Magnetic quantum criticality

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$T=0$

SDW

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$$

Collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 = 0$

Non-collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 \neq 0$

$$\langle \boldsymbol{S}_j \rangle = 0$$

Pressure,
carrier concentration,….

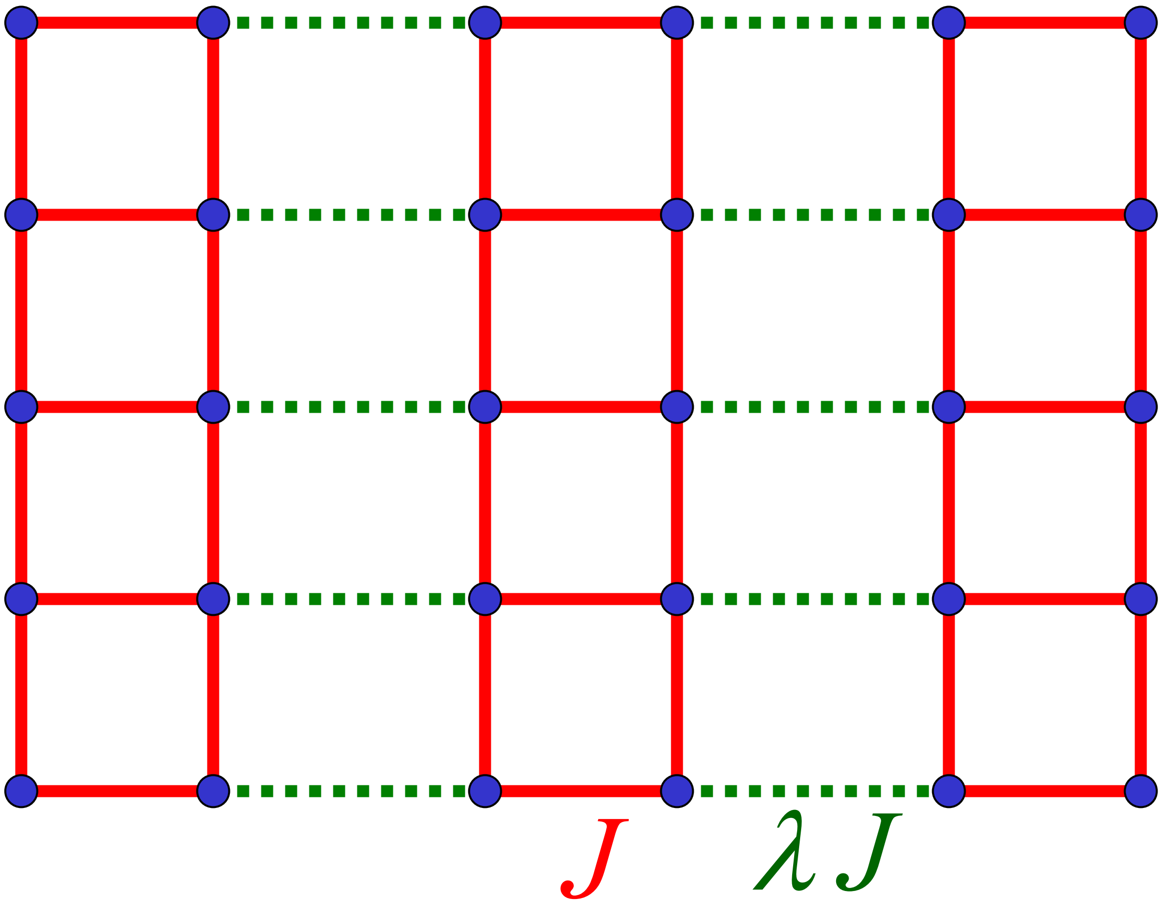
Quantum critical point

States could be either (a) insulators
(b) metals
(c) superconductors

(a) Insulators: coupled ladder antiferromagnet

N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev*. B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev*. B **65**, 014407 (2002).

S=1/2 spins on coupled 2-leg ladders



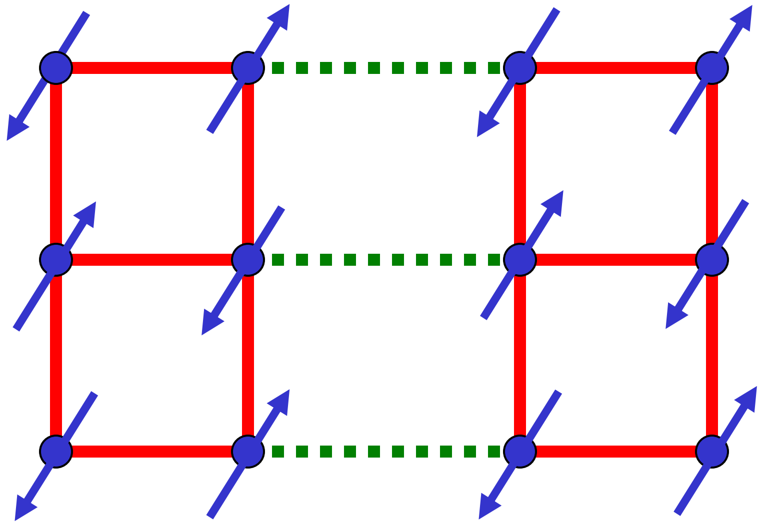
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



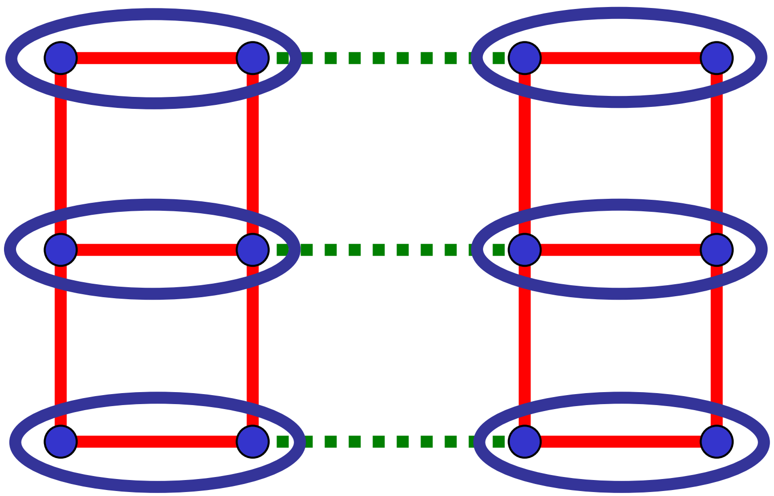
Ground state has long-range collinear magnetic (Neel) order

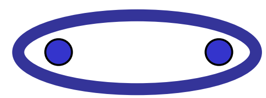
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x+i_y} N_0 \neq 0$$

Excitations: 2 spin waves $\varepsilon_p = \sqrt{c_x{}^2 p_x{}^2 + c_y{}^2 p_y{}^2}$

λ close to 0

Weakly coupled ladders



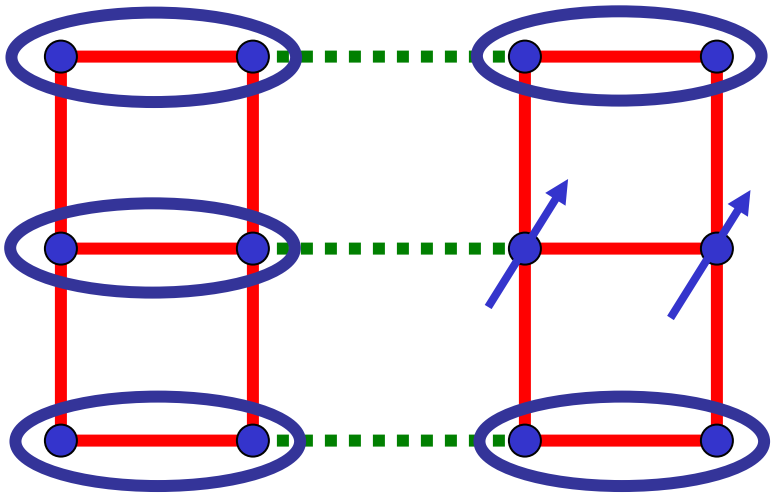
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Paramagnetic ground state

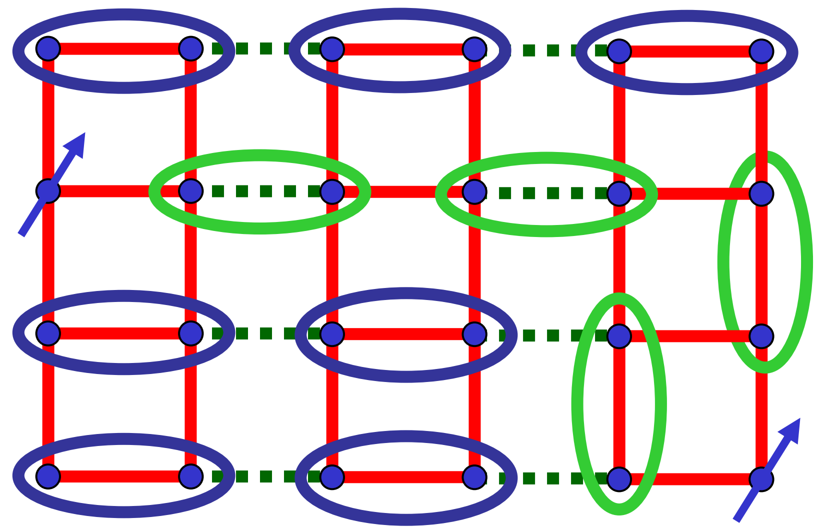
$$\left\langle \vec{S}_i \right\rangle = 0$$

λ close to 0

Excitations

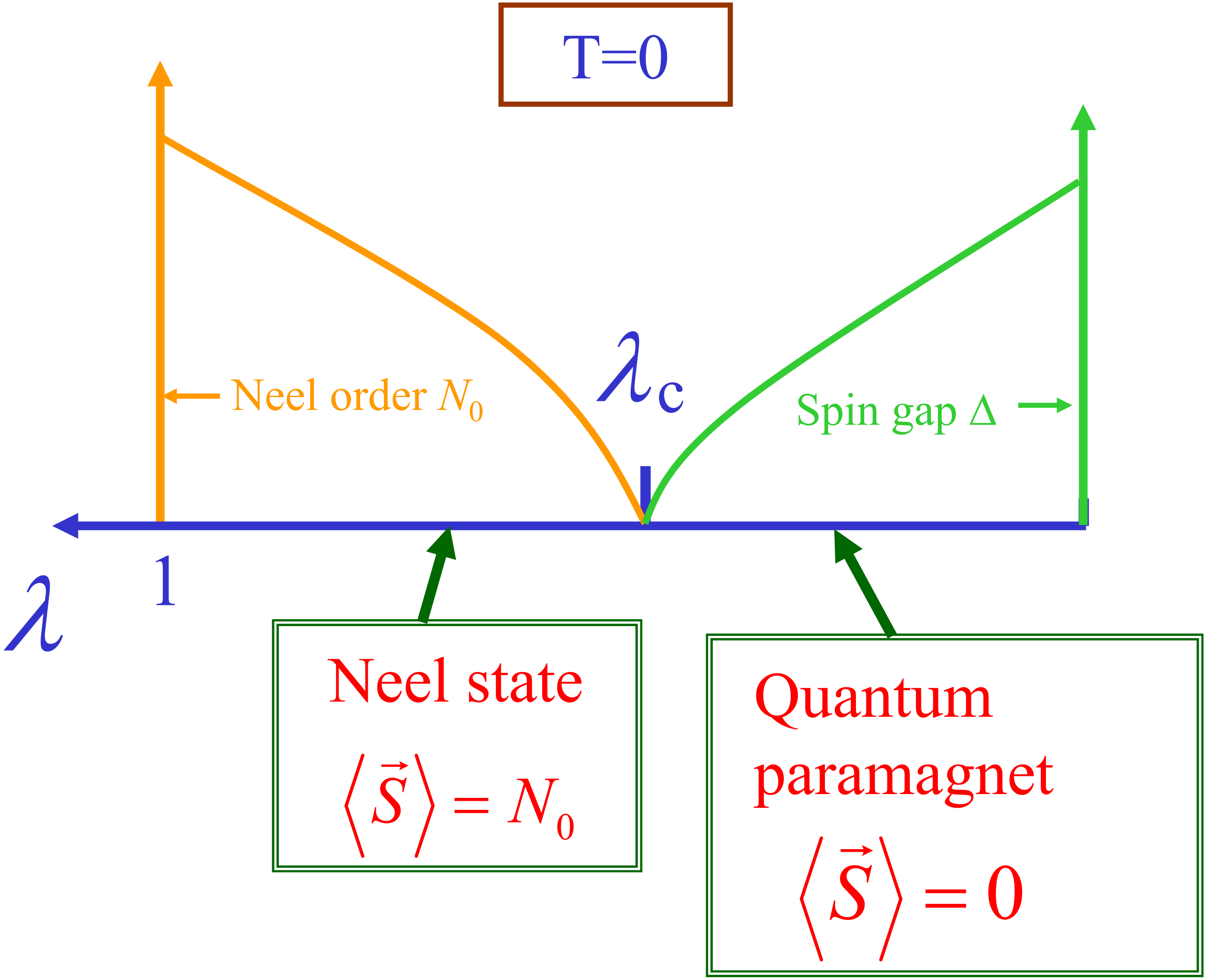


Excitation: S=1 ***exciton***
(spin collective mode)



S=1/2 spinons are ***confined*** by a linear potential.

Energy dispersion away from antiferromagnetic wavevector $\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$

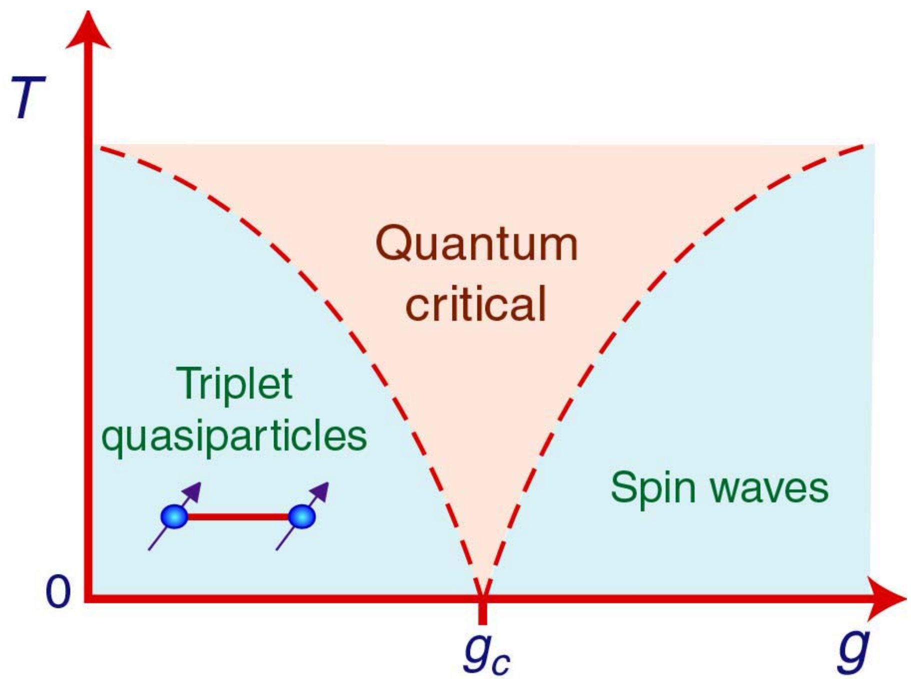

T=0
Neel order N_0
λ_c
Spin gap Δ
1
λ
Neel state
$\langle \vec{S} \rangle = N_0$
Quantum paramagnet
$\langle \vec{S} \rangle = 0$

Field theory for quantum criticality

λ close to λ_c : use "soft spin" field

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + \left(\lambda_c - \lambda\right)\phi_\alpha^2 \right) + \frac{u}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

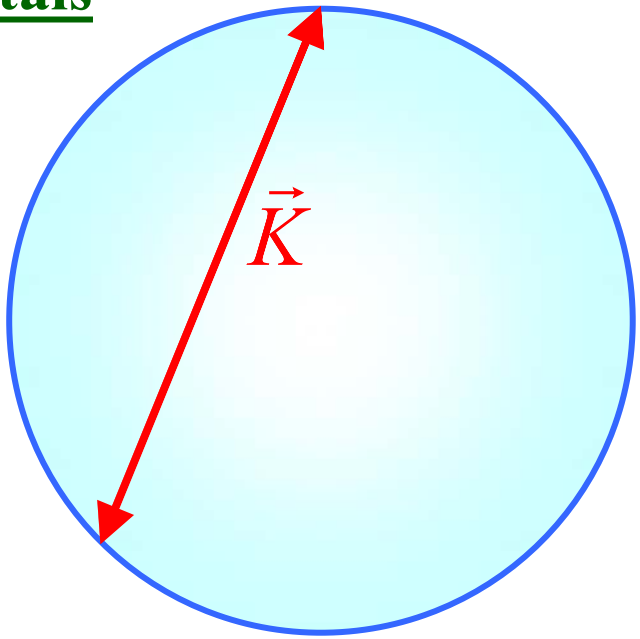


Quantum criticality described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar\omega / k_B T$

Exciton scattering amplitude is determined by $k_B T$ alone, and not by the value of microscopic coupling u

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

(b) Metals



Low energy "paramagnon" excitations near the Fermi surface

Damping by fermionic quasiparticles leads to

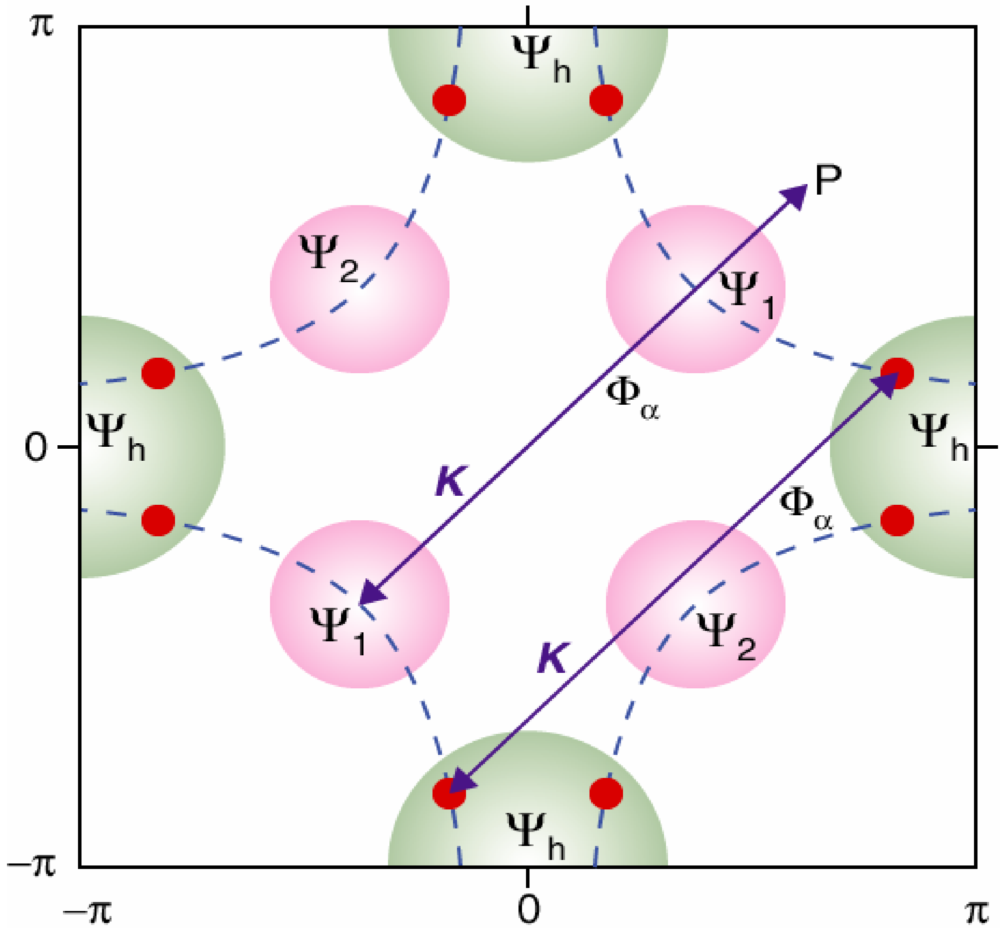
Action: $$S = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} \left|\vec{\phi}(q,\omega)\right|^2 \left(q^2 + |\omega| + \Gamma(\delta, T)\right)$$

Characteristic paramagnon energy at finite temperature $\Gamma(0,T) \sim T^p$ with $p > 1$.

Arises from non-universal corrections to scaling, generated by $\vec{\phi}^4$ term.

J.A. Hertz, *Phys. Rev.* B **14**, 1165 (1976), A.J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

(c) Superconductors



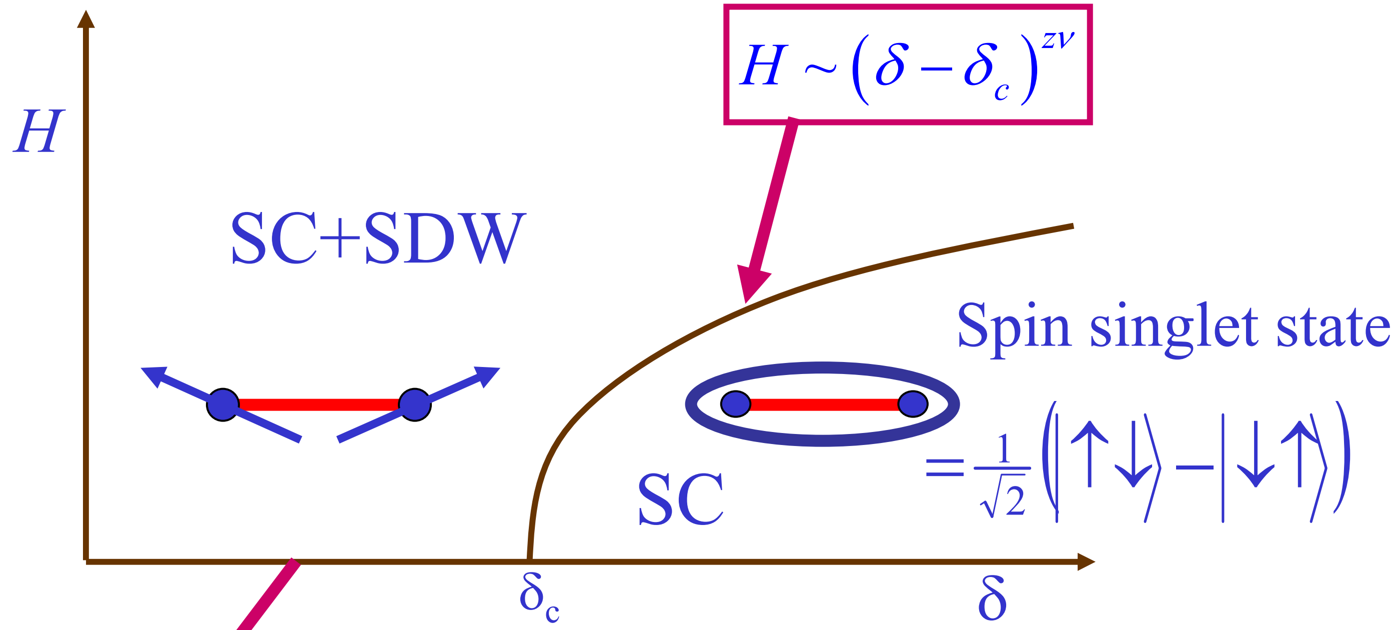
If $\vec{K}$ does not exactly connect two nodal points, critical theory is as in an insulator

Otherwise, new theory of coupled excitons and nodal quasiparticles

L. Balents, M.P.A. Fisher, C. Nayak, *Int. J. Mod. Phys.* B **12**, 1033 (1998).

Influence of a weak magnetic field

(a) Insulators: (also, double layer quantum Hall systems)



Elastic scattering intensity

$$I(H) = I(0) + a\left(\frac{H}{J}\right)^2$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

(b) Metals:

Similar, but no anomalous exponents; all corrections $\sim H^2$

(c) Superconductors:

$T=0$

The suppression of SC order appears to the SDW order as an effective δ:

$$\delta_{\text{eff}}\left(H\right)=\delta-C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

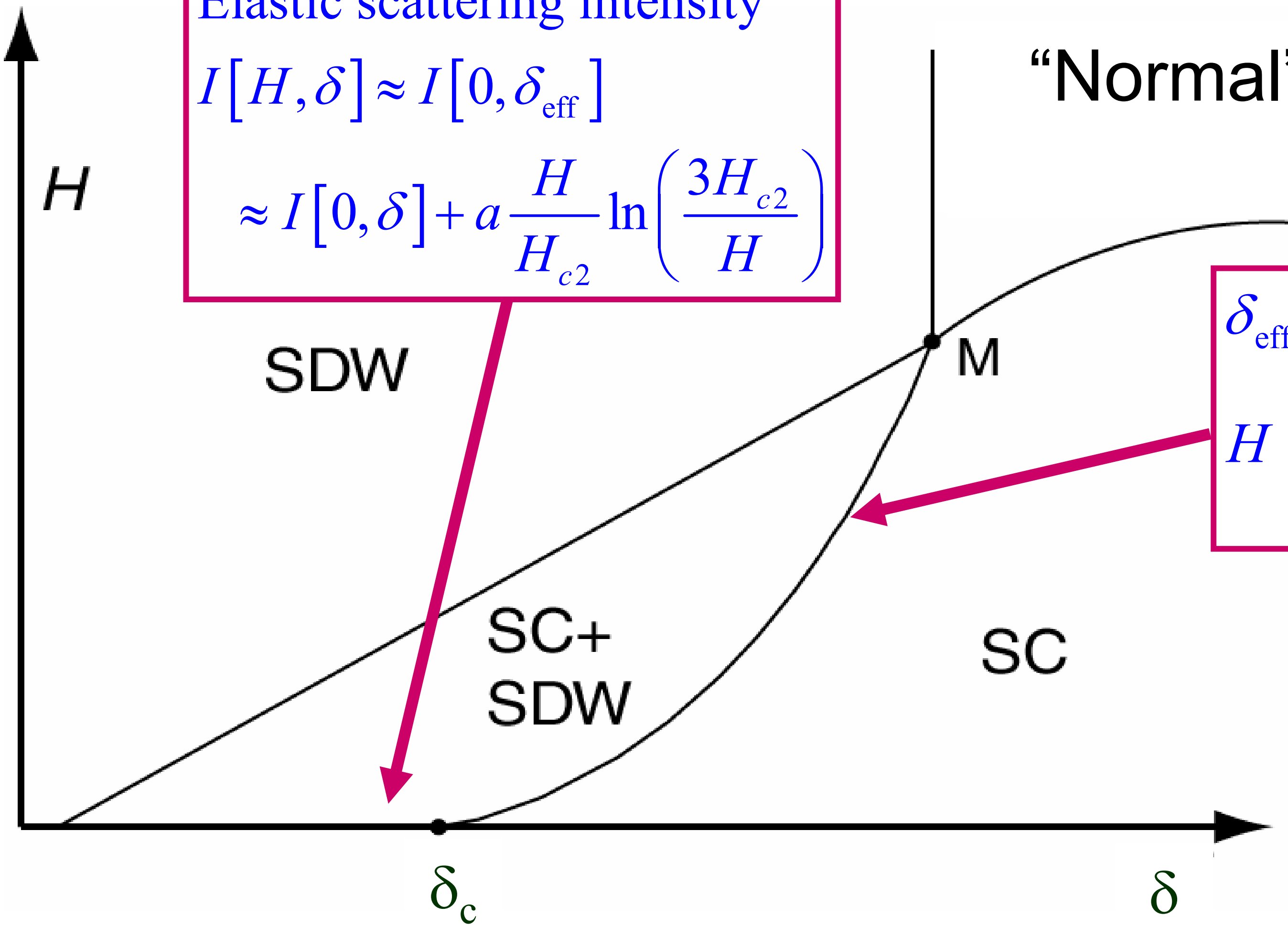
Elastic scattering intensity

$$I\left[H,\delta\right]\approx I\left[0,\delta_{\text{eff}}\right]$$

$$\approx I\left[0,\delta\right]+a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$\delta_{\text{eff}}\left(H\right)=\delta_c \Rightarrow$$

$$H\sim\frac{(\delta-\delta_c)}{\ln\left(1/(\delta-\delta_c)\right)}$$



E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Influence of a strong magnetic field

Metamagnetic transition: change in character of average (ferromagnetic) moment

- Conventional SDW order: metamagnetic transition is generically first order, and second order transition requires an additional tuning parameter.

- "Exotic" order parameters: metamagnetic transitions can be generically second order.