From the Sachdev-Ye-Kitaev model to A Universal Theory of Strange Metals

Subir Sachdev

International Congress of Basic Science
Beijing Institute of Mathematical Sciences and Applications (BIMSA)
July 15, 2024

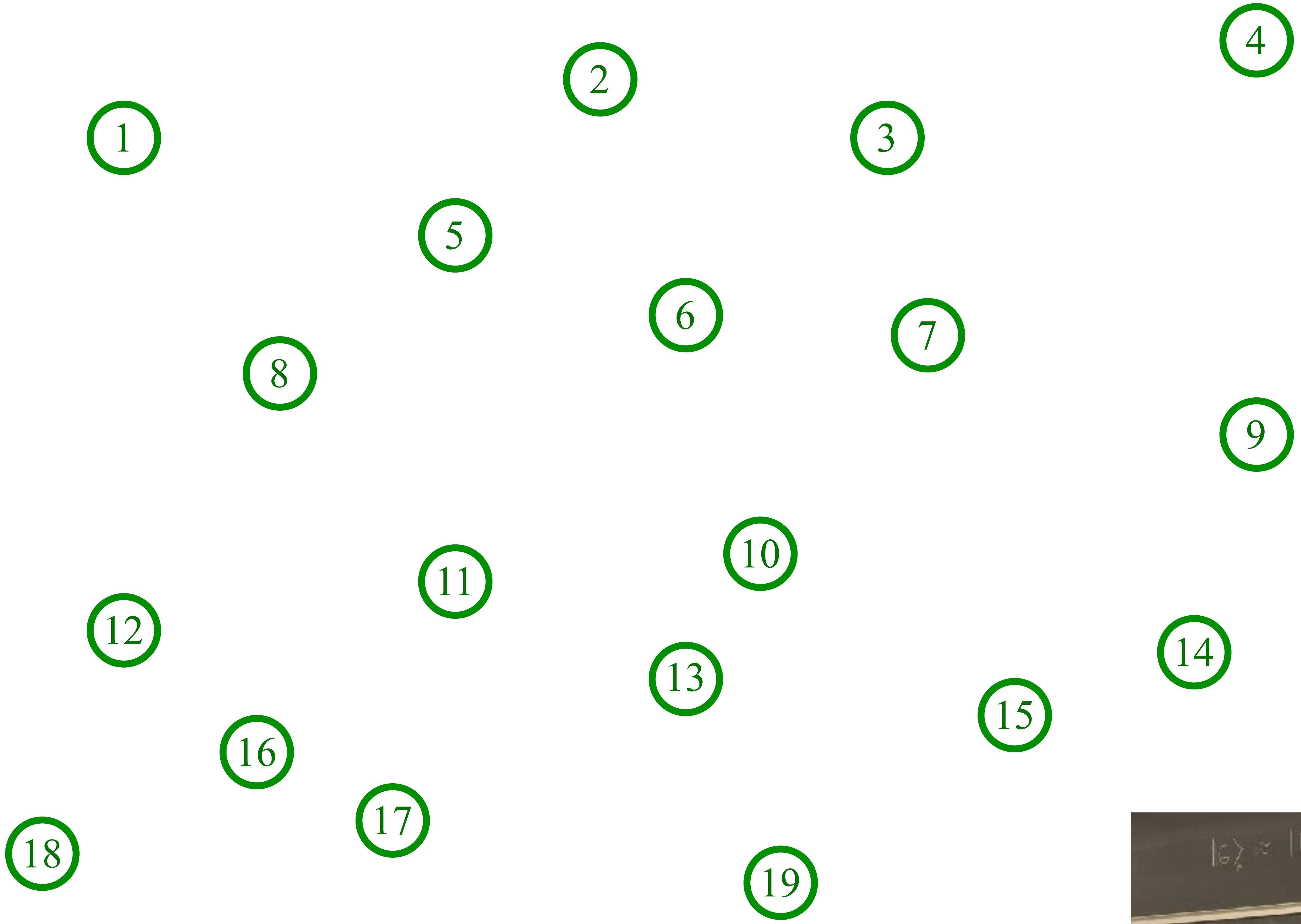
PHYSICS

HARVARD

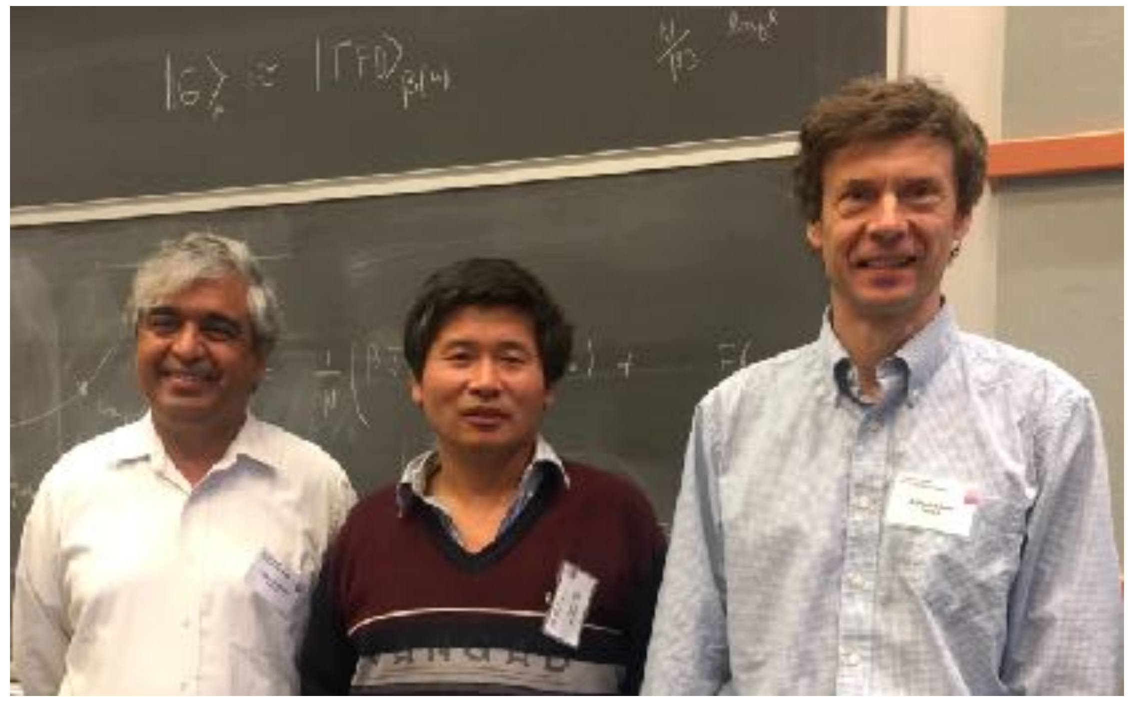
SYK model

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

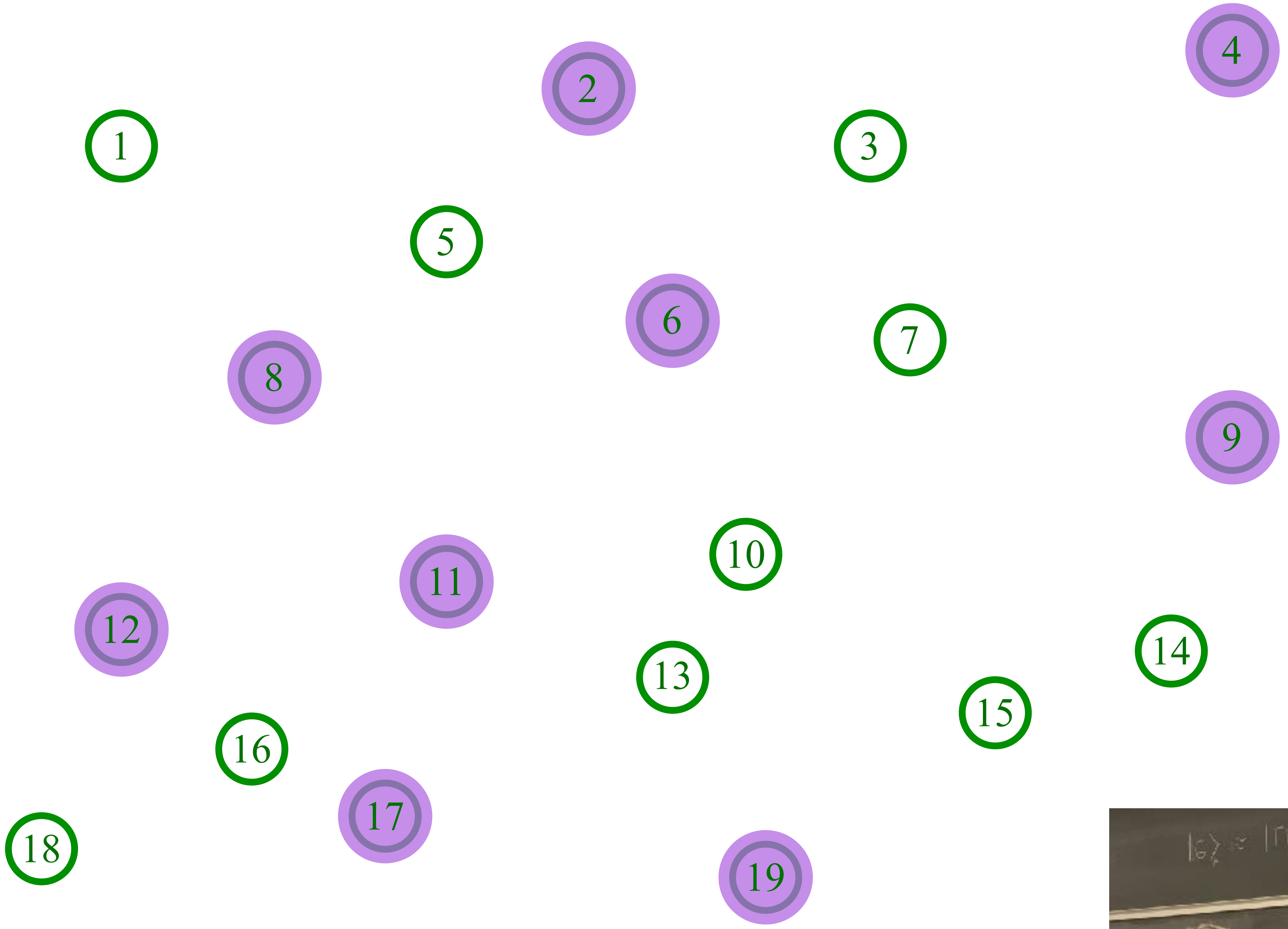


Pick a set of random positions

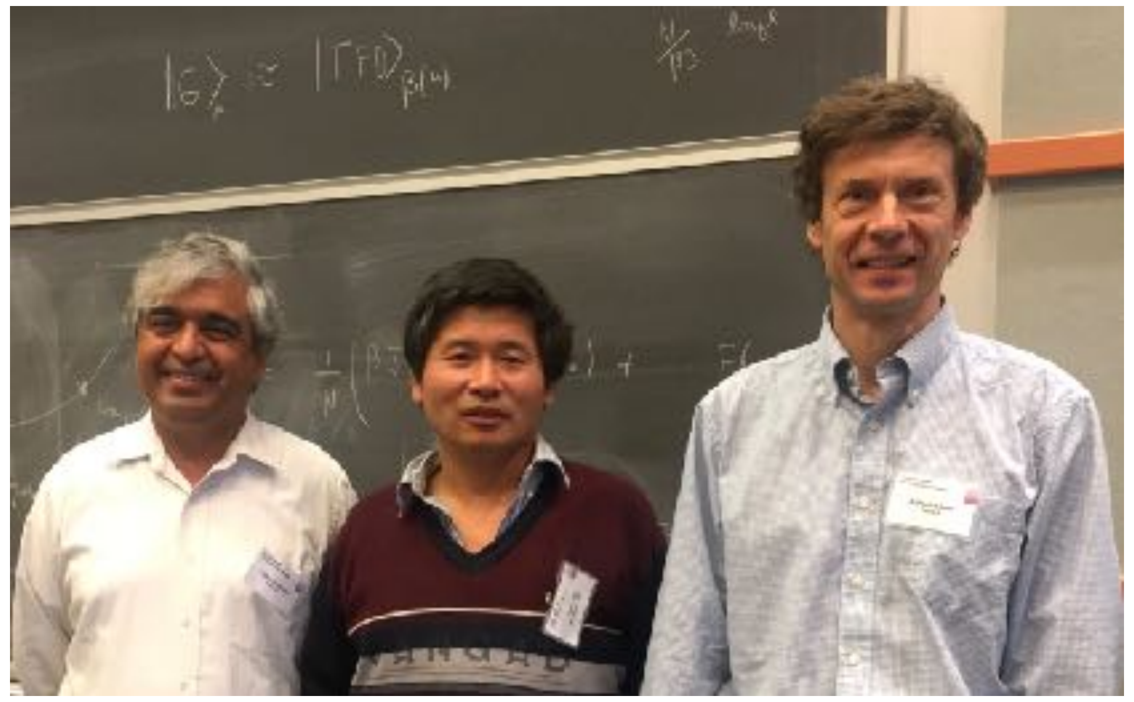


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Place electrons randomly on some sites



The SYK model

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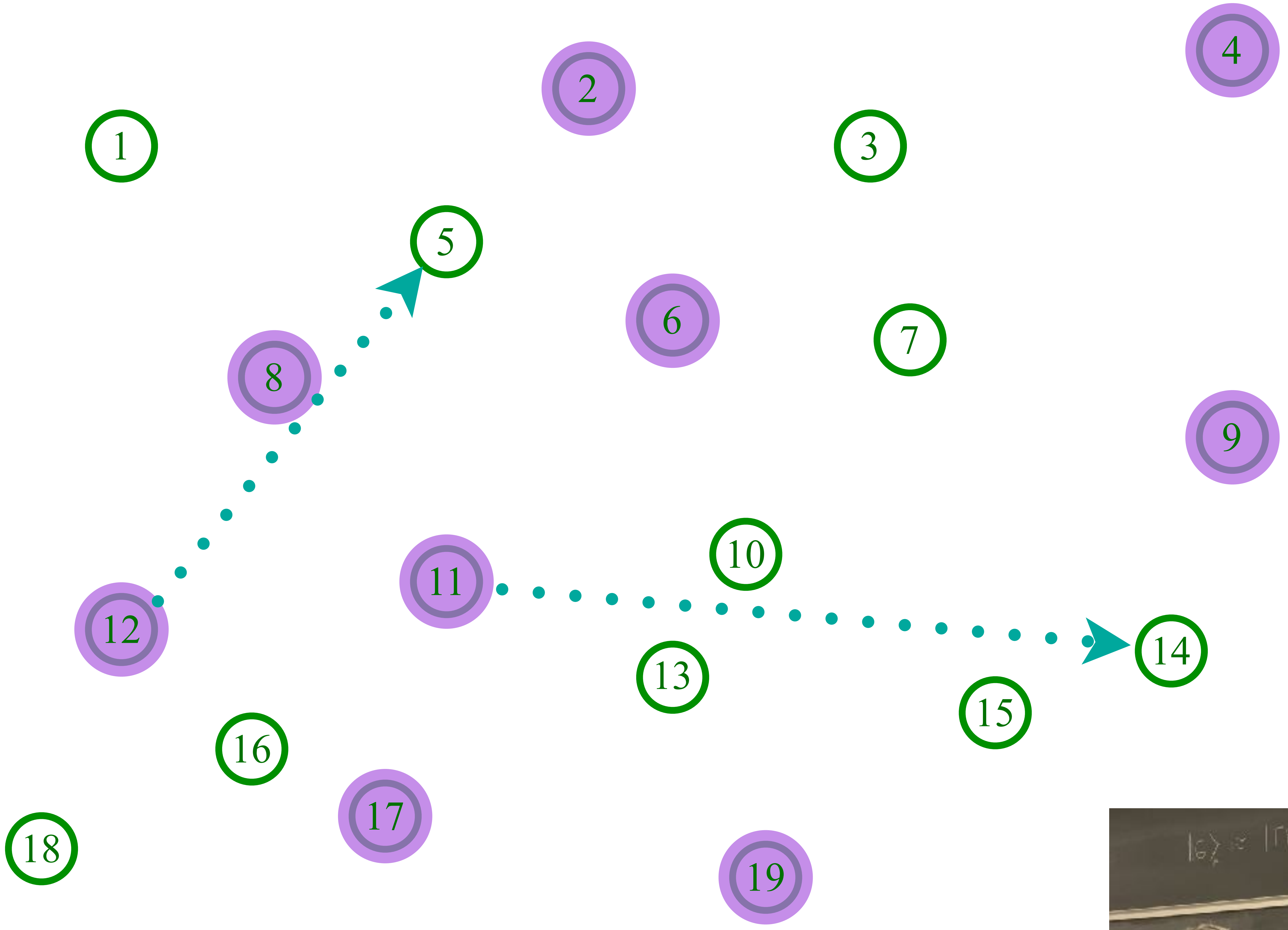
$U_{11,12;5,14}$



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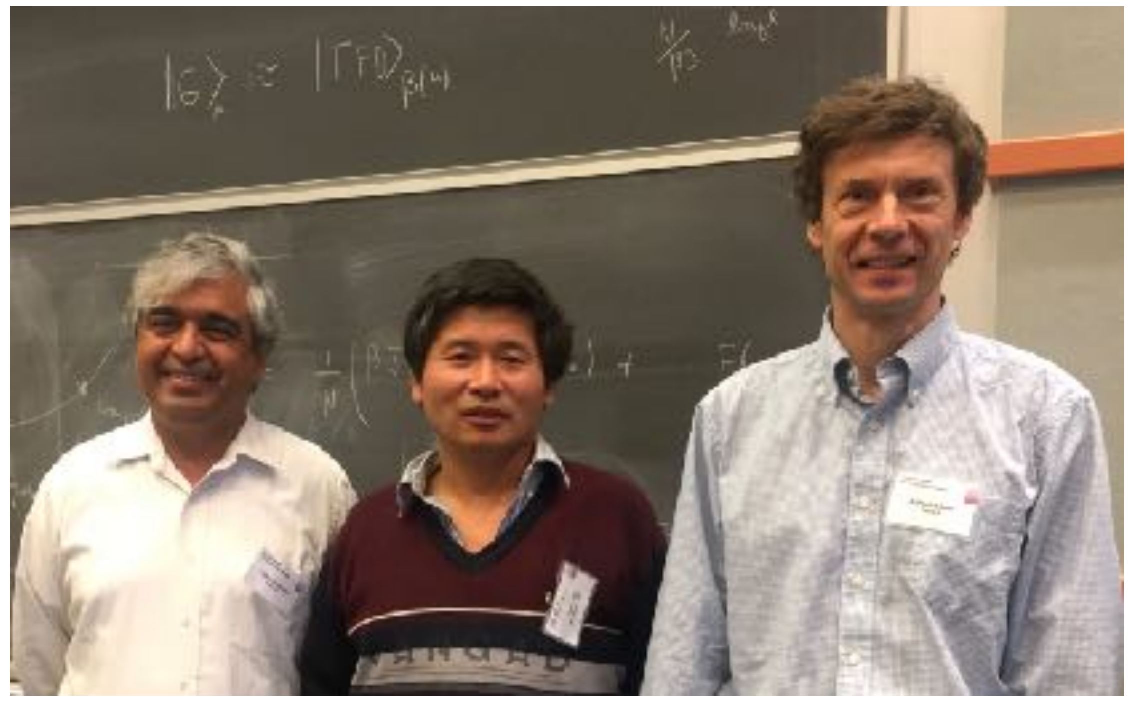

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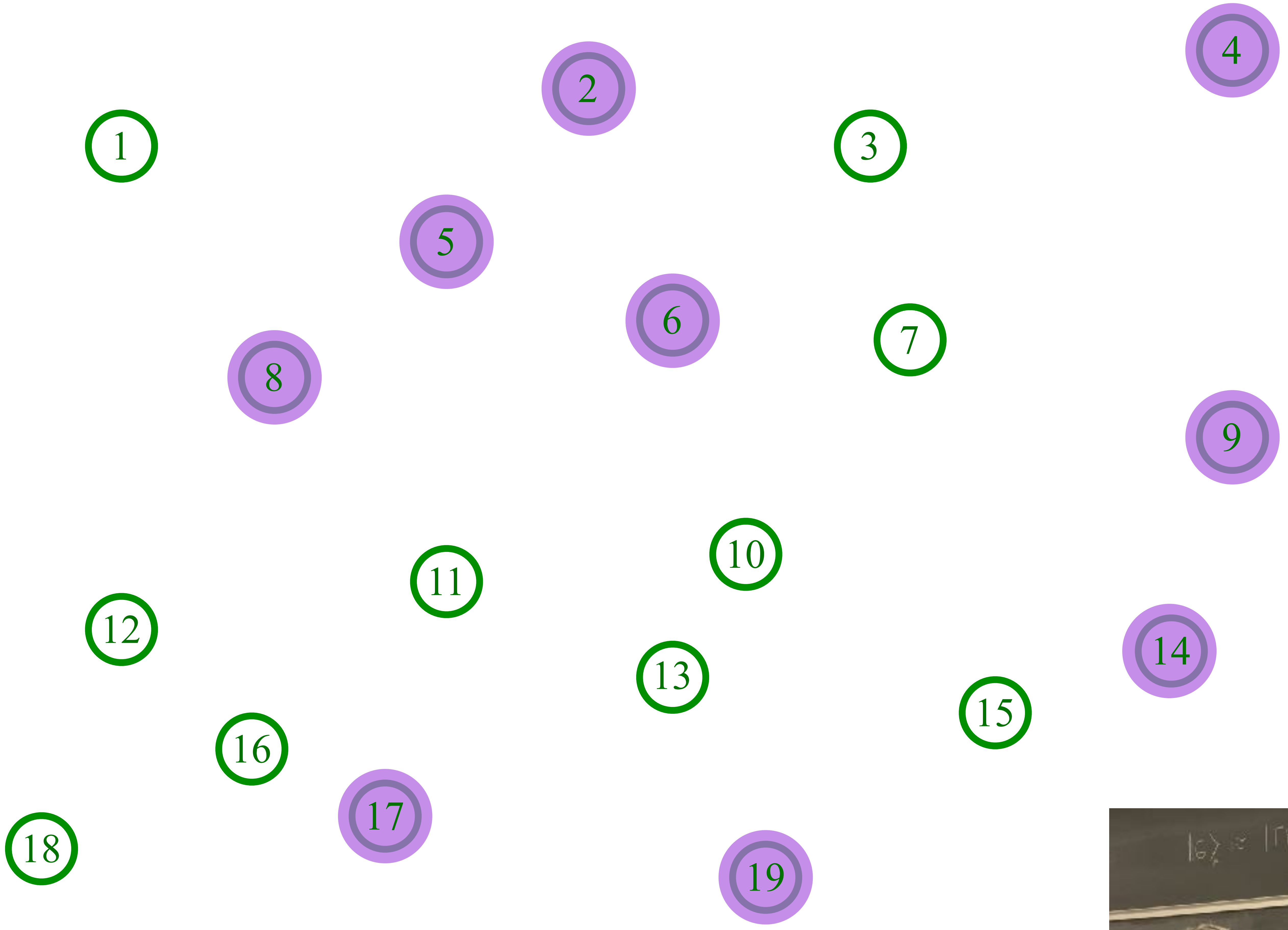
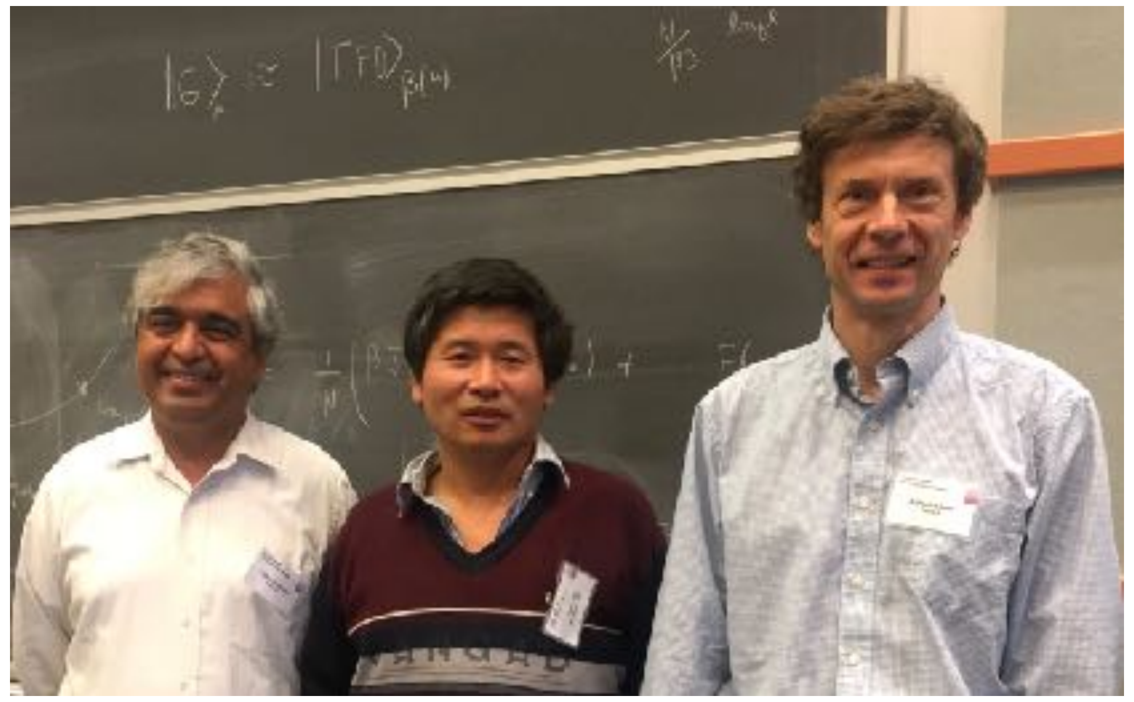


Entangle electrons pairwise randomly



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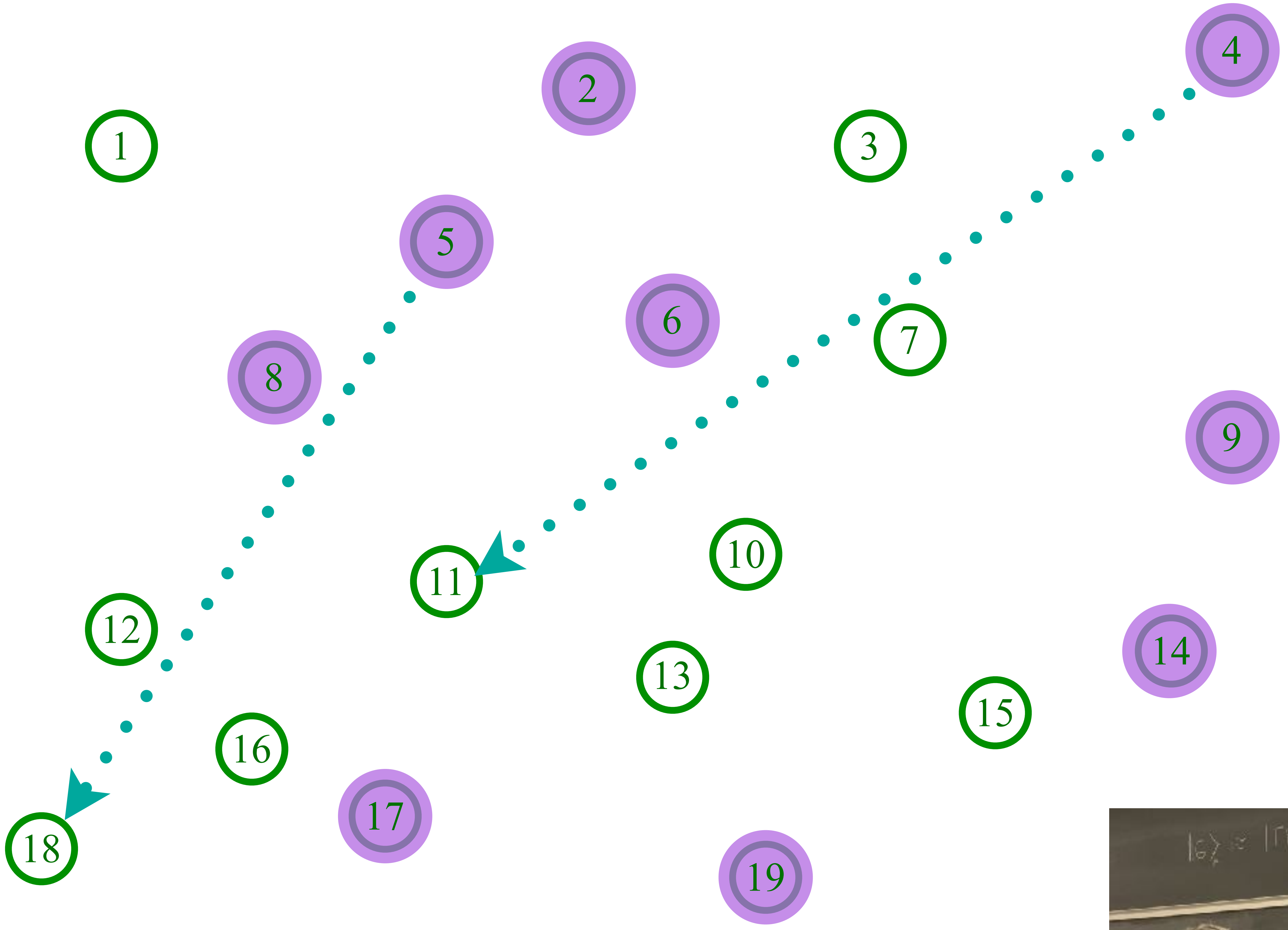
$$U_{4,5;11,18}$$



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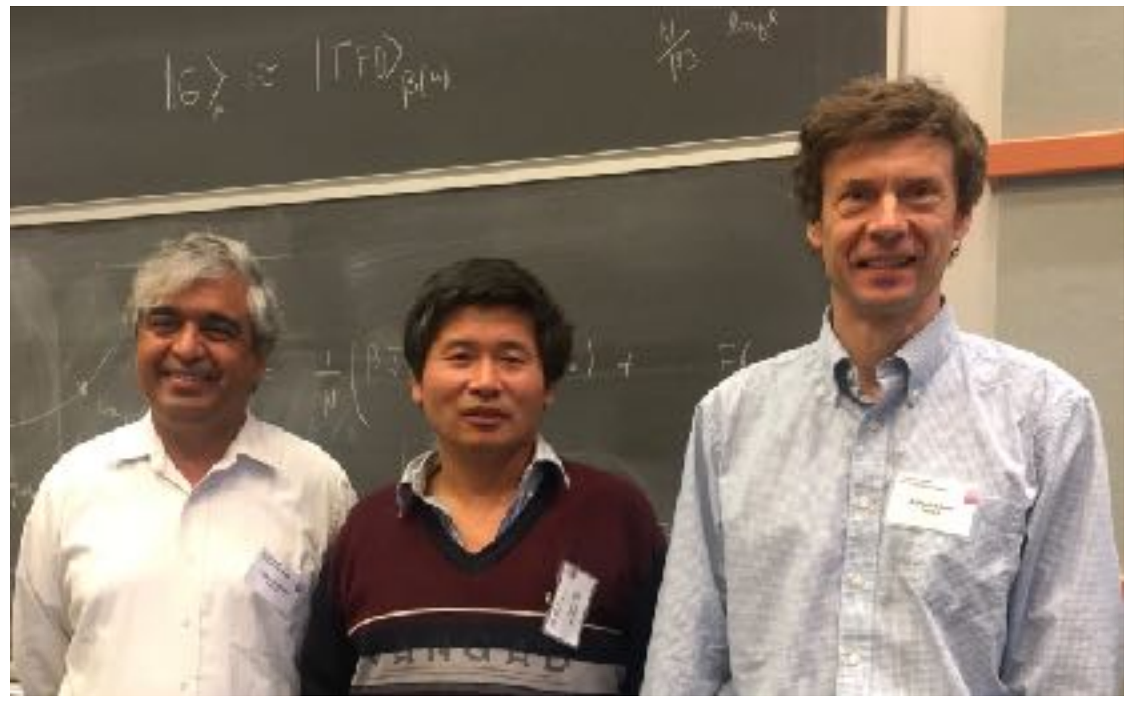

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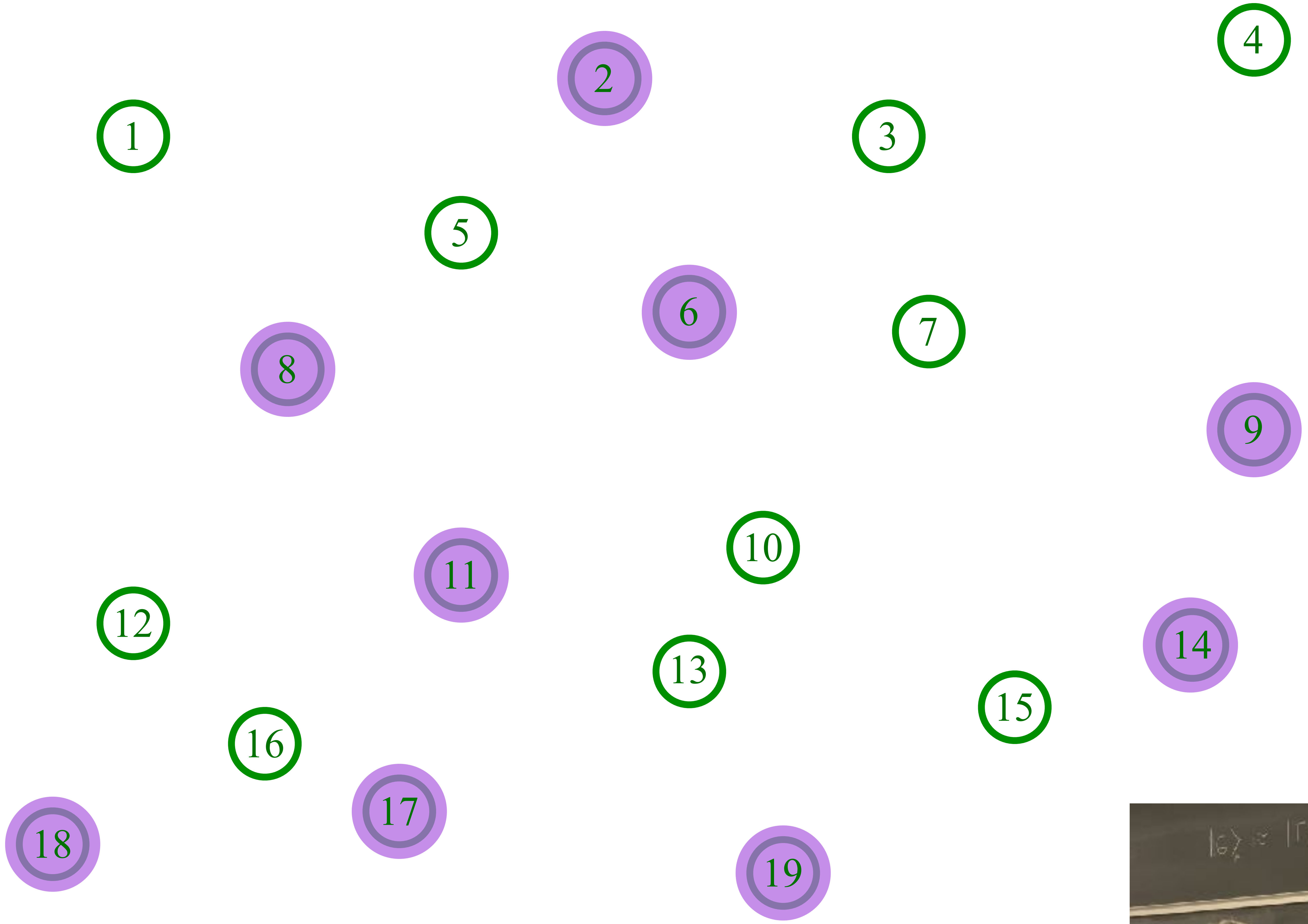

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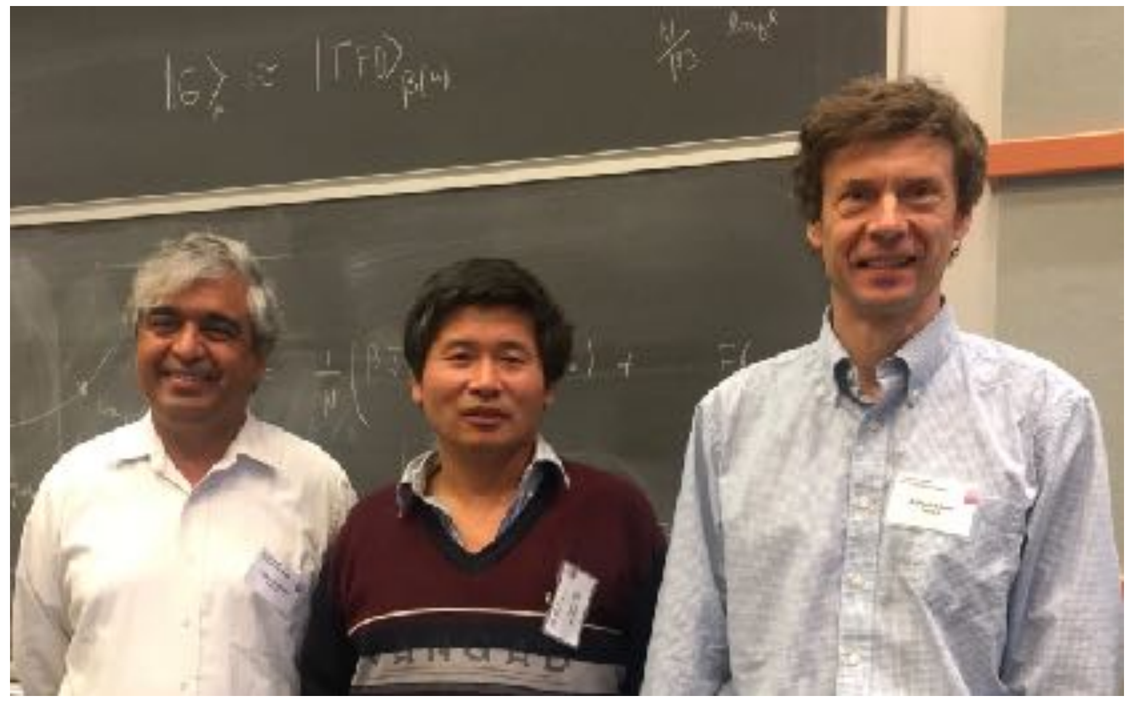

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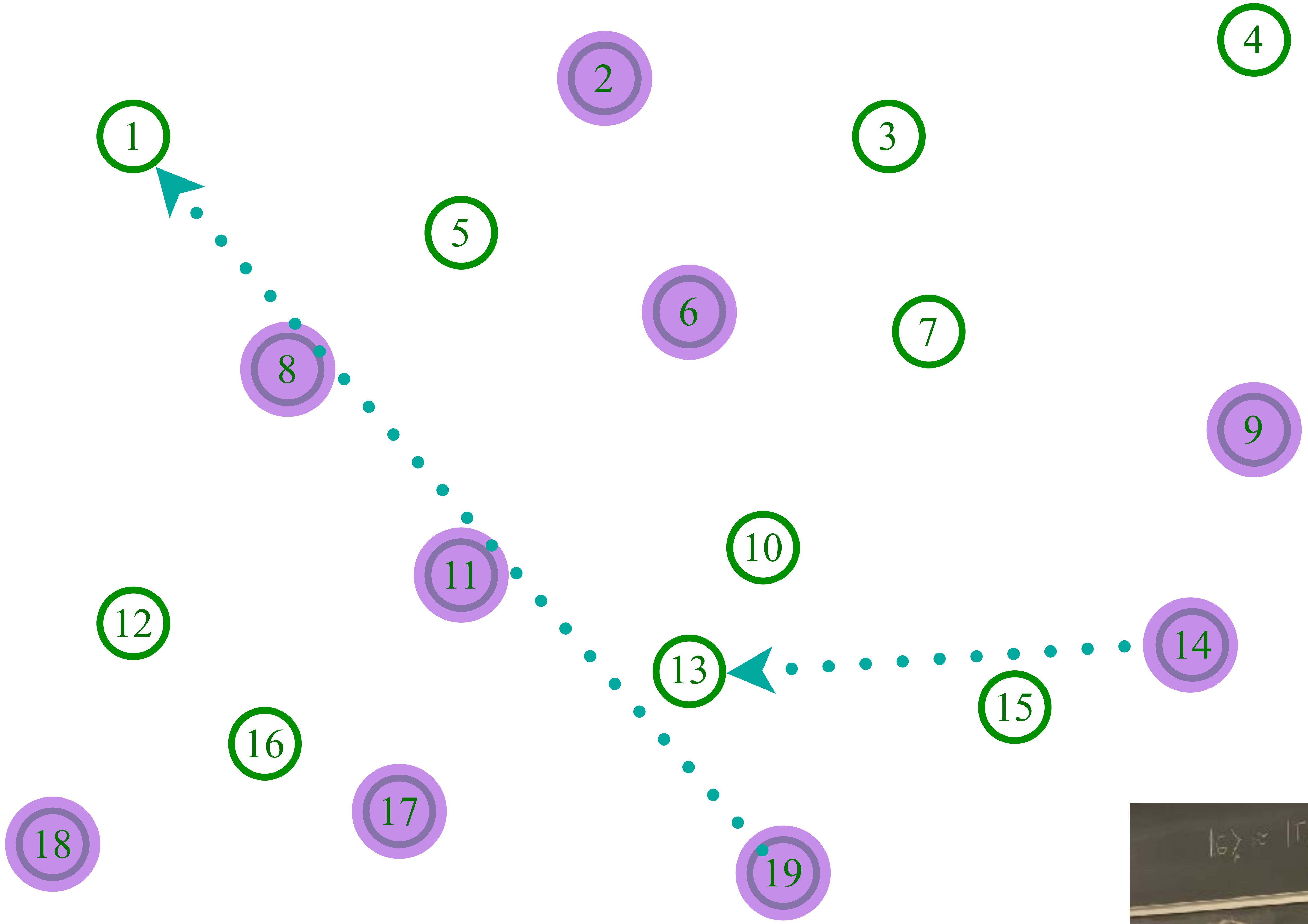

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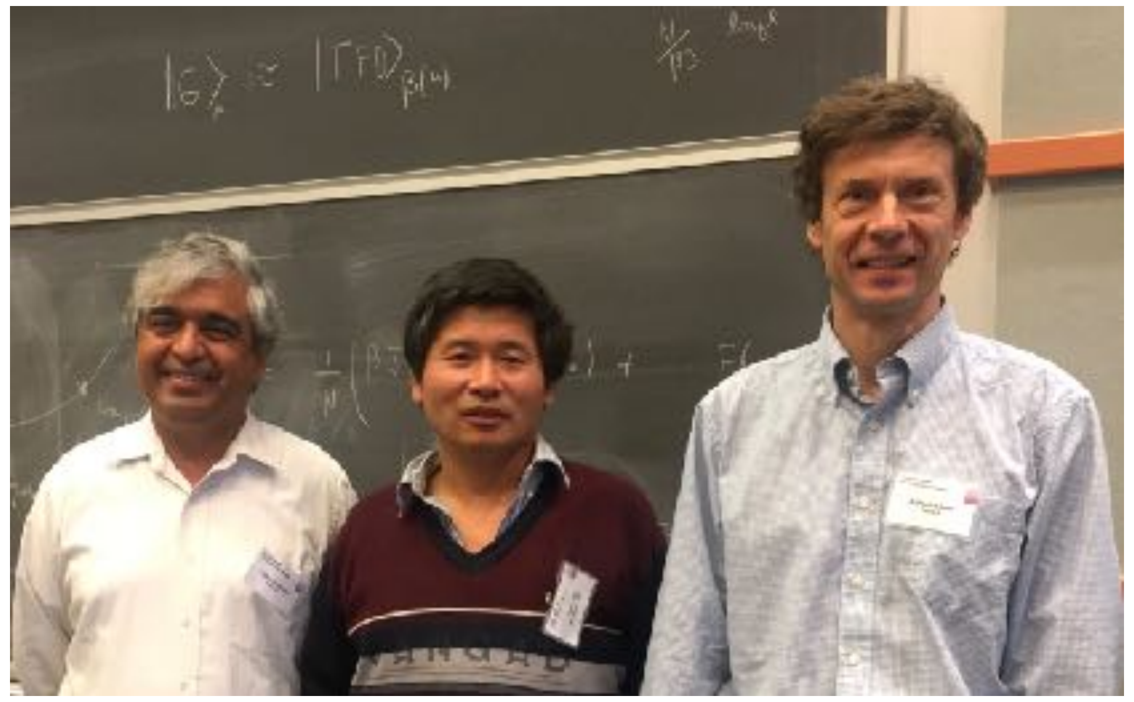

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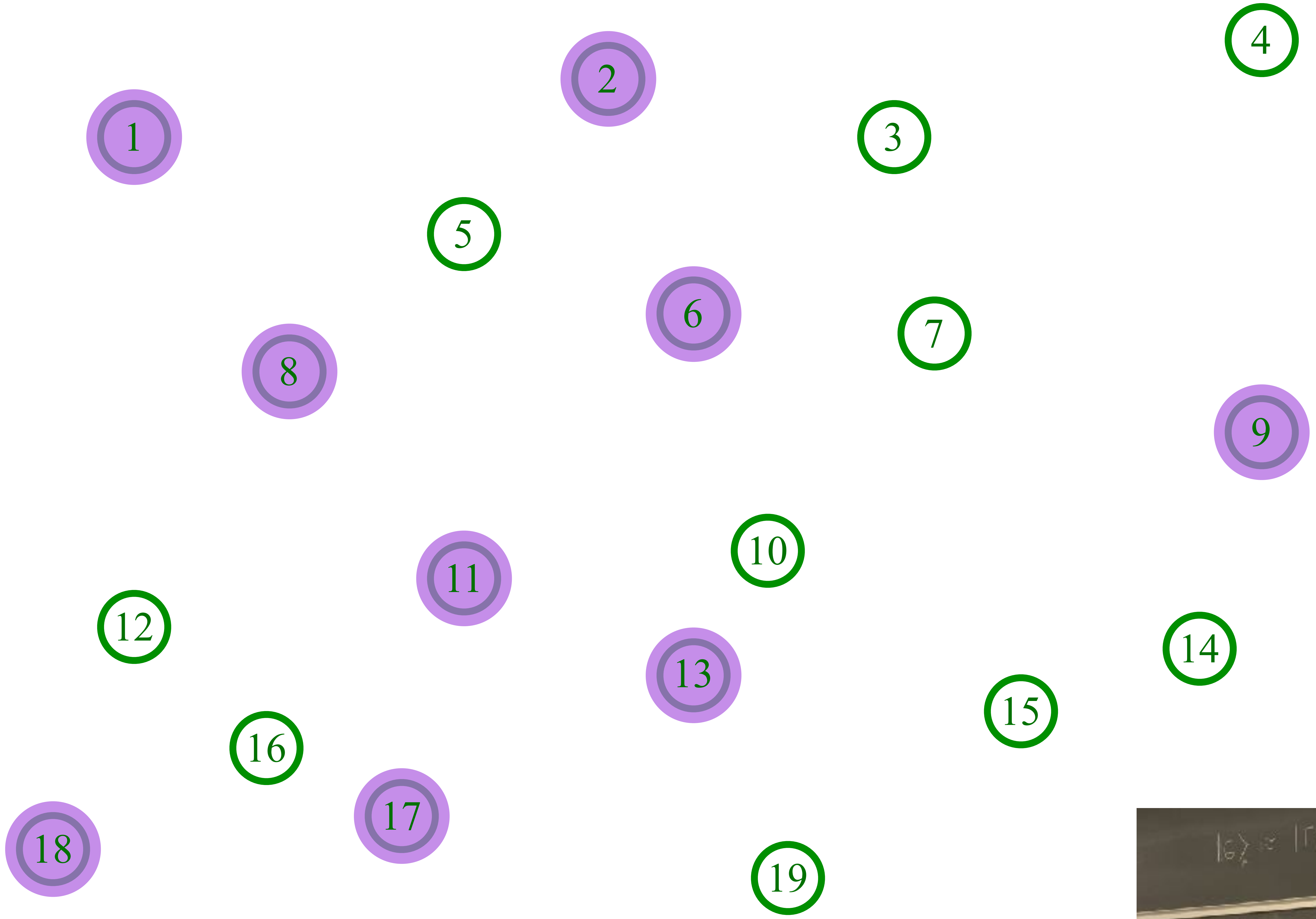

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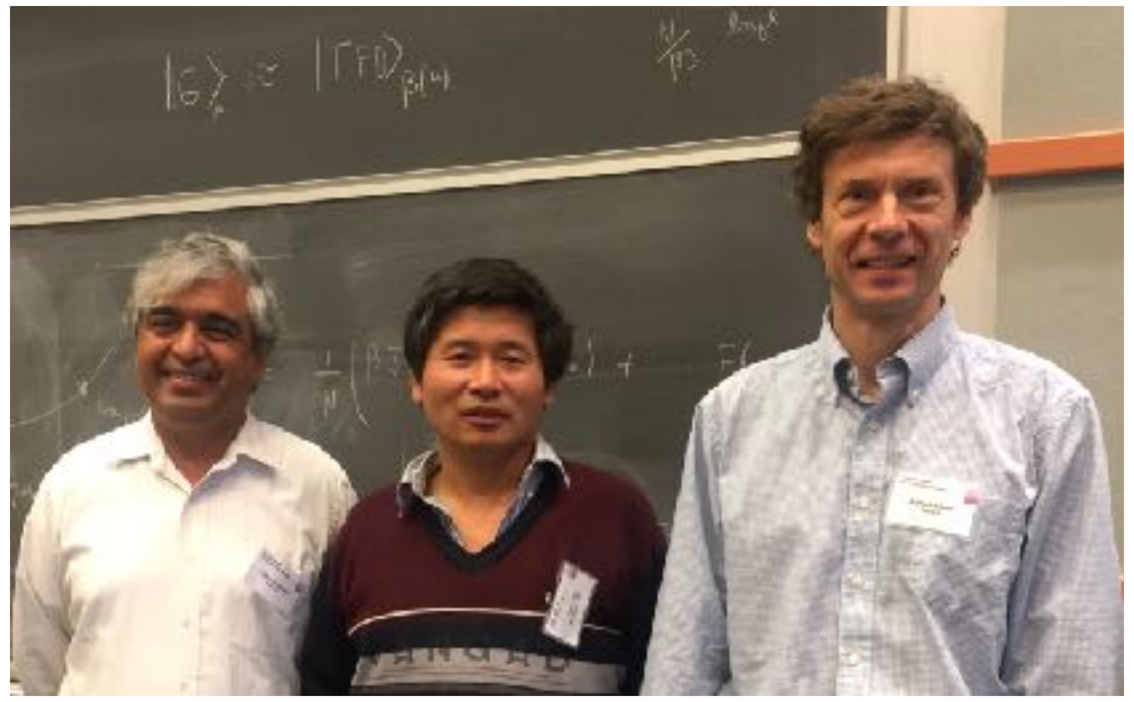

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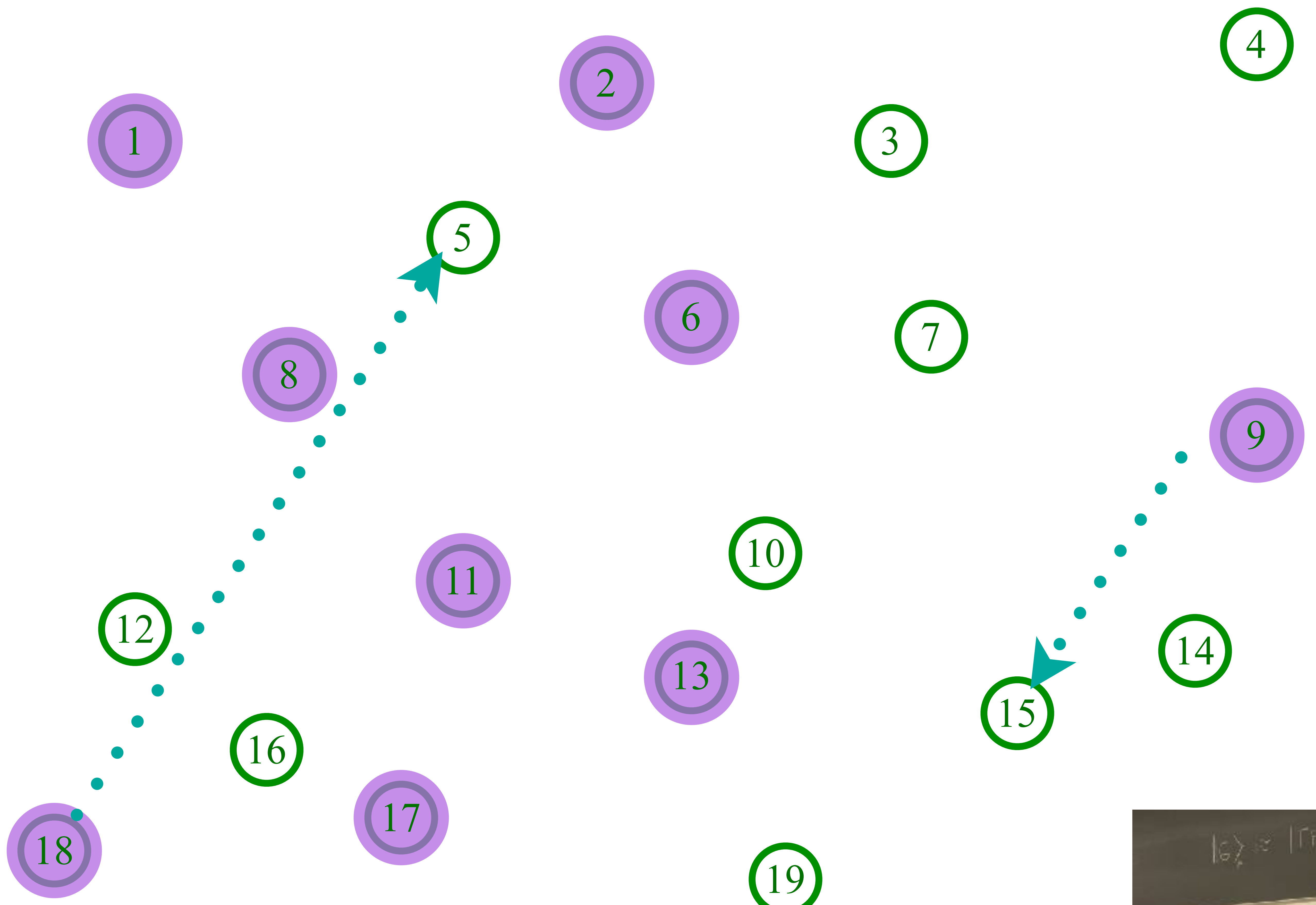
$$U_{9,18;5,15}$$



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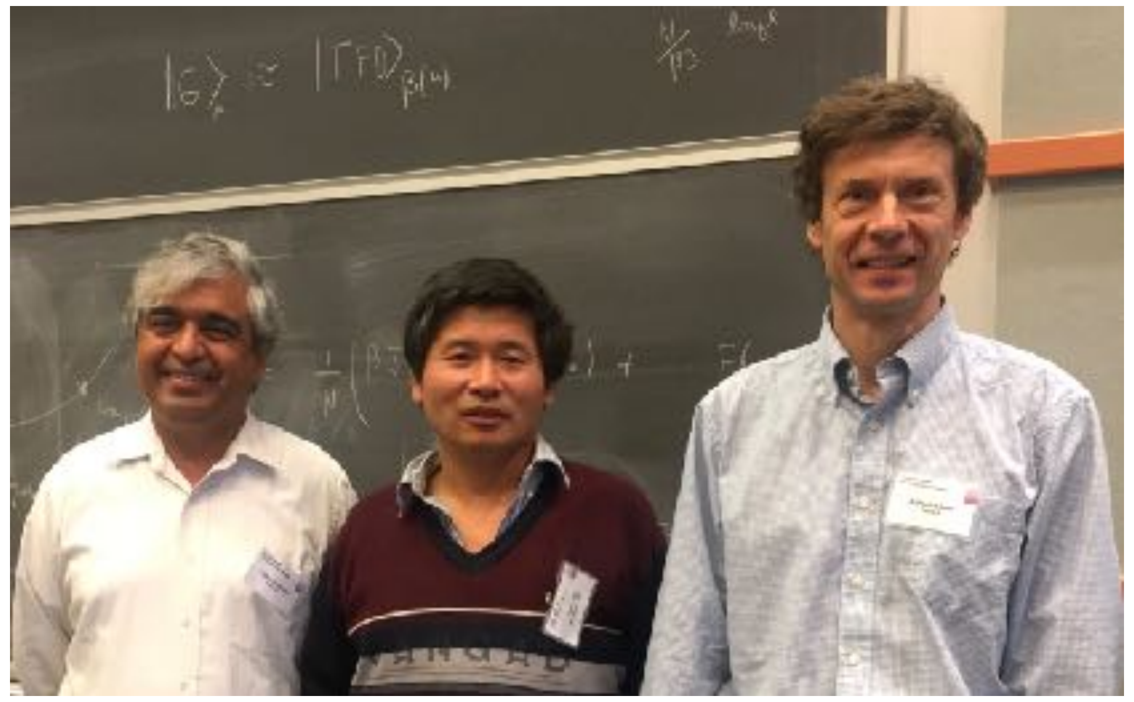

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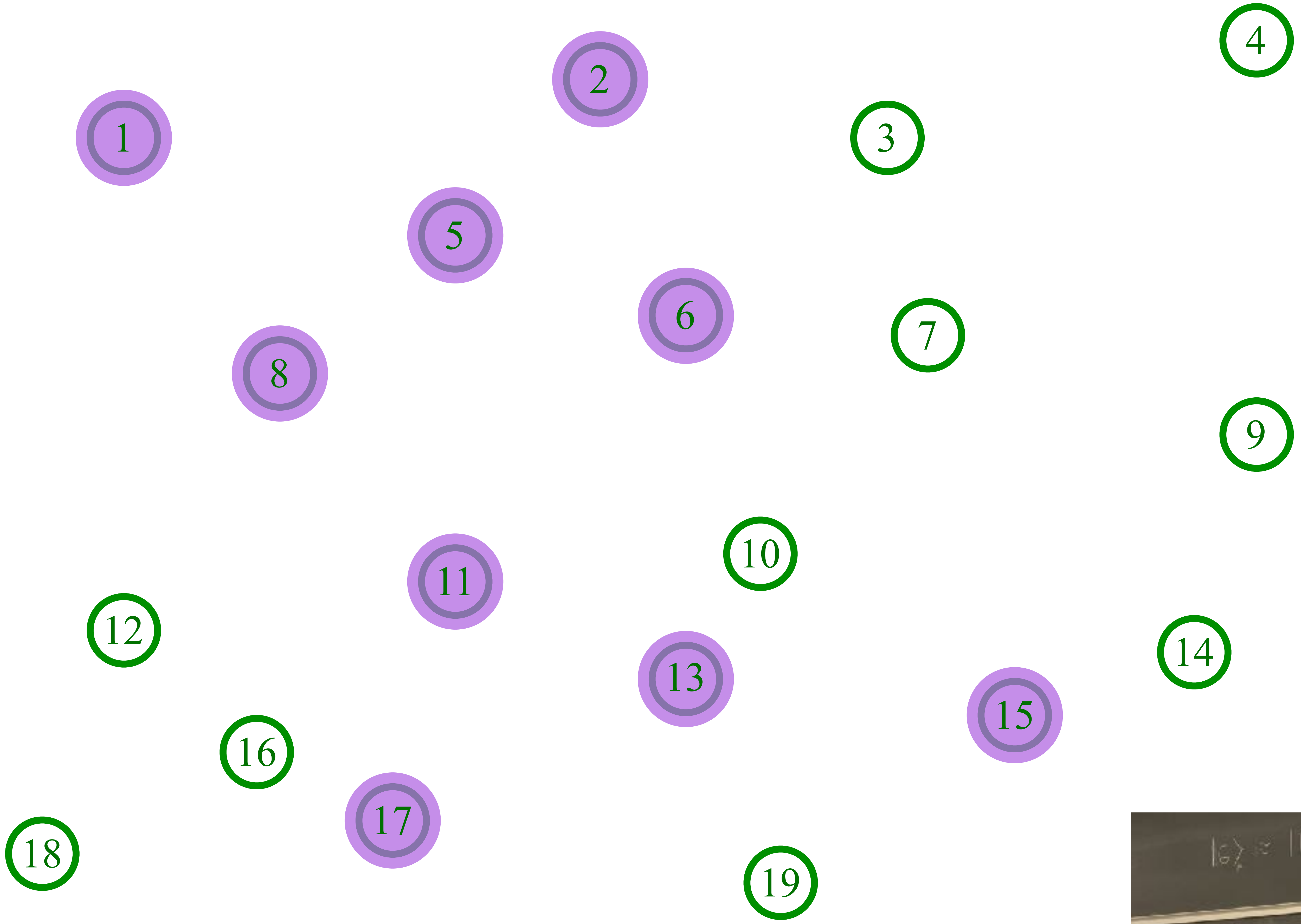

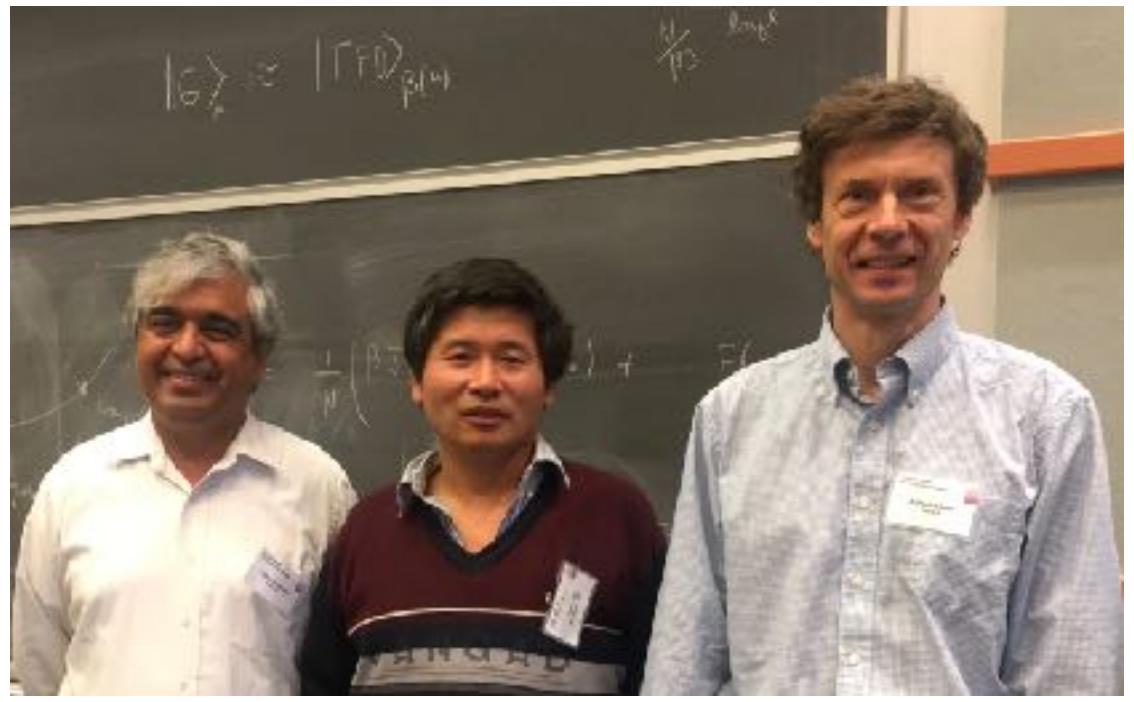

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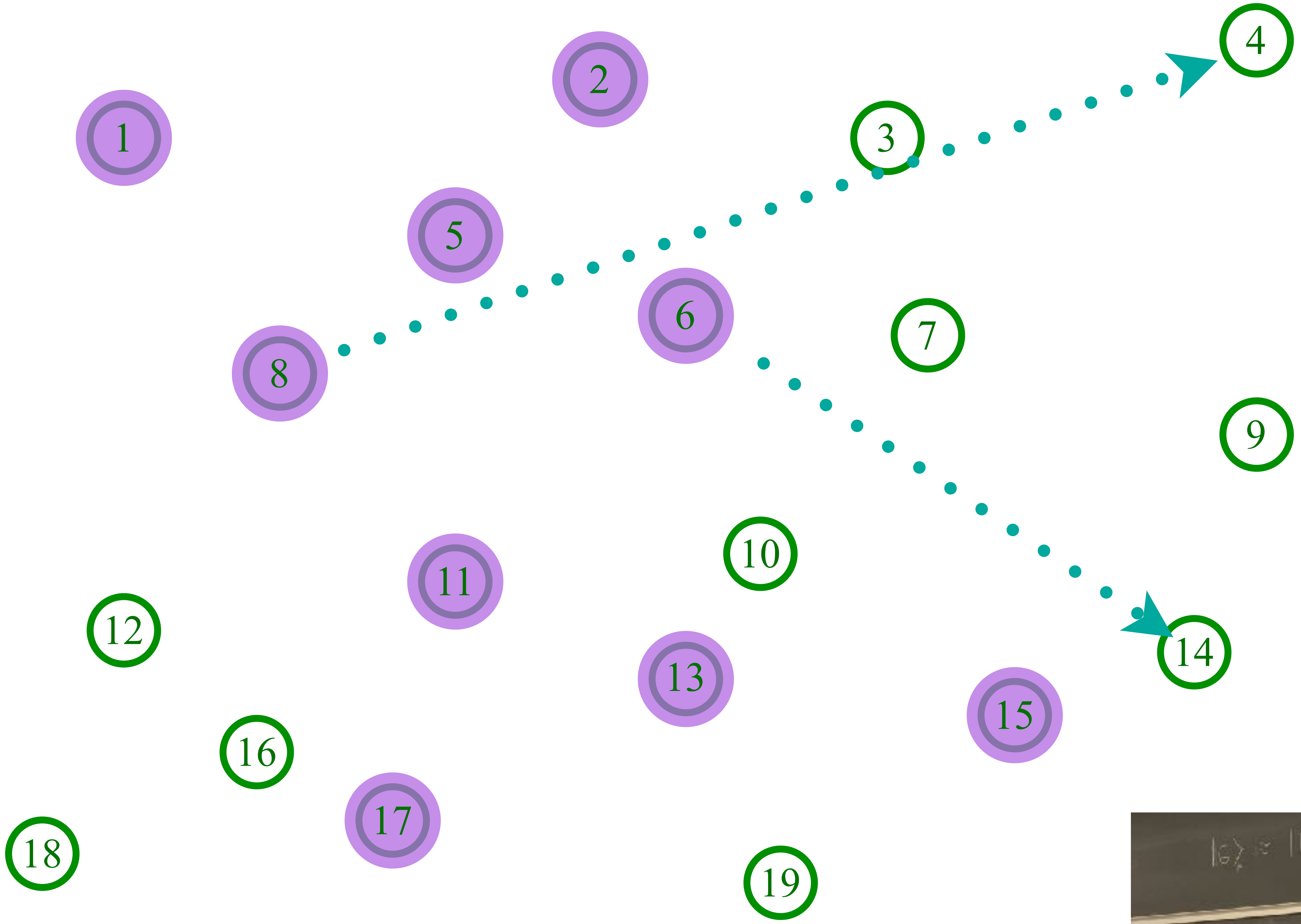
$U_{6,8;4,14}$



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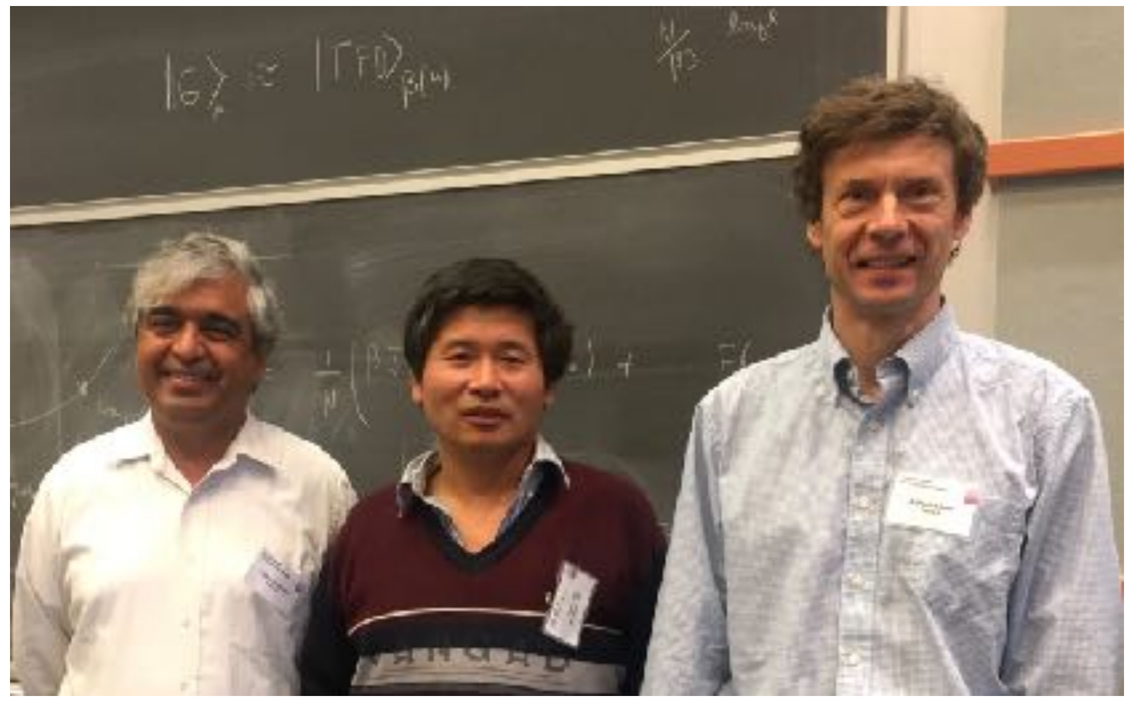

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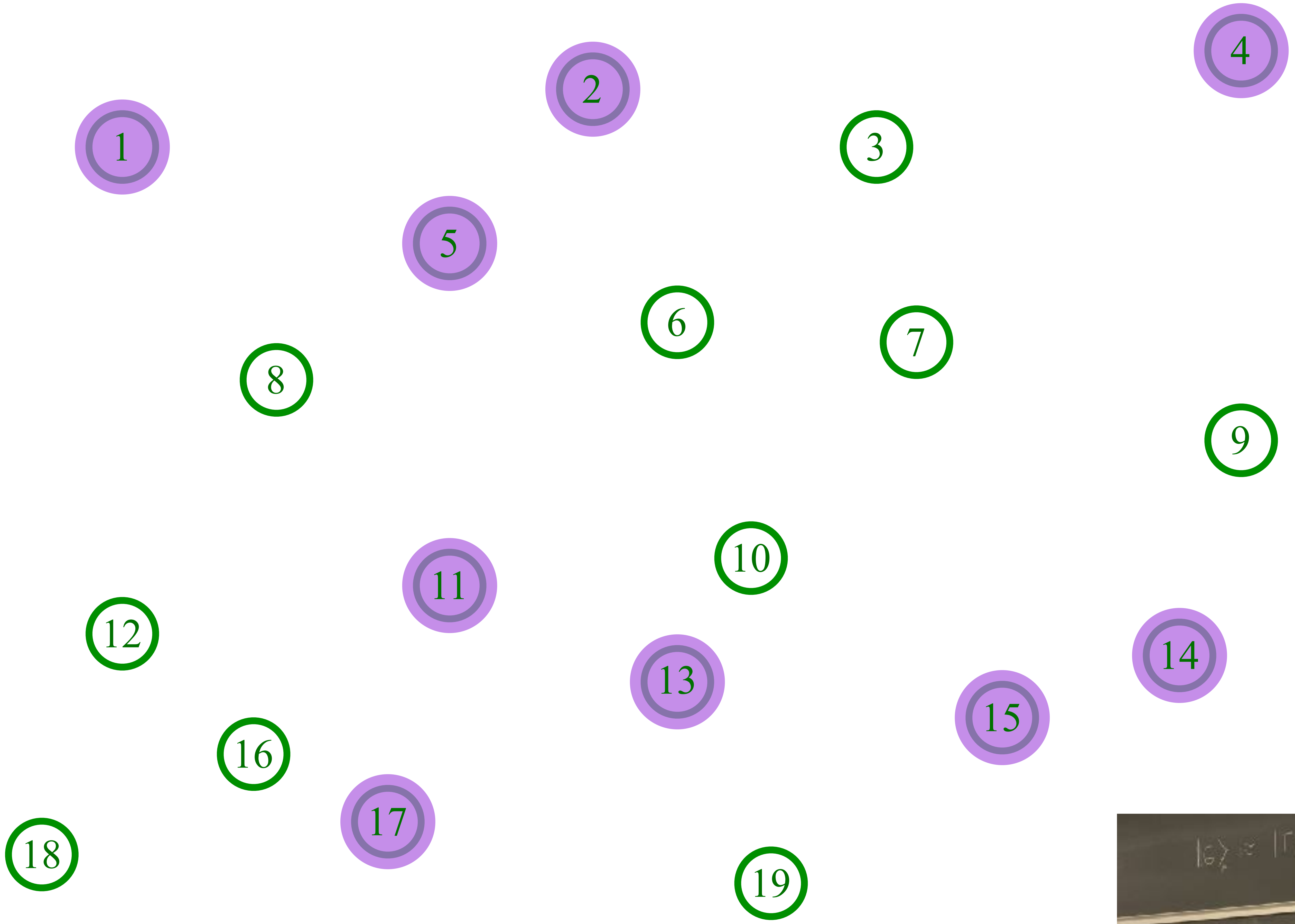

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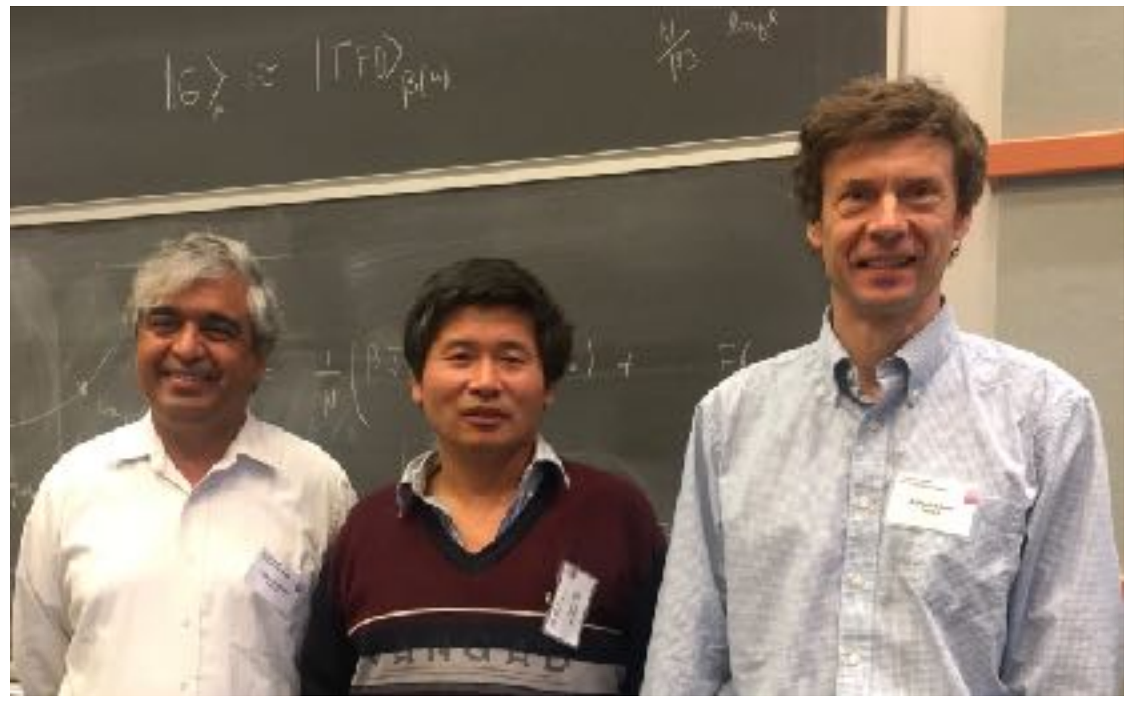


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

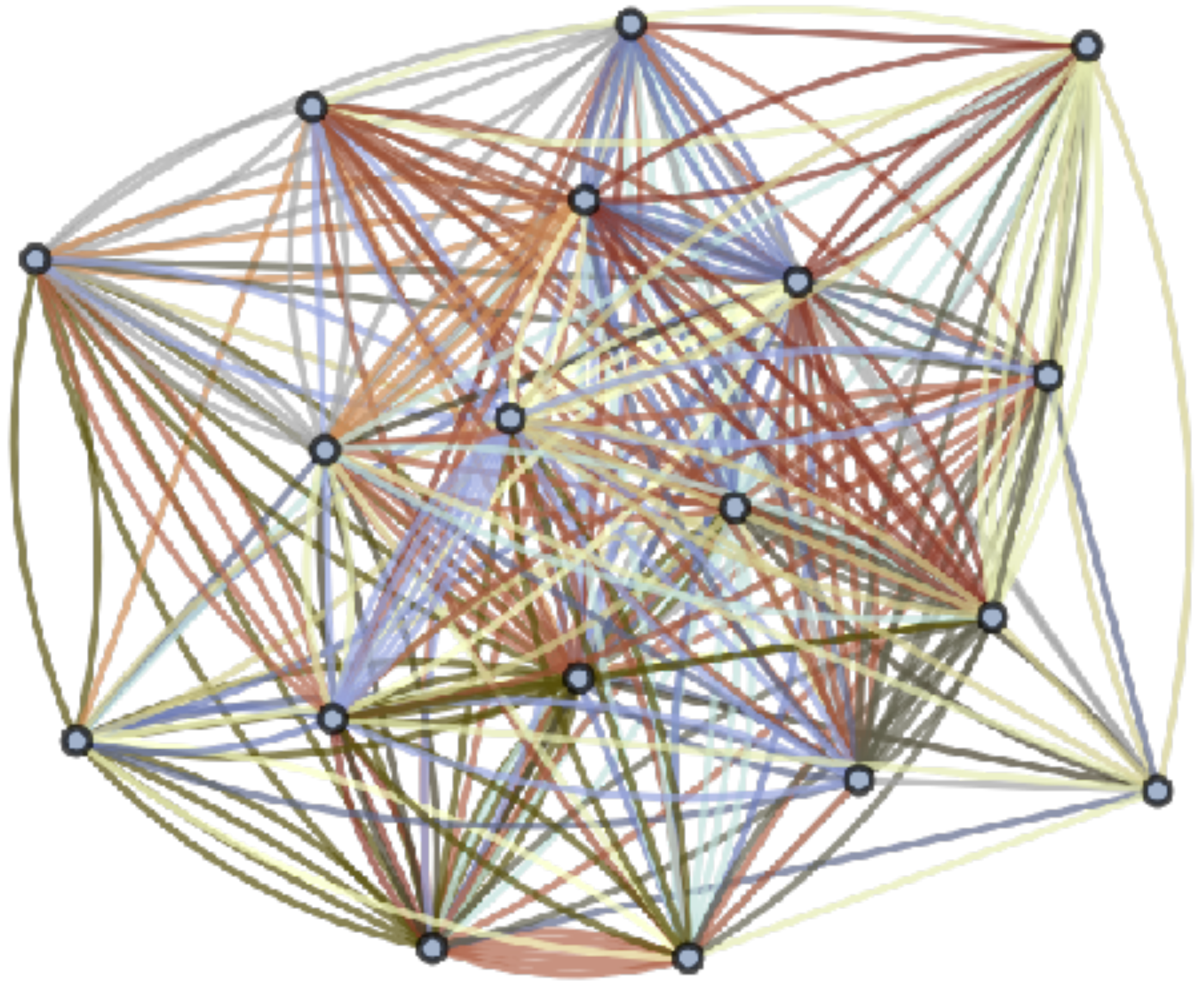
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

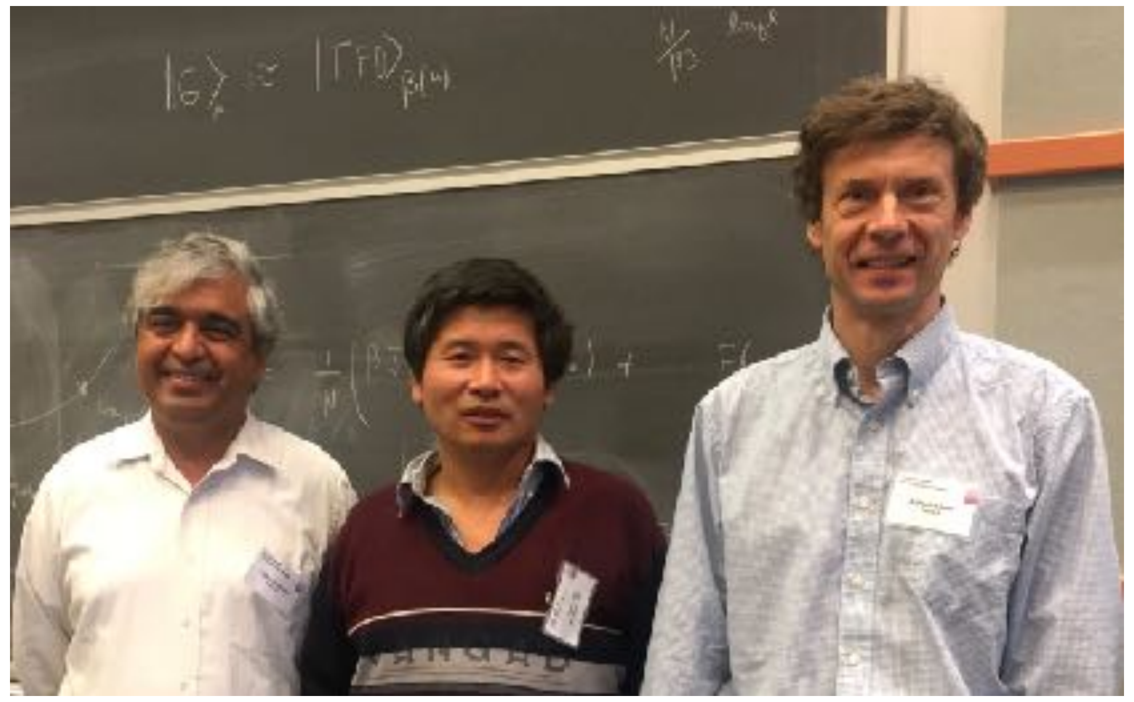
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





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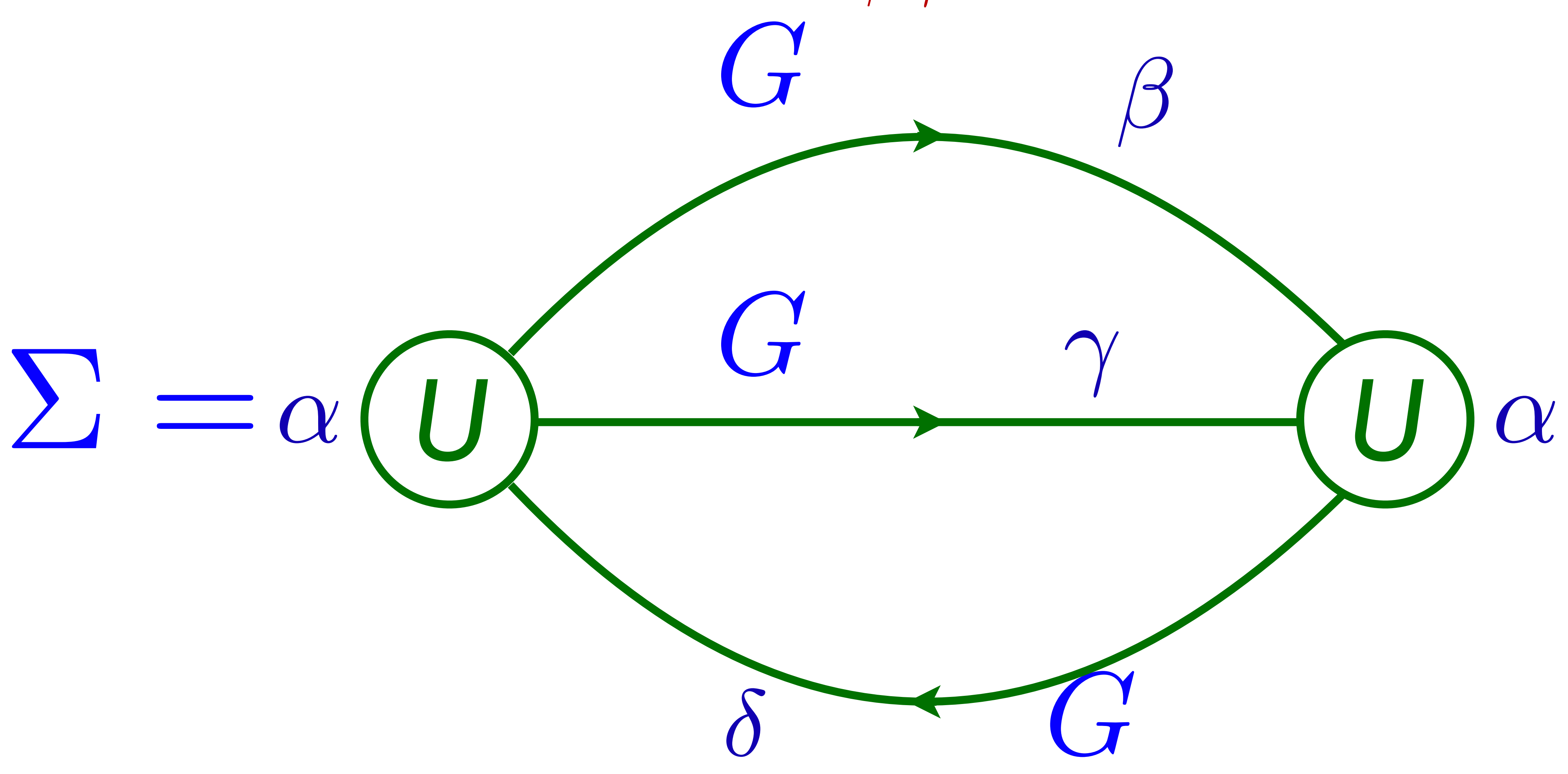
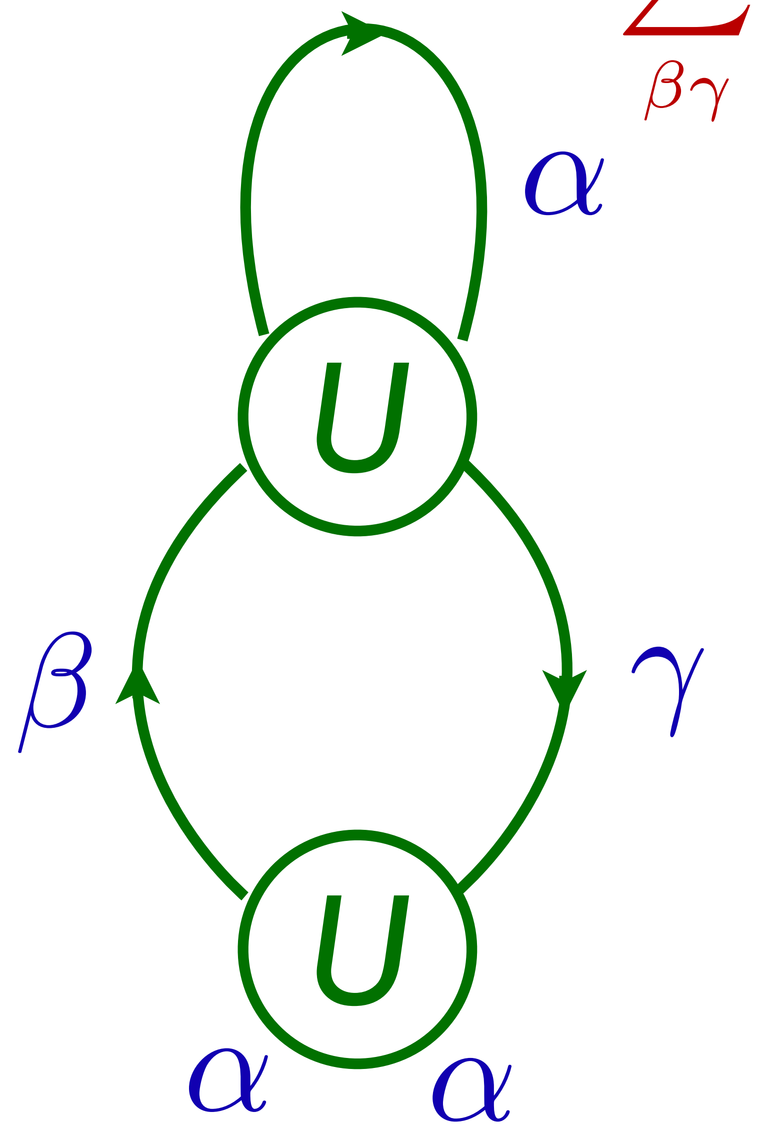
Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$\sum_{\beta\gamma\delta} \frac{U^2}{N^3} = U^2$$



$$\sum_{\beta\gamma} \frac{U^2}{N^3} = \frac{U^2}{N}$$



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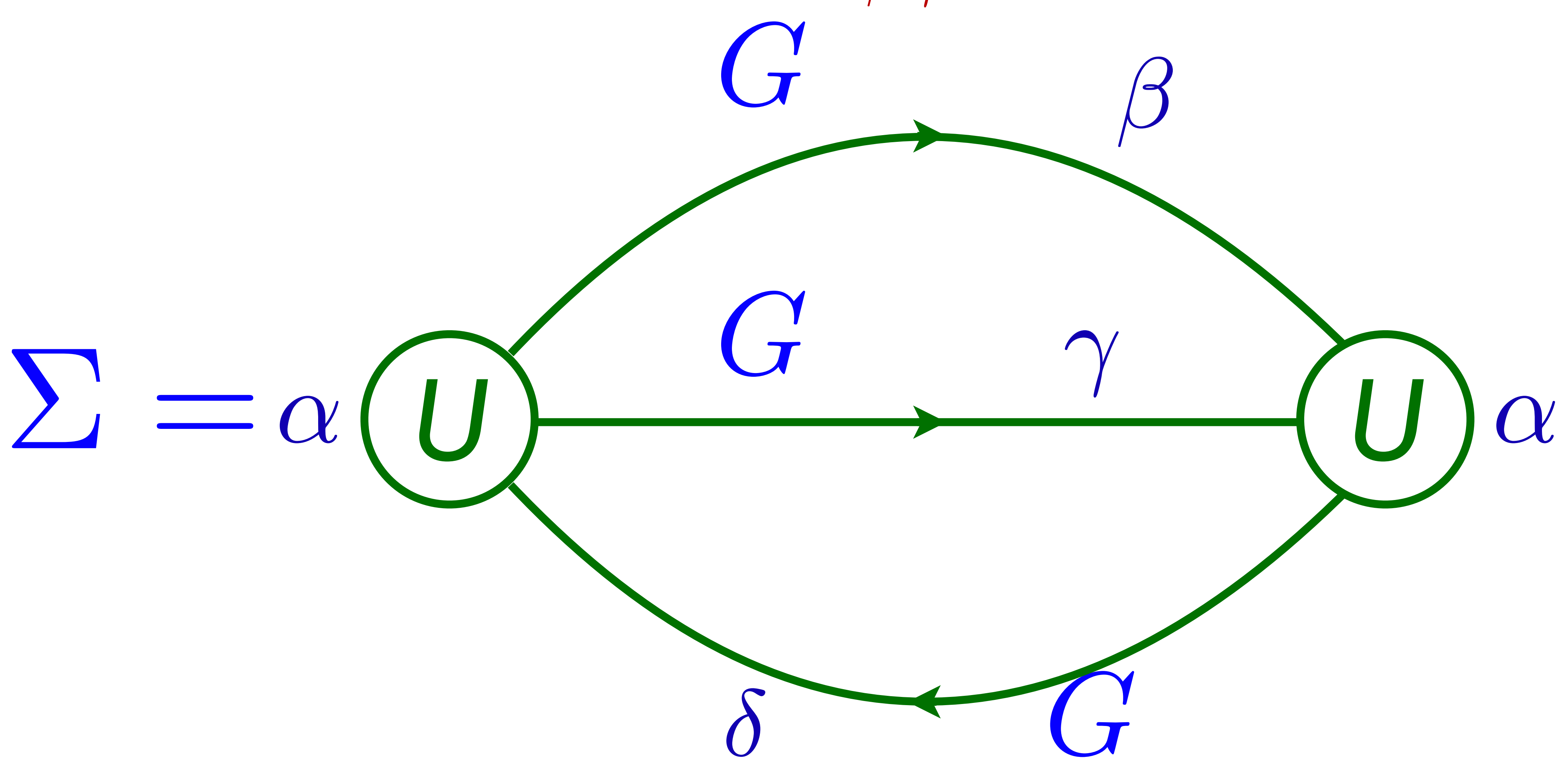
Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$$\sum_{\beta\gamma\delta} \frac{U^2}{N^3} = U^2$$



S. Sachdev and J. Ye,
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The complex SYK model

Solution of these equations, and of the free energy, yields universal results for the SYK model:

- At long times, and at $T = 0$, $G(\tau) \sim |\tau|^{-1/2}$ ($\Rightarrow$ indication there are no quasiparticles)

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- At general charge $\mathcal{Q}$, there is a particle-hole asymmetry determined by a parameter $\mathcal{E}$:

$$G(\tau) \sim \begin{cases} -\tau^{-1/2} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-1/2} & \tau < 0 \end{cases} \quad , \quad T = 0$$

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- There is a universal 'Luttinger relation' between $-\infty < \mathcal{E} < \infty$ and the total charge $0 < \mathcal{Q} < 1$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$\mathcal{Q} = \frac{1}{2} - \frac{\theta}{\pi} - \frac{\sin(2\theta)}{4}$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The SYK model

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

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Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

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The complex SYK model

$$G_*(\tau) = -C\frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2}.$$

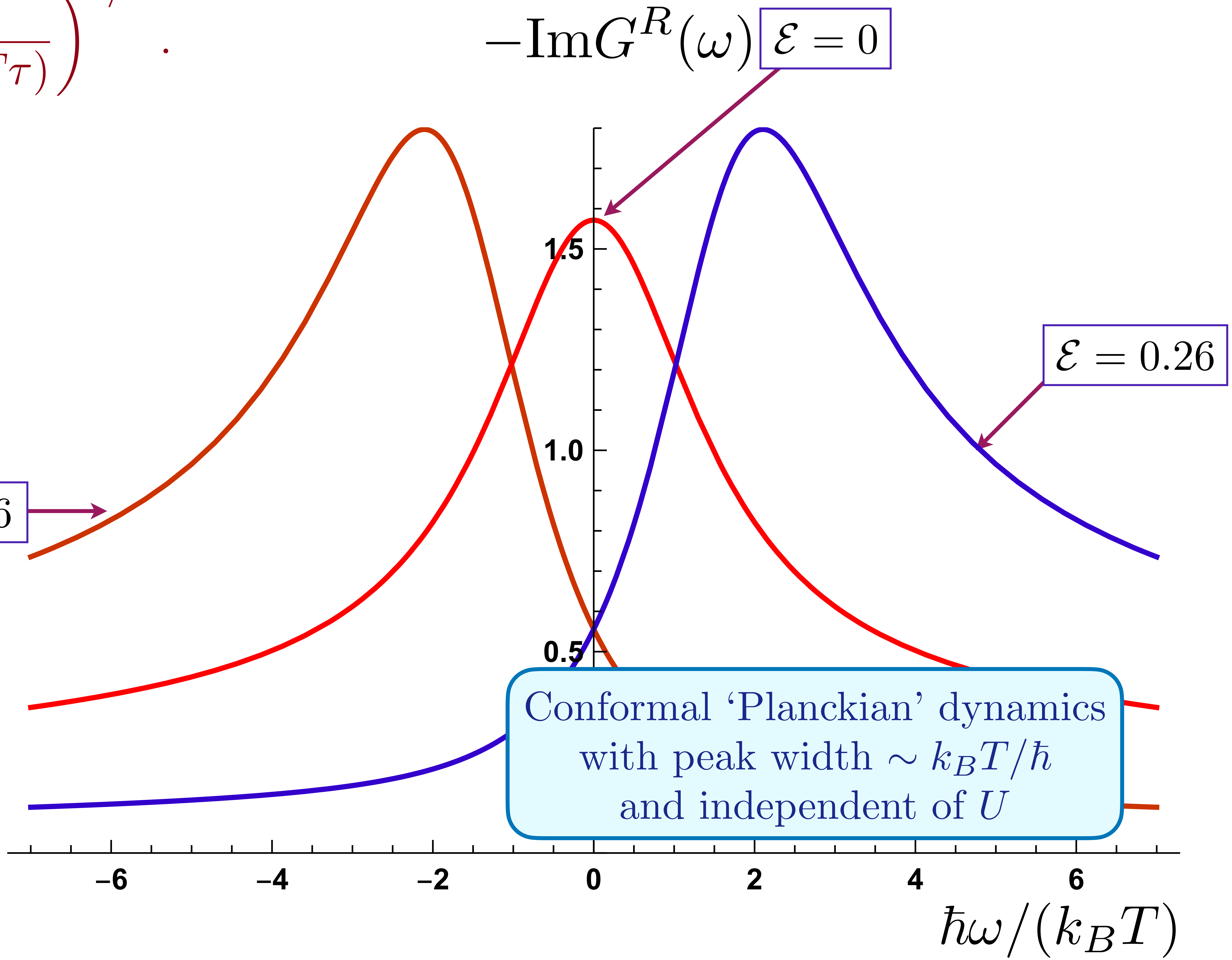
$$G^R_*(\omega) =$$

$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}}\frac{\Gamma\left(\frac{1}{4}-\frac{i\omega}{2\pi T}+i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4}-\frac{i\omega}{2\pi T}+i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$

$\mathcal{E}$ is a known function of $\mathcal{Q}$ (Luttinger relation)



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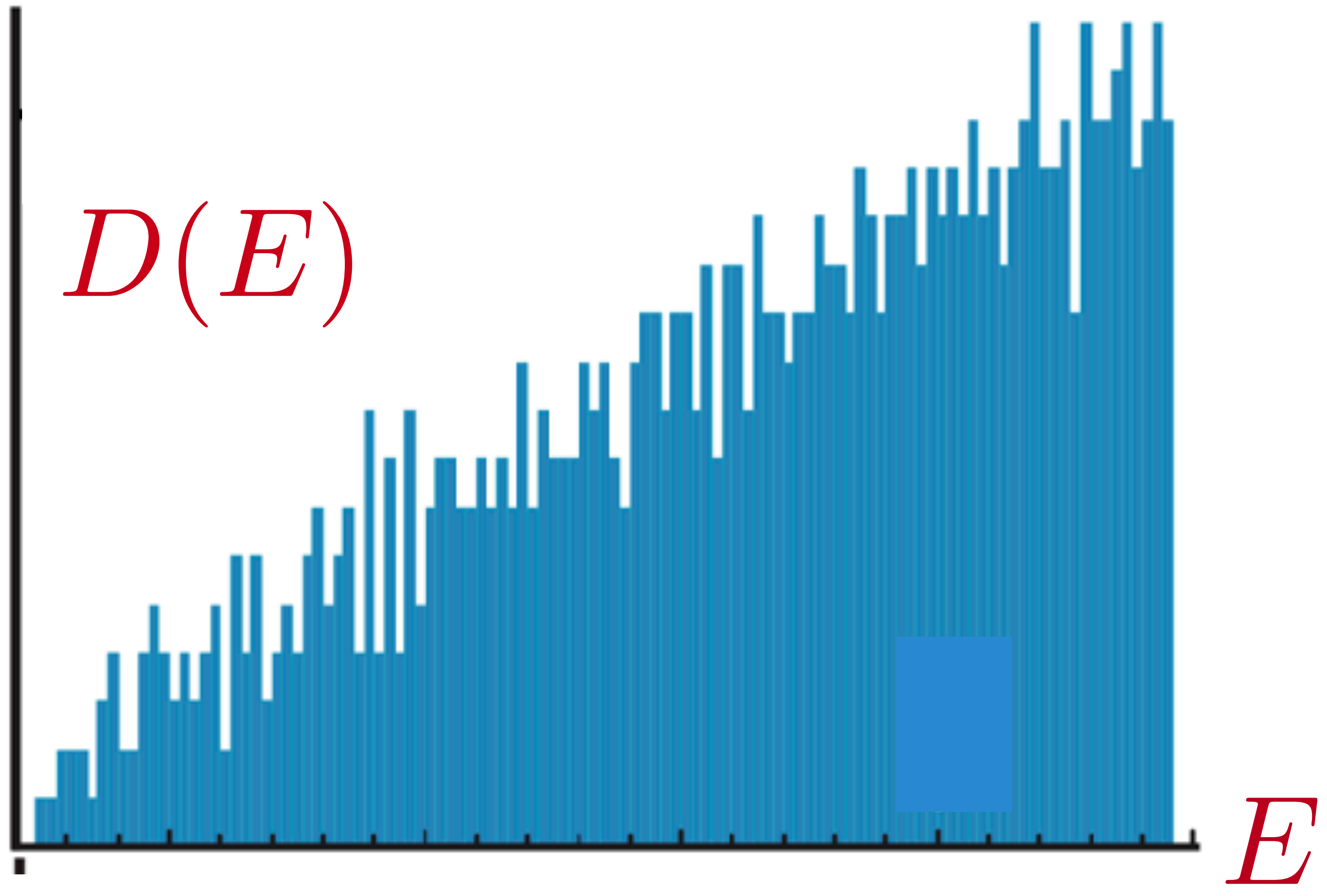
2. Zero temperature entropy without exponential ground state degeneracy!

$$\lim_{T\to 0}\lim_{N\to\infty}\frac{1}{N}S(T) = s_0 \quad , \quad D(E\to 0) \sim e^{N s_0} f_{\text{smooth}}(E)$$

$$s_0 = 0.464847699170805107\!49\ldots \text{ for } \mathcal{Q} = 1/2.$$



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The SYK model and black holes

Semiclassical connection first proposed by S.S. in Physical Review Letters **105**, 151602 (2010): SYK model and charged black holes exhibit Planckian dynamics and zero temperature entropy.

Fully quantum connection established in 2015 by A. Kitaev, J. Maldacena, D. Stanford....

Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

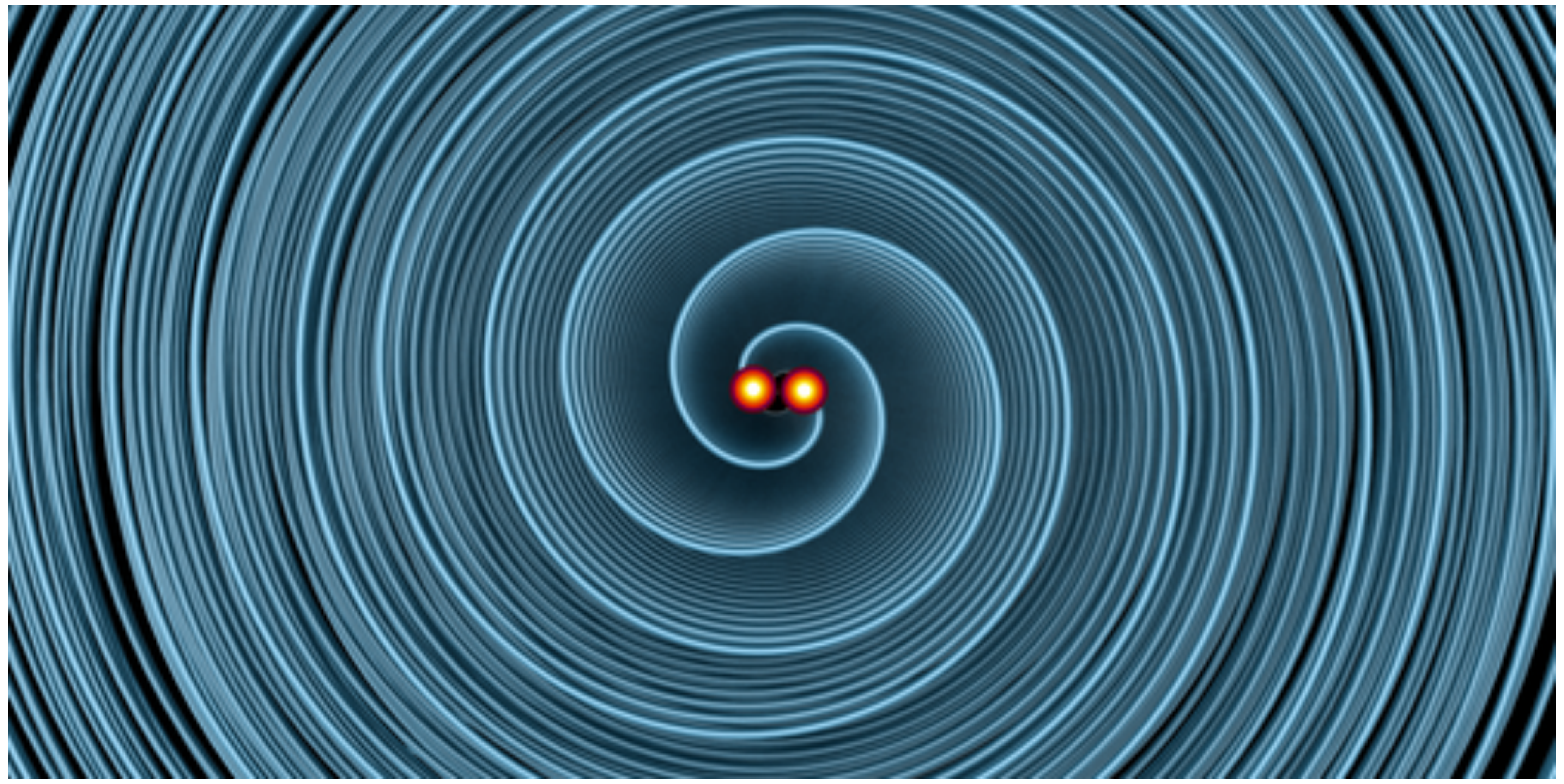
An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.

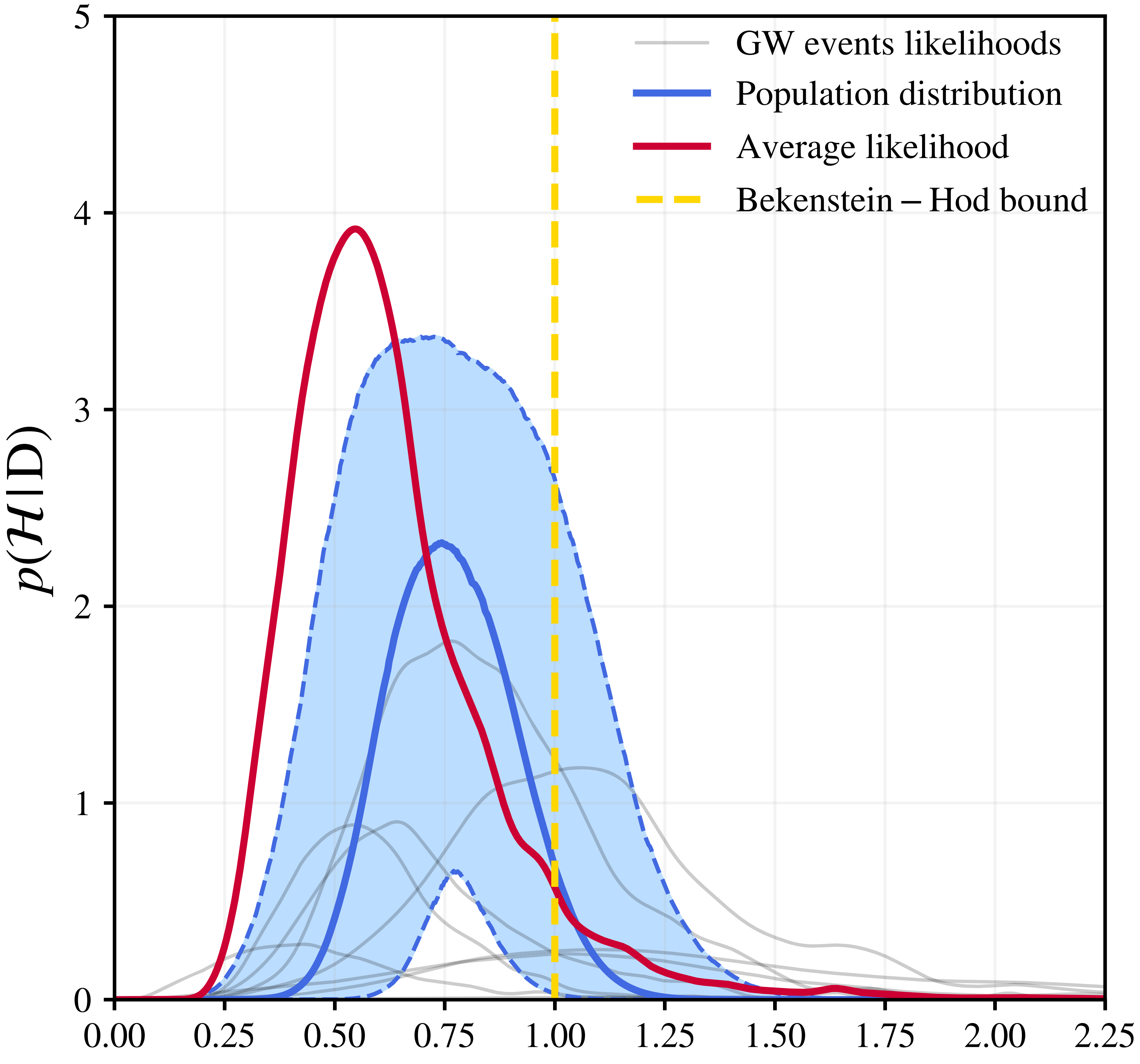
Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$

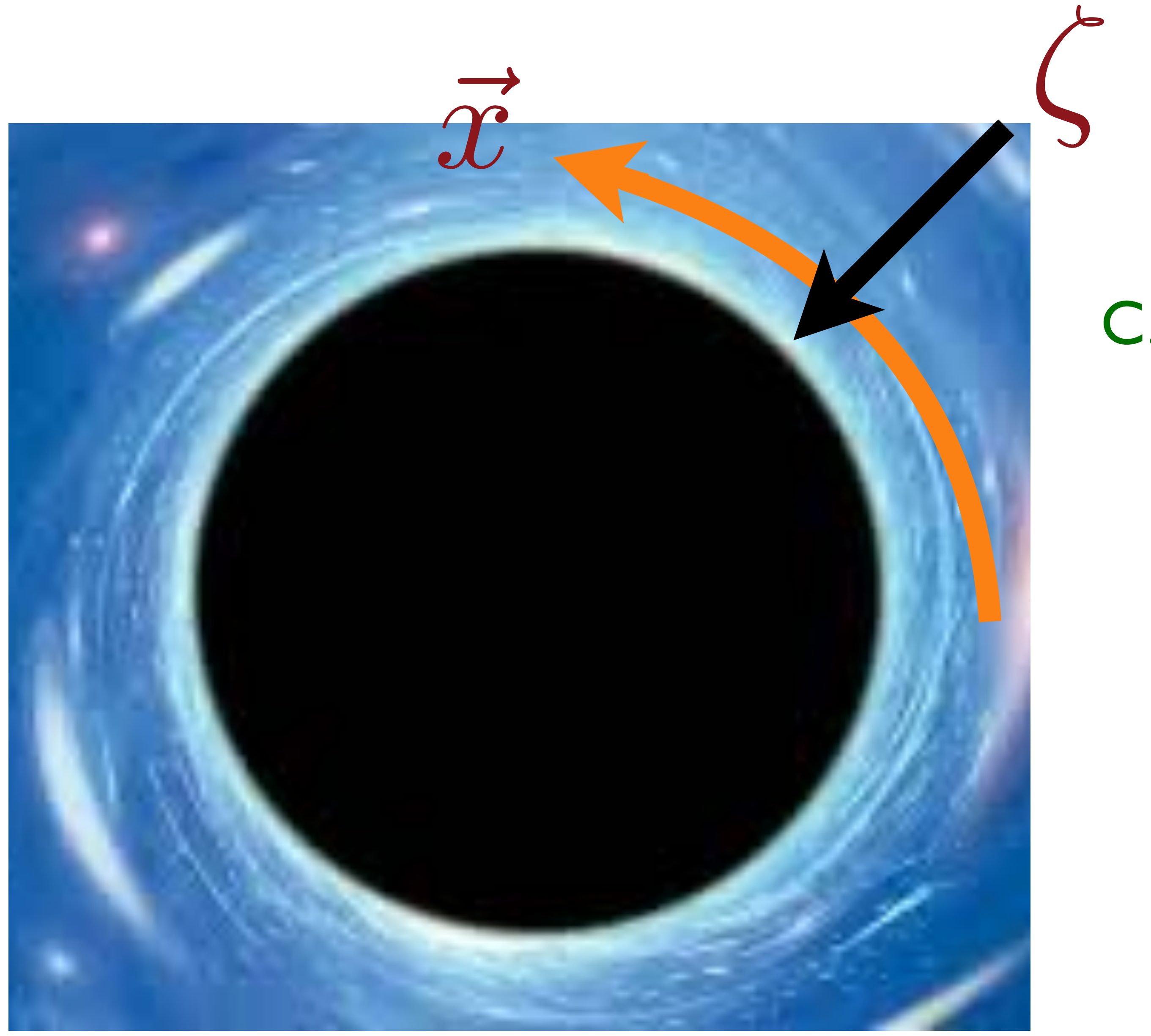




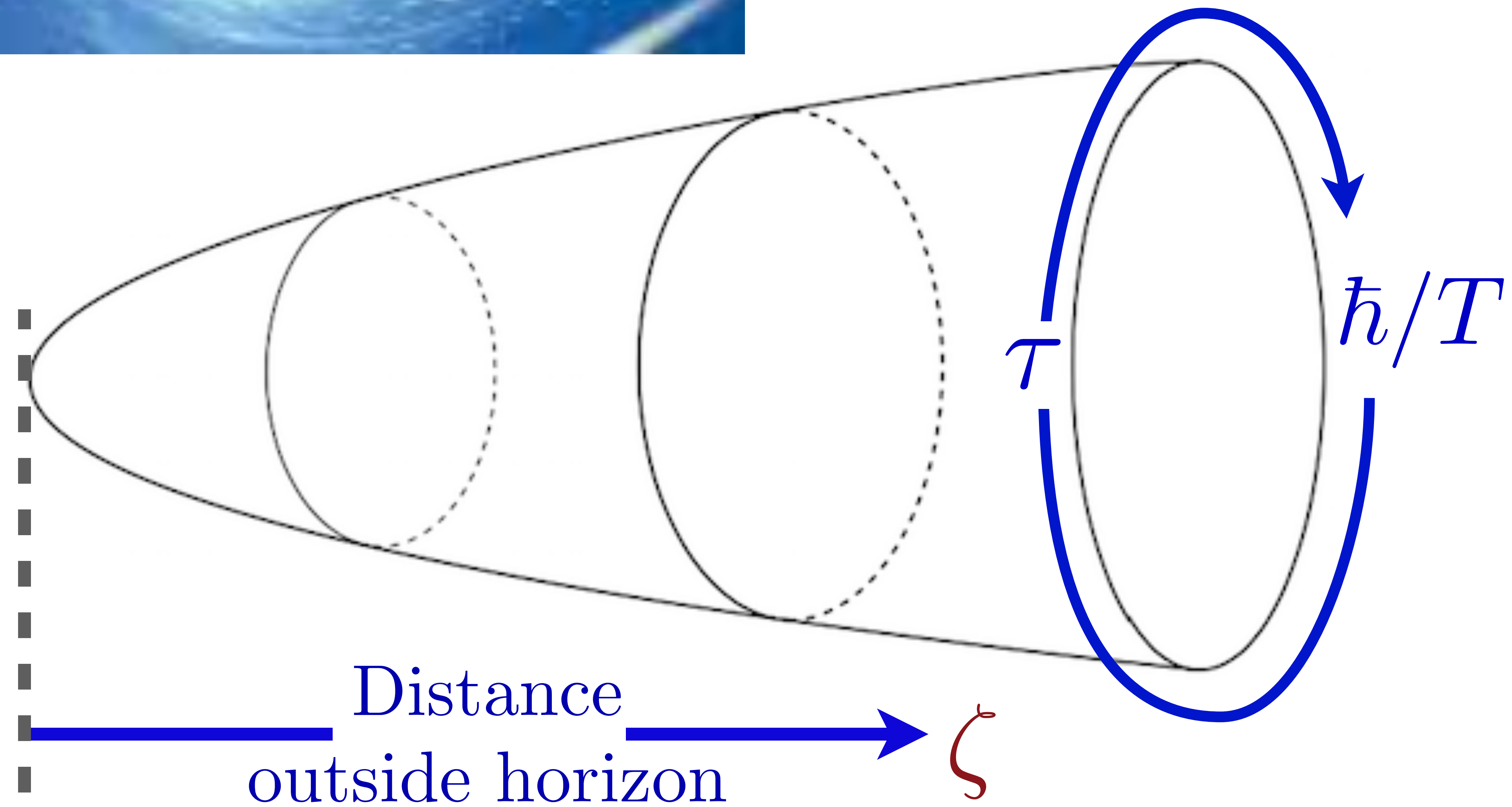
$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$



A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)



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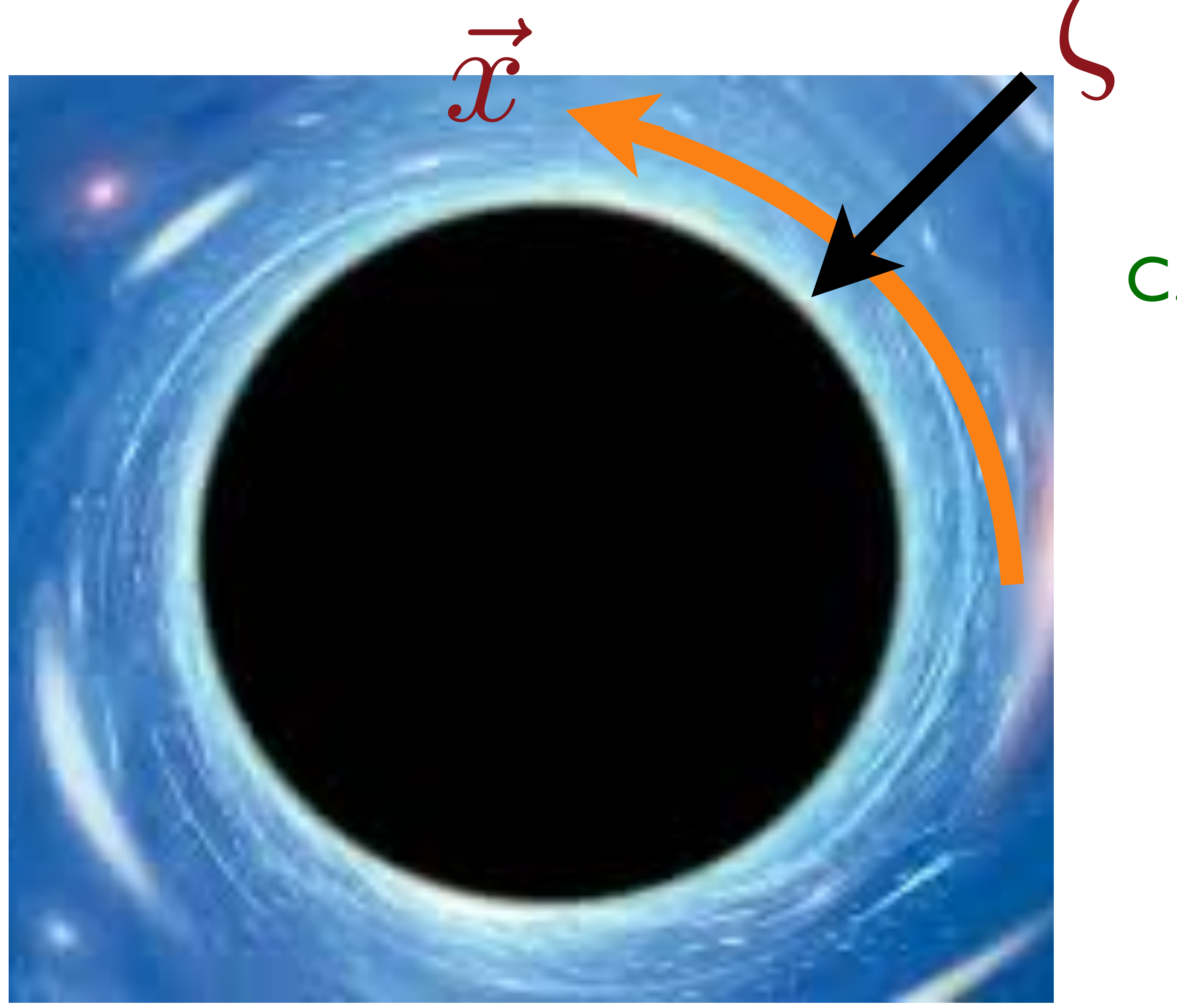
$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)$$



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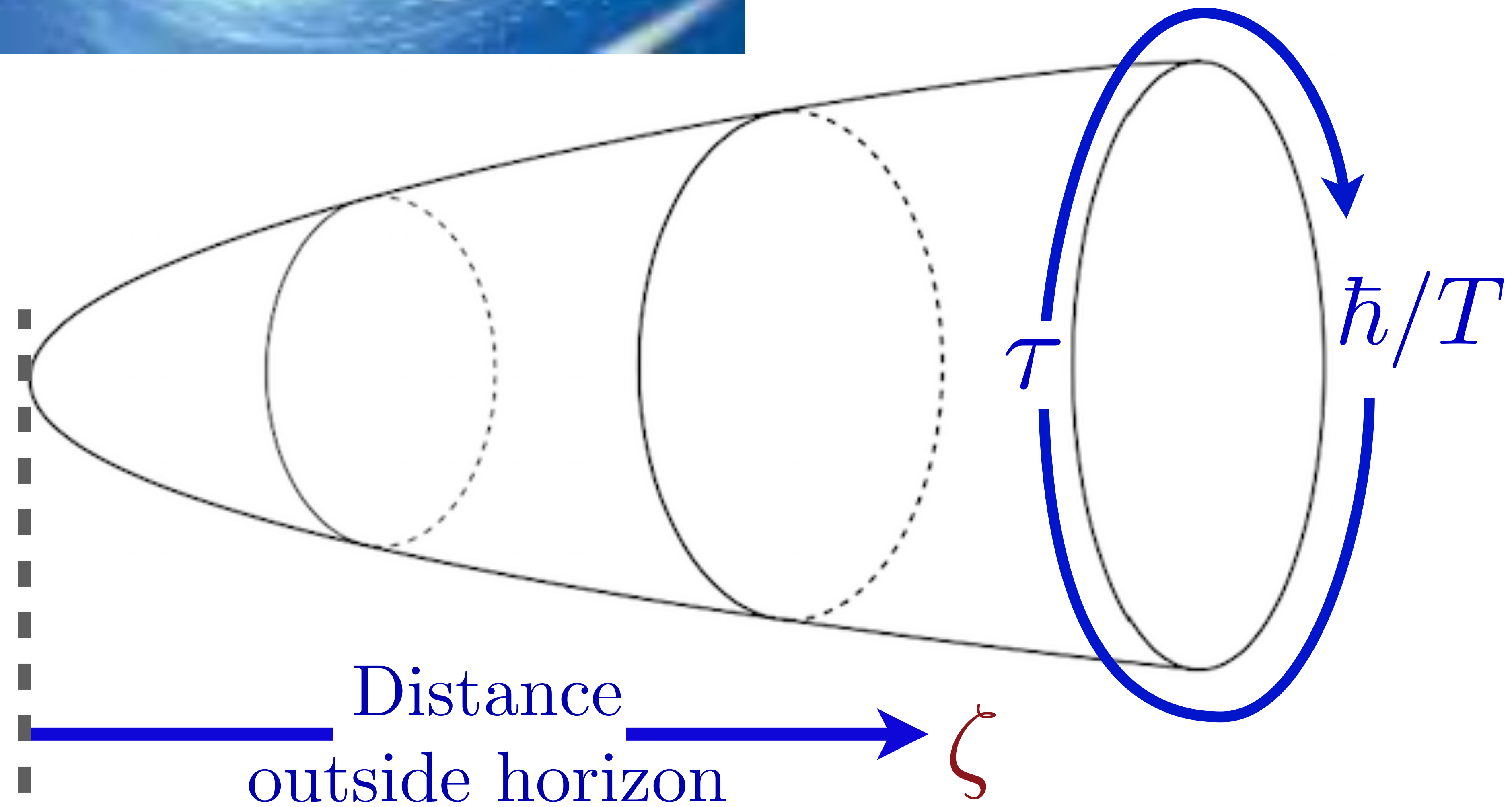
$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T = 0$. The Bekenstein-Hawking entropy $A_0 c^3/(4\hbar G)$ is the analog of the $T \to 0$ GPS entropy, $N s_0$, of the SYK model.



Sachdev (2010)

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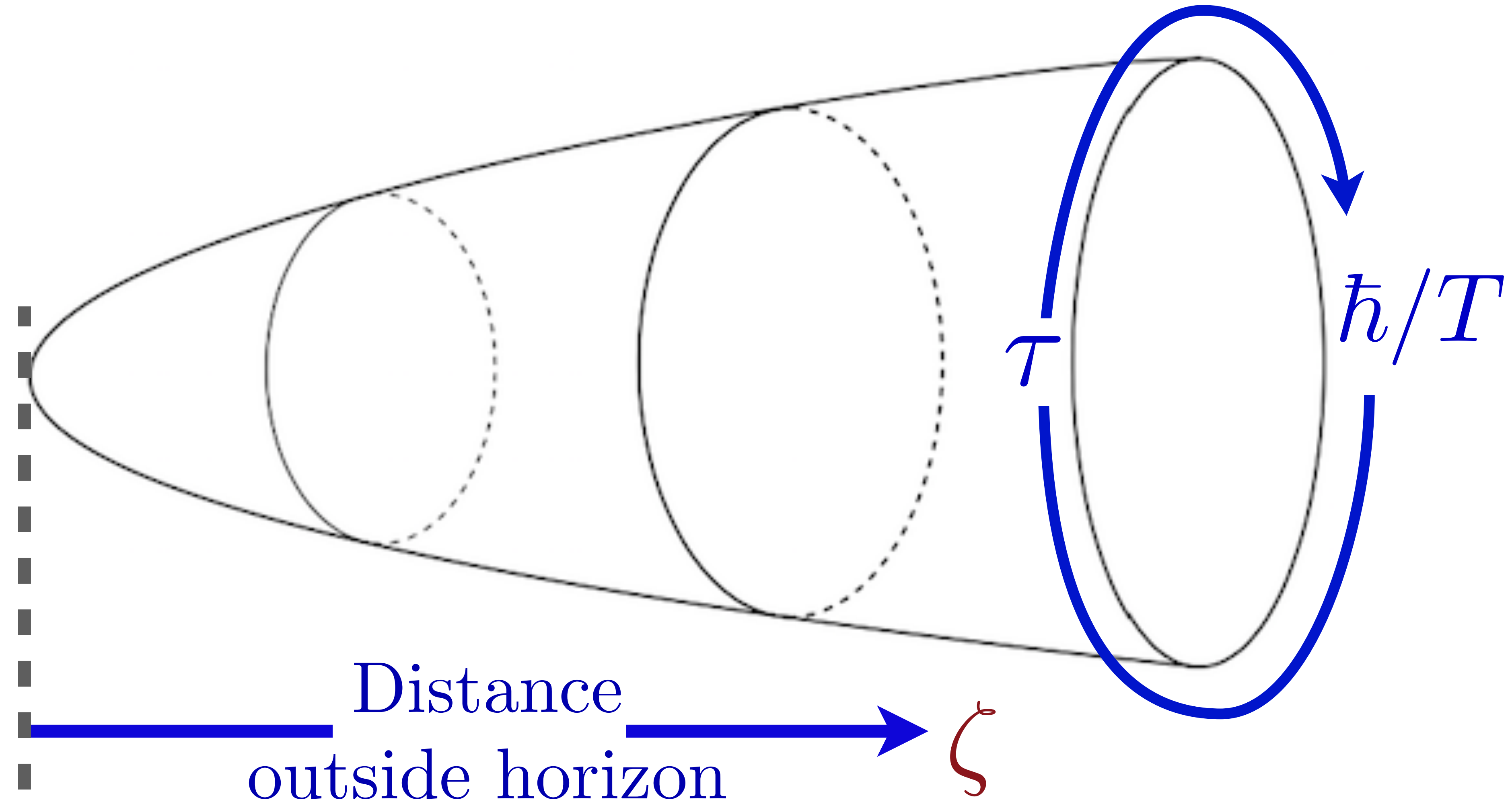

Kitaev (2015); Maldacena, Stanford, Yang (2016)

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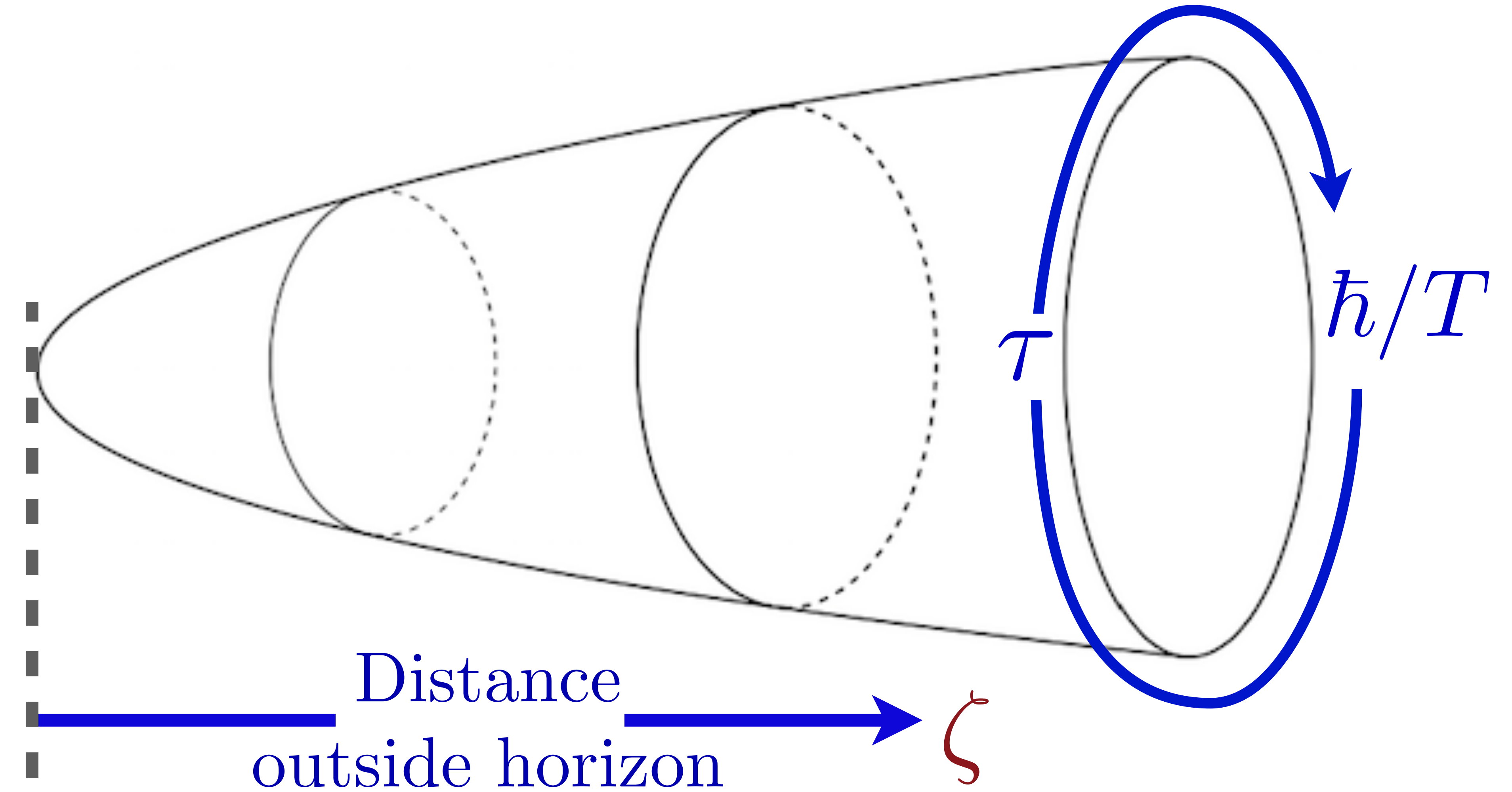
$$= \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) \exp\left(-\frac{1}{\hbar} I^{(0+1)}_{\rm SYK}[\text{time reparameterizations } f(\tau), \text{ phase } \phi(\tau)]\right)$$

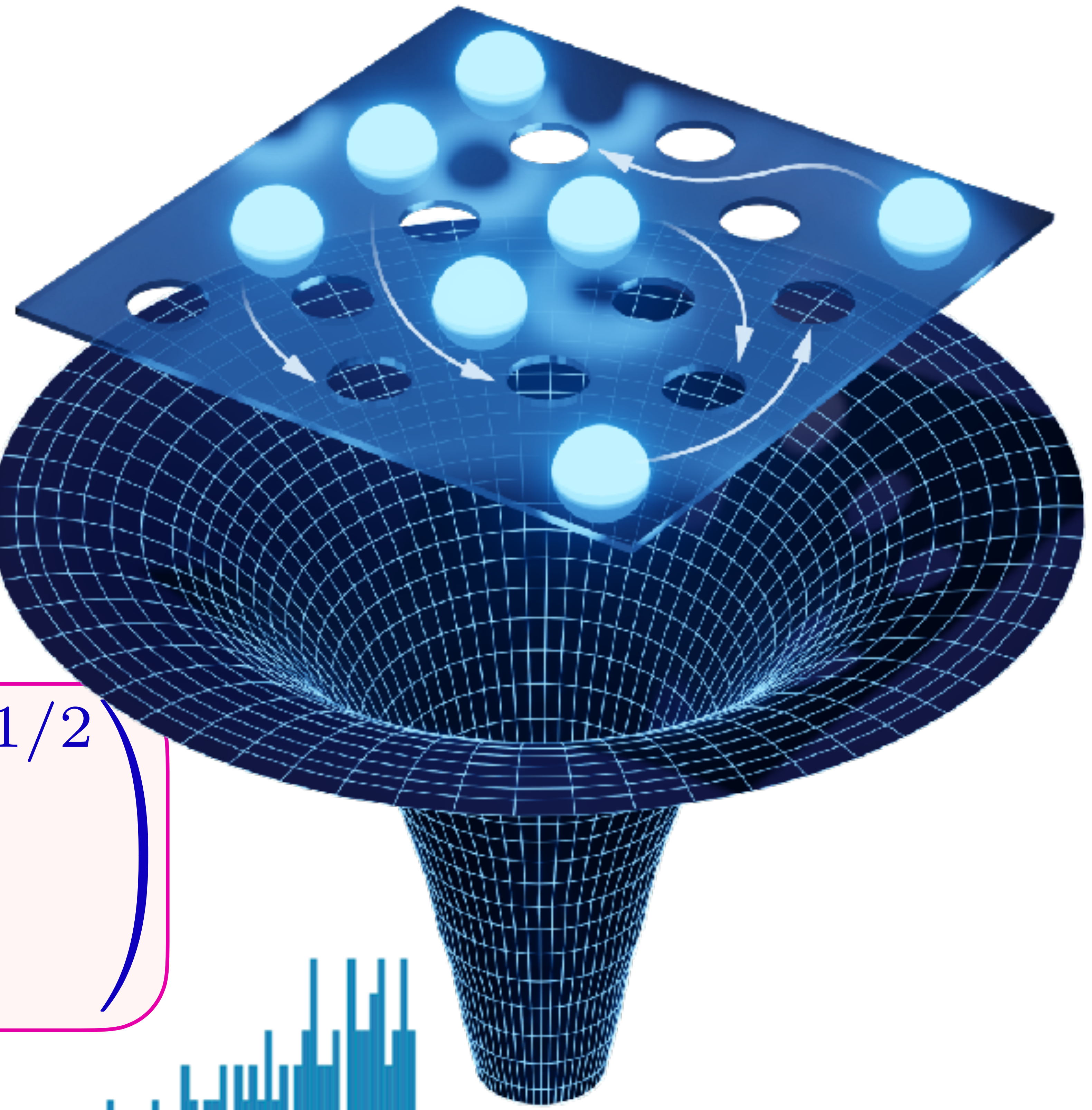
The path integral over the action $I^{(0+1)}_{SYK}$ can be evaluated exactly,
and leads to a computation of $D(E)$

$$\mathcal{Z}(\mathcal{Q},T) = \int dE D(E) \exp\left(-\frac{E}{k_B T}\right)$$



Kitaev (2015) Maldacena, Stanford, Yang (2016); Cotler *et al.* (2017)

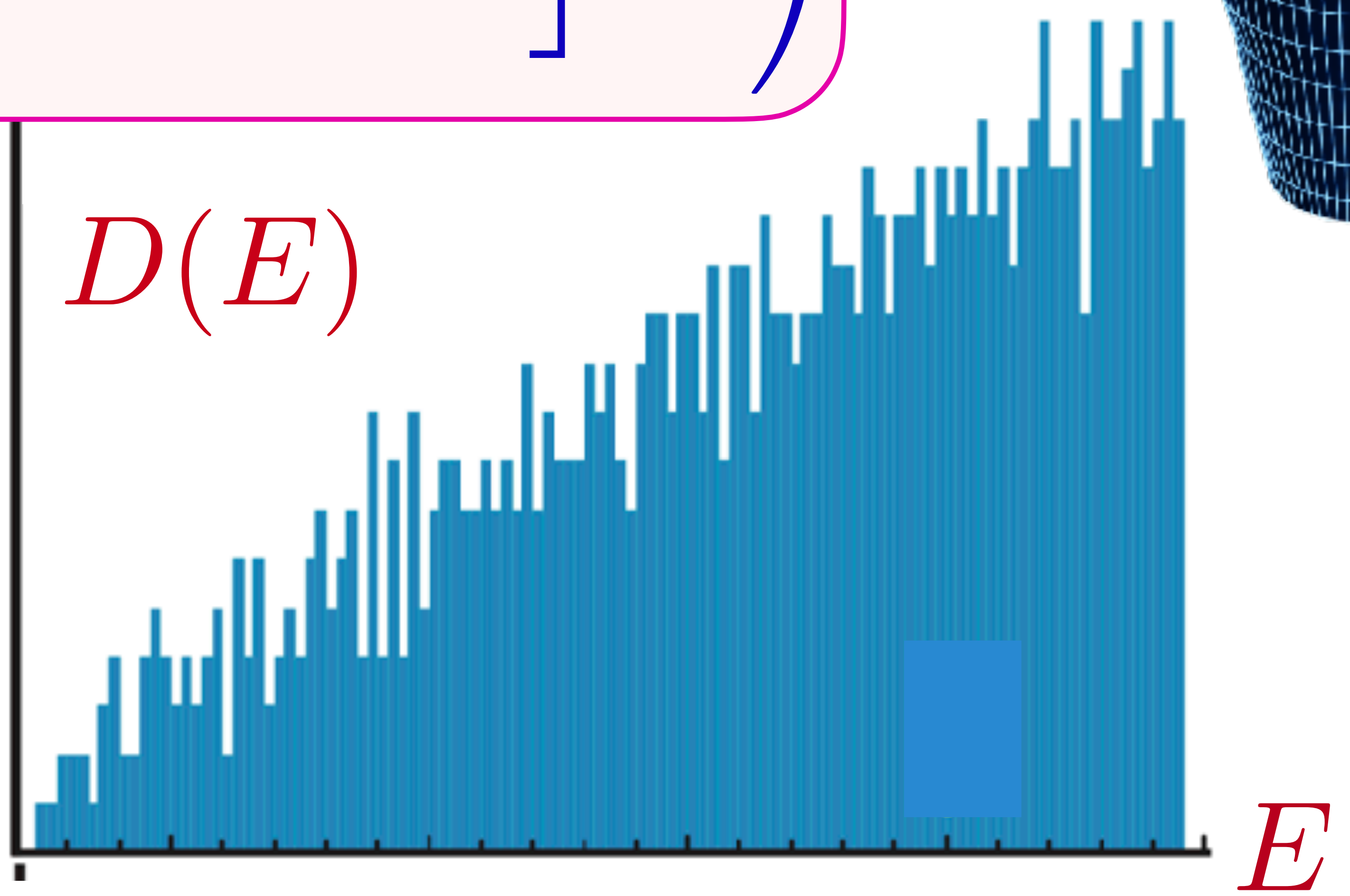
D(E) of charged black holes from the SYK model



- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0{}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$



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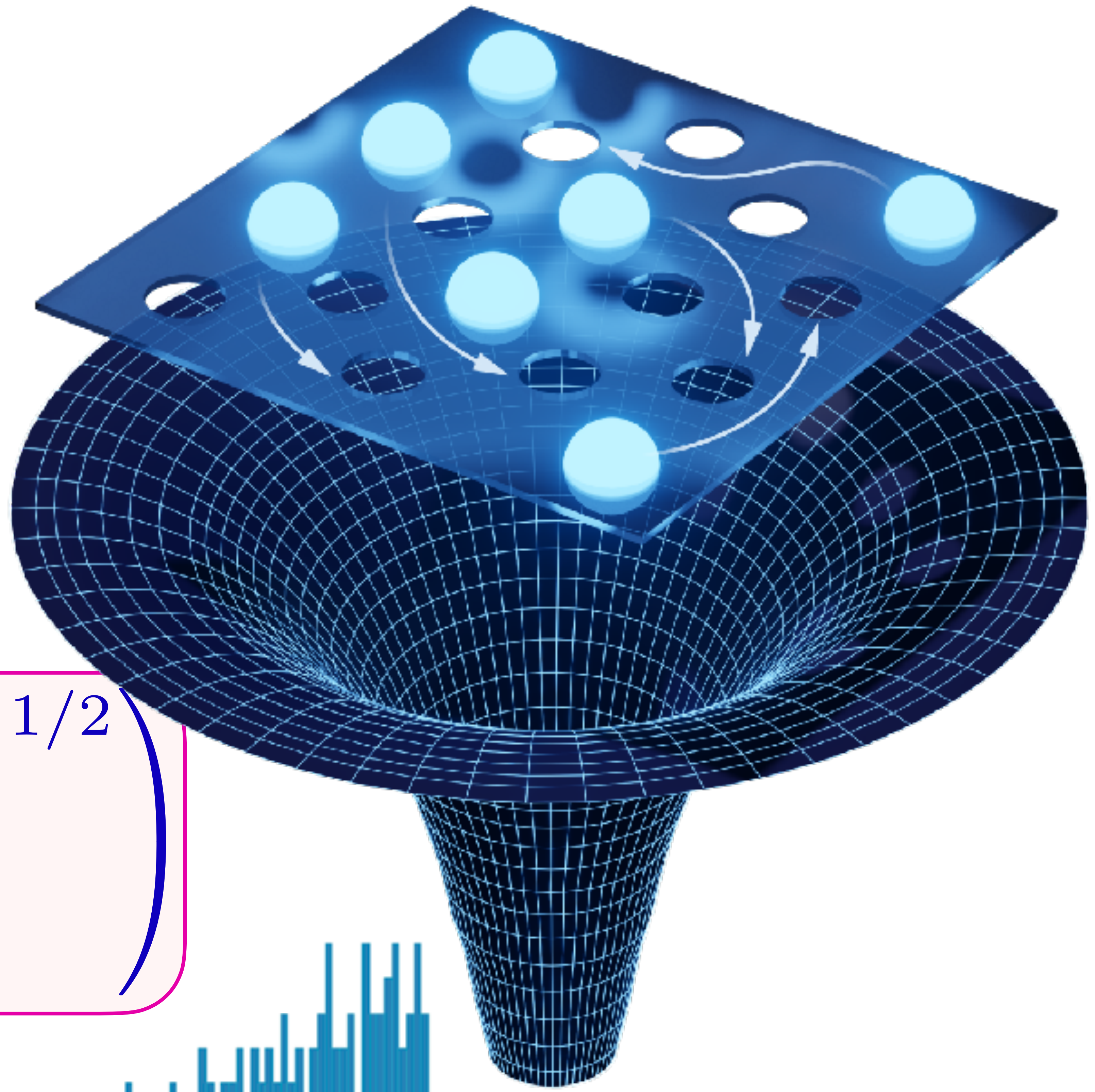

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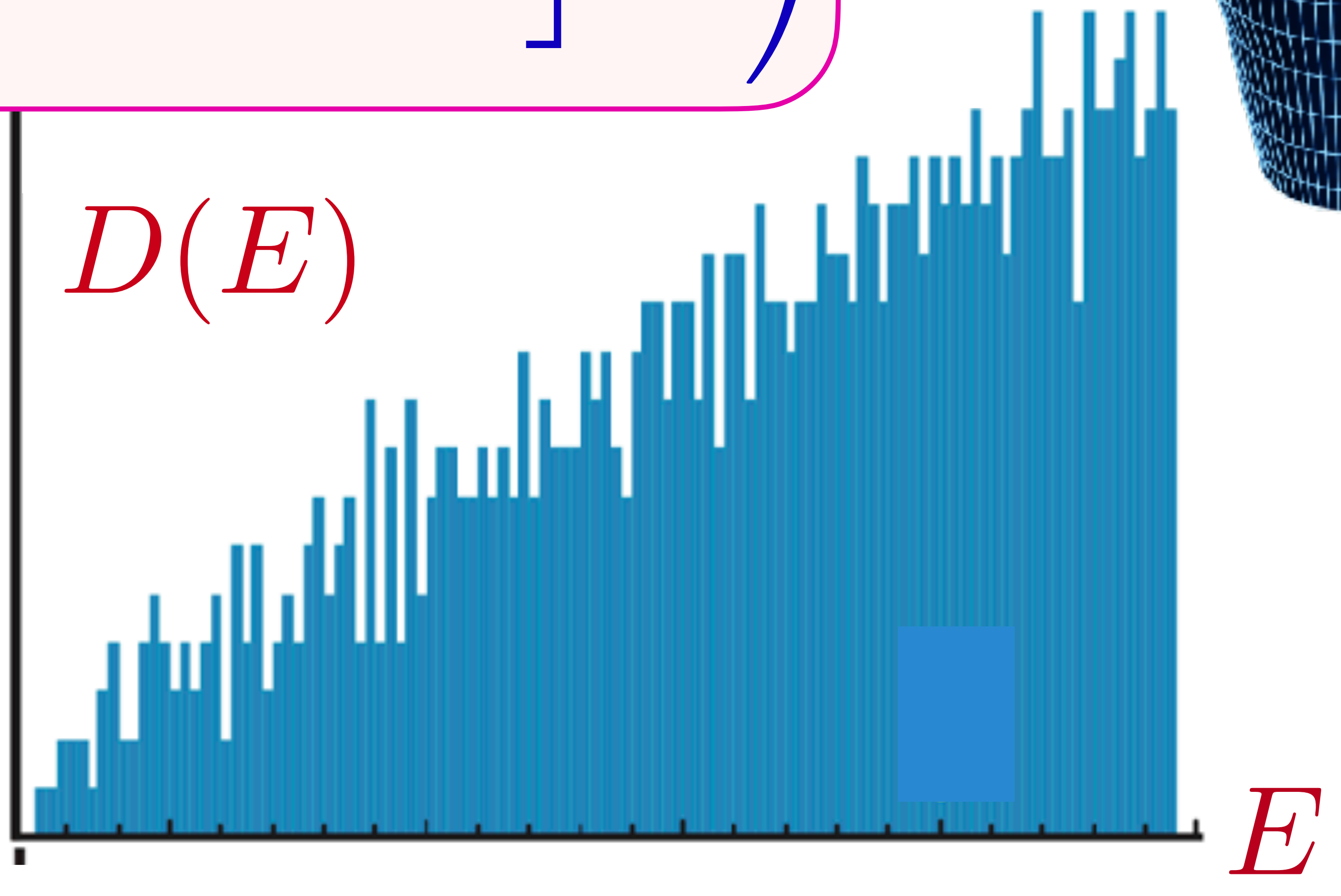
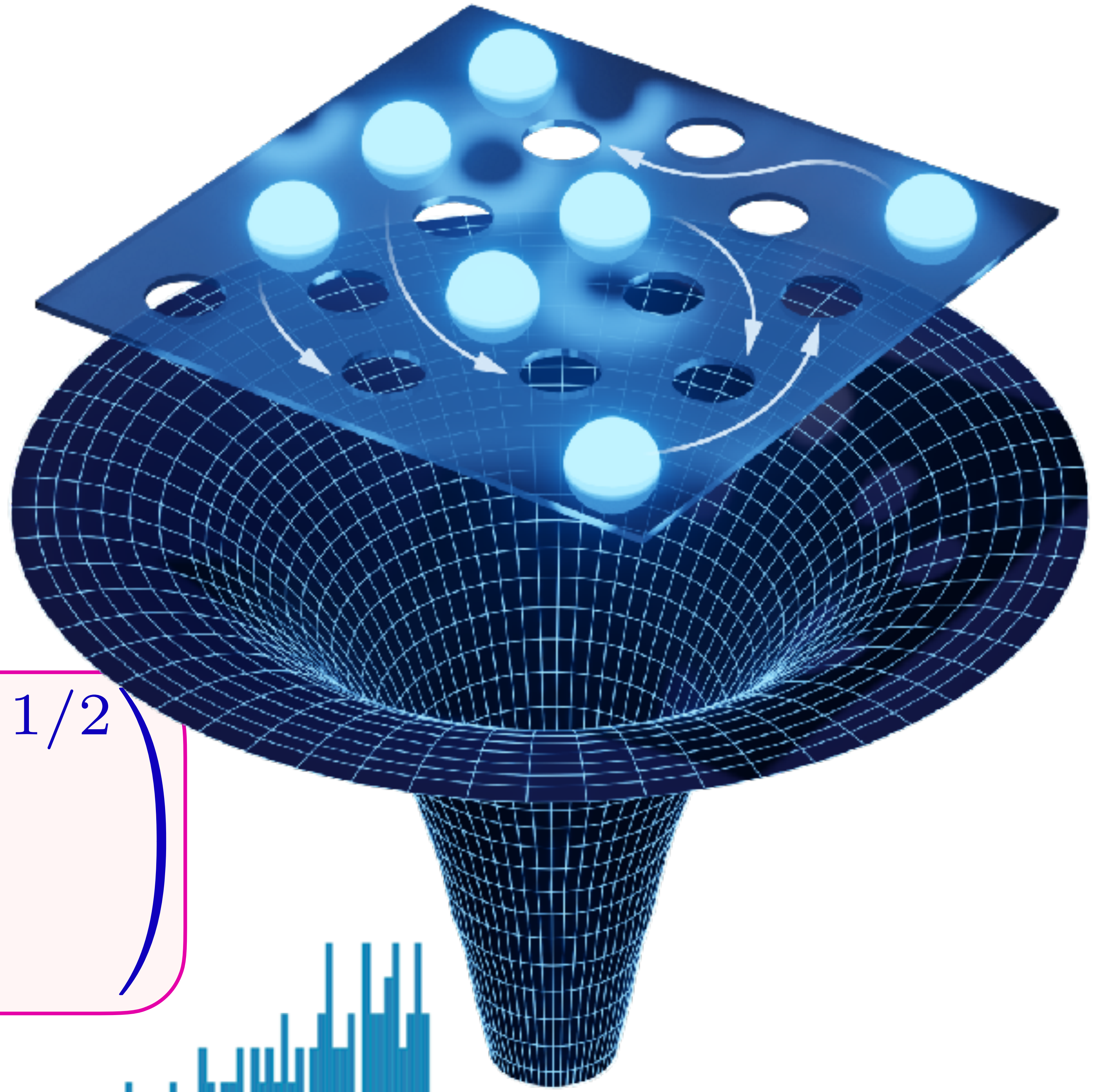
Developments from the SYK model



D(E) of charged black holes from the SYK model



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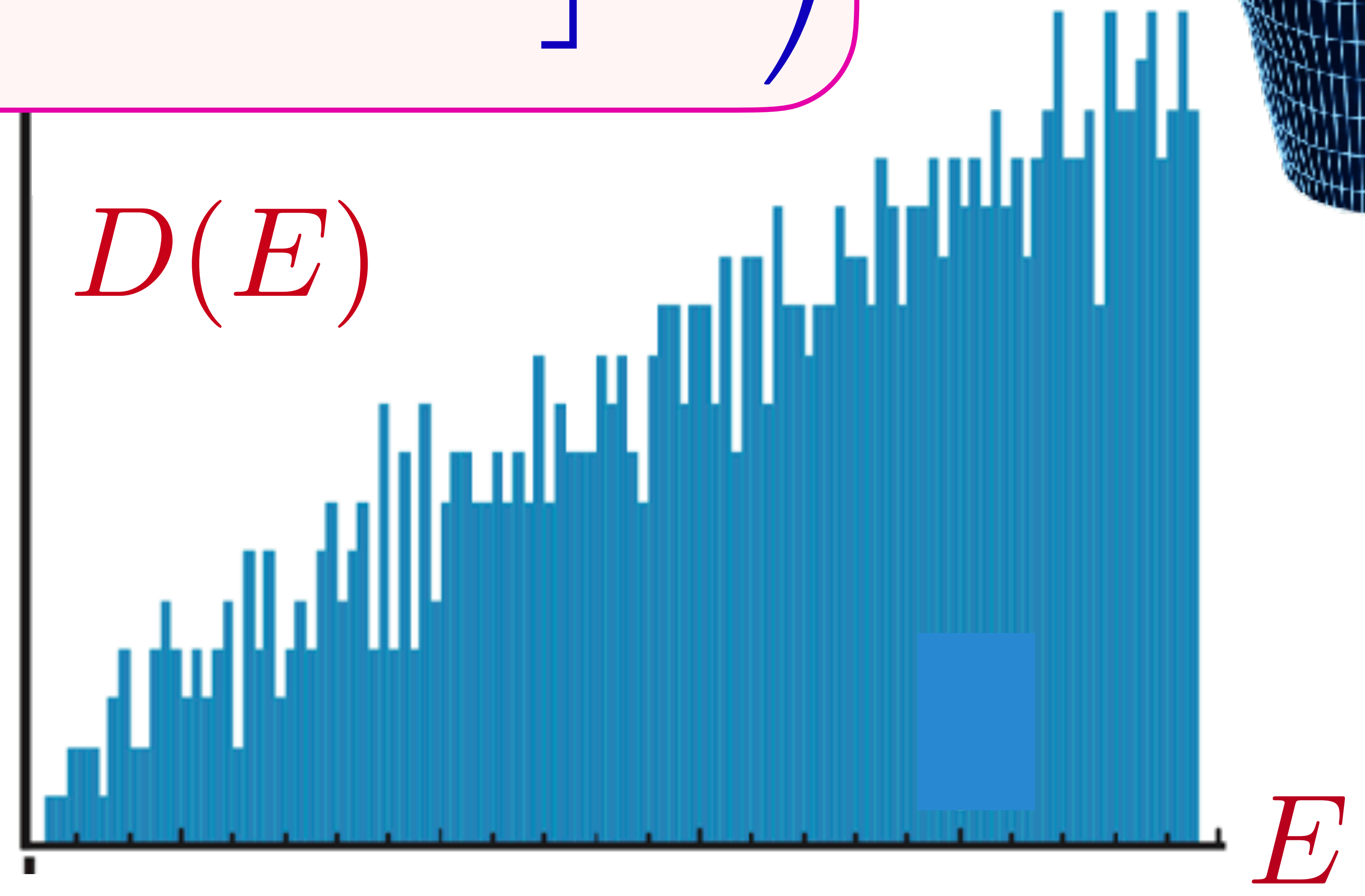
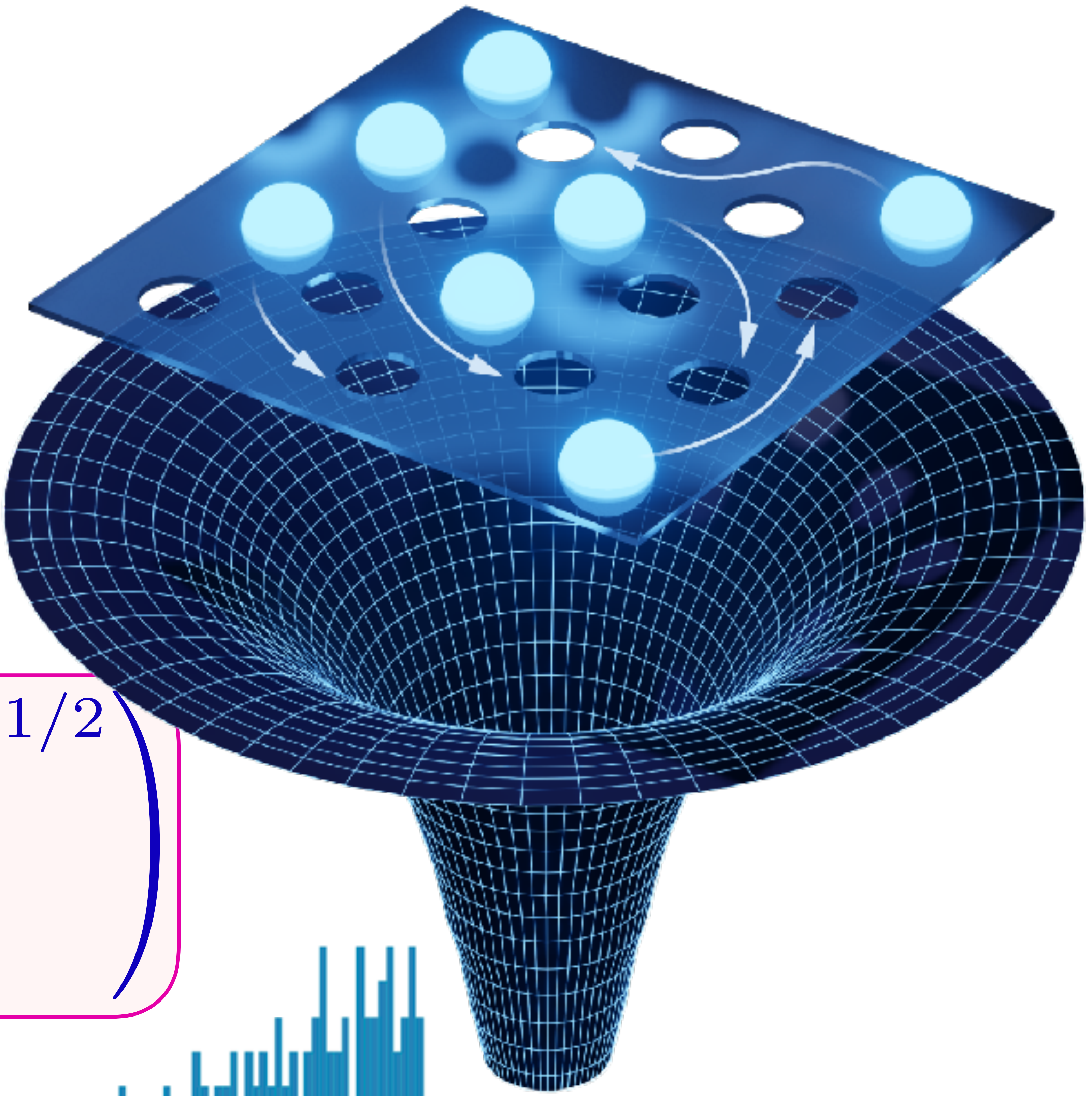
Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



D(E) of charged black holes from the SYK model



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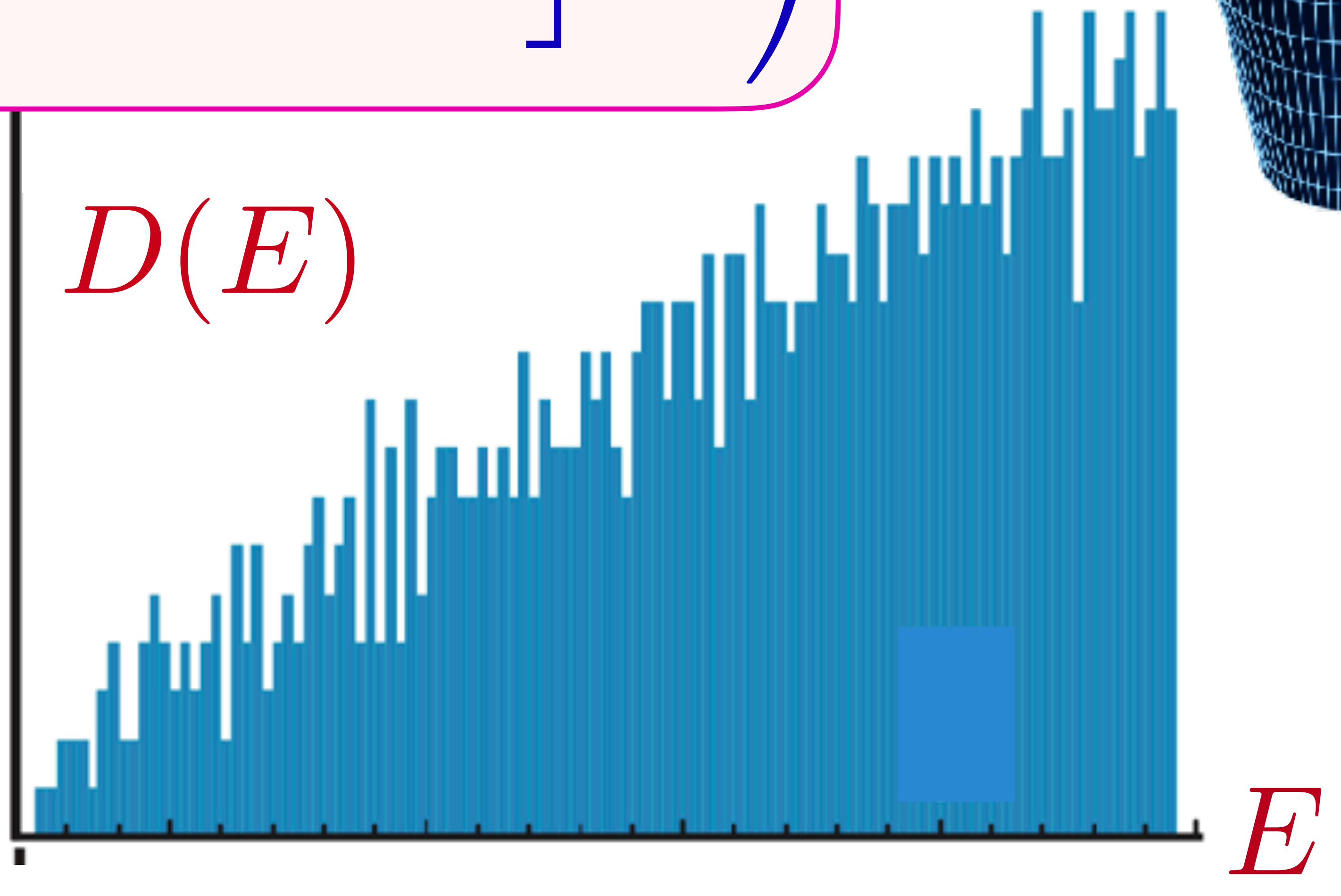
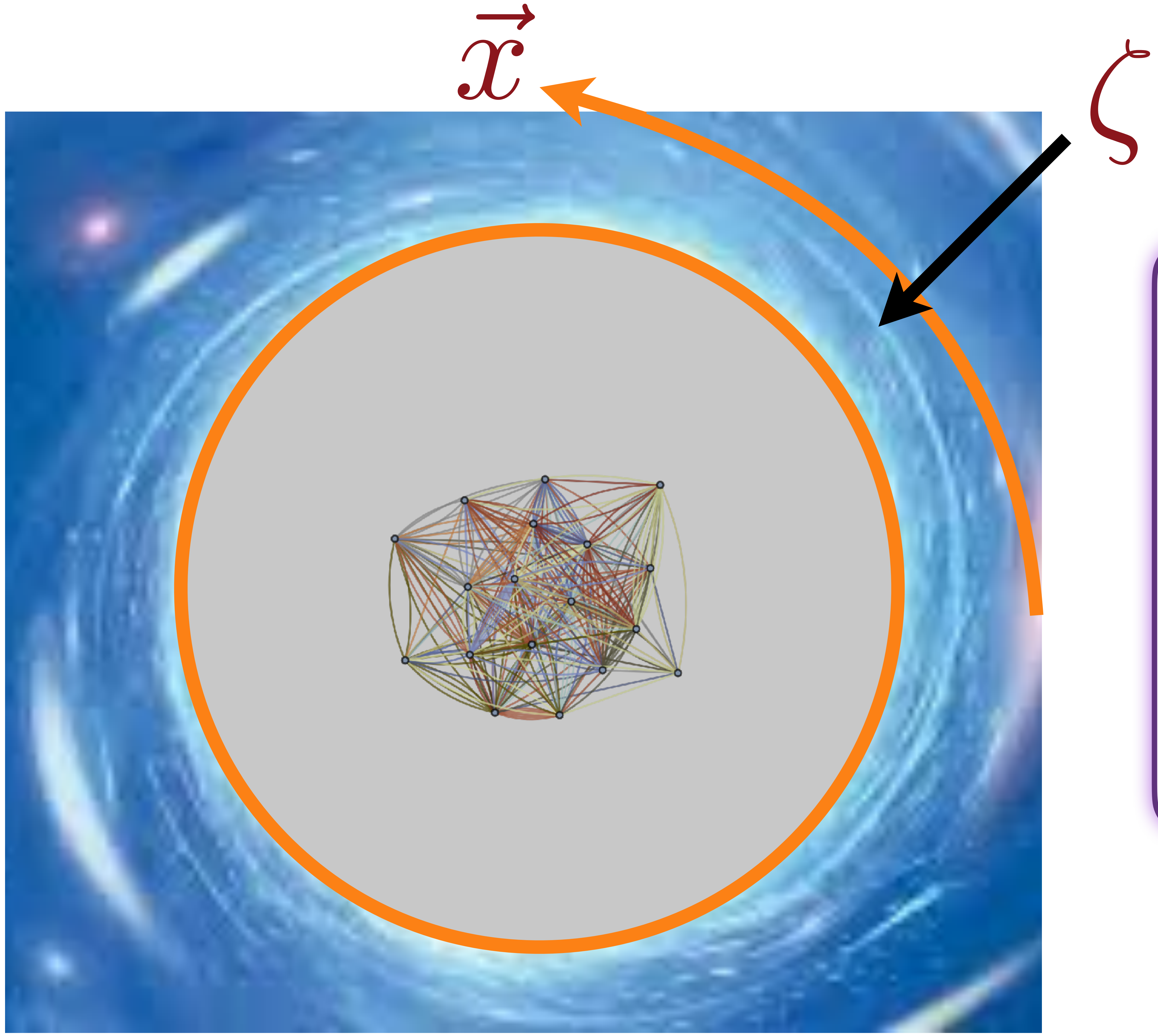
$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0{}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

For supersymmetric charged black holes or SYK models: $D(E) = e^S \delta(E)$
i.e. exponential ground state degeneracy.



Quantum simulation of charged black holes by the SYK model



The SYK model simulates the low energy properties of the interior of the black hole for an outside observer in ζ-τ co-ordinates.

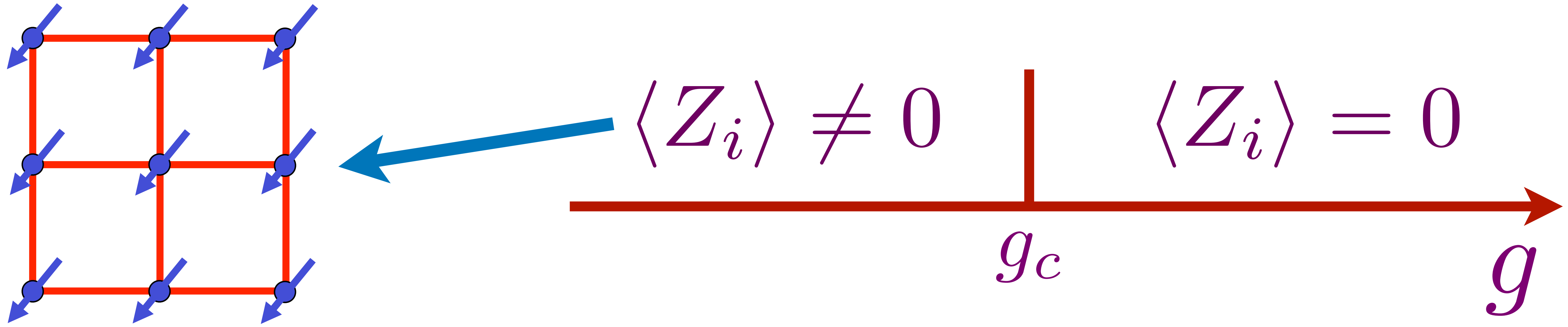
Planckian dynamics 2+1 dimensions

Quantum phase transitions of qubits

Quantum Ising model

$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - g \sum_i X_i$$

Quantum phase transition from a
“ferromagnet” with $\langle Z_i \rangle \neq 0$ for $g < g_c$
to a “paramagnet with $\langle Z_i \rangle = 0$ for $g > g_c$



Related models describe numerous recent quantum simulators:
Ebadi *et al.*, Rydberg atoms in optical tweezers, Science (2022)
King *et al.*, Superconducting Qubits, D-Wave Systems, Nature (2023)
Maciejewski *et al.*, Superconducting Qubits, Rigetti (2023)

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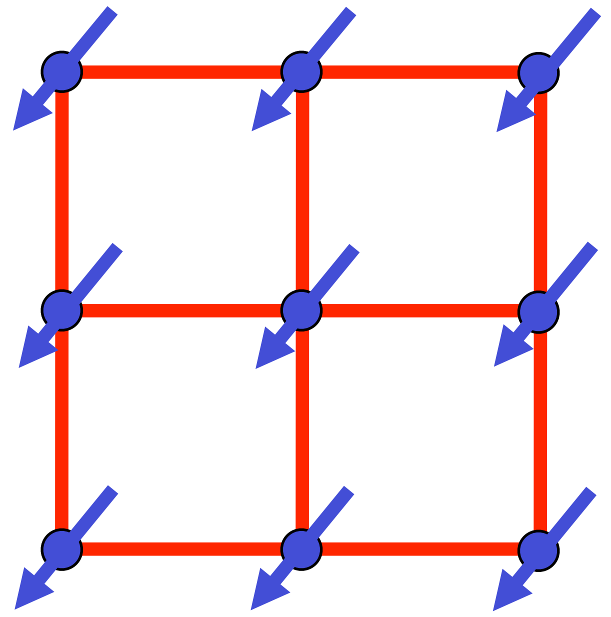
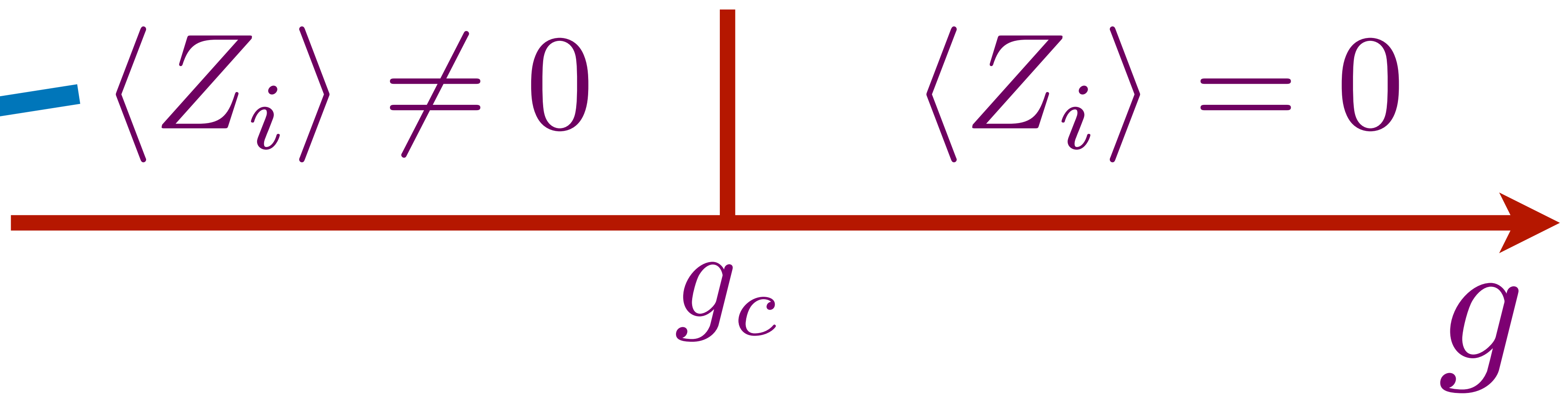
Wilson-Fisher
conformal field theory

$$\mathcal{L} = K_\tau \left[\partial_\tau \phi\right]^2 + K \left[\nabla_{\boldsymbol{r}} \phi\right]^2$$
$$+(g - g_c)[\phi]^2 + u\,[\phi]^4$$

with $\phi \sim Z$





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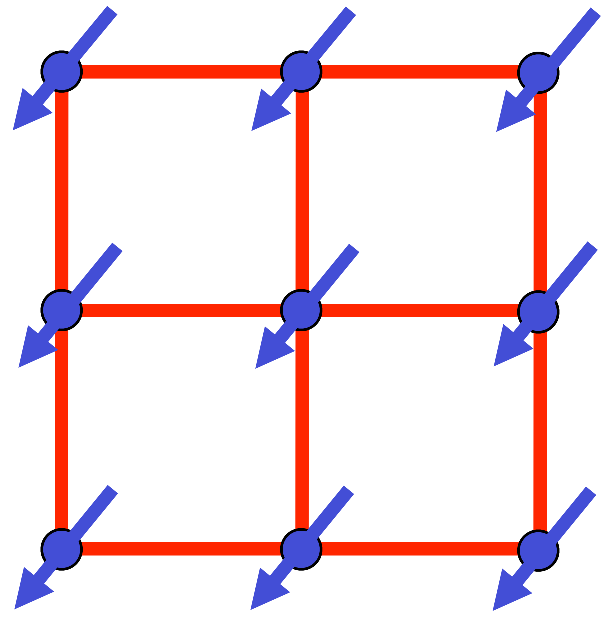

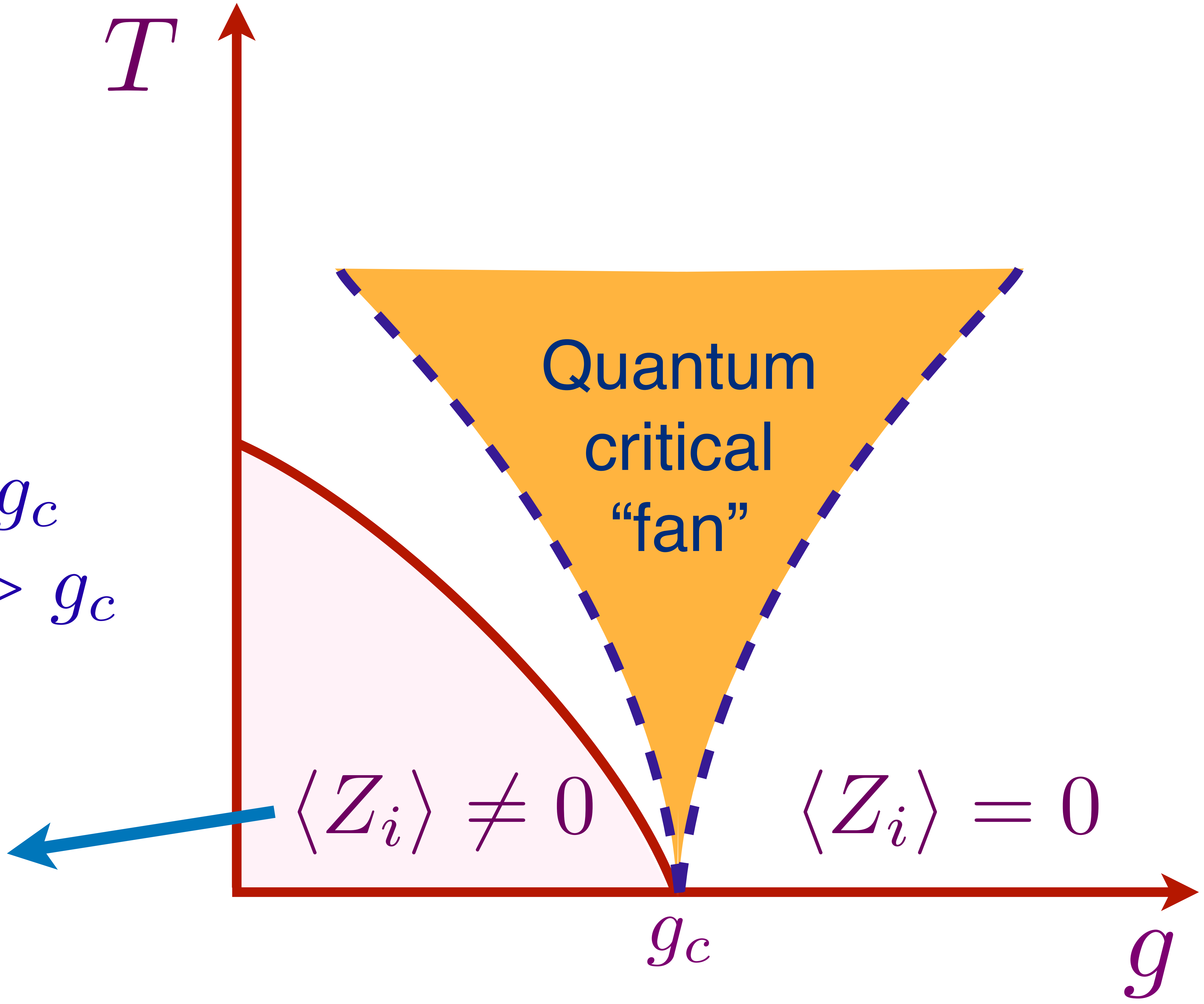

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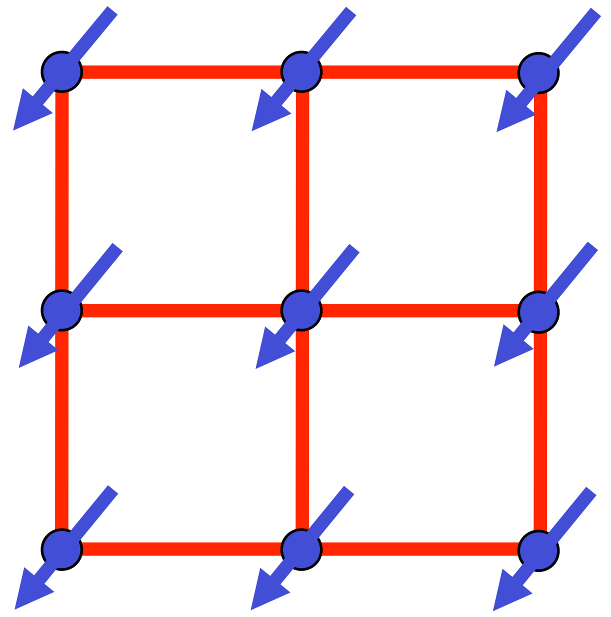

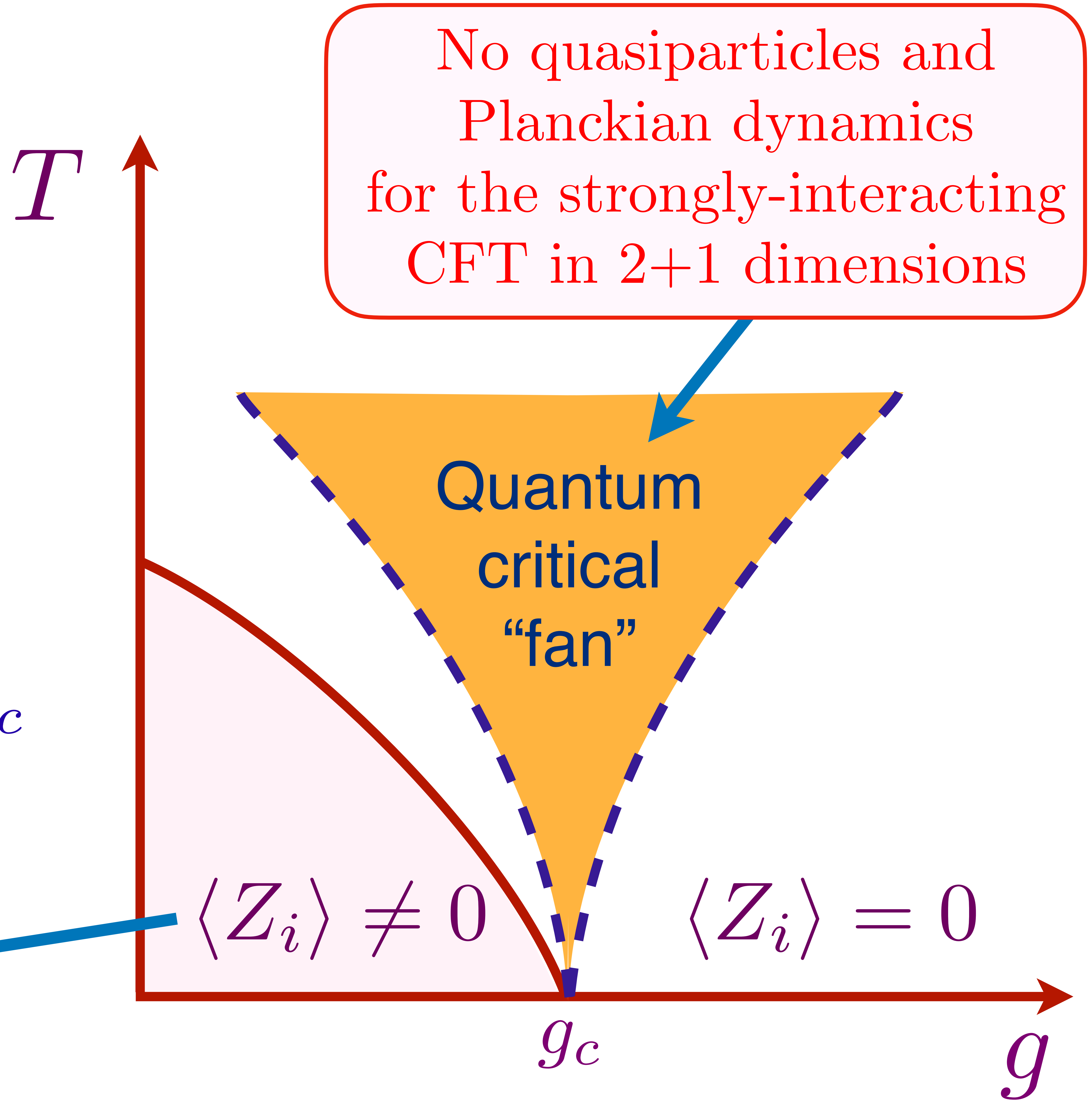

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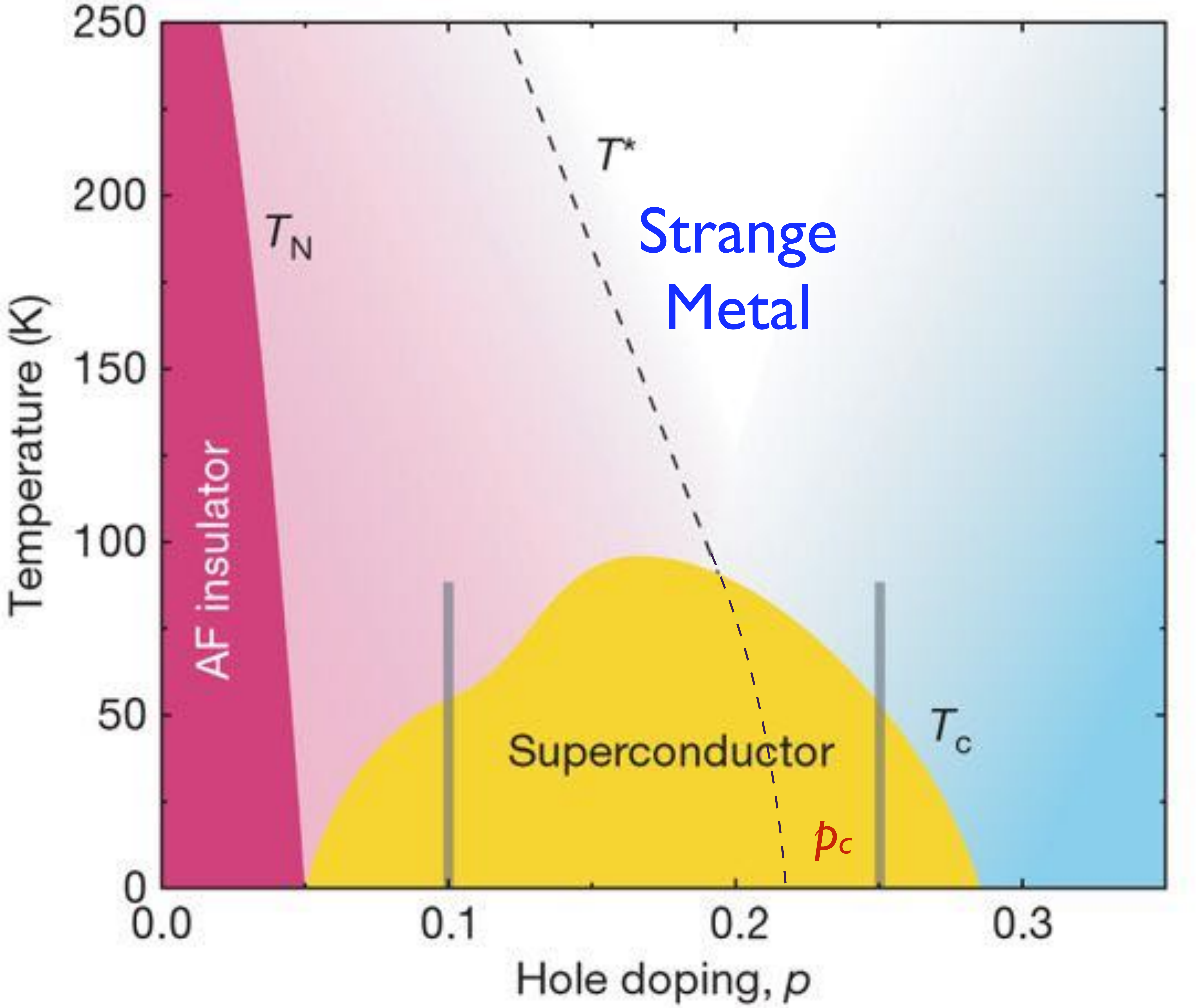
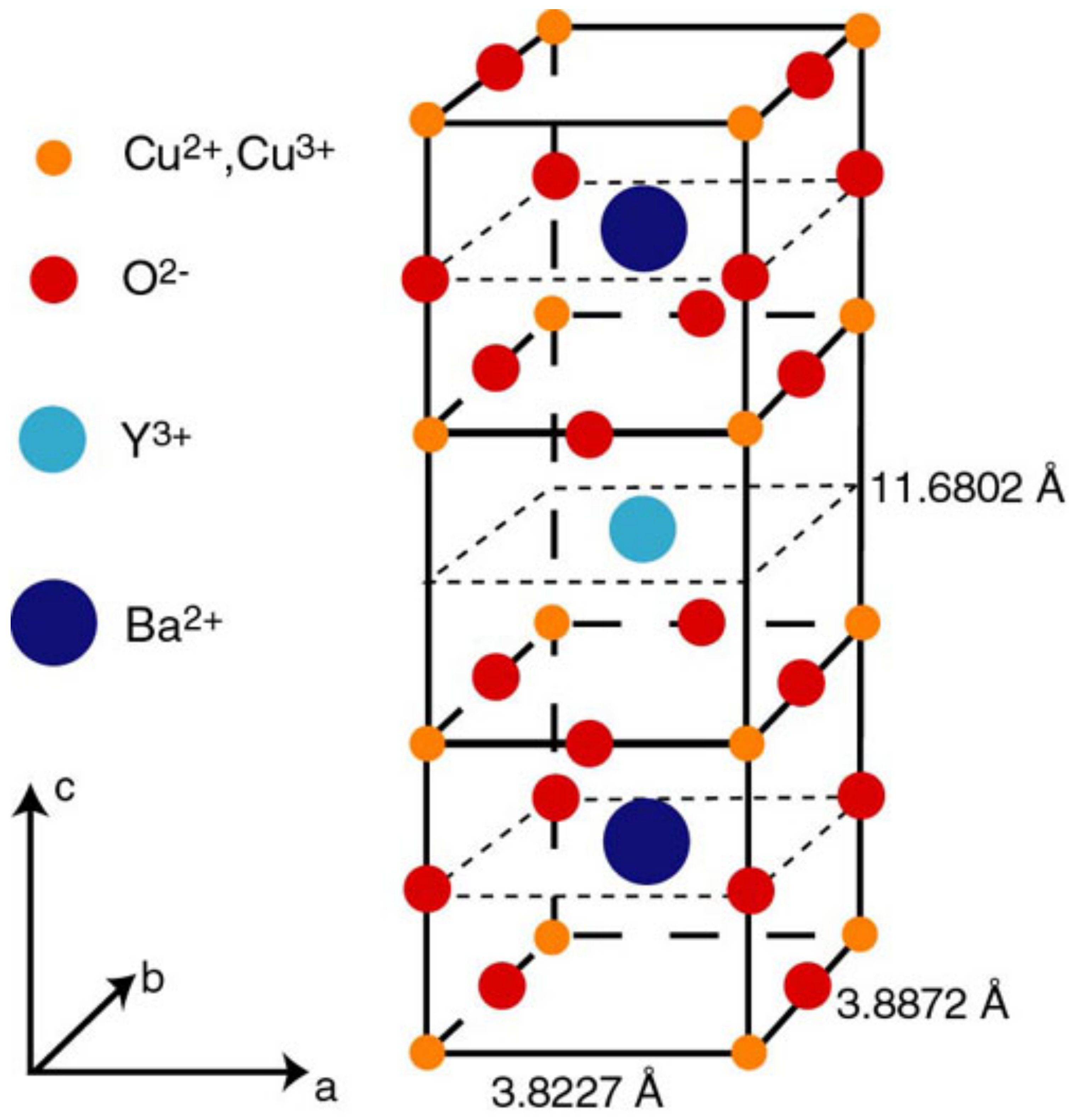
Quantum phase transitions of metals

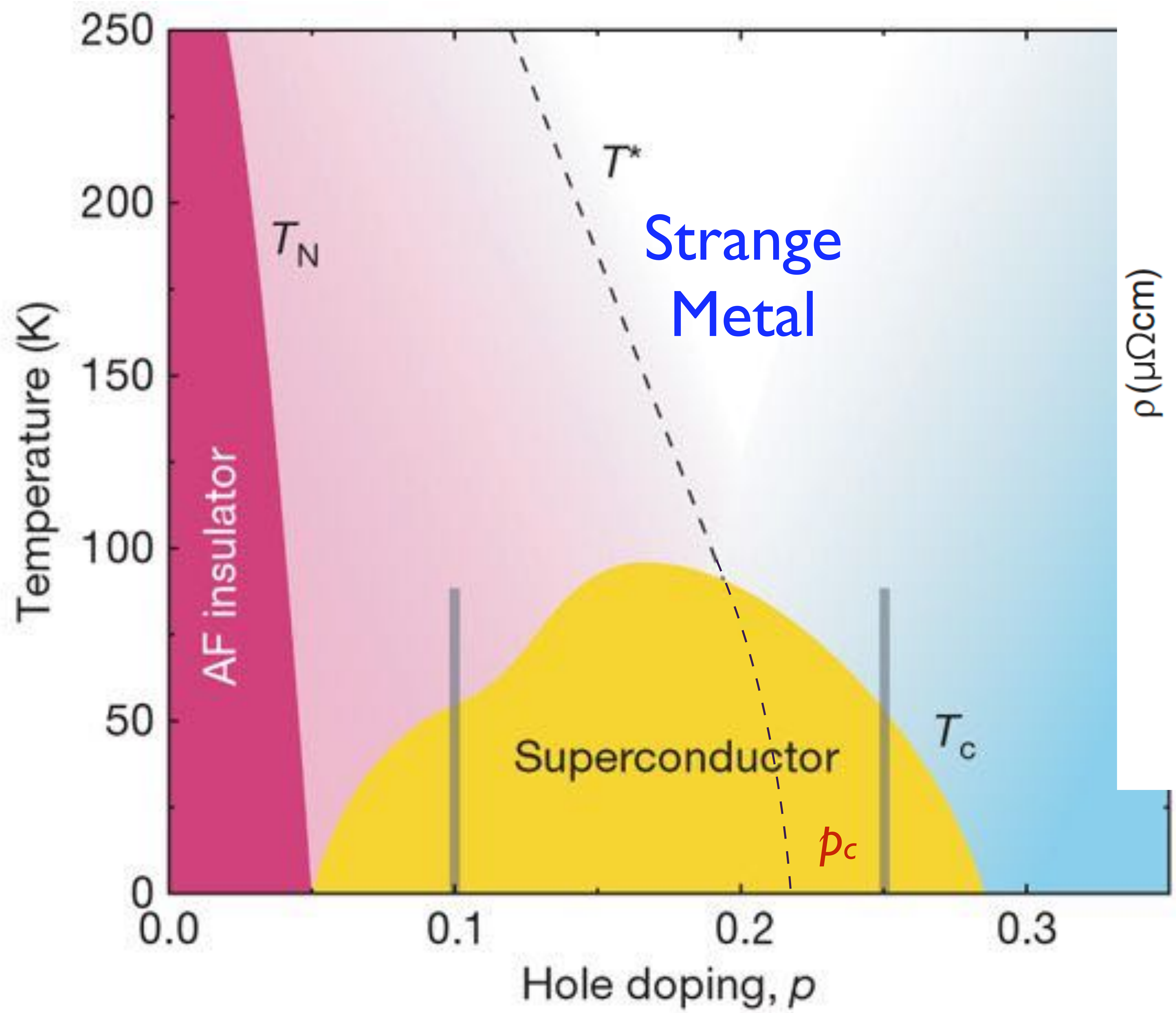


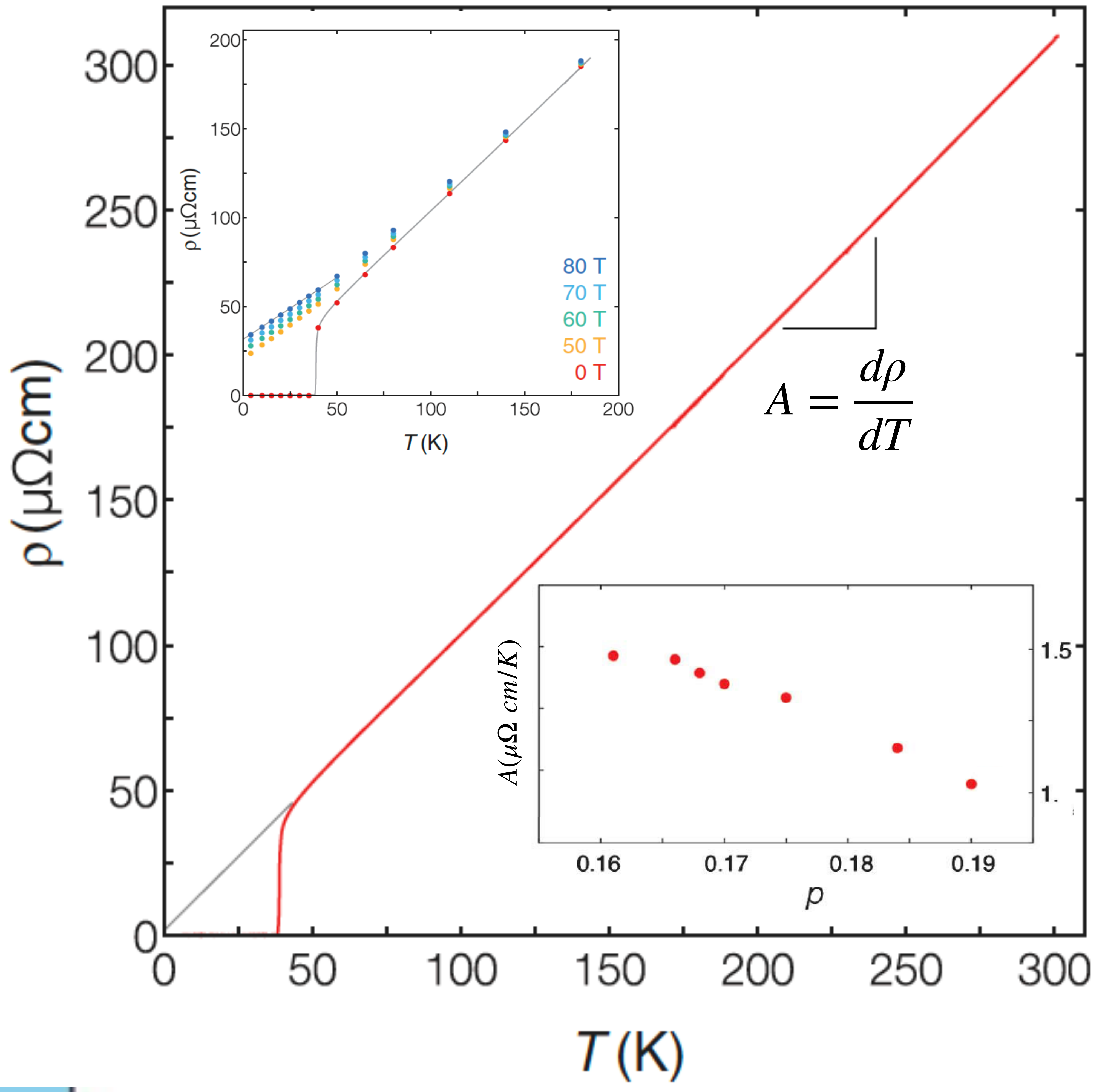
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
T*
T_N
Strange
Metal
AF insulator
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$





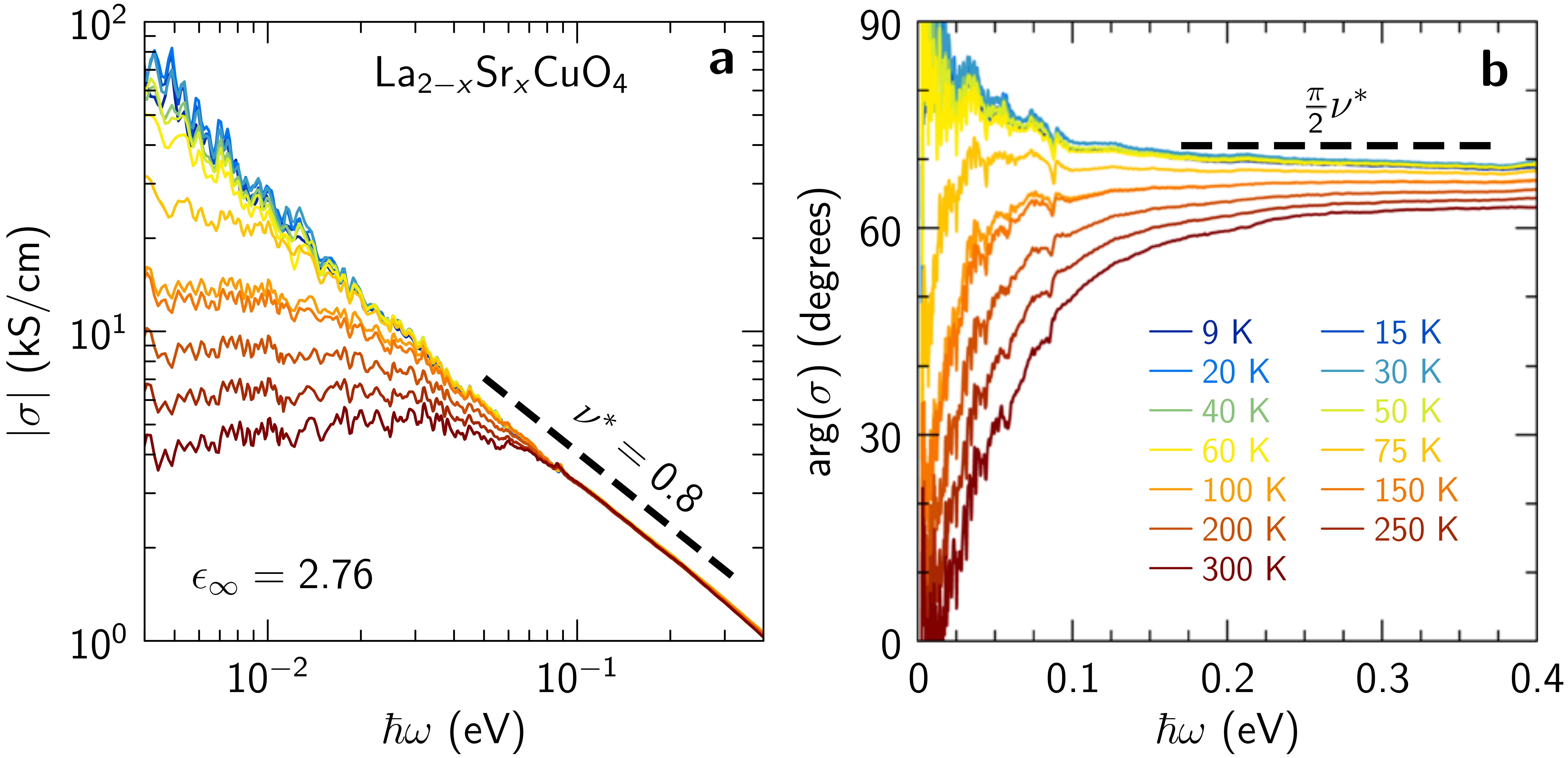
LSCO: Giraldo-Gallo et al. 2018

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$



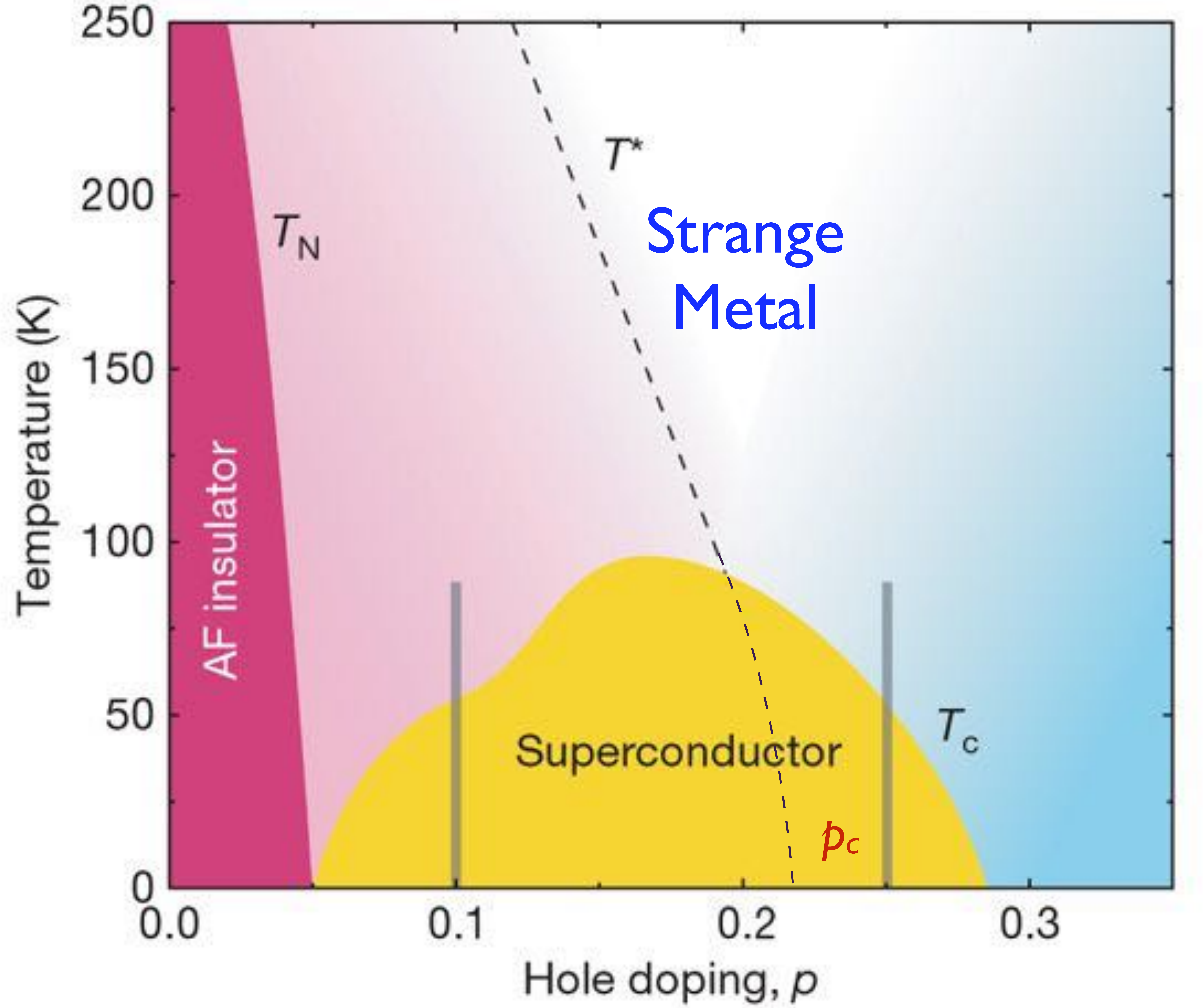
Planckian dynamics!

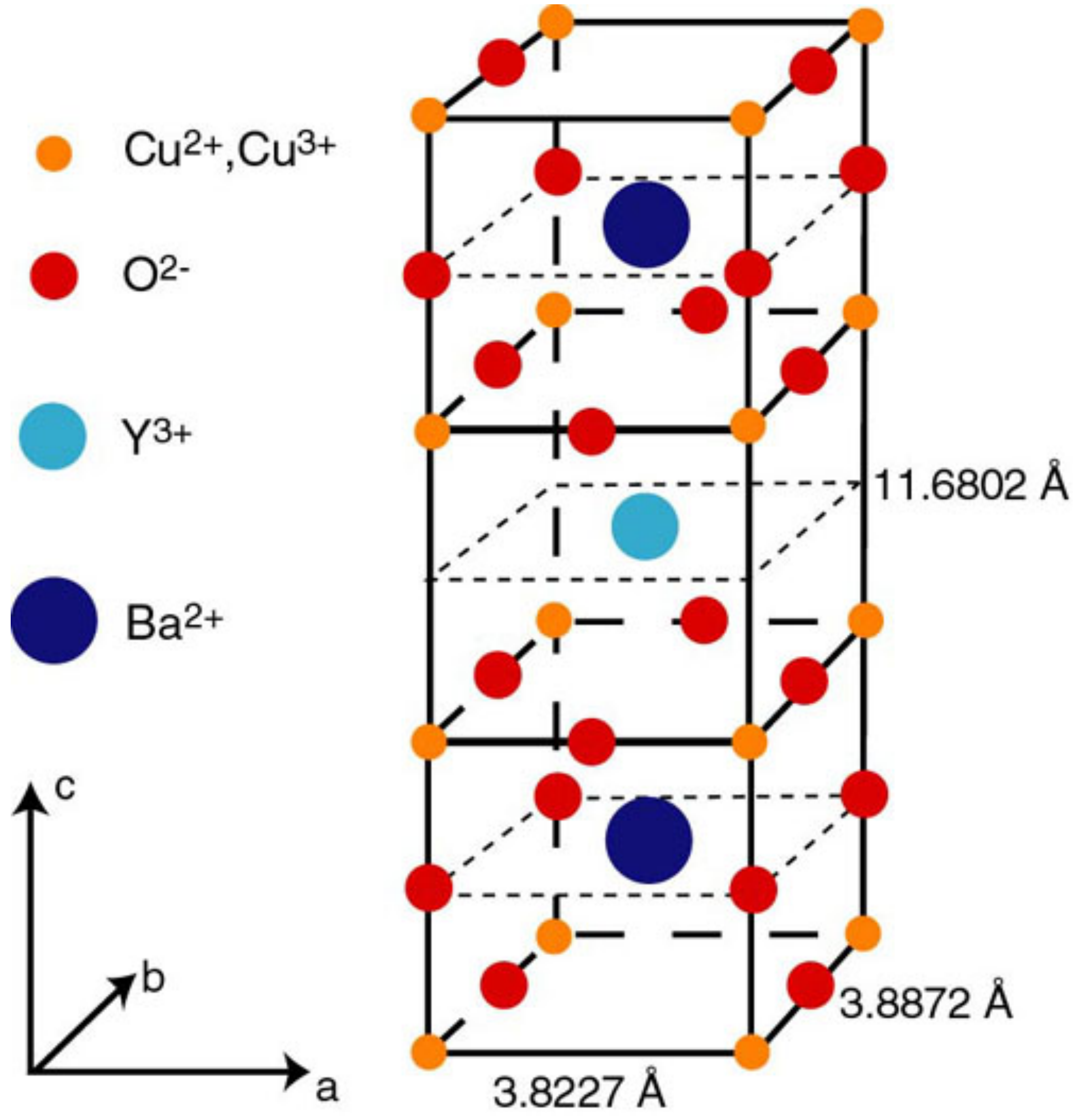
$$\tau(\omega) = \frac{\hbar}{k_BT}F\left(\frac{\hbar\omega}{k_BT}\right)$$

and entropy

$$S(T\to 0) \sim T\ln(1/T).$$

$La_{2-x}Sr_xCuO_4$
$p = 0.24$
$T_c = 19$ K


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
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c
b
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11.6802 Å
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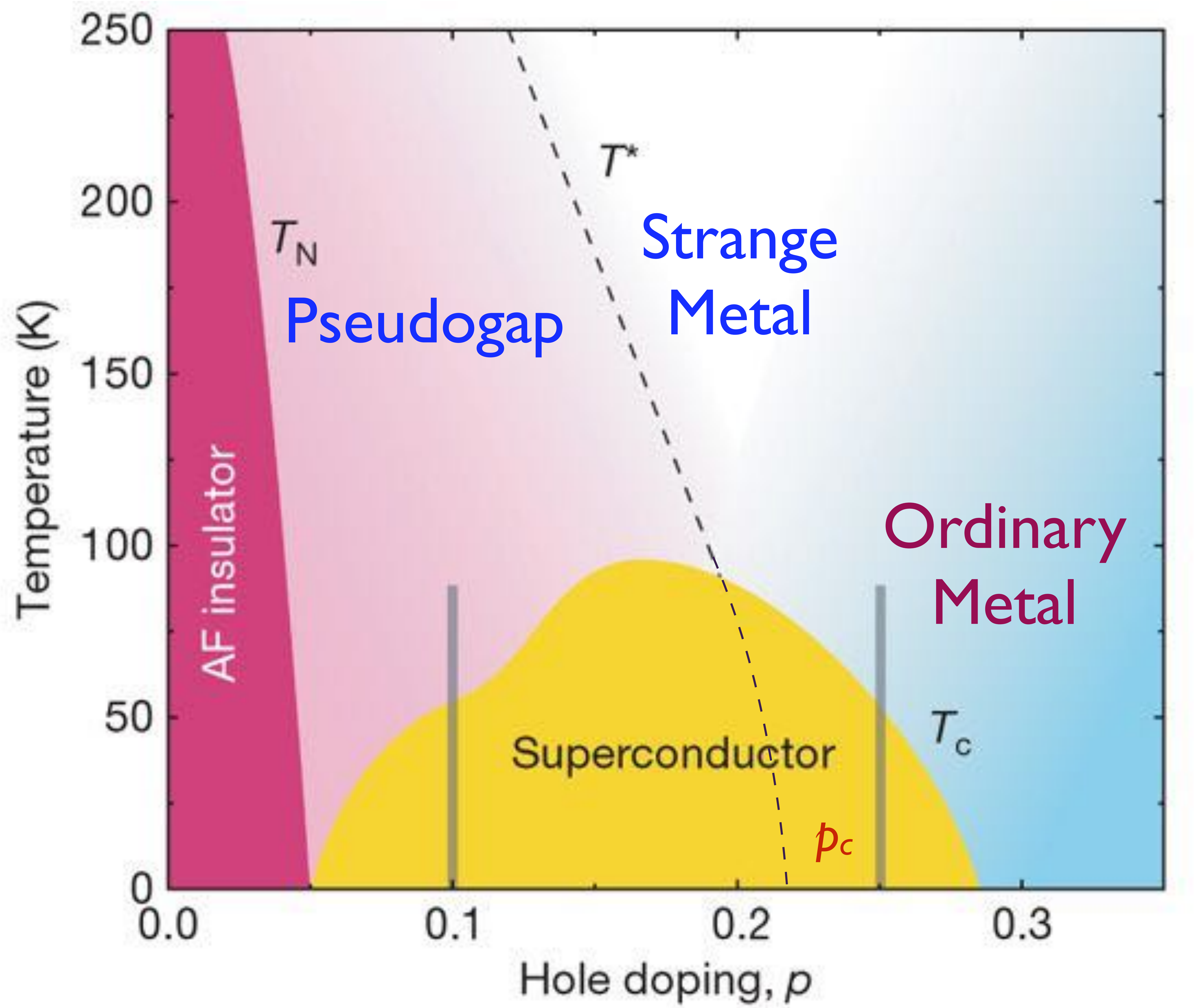
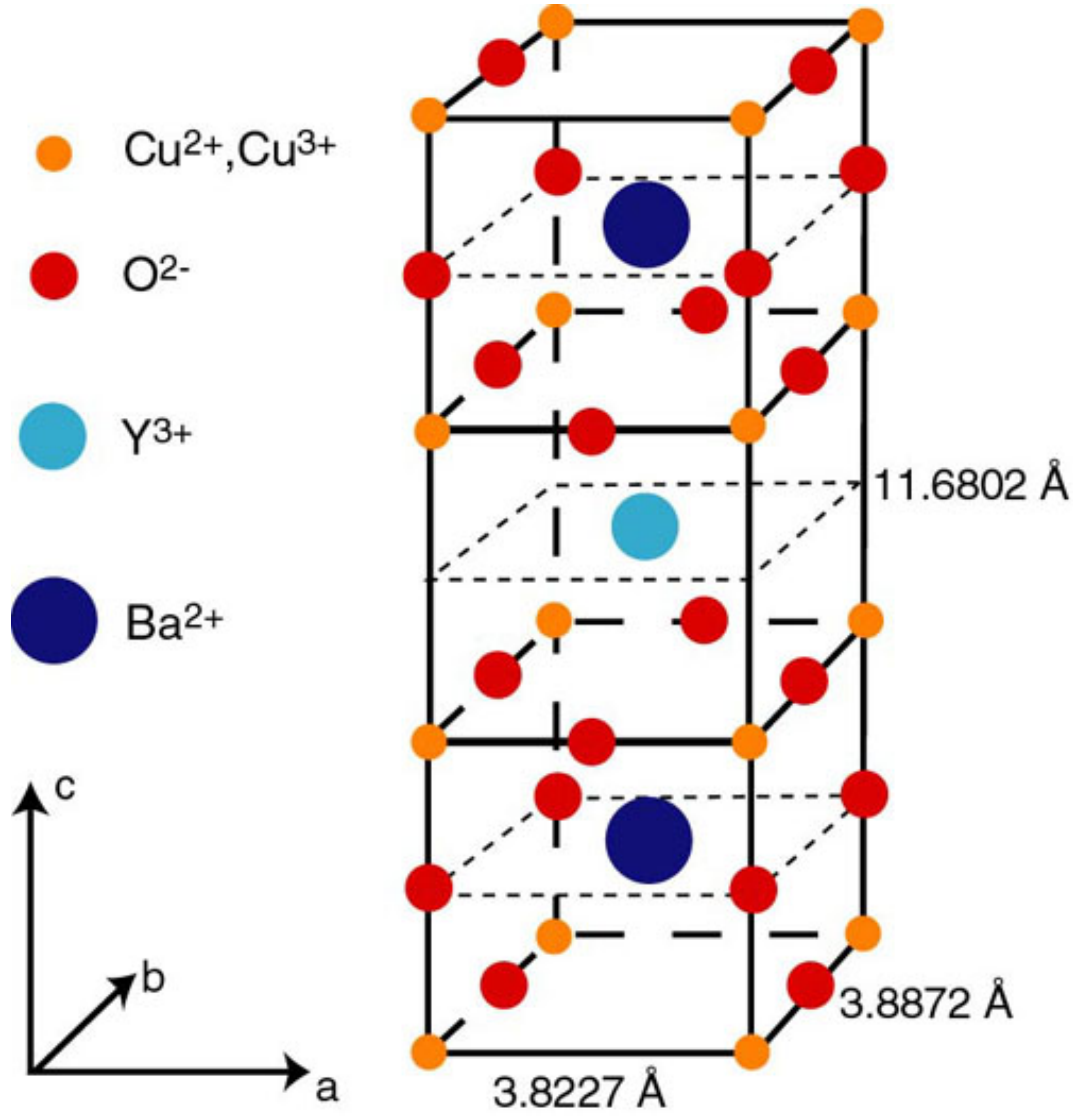
$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
Pseudogap
T*
Strange
Metal
Ordinary
Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
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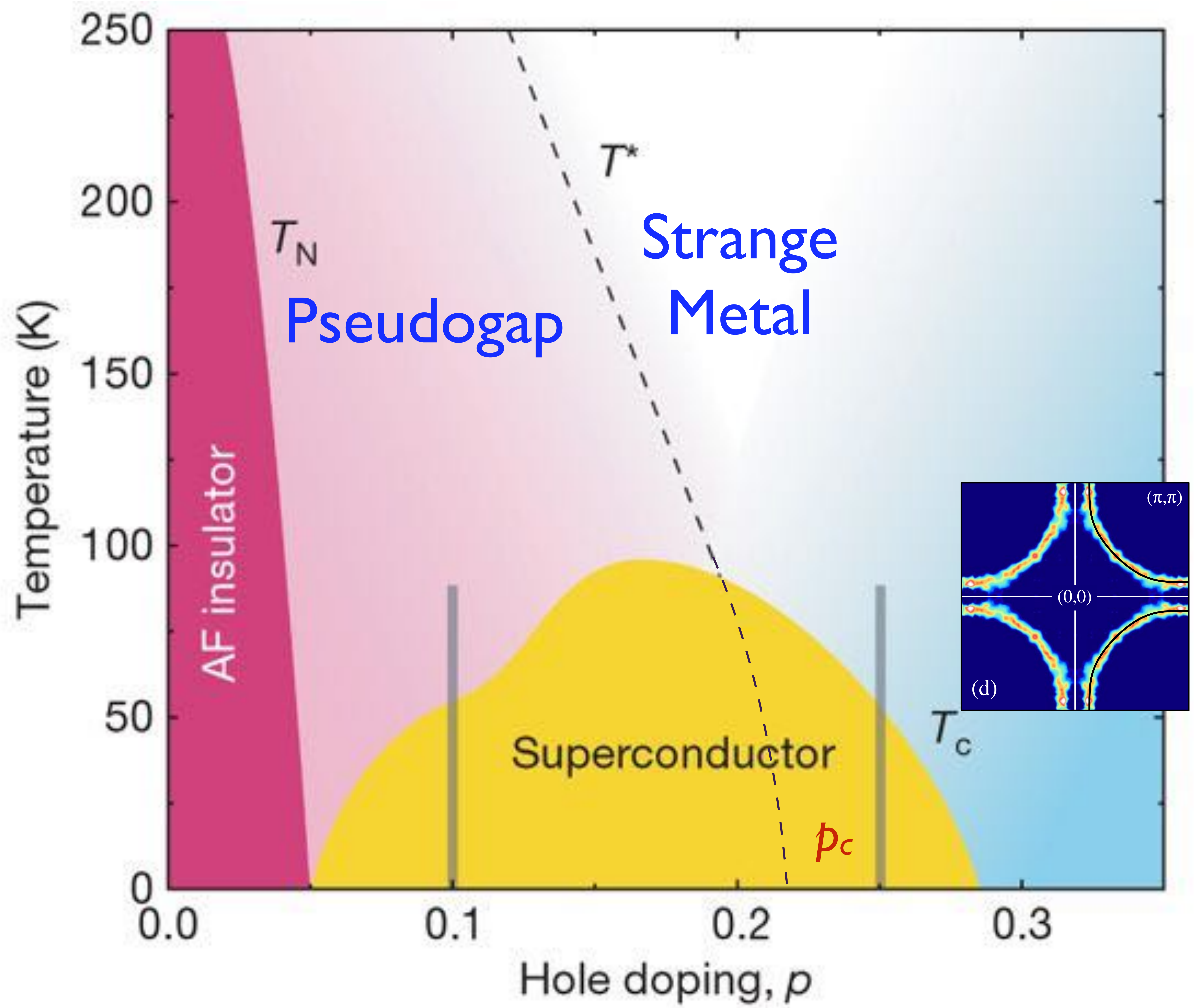
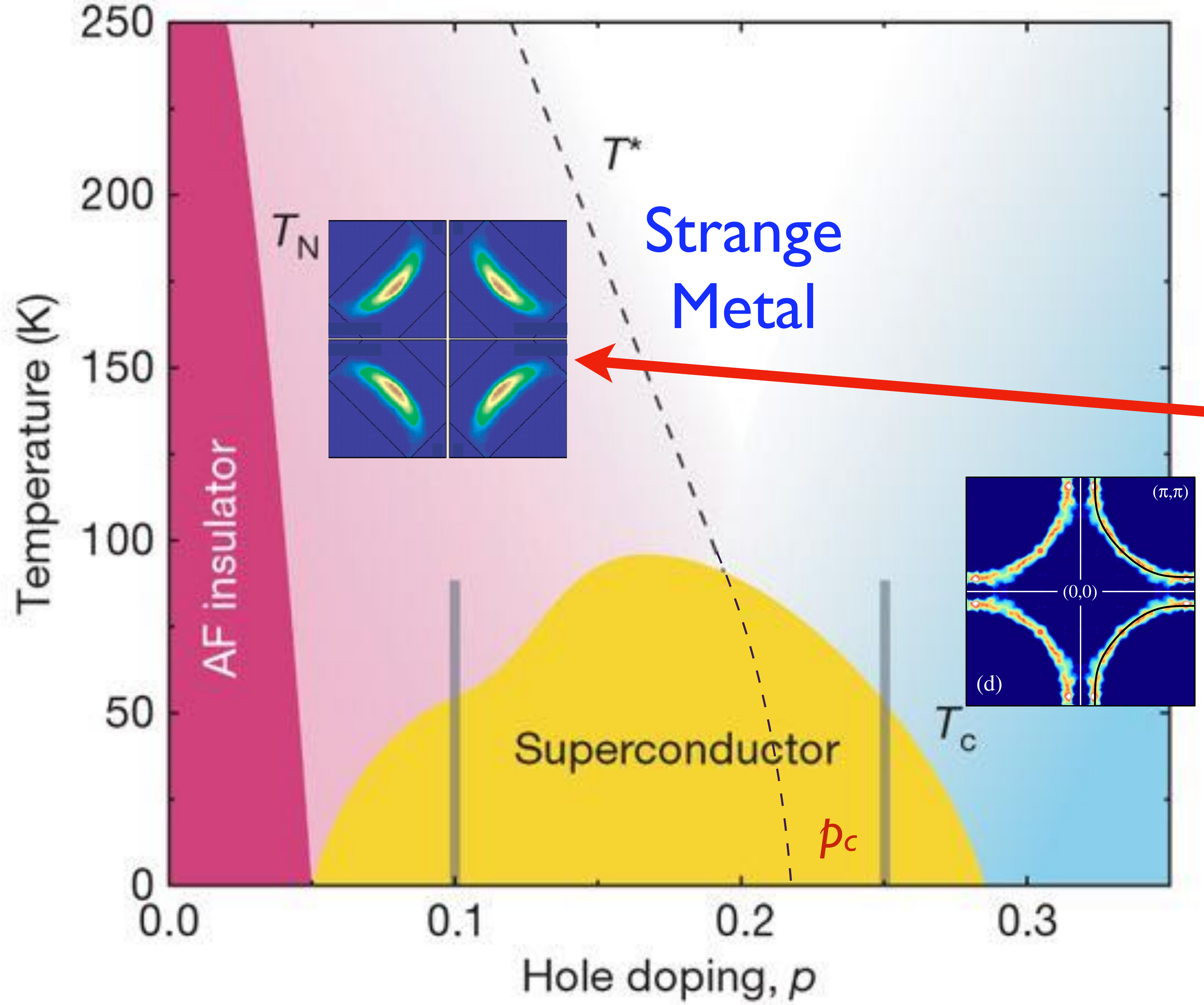
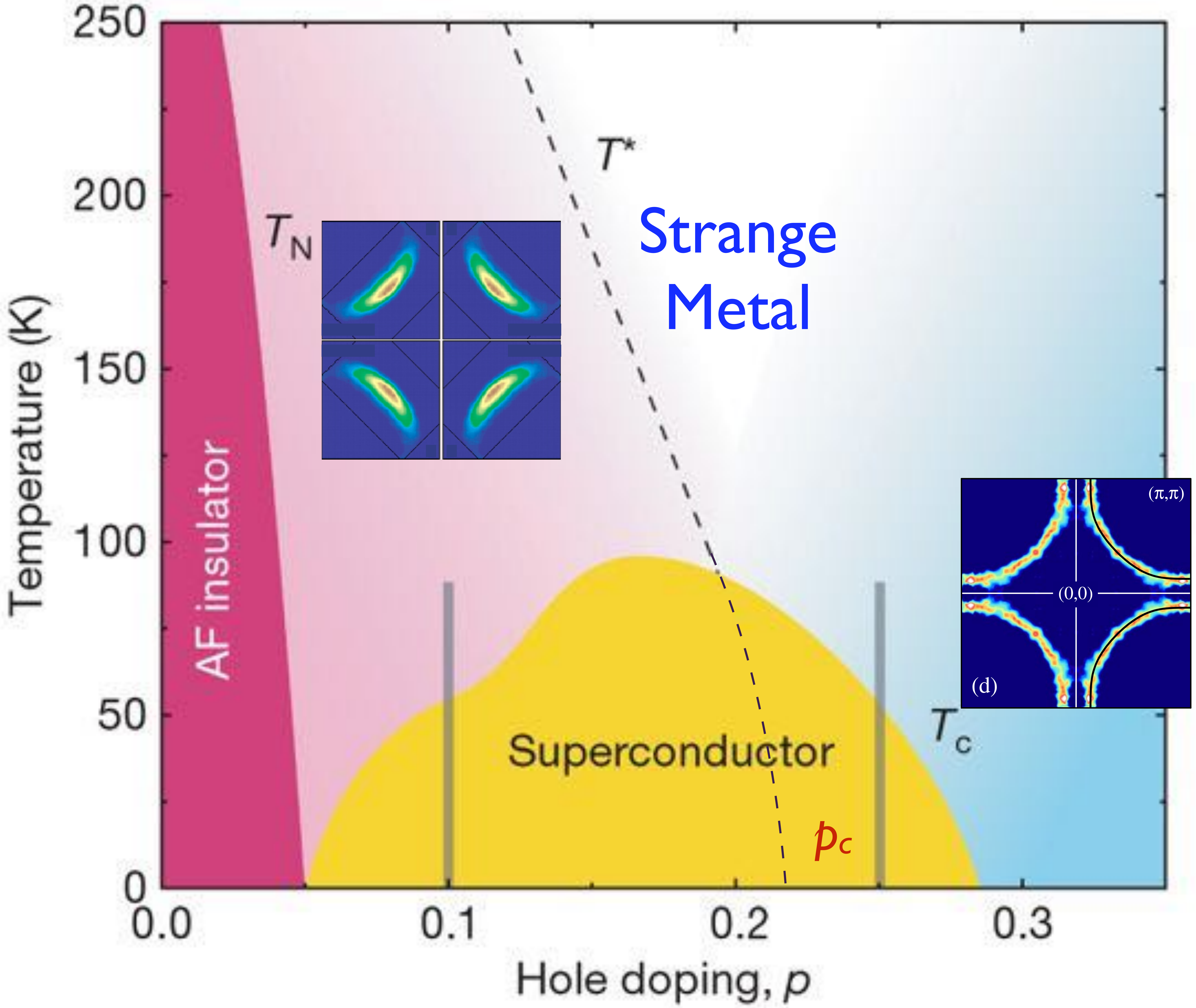


Fermi surface
as expected
in a model
of free electrons



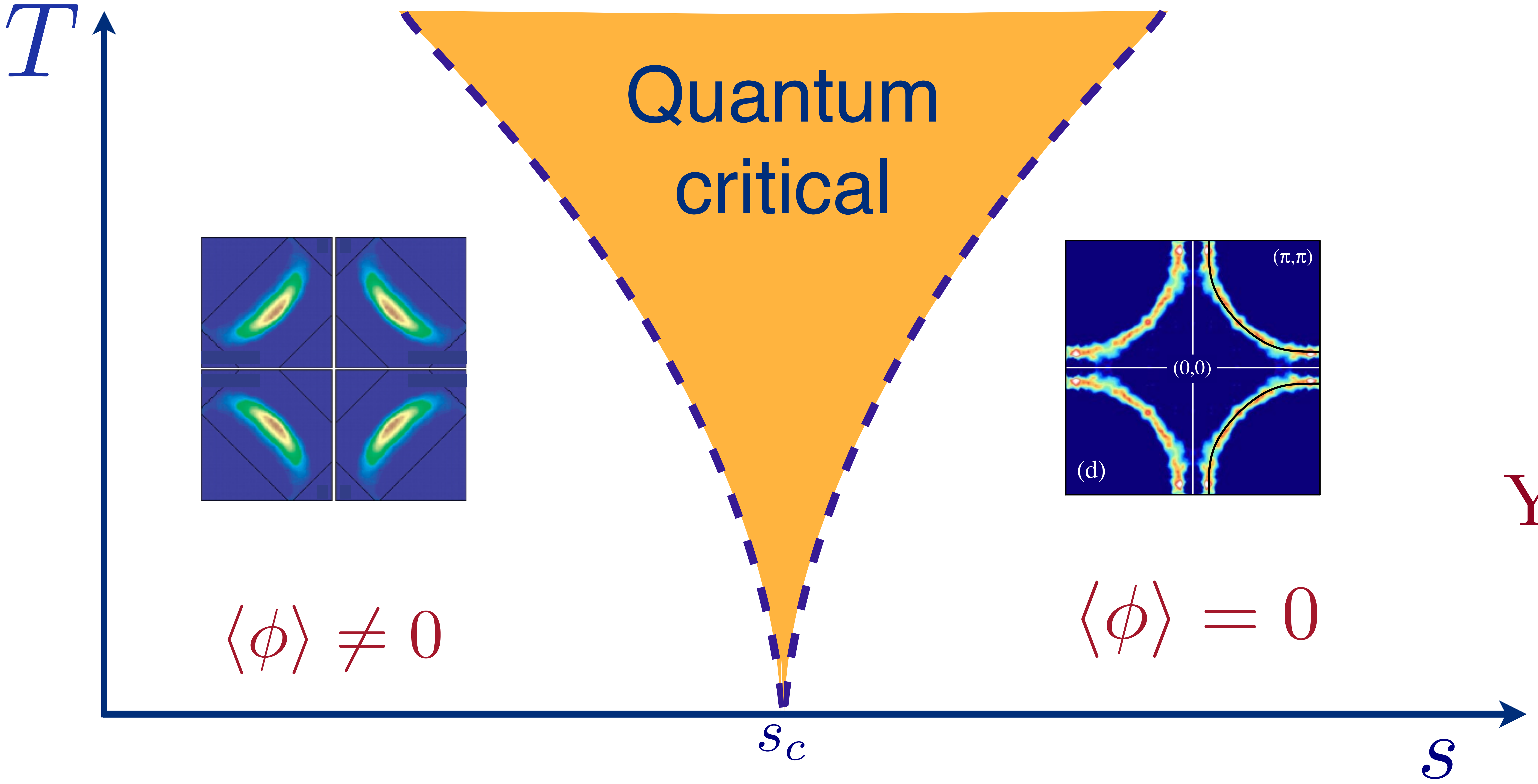
"Pseudogap metal"
Fermi surface
modified by
electron-electron
interactions



View the strange metal as a property of a $T = 0$ quantum phase transition involving change in the Fermi surface.

The onset of superconductivity may “hide” this quantum transition.

Quantum phase transitions of Fermi surface change



Fermi surface

$+$

scalar field ϕ

with a 'mass' s

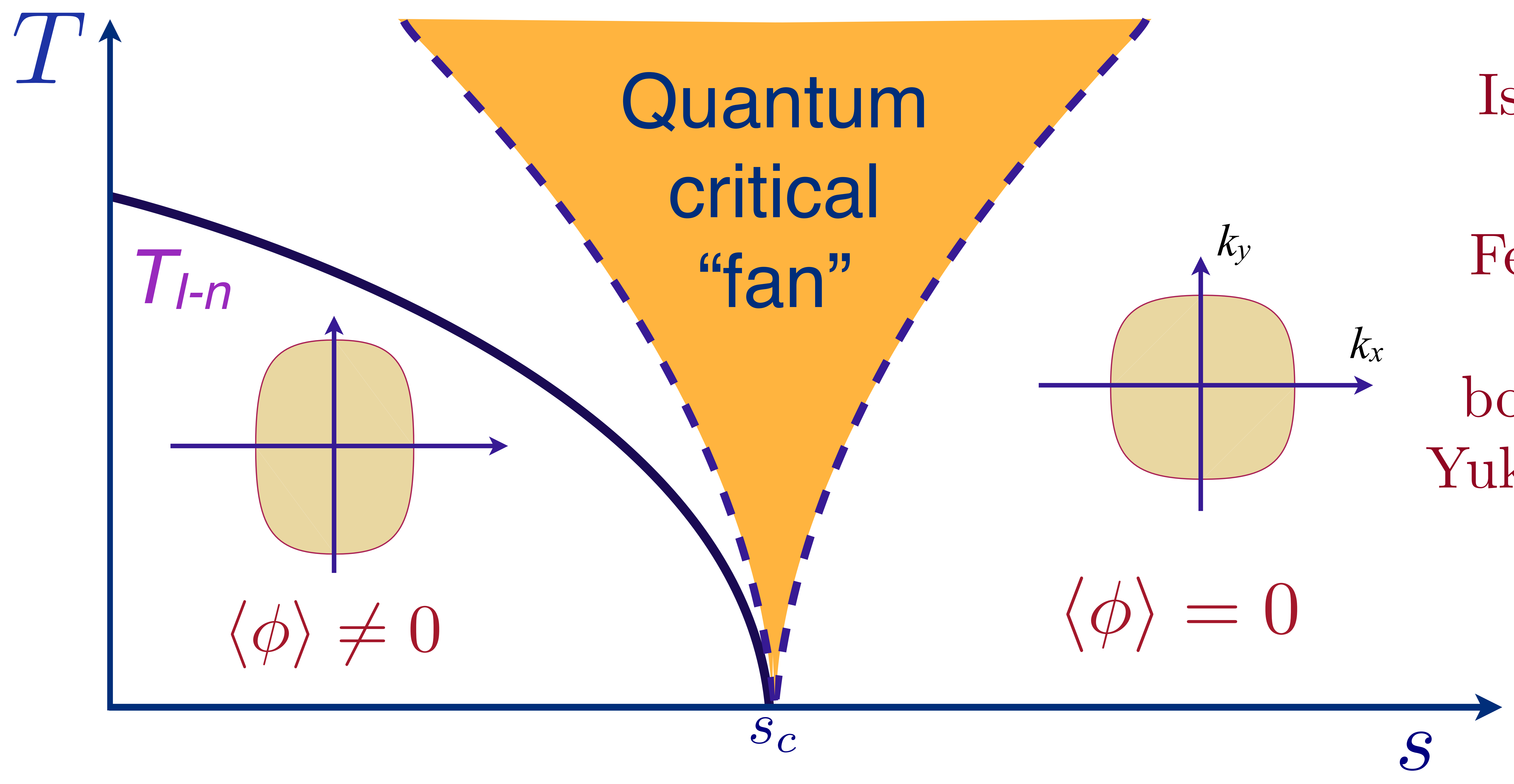
and

a boson-fermion

Yukawa coupling g.

Quantum phase transitions in two-dimensional metals

Type I: Fermi surface deformation



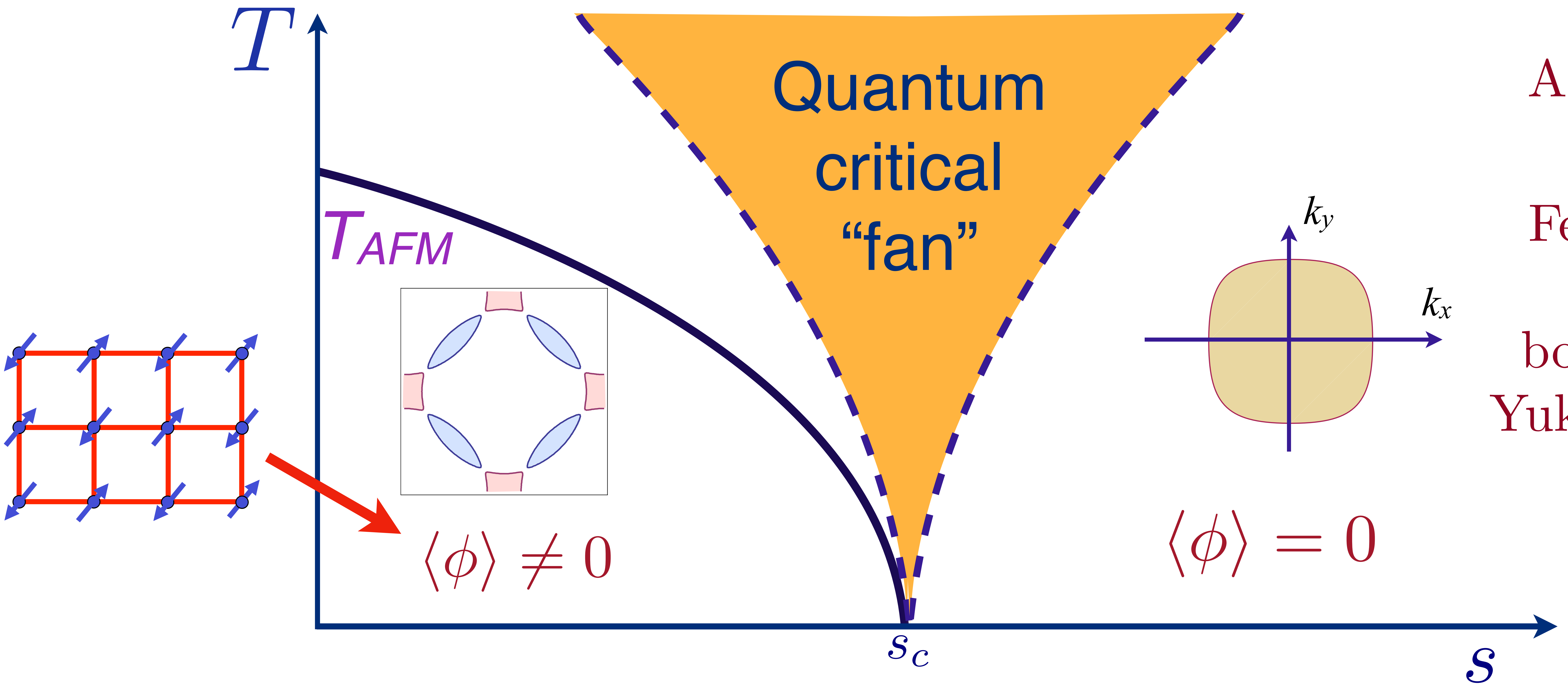
Ising field ϕ

$+$

Fermi surface
with a
boson-fermion
Yukawa coupling

Quantum phase transitions in two-dimensional metals

Type II: Fermi surface reconstruction



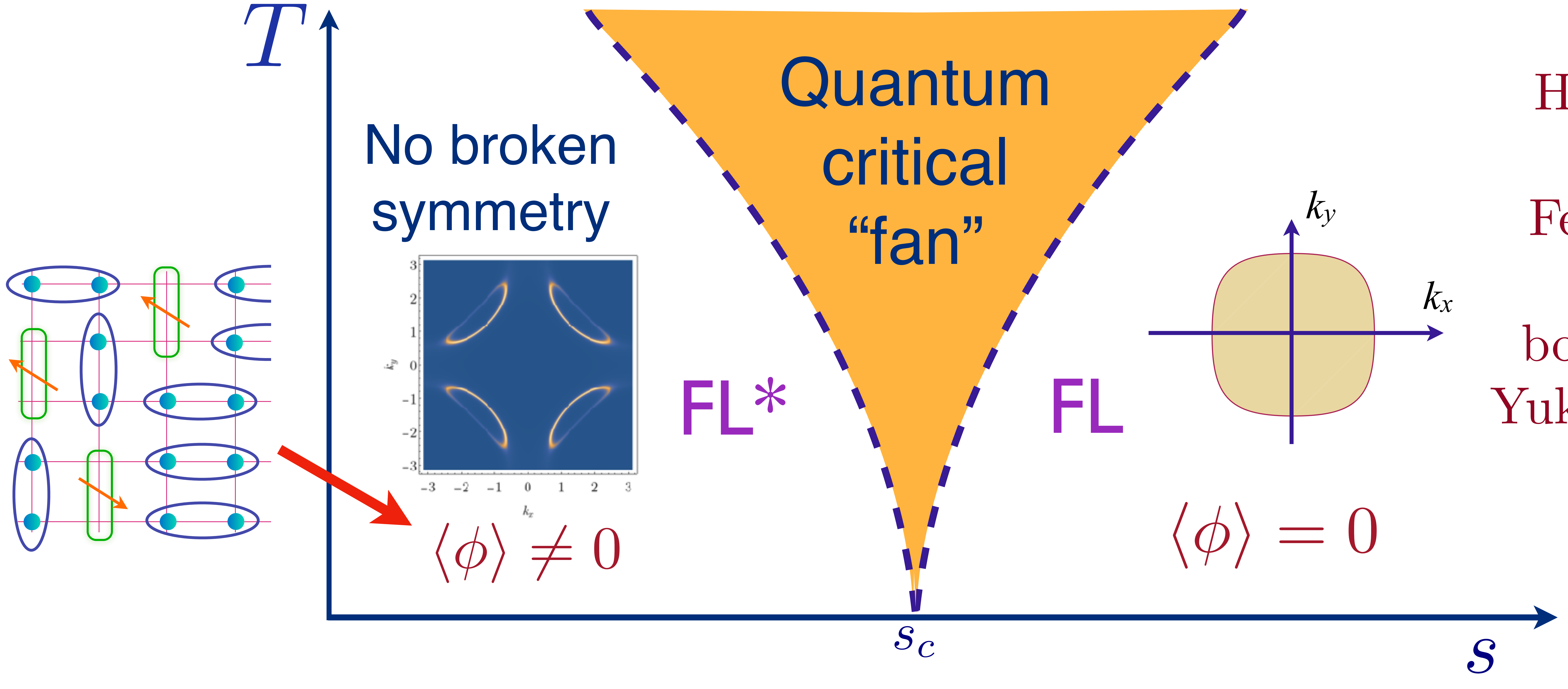
AFM field ϕ_a

$+$

Fermi surface
with a
boson-fermion
Yukawa coupling

Quantum phase transitions in two-dimensional metals

Type III: Fermi surface jump



Spin liquid
$+$
Higgs field ϕ
$+$
Fermi surface
with a
boson-fermion
Yukawa coupling

Applies to hole-doped cuprates

Quantum criticality in clean metals

Critical boson with no spatial disorder

Type I: a critical boson ϕ

e.g. Ising ferromagnetism

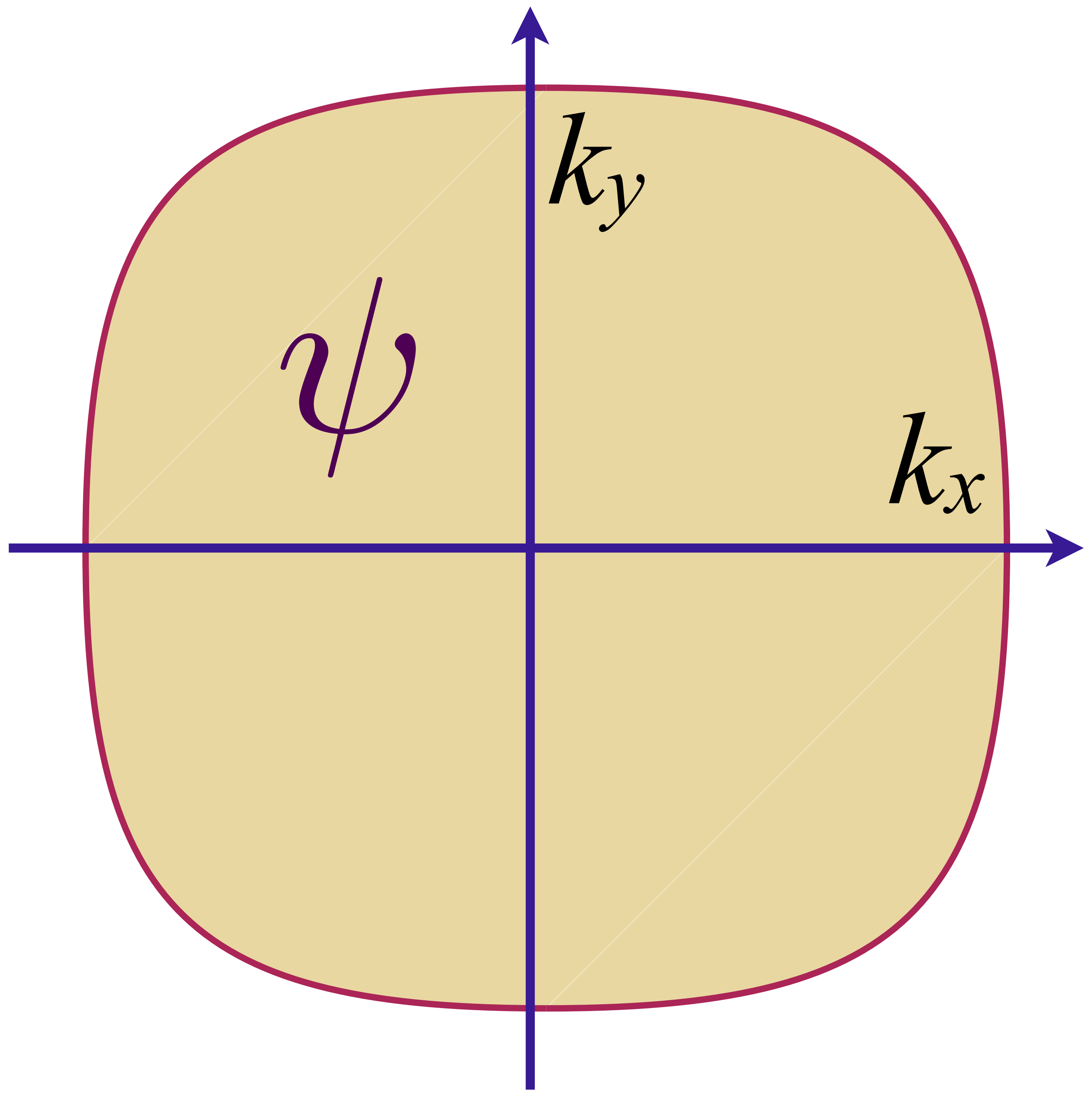
$$+s\,[\phi(\boldsymbol{r})]^2$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Type I: a critical boson ϕ
e.g. Ising ferromagnetism



$$+s \left[\phi(\boldsymbol{r})\right]^2$$

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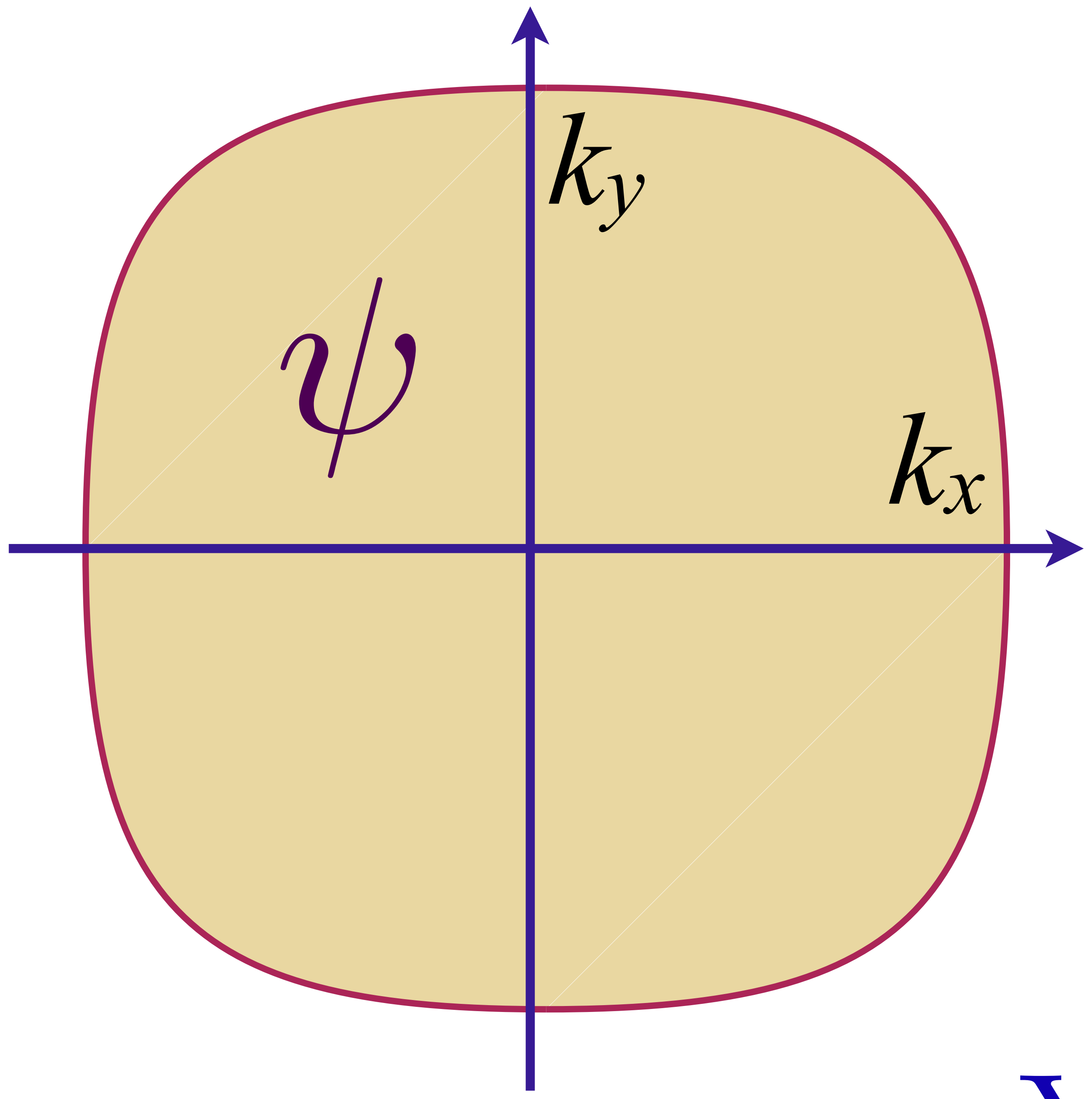

$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Yukawa coupling g between fermions and bosons

Fermi surface + critical boson with no spatial disorder

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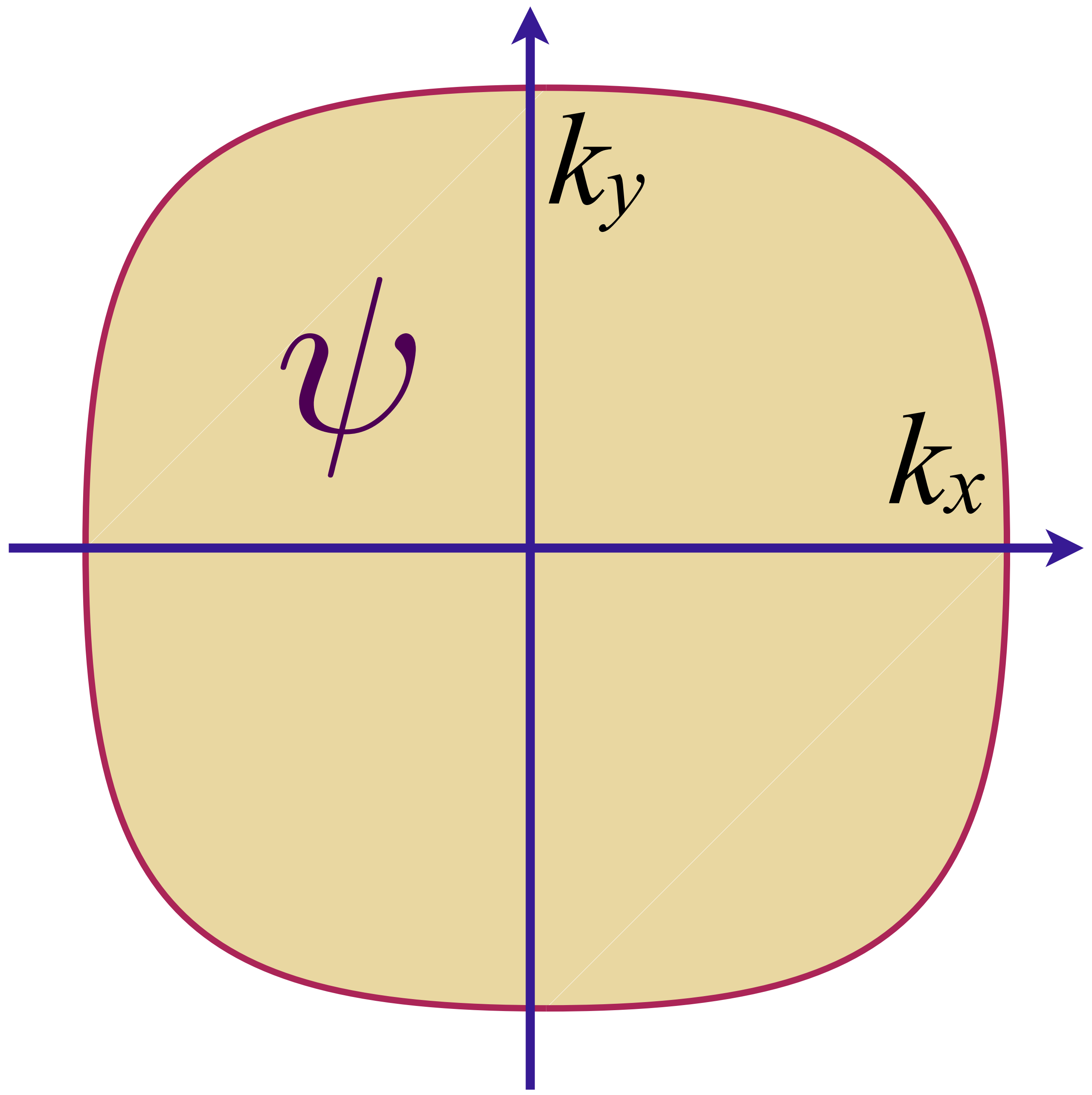

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$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Large N limit of:

$\dfrac{g_{\alpha\beta\gamma}}{N}\,\psi^\dagger_\alpha(\boldsymbol{r})\psi_\beta(\boldsymbol{r})\,\phi_\gamma(\boldsymbol{r})$

with $\alpha, \beta, \gamma = 1 \ldots N$

and $g_{\alpha\beta\gamma}$ random in flavor space,

as in *Yukawa-SYK* models

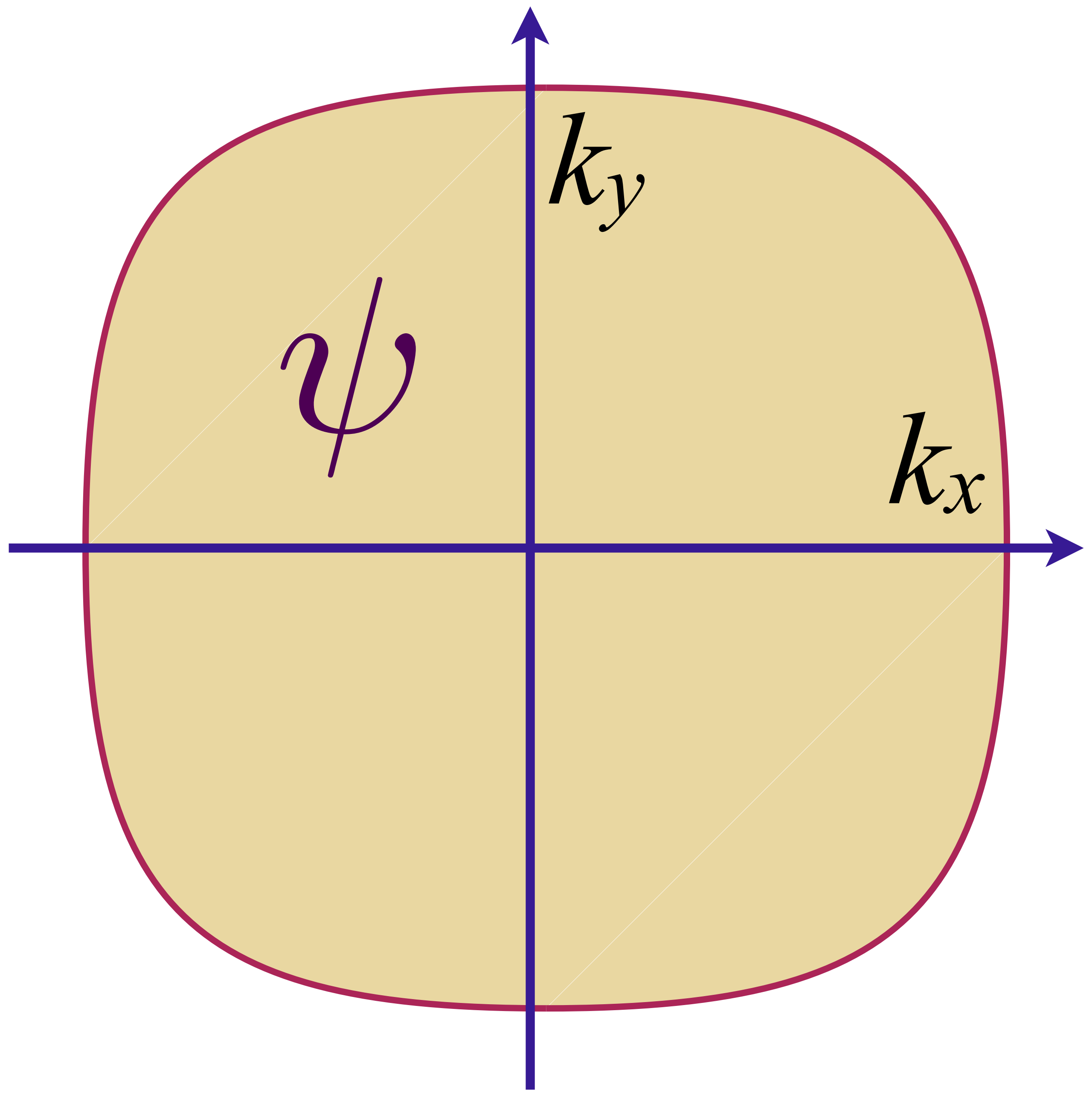
of fermions and bosons

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Type I: a critical boson ϕ

e.g. Ising ferromagnetism



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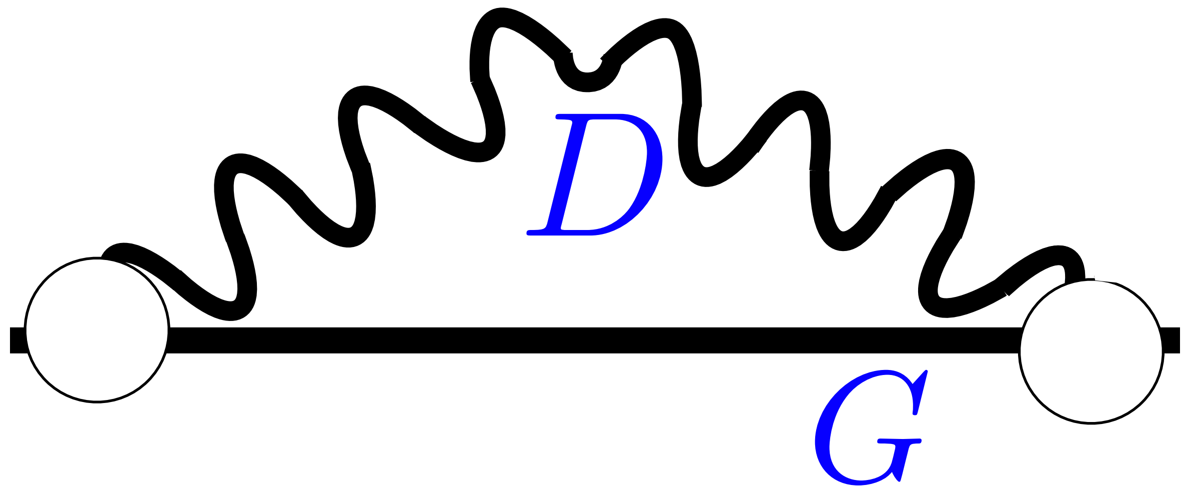
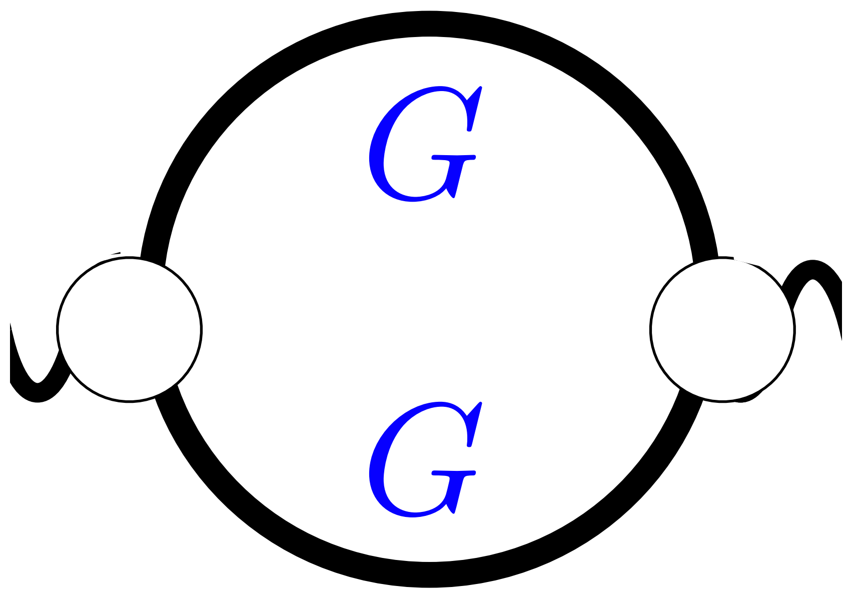
$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

$\Sigma =$



$\Pi =$



Fermi surface + critical boson with no spatial disorder

SYK-type self-consistent equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r})$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r})$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Fermi surface + critical boson with no spatial disorder

SYK-type self-consistent equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r})$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r})$$

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$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Solution of Migdal-Eliashberg equations for electron (G) and boson (D) Green's functions at small ω:

P.A. Lee (1989)

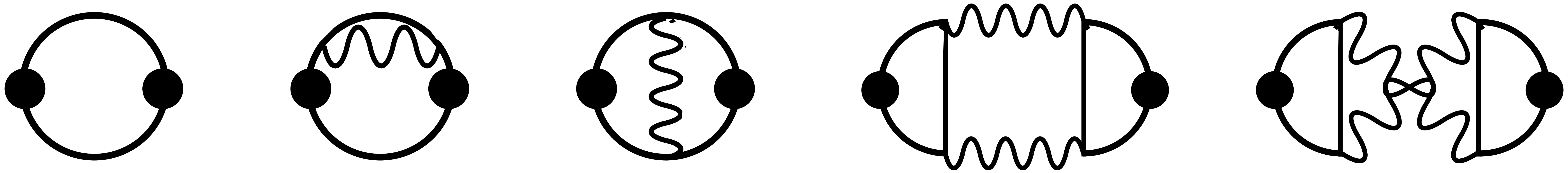
$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\text{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

Entropy $S(T \to 0) \sim T^{2/3}$

Sharp Fermi surface but no quasiparticles

Fermi surface + critical boson with no spatial disorder

Optical conductivity—Diagrams



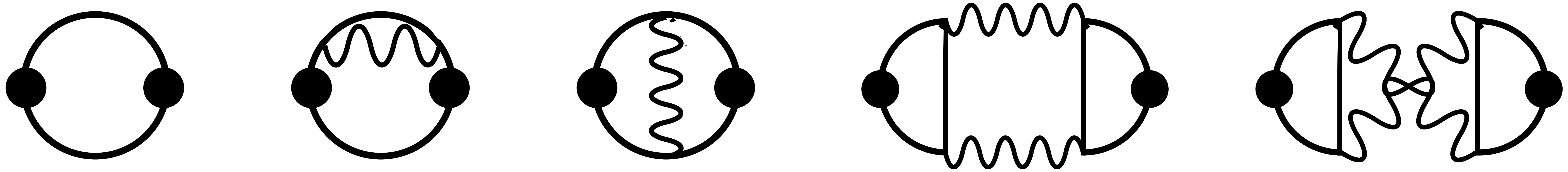
$$\mathrm{Re}\left[\sigma(\omega)\right] = C\,|\omega|^{-2/3}$$

$$\rho(T) \sim T^{4/3}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, and P. A. Lee, PRB **50**, 17917 (1994).

Fermi surface + critical boson with no spatial disorder

Optical conductivity—Diagrams



$$\mathrm{Re}\left[\sigma(\omega)\right] = C\,|\omega|^{-2/3}$$
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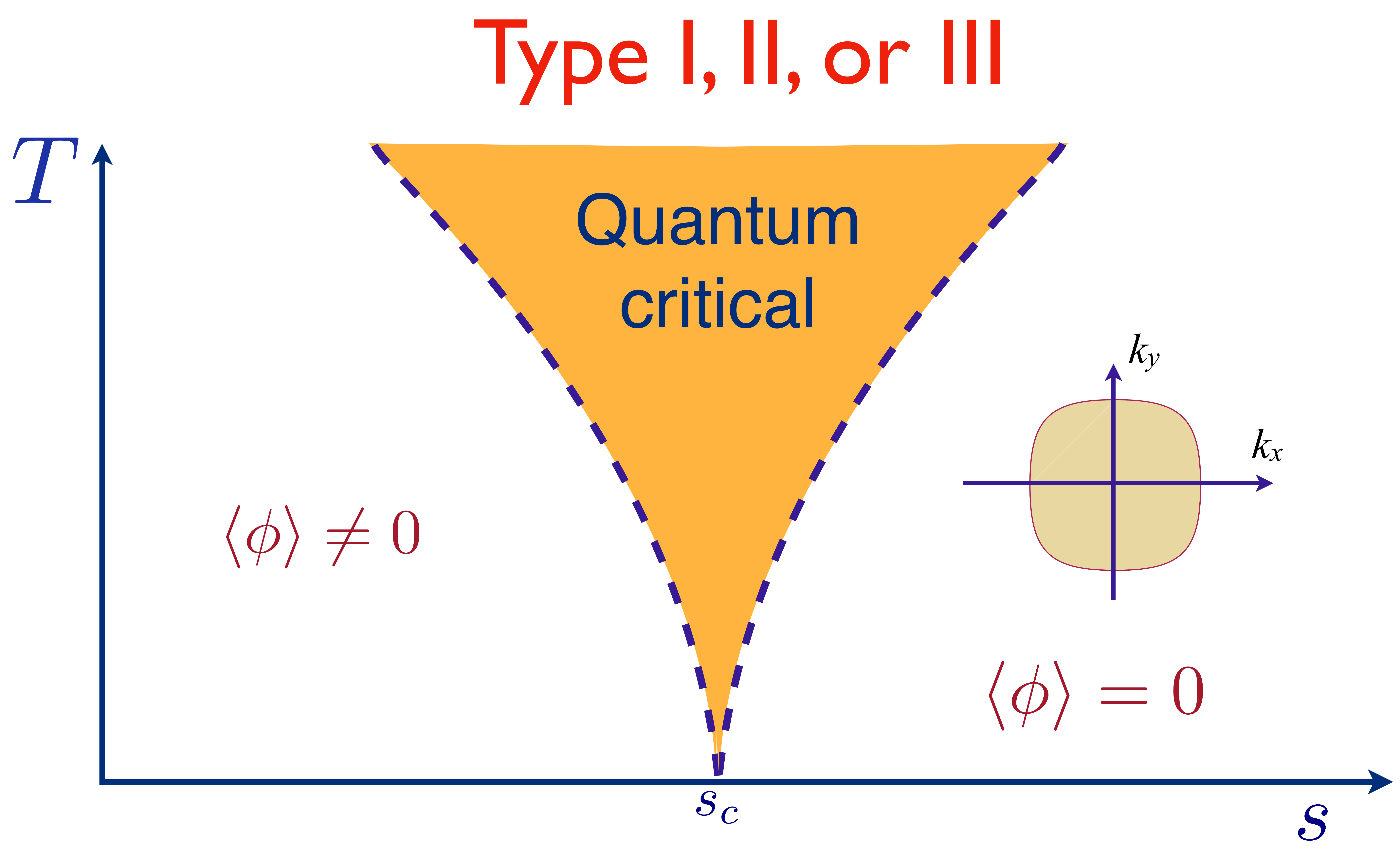
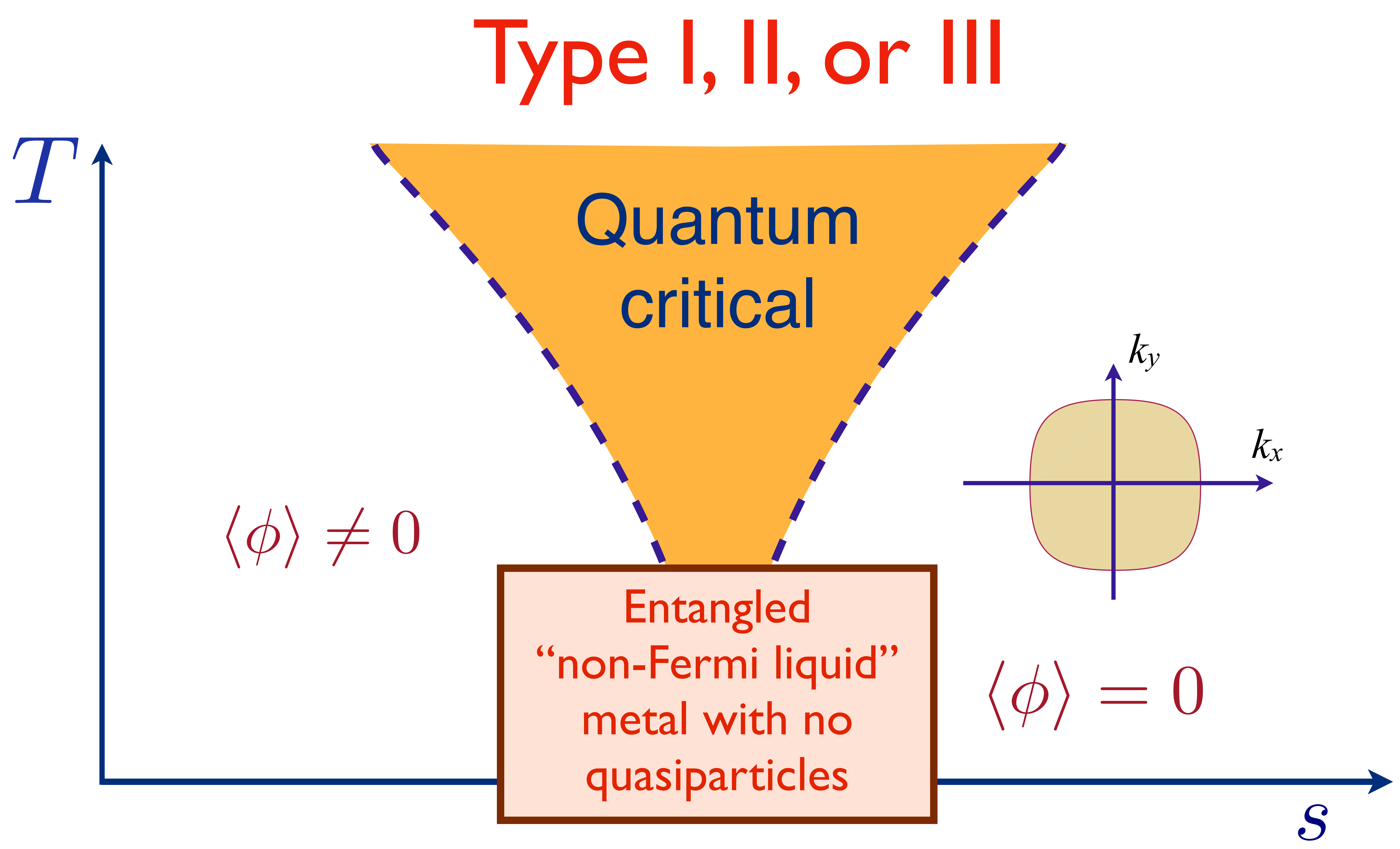
$$C = 0\,; \quad \sigma(\omega) = iD/(\omega - \omega_c) + \omega^0 + \ldots$$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022);
Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024);
D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017);
Zhengyan Darius Sh Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023);
Haoyu Guo, arXiv: 2406.12967

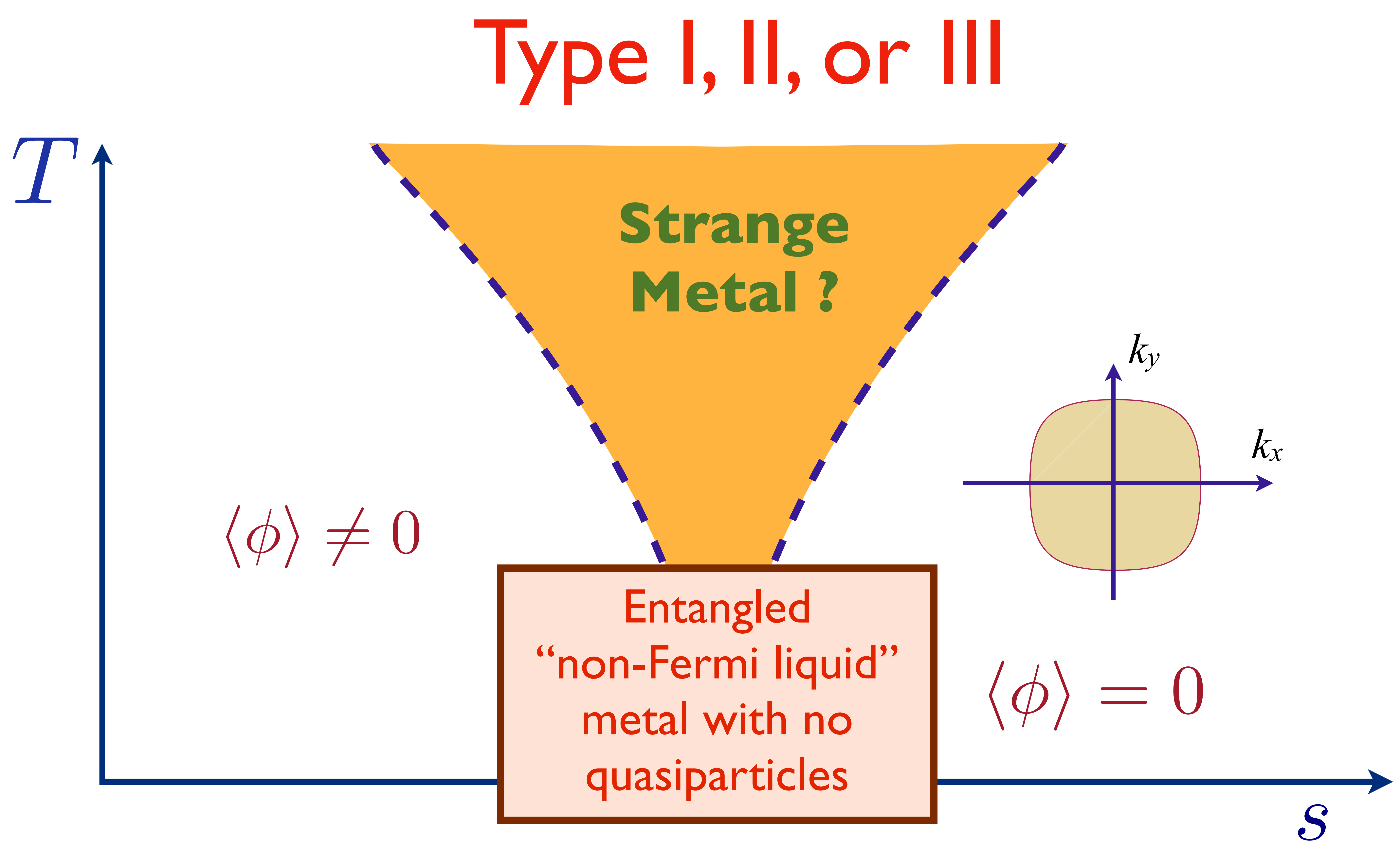
Fermi surface + critical boson with no spatial disorder



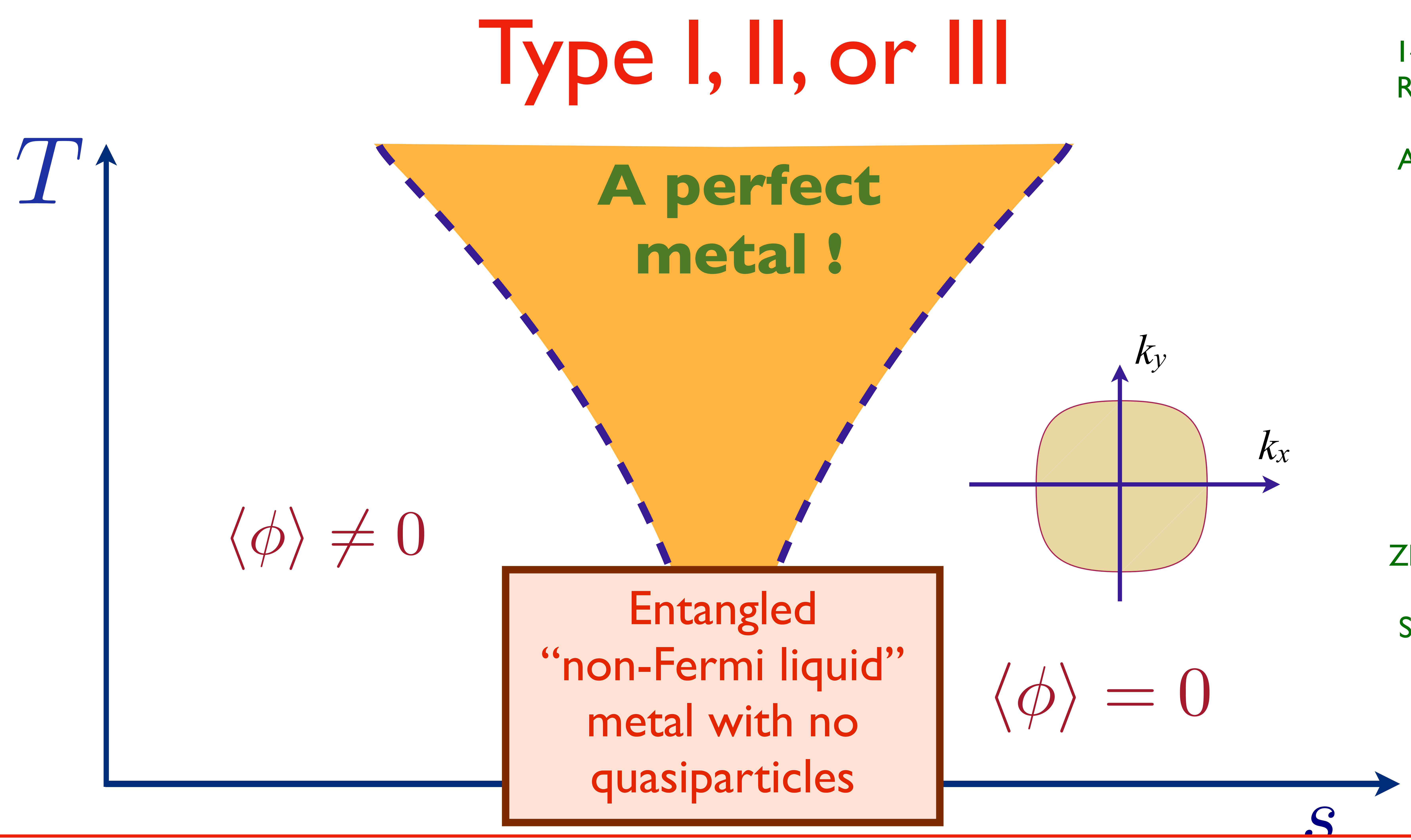
Fermi surface + critical boson with no spatial disorder



Fermi surface + critical boson with no spatial disorder



Fermi surface + critical boson with no spatial disorder



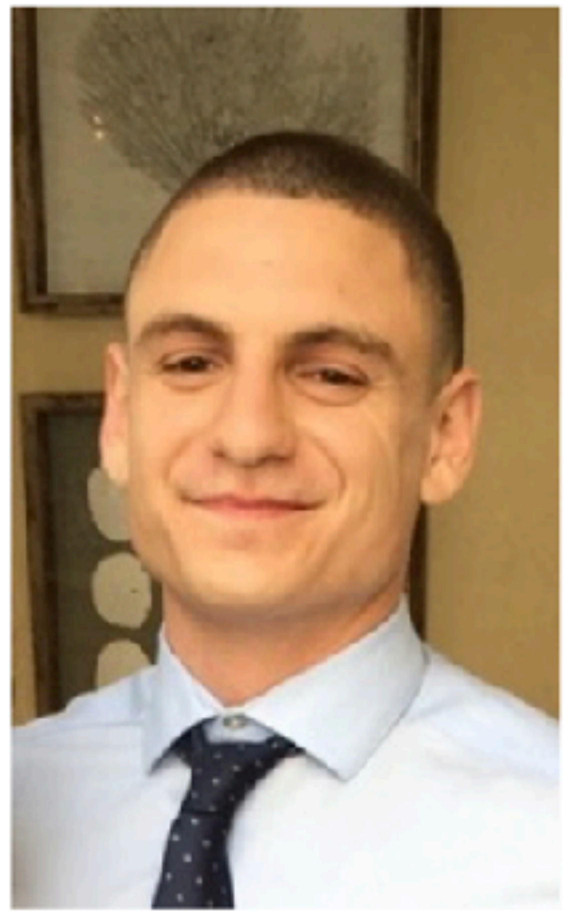
S.A. Hartnoll, P. K. Kovtun, M. Muller, and S.S., PRB **76**, 144502 (2007); S.A. Hartnoll, R. Mahajan, M. Punk, and S. S., PRB **89**, 155130 (2014); Aavishkar Patel and S.S., PRB **90**, 165146 (2014); Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022); Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024); D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017); Zhengyan Darius Sh Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023); Haoyu Guo, arXiv: 2406.12967

Extreme drag: the fermions ψ "drag" the bosons ϕ as they move, and so electrical current does not relax, even though strong ψ-ϕ scattering leads to absence of ψ quasiparticles.

Universal theory of strange metals:

Quantum phase transitions in inhomogeneous metals described by the two-dimensional Yukawa-SYK model

Theory applies for types I, II, III, with only minor differences.



Ilya Esterlis
Wisconsin

Haoyu Guo
Cornell

Aavishkar Patel
Flatiron

Chenyuan Li
Harvard → Rice

Davide Valentinis
KIT

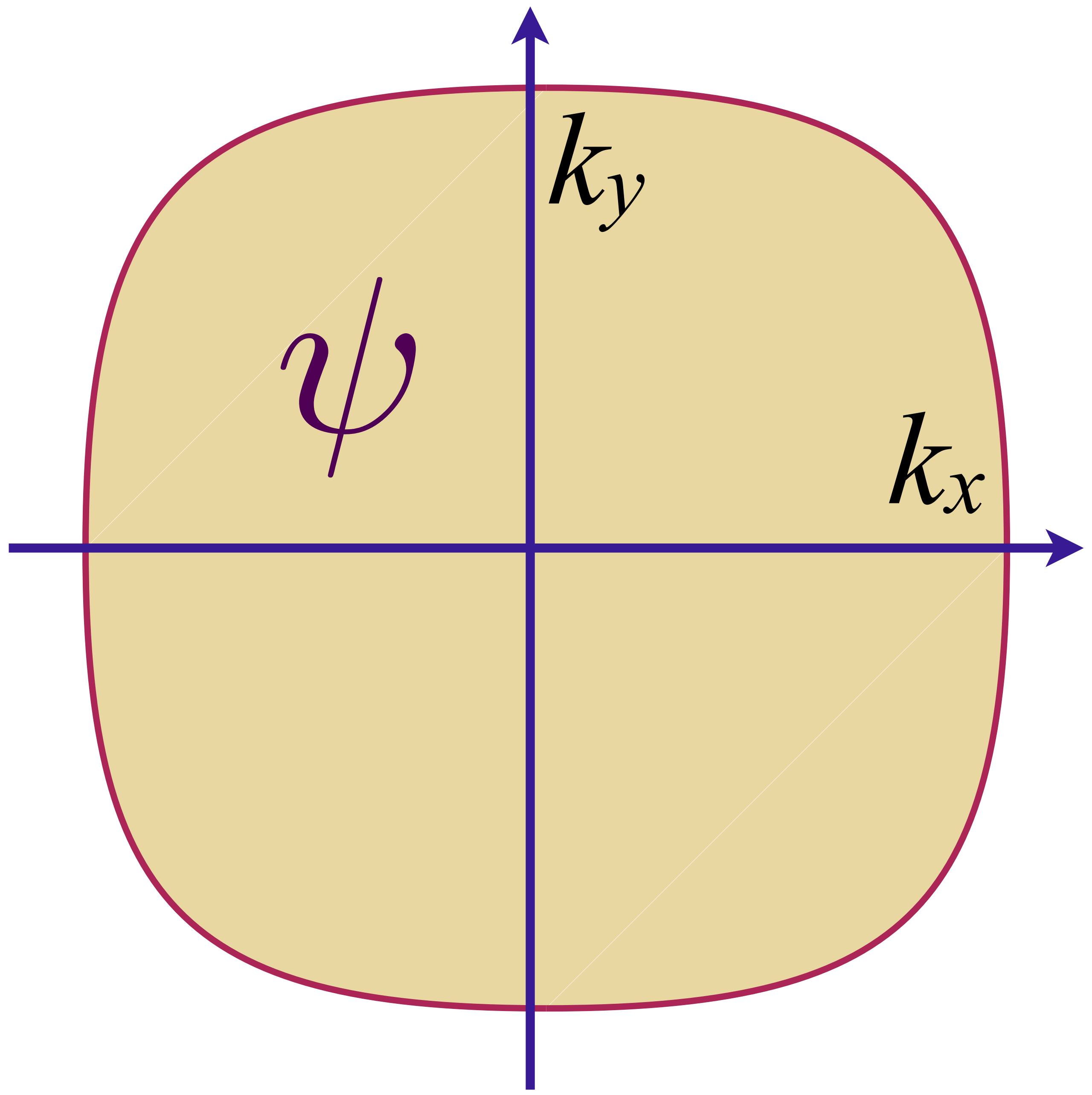
Joerg Schmalian
KIT

Peter Lunts
Harvard

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)
Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, arXiv:2406.07608

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



Type I, II, III: A critical boson ϕ
e.g. Ising ferromagnetism,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

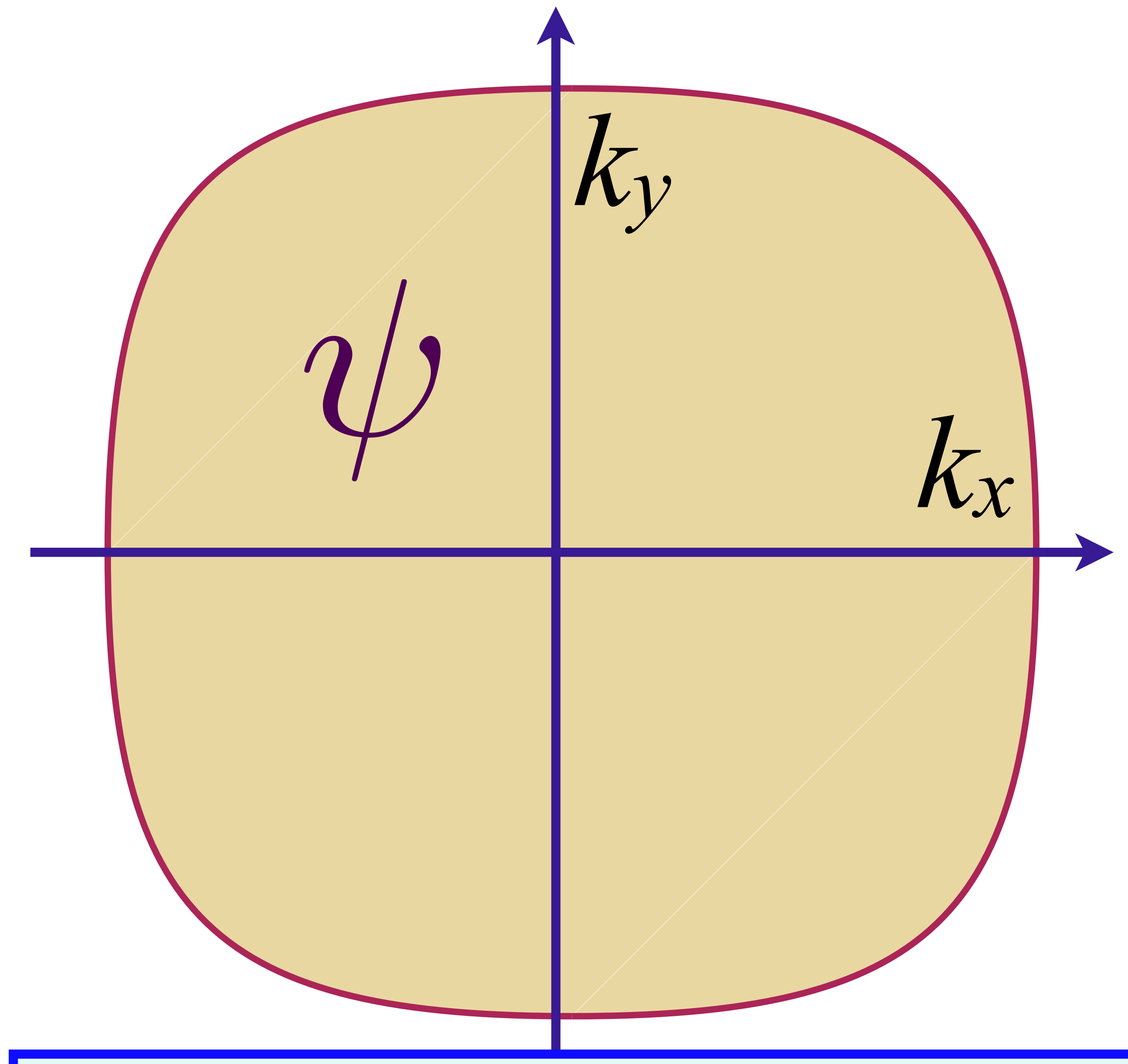
$$+s\, [\phi(\boldsymbol{r})]^2 \qquad +g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+K\, [\nabla_{\boldsymbol{r}} \phi(\boldsymbol{r})]^2 + u\, [\phi(\boldsymbol{r})]^4$$

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Type I, II, III: A critical boson ϕ
e.g. Ising ferromagnetism,
spin-density wave order,
Higgs boson for Fermi-volume changing transition



$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 \; +v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

$v(\boldsymbol{r})$ leads to elastic scattering of ψ;
localization of ψ only at long length scales, not relevant for experiments

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Type I, II, III: A critical boson ϕ
e.g. Ising ferromagnetism,
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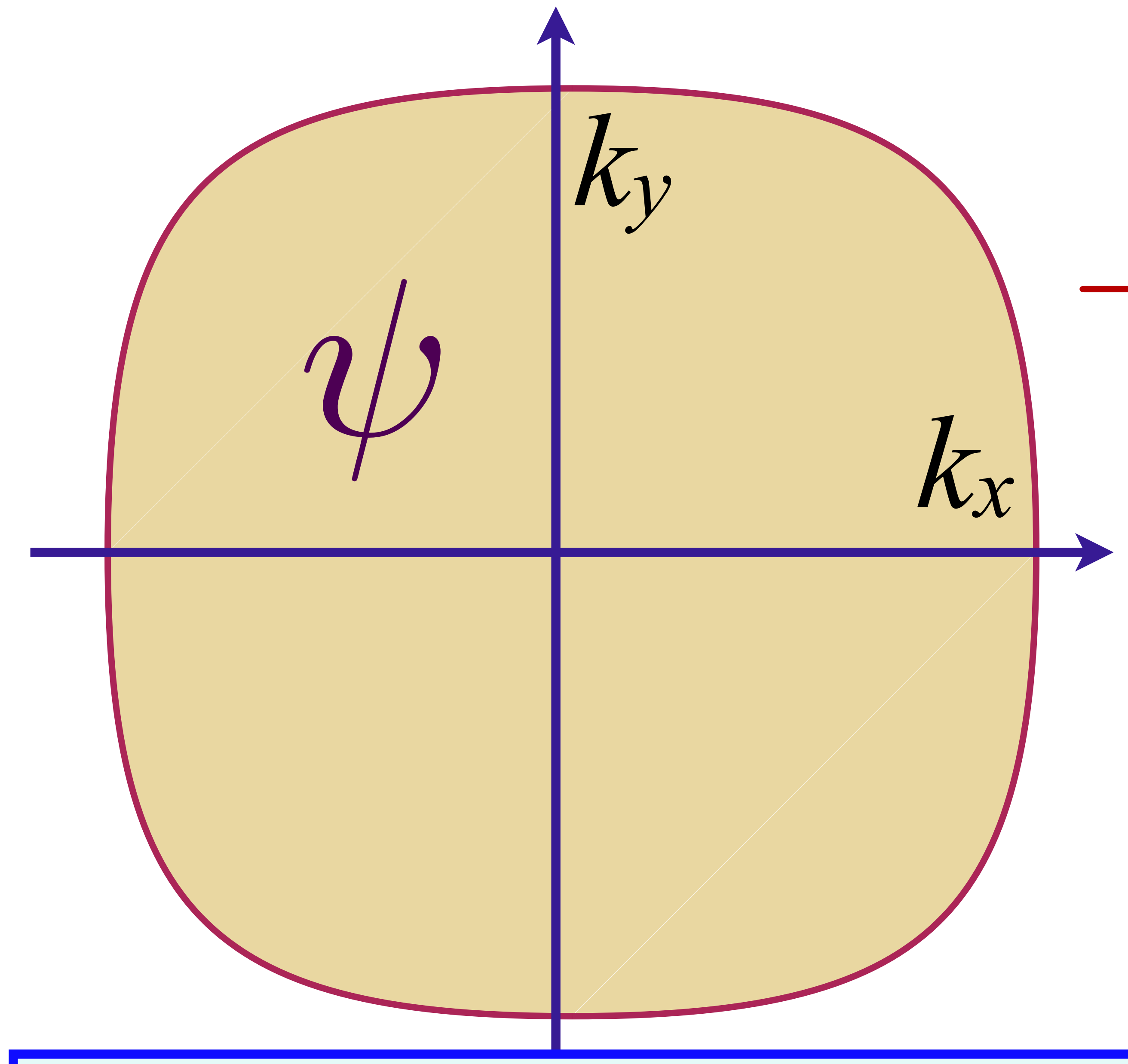

$$+[s + \delta s(\boldsymbol{r})]\,[\phi(\boldsymbol{r})]^2 + \qquad + g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 \; + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})} = 0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')} = \delta s^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

$\delta s(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder):
Very important and should be accounted for first.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

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e.g. Ising ferromagnetism,
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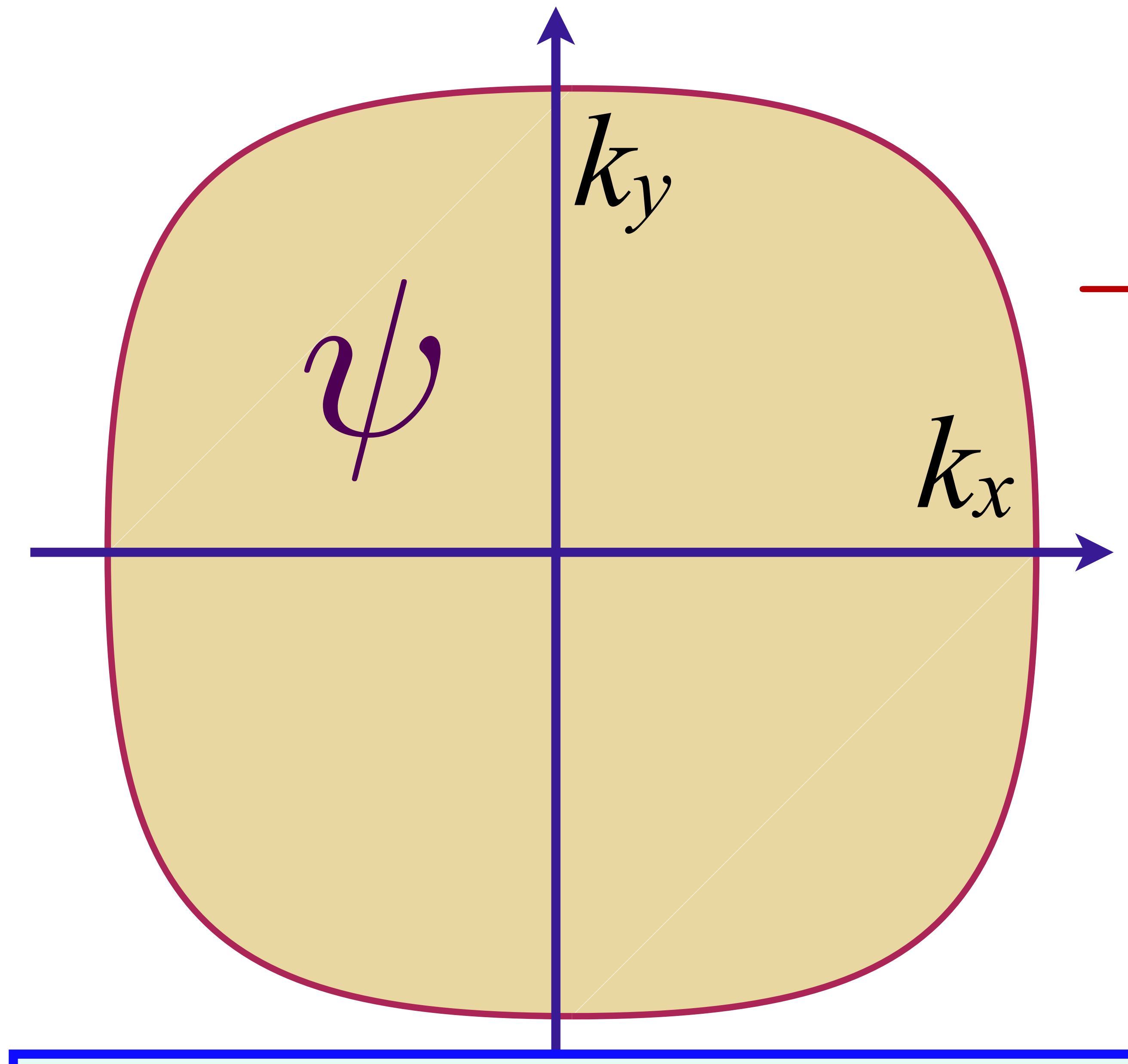

$$+[s + \delta s(\boldsymbol{r})] [\phi(\boldsymbol{r})]^2 + \quad +g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

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Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

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Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})} = 0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')} = \delta s^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

$\delta s(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder):
Rescale $\phi(\boldsymbol{r})$ to obtain a theory with $\delta s(\boldsymbol{r}) = 0$.

2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Type I, II, III: A critical boson ϕ
e.g. Ising ferromagnetism,
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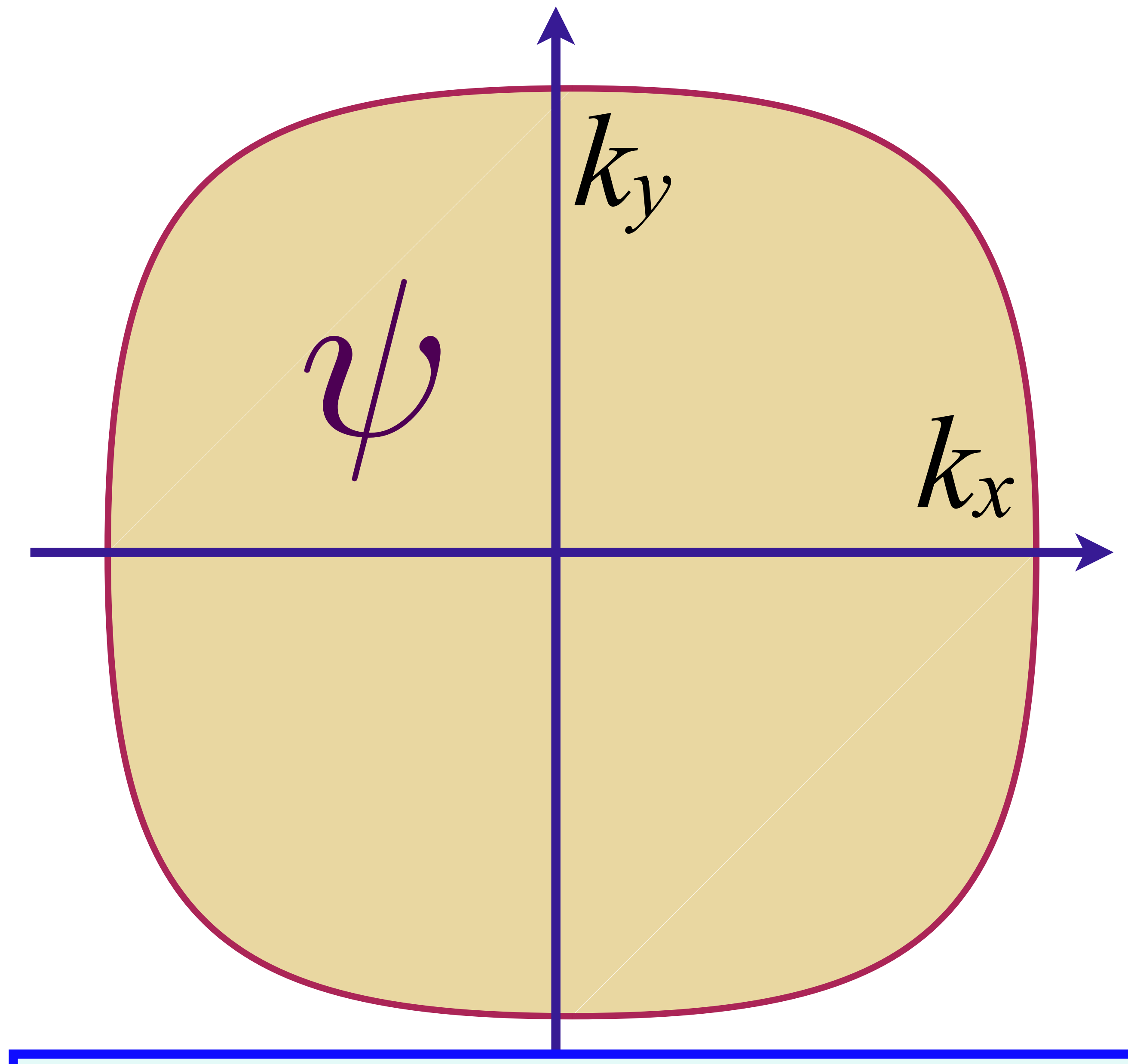

$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

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Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

$g'(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder):
the two-dimensional Yukawa-Sachdev-Ye-Kitaev model.

2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial\tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Type I, II, III: A critical boson ϕ
e.g. Ising ferromagnetism,
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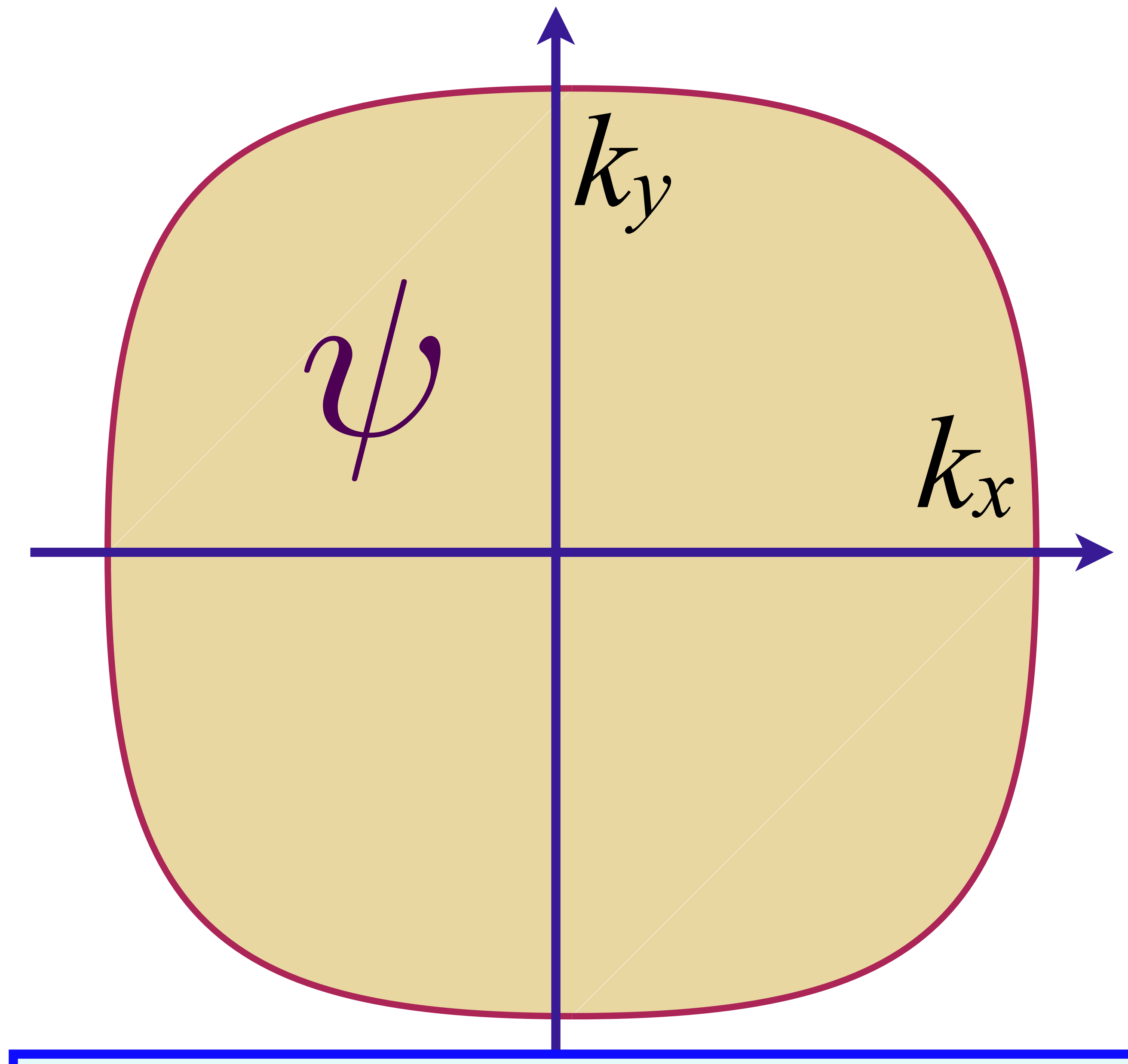

$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

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Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Analyze 2d-YSYK model in a self-averaging manner as in the SYK model.
Should be applicable as long as eigenmodes of $\phi(\boldsymbol{r})$ are extended.

2d-YSYK model: Fermi surface + critical boson with interaction disorder

SYK-type self-consistent equations

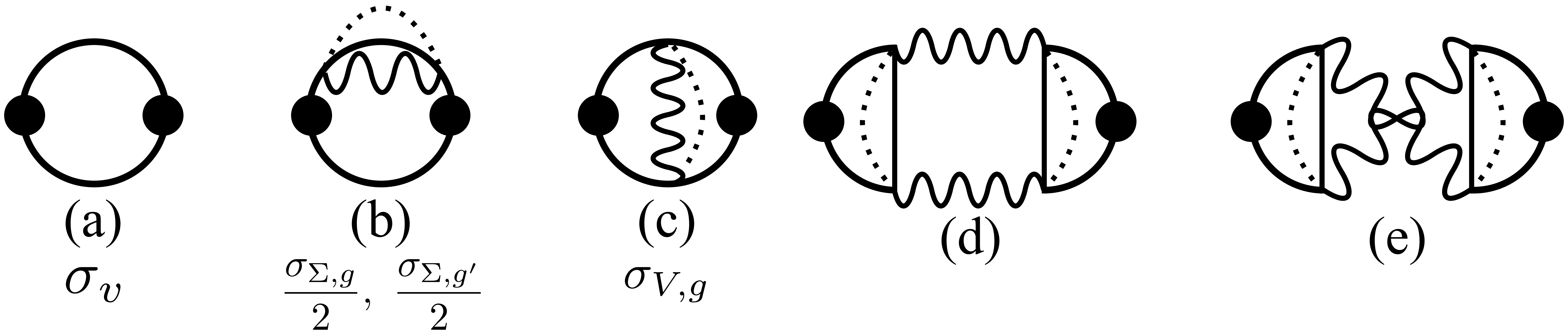
$$\Sigma(\tau,\mathbf{r}) = g^2 D(\tau,\mathbf{r})G(\tau,\mathbf{r}) + v^2 G(\tau,\mathbf{r})\delta^2(\mathbf{r}) + {g'}^2 G(\tau,\mathbf{r})D(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$

$$\Pi(\tau,\mathbf{r}) = -g^2 G(-\tau,-\mathbf{r})G(\tau,\mathbf{r}) - {g'}^2 G(-\tau,\mathbf{r})G(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$

$$G(i\omega,\mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega,\mathbf{k})},$$

$$D(i\Omega,\mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega,\mathbf{q})}.$$

Conductivity:



+ all ladders and bubbles…..

2d-YSYK model: Fermi surface + critical boson with interaction disorder

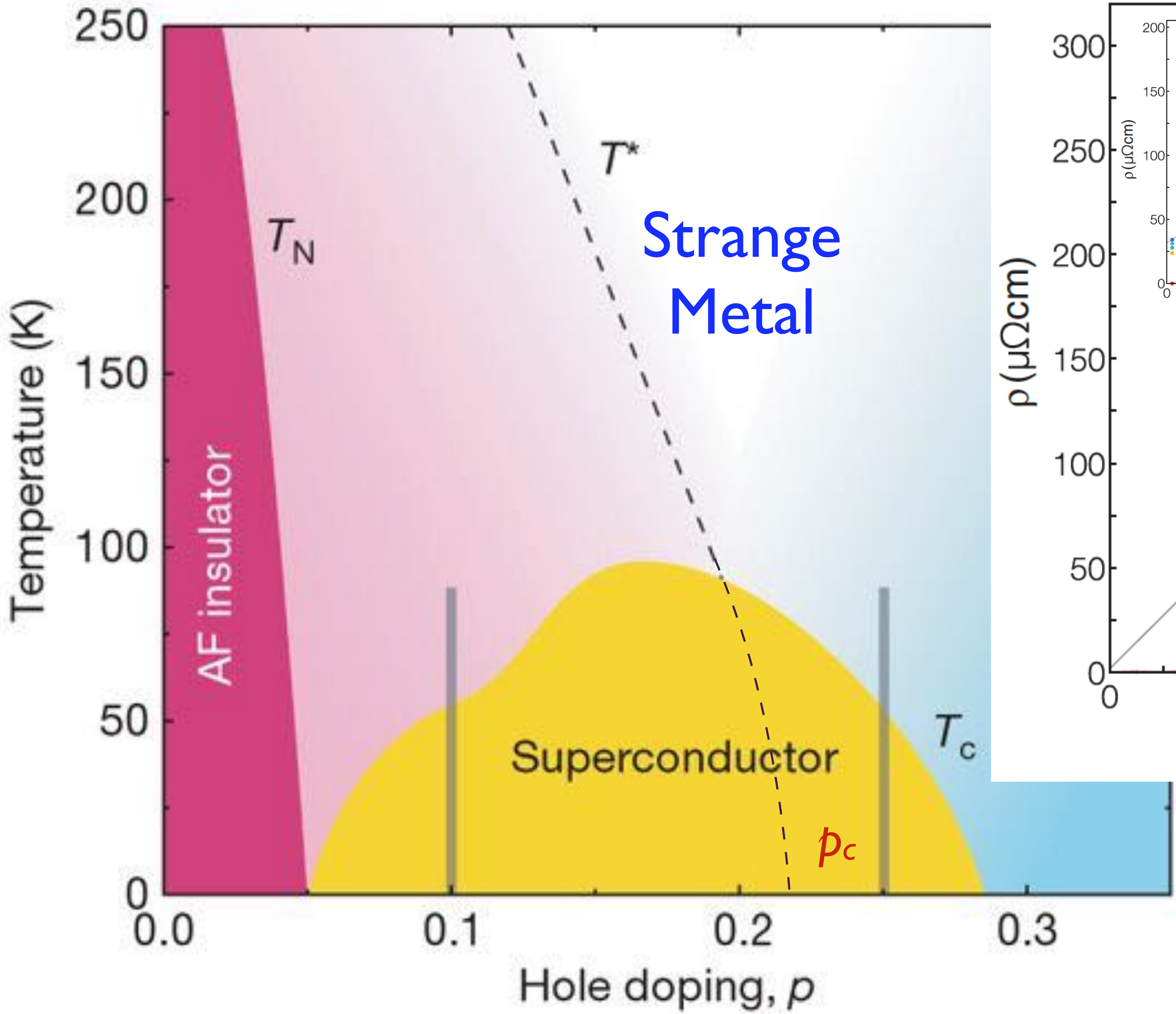
Electron Green's function: $G(\omega) \sim \dfrac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)}\right)\mathrm{sgn}(\omega)}$

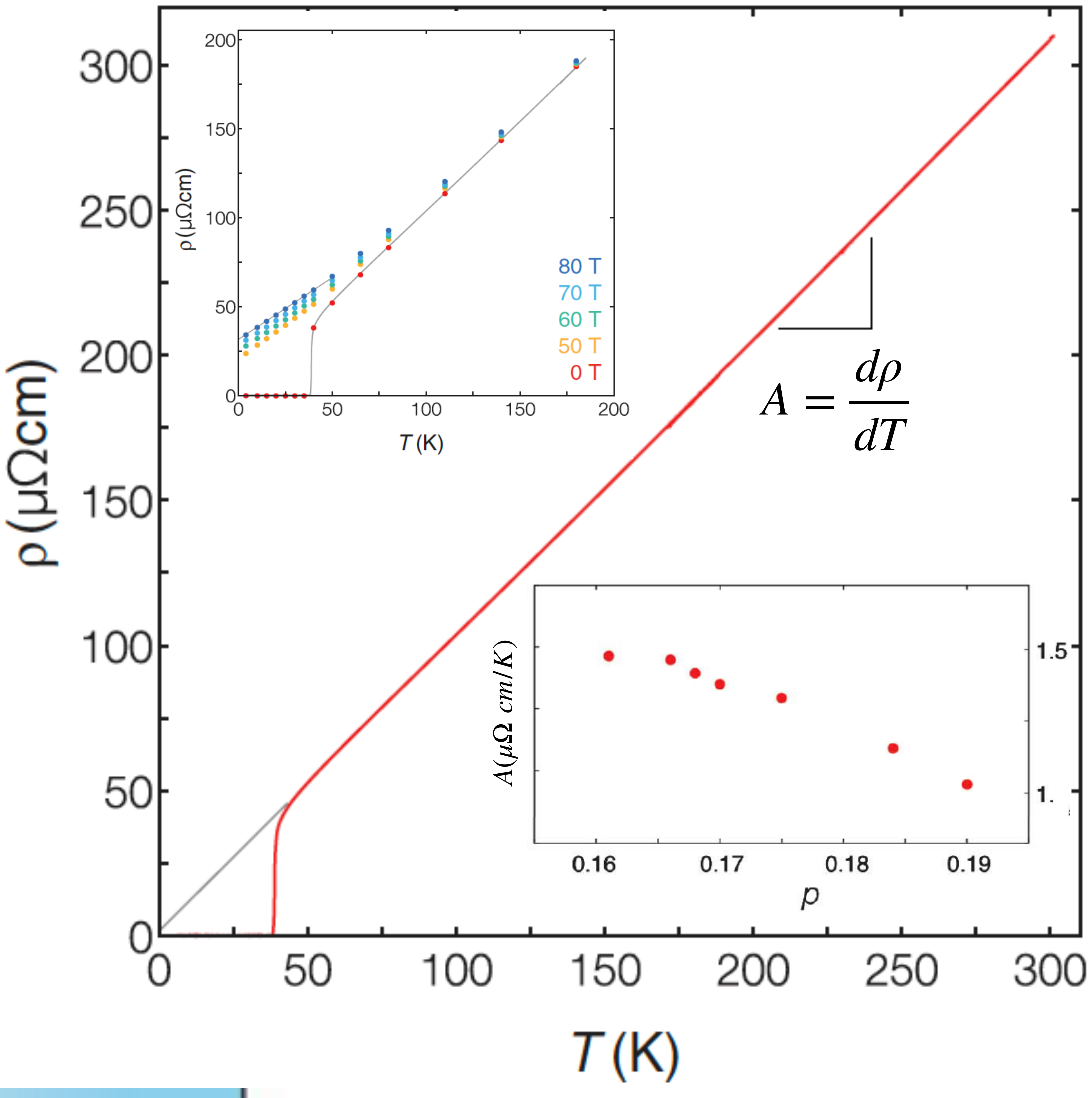
$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}}$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.



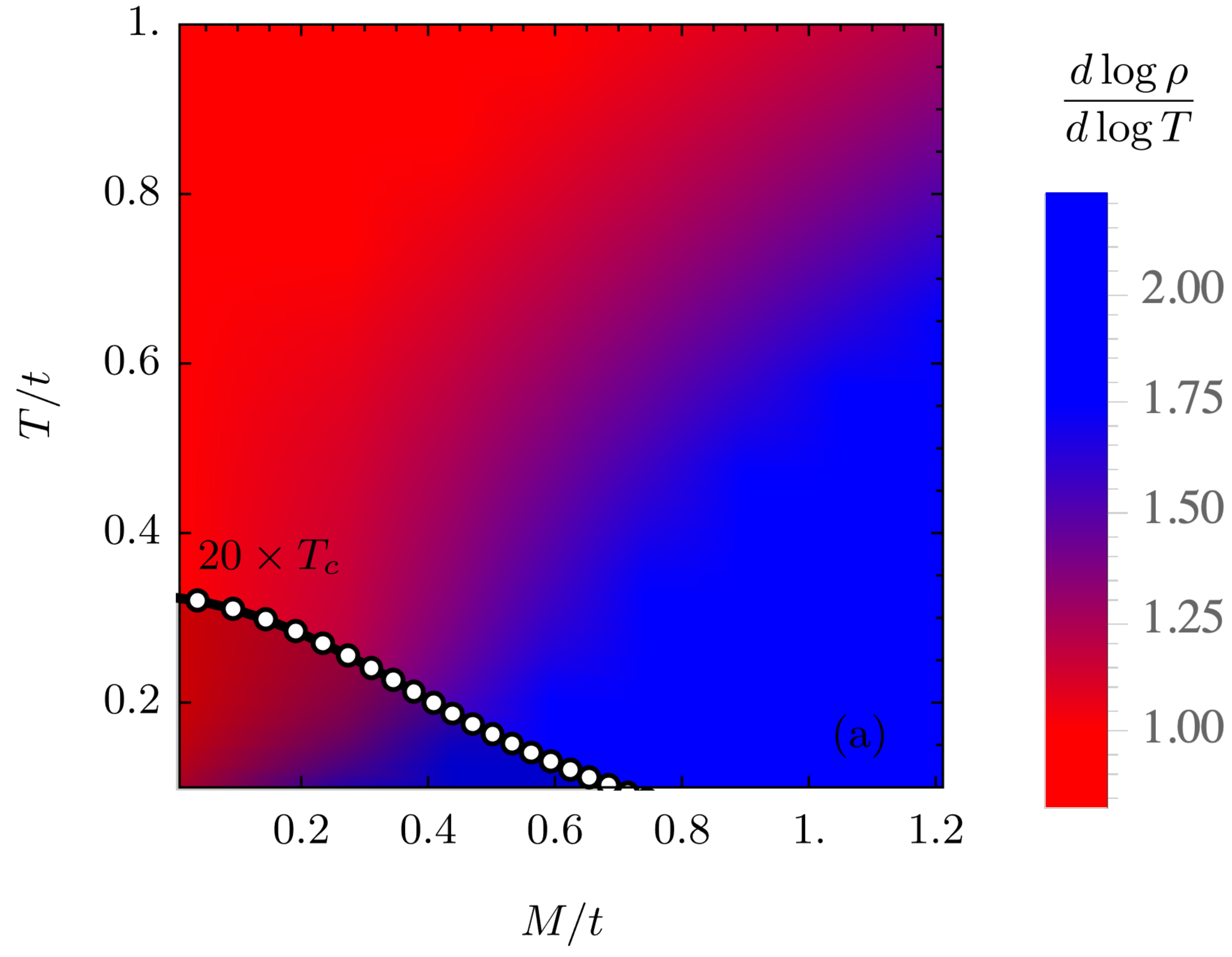


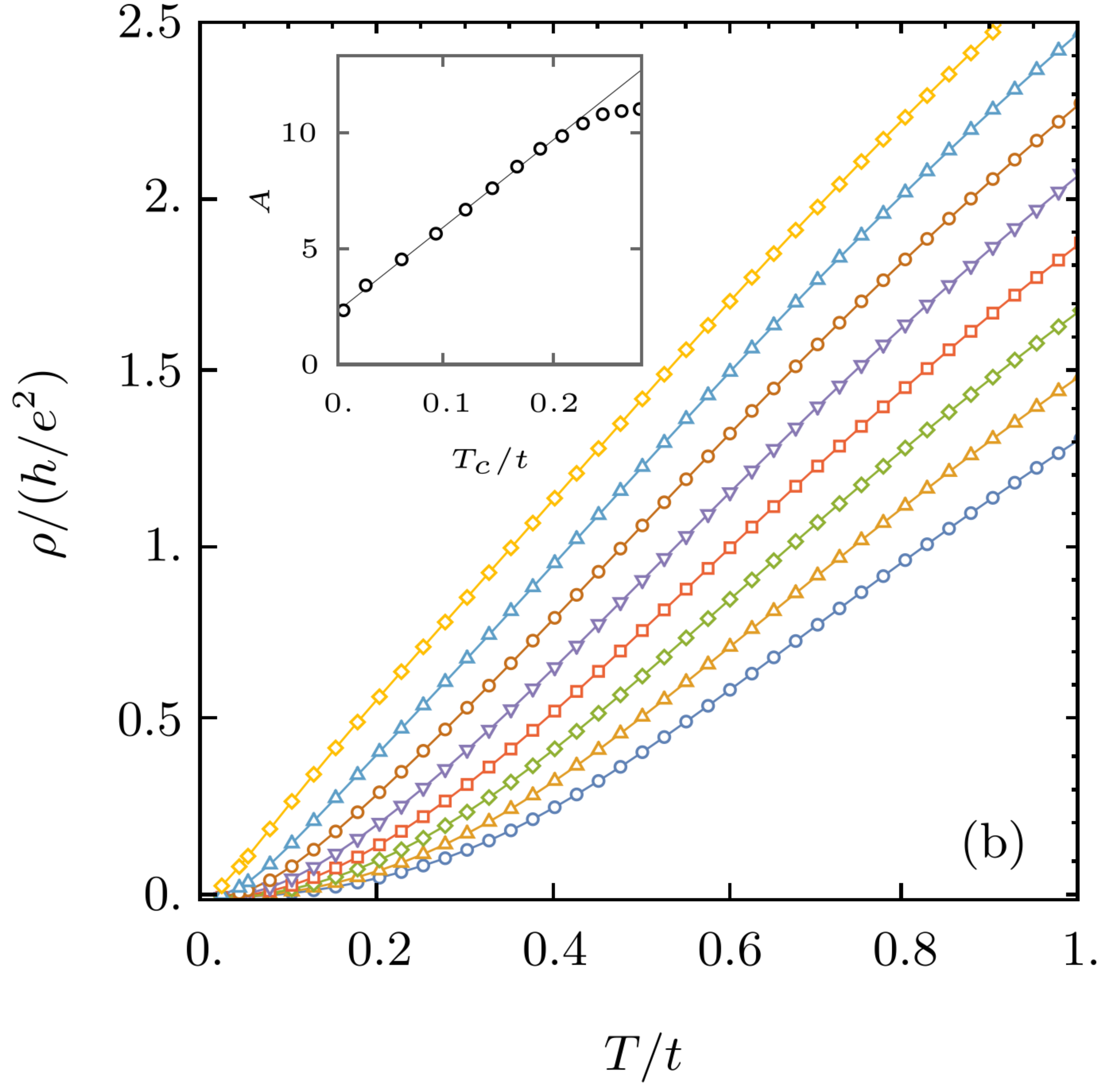
LSCO: Giraldo-Gallo et al. 2018

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, arXiv:2406.07608



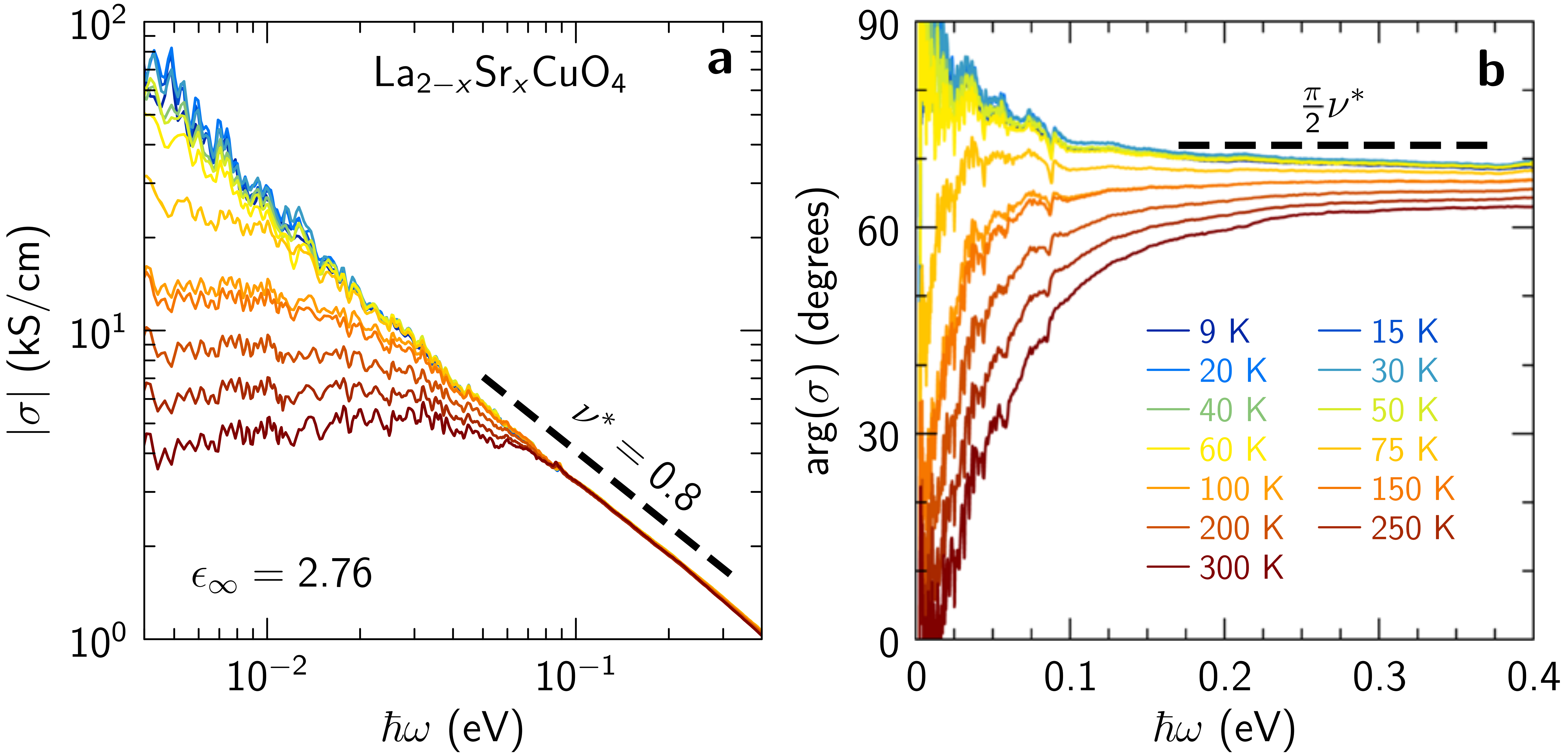


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$



$La_{2-x}Sr_xCuO_4$
$p = 0.24$
$T_c = 19$ K

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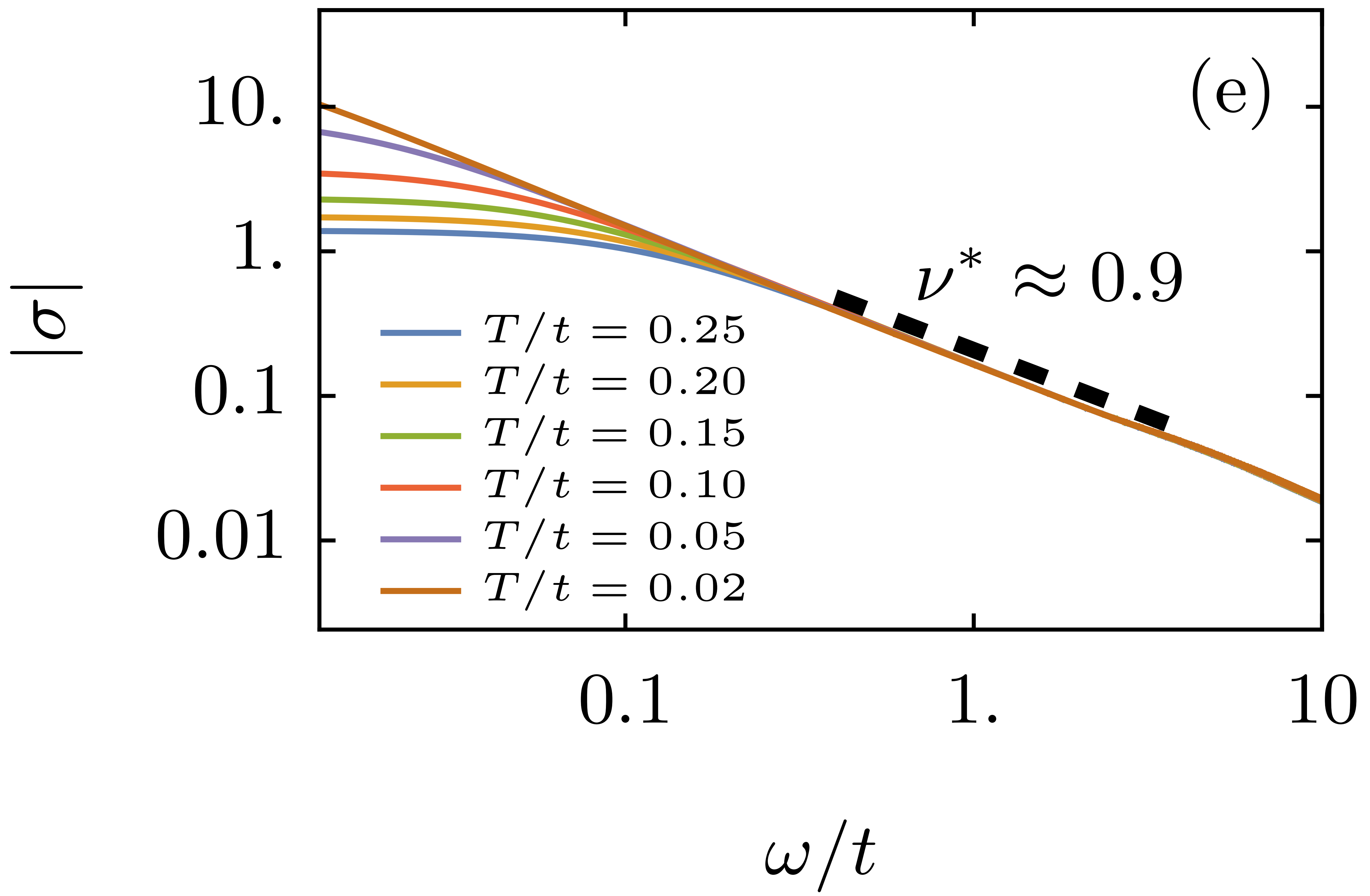

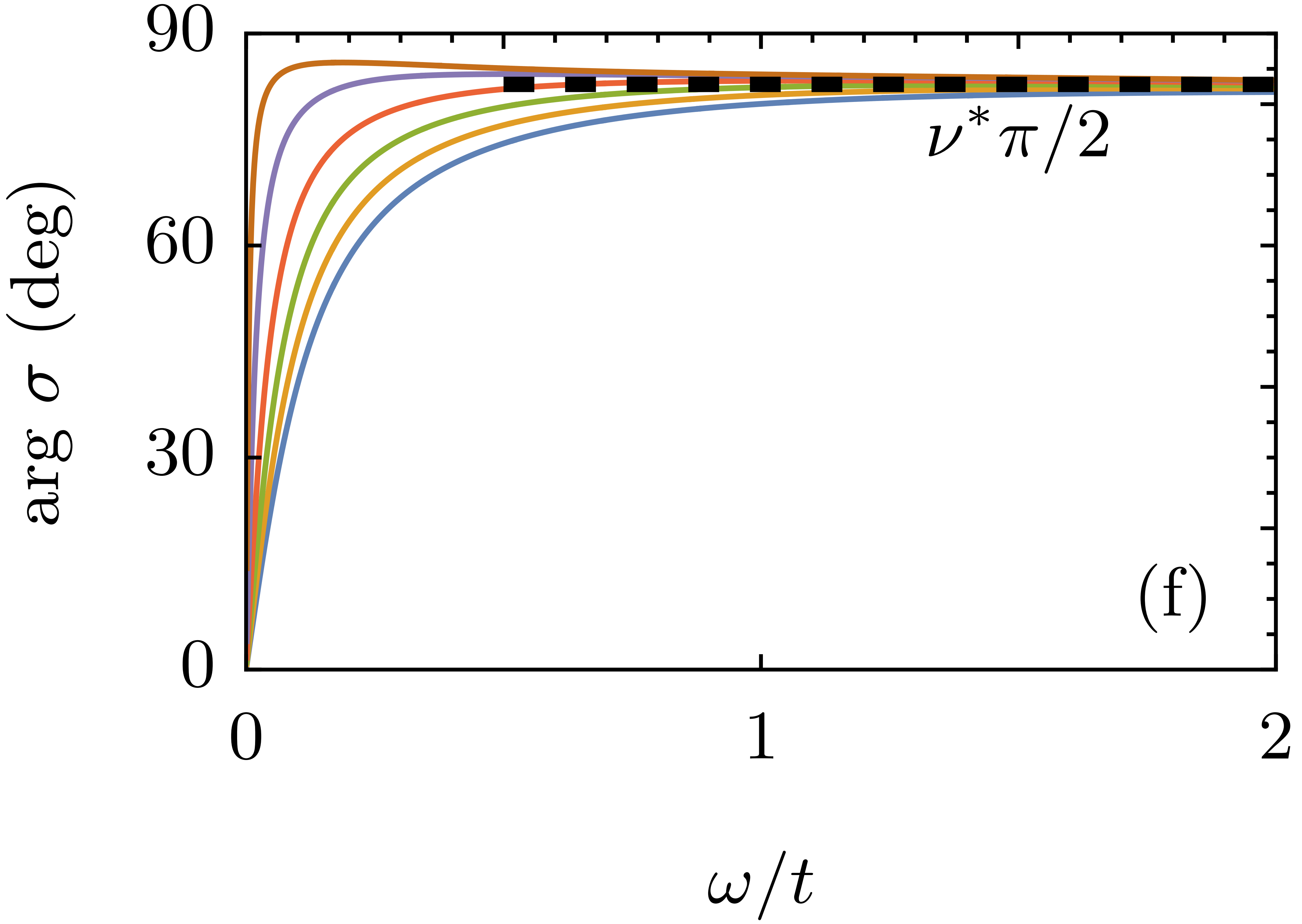

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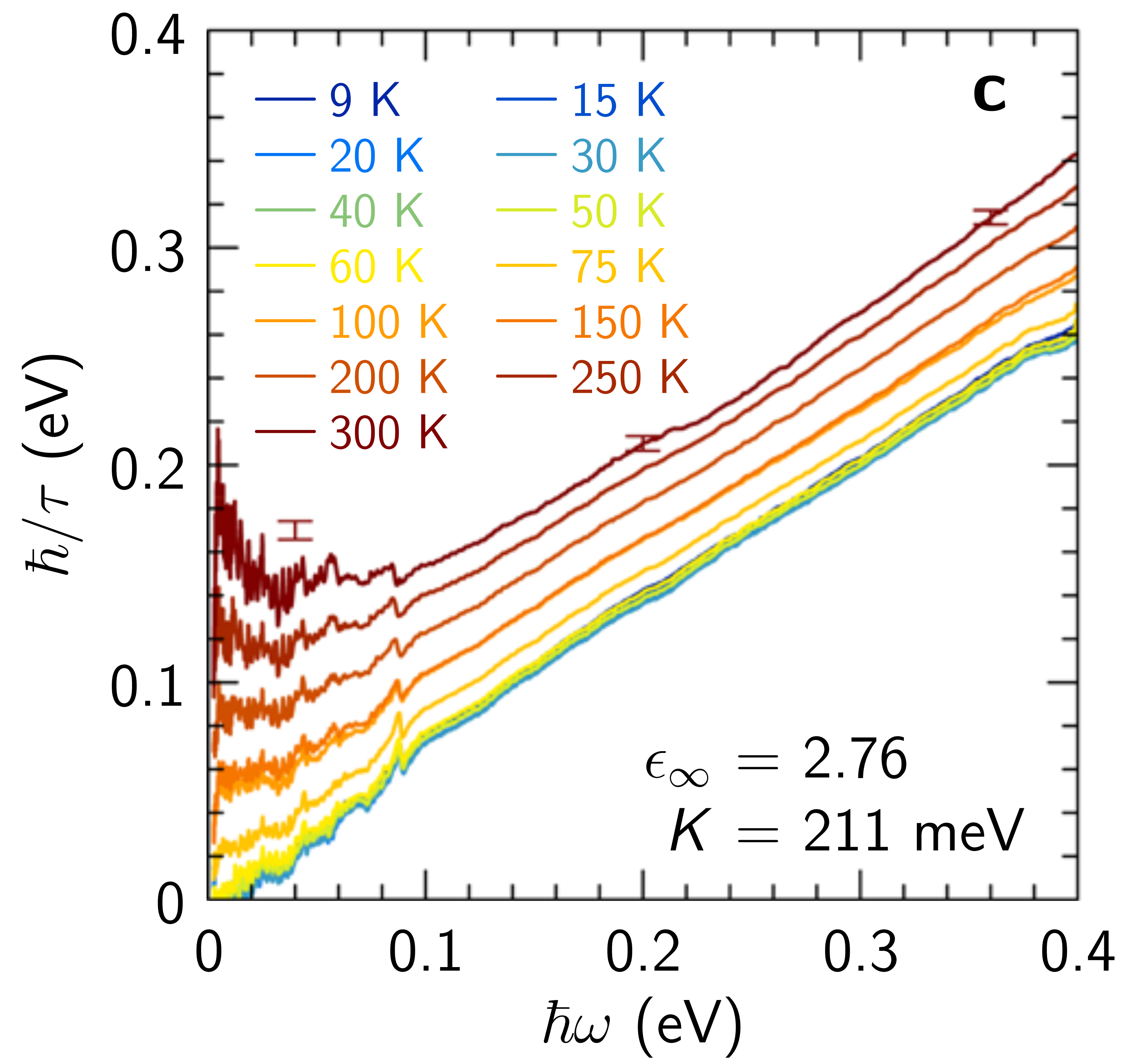

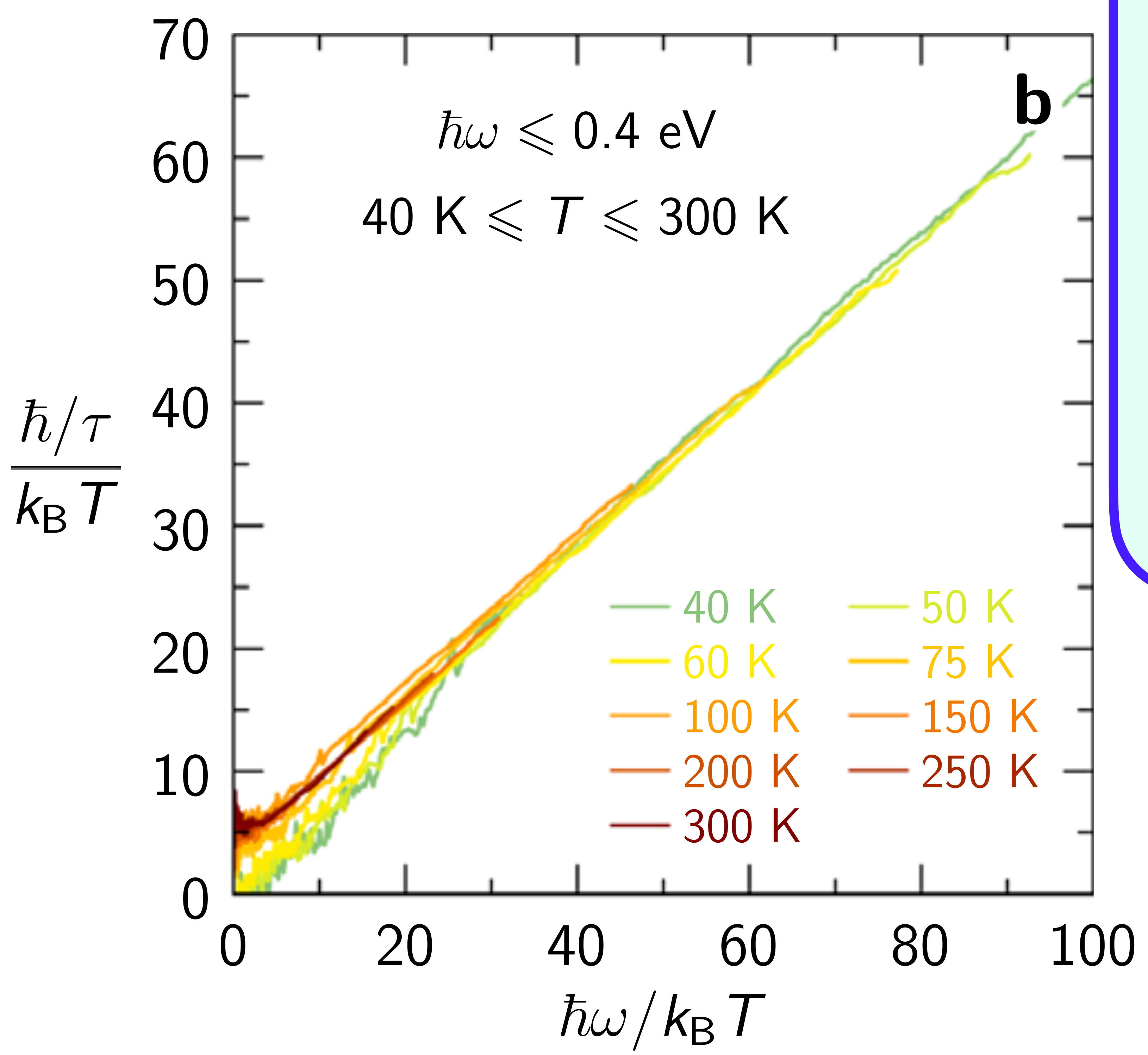

Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_BT}F\left(\frac{\hbar\omega}{k_BT}\right)$$

and entropy

$$S(T\to 0) \sim T\ln(1/T).$$

$La_{2-x}Sr_xCuO_4$

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Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

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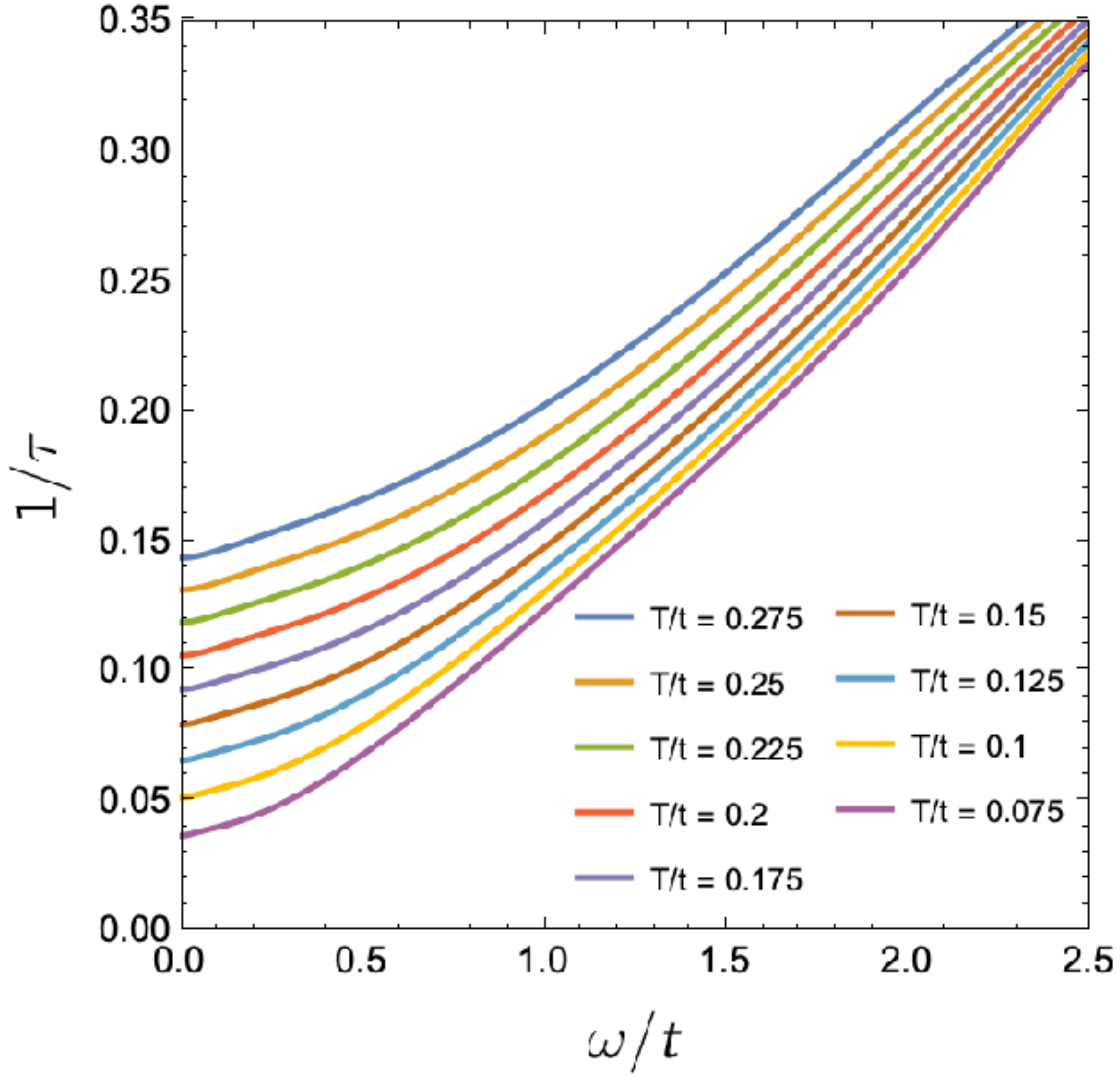

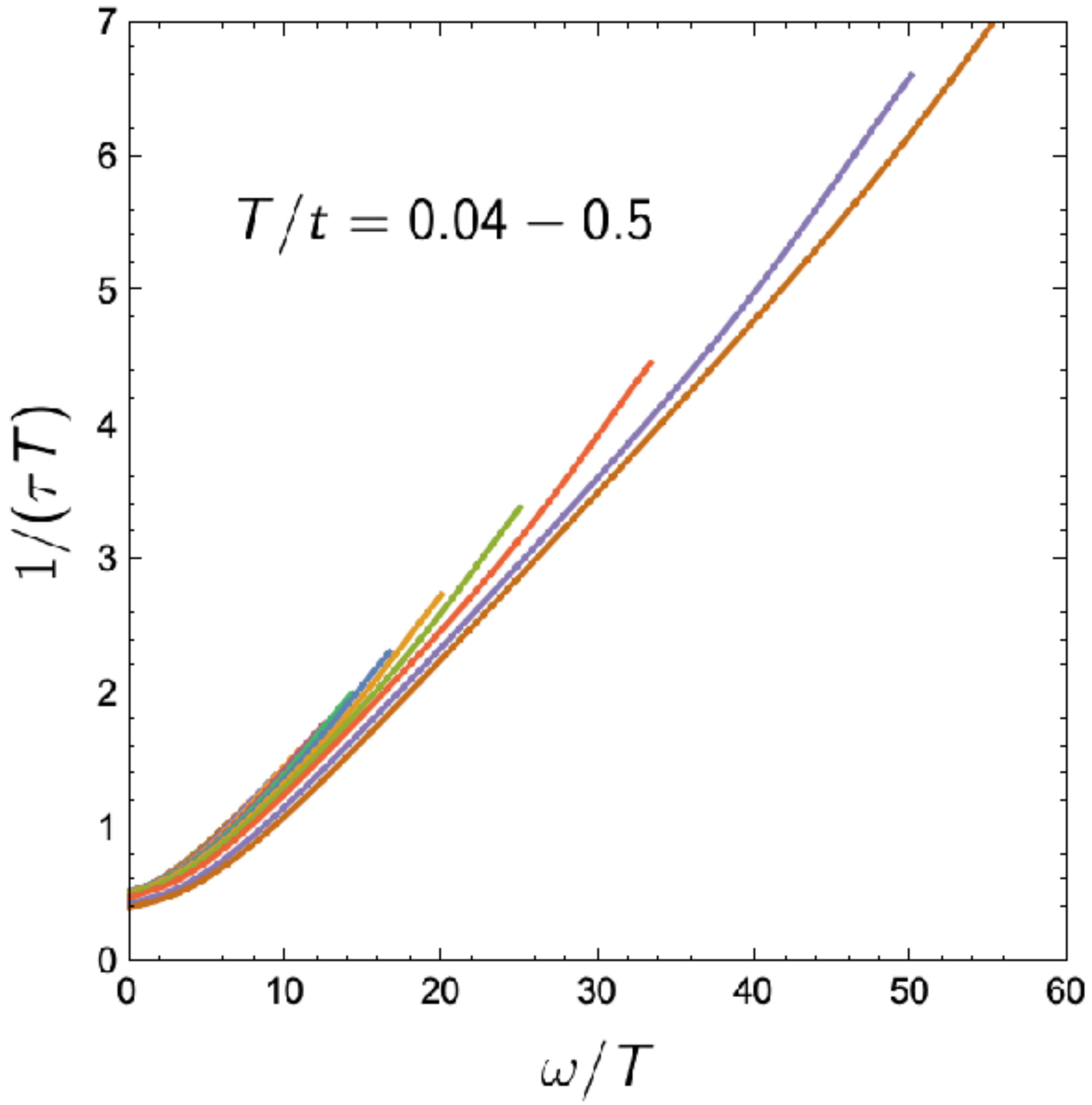

Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy $S(T \to 0) \sim T \ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Universal theory of strange metals:

Quantum phase transitions in inhomogeneous metals described by the two-dimensional Yukawa-SYK model

Theory applies for types I, II, III, with only minor differences.

Universal theory of strange metals:

Quantum phase transitions
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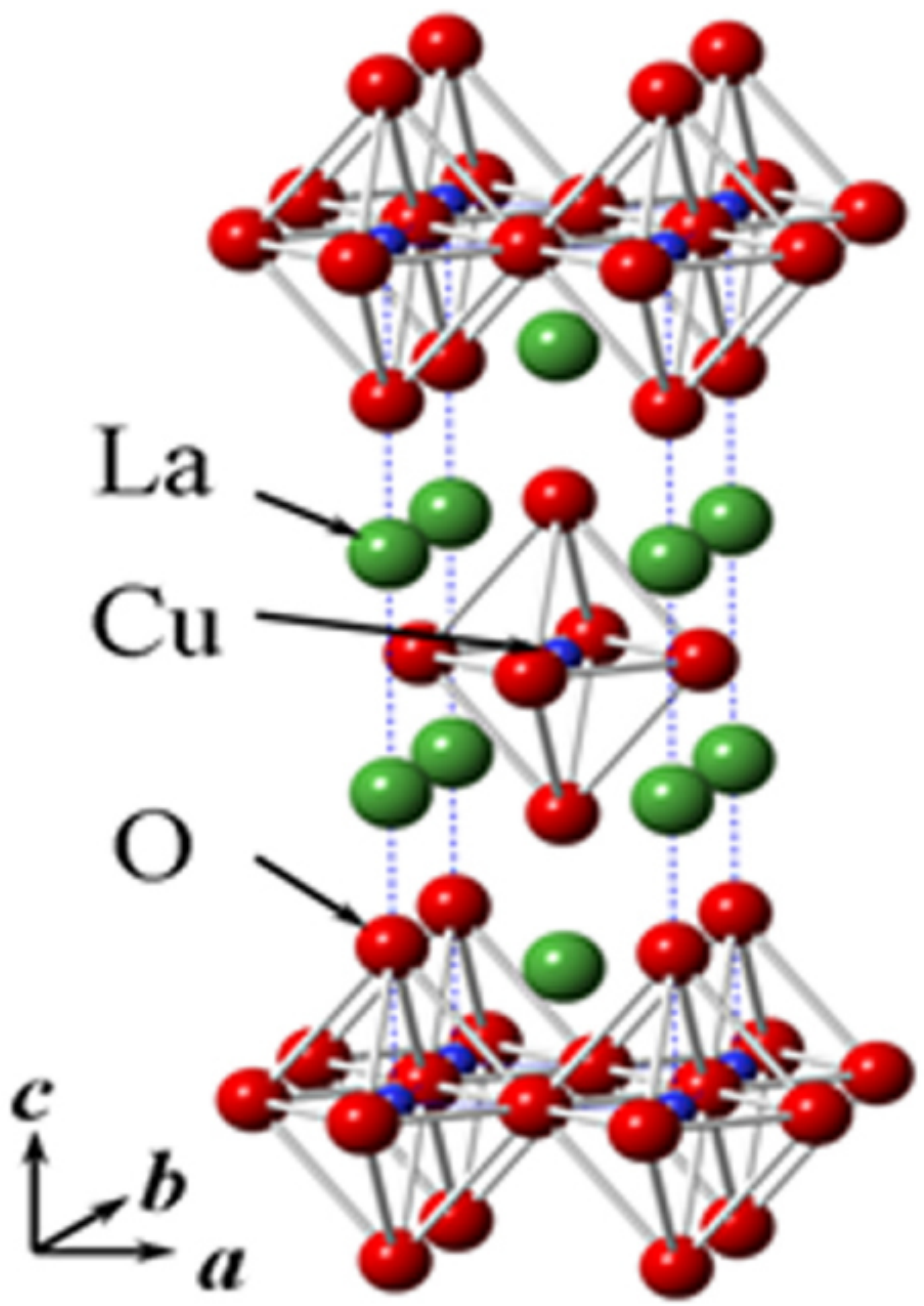
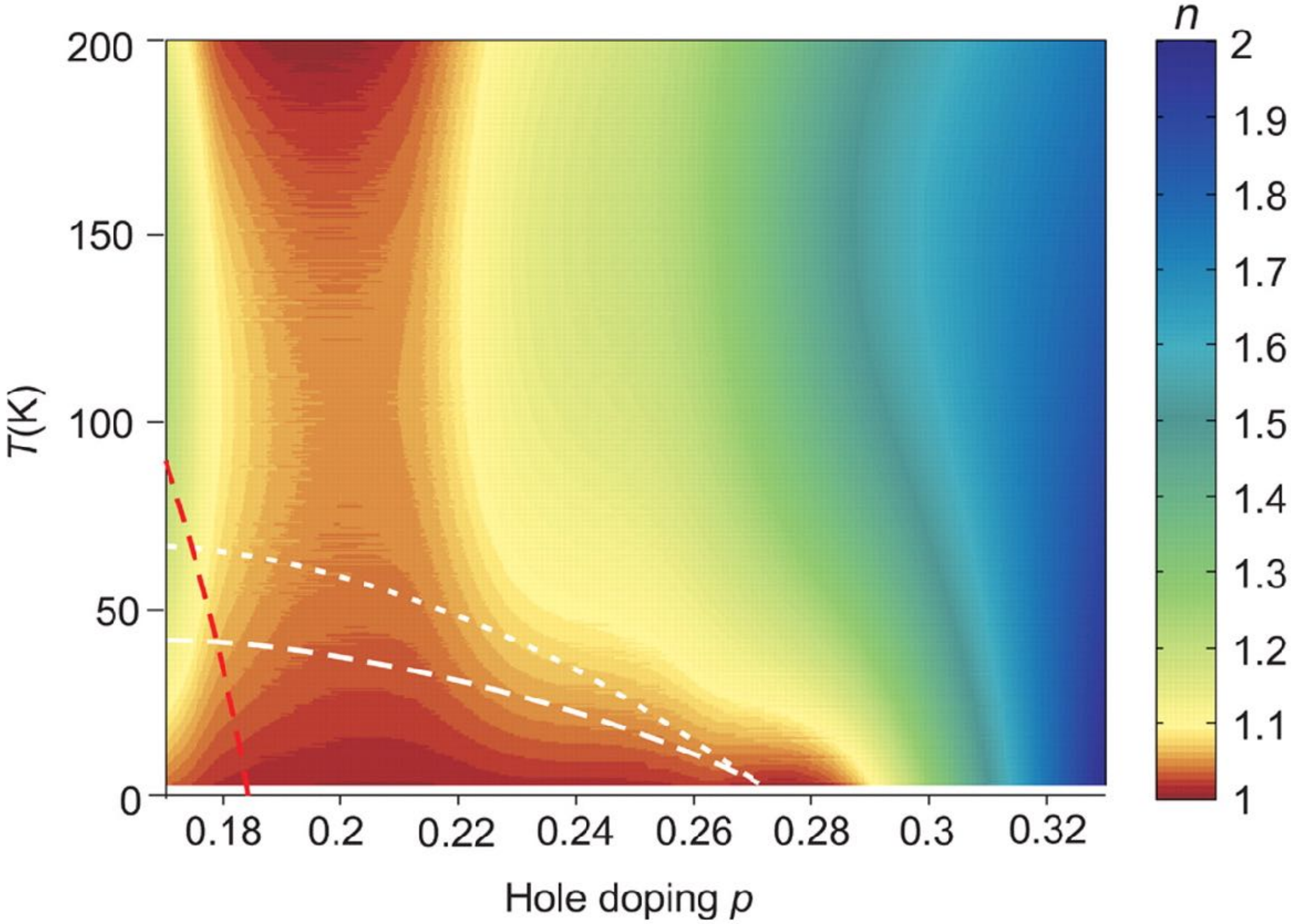
Boson localization and the "foot"

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009





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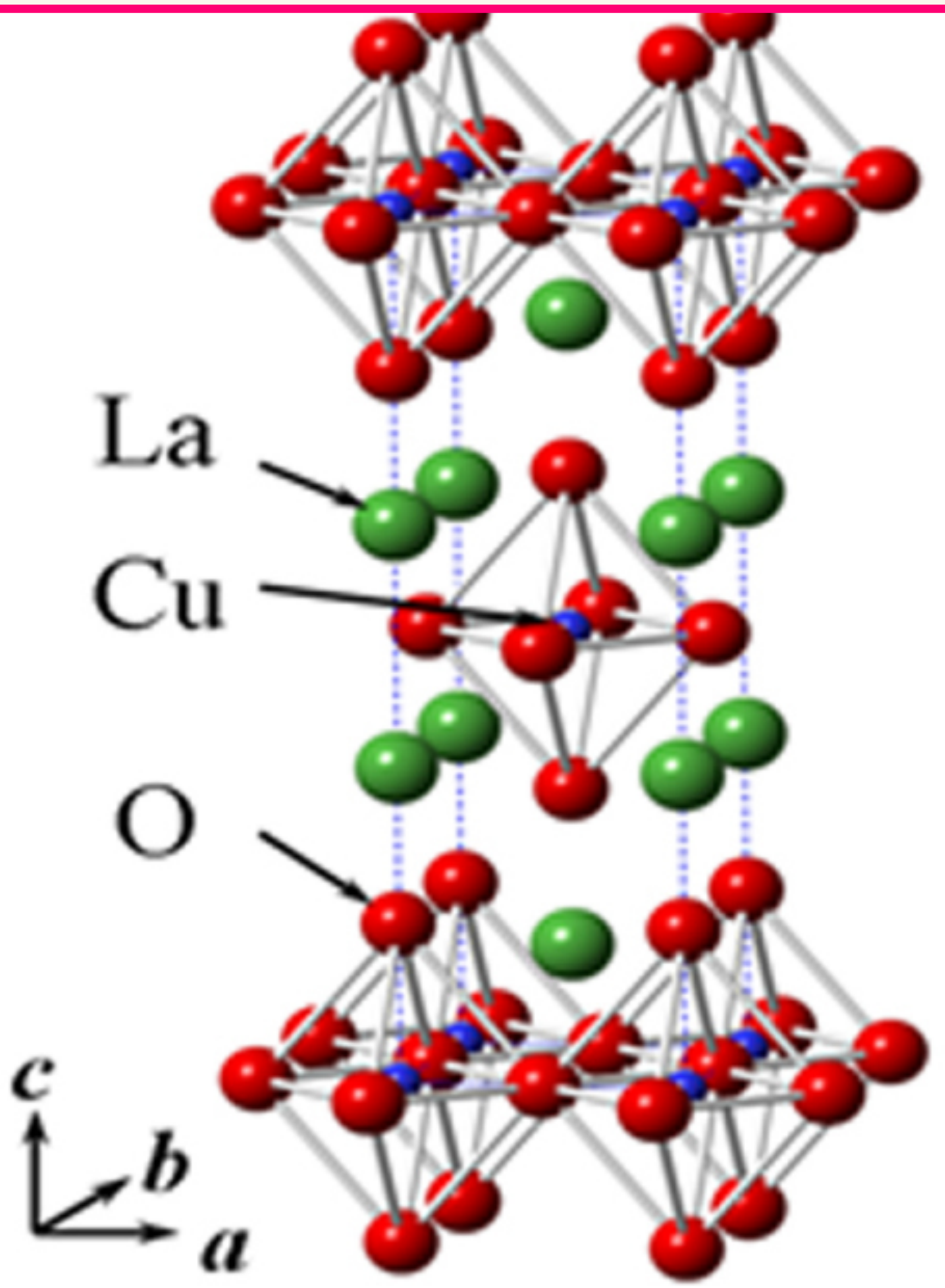
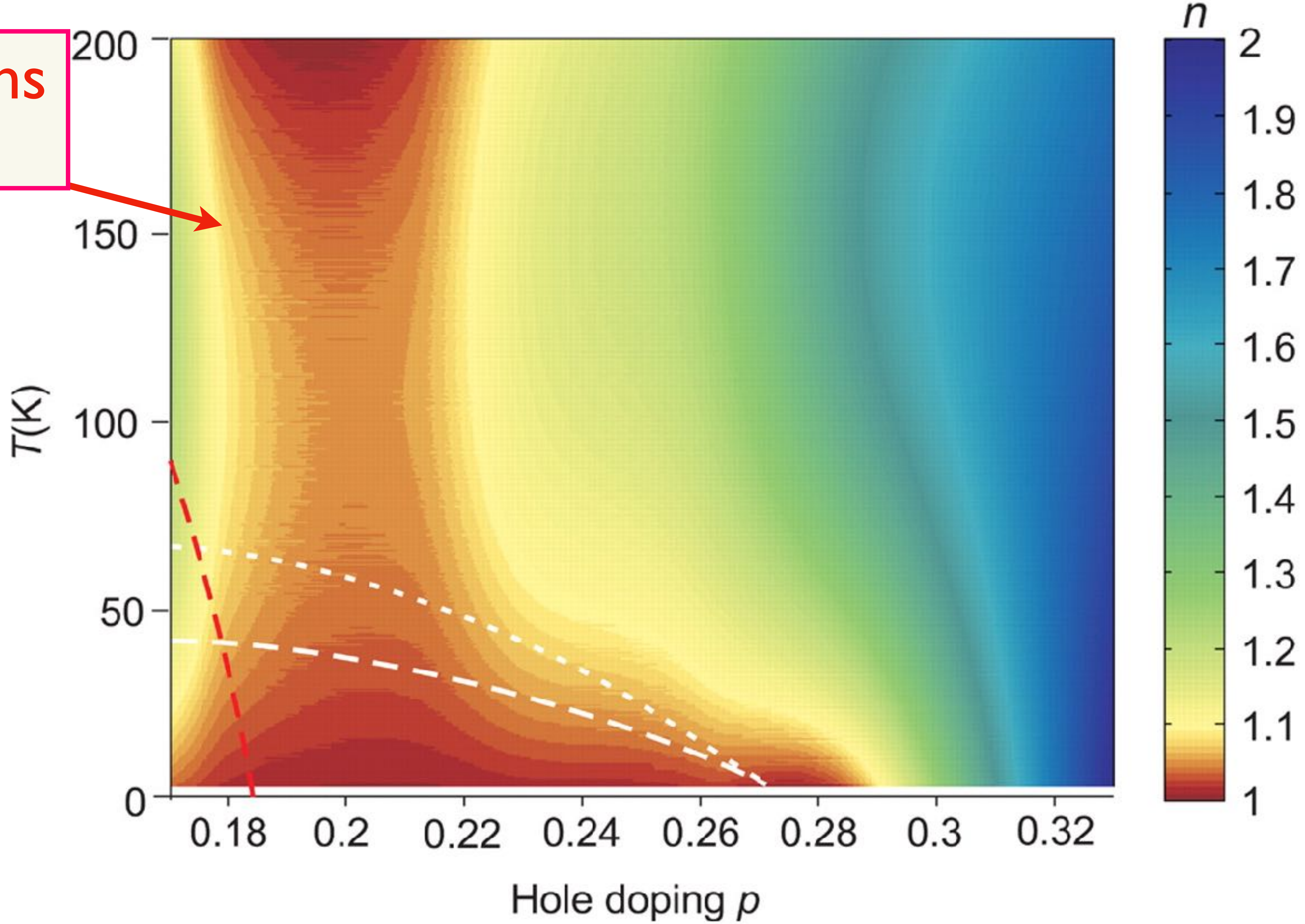
Extended bosons and fermions in "fan"





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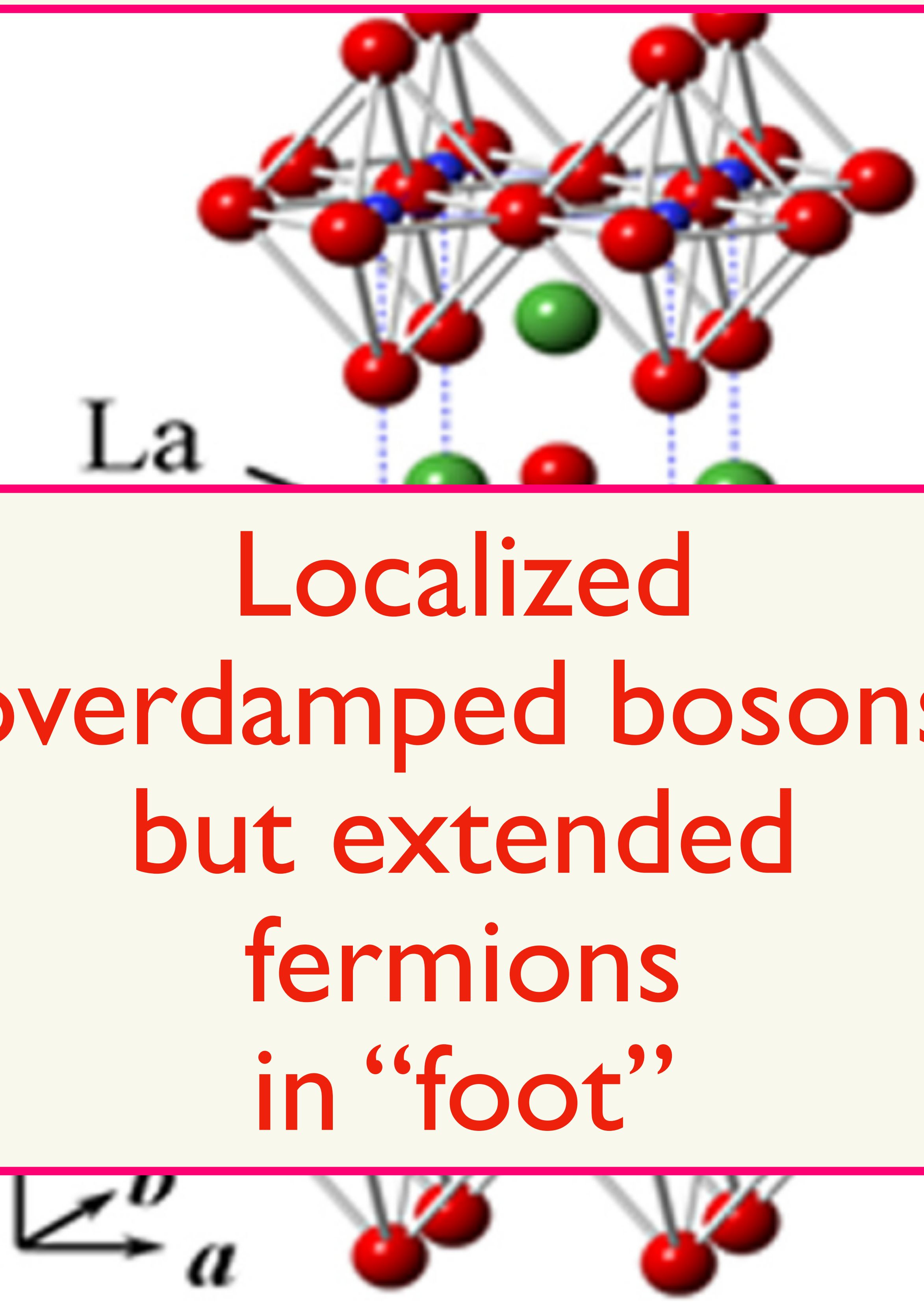
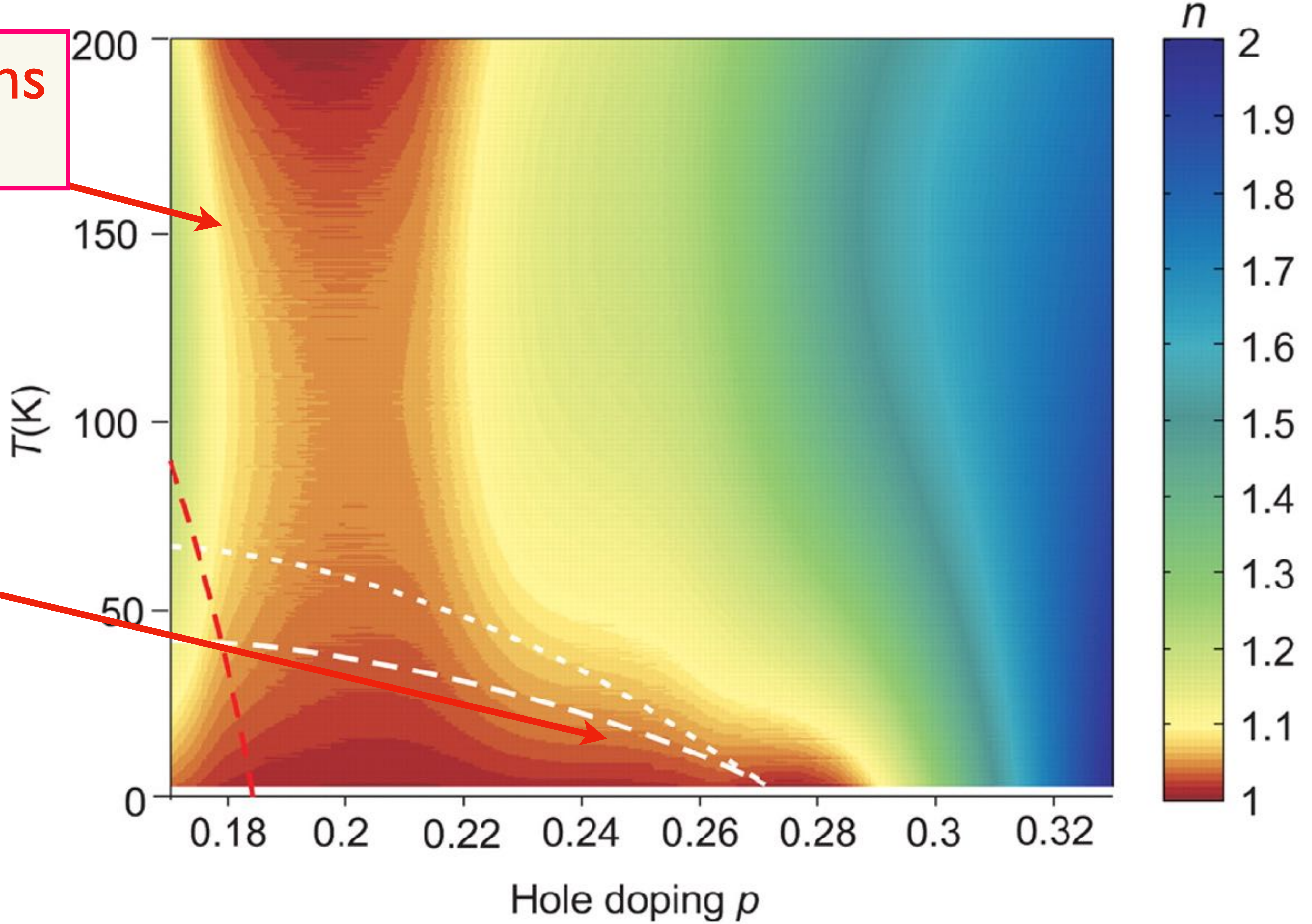
SCIENCE VOL 323 603 2009

Extended bosons and fermions in "fan"



Localized overdamped bosons, but extended fermions in "foot"



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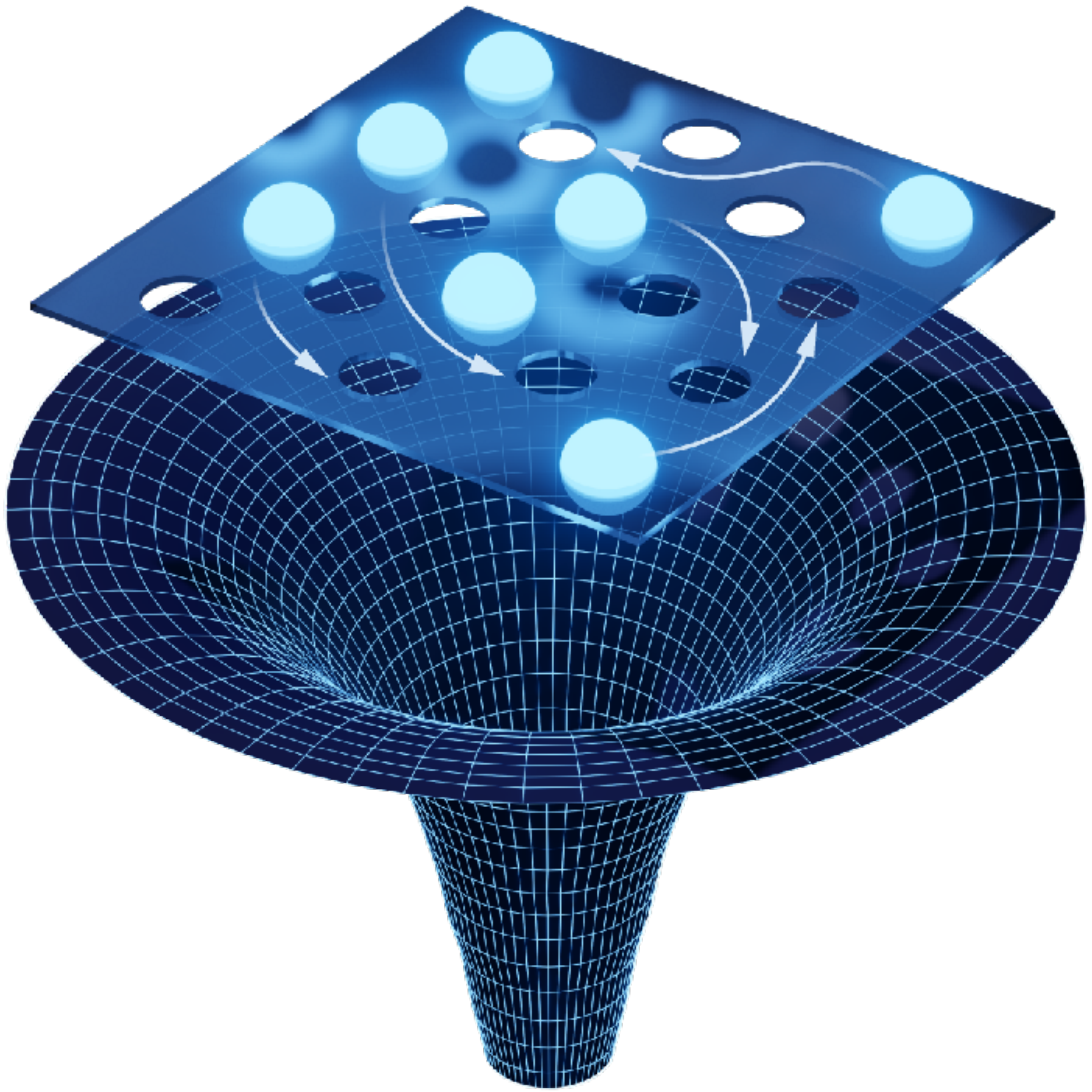

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Quantum phase transitions in inhomogeneous metals described by the 2d-YSYK model

- Universal theory of strange metals, good agreement with transport and optical data. The "foot" in phase diagram is associated with boson localization.