# Fermi surfaces and gauge-gravity duality

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Lecture notes
arXiv:1010.0682
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# Compressible quantum matter

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There are only a few established examples of such phases in condensed matter physics.
However, they appear naturally as duals of gravitational theories, and we want to interpret them in the gauge theory.

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All known examples of such phases have a *Fermi Surface*

(even in systems with only bosons in the Hamiltonian)

# The Fermi surface

This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge $\mathcal{Q}$.

$$G^{-1}_{\text{fermion}}(k = k_F, \omega = 0) = 0.$$

**Luttinger relation:** The toal "volume (area)" $\mathcal{A}$ enclosed by Fermi surfaces of fermions carrying charge $\mathcal{Q}$ is equal to $\langle \mathcal{Q} \rangle$. This is a *key* constraint which allows extrapolation from weak to strong coupling.

# Examples of compressible phases and their Fermi surfaces

1. The Fermi liquid (FL)

2. Fermions coupled to gauge fields

3. Fermion-boson mixtures

4. The fractionalized Fermi liquid (FL*)

5. Theories similar to $\mathcal{N} = 4$ SYM

6. Theories similar to ABJM

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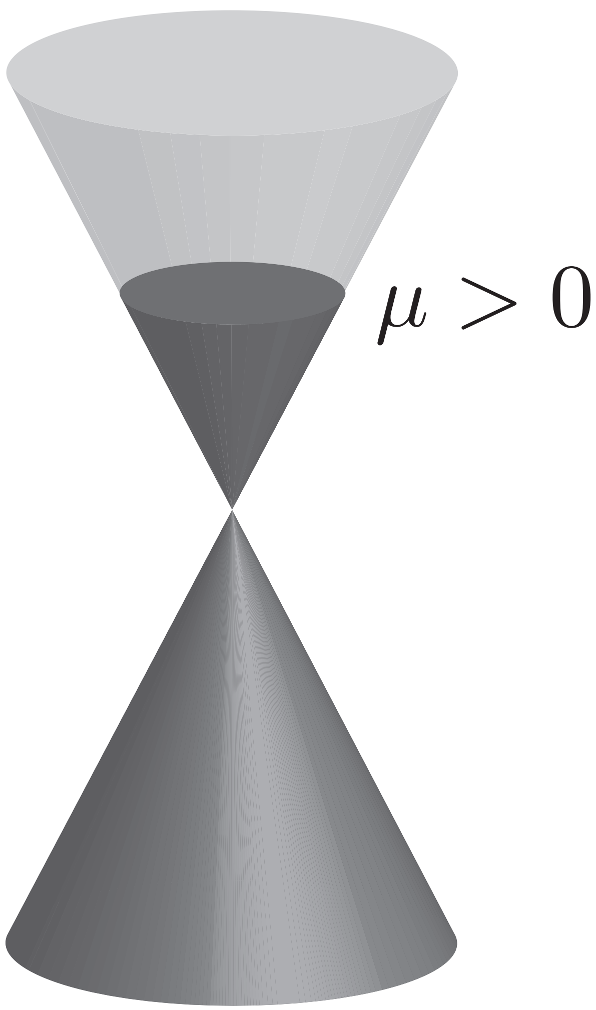
# The Fermi Liquid (FL)

Most common example: electrons with short-range interactions (or screened long-range interactions), which are adiabatically connected to the non-interacting limit. The electron Green's function $G_f$ has a pole which crosses zero energy at $k = k_F$, and the Fermi surface has the same area as the non-interacting case.

$$\mathcal{L} = \overline{f}\left(\partial_a - \mu\delta_{at}\right)\gamma^a f + 4 \text{ Fermi terms}$$



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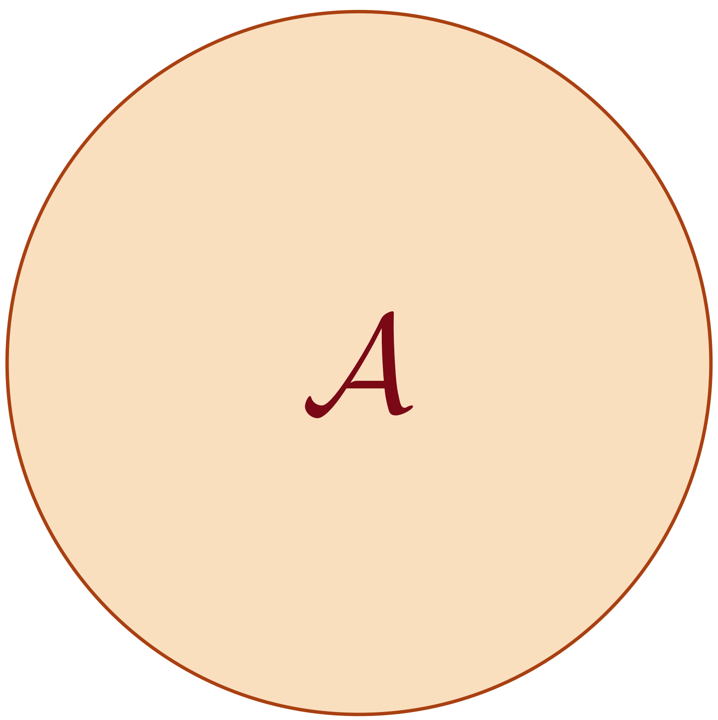
$$G_f = \frac{1}{\omega - v_F(k - k_F) + i\omega^2}$$



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S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
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- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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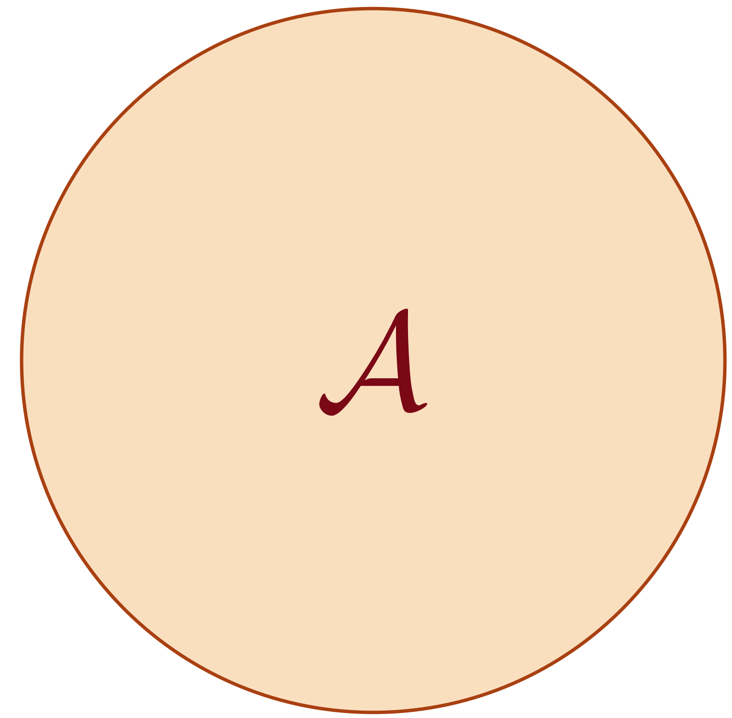


$$\mathcal{A} = \langle \overline{f} \gamma^t f \rangle = \langle \mathcal{Q} \rangle$$

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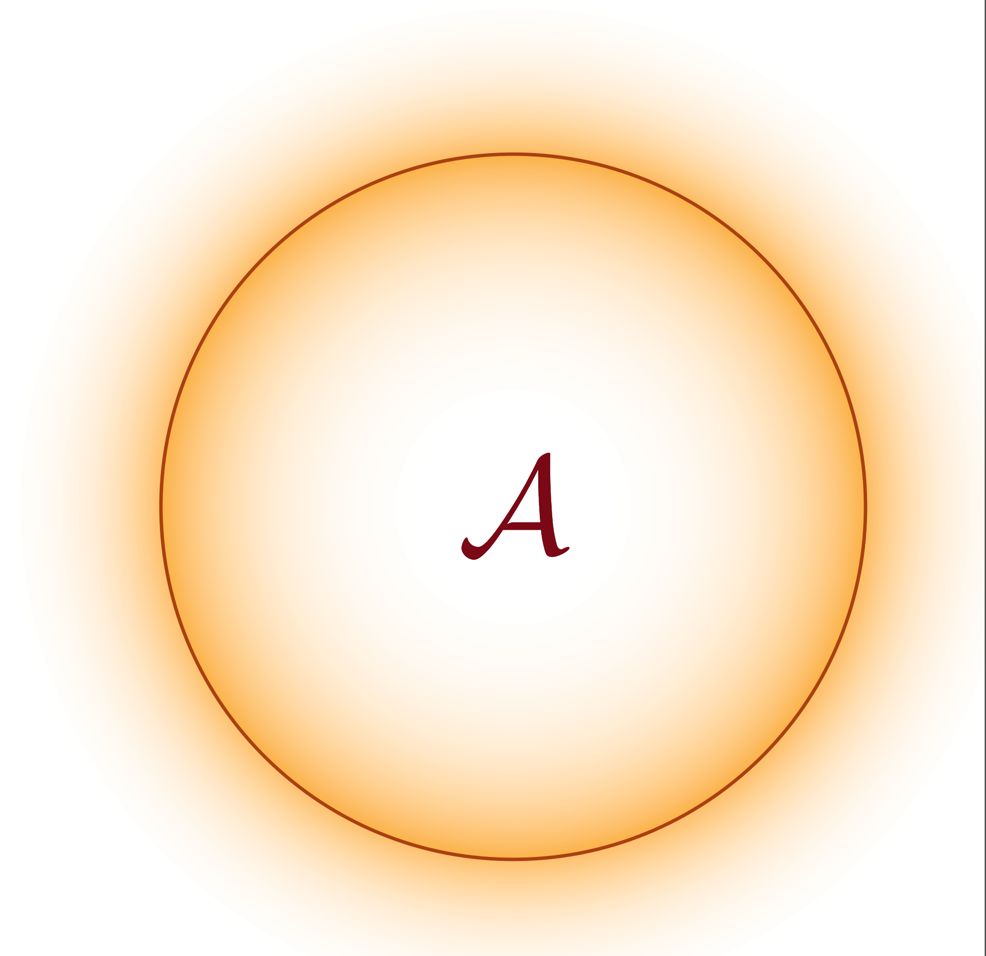


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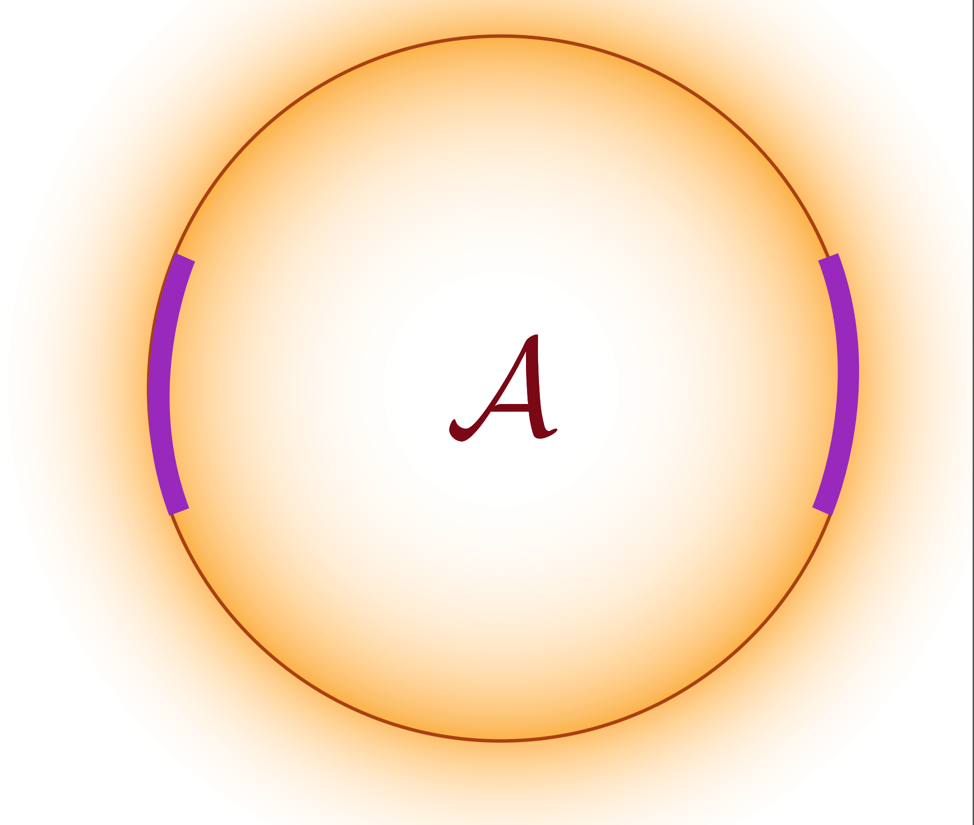


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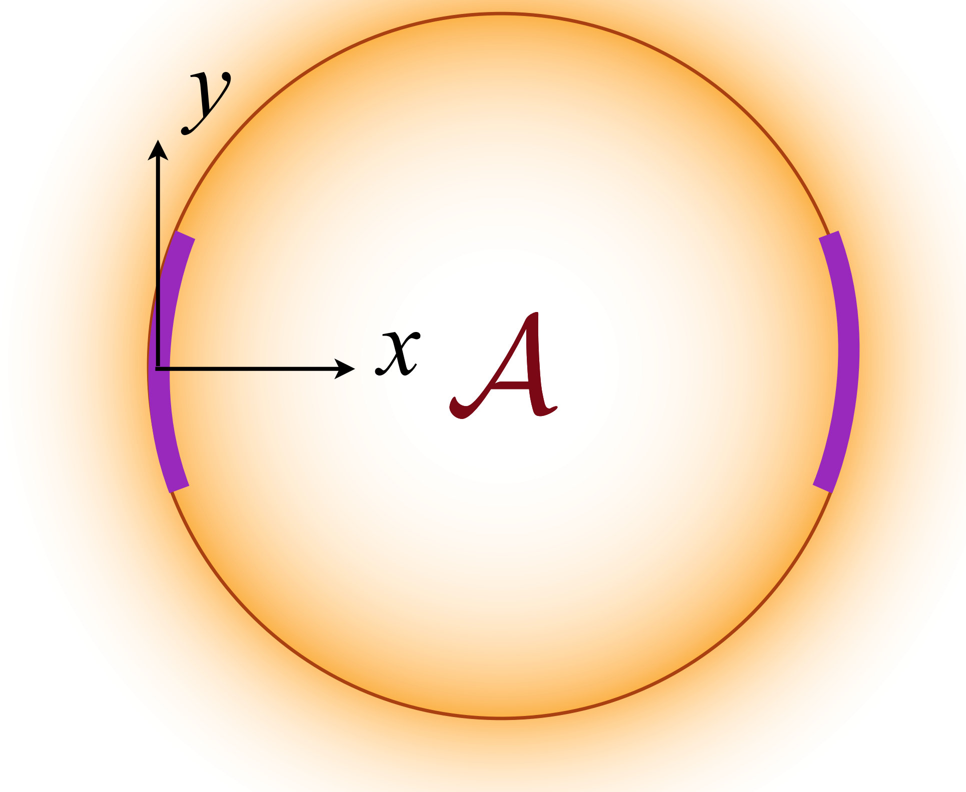
- The singularity of the Green's function upon approaching the Fermi surface is described by the scaling form

$$G_f^{-1} = q^{1-\eta} F(\omega / q^{z/2})$$

where $q_x = k_x - k_F$, $q_y = k_y$, and $q = q_x + q_y^2$, and $\eta$ and $z$ are anomalous exponents.

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Consider mixture of fermions $f$ and bosons $b$.

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$$+ \left|(\partial_a - \mu_b\delta_{at})b\right|^2 + s|b|^2 + \text{short-range interactions}\ldots$$

Consider mixture of fermions $f$ and bosons $b$.
There is a $\mathrm{U}(1)\times\mathrm{U}_b(1)$ symmetry
and 2 conserved charges:

$$\mathcal{Q} = \overline{f}\gamma^t f$$
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The 2 symmetries imply 2 Luttinger constraints. However, bosons at non-zero density invariably Bose condense at $T = 0$, and so $U_b(1)$ is broken. So there is only the single constraint on the $f$ Fermi surface. This describes mixtures of $^3$He and $^4$He.

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Superfluid: $\langle b \rangle \neq 0$
$\mathrm{U}_b(1)$ broken
$\mathrm{U}(1)$ unbroken

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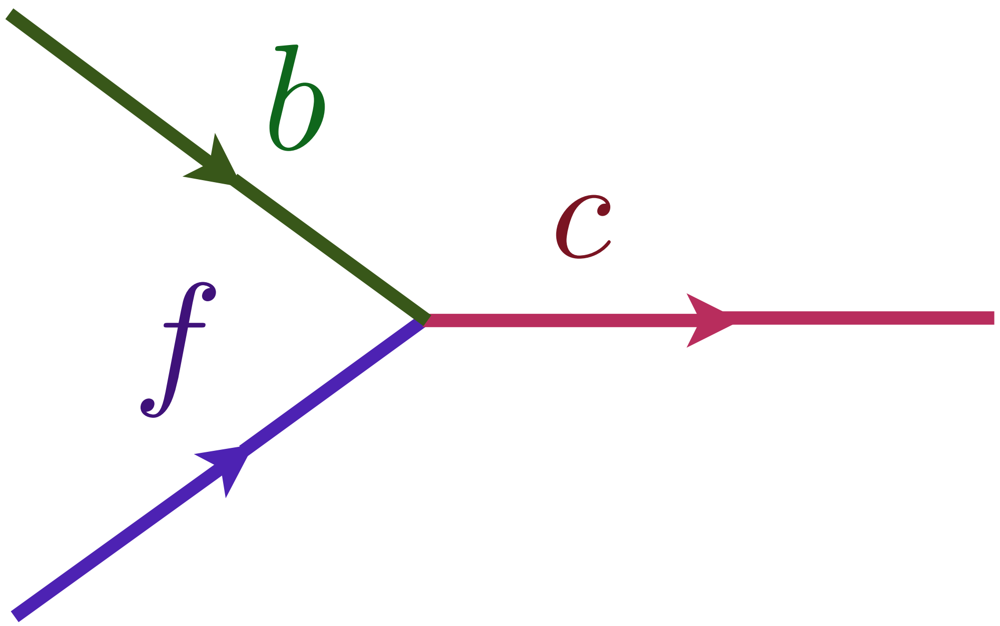
Suppose the boson $b$ fermion $f$ can bind into a 'molecule', the fermion $c$.

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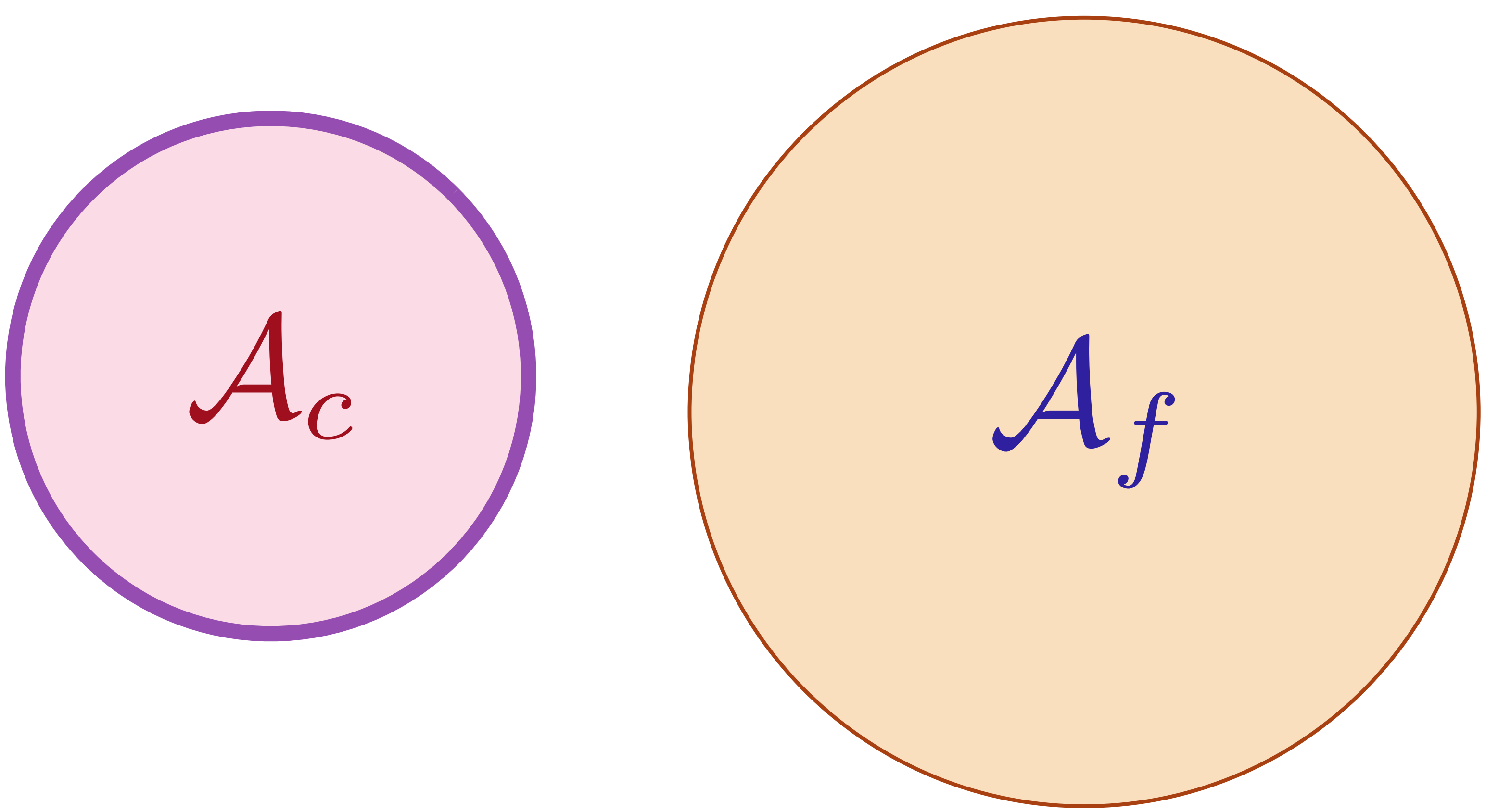
S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)

In a phase with $\mathrm{U}_b(1)$ unbroken, there is a Luttinger relation for each conserved U(1) charge. However, the boson, $b$ cannot have a Fermi surface in its Green's function, and so there is no area associated with it, although the boson density is *included* in the Luttinger relation

$$\begin{aligned} \mathcal{A}_c + \mathcal{A}_f &= \langle \bar{c}\,\gamma^t c \rangle + \langle \bar{f} \gamma^t f \rangle = \langle \mathcal{Q} \rangle \\ \mathcal{A}_c &= \langle \bar{c}\gamma^t c \rangle + \langle \bar{b}\, \overleftrightarrow{\partial_t}\, b \rangle = \langle \mathcal{Q}_b \rangle \end{aligned}$$




The $b$ bosons have bound with $f$ fermions to form $c$ "molecules"

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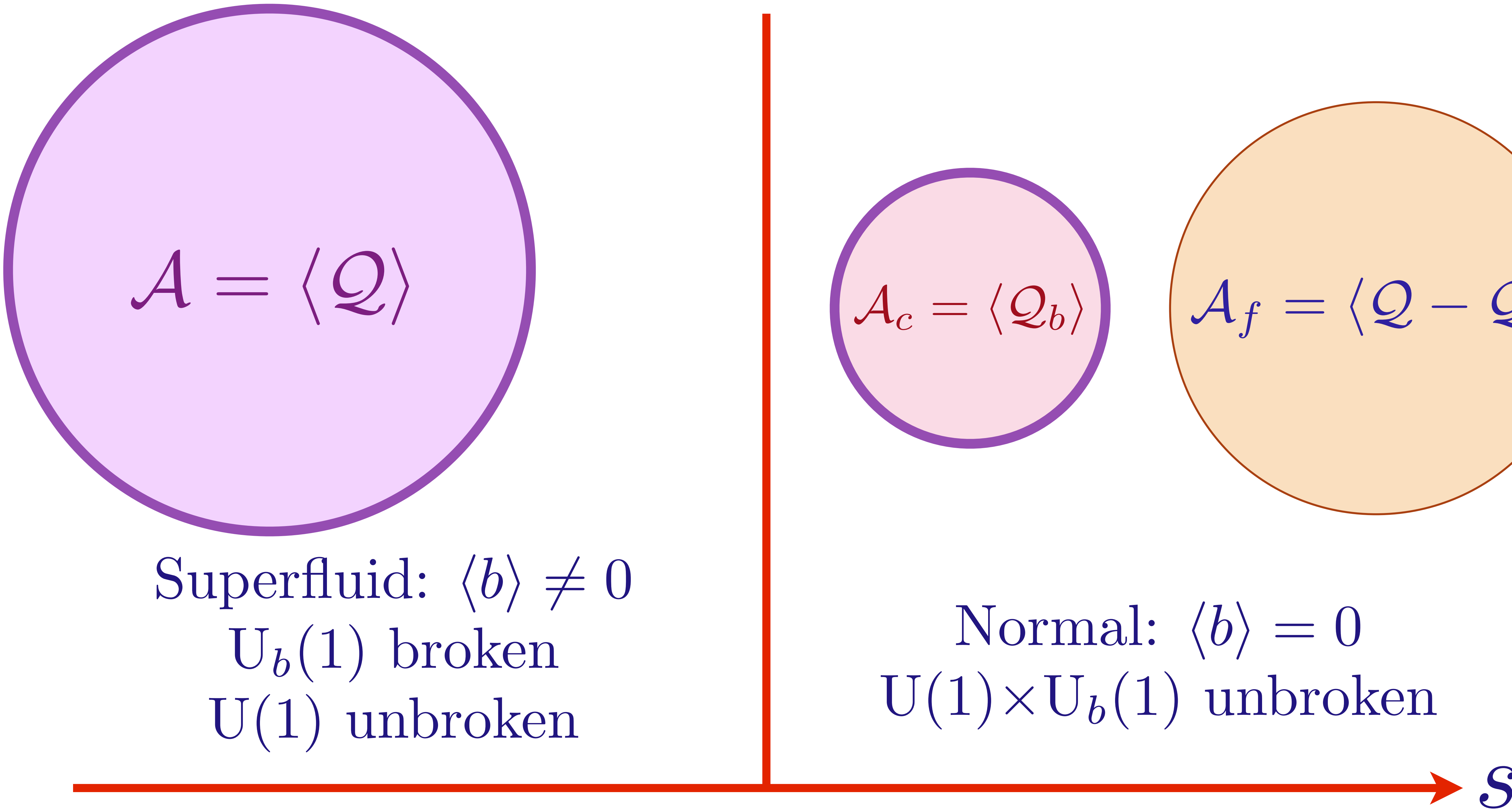
# Phase diagram of boson-fermion mixture

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(Need a background neutralizing charge)

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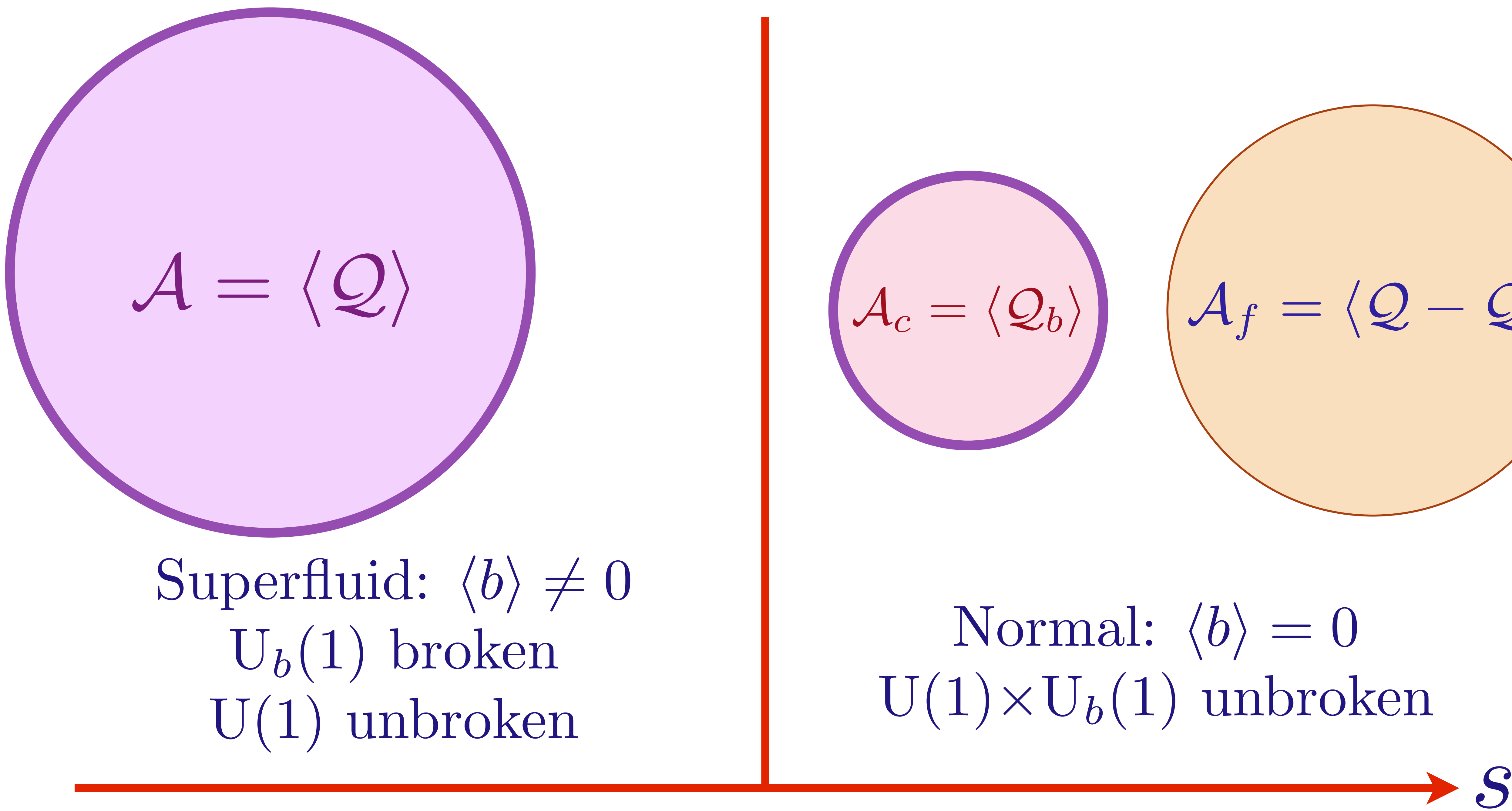


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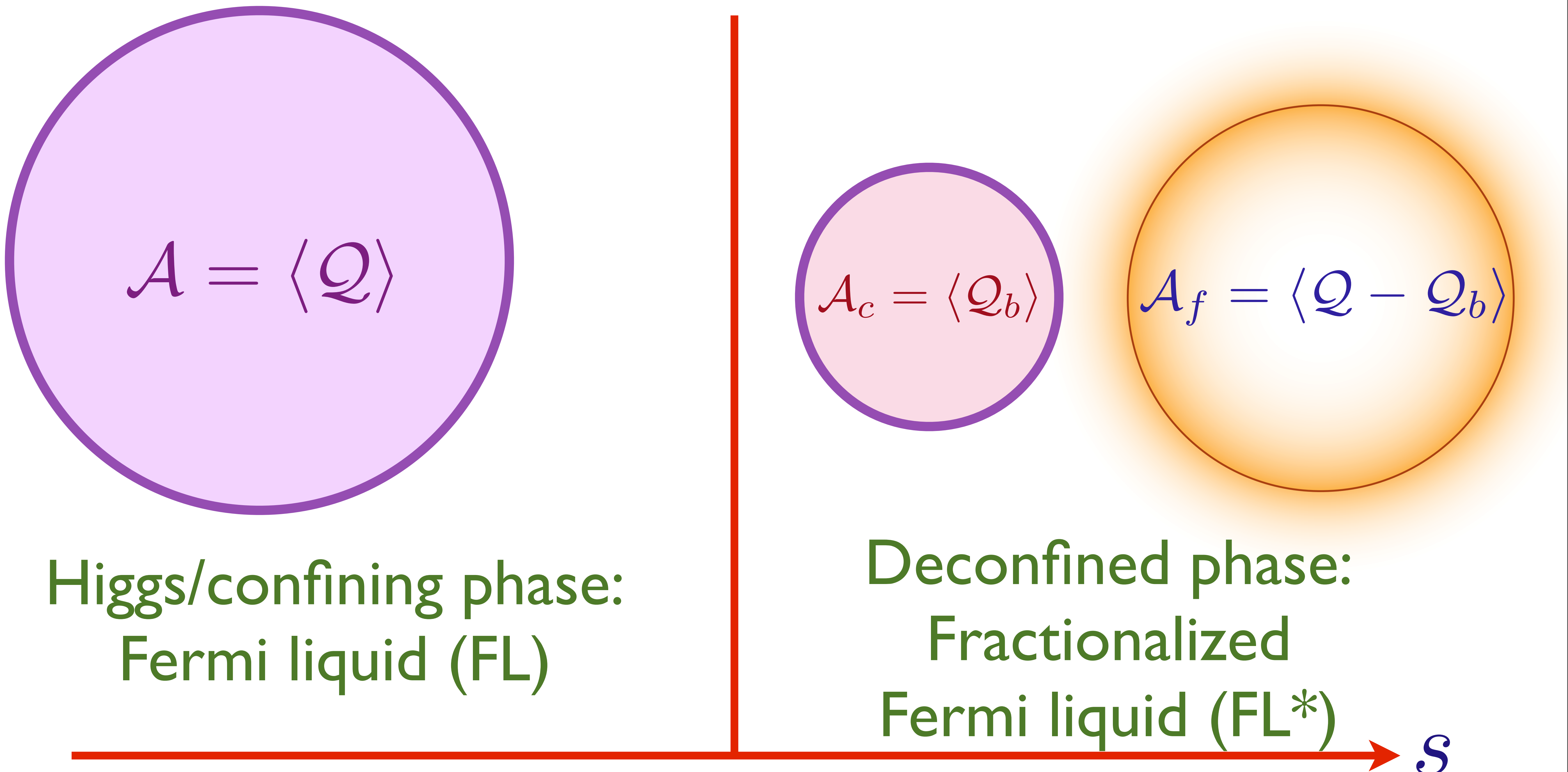


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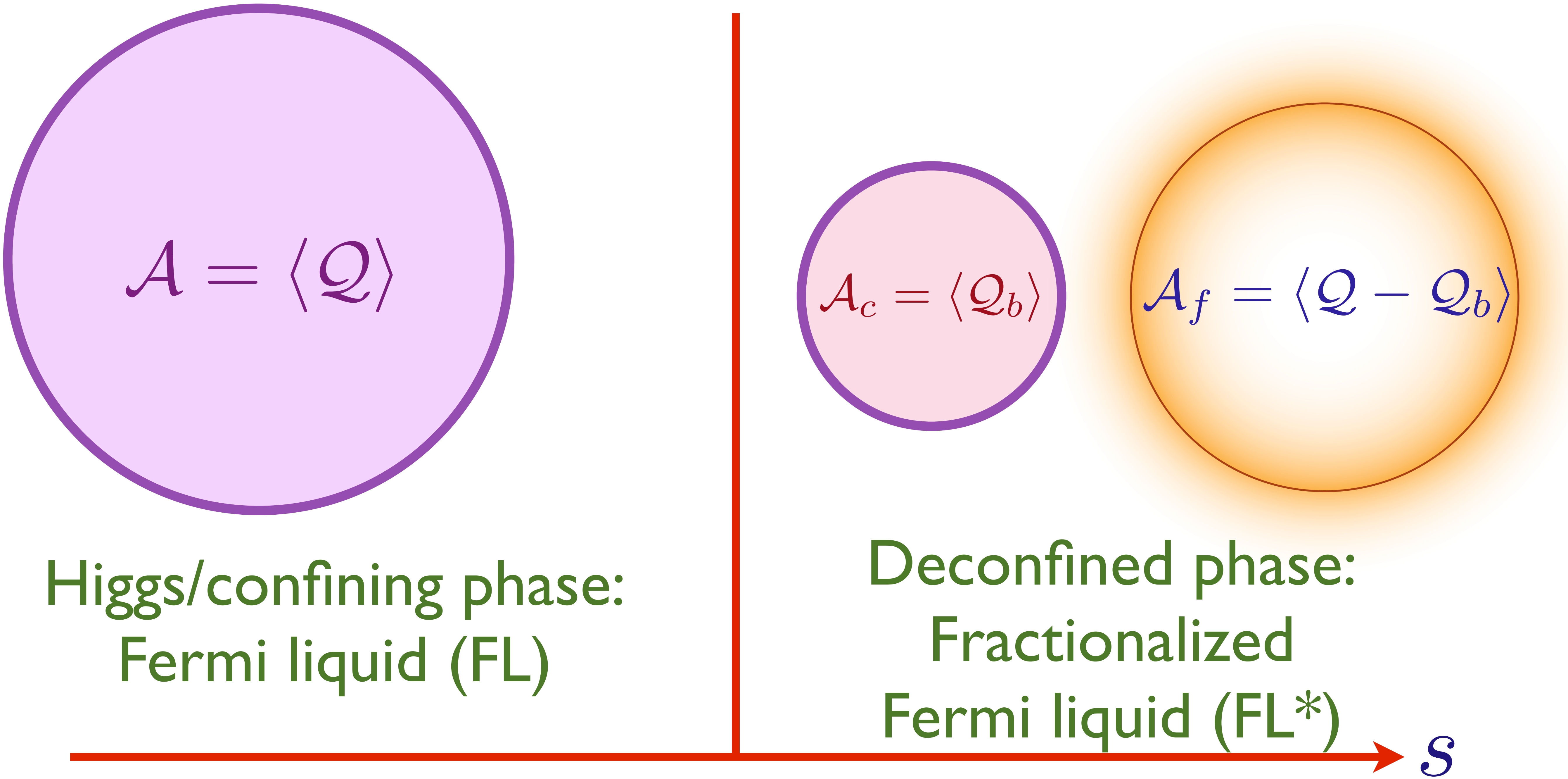


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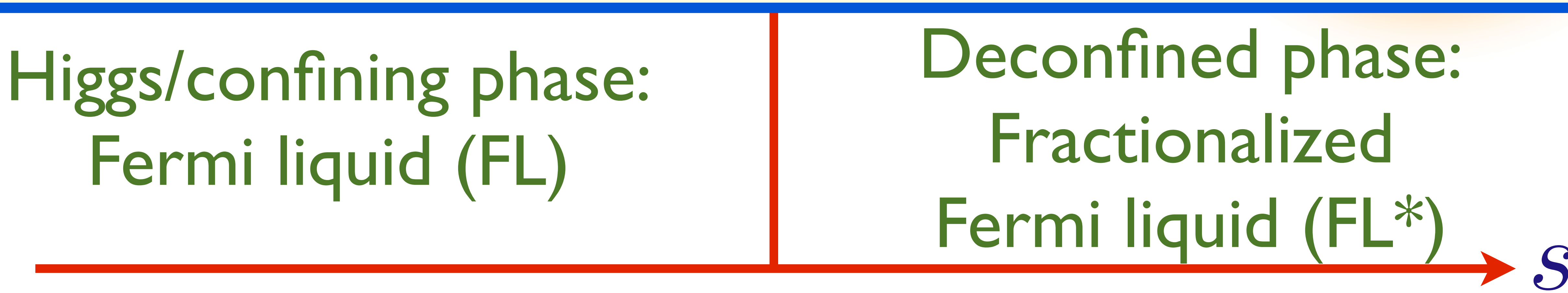
# Phase diagram of U(1) gauge theory

- FL phase: Fermi surface of gauge-neutral fermions encloses total global charge $Q$
- FL* phase: Fermi surface of gauge neutral fermions encloses only part of the global charge $Q$

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# $\mathcal{N} = 4$ SYM in D=3+1 dimensions

- SU($N$) gauge invariance and SO(6) global symmetry
- Fermions carry adjoint gauge charges and are SO(6) spinors
- Bosons carry adjoint gauge charges and are SO(6) fundamentals. Bosons are paired fermions.
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Adding a chemical potential coupling to a SO(6) charge breaks supersymmetry and SO(6) invariance

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- No supersymmetry
- Fermions, $c$, (analog of baryons), gauge-invariant bound states of $b$ and $f$, carry U(1) charge 3.

$$\mathcal{Q} = f_\alpha^\dagger f_\alpha + 2 b_\alpha^\dagger b_\alpha + 3 c^\dagger c$$

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$$H_f = \sum_{k,a} \frac{k^2}{2m_3} f_\alpha^\dagger f_\alpha - \mu \sum_k \left( \sum_\alpha f_\alpha^\dagger f_\alpha + 2 \sum_\alpha b_\alpha^\dagger b_\alpha + 3 c^\dagger c \right)$$

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# Theory similar to $\mathcal{N} = 4$ SYM in a chemical potential

$$H_f = \sum_{k,a} \frac{k^2}{2m_3} f^\dagger_\alpha f_\alpha - \mu \sum_k \left( \sum_\alpha f^\dagger_\alpha f_\alpha + 2 \sum_\alpha b^\dagger_\alpha b_\alpha + 3 c^\dagger c \right)$$

$$H_b = \sum_{k,\alpha} \left( \frac{k^2}{2m_1} + \varepsilon_1 \right) b^\dagger_\alpha b_\alpha + u \int d^d x \left( b^\dagger_\alpha b_\alpha \right)^2$$

$$H_c = \sum_k \left( \frac{k^2}{2m_2} + \varepsilon_2 \right) c^\dagger c$$

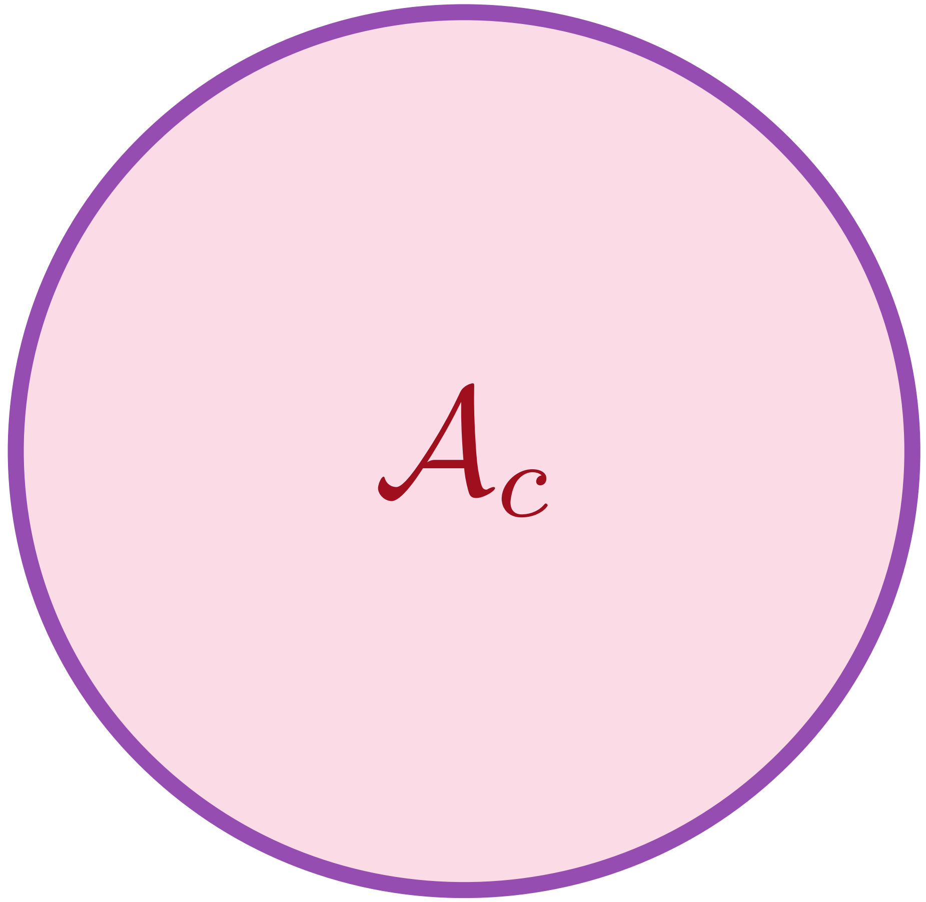
$$H_{\rm int} = g \int d^d x \left( \epsilon_{\alpha\beta\gamma} b^\dagger_\alpha f_\beta f_\gamma + \text{c.c.} \right) + \lambda \int d^d x \left( c^\dagger b_\alpha f_\alpha + \text{c.c.} \right),$$

The indices, $\alpha, \beta, \gamma = 1 \ldots N^2 - 1$, the structure constants of SU($(N)$ are $\epsilon_{\alpha\beta\gamma}$, and $\varepsilon_{1,2}$ are parameters tuning between possible phases. The SU($N$) gauge fields are not shown, and are included as usual by covariantizing derivatives.

$$\mathcal{Q} = f^\dagger_\alpha f_\alpha + 2 b^\dagger_\alpha b_\alpha + 3 c^\dagger c$$

# Phases of SYM-like theories

$$\langle b_\alpha \rangle = 0$$




$$3\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

## Fermi liquid (FL) of baryon-like particles

SU(*N*) gauge theory is in confining phase

# Phases of SYM-like theories

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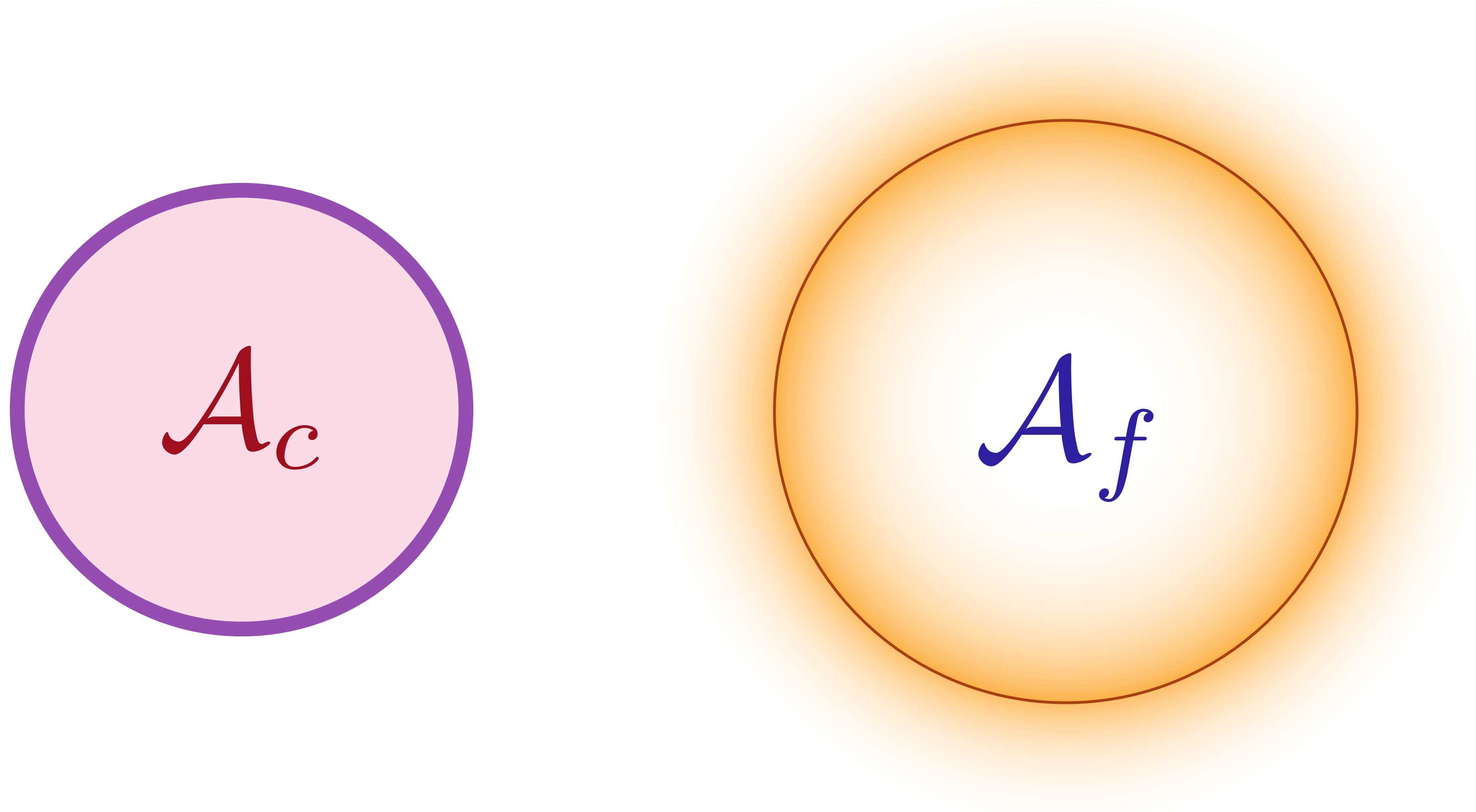


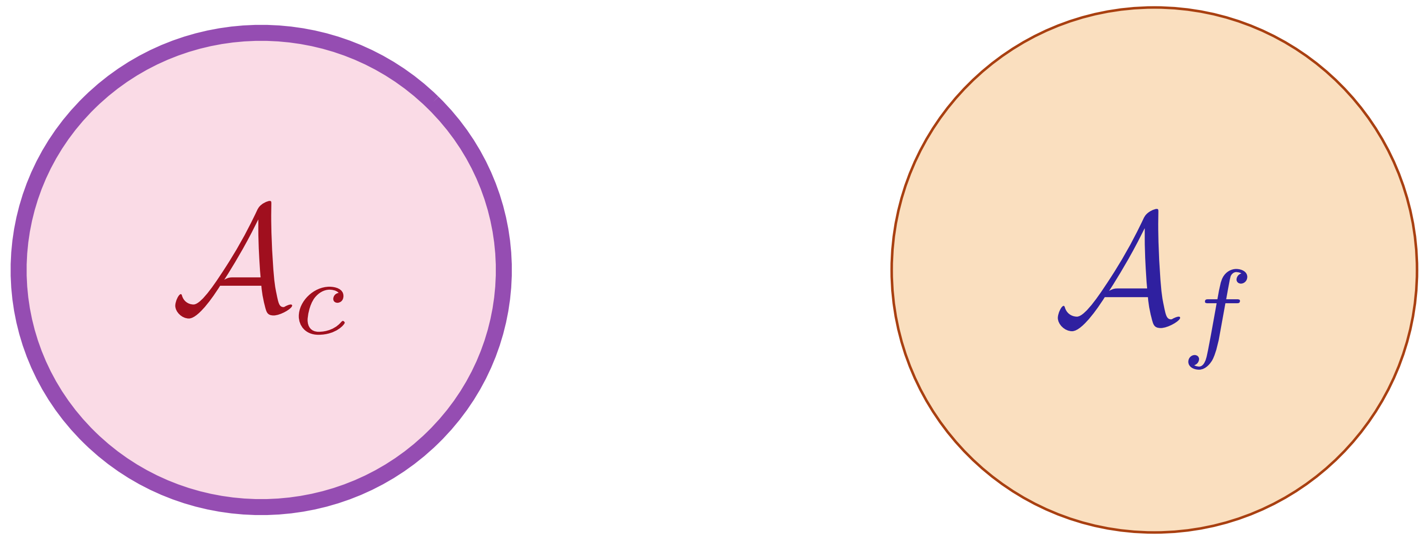

$$3\mathcal{A}_c + (N^2 - 1)\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

## Fractionalized Fermi liquid (FL*)

SU($N$) gauge theory is in deconfined phase

# Phases of SYM-like theories

$$\langle b_\alpha \rangle \neq 0$$




No constraint on Fermi surface areas

## Color Superconductor

SU(*N*) gauge theory is in Higgs phase

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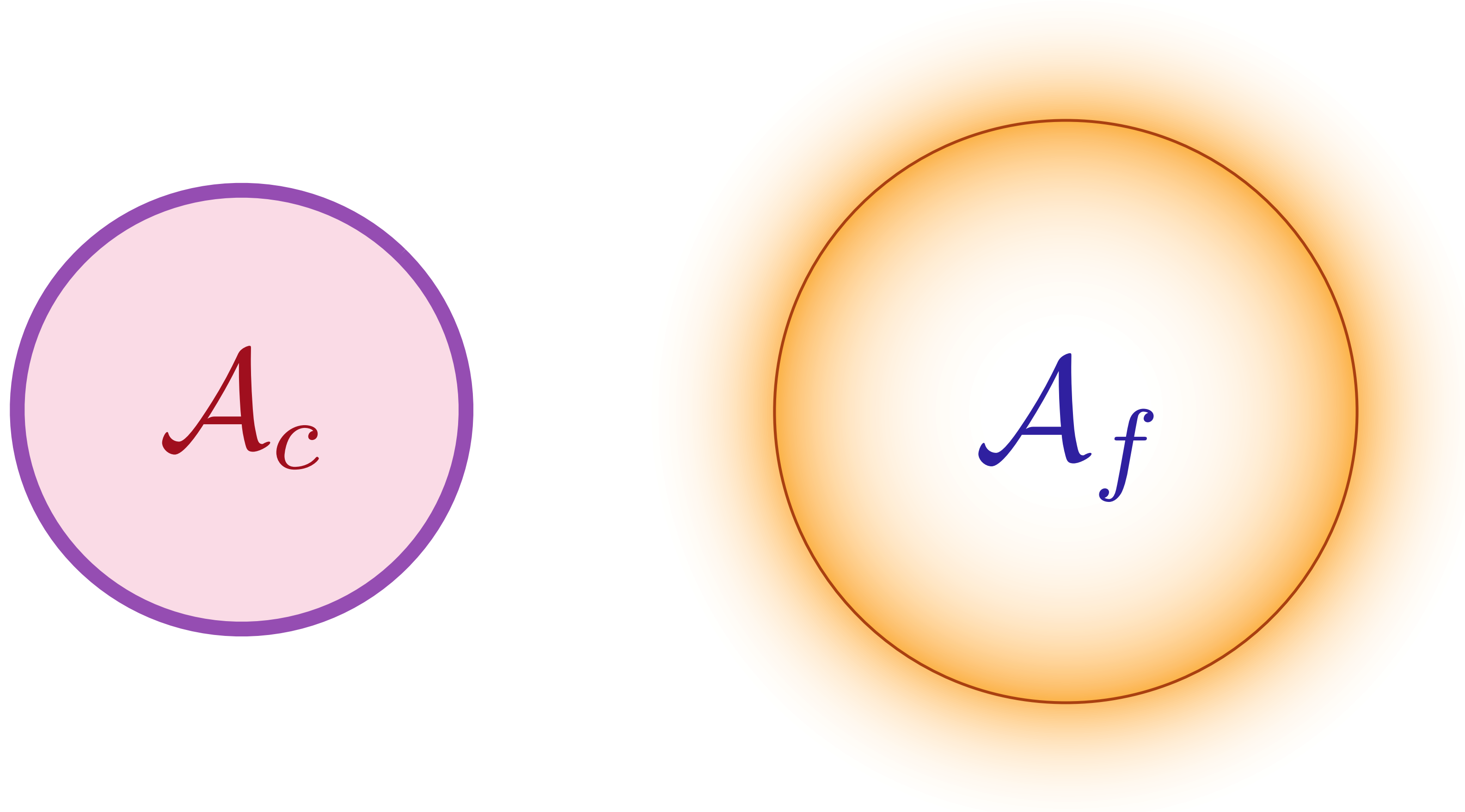
$$\langle b_\alpha \rangle = 0$$

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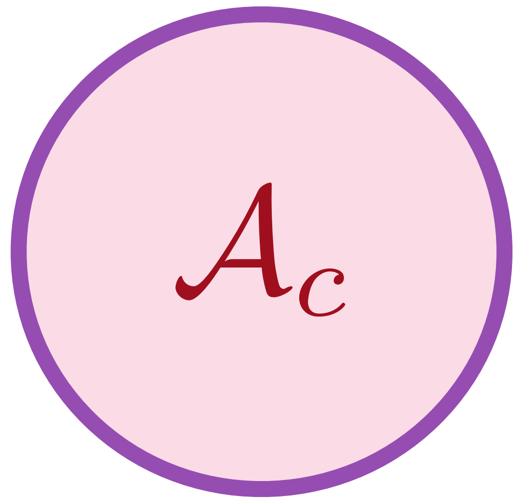


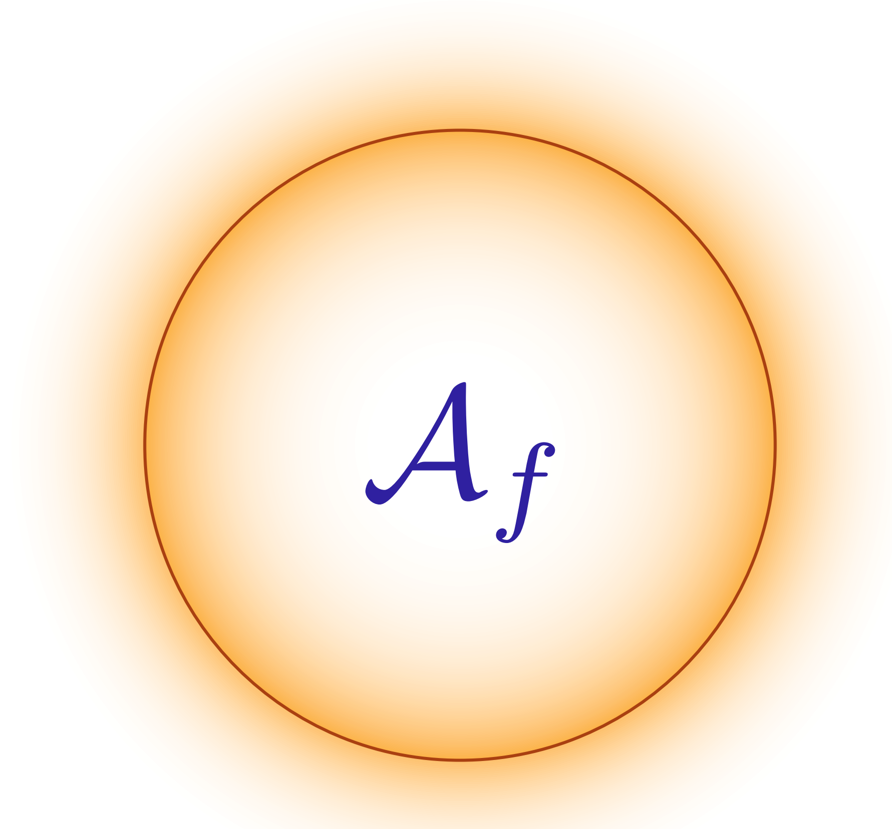


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## Fractionalized Fermi liquid (FL*)

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)

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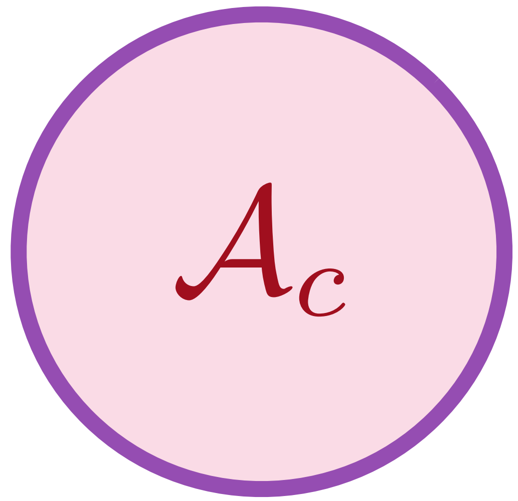


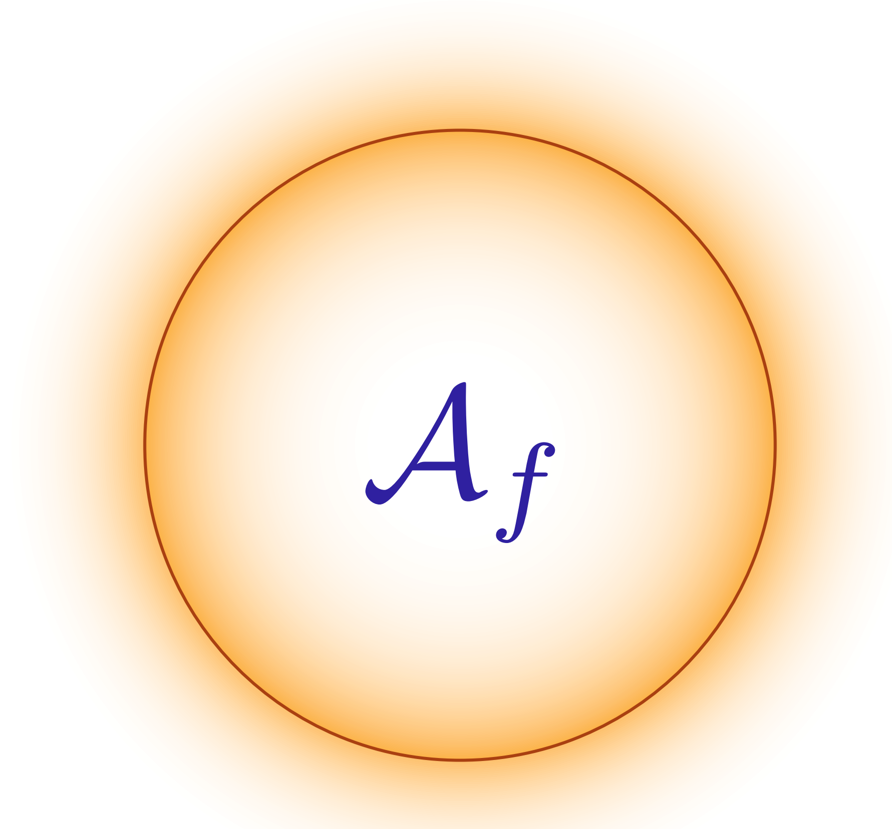


$$3\mathcal{A}_c + (N^2 - 1)\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

## Fractionalized Fermi liquid (FL*)

Claim: this is the phase underlying recent holographic theories of compressible metallic states.
However, a number of artifacts appear in the classical gravity approximation.

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# Phases of SYM-like theories

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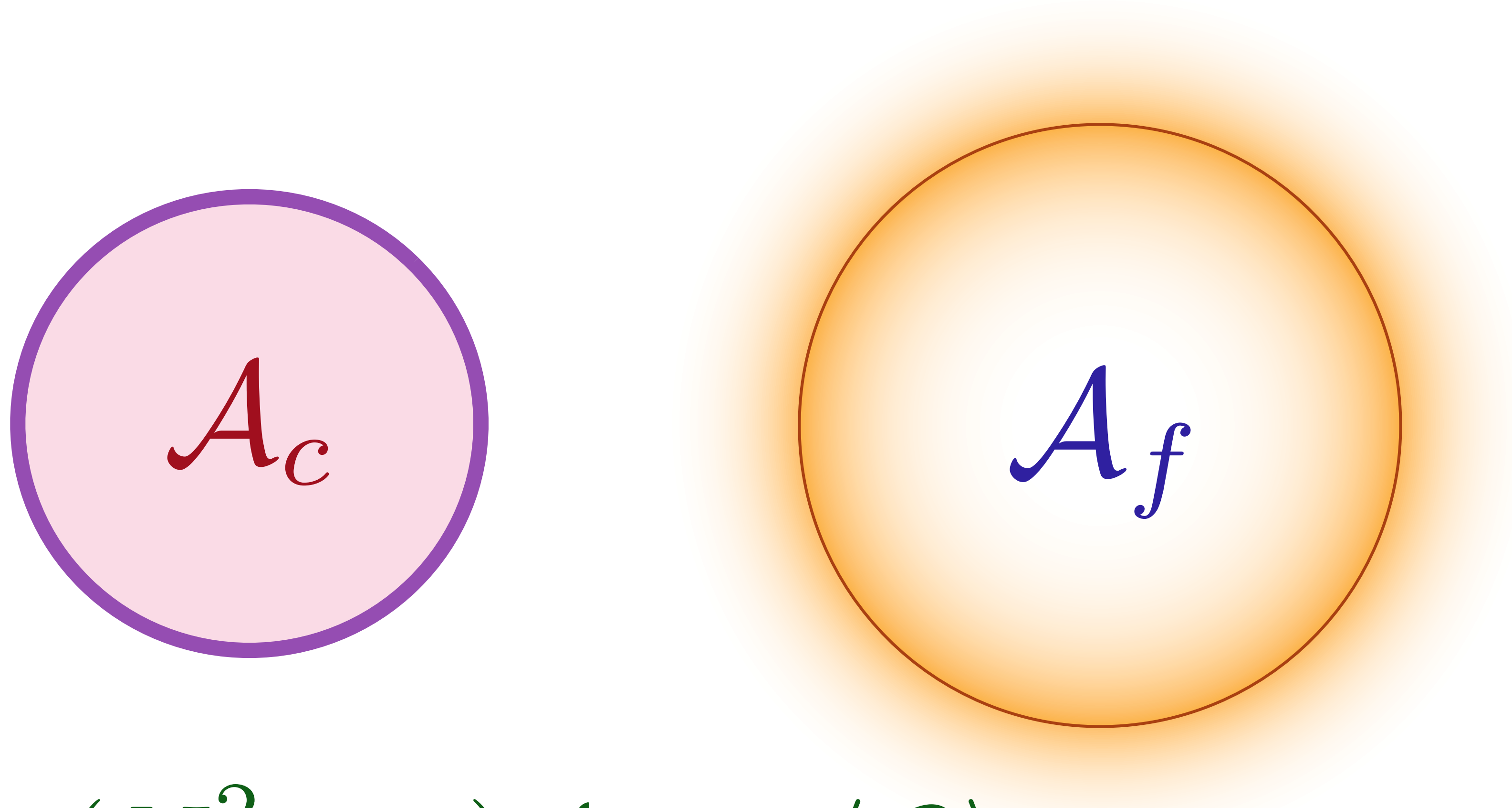


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## Fractionalized Fermi liquid (FL*)

The $f_\alpha$ Fermi surface is unstable to a pairing transition to a color superconductor, mediated by $b_\alpha$ fluctuations and SU$(N)$ gauge bosons at an energy scale $\sim \exp(-\sqrt{N})$.

D.T. Son, *Physical Review D* **59**, 094019 (2009)

M. Metlitski, D. Mross, S. Sachdev, T. Senthil, to appear

# Examples of compressible phases and their Fermi surfaces

1. The Fermi liquid (FL)

2. Fermions coupled to gauge fields

3. Fermion-boson mixtures

4. The fractionalized Fermi liquid (FL*)

5. Theories similar to $\mathcal{N} = 4$ SYM

6. Theories similar to ABJM

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6. Theories similar to ABJM

# ABJM theory in D=2+1 dimensions

- $4N^2$ Weyl fermions carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $4N^2$ complex bosons carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
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- $\mathcal{N} = 6$ supersymmetry

Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance

# Theory similar to ABJM

- Non-abelian gauge invariance (say $U(N) \times U(N)$) and U(1) global symmetry
- Fermions, $f_\alpha$ and $g^\alpha$, ($\alpha = 1 \ldots N^2$) carry fundamental and anti-fundamental gauge charges, and U(1) charge 1.
- Bosons, $a_\alpha$ and $b^\alpha$, ($\alpha = 1 \ldots N^2$) carry fundamental and anti-fundamental gauge charges, and U(1) charge 1.
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- Bosons, $a_\alpha$ and $b^\alpha$, ($\alpha = 1 \ldots N^2$) carry fundamental and anti-fundamental gauge charges, and U(1) charge 1.
- No supersymmetry
- Fermions, $c$, gauge-invariant bound states of fermions and bosons carrying U(1) charge 2.

# Theory similar to ABJM

$$
\begin{aligned}
H_{\text{fermion}} &= \sum_{k,a} \left( \frac{k^2}{2m_1} + \varepsilon_1 - \mu \right) \left( f^{\alpha\dagger} f_\alpha + g^\dagger_\alpha g^\alpha \right) \\
H_{\text{boson}} &= \sum_{k,a} \left( \frac{k^2}{2m_2} + \varepsilon_2 - \mu \right) \left( a^{\alpha\dagger} a_\alpha + b^\dagger_\alpha b^\alpha \right) + u \int d^d x \left( a^{\alpha\dagger} a_\alpha + b^\dagger_\alpha b^\alpha \right)^2 \\
H_F &= \sum_{k} \left( \frac{k^2}{2m_3} - 2\mu \right) c^\dagger c \\
H_{\text{int}} &= \lambda_1 \int d^d x \left( a^{\beta\dagger} b^\dagger_\beta f_\alpha g^\alpha + \text{c.c.} \right) + \lambda_2 \int d^d x \left( a^{\alpha\dagger} b^\dagger_\beta f_\alpha g^\beta + \text{c.c.} \right) \\
&\quad + \lambda_3 \int d^d x \left( c^\dagger \left( f_\alpha b^\alpha - g^\alpha a_\alpha \right) + \text{c.c.} \right)
\end{aligned}
$$

The $\varepsilon_{1,2}$ are parameters tuning between possible phases. The U($N$)×U($N$) gauge fields are not shown, and are included as usual by covariantizing derivatives.

$$
\mathcal{Q} = a^{\alpha\dagger} a_\alpha + b^\dagger_\alpha b^\alpha + f^{\alpha\dagger} f_\alpha + g^\dagger_\alpha g^\alpha + 2c^\dagger c
$$

# Phase of ABJM-like theories

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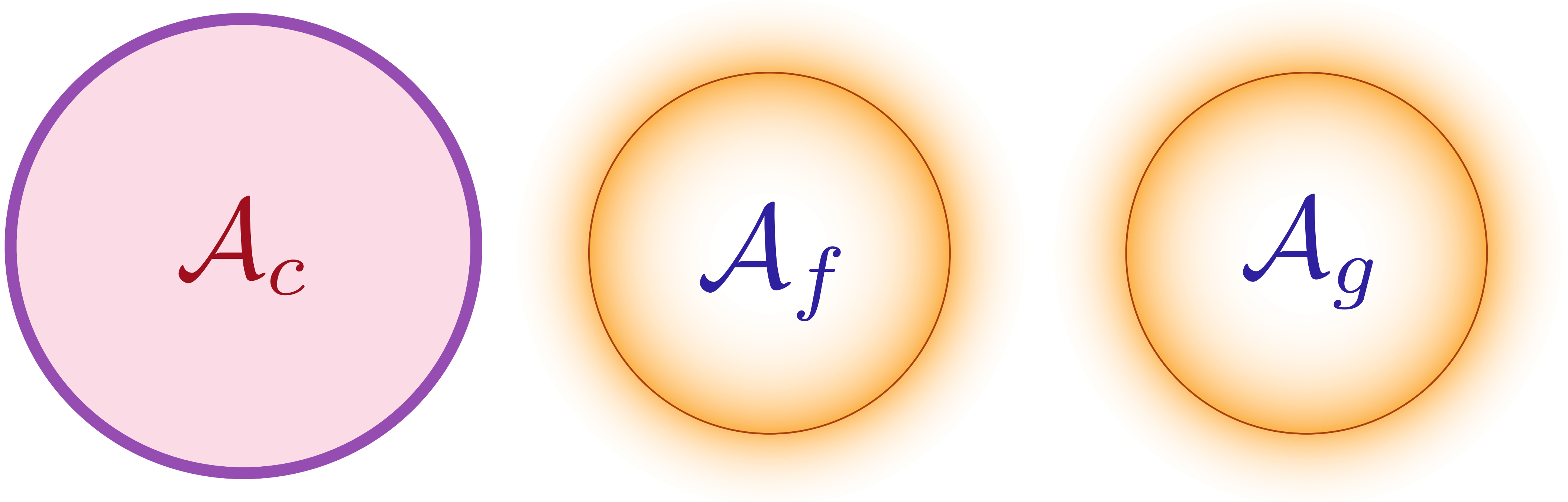


$$2\mathcal{A}_c + N^2 \mathcal{A}_f + N^2 \mathcal{A}_g = \langle \mathcal{Q} \rangle$$

## Fractionalized Fermi liquid (FL*)

SU(*N*) gauge theory is in deconfined phase

# Gauge-gravity duality
# and
# impurity mean-field theories

# Gauge-gravity duality

- Begin with a CFT e.g. the SYM theory with a SO(6) global symmetry

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# Gauge-gravity duality

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- In the Einstein-Maxwell theory, the chemical potential leads at T=0 to an extremal Reissner-Nordtrom black hole in the $AdS_5$ spacetime.
- The near-horizon geometry of the RN black hole is $AdS_2 \times R^3$. This factorization leads to finite ground state entropy density

# AdS theory of finite density quantum matter

Features of AdS Einstein-Maxwell theory of non-zero density quantum matter, not expected in the final theory:

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Features of AdS Einstein-Maxwell theory of non-zero density quantum matter, not expected in the final theory:

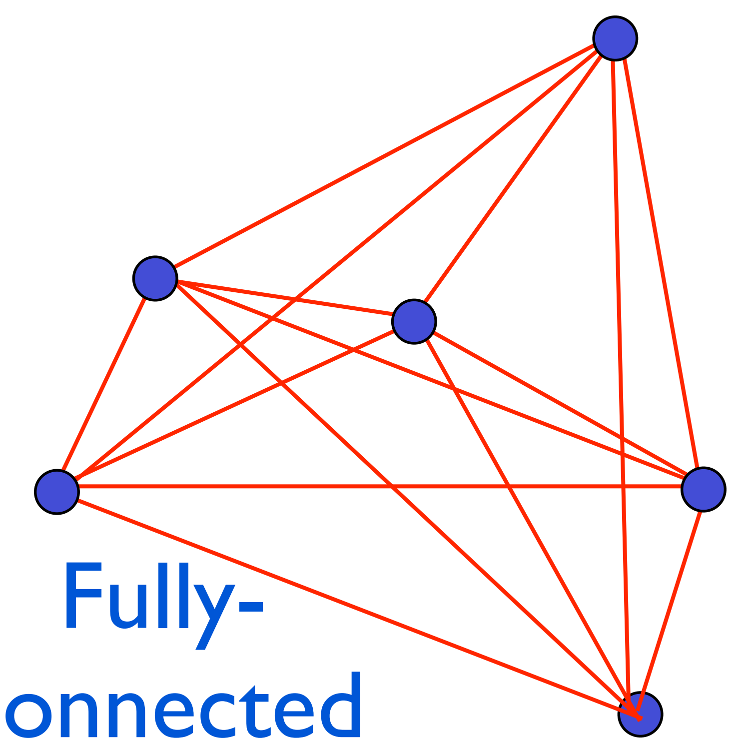
- Non-zero ground state entropy density.
- Single fermion self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
- Low energy singularities are described by "conformal quantum mechanics": a 0+1 dimensional defect in a $d$+1 dimensional CFT. This is linked to the factorization of the near-horizon metric to $AdS_2 \times R^d$,

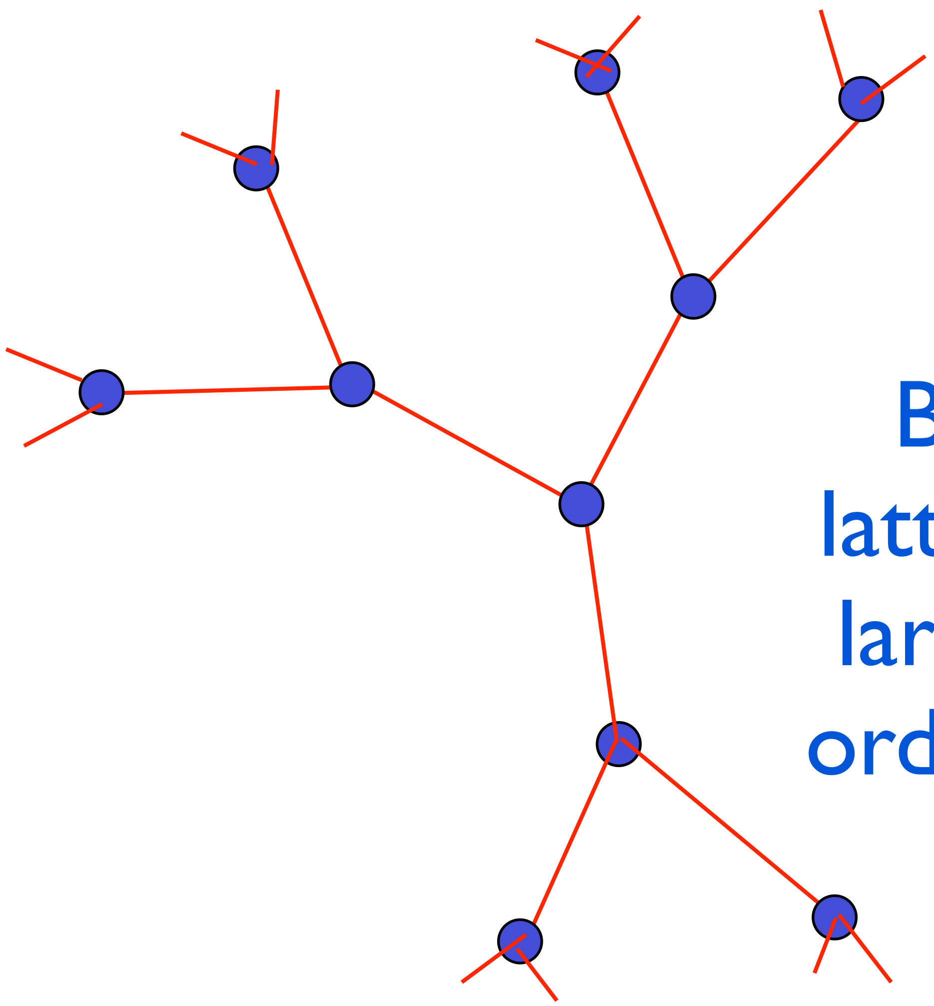
# Solution of lattice models

Place U(1) gauge theory theory on a lattice, integrate out $b$ and $A_a$, to obtain Kondo lattice Hamiltonian

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{i<j} J_H(i,j) \vec{S}_i \cdot \vec{S}_j + J_K \sum_i \vec{S}_i \cdot c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

where $\vec{S}_i = f_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} f_{\beta}$




S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001).
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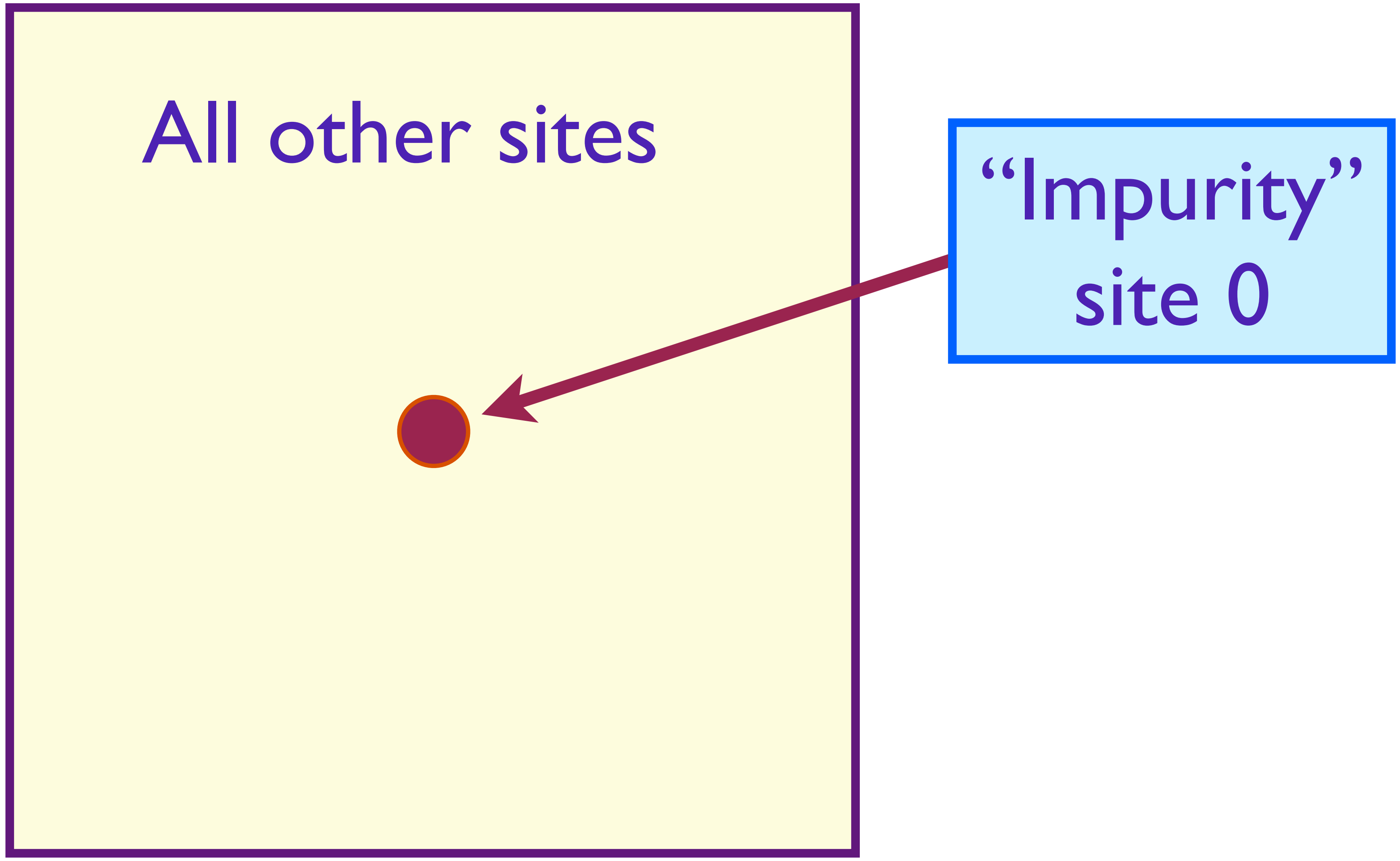


$$\mathcal{L} = \mathcal{L}_{\rm imp}[c_0, f_0] + c_0^\dagger F_{\rm bulk} + F_{\rm bulk}^\dagger c_0 + \mathcal{L}_{\rm bulk}$$

Has to be combined with a *self-consistency condition* between correlators on the impurity and the bulk.

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## Obtain both FL and FL* phases;

## properties of the FL* phase:

- The ground state has a non-zero entropy density


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- Imposition of the self-consistency condition between impurity and boundary yields the scaling dimension $\Delta = 1$, the 'marginal Fermi liquid' value.


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These features, and the resulting fermion correlator and transport properties, co-incide with those obtained (for general $\Delta$) using the holographic $AdS_2 \times R^d$ theory defined on the extremal horizon of the Reissner-Nordstrom black hole (T. Faulkner, H. Liu, J. McGreevy and D. Vegh, arXiv:0907.2694)

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# Conclusions

- Compressible quantum matter is characterized by Fermi surfaces.
- Fermi surfaces can be removed from the Luttinger count if the fermions acquire gauge charges
- Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

# Conclusions

- Mean field Kondo lattice models capture the physics of holographic metals with a $AdS_2 \times R^d$ geometry

- Needed: Holographic theory for FL* or related compressible phases, without a factorized geometry. Challenge: detect Fermi surfaces of fermions with gauge charges