Quantum matter and gauge-gravity duality

HRI, Allahabad
January 12, 2012

Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

1. The superfluid-insulator quantum phase transition

A. Field theory

B. Holography

2. Compressible quantum liquids

A. Field theory

B. Holography

1. The superfluid-insulator quantum phase transition

A. Field theory

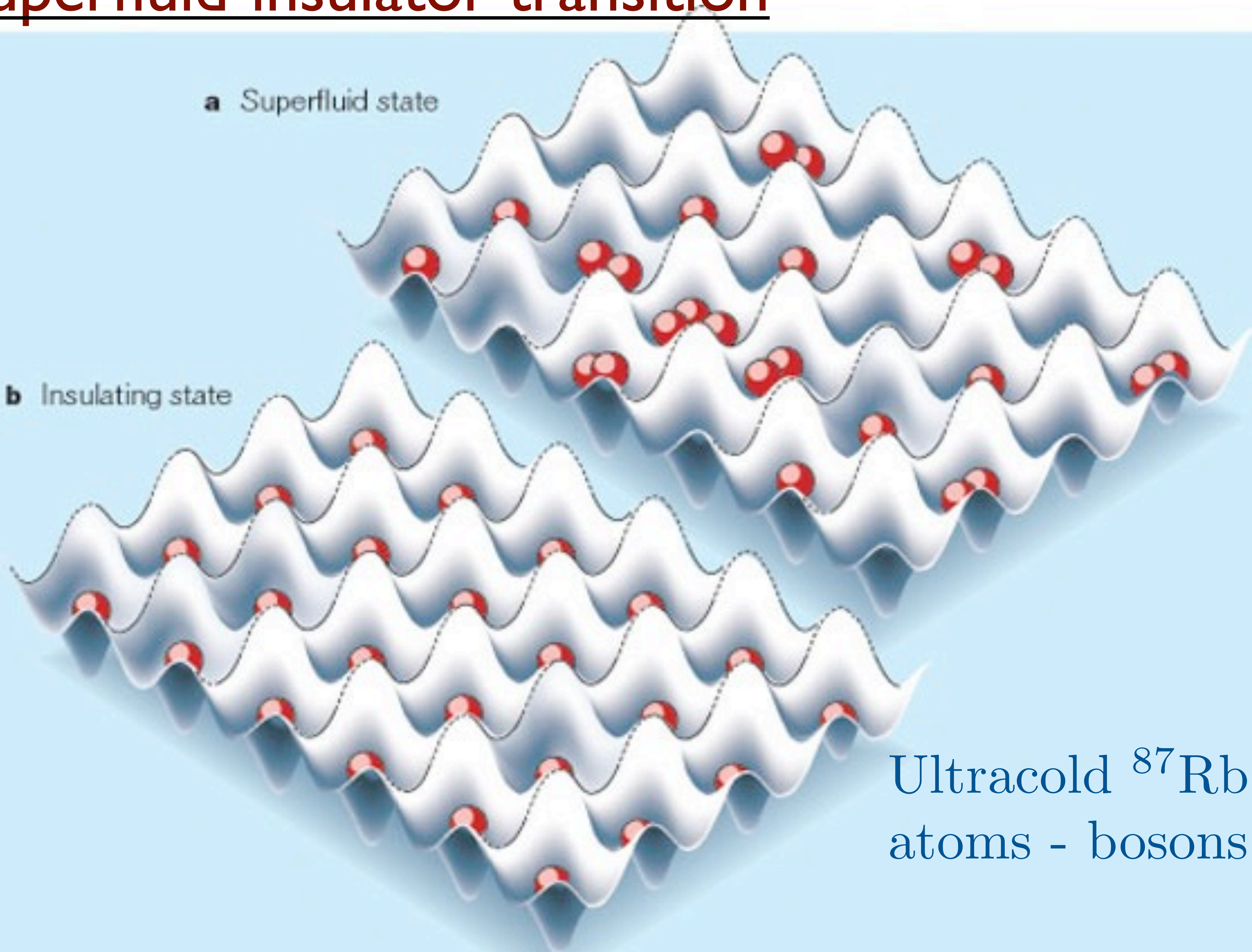
B. Holography

2. Compressible quantum liquids

A. Field theory

B. Holography

Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

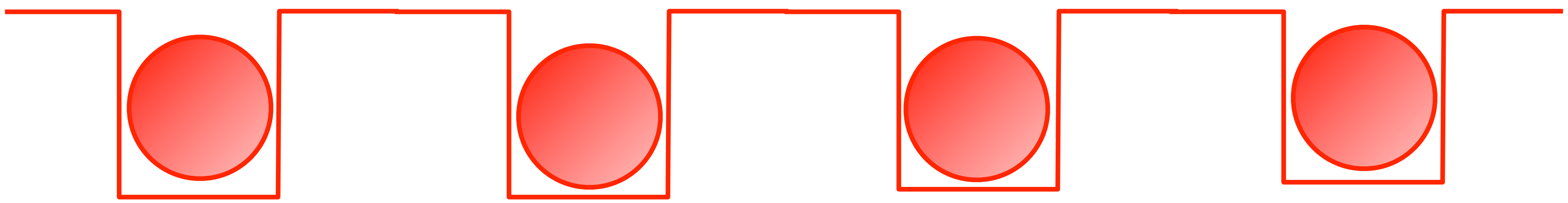
Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

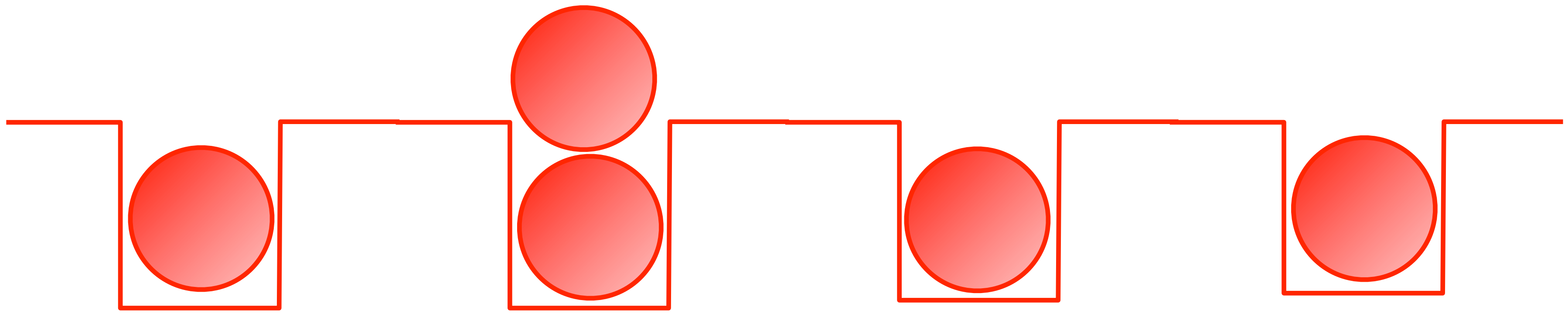
$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).



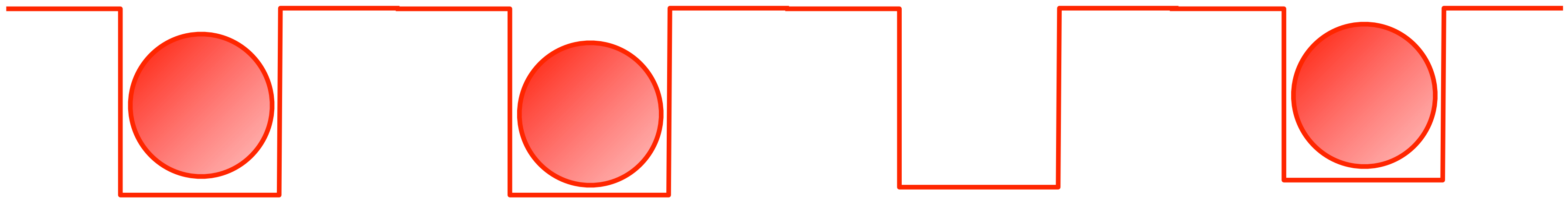
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



Particles $\sim \psi^\dagger$

Excitations of the insulator:



Holes ~ ψ

1. The superfluid-insulator quantum phase transition

A. Field theory

B. Holography

2. Compressible quantum liquids

A. Field theory

B. Holography

1. The superfluid-insulator quantum phase transition

A. Field theory

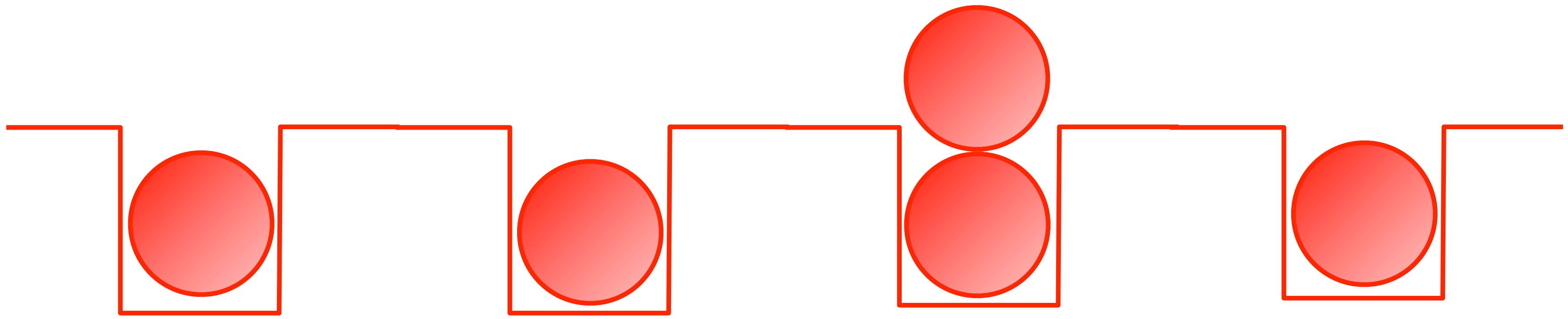
B. Holography

2. Compressible quantum liquids

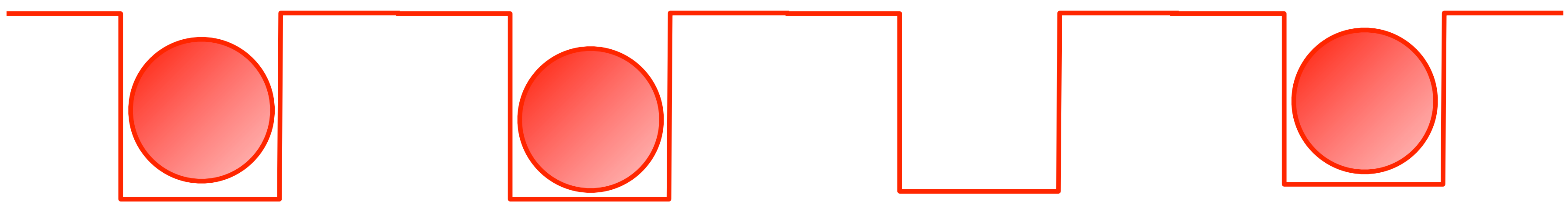
A. Field theory

B. Holography

Excitations of the insulator:



Particles $\sim \psi^\dagger$

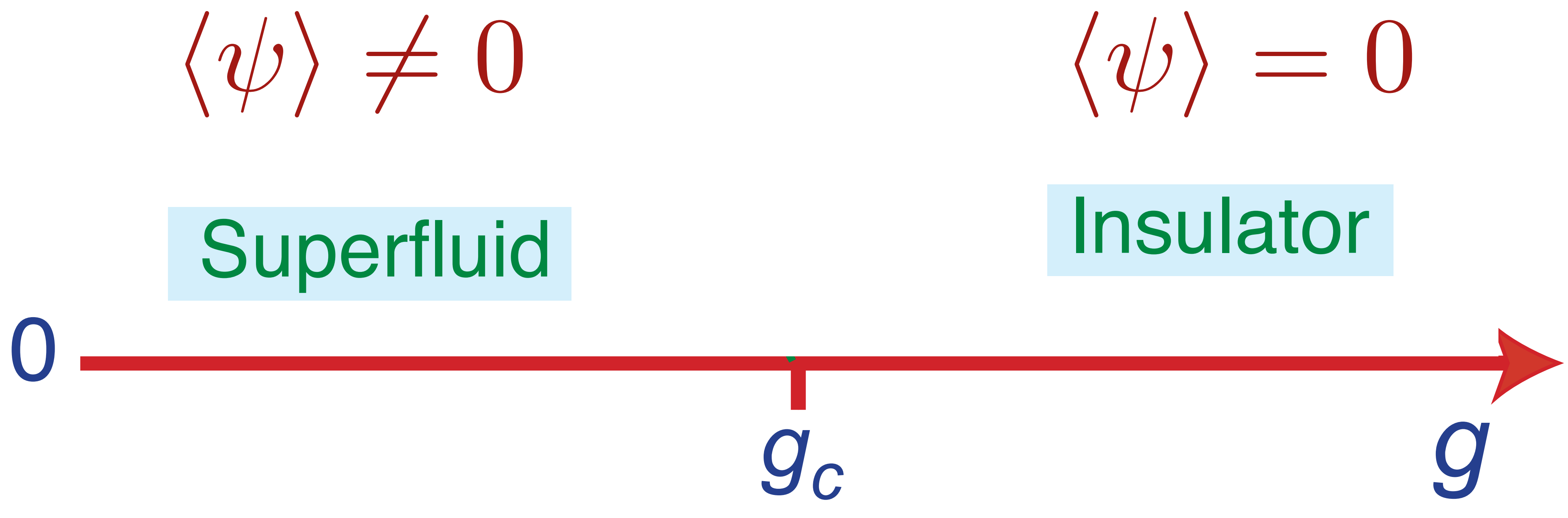


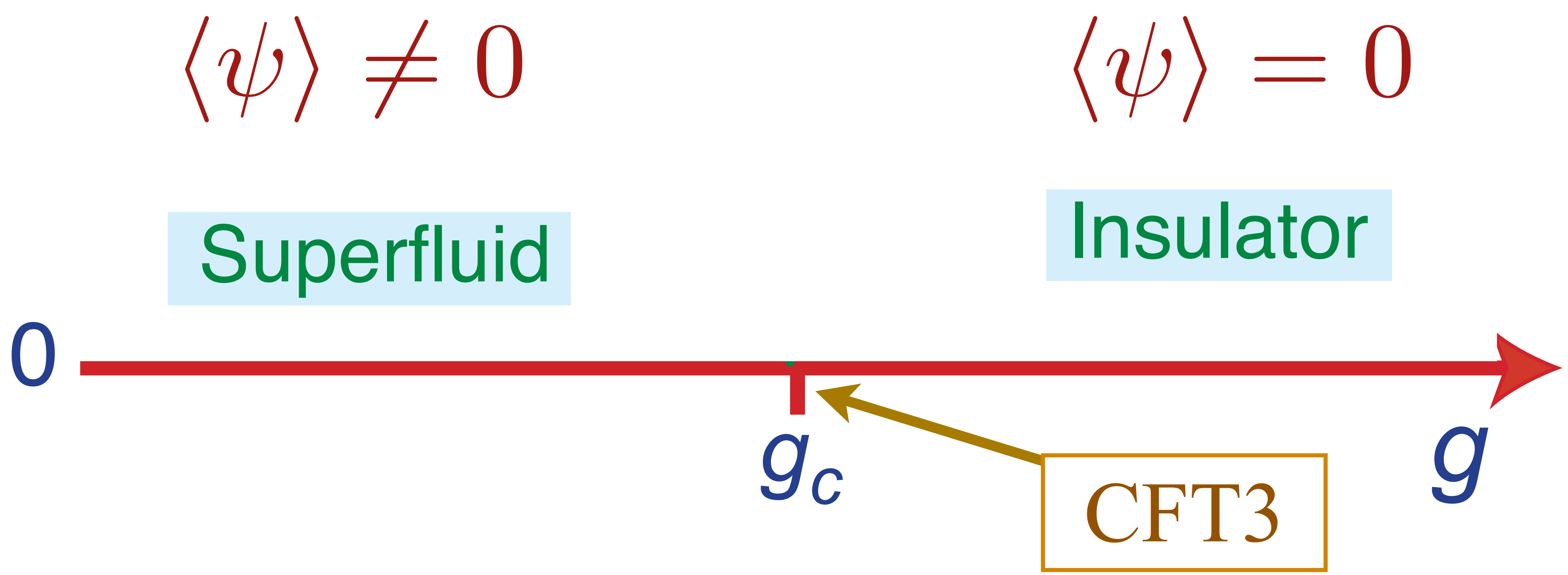
Holes $\sim \psi$

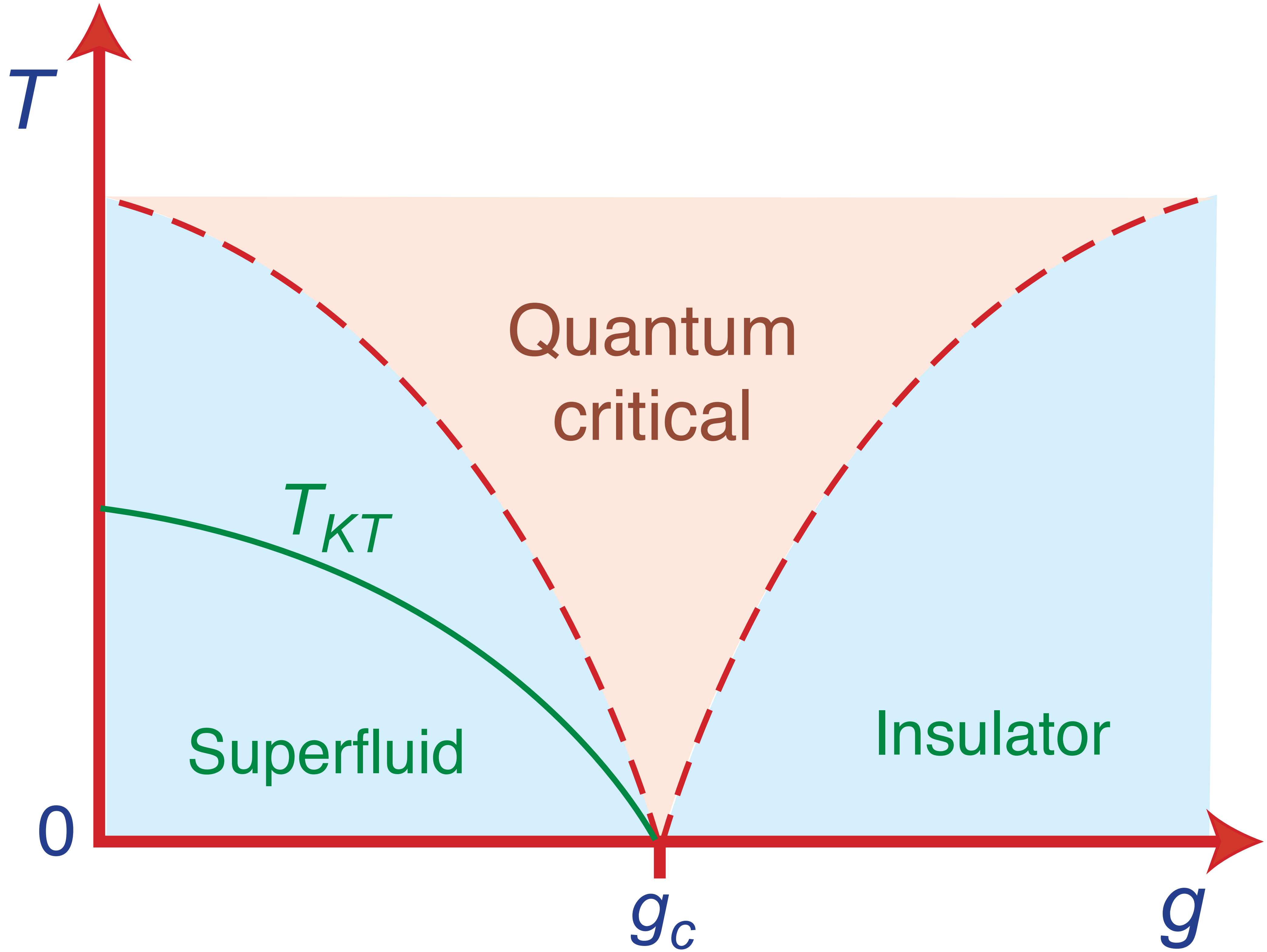
Density of particles = density of holes $\Rightarrow$
“relativistic” field theory for ψ:

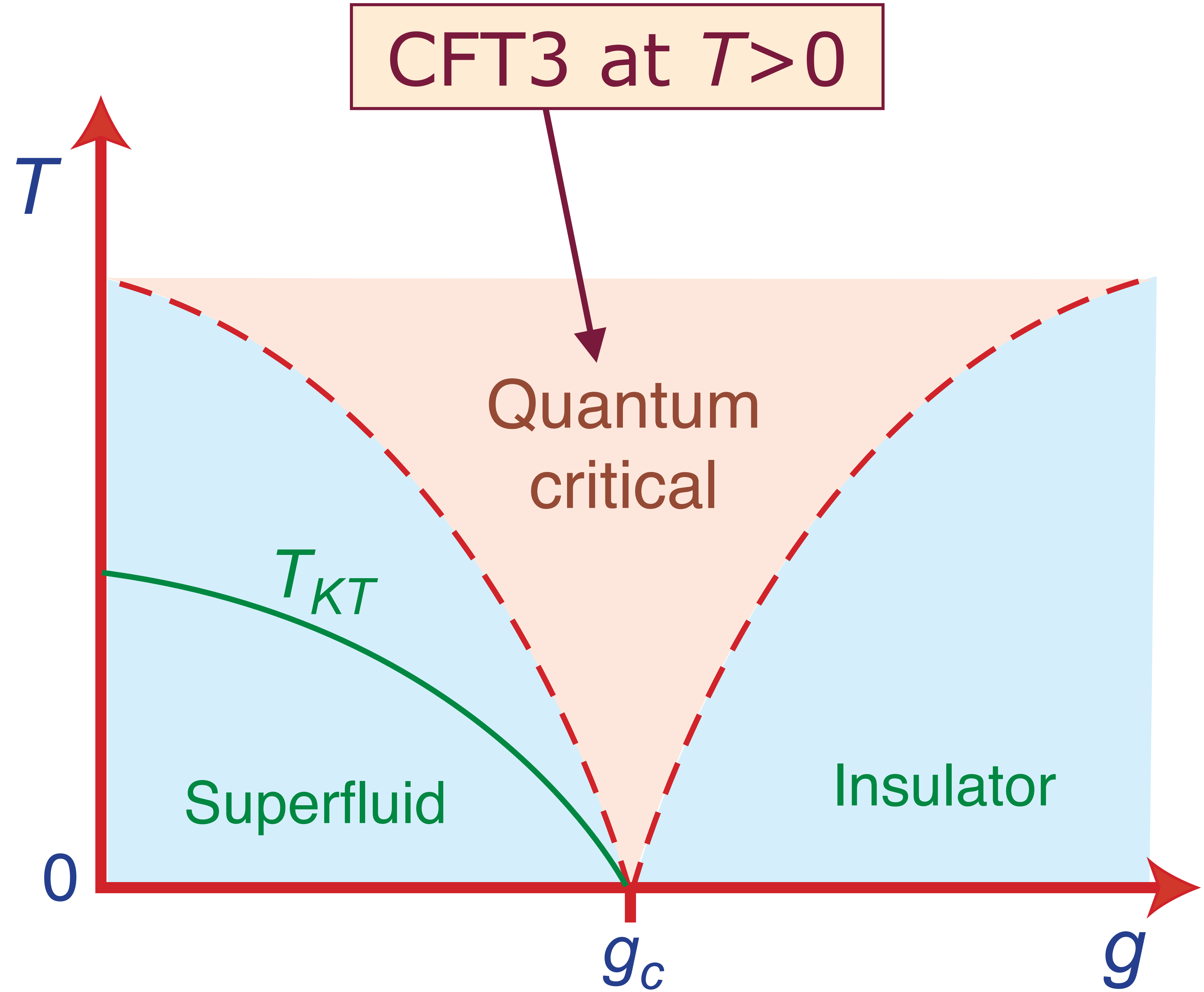
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).


$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g


$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g
CFT3


T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, $\tau_{\rm eq}$

$$\boxed{\tau_{\rm eq} = \mathcal{C}\,\frac{\hbar}{k_B T}}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\boxed{\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]}$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

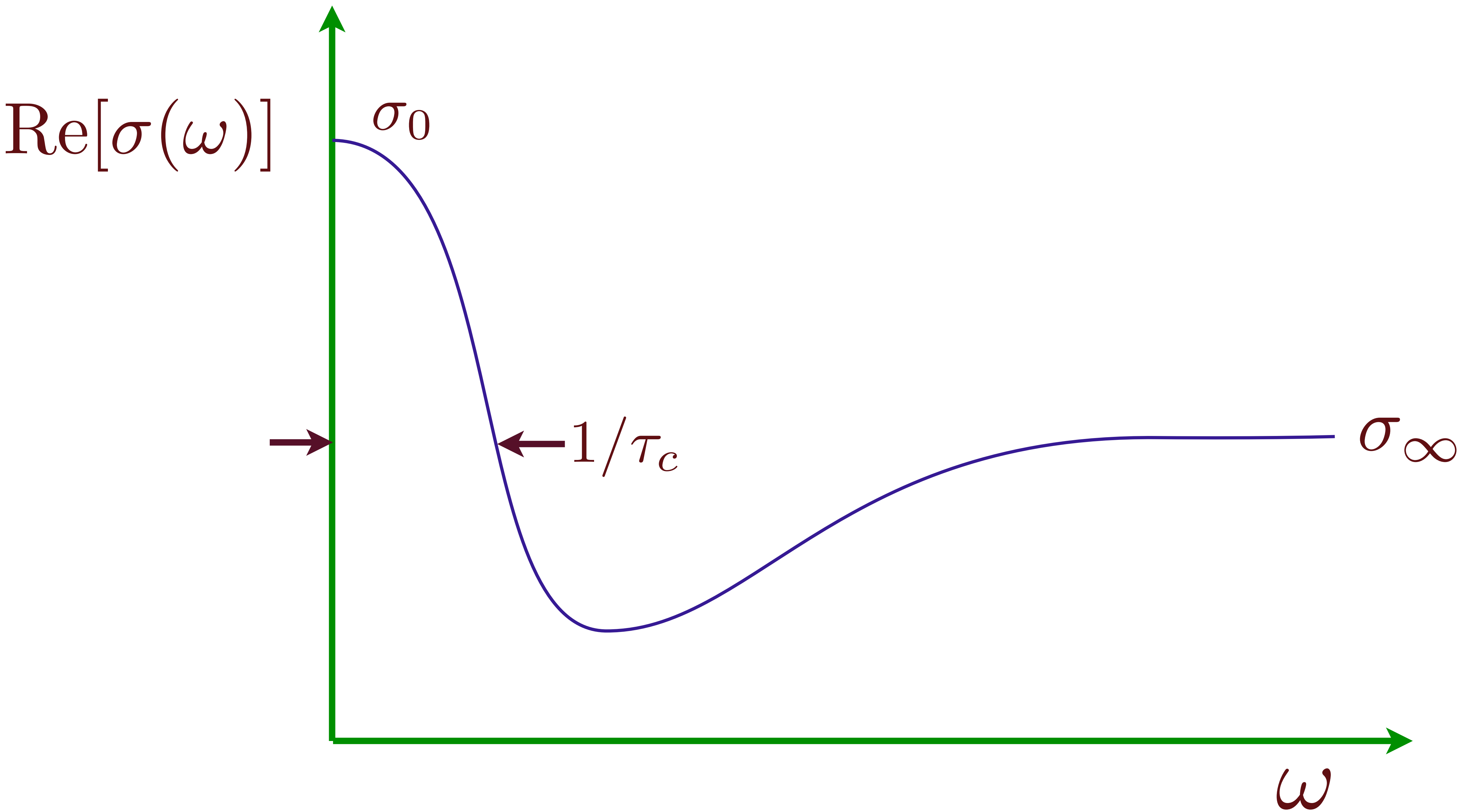
This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

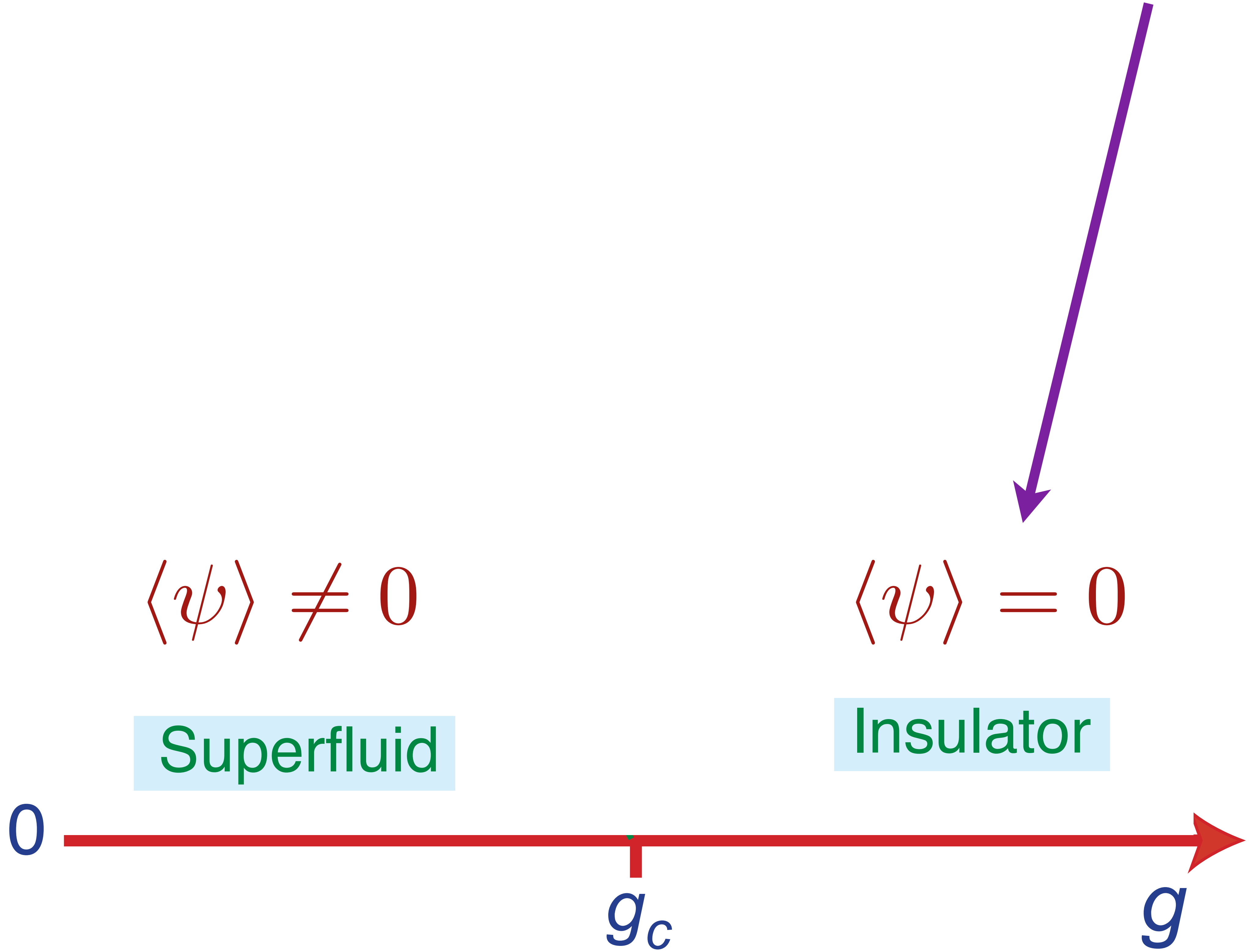
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

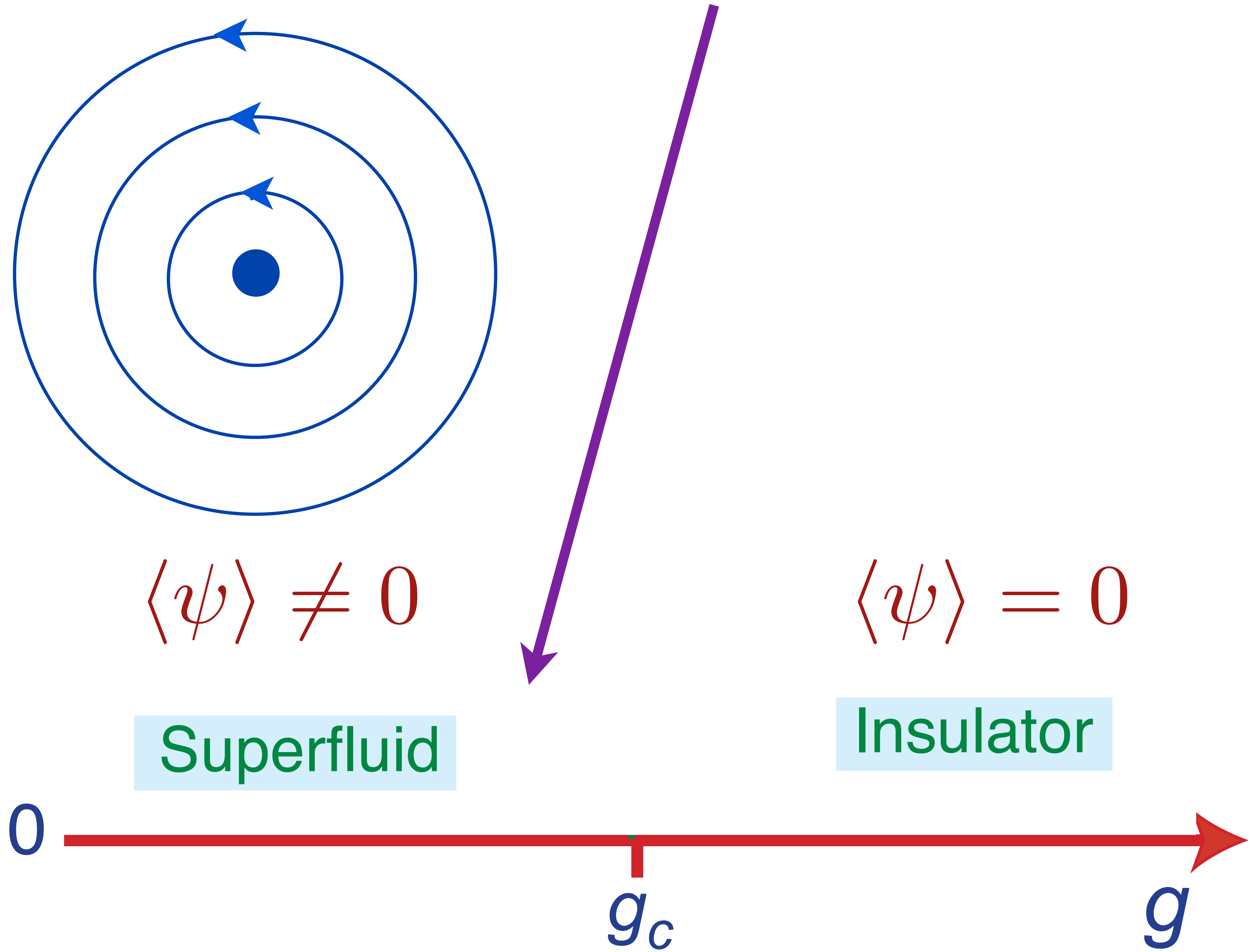
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



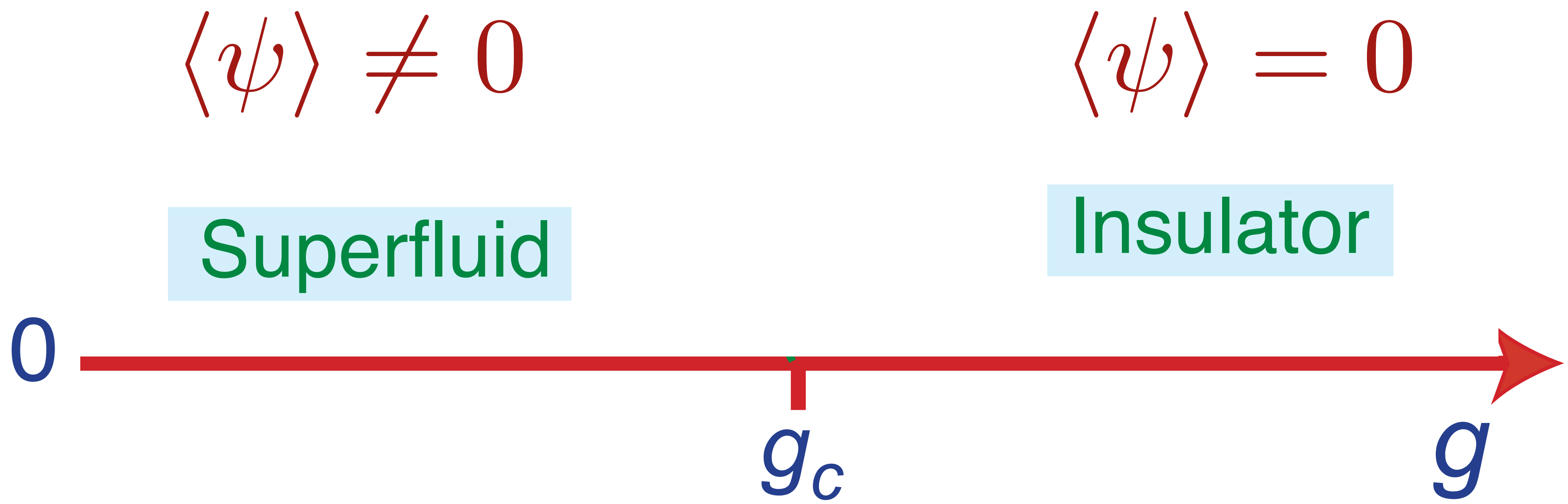
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



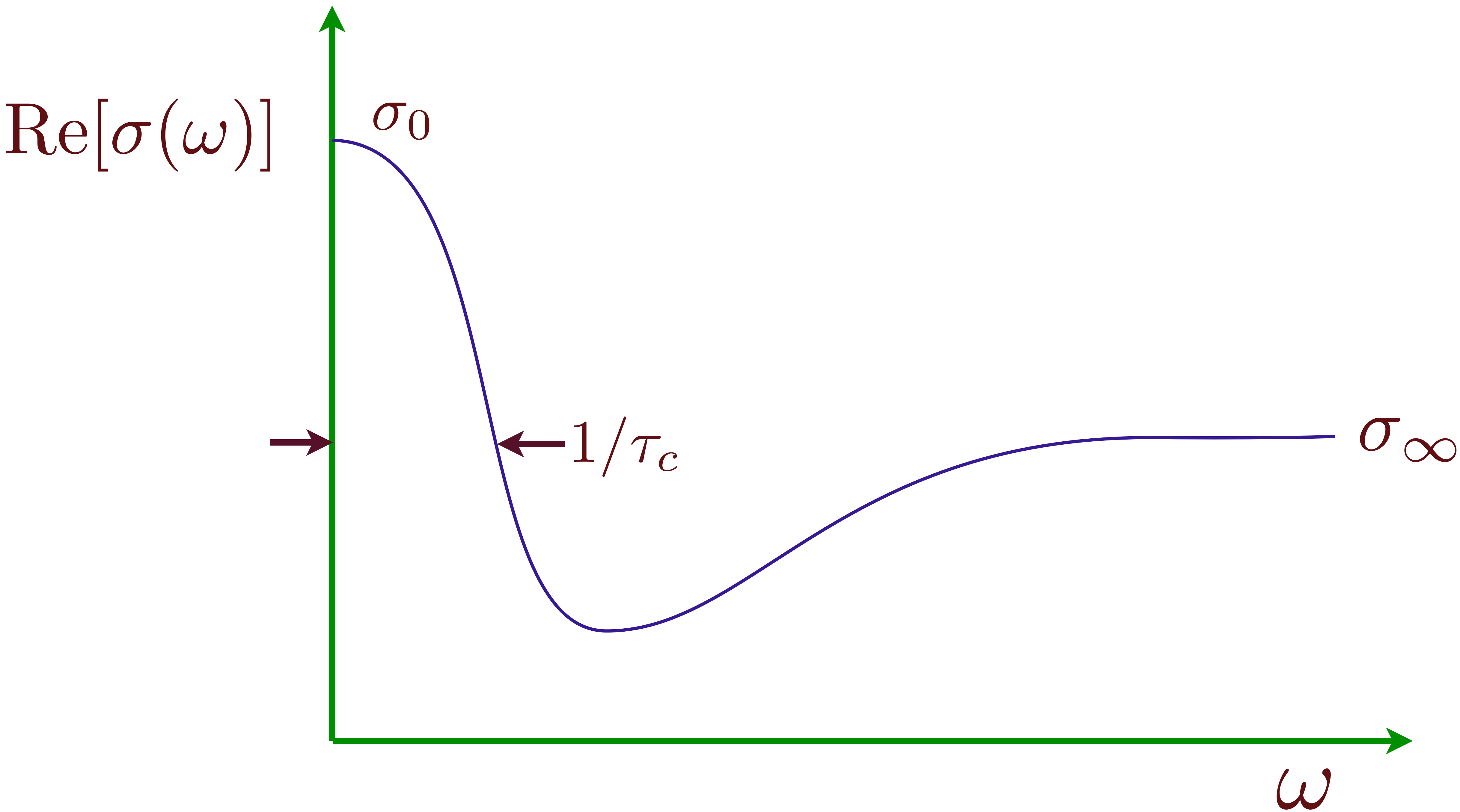
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

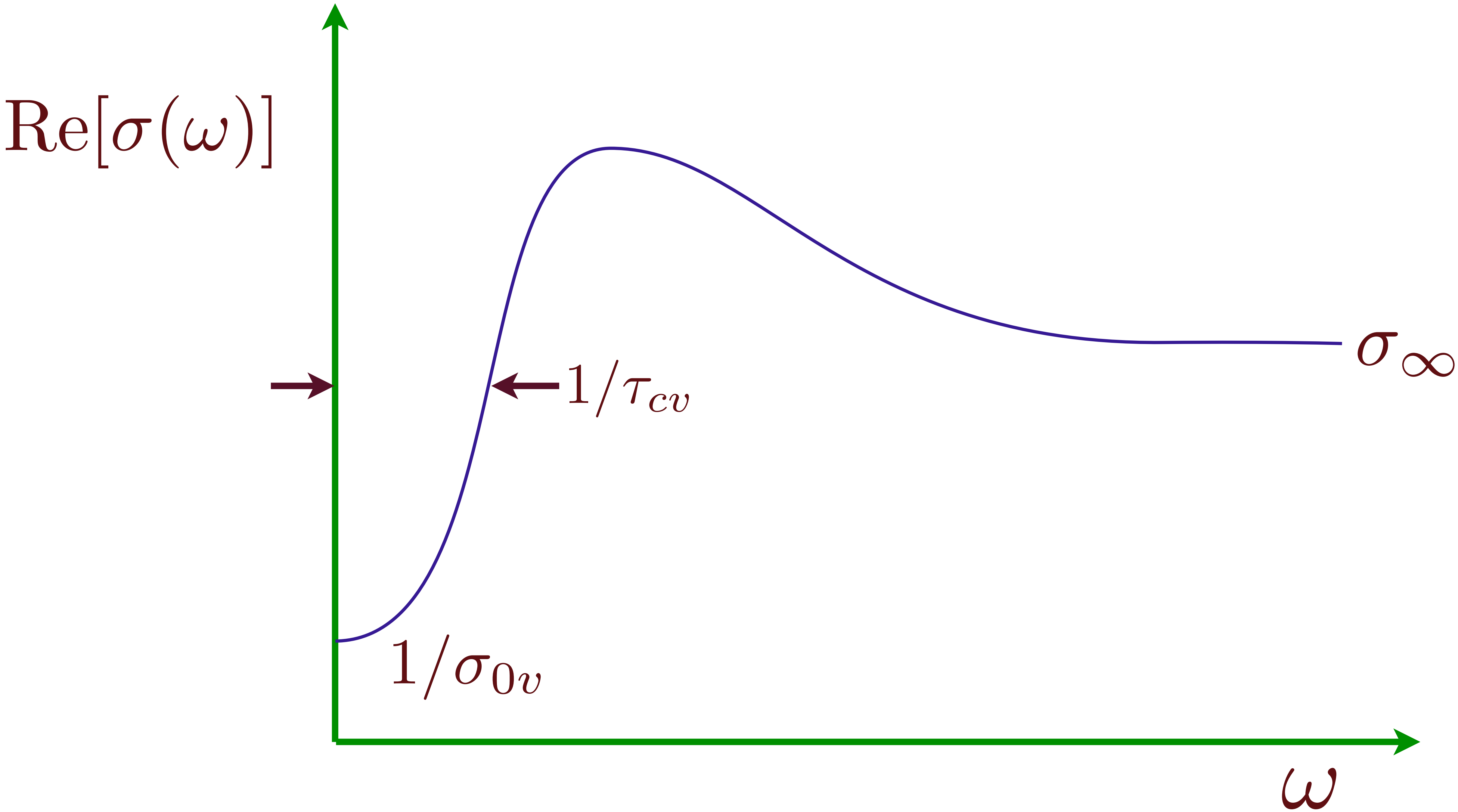
$$\text{Conductivity} = \text{Resistivity of vortices}$$



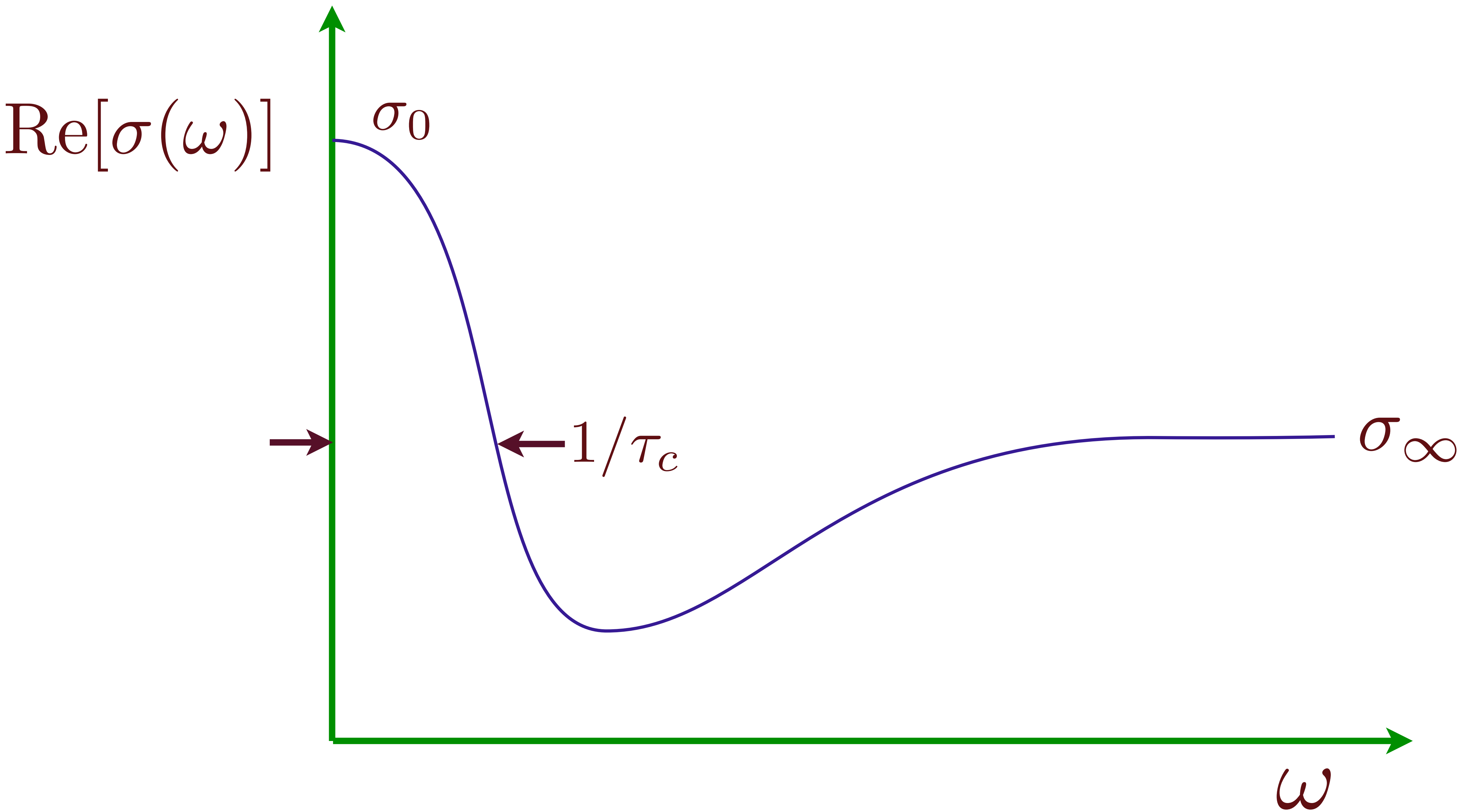
Boltzmann theory of bosons



Boltzmann theory of vortices

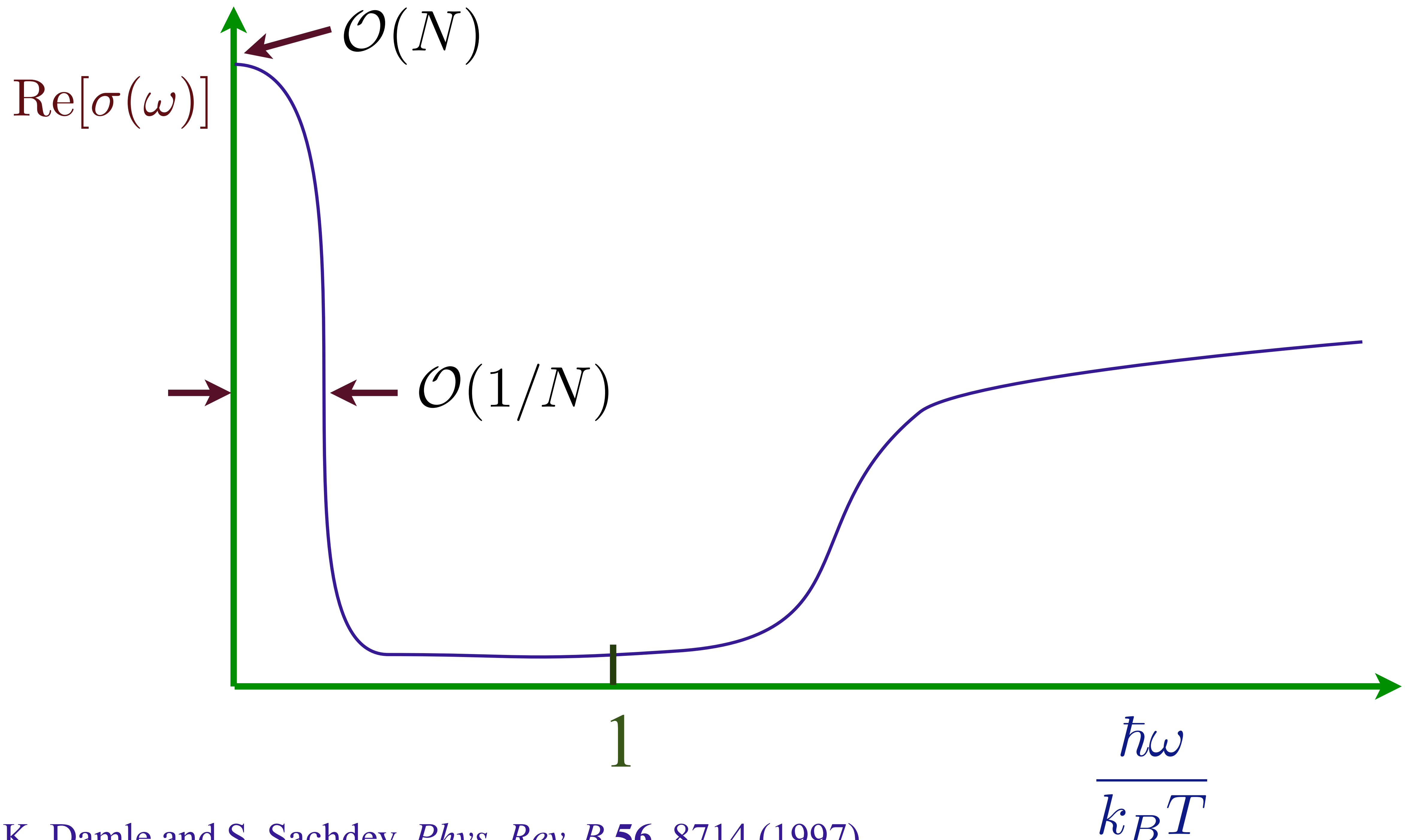


Boltzmann theory of bosons



Vector large *N* expansion for CFT3

$$\sigma = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right); \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

1. The superfluid-insulator quantum phase transition

A. Field theory

B. Holography

2. Compressible quantum liquids

A. Field theory

B. Holography

1. The superfluid-insulator quantum phase transition

A. Field theory

B. Holography

2. Compressible quantum liquids

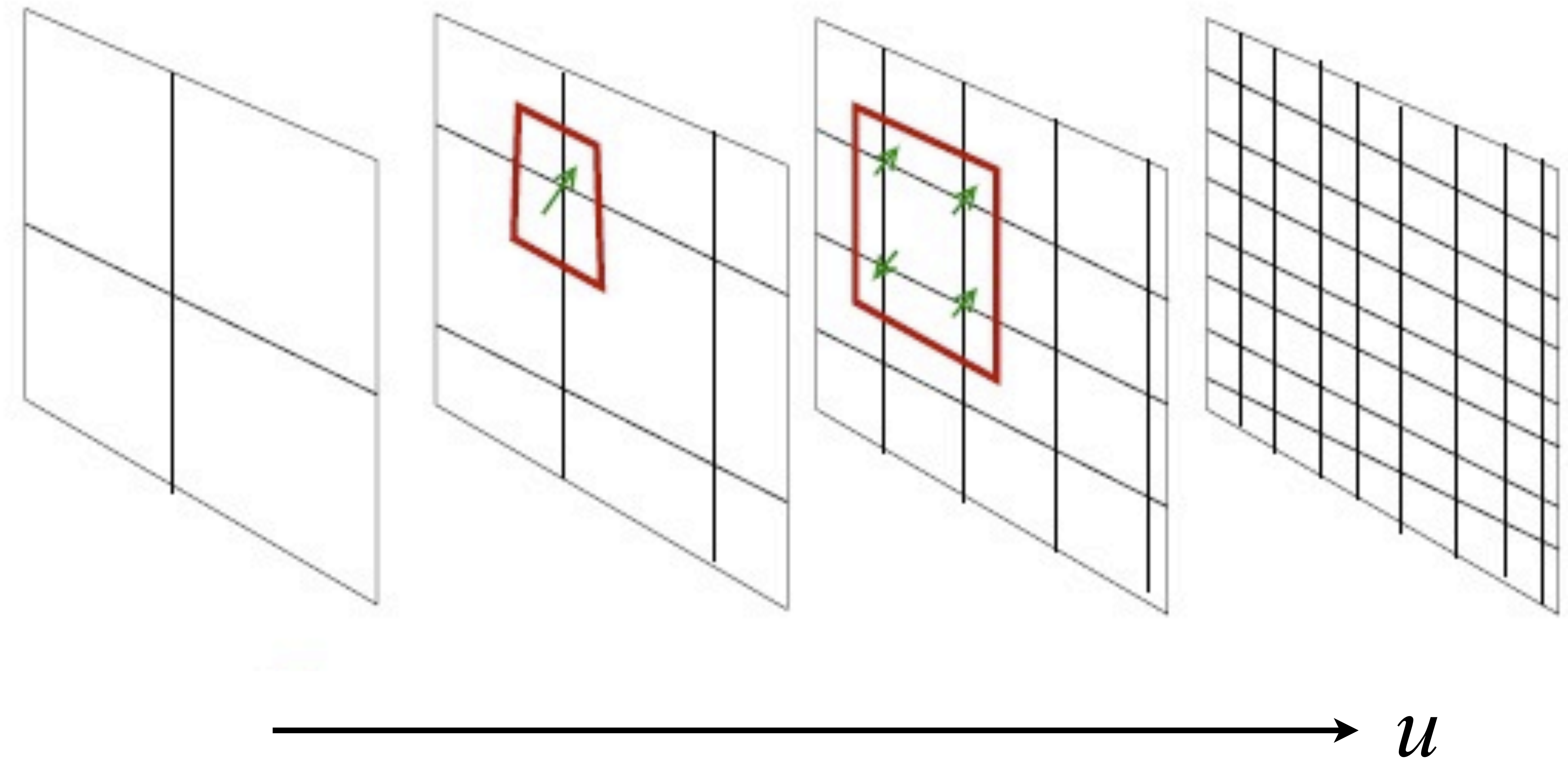
A. Field theory

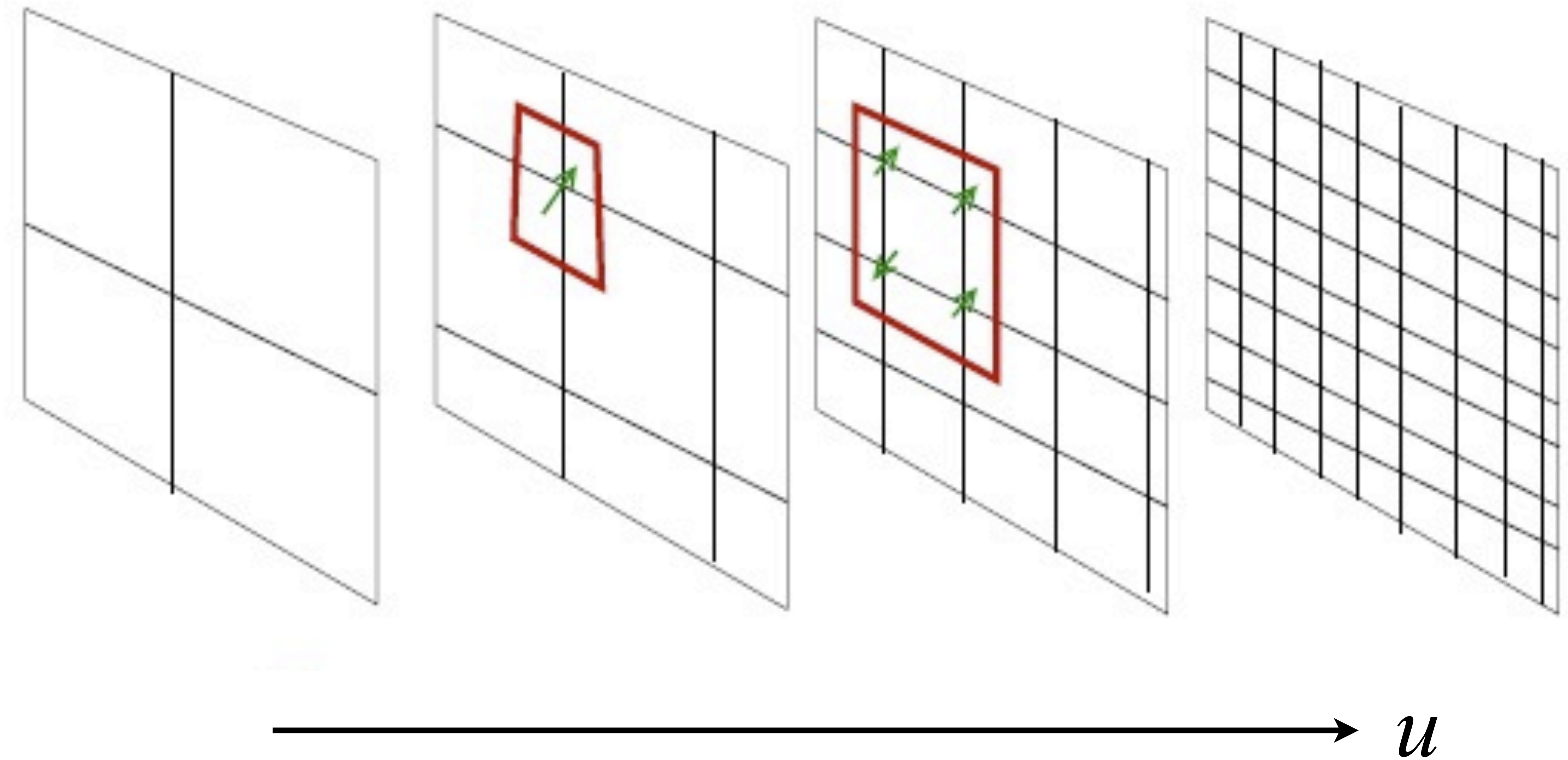
B. Holography

Field theories in $d+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.


u



Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

At the RG fixed point, $\beta(g) = 0$, the $(d+1)$-dimensional "relativistic" field theory is invariant under the scale transformation $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad u \to u/\zeta$$

At the RG fixed point, $\beta(g) = 0$, the $(d+1)$-dimensional "relativistic" field theory is invariant under the scale transformation $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad u \to u/\zeta$$

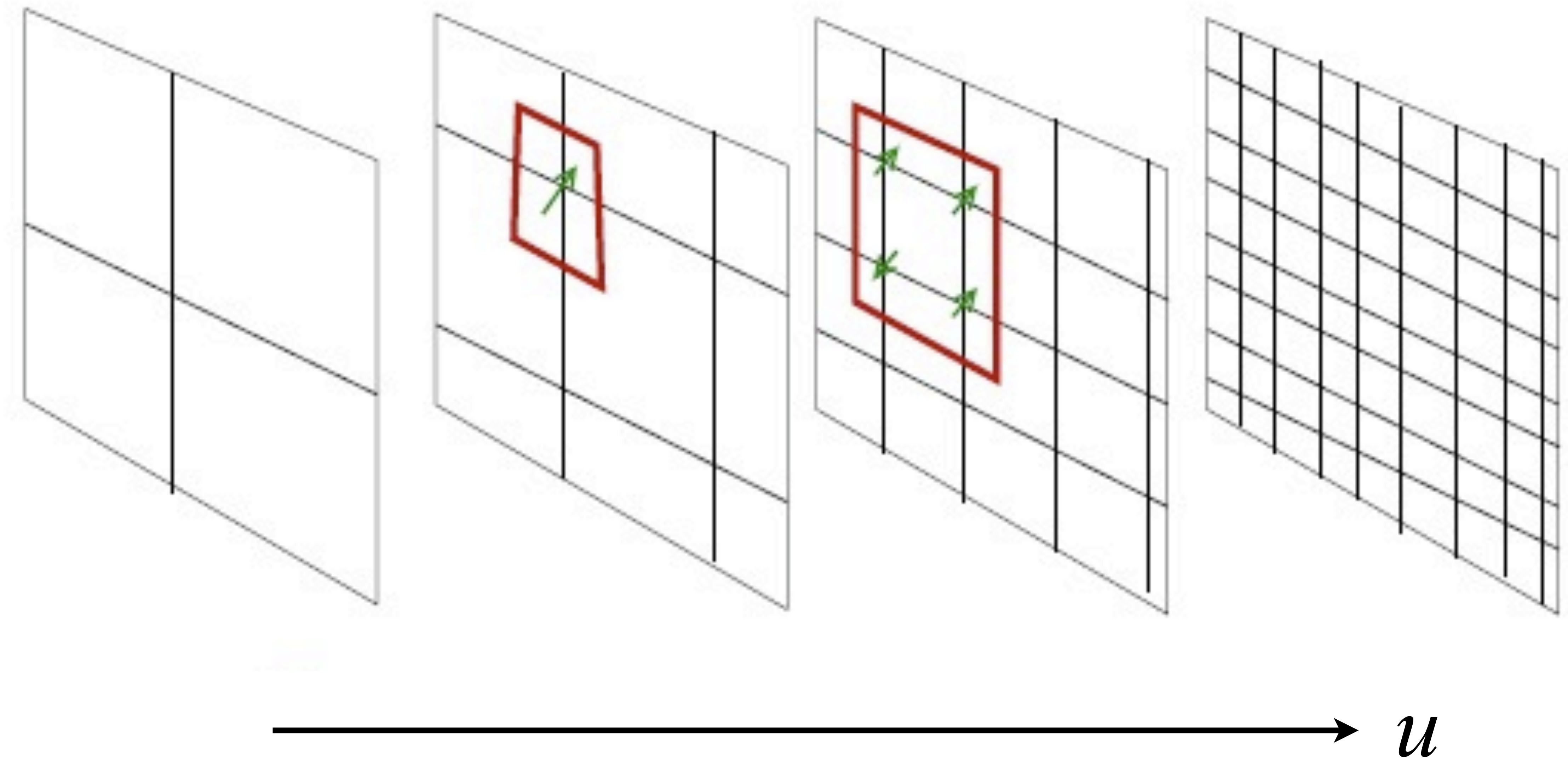
This is assumed to be an invariance of the *metric* of the theory in $d+2$ dimensions. The unique solution is

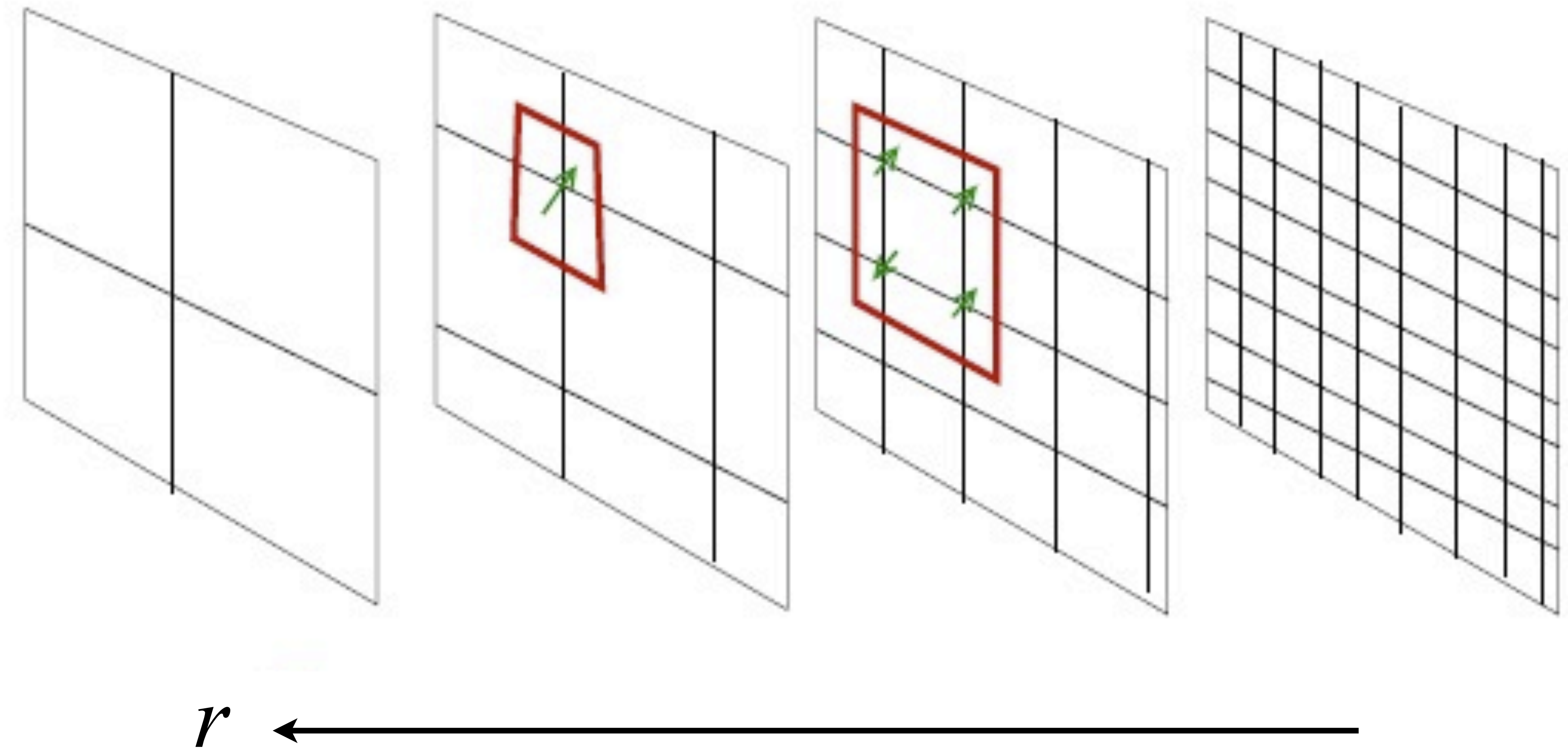
$$ds^2 = \left(\frac{u}{L}\right)^2 \left(-dt^2 + dx_i^2\right) + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $r = L^2/u$

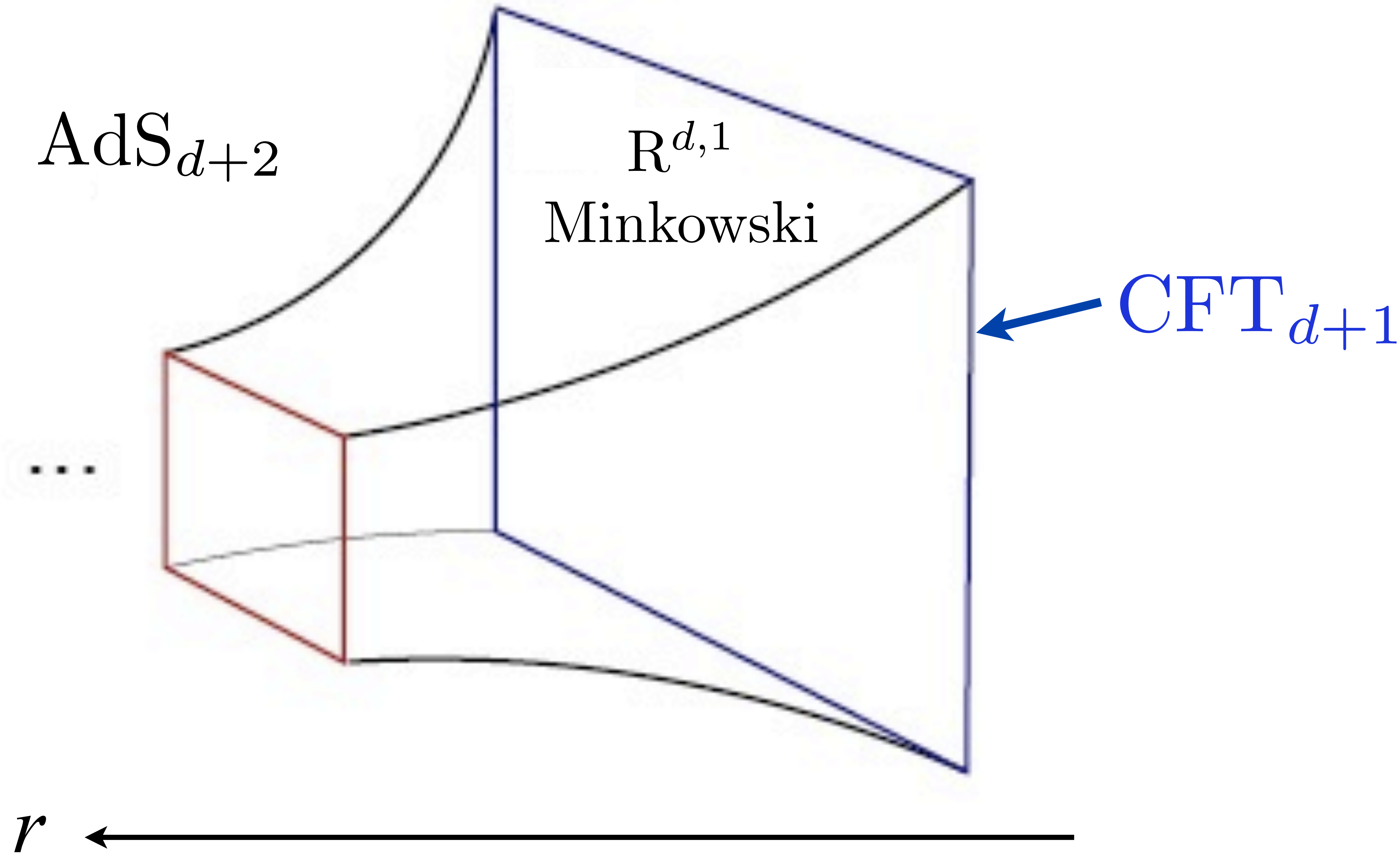
$$ds^2 = L^2 \frac{\left(-dt^2 + dx_i^2 + dr^2\right)}{r^2}.$$

This is the space AdS_{d+2}, and L is the AdS radius.


u

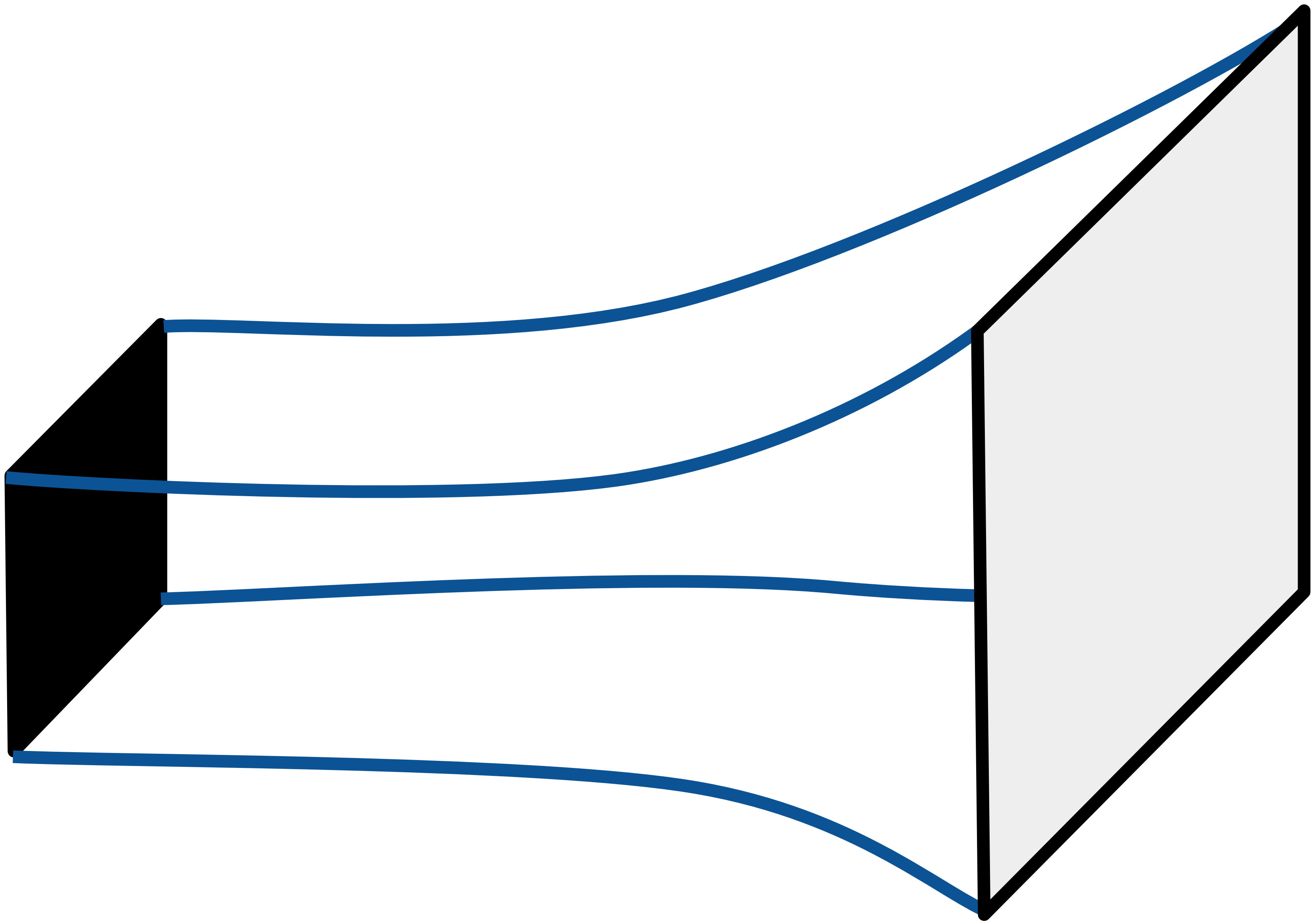


J. McGreevy, arXiv0909.0518



J. McGreevy, arXiv0909.0518

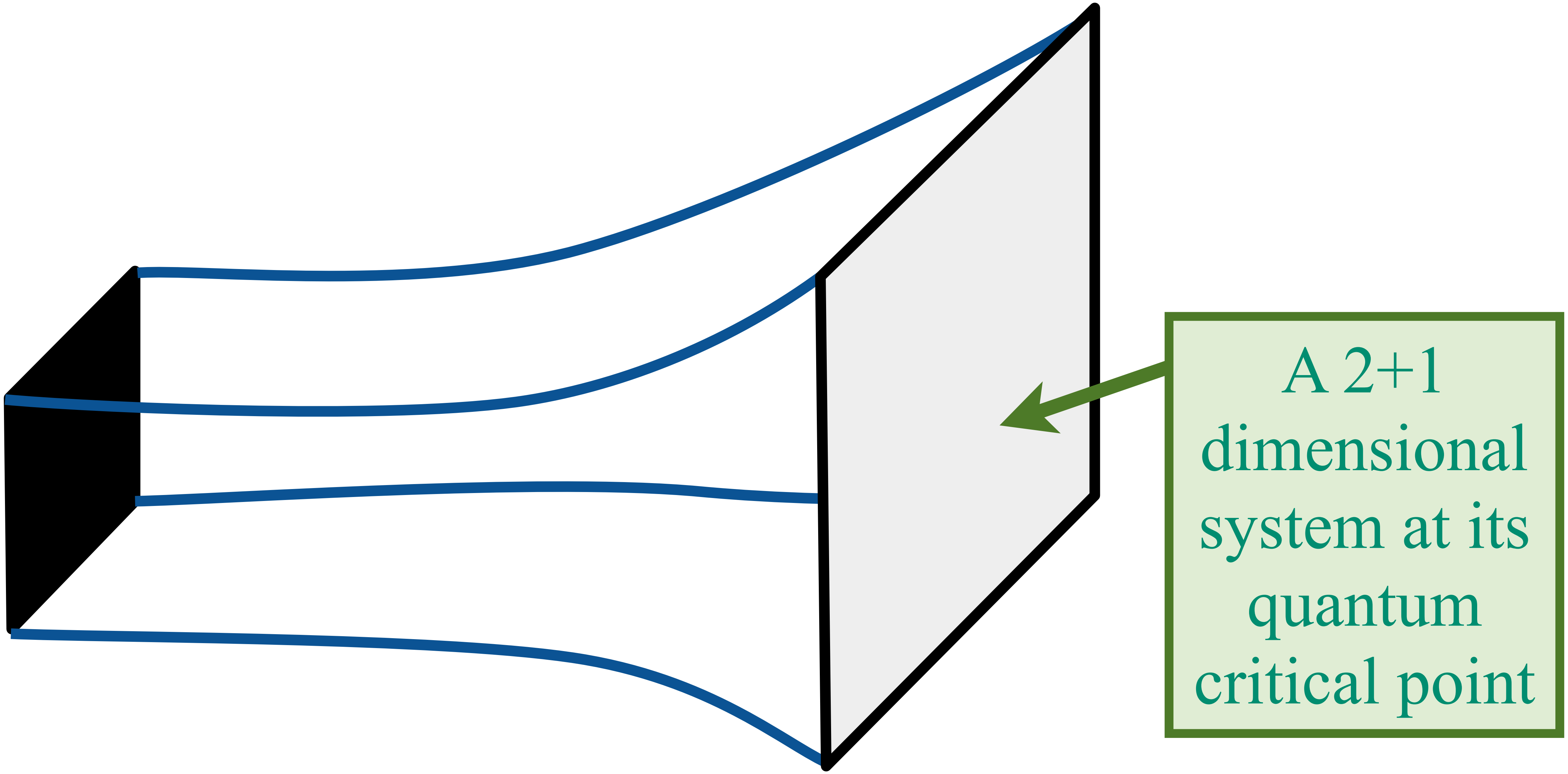
AdS4-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

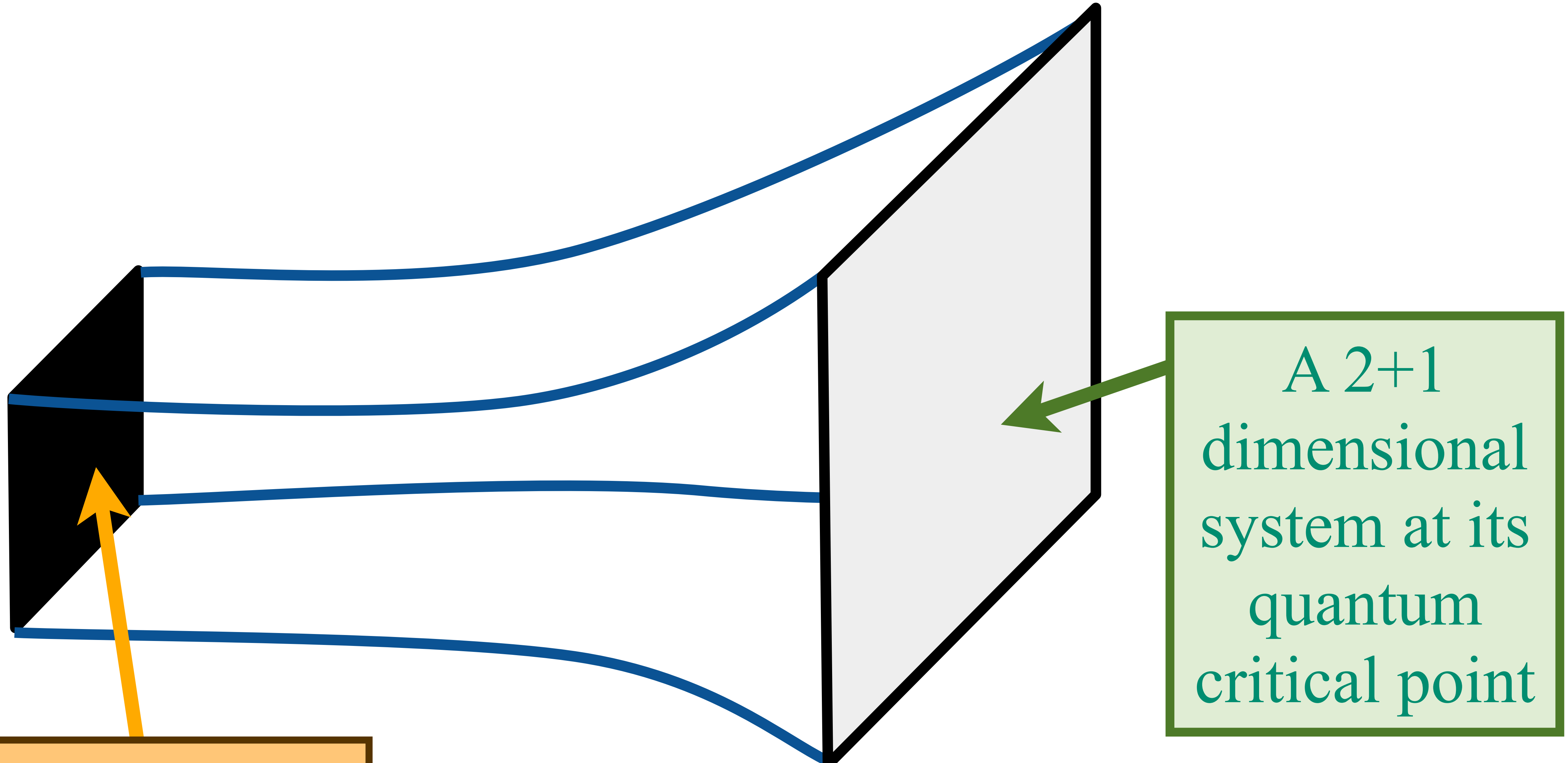
AdS/CFT correspondence

AdS_4-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

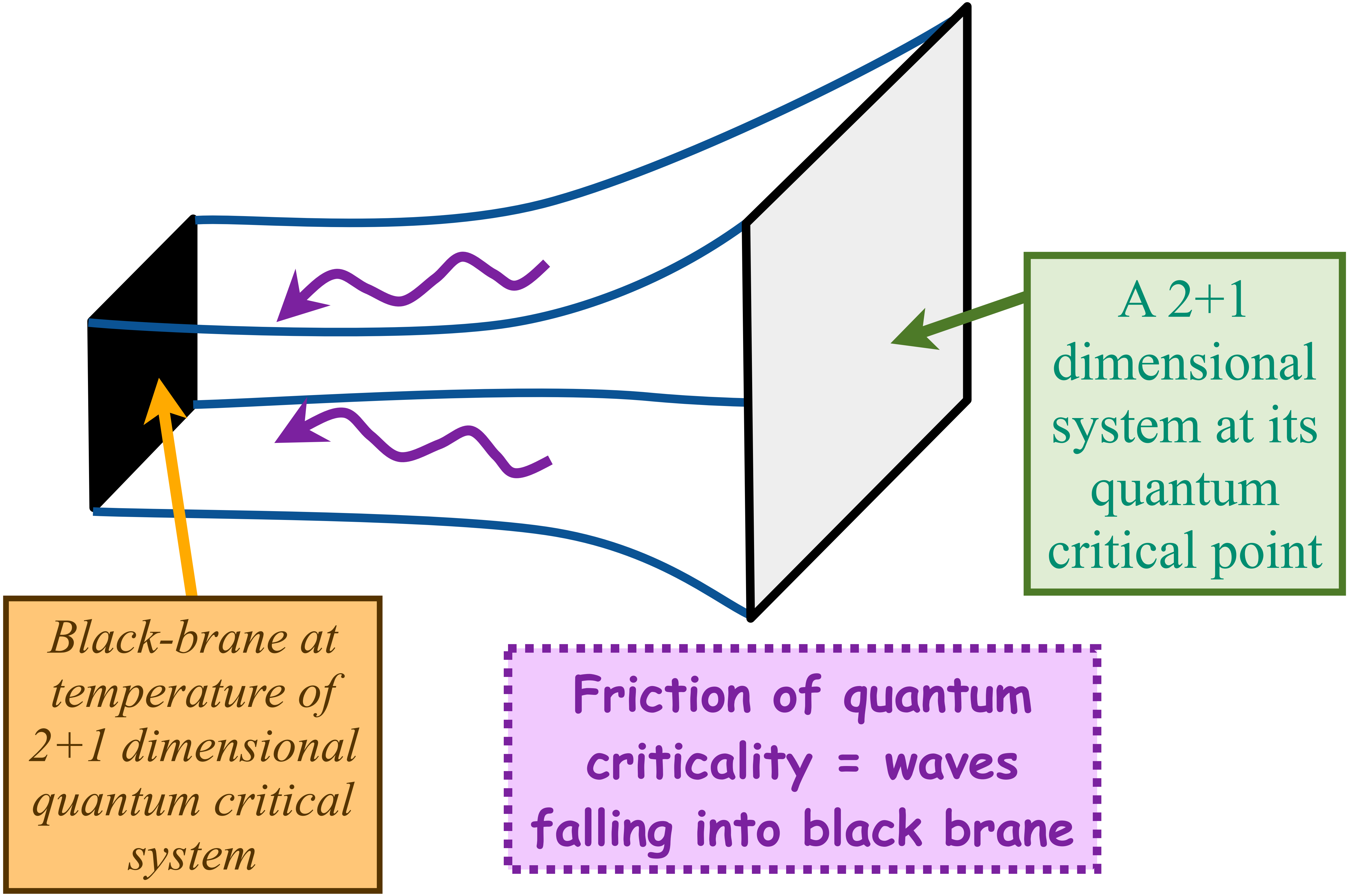
AdS4-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence

AdS$_4$-Schwarzschild black-brane



AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

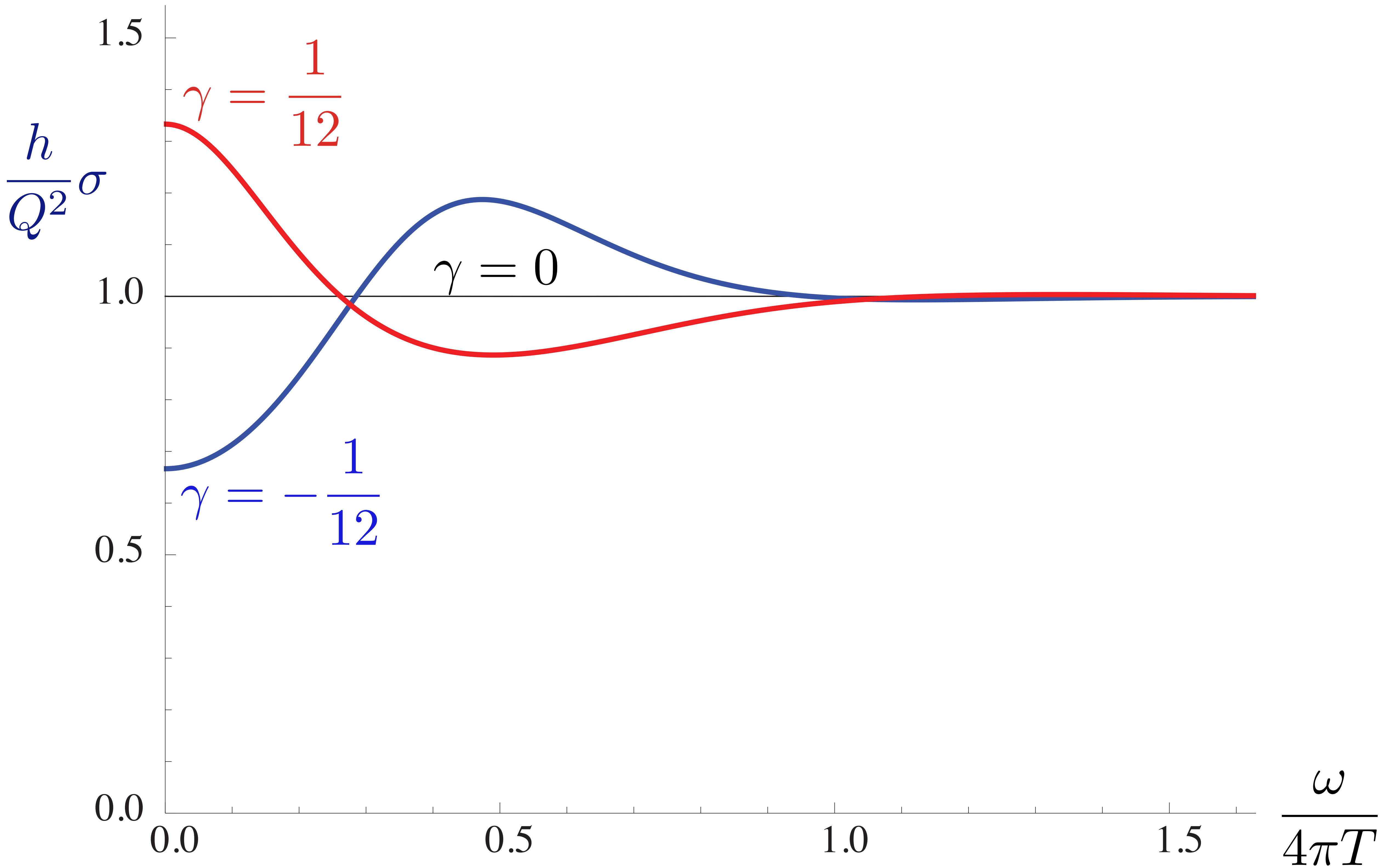
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

$$\mathcal{S}_{EM} = \int d^4x\sqrt{-g}\left[-\frac{1}{4e^2}F_{ab}F^{ab} + \frac{\gamma L^2}{e^2}C_{abcd}F^{ab}F^{cd}\right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

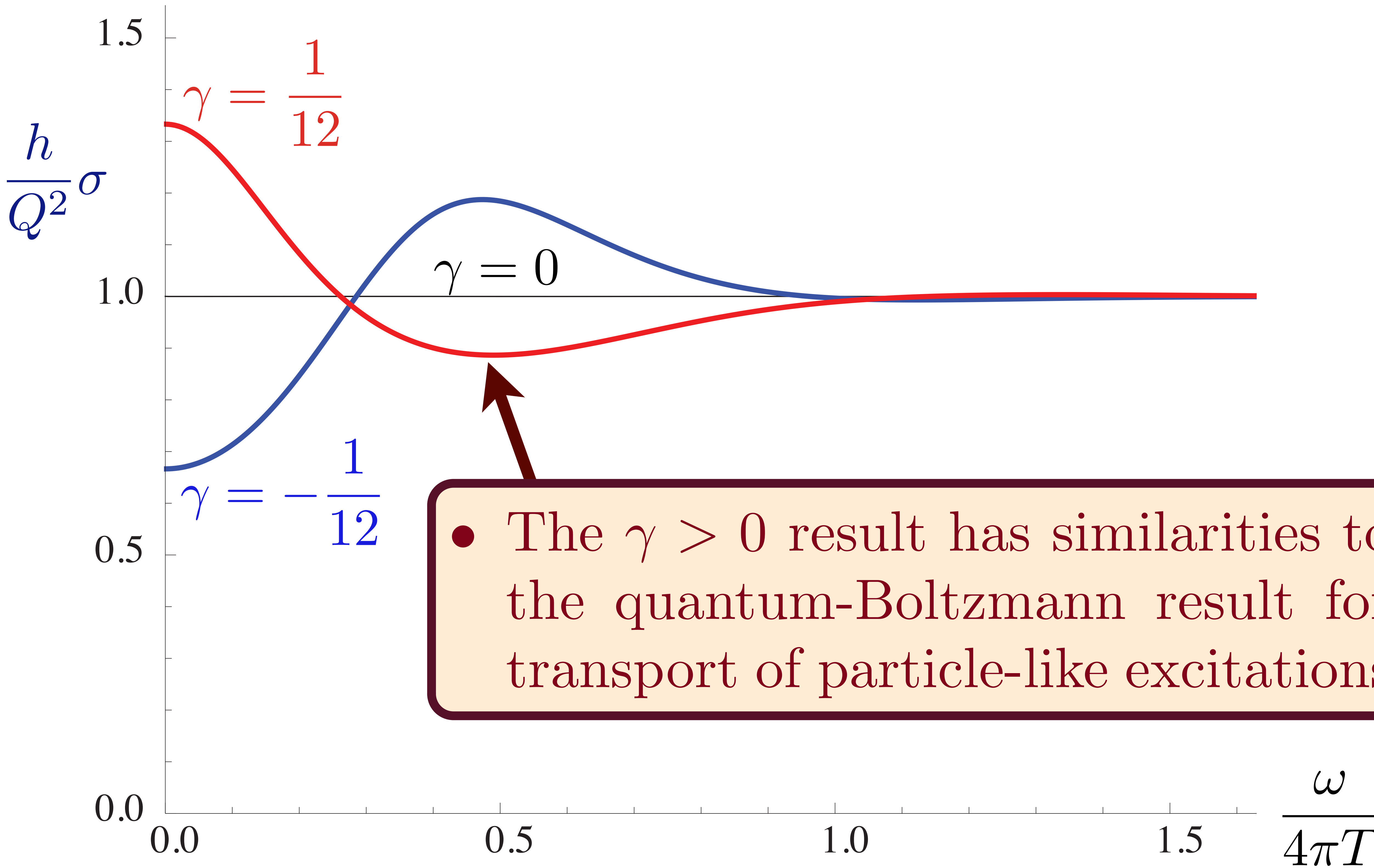
R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

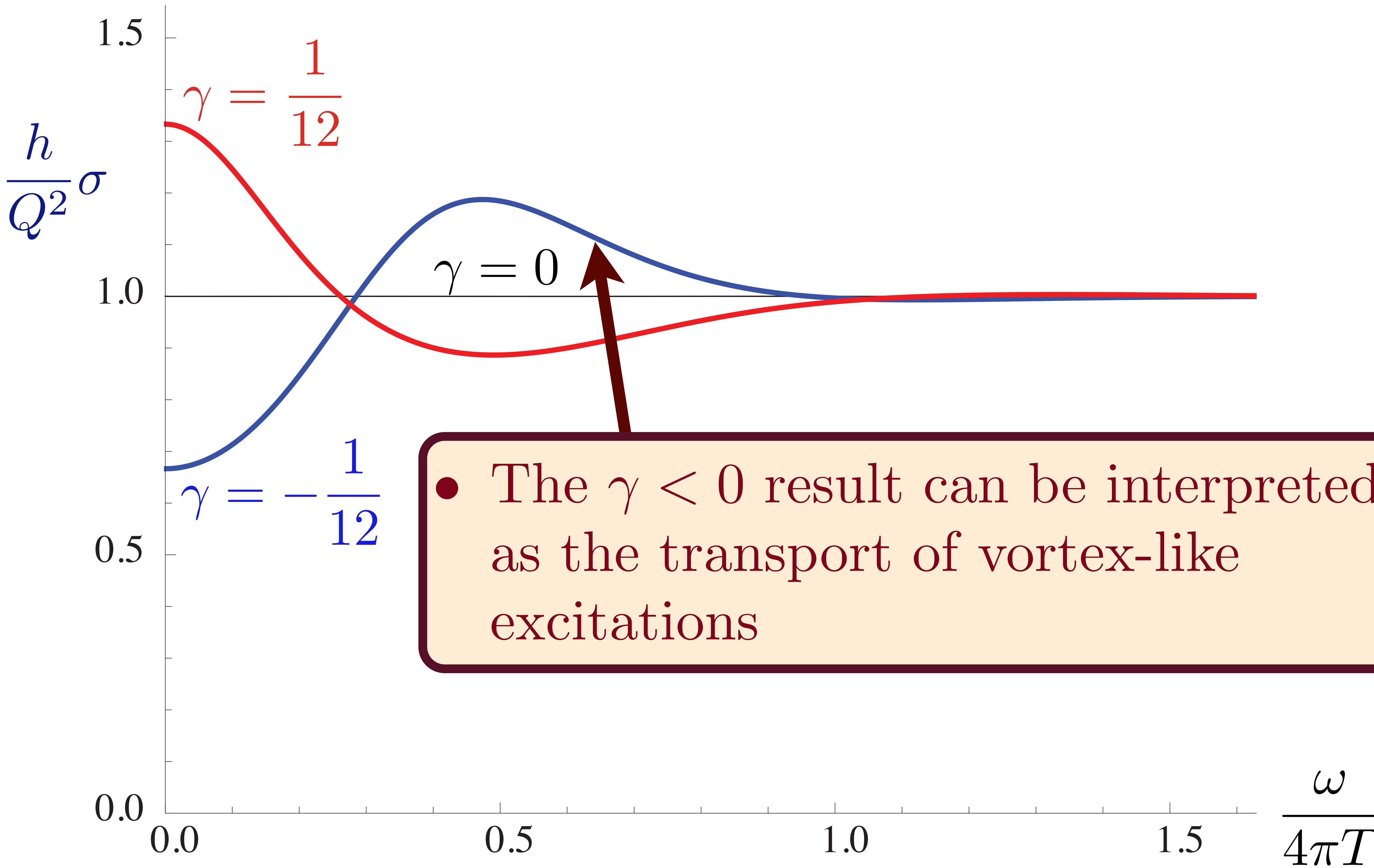
AdS$_4$ theory of strongly interacting "perfect fluids"



- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

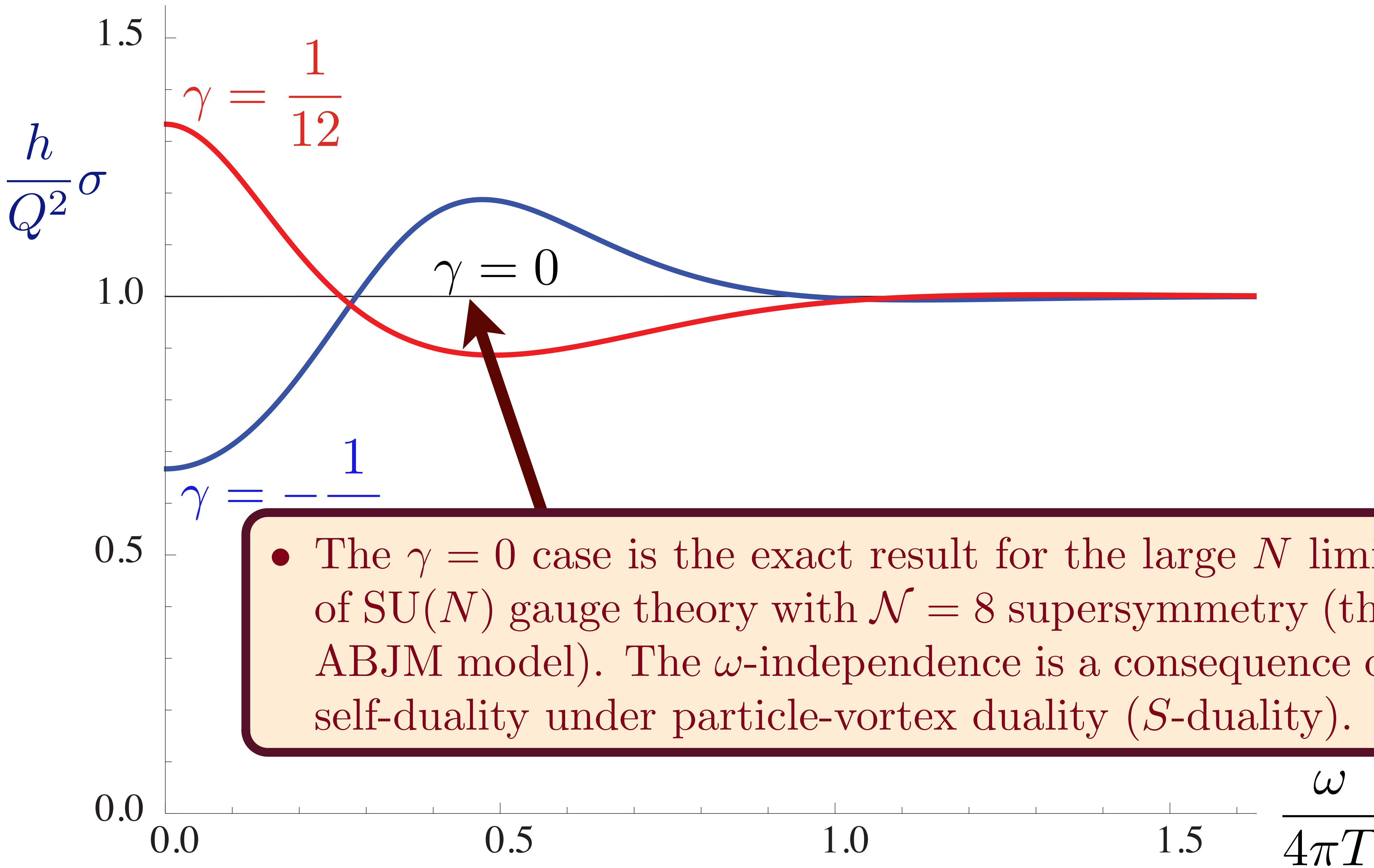
AdS4 theory of strongly interacting "perfect fluids"



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

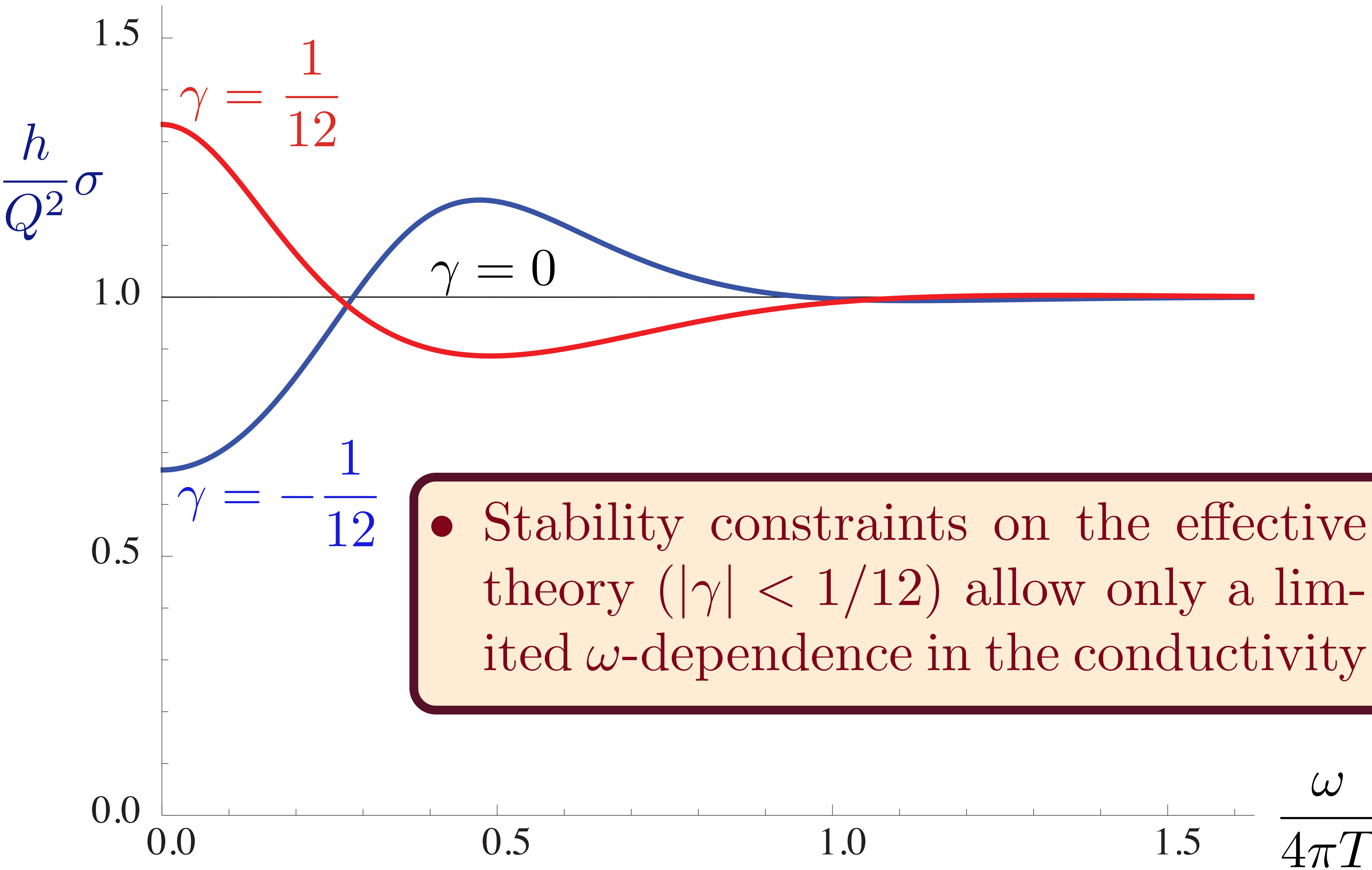
AdS$_4$ theory of strongly interacting “perfect fluids”



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of "nearly perfect fluids"

Theory for transport of conserved quantities in CFT3s:

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab}F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of "nearly perfect fluids"

Theory for transport of conserved quantities in CFT3s:

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab}F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.

General approach:

- Theory has 2 free dimensionless parameters: e^2 and γ. We match these to correlators of the CFT3 of interest at $\omega \gg T$: e^2 is determines the current correlator $\langle J_\mu J_\nu \rangle$, while γ determines the 3-point function $\langle T_{\mu\nu} J_\rho J_\sigma \rangle$, where $T_{\mu\nu}$ is the stress-energy tensor.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of "nearly perfect fluids"

Theory for transport of conserved quantities in CFT3s:

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.

General approach:

- Theory has 2 free dimensionless parameters: e^2 and γ. We match these to correlators of the CFT3 of interest at $\omega \gg T$: e^2 is determines the current correlator $\langle J_\mu J_\nu \rangle$, while γ determines the 3-point function $\langle T_{\mu\nu} J_\rho J_\sigma \rangle$, where $T_{\mu\nu}$ is the stress-energy tensor.

- We determine these $\omega \gg T$ correlators of the CFT3 by other methods (*e.g.* vector large N expansion), and so obtain values of e^2 and γ.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of "nearly perfect fluids"

Theory for transport of conserved quantities in CFT3s:

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right],$$

where C_{abcd} is the Weyl curvature tensor.

General approach:

- Theory has 2 free dimensionless parameters: e^2 and γ. We match these to correlators of the CFT3 of interest at $\omega \gg T$: e^2 is determines the current correlator $\langle J_\mu J_\nu \rangle$, while γ determines the 3-point function $\langle T_{\mu\nu} J_\rho J_\sigma \rangle$, where $T_{\mu\nu}$ is the stress-energy tensor.

- We determine these $\omega \gg T$ correlators of the CFT3 by other methods (*e.g.* vector large N expansion), and so obtain values of e^2 and γ.

- We use $\mathcal{S}_{EM}$ to extrapolate to transport properties for $\omega \ll T$. This step is traditionally carried out by descendants of the Boltzmann equation.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Conclusions

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

Conclusions

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

Conclusions

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport