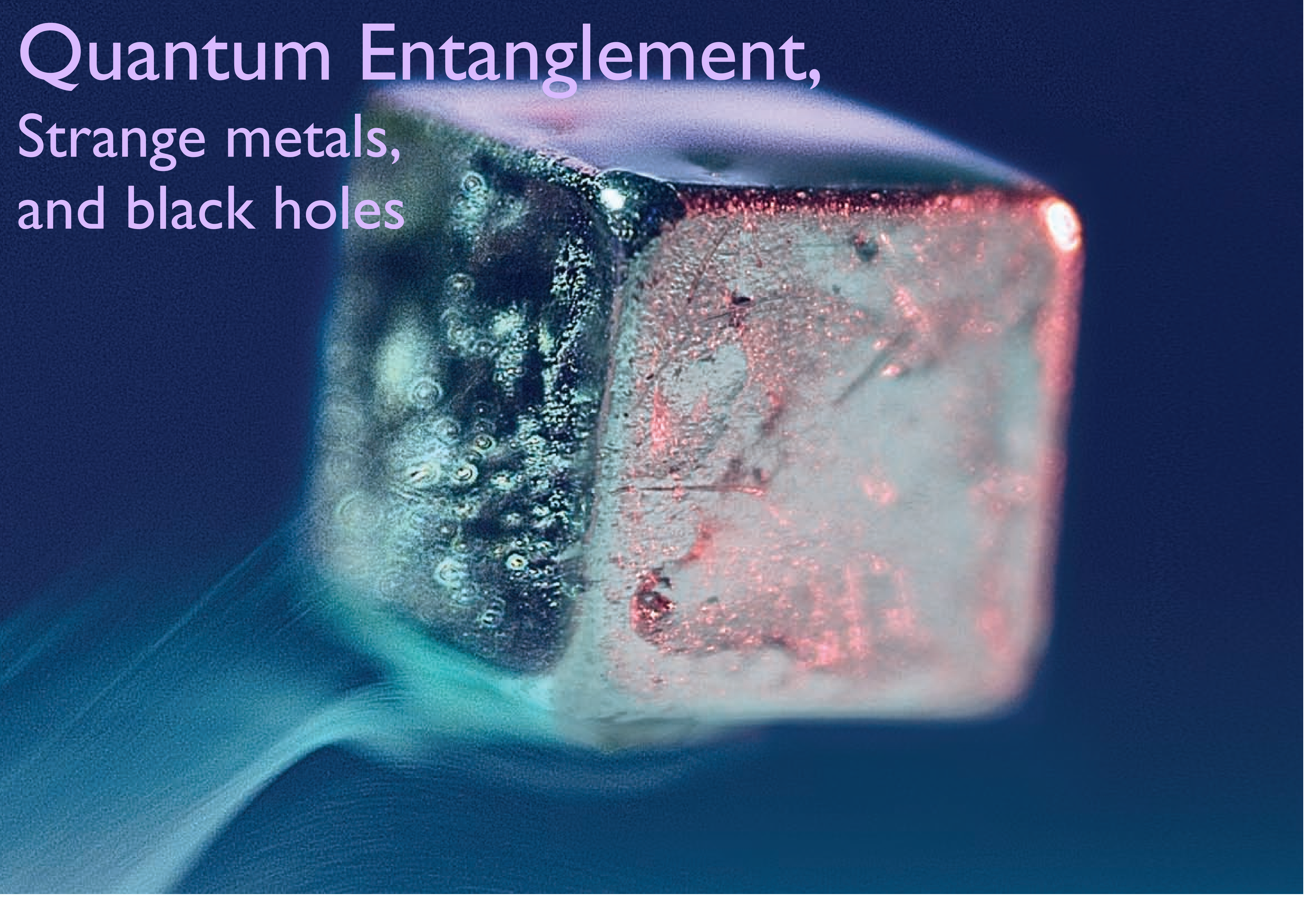
Quantum Entanglement,
Strange metals, and black holes

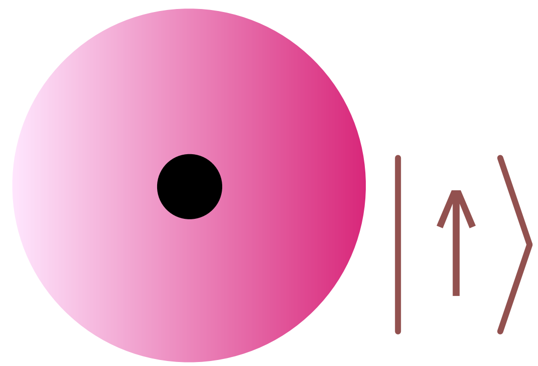


Subir Sachdev, Harvard University and Perimeter Institute

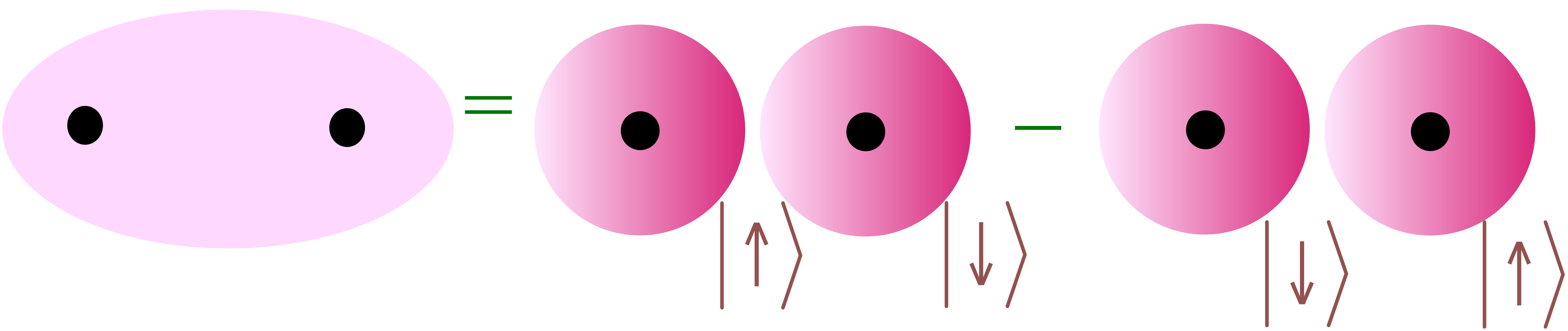
Quantum entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

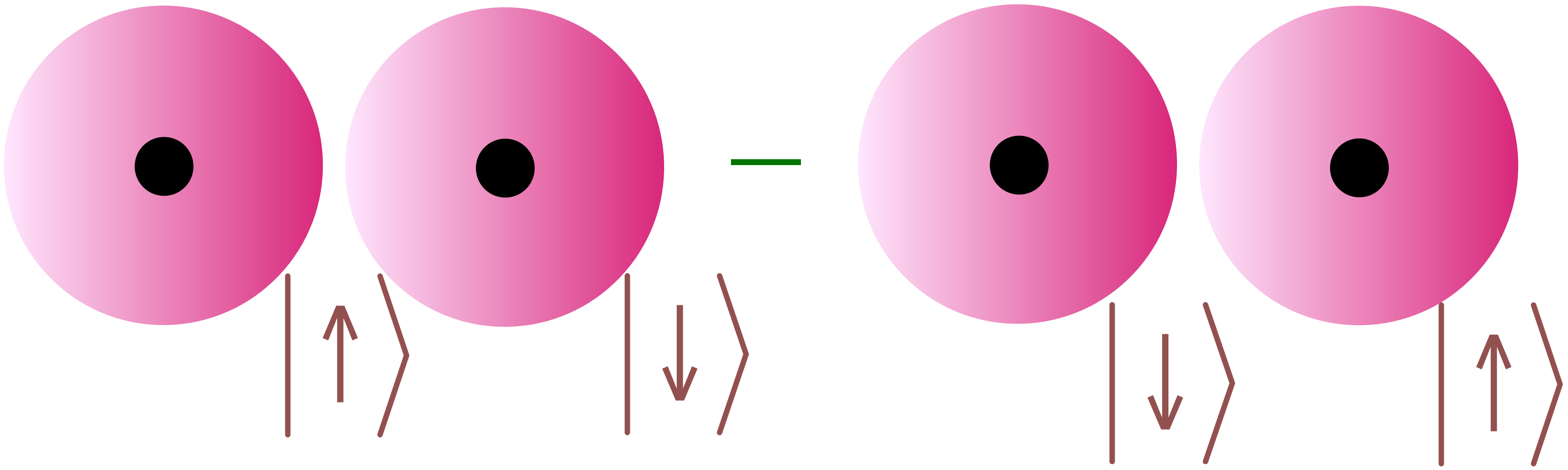


Hydrogen molecule:

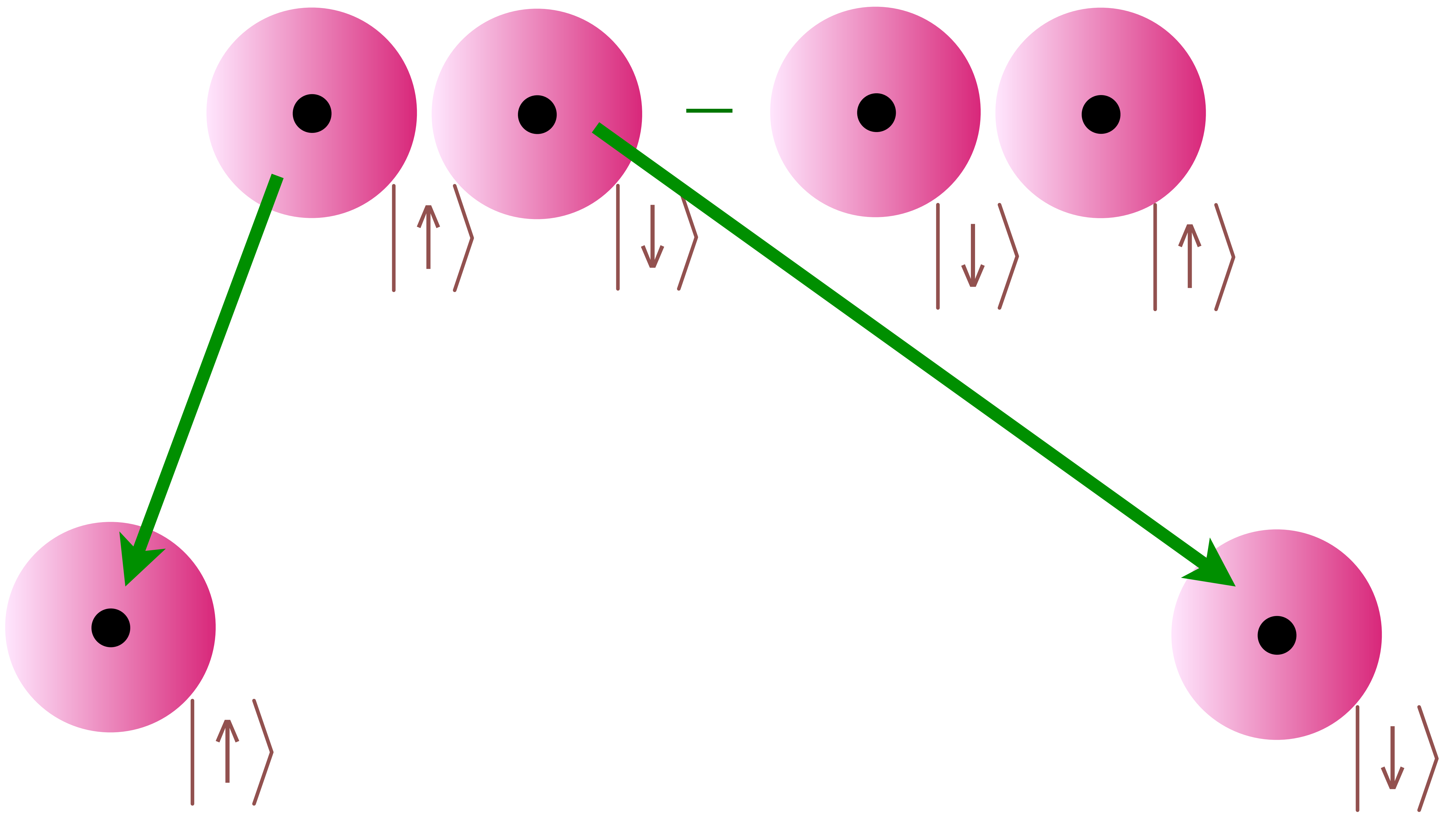


$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

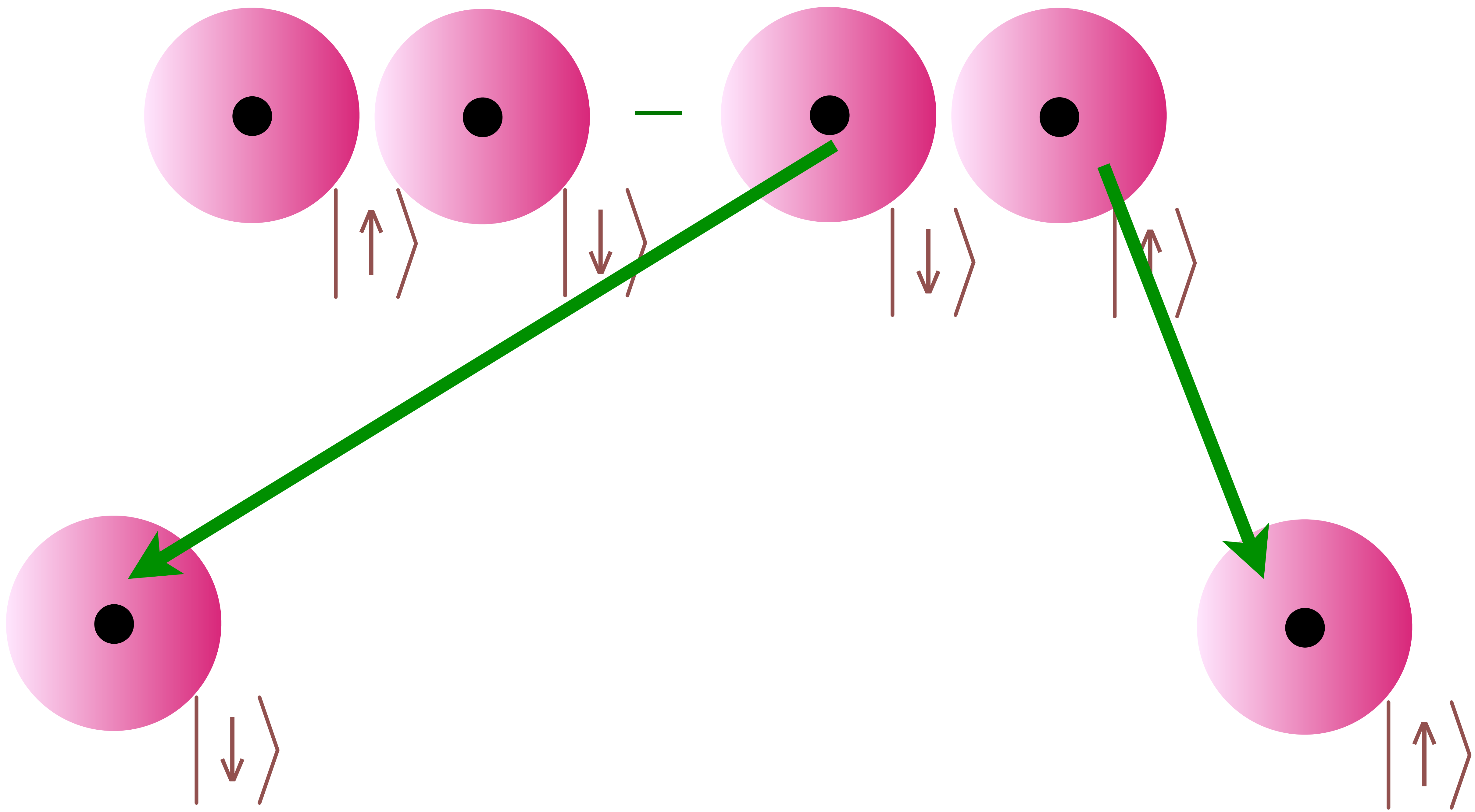
Quantum Entanglement: quantum superposition with more than one particle



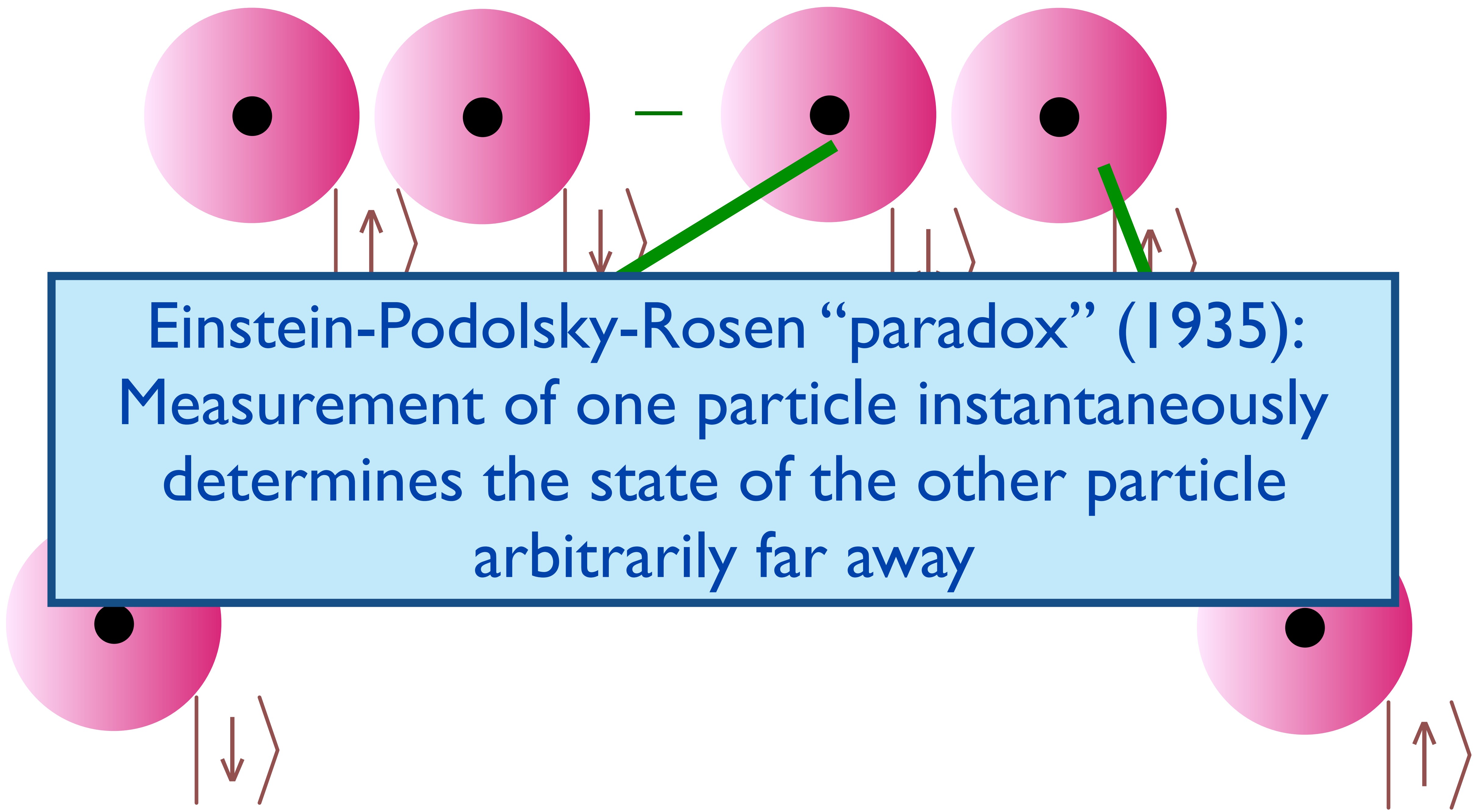
Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



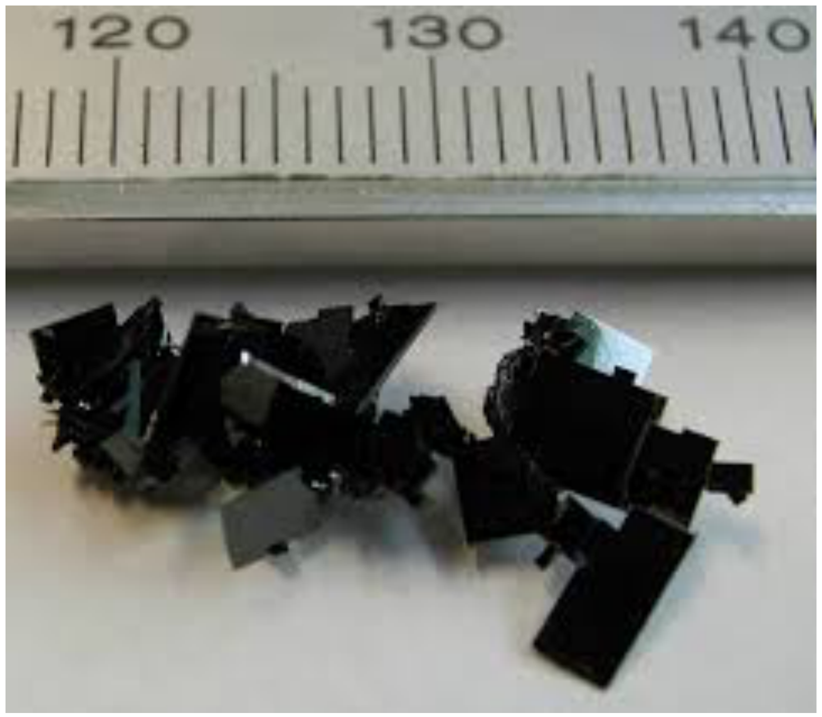
Einstein-Podolsky-Rosen “paradox” (1935):
Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

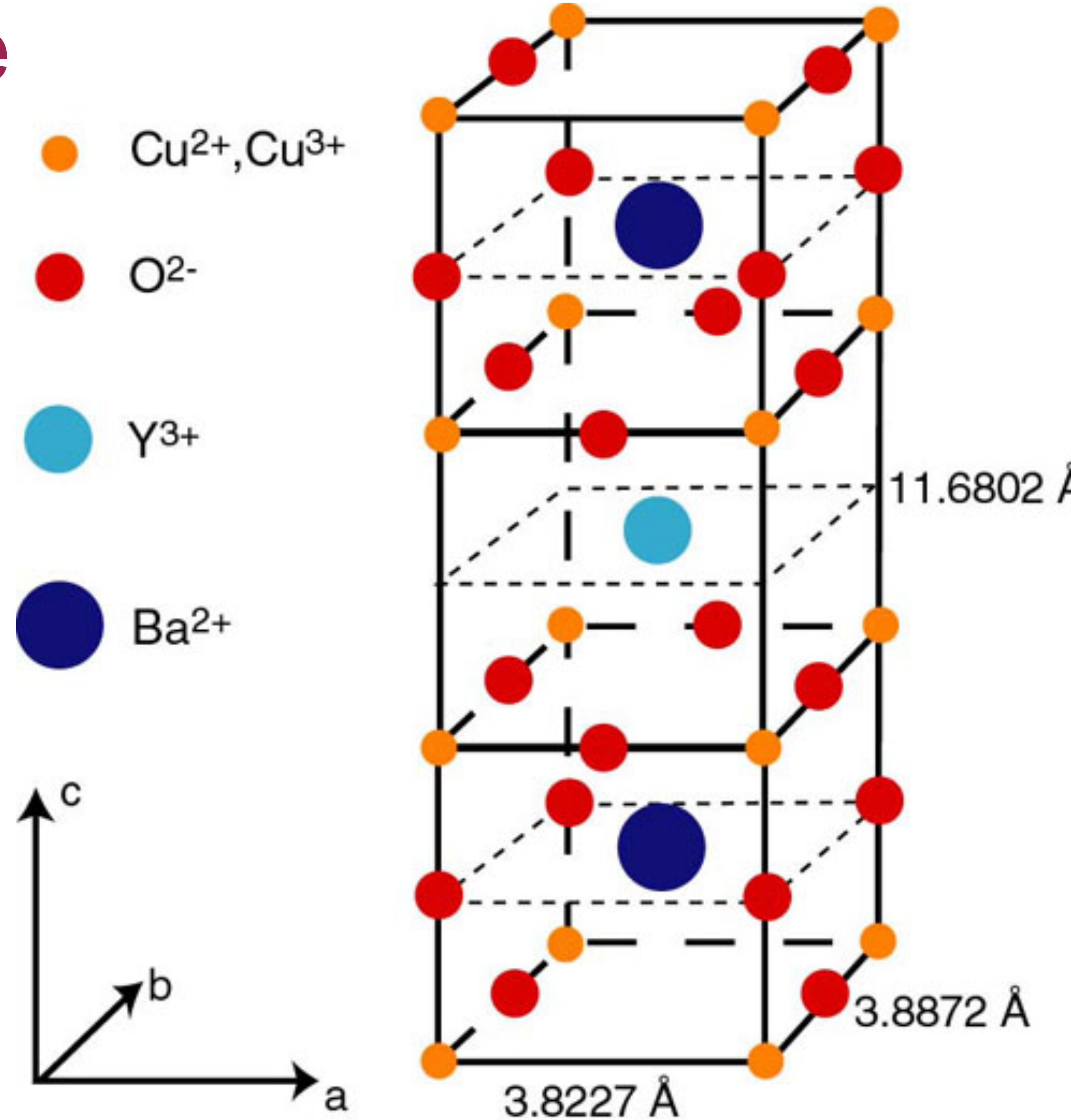
Quantum entanglement

Quantum entanglement

Strange metals

High temperature superconductors





$$YBa_2Cu_3O_{6+x}$$

Slide by J. C. Seamus Davis

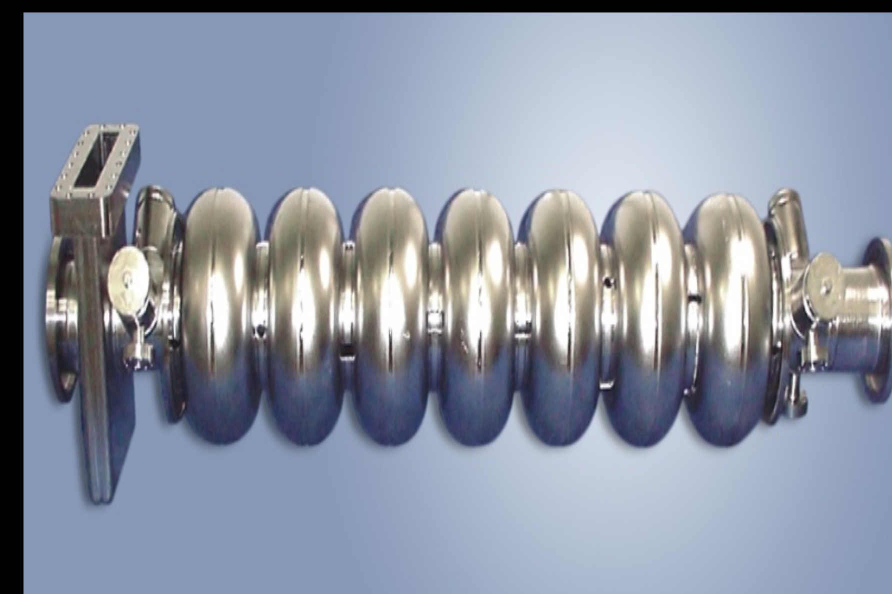

Power Efficiency/Capacity/Stability

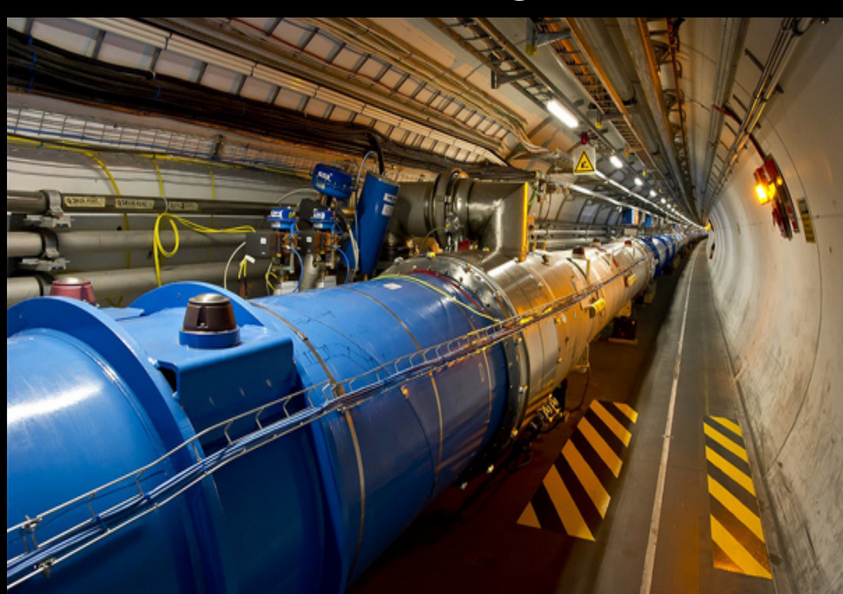

Power Bottlenecks


Accommodate Renewable Power


Efficient Rotating Machines


Information Technology

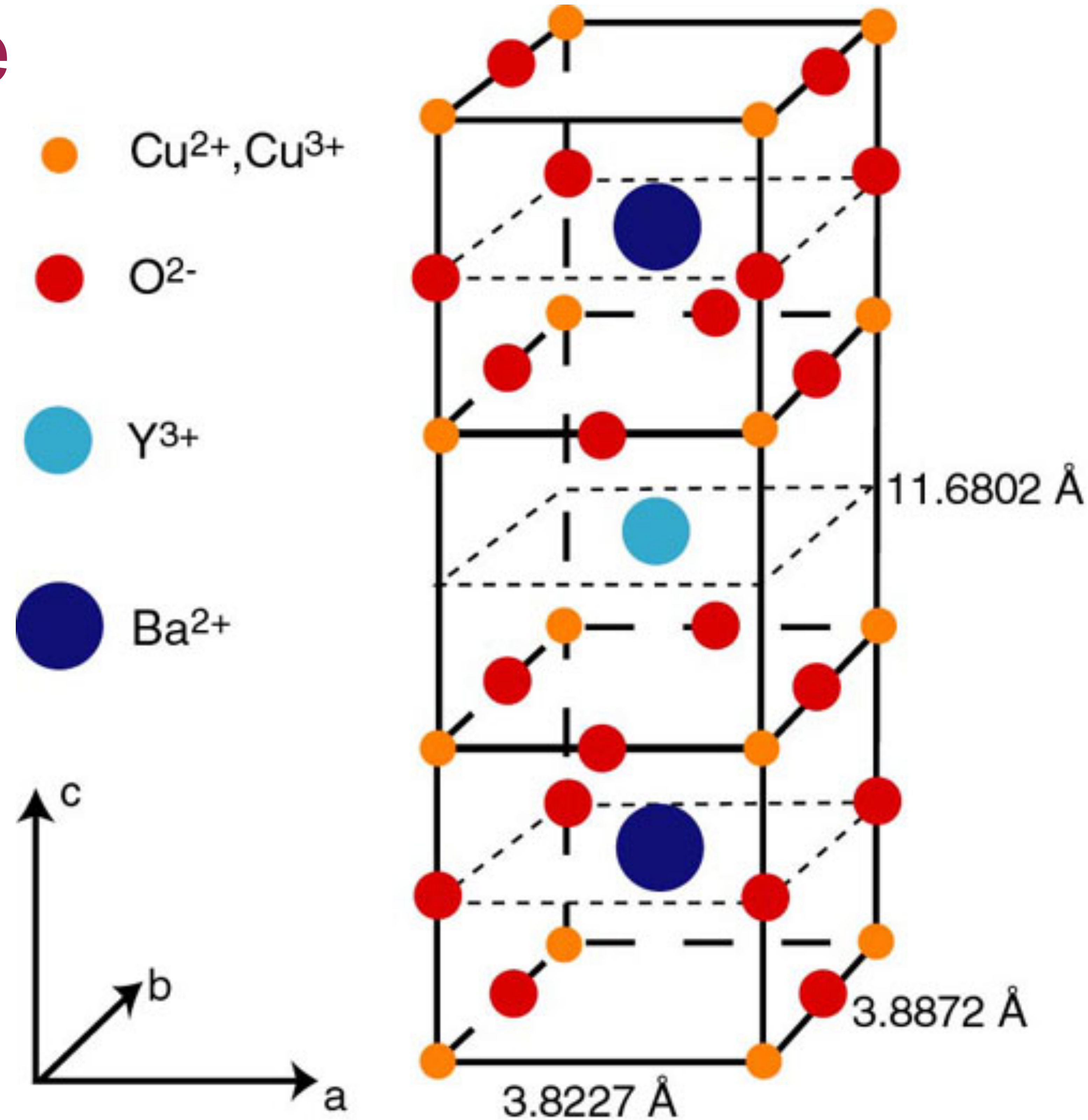

Next Generation HEP


Ultra-High Magnetic Fields


Medical


Transport

High temperature superconductors



$YBa_2Cu_3O_{6+x}$

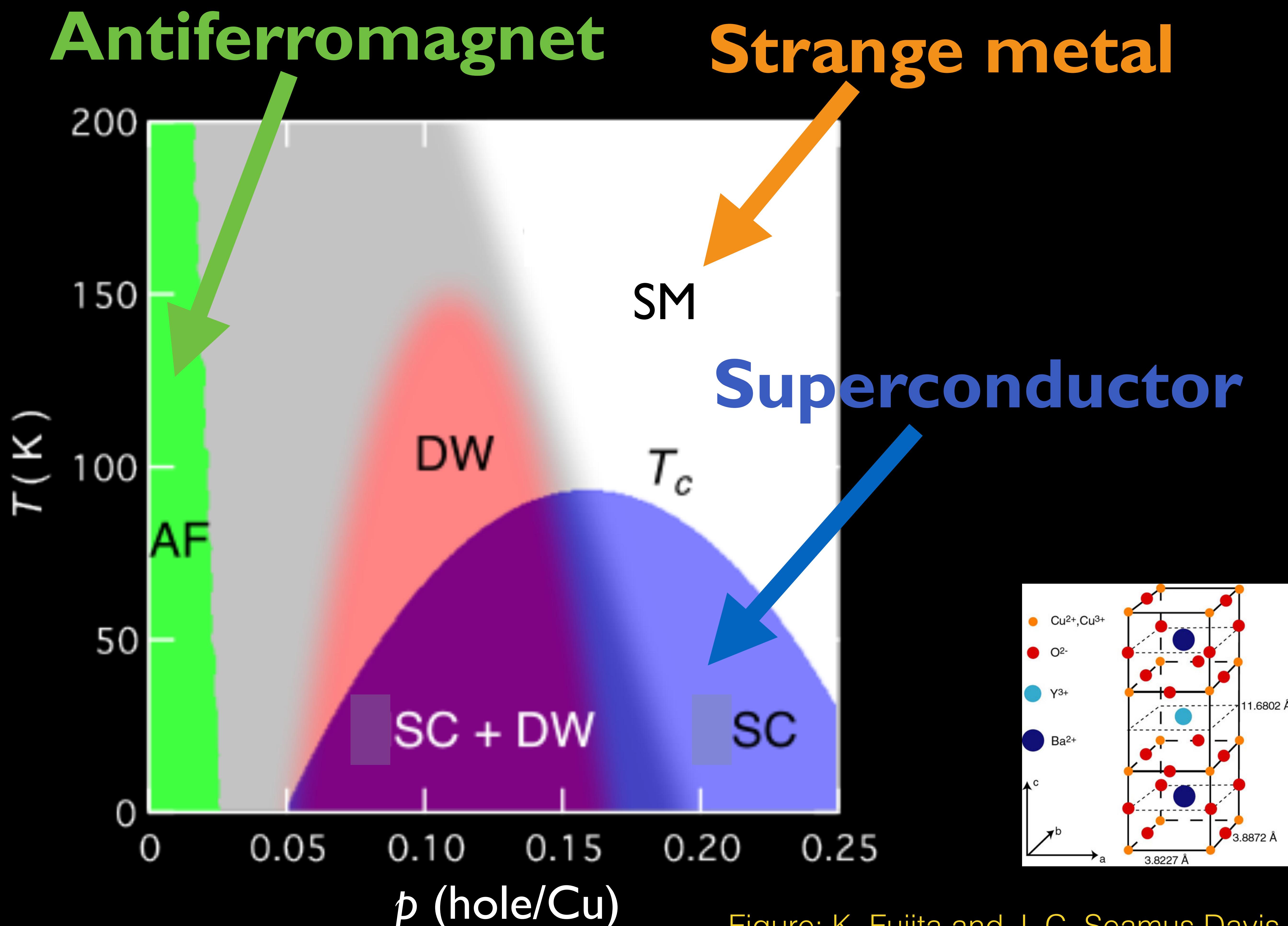
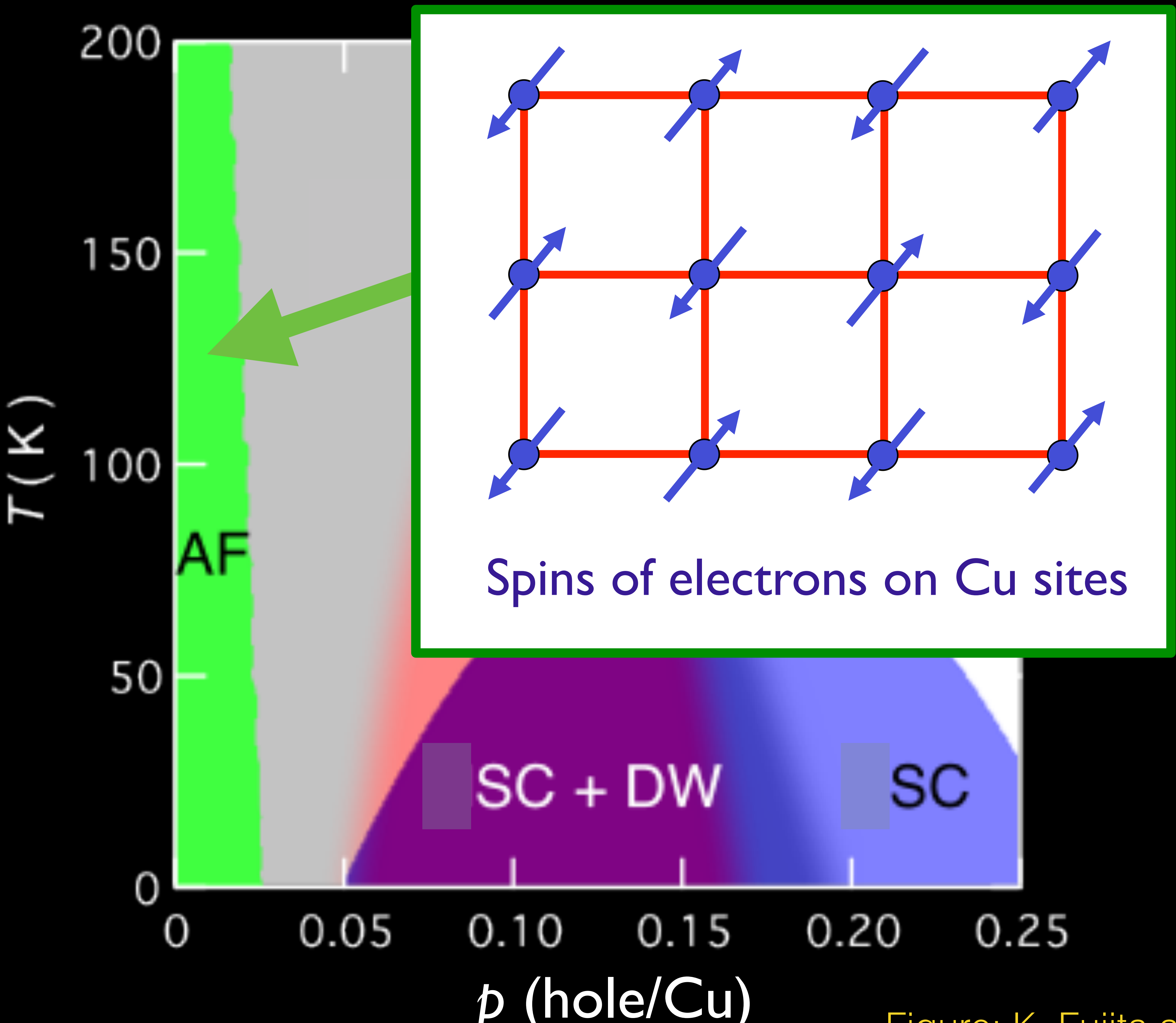


Figure: K. Fujita and J. C. Seamus Davis

Antiferromagnet



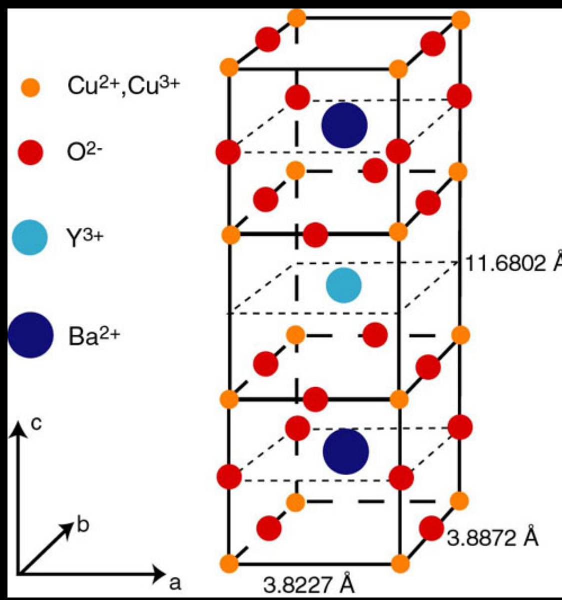
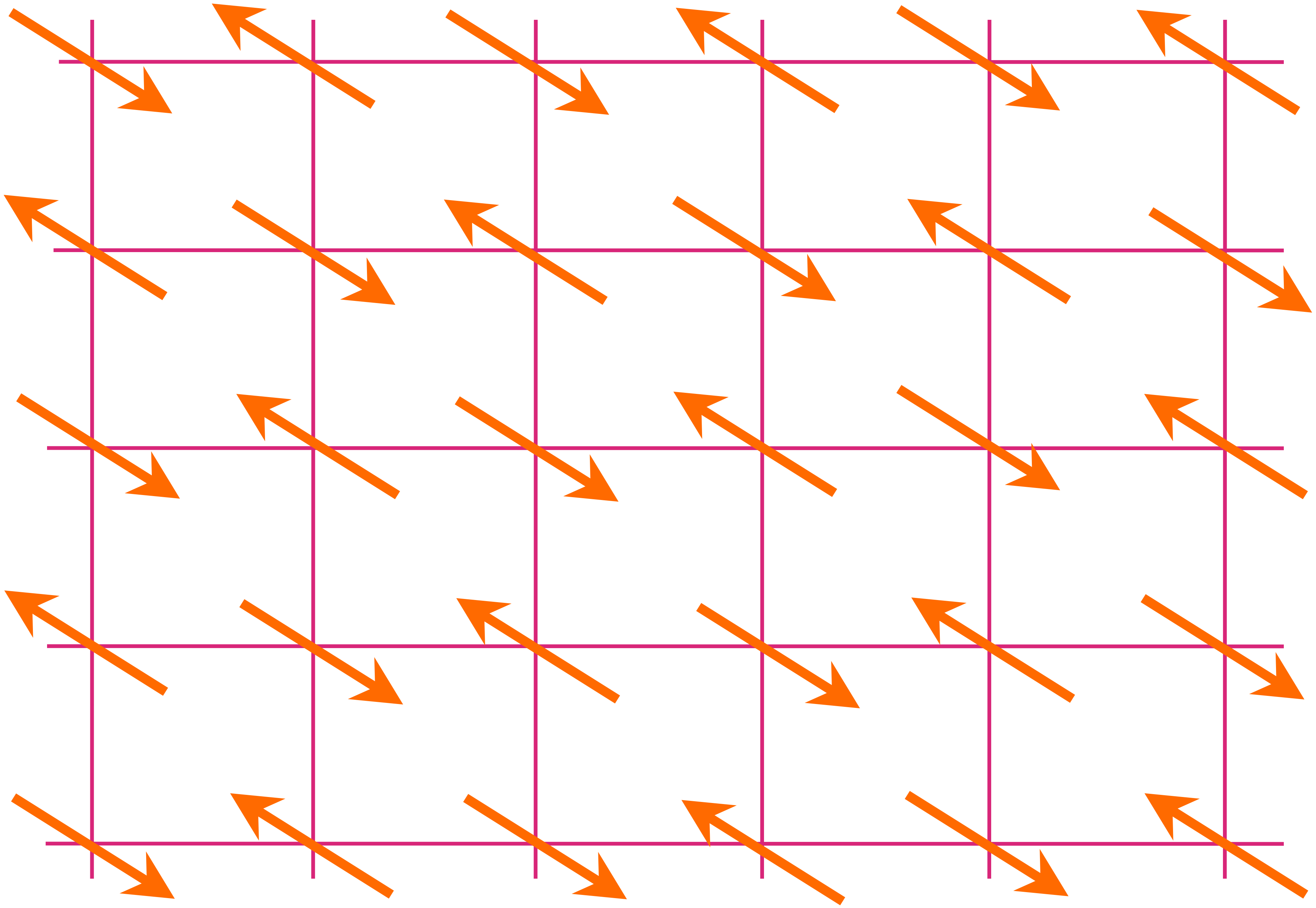
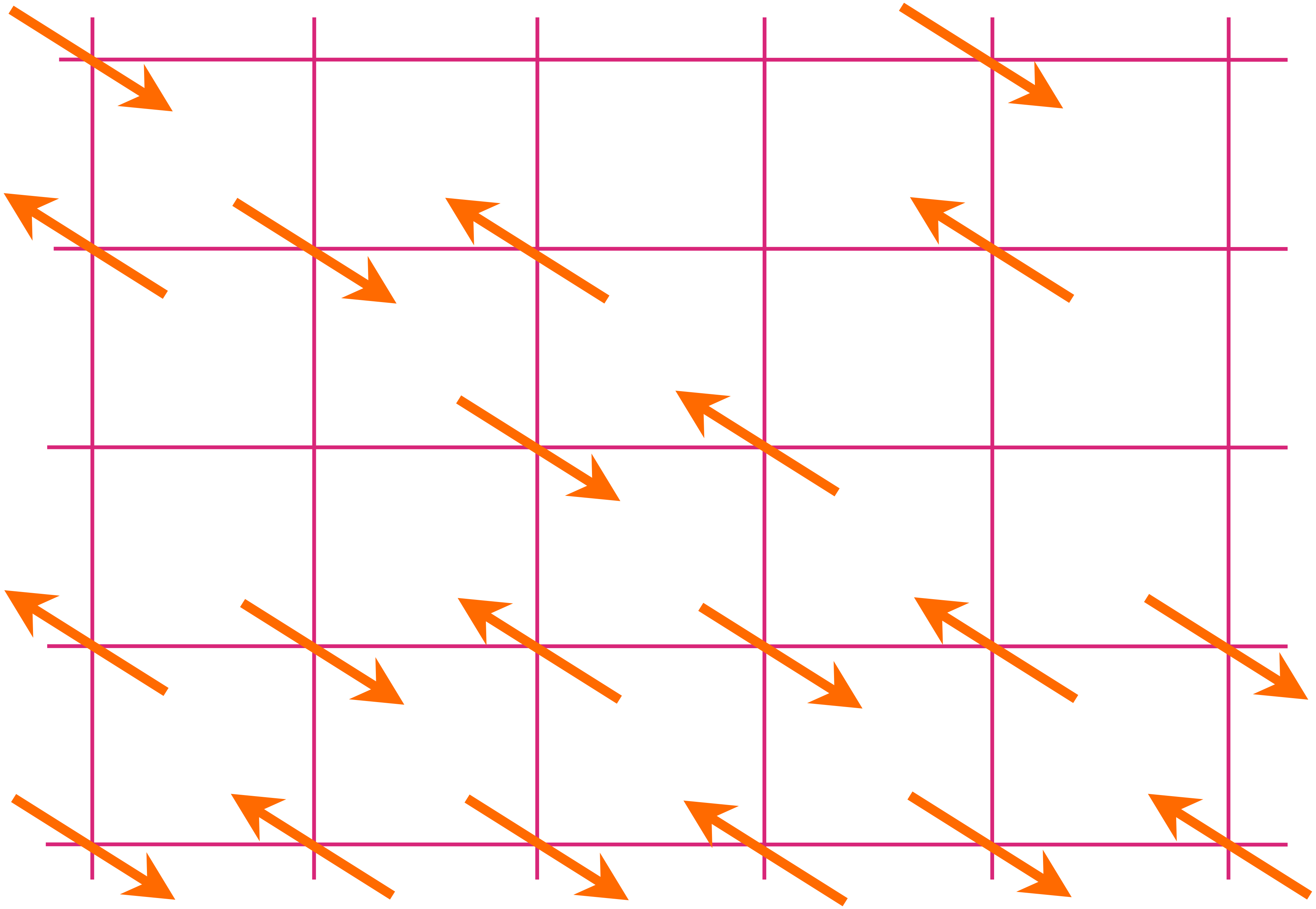


Figure: K. Fujita and J. C. Seamus Davis

Square lattice of Cu sites

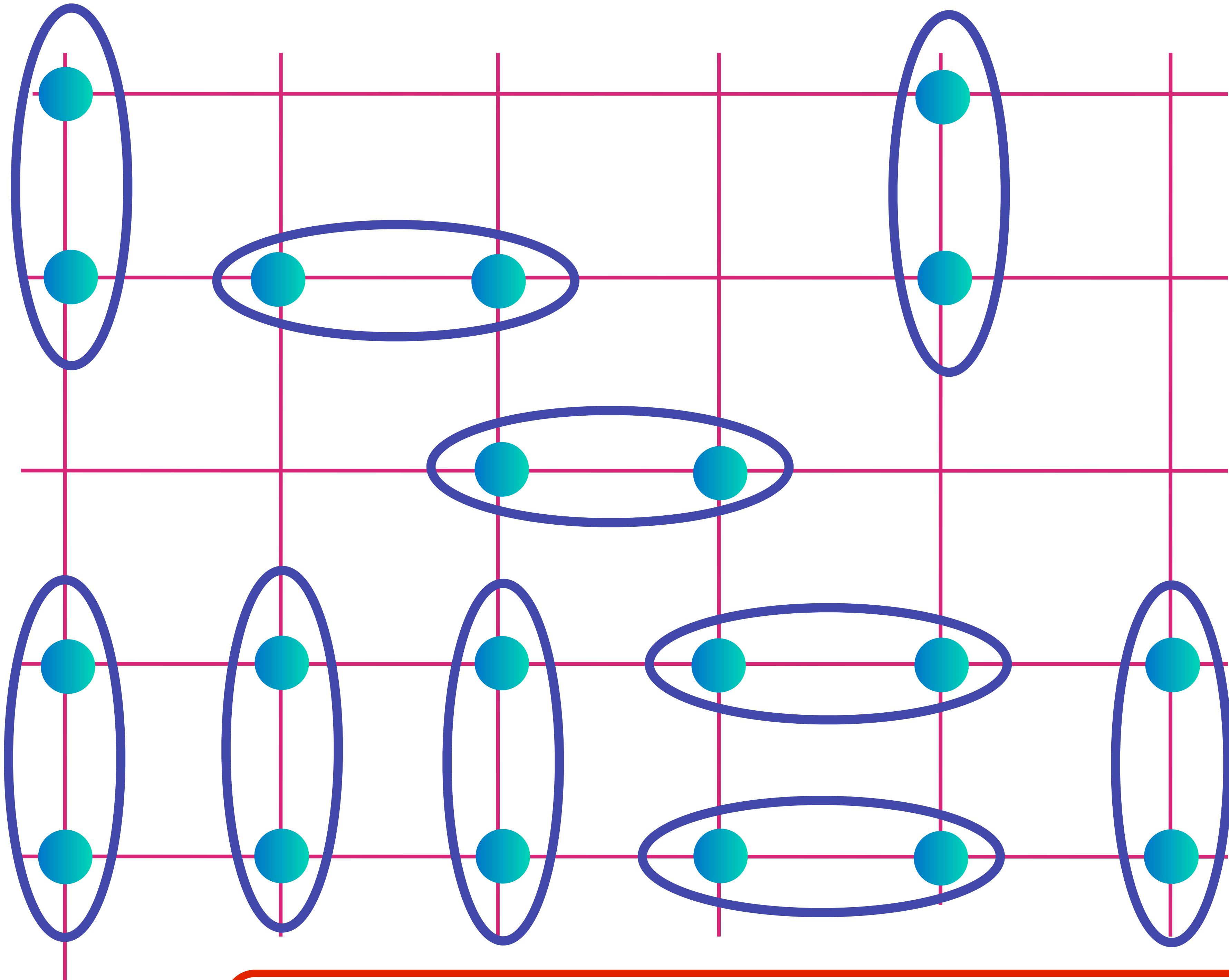


Square lattice of Cu sites

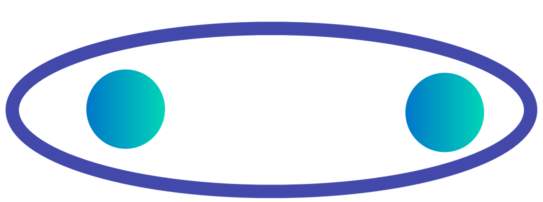


Remove density
p electrons

Square lattice of Cu sites

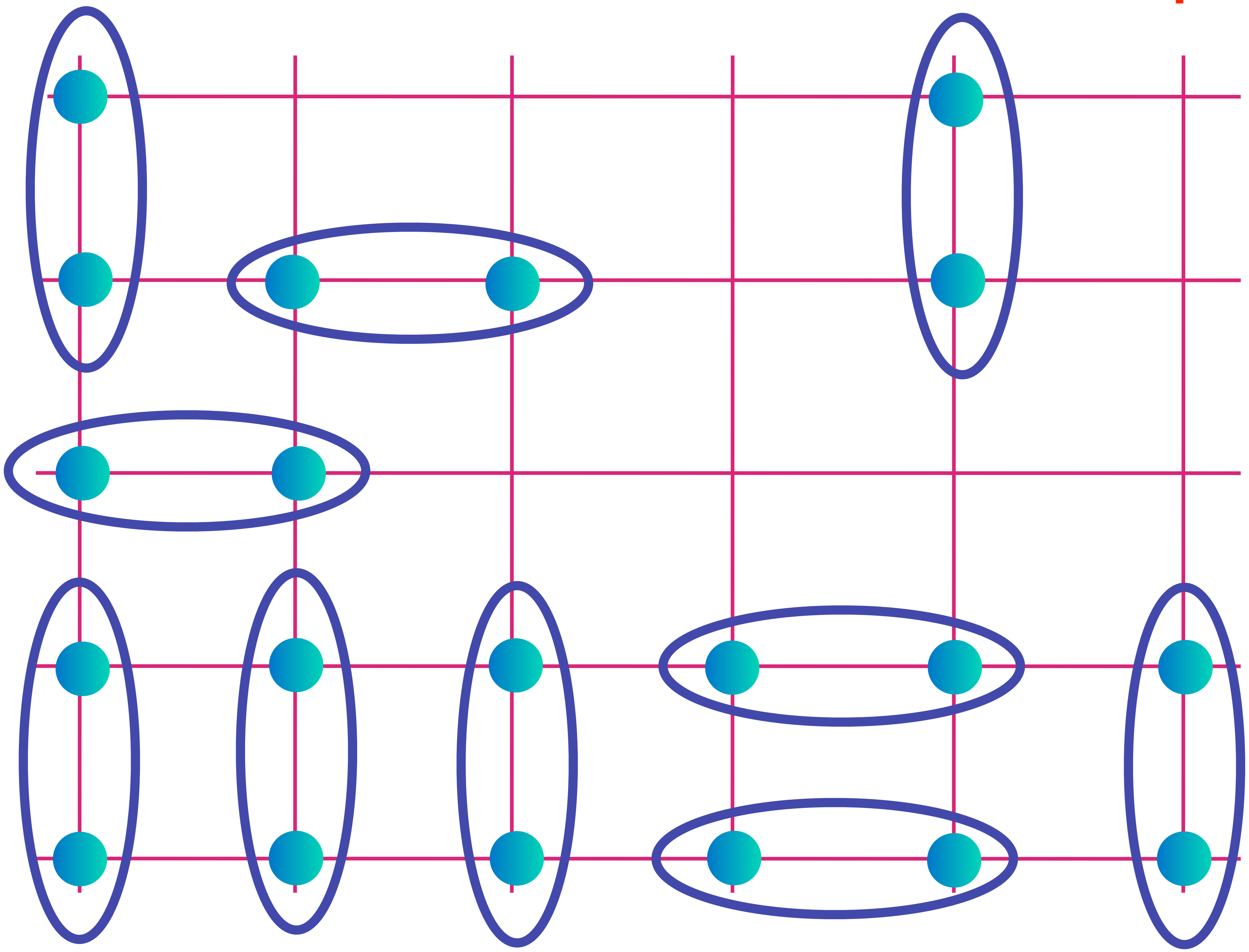


Electrons entangle in ("Cooper") pairs into chemical bonds

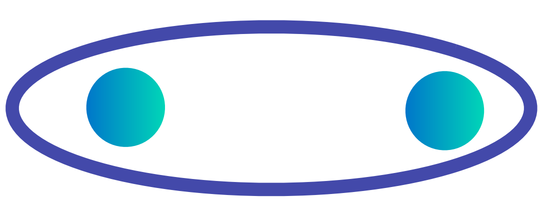
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

Superconductivity

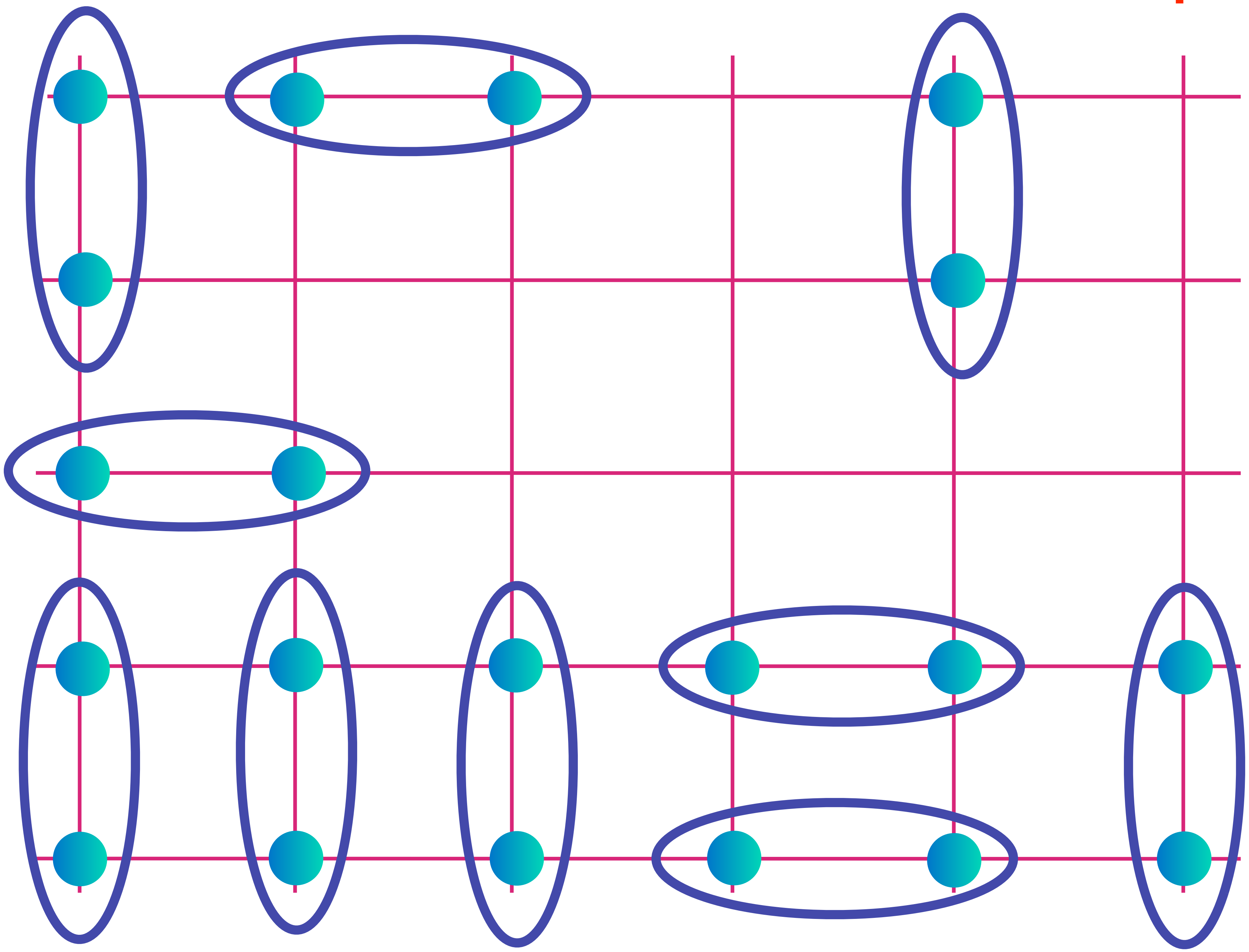


Cooper pairs form quantum superpositions at different locations: “Bose-Einstein condensation” in which all pairs are “everywhere at the same time”

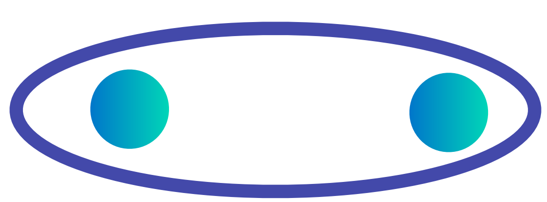
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

Superconductivity

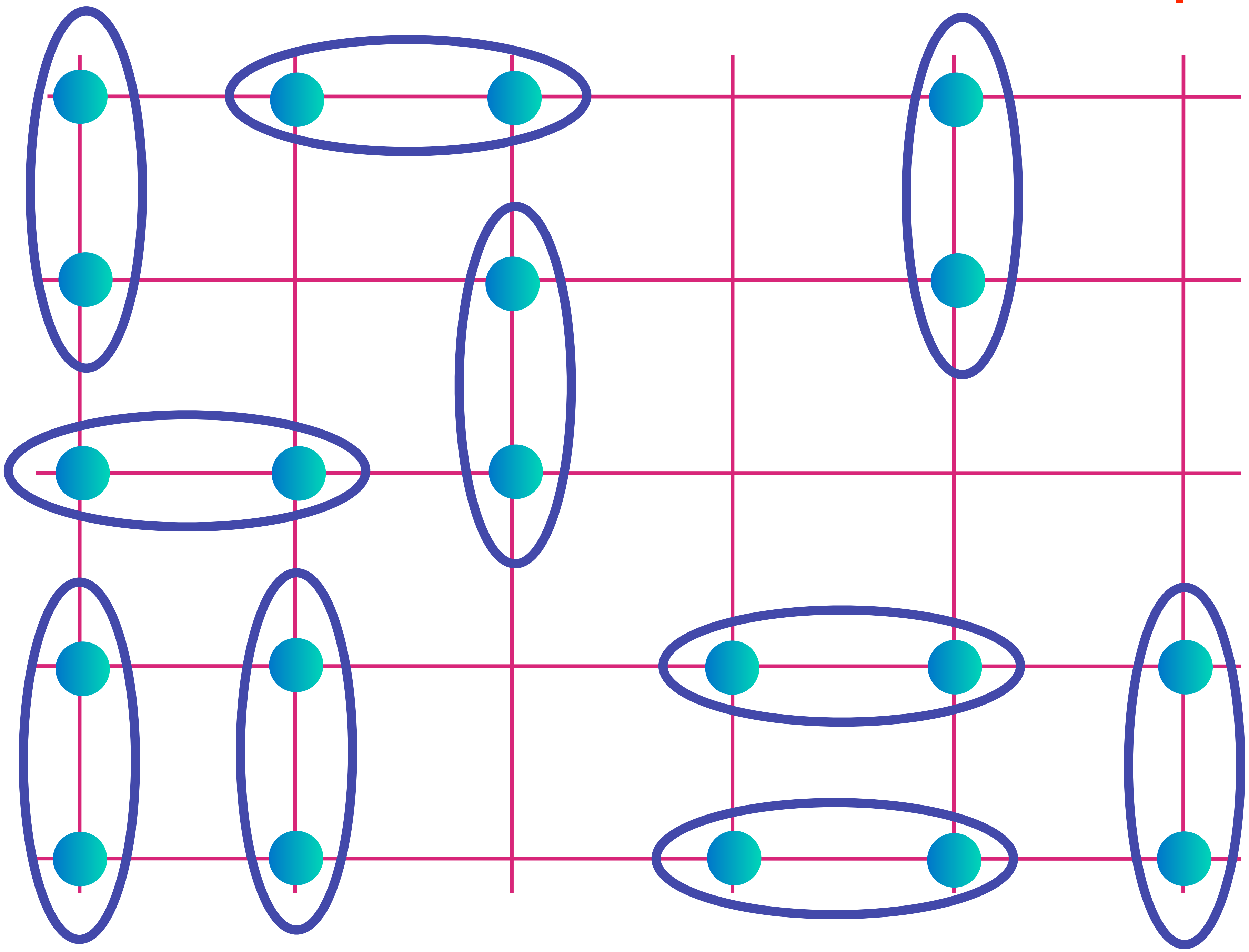


Cooper pairs form quantum superpositions at different locations: "Bose-Einstein condensation" in which all pairs are "everywhere at the same time"

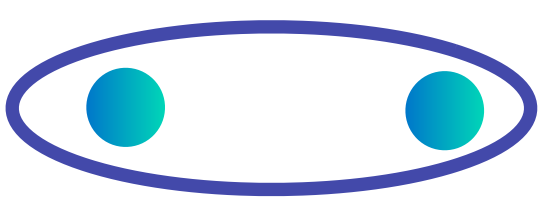
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

Superconductivity



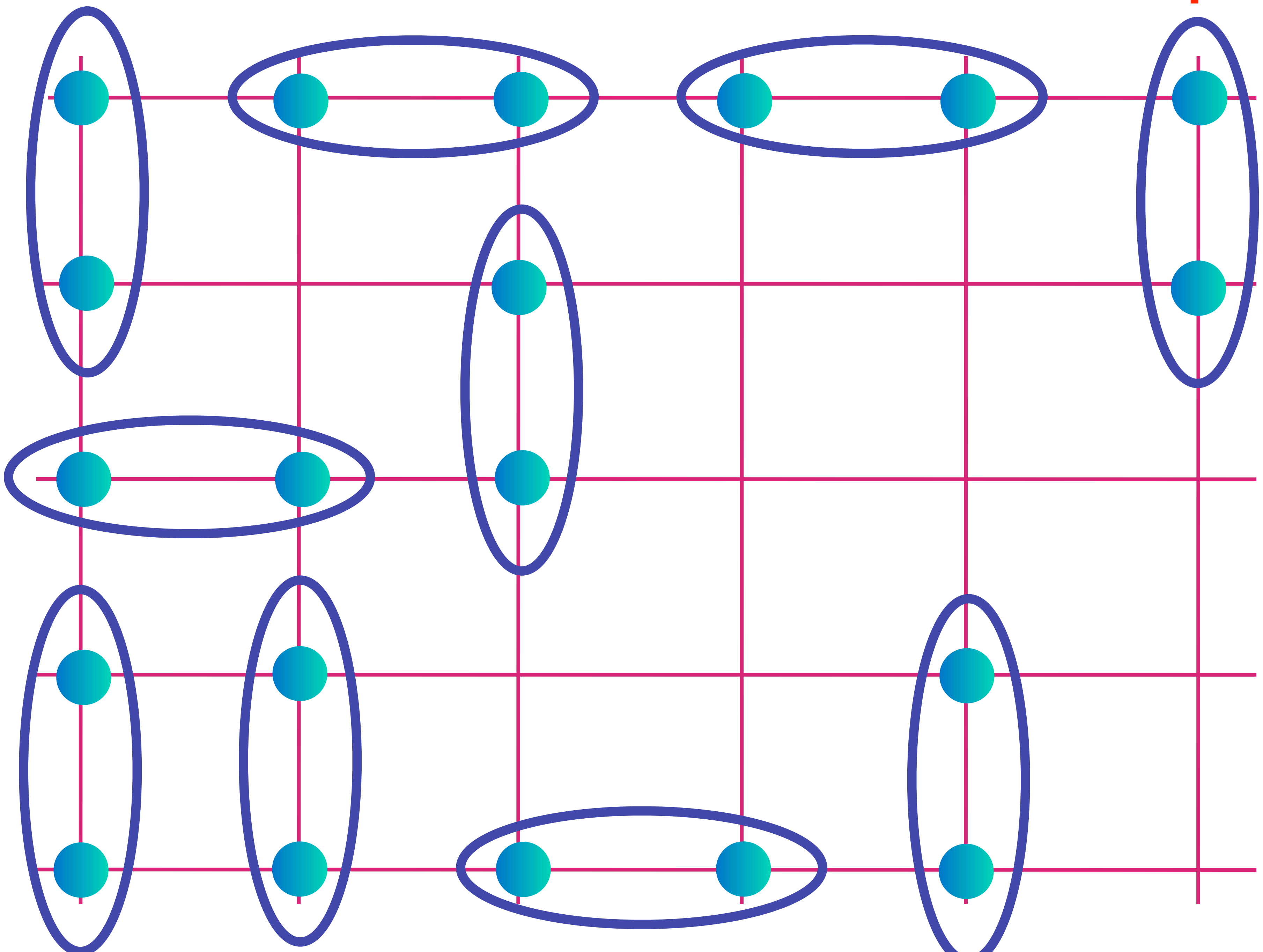
Cooper pairs form quantum superpositions at different locations: “Bose-Einstein condensation” in which all pairs are “everywhere at the same time”



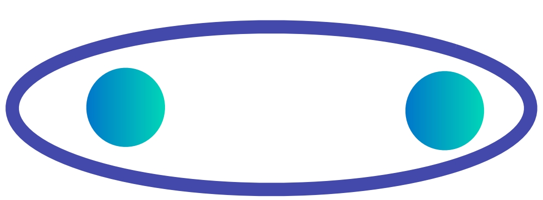
$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Square lattice of Cu sites

Superconductivity



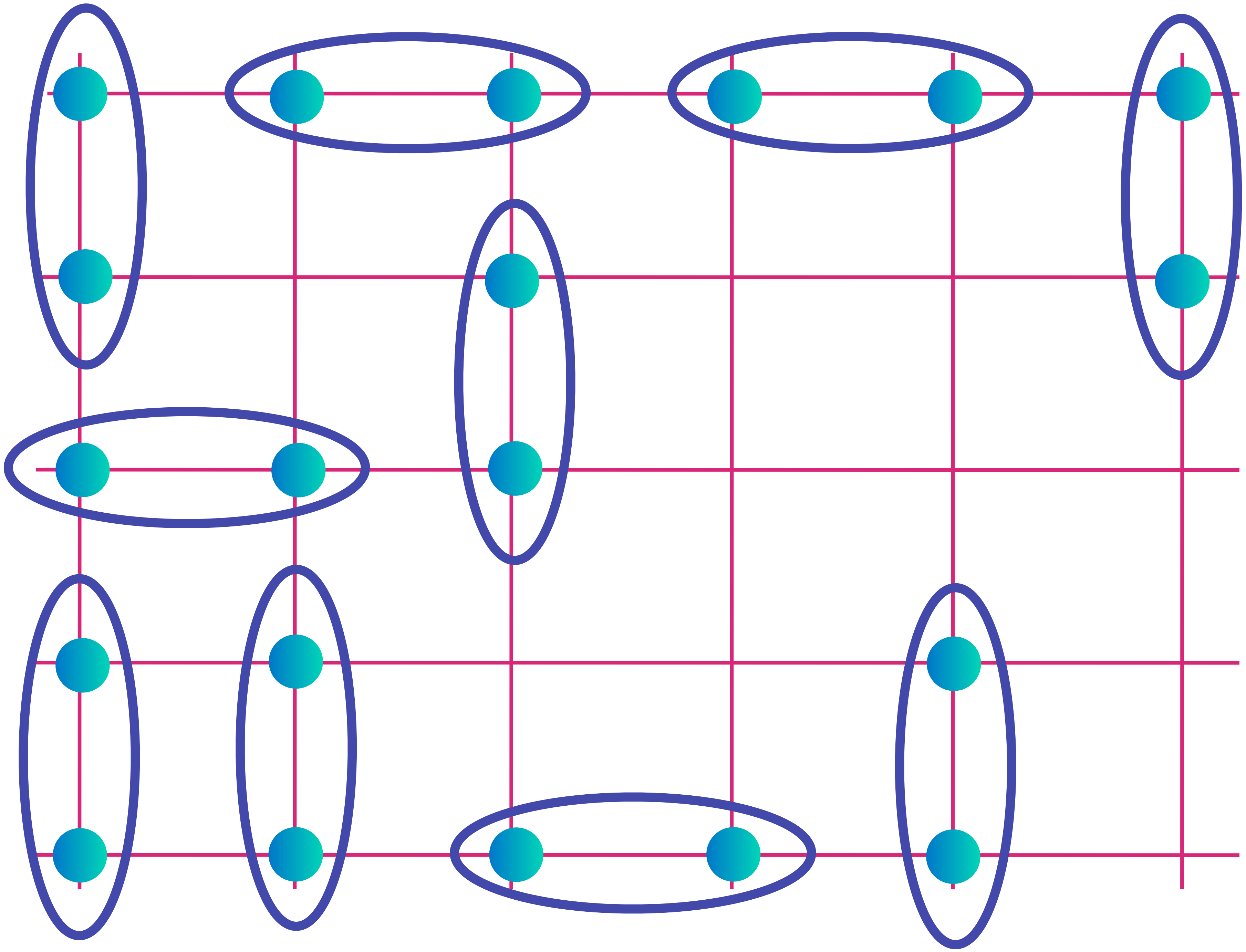
Cooper pairs form quantum superpositions at different locations: "Bose-Einstein condensation" in which all pairs are "everywhere at the same time"



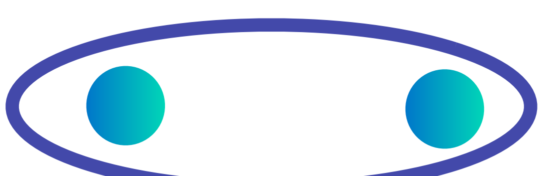
$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Square lattice of Cu sites

High temperature superconductivity !

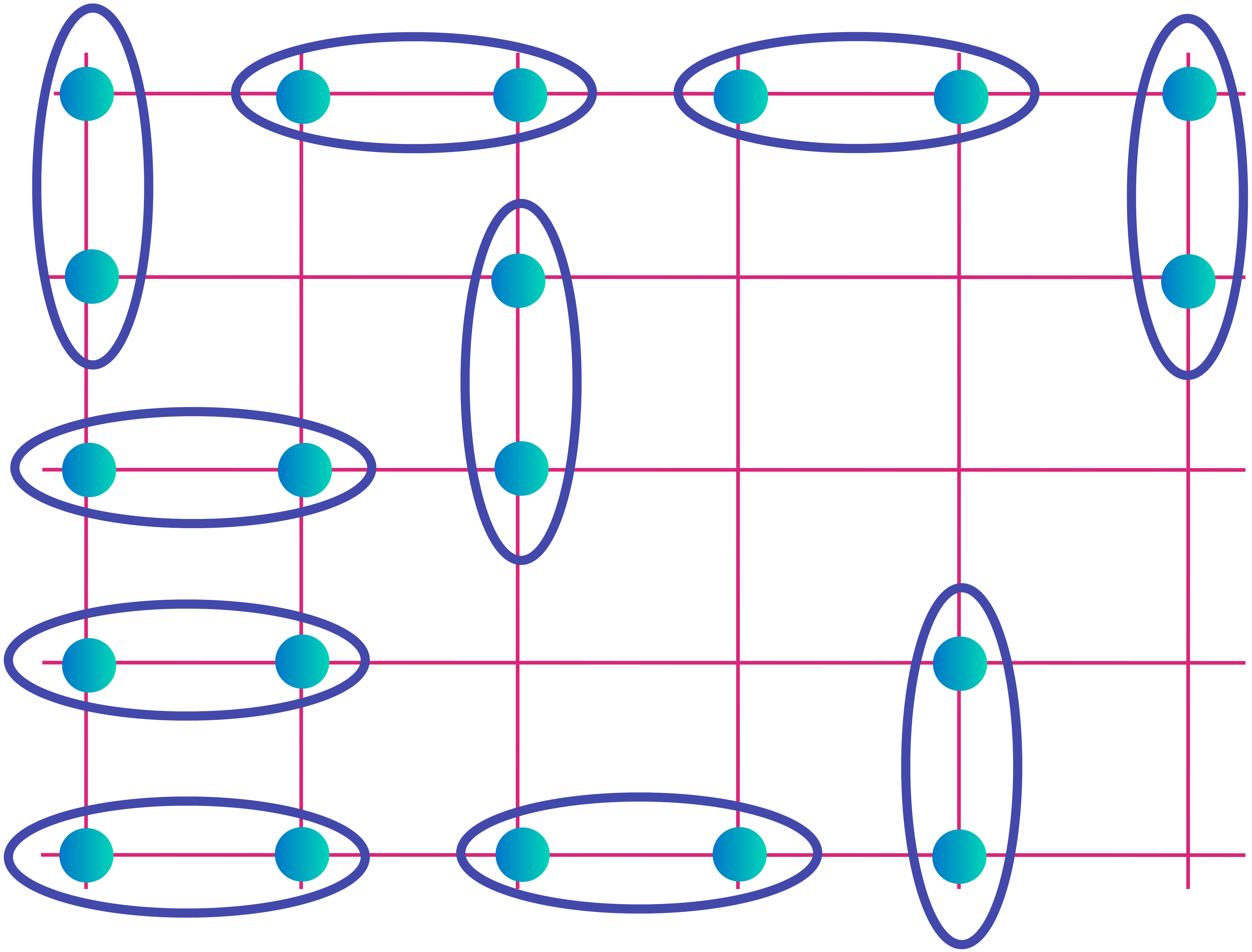


Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.

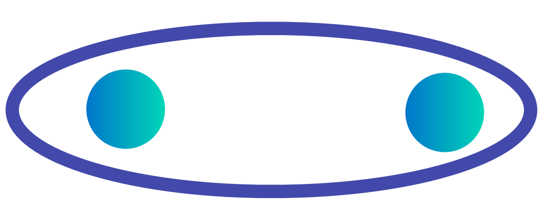
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

High temperature superconductivity !



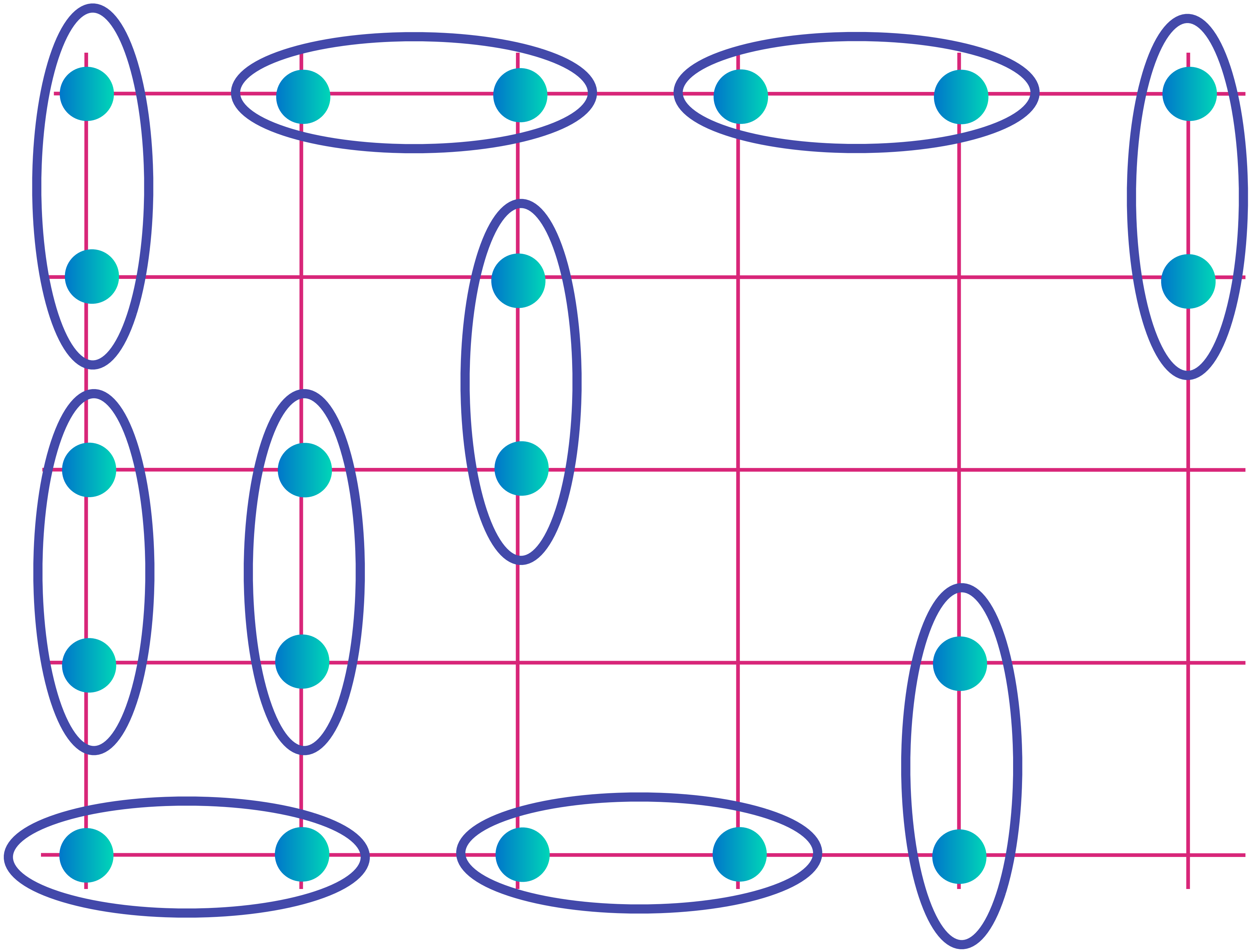
Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.



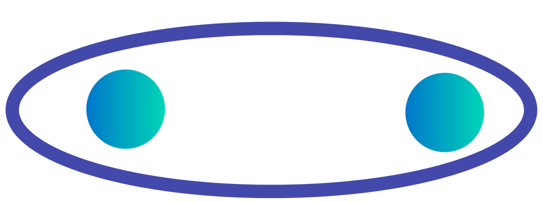
$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Square lattice of Cu sites

High temperature superconductivity !

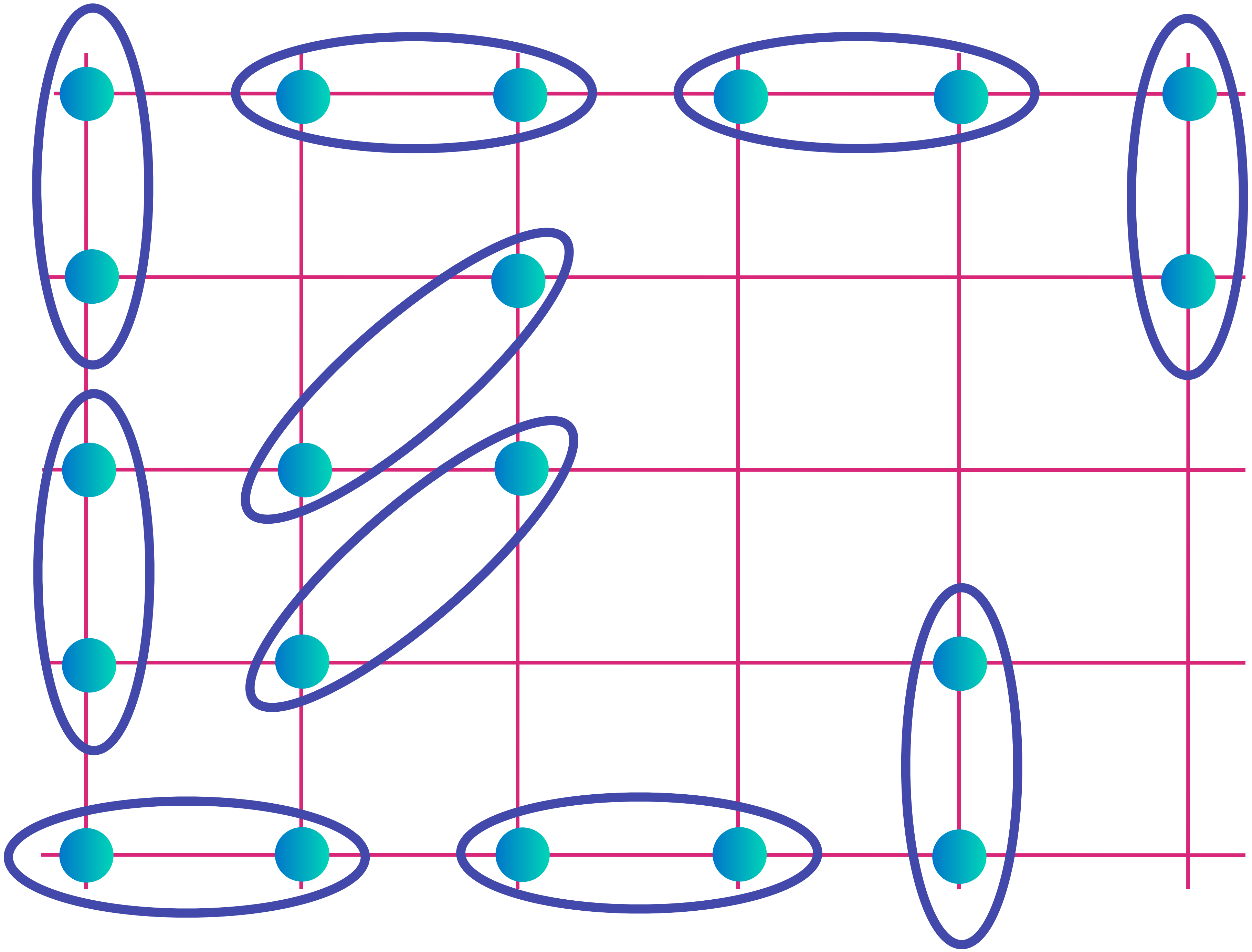


Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.

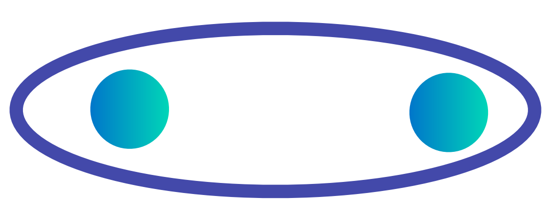
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

High temperature superconductivity !



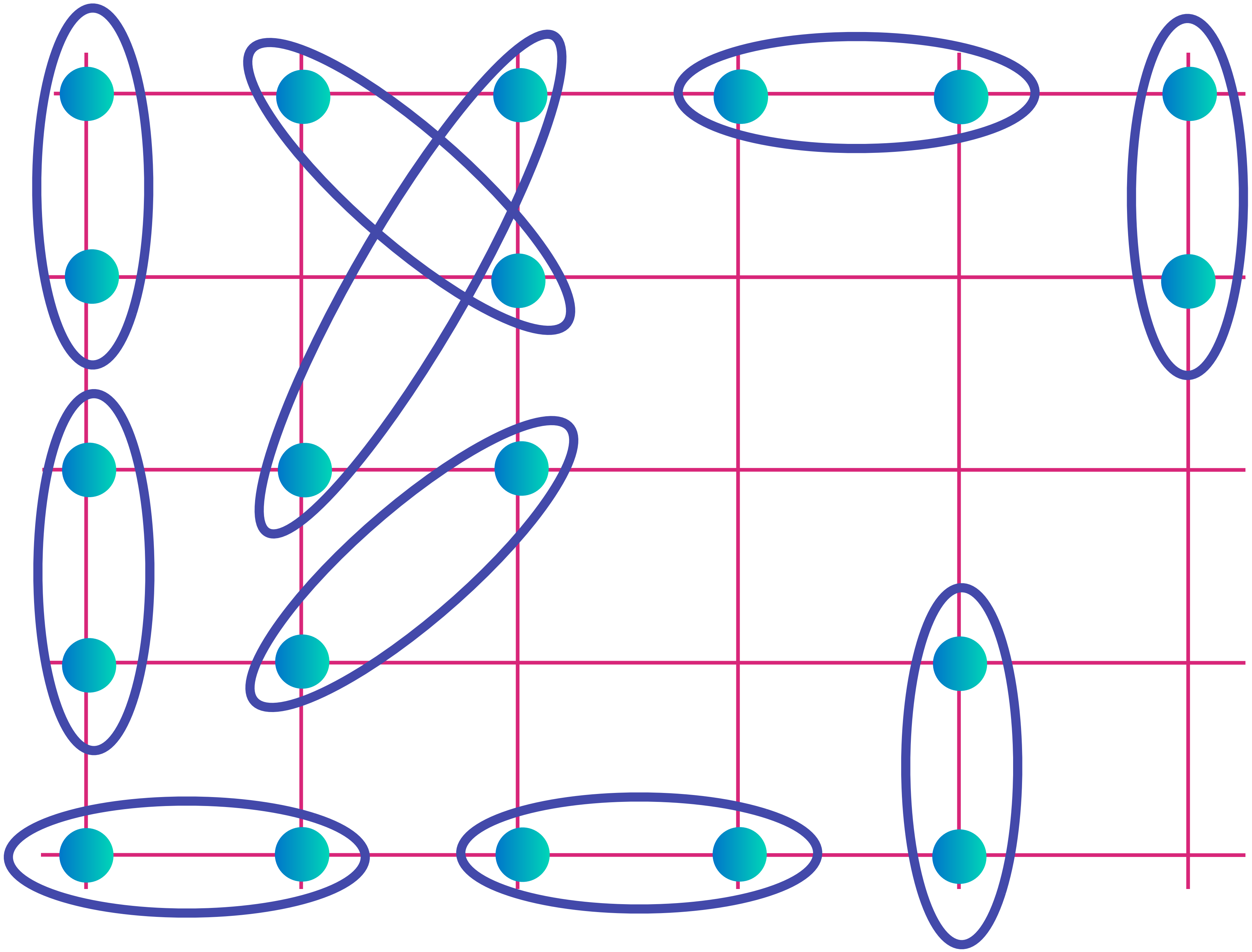
Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.



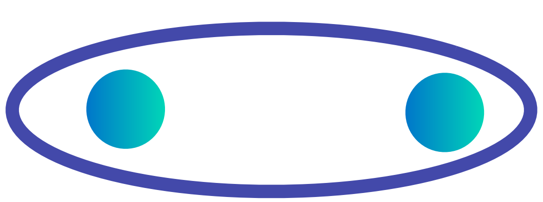
$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

High temperature superconductivity !

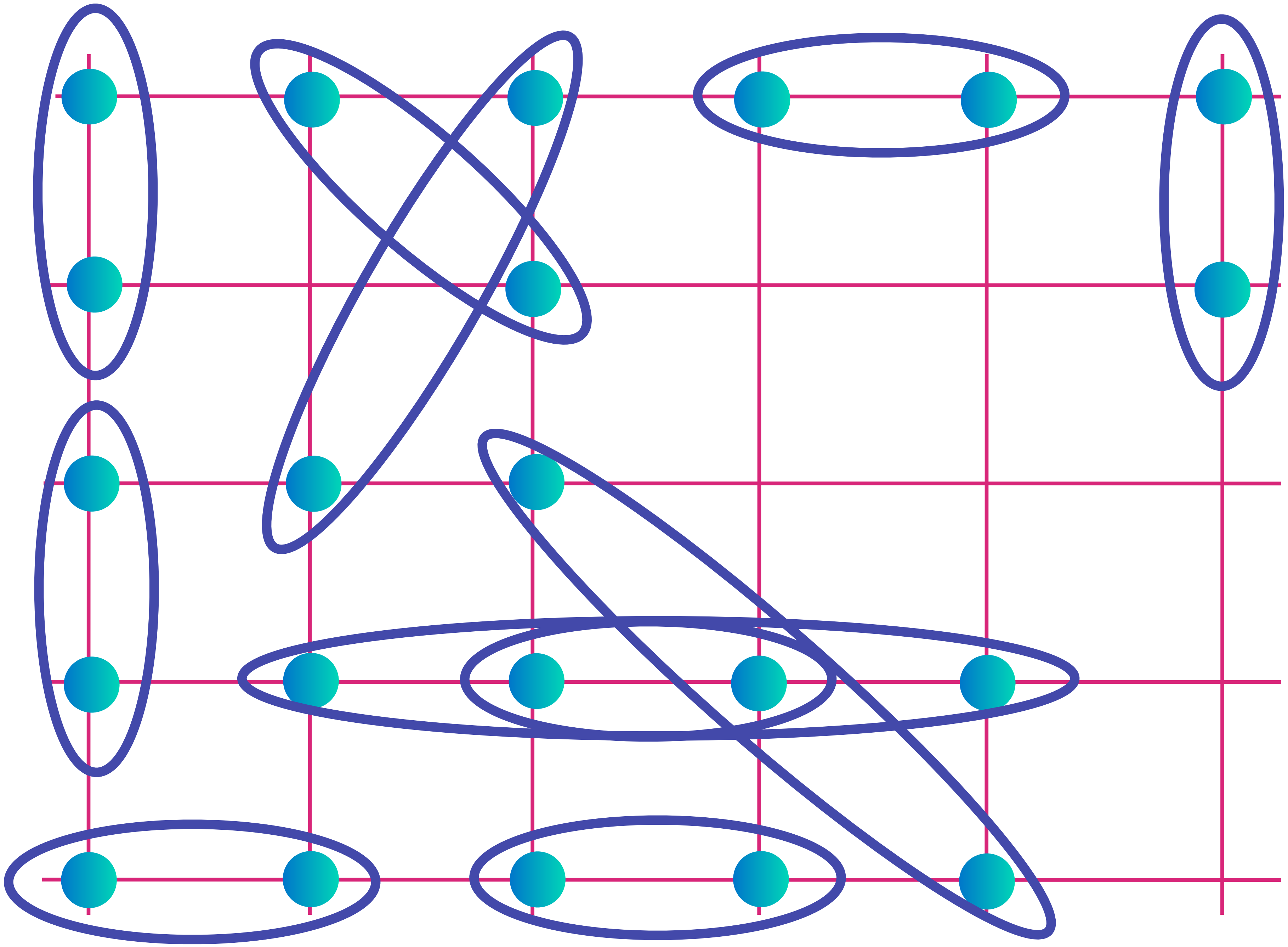


Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.

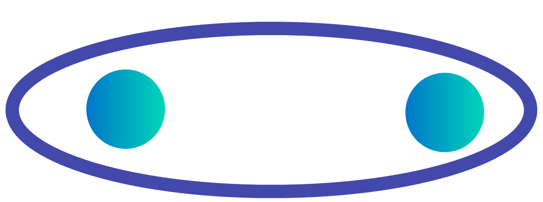
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

High temperature superconductivity !

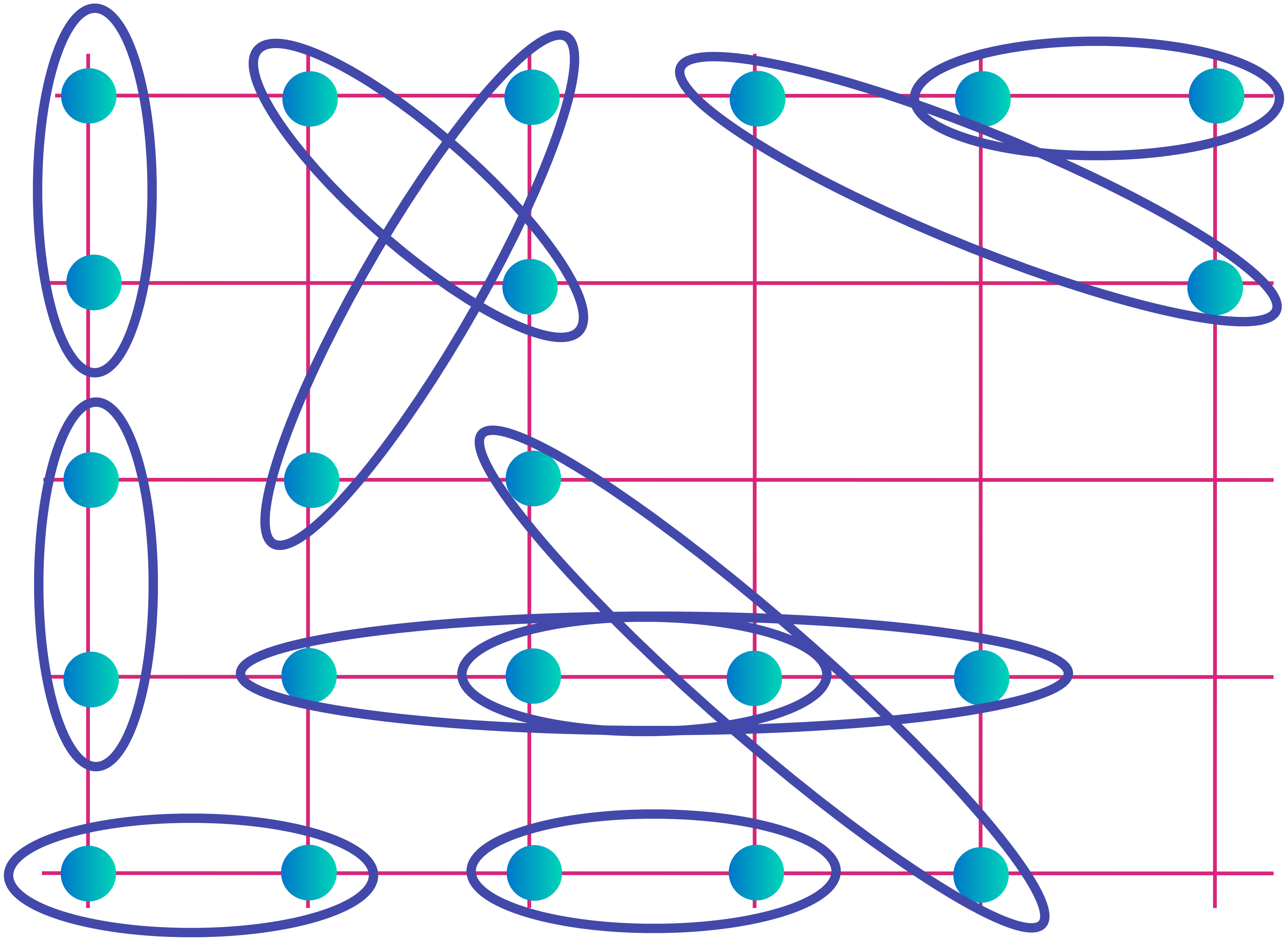


Electrons <u>entangle</u> by exchanging partners, and there is <u>long-range quantum entanglement</u> in the strange metal.

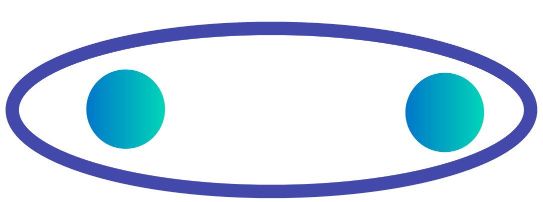
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

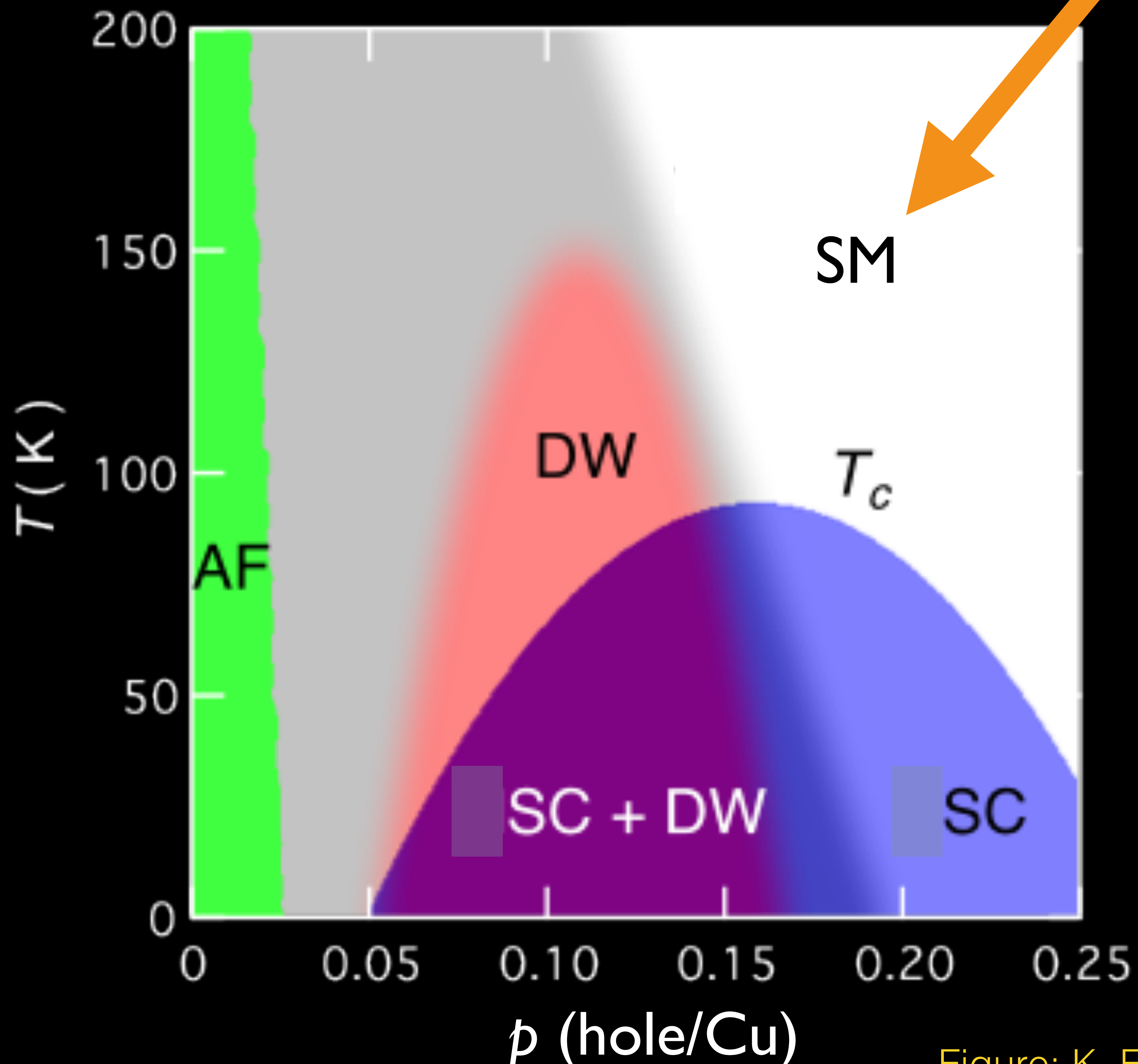
High temperature superconductivity !



Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.

 $= | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle$

Strange metal



Entangled electrons lead to "strange" temperature dependence of resistivity and other properties

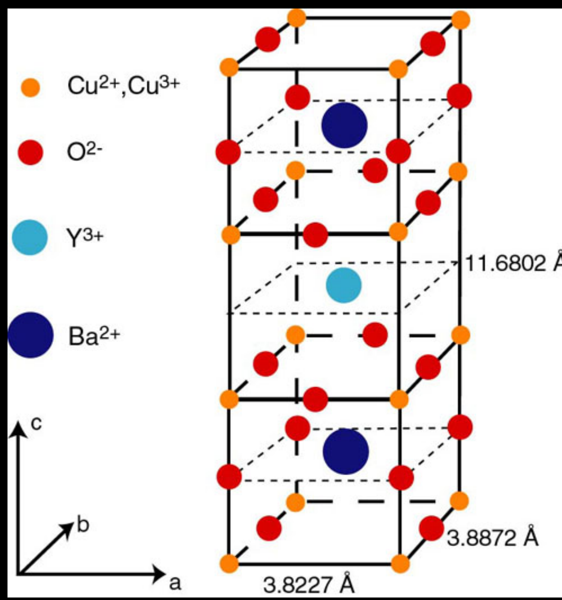
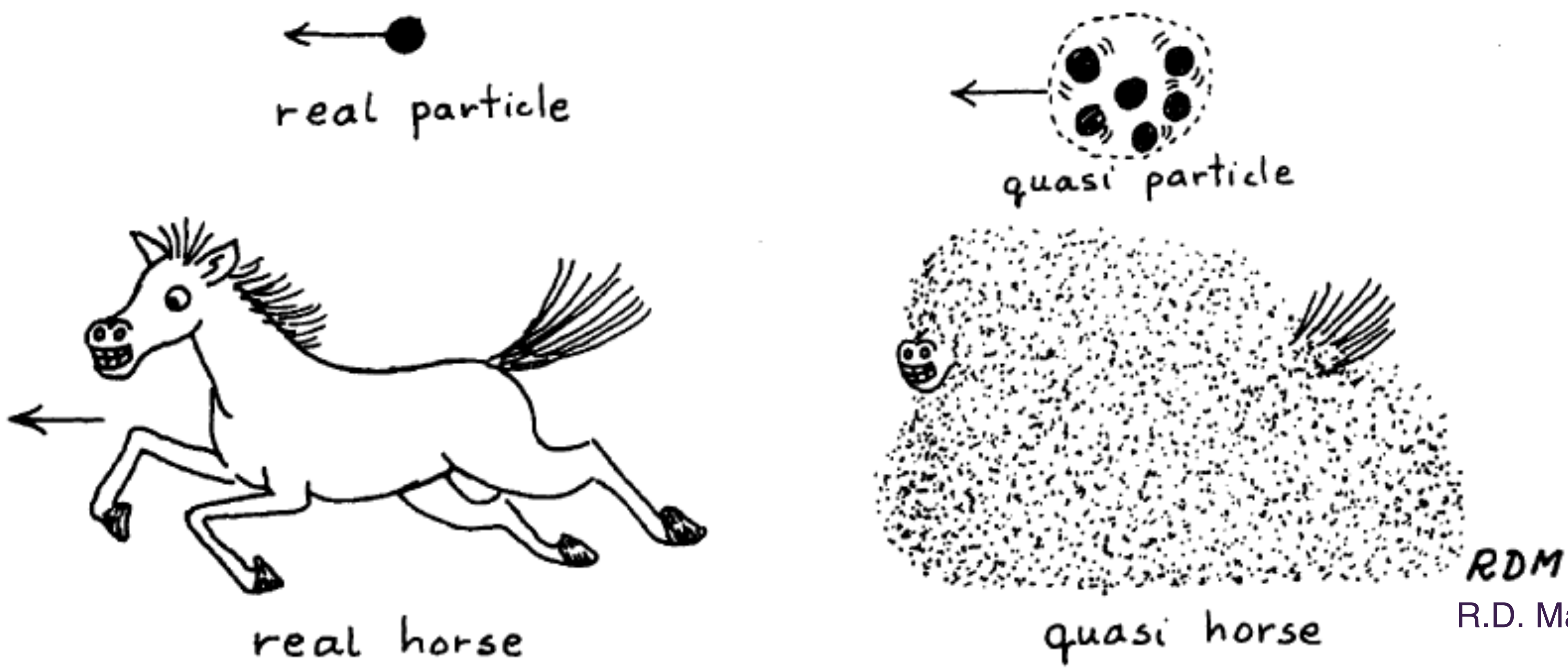


Figure: K. Fujita and J. C. Seamus Davis

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \dots$$

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

  - **Note:** The electron liquid in one dimension and the fractional quantum Hall state both have quasiparticles; however, the quasiparticles do not have the same quantum numbers as an electron.

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time is of order $\hbar E_F/(k_B T)^2$ as $T \to 0$, where E_F is the Fermi energy.

Quantum matter without quasiparticles

The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.

Quantum matter without quasiparticles

The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.

- Systems *without* quasiparticles, like the strange metal, reach quantum chaos much more quickly than those with quasiparticles.

Quantum matter without quasiparticles

The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.

- Systems *without* quasiparticles, like the strange metal, reach quantum chaos much more quickly than those with quasiparticles.

- There is an *lower bound* on the phase coherence time (τ_φ), and the time to many-body quantum chaos (τ_L) in all many-body quantum systems as $T \to 0$:

$$\tau_\varphi \geq C \frac{\hbar}{k_B T} \qquad \text{(SS, 1999)}$$

$$\tau_L \geq \frac{\hbar}{2\pi k_B T} \qquad \text{(Maldacena, Shenker, Stanford, 2015)}$$

So *e.g.* we cannot have $\tau_\varphi \sim \hbar/\sqrt{J k_B T}$ where J is a microscopic coupling.

Quantum matter without quasiparticles

The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.

- Systems *without* quasiparticles, like the strange metal, reach quantum chaos much more quickly than those with quasiparticles.

- There is an *lower bound* on the phase coherence time (τ_φ), and the time to many-body quantum chaos (τ_L) in all many-body quantum systems as $T \to 0$:

$$\tau_\varphi \geq C \frac{\hbar}{k_B T} \qquad \text{(SS, 1999)}$$

$$\tau_L \geq \frac{\hbar}{2\pi k_B T} \qquad \text{(Maldacena, Shenker, Stanford, 2015)}$$

  So *e.g.* we cannot have $\tau_\varphi \sim \hbar/\sqrt{J k_B T}$ where J is a microscopic coupling.

- In the strange metal the inequalities become equalities as $T \to 0$, and the time $\hbar/(k_B T)$ influences numerous observables.

Quantum entanglement

- Strange metals have no quasiparticle description.

Strange metals

- Their entropy is proportional to their volume.

- They relax to local thermal equilibrium in the fastest possible time $\sim \hbar/(k_B T)$.

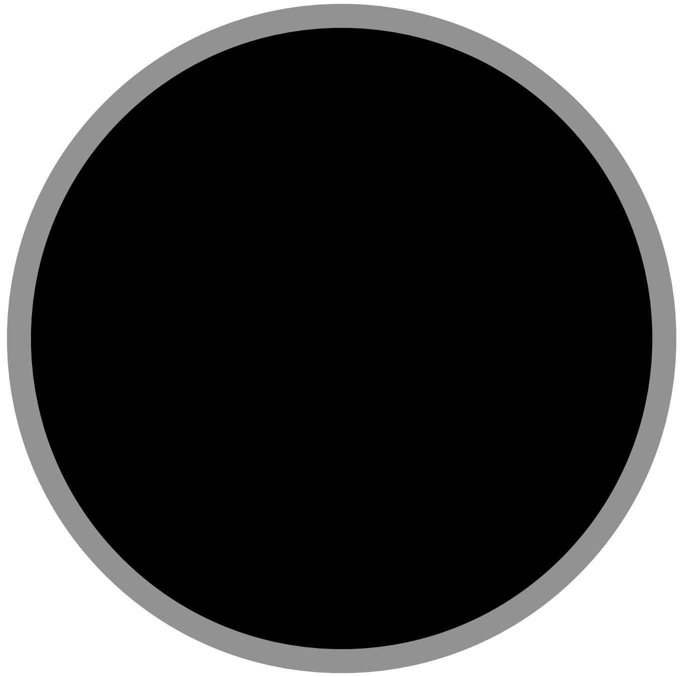
Quantum entanglement

Black holes

Black Holes

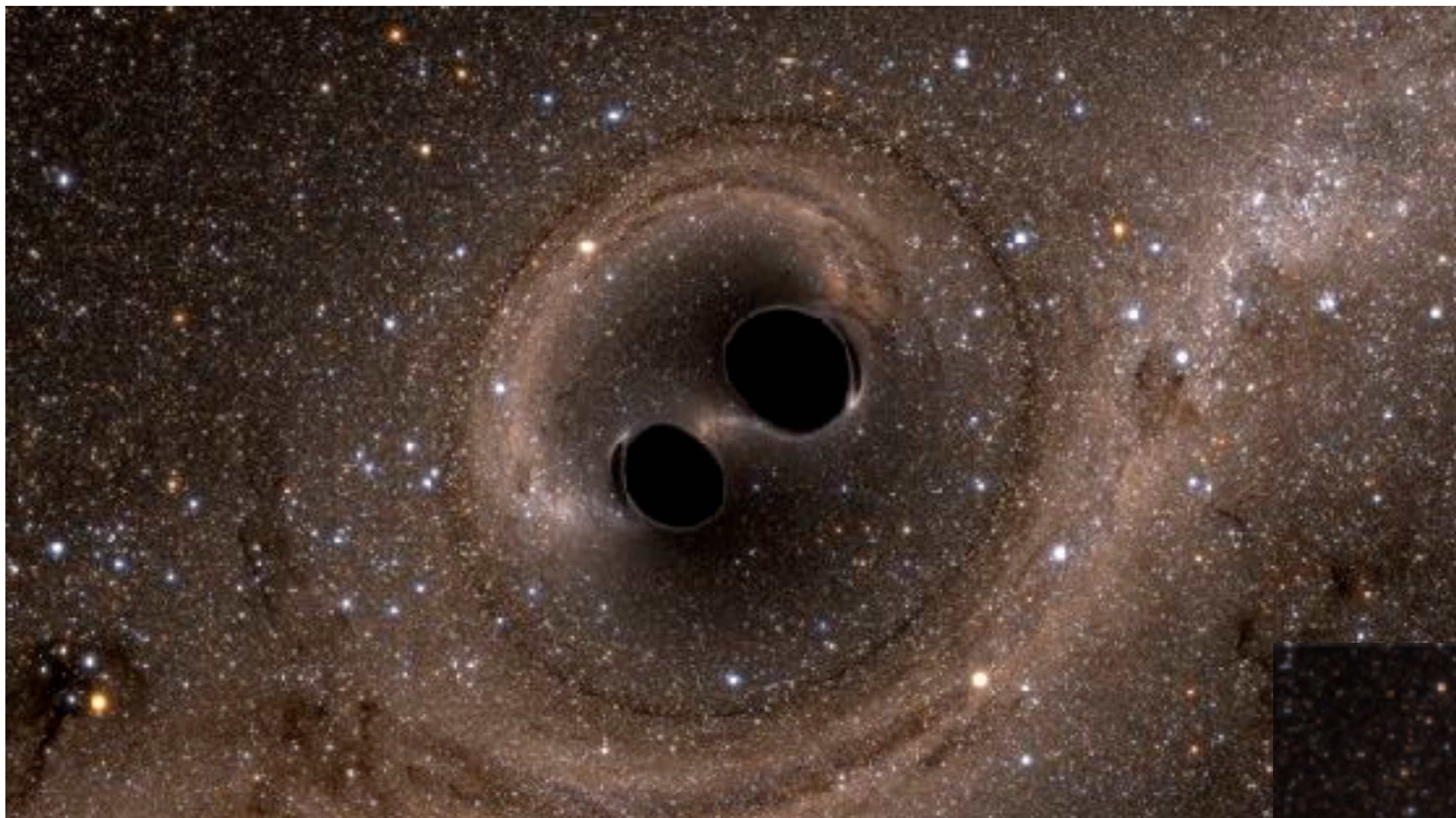
Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

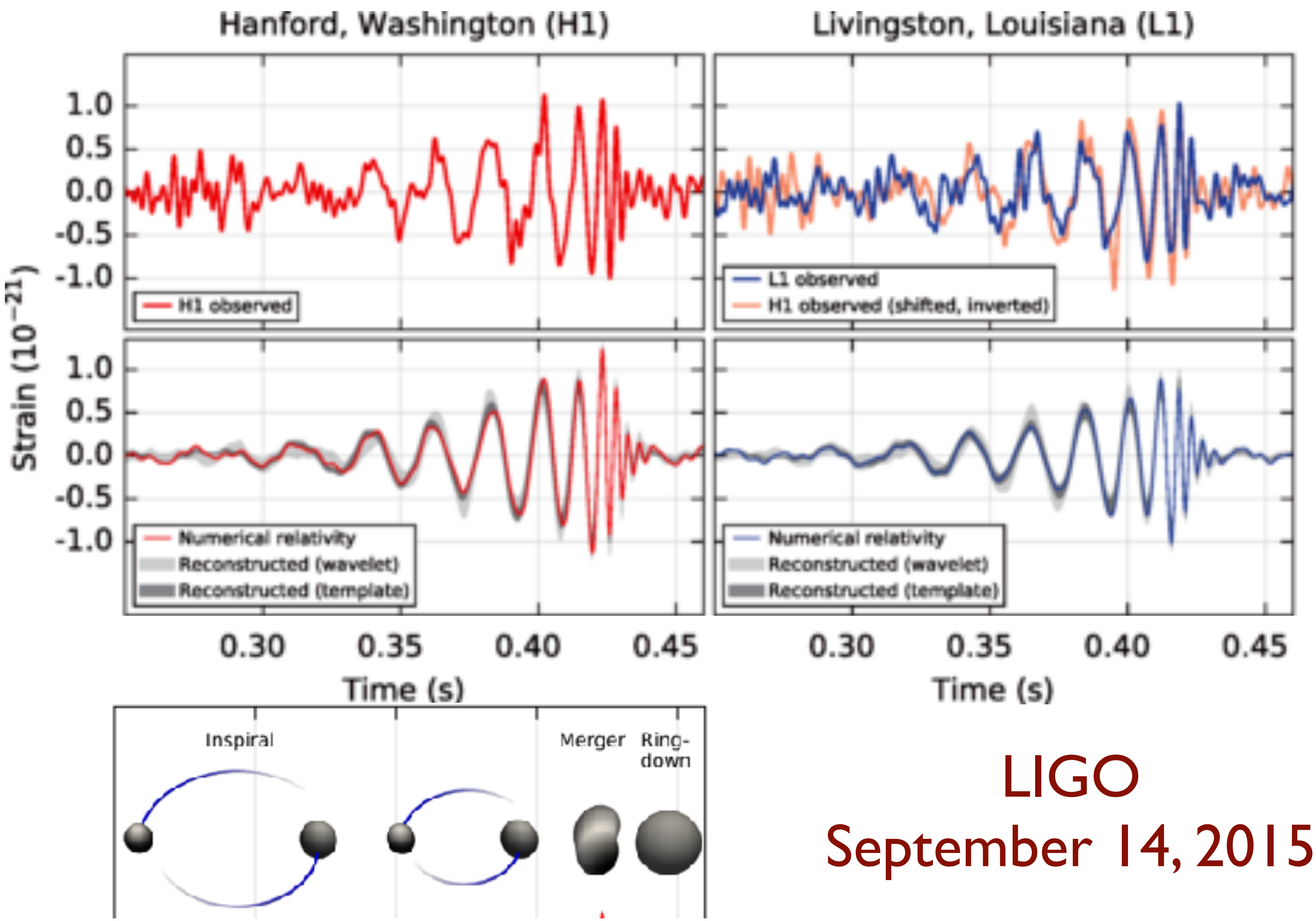


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



0.1 seconds later !

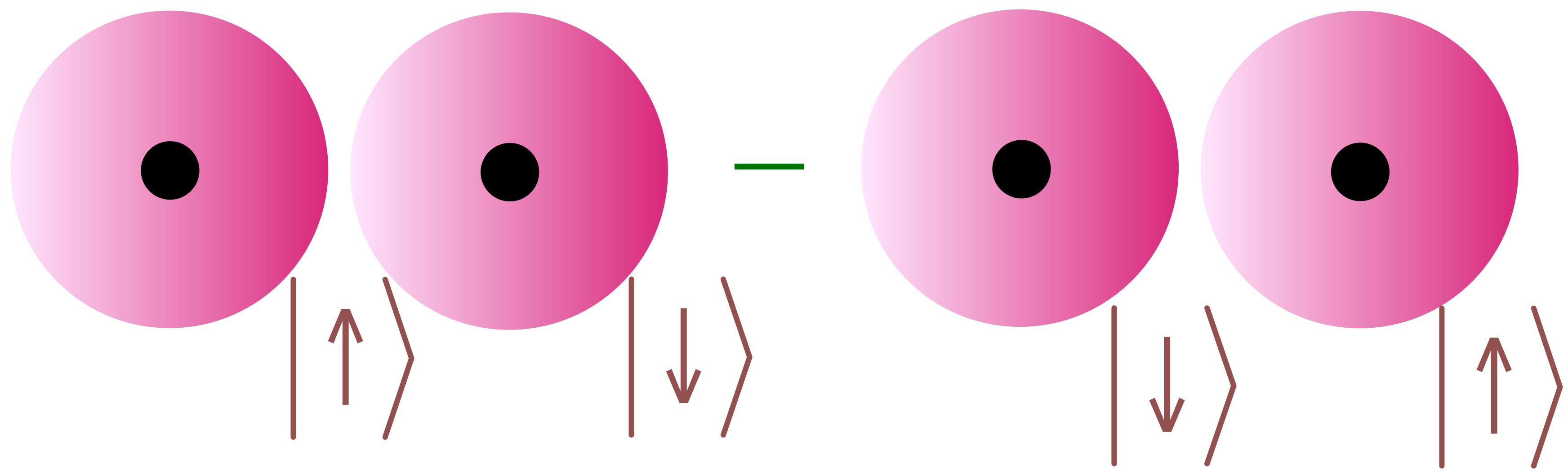


LIGO
September 14, 2015

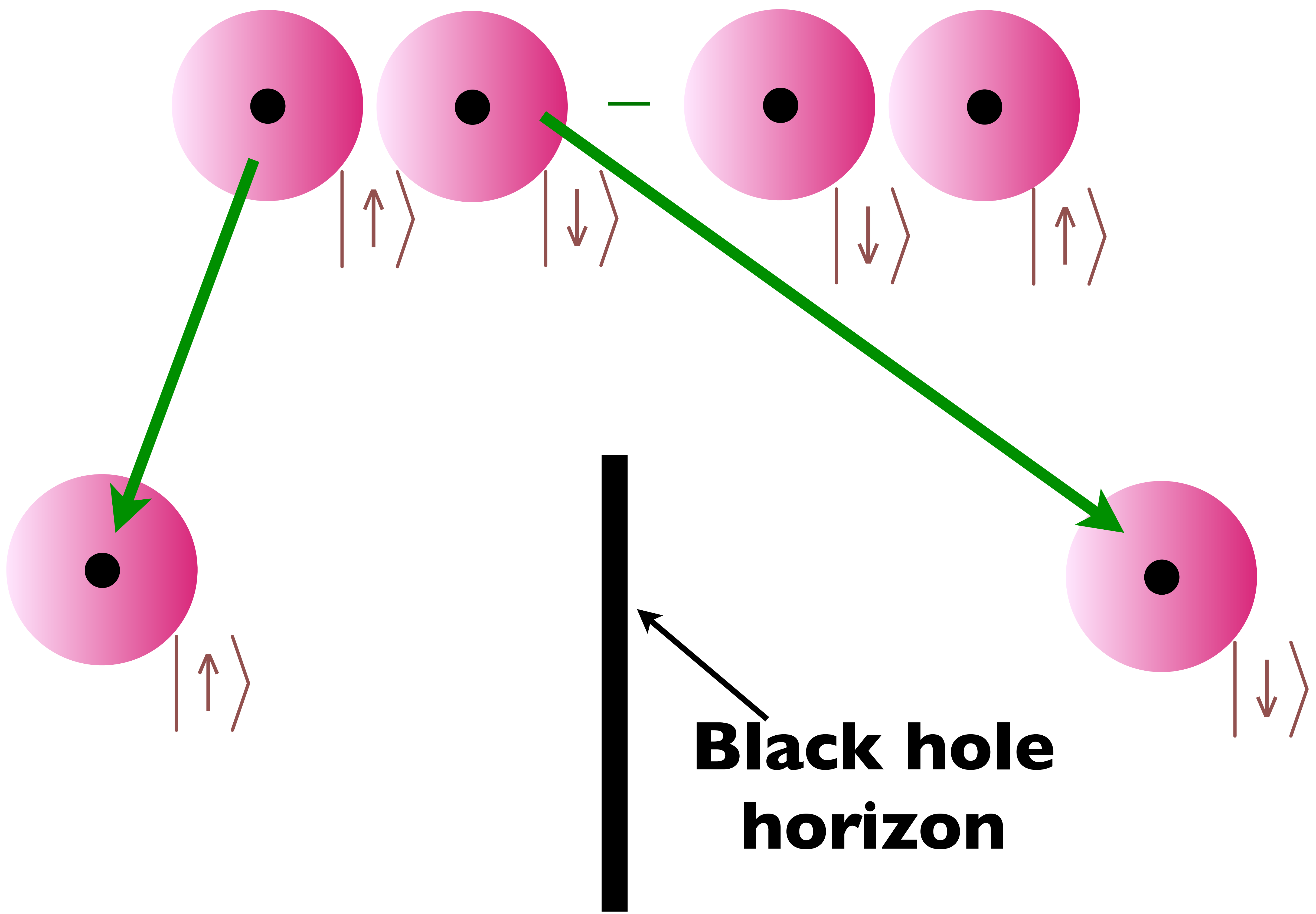
Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

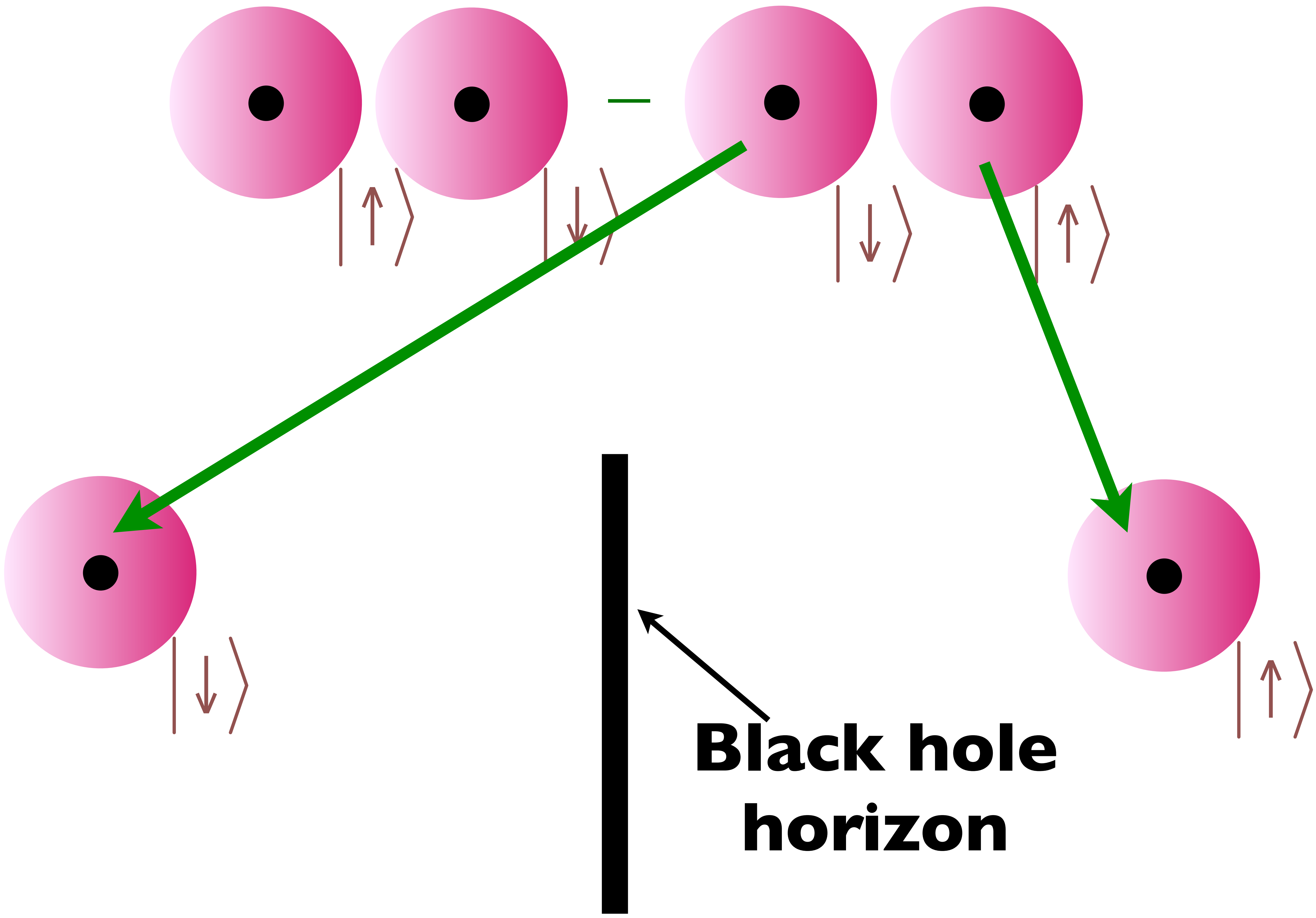
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

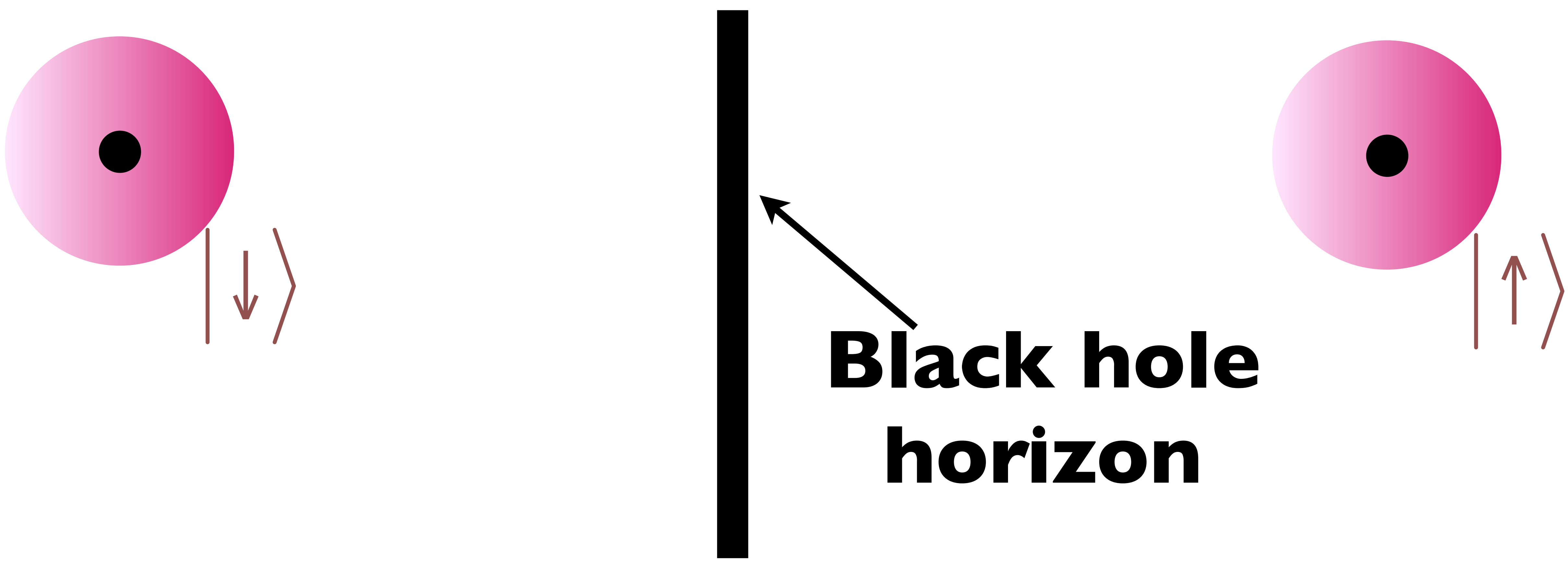


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

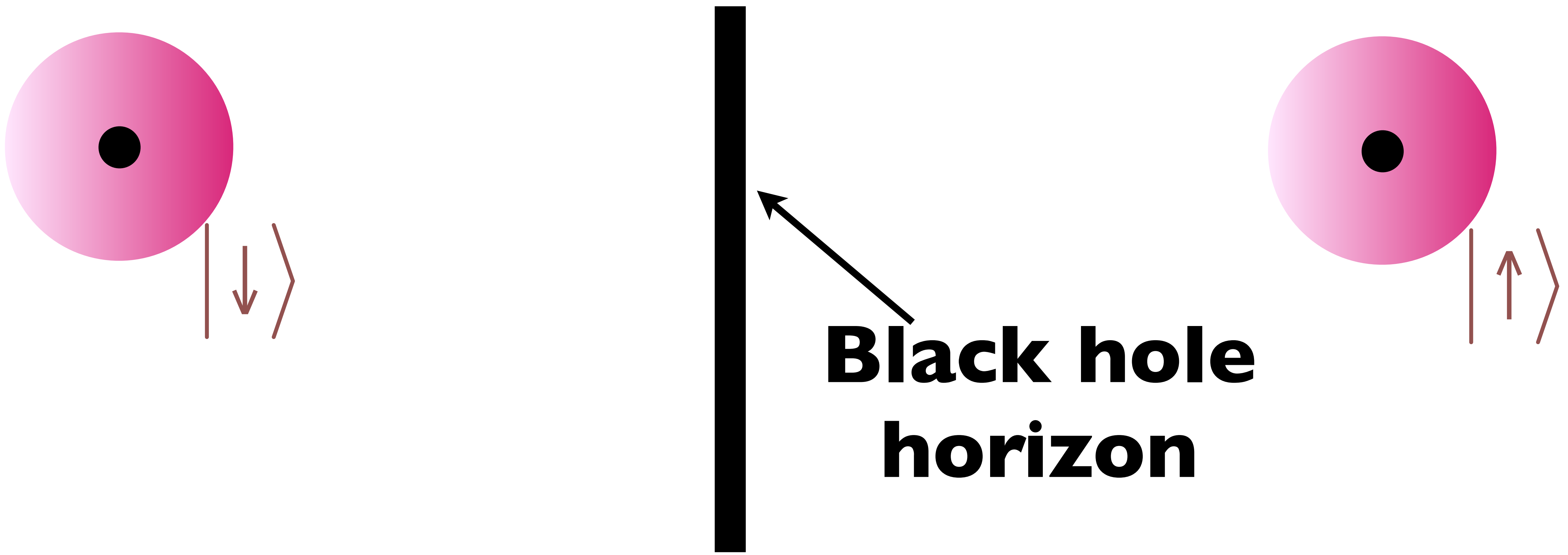
There is long-range quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

(because to an outside observer, the state of the electron inside the black hole is an unknown)



Quantum Entanglement across a black hole horizon

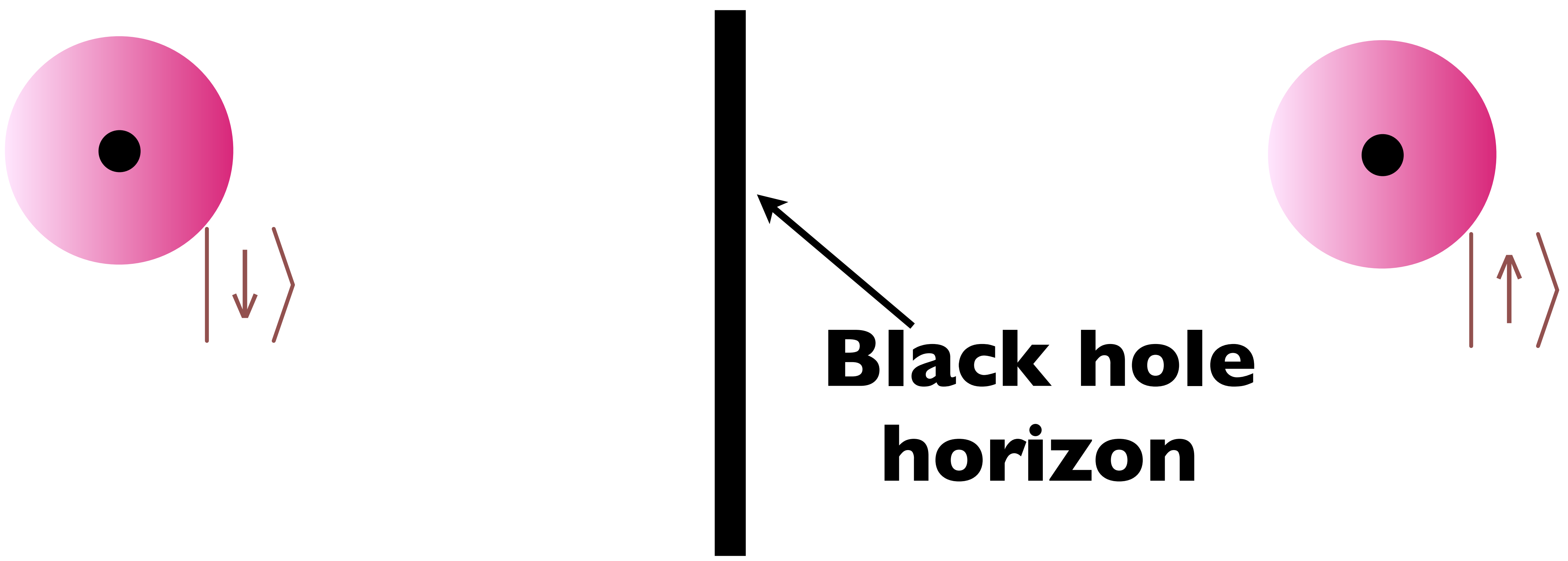
The Hawking temperature $k_B T_H = \dfrac{\hbar^2}{8\pi M \ell_P^2}$ and

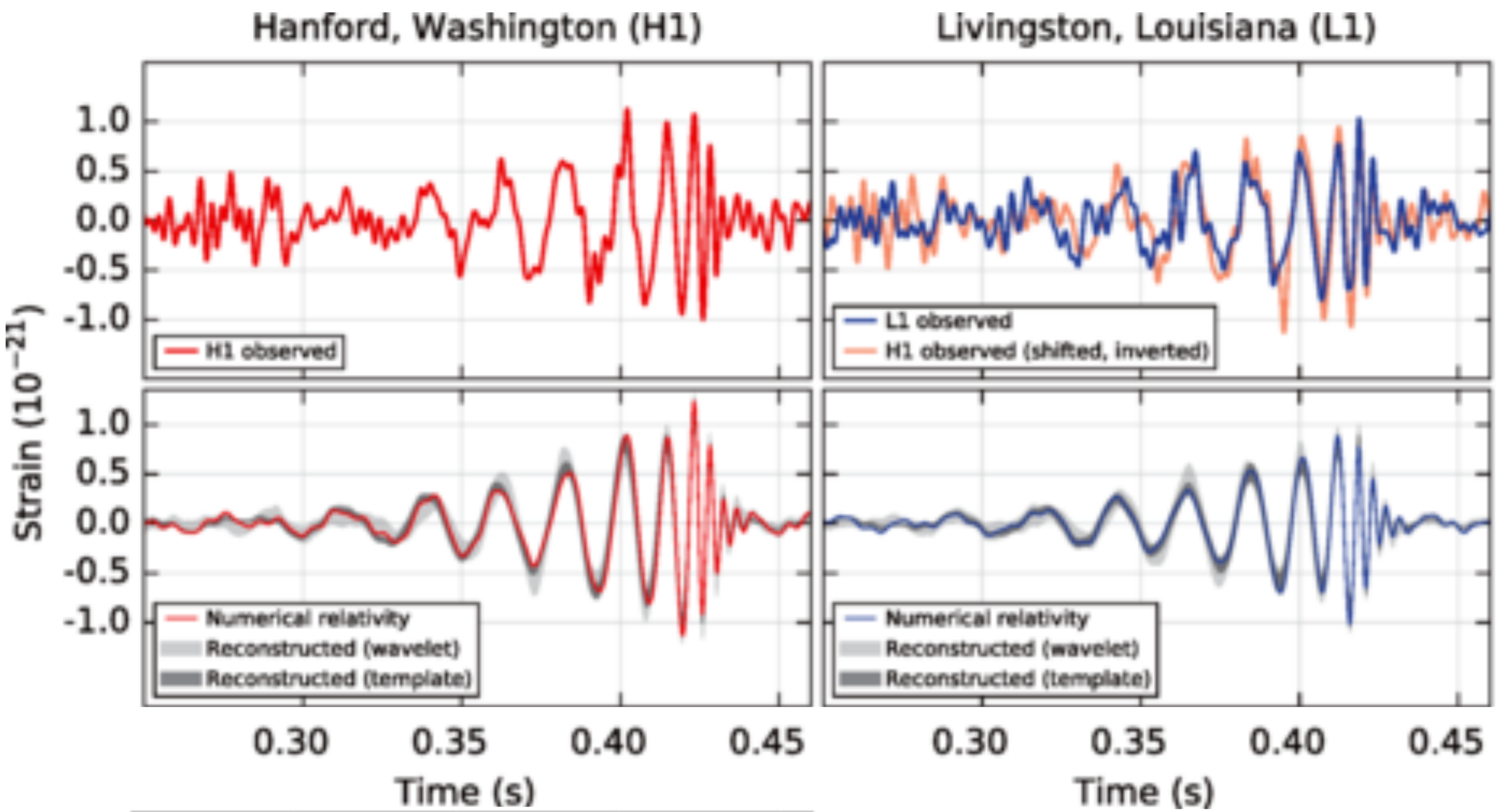
the Bekenstein-Hawking (BH) black hole entropy $\dfrac{S_{BH}}{k_B} = \dfrac{A}{4\ell_P^2}$

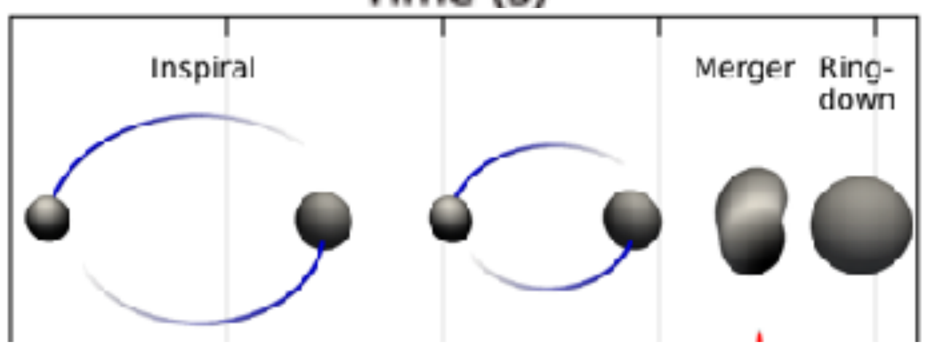
where $\ell_P = \sqrt{\hbar G/c^3}$ in the Planck length,

and A is the surface area of the black hole.

Note the entropy is proportional to the surface area rather than the volume.







LIGO
September 14, 2015

- The Hawking temperature, T_H influences the radiation from the black hole at the very last stages of the ring-down (not observed so far). The ring-down (approach to thermal equilibrium) happens very rapidly in a time $\sim \dfrac{\hbar}{k_B T_H} = \dfrac{8\pi GM}{c^3} \sim 8$ milliseconds.

Quantum entanglement

Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H)$.

Quantum entanglement

- Strange metals have no quasiparticle description.

- Their entropy is proportional to their volume.

- They relax to local thermal equilibrium in the fastest possible time $\sim \hbar/(k_B T)$.

Strange metals

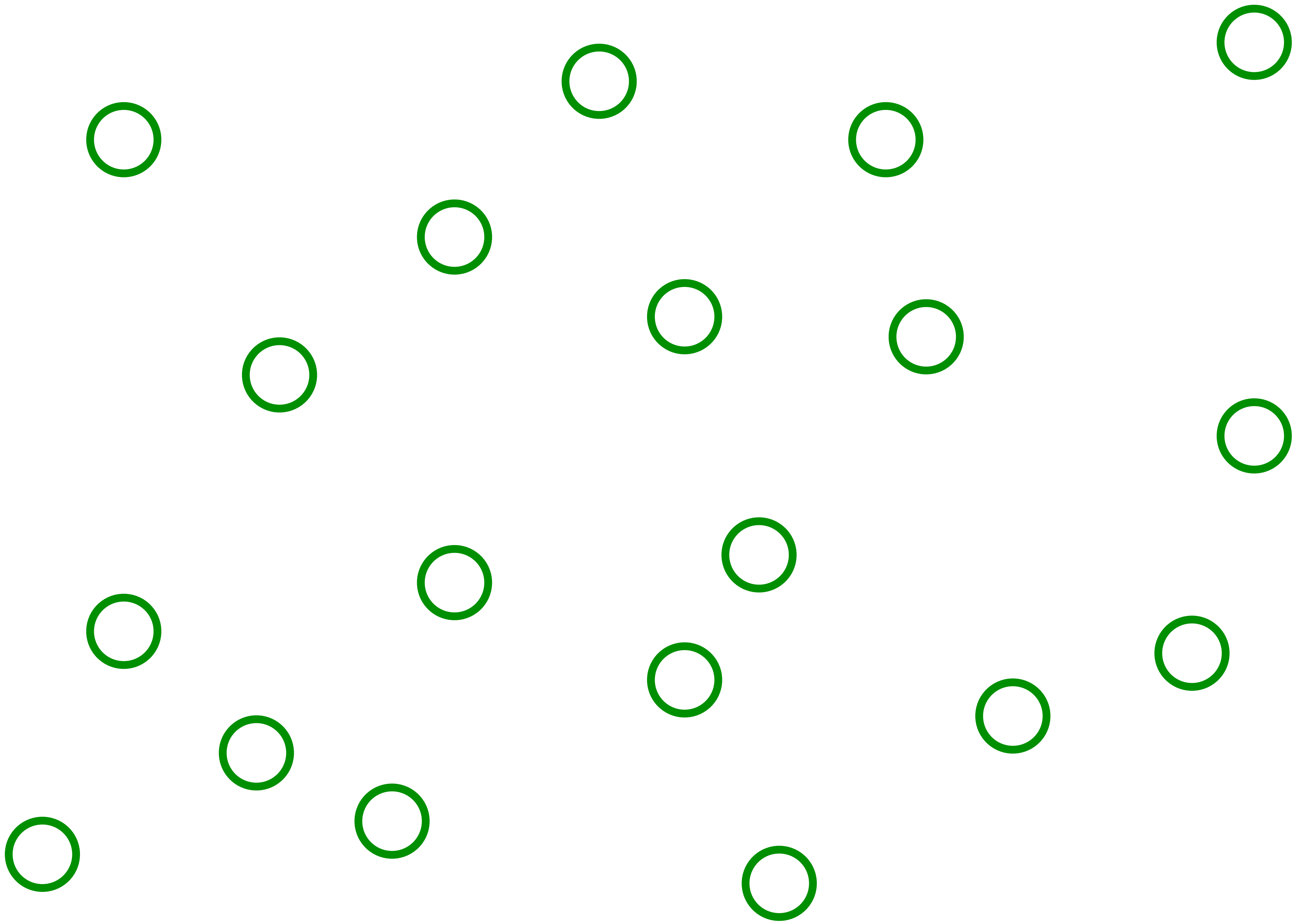
Quantum entanglement

Black holes

Strange metals

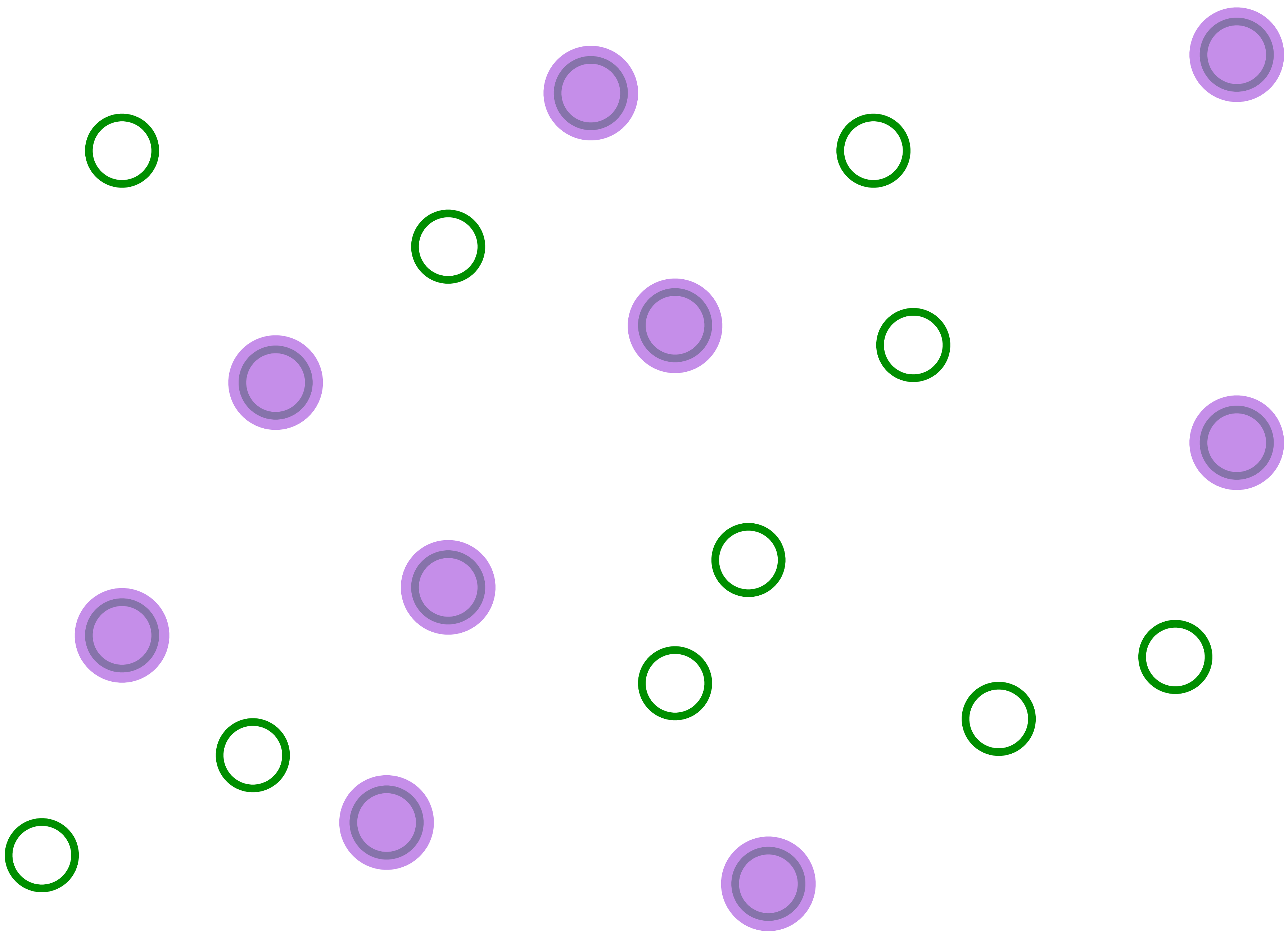
A "toy model" which is both a strange metal and a black hole!

A simple model of a metal with quasiparticles



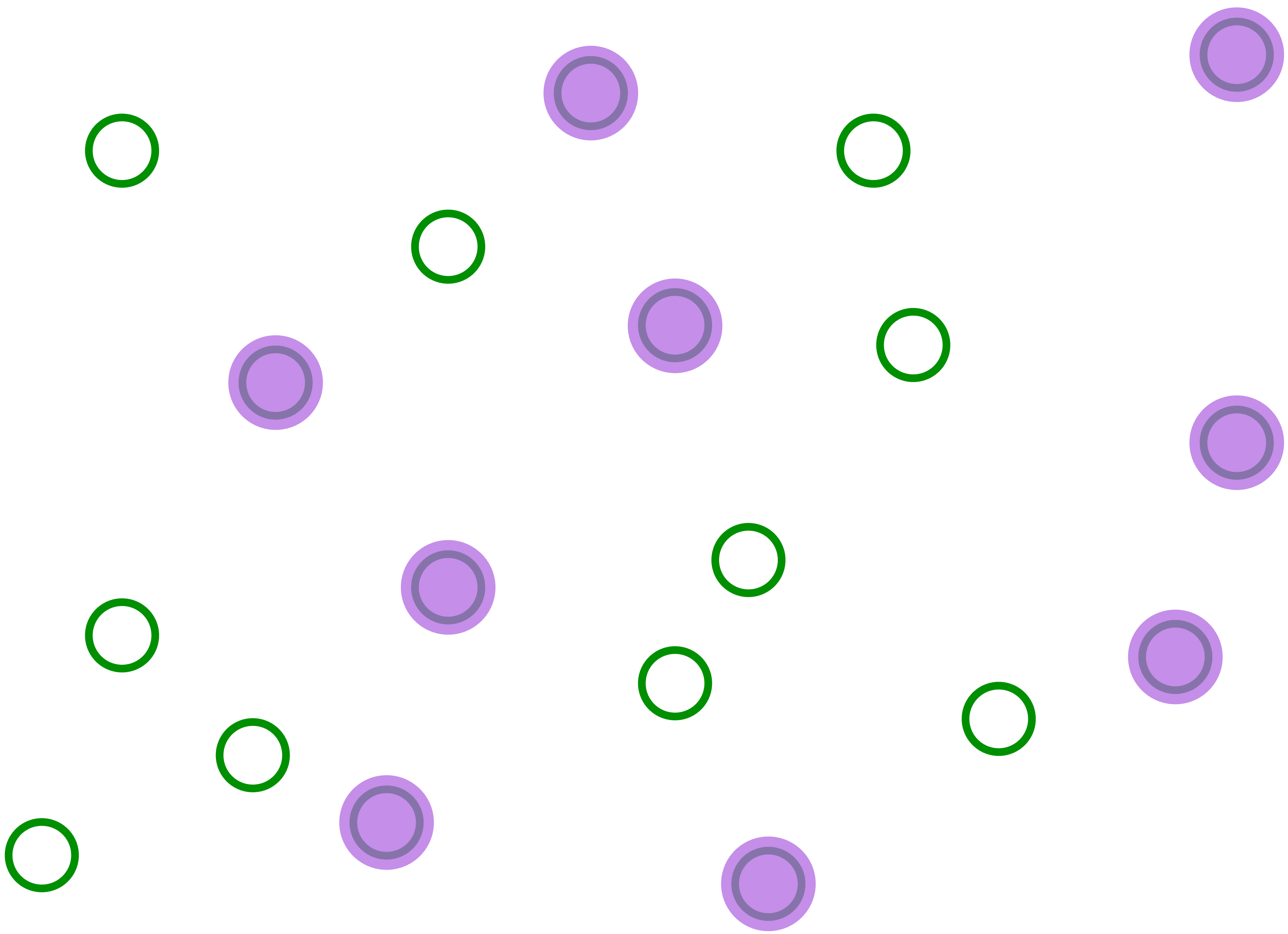
Pick a set of random positions

A simple model of a metal with quasiparticles



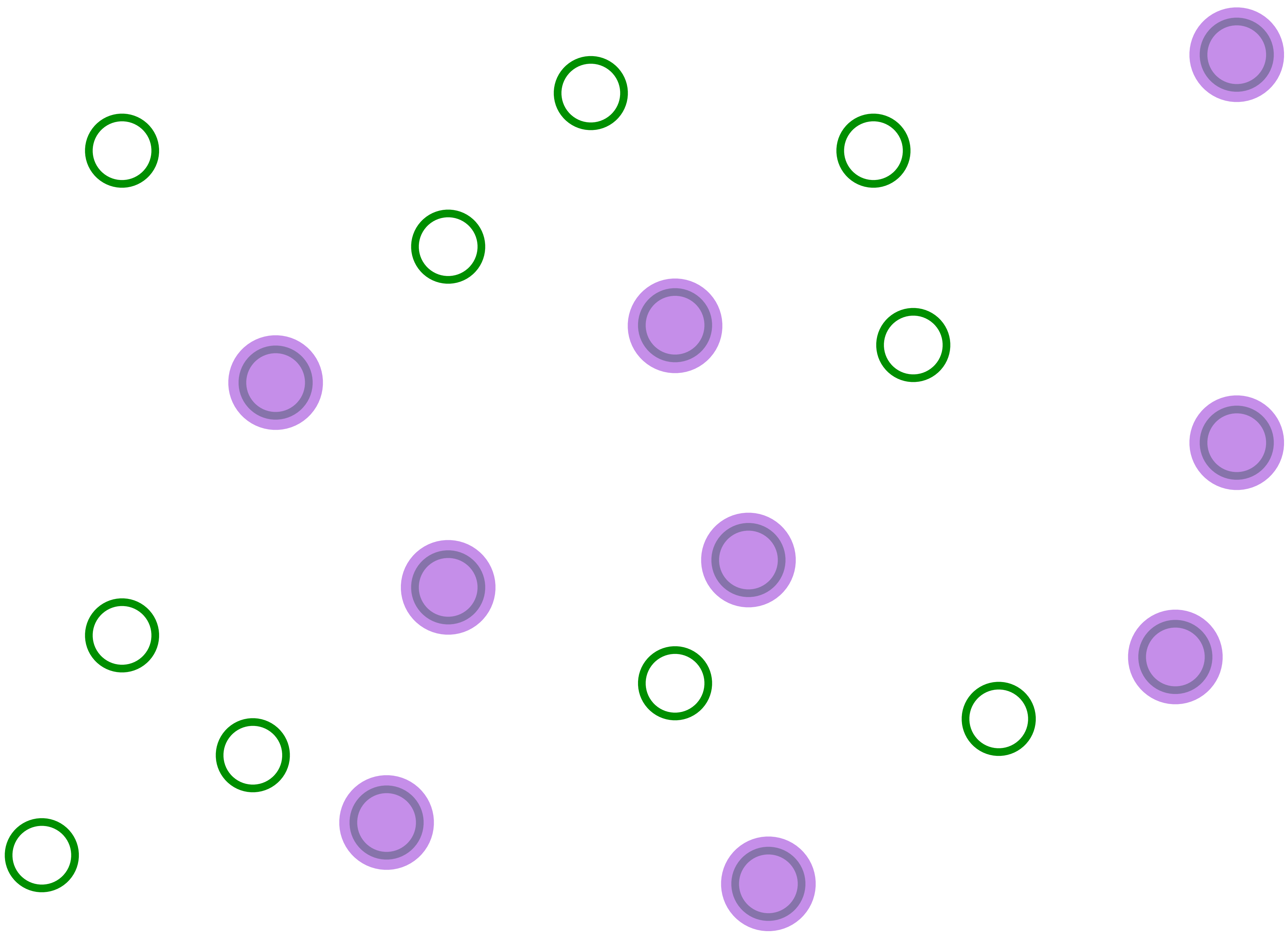
Place electrons randomly on some sites

A simple model of a metal with quasiparticles



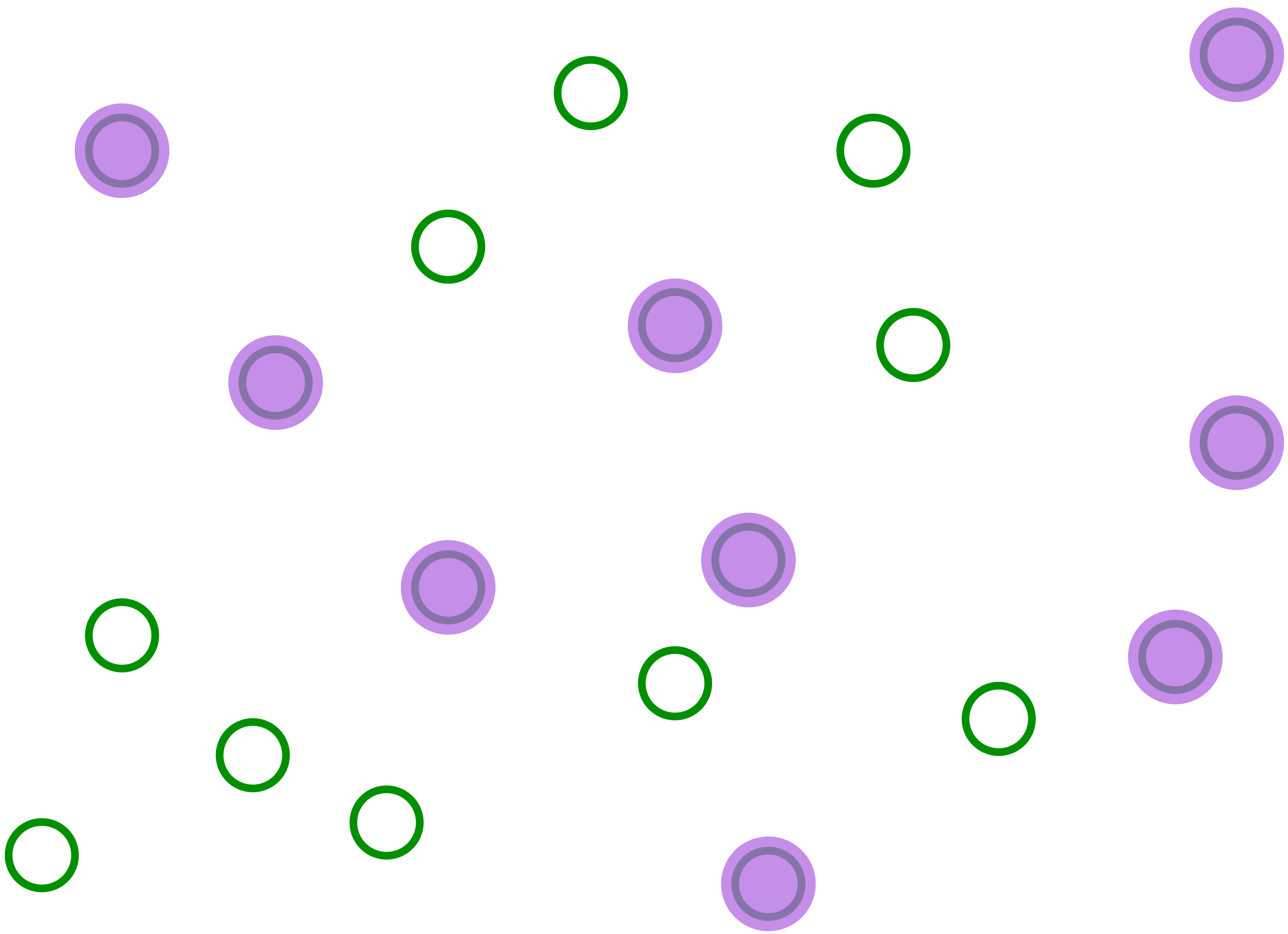
Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



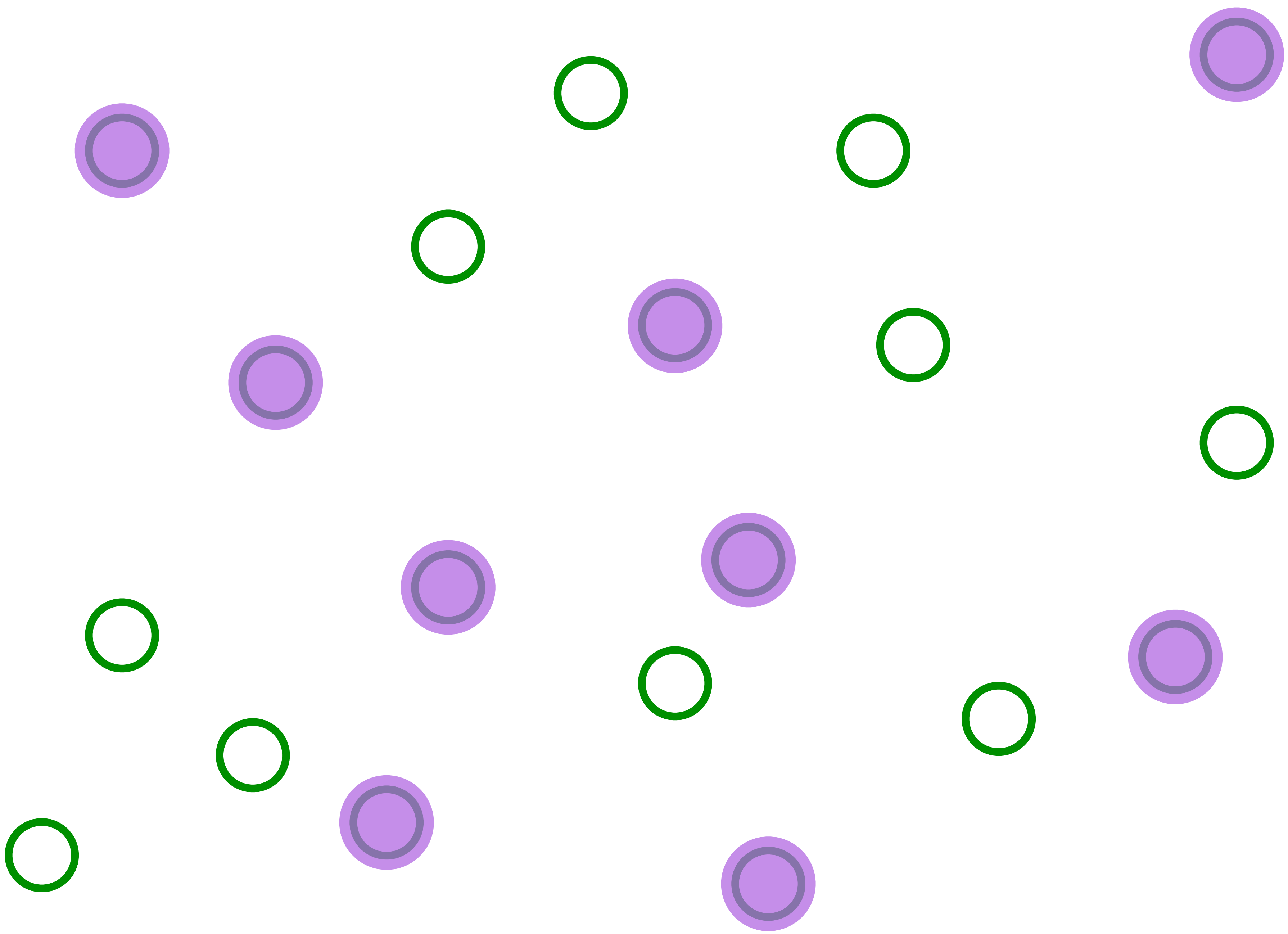
Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

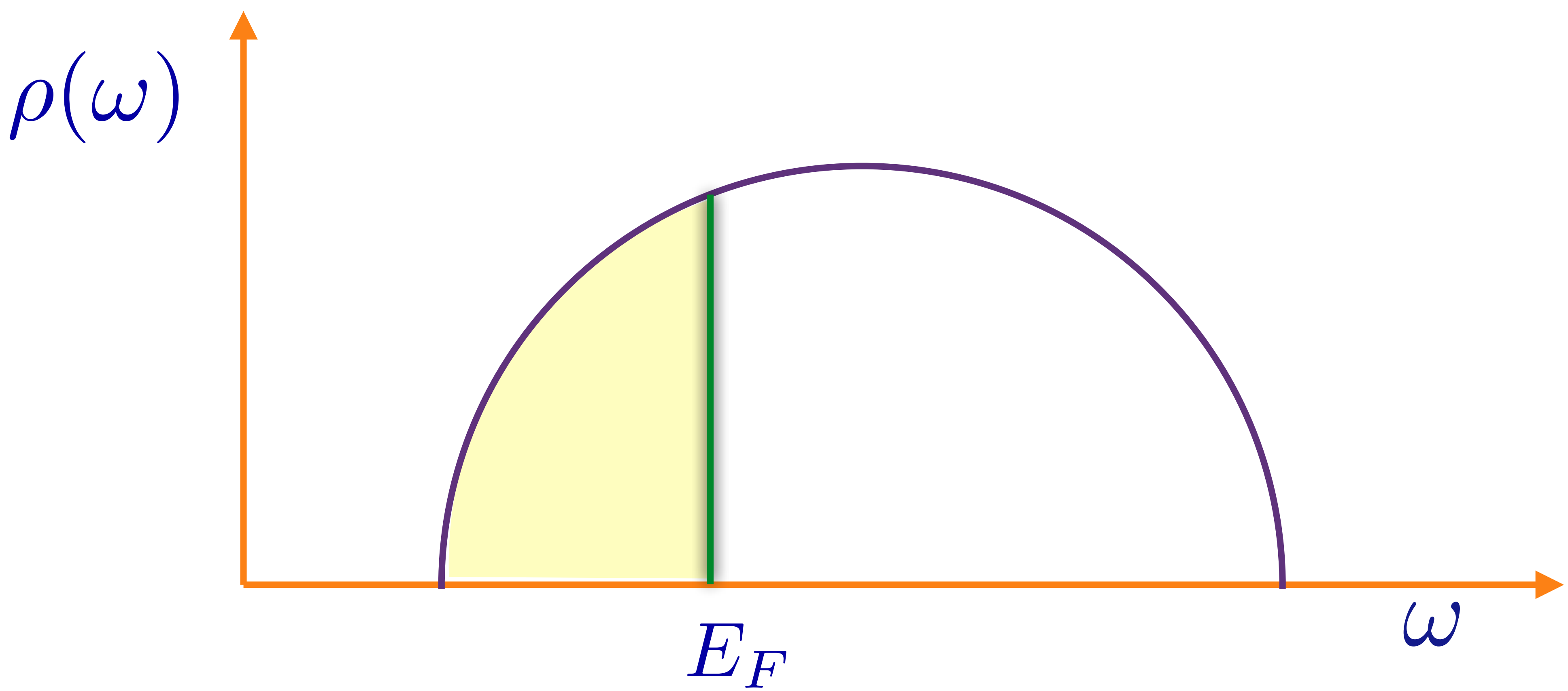
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

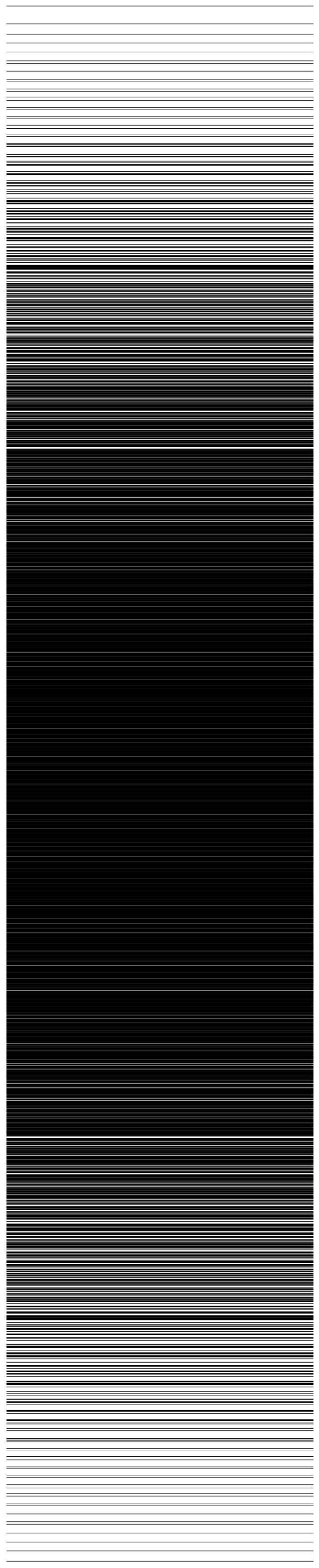
Fermions occupying the eigenstates of a
N x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



A simple model of a metal with quasiparticles



Many-body level spacing $\sim 2^{-N}$

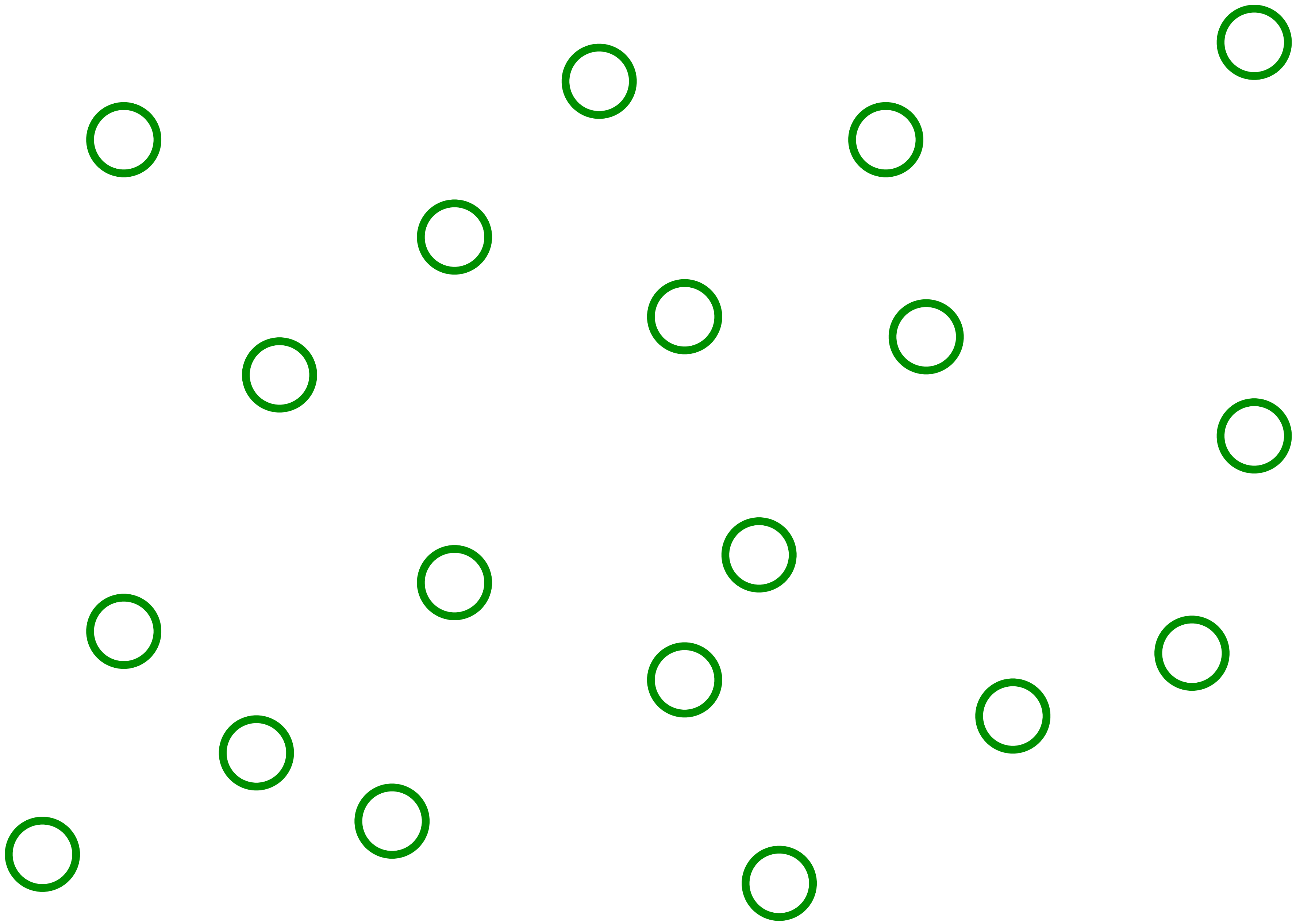
Quasiparticle excitations with spacing $\sim 1/N$

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

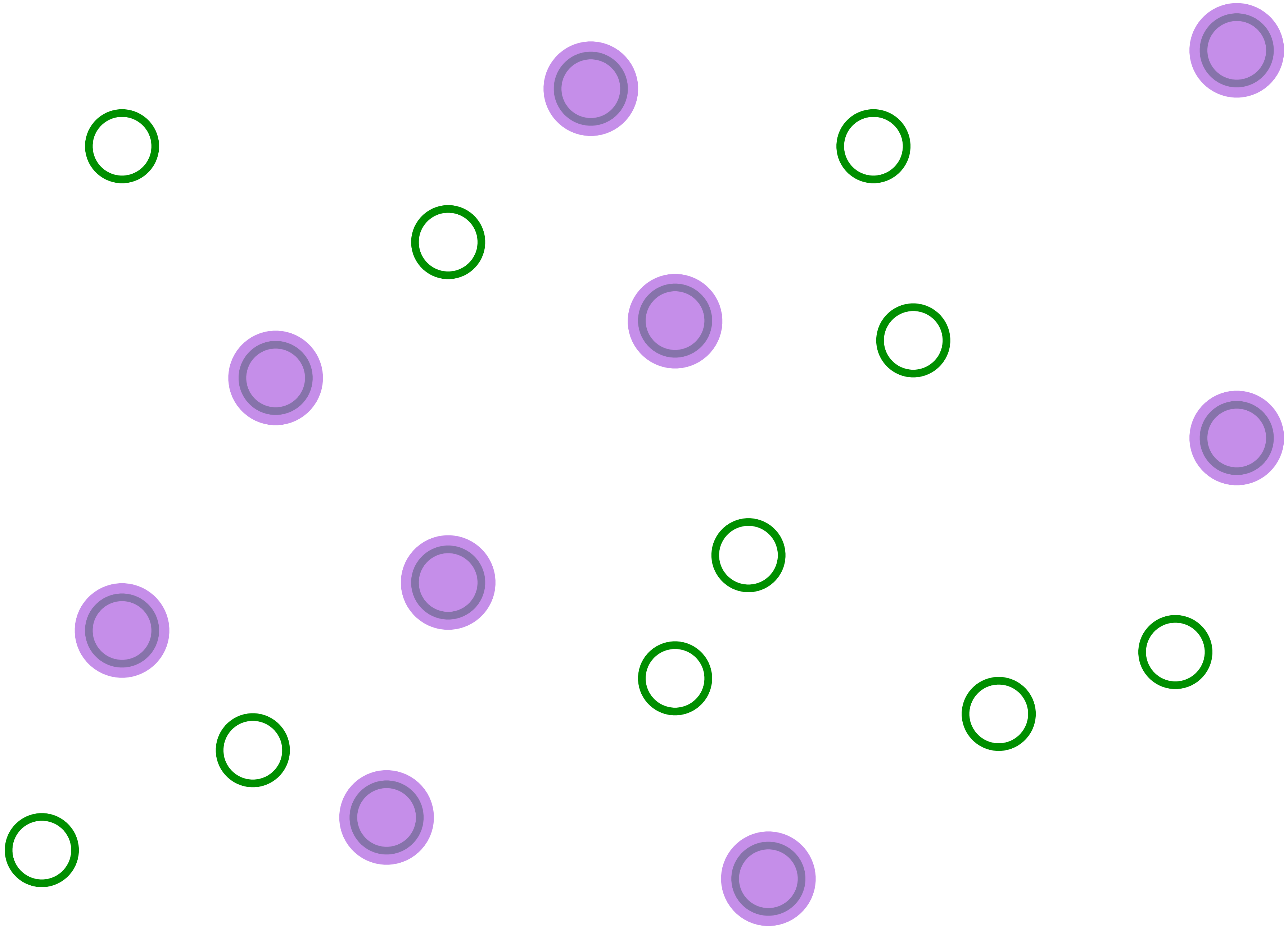
where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

The Sachdev-Ye-Kitaev (SYK) model



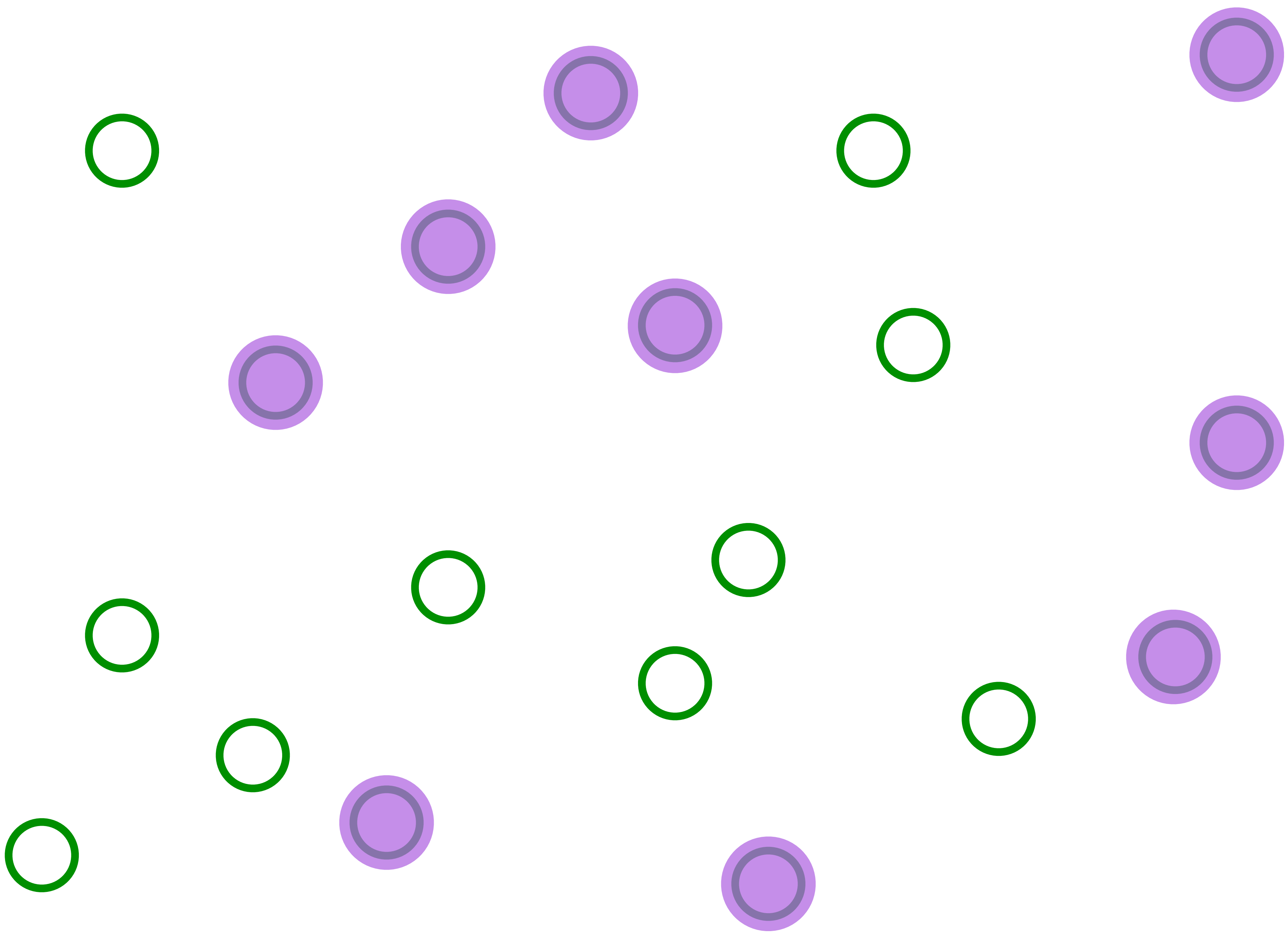
Pick a set of random positions

The Sachdev-Ye-Kitaev (SYK) model



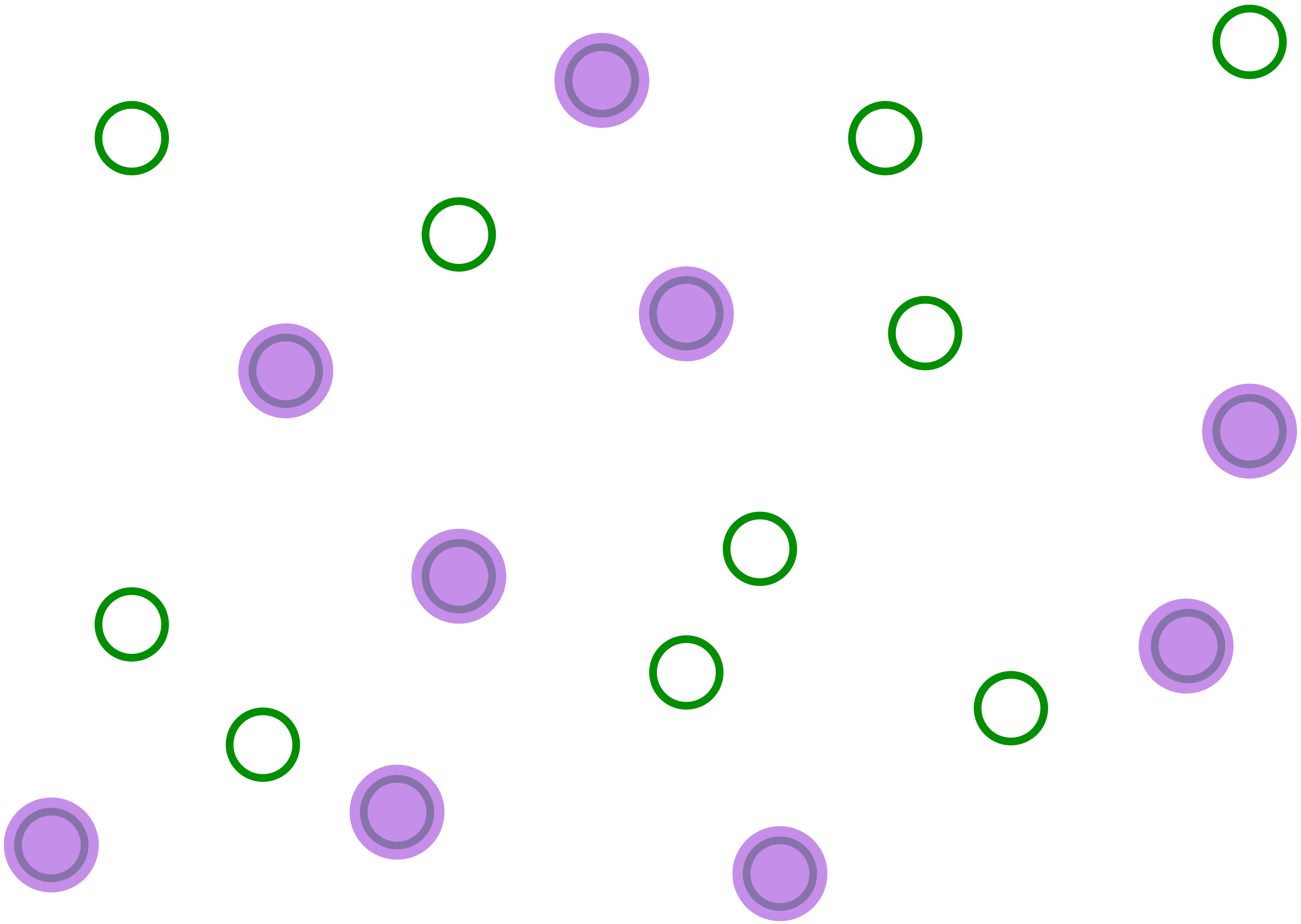
Place electrons randomly on some sites

The Sachdev-Ye-Kitaev (SYK) model



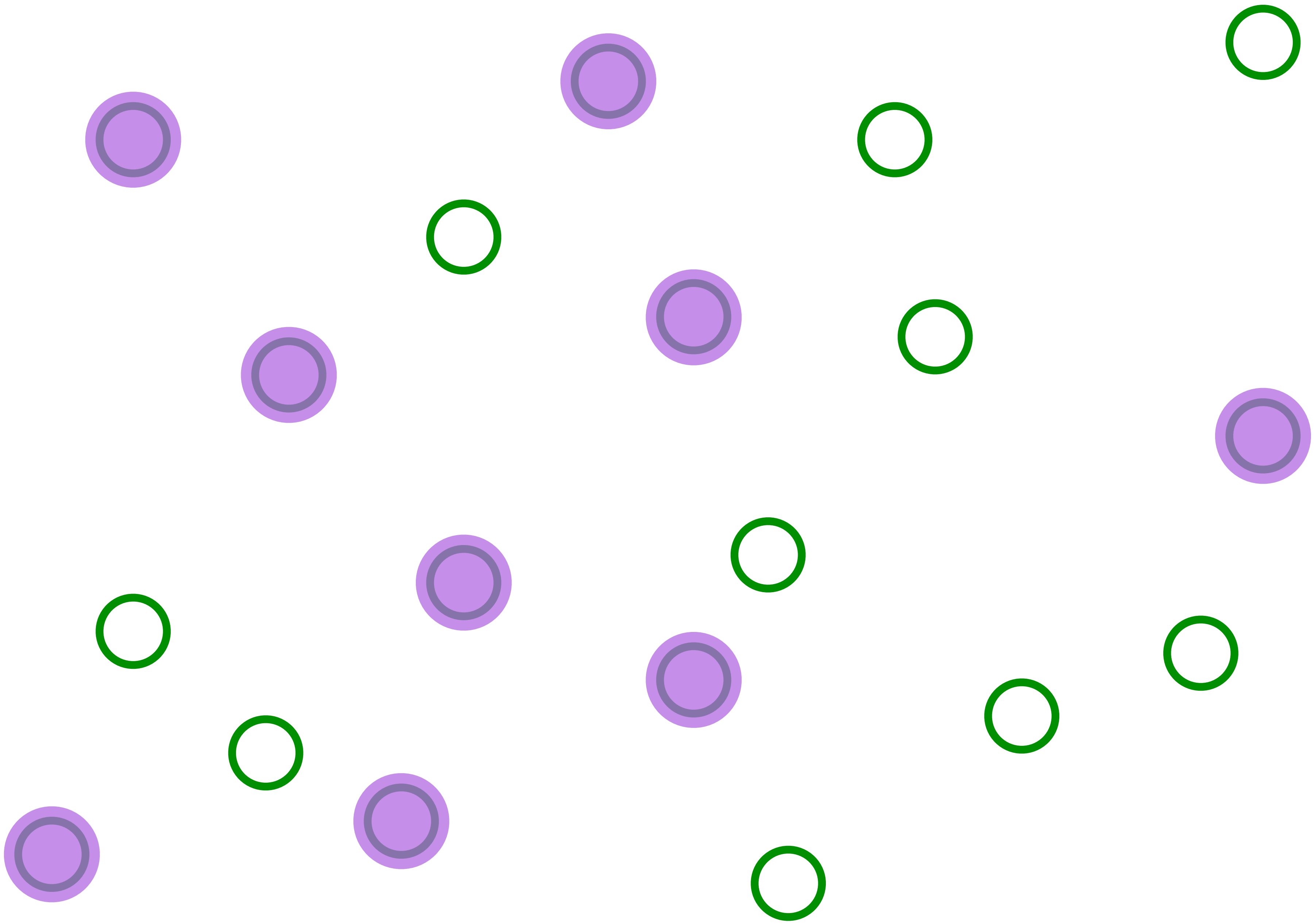
Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



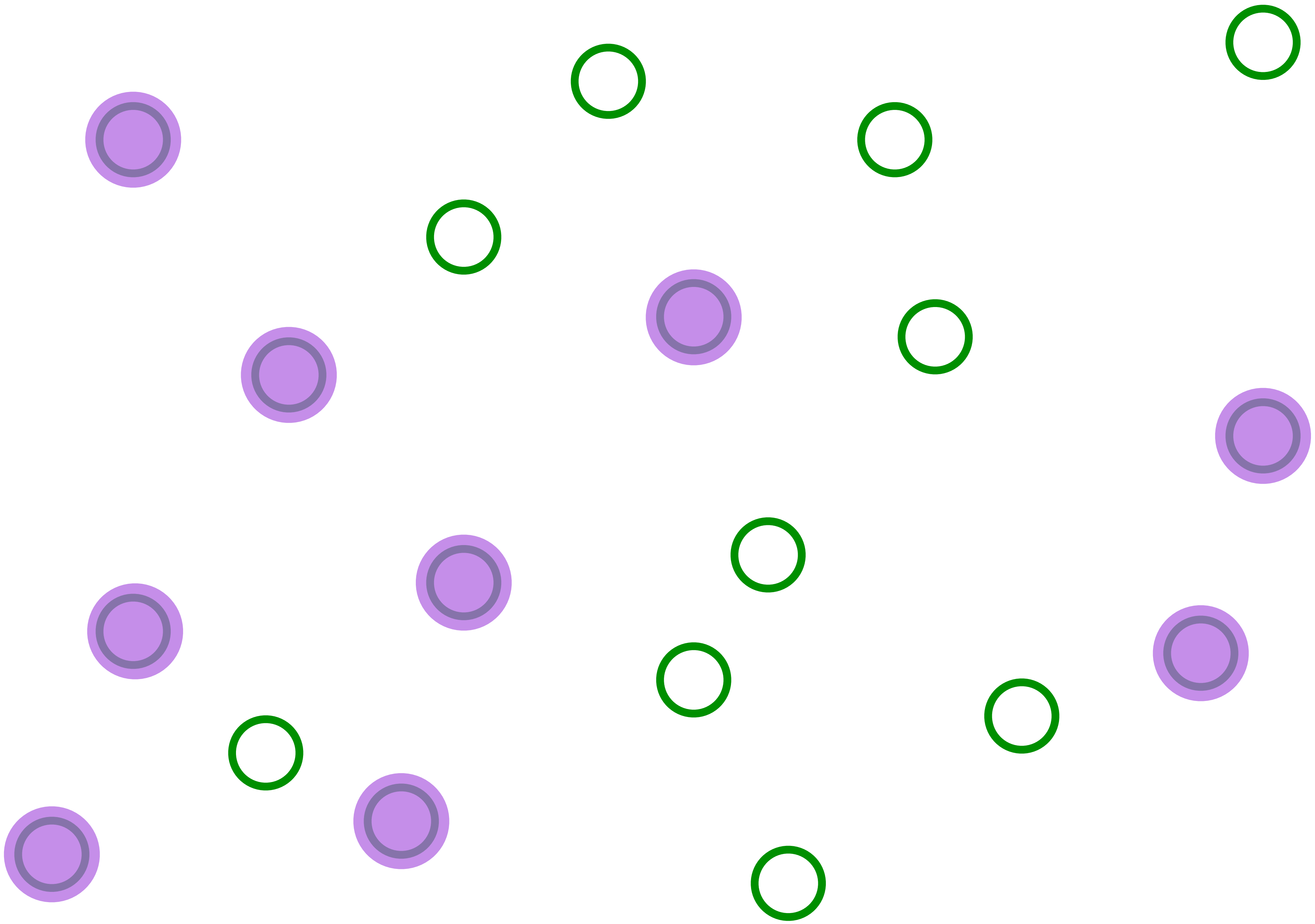
Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



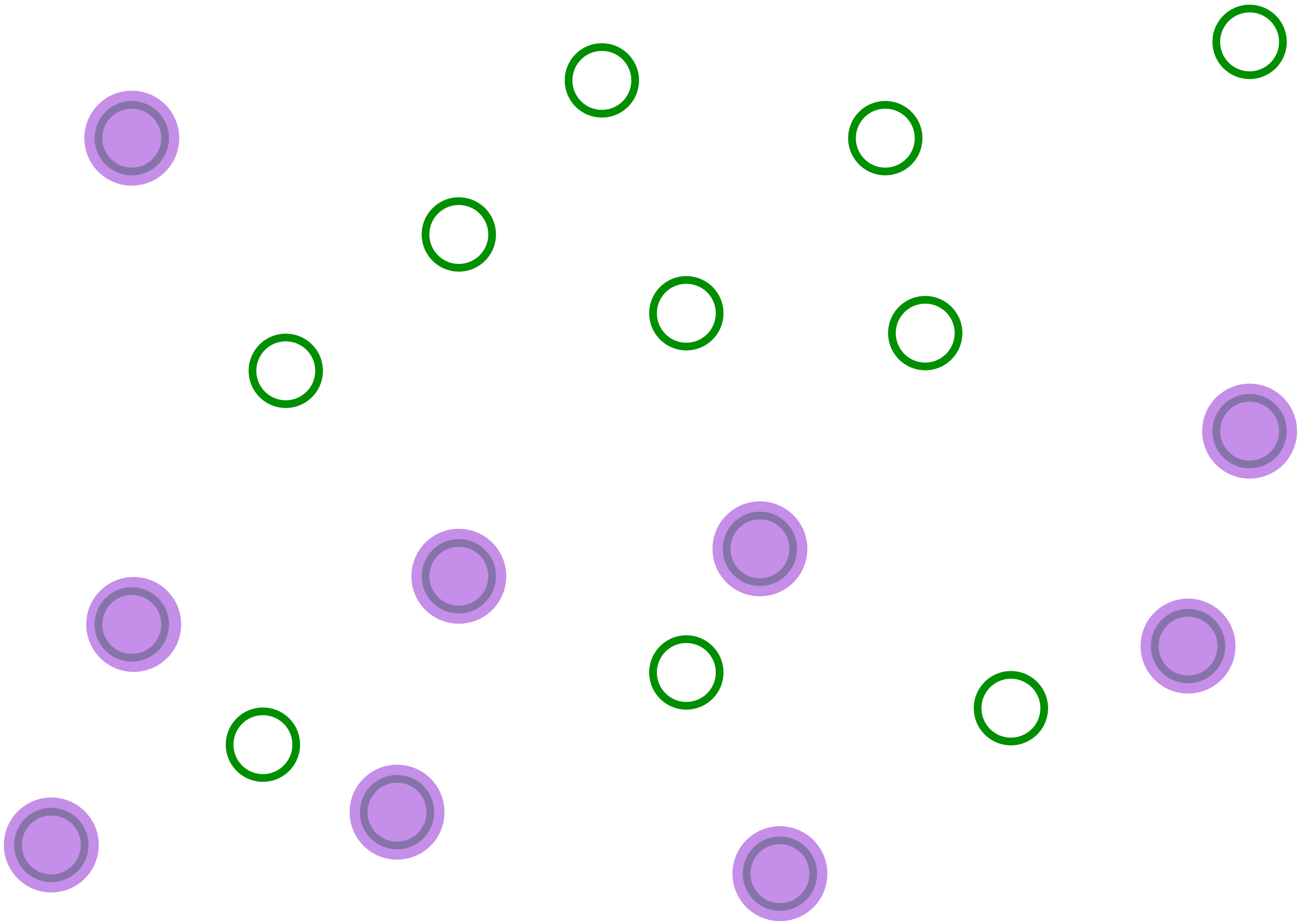
Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



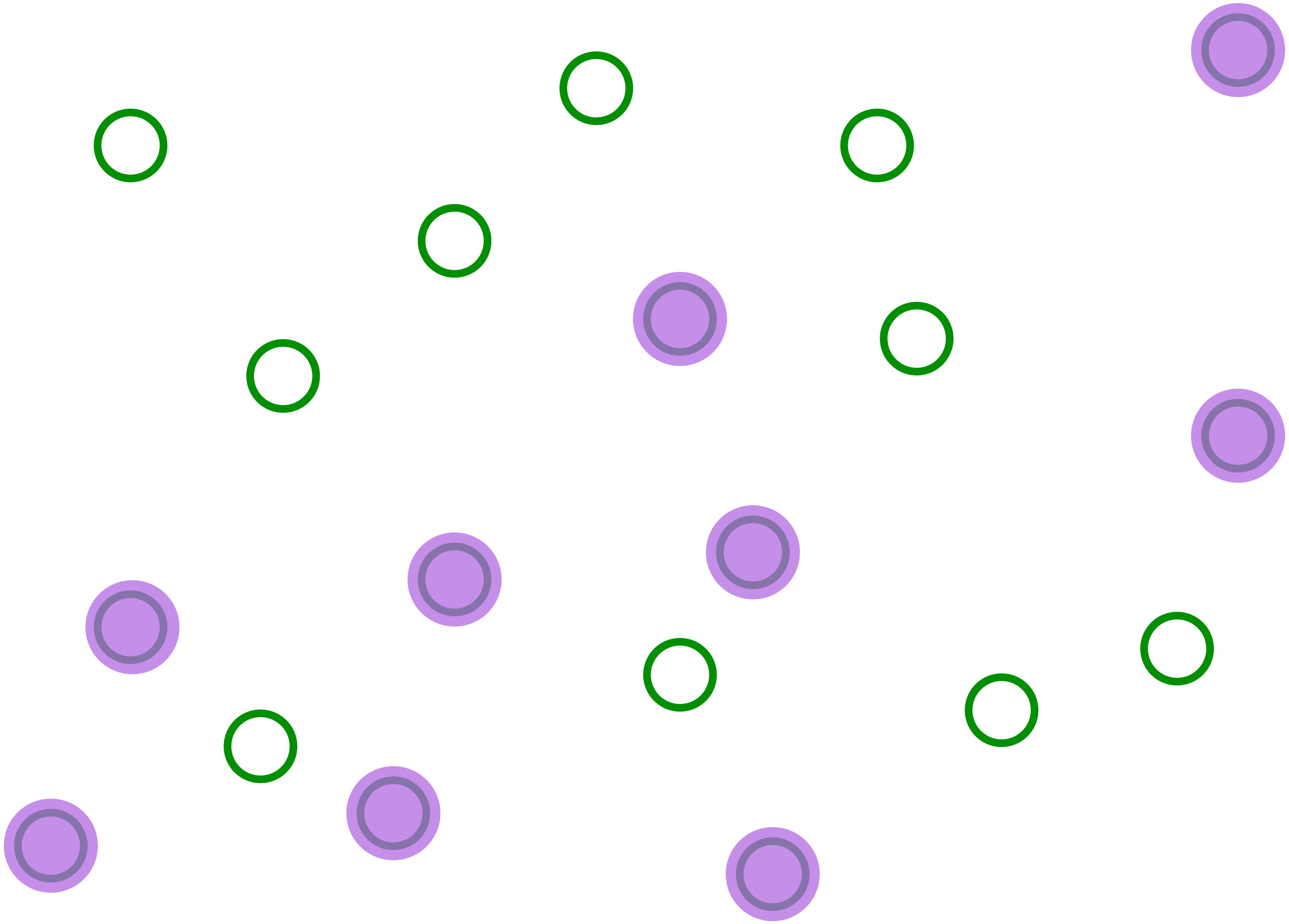
Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



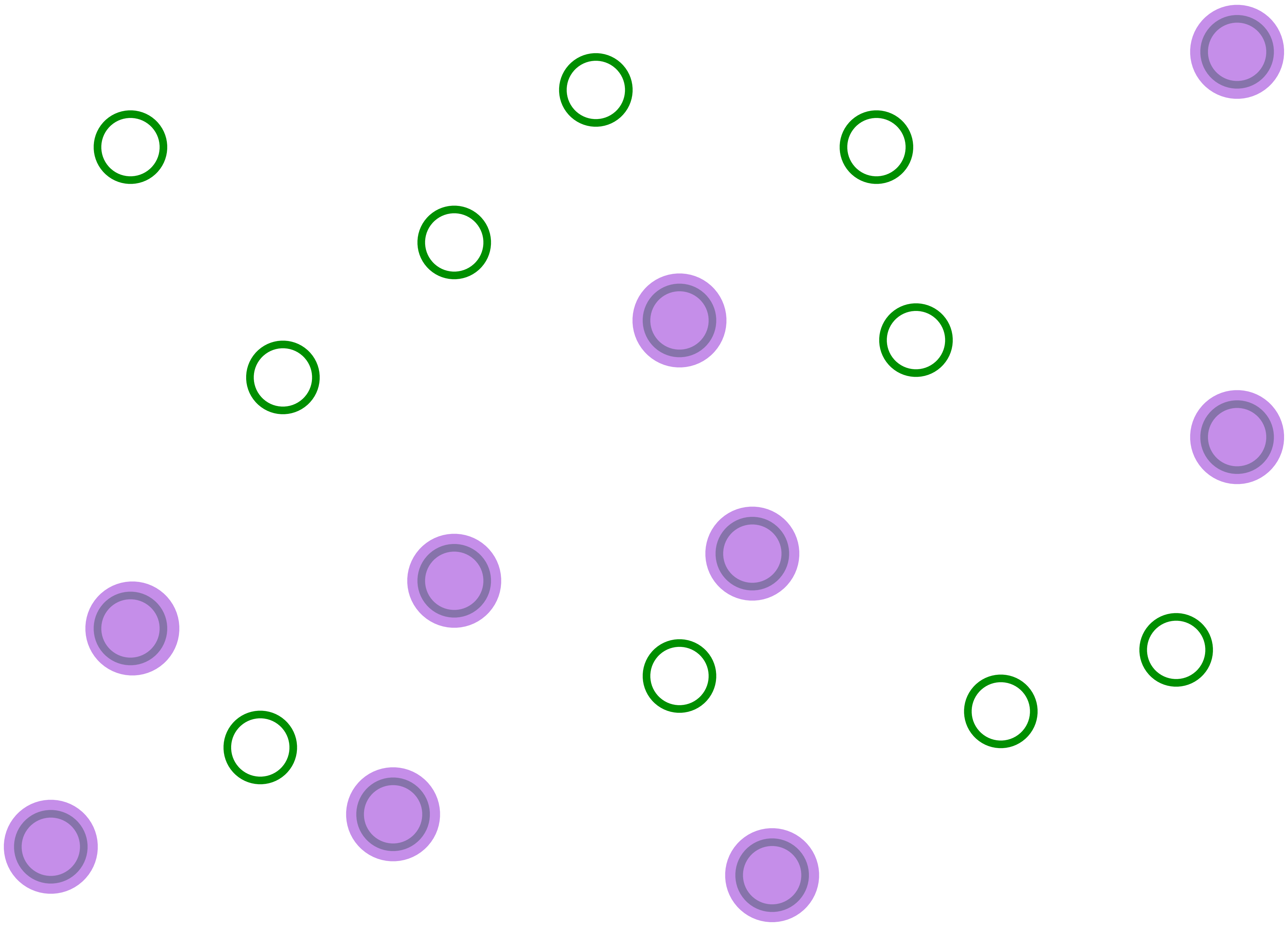
Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



This describes both a strange metal and a black hole!

The Sachdev-Ye-Kitaev (SYK) model

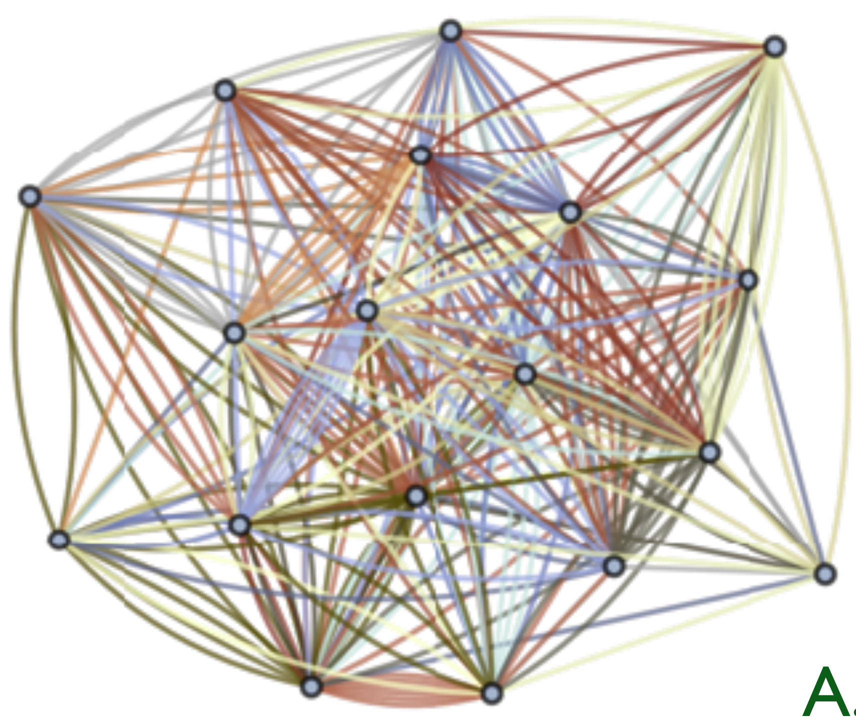
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

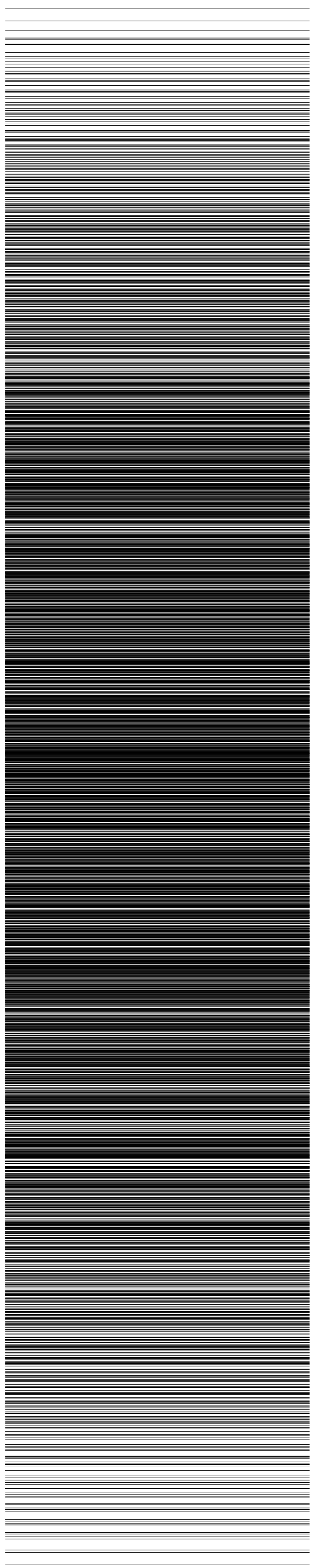
$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The Sachdev-Ye-Kitaev (SYK) model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-S_{GPS}}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy S_{GPS} with

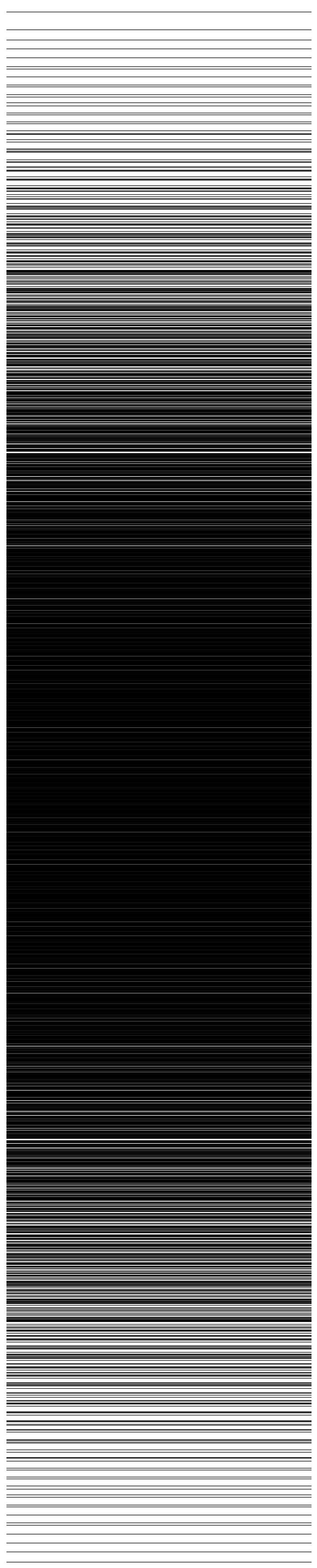
$$\begin{aligned} \frac{S_{GPS}}{N} &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\ &< \ln 2 \end{aligned}$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

A simple model of a metal with quasiparticles



Many-body level spacing $\sim 2^{-N}$

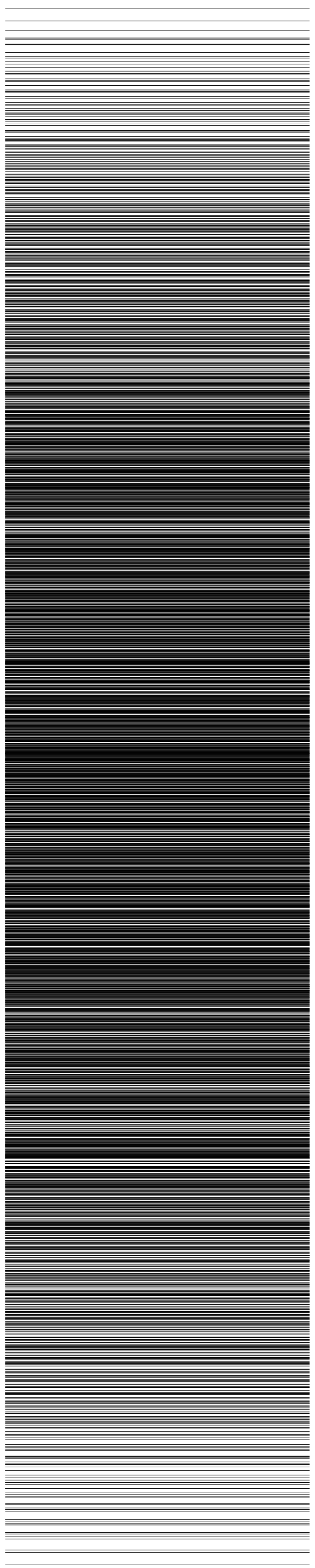
Quasiparticle excitations with spacing $\sim 1/N$

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

The Sachdev-Ye-Kitaev (SYK) model



There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy S_{GPS} with

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

$$\frac{S_{GPS}}{N} = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$
$$< \ln 2$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

Non-quasiparticle excitations with spacing $\sim e^{-S_{GPS}}$

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

SYK and black holes

- The SYK model has a non-zero entropy, $S_{GPS} \propto N$ as $T \to 0$.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

SYK and black holes

- The SYK model has a non-zero entropy, $S_{GPS} \propto N$ as $T \to 0$.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

- The SYK model has a phase-coherence time $\tau_\varphi \sim \hbar/(k_B T)$.

O. Parcollet and A. Georges, PRB **59**, 5341 (1999)

And $\tau_L = \hbar/(2\pi k_B T)$, the smallest possible value.

Kitaev, unpublished; J. Maldacena, D. Stanford, arXiv:1604.07818

SYK and black holes

- The SYK model has a non-zero entropy, $S_{GPS} \propto N$ as $T \to 0$.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

- The SYK model has a phase-coherence time $\tau_\varphi \sim \hbar/(k_B T)$.

O. Parcollet and A. Georges, PRB **59**, 5341 (1999)

And $\tau_L = \hbar/(2\pi k_B T)$, the smallest possible value.

Kitaev, unpublished; J. Maldacena, D. Stanford, arXiv:1604.07818

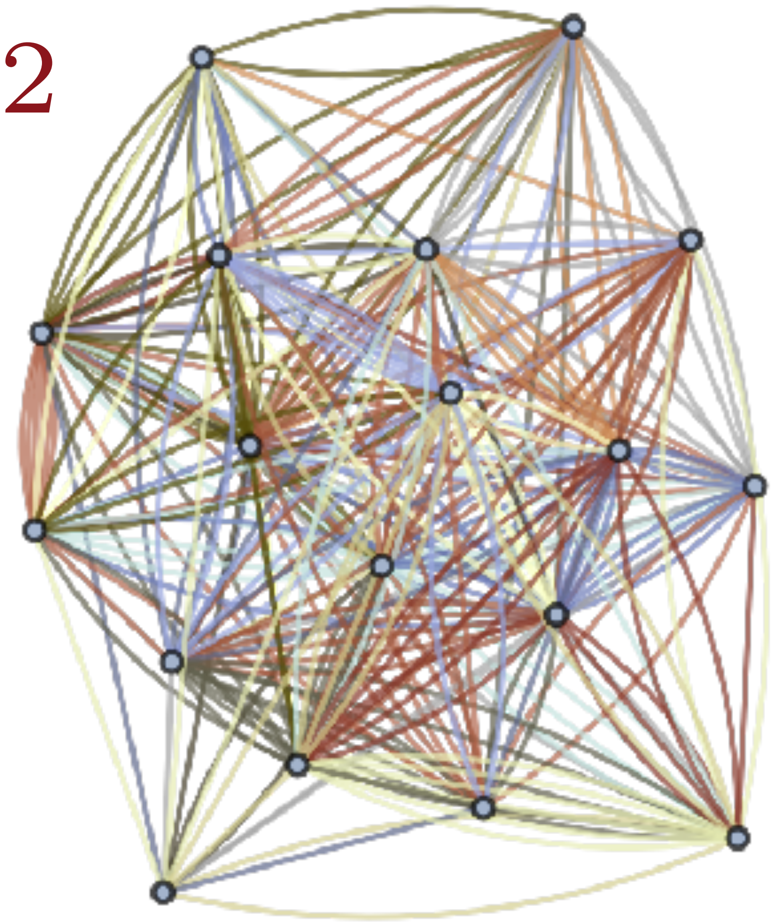
These properties indicate, as in the AdS/CFT correspondence, the SYK model is a holographic representation of a black hole, and the black hole entropy $S_{BH} = S_{GPS}$.

S. Sachdev, PRL **105**, 151602 (2010)

SYK and black holes

$\mathbb{T}^2$

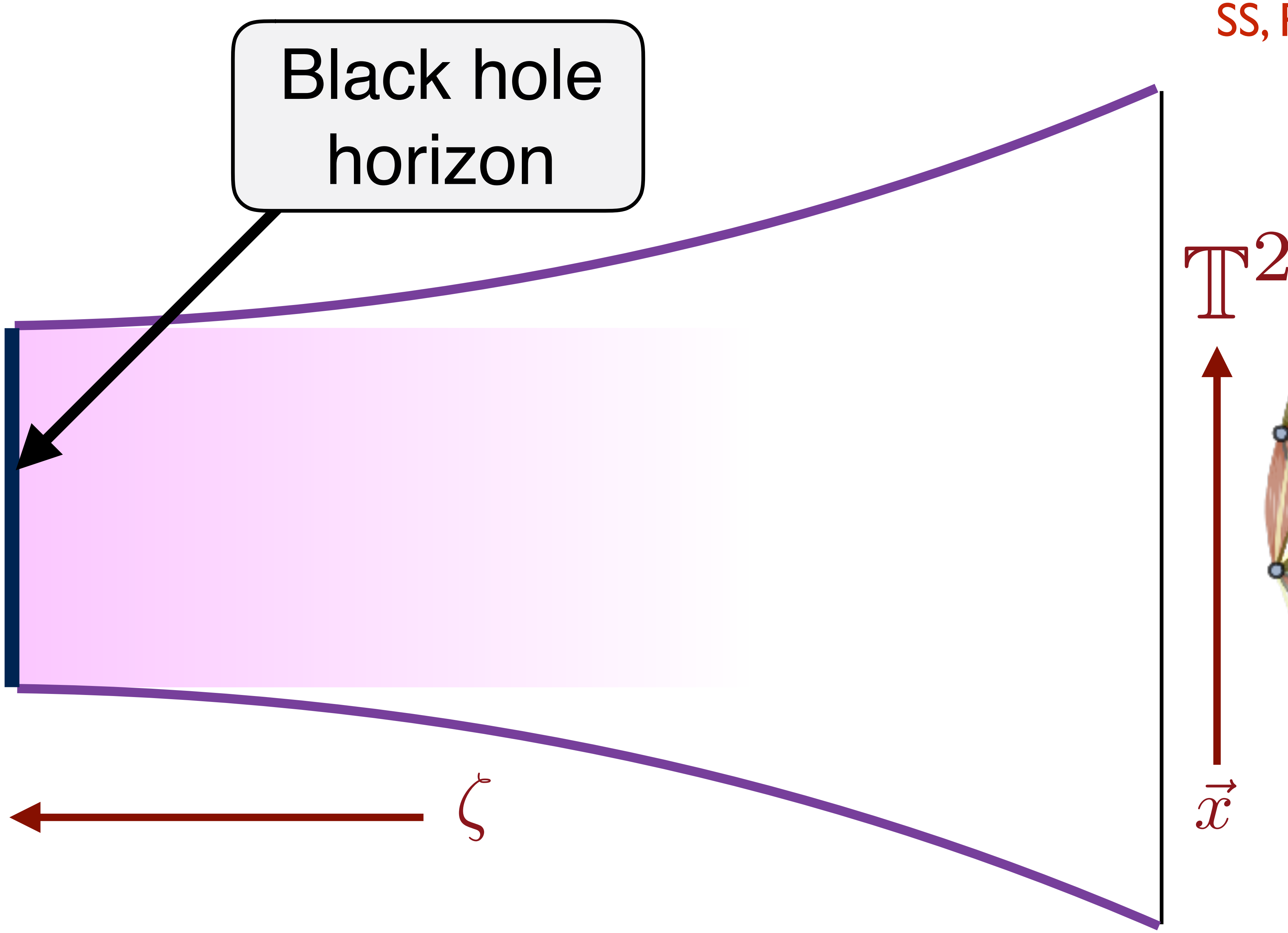
$\vec{x}$

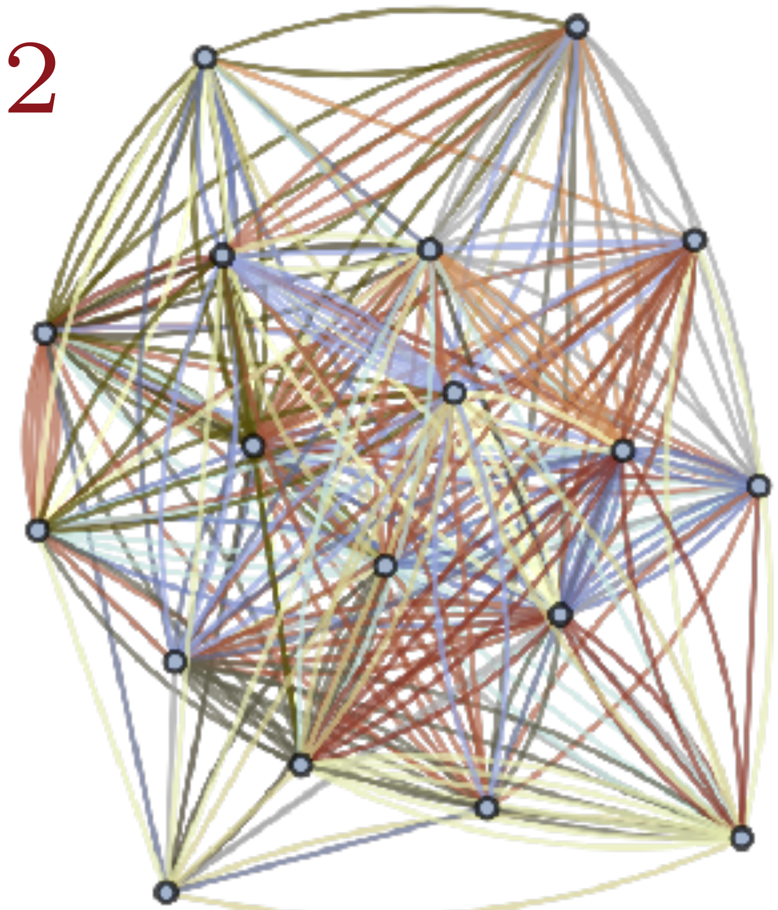


$\mathbb{T}^2 \Rightarrow$ two-dimensional torus

SYK and black holes

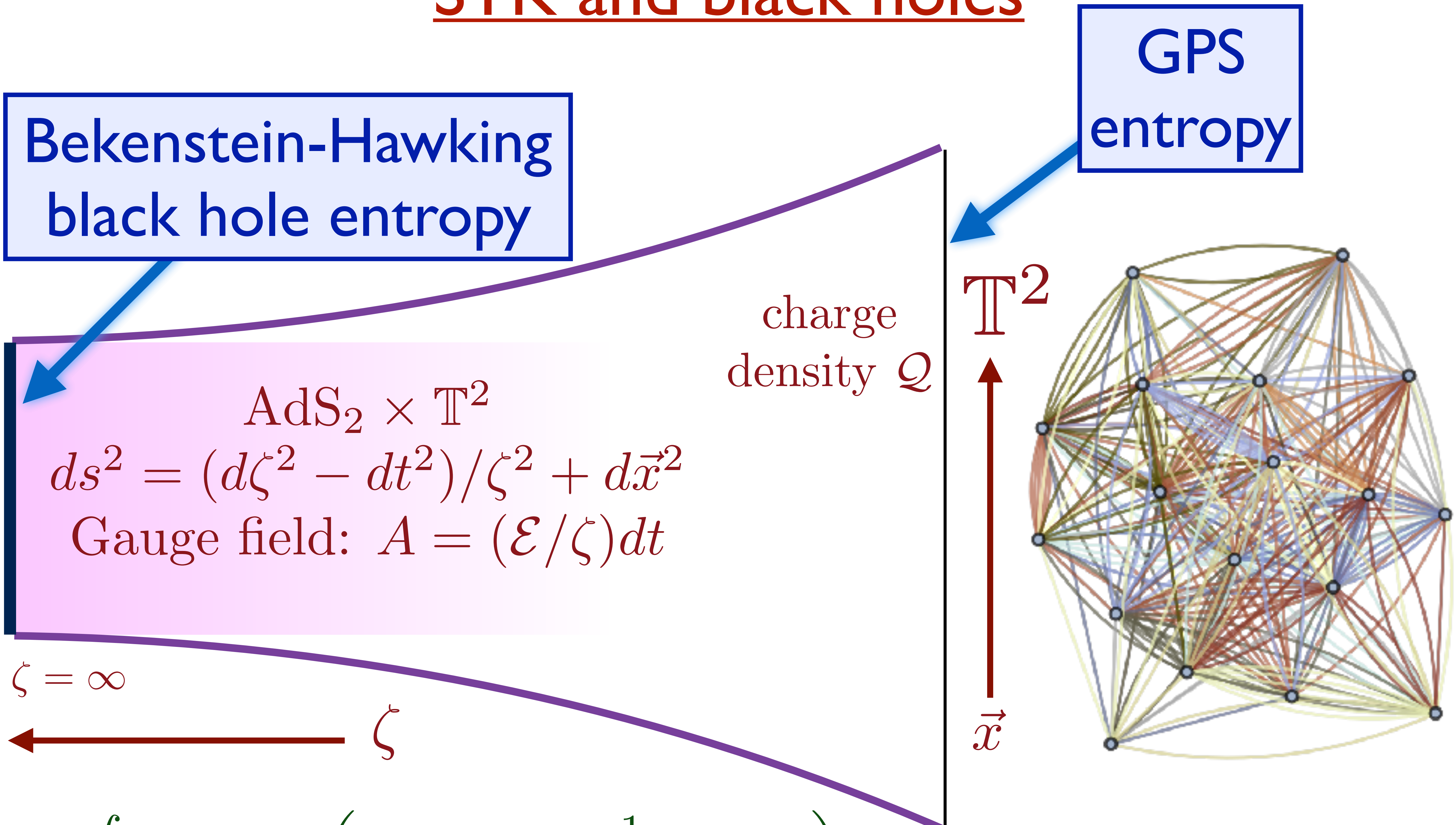
SS, PRL **105**, 151602 (2010)





The SYK model has "dual" description in which an extra spatial dimension, ζ, emerges. The curvature of this "emergent" spacetime is described by Einstein's theory of general relativity

SYK and black holes



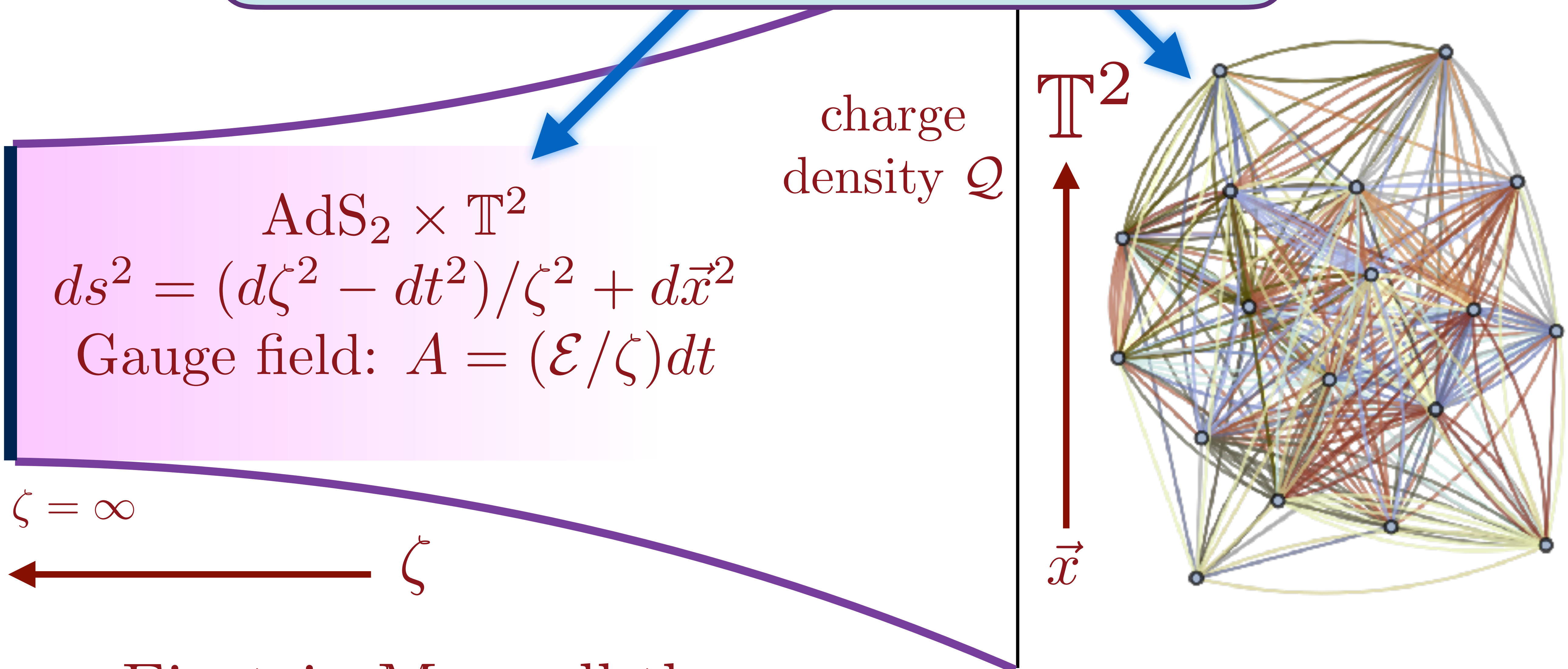
$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

SS, PRL **105**, 151602 (2010)

The BH entropy is proportional to the size of $\mathbb{T}^2$, and hence the surface area of the black hole. Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$.

SYK and black holes

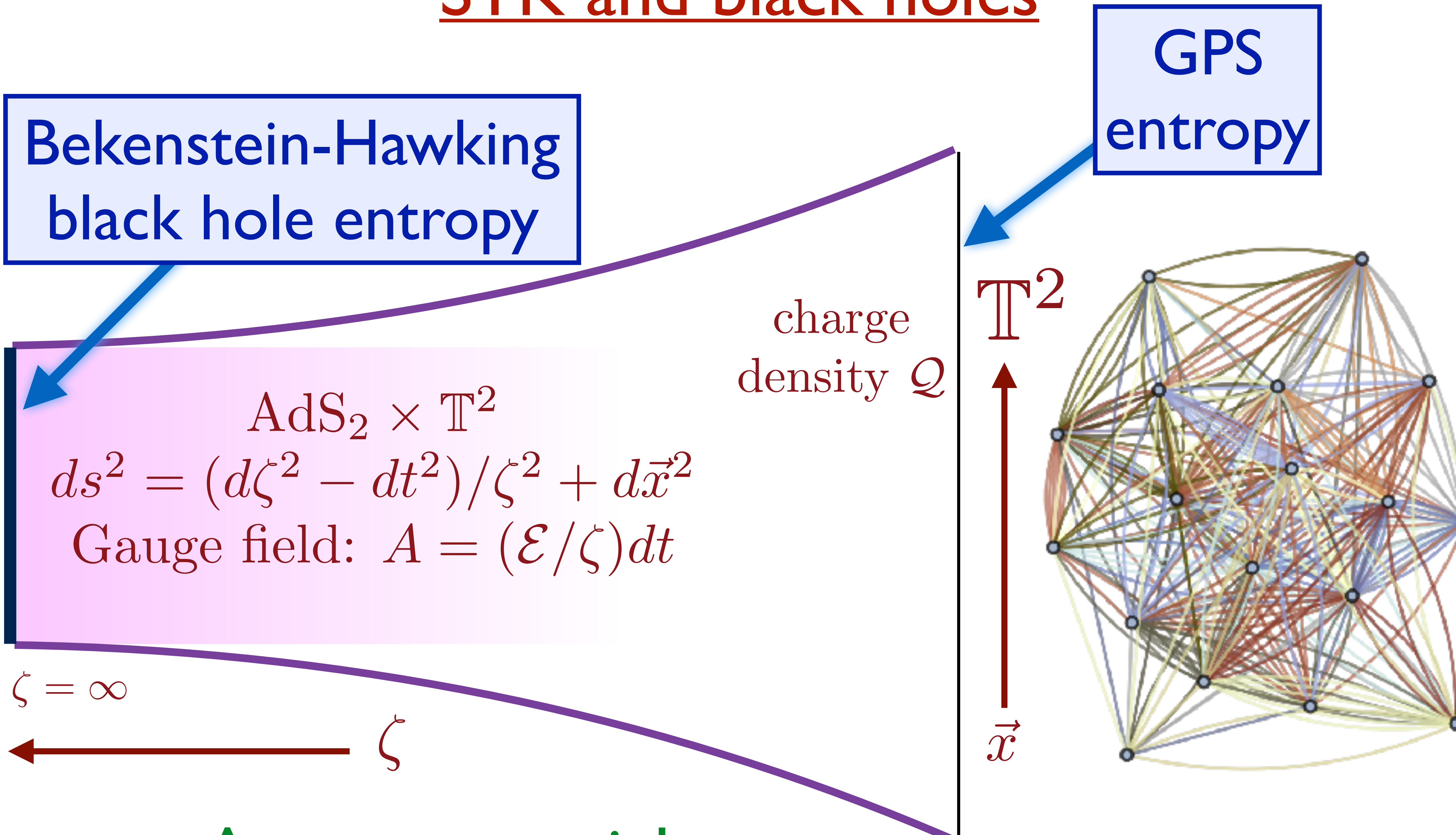
Same long-time effective action for energy and number fluctuations, involving Schwarzian derivatives of time reparameterizations $f(\tau)$.



Einstein-Maxwell theory
+ cosmological constant

A. Kitaev, unpublished, J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

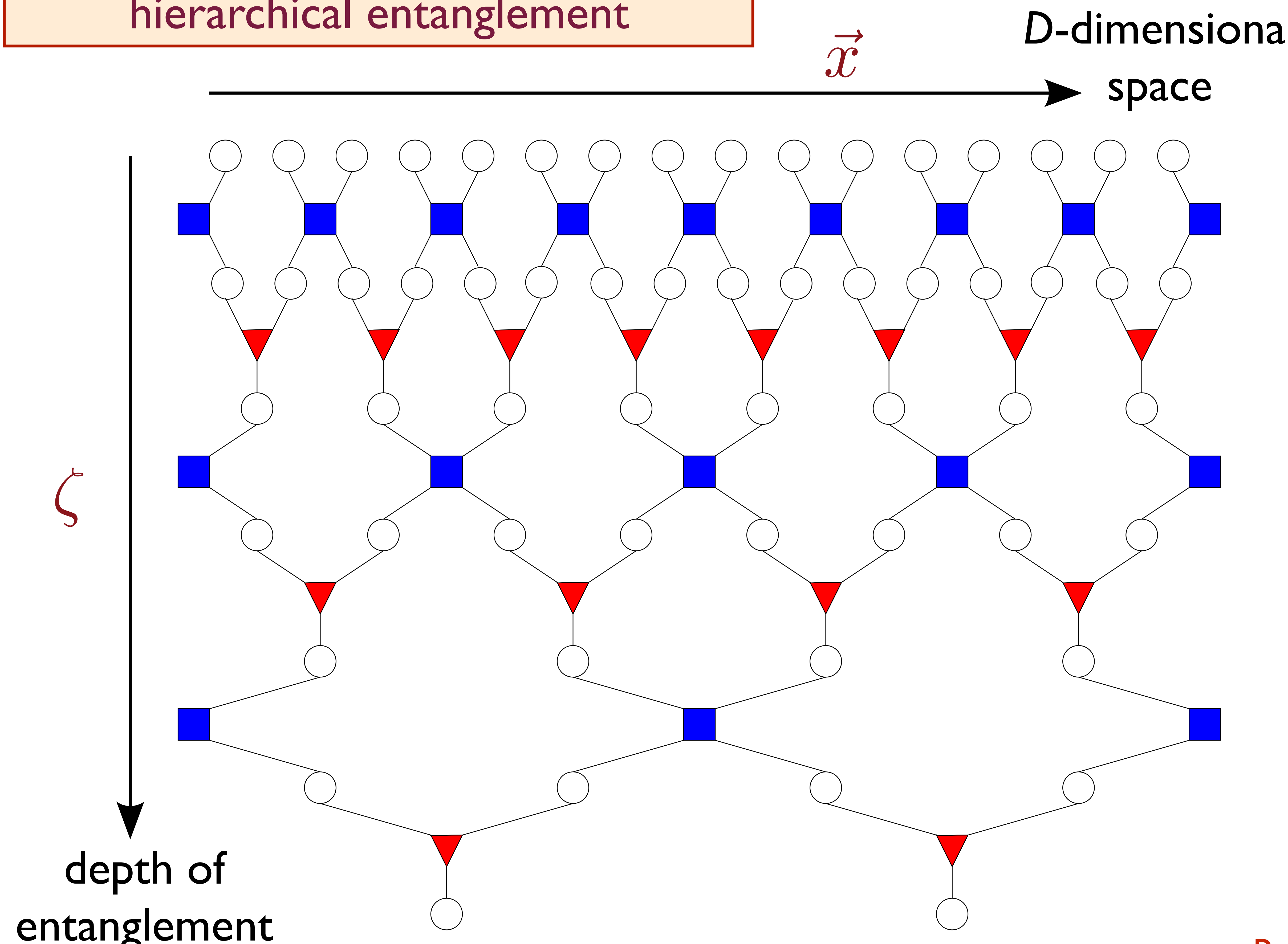
SYK and black holes



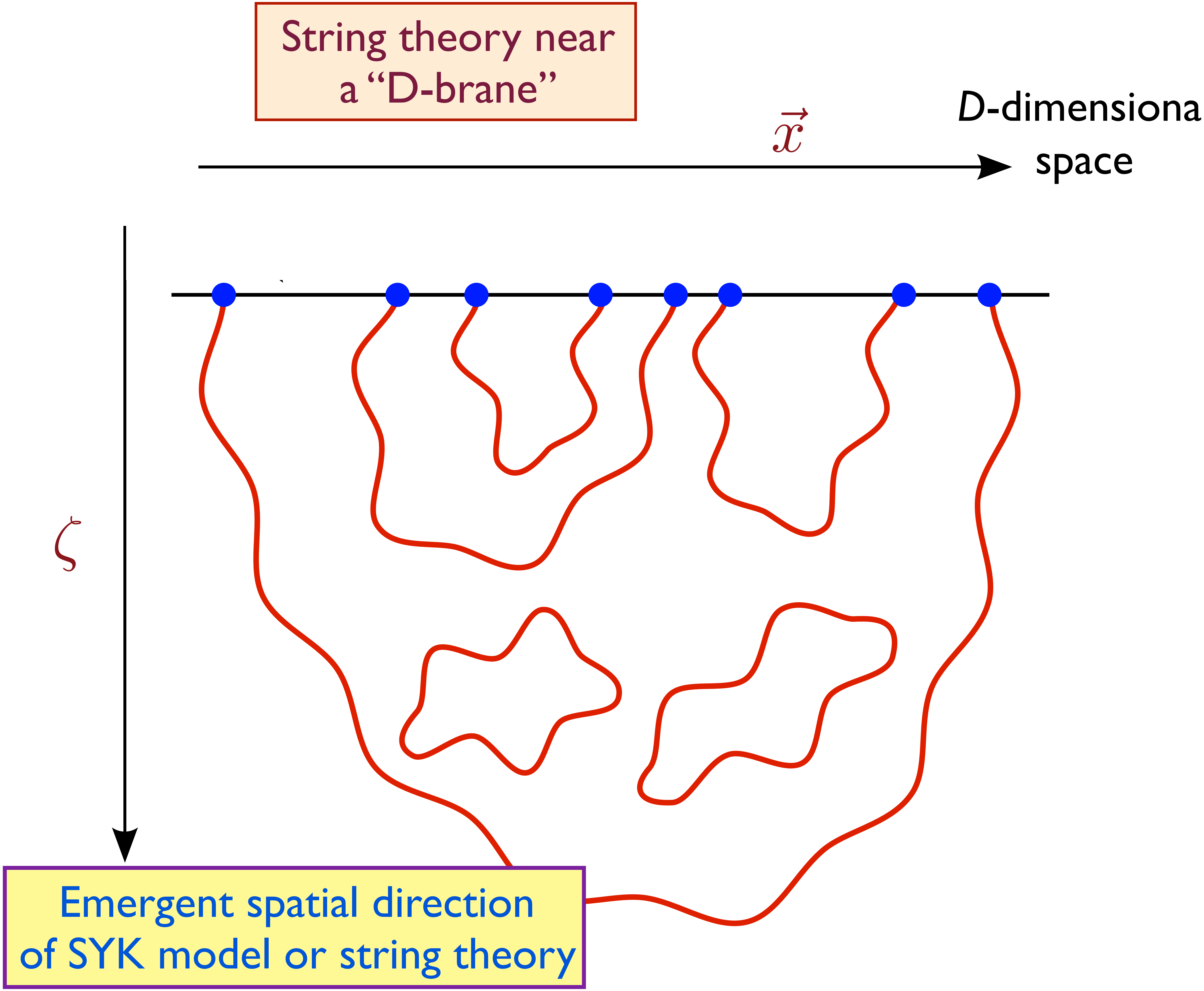
An extra spatial dimension emerges from quantum entanglement!

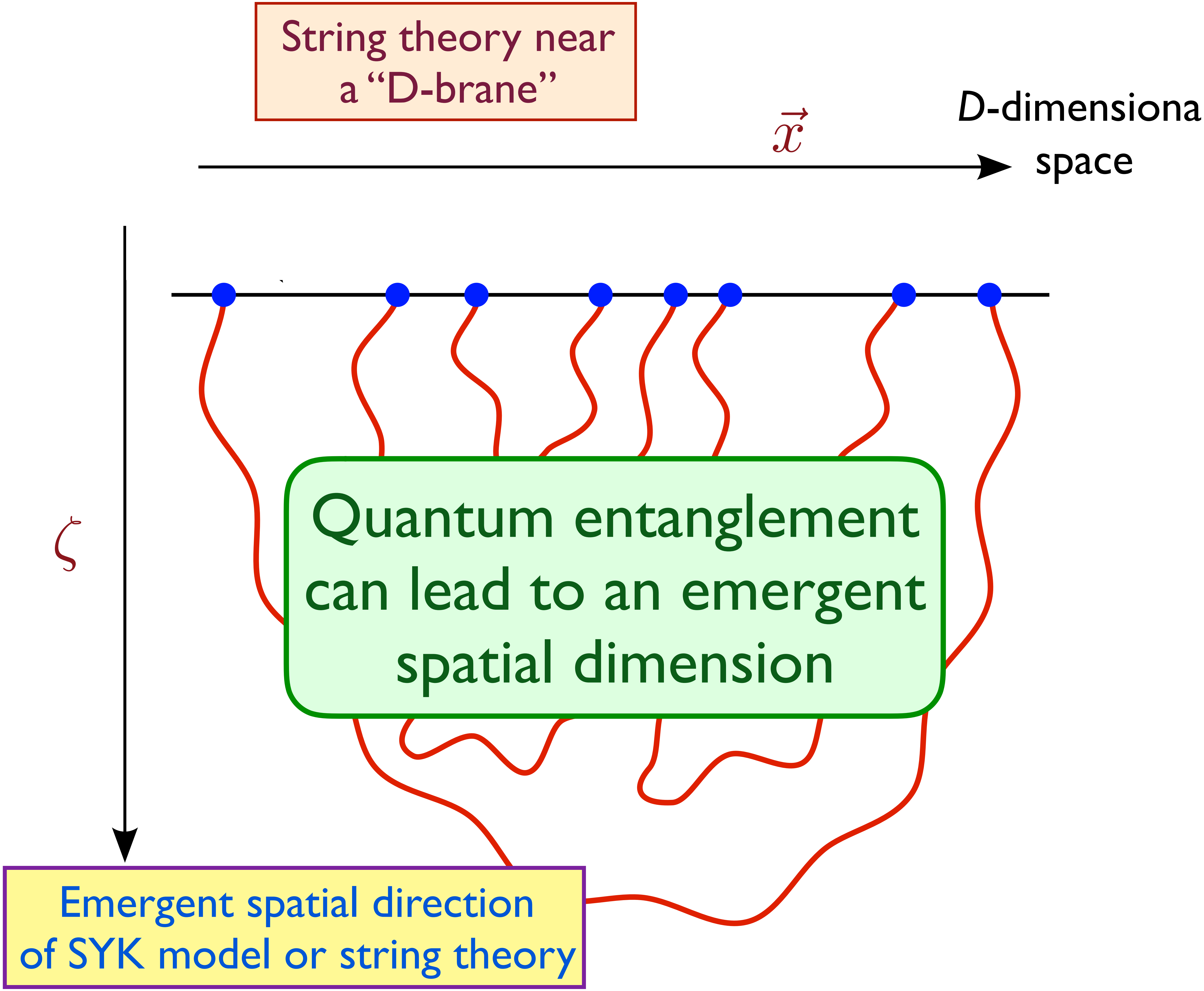
SS, PRL **105**, 151602 (2010)

Tensor network of hierarchical entanglement



B. Swingle


String theory near
a "D-brane"
$\vec{x}$
D-dimensional
space
ζ
Emergent spatial direction
of SYK model or string theory


String theory near a "D-brane"
$\vec{x}$
D-dimensional space
ζ
Quantum entanglement can lead to an emergent spatial dimension
Emergent spatial direction of SYK model or string theory

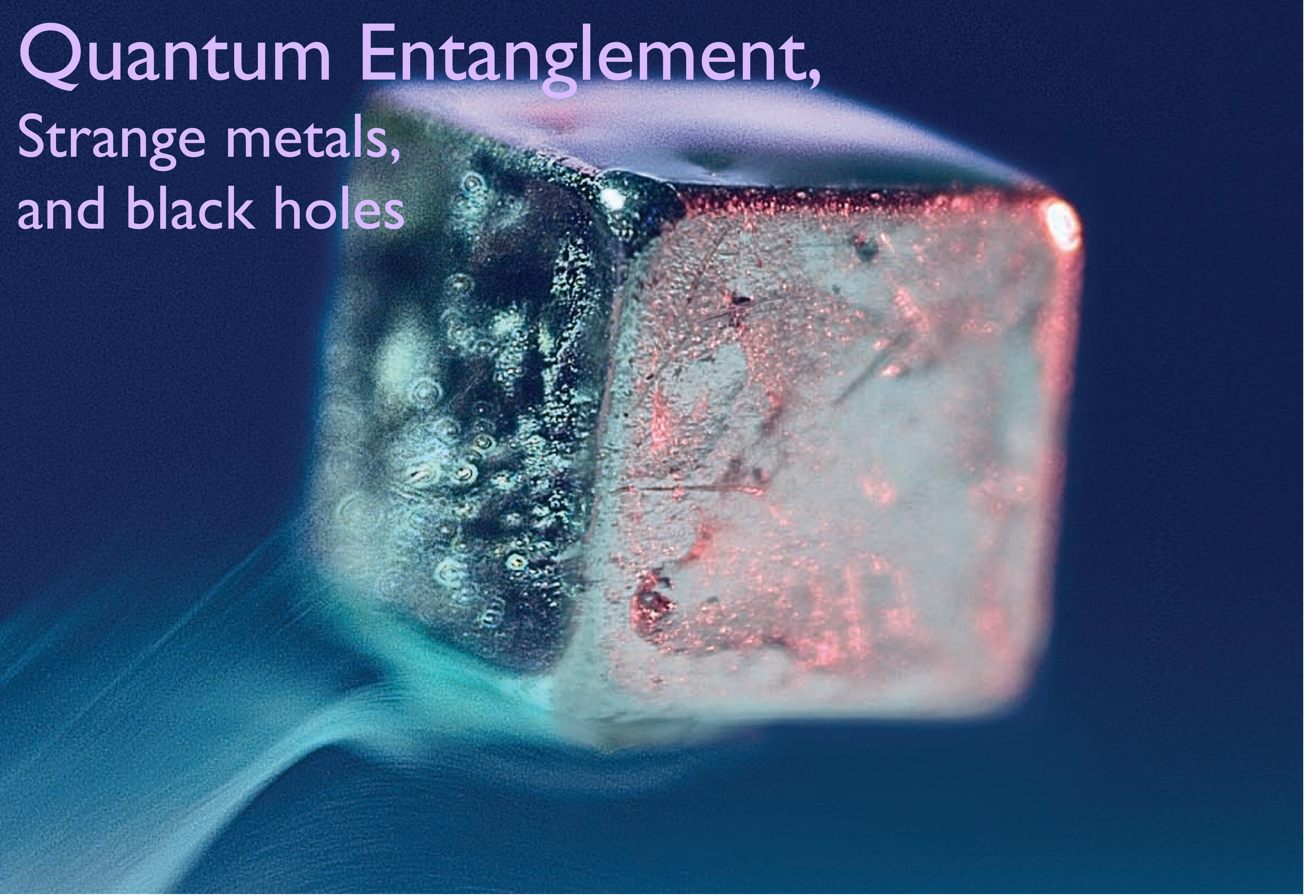
Quantum entanglement

Black holes

Strange metals

A "toy model" which is both a strange metal and a black hole!

Quantum Entanglement,
Strange metals, and black holes



Subir Sachdev, Harvard University and Perimeter Institute