

Quantum matter without quasiparticles

Topological Insulators and Mathematical Science

Center for Mathematical Sciences and Applications,

Harvard

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Talk online: sachdev.physics.harvard.edu

PHYSICS

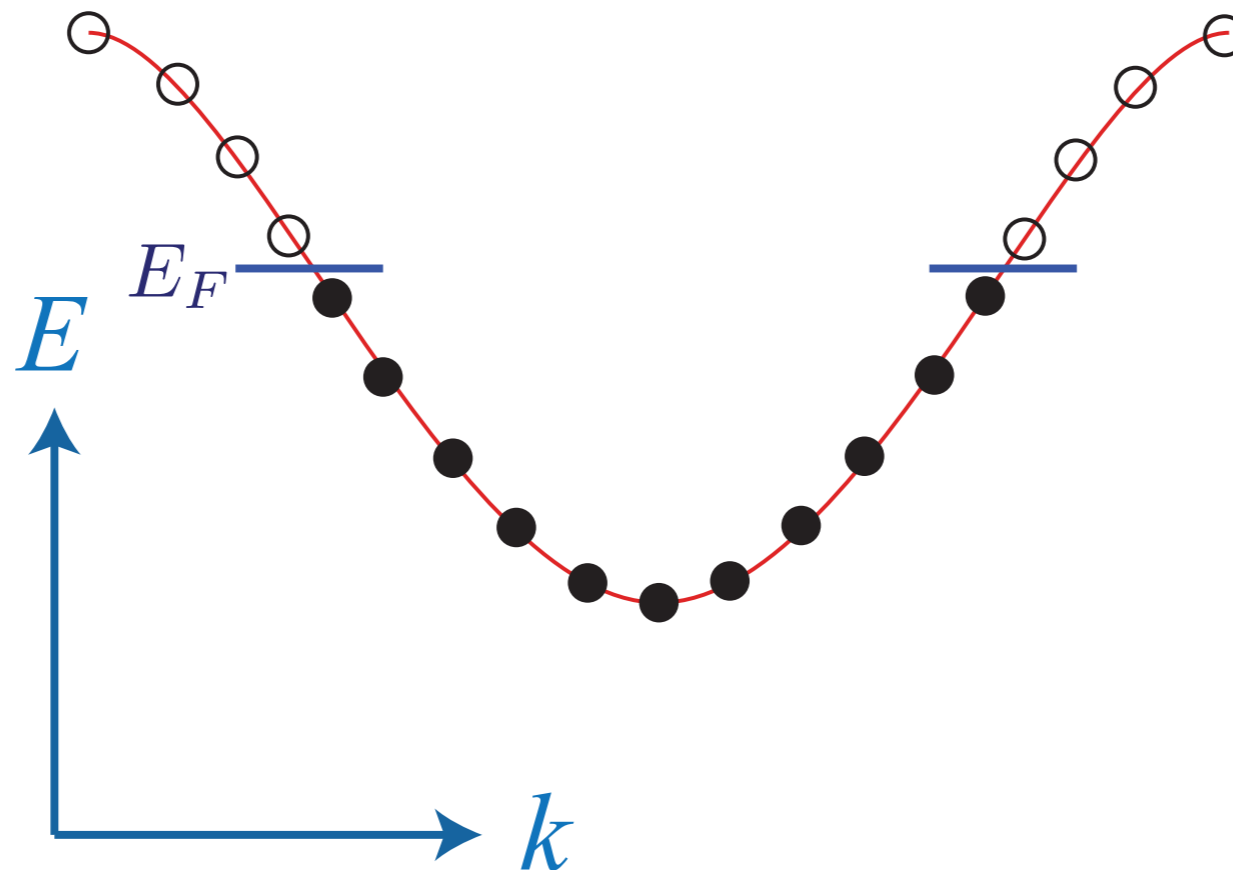


HARVARD

Foundations of quantum many body theory:

I. Ground states connected adiabatically to independent electron states

Metals

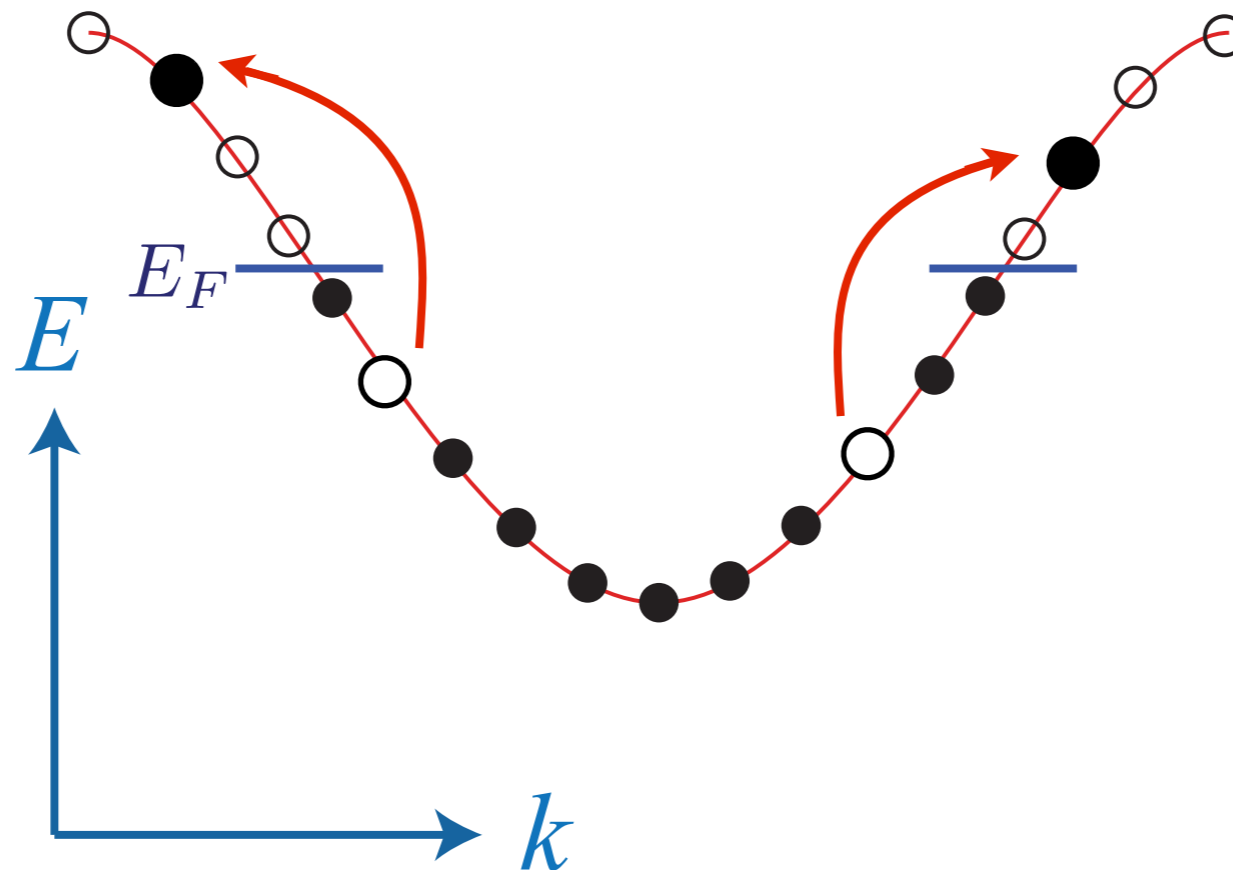


Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

2. Boltzmann-Landau theory of quasiparticles

Metals



Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. Boltzmann-Landau theory of quasiparticles*

Famous example:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. No quasiparticles**

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Only 2 examples:

1. Conformal field theories in spatial dimension $d > 1$
2. Quantum critical metals in dimension $d=2$

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1. Conformal field theories in spatial dimension $d > 1$
2. Quantum critical metals in dimension $d=2$



Emanuel Katz
Boston University



William Witczak-Krempa
Perimeter



Erik Sorensen
McMaster

Outline

1. A CFT in 2+1 dimensions
2. Boltzmann dynamics
3. Dynamics from the operator product expansion (OPE) for $\hbar\omega \gg k_B T$
4. Holography

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The Superfluid-Insulator transition

Boson Hubbard model

Bosons, b_j hopping on the sites j of a square lattice with Hamiltonian

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_j n_j (n_j - 1)$$

$$n_j \equiv b_j^\dagger b_j$$

The boson operators obey the commutation relation

$$[b_j, b_k^\dagger] = \delta_{jk}$$

We restrict attention to the sector of the Fock space with

$$\sum_j n_j = \text{integer multiple of the number of sites}$$

$$\underline{U \gg t}$$

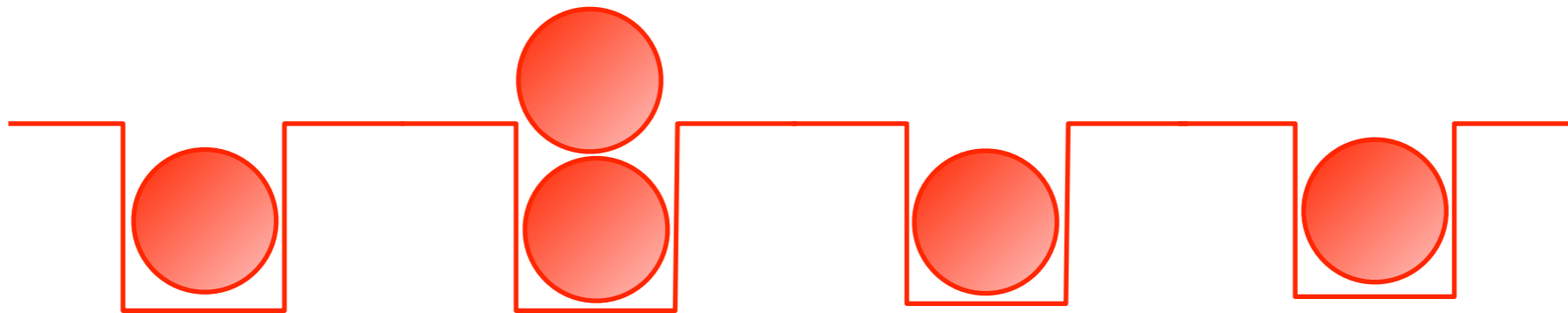


Insulator (the vacuum)
at large repulsion between bosons

$$|\text{Ground state}\rangle = \prod_i b_i^\dagger |0\rangle$$

$$\underline{U \gg t}$$

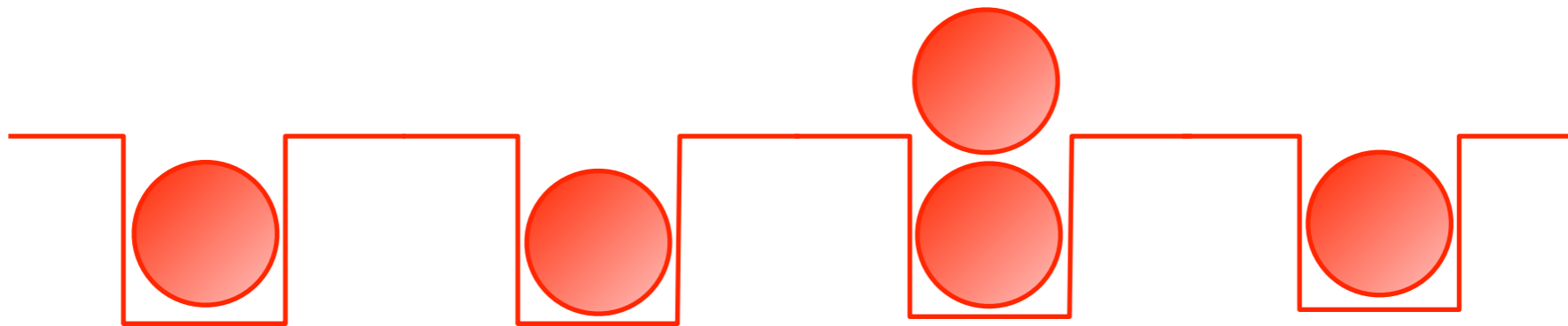
Excitations of the insulator:



Particles $\sim \psi^\dagger$

$$\underline{U \gg t}$$

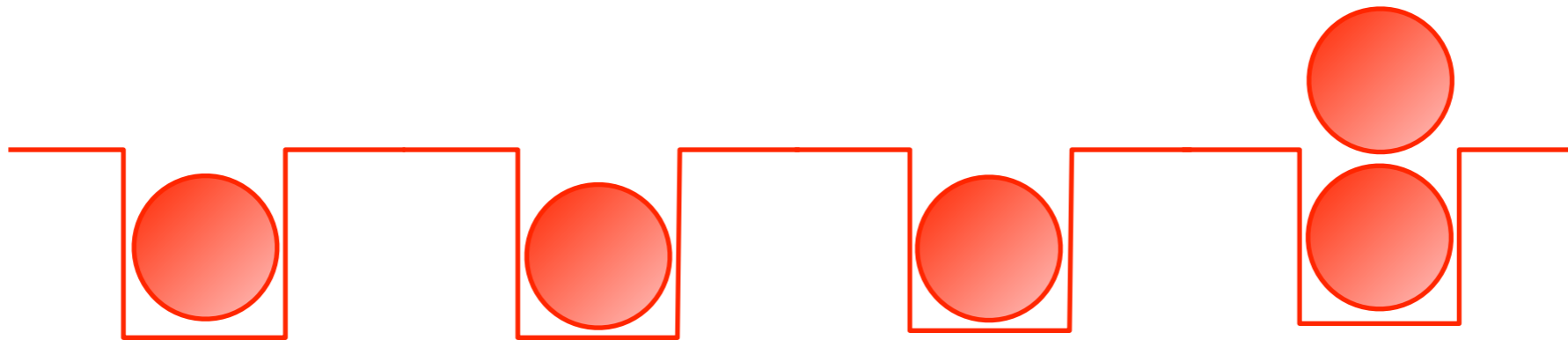
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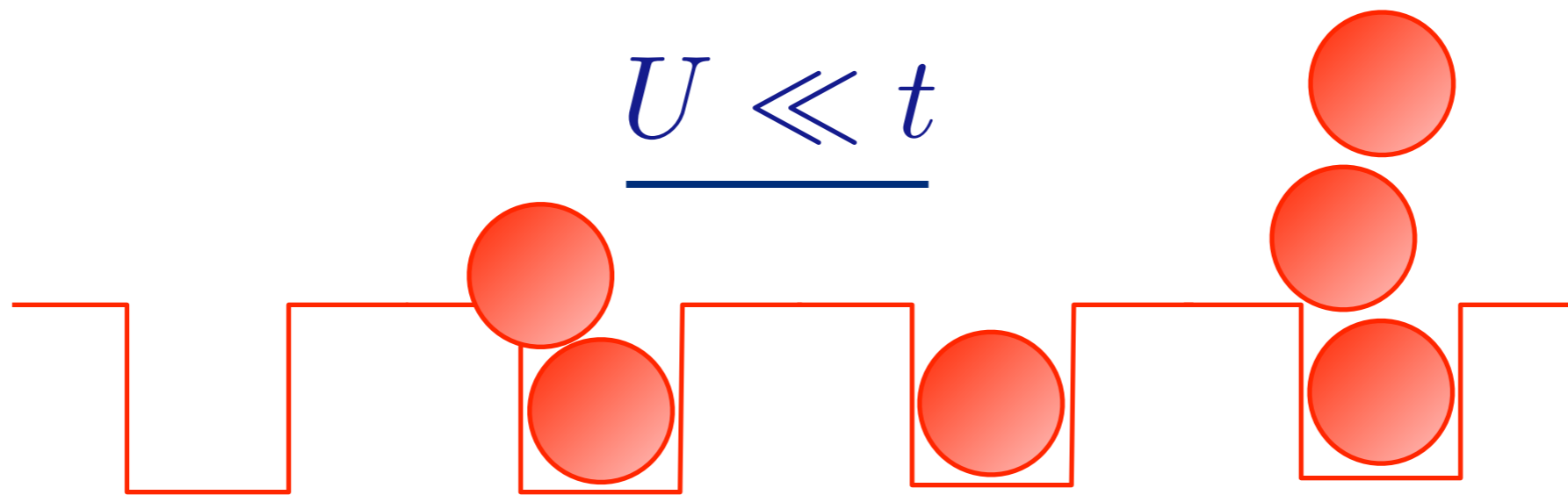
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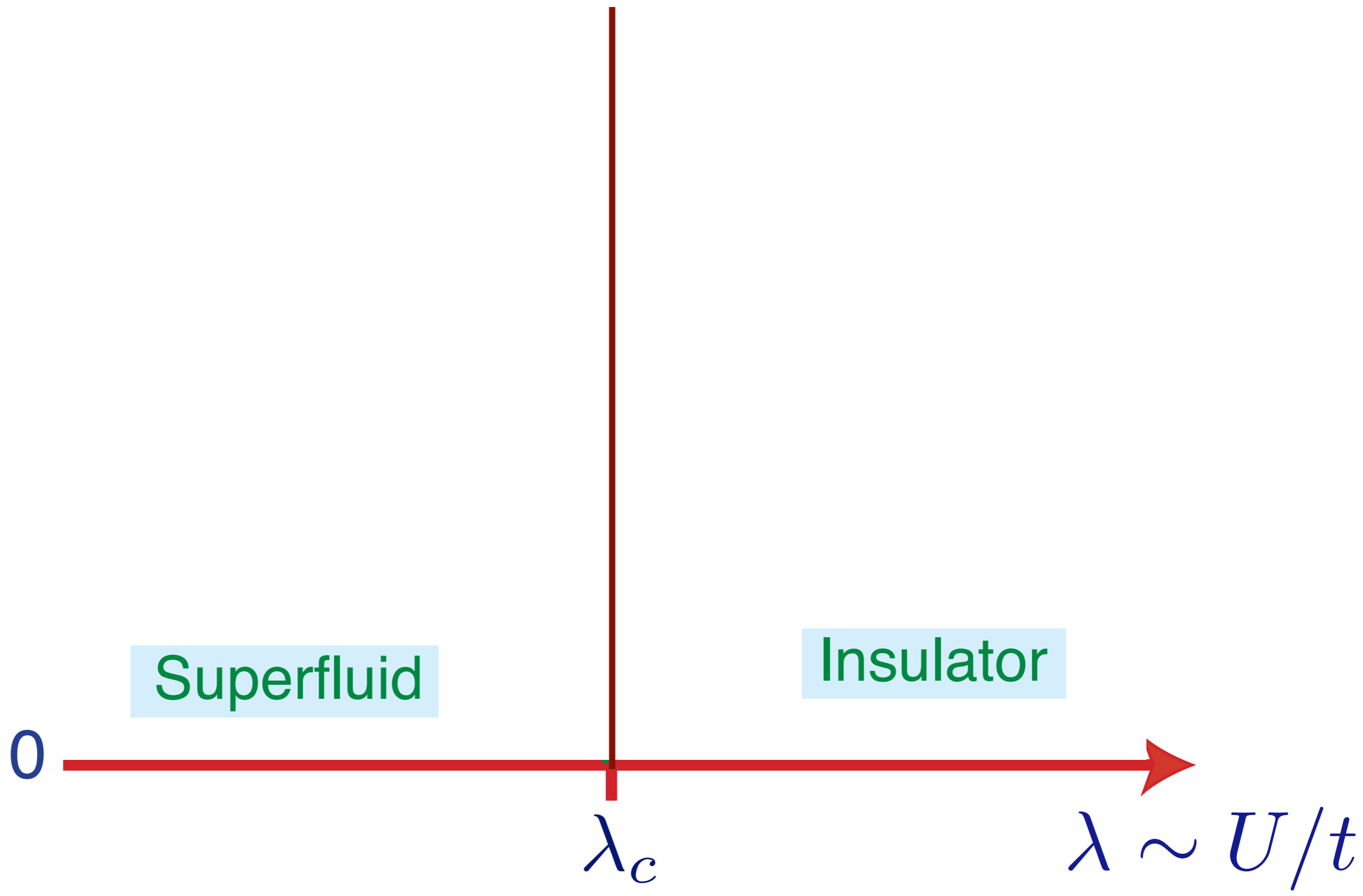
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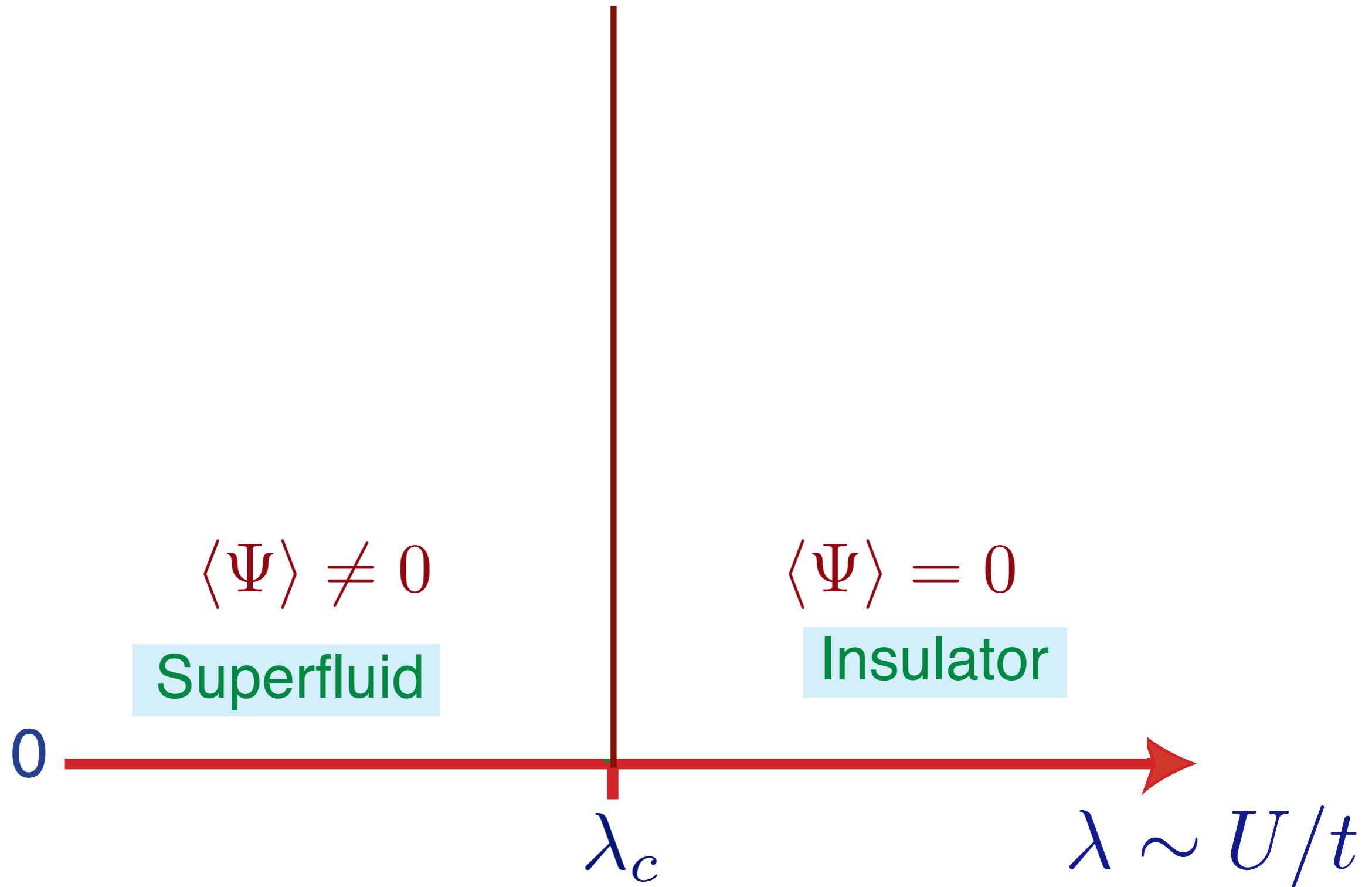


Superfluid
at small repulsion between bosons

$$|\text{Ground state}\rangle = \left[\sum_i b_i^\dagger \right]^N |0\rangle$$

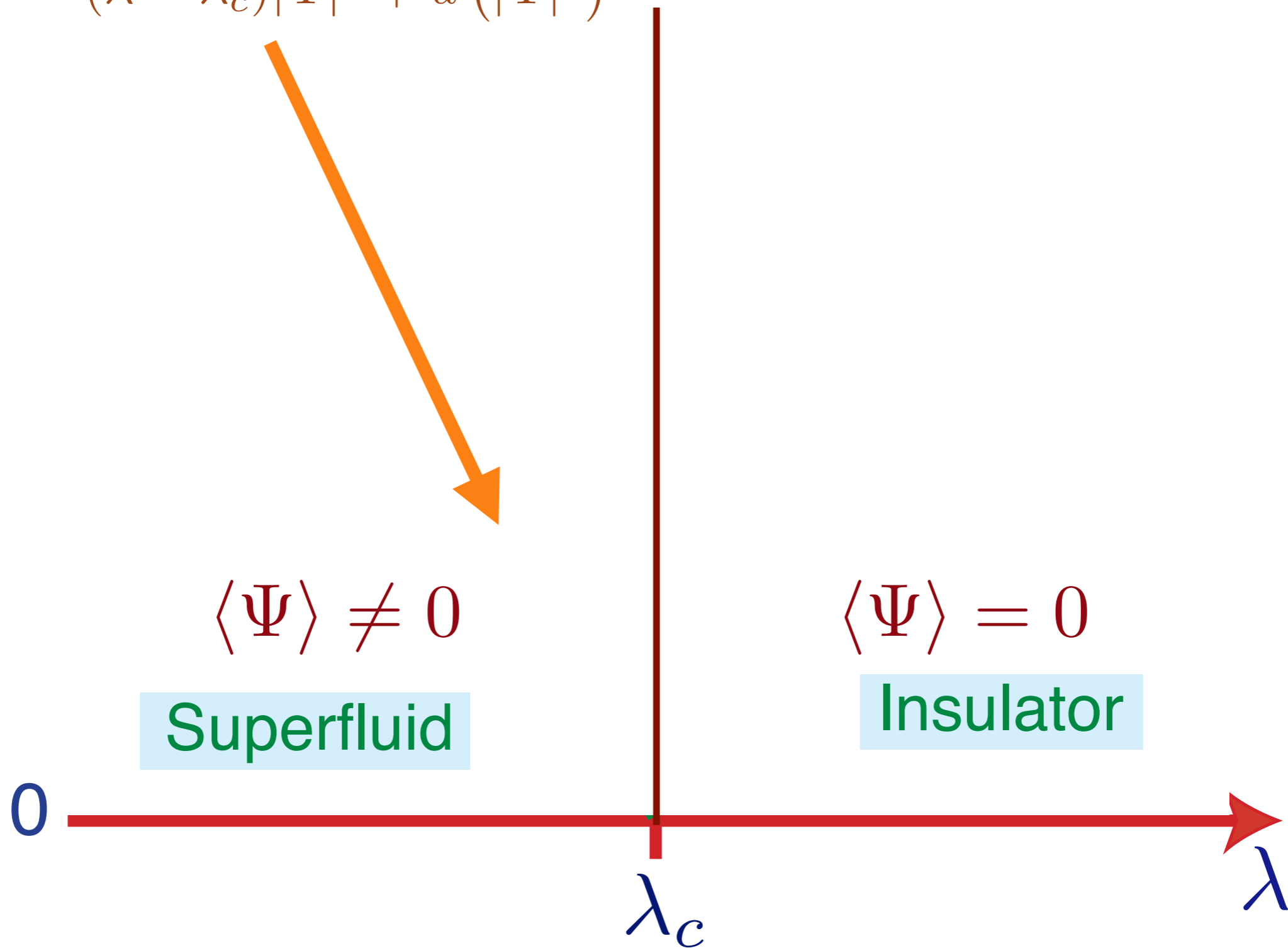


$\Psi \sim b_{k=0} \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{Z} = \int \mathcal{D}\Psi(r, \tau) \exp \left(- \int d^2r d\tau [|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi)] \right)$$

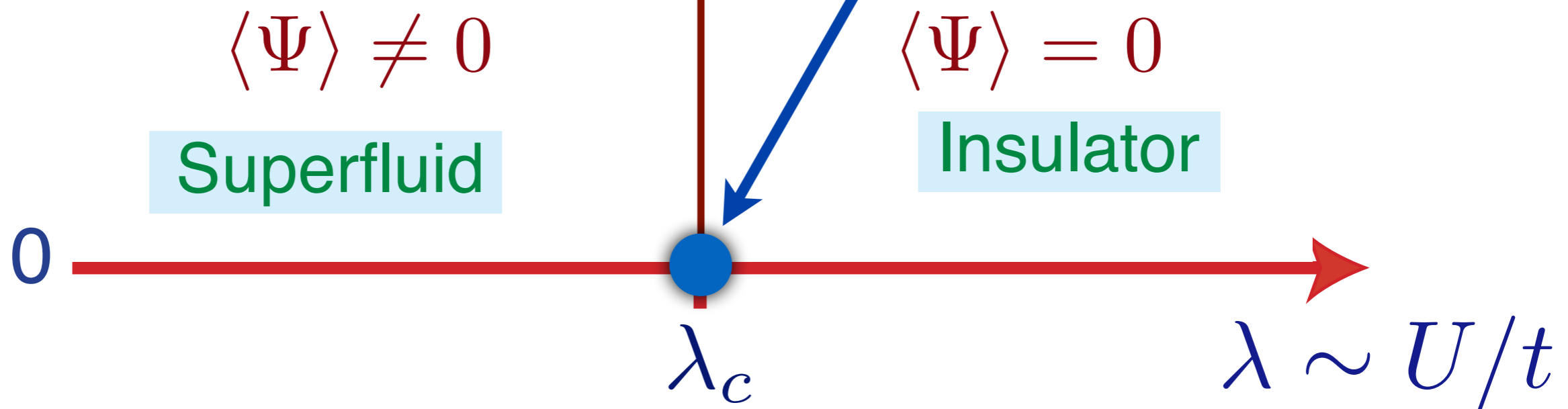
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



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A conformal field theory
in 2+1 spacetime dimensions (CFT3):
the O(2) Wilson-Fisher CFT3

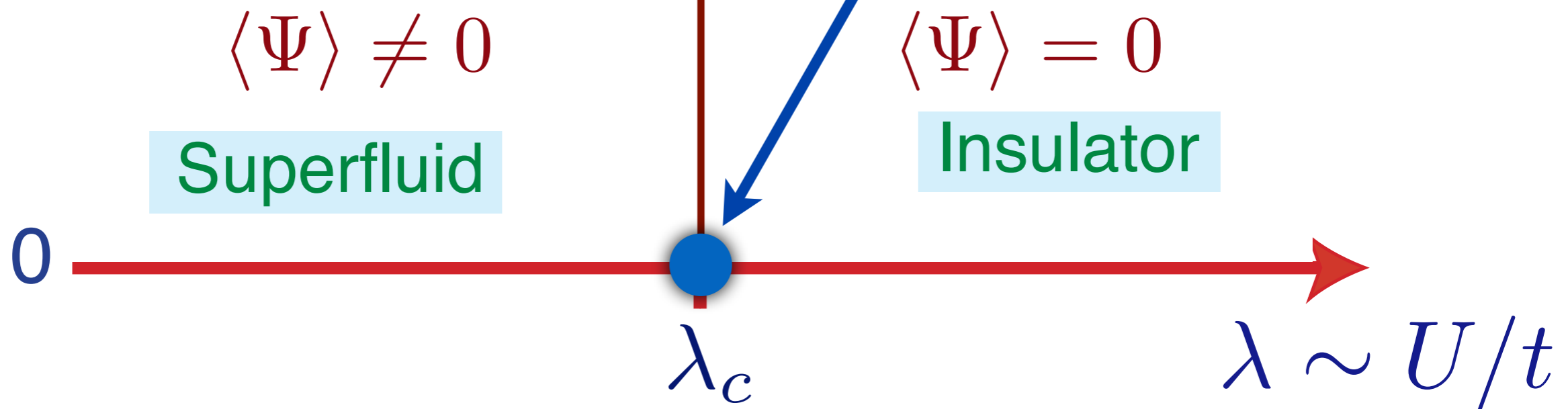


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The coupling $u \rightarrow u^*$,
the renormalization group
fixed point, for the CFT3.

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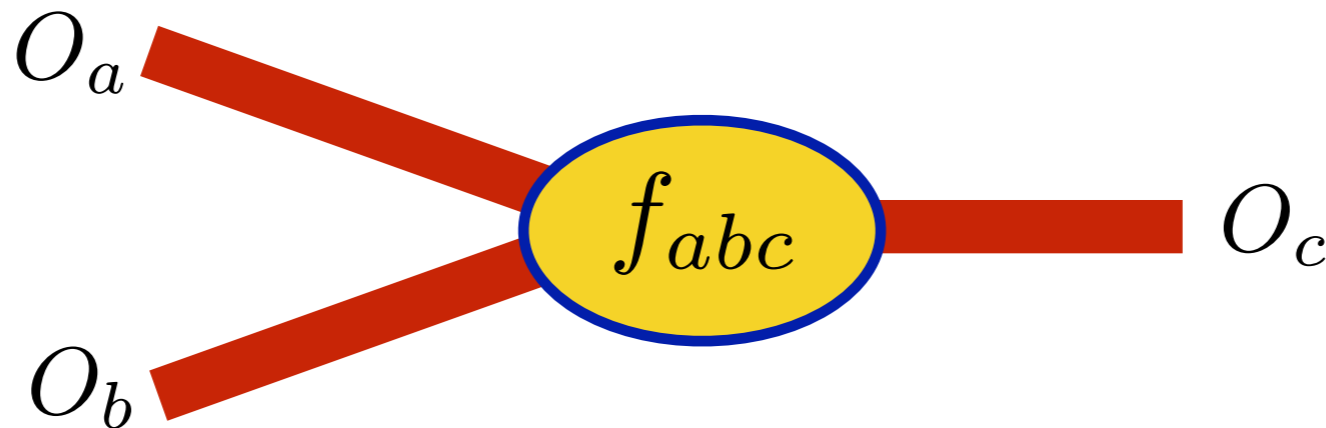
Basic characteristics of CFTs

Primary operators of CFT, $O_a(x)$, obey (at $T = 0$):

$$\langle O_a(x)O_b(0) \rangle = \frac{\delta_{ab}}{|x|^{2\Delta_a}}$$

where Δ_a is their scaling dimension. Their “interactions” are determined by the OPE (considering scalar operators only)

$$\lim_{x' \rightarrow x} \langle O_a(x')O_b(x)O_c(0) \rangle = \frac{f_{abc}}{|x|^{\Delta_a + \Delta_b + \Delta_c}}$$



The values of $\{\Delta_a, f_{abc}\}$ determine (in principle) all observable properties of the CFT, as constrained by conformal Ward identities. For the Wilson-Fisher CFT₃, systematic methods exist to compute (in principle) all the $\{\Delta_a, f_{abc}\}$, and we will assume this data is *known*. This knowledge will be taken as an *input* to the computation of the finite T dynamics

Basic characteristics of CFTs

Important sets of operators are the energy-momentum tensor $T_{\mu\nu}$, and conserved currents of continuous symmetries J_μ . For CFTs, the $T_{\mu\nu}$ obey the Virasoro algebra, while the J_μ obey the Kac-Moody algebra: in particular ($z = x + i\tau$)

$$\langle J(z)J(0) \rangle = \frac{k}{z^2}$$

where k is the (integer) central charge.

Basic characteristics of CFTs

Important sets of operators are the energy-momentum tensor $T_{\mu\nu}$, and conserved currents of continuous symmetries J_μ . For CFT2s, the $T_{\mu\nu}$ obey the Virasoro algebra, while the J_μ obey the Kac-Moody algebra: in particular ($z = x + i\tau$)

$$\langle J(z)J(0) \rangle = \frac{k}{z^2}$$

where k is the (integer) central charge.

CFT3s are much more complicated. In momentum space we have

$$\langle J_\mu(p)J_\nu(0) \rangle = -\sigma_\infty |p| \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

where σ_∞ is (almost certainly) an irrational number. From the Kubo formula, we can show that σ_∞ is equal to the *conductivity*, $\sigma(\omega)$, (in units of e^2/\hbar) of the CFT3. So the Wilson-Fisher CFT3 (and also the Bose-Hubbard model) has a universal, frequency-independent, conductivity.

Basic characteristics of CFTs

We will need higher-order terms in the OPE of 2 currents in CFT3s.
This has the general form

$$\lim_{|\omega| \gg p} J_x(\omega) J_x(-\omega + \mathbf{p}) = -|\omega| \sigma_\infty \delta^{(3)}(\mathbf{p}) - \frac{\mathcal{C}}{|\omega|^{\Delta-1}} \mathcal{O}(\mathbf{p}) \\ + \frac{\mathcal{C}_T}{\omega^2} \left[T_{xx}(\mathbf{p}) - T_{yy}(\mathbf{p}) - 12\gamma(T_{xx}(\mathbf{p}) + T_{yy}(\mathbf{p})) \right] + \dots$$

where \mathcal{O} is the scalar operator of dimension $\Delta = 3 - 1/\nu$ (it tunes away from the critical point), and \mathcal{C} , \mathcal{C}_T , γ are OPE coefficients. There is a conjectured exact bound $|\gamma| \leq 1/12$.

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1. A CFT in 2+1 dimensions
2. Boltzmann dynamics
3. Dynamics from the operator product expansion (OPE) for $\hbar\omega \gg k_B T$
4. Holography

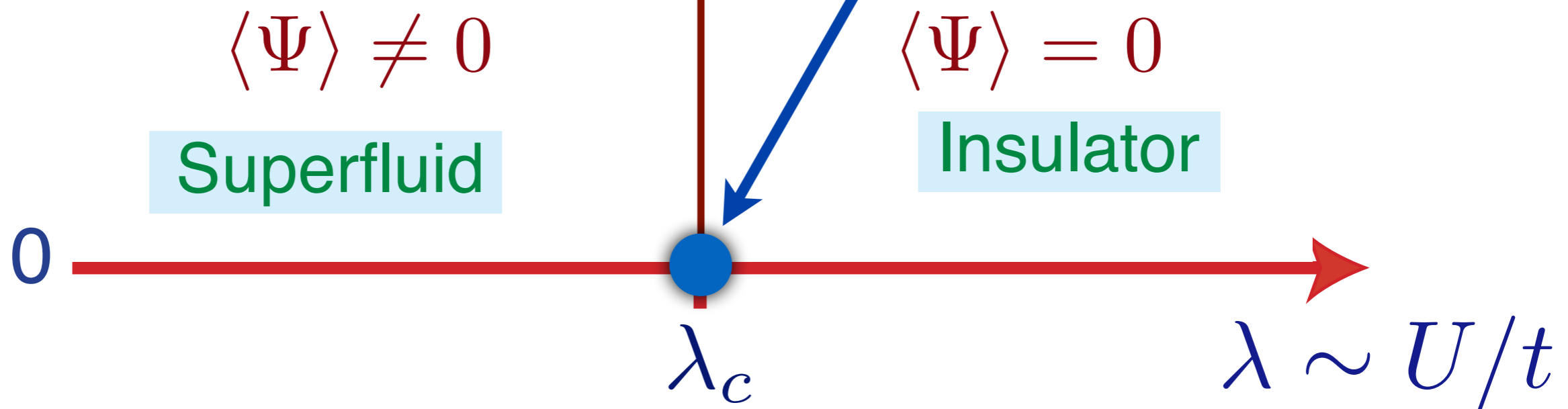
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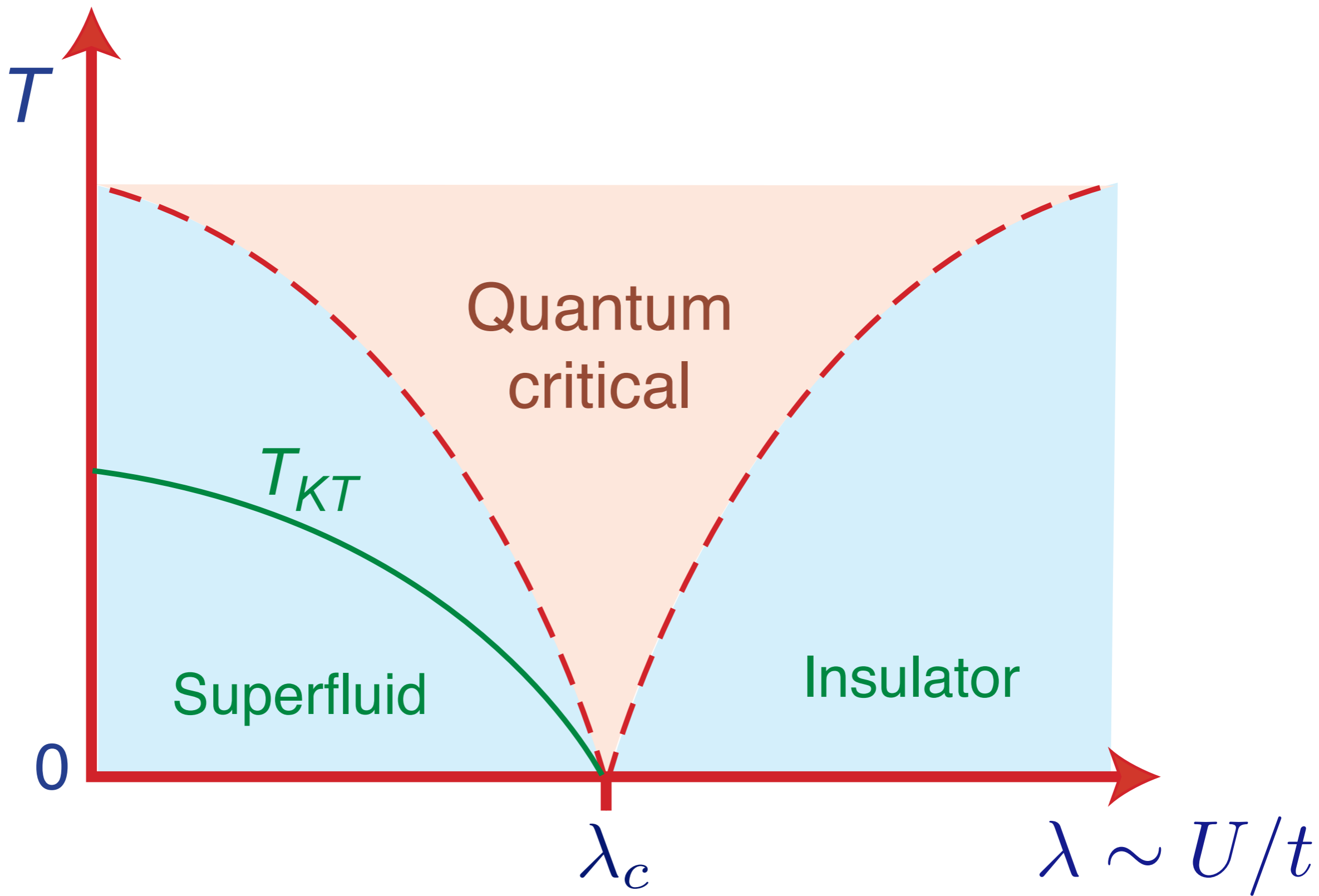
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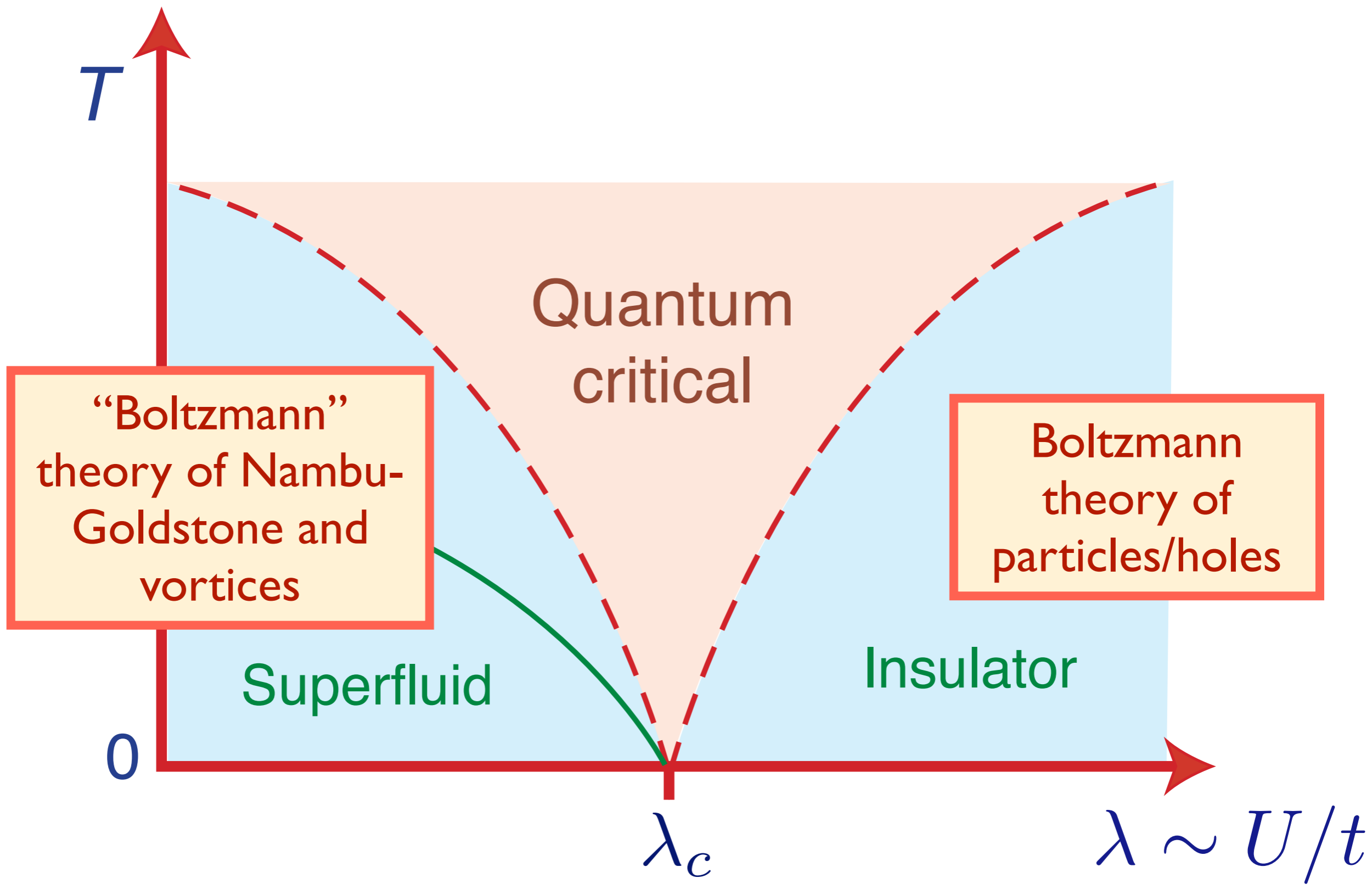
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T

0

Quantum critical

“Boltzmann” theory of Nambu-Goldstone and vortices

Boltzmann theory of particles/holes

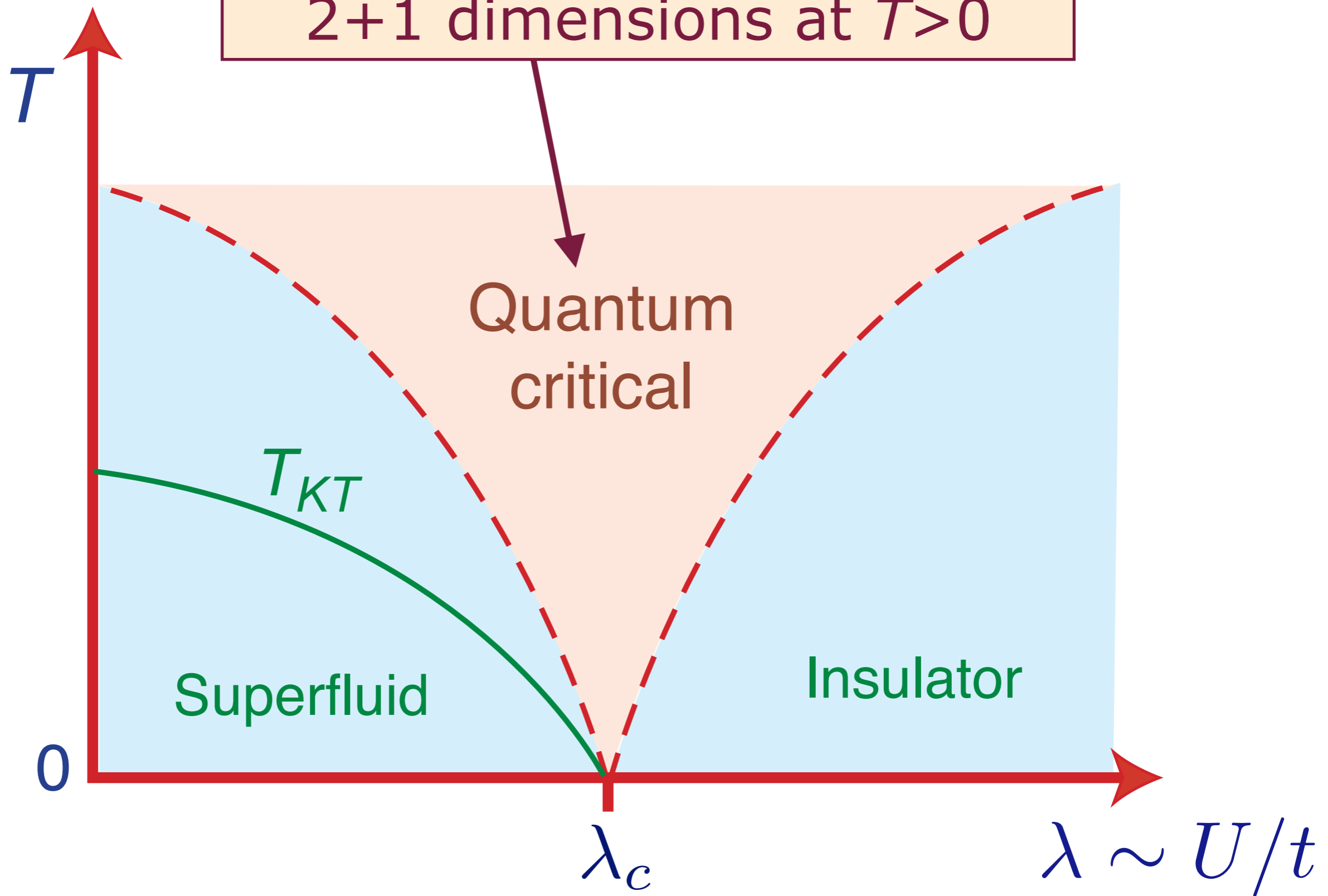
Superfluid

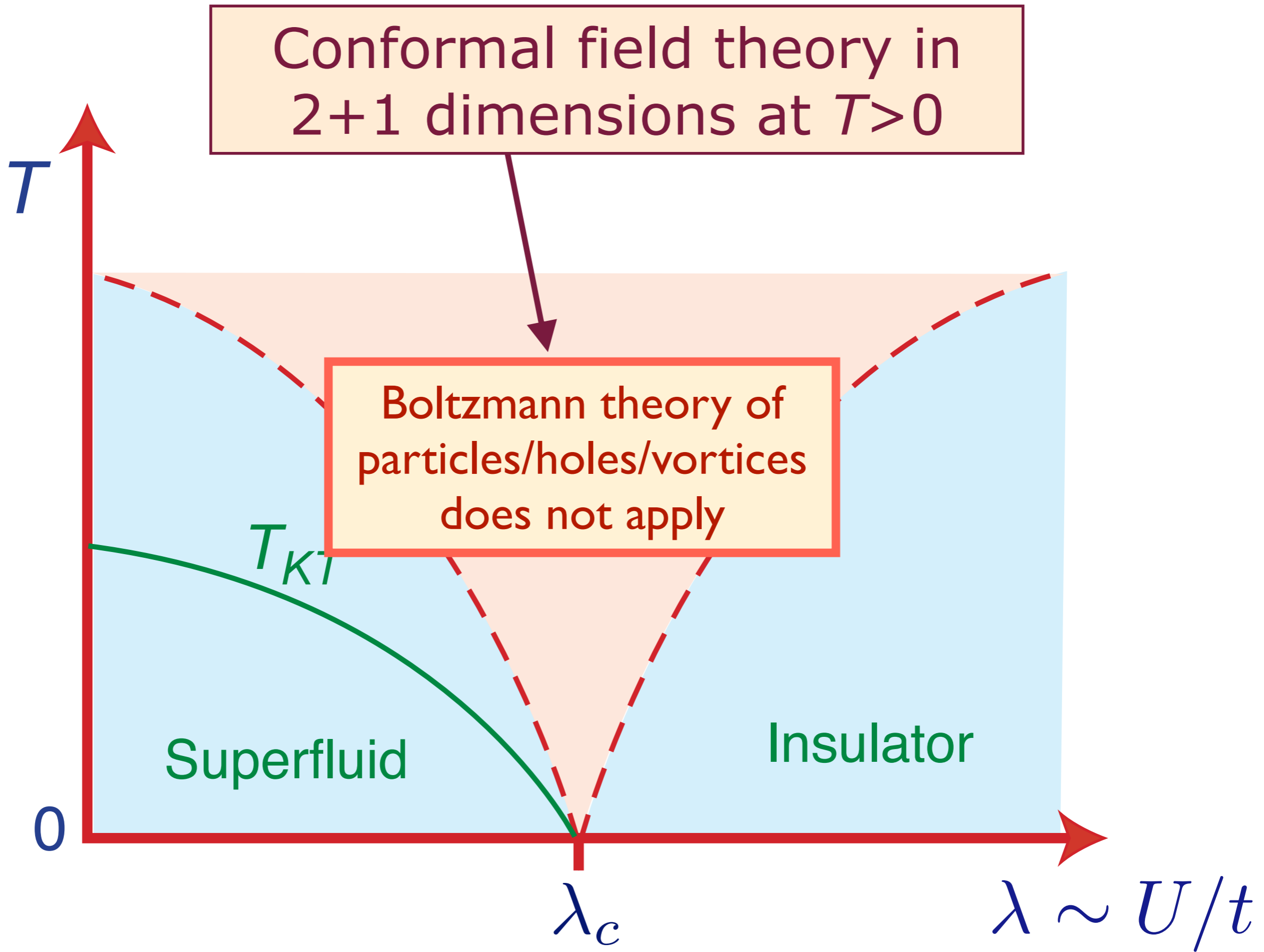
Insulator

λ_c

$\lambda \sim U/t$

Conformal field theory in
2+1 dimensions at $T > 0$





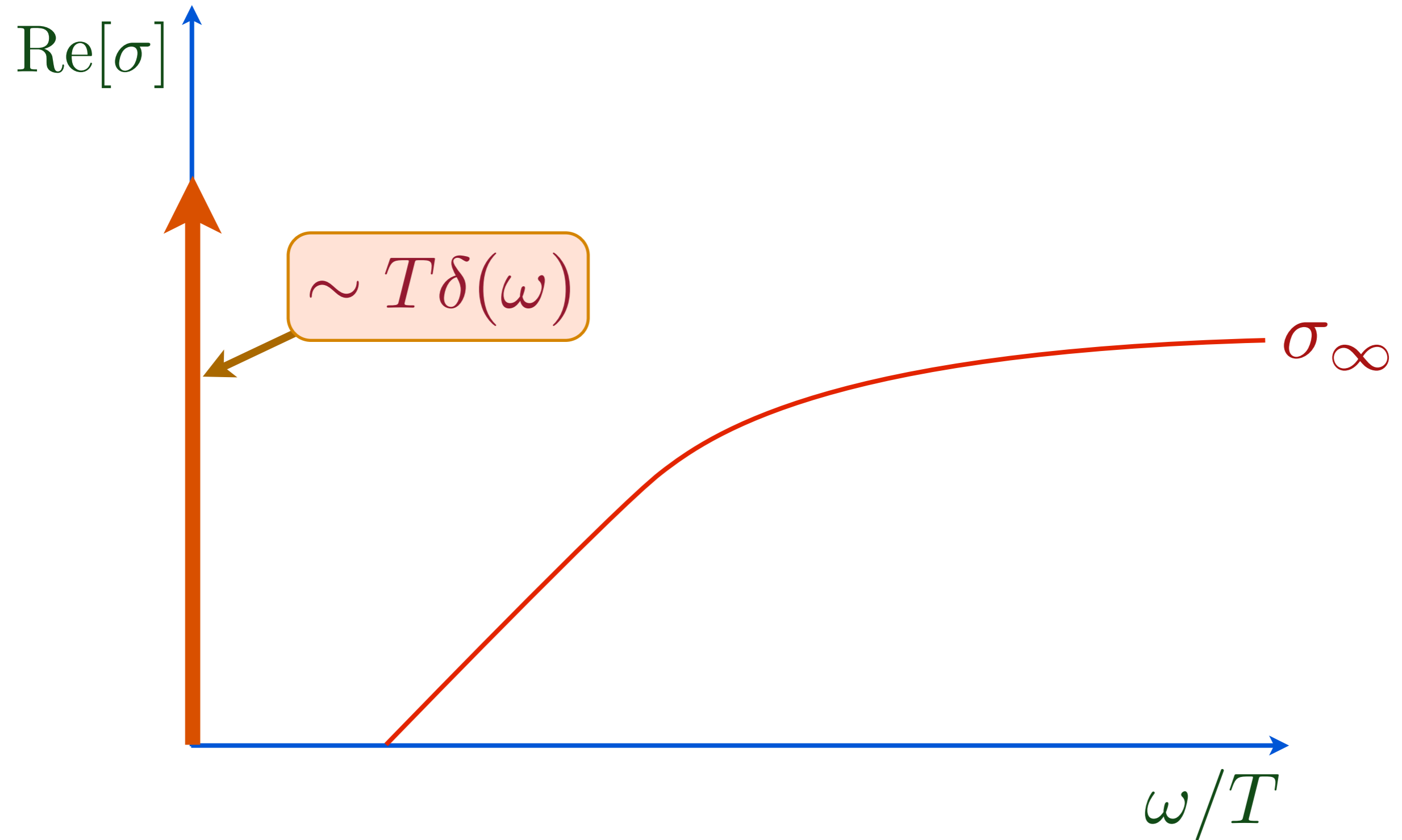
Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)

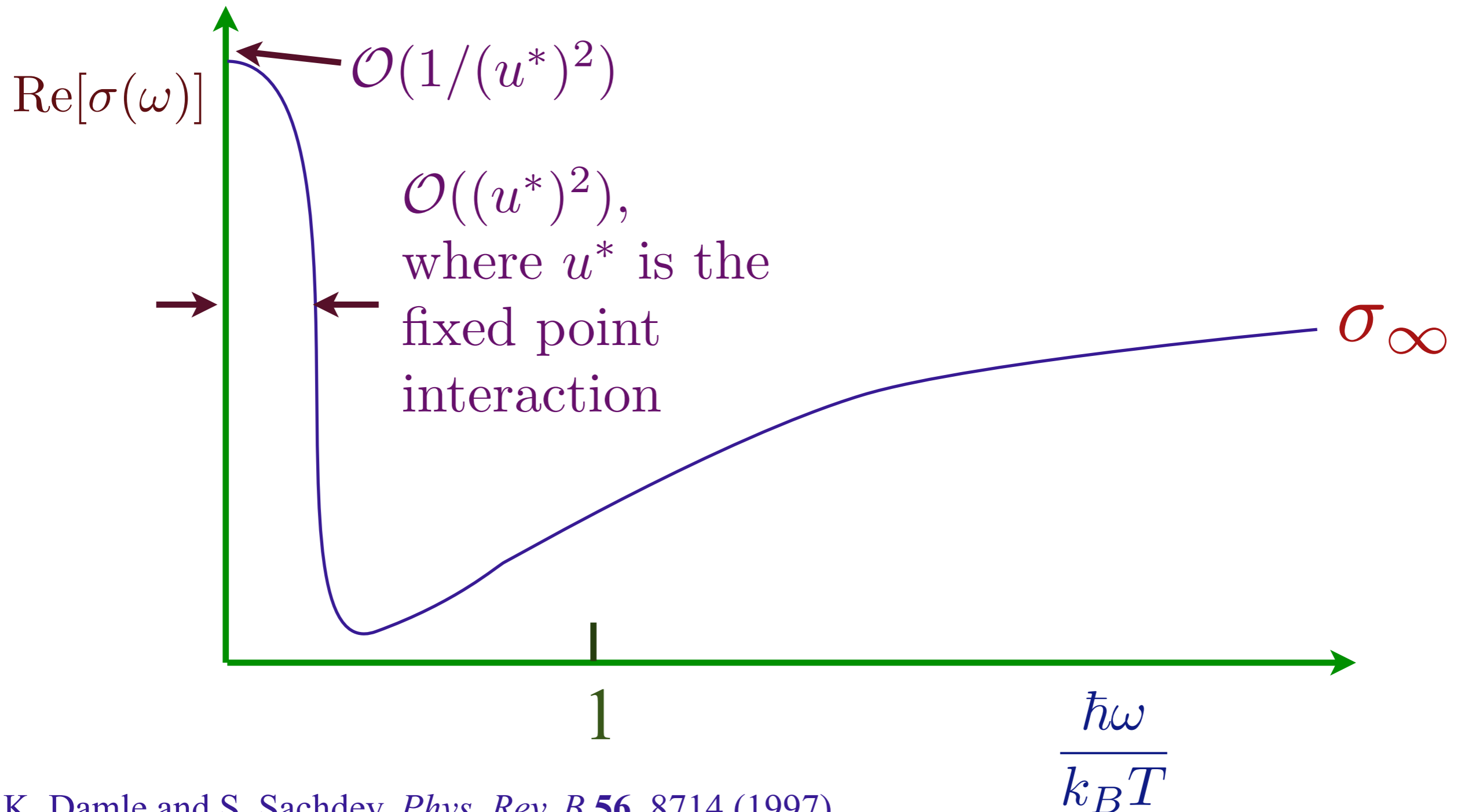
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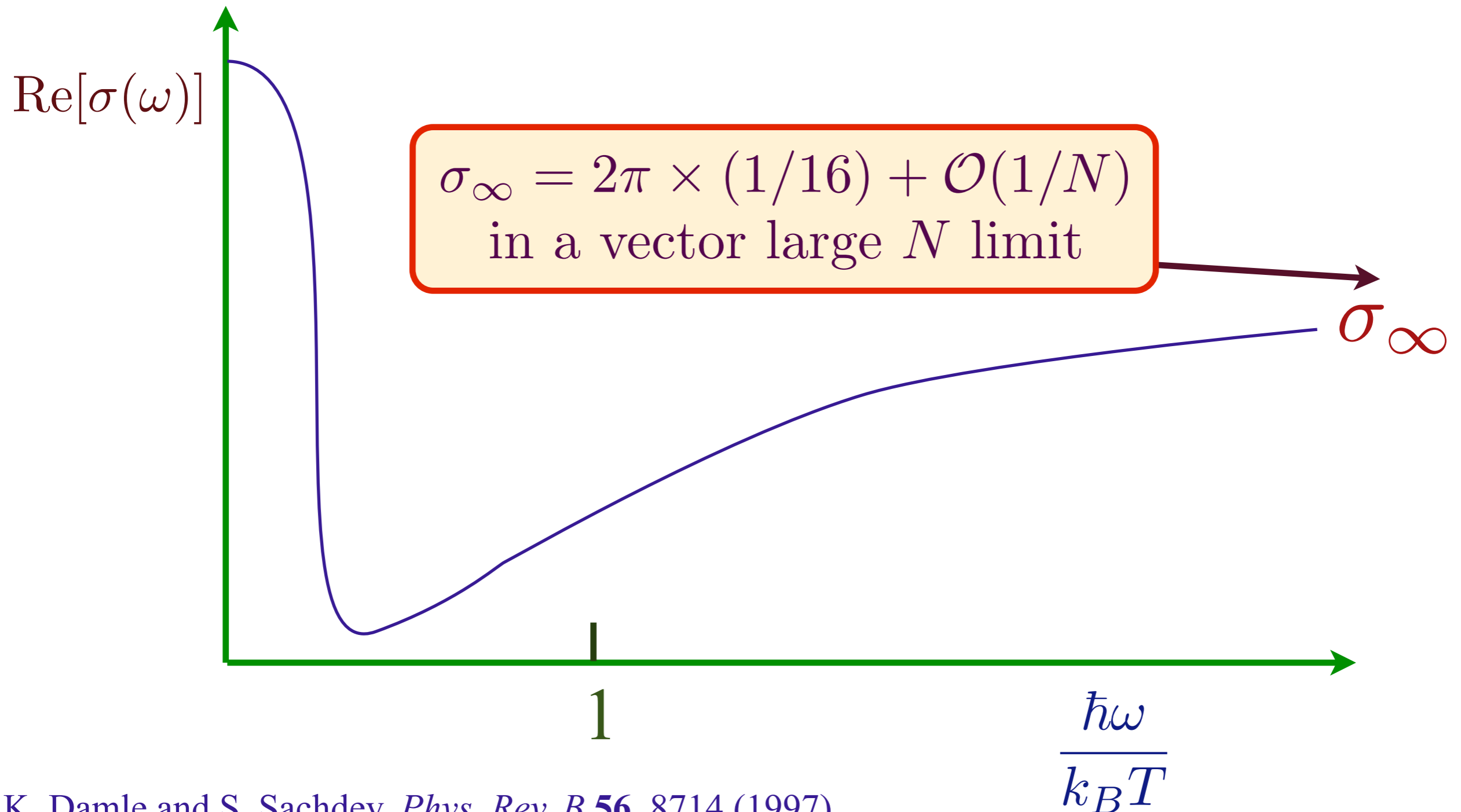
Quasiparticle view of quantum criticality (Boltzmann equation):
Electrical transport for a free CFT3



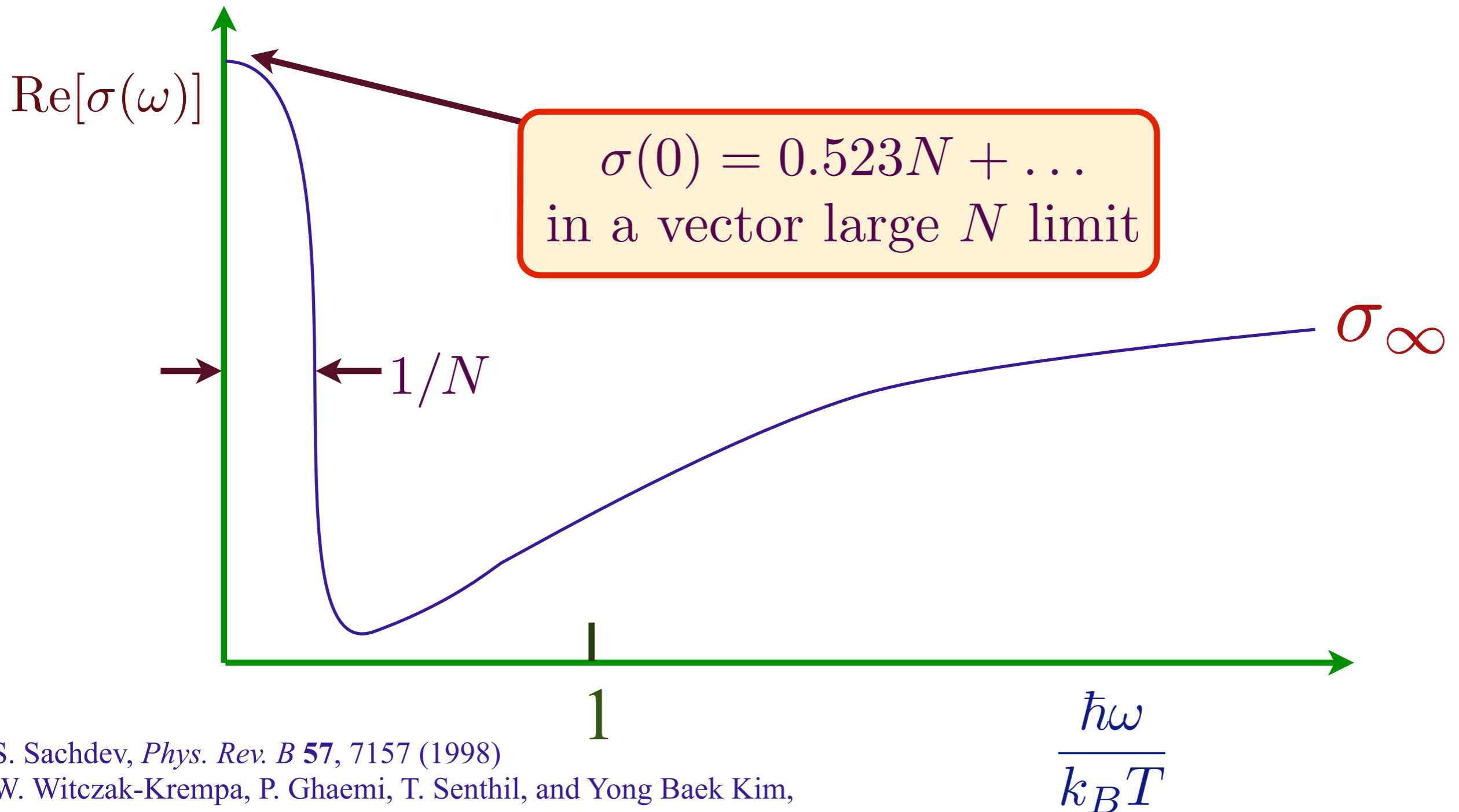
Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3



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S. Sachdev, *Phys. Rev. B* **57**, 7157 (1998)

W. Witczak-Krempa, P. Ghaemi, T. Senthil, and Yong Baek Kim,
Phys. Rev. B **86**, 24102 (2012)

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Dynamics without quasiparticles

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT

Basic characteristics of CFTs

We will need higher-order terms in the OPE of 2 currents in CFT3s.
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For $T > 0$, compute the conductivity by taking thermal average of the OPE.

Basic characteristics of CFTs

The thermal average of the OPE yields for $\omega \gg T$

$$\sigma(\omega) = \sigma_{\infty} + b_1 \left(\frac{T}{\omega}\right)^{3-1/\nu} + b_2 \left(\frac{T}{\omega}\right)^3 + \dots$$

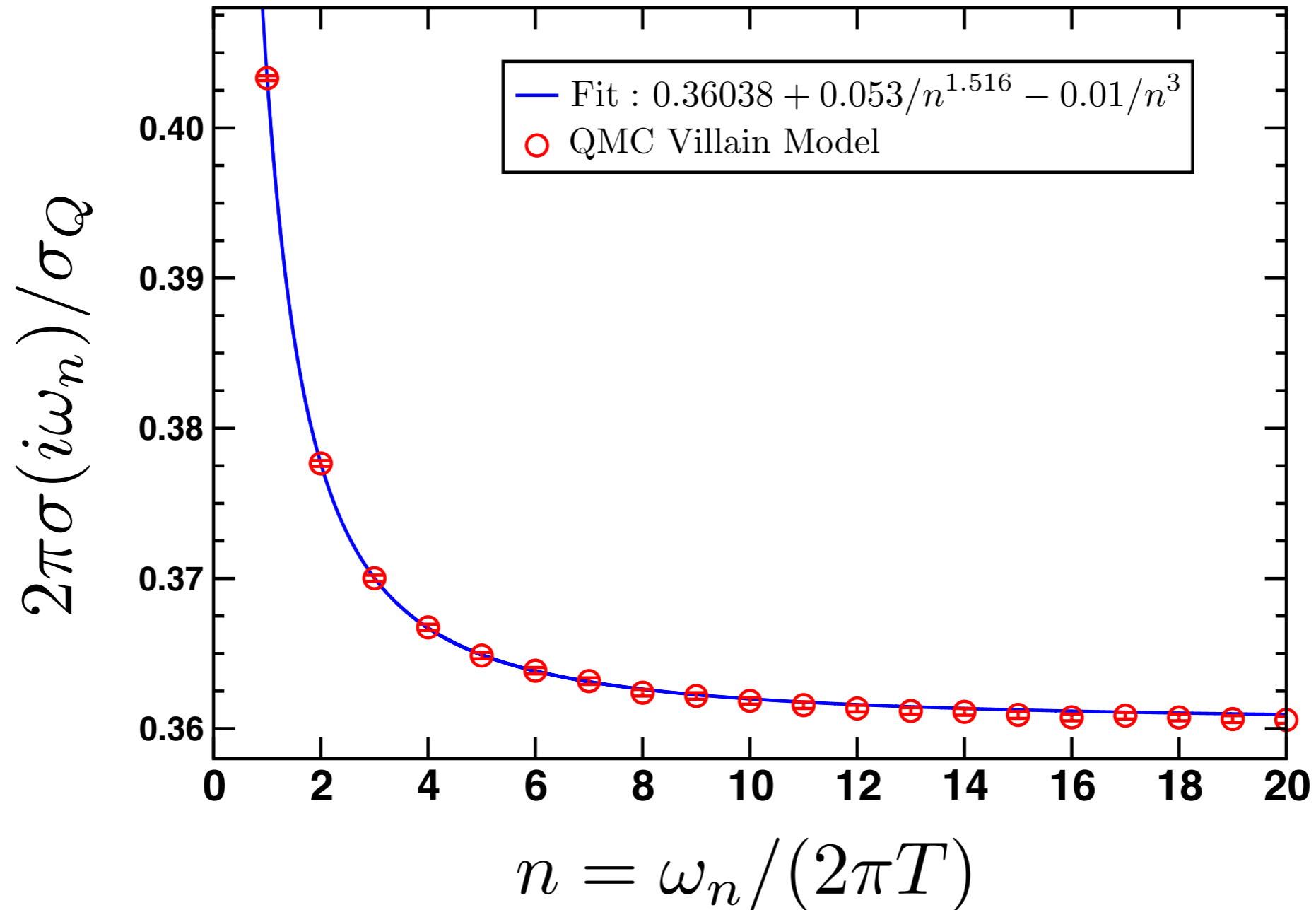
where $b_{1,2}$ are universal numbers.

The bare OPE expansion yields no information for $\omega \sim T$ or smaller.

For the $O(2)$ Wilson-Fisher CFT₃, $\nu \approx 0.6717(1)$.

For $T > 0$, compute the conductivity by taking thermal average of the OPE.

Quantum Monte Carlo for lattice model of integer currents (Villain model)



Excellent agreement with OPE

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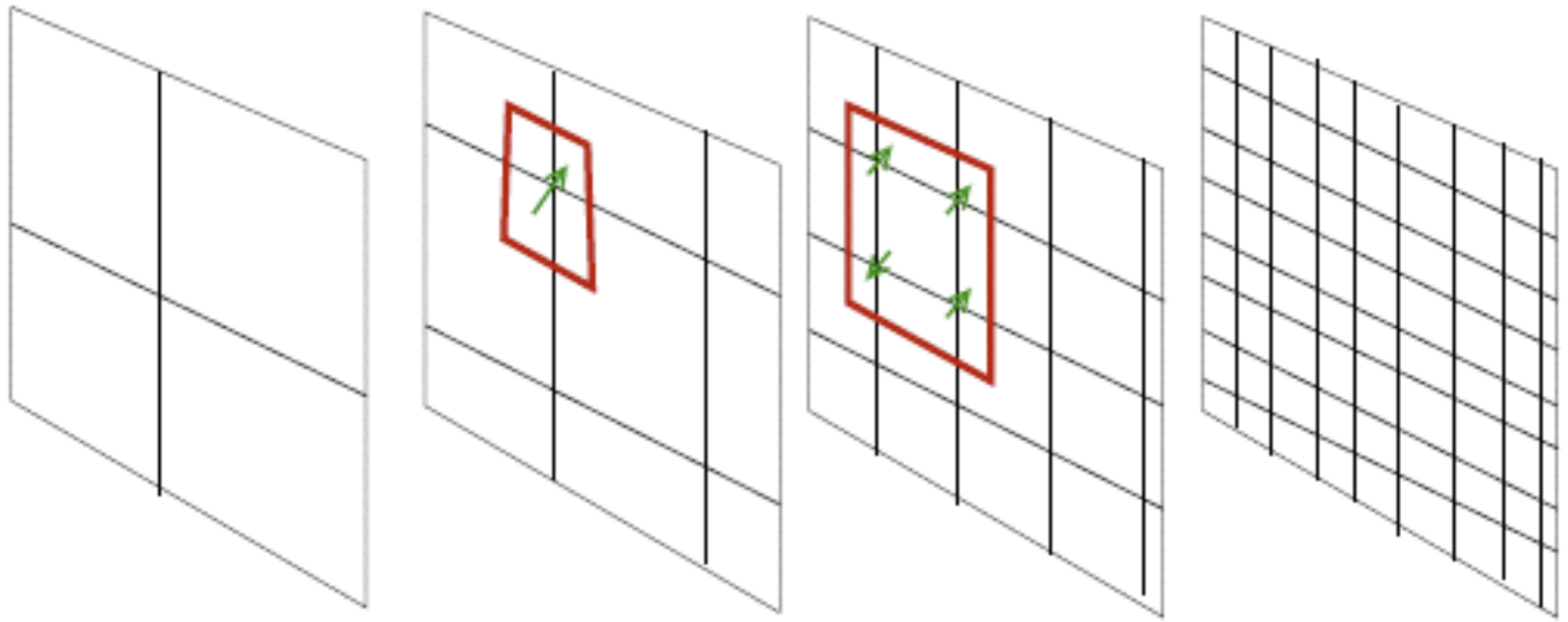
Dynamics without quasiparticles

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT
- Relate OPE co-efficients to couplings of an effective gravitational theory on AdS
- Non-zero T dynamics of CFT maps to dynamics of a “horizon” in (Einstein’s) gravitational theory

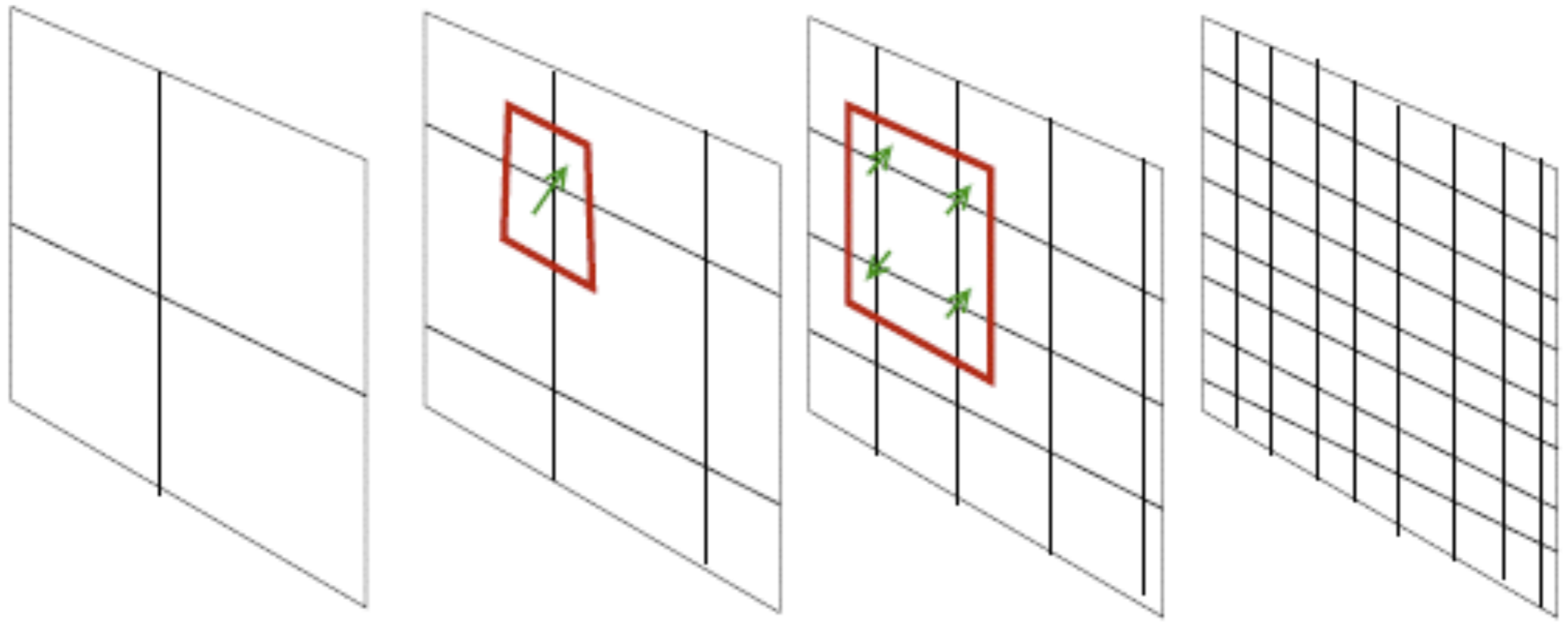
Field theories in $d + 1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

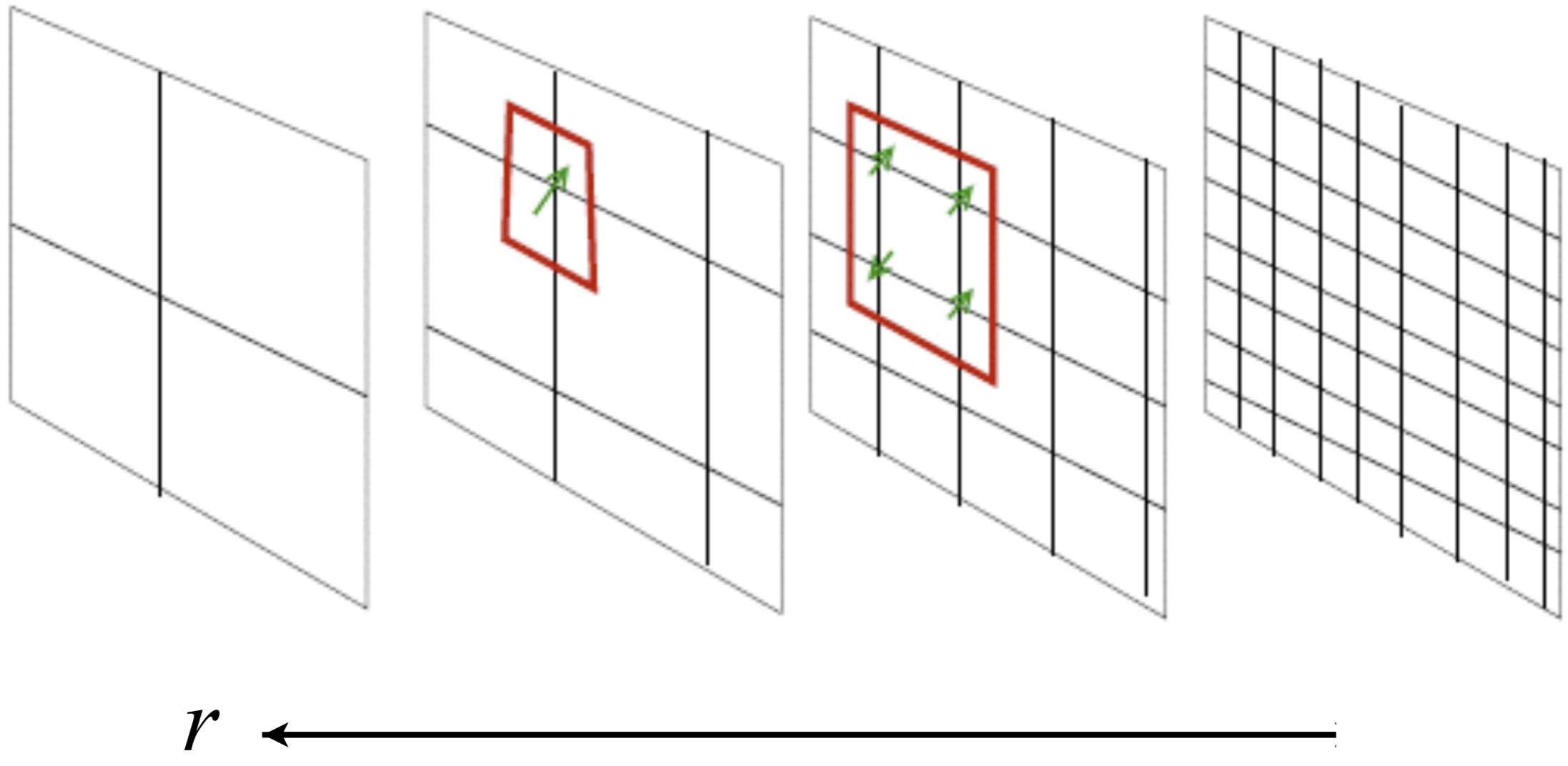
where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u .



→ u



r ←



Key idea: \Rightarrow Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformation ($i = 1 \dots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

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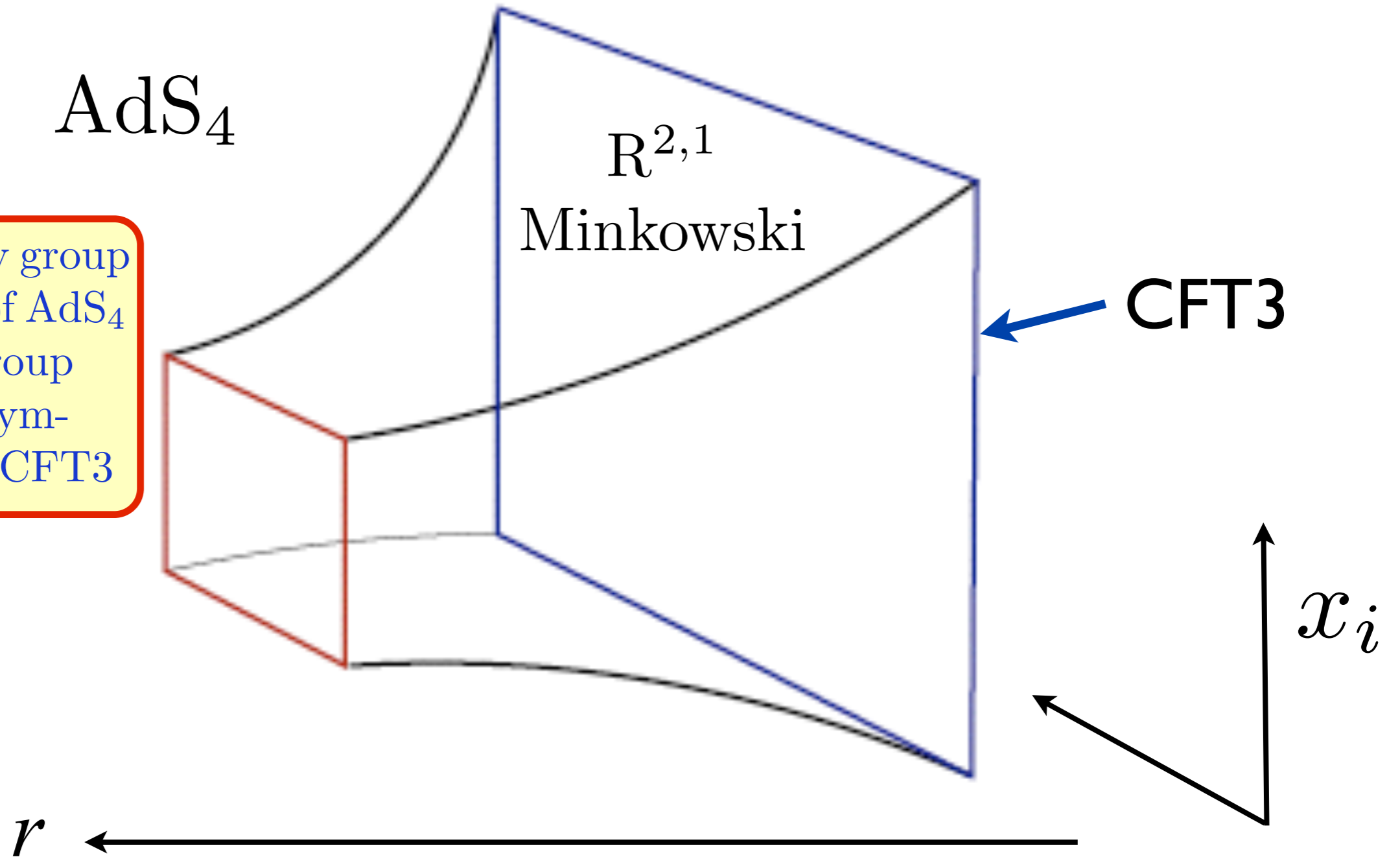
This gives the unique metric

$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \rightarrow \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2} .

AdS/CFT correspondence at zero temperature

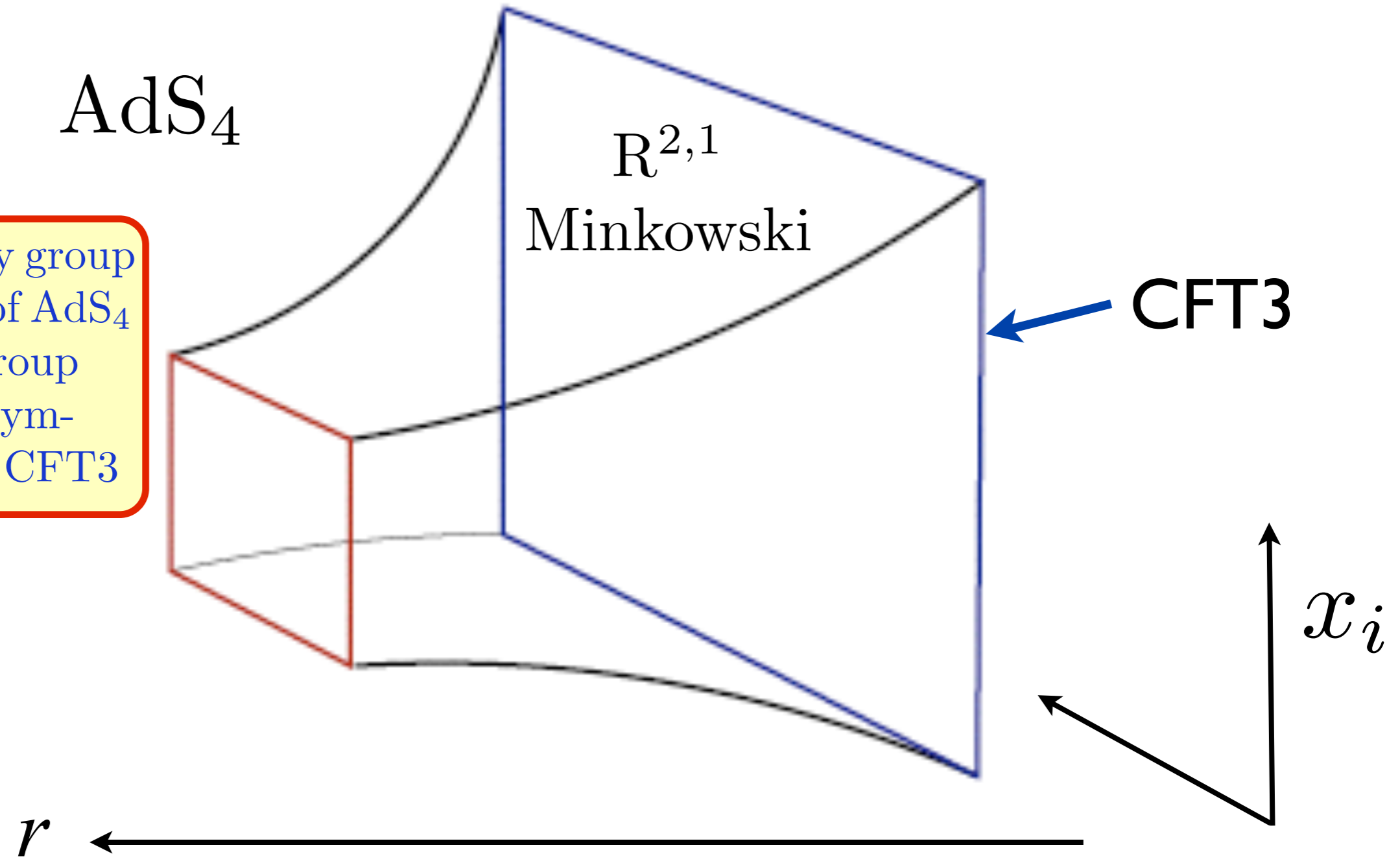
The symmetry group of isometries of AdS_4 maps to the group of conformal symmetries of the CFT3



$$ds^2 = \left(\frac{L}{r}\right)^2 [dr^2 - dt^2 + dx^2 + dy^2]$$

AdS/CFT correspondence at zero temperature

The symmetry group of isometries of AdS_4 maps to the group of conformal symmetries of the CFT3



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence at zero temperature

Consider a CFT in D space-time dimensions with primary operators $O_a(\mathbf{x})$ with scaling dimension Δ_a . This is presumed to be equivalent to a dual gravity theory on AdS_{D+1} with action $\mathcal{S}_{\text{bulk}}$. The bulk theory has fields $\phi_a(\mathbf{x}, r)$ corresponding to each primary operator. The CFT and the bulk theory are related by the GKPW ansatz

$$\int \mathcal{D}\phi_a \exp(-\mathcal{S}_{\text{bulk}}) \Big|_{\text{bdy}} = \left\langle \exp \left(\int d^D x \phi_{a0}(\mathbf{x}) O_a(\mathbf{x}) \right) \right\rangle_{\text{CFT}}$$

where the boundary condition is

$$\lim_{r \rightarrow 0} \phi_a(\mathbf{x}, r) = r^{D-\Delta} \phi_{a0}(\mathbf{x}).$$

AdS/CFT correspondence at zero temperature

For every primary operator $O(\mathbf{x})$ in the CFT, there is a corresponding field $\phi(\mathbf{x}, r)$ in the bulk (gravitational) theory. For a scalar operator $O(\mathbf{x})$ of dimension Δ , the correlators of the boundary and bulk theories are related by

$$\langle O(\mathbf{x}_1) \dots O(\mathbf{x}_n) \rangle_{\text{CFT}} = Z^n \lim_{r \rightarrow 0} r_1^{-\Delta} \dots r_n^{-\Delta} \langle \phi(\mathbf{x}_1, r_1) \dots \phi(\mathbf{x}_n, r_n) \rangle_{\text{bulk}}$$

where the “wave function renormalization” factor $Z = (2\Delta - D)$.

AdS/CFT correspondence at zero temperature

For a U(1) conserved current J_μ of the CFT, the corresponding bulk operator is a U(1) *gauge* field A_μ . With a Maxwell action for the gauge field

$$\mathcal{S}_M = \frac{1}{4g_M^2} \int d^{D+1}x \sqrt{g} F_{ab} F^{ab}$$

we have the bulk-boundary correspondence

$$\langle J_\mu(\mathbf{x}_1) \dots J_\nu(\mathbf{x}_n) \rangle_{\text{CFT}} = (Z g_M^{-2})^n \lim_{r \rightarrow 0} r_1^{2-D} \dots r_n^{2-D} \langle A_\mu(\mathbf{x}_1, r_1) \dots A_\nu(\mathbf{x}_n, r_n) \rangle_{\text{bulk}}$$

with $Z = D - 2$.

AdS/CFT correspondence at zero temperature

A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the change in metric of the bulk theory. We write $\delta g_{\mu\nu} = (L^2/r^2)\chi_{\mu\nu}$, and then the bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(\mathbf{x}_1) \dots T_{\rho\sigma}(\mathbf{x}_n) \rangle_{\text{CFT}} = \left(\frac{ZL^2}{\kappa^2} \right)^n \lim_{r \rightarrow 0} r_1^{-D} \dots r_n^{-D} \langle \chi_{\mu\nu}(\mathbf{x}_1, r_1) \dots \chi_{\rho\sigma}(\mathbf{x}_n, r_n) \rangle_{\text{bulk}},$$

with $Z = D$.

AdS/CFT correspondence at zero temperature

So the minimal bulk theory for a CFT with a conserved U(1) current is the Einstein-Maxwell theory with a cosmological constant

$$\mathcal{S} = \frac{1}{4g_M^2} \int d^4x \sqrt{g} F_{ab} F^{ab} + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right].$$

This action is characterized by two dimensionless parameters: g_M and L^2/κ^2 , which are related to the conductivity σ_∞ and the central charge of the CFT

AdS/CFT correspondence at zero temperature

To fully match the OPE of the current operators, we need an Einstein-Maxwell-Weyl-scalar theory

$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} [1 + \alpha \varphi(x)] F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) + g^{ab} \partial_a \varphi \partial_b \varphi + m^2 \varphi^2 \right],$$

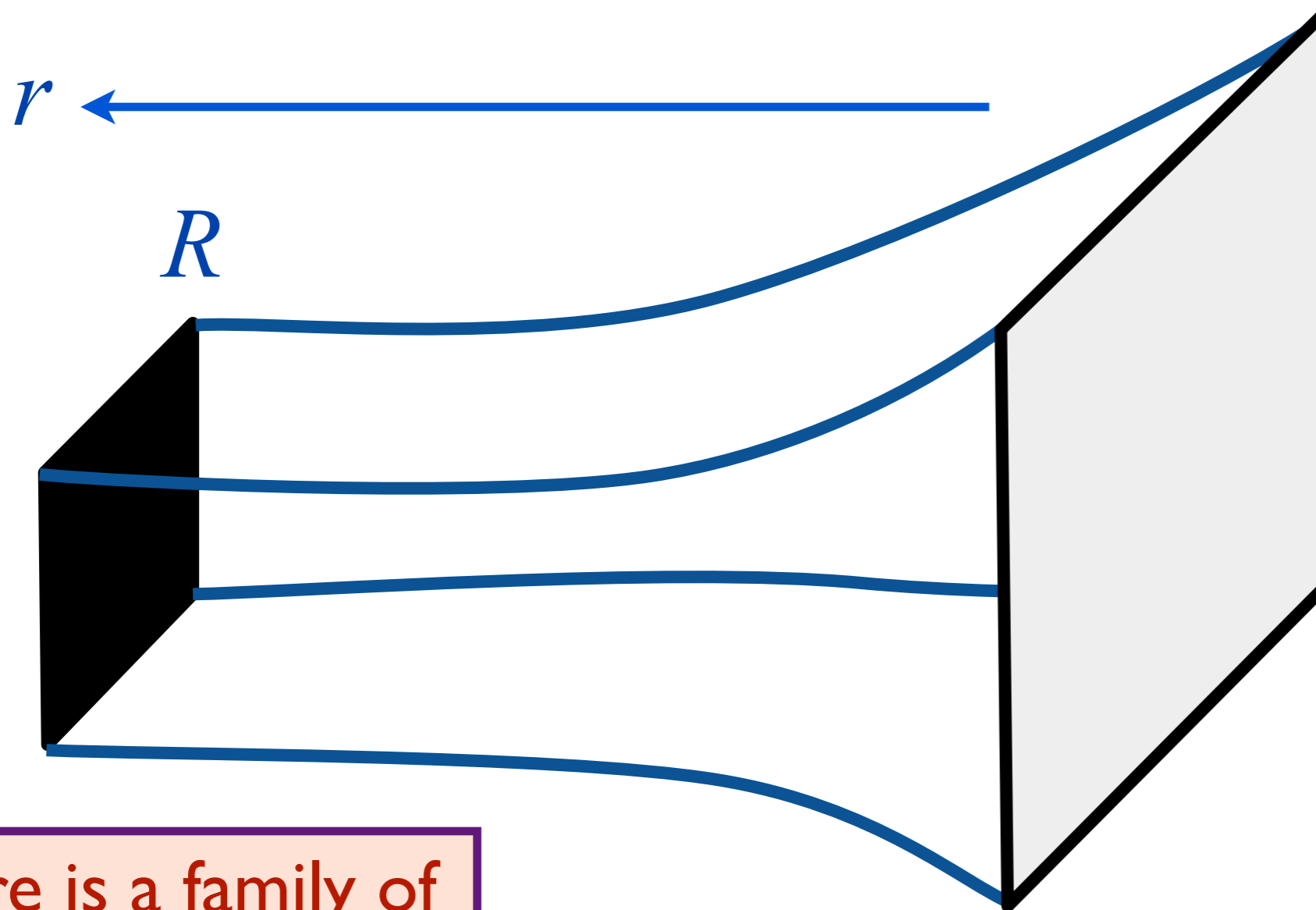
where C_{abcd} is the Weyl tensor. Stability constraints on this action restrict $|\gamma| < 1/12$, in agreement with results from the CFT3. The scalar field φ is conjugate to the CFT operator \mathcal{O} with scaling dimension $3 - 1/\nu$, which fixes its mass m . The coupling α is determined by the OPE of the currents with \mathcal{O} .

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Physical Review B* **87**, 085138 (2013).

E. Katz, S. Sachdev, E. Sorensen, and W. Witczak-Krempa, arXiv:1409.3841

AdS₄-Schwarzschild black-brane

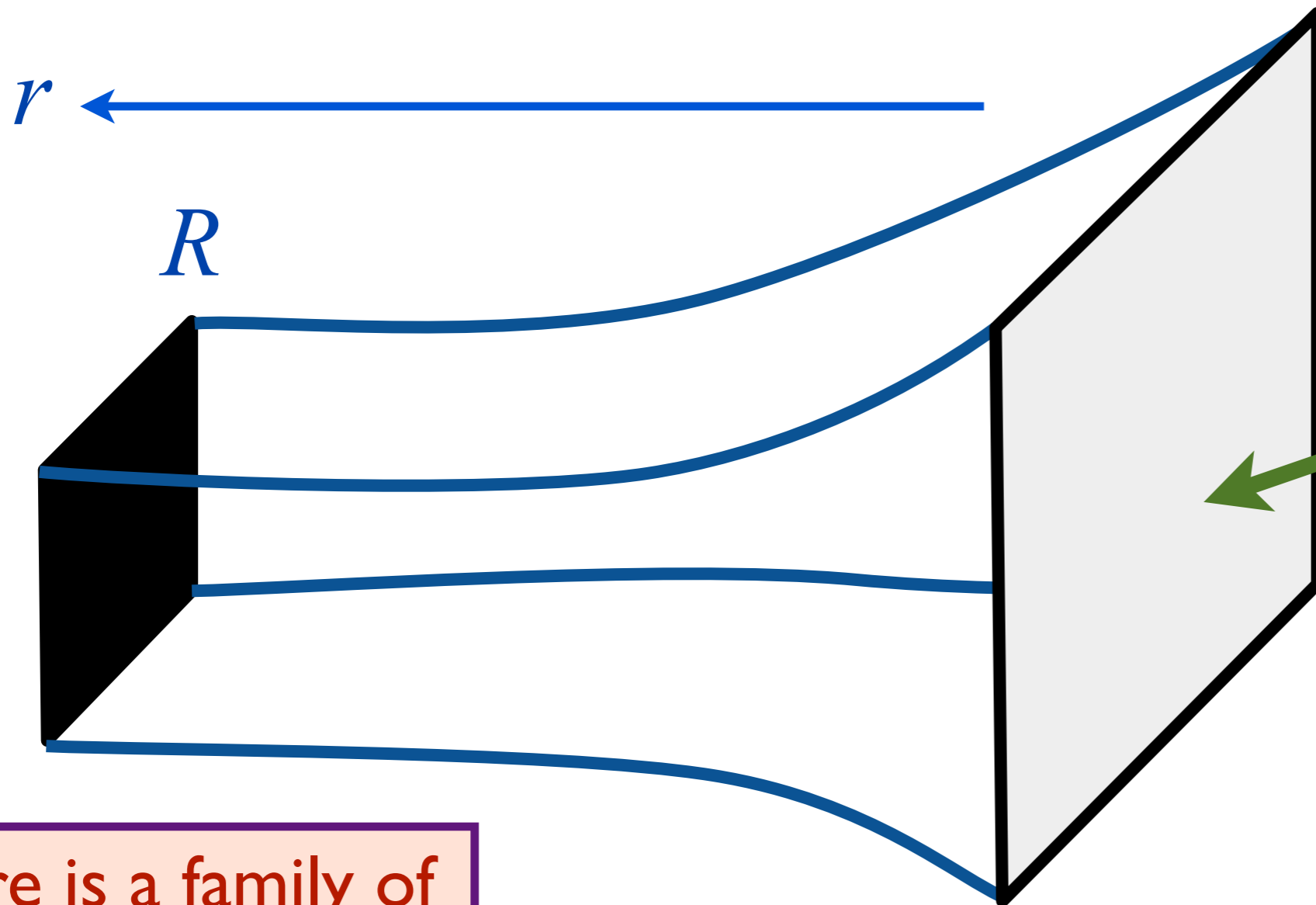


There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence at non-zero temperatures

AdS₄-Schwarzschild black-brane



A CFT₃
at a non-zero
temperature:
 $k_B T = \frac{3\hbar}{4\pi R}$.

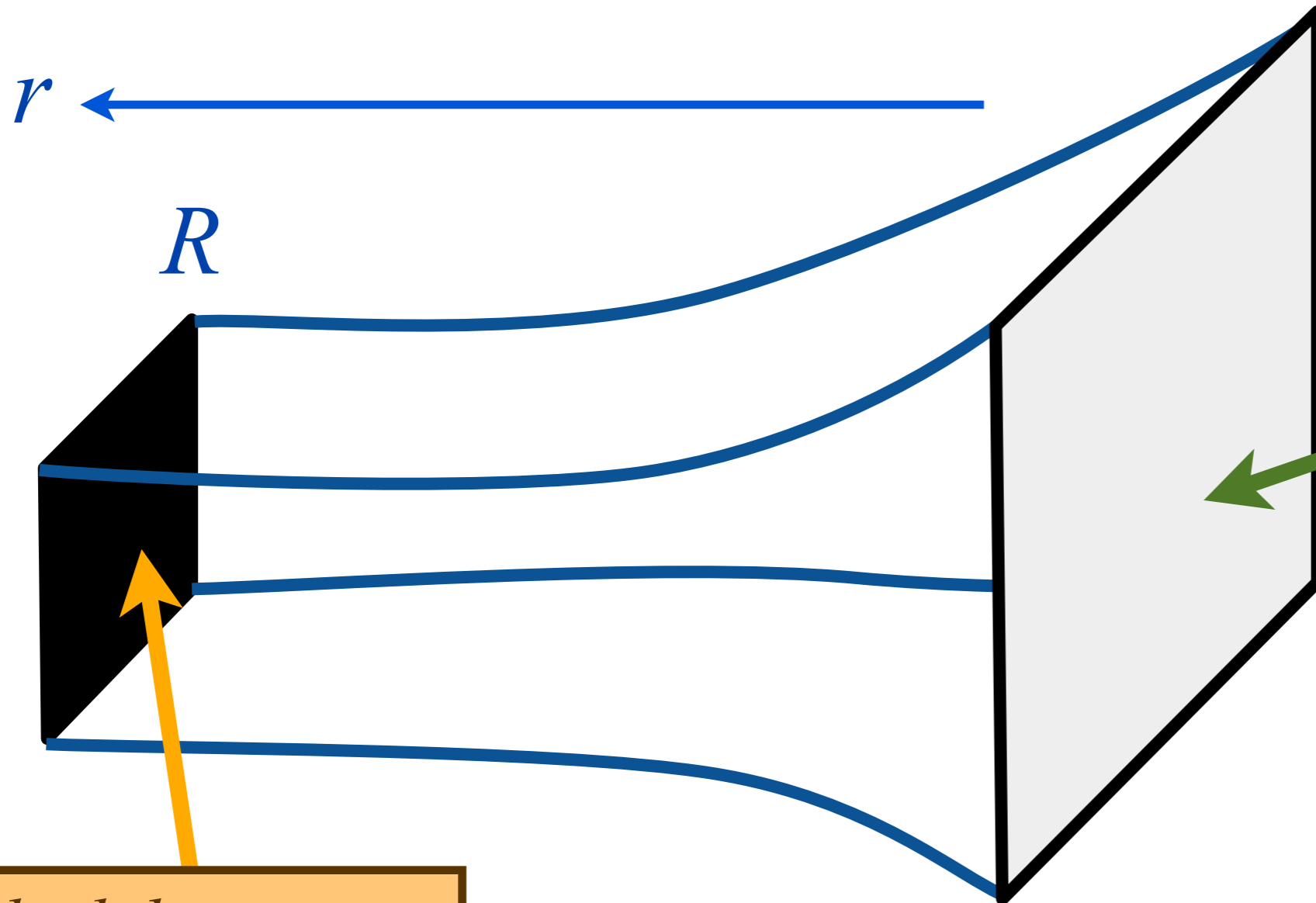
There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1 - (r/R)^3$

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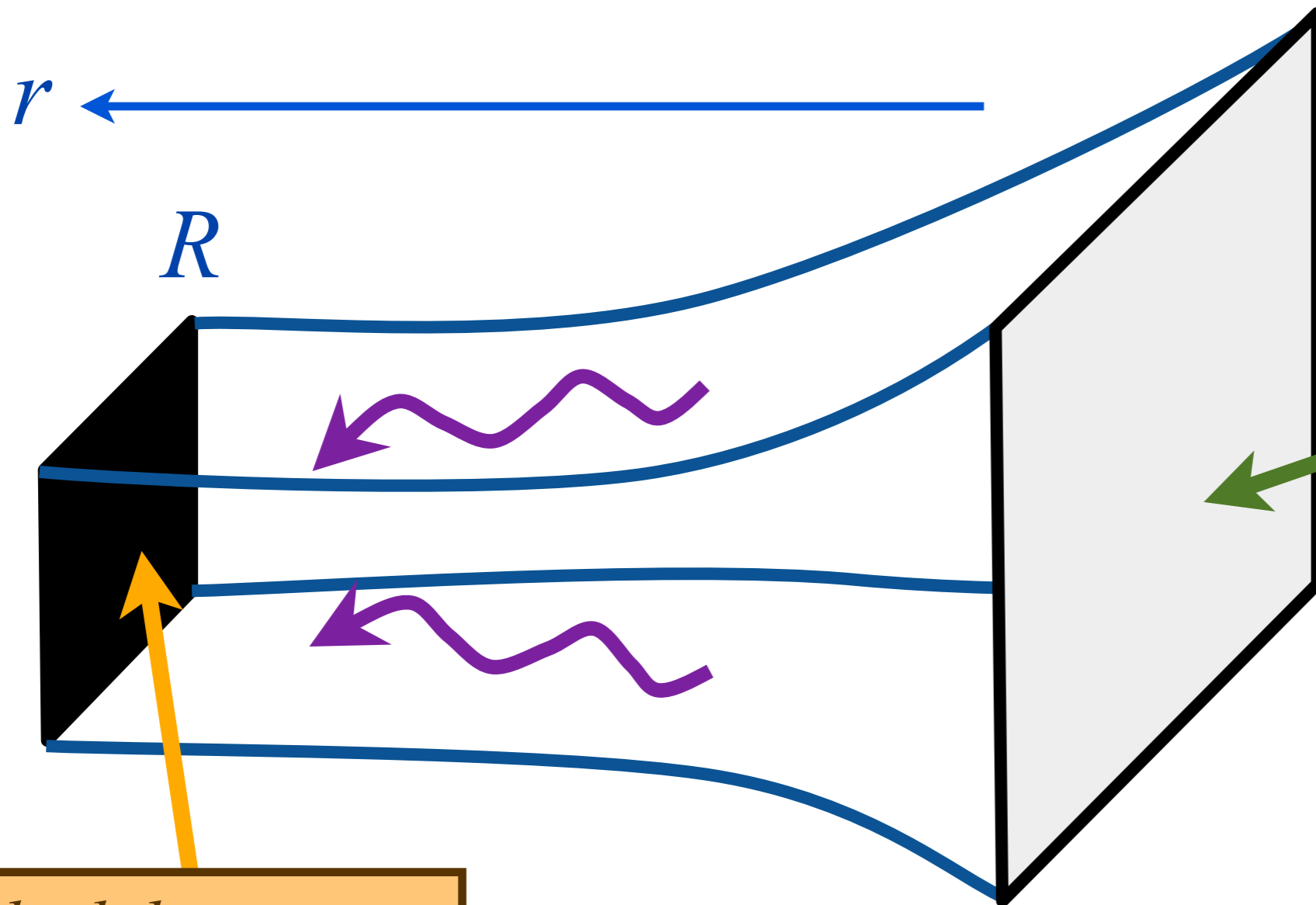
*Black-brane at
Hawking
temperature T*

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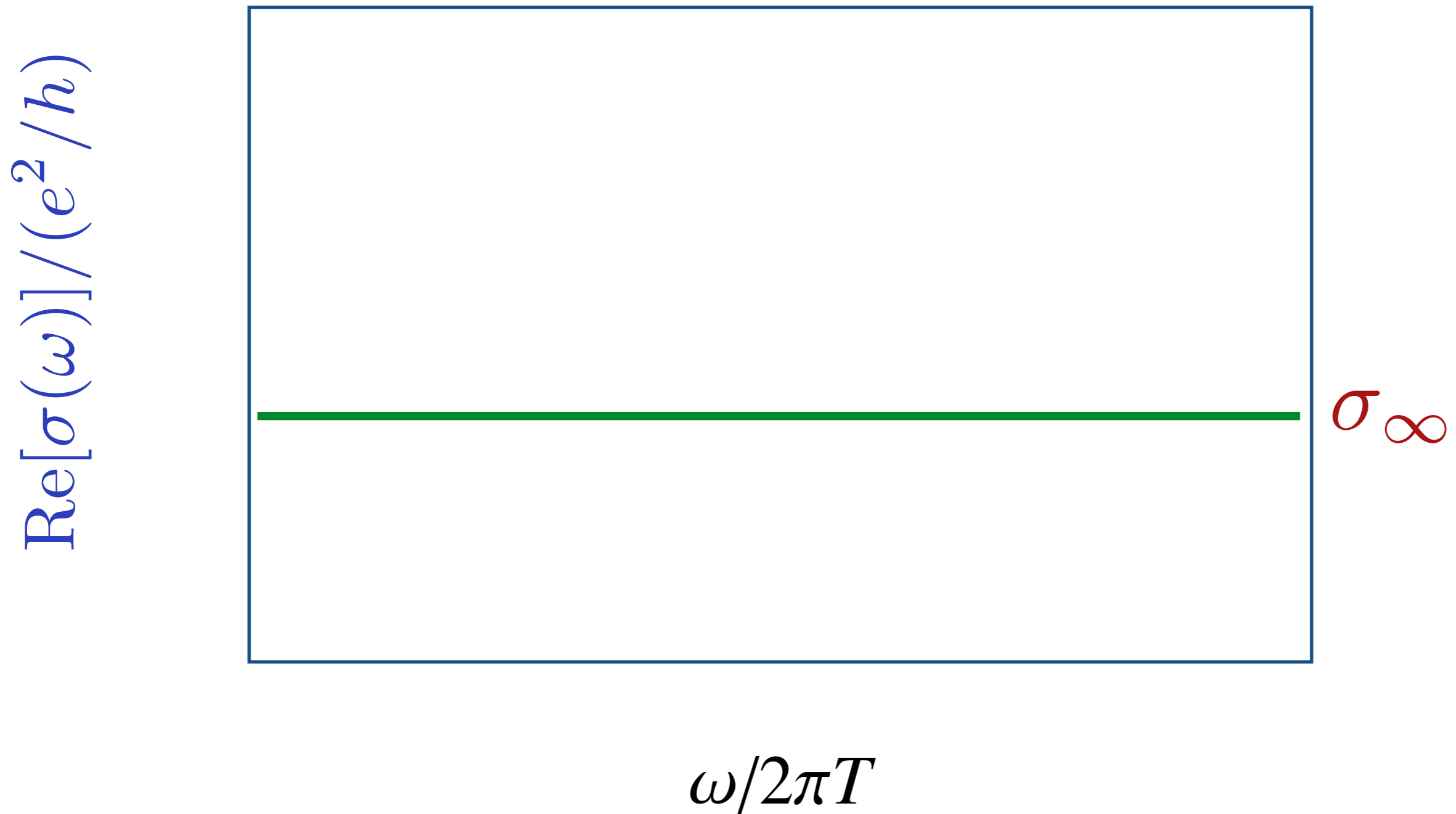


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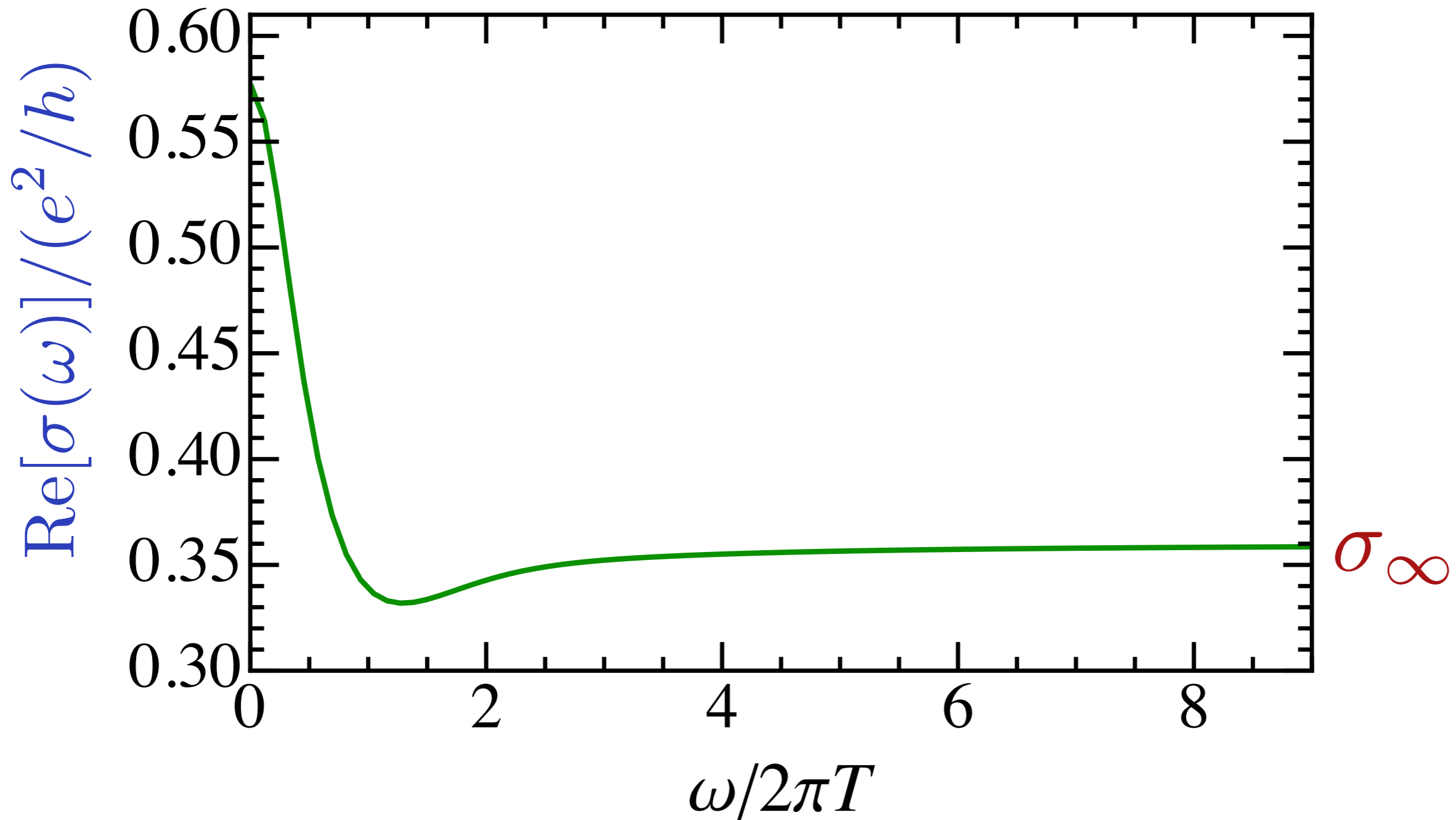
*Black-brane at
Hawking
temperature T*

Friction of CFT₃ =
waves falling into
black brane

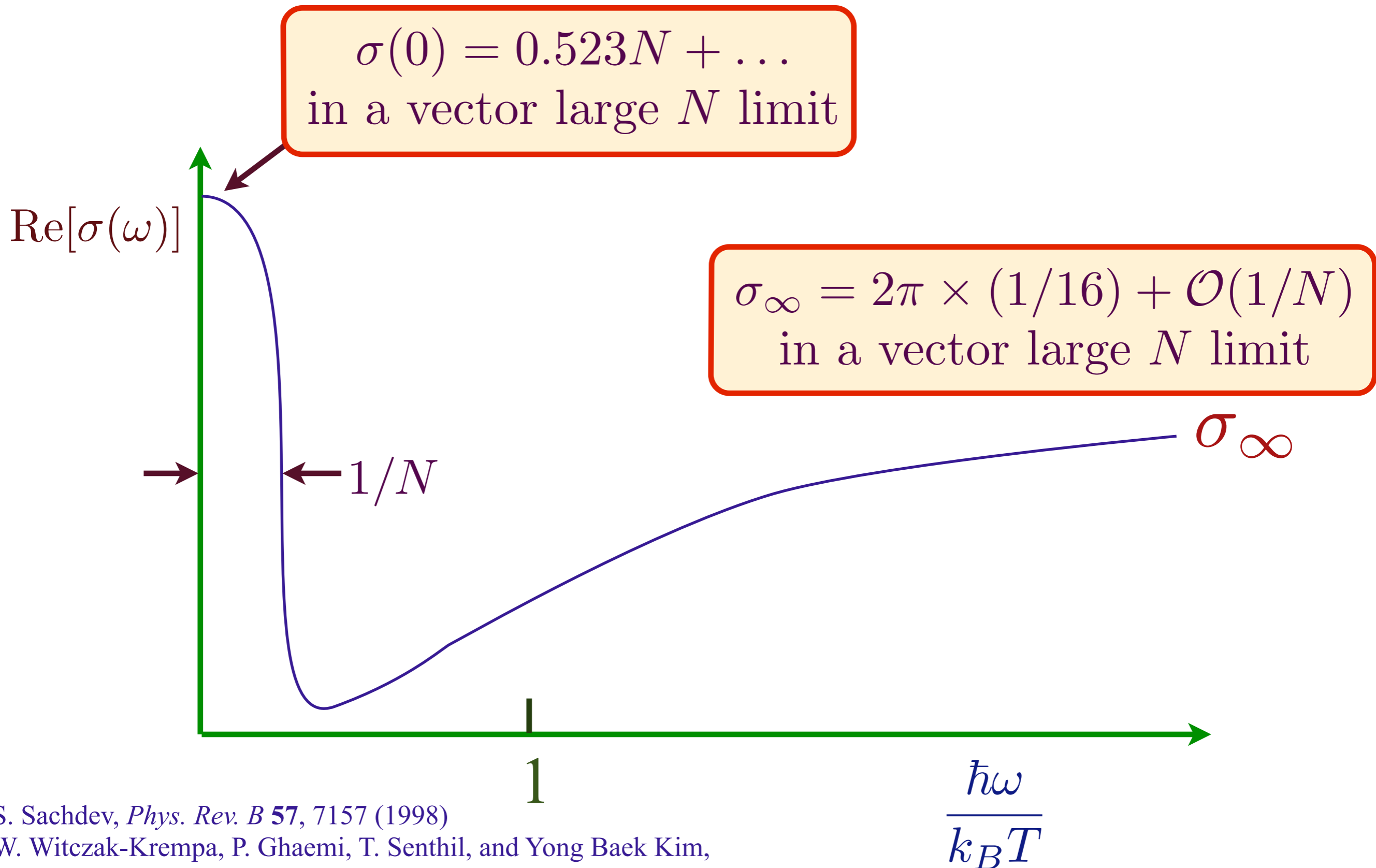
Conductivity of Einstein-Maxwell theory



Numerical solution of Einstein-Maxwell-Weyl-scalar theory + OPE info from QMC



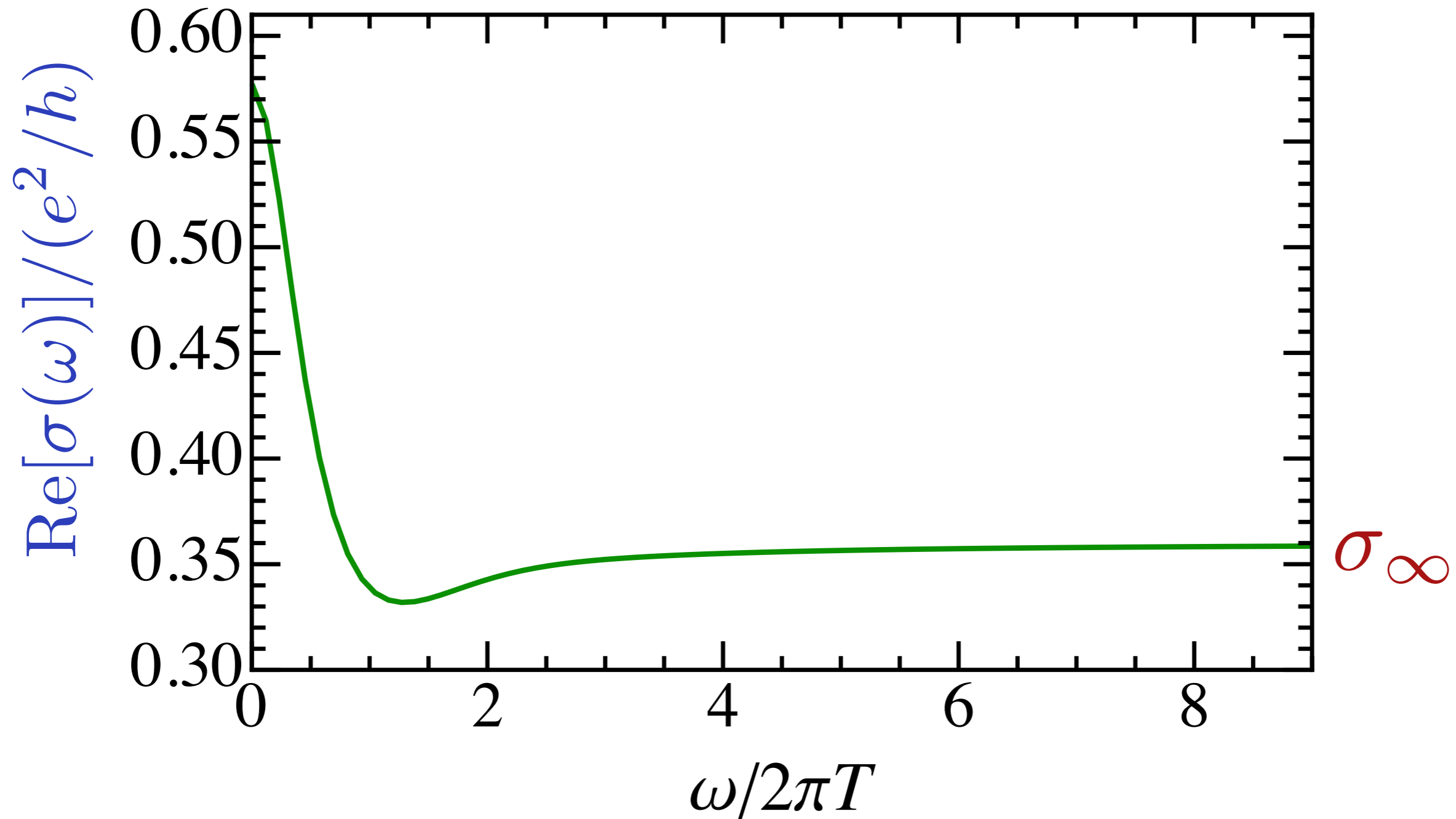
Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3



S. Sachdev, *Phys. Rev. B* **57**, 7157 (1998)

W. Witczak-Krempa, P. Ghaemi, T. Senthil, and Yong Baek Kim,
Phys. Rev. B **86**, 24102 (2012)

Numerical solution of Einstein-Maxwell-Weyl-scalar theory + OPE info from QMC



- Strongly-coupled quantum criticality leads to a novel regime of quantum dynamics without quasiparticles.
- The simplest examples are conformal field theories in $2+1$ dimensions.
- Quantitative predictions for transport obtained by combining the operator product expansion, quantum Monte Carlo, and the dynamics of black branes.