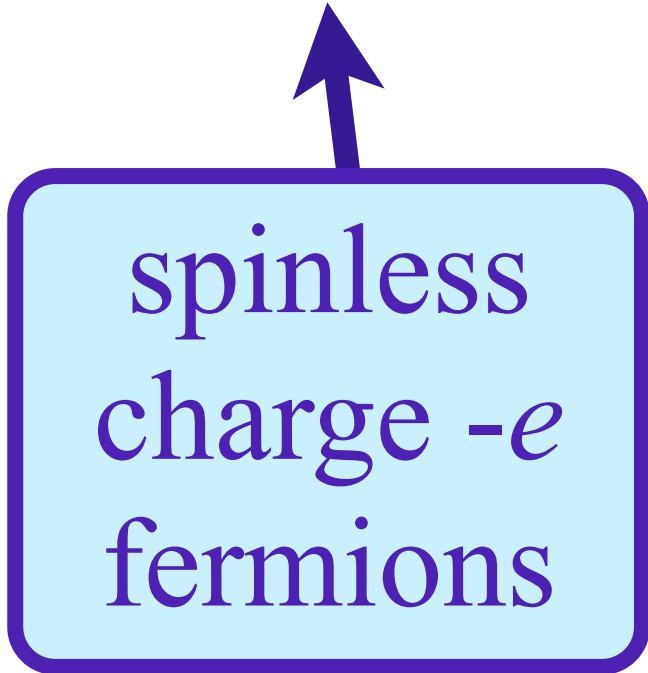
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neutral bosonic  
spinons which transform to a  
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spinless  
charge  $-e$   
fermions

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U(1)<sub>charge</sub>

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$U \times U^{-1}$   
Theory has  $SU(2)_s$   
gauge invariance

S. Sachdev, M. A. Metlitski, Y. Qi, and S. Sachdev *Phys. Rev. B* **80**, 155129 (2009)

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## B) Projected Fermi sea

X.-G. Wen, P.A. Lee,  
O. Motrunich, M. P.A. Fisher

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charge  $e$  spinless bosons (or  
“rotors”) whose fluctuations  
project onto the single  
electron states on each site

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neutral  
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$$\text{SU}(2)_{\text{pseudospin}} \supset \text{U}(1)_{\text{charge}}$$

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On the triangular lattice, this leads to a U(1) spin liquid with a Fermi surface of spinons which has been proposed to apply to  $\text{EtMe}_3\text{Sb}[\text{Pd}(\text{dmit})_2]_2$ .

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This spin liquid has a thermal Hall response which is not observed.

Complete fractionalization: separate excitations  
carrying spin, charge, and Fermi statistics

Cenke Xu and S. Sachdev, *Phys. Rev. Lett.* **105**, 057201 (2010)

# Complete fractionalization: separate excitations carrying spin, charge, and Fermi statistics

Decompose electron operator into real fermions,  $\chi$ :

$$c_{\uparrow} = \chi_1 + i\chi_2 \quad ; \quad c_{\downarrow} = \chi_3 + i\chi_4$$

Introduce a 4-component Majorana fermion  $\zeta_i$ ,  $i = 1 \dots 4$  and a  $SO(4)$  matrix  $\mathcal{R}$ , and decompose:

$$\chi = \mathcal{R} \zeta$$

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$$SO(4) \cong SU(2)_{\text{pseudospin}} \times SU(2)_{\text{spin}}$$

Cenke Xu and S. Sachdev, *Phys. Rev. Lett.* **105**, 057201 (2010)

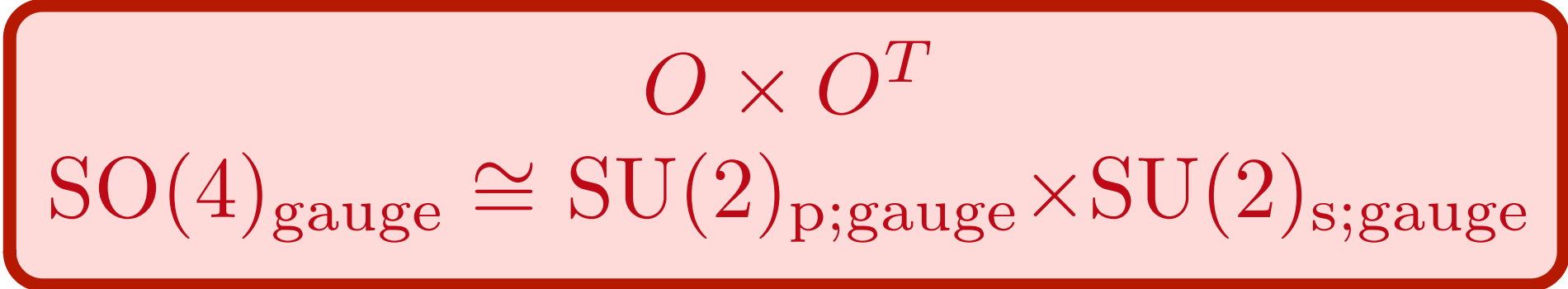
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$$SO(4)_{\text{gauge}} \cong SU(2)_{\text{p;gauge}} \times SU(2)_{\text{s;gauge}}$$
$$O \times O^T$$

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By breaking  $SO(4)_{\text{gauge}}$  with different Higgs fields, we can reproduce essentially all earlier theories of spin liquids. We also find many new spin liquid phases, some with Majorana fermion excitations which carry neither spin nor charge

Cenke Xu and S. Sachdev, *Phys. Rev. Lett.* **105**, 057201 (2010)

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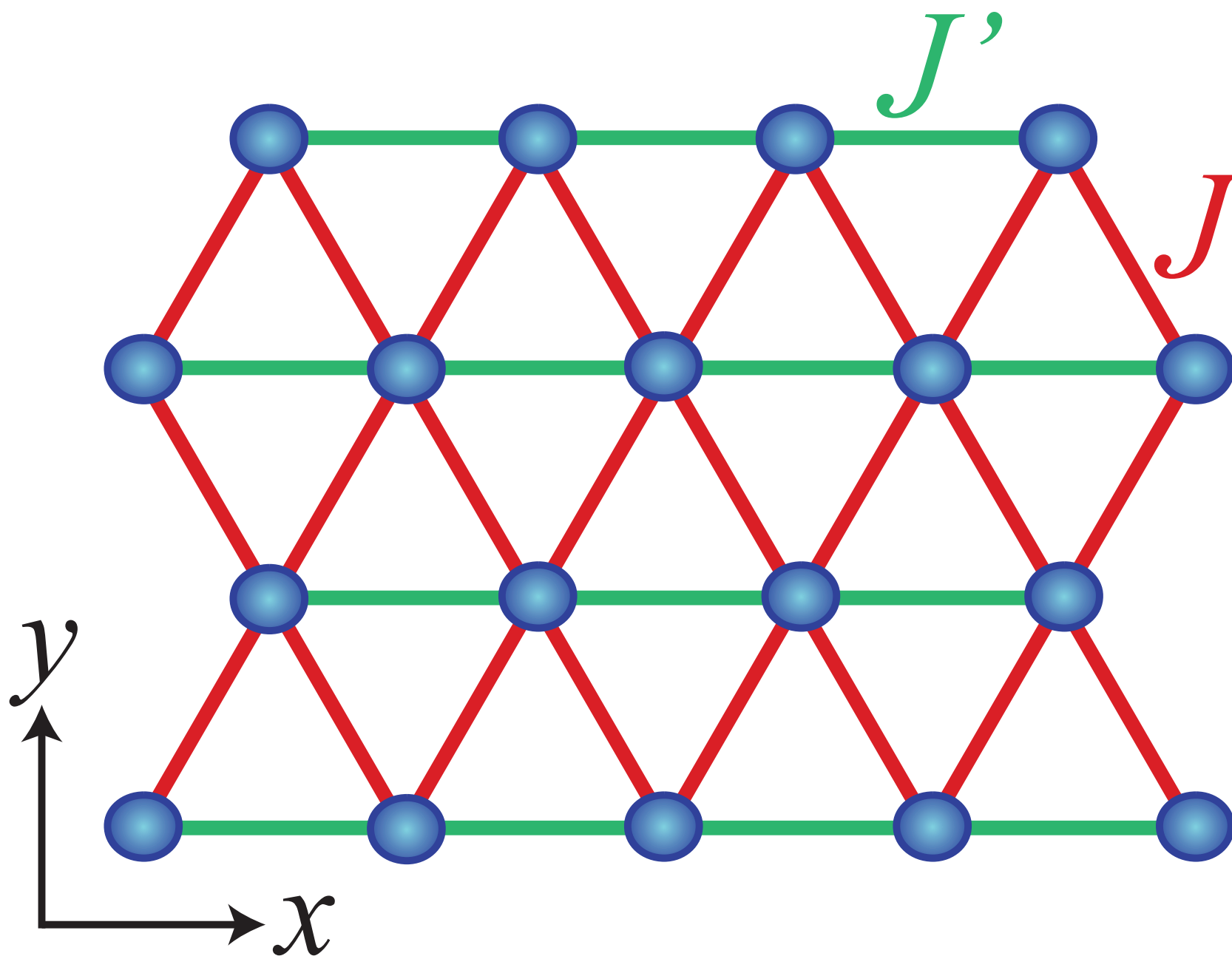
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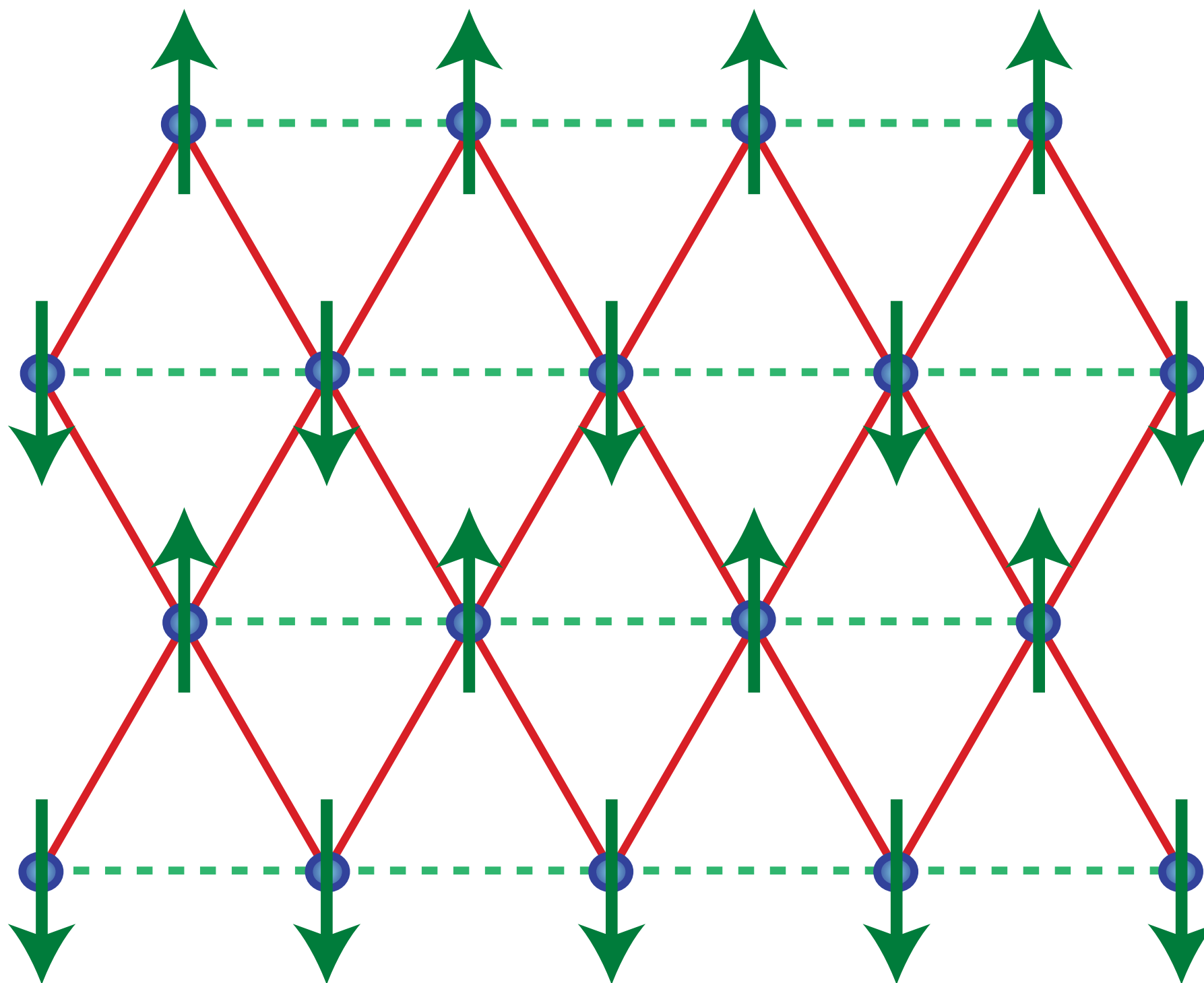
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$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \dots$$

$\vec{S}_i \Rightarrow$  spin operator with  $S = 1/2$



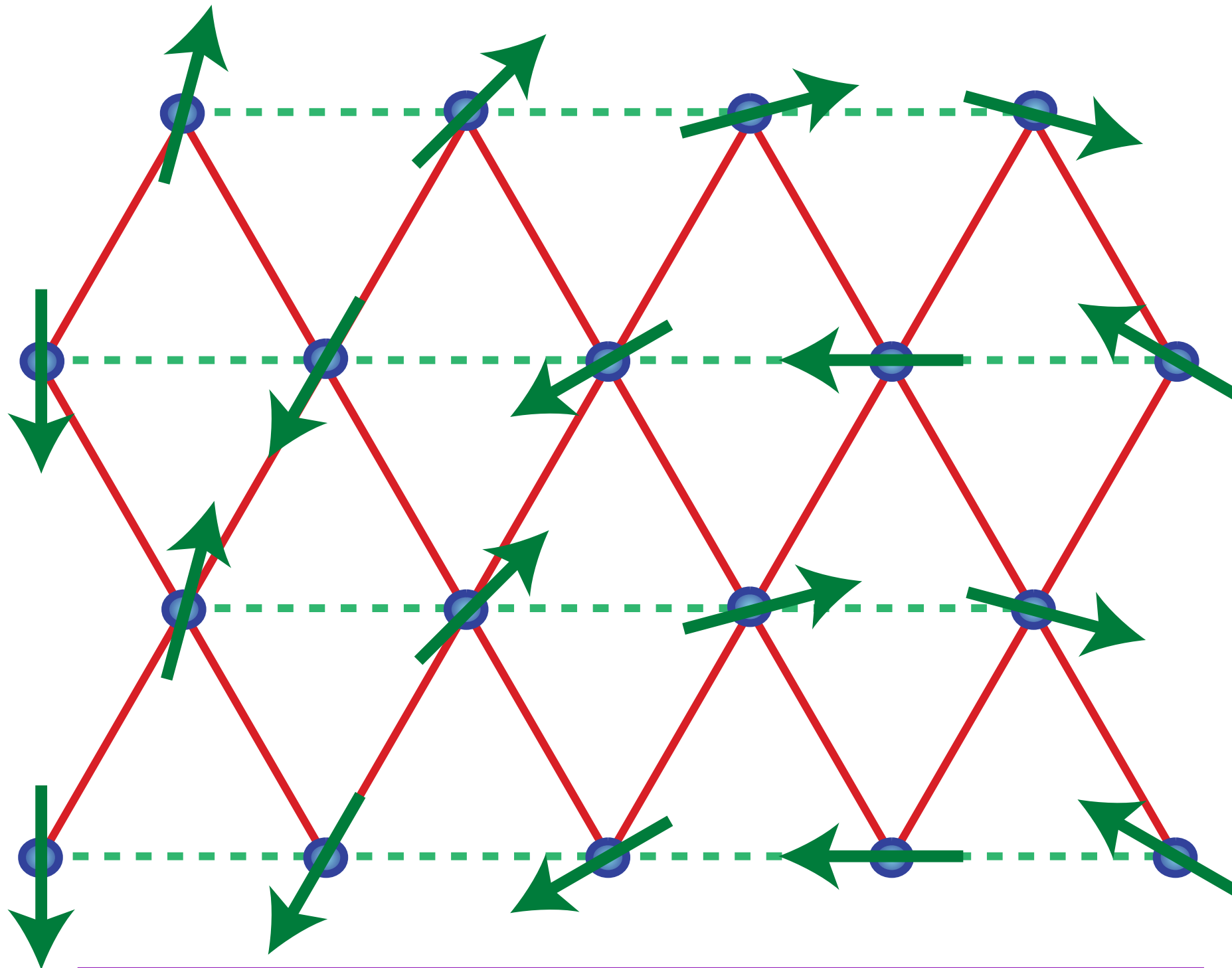
# Anisotropic triangular lattice antiferromagnet



Classical ground state for small  $J'/J$

Found in  $\kappa$ -(ET)<sub>2</sub>Cu[N(CN)<sub>2</sub>]Cl

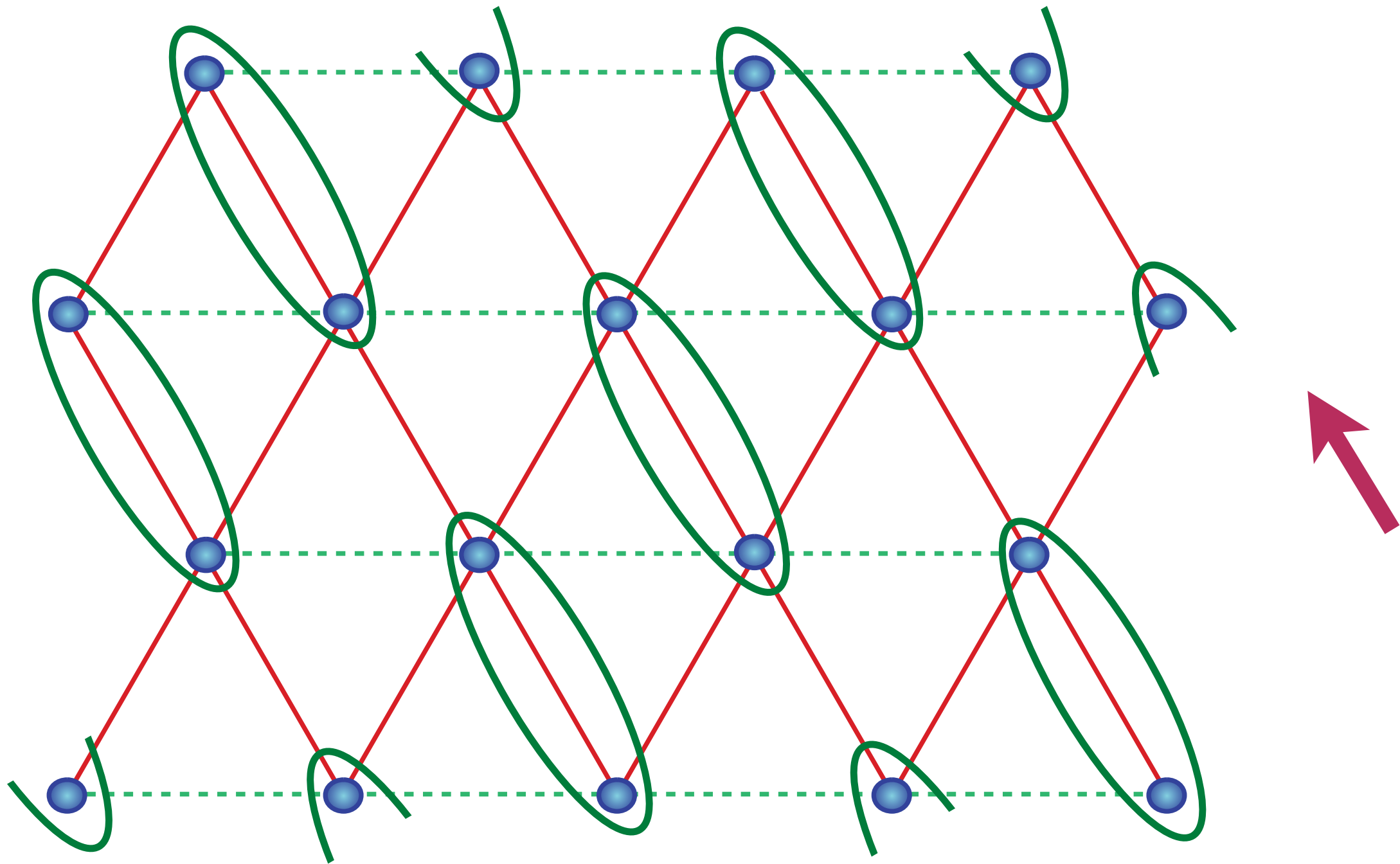
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Classical ground state for large  $J'/J$

Found in  $\text{Cs}_2\text{CuCl}_4$

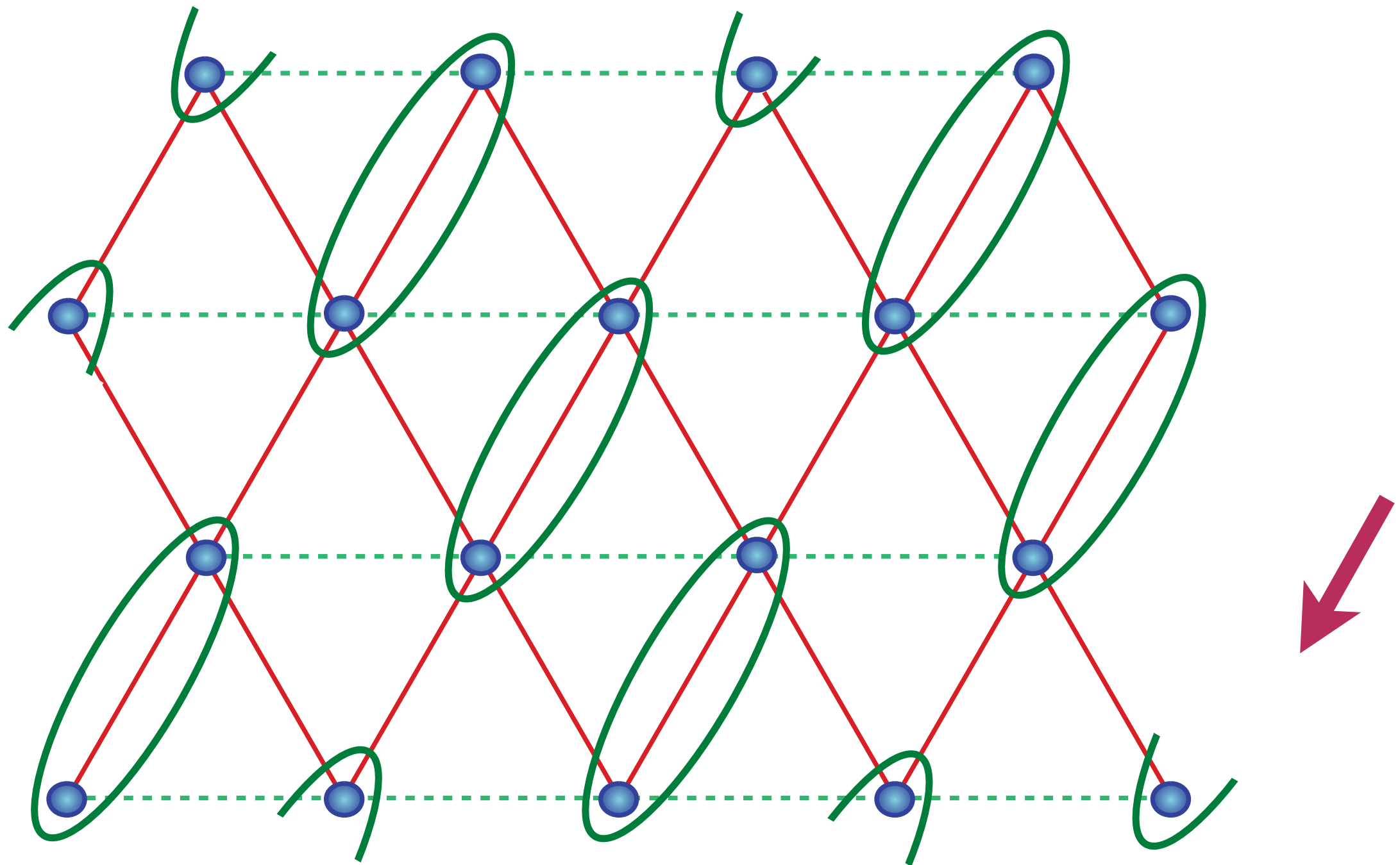
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## Valence bond solid

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

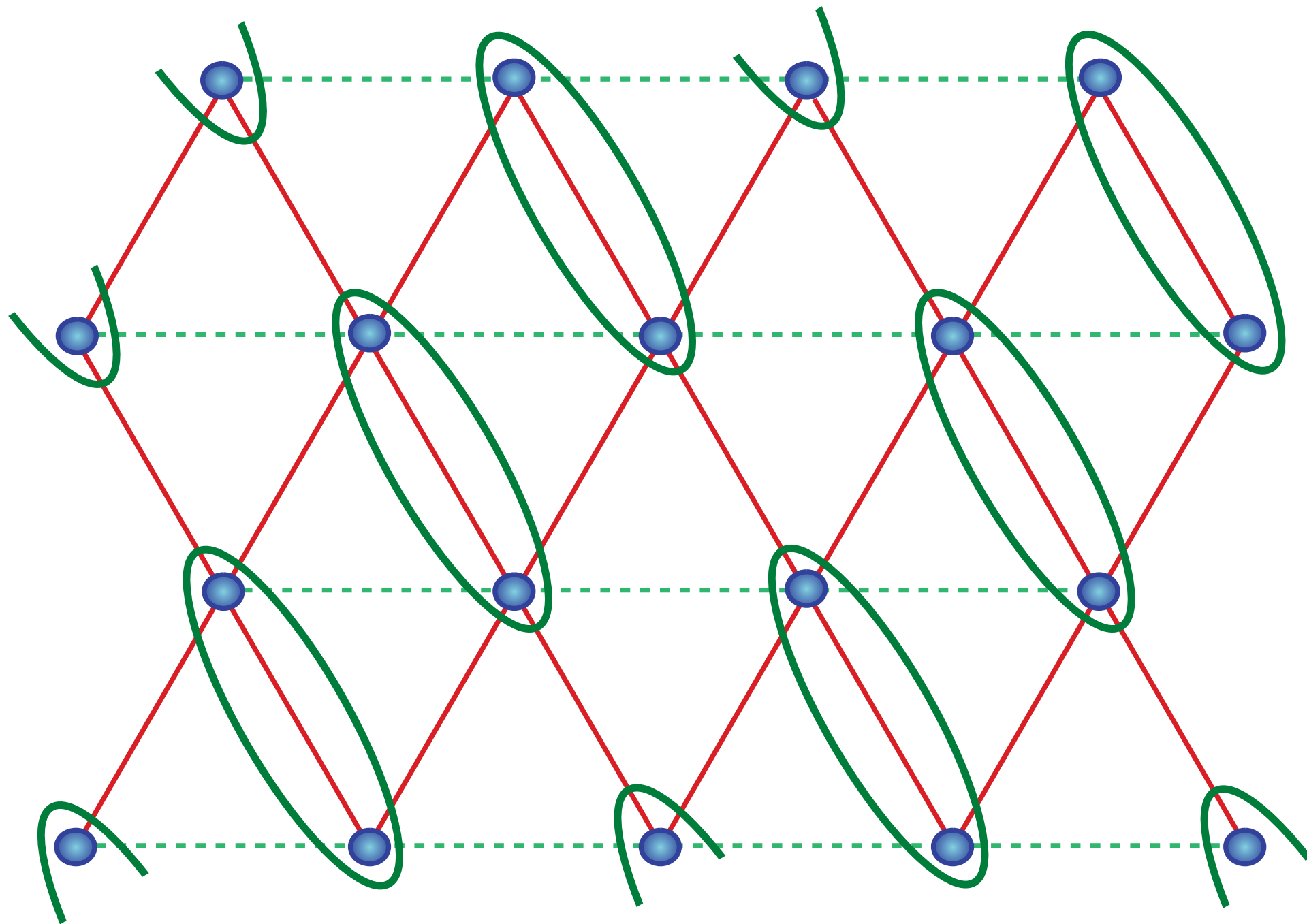
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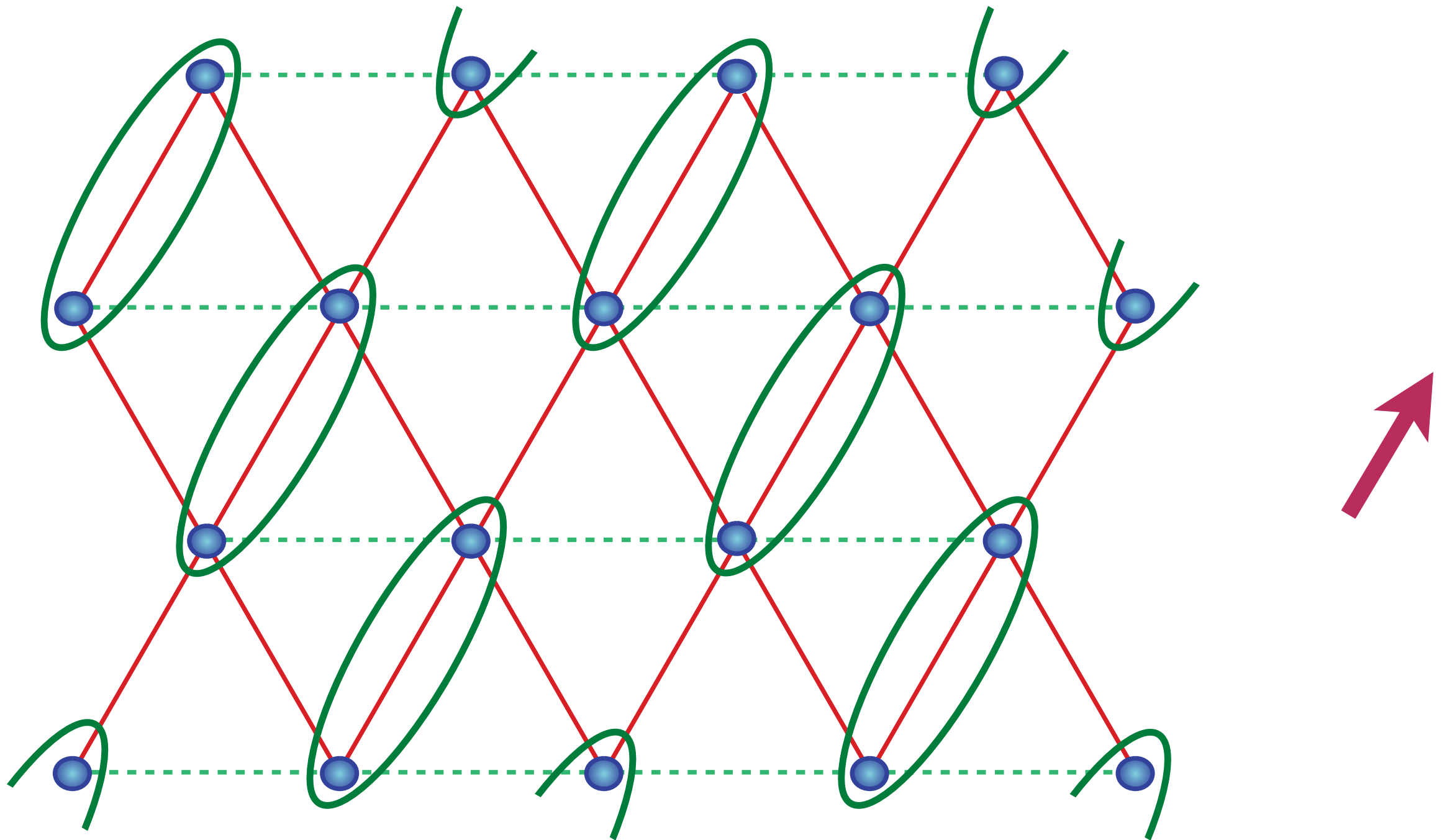
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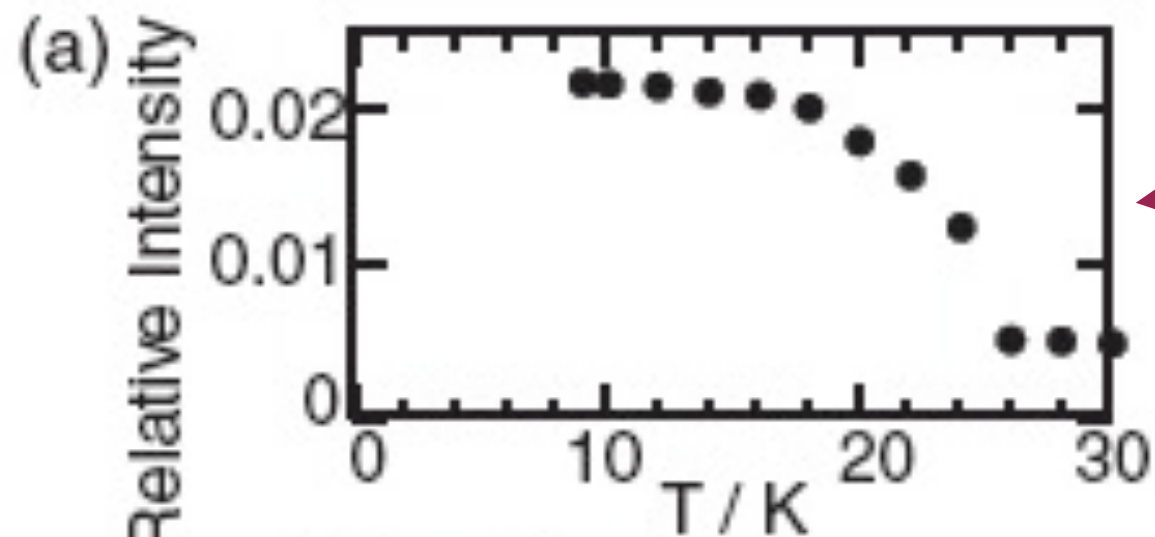


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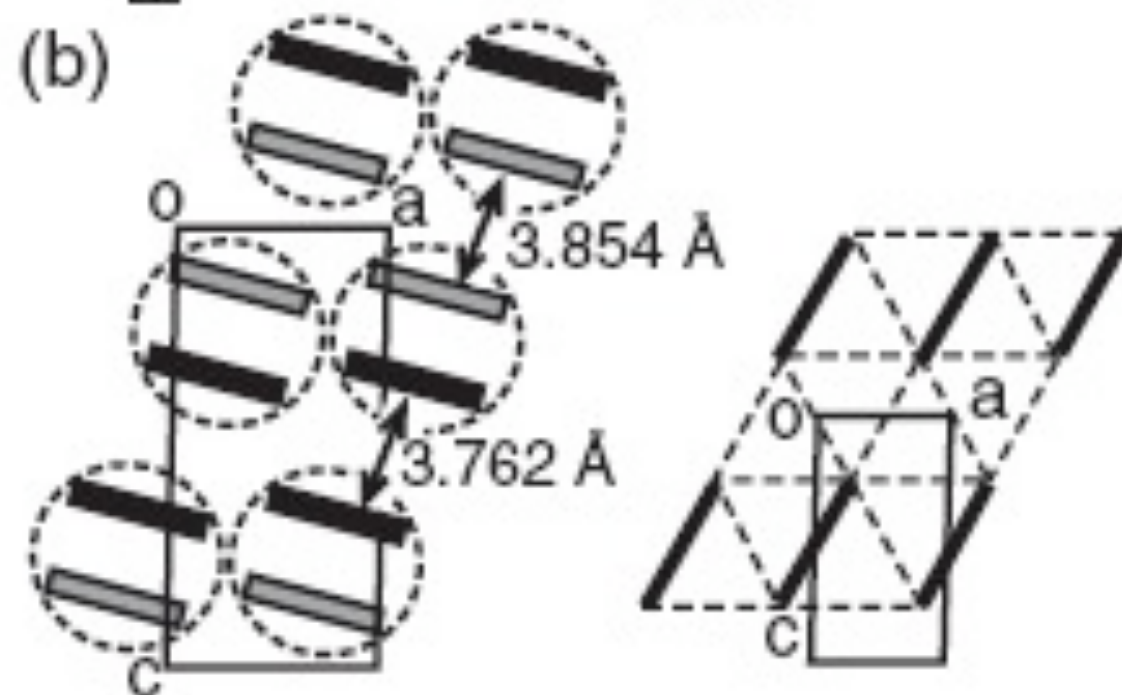
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# Observation of a valence bond solid (VBS) in $\text{ETMe}_3\text{P}[\text{Pd}(\text{dmit})_2]_2$



X-ray scattering




Spin gap  $\sim 40$  K  
 $J \sim 250$  K

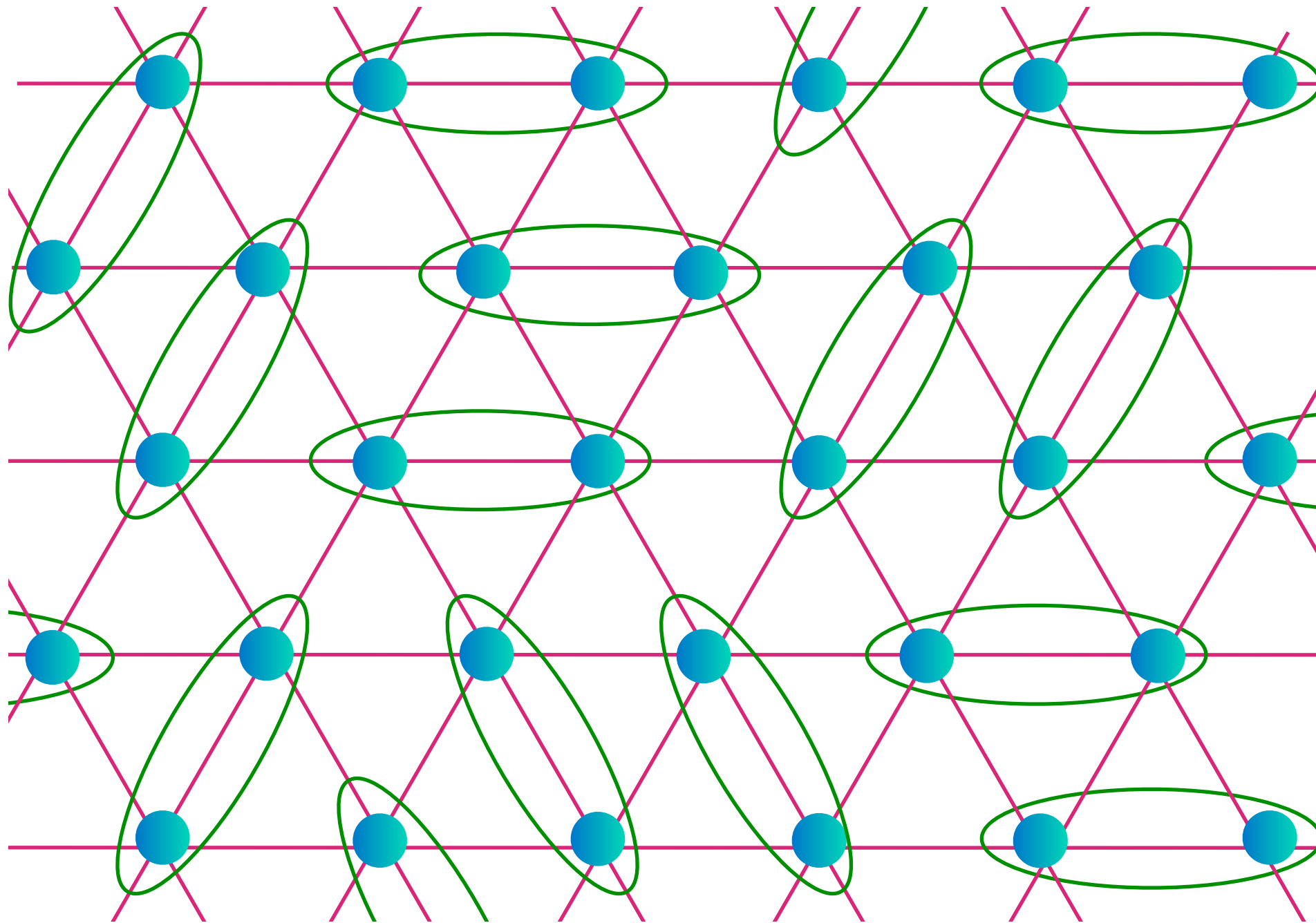
M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

# Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with  $S=1/2$  per unit cell



$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

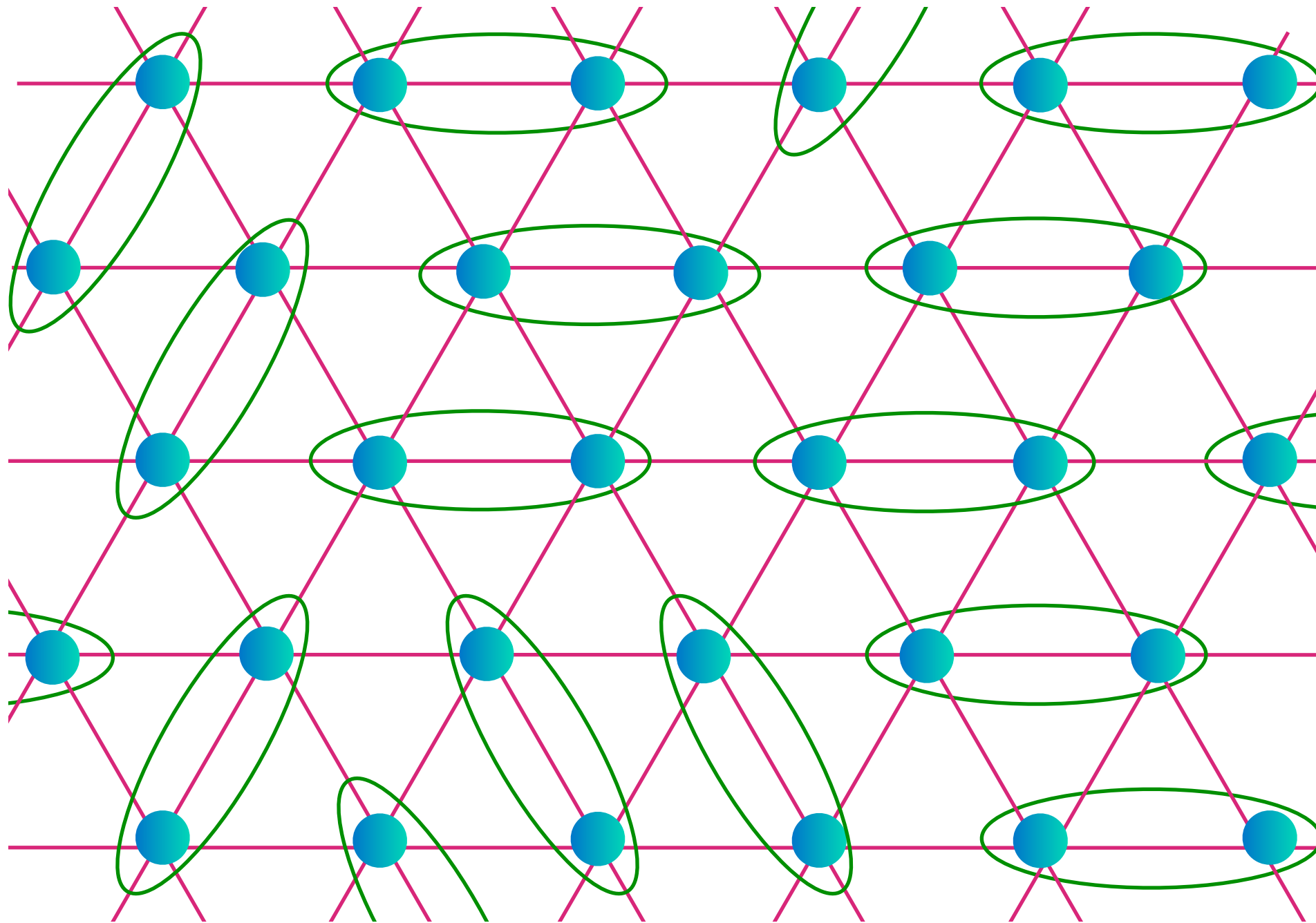


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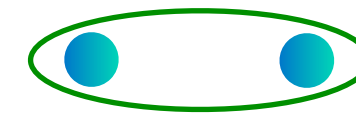

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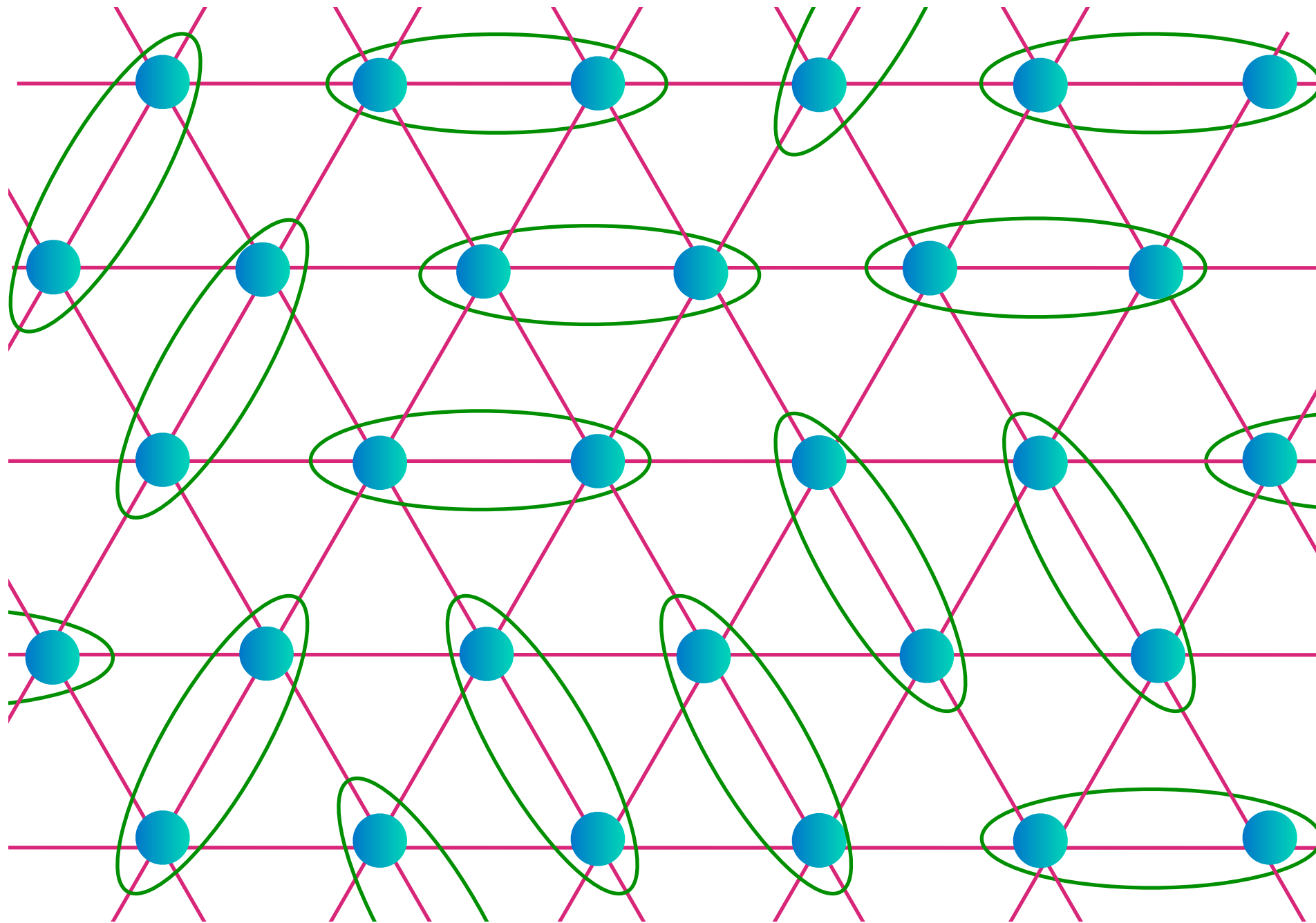


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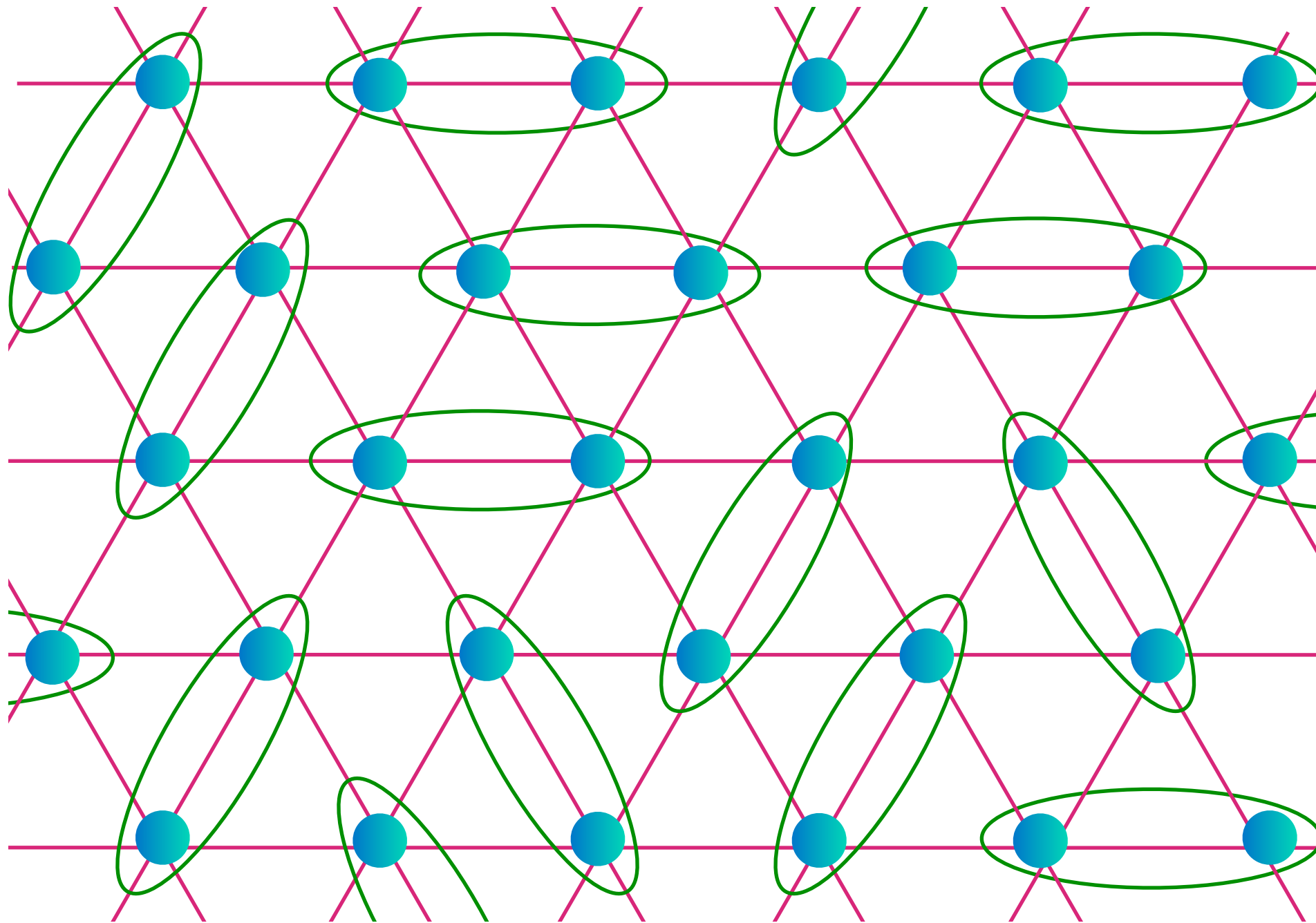


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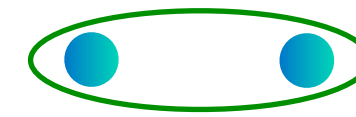
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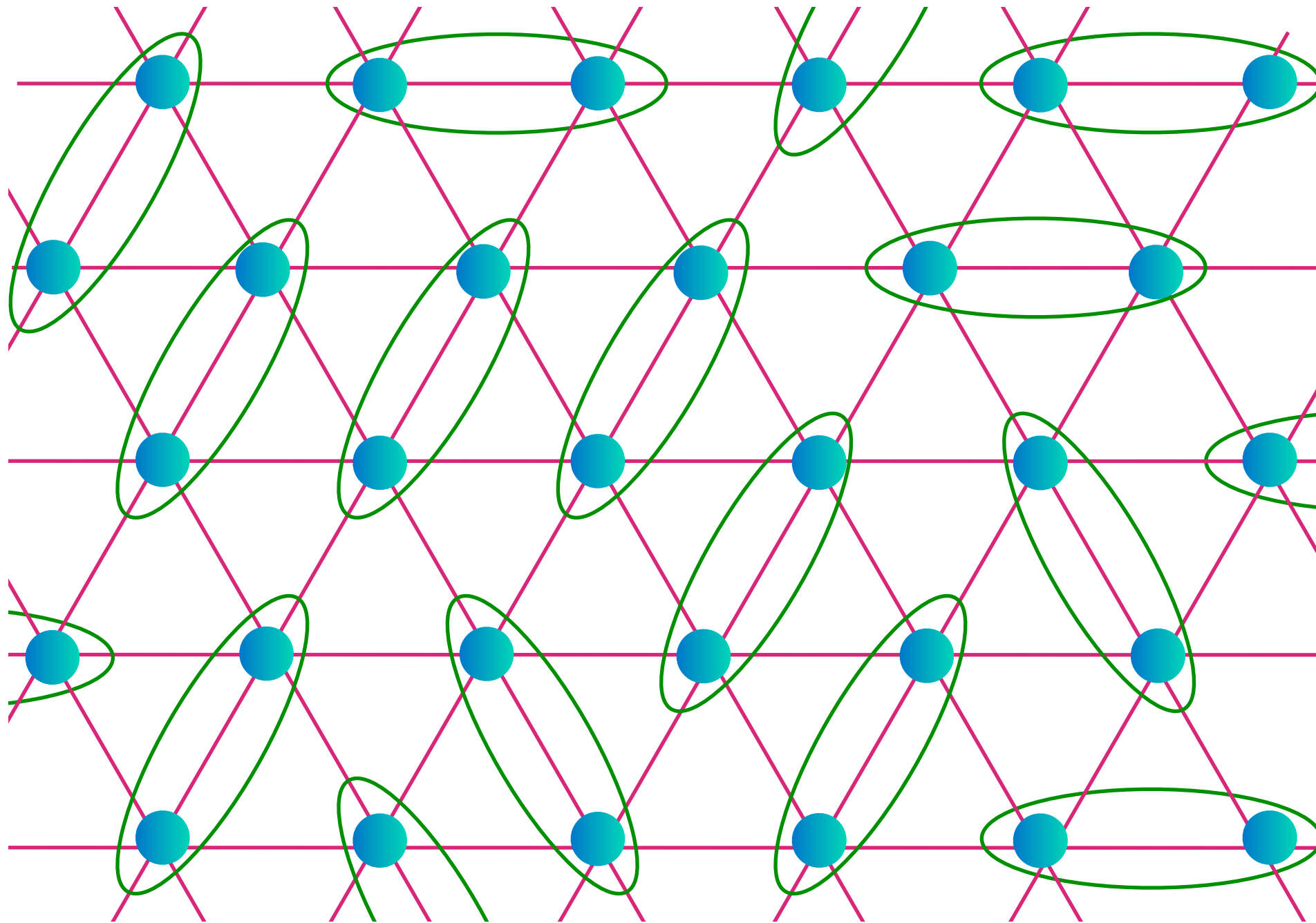
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Spin liquid obtained in a generalized spin model with  $S=1/2$  per unit cell



A diagram showing two blue spheres representing spins within a green oval representing a unit cell. The spheres are positioned at the two vertices of a triangle.

$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



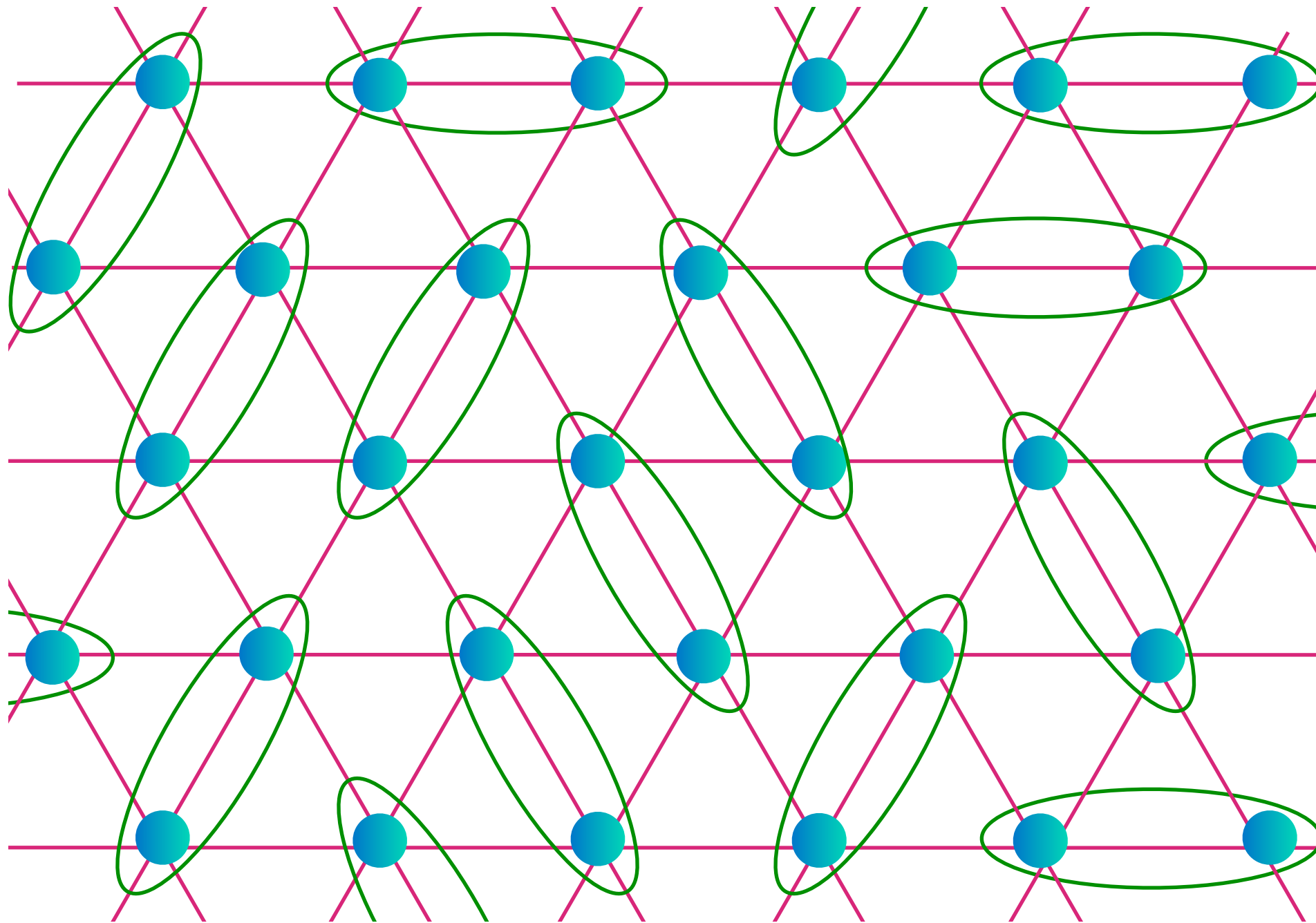
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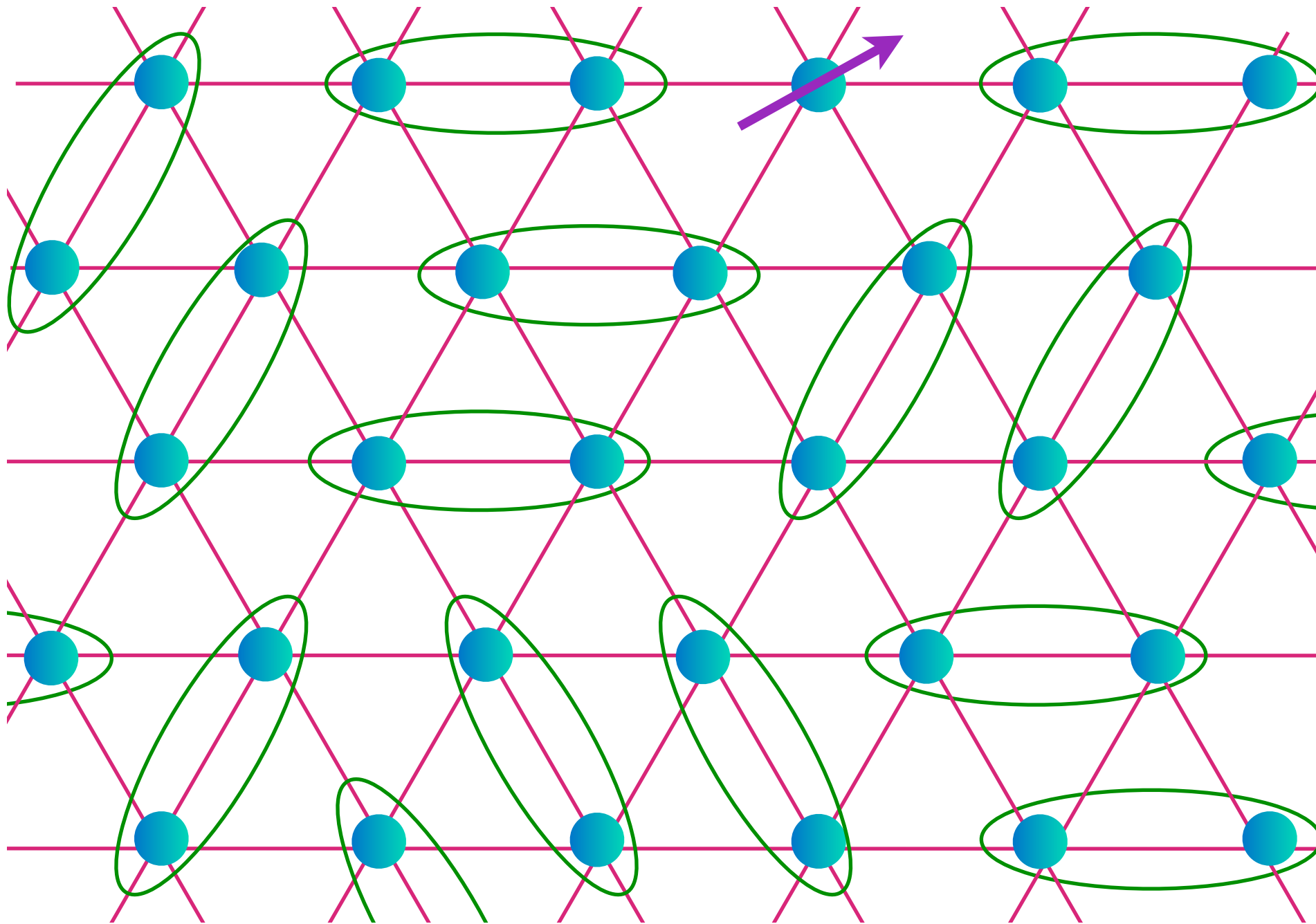


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# Excitations of the $Z_2$ Spin liquid

A spinon



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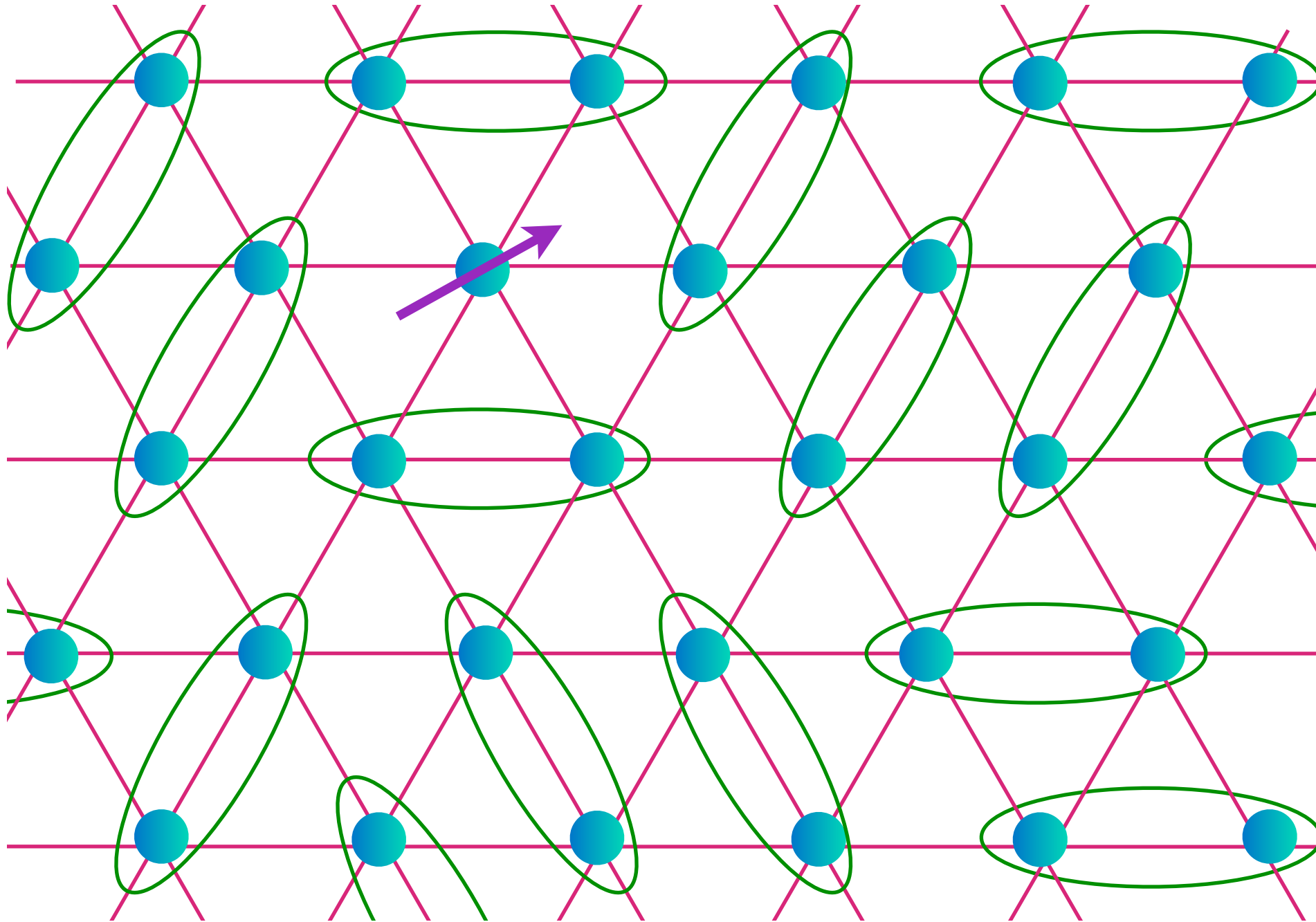




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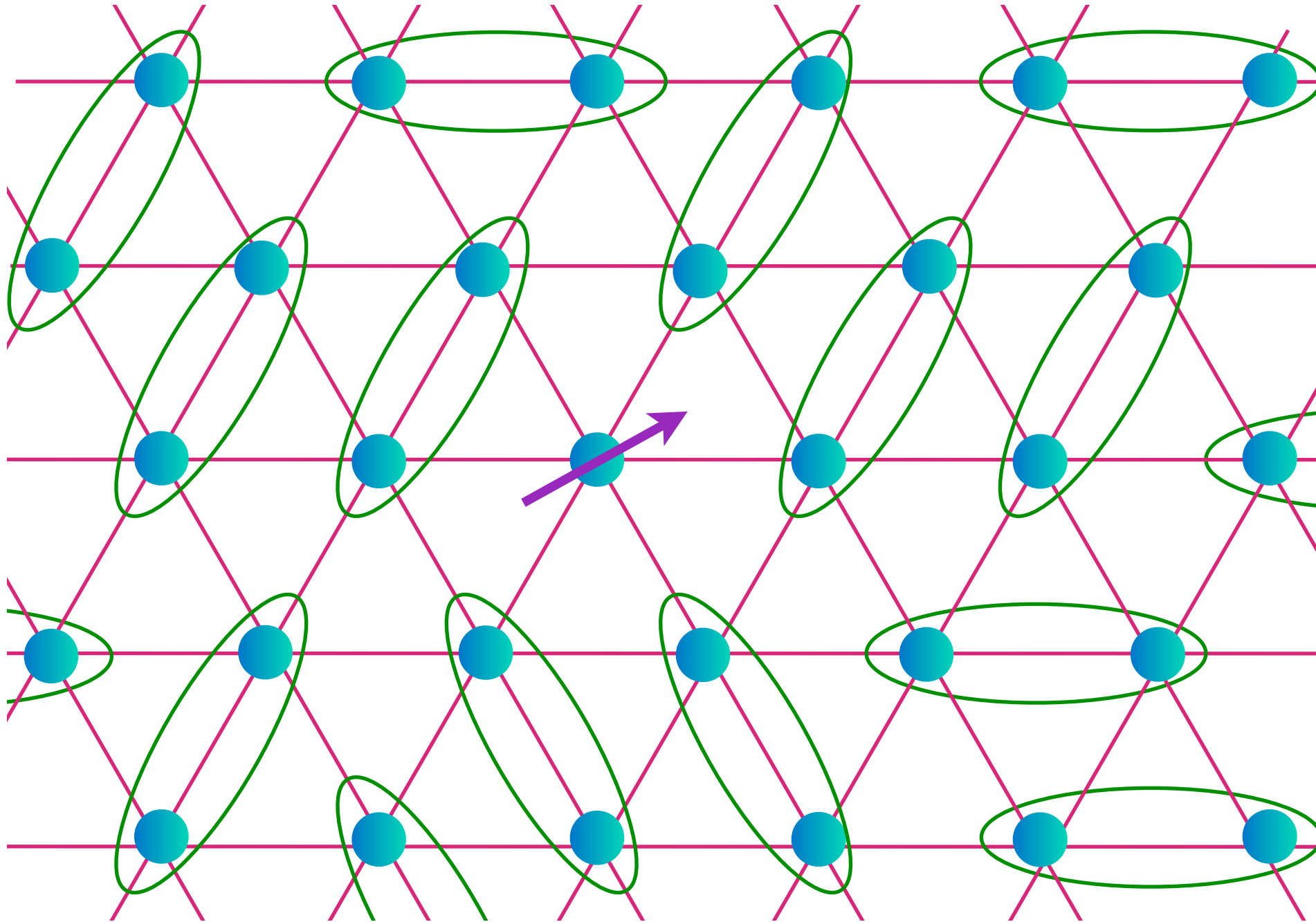

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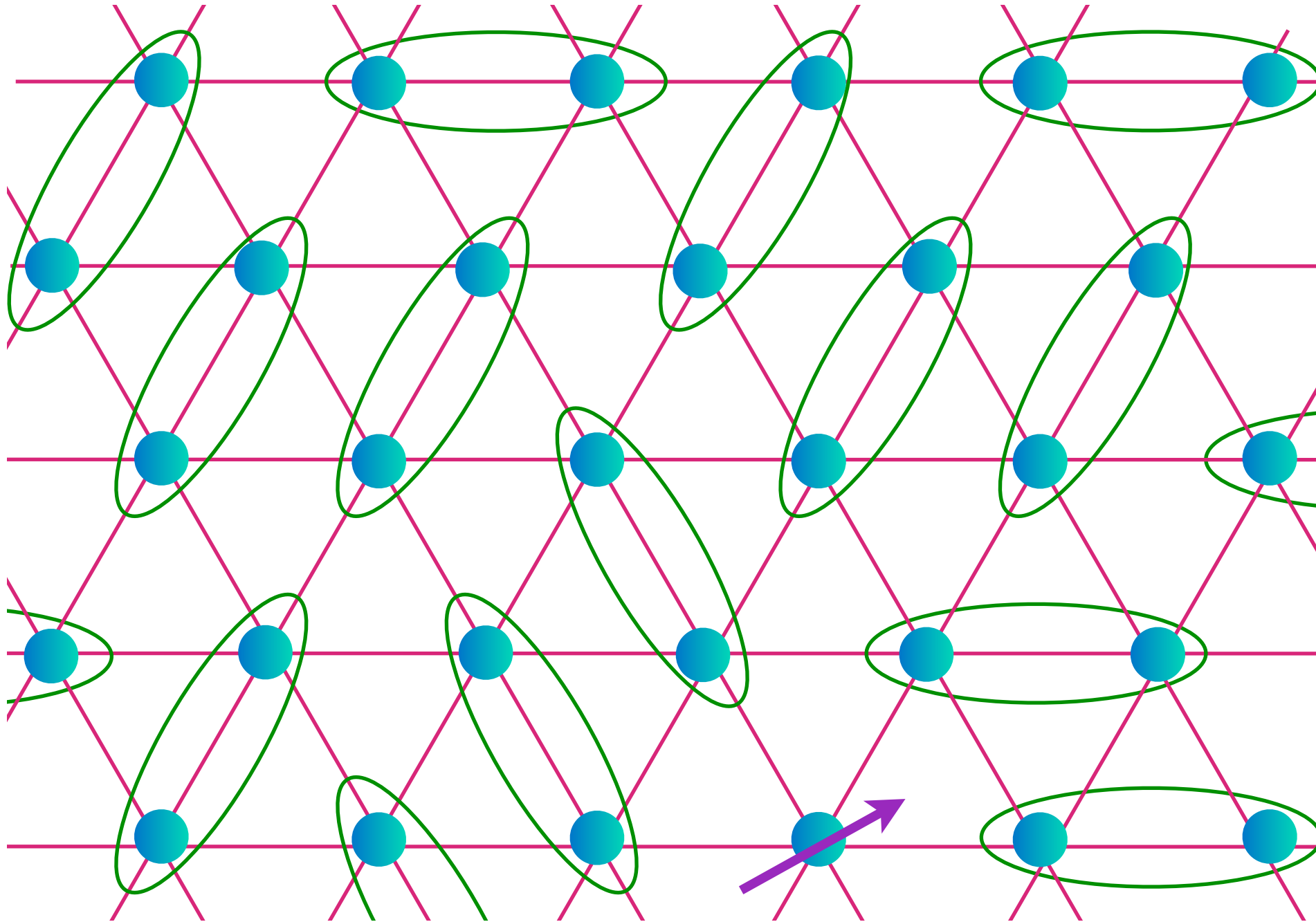

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# Excitations of the $Z_2$ Spin liquid

## A vison

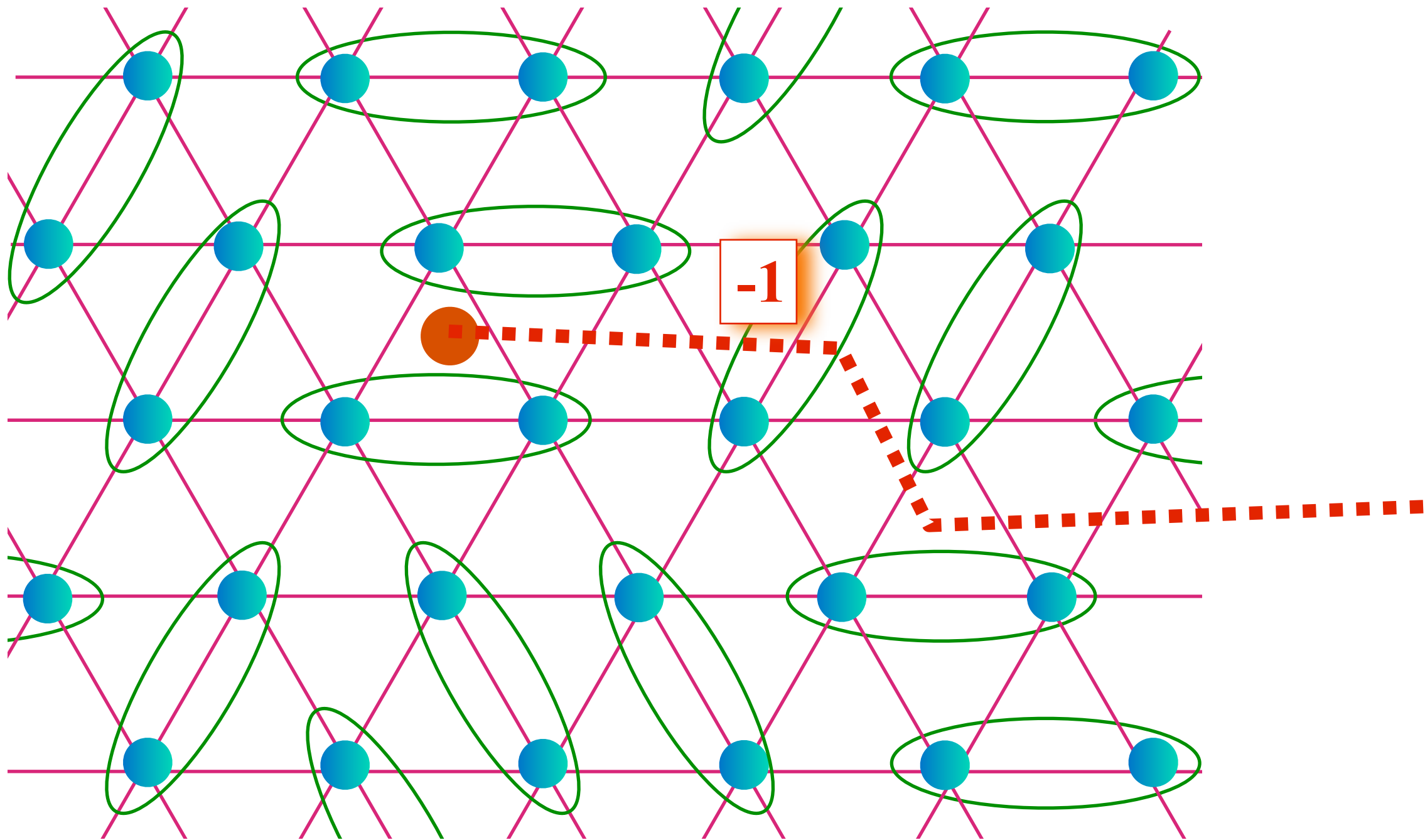
- A characteristic property of a  $Z_2$  spin liquid is the presence of a spinon pair condensate
- A vison is an Abrikosov vortex in the pair condensate of spinons
- Visions are the dark matter of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

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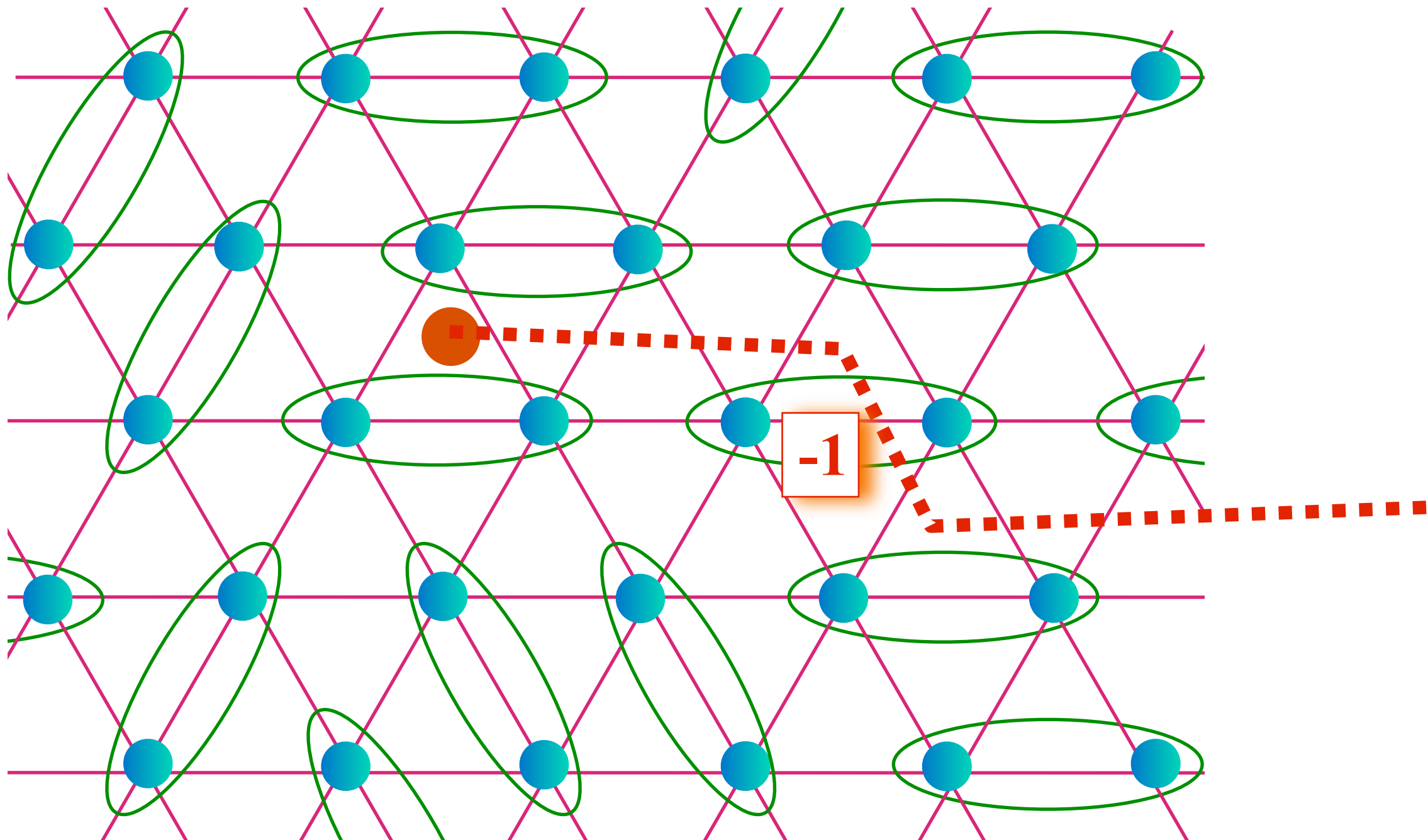


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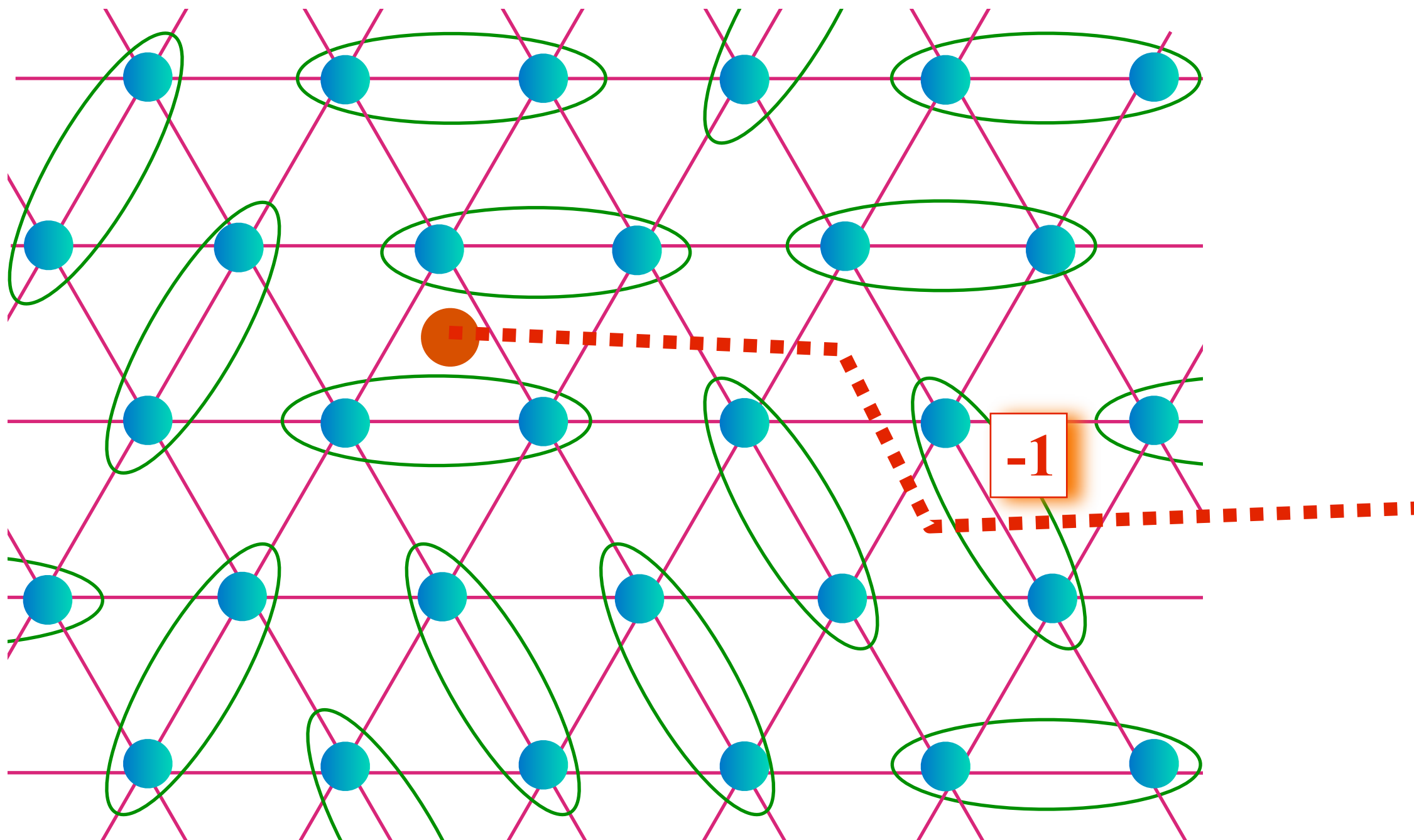


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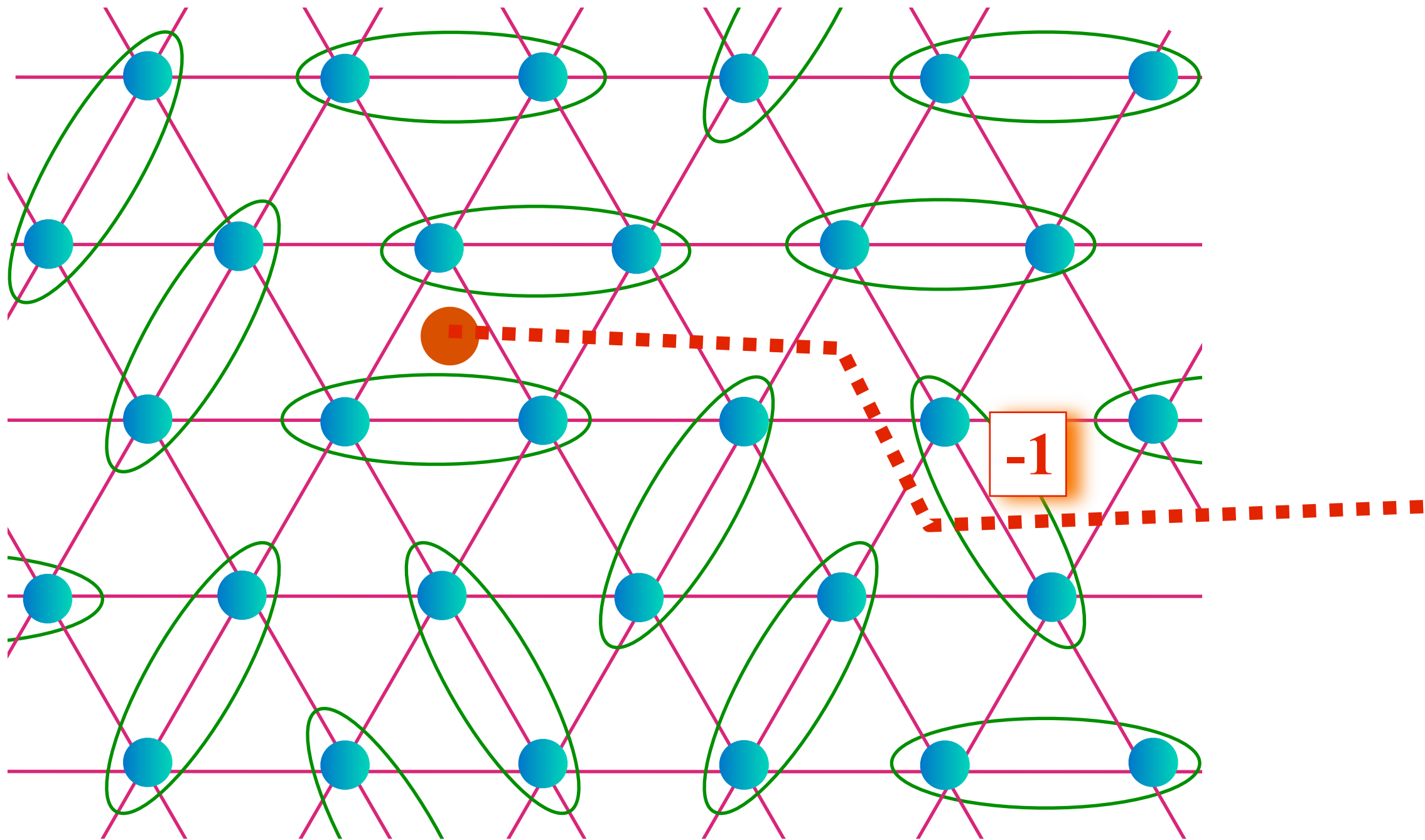
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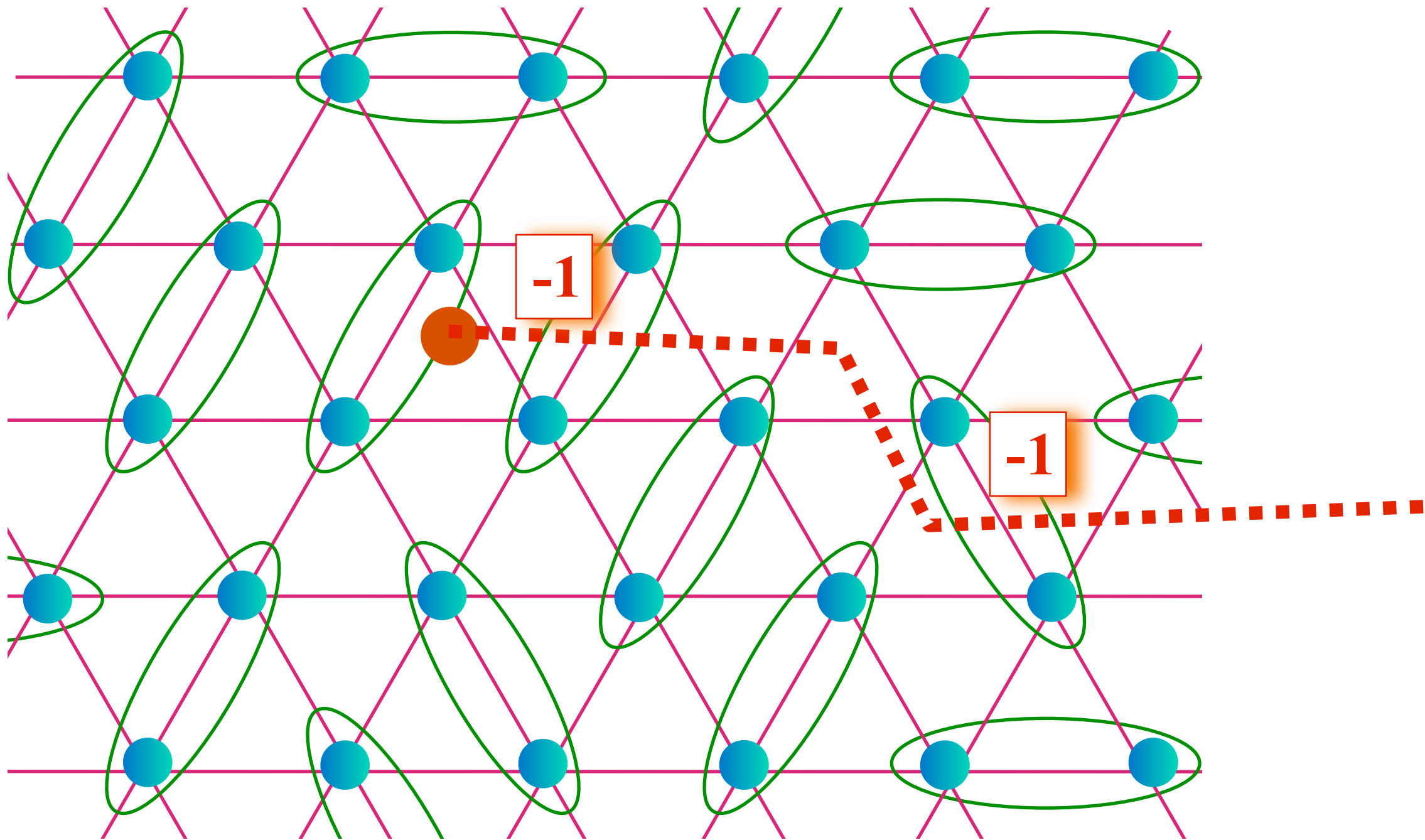
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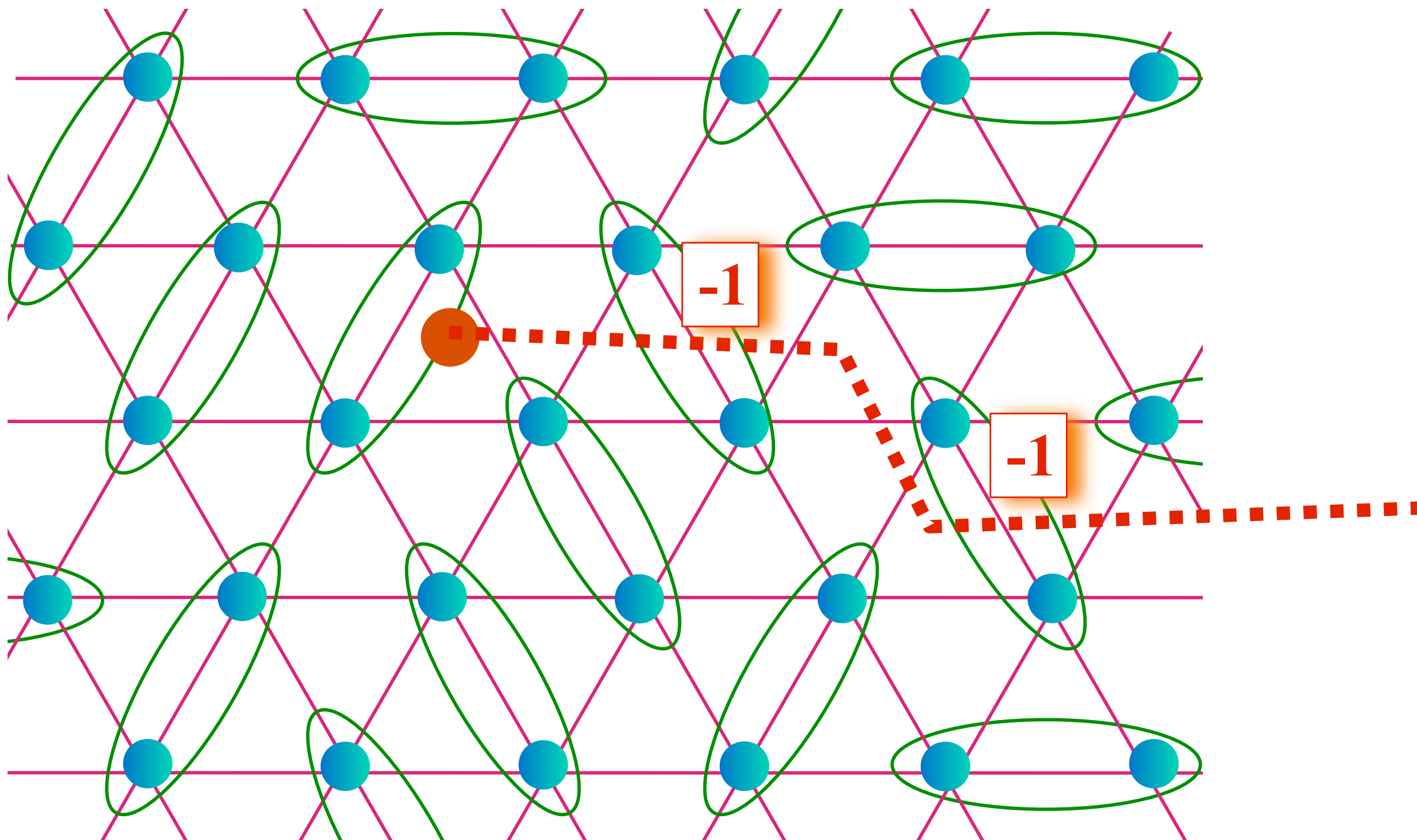


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# Mutual Chern-Simons Theory

Express theory in terms of the physical excitations of the  $Z_2$  spin liquid: the spinons,  $z_\alpha$ , and the visons. After accounting for Berry phase effects, the visons can be described by complex fields  $v_a$ , which transforms non-trivially under the square lattice space group operations.

The spinons and visons have mutual semionic statistics, and this leads to the mutual CS theory at  $k = 2$ :

$$\begin{aligned}\mathcal{L} &= \sum_{\alpha=1}^2 \left\{ |(\partial_\mu - ia_\mu)z_\alpha|^2 + s_z |z_\alpha|^2 \right\} \\ &+ \sum_{a=1}^{N_v} \left\{ |(\partial_\mu - ib_\mu)v_a|^2 + s_v |v_a|^2 \right\} \\ &+ \frac{ik}{2\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda + \dots\end{aligned}$$

Cenke Xu and S. Sachdev,  
*Phys. Rev. B* **79**, 064405 (2009)

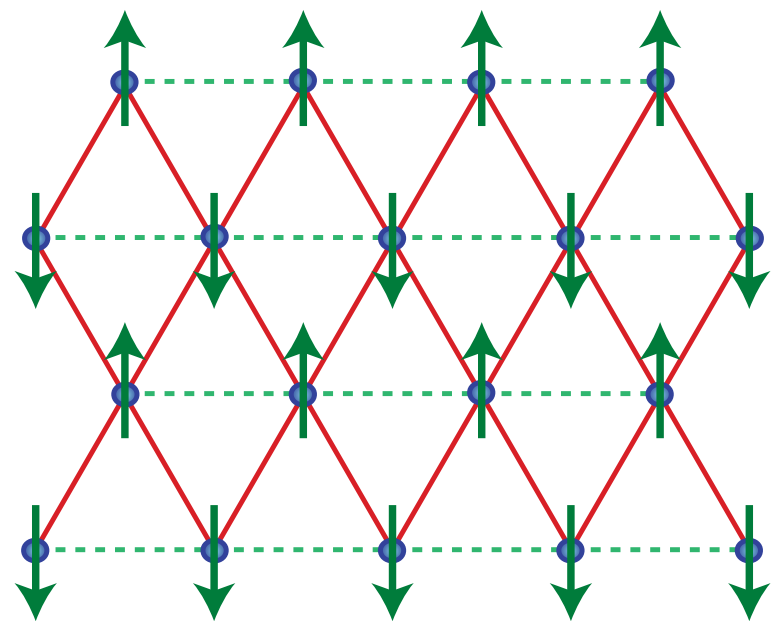
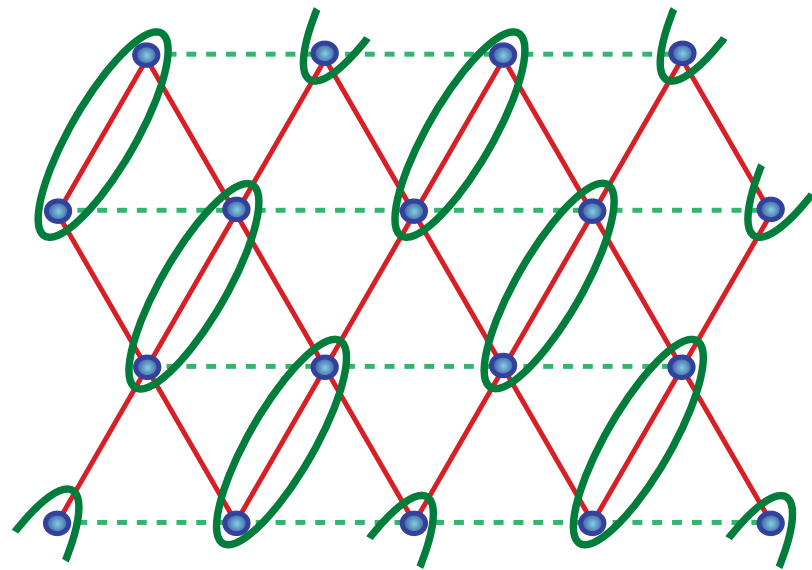
# Phase diagram of frustrated antiferromagnets

N. Read and S. Sachdev  
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Cenke Xu and S. Sachdev,  
*Phys. Rev. B* **79**, 064405 (2009)

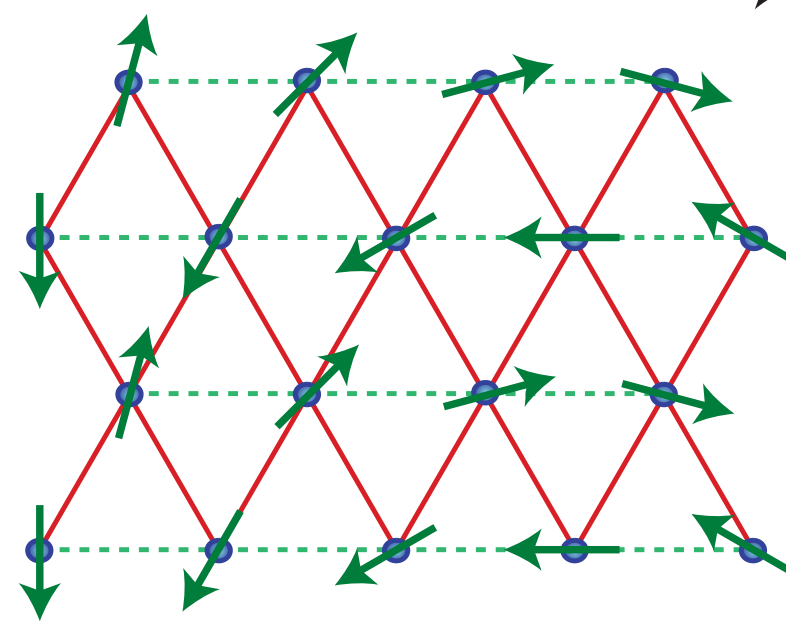
## $Z_2$ spin liquid

Valence bond solid  
(VBS)



Neel

antiferromagnet



Spiral

antiferromagnet

$S_z$

M

$S_v$

# Phase diagram of frustrated antiferromagnets

N. Read and S. Sachdev  
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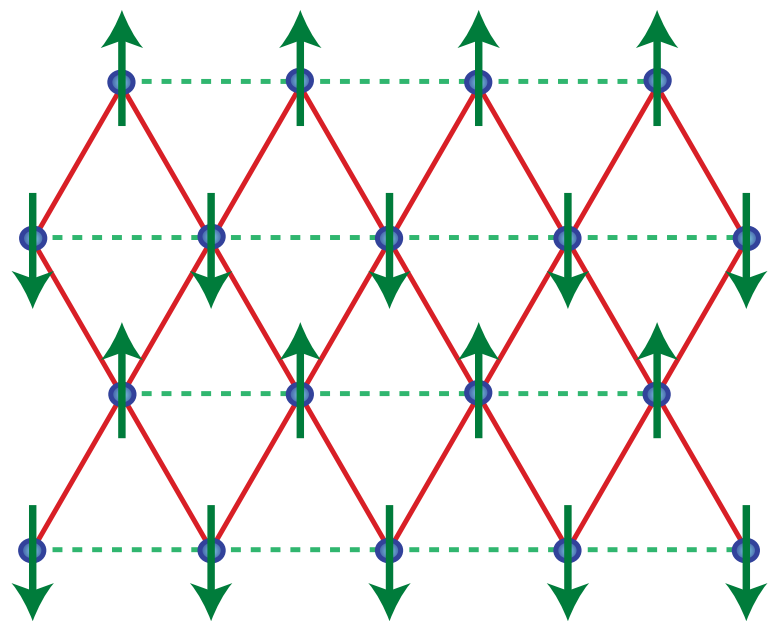
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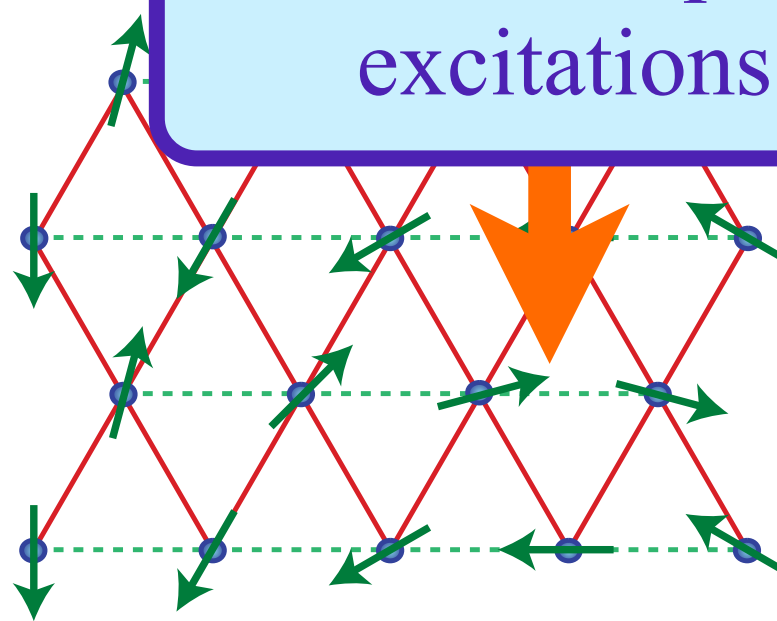
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Vanishing of gap  
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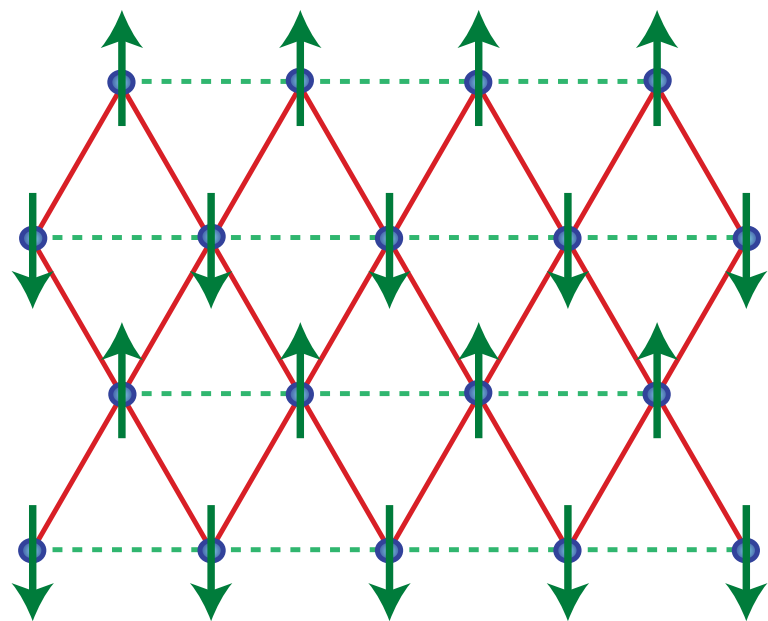
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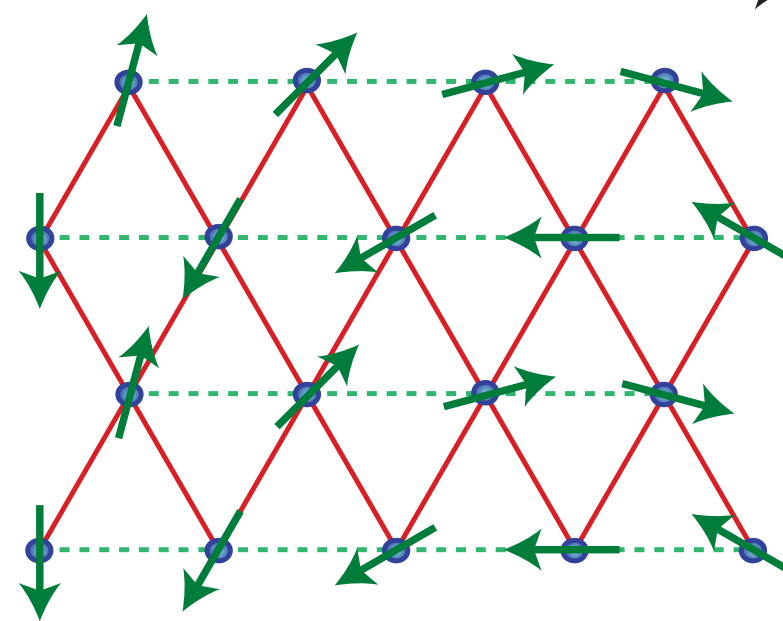


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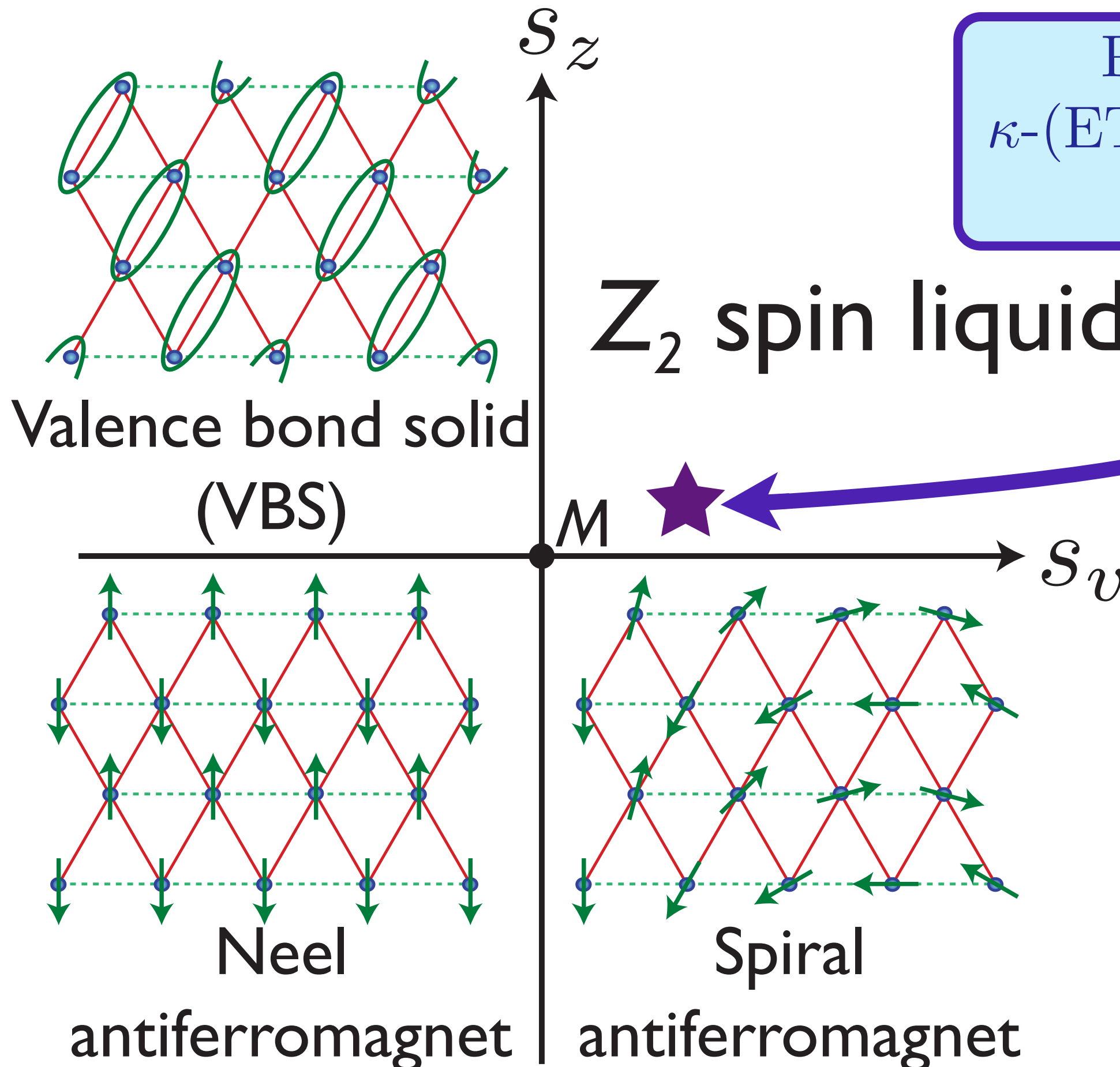
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# Phase diagram of frustrated antiferromagnets



Proposal:  
 $\kappa$ -(ET)<sub>2</sub>Cu<sub>2</sub>(CN)<sub>3</sub>  
is here

$Z_2$  spin liquid

Yang Qi, Cenke Xu  
and S. Sachdev,  
*Phys. Rev. Lett.*  
**102**, 176401 (2009)



Proposal:  $\kappa$ -(ET)<sub>2</sub>Cu<sub>2</sub>(CN)<sub>3</sub> is a  $Z_2$  spin liquid near a quantum phase transition to magnetic order

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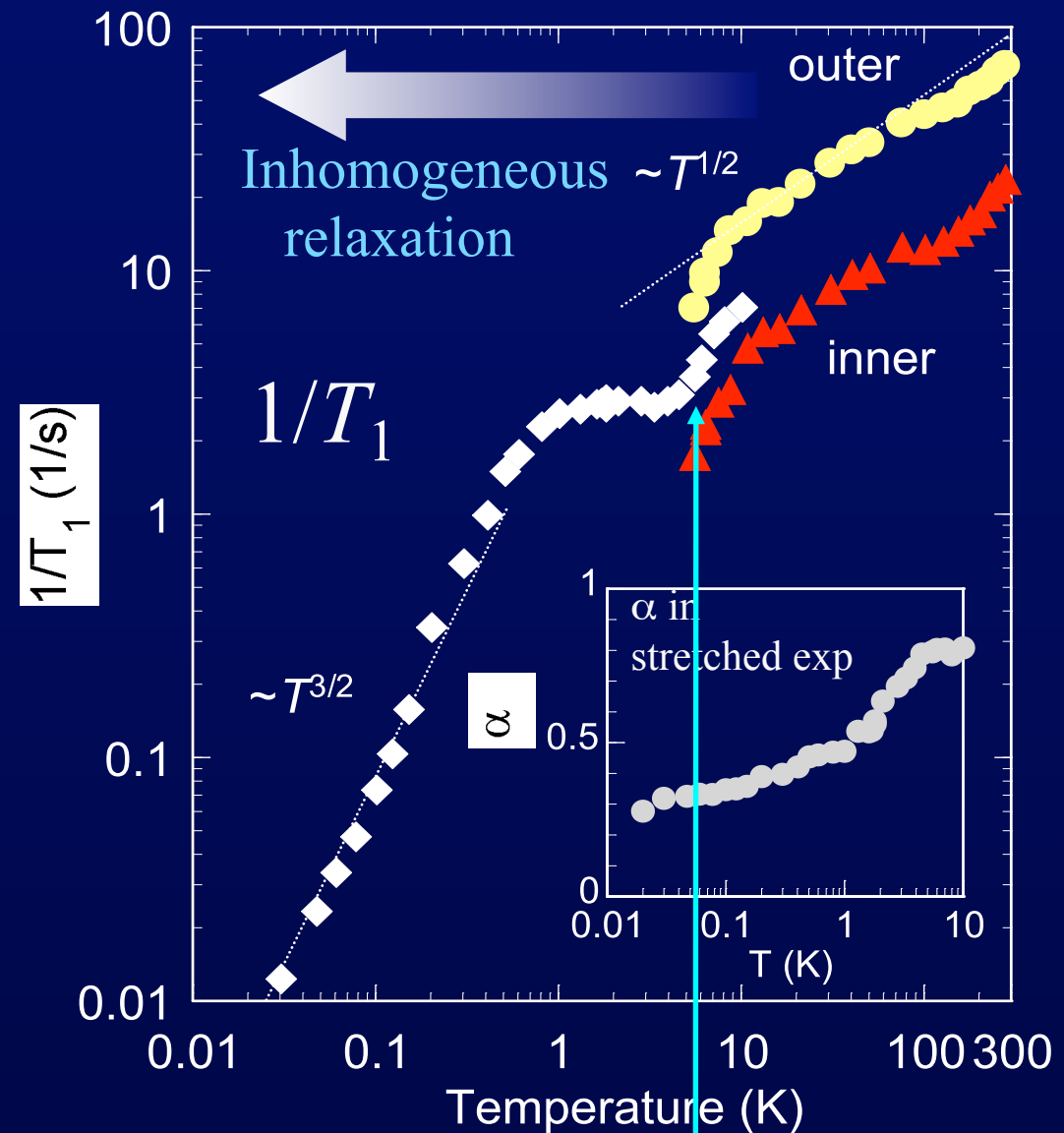
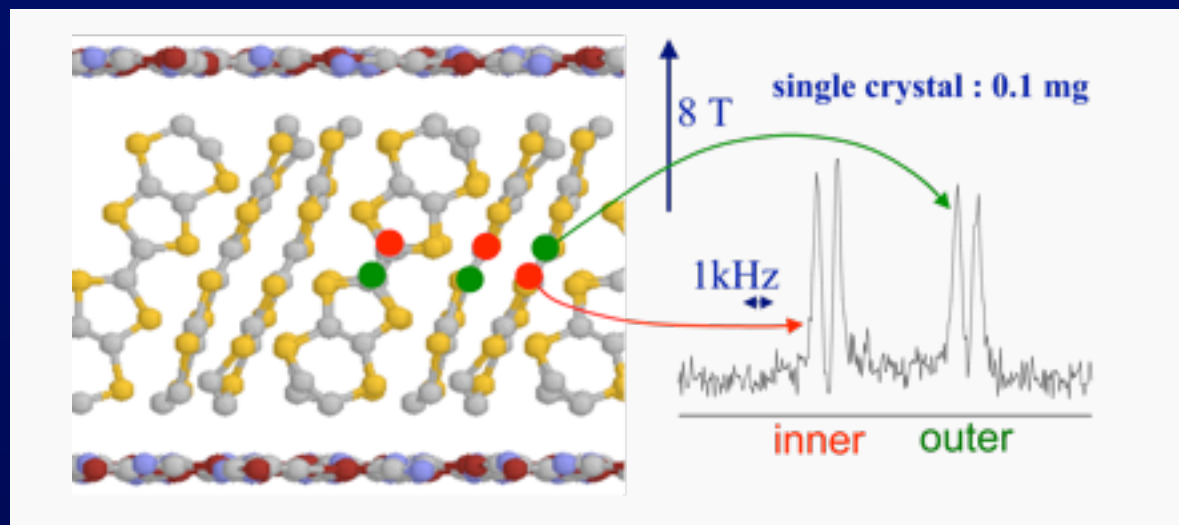
Yang Qi, Cenke Xu and S. Sachdev, *Phys. Rev. Lett.* **102**, 176401 (2009)



# Spin excitation in $\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$

$^{13}\text{C}$  NMR relaxation rate

Shimizu *et al.*, PRB 70 (2006) 060510



$1/T_1 \sim$  power law of T

Low-lying spin excitation at low-T

Anomaly at 5-6 K

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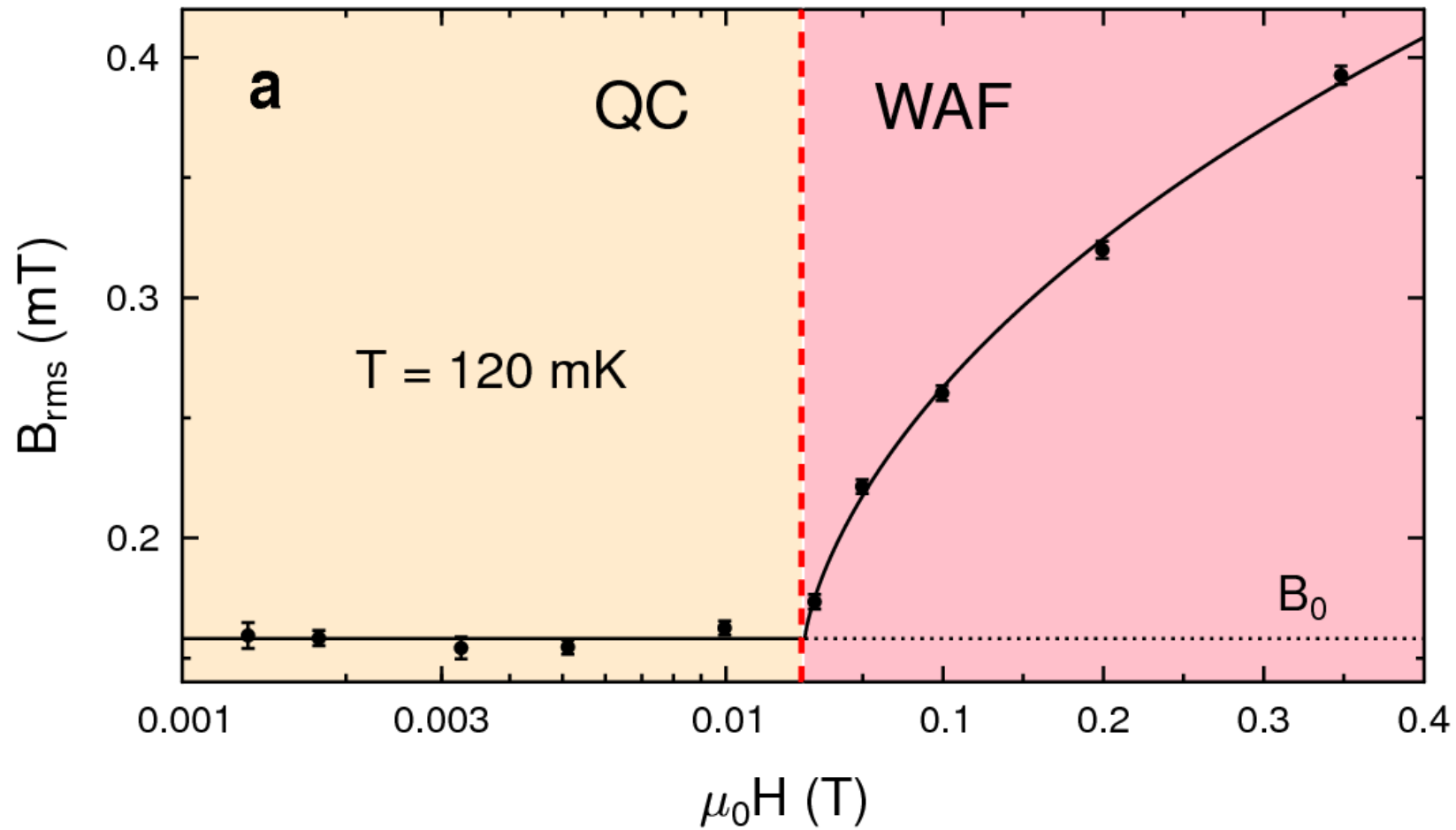
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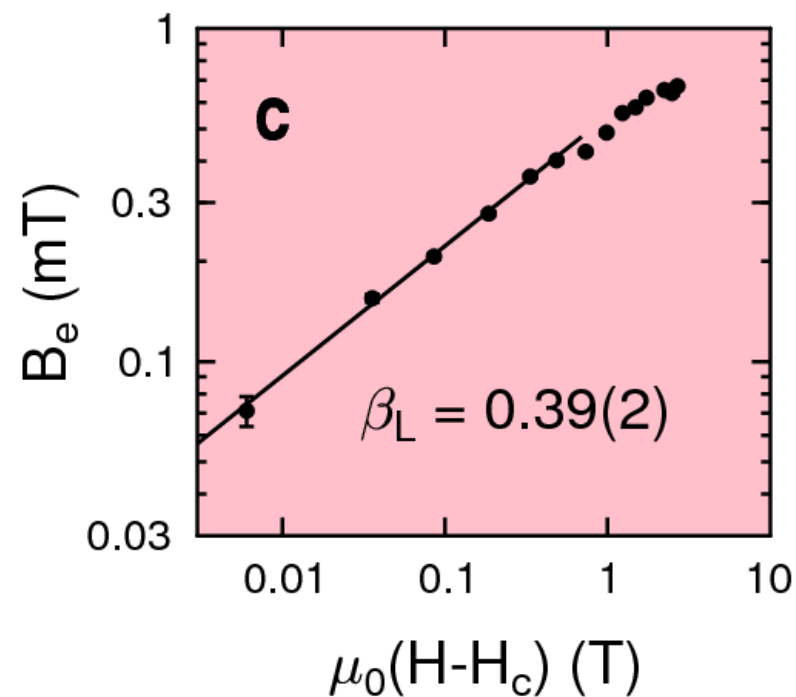
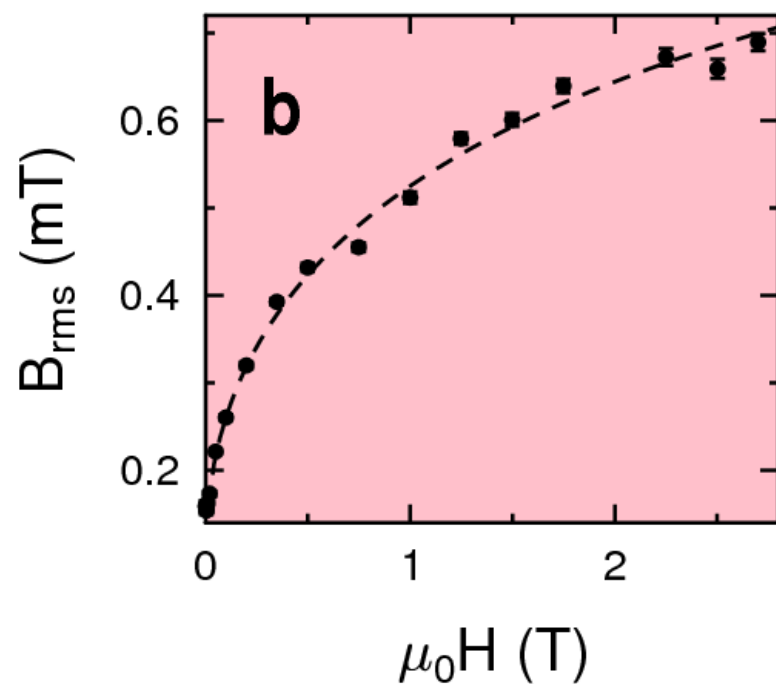
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Yang Qi, Cenke Xu and S. Sachdev, *Phys. Rev. Lett.* **102**, 176401 (2009)

# Field-induced QPT from $\mu$ SR Line Width

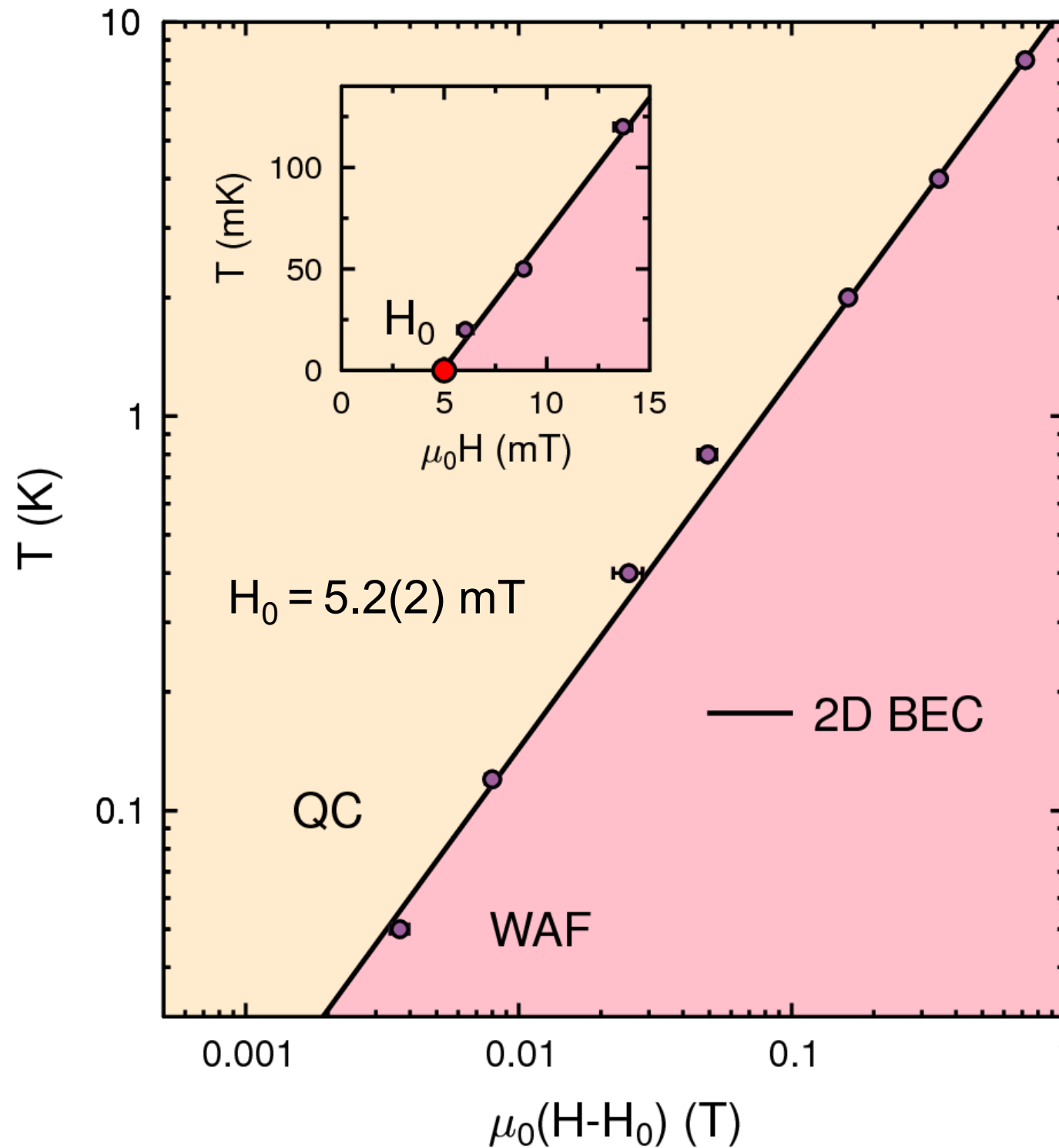


Consistent with high field  $^{13}\text{C}$  NMR broadening of Shimizu et al PRB 73,140407 (2006)



F. Pratt *et al.*  
(ISIS, UK)  
preprint

# Measured Phase Boundary of the QPT



2D BEC:

$$T_c \propto \mu \frac{\ln(t_{\parallel}/\mu)}{\ln \ln(t_{\parallel}/\mu)}$$

$$\mu \propto H - H_0$$

F. Pratt *et al.*  
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preprint

Tiny  $H_0$  implies that spin gap per spin 1/2 is  $\Delta_z \sim 3.5$  mK !

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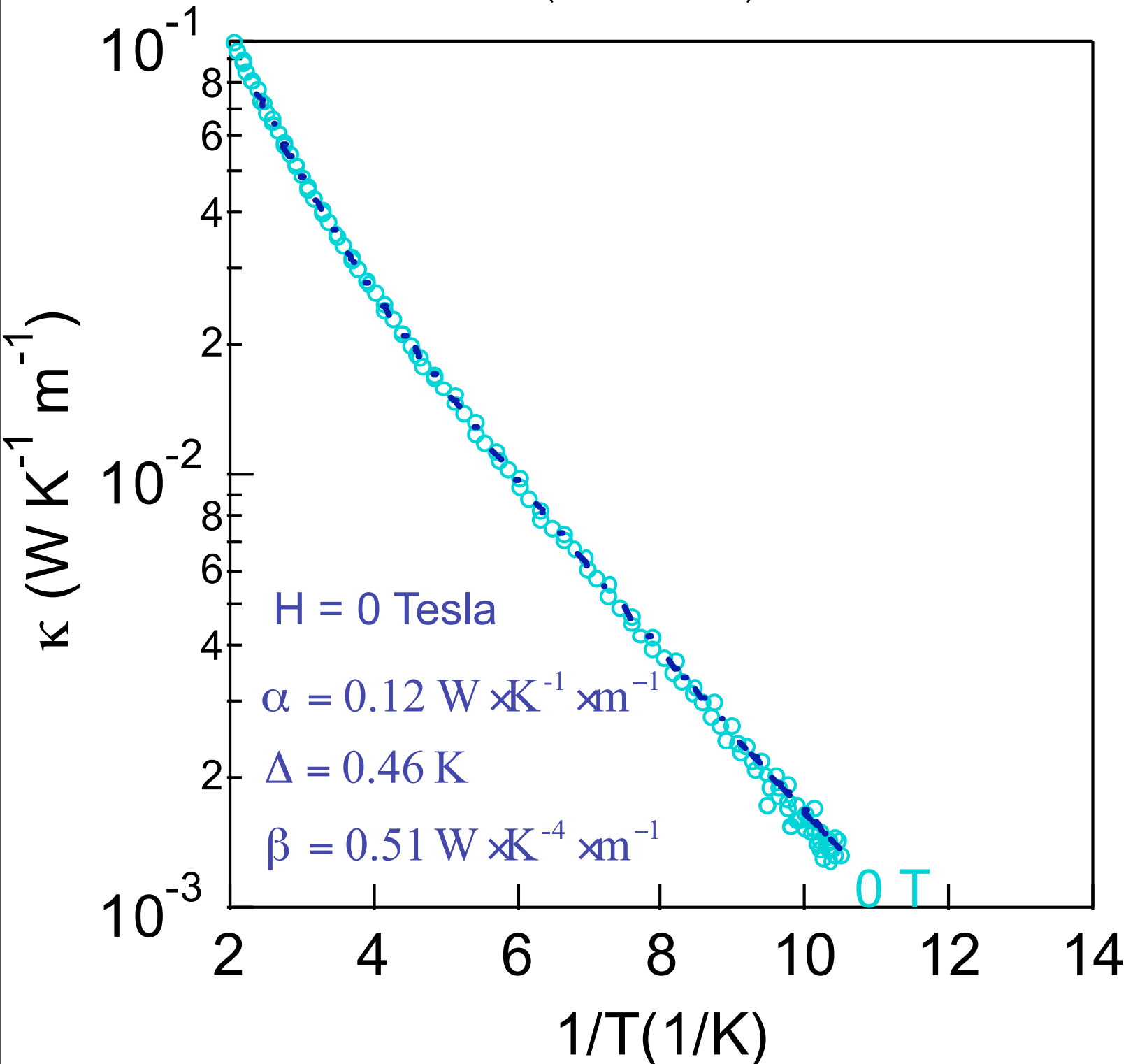
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Yang Qi, Cenke Xu and S. Sachdev, *Phys. Rev. Lett.* **102**, 176401 (2009)

# Thermal conductivity of $\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$

$$\kappa = \alpha \exp\left(-\frac{\Delta}{k_B T}\right) + \beta T^3$$



• Arrhenius behavior for  $T < \Delta$  !

• Tiny gap  
 ➤  $\Delta = 0.46 \text{ K} \sim J/500$

M. Yamashita *et al.*, *Nature Physics* **5**, 44 (2009)



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# Outline

## 1. Classification of spin liquids

*Quantum-disordering magnetic order vs.  
projected Fermi sea*

## 2. Quantum-disordering magnetic order

*Application to  $\kappa$ -(ET)<sub>2</sub>Cu<sub>2</sub>(CN)<sub>3</sub>*

## 3. Fermi surfaces of spinful Majorana fermions

*Candidate for EtMe<sub>3</sub>Sb[Pd(dmit)<sub>2</sub>]<sub>2</sub> ?*

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# Shastry-Kitaev Majorana representation

On each site, we introduce neutral Majorana fermions  $\gamma_{i0}, \gamma_{ix}, \gamma_{iy}, \gamma_{iz}$  which obey

$$\gamma_{i\alpha}\gamma_{j\beta} + \gamma_{j\beta}\gamma_{i\alpha} = 2\delta_{ij}\delta_{\alpha\beta}$$

We write the  $S = 1/2$  spin operators as

$$\begin{aligned} S_{jx} &= \frac{i}{2}\gamma_{jy}\gamma_{jz} \\ S_{jy} &= \frac{i}{2}\gamma_{jz}\gamma_{jx} \\ S_{jz} &= \frac{i}{2}\gamma_{jx}\gamma_{jy} \end{aligned}$$

along with the constraint

$$\gamma_{j0}\gamma_{jx}\gamma_{jy}\gamma_{jz} = 1 \quad \text{for all } j$$

# SU(2)-invariant spin liquids with neutral, spinful Majorana excitations

- Postulate an SU(2)-invariant effective Hamiltonian for physical  $S = 1$  excitations created by Majorana fermions  $\gamma_{ix}, \gamma_{iy}, \gamma_{iz}$ .

Rudro Biswas, Liang Fu, Chris Laumann, and S. Sachdev, to appear

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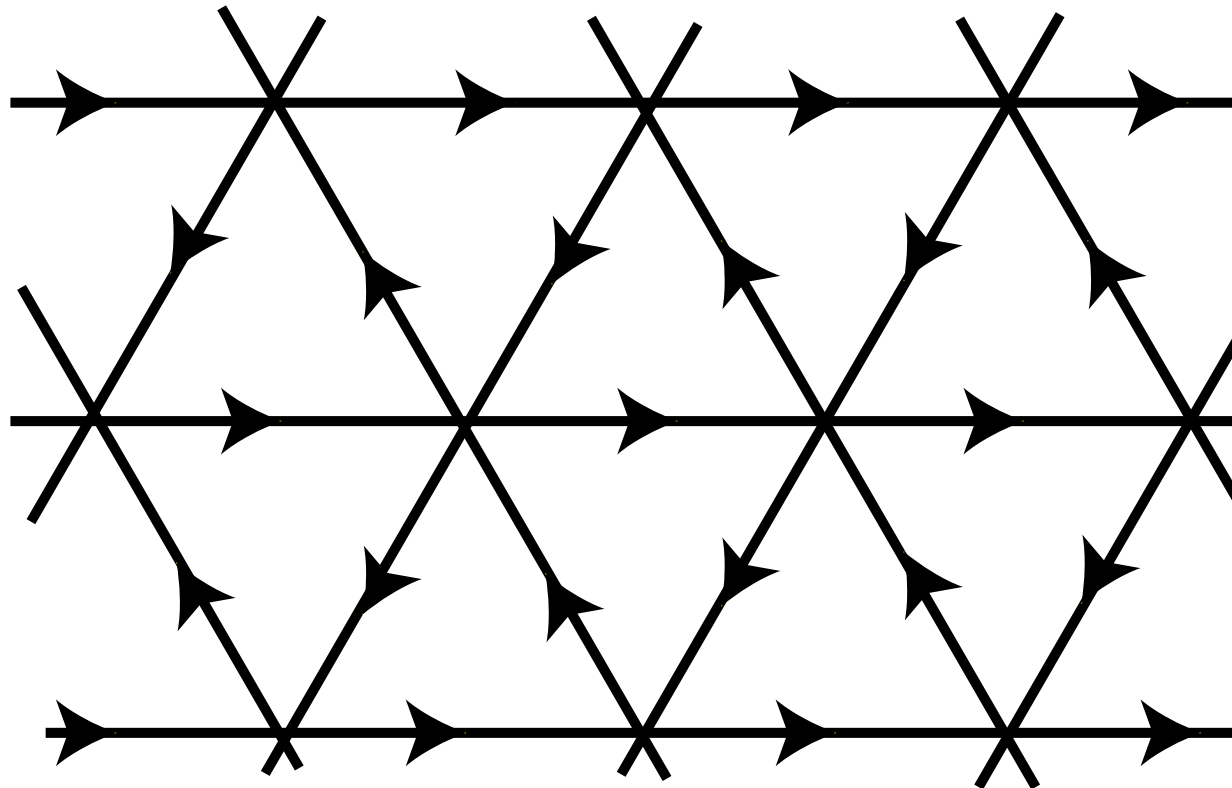
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- Ensure there is a Projective Symmetry Group (PSG) under which physical observables have the full translational symmetry of the triangular lattice.
- Examine stability of the effective Hamiltonian to gauge fluctuations.

Rudro Biswas, Liang Fu, Chris Laumann, and S. Sachdev, to appear

# Majorana fermions on the triangular lattice

$$H = -i \sum_{\alpha=x,y,z} \sum_{i<j} t_{ij} \gamma_{i\alpha} \gamma_{j\alpha}$$

where  $t_{ij}$  is an anti-symmetric matrix with the following symmetry

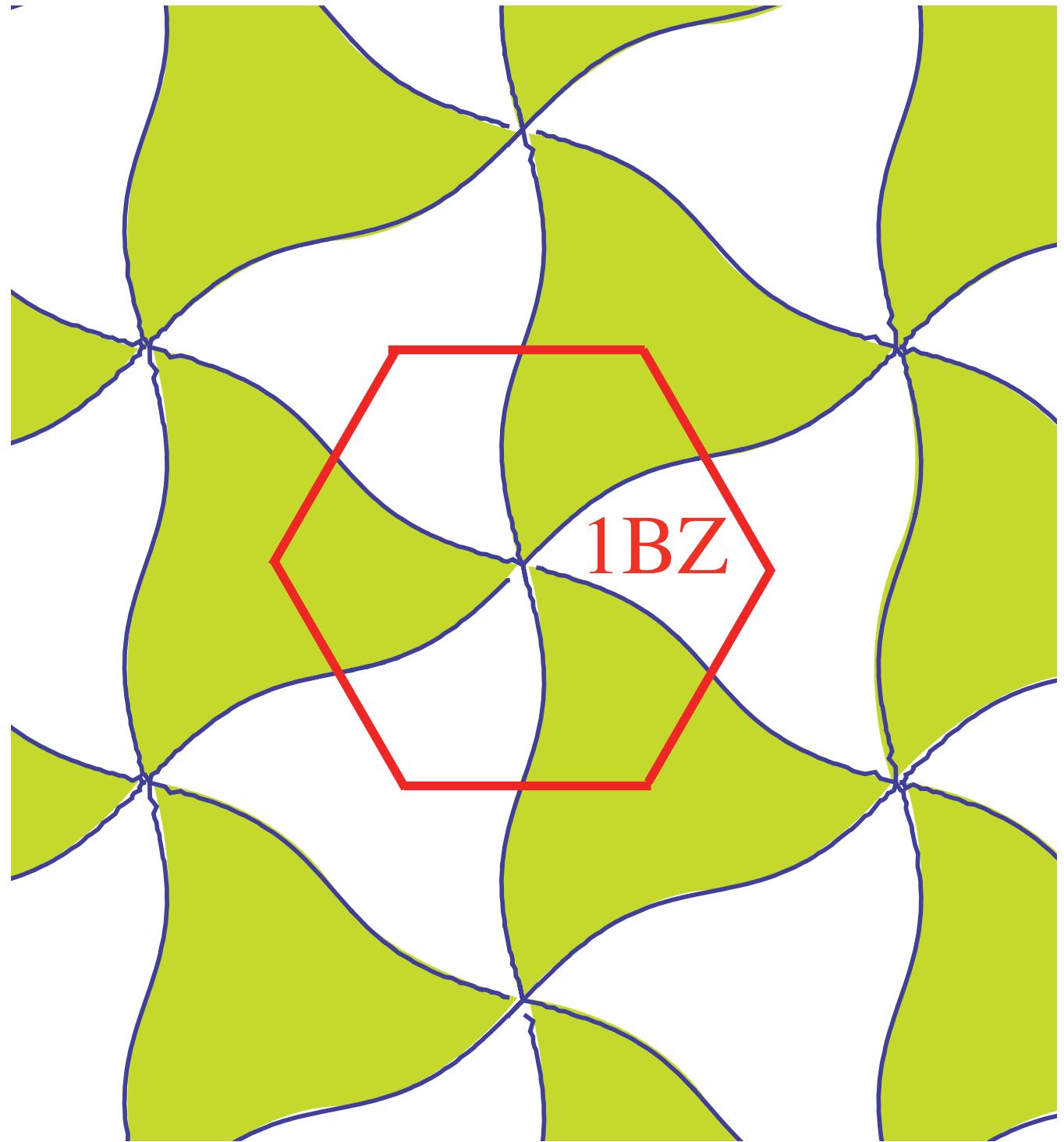


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# Fermi surfaces

The Majorana fermions generically have Fermi surfaces with the structure shown. 6 Fermi surface lines intersect at  $\mathbf{k} = 0$ , and the excitation energy  $\sim k^3$  as  $\mathbf{k} \rightarrow 0$ .

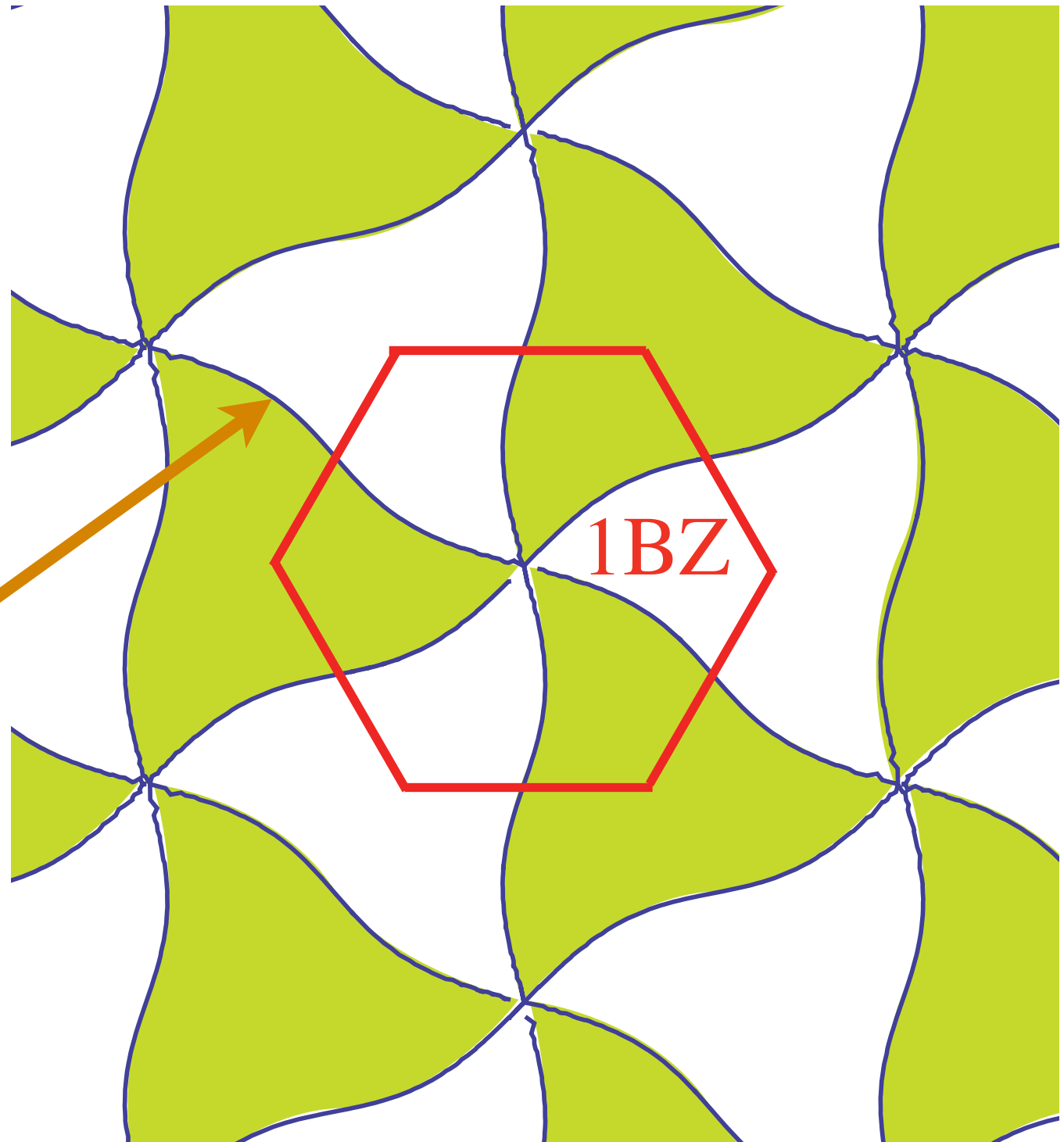


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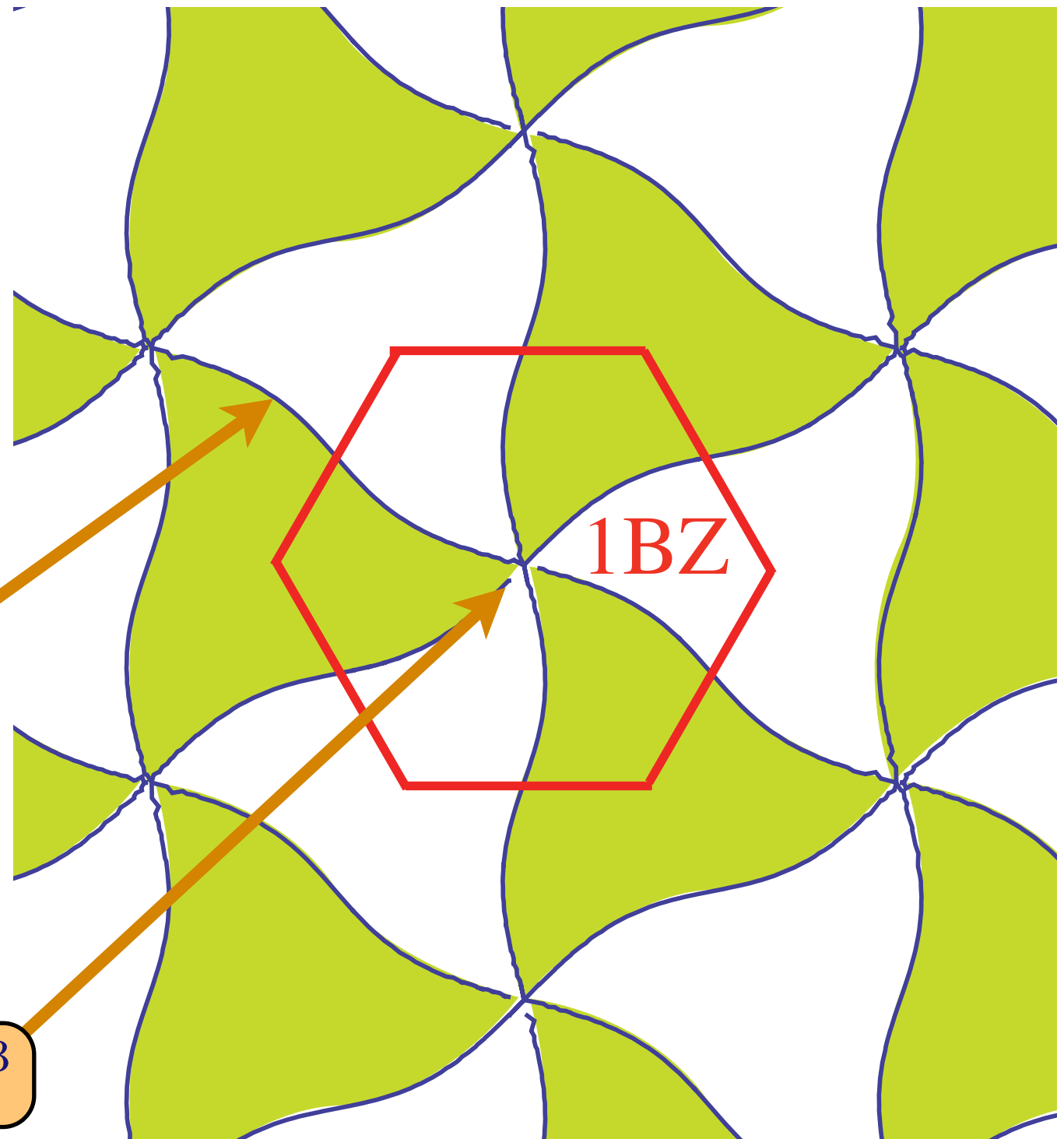
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- There is no thermal Hall transport.

Rudro Biswas, Liang Fu, Chris Laumann, and S. Sachdev, to appear



## Conclusions

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